Yamada Takumi

THE SPEED MATH BIBLE

Tranform your brain into an electronic calculator and master the mathematical strategies to triumph in every challenge

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written by Yamada Takumi, with the special collaboration of Danilo Lapegna

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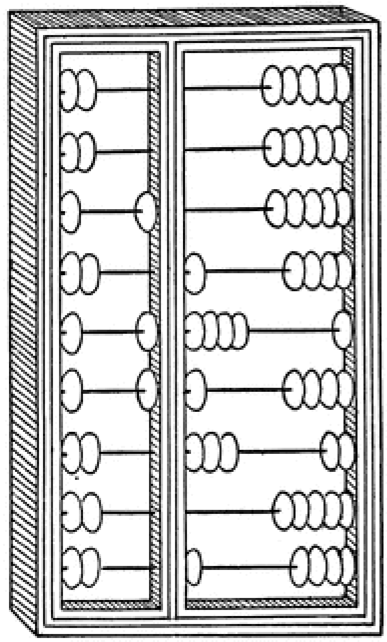
***"The 101 bibles"* series**

I - The Speed Math Bible



I'm quite sure we could all agree about a fact: *the traditional way to teach mathematics has a lot of structural problems*, and in most of cases it doesn't really help students to get confident with the subject. In fact, too many students finish their schooling still having a real "mathematical illiteracy", united with a *burning hatred of everything concerning numbers and operations.*

In particular, talking about classic method of teaching mathematics, I'm strongly of the opinion that:

- **It's really poor incentive for individual creativity:** in fact, too many times school will teach you that the proper method for performing a set of calculations is *"rigidly" one,* and that everything *should always be made in the same way*. This obviously can't do much more than *boring every student* and generating the feeling that the matter itself, rather than improving one's mental skills, actually shrinks them, *gradually transforming him/her into something that's more similar to an industrial machine.*
  This book, however, is designed to go far beyond this restricted vision and will teach you that the classic approach is not the

only possible approach and that *every set of mathematical calculations can be transformed into a deeply creative challenge.*

- **The idea of mathematical "trick" is unjustly "demonised":** very few people know that they could perform very complex calculations *just by using extraordinarily rapid and effective numerical tricks.*
  And although it's widely accepted that learning the right balance between tricks and rigour, creativity and structure, quick solutions and harder solutions would be ideal for any student, schools usually continue to prefer following a "complexity at all costs" that of course does not nothing but *alienating* people from the matter.

- **It's promoted very little self-expression:** too many people feel that they have "nothing to share" with some mathematical concepts even because the rigidity of the taught method *prevents everyone from expressing himself according to his talent, his wishes and his natural predispositions.*
  But this book presents a completely different approach, since everything among its pages will be *deeply self-expression oriented,* and the methods shown here will teach you not just one, unquestionable method to solve everything, but will give *you the freedom to act according to what you feel to be easier and more compatible with your natural attitude.*

- **It's promoted very little curiosity and "researching spirit":** math, as taught from some very bad teachers, is presented as a grey and squared world, made of endless repetitions and very few interesting things. But thanks to this book you will start *an original journey through the singularities and the curiosities of the numbers world* and you'll soon find out *how many interesting and beautiful things are hidden into its deep harmony.*

- **They speak very little about the real utility of mathematics**: beyond the ability to calculate your restaurant bill in an instant, the enormous advantage you will gain in every official test or exam, or the awesome progression of your logical and deductive faculties, you will discover that *mathematics has extraordinary strategic and creative applications* that will help you very often in your everyday challenges, giving you that extra oomph in the administration of your personal finance, your work, your studies, your health, your self-confidence and your strategic intelligence when dealing with any kind of challenge.

Math is also vital engine and fuel for every little technological wonder in the modern world. Computers and tablets could never exist today if two mathematicians like John von Neumann and Alan Turing didn't write the mathematical principles behind the first calculators. Internet would never have existed if no one had developed the mathematical principles the networks theory is based on. And the search engines and social networks could never be born without the equations and algorithms allowing each user to track, retrieve and organize documents, web pages and profiles directly from a set of chaotic data. Moreover, the inventors of Google and revolutionaries of the modern world, Larry Page and Sergey Brin, *are both graduates in mathematics.*

What I'm trying to tell with this? Well, I'm not sure that every reader of this book will end up founding the new Google, but I'm sure that a greater mathematics competence in a world tuned on these frequencies can help each of us to be *more a protagonist and less a bare spectator of it.*

In other words, this book will not simply be a set of strategies for impressing someone or increasing your academic performance, but will let you *start a journey through a pleasant and intriguing path of personal growth,* along which you'll learn to be more effective, creative, confident and, why not, more intelligent.

In addition to that, I would like to give you a last advice: *do not immediately try to learn every method explained here*, but go slowly, make notes, select the techniques you like most and train yourself calmly and always taking your time. This will help your mind to learn everything with much more ease and less effort.

Enough said: now I can't do anything but *wishing you all the best and, of course, to enjoy your reading!*

Yamada Takumi

About the authors:

Yamada Takumi and Danilo Lapegna are two Software Engineers working in London as software developers and freelance writers. Their series of books, "The 101 bibles", has been a great sales success in the self-publishing sector in Italy, engaging thousands of electronic readers even in a moment of economic instability for the Italian market. Now they're constantly expanding their series with new books about personal and professional self-development and their work represents an important reference point in the market.

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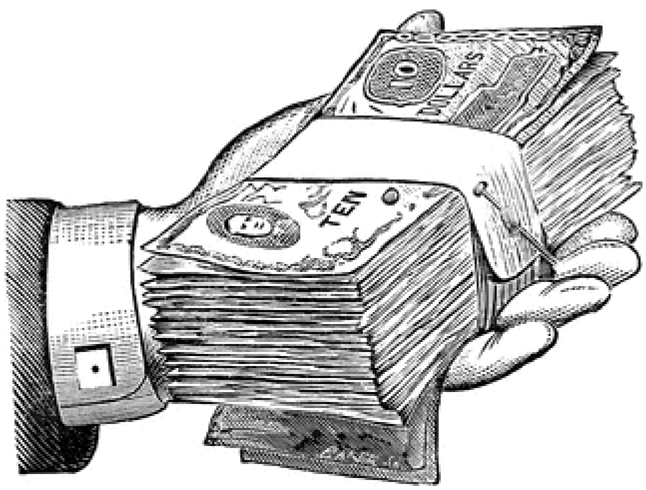
We also constantly improve our work with your feedbacks, so don't hesitate to send us anything about possible mistakes, typos or improvements suggestions. As always, we'll seriously take into consideration the opinions of our readers and we'll use them to enrich our next editions, adding a special mention for your help as well, if you like so. Turn even on the automatic Kindle Updates for your device to automatically receive our next updates for this ebook!

Thank you for buying "The Speed Math Bible", and thank you for becoming part of the "101 bibles" family!

"The moving power of mathematical invention is not reasoning, but imagination."

(A De Morgan)

II - 5 mathematical strategies that will seriously improve your life



We constantly count and make measurements in our everyday life: *how many hours must we sleep to feel really refreshed, what's our ideal body fat percentage, how much time do we need to finish examining those documents, after how many miles the gas tank of our car will be empty* ... these are all examples of everyday problems we constantly have to deal with, and that obviously would require a solution as more effective as possible.

At the same time, however, we too often *evaluate the factors involved in our problems from a purely qualitative point of view rather than a quantitative one*, giving them purely empirical solutions. That is, for example, instead of trying to understand the proper amount of sleeping hours for our body, we too often try to sleep enough to feel relatively fit. Rather than calculating our body fat percentage and adjusting our diet accordingly, we prefer some kind of homemade diet that does not really pay attention to our real physiological needs. And we do that despite it is intuitive that addressing these issues from a quantitative point of view, and so measuring, evaluating, calculating the quantities involved, would offer a *greater efficiency* to our actions, and so would give us *the ability to produce much more with many less expenses in terms of time, money and resources.*

Wait: this does not mean that you should run into some weird obsession for rational assessments or for measuring every single aspect of reality. *Your existence of course needs even impulsively done actions, lessons learnt from your mistakes and the coexistence with the inevitable unknown.*

However, when you actually feel the need to get more results in a specific field, a stronger mathematical "grip" on it could be a great strategy to enhance your personal improvement. After all, *the most successful companies are exactly those doing the best in retrieving and analysing data about the consequences of their behaviour*. But how could we do that? Here's some advices we could follow for the purpose:

- **Play giving marks more often.** No, this has nothing to do with school. If, for example, you really enjoy doing something, like, I don't know, eating a particular type of food or going traveling somewhere, try giving to these actions, or to the advantage you get from doing them, a *mark.* From 0 to 10, from 0 to 100, it doesn't matter! The point is playing making quantitative concepts out of qualitative ones, and starting making better analysis on their basis. For example, if dedicating yourself to jogging in the morning has for you a higher mark than going to gym, you could simply start to decide to ... go jogging more often!
  Quantity in most of cases *helps you to better focus on the most important things, and cleans your thoughts from the unnecessary stuff!*

- **Try to make measurements where nobody else would do it.** *"Not everything that can be counted really counts, and not everything that counts can be really counted"*, said Albert Einstein.
  This phrase not only confirms what I earlier said about the fact that mathematics and calculations sometimes must simply be put aside (luckily), but can help us to understand that, too much often, *the most commonly used measurement systems can deceive us,* and can divert our attention from the things that

really matter.
An example I love doing about this is *in the school report cards*: their obvious purpose should be giving the measurement of the performance of a student, but we often forget that, on the other side, there could be a teacher that simply has an awful teaching method.
And that's where *we should start to "make measurements" where nobody else would do it*: letting the students evaluate the quality and the goodness of their teacher's work could push everybody into doing better his/her work, and consequently into producing better results for everybody.
Making measurements, evaluating, focusing on the things that other people tend to ignore, *gives you that extra oomph that will produce a significant advantage in any kind of situation!*

- **Increase the quantity and improve the quality of your measurement instruments.** Imagine: it's a lazy evening and you decide to go watching a movie to the cinema. Now, you have to decide which movie buying the tickets for, but you're hesitating between that new action movie with a lot of old celebrities and that new horror Japanese movie about a dead woman who comes back from the grave. Now you could take your smartphone, read the movie reviews from some famous website and simply choose the movie with the highest rate. But of course you would better estimate the quality of those movies after *increasing the quantity* of your measurement instruments and, for example, reading movie ratings *from many different websites* instead than from only one of them.
  Now there is a concept that can really come in handy: the *mathematical mean* that, although is quite widely known, we'll explain in a few words, just to make sure that everybody knows what are we talking about.
  If you have multiple quantities, multiple measurements, multiple values (ex. 3, 4 and 5 stars rating for a movie on three different websites), you can sum those numbers (ex. 3 + 4 + 5 = 12) and then *divide the result by the number of values you considered* (three ratings in this case, so 12 / 3 = 4).

This will give you the *mean* of those quantities, which is universally considered as *a very precious value since*, as in the measurements theory, as in the probability one, it's said to be very close to the "real" value of something (if a real value can philosophically or scientifically exist, of course).
But now, let's add something and let's talk about *quality* of measurement instruments: how many times happened that a multi-award winning movie just left you asleep in your chair?
That simply means that those measurement instruments (the "official" movie reviews we used to estimate the goodness of the product) weren't as reliable as they were supposed to be. And this really happens a lot of times, doesn't it?
But what's the solution? How could we improve the *quality* of our instruments?
Well, we could, for example, *browse just among the opinions of the friends of ours who usually have similar taste to ours*. After adopting this strategy we'll automatically notice that even fewer "measurements" will be proven to be much more truthful, and so again that sometimes quantity is a poor concept (we should never trust an opinion only because shared by most of people, after all) and quality should have a heavier weight in our everyday reasoning.

- **If numbers rule over a situation, you must ally with them or you'll definitely be defeated.** Sun-Tzu, the most important strategic book of all times, repeatedly affirms that *victory can only be achieved if one harmonizes itself with the natural constants.*
  So, if we monthly spend more than how much we earn, bankrupt will soon be much more than an abstract concept, and there are no "qualitative factors" that can change this. If we build a shelving unit whose shelves are structurally made to sustain a heavier weight than the decorative object we just bought, *we should simply avoid putting it there.*
  All the fields strictly ruled by physical, economic, natural rules are clear examples of contexts in which you just can't escape from "numbers supremacy". So, making extra measurements for

your shelf, checking your monthly expenses, or being sure that your self-employing project will sustain your rent cost in the long-term are all *necessary actions,* that not only will definitely improve the quality of your work, but will also help you to avoid disasters.
Yes, this may sound trivial sometimes, but at the same time we should pay attention to the fact that *too often we tend to suffer for ignoring exactly the most obvious things.*

- **Doubtful? Use the power of the gain-loss ratio!** Suppose that you just received multiple job offers, with different pro and cons, and suppose to be totally doubtful about which one could be the best choice for your career progression. Or maybe you're looking for a new house, you have various options and you want to understand which one will be more suitable for your needs.
  Well, in these cases you could put yourself into the hands of the *gain-loss ratio:* try to make a list of the cons of each choice, and give to each disadvantage a *damage rating between 1* (almost annoying) *and 10* (death). And be careful: each choice must have at least a disadvantage with a damage rating greater than 0. For example:

  Flat 1:
  Very expensive: 7 (pretty big deal)
  No smoking allowed: 5 (annoying but almost acceptable)
  No pets allowed: 8 (this is going to be a serious problem if I want to buy a dog)

  Flat 2:
  Very dirty: 9
  Very far from the city centre: 8

  Now, for each choice, *calculate the mean of the damage ratings by its disadvantages,* and call the result "damage rating" of that specific choice. In our example it would be:

  Flat 1:

7 + 5 + 8 = 20 / 3 = *Damage rating:* 6.66

Flat 2:
9 + 8 = 17 / 2 = *Damage rating:* 8.5

Now it's time to start considering the bright side of the medal: make a list of the pros for each choice and give to each advantage a *"convenience rating".* Now, calculate the "convenience rating" of every specific choice by calculating the averages of the convenience ratings of its advantages. In our decision about which apartment going to live in, this could be:

Flat 1:
Very bright: 7
Rooms are pretty clean: 7
Very near to my workplace: 8
Convenience rating: 7 + 7 + 8 = 7.3

Flat 2:
Very cheap: 8
Pets allowed: 8
Convenience rating: 8

You're almost done. Now, for each choice, just divide the convenience rating by the damage rating, and you'll have your gain-loss ratio: *an approximate measurement of which choice is (possibly) the best to take:*

Flat 1:
Gain-loss ratio: 7.3 / 6.66 = 1.096

Flat 2:
Gain-loss ratio: 8 / 8.5 = 0.94

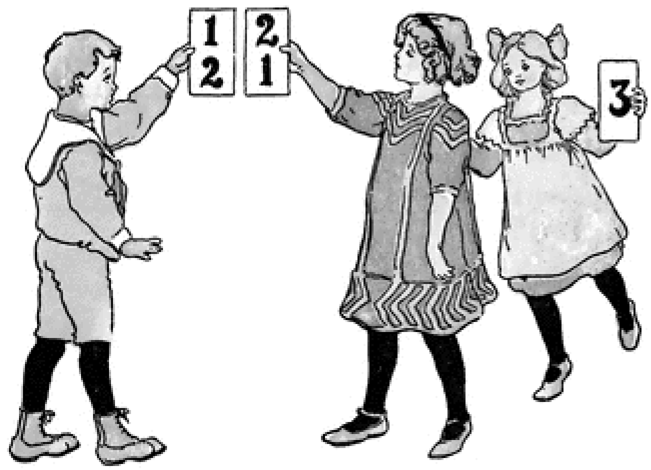
Flat 1 wins!

Of course this doesn't have to be taken as the ultimate truth, but it can be really useful to show you a brighter way in your decision making process. In addition to this, remember that more honest you will be with yourself in your rating, and more trustworthy will be this index in showing you the best choice to take.

Last important thing: when you're dealing with a situation in which fortuity has a strong influence, this index loses most of its usefulness, and in that case the best thing to do is taking your decision with the help of other factors we'll explain in Chapter XIX.

This chapter ends here, but your journey into the practical and strategic applications of mathematics just started. Most of calculation techniques shown in the next pages, in fact, will often come along with their more useful, and funny applications for everyday life, like probability theory applied to gambling or game theory applied to poker. So, if you continue reading, you will be page after page more surprised and wondering why your teachers only taught you the poorest side of this matter.

III - 4 basic steps to boost your calculation skills



Here I'll start explaining you *four helpful fundamentals* for improving the speed of your mathematical skills that you'll absolutely need to learn before starting to practise with any creative calculation strategy. Once you learnt them, the 80% of the hard work is done, so ... let's start!

1 - Take the right attitude

Yes, influence of the mental attitude in math is something as underestimated as powerful. If you subconsciously keep repeating yourself that math is not part of your world, that it's too difficult for you and that you will never truly use it in your everyday life, you can be sure that you will never do any progress as well. Rather:

- *Remember that, whatever is your opinion about your "innate predisposition" to mathematics, you're not obligated to follow it.* Yes, definitely there are people that are cut out for mathematics and logic since they were born, and people that had a panic attack in front of their first number written on a blackboard, but our nature doesn't have to be our curse. For example Gert Mittring, who won the gold medal at the MSO mental calculation for nine consecutive years, at school *was one of the* *worst math student in its class.* And, before him, many other geniuses like Albert Einstein or Thomas Alva Edison, achieved awful marks exactly in the same subjects today are considered as reference points for. Edison was even called a "mentally retarded boy" by one of his teachers.

So, 1) Our school grades will never be a real measurement of our real skills and, 2) From the adaptive nature of our brain inevitably follows that *we, and only we can decide which part of our mind to cultivate, and which part to leave withering and decaying.* And I could bet any amount of money on the fact that, if you properly work with the strategies I'll show you, you will develop skills you thought you never had!

- *Keep yourself motivated!* Motivation is the foundation of every personal growth and self-improvement path! In fact, if we keep ourselves motivated, every barrier becomes nothing but an opportunity to learn, build and improve our talent and creativity. But how do we keep motivated in front of a potentially tedious path made up of numbers and theorems? By *learning how to convert every arithmetical problem into a deeply creative challenge!* By constantly remembering ourselves that *backing out from a mental challenge means gradually losing our brightness and cleverness!* By understanding that mathematics *can sharpen and empower all those "mind tools" that over and over again will make the difference in our life between profit and loss, happiness and unhappiness, success and failure*. And, last but not least, by being aware that an improved ability to "play" with numbers *can even impress the people we know,* as we'll better see later. This will inevitably help us to build our self-confidence and will improve as our relationship with the others, as our connection with our inner self.

- *Train yourself.* Stop using your smartphone even to perform a "7 x 6" and use your everyday challenges to train your brain with mathematics! For example, try to mentally calculate your restaurant bill or your change after buying your daily newspaper! This advice may sound obvious, but at the same time is probably the most effective of all. After all, we should just remember what Aristotle, the ancient Greek philosopher, said: *"We are what we repeatedly do. Excellence, then, is not an act, but an habit!".*

- *Don't be stubborn!* Yes, keep yourself motivated and trained, but don't stubbornly insist on mathematical problems that you apparently cannot solve, because sometimes the solutions come *just as you think of something else.*
  In fact, even when we abandon a problem solving procedure, our brain often continues working "in background", expands its perspective and then it suddenly becomes able to give us the best solutions to our problems. Famous, moreover, was the expression "Eureka" that Archimedes, according to legend, shouted after finding a good solution for calculating the volume of solid objects while relaxing during a bath. Legend or not, in any case, when the solution to a mathematical problem or arithmetic just does not seem to arrive, *try to take a break and move on*. The act of coming back later on hard problems helps us to approach them from a new, wider perspective and, although it may seem counterintuitive, *it's the best way to help our brain to do its job.*

2 - Respect your brain

One thing we often forget is that *brain and body usually give us back nothing more than what we gave them.*

So we should just stop looking at them as *bare tools to squeeze the best possible result from*, and instead we should always see them as important elements of our self to be *fed as much as possible* in order to let them give their best fruits in return.

So what? A good brain, and therefore good calculation skills comes from the respect for your body, your health and your own biological rhythms first. In particular:

- **Make physical activity a habit.** Some constant workout, in fact, oxygenates our body and brain, improving our reasoning skills and making even more complex thoughts easier to process. No, you don't have to engage yourself in anything strenuous and even one hour a day walking in order to boost your metabolism will be fine.
  Oh, and always keep in mind that Dr. Marilyn Albert, a researcher in the field of brain function, has found that adults are more likely to keep a good brain in old age if they are physically active and keep a good cardiovascular health. So ... make a workout plan to keep you always young!

- **Eat properly.** There are dozens of theories about which the exact meaning of "eating properly" should be and, in addition to this, the habits and body of each of us and make us better suitable for specific kind of diets, so finding a way through the different voices is always a quite difficult challenge.
  Well, without necessarily having to consult a nutritionist, my simple advice here is to *never abandon some elementary common sense rules*: variety, don't overdo, always have a proper breakfast, have a good daily amount of antioxidants, don't omit or exaggerate with refined sugar and, most of all, don't forget to insert into your diet a good amount of foods containing phosphorous and B-complex vitamins, great substances to let your brain work at its best. Some examples of foods containing these nutritional substances? *Cereals, fish, nuts.*

- **Coffee? Yes, but in the right amount.** Yeah, coffee, canned drinks, black and green tea, and anything containing caffeine certainly enhances our cognitive skills, boosts our attention and improves the speed of our neurological reactions.
  However, if taken in high doses, these drinks just end up being counterproductive, since their hyper-stimulating effect starts compromising our ability to reason as best as we can.
  So be careful, and be *aware of what should be your daily limit of caffeine*. Of course, this limit can be difficult to calculate because it changes depending on factors such as our gender and age;

however, paying attention to the signals of your body can be really helpful, so *just try to notice after how many cups of coffee, tea or soft drinks you start to feel confused, distracted and nervous.*

- **Relax.** Stress causes our adrenal glands to produce excessive amounts of cortisol, a neurotoxic substance that can damage our synapses (the connections among the masses of neurons in our brain). So if you really want to improve your brain skills you always have to leave yourself some time to *work on your concerns*, to *enjoy the things you really like* and to eliminate (or at least learn to handle) as more sources of anxiety and stress as possible.

- **Sleep.** We can anywhere read advices about sleeping 8, 9 or 10 hours every night, but the truth is that *each of us can have different needs, depending on our physiology and state of fatigue or stress*. So do nothing but paying attention to your body signals and give a reasonable priority to your sleep time, since it will greatly boost your brain skills, your ability to focus and even your problem solving and stress management abilities.

3 - Understand the importance of your memory

Any calculation process is basically made up of two different phases: the first one *is the calculation itself*, while the other one is the *memorization of the previous results.* And the latter probably represents a crazily critical point for most of people.

Imagine yourself trying to solve without any pen or paper a long multiplication between three-or-more-figure numbers: probably the hardest thing for you will be exactly trying to retain all the digits coming out of the partial multiplications.

So, on the one hand you'll see that the "secret" behind many calculation strategies in this book is in the fact that *they're built in order to let us optimize the use of our short-term memory*, and on the other hand it follows that *your memory represents a key-skill to train if you want to improve your mathematical abilities.*

So, try to follow these advices:

- *Keep yourself focused:* This may sound obvious, but focusing on the tasks you're dedicating yourself to will seriously improve your ability to memorize everything involved. So, try to isolate yourself from any potential disturbance source, always force yourself to completely focus on the "here and now" and your memory will get an awesome boost!

- *Try to memorize only the essential things.* If for example you are mentally performing a long addition, don't repeat yourself any digit of the new addends you're getting, but *just memorize the partial results you obtain*. Same thing with subtraction, multiplication or division: memorize only the bare minimum amount of elements necessary to go ahead in your calculation. You'll realize by yourself that, after adopting this way of thinking, every arithmetic procedure will be *much faster and less tiring.*

- *Know yourself.* Somebody, for example, is capable to retain something much better into his memory after visualizing it, somebody else after *listening to it inst*ead, and so on. In other words, try to understand how, when and where your brain uses to store information in a faster and more efficient way, and then ... always do your best to help it to do its work! Imagine a big, coloured picture of the numbers you're working with if you prefer visualization, and repeat yourself their name if you are an "auditive" person: this will help you to remember them much more easily!

- *Divide.* Your brain works less hard while trying to memorize many combinations made up of few elements, than while trying to memorize one, single set made up of a lot of things. So, especially if you try to operate with very large numbers, try to split them in many shorter numbers made up of a few digits:

remembering them later will be as easy as the proverbial pie.

- *Use the mnemonic major system:* The mnemonic major system is probably the most famous and most commonly used mnemonic technique, due to its simplicity and power. In fact, thanks to it, you will be able to memorize even a really long number by turning it into a word or a phrase, which of course *is definitely much easier to remember than a sequence of digits.*
  This technique is made up of three phases:
  1. *Convert each digit in your number into a consonant* using a specific table.
  2. *Mix the consonants you got with the vowels you need* (or some "unassigned" consonants), in order to create a key-word or a key-phrase to remember.
  3. When you need to have your number again, you will just have to take your key-phrase, remove the vowels, and re-use your table in reverse. Oh, and by the way, this is your table:

0	s, z, soft c
1	t, d
2	n
3	m
4	r
5	l
6	j, sh, soft g, soft "ch"
7	k, hard c, hard g, hard "ch", q, qu
8	f, v
9	p, b
Unassigned	w, h, y, x

  But since I think that examples are always the best way to explain something, let's imagine you have to remember the number 3240191. You have M - N - R - S - T - P (or b) - D (or t).
  After mixing those consonants with some vowels you could obtain "Men are stupid". And more stupid or strange is the

resulting phrase, and easier it will be for you to remember!
On the contrary, let's imagine your key-phrase is "Black horse". You have B - L - hard C - R - S: your number is 95640!
Yes, this technique may sound difficult at the beginning, but a little training can give you awesome results. And consider that it's alone worth the cost of the entire book, since it won't give you only the ability to make any calculation without pen or paper, but will come in handy every time you'll need to remember dates or telephone numbers as well!

Now you have in your "mental toolbox" a lot of extraordinary instruments for improving your "numerical" memory. Treasure those which better work for you, and sharpen them to boost your mathematical skills at their maximum power!

4 - Strengthen your basis

Mathematics is like a "Lego" building structure: the simplest concepts can be combined together in order to shape more complex theories, those theories can be combined in turn to give rise to even more complex ideas, and so on.

So it's obvious that, if the starter bricks are tin-pot, the final structures will always be crumbling and unstable. In other words, if your mathematical basis is not well set in your mind, all the thinking that lays on that basis will be uncertain, slow and unreliable. And this is a hardly critical point, because it's quite underestimated by 99% of people. This, for example, reminds me about a lot of engineers-to-be I was studying with, who very often were reporting bad results in calculus and algebra tests, not because they actually were ignorant about calculus or algebra, *but because very, very often they were just making banal arithmetical mistakes.*

It's also intuitive that strengthening your basis is definitely a key-skill to accelerate your calculation making: if, for example, you train yourself to instantly recall from your memory all the results of the basic sums from 1+1 to 9+9, you won't lose anymore a single second while trying to perform a "7+8" you need to carry out during a seven-figure sum.

So, my advice here is to work on reinforcing the following concepts in your mind, as banal or obvious they may sound:

- **The basic number properties.** The basic number properties will be often used in this book in "creative" ways in order to build a lot of special speed math strategies, so you could find it very useful to brush up on them:

  ○ *Commutative property of addition and multiplication: changing the order of the operands in any addition or multiplication doesn't change the final result.*
  And so, for example: 3 + 4 + 5 = 5 + 4 + 3 = 5 + 3 + 4 = 12.

  ○ *Associative property of addition and multiplication: changing the order you perform your addition or multiplication with, doesn't change the final result.*
  So, for example, (3 + 4) + 5 which, after looking at the brackets, implies doing 3 + 4 first and then adding 5, won't be different from 3 + (4 + 5), which instead implies calculating 4 + 5 first and then adding 3.

  ○ *Distributive property of multiplication over addition.*
  That is: if I have an "a x (b + c)", the result will be the same as " (a x b) + (a x c)".

  ○ *Invariantive property of division: given an a / b division, if you multiply or divide both a and b for the same quantity, the final result won't change.*
  Let's make a simple example and let's imagine you have a 30 / 10 to perform: after dividing both numbers by 10 you'll have 3 / 1, which result obviously is the same as 30 / 10 and equals 3. Same thing if you multiplied both numbers by 2: 60 / 20 in fact still equals 3.
  The multiplication has a very similar property, but with one very significant difference: after performing any "a x b" multiplication,

the result doesn't change if you *multiply "a" by a generic quantity and divide "b" by the same quantity*, or vice versa.
So, given for example a 16 x 2, you can halve 16, but then to keep the exact result you'll have to double 2. So 16 x 2 = 8 x 4. Same thing with, for example, 100 x 9: it will give the same quantity as a result as 300 x 3, or 900 x 1.

○ *Identity property: Adding or subtracting 0 from a number doesn't change it. The same happens after dividing or multiplying it by 1.*

○ *Zero property:* Any number, if multiplied by 0 will still equal 0. And, in the same way, the result of a multiplication can never be 0 if neither of the factors is 0.
It may be also interesting to remember that this rule implies that you will never be able to divide any number by 0. Let's take in fact a non-null number, like 14. Division is the inverse of multiplication, and so trying to divide 14 by 0 means asking yourself: "Which number equals 14 when multiplied by 0?". And the zero property obviously tells us that this question has no answers.

- **Dividing or multiplying by 10, 100, 1000 or any other power of 10:** This is a pretty simple calculation as well, but brushing up on it is still a good practice, even because some fast multiplication strategies we'll see in the next pages are exactly based on the concept of transforming an "ordinary" multiplication into a multiplication by a power of 10. So:
○ *To multiply an integer number "a" by a power of 10, just add to "a" as many trailing zeroes as the power of 10 has.*
However, if "a" has a decimal point, then you have to shift the point on the right by as many positions as the trailing zeroes of the power of 10 are. So:
5 x 1000 = 5000 (just write three trailing zeroes)
1.28 x 1000 = 1280 (shift the point by two positions to the right and you get 128. But 1000 has three trailing zeroes, so you

have to write one more zero to the result in order to complete the multiplication)
567.3 x 1000 = 567300 (shift the point by one zero to the right and you get 5673. But 1000 has three zeroes, so you have to write two more trailing zeroes to the result in order to complete the multiplication)

○ *To divide a number "a" by a power of 10, just shift its decimal point to the left by as many positions as the power of 10 has. If the number is an integer, just think as if it had a ".0" after its rightmost digit. And if, during your operation, the point passes over the digit on the extreme left, then write a 0 on its left.*
So, for example:
345 / 1000 = 0.345 (shift the point to the left by three positions. The point passes over the 3, so put a 0 on its left)
2 / 1000 = 0.002 (shift the point to the left by three positions. You will have .2 at first, which will become a 0.2. Then, by shifting again, it will be .02 = 0.02. And so on.)
14 300 / 1000 = 14.300 = 14.3
700 000 / 1000 = 700.000 = 700
And a very interesting thing is that mastering this technique makes your life easier even when you have to perform *any kind of multiplication or division between numbers having one or more trailing zeroes.*
In particular, *in the multiplication case*, you can just remove any trailing zeroes, perform the multiplication, and then multiply the result by a power of 10 having the same amount of zeroes as those that were removed.
So, for example, 5677 x 300 = 5677 x 3, which must then be multiplied by 100 (two zeroes removed).
And, similarly, 350 x 2000 = 35 x 2, which must then be multiplied by 10 000 (four zeroes removed).
The division case is a little bit more difficult though. In this case, in fact, you must still remove any trailing zeroes, but then you have to *multiply* the result by a power of 10 having the same amount of zeroes as those removed from the *dividend*, and *divide* the result by a power of 10 having the same amount

of zeroes as those removed from the *divisor*.
So, if for example you must calculate 90 / 6, you can remove the 0 from the 90 first and then calculate 9 / 6 = 1.5. You just removed a zero from the dividend, so now you have to multiply the result by 10 = 15.
If you have to calculate 9 / 60 instead, you must remove the 0 from 60 first, then perform again 9 / 6 = 1.5, and at the end *divide* the result by 10, getting a 0.15 as a result.
Last case: let's imagine you must calculate 900 / 60. In this case you must remove two zeroes from 900 and one zero from 60, so, after calculating again your good old 9 / 6 = 1.5, you'll have to multiply the result by 100 and then divide it by 10. This obviously is the same as multiplying it by 10, and so *the final result is 15 once again.*

- **Multiplication and addition tables from 0 to 9:** probably everybody knows these tables, but at the same time very few people are able to recall them from memory instantly and without making any mistakes. So try to brush up on them, and examine in depth all the "harder cells". For example, it is proven that majority of people has problems with multiplication tables from 7 to 9, and the same happens with the sums between digits from 6 to 9.

Addition table from 0 + 0 to 9 + 9

+	0	1	2	3	4	5	6	7	8	9
0	0									
1	1	2								
2	2	3	4							
3	3	4	5	6						
4	4	5	6	7	8					
5	5	6	7	8	9	10				
6	6	7	8	9	10	11	12			
7	7	8	9	10	11	12	13	14		
8	8	9	10	11	12	13	14	15	16	
9	9	10	11	12	13	14	15	16	17	18

Multiplication table from 0 x 0 to 9 x 9

X	0	1	2	3	4	5	6	7	8	9
0	0									
1	0	1								
2	0	2	4							
3	0	3	6	9						
4	0	4	8	12	16					
5	0	5	10	15	20	25				
6	0	6	12	18	24	30	36			
7	0	7	14	21	28	35	42	49		
8	0	8	16	24	32	40	48	56	64	
9	0	9	18	27	36	45	54	63	72	81

So, this is where your "4 basic steps to improve your calculation skills" end. Just keep the right attitude, respect your brain, train your memory and strengthen your basis ... you'll enhance your skills in ways you could never believe. And this is just the beginning!

IV - The finger calculator

Everybody knows that one can perform little additions simply by using his or her fingers. It's one of the first things that they taught us in the school, after all, isn't it? But did you know that, by using your fingers, you could perform even some very ... *simple multiplications?*

If, for example, you have a shocking blackout during a complex calculation, and you can't remember a specific multiplication table, you can use two awesome tricks that will make you able to instantly perform a lot of possible single-digit multiplications.

But let's start with the first, really straightforward technique, which takes advantage of the fact that if you sum together the digits of any multiple of 9 (<90), you will always get 9 as a total. More specifically, it lets you immediately perform any multiplication from 9x1 to 9x10. And here's how you can use it:

- Turn both your palms towards your face, pointing the extremity of your fingers upwards.
- Now associate to each finger a number from 1 to 10, corresponding to the order they have. So, your left thumb will be

1, the left index will be 2, and so on, till the right thumb, which will be a 10.

- To perform 9 multiplied by a, simply lower the "a" finger.
- The first digit of your result will be equal to the number of fingers *before* the lowered one. The second digit of your result will be equal to the number of fingers *after* the lowered one instead.

So, if for example you want to calculate 9 x 2, the number 2 is associated to your left index. If you lower it, you will see that there is 1 finger before the index and 8 fingers after it: 18!
The same if you don't remember the result of 9 x 6: six is associated to your right pinkie, before the right pinkie there are 5 fingers, and so the first digit is 5. After it there are 4 fingers instead and in fact 9 x 6 = 54.

But let's immediately look at the second technique too, useful for retrieving the result of any multiplication from 6x6 to 10x10. Here it is:

- Turn again both your palms towards your face, but now point the extremity of your fingers laterally inwards, in order to let the two middle fingers touch each other.
- For both hands, associate 6 to the pinkie, 7 to the annular, 8 to the middle finger, 9 to the index, and 10 to the thumb.
- If you want to multiply two numbers, let the extremities of the corresponding fingers touch each other, making sure you keep the palms orientation I just described.
- The fingers under the point of contact, including the ones in the point of contact itself are the "under" fingers.
- The fingers above the point of contact will be the "above" fingers.
- Multiply the number of the "under" fingers by ten. Call this result "a".
- Multiply "above fingers on the left" by "above fingers on the right". Call this result "b".
- Sum a and b together.

But since this technique may seem a little bit more complex than the previous one (but still very easy to manage, anyway), let's introduce a little example and let's try to use it to calculate 6 x 7. We will act like this:

- Let our left pinkie touch our right annular (or vice versa).
- We will have 3 "under" fingers (the two touching fingers plus the pinkie). 3 x 10 = 30.
- "Above right" equals 3, while "Above left" equals 4. 3 x 4 = 12.
- 30 + 12 = 6 x 7 = 42

Second example, 8 x 7:

- Our right annular will touch our left middle finger (or vice versa).
- "Under" equals 5. Multiplied by 10 equals 50.
- "Above right" equals 2. "Above left" equals 3 instead. 2 x 3 = 6.
- 50 + 6 = 8 x 7 = 56

Here comes the fact that this technique can't be used properly if you cannot remember the lower multiplication tables. But this probably it won't be a big problem, considering that most of people have problems right with multiplications by the digits very close to 10, like 7, 8 or 9.

Through your fingers you can even use two little "mathematical" lifehacks that can come in really handy in everyday life:

- ***Measure the length of any object.*** It's quite simple: measure one of your fingers with a rule, always keep in mind the result and the next time you need to measure any object, and you don't have the proper tools with you, you don't have to do anything but counting *how many times the object is longer than your finger,* and then multiplying that result by your finger length.

- ***Measure the height of a tree, a building, or any other big object.*** This technique was used since ancient times: put a person you know the height of next to the big object you want to measure, place your finger next to your eye at such a distance that that it's exactly as high as that person ... and apply the previous technique! The multiplication here is going to be difficult to perform? Don't worry; *the techniques in the next chapters will exactly come to your aid!*

Now you just got the full basis to enter into the world of the speed math and so it's time to introduce a very fascinating topic, loved as much by the mathematics fans as by the Asian spirituality one: the Vedic Mathematics.

"Mathematics is a great and vast landscape open to all thinking men that adversely joy, but not very suitable for those who do not like the trouble of thinking."

(Lazarus Fuchs)

V - Mathematics and Hindu wisdom

Was the beginning of the 20th century, while an influential researcher of Hindu philosophy discovered into its personal copy of a sacred Hindu book, the *Atharvaveda*, a summarise of very original mathematical rules. More specifically, in that book he found *sixteen Sutras* (aphorisms of Hindu wisdom) and some corollaries presenting a completely original and *much more creative and flexible approach to various mathematical problems.*

Since then this "Vedic approach", which vas completely different from any other approach taught in the Occidental world so far, started spreading all over the world. For example today it's pretty renown among the most important USA schools, where it even represents the most prestigious chapter in their teaching plan.

Probably the popularity of the Vedic Mathematics is exactly due to the fact that *it commends everyone's creativity and inner expression*, making mathematics easier and more attractive for any kind of student. In fact, many of the mathematical stratagems we'll see in the next chapters will be exactly drawn from the Vedic Sutras, or their corollaries, often combining them in order to get fast, creative and extraordinarily effective calculation strategies.

But let's start immediately and let's explain the meaning and the purpose of the *"Nikhilam Sutra"*, which declaims: *"All from 9 and the last from 10".*

What? Well, it's not cryptic as it sounds, actually: this Sutra reveals nothing but a very fast and efficient method to *calculate the distance (or difference) between any number "a" and its higher power of 10* (which is, in the unlucky case you forgot it, that number made up of a "1" and as many zeroes as the digits of "a". So, for example, will be 10 for 4, 100 for 55, 1000 for 768, and so on).

So it will basically help you to rapidly perform calculations like "1000 - 658", "10 000 - 4530", "1 000 000 - 564 324", and so on. But its power, as we will soon see, doesn't end here.

In any case, to apply the Sutra in its essential form, you can do like this:

- Make sure that the subtrahend *(the number after the minus)* has as many digits as the zeroes of the power of 10 you want to subtract from. For example, you can use it to calculate 100 - 67, 1000 - 345, or 10 000 - 4589.

- Starting from left, subtract each digit of the subtrahend from 9 (so, be careful, *calculate 9 minus the subtrahend and not the other way round),* except the units digit, which must be subtracted from 10. Then, put all the results of these subtractions in order into the final result.

- Every time you get "10" as a result (like, for example, it happens when the units digit is 0), set 0 as current result digit and add 1 to the result digit on its left.

- That's the result of your subtraction!

So, for example, let's try to calculate 10 000 - 4350. We will have that:

After subtracting 4 from 9 the first digit in the result (on the left) will be 5

After subtracting 3 from 9, the second digit in the result will be 6

After subtracting 5 from 9, the third digit in the result will be 4

After subtracting 0 from 10, the fourth digit in the result will be 10. Put 0 in the result and carry 1 to the third digit that will then become a 5. Final result: 5650

With the classic method we learnt at school a subtraction like this would have required a much longer and more complex procedure. But here you can deal with the problem just through a few, simple steps, limiting so even the probability to run into an error.

But let's extend the Sutra now, and let's use it to perform different kinds of subtractions, too.

For example, you can use it to faster perform subtractions between numbers having the same amount of digits, and whose minuend *(the number before the minus)* is positive and made only up of zeroes except for the leading digit. So, for example, you can apply it to subtractions like "500 - 388", "6000 - 4567", or "70 000 - 43 200". In fact in this case you can:

- Apply the Sutra *"All from 9 and the last from 10"* in order to obtain the difference between the number and its greater power of 10.

- Subtract from the result of the Sutra the difference between that power of 10 and your minuend. So, if for example your minuend was 3000, then you will have to subtract 7000 from the Sutra to obtain your final result. If your minuend was 500, then you will have to subtract 500, and so on.

But another example of extension of the possible applications of the Sutra could be in using it to find the difference between a number and the power of 10 having *one more zero* than its amount of digits. So, for example you can use it to calculate 1000 - 28, 10 000 - 345 or 100 000 - 6 432. In that case, in fact, you will just have to:

- Apply the Sutra to the subtrahend.
- Add to the result a number made up of a 9 and *as many trailing zeroes as the amount of digits of the subtrahend itself* (and so, for example, 900 if the subtrahend has two digits, 9000 for three digits, and so on).

It's pretty simple, isn't it? So, if for example you want to calculate your 1000 - 28 you will have to apply the Sutra to 28, obtaining 72, and then banally add 900: 972!

At this point you may have noticed that those last two examples of Sutra extensions came basically from the same line of reasoning, which is, *given any kind of a - b, you can:*

- Apply the Sutra to b.
- Find out what's the higher power of 10 for b.
- If "a" exceeds that power by a specific amount, add that amount to the final Sutra result.
- If "a" is lesser than that power by a specific quantity instead, subtract that quantity from the Sutra result.

So ... 1002 - 456? Apply the Sutra to 456 and add 2 to the result!

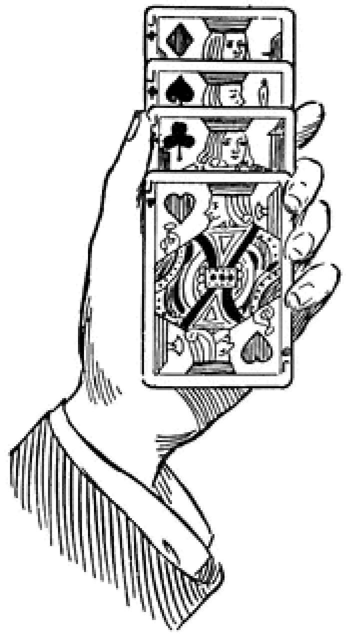
20 004 - 3498? Apply the Sutra to 3498 and add 10 004!

9998 - 4432? Apply the Sutra to 4432 and then subtract 2 from the result.

The first practical application of these strategies could consist in using them to *rapidly calculate expenses, starting from an initial budget,* which in fact is very often a "rounded" number. For example, if before starting a project you had 7000 dollars on your bank account and now, after a month, only 1288 dollars remain, you can apply the Sutra and then subtract 3000 from its result in order to rapidly calculate how much did you spend. So, in this case, the expenses were 8712 - 3000 = 5712 dollars.

But this was just an introduction to the topic and the applications of the Vedic mathematics of course don't end here. In fact, in the next pages, it will come in handy when we'll start talking about *fast multiplication.*

VI - Assemble, decompose and Blackjack



This chapter will teach you three basic techniques for more rapidly performing additions and subtractions. Then, at the end of the explanations, it will show you a definitely interesting practical application for all of them.

In addition, I'm pretty sure you've used some of these strategies in past yet, even if in an "intuitive" and "automatic" way. But, anyway, making you fully aware of them will help you to exactly understand when to use them, and so to unleash their full potential.

So, let's start explaining the first technique, which is very simple and useful, especially if you have to add (or subtract) among many small numbers. It banally consists in *pre-emptively summing up the addends, or the subtrahends, in order to get multiples of 10.*

For example, let's suppose that you have to calculate a 3 + 4 + 5 + 6 + 7 + 2 + 5 + 1 + 3 + 4 + 3

The first thing you can do is combining the first "three" and the fifth "seven", getting 10 + 4 + 5 + 6 + 5 + 1 + 2 + 3 + 4 + 3.

Now we take the penultimate "four", the last "three", then the third last "three". After being summed together they equal ten, so we'll have 20 + 4 + 5 + 6 + 2 + 5 + 1.

Again, after adding "four", "five" and "one" I get a third 10, and then 30 + 6 + 2 + 5 = 43.

Apart from the obvious practical simplicity, this method has even the advantage to be essentially "creative", especially for children. The "Quest for the ten", in fact, can also be fun and easily help the children to have fun with this specific strategy.

But let's rapidly go to the second technique, which is very similar to the previous one, can be used with additions or subtractions of any kind, and consists in "borrowing" an amount from one of the addends (or subtrahends) and adding it to another one of them in order to transform the latter into a multiple of 10.

Let's assume, for example, we want to calculate a 368 + 214. One thing that you can immediately do here is to simplify the calculations by borrowing a "2" from 214 and carrying it to the 368. So you will get a "370 + 212" that can be calculated in two seconds.

Same thing if we have a 967 - 255. We "borrow" a 3 from the 255, getting a much easier 970 - 258. As you may have noticed, since this is a subtraction, here the rules *change a little bit*: in fact, any quantity borrowed from the *subtrahend* must then be *added* to the subtrahend itself, while any quantity borrowed from the *minuend* must then be *subtracted* from the subtrahend instead. So ... be careful in this case!

Off topic: in order to perform 970 - 258 you could use a *Sutra* as introduced in the previous chapter, and this proves that the real point of strength of the "tools" contained into this book is the fact that they *can be freely recombined in order to create appropriate calculation patterns for any kind of situation.*

The "borrowing rule" also lets us build these easy simplification rules. They of course may seem obvious to somebody, but it's always a good practice to refresh in your mind even what's obvious, so that you will be able to even more quickly recall it from your memory anytime it's needed (exactly as we said in Chapter II).

- If you want to add 9, just add 10 and then subtract 1
- If you want to add 8, just add 10 and then subtract 2
- If you want to add 7, just add 10 and then remove 3
- If you want to subtract 7, just subtract 10 and then add 3
- If you want to subtract 8, just subtract 10 and then add 2
- If you want to subtract 9, just subtract 10 and then add 1

And of course the same reasoning can be applied to numbers like 19, 90, 80, 900, 800, and so on.

So, let's introduce the last technique, that's very simple as well, and will come in very handy anytime dealing with three-or-more-figure numbers: just *decompose one of the operands into a sum among its units plus its tens, plus its hundreds, and so on ... and then operate separately with these parts!*

So, when for example you have to calculate a 3456 - 1234, will be much easier for you if you decompose 1234 into *1000* + *200* + *30* + *4* and then calculate, in sequence:

3456 - 1000 =

2456 - 200 =

2256 - 30 =

2226 - 4 =

2222

The human mind in fact is much more rapidly and easily able to *perform a longer set of simple calculations than a single and more complex one*. And, as we will see in future, this is the basic principle a lot of speed math strategies are based on.

Now that we explained the three fundamental speed math strategies involving addition and subtraction, let's introduce a commonly known and quite interesting practical application for them: *the Blackjack "card counting".*

If you don't know what I'm talking about, let's start with a little premise: every gambling game is built in such a way that, in the long term, we will always lose our money while the Casino will always take everything dropping out of our pockets. This means that every gambling game *has a negative "expected value"*, as we will explain more in detail in Chapter XIX.

The only exception to this truth *is in the BlakJack* and in the fact that, if approached with the right strategy ("counting cards", for example), it can have a positive expected value and so it can lead us to a progressive enrichment in the long term.

Furthermore, Blackjack is a very simple game: you, the dealer and the other players are given an initial two-cards hand. It's each player against the dealer. Face cards are counted as ten points. Ace is counted as one or eleven points depending on the player's choice and every other card keeps its "standard" value.

So, in turn, each player can choose whether to "stay" with his/her hand or to have another card, trying to get as closer as possible to 21 points, but without exceeding this value. In fact, who exceeds 21 points automatically loses his/her bet, which goes to the dealer.

So, after each player still in game defined his/her score by deciding to "stay" with his/her hand, it's the dealer's turn. At this point he/she will have to make the same as the other players, deciding card after card if continuing to draw or keeping the hand. But the only difference here is that he/she's forced to continue drawing until he doesn't make a score of 17 at least. And if he/she will exceed 21, he/she'll have to pay to every player his/her bet. Otherwise he/she will simply have to pay the bet to any player with a higher score than his/hers, and take it from those having a lesser score.

Now that I'm sure you know what I'm talking about, let's introduce some good news about card counting:

- Counting cards is simple, or better, we'll talk about a very simple card counting method: the so-called "High/Low".
- Counting cards is perfectly legal.

- If you have been traumatized by some Hollywood movie in which the people who have been discovered counting cards were hardly beaten up, you can sit and relax because *it's just Hollywood fiction!*

And now, the bad news:

- Even if anybody will unlikely touch you when he/she finds out that you're counting cards, it's seriously possible that he/she could kindly invite you to leave the Casino.
- You will never be able to count cards when playing at some virtual Casinos: the virtual decks are virtually and repeatedly shuffled, with no chance to build any winning strategy.
- In the "physical" Casinos they actually do everything in their power to reduce your positive expected value to the minimum.

So, the question here could be: why should I ever still be interested in this kind of activity? Well, I can think about two answers:

- Because training your calculation skills by gambling *is infinitely funnier than doing it by reading books.*
- Because if you fly low and don't exceed in using this technique, in some little Casinos *it's still possible to earn some little cash without any risks or too much effort.*

Let's start, then! Let's imagine you're sitting at the Blackjack table with a mojito (hoping it's your first one or your mathematical skills definitely won't be at their best). We will start our count from 0 and, as long as we see the dealer drawing the cards from the deck, we will do like this:

- **When you see a 2, 3, 4, 5, or a 6, then add 1 to your count**
- **When you see a 10, Jack, Queen, King or an Ace, then subtract 1 from your count**
- **When you see 7, 8 or 9, then leave your count as it is**

As you can immediately notice, sure it's a very simple count, but it *must necessarily be done very fast*, and in order to achieve this you could use the first two strategies introduced in this chapter.

So, as far as we keep counting, we must consider that a high total (+6/+9) reveals that the deck is freight with high cards and figures and, as a consequence, it means that we have a strategic advantage against the dealer (who in fact will more easily exceed 21). A low total instead reveals that we have a higher probability to lose against him/her.

More in detail, this is an example of a both prudent and effective strategy. Decide how much a reasonably "basic" bet is and:

- **Count is negative or equal to 0: don't bet**
- **Count is +1: bet your "basic" bet**
- **Count is +2 or +3: bet the double of your basic bet**
- **Count is +4 or +5: bet the triple of your basic bet**
- **Count is +6 or +7: bet the quadruple of your basic bet**
- **Count is +8, +9 or higher: bet the quintuple of your basic bet**

So, here ends our digression about this eternally fascinating Casino game. Many more things of course could be said, but unfortunately this is not the right place. Anyway, never forget that you're lucky enough to be *born in the digital information age:* just a simple Google search and you'll be able to find a lot of interesting tricks to maximize the probability to beat the dealer while playing Blackjack. Good luck!

VII - The magic column

Let's repeat something very important we said in Chapter II: a calculation strategy becomes as faster and more efficient as *it's designed to let the brain store as less information as possible.*

Let's suppose, for example, we must perform this addition:

989+
724+
102+
670+
112=

If we had to calculate it through the classic column method we traditionally learnt at school, we should separately sum the units first, then the tens and the hundreds at last, trying to bear in mind every partial amount carried to get the final result. And trying to do that without any pen or paper would be totally frustrating, when not impossible.

So, as in mathematics as in life, when something doesn't work properly *we have to do nothing but changing our strategy*, and here is where the "Magic column strategy" comes in our help. But how can we apply it? Let's see it:

- Instead of starting from the units, start from the first digit on the left and sum all the numbers on the left column together, making sure you mentally repeat, while you calculate, only the results of the partial sums.
  So for example, if you look at the above-mentioned addition, you should mentally perform 9 + 7, then the result + 1, then + 6, etc., but only saying in your mind "9, 16, 17, 23, 24" (and here to faster perform all the partial sums you can easily use the techniques introduced in the previous chapter).

- Once you finished performing the sums on the left column, your partial result is 24. Now move to the next column on the right, take its first digit, put it next to your partial result (so now you will have 24_8) and, without touching the result of the previous column, add to the second number of your partial result the other digits of your new column, exactly like you did in the previous step. So, repeating the same procedure as before, you will have: "24_8, (after adding 2) 24_10, (after adding 0) 24_10, (after adding 7) 24_17, 24_18". And only if, like in this case, after performing your sums you get a number that's greater than 10, put in the new partial result only the units and sum the tens to the number on the left. So, in this case your partial result is 24_18 = 25_8.

- Move again to the next column on the right and put its first digit next to your previous result (in this case, so, you will have 25_8_9). Then, behave exactly as you did in the previous steps: don't touch the previous result, and sum the digits of the new columns to the number on the right. So you will have: "25_8_9, 25_8_13, 25_8_15, 25_8_17 = 25_9_7 (since we removed the tens digit)". There are no more columns, so *you can remove the underscores and 2597 is your final result.*

So we definitely got the result of a long sum in an extraordinarily fast and efficient way, keeping in mind only a very small amount of digits and without paying any attention to the partial amounts to carry.
Of course, in case we must put in column numbers made up of a different amount of digits one from each other, then we should:

- Align the units with the units, the tens with the tens, the hundreds with the hundreds, and so on.
- Using the same technique as the empty spaces (that inevitably will be on the left side of the smaller numbers) were instead leading zeroes. We could also *write* those zeroes, if it makes the things easier for us.

But let's have another example and let's say we have to calculate 1341 + 450 + 2451 + 888 + 9872. As a first thing, we must align the numbers in a column and, to make the things easier, we can write a leading zero next to each three-figure number:

1341+
0450+
2451+
0888+
9872=

- Starting from the left we have: "1, 3, 12". So our partial result is 12.
- After going to the right we have: "12_3, 12_7, 12_11, 12_19, 12_27". So our partial result is now 147.
- After going to the right again we have: "147_4, 147_9, 147_14, 147_22, 147_29". So the other partial result is 1499.
- After working on the last column we have: "1499_1, 1499_2, 1499_10, 1499_12". After carrying the 1 twice, the final result will so be 15 002.

Now, show this new ability to your friends, of course without explaining the strategy behind! I'm quite sure that, if you train yourself to perform any partial sum rapidly and without making any errors, the "scenic effect" will be absolutely awesome!

Do you want to pass to the "next level"? Train yourself to:

- Work even without aligning the numbers
- Work on two or more columns at the same time in order to perform even faster calculations.

But let's explain the second point. Let's imagine we have:

561+
343+
912+
134+
451=

We know how to operate column by column. But if we want to make the things even faster, we could:

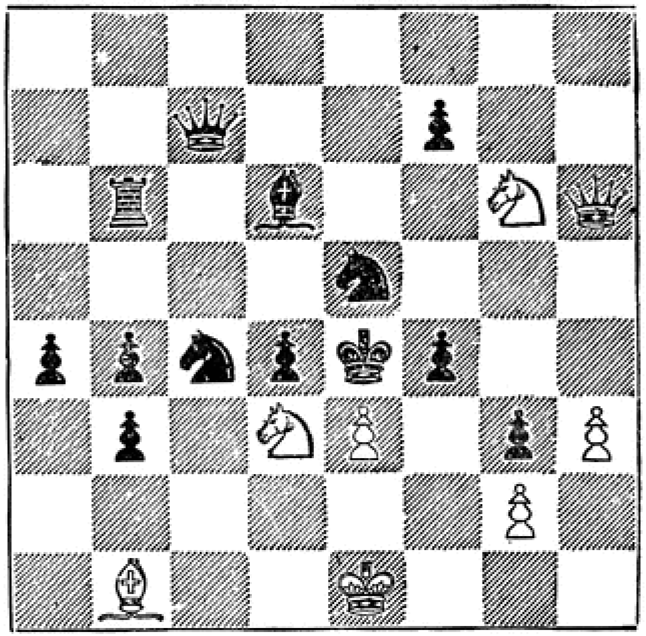
- Start from the left as before, but now, as a first step, sum directly the two-figure numbers taken from the first two columns. And so beginning with "56, (+ 34 =) 90, (+ 91 =) 181, 194, 239"
- Behave as before on the last column, calculating so: "239_1, 239_4, 238_6, 239_10, 239_11". The final result, in fact, is 2401.

This of course is just the next step, and you'll be confident with it only after properly training yourself to rapidly perform any two-figure number sums (even thanks to the strategies we introduced in the last chapter) and the "single-column" version of this technique as well.

"Mathematics is like checkers, in being suitable for the young, not too difficult, amusing, and without peril to the state."

(Plato)

VIII - Game theory pills



Now, since we just learnt quite almost anything about fast addition and subtraction, let's start to introduce an extremely useful, interesting and fascinating practical application of these operations: let's start talking about *game theory.*

Game theory, for those who don't know it, is nothing but a matter studying any kind of situations involving *people making decisions.* It's an extremely large and complex argument, and so we're definitely not going to have a complete dissertation about it. Anyway, giving you some hints about the topic will come you in extremely handy anytime you will face situations *in which you'll confront with other people making their own choices, potentially in conflict with yours.*

More in detail, I'll give you mostly some hints about the so called "2 x 2 games": all kinds of situations in which *the deciders are two,* and these two people can make their decisions in a range of *two specific choices.* And I'll do like that as because of the simplicity of the case, as because it's a quite recurring circumstance, as we'll soon see in our examples.

But let's start introducing some game theory notation, taking as an example the canonical "Prisoner's dilemma":

"Two members of a criminal gang, A and B, are charged and arrested because of an important bank robbery. Each of them can't speak to the other one. The police anyway haven't enough evidence to convict both of them on the principal charge, and so the officers try persuading them to confess, making both of them an offer:"

- If both confess, each of them serves just one year in prison.
- If A confesses, but B denies, A will be set free and B will serve 10 years in prison.
- If both deny, each of them will serve 5 years in prison.

Now, let's imagine we're "A", and let's try to schematise this situation through a table:

	The other one denies	The other one confesses
We deny	-1	-10
We confess	0	-5

As we can immediately notice, in this table the rows are our potential choices, the columns are the potential choices of the other person, while the corresponding cell indicates the years of prison we'll serve in case those choices are made at the same time (and the "minus" sign is due to the fact that the years of prison are obviously considered as a "handicap" score).

So, given this kind of table representation, it's time to understand, through some mathematical help of course, *which choice is potentially the more convenient for us.*

More specifically, the first thing to do would be trying to understand *whether we could successfully adopt something called "a pure strategy"*. In other words, we must evaluate whether there is a choice that's ALWAYS more convenient than the other one, no matter what our "opponent" does.

But how do we recognize the chance of successfully adopting a pure strategy? Well, it's very simple: we can adopt it in case EACH value on a specific row is bigger than the corresponding value on the other row.

In this case, for example, the pure logic indicates that *confessing is always more convenient than denying*, considering that 0 > -1 and -5 > -10. So, confessing is universally the best choice to minimize our risks, no matter what the other prisoner is going to say.

But let's take an example of "game" without any chance of pure strategies and, in order to understand what's the best thing to do in that case, let's examine a *quite recurring situation during the poker matches:* call or fold?

2000 dollars are in the pot, whose 300 dollars are our last bet. We changed two cards in our hand yet. Our last opponent in game raises by 1000 dollars and our hand, after changing, is good but nothing extraordinary. So, depending on whether our opponent is bluffing or not, we must decide whether to call his bet or not. Let's put this situation in our "game table":

	The opponent is bluffing	The opponent is not bluffing
We call	2000	-1000
We fold	0	300

In this case, as we may immediately notice, there are no values on any row that are clearly bigger than the corresponding value on the other row (2000 > 0, but -1000 < 300). So, the best way to act in this case would be:

- Take the higher value in each row
- Subtract the smallest value in each row from *the higher one in the same row.*
- Call this value DV (Difference Value), and put it next to the OTHER row.

So, in this case you'll have to go to the "call" row, calculate 2000 - (-1000) = 2000 + 1000 = 3000, and put it next to the "fold" row. At the same time you'll have to go to the "fold row", calculate 300 - 0 = 300 and put it next to the "call row". Then you'll have:

	The opponent is bluffing	The opponent is not bluffing	DV
We call	2000	-1000	300
We fold	0	300	3000

Now, what do those values mean? Simply that, given this specific situation, folding can be a situation statistically *ten times more convenient than calling.* More in detail, if we write the two Difference Values respectively as a numerator and as a denominator of the same fraction, and we *simplify this fraction until the smallest value is 1* (which is simply the same as dividing one value by the other one), *we get an evaluation of "how many times one choice is going to be more convenient than the other one"*. Which, returning to our case, in which one choice is specifically 10 times more convenient than the other one, means:

- If it's a recurring situation with exactly the same values, take the two choices in a 10-to-1 proportion. Which means, take in average the "10" choice 10 times, the "1" choice once and repeat it until the other variables don't change.

- If it's a one-case situation (which is likely nearest to reality in the poker case), sum the two DVs (in this case, 10 + 1 = 11) and do the "10" choice with a probability of 10 out of 11, and do the "1" choice with a probability of 1 out of 11. This could simply mean "just take the choice with a higher DV". Or, in case you want to do something mathematically more accurate (which is advised in case the ratio between the two choices is not so clear), use any tool capable to "extract" a random number in order to let it be in accordance with your exact probability. Oh, and you don't necessarily need a coin to flip or a die to roll here: for example, you could simply look at the second hand of your watch and then call only if it's *pointing at the first six seconds of the current minute.* In fact, if the seconds in a minute are 60, the probability we are in the first 6 seconds are exactly 6 out of 60 = 1 out of 10, which is quite near to 1 out of 11.

If the last reasoning is still not very clear to you *don't worry about it*, because we will explain more about probability theory later in the book. Even because, obviously, all of it starts with the assumption that you can't decipher your opponent's behaviour at all (as was happening in the prisoner dilemma, the fundamental premise is that there is no chance of "information leak" between the two counterparts). But if, for example, it's known that your opponent likes bluffing, you can refine your way of thinking through probability strategies we'll see later. By now, anyway, just be aware of the fact that that *the choice with a higher DV has a higher probability to be the right choice.*

Of course now somebody may notice that, in case of poker pots or years of prison, the procedure is simplified by having very clear quantities involved in the situation, while the reality is made up of hues and unknown variables. Well, in case of not clearly definable or computable quantities, the answer is in something we said in Chapter II: try to *give a mark* to what could happen as a consequence of specific choices. Of course this could not be very accurate, but it's still much more advantageous than a completely blind decision making.

For instance, let's suppose we are in a situation that's completely different from all of the previous ones: we must deal with an important oral exam at our university. We have really a little time to study, and we can choose to study only one between two particular arguments: a very simple one and a very difficult one. In addition to this, we're quite sure that our professor is going to ask us one between those two arguments. So, let's try to give a mark, from 1 to 10, to any potential situations, like:

- *We study the simple argument and the professor asks us about the difficult one*: 1, because in this case we will just ... make a very poor figure!

- *We study the simple argument and the professor asks us about the simple one:* 9, because in this case our examination will quite likely go very well.
- *We study the difficult argument and the professor asks us about the simple one:* 4, because our poor figure could be softened by the fact that our professor can appreciate the way we dedicated our time to something harder to learn.
- *We study the difficult argument and the professor asks us about the difficult one:* 8, because we could, despite everything, show some doubt due to the fact that that specific topic is quite hard to learn, difficult to handle and not very simple to talk about.

So, our "game table" could look like this:

	The professor asks for the simple argument	The professor asks for the difficult argument
We study the simple argument	9	1
We study the difficult argument	8	4

Here there is no chance to adopt a "pure strategy" as well. So, let's calculate our DV:

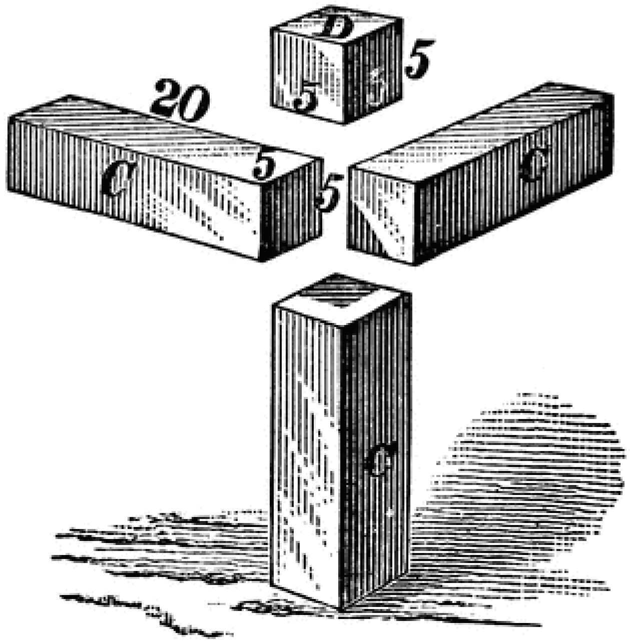
	The professor asks for the simple argument	The professor asks for the difficult argument	DV
We study the simple argument	9	1	4
We study the difficult argument	8	4	8

9 - 1 = 8 and 8 - 4 = 4.
4 / 8 = 1 / 2. So, the "difficult argument" choice is twice more convenient than the other one.
That means, considering that this is not a recurring situation, that *we can just study the difficult one*. Otherwise, for something more accurate, we can sum the two DVs (4 + 8 = 12), and take the 4 choice with a probability of 4 out of 12 (4 / 12 = 1 / 3), and the 8 choice with a probability of 8 out of 12 (8 / 12 = 2 / 3). And this can be easily done by tossing a die, and deciding to study the simple argument if 1 or 2 comes out, and the difficult argument otherwise. I'll repeat it: if the probability question is not very clear to you now, don't worry about it. It will be when we'll introduce it in the Chapter XIX. For now just be aware of the given instruments and of the fact that in any moment of work, study, leisure or relax, you'll likely face "2 x 2 games" circumstances. And so, learning how to take the best out of situations like these *means learning how to take the best out of an enormous amount of situations in your everyday life.*

But before leaving the chapter, let's put aside the "2 x 2" games in order to take a look to a wider topic. In the early 60s, a mathematician named John Nash, you probably have heard about because of the movie "A Beautiful Mind", formalised one of the most important mathematical concepts of the past century. Nash was living in a society that was grown and was prospering according to those economic principles expressed by Adam Smith, *telling that the maximum abundance in a group can be achieved only when everyone puts his best effort into pursuing his own personal profit*. However, in contrast with that, after analysing and studying the evolution of complex "games" in which even lots of decision-makers take decisions from within a wide range of choices, Nash mathematically demonstrated that "*the maximum abundance in a group can be achieved only when everyone puts his best effort into giving the biggest advantage as to himself as to the others "players"*. He made that small, but essential addition to what Smith said and found the mathematical proof that in any strategic situation the biggest profit for everyone can be gained *only by adopting a wider,* *group-oriented point of view*. And the truthfulness of this concept *has been actually proven in the further years by numerous studies, experiments and simulations.*

The "golden rule" one can learn from all of this is actually very obvious and seemingly trivial: whenever you take any decision, *learn to think and act in terms of win-win resolution for you, all the people involved and the whole environment in which you are*. Of course this is not always possible, but if you at least try to widen your perspective this way, you'll definitely have a great gain in terms of happiness, success and personal satisfaction. Dishonesty, on the other hand, enriches the individual but pauperises the environment in which he lives. An unbridled egotism can boost the overall capacities of a single human being, but at the same time it limits the development of the other people he has to deal with. And the right abundance and a strong self-development are sterile, useless and fragile for anybody if he's lacking of a proper human, social and structural environment that can adequately supports all of these things. Cooperation and win-win-oriented prospective after all seem to be the unique, real key to the future for any human group, from the smallest family until that quite large crowd living on this planet. It's much more than a set of banal, or even rhetorical rules of common sense: it's something as real and undeniable as the laws defining the orbits of the planets around the sun or the acceleration of free-falling objects: it's *mathematics*!

IX - Smash, simplify and save



In this chapter we'll start a long and complete dissertation about *fast multiplication and fast division.* As you'll in fact notice by yourself, all the mathematicians and the speed math students gave a strong importance to these two operations, developing a very big amount of strategies to calculate them in a very fast and efficient way. And, as we said before, *bigger the choice among the strategies, greater the freedom to approach the problem solving process in an entertaining, recreational and creative way.*

In addition to this, multiplication sure is a critical and fundamental operation into the everyday math: just think to its importance in basic economics, in percentage calculations, or (as we'll see soon), in any strategic decision involving *probability theory!*

So now, before moving toward our strategies, I want to give you just one, last advice: *don't try to learn all of them at the same time*, but read them slowly, exercise yourself and try to repeat my examples with pen and paper, trying at the same time to understand which kind of strategies are more fitting for you.

Now, let's pick up a principle we said in the previous pages: *your mind can much more easily perform a longer set of easier* *calculations than a smaller sequence of more complex calculations.* In fact, the purpose of the first three techniques we'll introduce will be exactly about *decomposing a harder multiplication into a sequence made up of very simple ones.*

And the first of these three techniques is the straightforward *"decomposition into addends"*, which can be used only on multiplications and can be applied like this:

- Decompose one between the multiplicands into a sum or subtraction. Typically the easiest thing to do is decomposing it into a sum between its units plus its tens plus its hundreds, etc. (for example: 456 = 400 + 50 + 6)

- Multiply separately these parts using the distributive property of addition. So, if for example you have 334 x 456, this will transform your multiplication into 334 x (400 + 50 + 6) = (334 x 400) + (334 x 50) + (334 x 6).
  So, instead of performing a multiplication of three-digit numbers that, if made through the classic column method, would have required a consistent amount of time, you'll have to perform a simple sum of single-digit multiplications, having, as an only annoyance, to add some trailing zeroes (you didn't forget how to perform the multiplication by 10, 100, 1000, right?).
  I clearly could also decompose the 334 into 300 + 30 + 4 and so transform the original multiplication into (456 x 300) + (456 x 30) + (456 x 4). The commutative property of multiplication in fact says that *no matter which number we apply the decomposition to, because the final result won't change*. And that's why it's up to us, from time to time, to choose the kind of decomposition that can lead us to the more immediate set of micro-operations.

Please notice that *the decomposition into units, tens and hundreds is only one of the options:* in fact, any number can be decomposed into infinite, different ways. If for example you want to perform 44 x 7, then instead of doing 44 = 40 + 4, you could apply this technique by decomposing 7 into (10 - 3), and thus 44 x 7 = 44 x (10 - 3) = (44 x 10) - (44 x 3) = 440 - 132 = 308. And decomposing into a subtraction instead of a sum is a strategy that greatly simplifies your life whenever you have to *multiply between numbers that are close enough to a multiple of 10*.

But let's have another example to clarify this concept once and for all: if you have to multiply 400 x 59, instead of decomposing the 59 into (50 + 9), you can do it as (60 - 1), and so 400 x 59 = 400 x (60 - 1) = (400 x 60) - (400 x 1) = (400 x 60) - 400 = 24 000 - 400 = 23 600, which was much easier to perform than the other option.

So, let's rapidly go to the second type of decomposition: *the factorization*, which can also be used for the division and requires some extra premises.

First of all, remember that decomposing a number into factors (or factorizing) means *transforming it into a multiplication among smaller integers, whose result equals the original number*. In addition, decomposing it down into its *prime factors* means transforming it into a multiplication *among prime integers* that returns the first number as a result. And, as you surely know, a prime integer is *an integer that's divisible only by one and by itself.*

For example, you decompose 16 into factors *when you say that 16 is equal to 4 x 4*. But you decompose 16 into *prime* factors instead *when you say that 16 is equal to 2 x 2 x 2 x 2.* In fact, 2 is a prime number, while 4 it's not.

Now, just think about the advantages of being able to decompose a number into factors, even better if prime. In fact if, for example, you have an 88 and you have to multiply it by 16, you will have that 88 x 16 = 88 x (2 x 2 x 2 x 2). The associative property of multiplication tells you that you can perform these tasks in any order, and so multiplying by 16 will be equivalent *to the much more rapid operation of doubling your original number four times.*

And a similar thing happens if you want to *divide a number by 16*, for example 256. In fact, 256 / 16 = 256 / (2 x 2 x 2 x 2) . The division does not have associative property and therefore we are obliged to proceed by applying the simple properties of the fractions and transforming the above expression into an equivalent ((((256 /2) / 2)

/ 2) / 2), that simply means that *you must divide 256 by 2 four times*. Similarly, if you want to divide a number by 96, and 96 = 2 x 2 x 2 x 2 x 2 x 3, you should first halve the number five times and then divide it by 3.

In other words, *dividing a number "a" by a number "b" is equal to dividing "a" by the sequence of factors of b.*

Of course the factorization is not an immediate procedure, especially in case of large numbers, and that is why it becomes necessary to learn, or at least brush up on, the so-called *divisibility criteria.*

Given that multiplication is the inverse operation of division in fact you can say that, *if a number "a" is divisible by another number b, then you can factorize "a" into a product "b multiplied something"*. Furthermore, it's obvious that this mysterious "something" is exactly equal to a / b.

For instance, suppose you want to factorize 256 step-by- step:

- After finding out that 256 is divisible by 4, you can immediately say that 256 can be factorized into a *"4 multiplied something"*
- You can more specifically get that "something" by performing 256 / 4 = 64. So, 4 x 64 = 256.
- You know that 4 is divisible by 2 and that it's equal to 2 x 2. Even 64 is divisible by 2, and being the double of 32 it's obviously equal to 32 x 2. Therefore, by replacing the 4 and 64 with their decompositions I got that 256 = 4 x 64 = 2 x 2 x 2 x 32 .
- And so on, stopping your factorization when you get all prime factors, or simply when the decomposition will allow you to work in an enough simple way.

Clearly after observing this procedure it could be immediately deduced that it no longer makes sense to apply it where a factorization starts to be too long or too hard to perform. And that's why knowing the *divisibility criteria* will let you operate with greater simplicity, and even help you to understand when the more appropriate thing to do is *discarding the method at all*:

- **Criterion of divisibility by 2:** This is probably the simplest and most well known: a number is divisible by 2 only if its units digit is 0, 2, 4, 6 or 8.

- **Criterion of divisibility by 3:** A number is divisible by 3 if, after summing its digits, you get a multiple of 3.
  Take for example 476. 4 + 7 + 6 = 19. If 19 is a multiple of 3, so will be 476, and to check the last condition you can apply the same strategy again: 1 + 9 = 10. 10 is not a multiple of 3 and then neither is 476.

- **Criterion of divisibility by 4:** A number is divisible by 4 if its last two digits are 00 or any two-figure number divisible by 4.
  Take for example 56 000 932. I can tell immediately that it's divisible by 4 because it ends in "32" and 32 is divisible by 4.

- **Criterion of divisibility by 5:** A number is divisible by 5 if its units digit is 0 or 5.

- **Criterion of divisibility by 6:** A number is divisible by 6 if it meets the criterion of divisibility by 3 AND it's even.

- **Criterion of divisibility by 7:** A number is divisible by 7 if is divisible by 7 the number given by ("number without its units" + "units digit you removed, multiplied by 5").
  So, suppose you want to understand whether the number 8422 is divisible by 7: you'll have to see if is divisible by 7 the sum of 842 (which of course is 8422 without its units) + (2 x 5) (units removed multiplied by 5). 842 + 2 x 5 = 852. To understand now if 852 is divisible by 7 you can apply again the same rule: (85 + 2 x 5) = 95. Then, again (9 + 5 x 5) = 34, which is not divisible by 7. Then neither was 8422.

- **Criterion of divisibility by 8:** A number is divisible by 8 *if the number made up of its last three digits is divisible by 8*. Or, if this is too difficult, you can use this alternative strategy: take its third digit starting from right, double it, add it to the penultimate one, double the result and sum it to the last one. If the final result is a

multiple of 8, so will be the original number.
So, if for example you have 865 341, take the 3, double it by getting a 6 and add the last result to 4, getting 10 as a result. After doubling 10 you will get a 20 and after adding it to the last 1 on the right you will get 21. 21 is not divisible by 8, then 865 341 is not divisible as well.

- **Criterion of divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

- **Criterion of divisibility by 10, 100, 1000:** A number is divisible by 10, 100, 1000 or any other power of 10 if it has an amount of trailing zeroes equal to those of the power considered.

- **Criterion of divisibility by 11:** Starting from right and going left, sum together the digits that take up an even place in your number. Then do the same with the digits in an odd place. Subtract the smaller from the larger and if the result is 0, 11 or a multiple of 11, then the examined number is also divisible by 11.
  So, for example, we have that 8 291 778 is divisible by 11 because:
  - Digits in an even place: 7, 1, 2 and (7 + 1 + 2) = 10.
  - Digits in an odd place: 8, 7, 9, 8 and (8 + 7 + 9 + 8) = 32.
  - The larger minus the smaller: 32 - 10 = 22.

- **Criterion of divisibility by 12:** A number is divisible by 12 if it's divisible by both 3 and 4.

- **Criterion of divisibility by 14:** A number is divisible by 14 if it's divisible by 7 AND it's even.

- **Criterion of divisibility by 18:** A number is divisible by 18 if it's divisible by 9 AND it's even.

- **Criterion of divisibility by 22:** A number is divisible by 22 if it's divisible by 11 AND it's even.

In addition to knowing when a number is divisible by another one, it can also be useful to know *whether a number is prime*, and therefore not decomposable into factors.
The search for a formula to help to understand whether a number is prime or not has been the subject of study by mathematicians for several years but, despite this, still has not been found anything sufficiently effective and efficient. Because of this, before you try to factorize a number, you should always try to apply the most immediate criteria first and, if none of them gives its fruits, check whether the number is written in the *prime numbers table* (and since I really care about you, you'll find a prime numbers table right at the end of this book).

The last type of decomposition we'll talk about is the so-called *"decomposition into expressions"*, which consists in decomposing a number *into a set of multiplications and divisions* that should make the original operation easier to perform.
For example, 50 is equal to 100 / 2. So, we can say that the multiplication of a number "a" by 50 can be just performed by "following the expression", and so by *multiplying our "a" by 100 and then halving the result*. The division by 50 instead can be performed by "inverting the expression" which means by swapping any multiplication with a division and vice versa, and so by *dividing "a" by 100 and then doubling the result.*

There is no specific criterion for decomposing a number into an equivalent expression useful for applying this strategy, so let's just say you can do it *after training your eye to recognize whether a number is a factor of 10* (or multiples) and it's not convenient to apply any other kind of calculation strategy. For example :

- **5 = 10 / 2.** And so multiplying by 5 is the same as multiplying by 10 and then halving the result. Dividing by 5 instead leads to *reversing the expression*, and so to dividing by 10 and then doubling the result.

- **75 = 300 / 4.** And so multiplying by 75 is the same as multiplying by 100, then triplicating the result and halving it twice. Instead dividing it is the same as dividing by 100, then doubling it twice and finally dividing it by 3.

- **250 = 1000 / 4.** And so multiplying by 250 is the same as multiplying by 1000 and halving the sum twice. And dividing is equivalent to applying the inverse strategy.

You could even *combine the decomposition into expressions with the decomposition into addends* and so simply solve, for example, a multiplication x26. 26 is in fact equal to (100 / 4) + 1. So, since "a" x ((100 / 4) + 1) = (a x 100 / 4) + a, you just got that multiplying a number "a" by 26 is the same as multiplying it by 100, halving twice the result and then adding "a" at the end.

I'll repeat once again that this is a procedure that greatly simplifies the operations, but that, at the same time, does not imply a precise pattern to follow and so it just requires a very well trained eye and intuition to be properly performed. For this reason, whenever you realize that trying to use it is complicating things too much, *just forget about it and use only the first two strategies.*

So, after facing the more complicated things, here's a very useful *fast multiplication and division table*, prepared exactly by applying all the shown decomposition strategies. Of course you don't have to memorize it, but my suggestion here is to *understand how was it built*, in order to easily become able to use its criteria to solve a large amount of operations:

- **Multiplication x4:** Double twice.
- **Division / 4:** Halve twice.

- **Multiplication x5:** Multiply by 10 and then halve the result.
- **Division / 5:** Divide by 10 and then double the result.

- **Multiplication x6:** Double and then triplicate the result or vice versa.
- **Division / 6:** Halve and then divide by one third, or vice versa.

- **Multiplication x7:** Multiply by 10 and subtract three times the original number (notice here that you cannot create any criterion of division by 7, because the decomposition into addends is not applicable to the division, factorization is not feasible since 7 is a prime number and a decomposition into expressions is not feasible as well.)

- **Multiplication x8:** Double three times the original number.
- **Division / 8:** Halve three times the original number.

- **Multiplication x9:** Multiply by 10 and then subtract the original number.
- **Division / 9:** Divide by 3 twice.

- **Multiplication x11:** Multiply by 10 and add the original number.

- **Multiplication x12:** Multiply by 10 and add twice the original number.
- **Division / 12:** Halve twice and then divide by 3.

- **Multiplication x13:** Multiply by 10 and add three times the original number.

- **Multiplication x14:** Multiply by 7 and then double the result.
- **Division / 14:** Halve and then divide by 7.

- **Multiplication x15:** Multiply the original number by 10. Then add the original number multiplied by 10 first, and then halved.
- **Division / 15:** Divide by 3 and then divide the result by 5.

- **Multiplication x16:** Double four times.
- **Division / 16:** Halve four times.

- **Multiplication x17:** Multiply by 10. Then add the same number multiplied by 10, but which you have removed its triple from.

- **Multiplication x18:** Double, then multiply by 10 and subtract from the result twice the original number
- **Division / 18:** Halve and then divide by 3 twice.

- **Multiplication x19:** Double, then multiply by 10 and subtract from the result the original number.

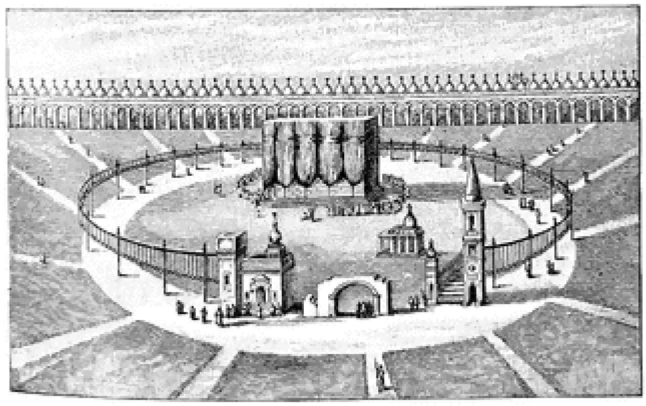
- **Multiplication x20:** Double and multiply by 10.
- **Division / 20:** Halve and divide by 10.

Now you can use these strategies to perform a mathematical trick that will come in very handy in your personal finance management and, more specifically, will let you understand *the annual impact of a daily expense:*

- Take your daily expense, for example those innocent 5 dollars you pay for dining out during your office lunchtime instead of bringing your own food from home.
- Multiply this amount by the days of the week you actually spend this money in. In this case, let's say they're the 5 days of an average workweek = 5 x 5 = 25 dollars, that will be the *weekly cost of your daily expense.*
- Now you have to do nothing but multiplying the weekly cost by 52 and you'll get your annual impact! But how can you do that in the fastest and most efficient way? Well, as a matter of fact it's very simple: you can split that 52 into 50 + 2 and then get 25 x 52 = 25 x (50 + 2) = 25 x 50 + 25 x 2.
- Multiplying by 50 is as easy as the proverbial pie: since 50 = 100 / 2 you just have to multiply it by 100 (which means adding two trailing zeroes) and then halve the result = 25 -> 2500 -> 1250.
- This last estimated expense is good enough yet to let us understand how big the annual impact of those "innocent" daily 5 dollars will be. But if you really want to be precise you can add those last two weeks to that result and getting then that 25 x 2 = 50, which added to 1250 equals 1300. Ouch! It's much more convenient to prepare your food by yourself at home, isn't it?

Ok, this chapter was a little bit more challenging, but first of all do always remember yourself that *bigger the effort, better the result.* And now just sit down and relax, because next two chapters will be much lighter, and will introduce you *some of the most curious and entertaining facts in the world of mathematics.*

X - Curiosities from Numberworld



Coffee break! If you have been exhausted by the previous chapters then it's time to relax and to enter into the unique world of mathematical and numerical curiosities. So ... click on "play", start your favourite music playlist, sit down and read: some of these facts will show you the deep and extraordinary harmony of the mathematical world. Others of them will project you among the many historical peculiarities of this discipline. While others again will be even useful for a better understanding of some simple but important events of your everyday life. Let's go!

- The odds of dying in a plane crash are the same as flipping a coin 21 times and always getting head. Try to do that and you will immediately understand why it's said that the plane is the safest way to travel.

- The Pythagoreans did not consider the number "1" to be odd nor even, but *both*. And that's because after adding "1" to a natural odd number you always get an even number, and vice versa. In this sense, it was considered to be the generator of all numbers, even and odd, and therefore of all things, finite or infinite.

- There is an EEC regulation (specifically, the n.55 of 21/11/1994) that formalizes how the multiples of a thousand must be called: in order, thousands, millions, billions, trillions, billiards and trillions). There are in fact, apparently, no "official" names for the larger numbers. And the strangest thing of all, perhaps, is that this definition was written ... in a law about the safe transportation of dangerous goods!

- In the Russian roulette game, if one does NOT rotate the drum after each attempt, who's at the end of the turn is much more likely going to die than the others before him. Otherwise each one has exactly the same probability to die as the others.
  This does not mean that you should risk life and limb in a game like that. But if it unfortunately happens that a mobster kidnaps you and takes you into his lair, in company of shady guys who force you to play, you've got a strategy for increasing the chances of saving your life. Yeah, this is not going to be very probable, too.

- The first number system has been probably invented by the Sumerians in 3000 BC. In this system, "one" also meant "man", "two" meant "woman" and "three" also meant "many".

- Some cicadas only hatch after prime numbers of years terms, in order to prevent predators with vital cycles equal to dividends of their hatching periods to evolve specializing in their catch.
  But this is not the only example of how deeply the laws of nature are written in the same alphabet as mathematics: several animals in fact have a quite clear idea of the concepts of quantity and size. Some bees, for example, are able to communicate and understand the exact distance between the hive and a pollen reserve.

- It is said that the inventor of the chess game, as a reward for his invention, asked for: "Put a grain of wheat on the first square, and double the quantity for each square coming next". A request like that, if accepted, would have required more than 18 billion billion grains of wheat. An amount far greater than the one could be achieved after using the entire planet for cultivating wheat and gathering several months of harvest.

- It looks like that the ability of the great mathematicians to be easily distracted from their everyday tasks is kind of legendary. It is said, for example, that Isaac Newton, during a lunch, went to take some wine and, after completely forgetting what he was doing, locked himself in his room to work, leaving the diners at the table waiting for him in vain.

- 1234567891, 1234567891234567891 and 1234567891234567891234567891 are all prime numbers.

- All the numbers obtained by removing one digit at a time from "197933933 ", starting from right, are prime numbers, except "1". In fact, in spite of the fact that "1" is clearly a number divisible only by 1 and itself, it's never considered as a prime one, because otherwise the premises of many important mathematical theorems would be completely lost.

- There are 7 math problems, the so-called "millennium problems" that, if solved, would provide a one million dollars prize because of their important implications in fields like physics and cryptography. Well, as a matter of fact one of them has actually been solved yet, but the winner of the prize, a Russian mathematician, refused the reward. Furthermore, there is actually the possibility that a solution for the other 6 problems doesn't even exist at all.

- If it was physically possible, you could fold a piece of paper just 23 times to let it be as high as Mount Everest. And 42 times to let it match the distance between the Earth and the Sun.

- There is a mathematical equation, called the "Drake Equation" as formulated by the American scientist Frank Drake, that should provide the number of extra-terrestrial civilizations should be possible to communicate with. In fact, he multiplied together:
  - The new-born star creation rate in the galaxy
  - The fraction of stars having planets
  - The average number of habitable planets per star with planets
  - The fraction of habitable planets life could *actually* be born on
  - The fraction of such planets this life could have become intelligent on
  - The fraction of planets where this intelligent life could have developed the ability (and the willingness) to communicate with other life forms.

  And obviously after basing himself on nothing more than conjectures, Drake seems to have come to the conclusion that the result of this equation *is equal to 10*. And despite the utter lack of mathematical precision of the result, it's probably nothing but the mathematical expression of our natural instinct to raise our heads and refuse to believe that we are the only living beings in such immensity.

- The Soroban is a special type of abacus, introduced into Japan from China in the fifteenth century, which lets anybody efficiently represent very large numbers and very quickly perform even complex calculations.

  Many Asian guys also train themselves since a very young age to mentally visualize a Soroban and perform operations on it, thus becoming able to achieve extraordinary mental calculations with extreme efficiency.

  The technique about mental calculations performed through the Soroban is called "Anzan" and can be properly managed only after training a lot with the abacus number representation.

  In fact, while the majority of the techniques explained in this book can easily be mastered in a few days and can give excellent results when combined with just a few memory techniques, the Anzan could even require years of training before giving satisfactory results.

However, if this is a topic you are interested in and you would like to launch yourself into this path, you can find a lot of interesting tutorials simply by searching "Soroban calculation" or "Anzan" on YouTube. Also, if you have any touchscreen device, you can find some interesting app that let you easily emulate operations on a real Soroban abacus.

- Through a formula, you can determine the day of the week of any day between the seventeenth and twenty-second centuries.
  As a first thing, let's introduce a table that assigns a specific number to each month. You'll need to memorize it if you want to use this strategy without any pen or paper:

  January = 0
  February = 3
  March = 3
  April = 6 or -1
  May = 1
  June = 4
  July = 6 or -1
  August = 2
  September = 5
  October = 0
  November = 3
  December = 5 or -2.

  Now, we'll need a second table that assigns a specific number to each century from the 17th until the 22nd. More specifically, we have that:

  Seventeenth century = 6
  Eighteenth century = 4
  Nineteenth century = 2
  Twentieth century = 0
  Twenty-first century = 6
  Twenty-second century = 4

So, take the last two digits of the year you want to know the day of the week from. Let's call it "a".
Divide it by 4 without considering the remainder. We call this number "b".
We call "c" the number taken from the "months table" above".
We call "d" the number of the day in the month.
And we call "e" the number taken from the "century table".
Now calculate (a + b + c + d + e) / 7 and consider the remainder of the division: 0 as a remainder means that the considered day is Sunday, 1 means Monday, 2 means Tuesday, and so on.

- The sum of N consecutive odd numbers, starting from 1, is simply squared N. So, for instance, you want to calculate 1 + 3 + 5? Squared 3 = 9. Want to instantly calculate 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17? Squared 9 = 81!

- Could you tell me how likely, given any class from any school, two boys studying there were born on the same day? 1%? 2%? 5%? None of this. According to the theory of probability, in fact, a class exceeding 23 people is a sufficient condition for that probability to be more than 50%.

- The Roman number system, which is perhaps the most widely known among the ancient ones, was incredibly difficult to manage and not very intuitive when it was necessary to perform any calculations. And this is the reason in ancient Rome the calculations were almost always made with the help of pebbles or schedules for. On the other hand, from the Latin, calculus = stone.

- There are several formulas for calculating the ideal body weight of a person, but one that's at the same time the most simple (requires no measurements apart from the height) and the most reliable (though still theoretical, as of course it does not take into account anybody's specific health and body structure case) is the so-called "Keys Formula", which says that the ideal weight of any person (expressed in kilograms) is equal to:

(Squared height in meters) x 22.1 for men
(Squared height in meters) x 20.6 for women
And if this calculation now seems complicated to you, don't worry ... because later you will learn all the necessary techniques for quickly and easily performing it mentally!

- Passionate about football or any other sports? And you want to know how many matches you must watch on TV until the end of a specific tournament in order to not to miss any of them? Well, remember that in any tournament whose matches are a 1 vs. 1 with knockout rule, the number of games is equal to the number of final losers (so, of course, it's equal to *contenders - 1*).

- The likelihood of having four of a kind right after the initial distribution of cards in a classic poker game is about 1 in 50 000.

- A mathematician named Charles Brenner has created a formula that can be used to find your soulmate.
  More specifically, he was inspired by a specific mathematical problem: there is a deck of face down cards on a table. On each card there is a number we cannot see. Now, our aim is to grab the card with the highest score, respecting the rule that *we can choose it only after having taken it*, and once decided to try a new one, *the card in our hands must be discarded*, of course taking the risk that in the deck are remaining only cards having a lower score.
  This can bring us to ask ourselves a specific question: *"How much large must be the sample of cards to draw, in order to be sure to get at least one of the higher scores?"*
  And here Brenner made his analogy with the relationship world, comparing the deck of cards to the "golden period" in which anybody can actually try to achieve the highest number of conquests, and the score on each card to *the level of connection and empathy* you can feel in a relationship with somebody.
  Also Brenner added that the best time to get an enough large

sample to choose the ideal partner is equal to 1 / e (e = Euler number = 2.718 ...) multiplied by the "golden period". Assuming, then, for example, that a man's golden period goes approximately from the age of 20 to 45, the best sample is reached at the age of about 29 and, once reached that age, it should become typically convenient to choose as a partner *the first person one feels a stronger connection with*.

Well, clearly I think that love shouldn't be really chosen by messing with numbers, but apart from the moral of the story ("*be aware of your limits in order to get the best you deserve"*) it is always interesting to see how the applications of mathematics can go far beyond what anyone could imagine. And then, who could ever believe that the Euler number (for those who knew him) had any relation with love conquests?

An extra note: the utility of this formula, as a matter of fact, doesn't end here. In fact it can be applied anytime we have to take one specific decision among a wide range of choices: just multiply 1 / e (about 0.37) by the number of choices (or by the amount of time in which the choice can actually be taken) and as soon as you see that one of them is better than the other ones inside that specific range... that's your choice!

- The so-called "perfect numbers" are those numbers that equal the sum of their divisors. For example, 6 is a perfect number because it's equal to both 1 + 2 + 3 and 1 x 2 x 3. Saint Augustine even wrote that *"6 is a perfect number and not because God has created all things in six days. Indeed the opposite is true: God created all things in six days because this is a perfect number".*

  Even 28 is perfect (since 28 = 1 + 2 + 4 + 7 + 14) and even 28 has always had a strong symbolic meaning, being the number of days in the lunar month in the Jewish calendar and the duration of the women menstrual cycle.

- The "narcissist numbers" are those numbers that equal the sum of their cubed digits. For example, 153 is narcissist since 153 =

1 ^ 3 + 5 ^ 3 + 3 ^ 3 = 1 + 125 + 27.

- According to rumours, there would be no Nobel Prize for mathematics because Alfred Nobel once discovered her lover cheating him with a famous Swedish mathematician, Magnus Gustaf Mittag-Leffler.

- The "twin prime numbers" are those prime numbers that are "separated" in the natural numbers sequence just by a single number. For example, 5 and 7, or 11 and 13 are twin prime numbers. Also there are the so-called "cousins prime numbers" that differ by 4 (like 7 and 11), and the "sexy prime numbers" that differ by 6 (like 5 and 11).

- It has been reported that many prodigious mental calculators ended up gradually losing their extraordinary abilities for apparently no reasons. Enry Safford, for example, born in Vermont in 1836, at the age of 10 was able to instantly perform multiplications like 365365365365365 x 365365365365365365, but just after a few years its mathematical skills became absolutely ordinary. Even Zerah Colburn, born in 1804, at the age of 8 was able to calculate in a few seconds the fifteenth or sixteenth power of different numbers, but it looks like that at the age of 20 had lost most of its ability.
  And what if this is another proof that some skills are more the result of a set of circumstances than something that's "divinely" given from nature?

- The height of the Pyramid of Cheops is exactly equal to one millionth of the distance between the Earth and the Sun. Pure coincidence or the ancient Egyptians were in possession of extraordinary astronomical knowledge?

- The "0" symbol was introduced in the Occidental World only in the 11th century. Prior to that the concept was quite unknown in Europe, if not considered even "taboo", because of its relationship with the concept of "nothing" or "emptiness". In other cultures instead it was not being used simply because were represented through pebbles and therefore *no stones corresponded to no number.*

- The largest number expressible through 3 digits is not 999, but 9 rose to the 9th power rose to the 9th power, which is a number made up of 369 693 100 digits and that, to be written, would require a 924 km long line of paper. If you wanted to write it in a simple Notepad text file instead, it would occupy almost 2 gigabytes on your hard disk.

- In 2004 it was calculated that the asteroid Apophis would have had a 2.7% chance of colliding with Earth in 2029. Given the catastrophic event that would have followed, it's a rather high probability (about 1 in 30) and quite similar to the odds of launching twice a die and getting both times "1". Fortunately, subsequent calculations have lowered the probability from 1 in 30 to 1 in 230 000 , which is the same probability as rolling seven times a die and always getting "1" as a final value.

- Now I want to try to frighten you a little: launching seven times a die and always getting a "1" is an event which has exactly the same probability as *launching it seven times and getting any other combination of values.*
  Yes, even if you roll any die seven times just now, you'll get a combination of values that, if calculated before, would have had exactly the same chance to come out as the annihilation for all of us in 2029. So? Your rolling die has just demonstrated that the world is likely going to end soon?
  Not really. Attention in fact to the words "if calculated before", because they emphasize an important conceptual point in the probability theory.
  The point in fact is not the combination you've got. That event is now happened, it belongs to the past and so it's an event with a probability of 100%. *Everything happened in past has now a 100% probability of occurrence.*
  In fact it makes sense to speak about probability *only if related*

to future events no one knows with certainty the outcome of. Estimating a probability, after all, means *making a prediction* and then *giving a measure of the plausibility of that prediction*. But this is precisely the point we're talking about: before you rolled the dice, *you didn't take any prediction about the result of your launches*. And saying "the result of my rolling dice had the same probability that the asteroid hits the earth in 2029" has the same logic as the clever fortune-teller who, after extracting tarot cards out of his decks says, "You know, I knew that this exact sequence of cards would have come out."
So, it rather makes sense to say that *if you roll a die seven times and you guess all the results at your first attempt, then it happens something that's exactly as plausible as the end of the world.* Try that and let me know!

- If somebody gives you the opportunity to combine (only) the 7 classic notes of the musical scale in order to create a 12-notes long melody and you want to understand how many different melodies you can freely compose with preconditions like these, you can simply raise 7 to the 12th power. And the result is that *you can compose 13 841 287 201 possible melodies, all of them different by at least one note one from each other.*
  Generalizing, if you want to know how many are the possible n-long combinations of (also repeated) elements taken from a set of "m" choices, you have to do nothing but *raising m to the n-th power.*
  Another example: taken the letters of the Anglo-Saxon alphabet (m = 26), if you want to know how many 3-characters words are obtainable by combining those letters (including even the nonsense ones of course), you'll need to perform 26 ^ 3 = 17 576 combinations that, if sorted alphabetically, would go from "aaa" to "zzz". If the word length was 5 instead, the combinations would have been 26 ^ 5 = 11 881 376.
  And this is precisely the reason for which any website requires us to invent a password that's both composed by uppercase letters, lowercase letters and numbers, and that's as long as possible. In fact a password composed by a large set of

elements increases m, while lengthening the string increases n. So these choices *both increase the combinations necessary to let some malicious software to guess your password and hack your personal account.*

- If you have a poker deck and you want to know how many different combinations it can be shuffled into, the problem is slightly different from the previous one.
  Here, in fact, "n" (the combination length) is fixed and exactly equal to "m" (number of elements), with the extra rule that *there is no possibility of elements repetitions* (shuffling a deck of course doesn't change the cards, but just their order).
  In this case then the operation to be performed is different, and it's "n!" (in case you don't know, that exclamation mark means that you have to multiply together all the positive integers from 1 to n).
  But before performing this operation with our cards deck, let's have an easier example.
  Let's imagine you have two slips of paper on a table, one with an "a" written on it and one with a "b". Now you want to understand in how many different orders you can arrange them and having n = 2 you'll just have to calculate n! = 2! = 1 x 2 = 2.
  In fact, the possible arrangements of two cards are only 2: the "ab" order and the "ba" one. Other and different arrangements with these premises of course are not possible.
  If the slips of paper were three, and on each of them was respectively written "a", "b" and "c", the possible arrangements would have been more. For n = 3 in fact you have that n! = 3! = 1 x 2 x 3 = 6 and you can order them as abc, acb, bca, bac, cab, cba.
  So, let's go back to our poker deck: in this case you have that n = 52 and so, in order to calculate all the possible arrangements, you'd need to perform 52! = 1 x 2 x 3 x 4 x 5 x 6 x ... x 50 x 51 x 52.
  This number is equal to 80 658 175 170 943 878 571 660 636 856 403 766 975 289 505 440 883 277 824 000 000 000 000 and its impressive value means that it's almost sure that *no deck*

enough and truly shuffled in human history has ever had the same card order as another deck.

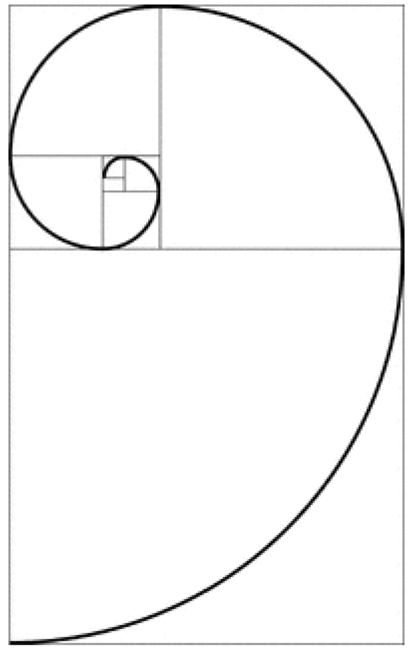
More specifically, taken a hundred billion stars, each with a trillion planets, each with one million inhabitants, each with one million decks of cards, and presumed that each inhabitant is able to shuffle a thousand decks per second for 14 billion years (the supposed age of the universe since the Big Bang), only today would start to appear two decks having the same card order, and it would take 14 billion more years of such a nonsense to let this happen again. The mathematical complexity originating from such a simple everyday item is quite impressive, isn't it?

This chapter ends here, but of course we won't stop here talking about mathematical curiosities! In fact in the next chapter we'll go back to the main topic of this book, facing some curiosities that are much more closely linked to speed math and examining the awesome harmony behind some very "special" numbers.

"The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful."

(Aristotle)

XI - 15 golden numbers



Now let's launch ourselves again in the speed math world, but still remaining at the same time within the orbit of the mathematical curiosities: let's introduce the so-called "golden numbers!". The "golden numbers" are all those numbers having a particular inner "harmony" that makes it ridiculously simple to use them to perform some specific kind of calculations. Of course some of them will be far less useful in the everyday life than others, but anyway even the most pointless one could be helpful in building some nice magic tricks. Enjoy!

3

Very often, in human history, to number 3 have been attributed various symbolic and esoteric meanings. Not to mention that it has often been considered to be the "perfect number" par excellence (although it has not anything to do with the mathematical definition of "perfect numbers" we saw in the previous chapter).
Beyond the symbolism and esotericism, however, what we want to examine in this chapter is a very fast method to instantaneously

square numbers only made up of recurrent "3" digits. In particular, to instantly perform this operation we must:

- Write as many "1" as the amount of "3" digits in our original number, except one.
- Write a 0.
- Write as many "8" as the "1" we wrote in the first step.
- Write a 9.

So, for example:
Squared 33 = 1089
Squared 333 = 110889
Squared 3333 = 11108889
Squared 33333 = 1111088889
And so on.

9

The proximity between numbers made up only of "9" digits and powers of 10 has always given a fascinating harmony to the results of the operations performed on them. For example, in order to add to any number "a" to another number made up only of recurrent "9" digits, you just have to subtract 1 from "a" and then write this "1" to the left of the number itself:
88 + 99 = 187
543 + 999 = 1542
2342 + 9999 = 12341

This strategy clearly has a lot of interesting practical applications, since it lets you quickly sum any number to another number that's very close to 99, 999, 9999, etc. For example, if you want to calculate 567 + 997, you can apply this technique first and then, considering that 997 is lesser than 999 by 2, you can subtract 2 to get the final result.

But it's also extremely easy to *multiply* between numbers made up only of recurrent "9" digits and any numbers having the same amount of digits. In fact, to perform this operation you can just:

- Subtract 1 from the number you want to multiply by 9 and write the difference on the "left side" of the result.
- Apply to the original number the *"All from nine and the last from 10"* Sutra we discussed in the chapter about Vedic Mathematics, and write it on the "right side" of the result (you do remember that the Sutra is about *subtracting from 9 all but the last digit of the number, which must be subtracted from 10 instead*, right?).

Now, let's have some examples in order to show the extreme simplicity of this technique:

89 x 99 = write 88 on the left and 11, which is the result of the Sutra, on the right = 8811
768 x 999 = write 767 on the left and 232, which is the result of the Sutra, on the right = 767 232
3451 x 9999 = write 3450 on the left and 6549, which is the result of the Sutra, on the right = 34 506 549

But another thing we can say about the properties of 9 is that multiplying it by any number made up of the same recurrent digit is very, very easy:

- Multiply 9 by the "recurrent" digit. Split the two digits of the result and put the units on the right and the tens on the left.
- Now, between those two digits, put as many nine as the digits of the original number, except one.

So, for example:

9 x 666 = 6 x 9 is 54. Split the two digits and put two "9" between them = 5994
9 x 7777 = 69 993
9 x 88 888 = 799 992

Another great property of this golden number? Well, it's even actually possible to almost instantaneously divide any number by it. In the case, for example, of a 2-figure number (let's say you want to calculate 68 / 9), you can do like this:

- The first digit of "a" is the result (in this example 68 -> 6)
- Sum together the first and second digit of "a". That will be the division remainder (for example, 6 + 8 = 14).
- If the resulting remainder is lesser than 9, it's all done. In the other case the result should be increased by 1 (becoming 7 in this case, since the remainder is 14 > 9), while the real remainder can be obtained by subtracting 9 from the actual one (so, since 14 - 9 = 5, in this case the final result of our division is 7 with remainder 5).

If your number is a 3-figure number instead (for example, 327 / 9), you can do like this:

- Take the first two digits of "a" and add them to the hundreds digit (in this case 32 + 3 = 35). That will be the result.
- Sum together the three digits of "a" (3 + 2 + 7 = 12). That will be the division remainder.
- If your remainder is lesser than 9, it's all done. If this sum is greater or equal than 9 instead (as happens in this case), you must subtract 9 from the remainder until it's lesser than 9 (and so, in this case, 12 - 9 = 3) and then you must add a "1" to the result for each 9 you have subtracted this way (so in this example, since you have subtracted only a 9, the result becomes 36).

Now, let's end this long dissertation about "9" by explaining how to instantly square any number made up only by "9" digits:

- Write as many 9 as the digits of your number, except one.
- Write an 8.
- Write as many 0 as the 9 you wrote in the first step.
- Write a 1.

So:

99 x 99 = 9801

999 x 999 = 998001
9999 x 9999 = 99980001
99999 x 99999 = 9999800001

11

Even the particular harmony behind 11 (and behind all the numbers made up of recurring "1" digits) certainly comes out from its proximity to 10. But let's start immediately to talk about practical math, and let's introduce the strategy for *rapidly performing a multiplication by 11.*

At first, a technique that could be used to perform this operation is the *decomposition into addends* and would simply consist in multiplying a number by 10 and adding the number itself. But, as a matter of fact, you can even use this alternative and probably more elegant technique, which is directly taken from the Vedic Mathematics and works like this:

- Take the first and last digit of the number you want to multiply by 11 and write them as first and last digit of the result.
- Sum, starting from left and going right, two digits at a time, the digits of the number you want to multiply by 11.
- Write, from left to right, the results of these sums. And if any of them was greater than 10, write only its units digit and add "1" to the digit immediately on its left.

Here are some examples:

- 11 x 23 = write 2 on the left and 3 on the right, getting so a partial 2_3. Now sum together two at a time, the digits of 23 (that of course are only 2, giving then only one sum to perform) and you'll get 5. Write it in the middle of your partial result and that's the final result of your multiplication: 253. That was easy, wasn't it?

- 11 x 387 = write 3 on the left and 7 on the right, getting so a partial 3_7. Now sum together two at a time the digits of 387:

start with 3 + 8 = 11. Write 1 and carry 1 to the next 3 on the left, transforming the partial result into 41_7. Now calculate 8 + 7 = 15. Write 5, carry 1 and get the final result: 4257.

- 11 x 4709 = write 4 on the left and 9 on right, getting so a partial 4_9. Now 4 + 7 = 11 and then the new partial result is 51_9.
  7 + 0 = 7 so the partial result is 517_9. And finally 0 + 9 = 9, getting 51 799 as a final result.

But let's level up and let's try to understand how to quickly multiply 111 by a 2-digit number. As a matter of fact the process is very similar to the 11 one, with just some little differences:

- Take the first and last digit of the number you want to multiply by 111 and split them exactly as you did for the multiplication by 11.
- Sum the two digits, multiply the result of the sum by 11 and write the result of the last product in the middle of your partial result. If the last one exceeds the hundreds, put in the middle only its tens and units and carry the hundreds digit to the first digit of the partial result.

An example? Here it is: let's try to calculate 111 x 76:

- Write 7 on the left and 6 on the right. Partial result 7_6
- 7 + 6 = 13. Multiplied by 11 it equals 143. The partial result would be 7_143_6, but we just said we must carry the hundreds digit if the result in the middle has one. So our final result is nothing but 8436.

Now, let's end the "11" topic with a very simple technique for squaring numbers made only up of recurrent "1" digits:

- Count. Yes, exactly, starting from 1, count the positive integers and step forward until you have an amount of digits equal to the digits of the number you're managing to square.
- Now continue to count, but stepping backwards until you go back to 1 again.
- That's your result.

Some examples? Here they are:

Squared 11 = 121
Squared 111 = 12321
Squared 1111 = 1234321
Squared 11111 = 123454321

37

37 multiplied by 3 equals 111. And, because of that, 37 can be multiplied by any multiple of 3 (< 30) and the result will always be a number consisting in a sequence of this multiple divided by three. So, for example:
37 x 12 = 444 (in fact, 12 / 3 = 4)
37 x 18 = 666 (18 / 3 = 6)
37 x 24 = 888
37 x 27 = 999

This also means that any 3-figure number made up of a sequence of the same recurrent digit (such as 555 or 777), if divided by the sum of its digits *always gives 37 as a result.* Try to build a magic trick based on this principle!

143

143 has a very singular property: it can be multiplied by any three-digit number simply after juxtaposing two copies of this three-digit number and dividing the last by 7.
Let's say for example you want to perform 143 x 887. In order to quickly perform this multiplication you can just "build" the number 887 887 and then divide it by 7. Of course a one-digit division is much more immediate than any three-digit multiplication and that's why even 143 is really perfect for *building some good speed math magic tricks.*

666

As a matter of fact it would be more appropriate to speak about *numbers only made up of recurrent "6" digits*, but the symbolism behind this number was too strong to not to let me decide to write it down exactly like this. More specifically, numbers made up of recurrent "6" digits have a curious property: just as happens with numbers made up of recurrent "1" or "9" digits, you can actually square them very rapidly. Just:

- Write as many 4 as the 6 are, except one.
- Write a 3.
- Write as many 5 as the 4 are.
- Write a 6.

So, for instance:
Squared 66 = 4356
Squared 666 = 443556
Squared 6666 = 44435556
Squared 66666 = 4444355556

1089

Take any 3-figure number, with the only condition that *the hundreds digit must be greater than the units one by 2 at least*. For example 542.
Now subtract from this number the one you obtain by swapping the units digit with the hundreds digits (in this case then 542 - 245 = 297).
Then take the result and sum it to the number you get by swapping units with hundreds again. So 297 + 792.
You've just got 1089, right?
The result of this procedure is in fact always 1089, no matter what's the 3-figure numbers you started from. And that's why sometimes 1089 is considered a "magic" number and it's the perfect base for some good number-based magic tricks. What if, for example, you

just *ask to a friend of yours to repeat the previous steps and then you show him you can guess the result?* Yeah, do that only once!

2025, 3025, 9801

These three numbers can be counted among the "golden numbers" because it's possible to calculate their square root in an almost immediate way. The "secret" here in fact is just to split them into two groups made up of two digits and to sum them. In fact:
$\sqrt{2025} = 45 = 20 + 25$
$\sqrt{3025} = 55 = 30 + 25$
$\sqrt{9801} = 99 = 98 + 1$
It would be really great if calculating square roots was always so easy, wouldn't it?

3367

3367 has very similar properties to 143. More specifically you can know the result of 3367 multiplied by any two-digit number simply by juxtaposing three copies of the last number and then dividing it by 3. So, for example:
3367 x 98 = 989 898 / 3 = 329 966
3367 x 55 = 555 555 / 3 = 185 185
Now, a question to stimulate your mind: *why the result of such a division will always be an integer number?* A hint: you can find the answer in Chapter IX.

37 037

37 037 is very similar to 37. In fact, if multiplied by 3 equals 111 111 and because of that, if multiplied by any multiple of 3 (< 30), the result will always be a sequence of that multiple divided by 3. So:
37 037 x 6 = 222 222
37 037 x 15 = 555 555

37 037 x 24 = 888 888
37 037 x 27 = 999 999

142 857

If you multiply 142 857 by any number between 2 and 6, the result will always keep exactly the same digits, with the only difference that they will be "shifted". In fact:
142 857x2 = 285 714
142 857x3 = 428 571
142 857x4 = 571 428
142 857x5 = 714 285
142 857x6 = 857 142

In addition to this, if you multiply this number by 7, the result will be 999 999. This still could not be very helpful in the everyday math, but again could be useful for building some nice mathematical magic tricks. *Just use your imagination!*

12 345 679

This number that's nothing but the natural numbers sequence from 1 to 9 without 8, has very similar properties to 37 and 37 037. In fact, if multiplied by a number "a" *multiple of 9 and lesser than 90*, it gives back as a result a number made up of a sequence of "a / 9" repeated 9 times. So, for instance:

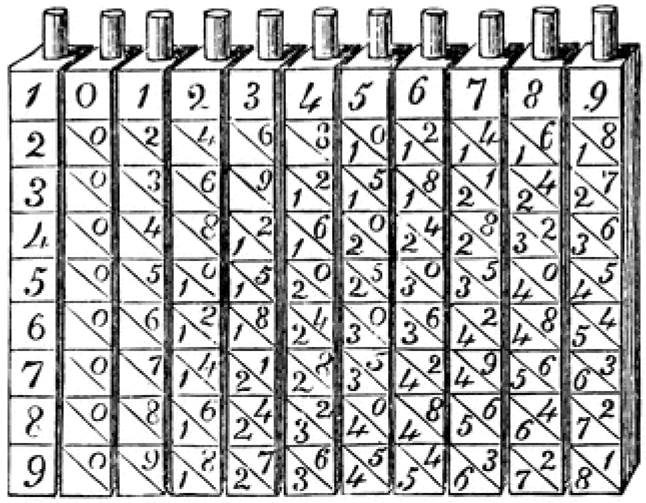
12 345 679 x 9 = 111 111 111
12 345 679 x 36 = 444 444 444
12 345 679 x 54 = 666 666 666
12 345 679 x 63 = 777 777 777
12 345 679 x 81 = 999 999 999

1 016 949 152 542 372 881 355 932 203 389 830 508 474 576 271 186 440 677 966

I seriously doubt you'll ever come across the number 1 016 949 152 542 372 881 355 932 203 389 830 508 474 576 271 186 440 677 966 in your life. Nevertheless, if one day a king promises his kingdom to all those who are able to multiply it by 6 within three seconds, simply delete the last 6 on the right and put it on the left. Yes, while the 9 and 11 were probably the most useful golden numbers, this is definitely the most useless of all.

The section related to the mathematical curiosities ends here ... without ending here at all! In the next chapter, in fact, we will talk about fast multiplication by introducing a multiplication table that's completely alternative to the one introduced at the end of Chapter IX and was written by a Russian mathematician during their stay in a Nazi concentration camp.

XII - Multiplication tables from hell



Was the Second World War while Jakow Trachtenberg, Jew Russian mathematician always hardly critical against the Nazism, was imprisoned in a German concentration camp. So, in order to try to keep his mind away from the horrors of that hell, he tried to mentally "escape" into the world of numbers and managed to build day by day an innovative and unique fast calculation method. And the most extraordinary thing was that he developed his set of strategies without writing anything but some lines on the occasional scraps of paper he could fortunately put his hands on. Fortunately then, in 1944, Trachtenberg was able to escape from the prison where he had just been transferred with the help of his wife who pledged all her jewellery and corrupted some guards. So he fled to Switzerland, where he ended up teaching his mathematical method that won several awards and became really popular, especially among those who had problems with the traditional calculation methods.

In particular, in this chapter we will analyse Trachtenberg's method of rapid multiplication by 5, 6, 7, 9, 11 and 12, while later we'll face his more general "graphic multiplication" that has the strong advantage to reduce the number of digits to keep in mind if compared to the "classic" column method.

As a matter of fact, Trachtenberg also wrote methods for rapid multiplication by 8, 3 and 4, but I deliberately decided to not to talk about them because they're quite cumbersome and complex, and if the goal of our entire book is to make math faster, easier and more efficient, I firmly believe that the nature of these methods would completely be in contradiction with our fundamental intentions.

So, which exactly are the main advantages of this method? Well:

- Except for some trivial operations like "halve" or "double", every multiplication is transformed into *a sequence of simple subtractions and additions*. And apart from the obviously reduced overall complexity, this will be very helpful if you still have some little problems with the "classic" multiplication tables.

- Its speed compared to classic multiplication enormously increases as far as the multiplicand (the number that you want to multiply by 5, 6, 7, 9, 11 or 12) is larger.

- It only operates on the multiplicand digits. In fact if for example you want to multiply 567 345 by 12, all you have to do is performing sums on the internal digits of 567 345. Clearly *the method these sums must be performed through will vary depending on the multiplier*. But the important thing here is that, in any case, once you've seen which the multiplier is and you've understood what's the right strategy to use, you can also "completely forget" about it. And this of course reduces the number of digits to keep in mind and so, as we should well know, it further makes the whole mental calculation procedures easier to perform.

- Apart from everything else, this is an extra instrument for your mental toolbox and, as we should well know, *having a choice among various tools for solving the same problem represents a tremendous creative and strategic resource.*

So, let's start to enter into the heart of the matter, and let's do some fundamental premises:

- As already mentioned you *must only look at the multiplicand and only work on that.*

- You must start from the units digit of the multiplicand and proceed *doing the operations one digit at a time, going left*. We will see in detail HOW to do these things soon. For now just keep in mind the right mode and order to proceed with.

- The next digit to the right of another digit is called "its neighbour". And since every Trachtenberg strategy is based on *performing calculations between a digit and its neighbour*, this is a fundamental concept to keep in mind.
  An example? Here it is: taken 3792, the neighbour of 3 is 7, the neighbour of 7 is 9, and the neighbour of 9 is 2.

- The "neighbour" of the units digit is always to be considered 0.

- The leftmost digit of any number should be handled as if it was the neighbour of a "ghost" leading zero. And this "ghost leading zero", mind you, unlike the one we mentioned in the previous point, should be considered *as if it was a real digit of the multiplicand*. So, only after you "handled" it properly, the whole Trachtenberg procedure ends.

- An easy way to remind yourself about the two zeroes we talked about in the last two points? At least in a first moment, before you operate, you may actually write them into the multiplicand, taking care of writing the trailing one after a decimal point. You can rewrite so, for example, 1234 as 01234.0 or 452 as 0452.0. Two zeroes written like that, in fact, won't alter the original value of the number and at the same time will help you to remember exactly when the method starts (*to the left of the decimal point*) and where does it end (*at the leading zero*).

- Anytime is required to halve a digit, and that digit is odd, you should always round the result down to the nearest integer. So when you are asked to halve 1 write 0, write 1 when asked to halve 3, 2 when 5, 3 when 7, and so on.

- Identically as happened in the case of some techniques discussed in the previous chapters, any sum represents a digit in the final result and so, if one of them gives a result greater than 10, only the units digit must be put in the result, while the tens must be carried to the next digit on the left.

But now let's go to the real table:

Multiplication x11

After the method by decomposition and the Vedic one seen in the chapter about Golden Numbers, here's a third way to multiply by 11: starting from the units digit and going left, *"add each digit to its neighbour".* That's it!

Well, let's make an example to better explain what does that mean and let's try to calculate 56782 x 11:

At first, if you want, write 56782 as 056782.0. As we said, the value does not change if written like that, and at the same time it will help you to remember how to act. Attention: we'll use this "simplified" notation only in this first example and we'll try to work with the original numbers later in order to get used to them.

05678**2**.0
Start from the units digit. Its neighbour, as mentioned before, is 0. So the first thing to do is summing 0 + 2. And here, very simply, you get that the units digit of your result is 2

0567**8**2.0
Now go left and add 8 to its neighbour. So 8 + 2 = 10. The second (tens) digit of the result is 0, with a 1 to carry to the next sum you'll perform.

056**7**82.0
7 + 8 = 15 + the 1 we carried from the previous sum = 16. The third (hundreds) digit in the result is 6. 1 must be carried again.

05**<u>6</u>**782.0
6 + 7 = 13 + the carried 1 = 14. The fourth digit of the result is 4. 1 must be carried.

0**<u>5</u>**6782.0
The next 5 + 6 = 11 + the 1 we carried = 12. The fifth digit of the result is 2. 1 must be carried.

<u>0</u>56782.0
We said we must consider, as the leftmost digit in the multiplicand, a "ghost leading zero". So, 0 + its neighbour 5 = 5, plus the 1 carried from the previous sum = 6. So the sixth and last digit of the result is 6.

Final Result = 624 602

As you can see this method, except for some details, is almost the same as the Vedic one. And since the time Trachtenberg lived, Vedic Mathematics wasn't absolutely known in the Occidental world, it's very fascinating to notice how people who lived in completely different times and places came basically to build exactly the same theories.

Multiplication x12

If you need to multiply by 12 you must *"Double each number and then add the neighbour"*. This means, more simply: just *operate like you did with the 11, but double any number before performing the necessary sums.*

A curiosity: like happened with the multiplication by 11, there is a specific Vedic Sutra saying *"Ultimate and twice the penultimate"* and reveals, except for some little details, exactly the same method. But let's jump to the practice and let's use this technique to multiply 829 x 12:

82**<u>9</u>**
Doubled 9 equals 18. Its neighbour is 0 + 18 = 18. Write 8 in the result and carry 1.

8**<u>2</u>**9
Doubled 2 equals 4. After adding the neighbour 9 it equals 13 and after adding the carried 1 it equals 14. Write 4 and carry 1.

<u>8</u>29
Doubled 8 equals 16. Plus the neighbour 2 and the carried 1 equals 19. Write 9 and carry 1.

<u>0</u>829
You are now at the "ghost leading zero". After doubling it, it still equals zero, plus the neighbour 8 = 8 and plus to the carried 1 = 9.

Final Result = 9948.

Multiplication x6

In order to multiply by 6 you have to *"add to each digit half of its neighbour, and add 5 if the number is odd".* So the procedure is basically the same as the 11 one again, but this time you must *halve your neighbour before adding it to your current digit* (remembering to round it down to the nearest integer if odd, as I said in the introduction). In addition to that, if your *current* digit is odd, you have to add 5 to the sum before moving to the next one. But let's have a demonstration of the method by trying to multiply 6821 x 6:

682**<u>1</u>**
The neighbour is 0 here, but you must add 5 to the final sum, too, because 1 is an odd number. So the first digit in the result is 1 + 5 = 6.

68**<u>2</u>**1
The neighbour of 2 is 1, which halved equals 0.5. 0.5 rounded down to the nearest integer equals 0, which does not affect the sum. In addition, 2 is an even number and so you don't have to add anything

else. The second digit of the result is simply 2.

6**<u>8</u>**21

You must add to 8 the half of its neighbour 2: 8 + 2 / 2 = 8 + 1 = 9. 8 also is an even number, therefore you don't need to add anything else. So the third digit in the result is simply 9.

Attention: many people here get confused and say *"oh, ok, 9 is an odd number, so I must add 5 at the end"*. Try to avoid this error and always remember you must check whether is odd only *the initial digit you started operating with.*

<u>6</u>821

After adding 6 to the half of its neighbour you get 6 + 8 / 2 = 6 + 4 = 10. Write 0 and carry 1. Don't add anything else because 6 is an even number. So, the fourth digit of the result is nothing but 0.

<u>0</u>6821

So we arrived at the "ghost leading zero". Add half of its neighbour 6, which is 3, plus the 1 carried from the previous sum = the fifth and last digit of the result is 4.

Final result = 40 926

Multiplication x9

Here is the procedure to multiply by 9:

- For each digit (except the ghost leading zero), subtract it from 10 if it's the units digit and subtract it from 9 if it's one of the other digits. Don't subtract anything if you're at the "ghost leading zero" and perform this subtraction *separately on each digit* and *before every step* instead of doing it on the whole number at the beginning. This precaution will help you to avoid some bad mistakes.

- Add each digit to its neighbour. Attention: the neighbour *will always be the digit that originally was in the multiplier before doing the subtractions explained in the previous point.*
- When you arrive at the ghost leading zero, instead of adding the neighbour, add the neighbour - 1.

In this technique, as you may have noticed, there is some little "shadow" of a Vedic Mathematics Sutra explained in the last pages. Pure coincidence or inevitably recurrent patterns?

In addition, this method may seem a bit more complicated than the others, but as a matter of fact you'll notice it's much more immediate than any classical multiplication by 9. So, let's calculate 9 x 3825:

382**5**
(10 - 5) + 0 = 5

38**2**5
(9 - 2) + the neighbour 5 = 12. Write 2 in the result and carry 1.

3**8**25
(9 - 8) + the neighbour 2 = 3. Plus the previously carried 1 = 4.

3825
(9 - 3) + the neighbour 8 = 14. Write 2 in the result and carry 1.

03825
0 + the neighbour 3 - 1 = 2 + the carried 1 = 3.

Final result: 34 425

Multiplication x7

The rule here is: "*double each digit, add half of its neighbour, and if the digit you started operating from was odd, add 5*"

So, let's immediately show how the strategy works, by multiplying 2894 x 7:

289**<u>4</u>**
Doubled 4 equals 8, plus halved 0 that is still 0 = 8, which is the units digit in the result.

28**<u>9</u>**4
9 x 2 + 4 / 2 = 18 + 2 = 20. Add 5 since 9 is an odd number = 25. Write 5 in the result and carry 2.

2**<u>8</u>**94
8 x 2 + 9 / 2 = 16 + 4 = 20. Plus the carried 2 = 22. Write 2 in the result and carry 2 again.

<u>2</u>894
2 x 2 + 8 / 2 = 4 + 4 = 8. After adding the carried 2 equals 10. 0 so is the fourth digit in the result and I'll have to carry 1 to the next sum.

<u>0</u>2894
0 x 2 + 2 / 2 = 0 + 1 = 1. After adding the carried 1 equals 2. 2, so, is the last digit in the result. 20 258 in fact is the final result.

Multiplication x5

Let's go back to a process that's slightly easier to perform than the previous ones. As we should know, a first method to rapidly multiply by 5 consists in halving a number and then multiplying it by 10, or vice versa. The Trachtenberg method actually is not that much different, despite being much easier to keep in mind. In fact, to multiply by 5 by using this technique you have to:

- Halve the neighbour.
- If the digit you began to operate with is odd, add 5.

A curiosity: for each extra kilogram (= 2.20 pounds) of fat, our body builds an average of 5 extra kilometres (= 3.11 miles) of blood vessels. So if you've recently gained weight, you can halve the amount of extra kilograms, apply this technique for seeing how many more kilometres of blood vessels your heart is forced to support ... and motivate yourself to immediately start to lose weight!

But let's have a demonstration of how this method works and let's try to calculate 24568 x 5:

2456**8**
The neighbour of 8 is 0. Halved 0 still equals 0, and so 0 is the first digit in the result.

245**6**8
The neighbour of 6 is 8. Halved 8 equals 4. Then the second digit of the result is simply 4.

24**5**68
The neighbour 5 is 6. Halved 6 equals 3. 5 is an odd number however, so you must add 5. 3 + 5 = 8, which is nothing but the third digit of the result.

2**4**568
The neighbour of 4 is 5. Just after halving and rounding it you get the fourth digit, which is 2.

24568
The neighbour of 2 is 4. After halving it you get the fifth digit of the result that's 2 again.

024568
The neighbour of the leading 0 is 2. After halving it you get the sixth and last digit in the result, which is 1.

Final result: 122 840

Do you think this chapter was challenging? Here is a little mnemonic table, useful for memorizing all of the explained methods:

- *The 5 and 11 strategies are the easiest: the first requires halving the neighbour and adding 5 if the digit is odd, while the second* *just requires adding each digit to its neighbour.*

- *The 12 multiplication strategy is the same as the 11 one, with the difference that you must double the digit before adding it to its neighbour.*
- *The 9 multiplication strategy is the same as the 11 one, with the difference that you must apply the Vedic Sutra first, and add the neighbour - 1 to the "ghost leading zero".*
- *The 6 multiplication strategy is the same as the 5 one, with the difference that you must add the digit itself to half of its neighbour.*
- *The 7 multiplication strategy is the same as the 6 one, with the difference that the digit must be doubled before performing any sum.*

But relax now and take a breath, because in the next chapter we will introduce two fast multiplication strategies so immediate to learn and to perform that you'll soon start wondering how it can be possible that nobody ever taught them at school.

XIII - Move, invert and amaze

The multiplication method we'll introduce, and that's extremely faster and more efficient than the classic "column" method, directly comes from the third Vedic Mathematics Sutra, the "Paravartya - Yojayet" that in our language means "Move, invert and apply". As a matter of fact it also has a precise demonstration, which could be performed using our polynomial algebra, but since we love practice much more than theory we will put it aside and we will directly show how this calculation strategy works. So, let's explain the two-figure numbers version of this method right by showing how to multiply two two-figure numbers, for instance 28 x 45:

- Write the product of the two tens digits in the left part of the partial result. So, in this case we have 2 x 4 = 8____
- Write the product of the two units digits in the right part of the partial result. So, in this case is 5 x 8 = 40 and the partial result becomes 8___40
- Now multiply the tens digits of each number by the units digit of the other one, sum the results of the two products and write the last in the middle part of the partial result. So in this case it will

be (2 x 5) + (4 x 8) = 10 + 32 = 42. So the partial result will be 8__42__40

- Now you just have *to carry the "extra digits":* as seen in most of the multiplication methods discussed so far, only the units digits of each number must be kept, while the tens digits must always be carried to the next number on the left. So here we have that:
  The 4 in the 42 must be carried to the 8 on the left = 12_2_40.
  We still have a two-figure number in the right part of the partial result (of course the 12 mustn't be considered since it's the leftmost number and so it can't be carried anywhere), so now the 4 in the 40 must be carried in turn to the 2 on its left = 12_6_0

- When the last two numbers are only made up of a single digit you're done and here's your result: 1260!

Notice you just obtained 1260 in a quarter of the time it would have taken to use the column or the decomposition method, and with the need to store in memory many less digits. In fact, this method is perfectly suitable for a purely "mental" calculation, performed without writing any partial result.
Well, as a matter of fact, if you want to perform a two-figure multiplication like that without any pen or paper, there is an extra trick you can do to optimize the usage of your memory: instead of starting from the first step, *start from the last one* (the one consisting in summing the results of the two products). In fact, if you perform the most complex operation first, then the brain will be completely free to rapidly and efficiently perform the easiest ones. But let's make an example of "pure" mental calculation performed exactly by following this procedure:

- Problem: multiply 78 x 44

- **First step:** Multiply the tens digits of a number by the units digits of the other one and vice versa. Or, if it's more immediate for you, multiply the "internal digits" and do the same for the "external digits". Of course the last is just a different, more

intuitive way to say exactly the same thing. In fact:
7**8** x **4**4: the units digit of the first number and the tens digit of the second one are on the "inner side" of the operation.
78 x 4**4**: the tens digit of the first number and the units digit of the second one are on the "outer side" of the operation.
Oh, and of course, at the end sum the results of the two products. In mind then calculate 7 x 4 = 28 and 4 x 8 = 32. After adding 28 and 32 we get 60 and we'll keep it in mind. That was easy, wasn't it?

- **Second step:** multiply the two tens digits: 7 x 4 = 28. Put it on the left and keep in mind "28_60".

- **Third step:** multiply the two units digits: 8 x 4 = 32. Put it on the right and keep in mind " 28_60_32".

- Now you have to do nothing but the carrying the extra digits, so:
  Carry the 3 from 32 to the 60 on the left, "transforming" your sequence into "28_63_2 "
  Carry the 6 from 63 to the 28, finally obtaining "34_3_2". Here is your result = 3432!

Here you can notice by yourself that if you started from the second or the third step, then doing the first one would have been much harder due to the bigger sequence of numbers to keep in mind. Just try that and you'll immediately understand what I'm talking about.

It may also be worth observing that, in case of operands with identical digits, you can take some little, extra shortcut. In fact:

- **If the numbers have the tens digits in common** you can replace the step involving the multiplication between the tens digits with *a simple square*, and the one about the multiplication between "internal and external digits" with a "*sum of the units digits, then multiplied by the tens one*". So, for example, if you want to multiply 76 x 72 you can do like this:
  -Squared 7 = 49. Write it on the left.
  -Sum of units multiplied by the tens: 6 + 2 = 8, which multiplied

by 7 gives 56 as a result. Write it in the middle.
-Units by units: 6 x 2 = 12. Write it on the right.
-After carrying tens from 49_56_12 you get 5472, that's your final result.

- **If the numbers have the units digits in common** you can replace the step that consists in the multiplication between "internal and external digits" with "*sum of the tens digits, then multiplied by the units*" and the units multiplication with *a simple square*. For example, if you want to multiply 28 x 38, you can do like this:
  -2 x 3 = 6. Write it on the left.
  -Sum of the tens multiplied by the units digit: (2 + 3) x 8 = 5 x 8 = 40. Write it in the middle.
  -Squared units: 64. Write it on the right.
  -After carrying the necessary digits in the 6_40_64 sequence you'll get the final 1064.

Now let's extend this method to the *multiplication between three-figure numbers*, which can be done like this:

- Multiply the first two digits from one number by the first two digits from the other one and write the result on the left. So if for example you have to multiply 143 x 388, multiply 14 x 38 = 532_

- Multiply the first two digits of one number by the units digit of the other one and vice versa. Then sum the results of the two products and put it to the right of the number you got in the previous step Thus, in case of 143 x 388, you'll have to perform:
  38 x 3 = 114
  14 x 8 = 112
  114 + 112 = 226
  So now the partial result is 532_226

- Multiply the two units digits and write the result of the product in the right side of the partial result. So, here you have to do 3 x 8 = 24 and the partial is 532_226_24.

- After carrying the extra digits as usual, you have:
  -532_228_4 (after carrying the 2 from 24 to 226)
  -554_8_4 (after carrying the 22 from 228 to 532)
  There are no more digits to carry, so 55 484 is exactly the result of the multiplication we were looking for.

And here one could rightly argue that the last procedure is more difficult and completely unsuitable for a "pure" mental calculation. But we must anyway consider that:

- We can simplify the mental procedure, as happened in the first case, simply by starting with the more complex steps (so, in this case, it's convenient to operate exactly in the order as shown before).

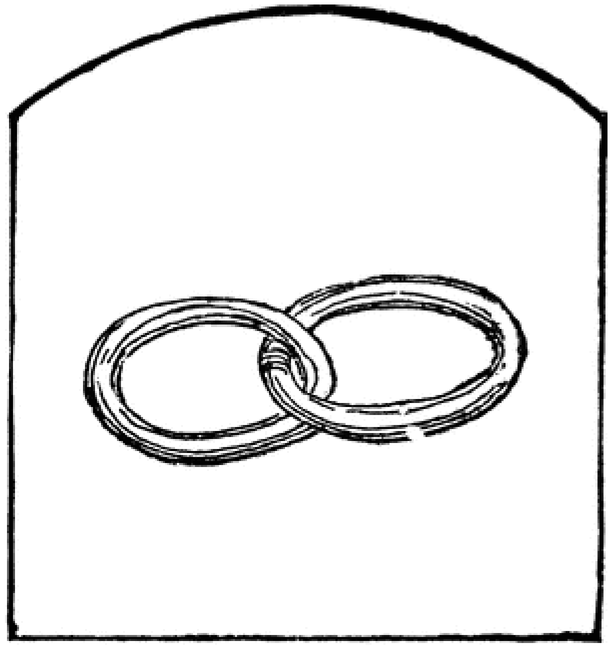
- More we'll train our memory and our confidence with techniques like the "phonetic conversion" and more easily we'll be able to use this technique without any writing supports.

- The quicker we learn to multiply two-digit numbers and the better we'll be able to manage this technique at its best. Its "harder challenge" in fact just into the first step. So, once we learn to do that rapidly and efficiently, the 80% of the hard work is done.

- *Hey, after all you don't necessarily have to use it without writing anything*, and among the written multiplication techniques this is still the fastest and most efficient at all!

Last thing before moving to the next chapter: as a matter of fact, this fast multiplication strategy can be extended in order to multiply even 4 or 5 figure numbers, but we will discuss that later because the basic concept of this extended version of the technique is slightly different. In fact what we'll be talking about is basically a *graphical technique*, that can be performed according to a precise visual pattern and that will let you complete even very complex calculations in the blink of an eye.

XIV - The prodigious connection



After having introduced a method to perform any kind of multiplication between two-figure numbers, let's go back to some "non universal" techniques. In this chapter, in fact, we'll talk about strategies that will let you rapidly and easily multiply two numbers together, but only at the condition that *they are linked by some "special" mathematical connection*. And even if this connection may sometimes be a bit 'restrictive', these strategies are as a matter of fact so immediate to learn and execute that anytime you're lucky enough to find them, *you'll become able to perform extremely complex calculations in a bunch of seconds*. Furthermore, as you will notice, from these specific cases it will be possible to deduce more general ones and so the preconditions will never actually be too "strict" for you! Well, enough said. It's time to start!

Two-figure numbers having the tens (or units) digit in common and whose other digits, when summed together, equal 10.

This technique is really simple to apply, even if it's necessary to distinguish between two cases:

- *Case A:* The tens are the same and the units, if summed together, give 10 as a result. This happens, for example, if you want to calculate 58 x 52 (2 + 8 = 10).
- *Case B:* The units are the same and the tens, if summed together, give 10 as a result. This happens, for example, if you want to calculate 23 x 83.

In particular you have that, in order to rapidly multiply numbers as described in "Case A" (58 x 52), you must:

- *Multiply the tens by themselves + 1* (In this case 5 x (5 + 1) = 5 x 6 = 30)
- *Multiply the units digits* and write the result to the right of the number you got in the previous step. (In this case, 2 x 8 = 16. Final result: 3016, obtained in few seconds and through extremely simple calculations)

In the "Case B" (for example 23 x 83), you have instead to:

- Multiply the tens together and add the units digit to the product (In this case 2 x 8 = 16. Add the units digits and it's 16 + 3 = 19)
- As in case "A", multiply the units together and write the result to the right of the number you got in the previous step (in this case, 3 x 3 = 9, and so the final Result is 1909)

Be careful: in both cases, follow this rule to avoid errors: *if any of the previous steps gives a single-digit result*, write a 0 as its tens digit before writing it into the partial result. Otherwise, for example, in the last example you could have written 199 instead of 1909, getting so a completely wrong number as a result.

As you can see these methods are extremely simple and immediate to perform. But how could we reduce the effects of the restrictions imposed by their preconditions in order to use them *to perform even more general multiplications?*

The answer here is right in Chapter IX, when we talked about decompositions. In fact, all we have to do is to use the decompositions we know in order to transform our general multiplication into a multiplication respecting one of our previous preconditions. For example, suppose you want to calculate 58 x 54:

- 54 is very close to *52*, the number having the same tens digit as 58 and whose units digit, if summed to 8, gives 10 as a result. But how can we apply the technique if we have 54 and not 52?
- We may use the "decomposition into addends"! In fact here we can decompose 54 into (52 + 2), and so:
  58 x 54 = 58 x (52 + 2) =
  = (58 x 52) + (58 x 2).
  58 x 2 is extremely simple to calculate, while 58 x 52 can be instantly resolved using our method.

The secret, then, is *to recognize when it's possible to decompose the multiplication in such a way that one of the two factors has the same tens (or the same units) of the other one, while the other digits, when summed together, give 10 as a result.* And this can generally be done when:

- The numbers have the same tens digit or are one very close to each other
- The numbers have the same units digit
- The numbers are such that their tens digit, if summed together, give 10 as a total
- The numbers are such that their units digit, if summed together, give 10 as a total

Note: this method can be extended to three, four or higher-figure numbers:

- *Extension of "Case A":* This method can be applied if the two numbers have the same leftmost digit, while the numbers made up of the remaining digits, if added together, give a power of 10 as a result. For example, it could be applied to 567 x 533, since 67 + 33 = 100.
  In this case, the procedure to apply is exactly the same: you must multiply the leftmost digit by itself + 1, and the product goes on the left in the partial result: (5 x (5 + 1) = 30_).

Then you have to do nothing but multiplying the remaining numbers and write the result of this product to the right of the previous number: (67 x 33 = 2211).
Final result: 302 211

- *Extension of "Case B":* You can apply this method if the two numbers have the same rightmost digit, while the numbers made up of the remaining ones, if summed together, give a power of 10 as a result (for example you can apply it to 441 x 561. In fact, 44 + 56 = 100).
  Here the procedure is very similar to the one adopted in the basic case as well. As happened there in fact, *the first step consists in multiplying the numbers without their units digit* (i.e. 44 x 56 = 2464) *and writing the product on the left.*
  Now be careful: there is a new, "extra" step to perform: take the next lower power of 10 to the one given by the sum of the first digits (in this case the sum of the first digits equals 100 and then its next lower power of 10 is simply 10) and add it to the previous product. In this case, 10 + 2464 = 2474.
  Finally, multiply the units together and trivially write the result of this product to the right of the previous one, as happened in the basic method (then in this case it's 1 x 1 = 1). Even here, however, be careful and *if you see that the product of the units is made up of a single digit only,* write a 0 as its tens digit. In fact, in this case, the result is 247 401 and not 24 741!

Of course, whatever the power of 10 considered, the discussion about using the decompositions in order to further extend the methods still remains valid, but since the procedure is a little bit more complicated, the use of the last trick should be evaluated with the proper caution.

Numbers between 11 and 19

This method is very simple to apply and learn. Let's imagine for example you want to multiply 17 x 18: here you simply need to:

- Sum a number to the units digit of the other one. So 17 + 8 = 18 + 7 = 25.
- Multiply the previous total by 10. So 25 x 10 = 250.
- Add to this result the product of the units digits. So, in this case, 7 x 8 = 56, which added to 250 equals 306,that's the final result of our multiplication.

Unlike the previous method, this is quite limited and does not have any special variants, except for the universal possibility of getting a multiplication of this type through the *decomposition into addends*. For example, if you want to calculate 12 x 21, you could decompose 21 into (19 + 2). Then 12 x (19 + 2) = (12 x 19) + (12 x 2). The first multiplication can then be calculated right by applying this method, while the other one is quite immediate.

But another extension can be obtained by using the *factorization*, which can be applied quite often, since this method is based on the multiplication between essentially low numbers, which can very easily be seen as the factors of higher two-digit numbers.

For example if you want to calculate 26 x 28 you can factorise 26 into 13 x 2 and 28 into 14 x 2. Then you would have 26 x 28 = 13 x 2 x 14 x 2. At this point, if you use the commutative and associative properties of multiplication it will become = (14 x 13) x 2 x 2. So now you have to do nothing but applying this multiplication method to 14 x 13 and then doubling the result twice. Of course transforming a general multiplication into a multiplication between numbers between 11 and 19 requires a well-trained mathematical eye, but with some time and experience you'll immediately be able to understand when it's the right moment to do it.

Numbers equidistant from an integer

This method basically consists in "replacing" a multiplication with a square calculation. Needless to say, therefore, that in order to let it be concretely useful, you'll have to *memorize or at least to learn how* *to very quickly square some two-digit numbers*. But we'll see that later, so let's do one thing at a time.

As first, let's specify that two numbers are considered as "equidistant" (=have the same distance) from an integer *if they're both even or both odd.* And this integer we're talking about is nothing but their *mathematical mean.* In fact, if we imagine all positive integers arranged in sequence on a straight line, we have that the mathematical mean of any couple of numbers "a" and "b" corresponds to the point that's exactly "in the middle" of them.

For example, 34 and 36 are equidistant from an integer because they're both even. And, remembering that *the arithmetic mean of two numbers is obtained by summing the numbers and dividing then by 2*, we have that their mean is nothing but 35. Or, equivalently, we can say that they are *equidistant from 35* and that their distance from 35 is 1, since 35 = 36 - 1 = 34 + 1.

Yes, the distance is nothing but the *absolute difference between a number and its mean.* Oh, and in case you don't know it, "absolute" in short means that if the value is negative, you just must ignore that and always write is as a positive value. A "negative" distance, in fact, would make no sense, even in its more commonly used meaning.

67 and 89 are also equidistant from an integer because they're both odd. In fact, after calculating the mean I have that 67 + 89 = 156, which divided by two equals *78.* And their distance from 78 is 11, since 78 = 67 + 11 = 89 - 11.

55 and 34 on the other hand would not fit the preconditions. Of course they're still equidistant from "something" and of course their mean could be calculated as well, but it would not be an integer number. In fact 55 + 34 = 89, which divided by two equals 44.5.

But now, how can we use all this information to perform our multiplication? Well, the multiplication of two numbers "equidistant from an integer" is equivalent *to their squared mean minus the squared distance from their mean.*

Confused? Let's get an example and let's take 34 and 36 again. We said that their mean is 35 and their distance from the mean is nothing but 1. The squared mean is 1225, while the squared distance is still 1, so the result of the multiplication is 1225 - 1 = 1224!

Another example: 67 x 89. That their mean is 78 and their distance from 78, as mentioned, is 11.

Their squared mean equals then 6084, while their squared distance from the mean equals 121.

6084 - 121 = 5963, which is the result of 67 x 89.

Before concluding this point, we'll explain *why two numbers equidistant from a non-integer number don't fit.* As first: *the formula "the square of the mean - square of the distance" works fine with them as well.* Why then have we excluded them from the beginning? The answer is actually simple: because if they are both even or both odd, their mean (as well as their distance) *turns out to be a non-integer number and this makes our calculations more complicated.*

For example, given 55 and 34, their mean is 44.5, while the distance between them is 10.5. Trying to square three digit numbers can be definitely cumbersome, and so the whole method would of course lose in terms of speed or efficiency.

Numbers that are close enough to a power of 10

This multiplication technique borrows its power from the Vedic Sutras. In particular it would be good to brush up on the one introduced in Chapter V, the *"All by 9, the last from 10"* that consists in subtracting all the digits of a number from, 9 except for the last one that must be subtracted from 10, in order *to let us calculate the difference between that number and its next higher power of 10*. So, if applied to 87 it gives the result of 100 - 87, if applied to 343 it gives the result of 1000 - 343, if applied to 7384 it gives the result of 10000 - 7384, and so on.

Furthermore it's important to specify that this technique can only be applied to multiplications between numbers *having the same number of digits*. In addition, it has the clear advantage of having a non-restrictive limitation: namely, the fact that the two numbers should be close enough to a power of 10 *is not a mandatory prerequisite*. However, as we'll see more in detail later, if the numbers are not close enough to a power of 10, the consequent calculations can be really complicate, and the whole procedure can become *completely inefficient.*

At first, let's distinguish between the three cases in which this technique can be applied:

- *Case 1:* The two numbers are both slightly smaller than a power of 10. For example, 9948 x 9975.
- *Case 2:* The two numbers are both a little larger than a power of 10. For example, 1018 x 1023.
- *Case 3:* The two numbers are such that one is a little smaller and the other a little larger than a power of 10. For example, 1007 x 883.

So, let's start explaining how to behave in case 1, because it's the easiest and because the other ones are nothing but its little variants:

- As first keep in mind that in this case the result should have a number of digits equal to the sum of the number of digits of the original operands.
- Calculate the differences of the two operands from the next higher power of 10. If not immediate, apply the Sutra *"All from 9, the last from 10".*
- Calculate the product of these differences.
- Subtract any between the two numbers from the difference of the other one (from its next higher power of 10) and write the result to the left of the product you got in the previous step.
- If, after juxtaposing the two results, the total number of digits you got is smaller than the one calculated in the first step, then you need to do nothing but writing as many "zeroes" between the two numbers as it's necessary to match that quantity.

But since it could look like a quite twisted procedure, let's see an example of how it can be applied by multiplying 9948 x 9975:

- The two numbers are made up of 4 digits and so the result must be made up of *8 digits.*
- The difference between 10 000 and 9948 is 52.
- The difference between 10 000 and 9975 is 25.
- Multiply 52 x 25 = 1300.
- Calculate 9975 - 52 or 9948 - 25 (it's the same thing), getting 9923, which must be put to the left of the previous number.
- Here is the final result: 99231300, obtained in a few seconds and without any effort!

Note that the extraordinary efficiency of this method is due to the fact that it transforms a 4-digits multiplication into a 2-digits one. And the more the two factors are close to their next higher power of 10, and the more the method is simplified. For example, a 9997 x 9998 multiplication would be reduced to a simple 2 x 3. And, in the same way, if the numbers start to be distant from their next higher power of 10, the actual multiplication will be transformed into a 3 or 4 digits one, losing then the whole advantage the technique was giving.

But let's immediately move to case 2, very similar to the previous one:

- Keep in mind that here the result should have a number of digits equal *to the sum of the amount of digits of the two operands, minus one*. So, for example, the result of 1002 x 1011 must be made up of *seven digits.*
- Calculate the difference between the numbers and their next lower power of 10. Here of course the calculation is straightforward.
- Calculate the product of these two differences.
- *Sum* to a number the difference of the other one from its next lower power of 10 (no matter which one you choose, the result will be the same) and write the result to the left of the number you got in the previous step.
- If the number of digits is larger than the one calculated in the first step, you must take the leftmost digit from the right section,

and carry it to the left one. For example, if you have 1345_4393, and you calculated that the result should have had 7 digits, you must take that 4 from 4393 and sum it to the 1345, getting so 1 349 393 as a result.

Now let's have an example of how this procedure works and let's calculate 1072 x 1048:

- Each multiplicand has 4 digits. 4 + 4 - 1 = the result must be made up of *seven digits.*
- The differences from their next lower power of 10 are obviously 72 and 48.
- Multiply them, getting 3456 as a result.
- 1072 + 48, or 1048 + 72 = 1120. Write so the partial result as 1120_3456.
- Attention, the total number of digits exceeds the one it should be. So, carry that 3 from 3456 to 1120, obtaining 1123_456. 1 123 456, in fact, is our final result.

Finally, case 3, in which *a number is slightly higher than a power of 10 and the other one is slightly smaller than the same power:*

- *Sum to the smallest one the difference of the highest from the considered power of 10, or subtract from the largest one the difference of the smallest from the considered power of 10.* These operations will give both the same number as a result, so just do the easiest one.
- Write, in the previous total, as many trailing zeroes as the considered power of 10 has.
- Multiply the two differences together and subtract the result of this product from the number you got in the previous step.

And since this procedure could look a little bit harder, let's show how it works by multiplying 989 x 1024:

- The two differences are 11 (989 + 11 = 1000) and 24. So here we can add 24 to 989 or subtract 11 from 1024, the result will always be 1013.

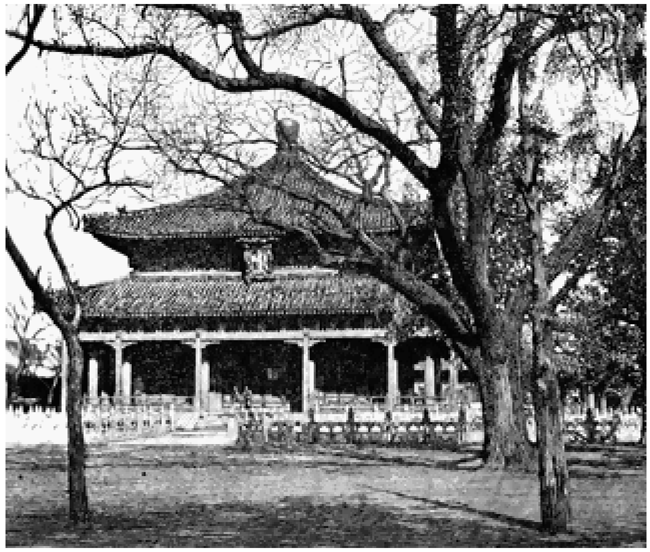
- The considered power of 10 here is 1000, so write three trailing zeroes: 1 013 000.
- Multiply 11 x 24, getting 264. Do you remember how to rapidly multiply by 11, right?
- Subtract 264 from 1 013 000. This could seem hard, but can be made very easily through a simple trick we saw in Chapter V: if 1 013 000 = 1 012 000 + 1000, you can calculate 1000 - 264 first by applying the proper Sutra, and then add 1 012 000. So we'll have:
  1000 - 264 = 736.
  736 + 1 012 000 = 1 012 736, which is also the result of 989 x 1024.

The chapter about the "prodigious connections" in the multiplication ends here. In the next two chapters we will start exploring two ways to *graphically perform any kind of multiplication*, and even if they cannot literally be considered as "rapid mental calculation" techniques, they're anyhow very interesting because of their *original structure and great simplicity of use.*

"To those who do not know mathematics, it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature. If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in."

(R Feynman)

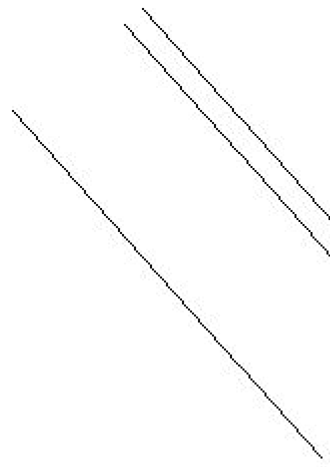
XV - Chinese multiplication



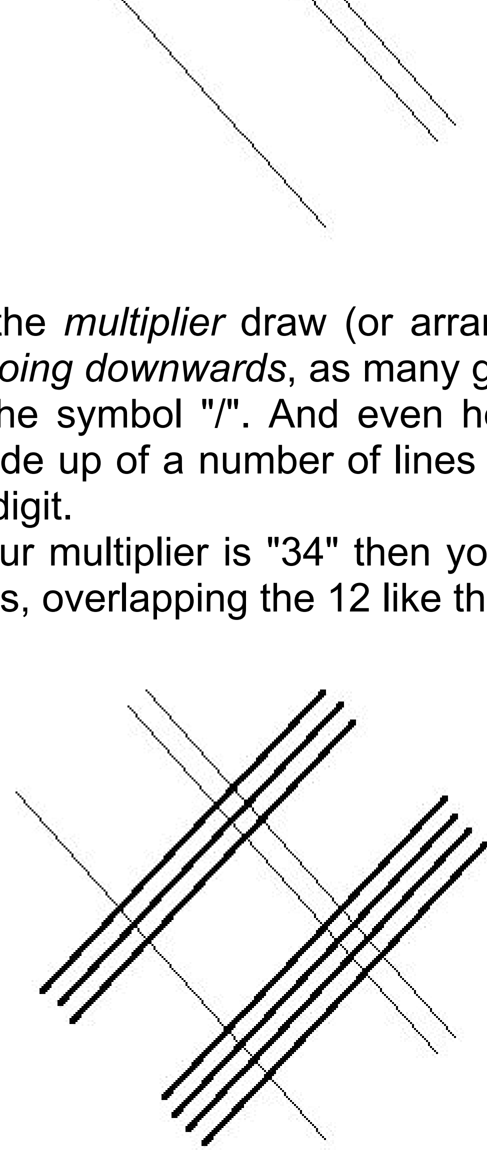
The first graphical technique for multiplication solving we'll introduce in this book is that so-called "Chinese Multiplication" that always has a certain charm for those who discover it for the first time. And this fascination is probably due as to its extraordinary simplicity, as to its great speed when compared to the classic multiplication methods, as to the fact that it lets one calculate any multiplication *without knowing a single multiplication table*. Also, if you do not have any pen and paper, you can even easily perform it by properly disposing *little toothpicks, matches or ... spaghetti sticks on a table!*

And here's how this curious technique of calculation works:

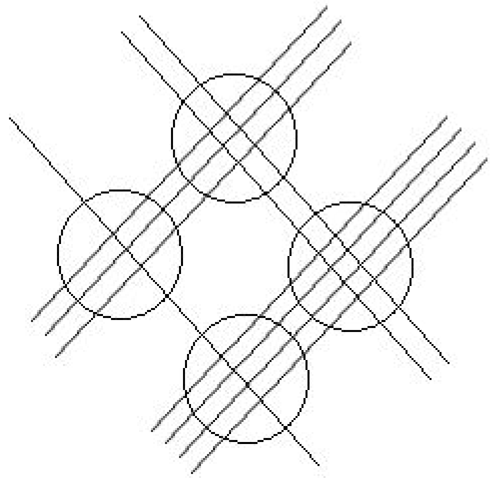
- For each digit of the *multiplicand* draw (or arrange), from the bottom going upwards, as many groups of oblique lines oriented as the symbol "\" (therefore, for example, two digits = two groups of lines, three digits = three groups of lines, and so on). Each group also should have a number of lines *equal to the value of the digit itself.*
  For example you can represent a "12" this way just by drawing a line and then two more lines arranged this way:

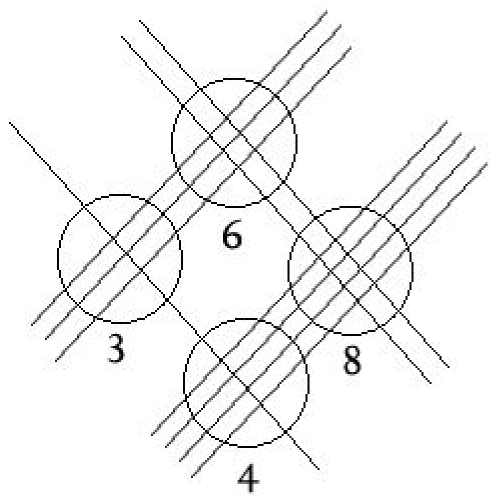


- For each digit of the *multiplier* draw (or arrange), now *starting from the top and going downwards*, as many groups of lines, but now oriented as the symbol "/". And even here of course any group must be made up of a number of lines equal to the value of the considered digit.
  For example, if your multiplier is "34" then you have to draw or dispose seven lines, overlapping the 12 like this:



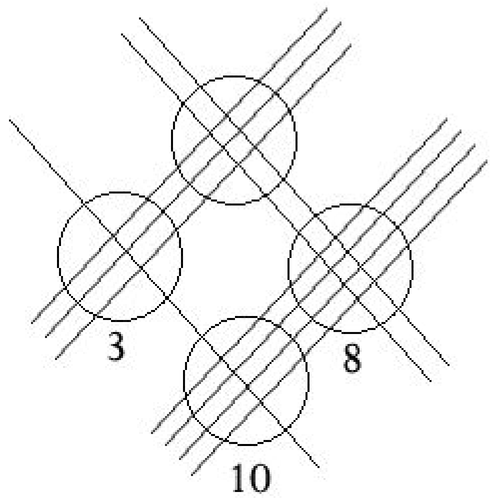
- You will immediately notice that in some sectors of the picture the lines *are crossing each other* and that, indeed, you *can group the nearest intersections*. Circle these groups and count *how many intersections there are in each group.*



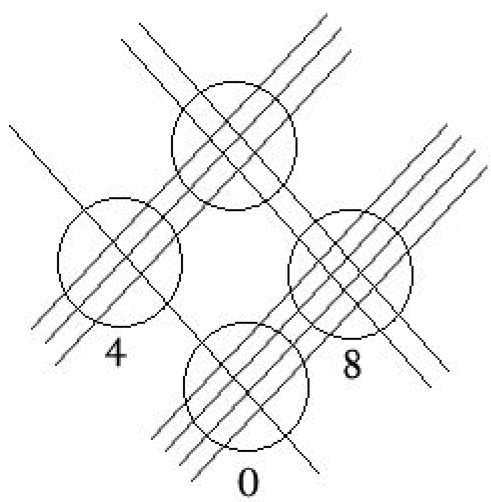


(Here you can actually choose whether to merely count the intersections or to multiply the number of lines coming into that group horizontally by the lines coming vertically. Of course do what's the easiest and most immediate to you.)

- You will notice that some of the resulting groups of intersections are vertically aligned. Then here you have to do nothing but *summing the amounts of the so aligned groups and writing them down your picture*. Here, for example, the two central groups are vertically aligned and their sum is 10.

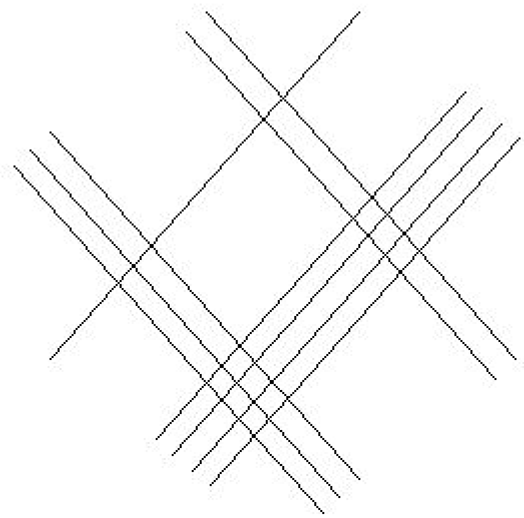


- Now, again, *it's time to carry the extra digits*. Wherever a product or a sum gave a number larger than 10 as a result, carry the tens digit to the left. In this case, for example, you will need to carry the 1 from 10 to the leftmost 3, getting so 4 as a result.



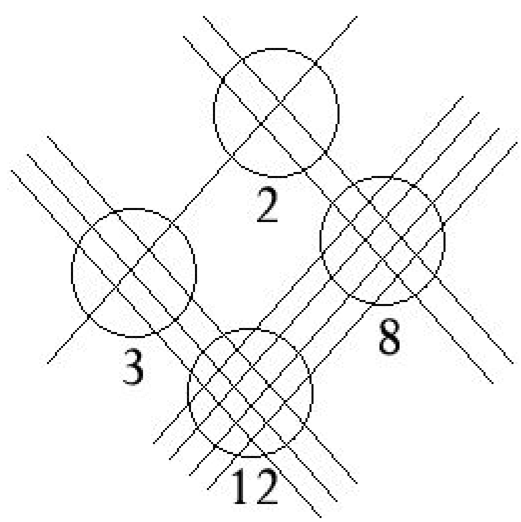
- Just put together the digits obtained this way and here is your final result: 408!

But since the best way to understand a graphical procedure at best is through *more visual demonstrations*, let's have an additional example. For instance, if you want to multiply 32 x 14, here's what do you have to draw:

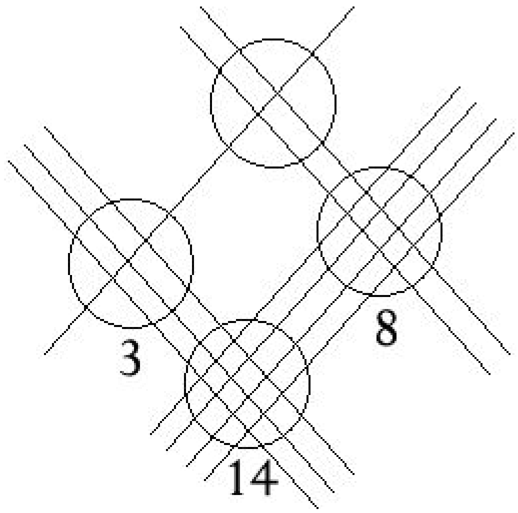


Here for the "32" we drew on the left three lines and then two more oblique lines from below going upwards, while for the 14 we drew a single line and then 4 more lines, from the top going downwards.

So let's look at the groups of intersections, let's circle them and let's count how many intersections there are in each group:

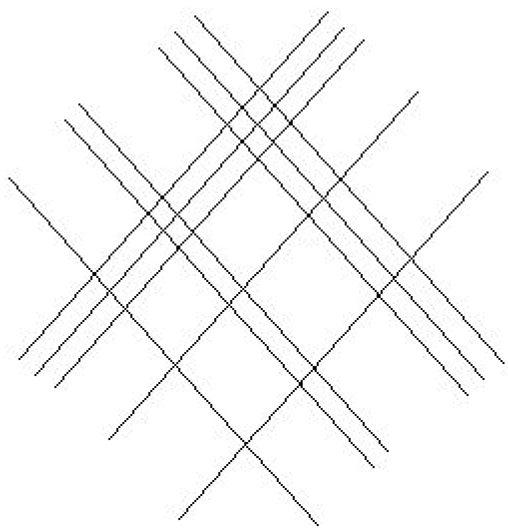


Now we must sum the numbers from the vertically aligned groups. In this case there are only 2 aligned groups again and 12 + 2 = 14. So:

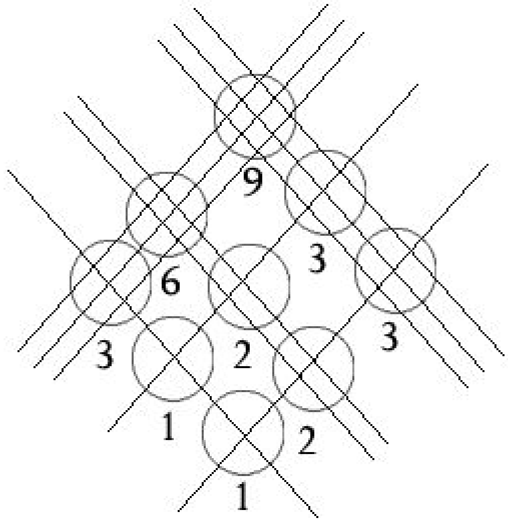


The last sum gave us a number larger than 10. So, after carrying the 1 from 14 we'll obtain 448, which is the final result.

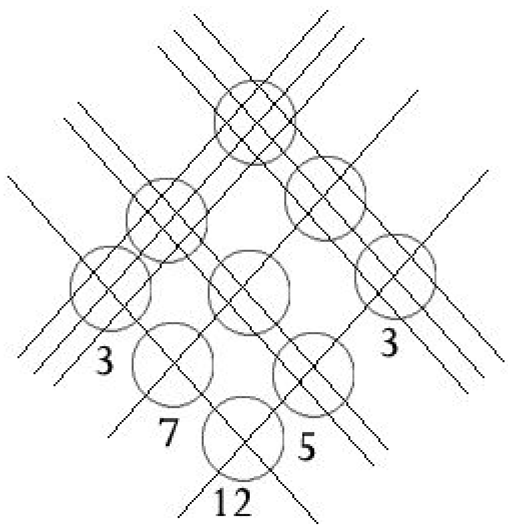
Let's have now a last demonstration of the method, but multiplying three-figure numbers. For instance, 123 x 311:



As you can see, the 123 is given by the groups of one, two and then three lines on the left, while the 311 is given by the groups of three, one and one last line on the right. Now highlight the groups made up of the closest intersections and count the intersections for each group.



It's time to perform the sums: sum all the numbers corresponding to the vertically aligned groups and write the totals down:



After carrying the 1 from 12, we get 38 253, that's our final result.

The method can be used even with 4, 5 or more-figure numbers. Clearly increasing the amount of digits will increase in turn the group of intersections to count and the consequent sums, thus *inevitably increasing the overall complexity and the length of the whole method*. Anyway, even in case of larger numbers, this method can still be considered as a very efficient, simple and even funny calculation strategy, especially for all those who want to make experiments with something completely different from what's more commonly used in everyday life.

XVI - The sliding cross

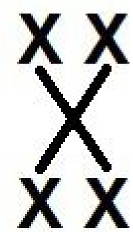
Let's return to a more "traditional" multiplication method and let's introduce the Trachtenberg's *crossing multiplication system* we mentioned a few chapters ago. This system is very simple and effective to learn and apply, even in the presence of very long operands, and unlike the "multiplication table" built by the mathematician, does not need you to memorize long or complicated procedures, but only to make some little "visual training", necessary to understand the "cross-shaped" graphical pattern to follow.

In order to simplify things, also, we'll only explain how to multiply two numbers *having the same quantity of digits.* In fact if you need to multiply numbers having differents amount of digits you'll still be able to use this technique, but in addition you'll have to write, in the smallest between the operands, as *many leading zeroes as they're needed to match the digits amount.*

Enough said, so let's start to show the **2 digits number x 2 digits number multiplication technique.**

Step 1:

Align the numbers in column, multiply their units and write the result of this product on the right in the partial result.



Step 2:

Multiply the units digits of a number by the tens of the other one and vice versa. Then, sum the two products and write the total to the left of the result you got in the previous step.

Step 3:

Multiply the tens digits and write again the last result to the left of the result obtained the previous step.

Step 4:

If any among the results obtained in the previous steps is bigger than 10, then carry the tens digit to the number on the left.

Yes, this is nothing but *the multiplication method introduced in Chapter XIII* and I wrote it again here because, as already mentioned, its basic pattern is the same as the one we'll see for multiplications between larger numbers, and therefore training your eye to work with this visual method since its basic case is definitely the best thing to do to become completely confident with it. So, let's see another example of this procedure and let's multiply 54 x 78:

54 x

78 =

Step 1:

Multiply 4 by 8 = 32 and write it on the right in the partial result.

Step 2:

Multiply 5 x 8 and add the result to the 4 x 7 one:
(5 x 8) + (4 x 7) = 40 + 28 = 68.
Write it on the left, obtaining 68_32.

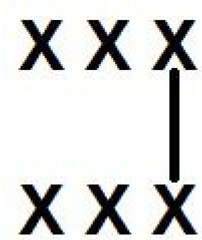
Step 3:

Multiply 7 x 5 and write it on the left again. You'll get so 35_68_32

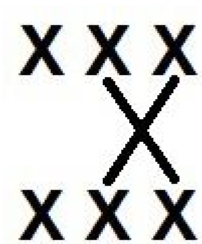
Step 4:

Carry the extra digits:
Carry the 3 from the 32 to the 68: 35_71_2.
Carry the 7 from the 71 to the 35: 42_1_2.
Your final result is 4212!

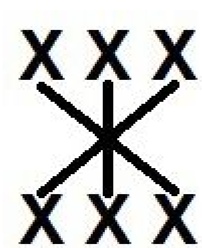
But since this was a well-known issue, let's go immediately to the next level, and let's introduce the graphical technique for the 3-digits number x 3-digits number multiplication:



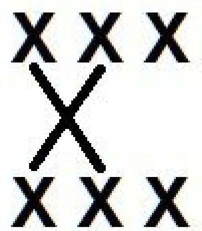
Step 1:
Multiply the units together and write the product in the right part of your partial result.



Step 2:
Multiply the units digits by the tens one and vice versa. Then, sum the two products together and write this result to the left of the one you got in the previous step.

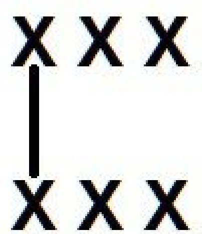


Step 3:
And here is a step we never faced before, even if the pattern to follow is always the same: *multiply the symmetrically opposite numbers and sum all the results of the products.*
In this case, then, calculate (Units of a x Hundreds of b) + (Tens of a x Tens of b) + (Hundreds of a x Units of b). Then write again the result of this step to the left of the previous one.



Step 4:

Now you're moving your "cross" to the left, and the procedure is almost finished. Multiply the hundreds by the tens and tens by the hundreds. Then sum the two products and write the result to the left of the previous one.



Step 5:

Multiply the hundreds digits together and write the result to the left of the previous one.

Step 6:

Carry the extra digits.

So, what visually happens, irrespective of the number of digits is that:

- You always start on the right side *by multiplying "units by units".*
- One digit at a time, going left, you go on by considering *an always larger piece of the two numbers.*
- For each "piece" considered like that, you have to *multiply the symmetrically opposite digits*, sum the results of these products and then write the total to the left of the results you got in the previous steps.

- When the "piece" you're actually considering exactly corresponds to the whole number, you must start taking in consideration smaller and smaller pieces of the two numbers, obtained by "cutting" the numbers on their right side one digit at a time. Then, you just have to repeat the same procedure as in the previous step.
- You have to stop when the "cross" has become a "line" and then you arrived *to the multiplication between the leftmost digit of a number and the leftmost digit of the other one.*
- Carry the extra digits anytime a step gave a number larger than 10 as a result *and ... it's done!*

This of course does not pretend to be a rigorous or formal description of the method, but only a *brief description of the visual scheme to adopt*. Once you learn that, you can easily apply it to any kind of number!

But let's have a practical example of the 3x3 procedure by trying to multiply 673 x 231:

673 x

231 =

Step 1:
Multiply the units: 3 x 1 = 3. Partial result _3

Step 2:
Units by tens and tens by units:
(7 x 1) + (3 x 3) = 7 + 9 = 16. Partial result 16_3

Step 3:
Units by hundreds, tens by tens and hundreds by units:
(6 x 1) + (7 x 3) + (2 x 3) = 6 + 21 + 6 = 33. Partial result 33_16_3.

Step 4:
Hundreds by tens and tens by hundreds:
(2 x 7) + (6 x 3) = 14 + 18 = 32. Partial result 32_33_16_3.

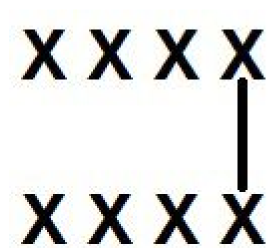
Step 5:
6 x 2 = 12. Partial result 12_32_33_16_3.

Step 6:
Carry-overs:
Carry the 1 from the 16 to the 33: 12_32_34_6_3.
Carry the 3 from the 34 to the 32: 12_35_4_6_3
Carry the 3 from the 35 to the 12: 15_5_4_6_3

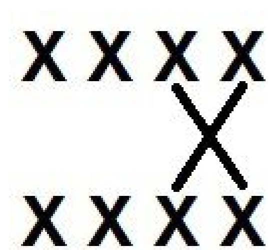
Final result: 155 463!

Now I'll conclude this chapter by showing you how to perform a **4x4 and a 5x5** digits multiplication through this method. Since the scheme is essentially always the same and since I think it's much more useful *to get it visually* rather than endlessly reading the same things, this time I'll leave you just with the images showing the pattern to follow, without describing it step-by-step again:

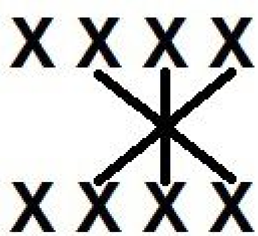
Multiply a 4-digit number x a 4-digit number



Step 1



Step 2



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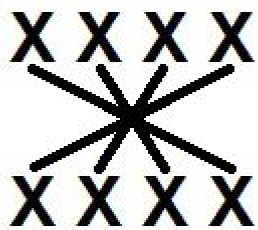
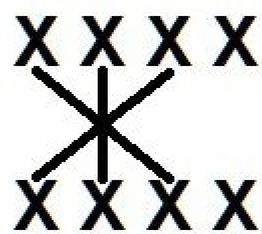


Step 4



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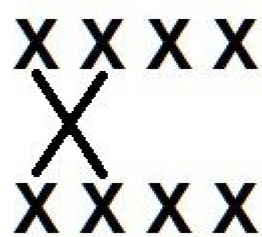
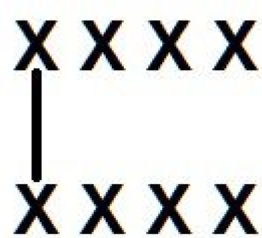


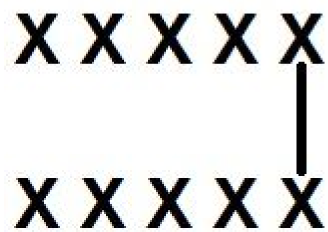
Step 6



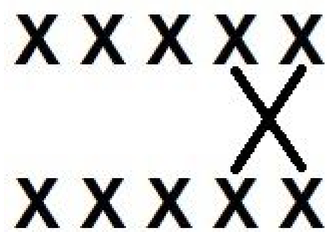
Step 7

Step 8: Carry-overs

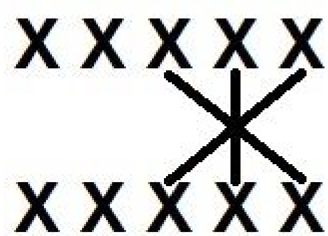
Multiply a 5-digit number x a 5-digit number



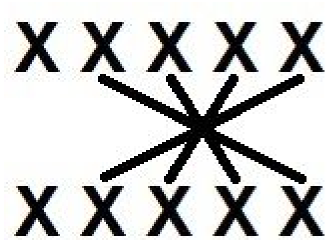
Step 1



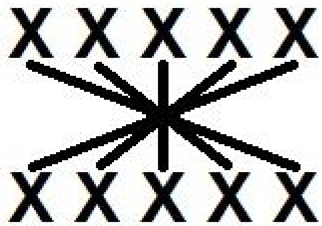
Step 2

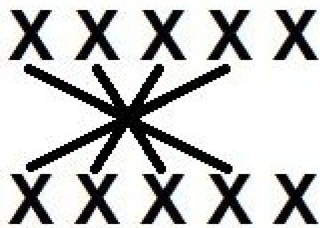


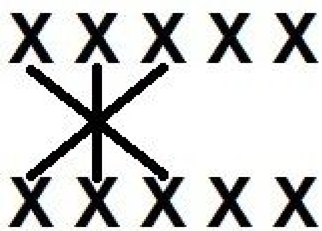
Step 3

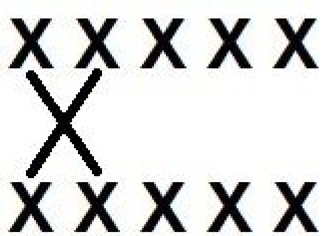


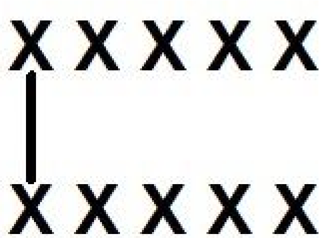
Step 4


Step 5


Step 6


Step 7


Step 8



Step 9

Step 10: Carry-overs

(*Here you may have noticed that the number of steps to perform simply equals the number of digits of the numbers you're operating with, multiplied by 2*)

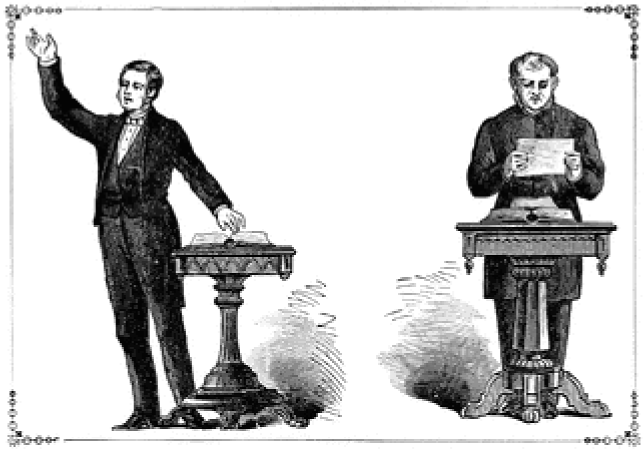
A tip for learning to mentally apply this technique consists in *using some mnemonic technique* as the "phonetic conversion" seen in Chapter III, in order to easily keep in mind the partial results and the carried amounts. With a little mnemonic training in fact you won't only be able to mentally perform very long multiplications, but you'll also learn how to calculate them faster and faster.

The "multiplication techniques" part of this book ends with this chapter. In the following chapter, however, we'll practically apply them, addressing two issues that are as a pain for lots of people, as absolutely indispensable in daily practice: *the percentages and the calculations involving numbers with a decimal point.*

"Even mathematics is a science created by human beings. And then, in each age, as well as in each community, has a different spirit."

(Hermann Hankel)

XVII - Discounts, bad politicians and international measurements



The percentages are a notation of the "parts" of something, represented as an answer to the question *"if this thing was made up of one hundred parts, how many of them we'd be considering?"* and since it's a very widely used concept, knowing how to work with them is really crucial in the everyday life: just think about the opinion polls, the simple store discounts, or to that *probability theory we'll just face in a couple of chapters!*

And despite it's a broadly intuitive concept (most of people are able to quite simply visualize the 50% or 25% of a pie, for example), the situation starts to be much less intuitive when it comes to require *more strict calculations of percentages*. An example can be found in this problem: *"If the price of a litre of petrol is 100, after a week is reduced by 10% and after another week increased by 10%, how much will eventually cost a litre of petrol?"*

The majority of people in fact here answers "100", forgetting that the percentage of something *should always be recalculated on the basis of its current value*. In fact, 100 - 10% = 90%. But then 90 + 10% =

99%, since now the 10% must now be calculated on 90 and no more on the initial 100!

In the same way, very often the media take advantage of this difficulty people have in concretely reasoning with percentages. And an actual example of this can be found in the words of a U.S. reporter who, as a result of the presidential election of 2008, declared:*"In 2004, 37% of voters had declared himself a Republican, 37% a Democrat and the rest an independent"*. He added then that, four years later, 39% of voters had declared himself Democrat and 32% Republican. So, the conclusion was that *"In four years the 5% of the Republicans had become democratic".*

Clearly to have become a Democrat was the 5% of *the total voters*, not the 5% of the Republicans, not to mention *that the number of voters had definitely changed over time,* and therefore that statement was lacking of any consistency.

From all of this we can deduce two fundamental truths:

- A mathematical one, which recommends to *always be careful to calculate the percentages on the actual and current value of something.*
- A less mathematical one, which recommends to *never believe to the percentages proclaimed by any politician.*

Given the right premise*s,* it's time to introduce some more practical tools for calculating percentages. First, as already said, the X% of a quantity means *dividing it by 100 and then multiplying it by "X".*
This, of course, can lead to simplifications that, after applying some principles from the rapid multiplication table we introduced in Chapter IX, can let you quickly calculate a lot of specific percentages. And here is a handy percentage table built right using these principles. Clearly you do not have to memorize it, but rather try to find out which specific criteria were used to get these results:

To calculate:

The 2% of any number = Divide that number by 100 and then double it (or vice versa, the order is not important).

5% = Divide the number by 10 and then halve it (or vice versa).

10% = Divide the number by 10.

15% = Triplicate the number, then halve it and divide it by 10 (in any order).

Another mathematical lifehack: these last two percentages are quite useful when you have to calculate tips for restaurant bills. For example, when you have to calculate a 15% tip on a 50 dollars bill, you can just:
- Triplicate it: 150
- Halve it: 75
- Divide it by 10: 7.5 dollars

It's very easy, isn't it?

20% = Calculate its 10% and then double it (or vice versa).

25% = Halve it twice.

30% = Triplicate it and then divide it by 10. If you don't care about having a too much accurate result, you may also approximate 30% to 33.3%, and then get the desired percentage just by dividing the original number by 3.

40% = Calculate the 10% and then double it twice.

50% = Just halve it.

60% = Double it, triplicate it and then finally divide it by 10 (in any order)

70% = There are no tricks here but multiplying it by 7 and then dividing by 10. But if you don't really care about having a too much accurate result, you may also *approximate 70% to 66.6%*, and so get the desired percentage just by *dividing the original number by 3 and multiplying it by 2.*

75% = Triplicate it and then halve it twice (or vice versa)

80% = Double it 3 times and then divide it by 10 (or vice versa)

90% = Triplicate it 2 times and then divide it by 10 (or vice versa)

So, for example, do you want to know how much you will pay if there is a 60% discount on that 2500 dollars car? Double the price to 5000, triplicate it to get 15 000 and finally divide by 10 to get your 1500 discount! Is it worth the cost now?

And wondering how much does it cost that bathtub having a 1955 dollars price and a 15% discount? Triplicate the price to get 5865, halve it to get 2932.5 and finally divide it by 10 by moving the decimal point to the left, getting your 293.25 dollars discount. Now that we got a result like "293.25", and since it's something deeply linked to the percentages topic, let's add some important information about speed math *involving numbers having a decimal point.*

Rounding and aligning in column

The first thing you can do if you want to more rapidly calculate with numbers having a decimal point is *rounding them:* just remove one or more trailing digits after the decimal point and if the leftmost digit among the ones you deleted was greater than 5, then increase by 1 the rightmost digit you "left" in the number. *It's easier done than said*: for example, if you want to round 5.489 by removing the last two digits (89), you'll transform it into 5.5 since the removed leftmost digit (8) was greater than 5.

In fact, as far as we delete digits after the decimal point the result *will be less accurate but at the same time will let you operate much faster*. And of course *it's up to you to decide how much precision is needed in your calculations and whether the precision loss is worth the ease gain.*

If, for example, you are measuring a wall in order to understand whether you have enough space to put your new wardrobe there, and you have an ultra-precise laser metre telling you that your wall is 1.784532 metres wide, eliminating the digits after the 4 won't create you any problem. For a problem like this, in fact, it's necessary to be accurate by the centimetre, by the millimetre at most, but the tenth or hundredth of a millimetre are of course completely irrelevant parameters.

Nothing special to say in case we must add or subtract numbers having a decimal point: you must just *align them in column properly before performing the calculation, handle any possible numbers without decimal point as if they had a ".0" next to their rightmost digit*, and then you can use all the normal techniques of addition and subtraction you saw in the previous chapters.

For multiplication and division between numbers with decimal point, on the other hand, we have to make some extra clarifications.

Multiplication and division - Decomposition into expressions

A first, "classic" way to multiply between numbers having a decimal point consists in:

- *Removing the point* (leaving the digits unchanged and so without confusing this with rounding them) *from all of the operands*
- *Counting how many digits there were after the decimal points in each of the operands before removing it*
- *Summing these quantities* (let's call this total "b")
- *Performing the multiplication*
- *Re-writing the decimal point* into the result in such a way that after it there are exactly "b" digits.

So if for example you have to multiply 7.76 x 9.31, you can multiply 776 x 931 first (and here you could use the technique for "numbers close to a power of 10" seen in Chapter XVI) to obtain 722 456. Then, since in both numbers there were 2 digits after the decimal point, and 2 + 2 = 4, you'll need to have *four digits after the decimal point* and your final result is nothing but 72.2456.

With the division instead you should simply remove the point from the operation by using its invariantive property (let's repeat it: *multiplying or dividing dividend and divisor by the same amount does not change the final result of the division*). So, you should just multiply both operands by a power of 10 such that it lets you remove the point by both of them. So, for example, if you have 1.24 / 2.33 you can multiply both of them by 100 in order to get 124 / 133. Same thing if for instance you have 133 / 0.002: in this case you can in fact multiply both numbers by 1000 and obtain that the result equals the 133 000 / 2 one.

But a technique that's much more efficient than the classic one is the "decomposition into expressions", very similar to the one we saw in Chapter IX. In particular, in order to apply this technique you can:

- *Decompose them into factors* ... like they didn't have the decimal point at all! For example you can handle a 4.5 like it was a 45 that, decomposed into factors, equals 3 x 3 x 5.
- Ok, you just behaved like the decimal point didn't exist, but of course multiplying a number by 3, 3 and 5 in sequence is the same as multiplying it by 45, but *not the same as multiplying it by the 4.5 we need.*
  So, how can we have back our 4.5? Well, it's very simple actually: you just have *to add to the previous decomposition a division by a power of 10 such that the decimal point reappears where it's supposed to be.*
  For example, in order to transform back the 45 into a 4.5 you will just need to divide it by 10 (division by a power of 10, do you remember?) and so, the full expression equivalent to 4.5 is nothing but 3 x 3 x 5 / 10.

So you just found an expression that's equivalent to your fractional number and that, as happened in Chapter IX, describes nothing but a simple sequence of tasks to perform. In fact, let's remember that *multiplying by a number decomposed into an expression is equivalent to "following that expression".* Then multiplying a number by 4.5 is equivalent to triplicating it twice, then quintuplicating it and finally dividing it by 10.

Dividing a number decomposed into an expression *is equivalent to "reversing the expression" instead,* which means replacing any multiplication with a division and vice versa. So, dividing by 4.5 is equivalent to multiplying by 10, then dividing by 3, then dividing again by 3 and at last dividing by 5.

Here is still valid what we said in Chapter IX: *check the divisibility criteria first (using the number without its decimal point, of course),* and if at least the fastest ones do not give any useful result, *check whether your number is prime or not.*

Decompositions into addends

Another calculation method we can apply to numbers having a decimal point is the *decomposition into addends* we saw in Chapter IX as well. This method on the one hand presents the usual limitation of not being applicable to the division, but the other one has the enormous advantage of being usable independently of the actual possibility to factorize a number. But let's imagine we want to use it to calculate *4 x 1.25: 1.25 here could be decomposed into (1 + 0.2 + 0.05) and at* this point we could separately multiply 4 by 1, by 0.5 and then by 0.05 and at the end sum together the results of the products. But of course this strategy would not be really efficient if compared to the "classic" one.

There is in fact a trick to make this technique infinitely more useful and effective. In fact 0.25 = 1 / 4 and multiplying by 1 / 4 means nothing but dividing by 4. So:
4 x 1.25 = 4 x (1 + 0.25) =
= (4 x 1) + (4 x 0.25) =
= 4 + (4 / 4) =
= 4 + 1 = 5

This means that if you can decompose a number with decimal point *into a sequence of key addends,* the multiplications coming from the decomposition *can be transformed into quite straightforward divisions and the whole process becomes ridiculously simple*. But which exactly these "key numbers" are? Let's see it:

- **0.1 - 0.01 - 0.001 - etc.** In fact multiplying by these numbers is the same as dividing by 10 - 100 - 1000 etc.
- **0.2 - 0.02 - 0.002 - etc.** In fact multiplying by these numbers is the same as dividing by 5 - 50 - 500 etc.
- **0.25 - 0.025 - 0.0025 - etc.** Multiplying by these numbers is the same as dividing by 4 - 40 - 400 etc.
- **0.33 - 0.033 - 0.0033 - etc.** Multiplying by these numbers is (almost) the same as dividing by 3 - 30 - 300, etc.
- **0.5 - 0.05 - 0.005 - etc.** Multiplying by these numbers is the same as halving the number - halving and then dividing it by 10 - halving and then dividing it by 100, etc.
- **0.66 - 0.066 - 0.0066 - etc.** Multiplying by these numbers is (almost) the same as subtracting 1 / 3 from the original number 1 / 3 - subtracting 1 / 3 and then dividing by 10 - subtracting 1 / 3 and then dividing by 100, etc.
- **0.75 - 0.075 - 0.0075 - etc.** Multiplying by these numbers is the same as subtracting from the number 1 / 4 - subtracting 1 / 4 and then dividing by 10 - subtracting 1 / 4 and then dividing by 100, etc.
- **0.8 - 0.08 - 0.008 - etc.** Multiplying by these numbers is the same as subtracting from number 1 / 5 - subtracting 1 / 5 and then dividing by 10 - subtracting 1 / 5 and then dividing by 100, etc.
- **0.9 - 0.09 - 0.009 - etc.** Multiplying by these numbers is the same as subtracting from the number 1 / 10 - subtracting 1 / 10 and then dividing by 10 - subtracting 1 / 10 and then dividing by 100, etc.

Of course you can't always decompose a number into a sequence of key addends, but using *some little approximation* can almost always help you to get what you're looking for.
For example, let's imagine you want to multiply 8 by 1.77. In that case approximate that 1.77 to 1.75 and then calculate:
8 x 1.77 = "almost" 8 x 1.75 =

= 8 x (1 + 0.75) =
= 8 x 1 + (8 x 0.75) =
= 8 + (8 minus 1 / 4) = "almost" 14

Oh, and of course even in this case *it's up to you to decide whether the loss in terms of precision is worth the gain in terms of speed and efficiency.*

Last thing before moving to the next technique: of course not all the multiplicands are "compatible" with the key addends. For example if in the previous operation we had 7 instead of 8, performing a "7 minus 1 / 4" of course would not have been so simple.

So, here you don't have to do anything but *remembering to apply the method whenever the resulting addends give back an easy division.* 25 x 0.8, for instance, can be quite simple because removing 1 / 5 from 25 is a straightforward calculation. On the other hand, 25 x 0.66 can be not so simple, since removing 1 / 3 from the same number is not so immediate. But apart from the specific cases, some good training and experience are the best way to let your eye immediately recognize when this technique can work at its best.

Decomposition into percentages

And here is where the percentages topic completely embraces the decimal point numbers one. So imagine, for example, you want to multiply a number by 3.6. By using the previous system you should have triplicated the number first and then you should have added 3 times 1 / 5 of the same number, which could definitely sound a cumbersome task to perform. But there is, as a matter of fact, a much easier way, which consists in *multiplying it by 4 and then subtracting the 10% from the result*.

In fact 3.6 is nothing but *4 we subtracted its 10% from,* and so we can say as a general principle that *if we want to multiply by a number having a decimal point* (like 3.6) *and the last equals an integer number we added or removed a specific percentage from* (like 4, minus its 10%)*, then we can simplify the multiplication by multiplying* *by that integer first and then by applying that specific percentage to the final result.*

This principle could still sound complex, and that's why it's better to write a couple of examples:

- **Multiplication by 1.8:** Double and eliminate the 10% from the result. In fact 1.8 = 2 - its 10%
- **By 2.5:** Double and then add 25% to the result. In fact 2.5 = 2 + its 25%
- **By 3.9:** Triplicate and then add the 33%. In fact 3.9 = 3 + its 33%.
- **By 4.4:** Quadruplicate and then add the 10%
- **By 7.2:** Multiply by 6 and then add the 20%
- **By 8.9:** Multiply by 9 and then remove the 10%

This method is certainly much *less intuitive, can't be applied to division and is less flexible than the previous one* but has its enormous utility in the fact that anytime you become able to "grab" the right percentage it's much faster than any other strategies seen so far. There is no general rule to understand when it's possible to apply it: it will just come automatically to your mind when you'll be confident enough with calculating percentages and recognizing them.

Integers multiplication through decimal point techniques

Any among the techniques for rapidly multiplying numbers having a decimal point *can be used to instantaneously multiply integers as well!*

Imagine, for example, you have to calculate 758 x 36. You could:

- Transform that 36 into a 3.6, which is equivalent to dividing it by 10.
- Applying the percentage multiplication technique! So multiply 758 by 4 (= 3032) and subtract the 10%. The result is 2728.8.

- You multiplied by 3.6 instead of 36, so that's not the result of your original multiplication, right? And how can we get the right one? It's simple! Since you divided the multiplier by 10 before calculating everything, you just had a result that's *ten times smaller* than the one you need. So you don't have to do anything but *multiplying it by 10 again in order to get the result of 758 x 36 = 27288.*

So, as a general principle, we can perform any multiplication between integers by:

- Checking whether one of the integers, if divided by a power of ten, *can become a number with decimal point you can easily apply a fast technique on.*
- If so, *perform that division by the necessary power of 10.*
- Perform the multiplication by *using that technique.*
- "Restore" the right result by *multiplying the previous product by the same power of 10 you divided by before.*

Let's have a last example of this in order to make it clearer. In case of **4844 x 12575,** which normally could be a very hard-to-perform multiplication:

- Let's divide 12575 / 10 000, obtaining 1.2575
- 1.2575 = 1 + 0.25 + 0.0075, which are all "key numbers". Let's apply their properties!
- 4844 x (1 + 0.25 + 0.0075) =
4844 + (a quarter of 4844) + (4844 - a quarter, divided by 100) =
4844 + 1211 + 3633 / 100 =
6055 + 36.33 = 6091.33.
- We divided by 10 000? Now we have to do nothing but multiplying by 10 000. Our final result, then, is 60 913 300.

Numbers for travellers

Inches, centimetres, pounds, gallons, litres ... what's the best thing to do when you start to be confused between quantities expressed in metric units and quantities expressed in the imperial ones? The first trick here, of course, could be to Google them, but what do you do when your smartphone battery is dead, you have no signal or you just want to quickly do it by yourself? The most efficient solution after all, due to its greater speed, reliability and potential, is always *to use the tool you have inside your head*. But how? *Simply by applying all the techniques seen so far!* Let's see more in detail how (taking the British imperial measurements as a reference):

From inches to centimetres: One inch is about 2.5 centimetres long. So, in order to convert your measure from inches to centimetres you can easily *take your inches, double them and add their half.* Done!

From centimetres to inches: Here you can use the decomposition into expressions: if 2.5 = 25 / 10 = 10 / 4, then you can easily divide the centimetres by 2.5 by "reversing" the 10 / 4, which means *doubling them twice first, and then dividing them by 10.*

From feet to metres: One foot is about 0.3 metres long. So here you can simply approximate it to 0.33 and divide by 3.

From metres to feet: Do the opposite and multiply by 3.

From miles to kilometres: One mile equals about 1.61 kilometres. If you approximate it to 1.60, you can double your miles and then remove the 20% from the product. In fact, 2 - 20 % = 1.60.

From kilometres to miles: If 1.6 = 16 / 10, you could get your miles by multiplying your kilometres by 10 and then by halving them 4 times. Alternatively, if you want something less precise but a little bit faster, you might consider the opposite equivalence: if a kilometre equals about 0.62 miles, you can multiply by 0.62. 0.62 can in turn be approximated to 0.66, and then you can simply remove one-third from the measure in kilometres in order to get it expressed in miles.

From pounds to kilograms: One pound equals about 0.45 kg. So given a weight expressed in pounds you can convert it into kilograms by halving it (multiplication by 0.5) and then removing its 10 % (0.50 - 10% = 0.50 - 0.05 = 0.45). That was simple, wasn't it?

From kilograms to pounds: One kilogram equals about 2.2 pounds. Then in order to make your conversion, you can simply double the kilograms and add the 10%.
Note: here the sequence of operations is the inverse of the one seen for the opposite conversion but, as a general rule, if you use the percentage technique for multiplications, *there are no fast mathematical rules to get the "inverse" expression for dividing by the same number*! Only the factorisation and decomposition into expressions are invertible operations!

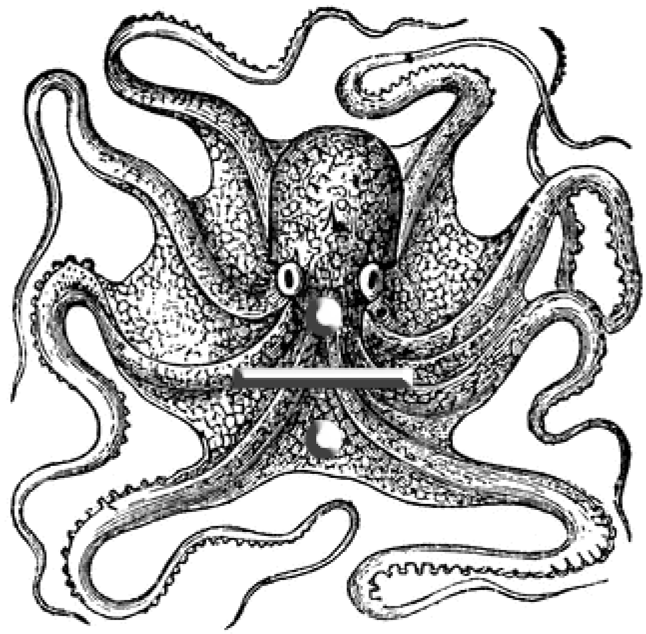
From gallons to litres: One gallon equals about 3.785 litres. Here you can easily approximate it to 3.80 and so multiply by 4 and then remove the 5% (4 - 5% = 4 - 0.20 = 3.80).

From litres to gallons: One litre equals about 0.26 gallons. If you approximate it to 0.25 you can just divide it by 4 and your problem is solved!

The techniques seen in this chapter can also be very useful for converting between *different currencies*. Obviously we can't write a conversion table for that since the conversion rates are constantly changing, but if you apply any of the techniques listed above you can immediately understand costs and expenses even if you are in any foreign country, without ... risking to be taken in from anybody!

The section about percentages and numbers having a decimal point ends here. Let's move to the next chapter and let's try to understand how we can make incredibly simple even *the more complex divisions.*

XVIII - How to make the division less dreadful ... and triplicate your capital!



Division is probably the most problematic among basic arithmetic operations, as for the expert mental calculators, as for those who recently started studying mental calculation. And this is definitely due to the fact that *it's a much less intuitive operation* than, for example, addition or multiplication, *has fewer properties* that can let us simplify it, and it *depends from a more complex calculation procedure.*

In this chapter, anyway, we'll find a way to approach this arithmetic operation from a new point of view, making it much more "intuitive" and "familiar". Of course we have already seen some useful techniques for this purpose in Chapters IX and XVII, but now we'll see probably the easiest one, always remembering that its effectiveness will increase anytime you'll be able to *combine it with the other techniques seen so far.*

This technique is called "The recursive subtractions strategy", is based on the fact that the division conceptually indicates *how many times a number can "contain" another number* and consists in replacing the calculation of the division itself with a *much easier set of subtractions.*

This method has the disadvantage of being not really so "fast", but it sure can make life easier to all those who know how to perform subtractions very quickly and / or are not very confident with division calculations yet.

And here is how you can perform it:

- As it happens in the classical division, if the dividend (note: the number *before* the division sign) is smaller than the divisor (remember: the number *after* the division sign), you must multiply your dividend by 10 and write a "0." in the partial result.
  So, for example, if the initial division is 7 / 15, you have to write 70 / 15 = 0. and write the next digits you calculate right after that decimal point.

- If despite the previous step, the dividend is still smaller than the divisor, write another 0 after the decimal point in the partial result and multiply again the dividend by 10, repeating this step as far as the number before the division sign is still larger than the divisor.

- If any between the operands has a decimal points, use the *invariantive property of division* in order to simplify the division and remove them.
  For example, if you have 56 / 0.23, multiply both numbers by 100 and transform the operation into a 5600 / 23. And the last one is exactly the division we'll apply the full technique to, in order to show how it works. In addition, we'll refer in the next steps to 23 as the *original divisor,* since it's the basic divisor before starting to perform the actual technique (these three steps were just "preliminar adjustments" to let the division be easier to calculate).

- And here is where the technique actually starts: *let the dividend and the divisor have the same number of digits, by multiplying the divisor by a power of 10 large enough to match them*. For example, in this case, in order to make sure that 23 has the same number of digits as 5600, just multiply it by 100 again,

having so 5600 and 2300.
Warning: *If after matching the digits this way the dividend is lesser than the divisor, remove a zero from the last before moving to the next step.*

- Now *subtract the dividend from the divisor*, and do it as long as the result is still a positive number.
  So, in the 5600 / 23 case you have: 5600 - 2300 = 3300.
  Subtract again: 3300 - 2300 = 1000.
  You can't perform subtractions anymore, or the result would become negative. Then *count how many times you've subtracted this way*: in this case it's 2 times and then 2 is the first number to write in the result (to the right of the decimal point, if you wrote it in one of the previous steps).

- Now take your last difference (in this case, 1000) and start subtracting from it, but instead of working with the divisor used so far, subtract *the same divisor divided by 10* (in this case then you'll have to use 230 instead of 2300), and continue doing that until the number is still positive.
  In this case, then, you have:
  1000 - 230 = 770.
  Still, 770 - 230 = 540.
  Then 540 - 230 = 310.
  and 310 - 230 = 80.
  Stop here because *otherwise you'd get a negative number.*
  You have subtracted 4 times and then you can write 4 in the partial result that now is *24.*

- If the result of your last subtraction (in this case, 80) is still greater than the original divisor (in this case, 23), repeat the previous step again using the divisor / 10 (instead of 230, then work with 23). So in this case you have:
  80-23 = 57
  57-23 = 34
  34-23 = 11
  11 - 23 would be negative and you subtracted just 3 times. So,

write 3 in the result, which now is 243. Attention, here: you just got a number, 11, which is even lesser than the *original* divisor (23). Then the partial result, 243, is also the final result and the number you stopped subtracting at is the final remainder of the division.

If instead of stopping at the remainder you want to continue and calculate the digits after the decimal point, you have to do nothing but dividing the divisor by 10 again and continuing with the subtractions as in the previous steps. For example, 23 / 10 = 2.3 and then:
11 - 2.3 = 8.7
8.7 - 2.3 = 6.4
6.4 - 2.3 = 4.1
4.1 - 2.3 = 1.8

You have subtracted four times and in fact 4 is the first digit after the decimal point. And you can still continue this way as far as you want. Of course sometimes the results can *have an infinite amount of digits* and then it becomes necessary to stop just when you have enough "significant" digits.

Now let's apply this technique to a formula that definitely will come in financially handy: do you want to (approximately) know after how long your capital invested at a certain rate of interest doubles? Then you just have to *divide 70 by the rate*. If you want to know after how long it triplicates instead, you have to *divide 115 by the same number.*

Let's assume, then, that you were lucky enough to deposit an amount of money in a bank account having an interest rate of 2.75%, that you want to triplicate that amount and that now you want to understand after how many years this "automatically" happens. So you'll need to divide 115 by 2.75 and this can be done exactly by applying this technique:

- Remove the decimal points: do not calculate 115 / 2.75. Apply the invariantive property of division instead, multiply both numbers by 100 and calculate 11500 / 275.

- Match the number of digits by writing enough trailing zeroes in the divisor. You'll get 11500 / 27500, but ... pay attention to the resulting operation! Here in fact, after matching the amount of digits, the divisor is greater than the dividend, so you must remove one trailing 0 from the divisor before starting, getting then 11500 / 2750.

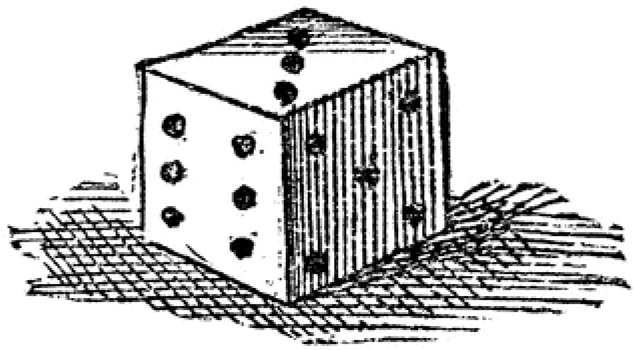
- Start performing the usual subtractions:
  11500 - 2750 =
  8750 - 2750 =
  6000 - 2750 =
  3250 - 2750 = 500.
  You've subtracted 4 times and you can't continue without getting a negative number. Then write 4 in the result.

- 500 - 275 = 225.
  You've subtracted once and have to stop. Write 1 in the result.

- 225 is lesser than the original divisor 275. This means that, if you want to continue, you must write a decimal point into the result and write the new digits you get right after that point.
  Without writing the whole procedure again, we have that 27.5 *can be subtracted from 225 eight times before getting a negative number*. So the partial result will be *41.8*. The procedure could continue again, but of course it's not necessary: we just got that, in order to triplicate a capital at the reasonable rate of 2.75%, we need more than 40 years. And this means that the only way to effectively triplicate an amount of money through an investment is *to have a lot of patience* ... or *to look for higher rates investments, accepting all the inevitable risks that will come with it!*

But speaking about risks and randomness: in the next chapter we will talk about one of the most interesting, exciting and even useful mathematical topics: the *probability theory.* Let's move on!

XIX - Gambling, guessing and gaining



Said Arthur Benjamin, a famous mathematician during a TED conference, that instead of converging to calculus, every mathematics programme in schools should converge *towards a study of the probability theory* instead, since it's probably the most charming, helpful and funny branch of mathematics. He added, also, that a greater common knowledge of statistics and probability would almost certainly have avoided the recent financial crash and helped everyone to take wiser economic and financial decisions, with better consequences for the whole modern society.

But apart from Benjamin's point of view, it's undeniable that understanding statistics and probability theory means acquiring powerful tools to put together data, conquering a greater understanding of the future ... and even increasing our chances to win some more money in gambling games!

So, after bothering you with division, percentages and multiplication between numbers having decimal point, in this chapter we will discuss their application in probability theory, and we'll start right by introducing one of the most useful and important probability indexes: the so-called *"index of expected value"*, or "expected value" we already mentioned in Chapter VI.

The expected value can be quite simply defined as *the average gain we get by doing an action, in case we keep doing it in the long term.*

It could be, for example the exact answer to: *"How much money I gain (or lose) each time on the average if I continue to bet three euro on red at the roulette?"*

The formula to calculate this index is very simple: *(Probability of gain x Gain) - (Probability of loss x Loss)* and deciding what to do once we calculated it is really straightforward as well: in fact, *if the calculated expected value is positive, then continuing to do that action will let us gain an advantage in the long term. If it's negative, will let us waste resources instead.* In short: *do if positive, don't do if negative!*

Now it could be legit to ask a question: *how can I actually calculate the probability of occurrence of an event?* Well, the probability theory explains that this can be essentially done in two different ways, each of them having its advantages and disadvantages:

- One consists in looking at the results of similar events happened in past and in calculating the probability as *the number of times the event has occurred, divided by the number of times that it "could have happened."*
  For example, it could be said that the probability that an aircraft has an accident during its flight is equal to *"Airplane accidents occurred" divided "Flights landed without any accident."*
  This method has the disadvantage of being clearly impractical if you do not have access to a sufficient amount of consistent data, and it may lead you into many dilemmas, really not worth dealing with here, about *what can actually be considered as "consistent"* (for example, does it make sense to use the flight data from airplanes built in 1920 to understand my chances to travel safely today? And those from planes from different airlines? And those of completely different models?).
  Clearly, on the other hand, this is a method that has the undoubted advantage of being very precise to calculate the probability of occurrence of *more complex phenomena.*

- Another method to calculate probability simply consists in saying: *"If an event can end up in x possible ways, the probability that any of those happens is 1 / x."*

Let me clarify it through an example: if you flip a coin, and heads or tails can come out (so, we're talking about an event that can end up in 2 possible ways), the probability that *one* of them happens, for example heads, is 1 / 2, or one out of two, or 0.5. Of course there is no need to have the data of all the coins tossed in the world from the ancient times till today to get that.
Same thing if you roll a six-sided die: at the end it can come out of a number from 1 to 6, so it's an event that can end up in 6 possible ways and the probability that ONE of them happens (the probability that a 4 comes out, for example) is 1 / 6. It's simple after all, isn't it?

Let's add something more: if we have an event that can end up in x possible ways and we want to know the probability that *any among two, three, four, etc. of those happen,* in that case the probability will be simply 2 / x, 3 / x, 4 / x, etc.
So if for example you have your die to roll and you want to know the probability that "one among 2, 3 *or* 4 comes out" (probability that an event with 6 possible evolutions ends up in one if 3 possible ways), the probability is 3 / 6 = 1 / 2. Yeah, *I just divided the number of "interesting" events by the number of total events.*
Same thing if, for instance, there is a lottery whose winner is drawn out of 90 total tickets and you bought one ticket: there your chances of winning would be 1 / 90. But if you bought 20 ticket instead, your chances then would drastically increase to 20 / 90.

This second probability calculation method has the clear disadvantage of being further away from reality and therefore much less able to describe complex phenomena. For example, if we go back to the above-mentioned airplane case, it would be absurd to say that a flight has 0.5 probability to end in an accident just because we have the two "Accident / no accident" choices. However, this method is much easier to use, and is well suitable for *representing with sufficient accuracy phenomena that are simple enough to let each case have the same probability of occurrence as the other ones.*

A mathematical note before moving on: in these strategies we talked about, the probability is expressed *in terms of fractions, whose value is between 0 (impossible event) and 1 (sure event)* instead of the more widely used *percentages*. And we decided to do like this because this notation will simplify our calculations. However, the conversion between the two representations (that are nothing more than two different ways to express the same value) is quite trivial: in order to switch from fractions to percentages you must *multiply the probability expressed in fractions by 100*, while in order to have the fractions back you must *divide the one expressed in percentages by 100*. And so, for example:

- "100% probability" in fractions will be nothing but "1", to be interpreted as "*anytime"*.
- "50% probability": after calculating 50 / 100 it will be 0.5. And since 0.5 = 1 / 2, it's the same thing as saying that "the event happens once in two times."
- Conversely, if you have an event that could happen "Five times a week" (five days out of seven), its probability will be 5 / 7 = 0.71, which multiplied by 100 = 71% probability.
- If you have an event that happens "Sixteen times out of seventy", the probability will be 16 / 70 = 0.23, which multiplied by 100 equals 23%.

Last two things worth knowing:

- **"AND" probability:** the probability that, given two (or more) *independent* events (where "independent" means that the occurrence of one of them cannot influence the other one), *both* of them happen, can be calculated by multiplying the *occurrence probability of an event by the occurrence probability of the other one.*
  For example, if the probability of having heads after flipping a coin is 1 / 2, *the probability of having heads twice after flipping two coins* (or flipping the same coin twice) is 1 / 2 x 1 / 2 = 1 / 4.

- **"OR" probability:** the probability that, given two or more independent events, *at least one of them happens,* is given by *the sum of the probabilities that the events individually happen*, from which we must subtract the probability that *they all happen* (and the latter can be calculated like we just said)*.*
  So if for example you want to flip two coins and calculate *what is the probability that heads comes out at least once,* you have to calculate:
  1 / 2 (the probability for heads to come out from the first coin flip) +
  1 / 2 (the probability for heads to come out from the second coin flip) -
  1 / 4 (the probability that heads comes out from both flips) =
  1 - 1 / 4 = 3 / 4 = 0.75 = 75%.

  Same thing if you roll a dice first, then you flip a coin and you want to know what's the probability that heads comes out from the coin flip *or* 6 comes out from the die roll. In that case, in fact, you'll have to calculate:

  1 / 2 (coin flip, probability of occurrence of any result) +
  1 / 6 (die roll, probability of occurrence of any result) -
  1 / 12 (the product of the previous ones, which is the probability that they both happen) =
  7 / 12 = 0.58 = 58%.

Now you have all the necessary tools to read the curiosities seen in Chapter X from a wider perspective and to understand all of them more in detail!

So, let's make an example of the practical utility of the expected value index by examining how the *American roulette* works*.* As most of people know, the American Roulette is a spinning disc divided into 38 numbered slots (18 reds, 18 blacks, a zero and a double zero). A ball is launched on the wheel and any player can bet an amount of money on the number (or colour, group, etc.) the ball will stop at.

Now we'll calculate the expected value of *repeatedly betting 1 dollar on the red.* Let's remember again that the expected value index formula is *(Probability of gain x Gain) - (Probability of loss x Loss)* and let's try to understand more in detail which exactly are the components of the formula:

- *Gain* = 1 euro, because this is the amount of money we'll win if the ball stops on a red slot.
- *Loss* = 1 euro, because this is the amount of money we'll lose if the ball stops on a black slot, on the zero or on the double zero.
- *Probability of gain:* being 18 red slots, a bet on red wins 18 times out of 38. We can use the second method of probability calculation and we have that it's 18 / 38 = 0.47 (or 47%).
- *Probability of loss:* we lose if the ball stops on one among the 18 black slots or the 2 green ones, so a bet on red loses 20 times out of 38 and the probability of loss is 20 / 38 = 0.53 (or 53%). And here you can start noticing the "suspicious" fact that the probability of loss is higher of the gain one.

The expected value then is (0.47 x 1) - (0.53 x 1) = 0.47 - 0.53 = -0.06.

So, what does this result mean? That *for each euro we bet on red, we'll get an average six cents loss if we keep playing.* And you can verify by yourself that the value will not change if you calculate it considering black instead of red. Therefore, inevitably, if we continue playing, no matter what result we bet on, the content of our wallet is meant to shrink more and more in the long term. And this is nothing but *the mathematical trick the Casinos get richer and richer with.*

But this precious tool can be actually used in various situations and not just in gambling. For example, we can use it *to improve our performances in multiple answering tests.*

For instance, let's imagine you have a test in which the questions can be answered by choosing among five answers: A, B, C, D, E. Let us assume that each correct answer will let us earn 3 points and that there is no penalty for wrong answers. Moreover let's imagine we don't know some of the correct answers and that so we just want to try guessing them.

So let's take the expected value formula and let's see how could we guess in order to maximise our final result:

- Gain = 3 points
- Loss = 0, because there are no penalties for wrong answers
- Probability of gain = since the choice is among five possible answers, the probability of guessing the right one is simply 1 out of 5 = 1 / 5 = 0.2 = 20%
- Probability of loss = the probability of guessing the wrong answer is 4 out of 5 = 0.8 = 80%, but since the loss is 0 that doesn't matter. In fact:

(1 / 5 x 3) - (4 / 5 x 0) = (1 / 5 x 3) - 0 = 3 / 5

The expected value index here is positive, which means we'll get an average gain of some points (3 / 5 of a point, which is still better than nothing) for each guessed answer. So the rule you can deduce from this reasoning is that *if there is no penalty for wrong answers in a multiple answering test, guessing in case we don't know an answer is always more convenient than leaving a blank box.*

But what happens if, in the same test, we introduce a penalty of 2 points in case of wrong answers? Well, we'll have that the formula becomes:

(1 / 5 x 3) - (4 / 5 x 2) = 3 / 5 - 8 / 5 = - 5 / 5 = -1

So, now, with such a penalty, the expected value index becomes negative, and for each guessed answer our risk is to lose one point. Therefore in this case *leaving a black box is the best thing to do.*

It would be different, however, if we could eliminate *three answers from the range of possible choices*, and start guessing just between the remaining two. In fact, in that case, the probability of guessing the right one would rise from it 1 / 5 to 1 / 2, while the probability to be wrong would drop from 4 / 5 to 1 / 2. Then we would have:

(1 / 2 x 3) - (1 / 2 x 2) = 3 / 2 - 2 / 2 = 1 / 2, which is a positive expected value again.

We just noticed then that the presence of a penalty is not enough to let us clearly take our decision. How to understand, then, more specifically, *whether guessing is more convenient than leaving a blank box?*

Just after applying some mathematical equivalence to the expected value formula (through steps we won't report here), we have that guessing is more convenient when:

Points for a correct answer > Penalty Points for a wrong answer x (Possible answers - 1)

This just means you must multiply your penalty points by (Possible answers - 1), and then check whether the resulting number is greater or lesser than the points for a single, correct answer. If so, guessing is convenient when you do not know an answer. Otherwise you just shouldn't answer at all. In addition, from this formula we can deduce than if we can *safely exclude some choices* from the possible ones, our expected value *increases,* the quantity after the "greater than" sign will be smaller, and so we could meet the case in which, even if "Points for a correct answer" is lesser than the other number, it won't be like that anymore after performing our exclusions.

Before ending this chapter, let's extend the last formula to the generic case, and let's transform the original question it was answering to, into a *"When should I just try to do something and continue doing it in the long term despite the random circumstances?"*

After applying to the general expected value formula some straightforward mathematical equivalence again, we have that we should continue doing something when:

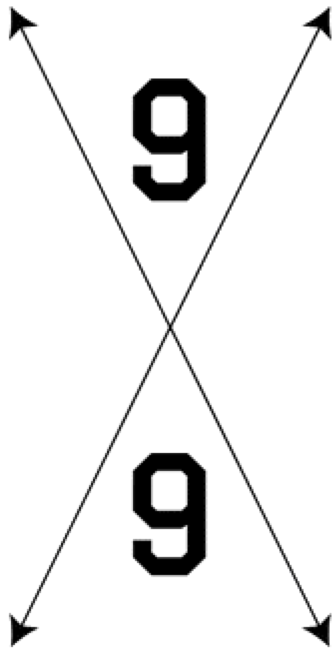
Probability of Gain x Gain > Probability of Loss x Loss

Despite its simplicity, this formula represents an amazing treasure because, if used in the right way, can help you to save thousands dollars and to maximize your results in many sectors, from your business to your interpersonal relationships. For example, when somebody says you that you have "nothing to lose" in asking that girl out, that means that it doesn't matter what's your probability of being actually refused: your courage *will always let you gain a consistent advantage in the long term.*

"I like to look at mathematics almost more as an art than as a science; for the activity of the mathematician, constantly creating as he is, guided though not controlled by the external world of the senses, bears a resemblance, not fanciful I believe but real, to the activity of an artist, of a painter let us say."

(Bocher)

XX - A game of time and precision



In this chapter we'll talk about *estimation* and *proof by 9,* two incredibly valuable tools for any mental calculator, since they respectively let anybody *perform more precise calculations in exchange for a loss of speed* or, vice versa, to *calculate more rapidly in exchange for a loss of precision.* More specifically, we have that:

- **The estimation** may be performed *before* any calculation and lets you know "more or less" and almost immediately its result.

- **The proof by nine** can be performed *after* any calculation and can *almost certainly tell* us if we made a mistake during the procedure.
  Curiosity: there is evidence of usage of this technique since the third century AD, known by the name of *abjectio novenaria.* However, since the Romans were performing calculations through pebbles, they quite unlikely were even able to *demonstrate why* it worked. The first documented explanation of its process in fact is much older, more precisely dated 1202, and was found among the pages of the *Liber Abaci by* Fibonacci, the same book that brought the Arabic numerals to Europe.

So, let's start talking right about *estimation* and let's start distinguishing between the two basic types of estimations you can apply to your calculations in order to make them extraordinarily faster:

Estimation by rounding

Rounding consists in nothing but *replacing with zeroes some trailing digits from the numbers you're calculating with.*
This, anyway, is not the only step to perform. In fact, in order to make the rounding a little bit more "consistent", you have to check *whether the leftmost removed digit was greater than 5* and, if so, then you have to *increase by 1 the rightmost non-null digit in your number.* Since it's easier done than said let's explain it through an example, and let's apply this strategy to a multiplication like 7845 x 9871:

- You can replace the last three digits with zeroes, getting 7000 and 9000.
- The leftmost removed digit in 7845 was an 8, which of course is greater than 5. So we'll have to increase the rightmost non-null digit in 7000 by 1, getting so 8000.
- Even the leftmost digit we removed from 9871 was an 8, so we must increase the second rounded number as well, getting 10000.
- 7845 x 9871 = approximately 8000 x 10000 = 80 000 000

Here you can notice that the actual result of 7845 x 9871 was 77 437 995, that's very close to 80 000 000. Obviously, in order to get a more precise result we could have applied rounding by replacing just the two trailing digits instead of the last three, calculating then a much "closer" 7800 x 9900 instead of 8000 x 10000. The result of the last operation is in fact 77 220 000, which is in fact even much more "similar" to the "real" result. So basically, *more are the digits you replace with zeroes, less the estimate is close to the real result, and faster/easier is the whole procedure.* And of course it's up to you to decide time to time how many digits it's convenient to replace, depending on *whether you need a faster result or a more precise one.*

Here you may notice that a similar technique, referring to numbers having a decimal point, was introduced in Chapter XVII. And it may be worth noticing that, in that case, *the rounding technique was much more "painless" to perform:* in fact, if we take any number and we just remove some digits after its decimal point (that's the same as replacing those digits with zeroes), we usually change it just by a *very small quantity,* and consequently *the approximation of a calculation involving that number uses to be much more "acceptable"* than what just happened with replacing zeroes "to the left" of the decimal point.

I'll explain myself better about the last observation: if we want to calculate 7.8793 x 6.4339 and we round both numbers to 7.9 x 6.4, we get that the estimated result is 50.56. The real one would have been 50.694628, which differs from the estimation just by 1 / 10, and this quantity is *much more irrelevant* than the 3 millions difference we had in our first "7845 x 9871" example. So, *always be careful to the "order of magnitude" of the approximation you get after rounding your numbers.*

Estimation by replacement with expressions

This technique consists in nothing but *replacing a number with a fraction of a multiple of 10* and, if compared to the previous one, gives one the chance to *get much more accurate results in exchange for a much faster and more simple calculation*.

However, it presents the same problem as the "decomposition into expressions" we saw in Chapter IX: *it's not easy to immediately understand which expression you should replace your numbers with*. However, once you trained yourself to work with large numbers, getting that will be kind of automatic for you. Also, trying to understand and memorise the following examples will definitely help you to get the hang of this kind of substitutions:

142 = about 1000 / 7. So if for example you have to multiply 148 x 344, you could approximate 148 to 142 and 142 to 1000 / 7. Then you can get an acceptably estimated result of the multiplication just by *adding three zeroes to 344 and then dividing it by 7.* The result of the last operation is about 49142, while the original result was 50912. As you can see, the distance between the two numbers is much smaller than the one we'd have had after using the rounding technique: in that case, in fact, we could have rounded 148 to 100 and 344 to 300, obtaining the quite imprecise estimation of 100 x 300 = 30 000.

333 = about 1000 / 3. Then if for instance you want to multiply 734 x 330, you could approximate 330 to 333, then perform the multiplication by adding three zeroes to 734 and at last divide it by 3 = about 244 666, very close to the "real" 242 220.

428 = about 3000 / 7.

666 = about 2000 / 3.

999 = about 3000 / 3.

1666 = about 10 000 / 6.

And so on.

Nothing else to add about the estimation but ... exercise, exercise, exercise! So let's go the other way round and let's try to understand how can we use the proof by 9 in order to *increase the precision and the accuracy of the calculations involving all four elementary operations,* in exchange for ... some little more time to perform it after our basic mathematical work*!*

Proof by 9 for addition and subtraction

First, here's how you can perform the proof by 9 for addition. The proof for subtraction, in fact, will require to execute the same procedure with just some little variants:

- For each summand, calculate the sum of its digits. Then sum together the digits of the last total until you get a single digit. We will call this operation the "Mod9" of the number since it's equal to the *remainder you would obtain after dividing that number by 9.*
  So, let's suppose you just calculated 237 + 344 = 581. In order to get the Mod9 of 237, you'll have to sum 2 + 3 + 7 = 12 first, and then 1 + 2 = 3. Same for 344: 3 + 4 + 4 = 11 and 1 + 1 = 2.

- Now sum the mods you got in the previous step and, if the total is made up of two or more digits, calculate its Mod9. In this case you just have that 3 + 2 = **5**.

- Now calculate the Mod9 of the result of your addition. In this case, 5 + 8 + 1 = 14 and 1 + 4 = **5, again**.

Did you notice that? The sum of the modules of the addends *equals the module of the total*. And this brings us in front of the basic concept behind the proof by 9, which namely is: *if the two final modules don't match, you can be sure that the result of your calculation is wrong.* However, *if the two final modules are the same, you can't be sure that your result is right.*

In short, the proof by 9 *can clearly let you know if you were wrong but can't positively tell you whether you've performed a calculation properly.* Anyway, we have that the results will match in case of a wrong calculation *1 time out of 9*. So, as far as this can concretely come in handy, we can say that it's quite *"rare"* to see two mods matching after performing a wrong calculation.

So, let's immediately go to the proof by nine for subtraction. As we said, it's very similar to the addition one, except for just some little differences. Let's see them:

- Calculate again the Mod9 of each of your operands. So if for example you just calculated 788 - 215 = 573, then you have to sum 7 + 8 + 8 = 23 and 2 + 3 = 5. After that, just perform the same calculation on the other number, getting 2 + 1 + 5 = 8.

- Now, instead of summing the modules you obtained in the previous step, just *subtract one from each other,* and *if you get a negative number, simply add 9.* Then in this case you have 5 - 8 = -3, and after adding 9 you get 6.

- Now take the result of your subtraction, and calculate its Mod9. In this case you have 5 + 7 + 3 = 15 and 1 + 5 = 6, which matches the previous result again.

But what if the result of the subtraction was negative? Well, in that case you should have *done the Mod9 as always*, but at the end of the procedure you'd have written *a negative sign next to the result of its module, and then add 9.* But let's have another example in order to clarify the last concept.

- Let's imagine you want to check the result of 45 - 388 = -343. The Mod9 of 45 is 9, while the one of 388 is 1. The first digit we need to perform our proof, then, is 9 - 1 = 8.

- Now let's calculate the Mod9 of the result: 3 + 4 + 3 = 10 and 1 + 0 = 1. Let's write a negative sign next to 1, getting -1 and let's then add 9, getting 8. Done!

Proof by 9 for multiplication

This is perhaps the most widely used kind of proof by 9, and here's how you can perform it. You'll notice that the procedure is very similar to the addition one as well:

- Calculate the Mod9 of your multiplicand. For example, if you just calculated 45 x 62 = 2790, add 4 + 5 = 9
- Calculate the Mod9 of the multiplier. In this case, 6 + 2 = 8.
- Multiply the results of the previous modules together and if the resulting product is made up of two or more digits, calculate its

Mod9. In this case, 9 x 8 = 72 and 7 + 2 = 9.

- Now calculate the Mod9 of your result. In this case 2 + 7 + 9 + 0 = 9 + 9 = 19, and 1 + 8 = 9.
- The verification to be made here is always the same: the first module *must be identical to the second one*. As usual, if so, we have *probably* performed our calculation properly, otherwise we *definitely* made a mistake.

Proof by 9 for division

The proof by 9 for the division is exactly the same as the one for multiplication, with the necessary premise to perform it *only in case of integer quotients.*

More specifically, since *the division is the inverse operation of multiplication,* in this case you have to do nothing but:

- Calculating the Mod9 of the *quotient*.
- Calculating the Mod9 of the *divider*.
- Multiplying the previous results and, again, if the product is a two-or-more-figure number, then calculate its Mod9.
- Comparing this result with the Mod9 of the *dividend* and drawing your usual conclusions.

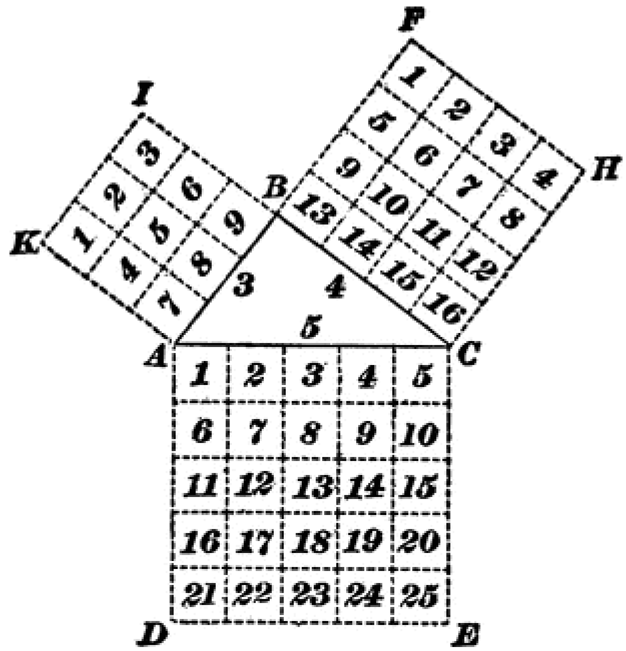
An useful but not necessary notion before ending this topic: sometimes the proof by 9 can be performed in union with a more complex but more accurate test, which is named *"proof by 11"* and consists in executing exactly the same steps as the proof by 9, but r*eplacing the "Mod9" calculation with the "Mod11" one.* And what's the "Mod11" about? Well, exactly as happens in the Mod9 case, it's nothing but the *remainder you get if you divide that number by 11* and can be calculated this way:

- Starting from right, sum together the digits in an odd place (so the first on the right plus the third on the right plus the fifth ... and so on).
- Now sum the digits in an even place (so the second on the right plus the fourth plus the sixth ... and so on)

- Subtract the last total from the first one. So, for example, the Mod11 of 341 is (1 + 3) - 4 = 0.

Since the Mod11 is not as immediate to calculate as the Mod9, the "Proof by 11" is clearly a bit more complex and slower to perform but wherever a greater accuracy in results is needed and you have enough time to perform all the additional calculations involved, it can be used alone or *in union with the proof by 9* in order to get a much more accurate result check. In fact, if you apply both tests to a calculation and you find out that the final modules are the same for both, you can be sure that the result will then be correct *in 99% of cases,* which is still not 100%, but *can be anyway much more than what we actually need.*

XXI - 10 strategies to square numbers ... and a strategy to save your life!



It's said that when Euler, mathematician who lived in the age of Enlightenment, ended up working on perfect squares, was so fascinated by their inner harmony that he started believing that they were *the unquestionable evidence of the existence of God*. Clearly the study of the philosophical beliefs that one can accrue from his personal approach to mathematics is not a topic this book is supposed to talk about, so I will just carefully observe that, as Euler noticed, squaring integers is often a *very simple calculation to perform* and, as seen in the "golden numbers" chapter, it often produces, as results, incredibly *symmetrical and harmonious numbers.*

But let's move immediately to practice and let's go through all the strategies useful for performing these calculations in the fastest and more efficient way. Maybe, unlike Euler, you won't see the hand of some almighty God there, but I'm sure they will extremely *simplify your life on more than one occasion.*

1 - Memorise the basic squares

Let's start this chapter right by introducing *the basic squares table.* Acquiring the ability to immediately recall from memory the squares of the 20 smallest positive integers can in fact be very helpful in speeding up the calculation of the squares for larger numbers:

Squared 1 = 1

Squared 2 = 4

Squared 3 = 9

Squared 4 = 16

Squared 5 = 25

Squared 6 = 36

Squared 7 = 49

Squared 8 = 64

Squared 9 = 81

Squared 10 = 100

Squared 11 = 121

Squared 12 = 144

Squared 13 = 169

Squared 14 = 196

Squared 15 = 225

Squared 16 = 256

Squared 17 = 289

Squared 18 = 324

Squared 19 = 361

Squared 20 = 400

A curiosity for gourmet lovers: did you just notice that squared 9 is almost twice bigger than squared 7 and that squared 12 is almost triple than the same quantity? Why am I saying that? Well, since the area of a circle exactly increases *with the square of its radius*, you just got that *a 7" pizza is big almost the half of a 9" one and one third of a 12" one.* And since you'll never pay a 12" pizza three times the price of a 7" one, this means that *buying a larger pizza is always the most convenient choice,* because it will let you have *much more pizza* in exchange for a much smaller price. In this case, again, math is your most precious ally!

2 - Using fast multiplication technique for numbers between 11 and 19

Yeah, sometimes memorising the squares of the first 20 positive integers can be just *a very hard task to accomplish*. Usually, in fact, most of people easily remember *only the squares of the first 10 integers* (essentially because they're in the multiplication tables they learnt at school) and so, in order to rapidly get the remaining squares when our memory doesn't really help us, we can simply use the *fast multiplication strategy for numbers between 11 and 19* we introduced in Chapter XIV. After adapting it to two identical numbers, in fact, it becomes:

- Sum the number to its units.
- Multiply the result by 10.
- Square the units of the number to square and add it to the product you got in the previous step.

So the procedure for rapidly squaring 17, for example, would be:

17 + 7 = 24

24 x 10 = 240

240 + 7 x 7 = 240 + 49 = 289

This technique can also be used to easily calculate the squares of numbers like 110, 120, 130, 140, etc. In fact, in this case, you have to do nothing but *repeating the same procedure as they were 11, 12, 13, 14 ...* with the only difference that you must *write two trailing zeroes in the result at the end.*

3 - Strategy for numbers ending in "1"

Squaring numbers ending in "1" is actually a very quick and simple calculation to perform:

- Take the number to square, remove its units digit and square what remains.
- Double the same number without its units and write it to the right of the result you got in the first step.
- If the last number is made up of two or more digits, carry the extra digit to the square you got in the first step.
- Write a "1" at the end of the partial result.

Let's imagine for example you want to calculate the square of 41.
You will need to:

- Remove the units digit, get 4 and square it = 16.
- Double 4 = 8. Now write it to the right of your previous result = 16_8.
- Write 1 at the end = 1681.

Same thing can be done with numbers made up of three, four or more digits. The only problem in that case will be that obviously *squaring and doubling the number without its units won't be as immediate as in the two digits case*. Anyway using some of the fast multiplication techniques we saw in the previous chapters, like the Trachtenberg one, should definitely help us to simplify our calculations even in case of longer numbers.

4 - Strategy for numbers ending in "5"

The strategy for squaring numbers ending in "5" is probably even easier to apply than the previous one. In fact, in cases like these you just have to:

- Take the number, remove its units and multiply what remains by *itself + 1*.
- Write 25 at the end.

So, if for instance you want to rapidly square 75, you can:

- Remove the units from 75 and multiply 7 x (7 + 1) = 7 x 8 = 56
- Write 25 at the end: 5625

Even in this case the rule can be applied to three, four or more figure numbers as well, with the only, usual inconvenient of longer and more complex calculations. But even here the techniques seen in the previous chapters can come to our help. In fact if for example you want to square 335, you can:

- Multiply 33 x 34. Here you can use the technique seen in Chapter XIII and easily get that the result is 1122
- Write a 25 at the end: 112 225.

5 - Strategy for numbers ending in "25"

Even rapidly squaring numbers ending in "25" is quite an elementary task. More specifically in this case you can:

- Take the number you want to square, remove the "25" and square what remains.
- Now add to the previous square half of the same, original number without its final "25".
- Multiply the previous total by 10.
- Write 625 at the end.

Suppose you want to calculate the square of 825. In this case you'll just have to execute these simple steps:

- 8 x 8 = 64.

- 64 + 8 / 2 = 64 + 4 = 68.
- 68 x 10 = 680.
- Write 625 at the end = 680 625.

6 - Strategy for numbers beginning with "5"

This is another calculation strategy very simple to learn and apply. Let's start looking at the procedure to rapidly square a number between 51 and 59 first, then we'll extend the topic to three or more-figure numbers:

- Square the units digit.
- Subtract 25 from the number you want to square and write the difference to the left of the previous square.
- ... done!

So if for instance you want to rapidly square 56, you can:

- Square 6 = 36.
- Calculate 56 - 25 = 31.
- The final result is simply *3136.*

As you can see, it's a quite straightforward procedure. So, which exactly are the differences in case of three, four or more-figure numbers? Well, let's see them:

- Square the number without its leading 5.
- Take 25 and write next to it *as many trailing zeroes as you need to match the number of digits of the number to square* (so you'll have to take 250 if you want to square 512, 2500 if you want to square 5887, 25 000 if you want to square 54 321, and so on).
- Subtract the multiple of 25 you got in the previous step from the number you want to square and write the result to the left of the square you calculated in the first step.

This strategy, applied for instance to 531, would therefore consist in the following steps:

- Squared 31 = 961 (and here you can use the strategy for rapidly squaring numbers ending in "1").
- Calculate 531 - 250 = 281 and write it to the left of the previous result.
- The final result is exactly 281 961.

7 - Using the "equidistant numbers" multiplication technique

In Chapter XIV we explained that it's possible to multiply two numbers by calculating *their squared mean minus their squared distance from that mean.*

So now we can just *reverse that formula* and therefore say that it's possible to square any number just *by multiplying two numbers whose mathematical mean is the number to square, and then by adding the square of their distance from that mean.* Transforming in fact a potentially hard-to-perform square into a sequence of easier tasks can seriously simplify our life if we do it properly.

Well, the first question one could ask here is: *"how can I find two numbers whose mathematical mean is the number I want to square"*?

It's quite straightforward actually, since you can just choose *the previous and the next integer*, or those numbers you obtain by adding to the number +2 and -2, -3 and +3, +4 and -4, and so on. For example, if you want to find two numbers whose mathematical mean is 38 you can just choose *37 and 39, 36 and 40, 35 and 41, and so on*.

More specifically since, taken an integer, there are *infinite numbers* whose mean is that specific integer, the best rule to make this method as easier as possible is: *take two numbers that are easy enough to multiply together.* And this can be achieved for example by *choosing a couple in which at least one number is a multiple of 10* (for example, by choosing *36 and 40* in the 38 case).

So if you want to rapidly calculate the square of 38 you can:

- Multiply 37 x 39 and add 1 (distance = 1 then the square of the distance = 1).
- Multiply 36 x 40 and add 4 (distance = 2 then the square of the distance = 4).
- Multiply 35 x 41 and add 9 (distance = 3 then the square of the distance = 9).
- ... and so on.

In this case, like we just said, the easiest strategy is definitely to calculate 36 x 40 = 1440 and then add 4, resulting in 1444, which is just the square we were looking for.

8 - Taking advantage of the proximity to another square

Simply by using a variant of the formula we saw in the previous point, we can easily calculate the square of a number *n in case we know* (or we can easily calculate) *the square of another "reference number" r that's close enough to our n*.

More in detail, in order to square our n, we can:

- Sum n and r
- Multiply this total by the distance between n and r (remembering that the distance is obtained by *subtracting the lesser number from the greater one*).
- Take the square of r you already know (or you could easily calculate) and *sum it* to the result of the previous product if your "reference" r is smaller than n. If r is greater than n instead, you have to *subtract* that result from the known square.

The strong point of this technique is in the fact that, as a reference, you can take any incredibly-easy-to-square number, like *the closest multiple of 10*, or *the nearest number ending in 1 or 5*. But let's have an example of this technique and let's use it to calculate the square of 58:

- Let's take 55 as a reference. Its square in fact, which is immediately calculable through the strategy for the numbers

ending in "5", is 3025.
- 55 + 58 = 113.
- Let's multiply 113 by 3, which is the distance between 55 and 58, obtaining 339.
- 55 *is smaller than 58*, so we have to *add* the last product to the first square*:* 3025 + 339 = 3364, which is nothing but our squared 58.

9 - Using the crossing multiplication technique for two and three-digit numbers

The "crossing" multiplication technique we saw in Chapter XIII can come to our aid during our square calculations as well. As happened in fact in the *multiplication technique for numbers between 11 and 19* case, we don't have to do anything but *adapting the multiplication procedure to identical numbers*!

It can also be useful to notice that this technique, since it's "universal" and doesn't have any specific prerequisite to be performed, represents an excellent alternative to the two previous ones, and so it's up to you, time to time, to choose the best technique to calculate with, depending on what's more congenial to you.

Let's start with the two-digits version:

- Square the tens digit.
- Multiply the units by the tens, double the result and write it to the right of the square you got in the previous step.
- Square the units.
- Carry all the extra digits to the left in case any of the previous results was made up of two or more digits.

So if for example we want to calculate the square of 87, we can:

- Square 8 = 64.
- Multiply 7 x 8 = 56 and double it = 112. So the partial result is now 64_112.

- Square 7 = 49. Partial result = 64_112_49.
- Carry the extra digits to the left:
  64_116_9 (added the 4 from 49 to 112)
  75_6_9 (added the 11 from 116 to 64)
  No more digits to carry, and in fact 7569 is exactly the square we were looking for!

When you come across three-digit numbers you can group the tens and the hundreds digits together, handle the resulting number like it was the tens digit in the previous version, and repeat the same procedure, which of course becomes a little more complicated, but sure is still a definitely doable challenge. In fact in this case you can:

- *Square the number made up of the first two digits of your number*. So, for example, in the case of the square of 458, you must simply square 45 = 2025.
- Now multiply the same number you started from in the previous step by the units digits and double the result. So in this example you must calculate 45 x 8 x 2 = 720. The partial result is then 2025_720.
- Then just square the units and write the result to the right of the number you got in the previous step. In this case you have 8 x 8 = 64, and the result so far is 2025_720_64.
- Carry the extra digits:
  2025_726_4
  2097_6_4 = 209 764, which is the number we were looking for.

10 - Using the multiplication technique for numbers close to a power of 10

Like any other multiplication technique, even this one can be extremely useful for rapidly squaring even very large numbers, at the condition that they are *close enough to a power of 10*. And since when we were talking about that technique we had to distinguish between two specific cases, we must do the same here:

How to square numbers slightly lower than a power of 10:

- Keep in mind that the final square should have a number of digits equal to *twice the digits of the original number.*
- Calculate the difference between the number to square and the next higher power of 10. If it's not immediate to perform, *apply the sutra "All from 9, the last from 10"*. Then square the result of your subtraction.
- Now subtract from the number to square its distance from the next higher power of 10 and write the difference to the right of the result you got in the previous step.
- If the total amount of digits in your partial result is lesser than the one you calculated in the first step, you just need to write *as many zeroes between the two numbers you got, as it's necessary to match that amount.* You just got your result otherwise.

So, let's immediately have an example, and let's calculate the square of 9987:

- The result will be made up of 8 digits, since 4 x 2 = 8.
- Its difference from 10 000, the next higher power of 10, is 13.
- Squared 13 = 169.
- Subtract 13 from 9987, getting 9974 as a result. Write it to the left of the previous result.
- You just got 9974_169. The total digits here are 7, and since we found out that the result should be made up of 8 digits instead, we must *write a zero between the two numbers*: final result 99 740 169.

As noticed in case of multiplication, this technique lets us square even huge numbers with amazing speed, but the more we move away from a power of 10 and the more the things start to get complicated. And this of course happens because *the differences calculated in the first step start to be larger numbers*, and the consequent square starts in turn to be a much longer calculation to perform.

How to square numbers slightly larger than a power of 10:

- Keep in mind that the final square should have a number of digits equal to *twice the digits of the original number, minus one.*
- Calculate the difference between the number to square and the next lower power of 10. And this should be straightforward.
- *Sum* to the number to square its difference from the next lower power of 10 and write the total to the left of the number you got in the previous step.
- If the total amount of digits in your partial result is *greater* than what you calculated in the first step, you just need to *take the leftmost digit in the number you got in the previous step and sum it to the number you got in the first step.*

But let's clarify this procedure once and for all through an example, and let's square 1094:

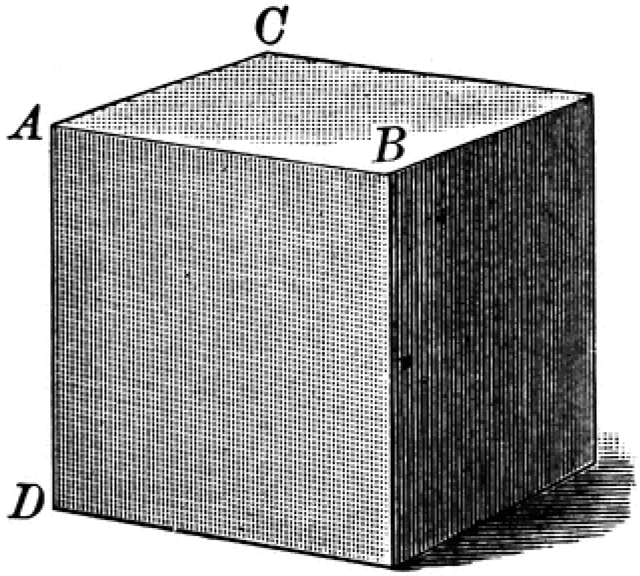
- The result will be made up of 7 digits (4 x 2 - 1 = 7).
- The difference from the next lower power of 10 is of course 94 and its square can be easily calculated through one of the previous techniques = 8836.
- Now sum 1094 + 94, getting 1188, and write it to the left of the previous result. Now your have 1188_8836.
- We said in the first step that the real result must be made up of 7 digits, so we must take the left-most digit from the right side number (the last "8" from 8836) and the sum it to 1188. So we get 1196_836 = 1 196 836, which is our squared 1094.

Now that you're fully aware of how to rapidly square a number, here's another curiosity: we saw that if you square *55* you get 3025 as a result. Now let's take a number slightly higher than 55: 75, for example. Its square equals 5625, which is almost *double than the previous one*. Like we saw in the pizza example, *a little increase in the number to square produces a big increase in the corresponding squared number*.

Now let's dig up some physics: we have that the kinetic energy (*the energy associated to a moving body)* of any object *increases with the square of its speed*. And you just calculated that the energy associated to a car running at 75 mph is almost double than a car running at 55 mph. Little increase in speed, *double the energy*, and double, if not infinitely heavier, *the impact damage in case of accident*. And that's why you should never run full-speed while driving your car: even small increases in speed enormously exacerbate the potential damage in case accident, as well as keeping the speed lower *reduces by several orders of magnitude any risk during your journeys.*

Meanwhile, *this* specific journey is very near to an end, and in the next, and final chapter, we'll discuss some tricks to *quickly extract the square and cube roots of any number.*

XXII - Square root, cubic root and ... thirteenth root!



Extraction of square root from non-perfect squares (*numbers whose square root is not an integer quantity*) can be very often a quite cumbersome and frustrating operation. And that's why, as good fast calculators, always looking for the faster and most efficient strategy to accomplish our tasks, even in this case we'll do our best to avoid tedious procedures and get, rather, a good result in as less time as possible. Let's start!

How to check whether a number is a non-perfect square

First, let's give a look to a very simple "trick" we can use to immediately identify a non-perfect square: *a perfect square always ends in 4, 5, 6, 9, or 0.*

The opposite, of course, is not always true (*if a number ends in 4, 5, 6, 8 or 0 we can't automatically say whether it's a perfect square or not*) but at the same time, thanks to this rule, whenever we see a number that *does not end* in one of the above-mentioned digits, we can be 100% sure that its square root won't be an integer number and so we can prepare ourselves to work on the calculation of its *approximation*, as we'll see later.

As a matter of fact there is another, more complex rule, which lets us positively identify whether a number is a perfect square or not: *a number is always a perfect square if, after being decomposed into factors, all of its factors have an even exponent* (or are repeated an even number of times, which is exactly the same thing).

If you remember what we said about factorisation in Chapter IX, you'll immediately understand what I'm talking about and in which cases this rule could come in handy. 36, for example is a perfect square since it's equal to 3 x 3 x 2 x 2. Same thing 1764, since it equals 7 x 7 x 3 x 3 x 2 x 2. Of course if you're not very confident with divisions and decomposing numbers into factors (despite the techniques we introduced in the last chapters) this can be a quite complex operation to perform, so my advice here is *to use it only when strictly necessary*.

Two ways to rapidly approximate the extraction of a square root

A first technique for approximating the calculation of a square root is based on taking advantage of a specific mathematical relationship involving any square roots. More specifically, given a number "a" we want to extract the square root from, and a number "b" we *already know the square root of*, we have that:

√a approximately equals √b - [b - a / (2 x √b)],

where the approximation is *as closer to the real square root as "a" and "b" are one near to each other.*

But since the mathematical formulas without any examples can only create confusion, let's immediately show a practical application of the technique and let's try to *calculate the square root of 5*. In this case we can of course take *4 as a reference number,* since its pretty widely known square root is two, and so we can do:

- √4 = 2

- From the previous result I have to subtract the fraction (4 - 5) / (2 x 2)
- Fraction denominator: 4 - 5 = -1
- Fraction numerator: 2 x 2 = 4
- -1 / 4 equals -0.25 (remember that *a negative number divided or multiplied by a positive one gives a negative number as a result*)
- Now 2 - (-0.25) = 2 + 0.25 = 2.25.
- We thus obtained that the approximated square root of 5 is 2.25. The real one is actually 2.236 but, as you can see, since 4 and 5 are quite close one to each other, the *resulting approximation is quite acceptable.*

If we want a greater accuracy in our result, anyway, we must use another *gradual* strategy*,* which works *as better as we perform more steps of it,* and *can be applied this way:*

- Taken the number we want to extract the square root from, let's take its *next lower perfect square* again (for example, if we want to extract the square root of 38, let's take 36).
- Now let's calculate the *mathematical mean* between our number and the considered perfect square (for example, in this case, (38 + 36) / 2 = 37) and let's divide it by the "known" square root of the perfect square (so, in this case, we'll calculate 37 / 6 = 6.1666). This is a first approximation of the square root of 38.
- But if we want to get an even more accurate result we *can perform one more step*: let's take the number we're extracting the square root from and let's divide it by the approximation we got in the previous step. So, in this case, we had that our previous approximation was 37 / 6 = 6.1666. So now we have to calculate 38 / 6.1666 = 6.1621621621
- Do we want the most accurate result of all? Let's add a last step and let's calculate *the mean between the two previous results!* Here the mean between 6.1666 and 6.1621621621 is 6.164411, while the real square root of 38 was 6.164414. So, as we can notice by ourselves, despite the more steps to perform, the result *is really close to the real one*. And if we don't care about that 1 / 100 000 imprecision level, this technique is a great, fast

replacement for the "classic" procedure everybody learnt at school.

Tips for extracting cube root and fifth root

The extraction of the cube root of a number is definitely not a simple task to perform, and that's why here we will introduce just a specific trick to *quickly extract the roots of perfect cubes* (numbers whose cube root is an integer) *made up of 4, 5 or 6 digits.*

The first thing that must be done in order to apply this trick is to learn the fundamental cubes of numbers from 1 to 10:

Cubed 1 = **1**

Cubed 2 = 8

Cubed 3 = 27

Cubed 4 = 64

Cubed 5 = 125

Cubed 6 = 216

Cubed 7 = 343

Cubed 8 = 512

Cubed 9 = 729

Cubed 10 = 1000

So, after helping yourself in the memorization process with some mnemonic technique remember that, unlike what happens in the square root case, *the last number of a perfect cube can immediately let us determine which is the last digit of its cube root*. In particular we have that:

- If the cube ends in 0, its root ends in 0
- If the cube ends in 1, its root ends in 1
- If the cube ends in 2, its root ends in 8

- If the cube ends in 3, its root ends in 7
- If the cube ends in 4, its root ends in 4
- If the cube ends in 5, its root ends in 5
- If the cube ends in 6, its root ends in 6
- If the cube ends in 7, its root ends in 3
- If the cube ends in 8, its root ends in 2
- If the cube ends in 9, its root ends in 9

And this table can be easily memorized by recognising that *the cube always has the same final digit as its root*, except when it ends in *2, 3, 7 or 8*. In these cases, in fact, the root, instead of having the same final digit as its cube, has *its difference from 10.*

Now you have everything you need to calculate the root of your perfect 4, 5 or 6-figure cube. In fact:

- Its root will be a two-figure number.
- The units digit can be easily found after looking at the above table.
- In order to find the tens digits instead, as a first step, *remove the three rightmost digits from your perfect cube.* Now you got *a number "a" made up of 1, 2 or 3 digits.*
- Take the perfect cubes table and, going from the top downwards, take *the last (= the larger) cube that's still lower than your "a".*
- Its cube root will be the tens digit you were looking for.

But let's have an example to clarify how this technique works and let's imagine *you want to extract the cube root of 85184*. The units digit can be obtained immediately and it's 4. In order to get the tens one instead we will have to *remove the three rightmost digits of the cube, getting an 85.*

So, let's take a look to the perfect cubes table and we'll see that *the last lower cube than 85 is 64*; its cube root is 4 and then our tens digit is nothing but 4. We found that the cube root of 85184 is 44!

Last trick I want to teach you about the cube root: you can actually approximate its calculation just by using *any classic non-scientific* *calculator* having only *the square root button.* In particular, here's what you have to do in this case:

- Type the number you want to extract the cube root from.
- Press the square root button once.
- Press the "multiply" button.
- Press the square root button twice.
- Press the "multiply" button.
- Press the square root button four times.
- Press the "multiply" button.
- Press the square root button eight times.
- Press the "multiply" button.
- ...

And so on, doubling the number of times you press the "square root" button until pressing the "multiplication" one does not change the result anymore. At that point, just *press the square root button one last time* and you'll see your cube root.
A curiosity: do you want to approximate the calculation of the *fifth root* instead of the cube one? *Repeat the same procedure, but by replacing the "multiply" button with the "divide" one!*

Thirteenth root?

As far as the root degree increases, the related operations start to be more and more complicated and at the same time *to move further away from the daily chores*, and that's why we'll just conclude this book with *some little curiosity about thirteenth root*.

The extraction of thirteenth root, in fact, is actually a quite popular procedure for mental calculation world records. The first world record ever, for example, was achieved in 1974 by Herbert B. De Grote, who calculated the thirteenth root of a 100-figure number in 23 minutes. And if this looks like a titanic accomplishment to you, just be aware that nowadays Gert Mittring, a German mental calculator we talked about some chapters ago, *is able to perform the same calculation within 11.8 seconds*. Of course Mittring, like the others who tried to achieve that result before him, performs this calculation through a specific algorithm, which is highly complex and only partially known. In addition, it is always assumed that the number the potential record-breakers try their challenge with is *an integer thirteenth root,* which slightly simplifies the procedure. In fact, without going into detail:

- The integer thirteenth root of a number *always ends in the same digit as the number itself.*
- The integer thirteenth root of a 100-figure number *is always an 8-figure number between 41 246 264 and 49 238 826*. Because of this, it's generally considered as a "quite easy" task, based on the calculation of a logarithm and some consequent multiplications.
- The integer thirteenth root of a 200-figure number *is always a 16-figure number between 2 030 917 620 904 736 and 2 424 462 017 082 328*. This, on the other hand, is considered as a "quite complex" task to perform. The world record here in fact, is about 5 minutes for a thirteenth root calculation, a time 30 times higher than the record for the 100 digits one!

Needless to say, some people also use to associate this calculation to mystical and esoteric meanings, and this seems to be due to the strong symbolism behind the number 13, which is associated as to the revolt of Lucifer against the heavens, as to Hebrew word for "One", referring to God. Again, the occult meanings behind numbers and calculations is a topic far beyond the actual purpose of this book, but I anyway hope that this chapter made spark off something in you: who knows, *maybe you'll be the genius breaking the next speed math-related world record!*

"Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. "Immortality" may be a silly word, but probably a mathematician has the best chance of whatever it may mean."

(G. H. Hardy)

In conclusion ...

Your personal journey into the world of creative, fast and useful mathematics ends here.

I hope that reading this book has been for you charming, funny, and most of all that it gave you that "extra boost" that every good book should give to its reader.

I want also take this opportunity to thank Danilo for having helped me once again with the editing, the translation and the layout of this edition, with great patience, accuracy and professionalism.

But, most of all, *thank you reader*, for all the help and affection that you constantly give to *"The 101 bibles"* publishing project, which is constantly growing thanks to the support of all of you!

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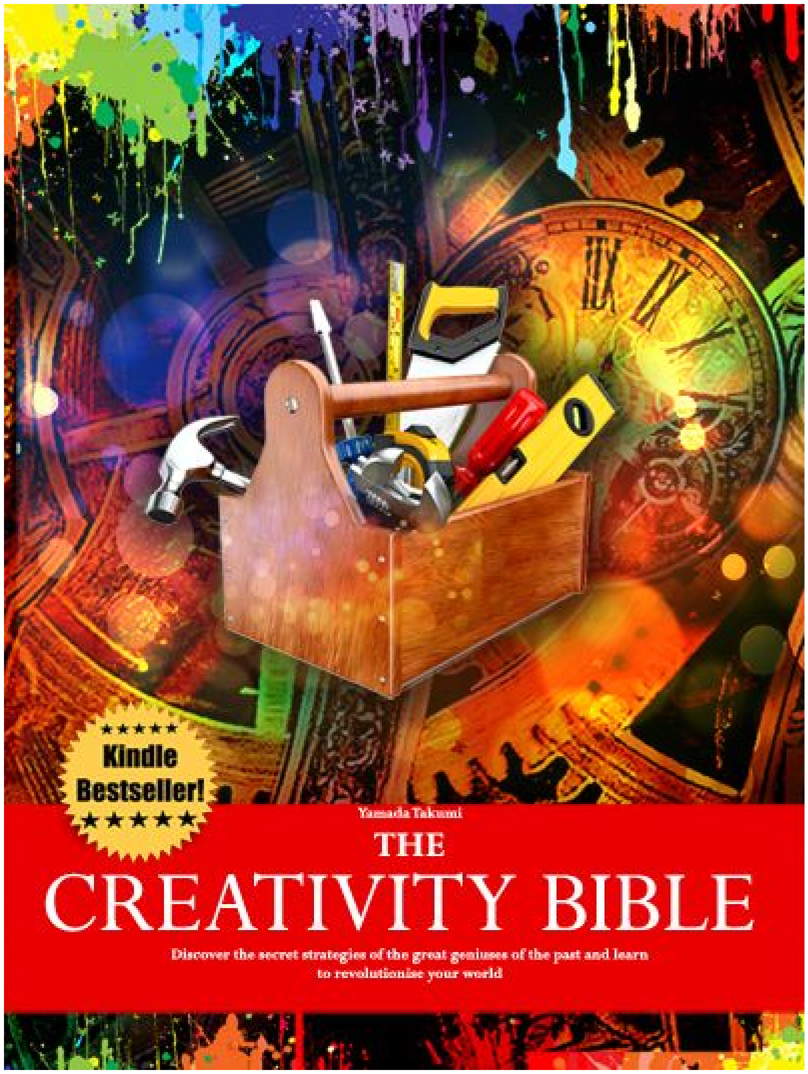

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Appendix: Prime numbers from 2 to 5000

2 3 5 7 11 13 17 19 23

29 31 37 41 43 47 53 59 61 67

71 73 79 83 89 97 101 103 107 109

113 127 131 137 139 149 151 157 163 167

173 179 181 191 193 197 199 211 223 227

229 233 239 241 251 257 263 269 271 277

281 283 293 307 311 313 317 331 337 347

349 353 359 367 373 379 383 389 397 401

409 419 421 431 433 439 443 449 457 461

463 467 479 487 491 499 503 509 521 523

541 547 557 563 569 571 577 587 593 599

601 607 613 617 619 631 641 643 647 653

659 661 673 677 683 691 701 709 719 727

733 739 743 751 757 761 769 773 787 797

809 811 821 823 827 829 839 853 857 859

863 877 881 883 887 907 911 919 929 937

941 947 953 967 971 977 983 991 997 1009

1013 1019 1021 1031 1033 1039 1049 1051 1061 1063

1069 1087 1091 1093 1097 1103 1109 1117 1123 1129

1151 1153 1163 1171 1181 1187 1193 1201 1213 1217

1223 1229 1231 1237 1249 1259 1277 1279 1283 1289

1291 1297 1301 1303 1307 1319 1321 1327 1361 1367
1373 1381 1399 1409 1423 1427 1429 1433 1439 1447
1451 1453 1459 1471 1481 1483 1487 1489 1493 1499
1511 1523 1531 1543 1549 1553 1559 1567 1571 1579
1583 1597 1601 1607 1609 1613 1619 1621 1627 1637
1657 1663 1667 1669 1693 1697 1699 1709 1721 1723
1733 1741 1747 1753 1759 1777 1783 1787 1789 1801
1811 1823 1831 1847 1861 1867 1871 1873 1877 1879
1889 1901 1907 1913 1931 1933 1949 1951 1973 1979
1987 1993 1997 1999 2003 2011 2017 2027 2029 2039
2053 2063 2069 2081 2083 2087 2089 2099 2111 2113
2129 2131 2137 2141 2143 2153 2161 2179 2203 2207
2213 2221 2237 2239 2243 2251 2267 2269 2273 2281
2287 2293 2297 2309 2311 2333 2339 2341 2347 2351
2357 2371 2377 2381 2383 2389 2393 2399 2411 2417
2423 2437 2441 2447 2459 2467 2473 2477 2503 2521
2531 2539 2543 2549 2551 2557 2579 2591 2593 2609
2617 2621 2633 2647 2657 2659 2663 2671 2677 2683
2687 2689 2693 2699 2707 2711 2713 2719 2729 2731
2741 2749 2753 2767 2777 2789 2791 2797 2801 2803
2819 2833 2837 2843 2851 2857 2861 2879 2887 2897
2903 2909 2917 2927 2939 2953 2957 2963 2969 2971
2999 3001 3011 3019 3023 3037 3041 3049 3061 3067
3079 3083 3089 3109 3119 3121 3137 3163 3167 3169

3181 3187 3191 3203 3209 3217 3221 3229 3251 3253
3257 3259 3271 3299 3301 3307 3313 3319 3323 3329
3331 3343 3347 3359 3361 3371 3373 3389 3391 3407
3413 3433 3449 3457 3461 3463 3467 3469 3491 3499
3511 3517 3527 3529 3533 3539 3541 3547 3557 3559
3571 3581 3583 3593 3607 3613 3617 3623 3631 3637
3643 3659 3671 3673 3677 3691 3697 3701 3709 3719
3727 3733 3739 3761 3767 3769 3779 3793 3797 3803
3821 3823 3833 3847 3851 3853 3863 3877 3881 3889
3907 3911 3917 3919 3923 3929 3931 3943 3947 3967
3989 4001 4003 4007 4013 4019 4021 4027 4049 4051
4057 4073 4079 4091 4093 4099 4111 4127 4129 4133
4139 4153 4157 4159 4177 4201 4211 4217 4219 4229
4231 4241 4243 4253 4259 4261 4271 4273 4283 4289
4297 4327 4337 4339 4349 4357 4363 4373 4391 4397
4409 4421 4423 4441 4447 4451 4457 4463 4481 4483
4493 4507 4513 4517 4519 4523 4547 4549 4561 4567
4583 4591 4597 4603 4621 4637 4639 4643 4649 4651
4657 4663 4673 4679 4691 4703 4721 4723 4729 4733
4751 4759 4783 4787 4789 4793 4799 4801 4813 4817
4831 4861 4871 4877 4889 4903 4909 4919 4931 4933
4937 4943 4951 4957 4967 4969 4973 4987 4993 4999

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