MENTAL MATH

HOW TO DEVELOP A MIND FOR NUMBERS, RAPID CALCULATIONS AND CREATIVE MATH TRICKS (INCLUDING SPECIAL SPEED MATH FOR SAT, GMAT AND GRE STUDENTS)

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Chapter 1: Meaning and Relevance of Mental Math

Mental math is a skill people practice on a daily basis, often subconsciously, which involves doing calculations in your head. In mental math, you don't have to write down elaborate details concerning the variables involved.

If, for instance, you intend to drive to a particular shopping center that normally takes you half an hour to walk to, you may deduce it will take you around seven minutes to drive there, considering a car is bound to move much faster than you, and the road you're going to use is unpaved. In short, to be good at mental math, you need to be able to quickly picture the relevant variables and their positions, and then make quick calculations or deductions.

Children are usually encouraged to learn mental math skills early in school, because being good at mental calculations can make a person successful in many other fields. Please note that even if being good at mental math does not necessarily signify high intelligence, people usually think it does, and that perception can help you obtain opportunities for advanced study or career development.

Also, although competence in mental math is not directly related to level of intelligence, you can only do quick, accurate calculations when your mind is alert.

Consider young people, for instance. When given a few minutes to enter a shopping mall during a school outing, they pick the items they want from the shelves and do quick mental calculations as they walk to the checkout register. By the time the cashier's total is displayed on the cash register, they will have the required cash ready in their hand. If such students stick to their allotted time during all excursions, their teachers or caretakers may be encouraged to allow them such luxuries whenever possible.

That is the same kind of advantage that adults enjoy when they are good at mental math. When eating out, for example, people enjoy the company of someone who is good at mental calculations, because, in a couple of minutes, such a person will have calculated how much everyone at the table needs to fork over, even if the meal orders are all on one bill. Conversely, people may be less than happy with someone who insists on slowly doing paper-and-pen calculations to make sure they are not being asked to pay more than their share.

Clearly, mental math proficiency is not just good for academic pursuits but also helps make life easier overall.

Main Benefits of Mental Math

1. Mental math makes the mind sharper

At the same time, there is a tendency for students' ability to understand concepts to grow when they become used to doing mental math. Students who are initially good at thinking mathematically soon become interested, and those who were already in the category of 'math people' become even faster at coming up with facts.

2. Mental math enhances comprehension as opposed to memorization

When a student learns how to calculate equations, such as 29 + 23, by using the long method where 9 + 3 is 12 and you are supposed to write 2 and carry 1, the student becomes equipped with a series of steps for handling a math problem. Although this is a very common method of doing math, and one which has been taught in schools for many years, it sometimes poses a problem of what to write and what to carry, and can get confusing.

Conversely, students learning how numbers are broken down in mental calculations learn to appreciate the value of the figures just by glancing at them. A quick glance at 29 + 23 gives the students the idea of 20 and 20, which, when added together, produces 40. In the meantime, and almost concurrently, the mind will have picked up 9 and 3 and added them to give 12. Then, quickly, the mind looks at 40 and 12 and adds 40 + 10 to get 50. Instantly, the mind adds up 50 + 2 and visualizes 52.

Essentially, as students learn and practice mental math, their minds adjust to appreciating numbers according to their real values. For example, when a student looks at 29, what comes to mind is not a combination of two numbers, but the value of 20 and more, because of the position of the 2 in 29. In short, when one is good at mental math, there is a better appreciation of the value of numbers than when one mechanically calculates individual numbers.

3. Mental math enhances logical thinking

Granted, there is room for working out math with pen and paper, using the method of carrying and adding to the next digit to the left, but if children learn mental math from an early age, you'll see them start to solve complex equations when they're as young as third grade.

In fact, children who are introduced early to mental math are able to mentally solve problems such as 543 + 362. They just pick the values of 500 and 300 and quickly get 8 as an 800 value, then pick 40 and 60 and picture 100. They add that onto the 800 to get 900, and the mind then fixes the 5 from 3 and 2 at the end. So, within a matter of seconds, a third-grade pupil can tell you that 543 + 362 = 905.

The point here is how much more practical the mental calculation method is. Because you become used to interpreting digits as values, as opposed to independent integers, it is very easy to detect when you or someone else has the wrong answer. You can even quickly figure out when the teacher has made a careless error, which less proficient students might assume to be correct.

Consider, for instance, an operation like 732 + 101. The teacher might know the answer is 631, but no sooner has she written 63 than she is distracted by a loud bang outside the classroom, where the school handyman has accidentally dropped his toolbox. Students mechanically copying off the board are likely to jot down the wrong answer, but your mind, which will have been trained to think logically, will notice there is no way subtracting 100 from 700 will have robbed 732 of more than 600. Just seeing 732 – 101 will have your mind preparing for an answer that is more than 600. Thus, looking right and sounding right both become key to ascertaining the correct answers in mental math.

Students who have been in large classes, or teachers who have handled classes that aren't used to mental math, are likely to have experienced students writing down ridiculous answers to easy questions. This is because they have gotten so fixated at the carrying business that logic is relegated to the backburner. Their method addresses the problem digit by digit, so they cannot see the overall picture. Instead, the moment they finish dealing with the leftmost digits of the equation, they move on to the next problem.

Examples of ridiculous errors are 340 – 150 = 410 or 230 + 103 = 233. Logically, how can 340 – 150 result in anything around 400? Students mentally working the math problem are unlikely to make such an embarrassing error, because they see 340 in terms of 300 and more, as opposed to a combination of 3, 4 and 0 from which to subtract other digits. This is not to say that it is impossible to make an error when doing mental math, but it is to highlight the likelihood of immediately noticing an error if it occurs.

4. Mental math makes other disciplines easier to learn

Sometimes students don't understand why certain degrees or certain universities require excellent math grades for purposes of admission. One of the main reasons is that math performance is an indication of how well you will do in other disciplines, based on your ability to think logically. Even when studying history, a subject which is fundamentally theoretical, it is helpful to have a mind that can detect when a story does not line up. For example, you might listen to a narrative about a vessel that was somewhere within the Atlantic at a given time and was then spotted at a location within the Mediterranean after a given number of days. If the ship's speed limit makes it impossible for it to have covered such a distance within the estimated number of hours, logic may lead you to question the truth of the historical account

In fact, there is no limit to the areas where mental math can be of help, including everyday situations and decision-making.

5. It is fun to engage in mental math

When you do a mental calculation and you get it right, you feel excited and engaged. Needless to say, you'll probably perform better if you are energized, as opposed to when you're bored by writing numbers in a rather mechanical manner.

Essentially, mental math has a way of making you like the subject. That helps teachers as much as students. As educators ride the wave of excitement, they automatically transmit that positive energy to their students. Consequently, math becomes exciting and math scores gradually begin to soar.

The reality is that the usefulness of mental math skills acquired in school or later does not end with exams. It extends to everyday life, at home, in careers and virtually everywhere.

How Mental Calculations Help In Everyday Life.

We are faced with many situations in our everyday lives that require quick calculations to save us both time and money. We often fail to see the tremendous benefits that come with keeping our mental math skills sharp. Let us look at a couple of scenarios to understand how these skills can come in handy when going about our daily lives.

At a store

Imagine that there's a power outage at a shop and not a single manual calculator available. There are still customers who are still waiting in line. In order to keep the line moving, the cashier needs to use his mental math abilities until the backup power kicks in. Without this skill, the store stands to lose both money and time

At the shopping mall

When you are in a shopping mall and decide to go into a shop that is advertising sales, you can figure out the discount percentages being offered without having to look at the sales price tag or having to wait for the cashier to tell you the final price.

At the gym

You can also use your math skills while working out by calculating the number of sets, repetitions and variations you need to do, as well as how long you need to last on the treadmill, for instance, in order to attain your intended fitness goal.

In the kitchen

When baking a piece of cake, you can do some basic mental math to determine how much of each ingredient you need. You can do the same thing for other recipes in the kitchen that may need tweaking depending on servings needed.

Summary

We are faced with numbers in every aspect of our lives and should use them as an opportunity to exercise our minds. This not only helps to improve GMAT test scores but also makes life easier since you don't have to worry about having a calculator available and numbers become less intimidating. You will find that, as a result of consistently practicing mental math, you make decisions faster and become more efficient all-around.

Why Speed is Important in Math

Often, teachers emphasize the importance of accuracy when doing math problems, but they rarely discuss speed. However, the reality is that speed is also important, not only because if you complete a whole math paper you'll get a better grade than if you run out of time and can't finish all the questions, but also because that speed impacts how you respond to real-life situations.

(1) The compound effect of acquiring math skills

Whenever you acquire math skills, you build on other skills which you had previously learned. This is because the subject has a cumulative effect, where no single skill is meant to serve an independent purpose, exclusive of all others. Yesterday's skills become a stepping stone to help you learn and understand new skills tomorrow.

That's why you need to go back to the beginning of a topic when you're stuck, because one missed step can make it difficult for you to understand new math skills.

Any student who hasn't mastered the ability to quickly and accurately do basic math is likely to find it difficult to grasp new math skills, especially those that are advanced, such as algebra, geometry and the like. The frustration they feel may then discourage them and lead them to decide they're not good at math, which isn't necessarily true.

The opposite thing happens when a student is comfortable with basic math concepts such as quickly and easily doing mental addition, multiplication, subtraction and division. Such students are at ease when new math skills are introduced. Thus, they are eager to learn more complex concepts, even when seeing them for the first time. The fact that they have mastered the basic skills and are able to work with them and get answers fast makes them confident that math is doable, and even interesting.

(2) Speed compensates for low aptitude

There are some people who are naturally gifted at math. However, since everyone has a level of math ability, even those whose natural aptitude is not very high can be at the top of the class once they have practiced and mastered basic math skills. Not only do such skills boost confidence but they also enable students to learn more complex skills.

(3) Speed is helpful in high schools as well as university exams

Every test has a time limit, and therefore, students who are used to working fast and accurately are at an advantage. On the other hand, students who work more slowly risk not completing their exams. This is unfortunate, especially when a student knows how to solve more problems, but just doesn't have a chance to complete them.

The only time it is reasonable for a student not to worry about speed is when learning new skills or concepts. By reducing their learning speed, they ensure they cover every step carefully, proceeding to subsequent steps only after mastering the earlier ones.

Times when speed needs to be curtailed:

If you are making careless mistakes. Careless mistakes can cost you marks.

If your work is turning out messy. Work should be neat and organized.

Summary

It is important that you learn to swiftly tackle math problems, but you should keep your work neat in order to avoid careless mistakes. The balance between speed, accuracy and neatness is regular practice. The more you practice, the more proficient you will be.

What Scientists Think About the Brain and Math

Every person is born with the ability to do math. Everyone has neural networks which enable them to do basic math and those same networks are required when processing complex math concepts. What scientists are still researching is whether the brains of math wizards have unusual capabilities. Scientists also are seeking to understand why some people (i.e., Albert Einstein, Stephen Hawking, Alan Turing and John Nash) appear to be average at math but then progress to understanding advanced and complex mathematical concepts.

In particular, scientists would like to establish whether the ability to comprehend high-level math concepts is linked to the sections of the brain that process language. If that is the case, it will mean that processing abstract mathematical concepts calls for linguistic representation and command of syntax (how words and phrases are organized to form correct, coherent sentence structures). Scientists also want to know whether, instead of math skills being associated with the language processing areas of the mind, they depend on regions of the brain that are independent and deal specifically with numbers and/or spatial reasoning.

Research findings by France's INSERM-CEA Cognitive Neuro-Imaging Unit indicate that the areas of a person's brain responsible for solving math problems are different from the areas of the brain that deal with complex problems of a non-mathematical nature. These findings are detailed at the National Academy of Sciences.

INSERM-CEA's experiment was interesting and revealing. The team of researchers used a functional magnetic resonance imaging, otherwise known as fMRI, to scan the brains of fifteen professional mathematicians. They also scanned the brains of fifteen non-mathematicians whose academic standing was as high as that of members of the other group. While being scanned, both groups were made to listen to 72 complex math statements in a series, which had segments from algebra, topology, analysis and geometry. They also listened to 18 complex statements that were non-mathematical, most of them of a historical nature.

The research subjects had four seconds to ponder each statement and respond with one of three answers: true, false or meaningless.

The researchers observed that when the subjects listened to a statement that was exclusively of a mathematical nature, there was a section of the brain that was activated, comprising a network that involved bilateral intraparietal and inferior temporal regions as well as the dorsal prefrontal area of the brain. The scientists noted that those areas comprised a circuit usually not linked with brain sections which deal with the processing of language, including semantics.

When both the mathematicians and non-mathematicians listened to the non-mathematical statements, regions of the brain that normally deal with language processing were activated. There are some regions of the brain, one of the researchers, Marie Amalric, said, which have always been associated with knowledge of numbers and space, and those are the areas of the brain that are activated when a person reflects upon something mathematical.

Earlier research indicates that non-linguistic regions of the brain, activated when mathematical statements are heard, become activated even when the calculations involved are very basic. It does not matter if you are required to do simple arithmetic or just look at some written numbers. Those sections are activated in the same manner as when you are presented with complex mathematical problems. That is a clear indication that there is a relationship between processing basic mathematical concepts and processing advanced mathematical concepts.

According to one researcher, Stanislas Dehaene, some people are born possessing an intuitive sense of arithmetic and quantity manipulation, otherwise referred to as number sense, and that is closely associated with spatial representation. However, scientists have not found the real connection between concrete number sense and ability to do high-level math. One indisputable thing, though, is that every person has a natural ability to recognize basic differences in quantities. For example, even a child who has not gone to school can tell that two balloons are more than one.

That is the natural ability you need to capitalize on to improve your proficiency in quickly and efficiently tackling mental math problems.

The Uniqueness of Mental Math

In practicing mental math, you do not need a calculator, computer, pencil, pen or any such equipment. You also don’t need any paper. All you need is yourself. Nevertheless, to be really good at mental math, which requires that you provide accurate figures, good estimates to questions given or realistic solutions to instant challenges, you need to be good at arithmetic, particularly multiplication and division.

Why Some People Find Mental Math Difficult

Some people have difficulties mastering mental calculations no matter how much they practice, due to a brain disorder known as dyscalculia. Other people are not good at mental calculations because they have a problem focusing. This could be a problem which affects them in other aspects of their life.

Some Signs of Dyscalculia

Some people may not recognize numbers once their normal order is altered.

Some people may not comprehend the concepts underlying a math problem.

Some people may find it hard to round up or to understand which rounded number is appropriate for a certain math problem. For example, such a person may not think of 120 as a reasonable answer as opposed to 92, when asked for an estimate answer for 90+22.

Some people are unable to tell, at a glance, which quantities are greater or smaller, for example 66 vis-à-vis 34.

Some people who count on their fingers, as opposed to doing mental calculations, may not appreciate that it is easier to add a small number onto a bigger one than to add a big number to a smaller one. For example, even if you’re using fingers to count, when given 6+4 to calculate, it is easier to add the 4 to the 6 in order to get the answer of 10. The calculation becomes shorter and takes less time than if you count 1 to 4, then 1 to 6 and then put them together and once more count 1 to the end to find 10.

In serious cases of dyscalculia, a person may even mix up the idea of a minute comprising 60 seconds and a dollar comprising 100 cents.

Without such problems, there are ways one can master mental math, even for people who normally think mathematics is hard. There are skills that you can master so that, through practice, mental math ends up coming naturally to you.

The ability to do basic arithmetic in your head is only one of the skills required to succeed in mental math. Other skills that you need include the ability to quickly round up numbers. For example, if someone asks you to add 299+199, why would you need to pick up a pen and paper to add the units and then carry the tens to the other tens on the second column and so on?

Instead, you can do the calculation mentally by rounding off 299 to 300 and 199 to 200. And since, at a glance, you can see you added an extra 1 to 299 and an extra 1 to 199, you simply subtract the two units from your answer. Your mental formula will, therefore, be 300+200-2 = 498.

To enhance your mental math skills, it is important to remember number-related concepts and to retrieve mathematical facts. You need to remember the sequential steps required in order to solve certain types of mathematical problem.

Parents can help their children find some easy ways of obtaining solutions through simple patterns. For example, in a math question that requires adding 9 to another number, simply substitute the 9 with a 10, so that you will be adding a 10 instead of a 9 in the equation. Immediately after this addition, subtract one from the answer.

As you will soon see, dealing with a 10 in math calculations is always easier than dealing with other numbers. When you are given 17+9, for instance, you will be faster if you mentally visualize 17+10 because 27 will automatically come to your mind as the answer. After this, subtracting the 1 you added will take just a second, and you'll have your answer, 26, in moments.

Teach your child other simple math patterns such as multiplication by 5, where the answer either ends with 5 or 0. Show your child that if such calculations are in a sequence of 5, the answers will automatically alternate in ending with 5 and 0. For example: 5 x 5 = 25; 5 x 6 = 30; 5 x 7 = 35; 5 x 8 = 40 and so on.

How ADHD Adversely Affects the Ability to Do Mental Math

ADHD stands for attention deficit hyperactivity disorder, and it is a disorder of the brain that makes it difficult for a person to remain focused on a subject or task. Although the disorder usually affects children, sometimes a person can suffer the condition into adulthood. One of the symptoms of ADHD is hyperactivity and impulsivity in children. The working memory of a person with ADHD tends to operate below par.

The connection between ADHD and mental math is that someone with ADHD is not able to relate newly-learned skills with those learned before. Yet, to be proficient in mental math, one needs to utilize basic skills built upon one at a time.

It has also been observed that people with ADHD often have a problem committing information to long-term memory, so there are gaps in basic skills. This is problematic because in math, many skills are interrelated. Even when rounding up numbers, for example, there are times subtraction is easier than addition and vice-versa, and if someone is only good at one of the skills, such a person will be at a disadvantage. Take, for example, 11+32. It is easier and faster to consider the numbers as 10+30 and then add the result of 1+2. This give you 40+3, which gives you 43 in just a few seconds. To round 11 and 32, you rounded your figures down. In other words, you subtracted the units to get nice rounds of tens.

Notice how much more complicated it would have been if you rounded off the two numbers by adding some units. To 11, you would have to add 9, and to 32, you would have to add 8. Adding 20+40 would then give you 60. You then would have to add the two big units, 9+8, and then subtract the answer from 60. This option is, obviously, not making your mental math any easier. Thus, mental math gets complicated for someone who has gaps in their basic skill set .

Someone who has ADHD may also find it hard to recall facts and procedures, and in mental math there are some facts one needs to remember. Some helpful facts are simply stored in the memory in the manner in which they were first learned, while others are stored in short forms, referred to as mnemonics.

One example is SOHCAHTOA, a mnemonic that helps a lot in trigonometry. It helps you remember the formulas for calculating angles of a triangle once you have some details provided. SOHCAHTOA stands for:

Sine θ = opposite divided by hypotenuse

Cosine θ = adjacent divided by hypotenuse

Tangent = opposite divided by adjacent.

Opposite, adjacent, and hypotenuse are the three lines of the triangle named in relation to angle theta (θ).

There are other challenges that children with ADHD encounter when trying to learn and practice mental math, and although such children are in the minority, it is important to be able to identify them so that they can be helped in other ways.

Why Mental Math is Precious

If you are still wondering how helpful it is to be adept at mental math, consider a musician who looks at some music notes when seated before a piano. After many months or years of sight reading, just a glance will make it clear what kind of melody the notes will produce. A good musician, who has had lots of practice, can also quickly tell, by looking at a given key signature, the combination of music notes and chords involved, noting those that are bound to be sharp, natural or just flat.

Likewise, once you have learned the fundamentals of math, you can sight-read and recall the relevant formula or sequence for required calculations. Sight reading is also employed in speed reading. Just as you are able to read many words per second, you will be able to do mental math just as quickly. Incidentally, the reason you are able to read fast is that you pick up a chunk of the text at once, which is equivalent to picking up an image. Presumably, then, you are able to visualize mathematical formulas and facts fast because when you look at a situation or listen to it explained, you are able to quickly retrieve images of relevant formulas and facts from your memory.

Once these mental calculation aids become obvious to you, your mind is left with enough room to process more complex, and more difficult, mathematical problems. One cognitive scientist, Daniel T. Willingham, wrote a book on the reason students do not particularly like school. He emphasizes the limitation of the working memory, the short-term memory employed in thinking about concepts and applying them. Willingham says it is important to think of this memory as one tiny room where you store knowledge. As you add more knowledge, the free space becomes smaller.

However, once you have mastered the skills of mental math, you will not require anymore space for every new mathematical problem you want to solve. This is because you will not consume lots of your short-term memory assembling new information and processing it. Instead, you will quickly do your mental calculations. Therefore, experts affirm that when you are good at mental math, you leave your short-term memory free to handle more complex issues.

From years of study, experts have noticed that people who are great at mental math are also:

(i) Focused

The reason they are focused is because large numbers do not overwhelm them. Instead, once they look at a math problem, they identify the logic in the particular problem, and then use the mental skills they've already mastered to solve the problem.

(ii) Efficient

A person proficient in mental math is able to do calculations faster, and that ability leaves them with ample time to tackle difficult problems.

(iii) Bold

People who are great at mental math, especially students, become more confident in their abilities, even in other fields. Sometimes this happens because of the response they receive from other people who are impressed by their ability to do calculations quickly and accurately.

A good deal of practice is required in order to be able to master mental math, which mostly involves addition and subtraction, multiplication and division. However, it is important to guard against over-drilling. The reason, as experts like Willingham have mentioned, is that over-drilling can gradually drain you of any motivation to learn, especially as a student using mental math as a basis for greater challenges. To succeed in math, and everything else in life, you need sufficient motivation.

Chapter 2: Different Ways You Can Polish Your Mental Math

Calculators are useful , obviously. However, there is a time for everything, and if you are going to reach for a calculator every time you want to do a simple math problem, you will quickly get frustrated.

For instance, you may want to find out how many watermelons are due to each of your 3 friends when you have 48 watermelons to share among yourselves equally. If the 4-times, probably learned from your multiplication tables, have escaped your mind, do a quick mental division by 2 and again by 2. Thus, easily, you will know how many watermelons to allocate, because it is easy to divide mentally by 2. And of course, 4 is simply 2 times 2. The answer in this watermelon-sharing case is 12. No need for a calculator!

As you learn the basic techniques of mental math, it is important to adopt a mindset that mental math is not difficult, and that anyone can master it. From that standpoint, you will find it easy to appreciate small tricks as they are laid out, and gradually, you will be amazed at how calculations that were seemingly difficult become easy.

Easy Rules to Help You Crunch Numbers Mentally

Do additions and subtractions beginning left, going right

Why should you change the convention of beginning on the right end and moving to the left as normally taught in elementary school? Remember the routine of adding the ones, then carrying the extras to add to the tens and so forth? Well, the conventional method taught in basic math is fine when the math problems are being tackled in writing, so you work with one number at a time.

However, when doing mental sums, you need to visualize the entire value of the number at once. Hence, you want to see 28 as a value that's almost 30, and not a number with 8 ones and 2 tens. Once you visualize 28 as a value much greater than 20, you will appreciate that it is more logical to round it up to 30 and not down to 20 when doing your mental math. In short, the reason the rule says you begin adding or subtracting from the left is so that you can figure out the greatest value instantly.

This understanding can help you solve practical problems extremely fast. If, for instance, boxes of books are being distributed to colleges so that each student gets a copy, and each box contains 100 books, by the time 10 boxes are offloaded from the truck, you can tell that a particular college has 3,000 students. How? You quickly add a zero to 100 to get 1,000, and you know that 1,000 is far less than 3,000. You can make a similarly good deduction when you notice there are 22 offloaded boxes, at which time you will instantly realize 2,000 students have been catered for. At this juncture you will know that with just a few more boxes offloaded from the truck, there will be enough books for 3,000 students.

Overall, counting your numbers from the left makes the grasp of numerical values more intuitive and faster to work with.

Take it easy

Although you are aiming at being able to do math calculations in seconds, do not put too much pressure on yourself. If you feel overwhelmed when doing mental math, you are likely to be completely derailed and miss even the easiest of the calculations.

If you have a number that looks rather large, break it down before you begin to work with it. Say you have 720+290. Simply subtract 20 from 720 to remain with 700, and when you have 700+290 your mind will easily picture the answer as 990. From there it is easy to figure out what to do about the 20 you removed from 720. You can quickly split it into two, and you will use one of them to top up the 990 answer to 1,000. Then you will leave the other ten on top so that you have 1,010 as your answer. Once you break down your large numbers in this manner, you will be confident of getting the correct answer, and although the method may appear a little lengthy, with practice, you will soon find yourself doing such mental calculations in seconds.

This simplification can also be applied to multiplication. Take a problem such as 59 x 7 as an example. Simply add 1 to 59 to get a round figure, 60, and then proceed to do the mental multiplication. 60 x 7 = 420. Since you have multiplied an extra 1 by 7, subtract 7 from 420. 420 – 7 = 413.

Remember that becoming good at mental addition, subtraction and multiplication will help you tackle math problems that are more complex, including those involving division. So, these skills are not entirely an end to themselves.

Adopt some fancy multiplication tricks

These multiplication tricks can make the difference between completing an examination paper and not finishing your exam on time. Consequently, such tricks can make the difference between passing and failing a test. In short, when you know some interesting tricks to use in multiplication, not only will you find it easy to do multiplication calculations mentally, but you will also find them easy to remember. Such tricks will be tackled more in depth later in the book, but for now, we'll look at a few examples.

(a) Multiplication by 10

When you are given a number to multiply by 10, simply add a zero to it and that will be your correct answer. For example, 70 x 10 = 700; 107 x 10 = 1,070; and the trend continues. Later in the book, we'll explore multiplication tricks involving other numbers.

(b) Multiplication by 5

The answers you get from multiplying a number, with 5 however big or small, will always end either with 5 or 0.

(c) Multiplication by 12

When called upon to multiply a number by 12, go the easy way of using 10 times. Then you can separately handle the smaller number on top, specifically 2. For example, when calculating 4 x 12, do the split in this manner:

4 x 10 = 40

4 x 2 = 8

40+8=48

In short, 4 x 12 = 48

(d) Multiplication by 13

When you want to calculate 13 x 4, split 13 so that you have 10 and 3 to multiply separately.

4 x 10 = 40

4 x 3 = 12

The next step is to solve 40+12, and you have already learned the easiest way to do such an addition, counting from the left.

So, 40+10+2 = 52. Again, the explanations may be lengthy, but that is simply for the sake of clarity. However, once you practice, these mental math tricks will come to mind automatically and easily.

(e) Multiplication by 15

Here is another one of those instances when you can use the 10 times trick. Looking at 15, you can see it is made up of 10 and 5. So you can multiply any number by 10 first. Then you multiply that same number by 5.

But hold on. 5 happens to be half of 10. Why tire yourself dealing with 5 when you can simply multiply the number involved by 10 again and then divide the answer by 2? Then you simply add the two products that you get. Here is an example:

6 x 15

6 x 10 = 60

6 x 10 = 60; 60 ÷ 2 = 30

60+30 = 90

In short, you have simply and easily established that 6 x 15 = 90.

(f) Multiplication by 16

16 may appear to be much more complex to work with than numbers like 5, 10 or 11, but there’s also a trick to work with it. Suppose you have been given a number to multiply by 16. How do you do your calculations quickly, accurately and without lots of writing?

Begin by multiplying that number by 10. Then take half of that same number and multiply it by 10. Now add those two products. Finally, add that same number to the sum you found.

Here is an example:

4 x 16

4 x 10 = 40

4 ÷ 2 = 2; 2 x 10 = 20

40+20+4 = 64.

In short, 4 x 16 = 64.

This trick will work wonders for you, irrespective of how big the number you are multiplying by 16 is. Take a math problem like 36 x 16, for example.

36 x 10 = 360

18 x 10 = 180

360+180+36 = 576

Therefore, 36 x 16 = 576

You multiplied 36 by 10 instead of multiplying it by 16 straightaway, then you halved it and multiplied the product, 18, by 10, and then you finally added together your two products, 360 and 180, and added the number itself, 36, to the result to establish your final answer.

(g) Use Squares to Make Your Multiplication Easier

You might be wondering if using squares will complicate matters, considering that a square is a number multiplied by itself. Granted, squares are often more complex than simple numbers, but in this case you are required to memorize the squares of numbers from 1 to 20. It does not matter if you are going to use a calculator to find out what those squares are. Once you know them by heart, they will go a long way in helping you do complex multiplications.

Here is how you go about it:

You have two numbers, x and y.

What is the average between these two numbers? It is (x + y)/2.

So if your problem is 10 x 4, you are going to find the average of 10 and 4. You will, therefore, have (10+4) ÷ 2, which gives you 7.

What is the square of that difference? In this case, 7 x 7 = 49. Does that really give you the answer to 10 x 4? No, it does not, but your simple 10 times tells you that 10 x 4 is simply 40, so you can tell that the square you found is a little far off.

How do you rectify that problem, or rather, what is the next step you take to ensure your method gives you the correct answer?

Find the difference between each number and the average of the original numbers.

Remember that the average was 7, and your two numbers are 10 and 4.

10 - 7 = 3

7 - 4 =3

Don't pay any attention to the order of your numbers in these calculations, because it will not matter whether your answer is positive or negative.

Now, square that difference. In our case, 3 x 3 = 9.

Go back and subtract 9 from 49. The answer you get, in this case, 40, will be the answer to the multiplication problem.

You have just used squares to find 10 x 4. It is true that you could have solved the problem faster by just adding a 0 to 4 to get 40, but using a simple calculation helps you understand the concept of using squares, which can come in very handy when you are faced with multiplications of more complex numbers.

In summary, you established the average of the two numbers, and then you established its square. You then established the difference between the average and each number, which happened to be the same for each of your two numbers. You then established the square of that difference and subtracted it from the resulting square of the numbers' average.

Squares of 1 to 20

An example involving bigger numbers:

Use squares to find out 15 x 11.

Your two numbers are 15 and 11. What is their average?

It is (15+11)/2 = 13

What is 13 x 13? It is 169.

Remember that by now you will have memorized the squares of numbers between 1 and 20.

What is the difference between 13 and 15? It is 2.

What is the difference between 13 and 11? It is 2.

Now square 2; 2 x 2 = 4

Subtract 4 from 169; 169-4=165

You are correct to say that 15 x 11 = 165.

With practice, you will realize this method does not tire your mind, and you will be certain your answer is correct even when your calculations are fast.

NB

The fact that we are dealing with mental math does not mean you cannot scribble something down. Like in our just concluded example, you can note down 169 as the number to come back to after you have squared your difference.

Experts say it is fine to do an approximation when dealing with mental math. In real life, you can benefit from making an informed estimate. During WWII, when the US embarked on a bomb-making project dubbed the Manhattan Project, one of the physicists, Enrico Fermi, is said to have sought a rough estimate of the ultimate power of the atomic bomb, even before the team could get the diagnostic data. During testing and from a safe distance, the scientist dropped some pieces of paper in the way of the blast wave. Seeing how far the pieces of paper traveled, he deduced that the strength of the bomb blast would be around 10 kilotons of TNT. In reality, the bomb blast strength ended up being 20 kilotons of TNT, which made Fermi's estimate reasonable.

Approximation Is Acceptable and Helpful In Mental Math

There is a term given to close approximations, adopted from the name of the aforementioned Manhattan Project physicist, Enrico Fermi—the Fermi Estimate. As such, if you estimate the number of students in a given state to be 10,000,000 and then you estimate the average number of students per school to be 1,000, it is easy to tell that the number of head teachers in the school is around 10,000. Although the answer you get from the Fermi Estimate may not be exact, it will be close enough to help you in your decision-making.

(h) Rearrange your equation for simplicity

Don't be overwhelmed when you see an equation that looks complex. You can always rearrange it in a way that is simple and easy to solve.

Here is an example:

5 of (14+20+43)

Following the basic order of operations where you begin by removing the brackets and then the 'of' in the equation, you are required to expand the equation in this manner:

5 of (14+20+43) = (5 x 14) + (5 x 20) + (5 x 43); better still:

(5 x 14) + (5 x 20) + (5 x 40) + (5 x 3)

Ensure you take advantage of simple tricks you have already learned, and your calculations will become easier every time. In our case:

5 x 14 can be (5 x 10) + (5 x 2 x 2) = 50+20 = 70

5 x 20 can be 5 x 10 x 2 = 100

5 x 40 can be 5 x 10 x 4 = 200

5 x 3 = 15

So, 5 of (14+20+43) = 70+100+200 = 385

As has been explained before, this elaborate writing is for the sake of explaining the procedure clearly, so that even those who are put off by huge numbers or complex equations can see it is possible to break them down to simple numbers that are easy to work with. Otherwise, when it comes to actual calculations, you end up capturing each part of the simplified equation as an image, and you can always note down the answer somewhere to await the result of subsequent mental calculations.

The secret to handling complex mathematical problems lies in being able to break them down into smaller problems. In short, as you solve each sub-problem, you solve one portion of the problem and soon you have the entire problem sorted out.

Chapter 3: How to Multiply Big Numbers Mentally

You may have heard the cliché that some people are simply good in math. While it is true that some people solve mathematical problems like magic, a good number of them have simply learned tricks that help them be fast and accurate. Even in ordinary life, if you know just one route to a location, you are likely to be stranded if there's suddenly an obstacle like a fallen tree branch and you don't have an alternate route in mind. However, if you know a different route (or a shortcut, which is even better), you can easily redirect and reach your location.

How to Multiply Big Numbers by Rounding Them Up

You can round both numbers up to have easy numbers to work with.

Let us try 97 x 96.

97 becomes 100 after you have added 3 onto it

And 96 becomes 100 after you have added 4 onto it

100 x 100 = 10,000

How to sort out the rounding inaccuracy:

Deal with the difference by which you increased the numbers first.

You added 3 to 97 to get 100 and 4 to 96 to get 100.

(3 x 100) + (4 x 100) = 700

Therefore, subtract 700 from 10,000.

10,000 - 700=9,300

However, you are not yet done with the calculation.

Take the 3, multiply by 4 and then add the product to 9300.

9,300 + (3 x 4) = 9,300+12=9,312

Can we recap that?

(i) We rounded each number in the equation up, and then we multiplied the resultant numbers.

(ii) We then took the figure we added to each number and multiplied it as we did with the bigger number (e.g. 3 x 100 and 4 x 100 above)

(iii) We then subtracted the total from the product of the large rounded numbers (for instance, 10,000-700).

(iv) Finally, we took each figure that we added in the rounding process and multiplied them together, and then we added that product to the net rounded figure; in our example, adding 3 x 4 to 9,300, which gave the result of 9,312.

Exercise 1:

(a) 99 x 23

(b) 86 x 37

(c) 119 x 99

(d) 255 x 19

(e) 1098 x 99

Additional Questions to Make the Exercise Longer

Quick Way of Doing 9 Times

This is unbelievably easy. When you want to know what 1 x 9 or 2 x 9 up to 10 x 9 is:

Write down your equations.

Begin writing answers in sequence: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Do not stop there. From the bottom this time, write the same sequence of 0, 1, 2 up to 9.

Look at the column you have containing the answers to the 9 times. It will show that 1 x 9 = 9 and so on, until you get to 10 x 9 = 90. Your 9 times table will look like this:

Of course, 9 x 11 = 99, because numbers double when multiplied by 11.

Essentially, therefore, the only solution you need to ensure you recall is 108, which is the product of 9 x 12.

How to Conveniently Add and Subtract Fractions the Butterfly Way

Say you have a problem like ¾ plus ^{2} / 5 .

Step 1:

Take the numerator of your first fraction and multiply it with the denominator of your second fraction.

Take the numerator of the second fraction and multiply it with the denominator of your first fraction.

Now add the two products.

In ¾ + ^{2} / 5:

Multiply 3 by 5 and you will get 15.

Then multiply 2 by 4 and you will get 8.

Now add 15 and 8 and you will get 23.

Step 2:

Look at your original fractions and multiply their denominators. In our current problem:

4 x 5 = 20

Step 3:

Now divide the first product with the second product. In this instance, you will have:

^{23} / 20 whose result is ^{13} / 20 (one and three twentieths).

Subtracting by the butterfly method:

When you have ¾ - ^{2} / 5 :

3 x 5 = 15

2 x 4 = 8

15 - 8 = 7

5 x 4 = 20

So your answer is 7/20.

Have you noticed what we have done?

We began by multiplying the numbers of the fractions diagonally just as we did in the addition problem.

Then we subtracted the second product from the first product.

We then multiplied the denominators of the two fractions.

Then we made a fraction out of the two products with the result of the denominators serving as the denominator of the resulting equation.

Math Skills in Multiplying By 11

The 11 times fall among the easiest to remember. When you consider 2 times, 5 times and 10 times easy, 11 times is in that list. However, people are usually very comfortable up to 9 x 9 because they rely on the principle of doubling the number being multiplied by 11.

Examples:

33 x 11 = 33

7 x 11 = 77

9 x 11 = 99

People also find it easy to tell what 10 x 11 is because any number multiplied by 10 only requires a zero at the end to get the answer. As such:

10 x 11, which is the same as 11 x 10, = 110

What, then, makes 11 x 12 easy, since it is also not considered so difficult?

Easiest Way To Multiply A 2-Digit Number By 11

Consider those two digits to be apart, leaving space for one digit in between, if you have 11 x 11, consider the answer to be something like 1_1. So now you have a problem of filling in the gap.

Add the two digits that you have. In this case, 1+1 will give you 2.

Hence, 11 x 11 = 121.

This is how you get 12 x 11:

12 x 11 = 1_2

Once you add 1+2, which are the two digits that make up 12, you get 3.

Hence, 12 x 11 = 132.

Tricks to Help Multiply Big Numbers by 11

(i) 35 x 11

You first have 3_5.

Then, when you add 3+5, you get 8, which you insert in the gap.

Hence, 35 x 11 = 385

(ii) 31 x 11

You first have 3_1.

Then, when you add 3+1, you get 4, which you insert in the gap.

Hence, 31 x 11 = 341.

Exercise 2

(a) 81 x 11 = ________

(b) 72 x 11 = ________

(c) 15 x 11 = ________

(d) 22 x 11 = ________

(e) 26 x 11 =_________

(f) 33 x 11 = _________

(g) 18 x 11 = _________

(h) 34 x 11 =_________

(i) 24 x 11 =_________

(j) 36 x 11 =_________

(k) 41 x 11 =_________

(l) 52 x 11 =_________

(m) 63 x 11 =_________

(n) 30 x 11 =_________

(o) 45 x 11 =_________

(p) 71 x 11 =_________

(q) 70 x 11 =_________

(r) 62 x 11 =_________

(s) 61 x 11 =_________

(t) 80 x 11 =_________

Easiest Way To Multiply A Big 2-Digit Number By 11

Sometimes you may have a number to multiply by 11, whose two digits give a sum of two digits. An example of such a number is 47, where you have 47 x 11. How do you proceed?

(i) 47 x 11

Begin as usual, spacing out the two digits to leave a space of one more digit. In this case, you will have 4_7.

However, when you add 4+7, you get 11, a number that cannot fit in the gap.

Write the 1 representing tens in the gap you have, then add the 1 in the tens to 4, to make it 5.

Now you have 517.

In short, 47 x 11 = 517, and the arithmetic is mentally doable.

Another example of two big digits:

(ii) 58 x 11

As usual, begin by spacing out your two digits, so that you have 5_8.

5+8=13.

Insert 3 in the gap.

Add 1 to 5 to make it 6.

Now you have 638.

Therefore, 58 x 11 = 638

Exercise 3

(a) 39 x 11 =

(b) 46 x 11 =

(c) 38 x 11 =

(d) 29 x 11 =

(e) 28 x 11 =

(f) 48 x 11 =

(g) 49 x 11 =

(h) 56 x 11 =

(i) 65 x 11 =

(j) 66 x 11 =

(k) 73 x 11 =

(l) 67 x 11 =

(m) 78 x 11 =

(n) 69 x 11 =

(o) 74 x 11 =

(p) 76 x 11 =

(q) 68 x 11 =

(r) 83 x 11 =

(s) 93 x 11 =

(t) 87 x 11 =

Easiest Way To Multiply A 3-Digit Number By 11

Suppose you have 243 x 11 . How would you deal with the problem without using a calculator?

(i) 243 x 11

Write down the first digit, in this case, 2.

Leave two spaces after the first digit and write down the last digit; in this case, 3. Now you have 2 - - 3.

To fill in the first gap from the left, add the first two digits of your original number. In this case, you need to add 2+4, which will give you 6.

When you insert 6 in the first gap, you will have 26-3.

Now add the last two digits of your original number, without caring that the middle number is being used a second time. In this case, you will have 4+3=7

Insert the 7 in the remaining gap (the second gap), and your number becomes 2673.

In short, 243 x 11 = 2673, and you have solved the problem mentally, without the aid of a calculator.

(ii) 123 x 11

Write down 1- - 3

1+2=3. This 3 is meant to fill in the first gap in 1- - 3.

2+3=5. This 5 is meant to fill in the second gap in 1- - 3.

Your complete figure should, therefore, be 1353.

In short, 123 x 11 = 1353.

Exercise 4

(a) 263 x 11 =

(b) 114 x 11 =

(c) 621 x 11 =

(d) 261 x 11 =

(e) 101 x 11 =

(f) 144 x 11 =

(g) 218 x 11 =

(h) 416 x 11 =

(i) 209 x 11 =

(j) 812 x 11 =

(k) 804 x 11 =

(l) 723 x 11 =

(m) 727 x 11 =

(n) 711 x 11 =

(o) 909 x 11 =

(p) 634 x 11 =

(q) 544 x 11 =

(r) 271 x 11 =

(s) 362 x 11 =

(t) 111 x 11 =

When You Need to Carry in 11 Times

A different situation arises when you are multiplying a number by 11 and, in one of the adjacent digits, the sum itself has two digits. For instance, in a case where you need to add 4+7, which gives you 11. What you do in such a situation is write down the ones in the gap; in this case, 1. Then carry the tens; in this case, adding 1 to the digit on your left.

Example

147 x 11:

As before, write down 1 - - 7, because the first and last digits of your number must be the first and last in your product.

Add 1+4 = 5. So now you have 15 - 7 in your mind.

Add 4+7 = 11. Here is where you write the ones down and carry the tens. You will end up with 1 filling the remaining gap, and 5+1 so that the 5 you had in the second position changes to 6.

In short, 147 x 11 = 1617.

Another Example

239 x 11:

Begin by visualizing 2- - 9

Add 2+3 and you get 5, which is meant for the second position in your product.

Add 3+9 to get the digit for the third slot. 3+9=12.

Therefore, in the third slot you will insert 2

Since you will carry 1 and add it to the 5 you had before, your second digit will be 6 and not 5.

239 x 11 = 2629

Exercise 5

(1) 174 x 11 =

(2) 182 x 11 =

(3) 148 x 11 =

(4) 277 x 11 =

(5) 267 x 11 =

(6) 264 x 11 =

(7) 357 x 11 =

(8) 275 x 11 =

(9) 368 x 11 =

(10) 349 x 11 =

(11) 337 x 11 =

(12) 329 x 11 =

(13) 369 x 11 =

(14) 419 x 11 =

(15) 429 x 11 =

(16) 438 x 11 =

(17) 447 x 11 =

(18) 459 x 11 =

(19) 519 x 11 =

(20) 728 x 11 =

When The Carrying Affects The First Digit

We have already dealt with instances when adding the middle number to the last digit produces two digits, making it necessary to carry one, a move that increases the second digit in your answer. How about instances when adding the middle digit to the first number produces two digits? How are you supposed to handle that so that the 11 times becomes fast and accurate?

If you add the middle digit to the first number and you get a sum with two digits, write down the digit in the ones position, and then carry the one in the tens position. Add the one you carry to your first digit, which means, for a change, that your answer to the 11 times will not start with the original starting number.

Example

372 x 11:

Ordinarily you would start like this: 3- - 2

However, 3 + 7=10

So, you are required to write 0 in the first gap and add 1 to the starting 3 to make it 4.

You now have 40 - 2.

7 + 2=9

So your product will be 4092.

In short, 372 x 11 = 4092.

Another Example

374 x 11:

The normal beginning for this trick is 3- - 4

However, 3+7 = 10

So you need to write 0 in the first space and carry one.

3+1 then becomes 4, so that you now have 40 - 4.

To fill in the second space, you calculate 7 + 4 = 11.

As usual, here you need to write down 1 to fill in the gap, and then carry 1.

When you add the 1 you carried to 0, you get 1.

Your complete figure is, therefore, 4114.

As you can see, you can be called upon to carry in two instances of adding digits. You just need to be alert so that you remember the digit you are adding the carried number to. Is it the original digit or a new one you have acquired after carrying once before?

Another Example

479 x 11:

As usual, visualize 4 - - 9.

Now do 4 + 7 = 11

You can fill the first gap at this juncture thus:

Carry 1 so that 4 + 1 = 5.

The other one is left to fill the gap.

You now have your incomplete answer as 51 - 9.

You now need to fill the second gap, so you do 7 + 9 = 16.

Again, here is a case where you need to carry something.

Write down 6 and carry 1

Since your incomplete answer is 51 – 9, 6 will fill in the gap.

Then you add the 1 you carried to the second digit, 1, to get 2.

Your final figure will now be 5269.

Hence, 479 x 11 = 5269.

Remember, the process looks long only because the explanation has been simplified for any person to follow. Otherwise, with just a little practice, you will be able to very quickly and accurately do the 11 multiplication mentally. With more practice, such math tricks will remain in your memory for a long time.

Exercise 6

(a) 190 x 11 =

(b) 392 x 11 =

(c) 381 x 11 =

(d) 475 x 11 =

(e) 483 x 11 =

(f) 477 x 11 =

(g) 582 x 11 =

(h) 732 x 11 =

(i) 831 x 11 =

(j) 851 x 11 =

(k) 883 x 11 =

(l) 828 x 11 =

(m) 568 x 11 =

(n) 662 x 11 =

(o) 670 x 11 =

(p) 191 x 11 =

(q) 292 x 11 =

(r) 391 x 11 =

(s) 731 x 11 =

(t) 733 x 11 =

Chapter 4: The Importance of Pi and How to Remember It

Did you know that many people who accomplish complex mathematical problems have simple formulas at their fingertips? Many others who are great at math happen to have mastered the simple tricks that help remember such formulas.

Some of these tricks come in the form of mnemonics, where you have terms like BODMAS and SOHCAHTOA reminding you of the correct sequential order of calculations or the relevant formula to use to solve the problem at hand. BODMAS stands for Bracket of Division, Multiplication, Addition And Subtraction and it reminds you what order to tackle your calculations in. SOHCAHTOA has already been explained elsewhere in this book.

Some people feel there is a disconnect between such formulas and real life, or even between them and their future careers. They think, why should I bother memorizing a formula I will hardly use outside of the math classroom? However, the reality is that virtually every formula learned in a math class can help solve problems in other spheres of life.

When students learn velocity in school, for example, it may sound quite abstract. Nevertheless, the concept is relevant in astronomy, in road traffic matters, in athletics, and in many other fields. This is the case with Pi, where students may imagine they only need to be able to work with Pi in order to pass their math exams. Yet Pi is useful in other fields.

What, exactly, is Pi?

Simply put, if you have a circle, the ratio of its circumference to the circle’s diameter is what is termed Pi. The actual value of Pi is 3.14159265359. This value remains constant, and a constant figure comes in handy whenever there is a challenge involving calculations.

Where in Life is Pi Helpful?

Besides geometry and trigonometry, Pi also comes in handy in other areas such as science. In trigonometry, it helps in establishing the value of sine, cosine and even tangent, and outside of a math lesson, such values are not an end to themselves. They help make other important deductions. In fact, they can come in very handy when designing a bridge.

In the industrial and related sectors, experts can use Pi to assess the circular velocity of mobile equipment such as the wheels of a truck, motor shafts, parts of an engine, vehicle gears and the like.

In electronics, Pi is used in calculating voltage between a capacitor and a coil. There is even a Pi Filter, which helps in the control of frequency, which, in turn, reduces noise that is essentially a voltage ripple.

In other fields of science, Pi is used in formulas that help to solve problems of a statistical nature and others relating to mechanics, thermodynamics, cosmology and others. Even in computer science, Pi is handy for evaluating the computing power and efficiency of a computer.

In nature, Pi can be useful to measure various waves such as ocean waves, sound waves and light waves, radioactive particle distribution and various other important aspects of nature.

What, Essentially, Is Trigonometry?

Trigonometry is one branch of mathematics that often utilizes Pi, and it basically stands for calculations involving triangles. It is imperative for students and other people trying to improve on their math skills to know that trigonometry is very valuable in real life. That way, they will be motivated to learn all the tricks there are to enhance their math competence, mental and otherwise.

How Trigonometry Helps in Real Life

The kinds of calculations you do in trigonometry involve lengths and heights, as well as angles. The triangles that bear the angles you calculate are of different types. Word has it that trigonometry evolved in 3 BC from the larger field of geometry and became even more important as scientists ventured into the field of astronomy. The field is now greatly relied upon by architects, engineers, surveyors, astronauts and scientists in general. In fact, trigonometry, which, as this book has indicated, utilizes Pi a lot, is helpful in some very unlikely disciplines such as that of investigation.

How Trigonometry Helps To Boost Health

Much as trigonometry is associated with serious subjects such as math, physics and engineering, it is also used in lighter fields like music. Computer music, which basically moves in varied patterns of waves, uses trigonometry. The sound engineer represents the kind of music needed in mathematical terms, and most of those that indicate the constituent sound waves are found in trigonometry. Needless to say, music is therapeutic, hence the reason trigonometry is helpful in boosting health. In fact, music can lift a person from depression, a condition that can sometimes disrupt one’s enthusiasm for work and even energy.

Trigonometry Can Help Deduce Structural Heights

If you want to know how high a building is without having to mount a ladder, or possibly sending someone up an apparently unstable wall, trigonometry can be used. Remember sine, cosine and tangent calculations all happen under trigonometry. In fact, even the height of a tall mountain can be determined through trigonometry.

For a building, for example, if you are standing some distance away looking at the top of the structure, you need to consider how far you are from the base of the building, a crucial distance in terms of units. The next important thing is to determine the angle of elevation, which is the angle created when you look at the top of the building from a distance. After that you have SOHCAHTOA to help you out, which you can use even when you know a distance other than the base.

Some of these distances in a triangle are calculated by surrounding the triangle with a circle, which gives room for the use of Pi. So as you learn how to remember the value of Pi quickly, it is for the sake of making other aspects of life easier.

Trigonometry is Helpful in Construction

Remember, the reason trigonometry is being highlighted here is because it utilizes Pi, which you need to recall quickly when there is a math problem which requires the constant. A field that relies on trigonometry may call upon the use of Pi. For example, how can trigonometry be used in the field of construction?

1) In the calculation of fields and lots

2) In designing perpendicular walls

3) In designing parallel walls

4) In ensuring ceramic tiles are properly fitted

5) In determining accurate roof inclination

6) In accurately calculating the sides of a building, ground surfaces, roof slopes and other parts of the structure under construction.

Trigonometry is Applicable In Flight Engineering

Flight engineering requires one to measure the speed and distance of the plane, as well as the direction and speed of the countering wind. In fact, the state of the wind determines the time the plane may take to reach its intended destination. Flight engineers use vectors to design triangles, which utilizes trigonometry, and that becomes the basis for calculating the required factors, including time required.

A good example is where a plane is flying at a speed of 377kph, and its direction is 45°N of E, under conditions where the wind is blowing south at a speed of 32kph. By using trigonometry, flight engineers can calculate the third side of the triangle, thereby pinpointing the correct direction the plane should take. In the calculations, the force of the wind is factored in as well.

Trigonometry is Applicable in Physics

Physicists use vectors, measure waves and calculate other physical electromagnetic components using trigonometry. Even oscillations and strength of magnetic fields benefit from trigonometry. The same case applies when calculating projectile motion.

Trigonometry is Applicable in Archaeology

Archaeologists also find trigonometry helpful, and by extension, the value of Pi, especially when they want to excavate sites in equal segments. These experts can also trigonometry to determine how deep some water systems are.

Trigonometry is Applicable in Criminology

There are many situations in criminology where trigonometry can come in handy, one of them being after a car accident. In such an instance, trigonometry can be used to assess a projectile's trajectory, and the calculated figure can then help to determine the factor that led to the car colliding with another. Trigonometry can also help determine the angle at which a bullet was aimed and shot, among other things.

Trigonometry is Applicable in Marine Biology

One of the reasons people keep a closed mind about mental math is that they do not envision real-life situations where such skills will be required. When they think about careers where such skills are helpful, for example, they sometimes find it difficult to go beyond two or three. Some students may even think that using mental math skills in trigonometry for marine biology is far-fetched.

Yet marine biologists fall back on trigonometry when they want to establish the size of living creatures underwater from a distance. They also use the mental math skills in trigonometry to find light levels at varying depths of water. They then determine how the intensity of light at each level affects algae's ability to photosynthesize. Marine biologists also make use of trigonometry as part of mathematical models that help to measure not only the size of sea animals, but also their respective behaviors. The mental math skills also come in handy when scientists are trying to calculate how far one celestial body is from another.

Trigonometry is Applicable in Marine Engineering

Marine engineers benefit from their proficiency in mental math when they apply it in the construction of marine vessels and/or their navigation. Trigonometry comes particularly in handy when engineers are designing a marine ramp, a surface that connects areas that are on different levels, one being lower than the other. Sometimes such connecting surfaces are in form of a slope and other times they require a staircase.

Trigonometry is Applicable in Navigation

If you are adventurous and imagine yourself traveling, you can use your mental math skills to determine how far some far-off features are. Professional navigators utilize their knowledge of trigonometry and mental math to set the direction for their vessel. It is trigonometry that helps to determine specific directions such as north, southwest and so on. In fact, it is trigonometry at work when you use a compass to keep the vessel straight.

While at sea, trigonometry is helpful in pinpointing a location, and it also comes in handy when calculating the distance between a vessel and the shore. The same mental math skills contained in trigonometry help to identify the horizon.

Other Areas Where Trigonometry is Applicable

There is no reason to dismiss mental math as a set of efforts confined to school because the skills learned are very helpful in real life. In fact, even when one is well established, career-wise, mental math skills make it much easier to make decisions and solve problems.

Some other professional fields where mental math is applicable under trigonometry include oceanography, cartography, the naval industry and the aviation industry. In oceanography, for example, the skills are used in the calculation of tide heights within the ocean, among other things, while in cartography the skills are helpful when creating maps.

Cosine and sine functions are very central to periodic functions, which are relied upon to describe sound waves as well as light waves. And, of course, calculus is a beneficiary of the mental math of trigonometry because, in addition to algebra, there is also trigonometry in calculus. Another area where trigonometry is central is in the design and operation of satellite systems.

Sometimes mental math under trigonometry is taken for granted, probably because people have mastered the relevant skills and made them part of their working pattern. A good example is the task of roof building, where a roof is required to be inclined. Trigonometry helps determine the height of the roof and the rest of the measurements that ensure the roof covers the required area and maintains the required inclination.

How To Recall Pi To The Sixth Decimal

Whenever you want to remember the beginning seven digits of Pi, you can simply say:

How I Wish I Could Calculate Pi.

The number of letters in each word represents a number. How has 3; I has 1; wish has 4; I has 1; could has 5, calculate has 9 and Pi has 2. Thus, the statement reminds you that the value of Pi to several decimals is 3.141592 .

Chapter 5: Math Strategies That Anyone Can Master

It is time to stop looking at math as an elite subject, or a subject for certain careers. You may not consciously think about it, but whether you like it or not, you are faced with problems requiring mathematical solutions on a daily basis. Why do you not tie the lace on one shoe and leave the other, even if neither is at risk of slipping off? That is probably a social issue—and some may say mental—but there is some math in it. How many tablets are fit for your nine-year-old child if you bought them over the counter for a cold? In short, even if you do not end up becoming a mathematician or pursuing a career in architecture, you will still need to be equipped with math skills.

In any case, what makes anyone think math is different from language? If the language of instruction in school is English, for example, teachers expect you to learn it in every imaginable way, from reading novels, doing crossword puzzles, playing scrabble and so on. At the end of the day, students end up communicating in English naturally, and it is rare that you hear anyone speak of how difficult English is. How can I complain about the tool I use to learn geography, history, science and every other subject? Instead, the tool is appreciated, and because of regular use, it gets sharpened by the day.

This is the mindset you need to adopt towards math. Appreciate that it is a tool to help you accomplish other things. Play around with numbers the same way you play around with letters or words. People who appreciate math view calculations the way musicians view varying musical chords and melodies. They do not see their complexity. Instead, they see an interesting challenge for them to form a song.

Practical Strategies That Simplify Math

If you find some math tricks which look close to others you have already learned, do not worry because the practice only makes you stronger in the skills concerned. After all, in English, the fact that you have learned the plural of ‘bear’ to be ‘bears’ does not prevent you from practicing how to form different words from the word ‘bears.’ Instead, when you are able to derive the words bear, ear and ears from the word bears, your language skills only become stronger.

Tricks to Enhance Your Math Skills

1) The trick of 9

This trick takes advantage of the number 9 being in close proximity to the number 10. As we’ve discussed earlier, the number 10 makes calculations easy. In multiplication, for instance, any number multiplied by 10 takes on a zero at the end and your problem is solved.

If you are dealing with a number slightly less than 10, just add the difference and subtract it quickly from the other number involved in the problem.

Example : 9 + 6

• Change 9 to 10 by adding 1

• Subtract that 1 from 6 and you get 5

• Your equation is now 10+5, which is very easy to calculate mentally.

• In short, 9 + 6 = 10 + 5 = 15.

This trick can be expanded to embrace bigger numbers like 99, which you can round to 100 by adding 1, and you will still succeed in making your equation easier to solve. Even a number like 980 can be rounded to 1,000 for ease of calculation, while subtracting what you added, in this case 20, from the other number involved.

Example : 99 + 62

• Add 1 to 99 and you get 100

• Subtract that 1 from 62 and you get 61

• 100 + 61 = 161

• In short, 99 + 62 = 100 + 61 = 161.

Example : 198 + 52

• Add 2 to 198 and you have 200

• Subtract that 2 from 52 and you have 50

• 200 + 50 = 250

• In short, 198 + 52 = 200 + 50 = 250.

If it helps, you can opt to you round both numbers either upwards or downwards. You will love math if you allow yourself to embrace math tricks freely, making use of the one that suits you with any particular problem.

If, for example, you are given 27 + 18, can you not make the problem easier by rounding 27 upwards to become 30 and 18 to become 20? What have you just done? You have added an extra 3 + 2 to your addition equation. After easily solving your problem of 30 + 20 = 50, remember to subtract 5 and your answer will be 45.

In short, 27 + 18 = 45 and you did not even tire your head trying to add up irregular numbers.

Feel free to add numbers onto both of your numbers if that makes your calculation easier. The important thing is to remember to subtract the amount you added when rounding.

Example : 268 + 197

• You can round up both numbers so that you have 270 + 200.

• This means you have added 2 to 268 and 3 to 197, a total of 5, to make smooth numbers that are easy to work with mentally.

• 270 + 200 – 5 = 470 – 5 = 465

• So, 268 + 197 = 465.

Exercise 7

(a) 8 + 7 =

(b) 7 + 6 =

(c) 19 + 12 =

(d) 17 + 18 =

(e) 22 + 31 =

(f) 257 + 13 =

(g) 2,998 + 100 =

(h) 10,098 + 198 =

(i) 100,299 + 16 =

(j) 200,998 + 999 =

2) Double and then add 1

To be comfortable using this math trick, it is imperative that you be skilled at doubling numbers. That is why it is important to encourage children to practice 1 + 1, 2 + 2, 3 + 3 and so forth. Gradually, even 20 + 20, 30 + 30 and the like will be easy to calculate mentally. Then you will find this trick easy to use.

For example, instead of struggling to use fingers to count or objects when given 4 + 5, you can simply say it is 4 + 4 + 1 or simply double 4 and add 1. The answer will be 9.

The same case applies to 7 + 8. You simply double 7 and then add 1.

In short, 7 + 8 is the same as one more than 7 + 7.

Exercise 8

(a) 8 + 9 =

(b) 6 + 7 =

(c) 30 + 31 =

(d) 200 + 201 =

(e) 300 + 301 =

(f) 1,000 + 1,001 =

(g) 900 + 901 =

(h) 10,000 + 10,001 =

(i) 7,000 + 7,001 =

(j) 100,000 + 100,001 =

3) Use small number addition skills to add more complex numbers mentally

Just to recap, you have learned that when you have 7 + 8, all you need to do is double 7 and add 1. 7 + 7 + 1 = 15.

Now, 7 and 8 in 7 + 8 are both ones in terms of value placement. Suppose you are given 70 + 80? 7 and 8 are tens in terms of value placement, and so you should ask yourself: How many tens are there in each of these numbers?

• There are 7 tens in 70 and 8 tens in 80.

• It follows then that your equation has 15 tens. Hurrah! Numerically, you know a number is a ten because there is either a zero in the place for the units or there is another digit there. Or, if it does not complicate matters for you, you can look at 15 tens as 15 x 10. Either way, you end up with 150.

• So, 70 + 80 = 150.

Another Example

90 + 80 = ?

• Using the double + 1 skill, you can quickly tell that 9 + 8 = 17

• Since 90 + 80 means 9 tens + 8 tens, you can tell automatically that 90 + 80 = 170.

This method is helpful even when the numbers given are not rounded as in 10, 100, 1,000 and so on. See how you can do more irregular numbers here below.

37 + 8 =?

• Mentally separate 7 + 8 and you get 15 by using the trick of doubling plus one.

• Since 37 has 30 tens and 8 has none, you will now deal with 30 + 15, which is a pretty easy equation to handle. Number 30 has 3 tens and number 15 has a ten and a five, so you have 30 + 10 + 5 = 45.

In reality, just by looking at 30 + 15 as an equation, the number 45 crops up. The calculation does not unfold slowly and systematically as it is in the explanation provided here.

The mind has a way of storing information in a compact but clear form, and the math skills you learn are part of that information. So when you look at an equation, your mind quickly retrieves the relevant math trick and deciphers what it represents, faster than you can speak. Remember, your mind works with images when it comes to information storage and images are faster to capture, store and retrieve than sequential letters and numbers.

Another example

49 + 6 =?

• 40 + (9 + 6) =?

• Here you can use the method of subtracting 1 from 6 to add it to 9 so that you have 40 + (10 + 5), a trick you have already come across in this book.

• You now have 40 + 15, which is much easier to deal with. You can break it down further to 40 + 10 + 5 or simply have confidence in your mind when it gives you 55 straightaway.

In short, 49 + 6 = 55.

Another Example

27 + 16 =?

• 20 + 10 + (7 + 6) =?

• 20 + 10 + 13 =?

• 30 + 13 = 43

In short, 27 + 16 = 43.

As you continue reading the examples, you will notice there is no restriction as to which trick you should use to simplify your problem. Whether you want to double a number and then add one, or subtract some from one number to add to another to have at least one round number to work with, it is up to you. That is the kind of freedom that makes math interesting. Feel free to discover as many safe shortcuts as you can, just like in real life.

Practice will help you a great deal, especially if you do lots of it as soon as you learn a particular trick. After that you are likely to remember it automatically, the way those who learned BODMAS do. As discussed earlier, BODMAS is an acronym for Brackets Of Division, Multiplication, Addition and Subtraction, and it serves as a guide on how to approach a mathematical problem in order to get the correct answer.

Exercise 9

(a) 30 + 20 =

(b) 400 + 300 =

(c) 405 + 24 =

(d) 64 + 25 =

(e) 116 + 105

(f) 500 + 405 =

(g) 7,000 + 8,000 =

(h) 2,500 + 2,400 =

(i) 57 +26 =

(j) 25 + 14 =

(k) 44 + 23 =

(l) 99 + 11 =

(m) 107 + 26 =

(n) 134 + 13 =

(o) 267 + 25 =

(p) 1,111 + 1,112 =

(q) 83 + 12 =

(r) 833 + 122 =

(s) 105 + 104 =

(t) 17 + 16 =

4) Think plus in a minus situation

Adding is usually the simplest of all math calculations, and so if there is a way you can use it to solve a subtraction, multiplication or other problem, go ahead. Just the same way you can think of 2 x 2 as adding two and two because you are having 2 twice, so you can think of subtraction in terms of addition.

Example

8 – 3

Having mastered addition, you can think of the problem as _ + 3 = 8. What do I add to 3 to get 8? Something plus 3 equals 8. Since, when doing additions, the order of numbers does not matter, you can also frame your question as 3 + _=8. Automatically, 5 will crop up in your mind.

And so, 8 – 3 = 5

This is a trick that comes in very handy when the number you are looking for falls below 20. Nevertheless, it is also helpful in other situations, including when you are looking for bigger numbers.

Example

66 - 20 =?

• What can you add to 20 to get 66? That number will be your answer.

• 20 + _ = 66

• That number is 46.

• As such, 66 – 20 = 46.

This trick has taught you how to manipulate your numbers so that you end up searching for the missing number, rather than racking your mind with subtraction.

Exercise 10

(a) 20 – 6 =

(b) 20 – 13 =

(c) 17 – 4 =

(d) 18 – 12 =

(e) 19 – 2 =

(f) 12 – 9 =

(g) 29 – 20 =

(h) 68 – 18 =

(i) 688 – 608 =

(j) 510 – 400 =

(k) 475 – 100 =

(l) 888 – 800 =

(m) 919 – 19 =

(n) 120 – 90 =

(o) 10,000 – 8,000 =

(p) 12,000 – 5,000 =

(q) 27 – 5 =

(r) 32 – 30 =

(s) 680 – 180 =

(t) 100 – 30 =

5) In times of 5, double your number first

Why do you double when you are supposed to multiply by 5? Simple. You want to be able to work with easy number 10. Once you double 5, you get 10, and if you were initially supposed to multiply a given number by 5, you just need to halve your product to get the correct answer.

This is a long explanation for a very short process. See:

Calculate 15 x 5

• Change your equation to 15 x 10 after doubling your 5.

• 15 x 10 = 150.

• But you do not want the answer to an equation with a double 5 but one with a 5.

• So, calculate 150 ÷ 2 or simply ask yourself what ½ of 150 is.

So, 15 x 5 = 75.

Another example

125 x 5 =

• 125 x 10 = 1,250

• 1,250 ÷2 = 625

So, 125 x 5 = 625

Exercise 11

(a) 85 x 5 =

(b) 27 x 5 =

(c) 35 x 5 =

(d) 17 x 5 =

(e) 250 x 5 =

(f) 112 x 5 =

(g) 42 x 5 =

(h) 30 x 5 =

(i) 19 x 5 =

(j) 90 x 5 =

6) In times of 4, double twice.

When a question is more complicated in that you are supposed to multiply a number by 4, you can still use the times 2 principle, because 4 = 2 x 2. In short, whichever answer you get when you say times 2, if you say times 2 once more, you will get the answer you would have gotten if you had said times 4 to the original number.

Here are some simple examples:

(i) 2 x 4 =_

Double 2 first: 2 x 2 = 4

Double 4 next: 4 x 2 = 8

So, 2 x 4 = 8

(ii) 3 x 4 = _

Double 3 first: 3 x 2 = 6

Double 6 next: 6 x 2 = 12

So, 3 x 4 = 12

(iii) 25 x 4 = _

Double 25 first: 25 x 2 = 50

Double 50 next: 50 x 2 = 100

So, 25 x 4 = 100

(iv) 41 x 4 = _

Double 41 first: 41 x 2 = 82

Double 82 next: 82 x 2 = 164

So, 41 x 4 = 164

(v) 122 x 4 = _

Double 122 first: 122 x 2 = 244

Double 244 next: 244 x 2 = 488

So, 122 x 4 = 488

Exercise 12

(a) 5 x 4 = _

(b) 6 x 4 = _

(c) 12 x 4 = _

(d) 21 x 4 = _

(e) 35 x 4 = _

(f) 105 x 4 = _

(g) 115 x 4 = _

(h) 410 x 4 = _

(i) 531 x 4 = _

(j) 331 x 4 = _

7) In times of 8, double three times

What you have seen is that when you double a number twice. it gives you the answer to times 4, so 2 x 2 x 2 is like saying 2 x 4. When you want to multiply by 8 the easy way, multiply the number by itself one more time. You are, therefore, going to have 2 x 2 x 2 x 2 to represent 2 x 8. 2 x 2 = 4 is the first doubling; 4 x 2 = 8 is the second doubling and 8 x 2 = 16 is the third.

Here are some examples:

(i) 3 x 8 = 3 x 2 x 2 x 2 = 6 x 2 x 2 = 12 x 2 = 24

(ii) 5 x 8 = 5 x 2 x 2 x 2 = 10 x 2 x 2 = 20 x 2 = 40

Exercise 13

(a) 25 x 8 =

(b) 31 x 8 =

(c) 13 x 8 =

(d) 45 x 8 =

(e) 102 x 8 =

(f) 71 x 8 =

(g) 21 x 8 =

(h) 222 x 8 =

(i) 300 x 8 =

(j) 112 x 8 =

7) In times, double one and half the other

This case imagines a scenario where you are supposed to multiply two numbers that are somewhat complex, or numbers that do not involve multiplication by 2, 4 or 8. You may be able to simplify your equation to make mental work easier by multiplying one of the numbers by 2 while dividing the other one by 2. Those two actions have the impact of canceling one another, so at the end you do not have to adjust your answer in any way.

Example

(i) 25 x 8 = _

Suppose you decide to double 25 and get 50.

In the meantime, you are called upon to halve 8 to get 4.

50 x 4 = 200. These are very easy numbers to work with, and they give you the same answer you were looking for.

25 x 8 = 200.

(i) 45 x 4 = _

Double 45 and halve 4

(45 x 2) x (4 ÷ 2) = 90 x 2 = 180

45 x 4 = 180.

Exercise 14

(a) 35 x 8 =

(b) 60 x 8 =

(c) 17 x 20 =

(d) 92 x 20 =

(e) 24 x 20 =

(f)   55 x 4 =

(g) 110 x 4 =

(h) 15 x 8 =

(i)   202 x 20 =

(j)   22 x 20 =

8) Work in parts

This method entails looking at your equation as a consolidation of different simple parts. You then calculate each simple segment separately and add up the different totals.

For example, if your math problem is 68 x 2, you can break it down to (60 x 2) + (8 x 2). You then get 120+16 = 136.

Other Examples

(i) 94 x 3 = _

(90 x 3) + (4 x 3) = 270 + 12 = 282

So, 94 x 3 = 282

(ii) 43 x 4 = _

(40 x 4) + (3 x 4) = 160 + 12 = 172

So, 43 x 4 = 172

(iii) 112 x 3 = _

(100 x 3) + (10 x 3) + (2 x 3) = 300 + 30 + 6 = 336

So, 112 x 3 = 336

(iv) 207 x 5 = _

(200 x 5) + (7 x 5) = 1,000 + 35 = 1,035

So, 207 x 5 = 1,035

(v) 87 x 6 = _

(80 x 6) + (7 x 6) = 480 + 42 = 522

So, 87 x 6 = 522

Exercise 15

(a) 76 x 2 =

(b) 83 x 3 =

(c) 123 x 5 =

(d) 304 x 6 =

(e) 661 x 3 =

(f) 252 x 4 =

(g) 117 x 7 =

(h) 91 x 8 =

(i) 82 x 4 =

(j) 344 x 5 =

Remember in math, as in ordinary life, practice makes perfect. So the more you employ these tricks, the more proficient you will be at mental math. It would also be a great idea to take advantage of any number games you come across. They will make more comfortable with numbers and help you learn to enjoy them.

Chapter 6: How to Attain a Great Score in SAT Math

SAT math is something you cannot afford to fail if you want to get into a good university. You need a minimum score of 800 if you want to join one of the top institutions.

In math, even when you are allowed to use a calculator, the very method you are trying to use to pass may result in a high grade, simply because it is more time consuming than doing your math mentally. Is it not faster, for instance, to add the two digits that make up 42 when multiplying it by 11 to get the middle number of the answer than to use a calculator to get the solution? 42 x 11 = 462, and all you need to have in mind is your simple math trick of retaining your end numbers and adding them to form your answer's middle number.

Mental Math Tricks You Need As You Prepare for SAT

Life with math seems rosy until the questions begin to look strange and disorienting. For example, when you see 2 x 2, you are fine, and even when some variables come in double digits or more, such as 24 x 2 or 311 x 10, you are not concerned. However, when questions begin to incorporate figures that look awkward, or, worse still, include powers, math begins to dull for many people.

That is why this book has introduced you to tricks to circumvent the awkwardness of the questions while helping you come up with the correct answer. Remember the trick for calculating a funny-looking number with 11, such as 311 x 11?

You begin by separating the end numbers; in this case, 3_ _1

You then add the first two numbers and insert the result in the second slot.

3 + 1 = 4, so you now have 34_1

You then add the last two numbers and insert the result in the third slot.

1 + 1 = 2, so you now have 3421

Even if you test this calculation using your calculator, you will still find that 311 x 11 = 3421.

The topic of powers sounds interesting as long as you are dealing with simple squares, such as 22 or 42, because you can easily say 22 is the same as 2 x 2, while 42 is the same as 4 x 4, which give 4 and 16 respectively. More complex problems involving powers can be worrisome if you are not familiar with the best way to tackle them. However, once you have acquired those skills, you will smile when you see such problems, because you will be assured of easy marks, even when the questions are the SAT type.

Tricks in Addition and Subtraction

It is advisable to use the column method, old school as it may seem, because it is quicker than trying to do your calculations horizontally. However, if you can do your additions and subtractions mentally, you'll complete your exam faster.

For example:

When dealing with 2,568 - 1,072, it is easier to do it vertically, or using the column method as demonstrated here below:

2568 2568

- 1072 - 1072

3640 1496

There are numerous other methods that you can apply to solve problems even in multiplication and division, but what you need right now are those that will help you leave the ordinary league and reach the high-scoring league in SAT.

There are many people who are almost guaranteed scores between 600 and 750 in SAT math, but they will be the happiest if they know they can score 800. Attaining 800 is achievable, as there are students who have done it.

What you require in order to excel in SAT math are concrete tips on how to go about tackling the questions, and that is what this book provides.

Skills In Adding And Subtracting In Powers

When you have a math problem that has numbers in powers and those numbers need to be added together or subtracted from one another, there are some basic rules to follow.

(i) The variables must be the same.

The variable in a math problem is an unknown value, usually represented by a letter. For example, you may have 2x, which means 2 times an unknown value. If you discover that x = 4, then you will know that the 2x you have been given in your math problem is 2 x 4, which is 8.

(ii) The required operation, either plus or minus, is performed on the coefficients.

This means the variables and exponents remain unaltered. In short, after your calculations, your answer has the same variables as the question, and similarly, it has the same powers.

(a) Calculate x + x + x

It is assumed that x on its own means 1x or a single x.

And since, as you have read, you only need to do the operation on the coefficients, in this case adding them and leaving the variable unaltered, this is how you do the calculation:

x + x + x = 1x + 1x + 1x.

But writing 1 before any number looks ridiculous because it is common sense that an x means just one 1x. Therefore, owing to the redundancy of 1, it is left out.

x + x + x = 3x

(b) Calculate 2x + x + 5x

Remember, the variable remains as it originally is

Only add the coefficients of the variable, in this case, 2, 1 and 5

2x + x + 5x = 8x

(iii) The exponents must be the same.

An exponent is that tiny number whose font is in superscript, which is written to the right side of the large number and slightly above it. That large number is called the base value and is what you are required to multiply by itself. The question is: how many times do you need to multiply the base by itself? Well, as many times as the exponent indicates.

In short, when people speak of a number raised to a certain power, what they mean is that some base value has been raised to the power of a given exponential value.

Note that the base can be any number.

The exponent, too, can be any number.

The exponent can be a plus, a minus or even a fraction.

The exponent can be a radical (√)

Calculate x + x + x

(c) Calculate 2 x 2 + x2 + 5 x 2

In a case like this, the base and its exponent do not part ways. This means x^2 remains unaltered.

However, you need to add the coefficients, to show by how much x^2 is increasing.

$2x^2 + x^2 + 5x^2 = 8x^2$

(d) Calculate 4x – x

Whether a variable has an exponent or not, it does not change. So, in this case, too, variable x will remain unaltered.

4x – x = 3x

As you may have noted, 1 x has been subtracted from 4 exes. In short, only the coefficients have been influenced by the subtraction.

(e) Calculate 4 x 3 – x3

Per the rule, you need to keep variable x and its exponent 3 exactly as they are.

Subtract the unseen coefficient, 1, from coefficient 4.

Hence, $4x^3 – x^3 = 3x^3$.

Rules for Working With Powers

(i) The meaning of y n = y x y x y x y x y x y...n times.

y in this equation is the base and can represent any real number.

n in the equation is the exponent, the one commonly referred to as the power. It can represent, or substitute for, any real number.

One caveat you need to keep in mind is that although it is acceptable for y to be zero (0), and it is also acceptable for n to be zero (0), it is not acceptable to have them both be zero (o) at the same time. While it would be true to say that zero to the power of zero is zero, the whole thing is really meaningless in algebra.

Another caveat is that if your y or your variable is zero, then its exponent cannot be negative.

Do not rule out the possibility of finding a problem on an SAT test where you are required to give the answer to a zero written to a negative power. While some students might try to work out the problem, you can just check the answer for 'none of the above' because problems where the exponent of zero is negative are not admissible.

(i) To evaluate an exponential expression, you need to multiply the base with itself n number of times. Remember, n is the exponent.

(a) Calculate 2^4

$2^4 = 2 x 2 x 2 x 2$.

The answer is 16.

The value of 2^4 is 16.

(b) Calculate 3^5

$3^5 = 3 x 3 x 3 x 3 x$.

The answer is 243.

The value of 3^5 is 243.

(c) Calculate 10^8

$10^8 = 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10$.

The answer is 100,000,000.

The value of $10^8 = 100{,}000{,}000$.

(i) When the base is 10, you can immediately tell the number of zeros you are going to need to add to get the answer.

In example c above, the answer to 108 has 8 zeros. The exponent of 10 was 4, so you would need to see 4 zeros in your answer, and so on. You will notice that rule holds true if you think about calculating 10 to power 2 or 102. The answer is 100, which has 2 zeros. So, in cases regarding 10, the exponent is a great guide.

Note that the exponent does not indicate the number of zeros to add to the base or 10, but the entire number of zeros present in the answer.

(ii) When there is an exponent outside, you solve the equation within the bracket first, before you can proceed to work out the power.

(a) Calculate $(a + b)^3$

Since a and b are different bases, they are supposed to remain as they are.

So $(a + b)^3 = (a + b) x (a + b) + (a + b)$

Polynomials and Their Importance

A polynomial is simply an expression which comprises variables as well as coefficients and which involves addition and subtraction. It also involves multiplication. The expression also involves integer exponents of variables that are non-negative. Note that exponents are what are often described as powers, and variables are sometimes referred to as indeterminates.

Here is an example of a polynomial that has just one indeterminate or variable:

• $x^2 + 4x + 7$

That indeterminate or variable is 'x.'

Here is an example of a polynomial that has three variables or indeterminates:

• $x^3 + 2xyz^2 – yz + 1$

The variables are x, y and z.

The term 'polynomial' is a product of two roots, the first one being 'poly,' derived from the Greek word meaning 'many,' and the other, 'nomen,' derived from the Latin for 'name.' The term was adopted in the seventeenth century, taken as a replacement for 'binomial,' which borrowed the root 'bi' from Latin, meaning 'two'. Basically, when you see the term 'polynomial, just think of 'many terms.' That is what the word is meant to convey.

Polynomials are one of the important topics tested on the SAT and they come in handy when doing advanced work in math and other fields of study such as chemistry. For example, polynomials appear a lot as polynomial equations within various science disciplines. They also appear as polynomial functions in chemistry, physics, economics and social sciences, among other fields. They are the basis on which other functions are determined in calculus and numerical analysis.

In advanced mathematics, polynomials are involved in the construction of polynomial rings, in algebraic varieties, algebra and related geometry.

As has been shown above, x is used in a polynomial to stand for a variable of an unknown value. Sometimes x is used in a polynomial that is considered an expression. However, although x is an indeterminate, the moment it appears in a function expressed in a polynomial, it stands for the function's argument and is termed a variable. Nevertheless, you should not think about these definitions in a very strict sense, because often, people use the two terms interchangeably.

In this regard, it is important to point out that when you are referring to an indeterminate in a strict sense, you need to denote it in capitals. On the other hand, when you want to express a variable in the strict sense, you need to use lowercase.

When you have a polynomial with just one indeterminate, x, you can always rewrite it to become:

$a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$

In this expression, a_0, ..., a_n happen to be constants, while x is considered the indeterminate. What this definition essentially means is that x does not represent any specific value, but at the same time, any value can be used to replace x.

A polynomial may comprise three terms, namely a constant, a variable and an exponent. One or more of these terms can come in multiples.

Constants include numbers like 2, 5, 8, - 21, ¼ and so on.

Variables include x, y and such other representative letters.

Exponents include integers that denote 'power of,' such as 3 in x^3 .

An important point to note is that a polynomial cannot be divided by a variable.

Let us look at the kinds of expressions that can be termed polynomials:

2x

x – 2

$-5y^2 – (49)x$

$4xy z + 3xy^2 z – 0.2xz – 300y + 0.5$

$612v^5 + 88w^5$

6

It may come as a surprise to you that 6 is included in the list of expressions that form polynomials, but it is acceptable. Just a single term can represent a polynomial, irrespective of the semantics of 'poly' denoting many.

Not every expression that has a small letter is a polynomial. In fact, you can have small letters of the alphabet such as x and y mixed in an expression with constants such as 2 and 3 and yet fail to form a polynomial.

Examples of Expressions not Qualifying as Polynomials

2xy -2 does not qualify as a polynomial because negatives are not acceptable as exponents.

2 ÷ (x + 2) does not qualify as a polynomial because division by a variable is not acceptable.

1 ÷ x is also not acceptable as a polynomial and for the same reason, division by a variable is not acceptable in a polynomial.

√x because √x translates to x ½ and is not acceptable.

Not all divisions of polynomials are disallowed.

Polynomial Divisions That Are Acceptable

x ÷ 3 is acceptable as a polynomial, because the division is by a constant

6x ÷ 10 is also acceptable as a polynomial, and that is essentially for the same reason that the division is by a constant and not a variable.

√3 is acceptable as well as a polynomial, just as other constants that are integers are acceptable. In this case, you need to view √3 as the result of root 3, which is 1.732.

Why You Need To Score Highly on the SAT

Once you have attained a 1530 and above on your overall SAT, you are as good as someone who has scored a perfect 1600. All the top colleges will be interested and are likely to consider you, depending on other factors reflected in your application form. The point here is that a 1550 and a 1580 score give you the same advantages, and there is no need to invest more time and resources trying to improve a 1540 to a 1580 or even to a 1600.

However, if your scores are in the 1400s and your aim is to be admitted to a top-10 U.S. college, you need to invest enough time to enable you to improve your score to 1540+. Aiming at a score of 1550 is wise, so that even if you slide backwards a little, you will still be within the threshold of the topmost colleges.

Taking the examples of Harvard and Princeton, a 1530 score does not make you competitive. Rather, it places you among the average students seeking admission.

Why You Need To Score Exceedingly Well In SAT Math

If you score highly on SAT math, such as an 800, your great performance will compensate for any areas of the SAT exam you may not have performed so well on. Although the tips being provided in this book are meant to help you pass math with good grades, they will also indirectly help you improve your overall score, which is what most colleges are interested in.

For example, if your SAT math score is 800, scoring 740 in the section on reading and writing gets you an overall score of 1540, which is impressive. It means that a wonderful performance in SAT math can be the reason you have vast flexibility when it comes to choosing a good college to join.

Still, some institutions are adamant that their potential students score highly in math. Massachusetts Institute of Technology, popularly known as MIT, is such an institution. It expects anyone wishing to join the institution to score nothing short of 800 in their SAT math.

If you are targeting math, physics, chemistry, statistics or any other subject of a quantitative nature at a reputable institution, your aim should be 800. That is the score Caltech and other institutions of such caliber expect from applicants, and that is normally because their courses are highly technical.

In any case, the better students are at appreciating math concepts, the easier it is for professors to teach them more advanced concepts and for the students to comprehend the more abstract concepts necessary to delve into deeper research. That is why institutions of higher learning make comparisons based on scores. It's why you need to work at improving your math skills so you can make the cut.

Skills That Anyone Can Use to Excel in SAT Math

Even when you are not a natural at math, you can always join the top league of mathematicians if you are equipped with the right skills. Not everyone who has attained a score of 800 in SAT math can claim to have found math easy from the start. Note that a high score in SAT math is not only a reflection of competence but also of diligence in study. You are accepted to a good college with the belief that you are capable of understanding more complex problems and diligent enough to put in the required hours to study, practice and do research.

The Uniqueness of the SAT

Although the SAT does not cover topics that students haven't already learned in school, the questions often seem strange to some students because they are presented in a different way. You will only do well on the SAT if you are able to think outside the box. If you can think of unconventional ways of solving ordinary-looking math problems, for example, you are on the right path.

Think of the problem 2448 ÷ 4. You can divide 2450 by 2 in two seconds and get 1224, and in another second you can divide 1224 by 2 to get 612. Is that method conventional? No. But is 2448 ÷ 4 = 612 correct? Yes. The conventional way might have involved you doing the division in a long column, and it would certainly have taken comparatively longer.

The point is, you should anticipate problems that are presented somewhat differently. In a way, it is like thinking of using the back door when you find the front door locked. In SAT math, your creativity is put to test, and you are better placed if you can think up creative solutions. In short, while the playing field is level for all high school students, only those who have learned additional methods and tricks beyond the conventional ones score impressively well.

The topics you should expect to see on an SAT math test include basic algebra, where you solve variables within equations; word problems; advanced algebra, which includes quadratic as well as exponential equations; geometry, which includes the one involving x/y co-ordinates, as well as circles, triangles and squares; and basic statistics.

As you can see, these are topics are all covered in high school. However, many students still find the SAT tough because the testing is done in ways that are unconventional. The good news is that you can enjoy the SAT as you do it and score as highly as you hope, but that is only possible when you take your time to prepare for it.

Example

Question : The radius of the circle is 5cm. What is the size of the shaded area in the diagram?

While this is a typical SAT question, what you may be used to doing in school is calculating the area of a square or the area of a circle. Yet, in the diagram above, you have a circle and a square, but nobody wants you to calculate their sizes. Instead, the examiner wants you to provide the area of some irregular segments within the diagram.

When some students see this, they panic. They do not mind being asked the area of any familiar shape, including a square or a circle, but a shape whose sides are straight lines and a curve has no definition and therefore the area has no clear formula. This is where you are called upon to think creatively, the way you would if you lost your keys and wanted to enter your locked house. Being able to solve math problems in unconventional ways is an indication that in real life you can find solutions in new or difficult situations.

Solution

Points to note:

Notice that your diagram comprises a square with a circle within it.

Notice that if you cut out the circle from the square, you are left with only the shaded area. That shaded area is the one whose size you have been asked to calculate.

Remember that you have been provided with the radius of the circle: 5cm.

Notice that the radius of the circle is equal to half the length of the square.

So the full length of one side of the square must be 5cm x 2 = 10cm

Find the area of the square

The area of the square is length x breadth, e.g. l x b.

In this case, the area of the square is 10cm x 10cm = 100sq cm

Find the area of the circle

To get the area of a circle you need to use the formula: πr^2

In this case, $A = \pi(5 \times 5) = 25\pi$.

Subtract the area of the circle from the area of the square.

Shaded area = 100sq cm - 25π sq cm

If you deem it necessary, you can proceed to do the full calculation, substituting Pi with its actual value.

Do you remember the trick for recalling the value of Pi?

How I wish I could calculate Pi: the number of digits in each word being the key. $\pi = 3.141592$

So you can say the size of the shaded area is 100 - (25 x 3.141592) sq cm = 21.4602 sq cm.

What To Expect in SAT Math

You need to practice questions formulated in non-familiar ways which may look odd.

It is important to be able to identify familiar formulas that can be applied correctly to solve the questions.

It is important to do a lot of practice so that you get a chance to notice likely mistakes or miscalculations and learn how to avoid them.

Many of the tests you do in class are straightforward, and it is easy to tell which formula is relevant to each problem. You are often able to tackle the math problem depending on how well you have understood the concept and how to apply the problem-solving techniques taught. That is not always the case with SAT math questions.

On the SAT, you often come across problems whose general outlook is familiar, yet by the time you finish reading the question, you still have no idea how to begin tackling it. The problems are often presented in a unique way. If you can tackle them successfully, it shows you are capable of adapting to situations that are new. It shows you do not always rely on memorizing concepts but on thinking and reasoning.

There are 12 questions below, and they cover the most common topics represented on an SAT math test: algebra, geometry and logic. As you will notice, solving the questions requires you to think beyond just a general understanding of relevant formulas.

How to Make Your SAT Practice Count

Have you seen students who look at their books all prep time long but hardly seem to improve their scores? Often it is because they do not stop to ask themselves what their takeaway has been. It is good to be able to say, “This prep time I have learned how to do times 11 in my head, the shortcut for dividing by 4 or something else that is helpful.”

Discover your mistakes

Identify the mistakes that you have made, in order to establish the reason you did not get a particular math question right.

Learn why you made each mistake

The next issue you need to tackle is how to ensure you do not repeat the mistake when dealing with a similar problem in the future. In short, you need to make your practice count. Much as you are encouraged to do lots of practice, it should not be quality over quantity. Rather, you need to do lots of practice so that you have room to tackle diverse kinds of questions and can identify as many of your weaknesses as possible.

Learn how to avoid mistakes you have made before

Once you have found out where you made a mistake on a math question, you need to learn how to avoid doing it again.

Failing just one math question on the SAT means a downward drop from the coveted score of 800. So it is important that you address every weakness you have in whatever math topic it is.

Do lots of math problems

Do specifically SAT-oriented questions.

Chapter 7: Tips to Help on SAT Algebra

The tips provided in this book touch on SAT topics that include variables, coefficients, algebraic equations, constants and even operators. In short, you can look forward to learning the fundamentals of tackling algebraic expressions.

1) Constants

A constant is any real number that you come across in arithmetic. It has a value that does not change wherever you use it. For example, 3 is a constant and whether you use it in division, subtraction or anywhere else, it will remain 3.

Constants come in various forms and not just whole numbers. For example:

(i) Numbers

In the example of 3, cited above, the value of a constant remains unaltered wherever it is written or used. There is no time when the value of 2 will be halved to 1, for example, or when 30 will increase to 32.

(ii) Variable

Variables' value can change depending on the values of other variables in an equation. For example, when you have an equation such as $2x = 4y$, x and y remain variables subject to change, depending on the values of one other.

However, if you receive information that the value of y is 6, you will be able to calculate 4×6 and you will get 24.

Now you can confidently say, $2x = 24$. As long as you can immediately tell the value of y, then you can comfortably term 4y a constant.

(iii) Fraction

Just like whole numbers, the value of a fraction does not change. A half is a half, whatever you are dividing. So, fractions are constants.

(iv) Other constants include square roots, numbers in powers, decimals and so on, as long as what you term a constant has no chance of having its value change.

2) Variables

Normally, variables are indicated as symbols like x, y, a, b and the like. Basically, though the symbols are used in algebraic expressions, in reality they stand for an undisclosed value.

Example :

John bought x number of books from the bookstore, and the price of each book was $5. So, in total, John paid x times $5, which is $5x. In this example, x is a variable.

Although you can use the symbols of your choice to express your information in the form of an equation it is helpful to represent your variables with relevant symbols. For example, if the equation is associated with distance, speed and time taken, t could stand for time; s for speed and d for distance covered.

3) Coefficient

A coefficient is the factor by which you multiply a given variable within an equation. For example, when you have an equation like $2x + 3y = 12$, 2 is the coefficient of x, x being a variable, and 3 is the coefficient of y, y being another variable. In short, wherever there is 2y it means 2 times y; 4z means 4 times z and so on, the number by which you multiply the variable always being the coefficient.

4) Algebraic Expression

When someone speaks of an algebraic expression, what they are referring to is an expression made up of constants, variables and mathematical operations, where mathematical operations refer to signs of division, addition, subtraction and the like. Below are some examples of algebraic expressions:

• $2x + 10$

• $y – 3$

• z2 + 5

• ¼ (3 + 9)

5) Equation

An equation refers to a statement that says each side is equal to the other. In math, those two sides consist of numbers or algebraic expressions. Equations are very helpful in algebra because they help us to learn the value of variables and also to solve relationships between variables and constants or among constants alone.

Here are some equations:

- x + 10 = 13
- 3 + 7 = 10
- 2x + 12 = 4 + 14y
- 10 + 12 = 24 – 2

Overall, algebraic expressions are helpful when you want to:

- expand expressions
- express mathematical terms in a simpler manner
- isolate a variable and establish its value as represented by a constant.

This book has more techniques to help you work out different algebraic problems, and you will have a choice to pick the one that best suits your individual problem. All you need to do is to practice each technique so that it comes to mind quickly when you see a related problem. That way, it will take you much less time to quickly and accurately tackle a math problem.

Example:

What is 4m + 5n when the value of m = -5 and the value of n = 1?

Your choices are:

A) 15

B) -15

C) 25

D) -25

E) 5

- What are the variables in the given expression? They are m and n.
- In the expression, the value of the two variables, m and n, has been given as -5 and 1, respectively.
- Cubstitute the variables with their respective values, as follows:

(4 x m) + (5 x n) = (4 x -5) + (5 x 1) = -20 + 5 = -15

Hence the answer is B) -15

NB

When you are adding numbers, it does not matter which one comes before the other, as the answer will always be the same. Let us use the same equation given in the above example.

What is 5n + 4m when the value of m = -5 and the value of n = 1?

- Substitute the two variables, n and m, with their respective values.
- (5 x 1) + (4 x -5) = 5 + -20 = -15
- So, likewise, 5n + 4m = B) -15.

In short, whatever the order of numbers in the equation, the answer should always be the same, because each number retains its plus sign or minus sign.

How to Work With Expanded Expressions

Usually on your SAT test, you are not given expanded expressions, so it is up to you to expand the equations you are provided with in order to make your calculations easier. In fact, expanding algebraic expressions is often necessary, especially because many of those problems involving parentheses or brackets. For instance, 3 + (5x + 10) ÷ (15x – 5) is an algebraic expression with parentheses, and you need to expand the expression as you remove the brackets.

In this case, you would be expected to apply one of the most important properties algebraic expressions have, the distributive property. Algebraic expressions have three very important properties:

(1) Distributive property

(2) Associative property

(3) Commutative property

How the Distributive Property Works

It is important to understand what the distributive property is and how it is used in math. Not many exams omit a question that calls for distribution skills.

Distributive property is used in multiplication, where one, two or more terms set within some parentheses are multiplied.

A simple example is 2 (4 + 5)

Inside the parentheses, you have what you can call a binomial, which is '3 + 6.' This is the operation you are supposed to solve first, because that is what the Order of Operations (BODMAS) demands. In this case, you need to deal with the brackets before proceeding to multiplication.

2 (4 + 5)

So you will have 2 times (4 + 5) next

2 x 9 = 18

Mistakes to Avoid

For students who have not understood the systematic way of tackling problems in algebra and other areas, they may proceed to deal with the math problem in any order. Here is a common mistake:

2 (4 + 5)= ?

2 x 4 + 5

If you multiply 2 by 4 first, this goes against the required procedure and leads to 8+5, which gives you 13, which is the wrong answer.

Many students who do such a problem wrong leave the exam room believing they got it right, only because the answer they got was among the choices given. The reason such an answer is put there is because tutors and examiners anticipate such errors.

Let us look at a more complex problem:

2 (4 + 5x)

You know you need to begin sorting out the operation within the bracket before moving to anything else. However, there is a small complication, as there are unlike terms inside the bracket. You have a constant, 4, and a variable, x, with its coefficient, 5. Certainly, 4 and 5x are not alike and cannot be added together.

It is like having a hen and a cat in the house. They can never be counted as one type of animal. Instead they will always remain one hen and one cat.

When you had like terms inside the bracket, i.e., 2 (4 + 5), it was easy to simplify the terms within the brackets because they were similar. Hence, you simplified the operation to 2 (9).

In cases where the expression within the bracket cannot be simplified further, the distributive property comes in handy.

How to Distribute a Number

The distributive property allows the removal of the brackets as long as the term the polynomial will be multiplied by gets distributed to each one of the terms within the brackets.

Let us use 2 (4 + 5x) as an example.

When you have 2 (4 + 5x), you need to distribute 2 to the terms within the brackets. Hence, you will have:

2 x 4 + 2 x 5x

You have used the distributive property, and now each one of the terms inside the bracket will be multiplied by 2.

Once you simplify the expression, you will get 8 + 10x.

How to Distribute a Negative

We are going to look at a problem that has an expression within brackets, but has a single minus sign on the outside. These are the kinds of scenarios that call for remembering the rules of plus and minus.

Obviously, the two signs stand for different operations. A ‘+’ sign shows that the number that follows it immediately is positive and you are supposed to add that number to another. On the other hand, a ‘-’ sign indicates that the number that follows it immediately is negative and you are supposed to subtract it from another. A number that does not have a sign in front of it, e.g., 3 – 1, is presumed to be positive; there is an implicit ‘+’ sign before it.

Nevertheless, there are some instances that the plus and minus signs are not as straightforward as 3 – 1 or 2 + 1. Some cases that can cause confusion look like this:

-3 – 1; -3 (4 – 2); 3 (-2 – 4).

Luckily, there are some helpful rules to guide you in dealing with such math problems.

Rules Guiding Addition and Subtraction of Numbers

1) Adding a negative means you subtract the number.

For example, 3 + (-2) = 3 – 2 = 1

2) Subtracting a positive means you subtract the number

For example, 3 - (+2) = 3 – 2 = 1

3) Subtracting a minus means you add the number

For example, -3 – (-2) = -3 + 2 = -1

Here, by adding 2 to -3, you have reduced the deficit. So, instead of having a ‘debt’ of 3, you now have only a ‘debt’ of 1.

To remember the rules quickly, think of things this way:

(a) Whenever the signs are alike, add.

(b) Whenever the signs are unlike, subtract.

Examples:

20 + (+12) = 20 + 12 = 32

20 + (-12) = 20 – 12 = 8

20 - (-12) = 20 + 12 = 32

20 – (+12) = 20 – 12 = 8

Going back to distribution, here is an expression:

- (3 + x^2)

Ordinarily, we are supposed to get rid of the brackets first, so now we have:

-3 - x^2

However, since this section is about distribution, we are going to introduce a different method.

As you look at the expression, $-(3 + x^2)$, remember that in math, any term without a coefficient is presumed to have 1 as its coefficient. In short, when you see x, y or z, there is an implied 1 before each one of the variables.

In our case, where we have $-(3 + x^2)$, it is presumed that the minus sign outside the bracket represents -1. It is, therefore, correct to write the expression $-(3 + x^2)$ as $-1 (3 + x^2)$.

The distribution now begins when, in the process of removing the brackets, you multiply each term inside the bracket with the -1 that is on the outside.

Hence, $-1 (3 + x^2)$ becomes $-1 \times 3 + -1 \times x^2$.

$-1 \times 3 + -1 \times x^2 = -3 + - x^2$

What we have just accomplished is distributing a negative sign and a number, the minus and the 1. However, it is also possible to distribute a variable.

How to Distribute Variables

Here, the idea is to distribute a variable within a set of brackets, and this is done the same way a constant or a sign would be distributed.

Take the example of x (y + 1).

All you need to do in distributing the variable outside the parentheses is to multiply it with each term that is on the inside of the parenthesis.

We are going to use the asterisk (*) to represent the multiplication sign to avoid the similarity the multiplication sign has with the letter x.

Hence, x (y + 1) becomes x * y + x * 1

We can proceed to solve this expression as shown below.

x * y + x * 1 = xy + x

If an expression with parentheses contains variables or numbers, do the distribution and solve in the same manner as above.

Here is an example:

The expression is: $4x (x^2 + 7)$

The number outside the parentheses serves as the multiplier, and you are supposed to multiply it with each term inside.

$4x (x^2 + 7) = 4x * x^2 + 4x * 7$

The next step is to simplify the expression

$4x * x^2 + 4x * 7 = 4x^3 + 28x$

Example 1

b = (c + d)

Once confronted with an algebraic expression that has parentheses, you expand it by multiplying each of the numbers or variables inside with the number or variable immediately outside the brackets.

• In this case, multiply c with a and then d with the same a.

• Retain the plus sign. In any case + times + = +, and in the expression given, all the variables are positive. When there is no sign of a plus or minus before a number or variable, the number or variable is presumed to be positive. As such, in this example a, c and d are all positive. In fact, they could be written as +a, +c and +d, only that would be redundant.

• In summary, therefore, when you want to expand an expression that has parentheses, multiply all the terms on the inside with the terms on the outside.

• b = (c + d) = bc + bd

Example 2

(b + c) (e + f)

• Take each term in the first bracket and multiply it with each term in the second.

• (b x e) + (b x f) + (e x b) + (e x f)

• be + bf + eb + ef

This kind of distribution helps those who advance to do binomial multiplication.

Example 3

(3 + 4x) (x – 6)

• As usual, multiply the terms, or numbers and variables, in the first bracket with each of the ones in the second bracket.

• You may begin with 3 times x, which will give you 3x.

• Then multiply 3 with -6, and you will get -18.

• Multiply 4x with x and you will get $4x^2$.

• Multiply 4x with -6 and you will get -24x.

• So now you have $3x - 18 + 4x^2 - 24x = 4x^2 - 21x - 18$.

Hence, $(3 + 4x)(x - 6) = 4x^2 - 21x - 18$.

Regarding the commutative property, the order of integers and variables involved in the calculation does not matter in the final results, whether you swap one order with another.

How the Commutative and Non-Commutative Property Work

Commutative Operators

Normally, an operator is described as being commutative when it produces the same results, irrespective of the order of the operands involved.

Operands are the integers and variables being calculated, and an operation is a plus, minus and such. When you have an expression or operation that fulfills this property, then you can term it commutative whether the expression or operation is purely arithmetic or algebraic.

Multiplication and addition are great examples of operations that are commutative.

Examples

• 3 + 5 = 8 and 5 + 3 = 8

• 58 + 2 = 60 and 2 + 58 = 60

• 3 x 5 = 15 and 5 x 3 = 15

• 58 x 2 = 116 and 2 x 58 = 116

As you can see from the above examples, the order of operands does not matter when you are adding or multiplying numbers. You will always get the same result.

• If you wish, you can summarize a + b = b + a

• You can also summarize a x b = b x a.

• You can also state that addition and multiplication are commutative operands.

More On The Concept Of Commutativity

This is a concept that, when used on the right figures, can save time in the exam room. The method uses the idea that you can rearrange the order of a number and still solve the problem. This, however, does not work on all problems. It is mostly ideal for addition and multiplication questions.

Example:

4 + 5 + 6 =15, 6 + 4 + 5 =15, 5 + 6 + 4 = 15.

A different form is 4 + 2 + 5 =11, 4 + 2 = 6 + 5 =11. In this case, it helps to automatically know that 4 + 2 is 6.

It does not work in all cases. In the subtraction case below, the commutativity method is not ideal. 6 – 5 =11, 5 – 6 = -1

A multiplication problem would look like this, 2 * 2 * 25 =100, 25 * 2 * 2 =100.

The above method promotes flexibility which is one of the main practices that the GMAT test seeks to promote, rather than solely focusing on arithmetic skills.

Non-Commutative Operators

Examples of operations that are not commutative are subtraction and division. For example, subtracting 2 from 6 is not the same as subtracting 6 from 2.

Suppose you have 6 oranges and you want to get rid of 2 of them. How many will you be left with?

• 6 – 2 = 4

On the other hand, if you have 2 oranges and someone asks you to give 6 people each 1 orange, what will happen? You will only manage to give 2 people an orange each before you run out. If you must give the remaining 4 an orange, you will have to borrow from someone else, and something borrowed needs to be returned at a later time. That makes something borrowed a debt. In math, you represent a debt with the minus sign, which is the subtraction operand.

• In short, 2 – 6 = -4

The fact that 6 – 2 = 4 while 2 – 6 = -4 proves that the subtraction operation is non-commutative.

Using the same operands, you can see that division is also non-commutative.

• 2 ÷ 6 = ⅓

• 6 ÷ 2 = 3

• Obviously, ⅓ and 3 are entirely different results.

Associative Operators

An operator is said to be associative if it does not matter how you group your operands or numbers. If you group the numbers you have in a certain way and get your result, then dismantle the original arrangement and create a new arrangement, and when you do your operations you get the same answer, this means the operator you are using is associative.

Example 1:

(2 + 8) + 5 = ?

2 + (8 + 5) = ?

In the first case, you are supposed to add 2 and 8 first, and then add your result to 5.

2 + 8 = 10, and when you add 10 + 5, you get 15

In the second case, you are supposed to add 8 and 5, and then add the results to 2

8 + 5 = 13, and when you add 13 + 2, you get 15

As you can see, 15 is the answer, no matter how you group your numbers. That then tells you that the plus operation is associative. If you want to test whether an operation is associative, ask yourself:

Does it matter how parentheses are used in the operation? If it doesn't, your operation has an associative property, and the reverse is true.

Is Multiplication Associative?

As has been pointed out, an operator is said to be associative when it does not matter which side a group of operands is on when solving the operation. In short, whether you solve a particular cluster first and then the other, or do your calculations in reverse order, the answer you will obtain is constant.

Examples

(i) (a x b) x c = a x (b x c)

(ab) x c = a x (bc)

In short, abc = abc

(ii) (5 x 3) x 2 = 5 x (3 x 2)

(15) x 2 = 5 x (6)

In short 15 x 2 = 5 x 6 = 30

(iii) (2 x 8) x 5

2 x (8 x 5)

In the first case, where you are supposed to multiply 2 and 8 first, you will get:

16 x 5 = 80

Then in the second case, where you are supposed to multiply 8 and 5 first, you will get:

2 x 40 = 80

Again, as you can see, it does not matter which side of the numbers has parentheses or which operands you work with first, because all will give you the same result. You have just confirmed that the multiplication operator is associative.

In summary, whether you are doing arithmetic or algebraic operations, both the plus and times operators have associative properties.

The minus and division operators are non-associative

On the contrary, if you are dealing with minus or division, it matters a lot what cluster of operands you work with first. Let us begin with subtraction.

(8 – 2) – 5 = ?

Here you are supposed to subtract 2 from 8 first, and then you subtract 5 from that result.

(8 – 2) – 5 = (6) – 5 = 6 – 5 = 1

But suppose you change the order of work, so that you have 8 – (2 – 5)

8 – (2 – 5) = 8 – (-3) = 8 + 3 = 11

Are you wondering how minus has borne a plus?

When you have brackets like these ones, you are supposed to multiply whatever is inside them with the number immediately on the outside, in a bid to get rid of the bracket.

In the latest example, at the stage where you have 8 - (-3), you need to multiply +1 by -3. You see, 1 x 3 = 3, 1 x -3 = -3, but -1 x -3 = +3.

This example serves to show that subtraction is not associative, unlike addition.

Division is also not associative.

Try 8 ÷ 2 ÷ 5

(8 ÷ 2) ÷ 5 = (4) ÷ 5 = $^{4}/5$

8 ÷ (2 ÷ 5) = 8 ÷ ($^{2}/5$) = $^{8}/1$ ÷ $^{2}/5$ = $^{8}/1$ x $^{5}/2$ = 4 x 5 = 20

In short, the division operator is not associative.

Exercise 16

(1) Solve the expression below.

$-2x(-5 + 6y)$

Answer choices:

$-10x - 12xy$

$10x - 12xy$

$-10x + 12xy$

$10x - 12y$

(2) Solve the expression 5,327 x 7 and do not use a calculator

Answer choices:

35,928

35,289

35,298

37,289

(3) By using distribution, simplify the expression, $5(6y + -1)$.

Answer choices:

$30y + - 5$

$30y + 5$

$30y - 1$

$6y - 5$

(4) Using the distributive property, simplify the expression, $-3(2z - 4)$

Answer choices:

$-6z - 12$

$-6z + 12$

$-3z - 4$

$6z - 12$

(5) You have been shown a rectangular plot whose size is 12ft by 15ft, and you have been asked to increase the measurement of each side by x. What will the new plot measure in terms of area?

Answer choices:

$180 + 2x$

$x^2 + 27x + 180$

$12x + 15x + 27$

$27x^2 + 180$

(6) Solve the expression given below.

$-3y(x + 6y)(3x - 4y)$

Answer choices:

$-9yx^2 - 42yx^2 + 72y^3$

$-9yx^2 - 54yx^2 + 18y^3$

$-9yx^2 - 54xy^2 + 18y^3$

$-9yx^2 - 42xy^2 + 72y^3$

Chapter 8: Quadratic Equations and Inequalities

Quadratic equations are found within algebra and they are a favorite of examiners, particularly on tests that require students to think creatively. For some SAT students, such equations are the reason they do not get the full score of 800 in math, yet the topic is not too difficult. However, it can be tricky if you do not understand the basics. As mentioned earlier in the book, in math, if you miss one topic, you are bound to find the topics that follow extremely difficult because math topics build on one another.

For quadratic equations, it is important that you understand the basics of working with equations and the sign rules, including how the negative sign behaves in multiplication, division and addition. If you miss those fundamental elements, dealing with quadratic equations will seem like rocket science. Luckily, with a book like this one, whose examples are elaborate and simplified, you do not have to lose marks over quadratic equations.

Incidentally, the word 'quadratic' was derived from Latin's 'quadrates,' meaning 'square.' Ordinarily, an equation is termed quadratic when it is laid out like this:

$ax^2 + bx + c = 0$,

where letter x denotes an unknown value, but a, b and c are known numbers. In a quadratic equation, a, b and c are the equation's coefficients and cannot be equal to zero. In situations where the value of a is zero, the equation is termed linear, rather than quadratic.

When working with a quadratic equation, you seek to find the value of a single unknown, and for that reason it is termed 'univariate.' When it comes to powers, the power of x in a quadratic equation cannot be negative. For that reason, a quadratic equation is described as a polynomial equation. Another important point is that in a quadratic equation, x can only have a maximum power of 2, and therefore the equation is seen as a second-degree polynomial equation.

When solving quadratic equations, the process employed is called factoring. Sometimes one can express a quadratic equation as a product of another expression such as:

$(px + q)(rs + s) = 0$

In case the quadratic equation is presented as in the latter expression, and not in the form $ax^2 + bx + c = 0$, there is something called the 'zero factor property' which indicates that, to satisfy the quadratic equation, either $px + q = 0$ or $rx + s = 0$. These two are linear equations, which, if you manage to solve them, will give you the fundamentals of your quadratic equation.

Students often learn how to factor by inspection when they encounter quadratic equations, beginning from the premise that either:

$(px + q) = 0$

or $(rs + s) = 0$

Practically speaking, this is how it works:

If you have been provided with the expression $x^2 + bx + c = 0$, the factorization expected is in the form of $(x + q)(x + s)$, where you are required to establish the value of q and s. These two unknowns, q and s, in essence, add up to b in the quadratic equation, and their product makes up c in the same quadratic equation. This is sometimes referred to as Vieta's rule.

Here is an example of what has just been explained.

You can factor $x^2 + 5x + 6$ to form $(x + 3)(x + 2)(x + 3)(x + 2)$

In some cases, it can become a little tricky and require a bit of trial and error, when the value of a in the quadratic equation is not 1. At the same time, often, quadratic equations do not require factoring by inspection. Instead, this method is suitable in scenarios where either $b = 0$ or $c = 0$.

Sometimes there is a method used, termed 'converting the square,' and this is how it is done:

The quadratic equation you have is $ax^2 + bx + c = 0$.

Take the 'a,' which is the coefficient of the squared 'x,' and divide both sides with it.

Next, take the constant term, c/a, and subtract it from each side of your equation.

Take the current coefficient of x, basically half of b/a squared, and add it to each side of the equation.

What you have just done is converting to a square, and the left side of the equation is now a perfect square.

Write the square neatly on the left side and proceed to simplify the expression on the right, if possible.

You can now create two linear equations just by taking the left side of your equation and creating its square root, and then equating that side to the square roots on the right-hand side that are positive and negative.

At this stage, you can proceed to solve the linear equations.

Example:

See how the equation $2x^2 + 4x - 4 = 0$ is solved below:

$x^2 + 2x - 2 = 0$

$x^2 + 2x = 2$

$x^2 + 2x + 1 = 2 + 1$

$(x + 1) 2 = 3$

$x + 1 = +-\sqrt{3}$

$x = -1 +-\sqrt{3}$

What the combined plus and minus signs indicate before the square root of 3 is that there are two solutions to the quadratic equation:

(i) $-1 +\sqrt{3}$

(ii) -1 k $- \sqrt{3}$

Inequalities Simplified

In the language of math, an inequality is a relationship that exists between two separate values when they do not equal each another. To show the existence of an inequality, the values are written in this manner:

$a \neq b$

This means that a is not equal to b. The inverse is also true in that b is not equal to a. Incidentally, this notation does not indicate that one of the values is greater. The information ends at denying equality.

However, in case the elements affected are part of an ordered set like real numbers or integers, their sizes can be compared as in notations like the following:

$a < b$, denoting that ‘a’ happens to be less than ‘b’

$a > b$, denoting that ‘a’ happens to be greater than ‘b.’

Note that even in these latter notations, ‘a’ is still not equal to ‘b.’ Relationships like these, between values, are referred to as strict inequalities. As such, you are permitted to read $a < b$ as ‘a’ being strictly less than the other value, ‘b.’ The reason for this clarification is that there are other relationships involving inequalities, which are not so strict.

Here are some examples:

(i) $a \leq b$

The above notation means that a is either less than b or equal to b. Under no circumstances can a be greater or more than b.

(ii) $a \geq b$

This second notation means that a is either greater than b or equal to b. Under no circumstances can it be less than b. Those students who end up pursuing engineering may find these notations used with more emphasis. For example, when a person wants to convey that a is much smaller than b, the notation used is $a \ll b$. Conversely, when the message to be emphasized is how much more a is than b, the notation used is $a \gg b$.

As math is a very practical subject, it is always good to know the math signs one is likely to come across in different areas, including real-life situations.

Properties of Inequalities

Inequalities have a few properties, which, if you remember them, will help you accurately and quickly solve math problems relating to inequalities. These are the most important properties of inequalities:

(a) Transitive property of inequality

When you have real numbers as represented by a, b and c:

Whenever $a \geq b$ and $b \geq c$, $a \geq c$

Whenever $a \leq b$ and $b \leq c$, $a \leq c$

When you have either premise being a strict inequality, you can draw a final conclusion of strict inequality in the following cases:

When $a \geq b$ and $b > c$, $a > c$

When $a > b$ and $b \geq c$, $a > c$

Considering $a = b$ implies $a \geq b$, the implication, therefore, is that:

When $a = b$ and $b > c$, $a > c$

When $a > b$ and $b = c$, $a > c$

(b) The converse property

This regards the relationship between ≤ and ≥. The two are actually converse to each other. As such:

Where real numbers are concerned, say, a and b:

When $a \leq b$, $b \geq a$.

When $a \geq b$, $b \leq a$.

(c) The addition and subtraction property

When $x < y$, $x + a < y + a$.

Sometimes you may find an additional constant in your expressions, often a 'c.' Sometimes it is added to both sides of the inequality, and other times it is subtracted from both sides.

In case the inequality you are dealing with has real numbers as represented by a, b and c, it follows that:

Whenever $a \leq b$, $a + c \leq b + c$ and also $a - c \leq b - c$.

Whenever $a \geq b$, $a + c \geq b + c$ and also $a - c \geq b - c$.

(d) The multiplication and division property

Under this property:

When $x < y$ and $a > 0$, it follows that $ax < ay$

When $x < y$ and $a < 0$, it follows that $ax > ay$.

There are other properties associated with the multiplication and division of inequalities as shown below.

When dealing with real numbers, as represented by a, b and a c that is non-zero:

Whenever c is positive, multiplying or dividing by c does not alter the inequality:

When $a \geq b$ and $c > 0$, $ac \geq bc$ and also $a/c \geq b/c$.

When $a \le b$ and $c > 0$, $ac \le bc$ and also $a/c \le b/c$.

Whenever c is negative , multiplying or dividing by c inverts that inequality:

When $a \ge b$ and $c < 0$, $ac \le bc$ and also $a/c \le b/c$.

When $a \le b$ and $c < 0$, $ac \ge bc$ and also $a/c \ge b/c$.

(e) The additive inverse property

The properties that apply whenever you have a state of additive inverse are:

When your inequality has real numbers, as represented by a and b, having a negative inverts that inequality so that you have:

When $a \le b$, $-a \ge -b$

When $a \ge b$, $-a \le -b$

(f) The multiplicative inverse property

There are a number of properties guiding an inequality's multiplicative inverse state as you will see below.

Whenever you have real numbers, such as a and b, which are non-zero, and both are either positive or negative, these properties must be fulfilled:

When $a \le b$, $1/a \ge 1/b$

When $a \ge b$, then $1/a \le 1/b$

Whenever one of the real numbers, a and b, happens to be positive, and the other one is negative, the properties that must be fulfilled are:

When $a < b$, $1/a < 1/b$.

When $a > b$, $1/a > 1/b$.

Sometimes these inequality properties are written in the form of chained notation, as follows:

When you have real numbers, such as a and b, which are non-zero:

Whenever $0 < a \le b$, it follows that $1/a \ge 1/b > 0$.

Whenever $a \le b < 0$, it follows that $0 > 1/a \ge 1/b$.

Whenever $a < 0 < b$, it follows that $1/a < 0 < 1/b$.

Whenever $0 > a \ge b$, it follows that $1/a \le 1/b < 0$.

Whenever $a \ge b > 0$, it follows that $0 < 1/a \le 1/b$.

Whenever $a > 0 > b$, it follows that $1/a > 0 > 1/b$.

Chapter 9: How Algebra Helps in Day-to-Day Life

In algebra, letters and other symbols are used as representatives of numbers and quantities within formulas as well as equations.

An equation is a form of math sentence that contains an equal sign, which clearly states that two given expressions are either similar in meaning or stand for the same number. Individual equations usually have variables as well as constants. Equations are used to express factual math information in a short form that is easy to recall. They help solve math problems fast.

People use algebra on a daily basis without even considering it to be algebra. Consider:

A shopping environment

Example 1

Suppose you are shopping and have picked 15 items. When it comes to packing the items, you want to know how many bags you need.

You need to know how many items one bag can hold. Suppose you establish that each bag will hold only 5 items.

To know exactly how many bags you need, you have to do a quick mental calculation:

$15 \div 5 = 3$

In short, you need 3 bags to carry 15 items, with each bag carrying 5 items.

Example 2

Suppose you went to a shopping mall and picked 30 items. You want some bags to carry the items in and must determine how many bags you need.

In this situation, you can probably tell immediately that you need a big bag and establish that one of those bags can hold 7 items.

To find out exactly how many of such bags you will require, you need to do a quick mental calculation, following a suitable algebraic formula:

Number of items bought = x

One bag's capacity = y

Number of bags needed = $x \div y$

$x = 30$

$y = 7$

$x \div y = 30 \div 7 = 4\ ^2/7$

Are you really going to give your answer as $4\ ^2/7$? Is anyone going to provide $^2/7$ of a bag? Of course not! Practically speaking, you can only have an additional bag to carry the 2 items that remain after you have filled 4 bags with 7 items each.

In a SAT exam or any other exam, you need to think beyond the formulas. Think of the applicability of the answer you are giving.

If the exam paper has such a question in a multiple-choice section, you are likely to have 5 bags and $4\ ^2/7$ among the answers to choose from. If you are working mechanically, it is very easy for you to get excited at seeing $4\ ^2/7$ among the choices given, as that is the accurate answer that comes up in the division. However, when you think of what your answer represents, logic will tell you that you cannot use a fraction of a bag to carry items bought from a store or supermarket, so you can only take a fifth bag.

In this example, 5 bags is the correct answer.

Example 3

You are planning to drop by the store to buy some items and want to know how much money to take with you. You would like to buy a dozen eggs, 2 loaves of bread and 3 bottles of fresh juice. The price for each of those items is \$10, \$2 and \$5, respectively. How much money must you take with you?

Calculation:

Call the price of a dozen eggs a, the price a loaf, b, and the price of a bottle of juice, c.

The total cost can be expressed algebraically as $(a \times 1) + (b \times 2) + (c \times 5)$

For a dozen eggs, you will need 1 x \$10, which is \$10.

For 2 loaves of bread, you will need 2 x \$2, which is \$20.

For 3 bottles of juice, you will need 3 x \$5, which is \$15.

To find out the total amount you need you must add the money required for all the items, which is \$10 + \$20 + \$15 = \$45.

The answer is \$45.

Example 4

Suppose you are at a gas station and want to fill your tank. After checking your pockets, you realize you only have \$30 on you. If the price of gas is \$5 per gallon, how many gallons of gas will you afford to buy for your car?

Calculation:

To get the total cost of the gas:

Let the price of gas be p.

Let the number of gallons of gas bought be g.

Term the total cost of the gas c.

To find your answer you can form an algebraic equation:

c = g x p

You already know the total amount of money you plan on spending, which is $30. In short, c = $30.

You also know the unit price of gas, which is $5. In short, p = $5.

The only thing you do not know at this juncture is how many gallons you will buy, and that is represented by g. With only one unknown in an algebraic equation, it is easy to find the value.

In this case, when you substitute the known variables with their actual value, the equation reads like this:

$30 = g x $5

In order to leave the unknown alone on one side, divide each side by $5:

$30 ÷ $5 = (g x $5) ÷ $5

6 = g

In an algebraic equation, it does not matter how you arrange the constituent parts because either side is equal to the other. So you can simply say:

g = 6

Hence, since the question asks how many gallons of gas you can afford to buy for your car, you can give the answer as 6.

Exercise 17

Ten equations are given below, followed by some statements. Choose the equation that best represents each statement.

n / 6 = 3

n – 6 = 18

6n = 36

6 + 18 – 9 = 15

n – 18 = 6

¼n = 6

^{6} / n + 12 = 18

6n + 2 = 18

n + 6 = 18

6 + 15 – 9 = 12

(1) An unknown number plus 6 is equal to 18.

(2) 6 is a quarter of a certain number.

(3) The sum of 6 and 18 minus 9 is equal to 15.

(4) The sum of 6 and 15 minus 9 is equal to 12.

(5) An unknown number divided by 6 gives you 3.

(6) An unknown number times 6 is equal to 36.

(7) 6 is the difference between an unknown number and 18.

(8) A unknown number minus 6 is equal to 18.

(9) An unknown number times 6 plus 2 is equal to 18.

(10) 6 divided by an unknown number plus 12 is equal to 18.

How to Work Out Word Problems

How many times do you come across real-life situations that can solved mathematically? The answer is more often than you realize. A look at some examples will help you quickly notice a situation that can be clarified or resolved through math equations.

Example 1

You have the number 216.

The question is: What three consecutive numbers can add up to exactly 216?

First of all, you must know the meaning of consecutive numbers. They are numbers that follow one another. For example, 1, 2 and 3 run consecutively just as 97, 98 and 99 are consecutive numbers.

Have you noticed you only add 1 to a number to get the next consecutive number? This point is important in formulating a helpful equation.

Begin by identifying a variable to work with, such as x. In short, you want three consecutive numbers and the first one is x.

What do you expect the second consecutive number to be? You need to add 1 to x.

You now have the first number as x and the second consecutive number as $x + 1$.

How do you find the third consecutive number? You need to add 1 to the second number in order to have $(x + 1) + 1$.

You now have your three consecutive numbers whose sum is 216, represented in an equation which is $x + (x + 1) + ((x + 1) + 1) = 216$

It is now time to work out the equation. Begin by getting rid of the brackets.

$x + x + 1 + x + 1 + 1 = 216$

Add all the unknowns and also add the known values, all of which equal 216.

$3x + 3 = 216$

You now need to leave the unknown value alone on one side by getting rid of 3.

You can only eliminate 3 mathematically by subtracting it from either side of the equation

$3x + 3 – 3 = 216 – 3$

The left side is now without the 3, while 216 on the right side of the equation has been reduced by a similar amount, 3.

$3x = 213$

What is the value of x?

Divide either side by 3 to establish the value of x.

$3x ÷ 3 = 213 ÷ 3$

$x = 71$

Now that we know what the value of x is, and what the three consecutive numbers adding up to 213 are in terms of x, it is easy to find what each number is.

The first number is x, so obviously it is 71.

The second number is x + 1, so it is 71 + 1 = 72.

The third number is x + 2, so it is 71 + 2 = 73.

In fact, you might as well pick the very original equation you created:

$x + (x + 1) + ((x + 1) + 1) = 216$

$71 + (71 + 1) + ((71 + 1) + 1) = 216$

$71 + 72 + (71 + 2) = 216$

$71 + 72 + 73 = 216$

Hence the answer is: The three consecutive numbers that add up to exactly 216 are 71, 72 and 73.

You can test your answer by adding the three numbers, 71 + 72 + 73, and you will find the answer is truly 216.

Example 2

5 girls went to a concert as a group, and their tickets cost them $55 in total. They also bought an assortment of candy and it cost them $25.

Question : Calculate the cost per person using an algebraic equation.

Solution :

Let the cost of one ticket be x.

Total cost of 5 tickets = 5x

5x = $55

Therefore, what is x?

You need to divide each side by 5, and in the process the unknown will remain alone on one side.

5x ÷ 5 = $55 ÷ 5

x = $11

Let the cost of candy per person be y.

Total cost of candy for the 5 girls is 5y.

5y = $25

Divide each side of the equation by 5 so that you are left with the unknown alone on one side.

5y ÷ 5 = $25 ÷ 5

y = $5

Total cost per person = x + y

As such, the correct answer is $11 + $5, which is $16 .

If you want to be sure that you are right, calculate 5x + 5y to see if it gives you $80, the total of $55 + $25.

If x = $11 and y = $5, then 5x + 5y = (5 x $11) + (5 x $5) = $55 + $25 = $80.

That operation has just confirmed the calculation is correct.

Exercise 18

(1) What are the three consecutive numbers which, when added together, result in 156?

(2) It costs a kindergarten teacher $50 to feed her class of 20 in one day. If 5 children miss school one day, how much will she spend if she only

buys enough for those present?

(3) The area of a rectangle is $72cm^2$ and the width is twice its length. What are the dimensions of the rectangle?

(4) It takes John 9 hours to mow 3 lawns. How long, on average, does it take him to mow one lawn?

(5) Mary's team has spent more money this weekend than it has spent before for recreation over any one weekend. In fact, $1000 is just ⅔ of the total amount they spent. How much has Mary's team spent this weekend?

(6) In the equation 5x + 6 = 16, what is the coefficient of x? Pick the answer from the choices given below:

(a) 6

(b) 2

(c) 5

(d) 16

Chapter 10: How to Work with Proportions and Ratios

Are you preparing for the SAT, GRE or GMAT? Whichever exam you may be studying for, ratios and proportions will always feature. In fact, just as in real life, the skills you have gained with ratios and proportions come in handy in an indirect manner. In short, you do not even need to be preparing for an exam in order for you to benefit from the knowledge of how to apply ratios and proportions.

In the field of mathematics, a ratio is the relationship that exists between two numbers, one that indicates how many times one number can fit on the other. Think of a basket of fruit that you want to take to a friend who is recovering from a bout of the flu. You want to take her 2 watermelons, after all, she can't finish one of them in one sitting. In the same basket, you want to include some passion fruits, probably 20 of them, your estimate being that she is going to consume between 5 and 6 of them in a day.

The ratio of watermelon to passion fruits, once you have packed your basket with 2 watermelons and 20 passion fruits, is 2 : 20.

In math, it is always good to keep numbers simple whenever possible, especially because it helps you remember them and also makes working with them easy. Along that line of thinking, you can simplify the ratio 2 : 20 by dividing each side by 2, and you will end up with a ratio of 1 : 10.

Hence, the ratio of watermelons : passion fruits is 1 : 10

You can also have the same ratio arranged differently, so that you begin with the passion fruits, followed by the watermelon.

The ratio of passion fruits: watermelons is 10 : 1

You can also have the ratio of each fruits in proportion to the total number of fruits in the basket. The basket has 2 watermelons and 20 passion fruits, and so the total number of fruits is 22, which you get by adding 2 + 20.

The ratio of watermelons to total number of fruits is 2 : 22

Though the ratio is correct, it is best to simplify it, and very likely, any answer from a selection of multiple choice questions will be in its simplified form.

To simplify 2 : 22, you need to divide each side by 2.

2 : 22 = 1 : 11

Hence, the ratio of watermelons to total number of fruits is 1 : 11

The ratio of passion fruits to total number of fruits is 20 : 22

We need to simplify the numbers by choosing a common number to use for both sides. Whereas 10 can be used on 20, it cannot be used on 22 because there will be a remainder. You can also divide 20 by 5 and it will give you 4, but when you try dividing 22 by the same number, as you are supposed to, you get 5 remainder 2. However, you can divide 20 by 2 and 22 by 2 and there will not be a remainder on either side.

Hence, 20 : 22 = 10 : 11

What this ratio means is that for every 11 fruits you have in the basket, 10 of them are passion fruits, whereas the previous equation of watermelon : total number of fruits, 1 : 11, means that for every 11 fruits you have in the fruit basket, one of them will be a watermelon.

At this juncture, it is important to point out that ratios do not always stand for absolute numbers or absolute value. In the example about a basket with 2 watermelons and 20 passion fruits, the ratio 2 : 20 represents absolute numbers, while the ratio 1 : 10 does not. However, the simplified form does not distort anything, because in reality, if you multiply both sides by 2, you still end up with 2 : 20.

In short, the simplification is meant to show the proportion of one number to the other, and if you want to fill a bigger basket using the same ratio, you will find it easy to determine the actual number of watermelons to put in and how many passion fruits to include.

In ratios, units do not matter. You can have ratios of lengths, time, number of items, weights; any quantity or measurement.

Ratios as Fractions

When it comes to presentation, ratios can be in ordered pairs, as in the examples above, or they may be in the form of fractions. In fractions, when you intend to put things in a ratio of 2 to 3, for instance, you put 2 on top, making it the numerator, and you put 3 below, making it the denominator. In this case you would have $^{2}/3$ as the ratio, and you can also write the same comparison as 2 : 3.

In fact, a ratio is a mere comparison of two different things. You cannot have a ratio of the same things. If you have 5 buns baked the same way in all respects, you cannot create a ratio out of them. To create a ratio that the buns are a part of, you need to have another item such as muffins. You can then say, for example, the ratio of buns : muffins is 2 : 3, 1 : 2, or whatever ratio you want.

Otherwise, to have a ratio involving one item, you need to have a distinct difference to represent by way of the proportion. For example, you may want to represent the difference of burnt buns to good ones. If your boss is wondering why you did not distribute the buns you were meant to deliver from the bakery, you may wish to convincingly explain that there was a mishap in the kitchen and it was significant enough to warrant cancellation of the day's deliveries. To show that there were more messed-up buns than good ones, you may use a ratio and say the ratio of burnt buns : good buns is 5 : 1. The same ratio can be presented as good buns : burnt buns is 1 : 5.

You use the term proportion to show one part of the ratio. For example, only 1 part out of 6 parts in every lot of buns is good. This means the ratio of good buns to burnt buns is 1 : 5. When you see the ratio as 2 : 20 in the example given about fruits, it means all the fruits in the basket are 22 in total.

When you have a fraction to represent a ratio or proportion, the total parts are made of the sum of the numerator and the denominator when you are comparing two items.

To make sure you frame your answer correctly, you need to appreciate the language of ratios. In this regard, it is important to keep in mind that the ratio of x to y is represented as x : y, while the ratio of y to x is represented as y : x.

If you have a group of 50 people, comprising men and women, and only 20 of them are men, what is the ratio of men to women? It is 20 : 30. To find the number of women among the group, you just need to subtract the number of men from the total number of people: 50 – 20.

Suppose you want to describe the proportion of the group that is made up of women. You need to know how many women there are in the group in comparison to the total number of people.

There are 30 women in the group of 50 people.

The proportion of women in the group is $^{30}/50$.

Ratios as Percentages

Ratios can also be expressed as percentages. All you need to do is multiply the representative fraction by 100%. For example, when you say that the proportion of women in the group is $^{30}/50$, you would be correct to express the same information as a percentage by multiplying that fraction by 100%.

$^{30}/50 \times 100\% = 60\%$.

In short, you can confidently say from the above information that in the group of 50 people, 60% of them are women.

Exercise 19

Choose the correct answer from the choices given.

(1) There are 8 hens and 2 cocks in the compound. What is the ratio of cocks to hens?

(a) 2 : 10

(b) 8 : 10

(c) 2 : 8

(d) 8 : 2

(2) There are 8 hens and 2 cocks in the compound. Suppose you want to double the number of birds and, at the same time, change the ratio of hens to cocks to 9 : 1. How many cocks would you have?

(a) 1

(b) 9

(c) 18

(d) 2

(3) The school bookshop stocks fiction and non-fiction books, and many of the non-fiction books are biographies. At the beginning of the year, there were 500 titles of fiction books and 300 biographies. The total number of titles at the time was 1,200.

What proportion of the titles made up the non-fiction books?

(a) ^{300} / 1,200

(b) ^{700} / 1200

(c) ^{300} / 500

(d) ^{500} / 1200

(4) Reduce the ratio 250 : 750 to the simplest terms

(a) 1 : 4

(b) 250 : 1,000

(c) 1 : 3

(d) 25 : 75

(5) The school bookshop stocks fiction and non-fiction books, and many of the non-fiction books are biographies. At the beginning of the year, there were 500 titles of fiction books and 300 biographies. The total number of titles at the time was 1,200.

Which ratios represent absolute numbers?

(a) 300 : 1,200 and 1 : 4

(b) 500 : 700 and 300 : 900

(c) 400 : 1,200 and 4 : 12

(d) 4 : 12 and 3 : 5

(6) In the first test Mr. Kim gave his large class of 48 students this year, the proportion of students who passed to those who failed was 4 : 8. How many students failed the exam?

(a) 8 students

(b) 16 students

(c) 32 students

(d) 40 students

(7) In the first test Mr. Kim gave his large class of 48 students this year, the proportion of students who passed to those who failed was 4 : 8. How many students would you say understood the topics covered on the test?

(a) 8 students

(b) 16 students

(c) 32 students

(d) 40 students

(8) The school bookshop stocks fiction and non-fiction books, and many of the non-fiction books are biographies. At the beginning of the year, there were 500 titles of fiction books and 300 biographies. The total number of titles at the time was 1,200.

What was the percentage of non-fiction books in the school bookshop? Give your answer to 2 decimal places.

(9) There are 8 hens and 2 cocks in the compound. Suppose you want to double the number of birds and, at the same time, change the ratio of hens to cocks to 9 : 1, What will be the percentage of hens?

(a) 80%

(b) 16%

(c) 90%

(d) 10%

(10) Simplify the ratio 110 : 99

(a) 209 : 110

(b) 10 : 9

(c) 99 : 10

(d) 1 : 9

Best Way to Simplify Ratios

If you are asked to simplify a ratio, find the largest common factor by which you can divide each number that is part of the ratio, and the resultant ratio will be the simplest possible.

Example 1

Question : Reduce the ratio of $15 to $75 to its simplest form.

Solution :

Ask yourself: What is the biggest number I can use to divide both $15 and $75?

The answer is $15.

$15 : $75 divided by $15 gives you 1 : 5

Notice that the dollar sign has also canceled out. However, if you had the dollar sign on only one number and not the other, you would leave the dollar sign intact.

Example 2

Question : Reduce the ratio of $30 to $120 to its simplest fractional form.

Notice another important term here, in addition to 'simplest,' is 'fractional.' This means you are required to give your answer in the form of a fraction. When doing the SAT, GMAT or any other exam, it is important that you give the answer that has been asked. If you say things that are factual or true but they are not the ones that the examiner is looking for, you can fail. So you need to note an important keyword such as 'fractional' so that you do not leave your ratio as a plain proportion, but rather present it as a fraction.

Solution :

$30 : $120 = $30 / $120 in fractional form

To simplify the fraction, $30 / $120 , you can divide the numerator by $30 and the denominator by $30 as well, and the resultant fraction is ¼. This is because $30 ÷ $30 = 1 and $120 ÷ $30 = 4

Another point to note is that other unit signs can also cancel out the same way the dollar sign has canceled out, as long as the sign appears against both numbers of the ratio or percentage.

Sometimes the signs, which are often referred to as units, are referred to as designators. Although the unit is not a factor, it is treated in the same manner as a factor when it comes to division or cancelation within a fraction.

Example 3

A car consumes 30 liters of gasoline when it travels a distance of 330 kilometers.

Required: Reduce the ratio of 30l : 330km to its simplest fractional form.

You may notice the two numbers have different units, one being liters and the other being kilometers. In such a case, those units do not cancel out. On the contrary, they will be expected to surface in the final, simplified form of the fractional ratio.

Solution:

30l : 330km in fraction form is ^{30}l / 330km .

To simplify 30 and 330, you need to find a common factor, and in this case you can use 15.

30 ÷ 15 = 2 and 330 ÷ 15 = 22. So at this juncture, the simplified form of the fraction is ^{2} / 22. However, a cursory look at the fraction shows that the numerator and denominator can be divided further by a common factor, 2. This means the fraction you currently have is not the most simplified form of the ratio 30 : 330.

2 ÷2 = 1 and 22 ÷ 2 = 11

Hence, 30l : 330km in the most simplified fractional form is ^{1} / 11 ltr/km.

In words, this fraction can be expressed as one-eleventh of a liter per kilometer.

Of course, you could have used 30 as the largest common factor to get the most simplified form of the fraction, ^{30}ltr / 330km , which is ^{1} / 11 ltr/km.

Example 4

Here is the same question, framed differently. A car covers 330 kilometers using 30 liters of gasoline.

Required : Reduce the ratio 330km : 30ltr to its simplest fractional form.

Solution:

Divide both numbers with the same factors and the units will not cancel out since they are different.

The fractional form will, therefore, be ^{330} / 30 km/ltr

This fraction can be simplified to its utmost to become ^{11} / 1 km/ltr.

In real-life situations, the assessment on the rate of fuel usage by a car is reported in terms of number of liters per 100 kilometers. In our current example, if a car consumes one liter over a distance of eleven kilometers, how many liters will it be expected to have consumed after covering 100 kilometers?

Solution: 1/11 ltr/km x 100, which is 9 liters per 100 kilometers, or 9ltr / 100km.

This example demonstrates why it is useful to include the units when they are not the same in a ratio. If, for instance, you were to leave out the liter or the kilometer units, the ratio would be meaningless or confusing.

As has been stated before in this book, SAT questions test areas you should have covered in school, but they become tricky because they are set up differently from the way questions are framed in school exams.

On the SAT, GMAT, GRE and other such exams, you are often given questions based on practical scenarios or real-life situations, and it is up to you to think of the best formula or trick to use to solve the problem. You may have a short narrative followed by a question, but you are not told to use conversion factors, ratios, percentages or anything like that. It is up to you to make the appropriate deductions.

Consequently, it is important to know the kind of scenarios likely to be portrayed within a math context.

1) A shopping environment

This is a favorite for examiners, because they can ask about prices in different units of quantity and even in different units of currency. Given that ratios are mathematical relationships, they are used very frequently in grocery shopping.

Take, for instance, a situation where a 1-liter bottle of fresh juice costs $10 and a 2-liter bottle of the same fresh juice costs $18. Which is more fairly priced, or are they priced the same way?

To answer this question, you need to use ratios. The question does not mention ratios or proportions at all, and it is up to you to think broadly and determine which of the mathematical operations can help you solve the problem.

For the 1-liter bottle of juice, the ratio is 1ltr : $10

For the 2-liter bottle of juice, the ratio is 2ltr : $18, which you can simplify to 1ltr : $9

The ratios in their simplest forms show the difference in the pricing of the different bottles of fresh juice. People buying the 1-liter bottle are paying $10 for it, while people buying the 2-liter bottle are paying a dollar less for 1 liter of the same juice. In short, the 2-liter bottle is more fairly priced than the 1-liter bottle.

2) In the kitchen

Often, ratios are used in recipes. The amount of one ingredient to use in comparison to another is very important, and how well the appropriate ratios are used can mean the difference between producing a delicious meal and messing up everything.

Can you imagine, for instance, confusing the ratio of salt to sugar in a cake recipe, so that instead of measuring 1 teaspoon of salt to 2 cups of sugar you measure 2 teaspoons of salt to 1 cup of sugar? It would still be a mess if you mistook the ratio of 1 tablespoon of baking soda to 2 cups of sugar and instead measured 2 tablespoons of baking soda to 1 cup of sugar. You might not end up with an edible cake.

3) In travel

Ratios are often used to establish the time a car takes to travel from one point to another, and even how much fuel it requires to cover a given distance.

Example

If the car you are riding in is traveling at 80 kilometers per hour or 80km/hr., and from your starting point to your destination is 120 kilometers, you can expect to take one and a half hours on the journey. How do you arrive at this amount of time?

Working :

The car takes 60 minutes to cover 80 kilometers

How many minutes does the car take to cover 1 kilometer?

It takes (^{60} / 80) minutes to cover 1 kilometer.

How long will it, therefore, take for the car to cover 120 kilometers?

It will take (^{60}min / 80 km) x 120km

60min divided by 20 = 3min;

80km divided by 20 = 4km;

4km divide by 4km = 1

120km divide by 4km = 30

What you are left with, therefore, is ^{3}min / 1 x 30

3 minutes x 30 = 90min

You need to ignore the 1 at the bottom because any number divided by 1 is the same number.

How to Work With Simultaneous Equations

Simultaneous equations involve more than one variable, and they are all solved at the same time. This means that you are supposed to work out the entire expression at one go, rather than seeking one unknown at a time. If you are to pass your SAT math test with a high score, or even other tests like the GMAT and GRE, it is important that you recognize scenarios where simultaneous equations are applicable.

The most important step in solving a simultaneous equation, next to formulating it, is recognizing whether addition is required in solving the equation or if subtraction will work. Depending on the operator that you use, you will be able to solve one unknown after the other has canceled out, and subsequently, you will be able to deduce the value of the other unknown.

One thing you need to keep in mind is that if you are using subtraction, the negative sign should be distributed all across the equation, and not used on just one side of the equation. If you use the sign on only one side of the equation, the answer you get will be wrong. Unfortunately, if you are doing the SAT exam and similar ones that provide multiple choice questions, chances are that your wrong answer will be one among the choices given, because examiners know there is a likelihood some students will make that kind of mistake.

As you try to learn the correct way of solving a math problem, remember that just because the answer you get happens to be among the answers given does not make it right. If you are multiplying the numbers to make certain unknowns match, remember to do the multiplication all across the equation and not just on one side or on some operands.

If you want to know if you need to multiply your equation by 2 or 3, or even by a negative number, just place one equation above the other. That way, just a glance at the two equations together will let you know the best way to handle the equations, beginning with which variables you want to eliminate before proceeding to solve the final equation. Knowing how to organize equations in order to solve them simultaneously gives you a better chance of scoring highly on your SAT math.

In order to be certain that using simultaneous equations is necessary, check the question properly to establish what the examiner is looking for. Usually you will require such equations when the examiner wants you to solve a problem with two unknowns.

Chapter 11: How to Work With Conversion Factors

A conversion factor is the number that you use to convert a set of units to a different set of units. Such a change is done either by way of multiplication or division. Whenever conversion of a unit or a set of units is required, there must be a suitable conversion factor available. For example, if you have a math statement given in kilometers, yet the answer is required in meters, you can use the conversion value 1 kilometer equal to 1,000 meters.

Other examples of conversion values:

60 seconds equal 1 minute

12 inches equal 1 foot

1,000 milligrams equal 1 gram

An important point to note is that the different units of measurements used in the conversion express the same property. For example, when you speak of 1 hour, you are referring to the same thing someone else is calling 60 minutes. Or you could speak of 1 hour and 60 minutes in different sentences of the same paragraph when referring to the same thing.

As for distance, the units used in conversion can include meters, kilometers and miles, and length as a whole can include the units of distance and others like feet, inches, centimeters and so on.

In math, some students get panicky because of little things like solving a problem in a certain unit and then having the question demand an answer in a different unit. Often, the students who panic and think the problem is difficult are those who have not come across such questions before. In high school, it is usual for a math problem to have a certain unit consistently from the beginning to the end. However, in questions such as those found in the SAT, GMAT, GRE and other such exams, the units are mixed up.

This book contains questions of that kind, where sometimes distance is given in miles but the answer is required in feet or the question is detailed in minutes but the answer is required in fractions of an hour. With constant practice, the need for conversion of units comes to mind very fast just by looking at the problem once.

Example 1

Ryan would like to know the number of seconds in 3 hours and 36 minutes.

First of all, just as we saw regarding unknowns like x and y, if you want the answer in one particular unit, it is difficult to work with units that are different. As such, in this case, it is important that you change hours and minutes to one common unit.

1 hour = 60 minutes

Therefore, 3 hours = 60 x 3

So, 3hrs 36min = (60 x 3) + 36 = 180 + 36 = 216 minutes

You are now one step closer to the actual answer, but you are not there yet because the answer is about seconds and not minutes. Keep going back to the question, lest you choose the wrong answer, because if you have a multiple- choice question like this one, chances are that 216 will appear as one of the choices. If you are not careful, you may be tempted to choose it, not noting that the answer is required in much smaller units.

1 minute = 60 seconds

Therefore, 216 minutes = 216 x 60 = 12, 960 seconds

Hence, the answer is 12,960 seconds.

Pay keen attention to the question, to see if it states the answer should be given to a particular significant figure.

The Meaning of Significant Figures

A significant figure or significant digit refers to a digit that gives a number the closest meaning or significance. It is the figure that tells you with closest precision what the value of the entire number is.

If someone offers you a job and tells you the annual salary for that post is $63,240, you may not go into details when telling your closest friend the news. You just want your friend to have a good idea what the value of the offer is. As such, you may give the salary as one significant figure. Which number among the five in $63,240 gives the best idea about the amount of pay the company is ready to pay you? Obviously, it is 6, which is in the ten thousands place value slot.

$63,240 to one significant figure is $60,000.

On the other hand, you will wish to be as accurate as possible when making tax returns, so you are likely to report your salary to four significant digits, which is $63,240. In fact, the reason you cannot report the salary to five significant figures is that the fifth significant digit is a zero.

If the company promises to increase your annual salary to $70,000 after a probationary period of one year, the only way you can report that salary is to one significant figure, and that significant digit is 7 because the zeros do not add any value to the number.

Although you are expected to give your answer to the significant digit the question requires, it is important to keep in mind that the purpose for which you want to know the figure often influences the accuracy. For example, when timing professional runners, seconds are significant. Particularly in short races, even fractions of seconds matter a lot.

It is not unusual to find 2 athletes crossing the finish line almost at the same time, to the extent that a casual observer sees them as crossing the finish line simultaneously. Yet the timers for the two athletes read differently, one reading 10.55 seconds and the other reading 10.56. In this latter example, the runner with a time of 10.55 seconds has beaten the one whose time is 10.56 seconds by 0.01 seconds. This fraction of a second is very significant and can mean the difference between getting a gold medal and a silver medal or getting a medal and not getting one at all.

As such, even when the question has not required the answer in any significant digit, you need to be realistic when giving your answer, so that you do not round up numbers that serve best when given to the closest accuracy.

In case you forget which side to begin looking for the significance of a number, ask yourself what you would like to win in a lottery: $ 1,000 or 100,000. Obviously, you will be judging the value of these numbers from the left, and the minute you see 1 versus 100 and it is about money, you know 100 is always bigger and better. Both numbers have three zeros to the right but what matters to you is the left, as that is where the significance begins.

In astronomy and other fields of science, the significance of a digit helps scientists know how far to round their numbers. Knowing the significant digit helps researchers deal with very large numbers and very small ones while still portraying the intended information with relative precision. Mind you, very small numbers come in decimals that may not always be easy to work with. You may have observed that in math, for example, Pi, whose value is 3.141592, is often taken to be 3.14.

The Importance of Place Value

The place value concept is very important in understanding and using the number system. And, although digits from 1 to 9 are considered very important as far as value is concerned, zero also plays a significant role in the number system. Some people think of zero as nothing, and while that is one meaning of zero, it is sometimes used as a placeholder for another absent number or value. Think about a thousand. If you assumed zeros are valueless and tried to omit them, would the thousand you wanted to show be a thousand anymore? Of course, 1,000 minus the three zeros becomes one—an entirely different number of comparatively low value.

In the case of the zeros in 1,000, the zero next to 1 sits in the place of hundreds, because there are no ‘hundreds’ in that number but there is a place for it; the second zero sits in for tens because the number has no ‘tens’ but there is a place for tens and the last zero sits in for ones because there are no ‘ones’ though there is a place for them. Zeros are also very important within the decimal number system.

When learning how place value works, you can begin with the last digit on your right when dealing with whole numbers. The very first one is in the place of ‘ones.’ As you move one step to the left, multiply one by 10 to get the next place value, which is ten. So the second slot from the right represents tens. Any number in that slot is considered to be in the tens value.

For example, in the number 621, 1 is in the place value of ‘ones’ while 2 is in the place value of ‘tens.’ To get the other place values as you move left of the number, you are supposed to multiply every step by 10. That is how you get the third slot from the right, referred to as the ‘hundreds’ when speaking of place value.

It is usual to put a comma after every three digits from the right, and that gives the number easier readability. Often when numbers are written with commas, they are said to be written in standard form.

Why do people prefer to rely on knowledge of place values instead of writing out the digit values themselves? For one, once you have acquired the skills of recognizing place values, you will know the value of a number to the most significant digit in a fraction of a second. Conversely, reading other forms of number representation, especially in numerical form, can be clumsy and difficult to read.

Look at some numbers below as represented in different forms:

(i) One billion, eighty million, five hundred forty thousand.

1,080,540,000

(ii) Seven sixteen thousand, two hundred and thirty-one.

716,231

(iii) 9,000,000 + 50,000 + 30 + 1

9,050,031

Looking at the three examples immediately above, is it not correct to say the standard method of representing numbers is easier to work with?

Rules for Determining How Significant a Number Is

There are some standing rules when trying to identify significant digits in given numbers.

All non-zero digits are significant

All numbers from 1 through to 9 are significant digits. So whenever you see a 3, a 4 or any other number through 9, irrespective of the number’s position, consider it significant.

Examples:

You have been given five numbers: 258, 36.5, 139,248, 22 and 9.

How many significant digits does each number have?

Answers:

258 has 3 significant digits

36.5 has 3 significant digits

139,248 has 6 significant digits

22 has 2 significant digits

9 has 1 significant digit.

Any zero in between digits that are non-zero is significant

This means that if the zero you have in the number is sandwiched between two numbers from 1 to 9, the zero is significant. What is the importance of considering a zero significant? It means it is treated the same way you treat the numbers from 1 to 9 when counting significant digits. Look at some examples below:

How many significant digits are there in the numbers 6,702, 109, 892,007, 101,218,503 and 2,008?

(i) 6,702 has 4 digits

(ii) 109 has 3 digits

(iii) 892,007 has 6 digits

(iv) 101,218,503 has 9 digits

(v) 2,008 has 4 digits

Any zero appearing before non-zero digits is not significant

Have you ever considered that you do not see numbers written as 070, 0, 220, 121, and so on? Such a zero would be redundant as it would have no value, so whole numbers are never preceded by zeros. However, numbers in decimal form do have zeros at the beginning, but they, too, are not significant.

See the following examples:

(i) 7 x 1/1,000 = 0.007

(ii) 28 x 1/1,000 = 0.028

(iii) 542 x 1/10,000 = 0.0542

In the above examples, 0.007 has only one significant digit; 0.028 has two significant digits and 0.0542 has three.

Zeros appearing after an integer, but not in between integers, are not significant

If you have 20,000, 6,200, 40 and 101,000, the number of significant digits in these numbers is 1, 2, 1 and 3, respectively.

Exercise 20

(1) What is the value of 6 in 18, 627 and 020?

Answer choices:

Six twenty-seven

Six hundred thousands

Six hundred twenty-sevens

Six

(2) What is the value of 6 in 4,220,936,000?

Answer choices:

Six hundred

Sixes

Six thousandth

Six thousands

(3) What is the value of 6 in 1,060,888?

Answer choices:

Sixty thousands

Sixty-eight thousands

Six and eight thousand

Sixty

(4) How many significant digits are there in 0.00458?

Answer choices:

5

6

3

458

(5) How many significant digits are there in 0.971?

Answer choices:

971

3

9

4

(6) How many significant digits are there in 0.1088?

Answer choices:

4

5

2

88

(7) How many significant figures are there in 9,857?

Answer choices:

10,000

9,857

9

4

(8) How many significant figures are there in 63,021?

Answer choices:

63

5

6

63,021

(9) How many significant numbers are there in 10,000?

Answer choices:

1

5

10,000

10

(10) How many significant digits are there in 2,350?

Answer choices:

4

5

3

235

Chapter 12: Typical Mix of SAT, GMAT and GRE Questions

Exercise 21

1) A canteen has 40 tables of different sizes, which have the capacity to hold 600 people. Some of the tables can hold 10 people while the rest can hold 20. Find the ratio of tables with the capacity to hold 10 people to those with the capacity to hold 20 people.

Answer choices:

1 : 4

1 : 2

1 : 3

1 : 1

2) You need to build a sequence, and the sequence begins with m. For the terms that follow, every one of them is 5 more than ¹ / 10 of the term immediately before it. Keep in mind that m is not equal to zero.

Find the ratio of the term that follows the very first one.

Answer choices:

(m + 1) / 50m

(m + 50) / 10m

(m + 5) / 10

(m + 10) / 5

3) There are 2 cars traveling over a distance of 630km. One of the cars, x, moved at an average speed of 70 kilometers per hour, while the other one, y, moved at a speed of 90 kilometers per hour.

How many kilometers had car x traveled when car y reached the end of the journey?

Answer choices:

490km

140km

630km

700km

4) Deduce the answer from the table below.

Each letter in the table indicates the number of student players in each category. For example, a number of students are male who play basketball.

Which of the answer choices given below is equal to i?

a + d + g

g + h

a + b

a + d

5) The lawn in front of the office block can be mowed by 8 people within 6 hours. Suppose 3 people fail to report for duty and do not take part in the mowing of the grass. How long, in terms of hours, will each of the remaining people take to complete mowing the lawn?

Answer choices:

$6^{2} / 5$ hrs

$9^{3} / 5$ hrs

$2^{1} / 4$ hrs

$12^{1} / 8$ hrs

$5^{1} / 2$ hrs

6) Water flows through a medium-sized pipe at the rate of 20 cubic meters per minute. Suppose you want to fill two tanks that are cube-shaped, each of them 20m in length, and you are to use 4 pipes that are similar to the one that has just been described. How long, in terms of minutes, will it take to fill both tanks?

Answer choices:

40

200

100

50

20

7) Suppose you want to host a party, and the hall at your disposal provides for 1 person per 5 square foot area. This particular hall has a width of 60 feet and a length of 50 feet.

If you want the people attending the party to be comfortable, what is the maximum number of people you can invite?

Answer choices:

600

500

2,500

1,500

3,000

8) You have been given a 10-gallon water jug, and each gallon can hold 8 pints. Each of those pints, in turn, can hold 2 cups.

How many cups of water will it take to fill the 10-gallon jug?

Answer choices:

160

80

26

220

20

9) Paula was able to walk at a quick pace of 4 miles per hour. Considering that 1 mile comprises 5,280 feet, how long, in terms of feet, will Paula have traveled in 40 minutes' time?

Answer choices:

15,000

15,840

8/3

14,080

3

10) There is a box containing 240 marbles of different colors. Some are red, others are blue, while still others are green. The ratio in terms of the different colors of the marbles is red : blue : green is 5 : 2 : 1.

⅓ of the red marbles are removed from the box.

⅔ of the green marbles are removed from the box.

Out of all the marbles remaining in the box, what fraction of the marbles still in the box will be blue?

Answer choices:

1/3

6/13

7/18

1/2

6/17

Chapter 13: Median, Mean and Standard Deviation

Sometimes the easiest topics may appear hard for a number of reasons, and a student can end up failing to pick up easy marks on an exam. One reason is that the student may have listened to the topic in class without paying much attention because the terms sounded familiar. When the teacher mentioned that the meaning of mean was average and that median stood for something in the middle, it was tempting for the student to think that was all there was to the topic.

However, in order to be able to excel on an exam, it is important to have the exact meaning of the terms in a mathematical context. In an exam environment, you need to be able to calculate the mean given a variety of numbers and scenarios, and to be able to identify the median, or calculate it, if necessary.

This section of the book will show you what the terms median, mean and standard deviation mean, and what you need to do in order to give the right answer in an exam situation.

Meaning of Mean

To be able to identify the mean, you need to have a set of data. Once you have your data set ready, you get the mean by finding the numerical average of the data therein.

Mean Example :

Mr. Tom has been coaching 10 students in math. Here are their marks for the most recent test:

88, 95, 69, 75, 75, 84, 86, 92, 96, 80

Question :

What is the mean performance?

Solution

Ask yourself:

What do I get as the sum once I add all the values in the data set?

How many values are there in the data set?

Now divide the sum by the number of values in the data set.

In this case, to find the sum of marks scored by the students, you are supposed to add 88 + 95 + 69 + 75 + 75 + 84 + 86 + 92 + 96 +80

The sum of marks for all the students is 840.

Each score, e.g., 88, is a value. So in this case, the values are 10.

To find the mean, just divide 840 by 10.

840 ÷10 = 84

Hence, the mean of the data set is 84.

Finding the Median

Median Example:

Find the median in the data set from Mr. Tom's class.

To find the median, you need to begin by organizing the data values either in decreasing order or in increasing order.

Then find the value right in the middle, and that will be the median.

In the case of Mr. Tom's students' marks, the values are:

88, 95, 69, 75, 75, 84, 86, 92, 96, 80

When organized in order of magnitude, the values are:

69, 75, 75, 80, 84, 86, 88, 92, 95, 96

Oops! The values make an even number, 10, so no single value is left distinctly in the middle. What happens now?

When the situation is like this, where the data set splits in two equal halves without leaving a clear value in the middle, you have to pick the two innermost values; those next to each other in the middle. In short, you need to go to the middle and pick one value from either side of the two halves.

Add the two middle numbers and get their sum

69, 75, 75, 80, 84, 86, 88, 92, 95, 96

Divide that sum by two.

In the case of the marks from Mr. Tom's class, the middle values are 84 and 86.

84 + 86 = 170

170 ÷ 2 = 85

Hence, the median is 85.

(i) Suppose by the time Mr. Tom gives the next class there is an additional student who is very good at math but wants to attend a tutoring class to ensure she does not leave anything to chance. In the next test, the class scores are as follows:

77, 78, 88, 97, 88, 87, 90, 96, 97, 98, 100

The sum of the marks is 996

The class mean will be 996 ÷ 11

Notice that the marks have not just increased because of the students' improvement in performance, but also because of the new student's marks.

996 ÷ 11 = 90.55

The mean is 90.55.

What is the median?

(ii) Here you need to organize the values in the data set in order of magnitude, either in ascending or descending order. So for the marks scored by Mr. Tom's students, you will have:

77, 78, 87, 88, 88, 90, 96, 97, 97, 98, 100

This data set has 11 values, so you can leave 5 values on either side and pick the value right in the middle as the median.

77, 78, 87, 88, 88, 90 , 96, 97, 97, 98, 100

In this case, the median is 90.

Luckily, some terms in math do not need any memorization. Median, for example, evokes the thought of medium, and automatically the sound or thought of medium brings to mind the middle—not really large and not really small.

Standard Deviation

Standard deviation refers to a number that is relied upon to describe how a group's measurements are spread out from the mean or average. Sometimes, instead of describing how far the group's measurements are spread out from the mean, the number representing the standard deviation describes how far the group's measurements are spread out from the expected value. It is more or less like talking about deviating from the norm, only that standard deviation is measured with precision.

When the standard deviation is low, it is an indication that most of the numbers in the data set are in very close proximity to the average. On the other hand, when the standard deviation is high, it is an indication that the numbers comprising the data set under consideration are vastly spread out.

The Greek letter sigma is used in its lowercase to denote standard deviation: σ

The Importance of Standard Deviation in Real Life

Although standard deviation is included in the school curriculum and is tested on the SAT and other serious exams, often the importance of the topic is lost amid other more common math topics.

Often, in print and electronic media, when statistics are given about the structure and performance of a company, per capita income, price indices and the like, there is talk of the average and, sometimes, the mean, but rarely is there mention of standard deviation. Yet the standard deviation provides a closer picture of the scenarios reported on than the averages. This is just to point out that standard deviation is undervalued, and you need not follow suit. If you want to make a good decision based on statistics, do not end at the mean, the median or even the mode. Go a step further and you will get a truer picture of what is happening whether it is the market, a company or anywhere else.

If you make a point of finding the standard deviation, you get to see how varied the data set before you is. But if you rely on the mean alone, you get an incomplete picture. Hence, the decision you make based upon that statistic might mislead you. If you have an attractive median, you might be tempted to think the rest of the data is just as attractive, whereas it might just be a coincidence that the attractive number fell right in the middle. There are times, for example, where half of the class has not understood the topic of the test, and their marks reflect that. However, if one or two of the poor performers missed the test, the median is likely to fall on the side of the higher marks.

If a new teacher gives the lessons and relies on the median mark to determine whether the class has understood the topic or not, he will be misled by the median and will proceed to teach other topics when half of the class cannot use the concepts of the older topic to understand subsequent topics.

The crucial part of standard deviation is that it indicates how varied the data within the data set is. If, within a company, members of management earn a thousand times more than the average employee, you will get that image from the standard deviation, a picture the mean and the median do not portray. Standard deviation shows you how close the data available is from the average or mean. If it is close, then you can tell that even the average and median are reliable, but if it is far away, then you know there is more that needs to be explored regarding the data.

In short, the smaller the standard deviation, the more reliable the given data is. It confirms that the given average is representative of the entire data. This is important because usually there is not enough time to evaluate whole populations of data, and it is always good to know that the figure given as the mean provides a good image of the reality. Just as an example, you may witness a certain company release information, indicating that its average entry-level salary is $80,000. While that is pretty impressive, suppose you learn that the standard deviation for entry salaries is $25,000.

Needless to say, this is where you begin imagining the wide range of possibilities there are for your starting salary, including figures that are much lower than $80,000. $25,000 makes for a very wide gap when it exists within the same data set, in this case within the bracket of beginners' salaries. This means the figure of $80,000 is not helpful to a potential employee who is trying to assess whether a move to the company would be toward greener pastures or not.

If, on the contrary, the company's starting salary has a standard deviation of $3,000, you will feel more convinced that the figures at your disposal are a reflection of what you are likely to earn as a new employee. When the standard deviation is small, the general statistics available are appealing as these are an indication that there are no extreme variances.

Just so you can appreciate how important standard deviation is, you can have two data sets with the same average, yet internally, the terms are incomparable. The values of the two data sets can be so different that you might find it incredible the two even have a common average.

Compare these two data sets:

(i) 199, 200, 201.

The mean of this data set is 200.

The standard deviation of this set of data is 1.

Obviously, a standard deviation of one is very small.

(ii) 0, 200, 400

The mean of this data set is 200.

The standard deviation of this data set is 200.

An average of 200 here is very big.

Seeing two data sets with the same mean have standard deviations that are a whole world apart is an indication of how unreliable the average is on its own, and how useful standard deviation is in revealing disparities within a given data set.

Since the two data sets just analyzed are small, you can analyze individual terms within each data set and see how true the standard deviation is. For the first data set, you can see the individual terms or values are almost the same. Every one of them is very close to the others—the 199, 200 and 201. On the contrary, and true to the standard deviation, members of the second data set are varied. There are huge differences among them—the 0, 200 and 400.

You need to look at standard deviation as a statistical term referring to the measurement of variability or even dispersion around a given average. In technical terms, standard deviation effectively measures volatility. When you talk about dispersion, you are referring to the difference that exists between the actual values and the value held as average. The more this variability or dispersion increases, the higher the standard deviation rises.

Example 1

Suppose you were to invest in the stocks of Company X, which says its average year- on-year return is 25%. You may not even consider the merits of investing in another company whose year-on-year return is 20% on average. What you may not have considered while ignoring the stocks of other companies is that the average does not reveal the real situation for individual years. That attractive return of 25% might have been attained by averaging a few exemplary good years with some other years of extremely poor performance.

That is the kind of revelation that standard deviation discloses, so that you can see it has not always been rosy for this company, and there are probably going to be some years where the return will be very low. In this company whose average return is 25%, it will not be surprising to find one year that had a return of 2% or even no return at all.

However, without more disclosures or the benefit of standard deviation, investors are likely to commit their money to stocks whose performance is prone to high fluctuations. Serious investors are wary of committing their money in a volatile market, and a company whose performance is not steady or consistent can be termed volatile. Nevertheless, investors who do not know what else to look for in telling whether a company is solid or not are likely to be impressed by the single figure of a good average return.

Standard deviation enables potential investors to assess the risk involved in investing in the company, and even if it is not given outright in the company prospectus, serious investors can go the extra mile to have a professional calculate it before committing big monies to buying stocks. After all, just as stability is important in personal life, so it is important in business. Nobody wants to exit a company when the stock value is far lower than it was when they first invested, yet that is what happens when a company's returns keep dropping or having big fluctuations.

A company whose stock returns are 30% one year, 10% the following year and 15% the subsequent year, could drop to -2% the year that follows. The minute investors start questioning the stability of the company following its unsteady performance, they are likely to decide to sell before things become worse. If, for instance, investors try to offload their stocks onto the market after seeing a drop of the returns to 15%, this may lead to an excessive supply of the company stocks as compared to the demand. Then, like the sale of any other commodity, such a scenario is likely to lead to a drastic drop in the price of company stocks. Seeking to know the standard deviation from the beginning is very helpful because it preempts the chances of panic selling that leads to capital losses.

How to Calculate the Standard Deviation

Many students have missed marks from math problems with standard deviation just because they were not well-prepared. There are times teachers delve promptly into curves when discussing standard deviation, and that often makes the topic look complicated. Luckily, as you will see in the explanations below, calculations of standard deviation develop smoothly from calculations of the mean.

(i) Calculate the mean of the set of data you have.

(ii) Take each of the values in the data set and subtract the mean in order to find how much each value has deviated from the mean.

(iii) Next, square each of those deviations.

(iv) Add up those squares. You now have the sum of deviation squares.

(v) Divide that sum by the number of values or items in the data set.

(vi) Finally, find the square root of that average.

Is that not simple? Of course, the explanation is lengthy and might be somewhat discouraging, but you will be surprised to see how easy the calculation is. Below is a demonstration of how to find the standard deviation.

This data set has: 2, 4, 4, 4, 5, 5, 7 and 9

(i) Calculate the mean of the set of data you have.

(iii) (2 + 4 + 4 + 4 + 5 + 5 + 7 + 9) = 40 ÷ 8 = 5

(ii) , (iii) and (iv) Subtract the mean from each value and square the deviation.

____

(v) Divide that sum by the number of values or items in the data set.

Divide 32 by 8, because all the items in the data set are eight.

32 ÷ 8 = 4

(iv) Find the square root of the average you have just calculated.

$\sqrt{4} = 2$

Hence, the standard deviation of the given data set is 2.

Importance of Mean, Median and Mode in Daily Life

Mean happens to be one type of average among others like the median and the mode, but mean is the most commonly used. Usually when people refer to the mean, they are referring to the arithmetic mean.

This favored average, the arithmetic mean, is very simple, and the way to get it is to add the value of all the items you have been given, and then divide the total by the number of items. As has already been demonstrated in this chapter, if 4students have scored, for example, 80, 78, 90 and 92, to find the average you need to work the problem out as follows:

(80 + 78 + 90 + 92) ÷ 4

So you will end up with 340 ÷ 4, and the answer is 85

Therefore, the mean is simply 85.

In fact, whatever field you are in, including science, this is the method you will be called upon to use to find the mean. The challenge arises when the arithmetic mean you so ably calculated does not give you a good picture of the situation at hand. Any idea what those scenarios could be like?

Example 1

Suppose you had a different set of scores:

99, 96, 45, 46, 35, 97, 99, 100, 43, 99, 41, 100 =

The mean of this data set is:

(99 + 96 + 45 + 46 + 35 + 97 + 99 + 100 + 43 + 99 + 41 + 100) ÷ 12

900 ÷ 12 = 75

The mean, as you can see, is 75.

However, since you have the advantage of seeing the scores of the twelve individual students, you can tell with certainty that there are some students in that group of twelve—in fact, half of the group—who require remedial action because their scores are below 50%. Yet a new teacher, relying on the mean of 75%, might not think that group needs tutoring, citing their brilliance. Or the principal might recommend that the group be eliminated from the groups requiring extra assistance. In the meantime, 6 students from that group are on the verge of failure and are in dire need of attention.

Example 2

Suppose you are looking to change jobs and are at the stage of doing due diligence to check out companies that might suit you. You have been working for Company X for 5 years and your annual salary for the last 3 years has stagnated at $60,000.

You then discover that there is a small company with 9 employees, whose average annual salary is $72,000. That average attracts you, and when you ask around, you learn that the chances of you getting hired are very high, everything considered. You are excited and about to get busy compiling your résumé and sending your application, until you casually ask one of the company employees a question: How long how has your salary been in the range of $70,000?

Certainly, you are concerned about leaving your comfortable current job, where the only problem is the management's delay in reviewing employees' salaries, only to have your salary stagnate at a new company for years on end. Your question is shocking to the employee from the other company and he asks, "Who says I'm even at $50,000?"

The employee is gracious enough to give you the pay details of that company, and you learn that the mean does not in any way reflect what many of the employees earn.

For starters, you learn that the owner of the company is also on the payroll as he works there too.

You then learn that of the 10 employees, 9 of them earn an annual salary of $36,000 each.

At this juncture, you are wondering: How can this be if the mean of $72,000 is correct?

You are then informed that the company owner is the tenth employee, and his annual salary is $396,000.

In short, the data set for the employees' annual salary looks like this:

36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 396,000

The mean of this data set is ((36,000 * 9) + 396,000) ÷ 10

((36,000 * 9) + 396,000) ÷ 10 = (324,000 + 396,000) ÷ 10 = 72,000

This is how deceptive mean can be if you do not have any other pertinent information. In this company, you have the salary of a single person pulling up everyone else's numbers, thus raising the mean to a figure that is misleading for purposes of decision-making.

What this tells us is that the mean salary can be taken at face value without being misleading only if the disparity in remuneration is not very big.

Where the disparities are significantly big, the median might help to provide a better picture of the situation.

Let us see the answer you would have gotten if, instead of the average salary, you had been provided with the median. The data set is:

36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 396,000

This is one of those instances where you do not have a distinct figure right in the middle of the data set, as the terms within the data set form an even number. What you have here are two amounts at the center, which you need to add together and then divide by two to get the average, which will then serve as the median. Incidentally, these numbers happen to be equal—36,000 and 36,000— so, automatically, the median annual salary is $36,000.

Certainly, if you had known the median salary you would have known to rate the company's attractiveness lower, considering that your current salary of $60,000 is well above the company's median. In fact, with the company's median at $36,000, chances are that you would have to be in top management to earn a salary of $60,000. A picture then begins to form in your mind about what kind of company this is. For example:

This must be a very small company.

Probably, the company is just starting.

Maybe the company is experiencing tough financial times.

Probably, the company is under receivership.

Whatever the reason for the salary structure, your most important question has been answered: This company is not what you are looking for. And you are able to deduce that from the median salary.

Does that mean the median is the best number to use for decision-making? Well, not always. Sometimes you may have the mean and the median, yet you cannot make a credible decision.

Example 3

You are still doing due diligence to see which company is right for you since you want to leave the company where you currently work. Let us analyze one of the companies you have your eye on and see how their remuneration structure looks like. First of all, bear in mind that the company's mean annual salary is $144,000, and similarly, the median salary is $144,000.

The mean salary being attractive, and the median being equal to the mean, might give you hope that this company is what you're looking for. However, more details might distort that image.

The company has 10 general employees, 7 assistants, 3 managers and a single owner. In your case, everything considered, you can only apply for the post of general employee, and an entry-level salary for that position starts at $36,000. Can you see how far that starting salary is from the mean and the median? But can you really have the median at $144,000 when there is someone earning $36,000 per annum?

On further inquiry about the company's remuneration structure, you learn that the 10 general employees each receive an annual salary of $36,000; the 7 assistants each receive an annual salary of $144,000; the 3 managers each receive an annual salary of $324,000 and finally, the company owner receives an annual salary of $684,000.

See how it works:

36,000, 36,000, 36,000,36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 144,000 , 144,000, 144,000, 144,000, 144,000, 144,000, 144,000, 324,000, 324,000, 324,000, 684,000

The median here must be around the eleventh term, considering there are 21 items in this data set.

As such, the median is, as stated at the beginning, $144,000.

The mean, of course, is ((36,000 x 10) + (144,000 x 7) + (324,000 x 3) + 684,000) ÷ 21

((360,000) + (1,008,000) + (972,000) + 684,000) ÷ 21 = 3,024,000 ÷ 21

3,024,000 ÷ 21 = 144,000

So it is correct that the mean annual salary for this company, as indicated at the beginning, is $144,000.

Here is a scenario where both the mean and the median are impressive, yet your potential salary, according to the company's remuneration scale, is very poor. Luckily, there is yet another average you can use in such circumstances, and that is the mode.

Mode is the figure that comes up most often among all other given figures. It is referred to as the most frequent term.

In Example 1 above, where the data set is 99, 96, 45, 46, 35, 97, 99, 100, 43, 99, 41, 100, the mode is 99, because it is the term that appears most frequently among all 12 numbers—3times.

In Example 2, where the data set is 36,000; 36,000; 36,000; 36,000; 36,000; 36,000; 36,000; 36,000; 36,000; 396,000, the mode is 36,000, because it appears a whopping 9 times among 10 values.

And in this most recent example, Example 3, where the data set comprises 36,000, 36,000, 36,000,36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 36,000, 144,000 , 144,000, 144,000, 144,000, 144,000, 144,000, 144,000, 324,000, 324,000, 324,000, 684,000, the mode is 36,000, because it appears 10 times while no other salary appears beyond 7 times.

In Example 3, if you had known that the most common salary the company pays is $36,000 per year, you would not have considered the company as a potential employer, at least not in the short-term. You would compare that against your decent annual salary of $60,000, and you would move on to research other hopefully better-paying companies.

As you can see, all the averages you have learned—the mean, the median and the mode—are useful, but each is best suited for different scenarios. As such, it is good to be equipped with the skills for calculating each of them, so that when a situation arises where a particular one is most suited, you can apply it and make the best judgment call.

Exercise 22

1) Calculate the standard deviation of the data set comprising test scores for 5 math students.

The scores are 68, 92, 88, 52 and 80.

2) There is another data set comprising the following scores:

52, 92, 92, 52 and 92.

What is the standard deviation?

Chapter 14: Journey to Attaining 600+ on SAT Math

It is often said it is good to aim for the best, and in the case of SAT math, the best is a score of 800. Nevertheless, there are circumstances that can lead you to admit that the best you can hope for is a score of 600. This part of the book is dedicated to ensuring that you do not drop below 600, with the possibility of doing even better.

According to the marking and grading of SAT math, to attain a score of 600, you need to do 38 out of 58 problems correctly. That results in a score of 65%. Essentially, therefore, if you can only get two-thirds of the questions correct, you will be on the safe side. Although some students fret about SAT, the fact is that while 65% may not be impressive on a class test, on SAT math, it may be good enough to help you get the overall score you are aiming for.

The reason for discussing math scores in relation to an overall target score is that usually students have particular colleges in mind as they take the SAT exam, and you do not want your math score pulling your overall score down. At the same time, you shouldn't be too scared of attempting the SAT just because you think you aren't good enough for a score of 800. In any case, Harvard, MIT and institutions of that cadre can only admit so many students, so even if everyone scored the maximum possible, some students would still be left out.

In reading this book, hopefully you will take note of your preparedness right now and after the boost you get from learning simpler exam skills, then assess how much more you need to do in order to reach a 600. For example, if currently you can only attain 480 in SAT math, your aim should be to get 15 more questions right, or even 14, in order to get a 600. Armed with that knowledge, you can choose a few more topics that you usually wouldn't attempt because you have no idea how to tackle them, or learn better skills for handling the topics you aren't very certain about.

In short, there is no need to give up just because you do not rate yourself among the top math scorers. There is always a strategy you can adopt to attain a math score that will raise your overall SAT score to what you want. This section of the book has strategies that are easy to implement, so that you can study and prepare in a way that will help you improve your SAT math score to the maximum.

Easy Strategies to Enhance Your SAT Math Performance

(1) Ignore the most difficult problems at first

Remember, this section of the book is about helping you score at least 600 in SAT math, and, as has already been pointed out, you only need 38 correct answers out of 58. Do you see how much leeway you have when it comes to which questions to do and which ones to leave out? There are 20 questions you can play around with if you are certain you have done justice to the other 38. The point here is that you have room to compartmentalize the 20 questions you find most difficult which comprise 30% of the entire set of SAT math questions.

The reason this strategy is so helpful is that it makes you comfortable solving the problems you are good at, and also helps you tackle the ones you only have a little difficulty with. That way, you can have time to reread the questions you understand well, but whose style of presentation makes you doubtful about how to solve them. The extra time you get from ignoring the most difficult questions will allow you time to try out different methods of tackling those questions you are not sure about.

Students who are normally pressed for time will benefit greatly from this strategy if only they know how to go about it. You do not want a situation where you spend too much time reading questions in detail, only to abandon them untouched. Luckily, math questions are ordinarily organized in degrees of toughness. This means the easy ones come first and the toughest are last.

How best can you use this information to enhance your math performance? The answer is to allot your time optimally to the easy questions. In Section 3, for example, there are 20 math questions to be tackled within 25 minutes, and that is a big constraint on time for everyone except those who know all math topics inside and out. In reality, you only have 75 seconds to solve a full question correctly.

Can you imagine an average math performer trying to attempt every one of those questions, each within slightly more than a minute? If one does not panic and only tries to solve as many problems as possible, what usually happens is that such students make very many careless mistakes, missing questions they would otherwise have gotten right. Worse still is that, at the end of the day, the rush leaves them with too many minutes at their disposal, which they spend trying to unsuccessfully decipher the last questions. Since, as has been noted, the most difficult questions appear at the end, that’s not a good use of time.

Nine Best SAT Math-Scoring Strategies

A workable solution is to forget that the last 20% of the SAT math questions in every subsection exist. In short, focus your energy on the first 80% of each subsection, which you are likely to feel confident about.

When referring to subsections, bear in mind that sections 3 and 4 of the SAT math exam have two subsections each, the first part comprising multiple choice questions, and the second comprising questions that you answer by expressing yourself the best way you know how. It is the first 80% of each subsection that you are being encouraged to tackle . Then, of course, if you still have time in your hands you can look at a couple of questions from the relatively tougher ones in the second subsections. Remember, by virtue of reading this book you have already improved your proficiency in math, and you have tricks ready to use on questions that would have scared you before.

To see the reality of how simple it is to gain marks within seconds, as well as how easy it is to lose marks struggling with a problem you have little chance of solving, see the example below.

Example

If $(ax + 2)(bx + 7) = 15x^2 + cx + 14$ and that is true for all the values of x, and also $a + b = 8$, what are the possible values of c?

Answer choices:

3 and 5

6 and 35

10 and 21

31 and 41

Does this look like a problem you can solve easily within 75 seconds? Following the reasoning just presented about the merits of prioritizing the easiest math questions, you had better skip this one and try an easier one.

Incidentally, this was a real question presented on one SAT math exam, and it appeared towards the end of the multiple choice questions. Then, on the same exam, the opening question in the next subsection was so easy that you could smile while answering it:

If $t > 0$ and also $t^2 - 4 = 0$, do your calculations and say what the value of t is.

How much easier can a question be? Surely, if $t^2 - 4 = 0$, you can mentally switch 4 to the other side and leave t^2 alone on the left, so that you have $t^2 = 4$. What number can you square to get 4? That is all the question is asking in a very straightforward way.

Of course, that is either -2 or 2, but the question guides you by stating that the value you are looking for, t, is more than zero. Can a negative ever be more, or even equal to, zero? No. So, straightaway, you have your answer: $t = 2$.

It is a question, which, although it has taken a whole minute to write about, can take you less than the allocated 75 seconds to solve, and you can say with certainty that you got it right.

Here is a schedule you can use as a guide to show you how many questions you should target in every section to be able to clinch a minimum of 600 in SAT math. Such a schedule might look redundant for math gurus, but for those not very confident in math, it can mean the difference between having a great overall SAT score or a dismal performance.

Going by this schedule, you will have raised the time meant for one question to 104 seconds, which is around 1¾ minutes. With such ample time, it is possible to tackle the questions with medium difficulty and still get them right, while scoring the highest possible for the easy questions.

Do you remember why the schedule above recommends you skip a whole 12 questions? It is to let you work on questions you are assured of getting right. The schedule is meant to help you avoid wasting time doing problems, only to realize halfway through that you cannot solve them, and in the meantime you have wasted time you could have used to solve other questions with ease.

For that reason, if you feel confident about math, by all means, tackle everything as laid out. But you need to be confident enough that your math proficiency can get you anywhere from 700 to 800. If you keep practicing math, especially samples from past SAT exams, you will be able to rate yourself in a way where you can tell if you are a 600 or an 800.

However, do not limit yourself. By now you must have seen from the skills taught in this book that some math problems that appear strange and difficult are easy to solve as long as you are equipped with the right skills, many of which are contained in this book. In short, the advice directed to students aiming at 600 scores is not meant to put a ceiling on anyone’s potential. Rather, it is a case of preparing for the worst as you hope for the best. So, learn as many skills as you can to prepare you to reach a math score of 800, but if exam time arrives and you do not feel very confident, have the schedule as your fallback plan so that you do not slip below a score of 600.

Another point to note is that in the free-response subsections, you are not penalized for writing wrong answers. So write what you think makes sense, even if you are not sure about it. You may end up earning some additional points that you would have missed if you left those questions entirely blank. This includes the 20% portion we said you could ignore. You should only ignore that last part of each subsection as long as you have not solved the preceding 80% that can earn you easy marks.

(2) Strengthen Your Weak Areas

The first strategy explained above addresses the manner in which you need to handle your SAT exam as you take it. This second strategy addresses the time before you actually take the SAT. How do you best prepare? Getting the best book available in math and trying to revise from the first page to the last is not the best way, considering not many people seeking to do the SAT have enough time for preparation. The SAT is not ordinarily the only thing on a person’s mind. You might have soccer, basketball, chess, or another game you want to practice. There are parties to attend and other things to do with friends and family. Even household chores consume a chunk of a person’s time.

How do you then ensure you can still prepare optimally for your SAT exam with all the other commitments you have? Work smart. Eliminate your math weaknesses one by one, study the math areas you are weak in or the topics you are not confident about. Identify those areas are that you are weak in. Whether you do that by going through your past papers and seeing the problems you missed, or redo some timed past papers, or just use your personal knowledge, list those topics or sub-topics and use that list to guide your revision.

What some students do, unfortunately, is go straight to the familiar questions, those that they are almost certain they will get right. This strategy is very helpful once you are already in the exam room, but unhelpful when you are reviewing for the test.

The essence of revising and preparing for the exam is to ensure that, by the end of the session, you have learned something new, such as how to confidently solve a math problem you would not have tackled before. In short, you review for a math exam so that if you were originally set to attain a score of 500, you can now say for certain that you are 600 material, and if initially felt that the best you could earn was a 600, after reviewing you can say with confidence that the least you will score is a 700 or 750. For a person in the range of 700-750, your revision should prepare you for 800. In short, revise with a purpose, and be results oriented.

Practically speaking, if you are great at algebra but geometry scares you, ignore algebra in your revision unless you are doing a timed revision paper. Study geometry from the beginning even if it means getting an elementary level book. Let your revision be progressive, so that you are moving from one elementary step to the next until you reach high school geometry. You are likely to notice the point at which you began to lose your step, and that is the place you should concentrate on until you understand the concept and how it connects with subsequent geometry topics.

For any topic, once you have identified the point at which the topic begins to become unpalatable, investing time in that area and trying to study it from different angles is the best approach. Do lots of practice on the newly-learned topics, the areas you previously considered your weak links, just to ensure you know them as well as your favorite topics.

Remember: avoid the temptation to impress yourself during revision, solving problems from your favorite topics over and over again, just to marvel at your proficiency. Instead, treat your revision the way you would a leaking boat. You certainly would not go for the tiny holes in the boat while water was gushing in through the big holes. Prudence would have you plugging up those big holes first, and much as it would be a struggle, you would save your boat from sinking. In summary, in preparing for SAT math, mend your weakest links first.

Topics Covered in SAT Math

To be able to find out what you know or do not know in math, it is important for you to be aware of what is expected. This is especially important for students who do not have a guide or a coach, or those who probably left school several years ago. In terms of skills, SAT math covers 24. They comprise:

(i) Basic algebra: linear functions, single variable equations, systems of linear equations and absolute value.

(ii) Advanced algebra: manipulating polynomials, quadratic equations, dividing polynomials, exponential functions, function notation, solving exponential equations and systems of equations with nonlinear equations

(iii) Problem solving and data analysis: ratios and proportions, scatterplots and graphs, categorical data and probabilities, experimental interpretation, median, median, mode and standard deviation

(iv) Additional topics: coordinate geometry—lines and slopes; coordinate geometry—nonlinear functions; geometry—circles; geometry—lines and angles; geometry—solid geometry; geometry—triangles and polygons; trigonometry and complex numbers.

(3) Polish your math skills in order of importance

This is another strategy that deals with revising in a smart way. People who perform best are those who direct the most resources to where they are most productive. The situation is no different when it comes to revising for SAT. Why give all the topics equal time when questions are not proportionately derived from those topics?

It makes more sense to emphasize topics that are more likely to produce more questions. Obviously, if you are very skilled at topics that produce the highest proportion of SAT math questions, you can be guaranteed of a score of 600 before you even have a look at topics that contribute the fewest questions. With a smart strategy like this, scoring 750 should not be surprising. The big question here is: how can you tell where the most questions will come from? Well, history helps. Below is a list of topics from which most SAT math questions are derived, and another one where the least questions come from.

Topics That Contribute the Most SAT Math Questions

Once equipped with the right strategies, passing SAT math no longer seems a daunting task. The statistics and total information contained in the schedule above show that you only need to be fantastic at 33% of all the 24 skills tested in SAT math, for you to perform marvelously on 71% of the questions.

Do you realize that if you follow the guidelines in the above schedule and end up getting all 42 questions correct, you will be looking at a math score of 630? Of course, this argument is a little too simplistic because there are the 20% tough questions we recommended you ignore for a bit, but it still serves as a good guide in helping you emphasize the areas most likely to earn you high scores.

Topics Least Common In SAT Math

A careful look at this second schedule will show you that the questions from the rare topics comprise just 10% of the whole SAT math test. If you are in the category of students working towards a score of at least 600, you only need a 66% on the math test. Yet, ignoring the topics that contribute a minimal number of questions and polishing the ones that contribute the most can easily earn you a score of 730. Among the topics you do not need to pay much attention to are those that discourage many students, such as complex numbers and solid geometry.

What, in brief, have you learned from this strategy?

That not all the 24 math skills covered on the SAT are of equal importance as far as attaining a high score is concerned

Just to give you a practical example, you are likely to see an SAT math test that has algebra, requiring solutions to single variable equations, appearing a good 29 times more than questions with function notation, which are also under algebra. Thus, even within a single topic, not all skills are valued the same on SAT math. Knowing and appreciating this reality helps you to avoid wasting precious time on topics and skills with the least likelihood of being tested.

(4) Read high-quality preparation material

If you are going to invest your time preparing to score highly in SAT math, you had better learn from the best sources. Avoid relying on ancient material that has not been updated for ages. At the same time, see if the material you are relying on takes into consideration the changes that have taken place on the SAT way over the years. Note that not everything in print is credible, and a lot of material on the internet is not well researched. Like everything else, it is important to do your due diligence with regards to sourcing your SAT material.

Still, there is some great SAT preparation material available online, compiled by professionals, math gurus and people who struggled through SAT, learned the hard way and, over time, became great resources.

(5) Recognize your SAT math mistakes

It is important to identify your mistakes after you have done a test or even an ordinary exercise because having this information will direct you to the areas you need to improve on in order to score the highest you are capable of. Granted, failed questions are usually unpleasant to look at, but it is important to look at the bigger picture. What is the use of feeling good perusing through questions you have excelled in if they're not helping you improve any longer?

Exercises and tests should help you raise your score. If you are currently a 600 SAT math scorer, identifying failed questions on a test or exercise and working on the requisite skills will improve your capability to between 650 and 750 scores. Let every error that you make in an exercise count by marking the relevant topic and revising it. Otherwise, logically, what would make you get a question right, if when you failed a similar one in the past you did nothing about it? Working smart calls for identifying errors, revising the topics and skills associated with it and testing yourself again. In short, let exercises and tests help you improve your scores.

How to Avoid Repeating Math Mistakes

Once you have a paper ready during practice, mark each question that you feel you are unsure about, even if you are only 20% unsure.

Do your paper and have it marked, whether from an answer sheet or by someone else

Review your performance and as you do, put a mark any question you identified as being unsure about. Also put a mark by each question that you missed. You might wonder why you are being asked to mark questions you earlier identified as being unsure of, but it's because of that lack of confidence you have recognized. Even if you got such questions correct, it may have been by luck or just by making some educated guesses. When preparing for the SAT, you want to be sure that there are specific topics you can say with certainty you are able to tackle.

Pick a notebook, write down the specific skills you have not mastered, as reflected by your missed questions, and indicate in writing what you need to do to avoid repeating the same mistakes on later exercises or tests. Ensure you indicate not only the topics but also the subtopics. This will help you use your revision time optimally. For example, if you have missed a question involving fractions, do not just indicate number theory as the topic that requires revision. Instead, indicate number theory: fractions . If it is a question on equations that you have failed, write down algebra: how to solve equations.

Often, students make the mistake of assuming they will remember the topics they are weak in, and while they might remember some, the question is how many. If you do not refine your weak points and gain the necessary skills to do the questions correctly, you will still be ill prepared for the SAT math test. It would be very sad to see a question from a familiar topic which you cannot solve because you forgot to review the topic when you had a chance.

It may seem like this strategy is addressing the obvious, but the gist of it is that you should be reviewing in a structured manner in order to avoid repeating math mistakes you have made before. That running log you have in your notebook will be very helpful in guiding you to the topics and subtopics to revise.

Delve deeper into the reasons for missing the question. It is not enough to say the question was hard or that you did not know enough of the material. You need to go further and establish exactly what you missed. If it is the concept of how signs change under multiplication, identify it as such. Then you need to go to your notes or resource book and learn anew how the relevant signs work when they relate in multiplication.

In short, you are not just identifying your specific weakness, but you are also finding out what exactly you need to do the next time you come across a similar question.

The Most Common Reasons for Failing a Question

(a) Content

You identify your weakness as content related when you feel you have not acquired enough skills to handle the question, or you are not equipped with sufficient knowledge on the topic.

Step to take: Find out the exact skill you need to acquire or polish and determine how to go about it.

(b) Wrong approach to the question

If you are certain you had all the required information yet you did not answer the question correctly, then probably you approached the question in a way that did not show you had mastered the respective topic and the relevant skills. In fact, you may have known immediately that the approach was an issue. Sometimes you may look at a problem and know there is nothing new about it, but then you have no idea how to even begin the question. Or sometimes you can begin a question and lose your way halfway through.

Step to take: Find out how such problems are solved. Look for examples in your resource books and make sure you know how one approaches such a question, and how one pursues every step up to the end in order to get the correct answer. Do not abandon your review until you are sure you can solve such a question in future.

(c) Careless mistake

The time to categorize a problem as a careless mistake is when you misread a question and end up answering something that was not asked. Note that no matter how good your answer is, if it is not what the question required, you will not earn any marks. There are also times when students fail a question because even when they solved the problem in the right manner, they ended up indicating a wrong figure as the answer.

Step to take: Find out what you need to do to avoid such careless mistakes in the future. Probably you need to ensure you are calm as you begin the test, if you think nerves might have played a role in your missing the question.

The idea is to ensure you pick out precisely what went wrong, the exact reason you did not get the question right, and ensure you straighten that out. At the end of your analysis and revision, you should be able to tackle similar questions and get them all right.

In order to excel you must go the extra mile. Otherwise, you risk falling into the bracket of average performers whose chances of being admitted to college hang in the balance.

(6) Enter the exam room with a toolkit of techniques

Just the same way the adage goes that there are several routes to Rome, so should you have different skills to fall back on if the first one fails or temporarily escapes your mind.

In an exam environment, do not confine yourself to methods you learned in school. If a method works, use it, because getting a correct answer is what actually matters at the end of the day. Below is an example where you might forget the conventional way of solving a problem and still get the answer right.

Example

Question : Of the following choices, which one cannot be the answer to the given inequality? The inequality is: $3x - 5 \geq 4x - 3$

Answer choices

-1

-2

-3

-5

If you are looking at this question and, despite being good at algebra, are drawing a blank, you can decide to approach the question in a rather unconventional way as long as it leads you to the right answer. First, make sure you understand the question:

Which of the given numbers CANNOT BE a solution of the inequality given?

What this essentially means is that of the four choices given, three of them will produce a true statement if you plug them in as the answer. The remaining one will produce a false statement, and it is the one required as the answer to this question.

Let us plug in the given choices one by one and see which one produces a false statement.

The first choice among the answer choices is A.-1, so let us begin with it.

The given inequality is $3x - 5 \geq 4x - 3$

If $x = -1$, the inequality will read like this:

$3 (-1) - 5 \geq 4 (-1) - 3$

$-3 - 5 \geq -4 - 3$

$-8 \geq -7$

Now, that being the solution to the inequality after substituting -1 for x, do you believe that statement? What is bigger: negative 8 or negative 7? Which value is greater: a debt of 8 or a debt of 7? A debt is not a value, so the less you have, the better. So, really, this statement is false. If it had been an 8 and a 7 without the negatives, and the rest of the statement remained unaltered, the statement would have been right. But not now—now it is false.

You have, therefore, used an unconventional method where you are plugging in possible answers one by one to see which one is false. Luckily, the first one you tried happens to have been the correct one. However, in an exam situation you may wish to reassure yourself. So try choice B. -2.

The inequality is $3x - 5 \geq 4x - 3$

If we substitute -2 for x we shall get:

$3(-2) - 5 \geq 4(-2) - 3$

$-6 - 5 \geq -8 - 3$

$-11 \geq -11$

Now, is -11 more than -11? No. But is -11 equal to -11? Yes, it is. And the inequality gives you a choice: either more than or equal to. As such, choice B satisfies the inequality, so we do not want it as the answer asks for the choice that does not satisfy the inequality. At this point you should be confident choice A is correct, choose it as your answer and proceed with the rest of the test. You will get full marks for that question, and the examiner will not care how you got it.

If you think you have ample time, you can try out choices C and D, but it is not at all necessary as the question demands only one number that does not satisfy the inequality.

Do you know you can still solve the above problem the algebraic way? As already mentioned, the more routes to a place you know, the better. Here is how you would do it:

The inequality is $3x - 5 \geq 4x - 3$

$3x - 4x - 5 \geq 4x - 4x - 3$

$-x - 5 + 5 \geq -3 + 5$

$-x \geq 2$

$x \leq -2$

What answer choices have we been given as possible values of x?

They are:

A. -1

B. -2

C. -3

D. -5

Is $-1 \leq -2$? No, it is not. A smaller negative is greater in value than a bigger negative. ‘A’ must, therefore, be the answer required as it does not satisfy the inequality produced in the algebraic solution.

Just to be certain, how would B. -2 fare?

Is $-2 \leq -2$? Well, -2 is equal to -2, so choice ‘B’ satisfies the inequality. How about C?

Is $-3 \leq -2$? Yes, -3 is less than -2, so choice ‘C’ satisfies the inequality. How does D fare?

Is $-5 \leq -2$? Yes, -5 is, definitely, less than -2.

The choice that is false is A -1, and just as in the earlier unconventional method, it comes up as the correct answer.

You are, therefore, encouraged to try out different approaches to a question whenever you get stuck. We shall look at more of these unconventional techniques later on, because they are certainly liberating and will get you scoring high in SAT math even when you have forgotten the most common methods.

(7) Apportion your exam time prudently

Why is time allotment so important? There are instances when students get bogged down with one question, yet they end up getting it wrong. Supposing they get it right? Still, the time spent on that single question will not have been worth it if it prevented the student from tackling three more questions that were simple and would have been correct.

For people who excel at math, time may not be an issue, and by excel we mean the 800 scorers. However, for students struggling to reach or exceed the 600 score, time is, definitely, a constraint. So, if you want to rule out time as a constraint, learn the necessary SAT math skills and polish them with lots of untimed and timed tests, and you will be good to go. Otherwise, consider the total time allocated to the different sections of math SAT math, and then divide it by the total number of questions.

One thing you need to note is that marks and points are given equally to individual math questions whether the questions are deemed difficult or not. Hence, it is not reasonable to spend excessive time on one question at the expense of others.

The best way to go about apportioning time to individual questions is, first of all, to spend just half a minute on one problem, and if you find you are not making headway, abandon it and proceed to another one.

Then, in case you complete that math section early and have extra time, you can go back to any question that you skipped and give it more time beyond the initial thirty seconds. Imagine how much time you will have to resolve questions you think you should know, but which you are having problems with, if you have already ignored the very difficult questions at the end of the section as suggested earlier on!

(8) Fill in your answers at the same time

This strategy is all about saving the several seconds that you spend filling in the answer after you have completed every question. Think about it this way. Some people spend five or more seconds after they are done with a question, putting the answer sheet in a good position and then filling in their choice of answer. Imagine doing that for twenty questions: completing question number one, then switching from the question paper to the answer sheet and then back to the question paper. Then you work on question two and when you complete it, you switch from the question paper to the answer sheet and then back to the question paper to tackle the next question. If you stop wasting those in-between seconds, after 20 questions you will have saved a whole 100 seconds.

Concentrate on working out the math questions, and then, once you are through, draw your answer sheet close and fill in all the answers appropriately. Not only does this sequence save you time, but it also helps you avoid careless mistakes when filling in the answers. The presumption here is that there are some questions you may wish to skip first and then you may solve others that are ahead, and if you keep filling in answers one at a time, you may forget to skip the number for the undone question on the answer sheet.

There is also another problem that is often ignored when marking a single answer after every question and that is the problem of distraction. You may have noticed how fast you work when there are no distractions as opposed to when there are regular distractions. In fact, with distractions, one usually loses momentum, especially because the distraction is multi-pronged. There is the mental distraction, and there is also the physical distraction because your hands, arms and eyes keep moving from the question paper to the answer sheet and back.

The suggestion here is that you tackle all your math questions in the section first, and then fill in the answers on the answer sheet all at once. What you need to be alert about is the time you need to fill in all the answers. You don’t want to end up in a situation where you have solved your math problems but don’t have enough time to fill them in on the answer sheet. This is something you can work on during practice. As you do your timed practice tests, check how long it takes you to fill in your answers on the answer sheet, and then during the real SAT, ensure you begin filling in answers early enough. Do not try this strategy during SAT unless you have practiced it on full-length past SAT papers and have established how much time you need.

(9) Fill in answers to omitted questions by guesswork

For years, students avoided guessing answers in SAT math because every wrong answer on multiple choice questions would be penalized. In fact, filling in one wrong answer would have the student lose 0.25 points on the test’s raw score. The only way one could risk guessing was by way of eliminating various answers among the choices, till one was left untouched. Then the student would logically pick that one answer as the correct one. This is no longer the case, because the wrong answer penalty was scrapped effective 2016.

Today, there is no reason why anyone should leave a multiple-choice math question unanswered. Note that by marking an answer you stand a fifty-fifty chance of earning some marks, but by leaving a question unanswered you have no chance whatsoever of earning any marks. If you can do some elimination and rule out one or two answers, it means you will be guessing your answer from a smaller range of answers and you stand a higher chance of getting it correct than if you had not eliminated any answer, and, definitely, that if you had left the question unanswered. And even if you have no idea what the answer should or should not be, making a random guess still gives you a quarter chance of getting it right and earning a score, which is better than no chance at all.

Chapter 15: How to Pass Math By Plugging In Answers

It is important to point out the importance of having an open mind when taking a math test. There are many times that students lose marks just because they are rigidly thinking of formulas they learned in school, from resource books or from elsewhere. Yet there are several math problems that can be solved by your own intuition, logic or what is usually referred to as educated guesses.

In this section of the book, you are going to learn how you can pass given math questions even when you do not remember any conventional formulas.

Techniques Great for Passing the SAT, GMAT and GRE

The kind of questions you can easily tackle in this manner, incidentally, happen to be of the multiple-choice variety, and luckily, many serious tests, such as the SAT, has many of them. There a number of reasons why a student should be happy about multiple-choice questions and one of them is that the examiner does not expect to see your work. This means you can use your imagination and creativity without engaging any known formula, and as long as you get your answers correct, you will have earned full marks for the question. Many standardized questions actually give you that advantage, including the most prominent ones such as the SAT, GMAT and GRE.

These exams, and even the ACT, do not require you to show any work in order to earn marks. You just need to get your answer right, and it is presumed you understood how to solve the problem. The tests seem to be generally created to check how fast you can respond to situations in an accurate manner. You should keep this in mind as you take any of these very important exams, so that you shed the school mentality that tends to remain with many of us for a long time even after we have left school. It is not surprising to see someone reach for a pen to scribble work down even when they already have the answer ready in their mind.

The sections of this book that have introduced you to mental math serve well in helping any reader shed the mentality of writing down everything and detailing work. Marking the answer on a test question does not reduce the score allocated to that math problem; not even if 75 seconds are allocated to one question and you answered it in 25 seconds. For that reason, math experts and creative thinkers have come up with several ways of getting around certain math questions.

In this section, you are going to learn how to utilize such methods and earn marks in your math exams without having to engage in complex methods of problem solving. The simplest method that you will learn in this part of the book is referred to as plugging in given answer choices. This means you are going to learn to pass your test using just the information readily available during the test. The method has been used long enough now to acquire an acronym, PIA, where P stands for Plugging, I for In and A for Answers.

In order to make the best use of this method, it is important to learn why, exactly, it is recommended, how it is applied and when best to use it.

Why Use Plugging in Answers?

There are times exam questions look strange even when they are from topics you are familiar with. In short, there are times you can look at a math problem and no clue comes to mind as to how you should even begin dealing with it. Instead of giving up on such a question, you can PIA as your go-to method, one that can easily bail you out and see you earning marks you would otherwise have lost.

There are other times when you have a pretty good idea what is expected of a math question, but you wonder why such a question was included on the exam, considering how lengthy its workings normally are. Avoiding having to take too much time is another reason you may need to use PIA to solve your math questions. Granted, there are not many problems you can solve algebraically and extra fast. The writing and the analyzing always slows you somewhat.

In other situations, you may have solved the math question before you but are not certain whether you have missed something important such as a parentheses, a negative sign or anything else like that. If you are in doubt that you put in everything correctly, PIA can help you reassure yourself. You just plug in your choice of answers and identify the one that is correct.

Glance at Your Answer Choices First

Any time you have an exam paper to do that has multiple-choice questions, you need to know how to take advantage of the nature of the paper. Unfortunately, some students begin to tackle the questions immediately, reading the question and immediately starting work. If you approach your questions that way, you might find yourself spending time unnecessarily on a question whose answer you could have picked instantly if only you had looked at the answer choices.

In fact, just by quickly reading your given answer choices, you can determine what the expected answer should look like. For instance, if it is a percentage or ratio, you will know from the beginning. That way your calculations will be along relevant lines. Even when it comes to amounts, you cannot allow yourself to stray, because the answer choices will have given you an idea within what range the correct answer should fall.

For a practical scenario, when you are taking an ACT or SAT math test, one of the answer choices will always be 100% correct. Suppose your answer choices in a given question are 5, 16, 19, 27 and 29. Why would you or any other student pursue calculations that point to a figure like 30 or anything above? Would you even check if 4 or anything below is correct? Of course not! The range of answers here, 5 – 29, should serve as a good guide for you in trying to find the correct answer to the question.

Whereas such advice may not seem to have much weight in an ordinary environment such as when you are doing your day-to-day revision, the tension of an exam environment can confuse you. In a time of stress, what is logical does not always come to mind, and we often only think about it later. However, when you have read tips such as those provided in this book, something is bound to jolt you to reality, reminding you when you are about to fall prey to exam approach weaknesses.

Since even the most prepared students sometimes become tense and stressed in an exam environment:

Ensure you take a deep, healthy breath before you begin your paper and repeat that periodically throughout the exam.

Remember, you have other means at your disposal of solving math problems besides the routine ones, and which do not entail working at the question from the beginning.

Avoid becoming overwhelmed as much as you can. The more you can remain levelheaded, the better for your performance.

Make use of shortcuts whenever you can, because then you will save a significant amount of time that you can use to tackle questions that you may have skipped. Of course, by the time you finish reading this book, you will know many shortcuts.

The Best Way to Use PIA

Hopefully, you now have a positive view towards this rather unconventional method of plugging in the answers given, so we can proceed to learn how to go about implementing the skill:

(1) Take each answer and plug it into the given equation

You are supposed to do this for each of those answers until you get the correct one. This does not only apply to equations but also to other kinds of information, and the idea is to let you see how any of your given answers fits in.

If, for instance, you have x as a positive integer in an equation like this one, establish what its value is.

$(x + 1)/ 2 x = 3 / 16$

The answer choices you have been provided with are:

A) 1 B) 2 C) 3 D) 4 E) 5

In short, the question requires that you solve for the variable, x. At a glance, it looks like this math problem is among the complex ones, and as if it might take quite a long time to deal with it conclusively. To circumvent that lengthy procedure, pick each number given as a potential answer and plug it into the equation wherever x appears.

You will quickly solve for x using the answer given in A, then B and so on, and sometimes you may not even need to try out many of them before you find the one that fits exactly.

If you plug in the 1 you have in answer choice A, the equation will look like this:

$(x + 1)/ 2 x = 3 / 16$

$(1 + 1)/ 2^1 = 3 / 16$

$2/2 = 3/16$

$1 = 3/16$

When does one ever equal a fraction? The choice, A.1, is definitely, incorrect.

If you try plugging in the B.2 answer choice, this is what you will get:

$(x + 1)/ 2 x = 3 / 16$

$(2 + 1)/ 2^2 = 3 / 16$

$3 / 4 = 3 / 16$

This one is also wrong and, hence, x cannot, in this equation, be equal to 2. In short, answer choice B.2 is not the right answer.

If you plug in the 3 in answer choice C.3, the given equation will work out like this:

$(x + 1)/ 2^x = 3 / 16$

$(3 + 1)/ 2^3 = 3 / 16$

4/8 = 3/16

½ = 3/16

This equation is definitely wrong, which means x in the given equation cannot be 3. As such, answer choice C.3 is not right.

If we plug in 4 from the answer choice, D.4, this is how the equation will look:

$(x + 1)/ 2^x = 3 / 16$

$(4 + 1)/ 2^4 = 3 / 16$

5 / 16 = 3 / 16

This equation, too, is false, and so answer choice D.4 cannot be correct.

We are now going to try substituting 5 for x, being the number in the answer choice, E.5, and the equation will look like this:

$(x + 1)/ 2^x = 3/16$

$(5 + 1)/ 2^5 = 3/16$

6/32 = 3/16

Once we simplify the fraction 6/32, dividing the numerator and the denominator by 2, the resultant simple fraction is 3/16. So, the original equation, $(x + 1)/ 2^x = 3/16$, finally works out to 3/16 = 3/16, which is true.

So, through plugging in answers, we have established that answer choice E.5 is the correct one. There has not been any guesswork involved, so if you are in an exam setup, you will be sure to get the question right.

Of course, if you are pressed for time and A to D all have proven to be wrong, you might as well just consider the answer given in E to be the correct one. After all, it is the only number you will not have plugged in already. It would be different if among the answer choices there was one indicating 'none of the above', in which case you would need, of necessity, to plug in each and every answer as long as you have not identified one that is fitting.

After going through the entire process of plugging in five probable answers you might be wondering about the method's effectiveness in saving time, even if it is effective in getting the correct answer. However, not only does the method seem lengthy in the demonstration, but you also do not have to go all the way as we have done in this example. In short, you do not have to begin your plugging in of answers from the very first one. There is a way you are going to learn here that shortens the time taken to plug in answers when trying to solve for an unknown in an equation.

How to Use PIA in Seconds

The two most important objectives when one sits for a standardized test are:

(i) Getting the correct answer

(ii) Spending the minimum possible time to get that right answer

However, as we have seen, this method of plugging in answers, good as it is in getting you exactly the right answer, can be time consuming, something that nobody wants in an exam environment. It is, therefore, important for you to learn how to make use of the positive aspect of PIA, which is accuracy, while circumventing its weakness of consuming too much time.

In the example just completed, the testing began with the answer given as first choice, and it took the testing of all of the other answer choices to find the correct answer, which happened to be right at the end of the list of choices. There is a way we could have gotten the right answer while spending just a fraction of the time we spent.

How To Save Time In Answer Plugging

One important characteristic of many standardized exams, the SAT included, is that the answer choices are given in either ascending or descending order. This means that if there are five answer choices, answer choice C comes right in the middle. Get into the habit of using it as your starting point when plugging in the given answer choices. Can you guess why?

The reason is that if the answer choices are in ascending order, and the choice given in C is ridiculously low, obviously the answers in A and B are even lower, and there would logically be no reason to plug in the answer in either A or B. So you would go straight to D and E. The presumption here, of course, is that you are dealing with a situation where there is a direct relationship between the answer choices given and the equation that forms the question.

If we can use our example above to show how it helps to begin midway through the answer choices, look at the answer choice, C. 3. When you substitute 3 for x, eventually the denominator on the left side of the equation is 8. The denominator on the right is 16 and the fraction is already in its simplest form. Can you see the logic? There is, therefore, no way that the equation on the left will equal the one on the right. As we dismiss choice C, we acknowledge that the answer choices have been provided in ascending order, which means the list begins with the smallest number and increases progressively, so that the last answer choice is the one with the biggest number.

After having established that 3 in choice C is too tiny to fit the equation, should you really bother trying choice A or B? Their results are bound to be even more far off from what is required. Therefore, the best option is to begin plugging in the answers that are more than the one provided in C; meaning the numbers given in answer choices D and E.

If you are calm and thinking logically, you will have noticed that the answer choices given have a correlation with the equation. You may also have noticed that the higher the number given in the answer choice, the smaller the equation on the left becomes. In short, it is not only a direct relationship that exists between the equation forming the answer and the answer choices given, but it is one you can aptly describe as an inverse relationship.

Here you can see it is possible to notice an inverse relationship but think about how much easier it is when the relationship between the equation and the answer choices given is direct in the sense that if a given answer option is large, the resultant figure on the left of the equation also proves to be comparatively larger than when you plug in a smaller answer option.

However, answer choices are not always aligned in the manner we would like them to be, and so there are times you will test the answer choice in the middle, in this case, C, and the result still will not give you a clue as to which side of C you should prioritize. In such instances, do not fret. Make up your mind immediately to try one other answer option at random, without worrying whether it is on the lower side of C or the higher side.

In case you test D and it gives a result that is more adverse than that provided by the C option, this should point to the lower side as having the more suitable options. Without having to plug in E, you need to now try B or A, and most probably, one of the options will provide you with the correct answer after you have plugged it into the given equation.

This method is more time-saving than the instance where we tried all the options. As has just been explained, here you do not have to test all the five answer choices given, yet you are likely to quickly identify the correct answer.

In a nutshell, it helps if you make it a principle to always begin plugging in your answers in the middle, where option C is. Another question might still arise: How do we handle the situation when the answer option is negative? For one, if you plug in a negative answer and end up multiplying it with another number in the equation, you will end up with negative values. Yet, as you can see, the number you want to equate your answer to is positive.

So, such scenarios just make your work easier, because you outright ignore the negative, having ruled it out as a potential substitute for x. There are even some figures you can look at when they are provided as answer choices, and by the look of your equation you can deduce that the number is too tiny to serve as the x value. In short, when using the PIA method, eliminate from contention any number which, in your good judgment, is out of place. Of course, every instance you eliminate an answer option means you are left with fewer answer options to test and less time to spend on your math test as a whole.

Solved Examples in PIA

(1) Suppose you have x and y as positive integers in the equation given below. What is the answer choice with the values providing all the solutions of (x, y) in the equation? The equation is:

$3x + 2y = 11$

The answer choices are:

(1, 4)

(3, 1)

(1, 4) and also (2, 2)

(1, 4) and also (3, 1)

(2, 2) and also (3, 1)

Solution:

Begin with the middle option, C, which, in this case, has (1, 4) and also (2, 2). The answer option gives (1, 4) as the first options for x and y in that respective order.

When we plug 1 and 4 into their respective slots in the given equation, this is how the equation looks:

$3x + 2y = 11$

$3 (1) + 2 (4) = 11$

$3 + 8 = 11$

$11 = 11$

This is very true. The value on the left side is equal to the value on the right side of the equation. Can this truth help you shorten your list of answers to plug in? Yes. Since the equation that forms the question has to have 1 and 4 as one option of its x and y, respectively, it is reasonable to conclude that any answer option without that set of values, (1, 4) is not correct. In this regard, options B and E are out of contention, and we can henceforth ignore them.

The next step is to plug in the second option in answer choice, C, which is (2, 2). This set indicates that the value of x is equal to the value of y, both being 2. The solved equation will turn out this way:

$3x + 2y = 11$

$3 (2) + 2 (2) = 11$

$6 + 4 = 11$

$10 = 11$

This second answer option from C is certainly wrong, as 10 is not at all equal to 11. Following the same logic we used earlier to eliminate the answer options without (1, 4), we can conclude that if (2, 2) in C is wrong, then any other answer with that option is also out of contention. We could eliminate option E, had we not already eliminated it earlier.

So all we can do now is eliminate C and be left with options A and D. Remember, as far as the question goes, all the answers provided must be correct for the answer to be the right one. With B, C and E out of contention, clearly the correct option must lie with either A or D.

It has already been established that the set of values (1, 4) does work in the equation, and we can also see that the two remaining options have that set. The difference between the two options is that, while option A has only one set of values for the set of (x, y), option D has another set of values, which is (3, 1). What we need to know is if that second option will fulfill the equation as values of x and y.

$3x + 2y = 11$

$3 (3) + 2 (1) = 11$

$9 + 2 = 11$

$11 = 11$

Option D's second set of values for (x, y) has worked. In short, answer choice D is the one that has all its values satisfying the given equation.

Hence, D. (1, 4) and also (3, 1) is the correct answer.

(2) Suppose 13 is added to ½ of a given number and the results are 37. What would you say the original number was?

Answer choices:

24

40

48

61

Please note that the question asks for the original number, and that implies that the original number is among the answer choices provided. The most important thing to keep in mind is that the original number you are looking for was divided by a half. That tells you straightaway that there are some numbers that cannot stand a chance because of their smallness of size. At the same time, we want to adopt the principle we recommended of beginning our analysis or evaluation with answer C.

C's value is 48.

Let us begin simply by saying that 48, being the presumed original number, was divided by 2. $48 \div 2 = 24$.

What else happened? 13 was added to half of that presumed original number. $13 + 24 = 37$.

There! Already, we have $37 = 37$, so option C is the original number being sought. It is the number represented by the unknown, x.

The answer is, therefore, C. 48.

This is one of those lucky instances where the first answer plugged in has given us the correct answer. There is now no need to test any other value. By applying the PIA technique, you have saved a good number of seconds that you can now utilize on another question. As you can see, using the PIA method means you do not have to rack your brain, as doing a substitution of x with a ready value at hand is child's play.

However, not all questions turn out to be quick to solve with PIA, because sometimes you may begin the PIA with the middle option, C, and then end up discovering the correct answer is A or D, and they were probably among the last options you tested. Would it not, therefore, help a great deal if there was a way of finding out when this PIA technique is best suited?

Best Times To Use PIA to Solve Math Questions

To avoid the drawback of taking longer on a question than you would when using algebra, it is important to learn how to assess each question, so that you can tell the ones that can be best solved through PIA and the ones that can be better solved the algebraic way.

Let us solve the following math problem:

$3x + 2y + 2z = 19$

$3x + y + z = 14$

The question is: Supposing each of the stated equations is true, which of the choices below is the actual value of y + z?

The answer choices are:

-5

-4

0

4

5

This is one problem that you can solve either the algebraic way or the PIA style. The quandary is: Which of the two methods is quicker for this particular question? We are going to find out.

Solving the problem using PIA:

One thing to remember is that any answer choice you pick will be a replacement for (y + z)

Begin with answer option C, as recommended, which in our case has a value of zero.

Identify the unknown you want to substitute for, (y + z), as being in the second equation, so that is where we shall use the zero. See below:

$3x + y + z = 14$

$3x + (y + z) = 14$

$3x + 0 = 14$

3x = 14

At this juncture, we do not need to solve for x, because there is already a 3x in the two equations above. With such a workable scenario, why bother calculating the value of x only to end up with fractions, something we can deduce at a glance?

Let us now solve the first equation:

$3x + 2y + 2z = 19$

$14 + 2y + 2z = 19$

$14 + 2(y + z) = 19$

$14 + 2(0) = 19$

$14 + 0 = 19$

$14 = 19$

Obviously, an answer option that leads us to say that 14 = 19 cannot be correct. So, you can rule out option C as the answer.

It is possible to extrapolate from this math question that (y + z) is not going to be negative, because 3x is remaining constant while the solution is becoming greater when (y + z) is doubled. A look at the two equations will reveal that.

This essentially means that the value of (y + z) is bound to be positive. If that was not the case, it would mean that the solution to the first equation would be smaller than the solution for the second equation. Yet, as you can see, the first equation has 19 as its solution while the second one has 14 as its solution, indicating that the solution for the first equation must be bigger than that of the second one.

Let us see how the value in answer option D fares:

The value in D is 4, and so:

$3x + y + z = 14$

$3x + (y + z) = 14$

$3x + (4) = 14$

$3x = 14 – 4$

$3x = 10$

As before, there is no need to solve for x, so proceed to solve the first equation, substituting the values you have calculated.

$3x + 2y + 2z = 19$

$10 + 2(y + z) = 19$

$10 + 2(4) = 19$

$10 + 8 = 19$

18 = 19, is, of course, not correct.

As such, you can rule out D as the correct answer.

It is now time to try out the value in answer option E, which is 5.

$3x + y + z = 14$

$3x + (y + z) = 14$

$3x + 5 = 14$

$3x = 14 - 5$

$3x = 9$

Now, plug in the 3x value found in the first equation:

3x + 2 (y + z) = 19

9 + 2 (5) = 19

9 + 10 = 19

19 = 19

Finally, you have found the answer: E. 5

You have employed the PIA method and gotten the right answer, which is not surprising.

What we would like to do now is use the algebraic method to solve the same question, and find out if it will give us the correct answer, and better still, if we are going to spend less time while at it.

Algebraic Method of Solving Equation

The two equations used in the PIA method are the ones we are going to use here, so that the comparison is fair. The two equations are:

3x + 2y + 2z = 19

3x + y + z = 14

We can use the subtraction method, the one commonly used in simultaneous equations.

3x + 2y + 2z = 19

3x + y + z = 14

0 + y + z = 5

What we have done is subtract 3x from 3x and the result is zero, so there is no longer an x in the equation.

We have then subtracted y from 2y and we have been left with a single y.

We have also subtracted z from 2z and we have been left with a single z.

Finally, 14 subtracted from 19 has resulted in 5.

Therefore, y + z = 5

Remember what the question demands: the value from the answer choices given which matches the value of y + z.

The correct answer is E. 5.

Can you see how fast we have been using this algebraic method? Although both methods led us to the correct answer, using the algebraic method has proven much faster. And every candidate can always do with an extra minute, whether it is to complete the exam or to review the questions already completed.

Your takeaway point here is that you should use your discretion to use the method you deem fastest in solving each problem. Generally, it looks like more math problems can be solved faster by way of algebraic calculations than through plugging in answers. Whichever method you choose, you need to feel confident about its capacity to lead you to the correct answer and to help you save time.

Still, it is good to point out scenarios where PIA can be a real savior; times when no other method is likely to help you solve the problem. In short, having PIA skills is crucial because there are times when it is the only method that can save you from losing points for entire exam questions.

When PIA Is The Only Way Out

One thing we must emphasize is that if time is not on your side, it is not time to begin plugging in answers at random. If you know a faster way, get on with it and save precious time. However, there is no reason for you to miss marks just because you cannot recall how you went about tackling a problem. Use the PIA technique if:

(1) You do not know an alternative method

If, even after reading a question a couple of times, you cannot make heads or tails about what you are supposed to do, PIA can be of great help.

Such problems are those which you look at and cannot determine how to begin. Luckily, you do not have to give up right away, because PIA does wonders.

That is the same technique that is best used when you forget the rules used to solve a particular kind of question, or even the equation to use in certain circumstances. You can conveniently get to the right answer by using PIA.

There are several cases where you can successfully use PIA without having to bother about the rules that govern roots, shapes or even means. You just dive into plugging in the answers until you deduce the one that is right.

There are also cases where the use of PIA is inevitable, because trying to solve the problem differently would be too clumsy. A case in point is where you have a straight line AC on a plane, the kind of line whose single angle comprises 180°. If B is the midway spot on that line, and there are several angles making up the 180°, each with angles whose sizes are denoted in terms of degrees of x, the only way to know the size of each angle is to substitute x for the value given in the choice of answers.

Such answer choices could comprise a list like this:

30

35

40

50

55

In short, there are certain math questions where plugging in the answers is not an option among many but rather the only way out. Such are the cases where the information available is not sufficient to go by in order to solve the problem. And since plugging in answers is tantamount to working backwards, you cannot go wrong with it.

(2) In case you have been extra fast in other areas of the test

Basically, this point is emphasizing the need to ensure you are on the safe side, time-wise. So, in case you have been quite comfortable with earlier questions and are convinced you have ample time at your disposal to use the PIA method, there is no good reason why you shouldn't it. In short, whether you use PIA or not is your personal call and needs to be one of good judgment. It is always advisable to try and garner as many points as possible across the board.

(3) When your aim is to double-check your chosen answer

One good thing with plugging in answers is that the method is self-checking. This means that even as you use the technique so seek a solution to the problem, the technique is also serving to affirm the correctness of your selected answer. Nevertheless, students are always warned against overdoing the checking via PIA, because if you were to subject every question you to do PIA to ensure your answer is right, you will be unnecessarily spending too much time on questions you have already solved. Reassuring as it is, PIA should be left to questions that require it the most.

Example

You have been given an equation $x^2 - 2x - 8 = 0$, and the answer choices are as follows:

A) -8 B) -6 C) 0 D) 2 E) 4

The question then follows:

What can the value of x be?

Solution

You can always solve such a question through factoring, where you solve it as a quadratic equation. Your two equations would look like this:

$x2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

Suppose you have no clue how to begin tackling this question. You might look at answer option D and think it looks like the best fit, but that is because you have not recognized that there is a need to have each factor equal to zero in order to be able to calculate x.

All the same, something at the back of your mind might keep signaling that you are not certain D is right, although you are bent on picking it. This is the best time to make good use of PIA, to test whether D's value is really correct.

Choice D gives us 2, so let us plug in the value, 2.

$x^2 - 2x - 8 = 0$

$2^2 - 2(2) - 8 = 0$

$4 - 4 - 8 = 0$

$-8 = 0$

You see? You have just proven that answer option D, which gives the value of x as 2, is wrong in this case. You have just saved yourself unnecessary loss of points by proving that the answer you were about to submit as correct is wrong. You now have a chance to redo the question.

Let us see how you can complete this same question using algebra.

$(x+2) = 0$

$x = -2$

You can appreciate how easy it is to confuse answers when you are in a hurry. It is, therefore, understandable that one might wish to check the work before deeming it complete. As long as there is such checking to be done, the PIA method is great.

Let us observe the scenario involving the second factor.

$(x-4) = 0$

$x = 4$

The two values representing x are, therefore, -2 and 4. This is the correct answer for this question, E.4.

In case you still want to prove your entire calculation right, you can plug in the value of 4, which you have found to be the correct answer, and see if it works well. It is just a way of double-checking your answer.

Here you have $x^2 - 2x - 8 = 0$.

Substituting 4 for x, the equation becomes $4^2 - 2(4) - 8 = 0$.

$16 - 8 - 8 = 0$

$16 - 16 = 0$

Of course, 16 minus 16 is zero, and you have cleared up any doubt as to whether your answer is correct or not. Your PIA has confirmed that the value of x in the given equations is 4.

(4) In case you suspect the algebraic method you used led you to an incorrect answer.

What kinds of questions you will encounter on an exam is unpredictable, even when you know the relevant topics. In terms of the way the question will be framed or the approach it will require, it is quite difficult to tell in advance. That is why there are instances when students begin to solve a problem only to realize midway that they cannot complete it. Often there is a move that is needed for one to get to the required answer, but the candidate has no idea how to proceed.

Here is one example where the method of plugging in the answer appears as the next best option. Without it, the student is likely to give up on the question and its corresponding marks, or will waste time unnecessarily trying to figure out how to proceed.

Sometimes after successfully completing a math question, you may feel like you were a little off. PIA is the best tool to provide reassurance. You just plug in the answer you thought was right in the first instance, and at the end of your calculations, you will know whether your answer was right or wrong. If wrong, then you will have a chance of rectifying the problem.

Overall, if you feel like you are unsure of what you are doing with a particular math problem, and it is one of those doable through plugging in the answer, use PIA. As mentioned earlier, nobody will care how you got your answer. What will matter is what score you got, courtesy of the correct answers you marked on the answer sheet.

(5) If you reach the exam question range where you begin to make errors

Just like in many areas of life, historical experience has a lesson for you if only you care to analyze the information. You may, for example, be among the students who are only able to concentrate fully for the first hour in an exam. After that period, you become prone to errors, many of them careless mistakes. If you know that and have not managed to change it, then do the next best thing: use PIA an hour into your exam.

That way, you will benefit from the answers you will have gotten correct in the initial hour before you began losing concentration. Then, in the latter hour, you will benefit from the accurate PIA technique even if you may not be as fast as if you were using the algebraic methods of solving the problems.

One thing you should keep in mind is that you are the person best suited to make a good judgment. What we give in a book like this is guidance on how best to handle various math problems, tricks to make solving the problems easier and so on. However, you need to apply these pieces of advice according to how best you think they can help you in your specific circumstances.

Chapter 16: How to Pass Math By Plugging In Numbers

Many students who have heard of PIA wonder whether it is the panacea for all math problems. The answer is no. There are certain types of questions that cannot be solved by way of plugging in possible answers. Many of the questions best suited for the PIA technique have answer choices in numerical form. This means they are sometimes integers, other times fractions and sometimes even decimals. In short, just check to see if the answer choices provided are in numerical form, and if so, you can be assured that PIA will be applicable.

However, if you notice the choices given are in the form of variables, be aware that PIA may not be of much help, if any. Even in instances where the question itself has numerous variables, using PIA becomes a daunting task. Luckily, there is another technique that is close to PIA, but this one is termed plugging in numbers. It comes in handy in helping you solve questions you are not able to solve using other methods, and where PIA does not work effectively.

How to Apply the Plug In Numbers Technique

As has already been mentioned elsewhere in this book, examiners of many standardized tests, including SAT and ACT, do not concern themselves with how the students arrived at their answers. All they want to see is whether the student got the answer right or wrong. Then they can award marks accordingly.

It is for this reason that students need to equip themselves with techniques such as PIA. Unfortunately, even PIA has its limitations, and there are some math problems you cannot use it to check the accuracy of your answer, let alone help you find an answer to the question.

So, is there a fallback plan? Fortunately, there is. Just like there is PIA, there is PIN, an acronym for Plugging In Numbers. PIN, like other shortcuts introduced in this book, comes in handy in helping solve math problems accurately, raising math scores in a way you might not have thought was possible. As you will soon see, PIN is a pretty simple process that helps work out answers to math problems that would otherwise be termed difficult. One great thing about it is that it tackles the question directly.

In this section of the book, we shall explain the reason PIN is helpful, how it works and the best times to employ the technique.

Why Plugging In Numbers is Helpful

As is the case with PIA, PIN comes in handy when you are confronted with a math problem you have no idea how to solve. You might not even have a clue about the formulas involved in solving the problem, but when you know how to use PIN, you can solve the problem, get the answer right and earn yourself much-needed marks.

There also times PIN comes in handy because, in your view, the alternative methods of solving the problem, including algebraic ones, are too lengthy. Remember, you have no luxury of time during the SAT or any of the common standardized exams. You might also find that a particular math problem has many variables. In such an instance, you may start doubting your performance, wondering if you really got the value of all the variables correct. PIN then comes in handy because you can use it to verify whether every value you concluded was correct. In this regard, PIN plays a similar role to that of PIA. You just plug in your numbers where they fit and as you solve the problem, you can see from the results whether your chosen numbers are correct.

The instances imagined in this explanation are cases where the answer choices provided contain a range of variables, as opposed to one or two variables. Such scenarios can be intimidating, and a student might not even know where to begin finding a solution. It is even worse when the student remembers that the clock is ticking and there are marks and scores at stake.

When a question is dominated by variables such as x, y, z, a, k and the like, as opposed to constants, the problem often looks obscure and intimidating. However, if you know a way of changing those variables into numbers, you have an opportunity to make the problem look realistic and doable. That is what PIN is about: plugging in numbers where previously there were variables. Consequently, the problem becomes easier to solve.

Examples of Problems Solved Via PIN

Example 1

In a question with variables a and b, a . b are expressed by a . b = ab + a + b

Find which of the given answer choices is true for x, y and z.

x = . y = y . x

(x – 1) . (x + 1) = (x . x) – 1

x . y (y + z) = (x . y) + (x . z)

Answer choices:

(i) only

(ii) only

(iii) only

(i) and (ii) only

(i), (ii) and (iii)

Incidentally, it is not surprising, though it probably should be, to know that there are students who leave questions like these unsolved because, after thinking of possible algebraic methods to apply and not coming up with any, they consider the question undoable and they give up. That's why students are often advised to make a conscious effort to remain relaxed. It is easy for an extra technique to enter your mind when you are relaxed, one like PIN that will help you solve your problem even with several unknowns involved.

Always remind yourself that there are usually different ways of solving a mathematical problem. Luckily, it will not matter on the test which technique you used—as long as it is legal and does not involve cheating.

How Best to Use PIN

It is one thing to have a tool and another to utilize it effectively. In the case of PIN, it is important that you learn how to make optimum use of it in order to gain marks you probably would have lost on a test. As we mentioned when analyzing the use of PIA, if not prudently used, the tool can cause undue delay and cause you to lose more marks overall than the ones you earn by using the tool.

Fundamentally, PIN, as a mathematical tool, gives you room to insert real numbers in places where there initially were variables. Such variable replacements can work for any kind of math problem with unknowns, especially in algebra and geometry, where the unknowns occur in high frequency. In fact, there are instances where, in algebra and geometry, there are several unknowns in the same problem, a situation that sometimes overwhelms students. Luckily, once you know how to use PIN, you no longer have to panic, no matter how many variables a math question has.

You will get the first sign a question can be solved with PIN by quickly analyzing the question. Does it have two variables or several? If there are just two, simultaneous equations will probably work, but if the question has three or more variables, your surest way of solving the problem without feeling overwhelmed will be PIN. The other place you need to keenly observe is the list of answer choices. If they come with multiple variables, then PIN may be the best tool to apply.

As you get to understand how PIN works, consider this: You will be dealing with questions that have numbers, degrees, objects or such other terms with relationships that are constant. What this means is that, irrespective of the actual numbers you use in solving the problem, this relationship will remain constant.

Basically, what this means is that you can pick any number you want as long as it conforms to the rules contained in the question you are trying to solve, and it will work in that particular question. Whatever such number you choose will help you obtain the correct answer.

Once you have made your decision on the number you want to use as your plug-in, use it in solving the question before you. Notice that it is important for you to use your chosen numbers to substitute the unknowns contained in the original math question. At the same time, use the very same numbers to stand in place of the unknowns in your answer options. Once you have completed doing this, solve that original question and compare the solution you get with the values in the answer options provided. Remember, you will have changed your answer options to answers of real values, and as such, the answer you have gotten and the answer choices you have will be comparable.

Example:

When you have numbers represented by a and b, take a . b to mean a . b = ab + a + b.

When you have numbers represented by x, y and z, which of the expressions below would you say is/are true?

(i) x . y = y . x

(ii) (x – 1) . (x + 1) = (x . x) – 1

(iii) x . (y + z) = (x . y) + (x . z)

(i) only

(ii) only

(iii) only

(i) and (ii) only

(i), (ii) and (iii)

Remember, we have been informed that each of the numbers x, y and z will be solved using the number relationship stated above. In that case, you can pick any number that you wish to substitute for the unknowns x, y or z. The gist of the matter is that any number will work as well as the next.

Still, several variables exist in the given question, coupled with a series of equations that cannot be termed simple. To demystify the numerical relationships and equations, it is best to allocate each unknown a real number:

x = 2

y = 3

z = 4

We are now prepared to solve the problems contained in the math question in compliance with the rules provided. Then we shall confirm if the equations hold true.

For the very first situation, x . y = y . x

Beginning with the first half or the left side of the equation, replace the unknowns.

x . y then becomes 2 . 3

Going by the rules provided, this relationship translates to:

2 . 3 = (2) (3) + 2 + 3

(2) (3) + 2 + 3 = 6 + 2 + 3 = 11

So far we have established that the left side of the equation has 11.

It is now time to solve the remaining half of the equation; the right side.

y . x becomes 3 . 2

3 . 2 = (2) (3) + 3 + 2

(2) (3) + 3 + 2 = 6 + 3 + 2 = 11

We have established that the left side and the right side of the equation are equal, each having a value of 11.

The implication is that (i) is correct. As such, answer choices B and C that do not contain (i) are out of contention. We have to eliminate them.

Next we will try option (ii), whereby we plug in the same numbers we used in the first option, and they will serve as replacements for the variables.

(x – 1) . (x + 1) = (x . x) -1

Let us deal first with one side of the equation, beginning with the left side.

(x – 1) . (x + 1) will become (2 – 1) . (2 + 1)

(2 – 1) . (2 + 1) then becomes 1 . 3

1 . 3 = (1) (3) + 1 + 3

(1) (3) + 1 + 3 = 3 + 4 = 7

We can now say with confidence that the left side of the equation represents 7.

It is now time to find out what the right side of the equation represents by plugging in the same numbers we used for the left side.

(x . x) – 1

(2 . 2) – 1 = ((2) (2) + 2 + 2 – 1)

((2) (2) + 2 + 2 – 1) = 4 + 2 + 2 – 1 = 8 – 1 = 7

The right-hand side of the equation holds the value of 7, the same value the left side held. As such, we have established that option (ii) is correct.

We can also confidently rule out the possibility of answer choice A being correct.

It is now time to test whether the last equation holds true.

x . (y + z) = (x + y) + (x + z)

We shall begin with the left side of the equation, and the work will look like this:

2 . (3 + 4)

2 . 7

2 . 7 = (2) (7) + 2 + 7

(2) (7) + 2 + 7 = 14 + 2 + 7 = 23

Hence, we can say the value represented on the left side of the equation is 23.

We now need to find out what the right side of the equation holds, so that we can establish whether the two sides are equal.

(x . y) + (x . z)

(2 . 3) = (2) (3) + 2 + 3

(2) (3) + 2 + 3 = 6 + 2 + 3 = 11

2 . 4 = (2) (4) + 2 + 4

(2) (4) + 2 + 4 = 8 + 2 + 4 = 14

The requirement for this question is that we add the two values together. So let us do the required addition:

11 + 14 = 25

If you recall, the earlier side of the equation was 23, and we have now found out that the other one is 25. As such, since those two values that are representative of the two expressions are not equal, it means the original expressions themselves are not equal.

That leads us to the conclusion that option (iii) is not correct.

Since answer option E has (iii) as one of its correct answers, we can rule it out, after which we are left with only one choice of an answer—option D.

Hence, the correct answer as per the given math question is D. (i) and (ii) only .

What this means is that the equations in (i) and (ii) are the only ones among all the given equations that are correct with regards to the values of x, y and z.

As you can see, once more we have managed to use numbers of our own choice to solve a math problem. While this is exciting, we may not always be able to solve math problems in that manner. It is, therefore, up to you to make a wise decision after making a quick study of a particular question. There are times you have leeway to apply PIN, and there are times PIN cannot be used.

Also, there are times PIN can be used effectively to substitute all the variables in the question, and there are other times PIN can only be used for the substitution of a single variable among many within the same question. You will have the discretion to choose any of the options available. In order to be able to judge when best to use PIN effectively you need to do lots of practice before exam time.

You need to understand, for instance, the reason we were able to use the PIN method in the example we worked on above, plugging in our own numbers for each and every variable in that problem. The reason was that the question already stated that the equations involved held true for all the numbers. The importance of that statement was that whatever number we chose would fit into the rules that governed the math problem. Notice the use of the qualifying word, 'all.'

Consequently, by way of reading the guiding rules carefully, you will be in a position to tell if you should or should not use the PIN method. If the problem has multiple variables and it has a requirement that 'all numbers' must, of necessity, work where variables were before, then you will have room to do as we did in the example we worked out. Note that the question could refer to integers instead of numbers, but that does not change anything as far as the rules of operation go. The key word is 'all,' so if the question stipulates the same thing about 'all integers,' you are free to substitute all the variables for your chosen numbers.

As you must have noticed with the example we solved earlier on, we did not set a criterion for picking the numbers to plug in to replace the variables or the unknowns. We just picked our numbers at random.

To be on the safe side, as long as the question you want to solve has no mention of 'all' numbers or 'all' integers, be cautious and only work with one number of your choice. Note that, even in this case, you are not restricted by which number you can and cannot use. Once you choose the variable to substitute and have your number ready, work with that number to help you solve the problem and resolve the value of the other unknowns.

This need not be disappointing because that one number that you choose can easily open up an avenue for you to see how best to solve the question, whereas without one unknown less, you might be stuck. The most important point you need to remember is that you should, at all times, ensure the variables you are working with keep to the rules stipulated in the question; those that maintain the original relationship among the variables. We shall now work on one question as an example where you are not at liberty to pick a number for every variable, but where you can only choose one number to replace one variable among the ones in the question.

Example

In our problem, we have these equations:

$x = 3v$

$v = 4t$

$x = pt$

Question: For the system or series of equations above, suppose x is not equal to 0. What will the value of p be?

That is all the information you have been provided with, and there is no mention of how many integers or numbers should be used in place of variables. When the question is mute regarding whether you are at liberty to choose numbers for all the variables, proceed with caution and avoid choosing your own numbers for each and every variable. Instead, be content with choosing just one number to use in place of one of the variables in the question. Hopefully, you will be able to choose a variable that will open a quick avenue for you to solve the question.

So, to reiterate an important point, the guiding word in determining whether to plug in several numbers to match several variables in the math question, or to stick to just one, lies in the use, or lack thereof, of the word 'all.' If the question is silent on the issue of whether all variables can be replaced or not, choose to let them remain as variables, except for one. Then you can proceed to solve the problem and find the value of the other variables through calculations.

In this particular math question, we can replace v with any number of our choice. The question you may want answered is: Why are we choosing v and not any other variable for replacement, such as x? The reason is simply that v is located somewhere in the middle of the equation, and as such it has the potential to open up the equation better and help us find the value of other variables.

Solution

In the information provided, $v = 4t$.

Suppose you give variable v a number that is divisible by 4 in this example. Your calculations will certainly be easy. However, you do not have to make v divisible by 4. The advantage to plugging in a multiple of 4 where v would be is that you will not have to deal with fractions or decimals in subsequent calculations. It helps to have a quick overview of the question before you make up your mind about which number to select as a replacement for your variables.

Give v 8, so that you can have an equation, $v = 8$

Substituting 8 for every v in the question makes the first equation look this way:

$x = 3v$

$x = 3 (8)$

$x = 24$

You now have the value of x ready. You just used the PIN for number 8 to solve for x. Moving to the second equation:

$v = 4t$

$8 = 4t$ or $4t = 8$

$t = 8 \div 4$

$t = 2$

The new information you have acquired so far is that whenever $v = 8$ under the rules of our given question, $x = 24$.

Work on the second equation now.

$v = 4t$

Substituting 8 for v, you have $8 = 4t$

$8 = 4t$ or $4t = 8$

Therefore, $t = 8 \div 4 = 2$

Hence, $t = 2$

You now have plenty of information and that is $x = 24$, $v = 8$ and $t = 2$.

Now analyze and work on the last equation by using the numbers you have found as being values of the given variables.

$x = pt$

When you replace x with 24, the equation becomes $24 = pt$.

With $pt = 24$, which is the same equation as $24 = pt$, you need to replace t with 2, because you already know 2 to be the value of t.

$p (2) = 24$

$2p = 24$

Hence, $p = 24 \div 2 = 12$

There are some people who may assume that the value of p has turned out to be 12 only in this example where our chosen plug-in number was 8. However, this is not the case. As long as the rules of the question remain as they are and the equations provided remain as they are, with their respective variables, the value of p is bound to be 12.

Here is an example where v is not equal to 8 but is equal to 20:

$v = 20$

$x = 3v$

$x = 3 (20)$

Therefore, $x = 60$

The second equation will look like this:

$v = 4t$

20 = 4t

If 4t = 20, then

t = 20 ÷4 = 5

t = 5

The last equation will look like this:

x = pt

60 = p (5)

5p = 60

p = 60 ÷ 5

p = 12

As you can see, the value of p in this second example is 12, just like in the first example. The key is to leave the relationship between the variables the same way they were presented in the original statements, if you want the outcome for your variables to remain constant.

Remember to go back to the question to remind yourself of what was required or what the question demanded as an answer. Otherwise, you could do a lot of great work and then end up answering the wrong question. In this case, the question requires that you find the value of p.

P = 12 is the answer.

Helpful PIN Tricks

You are likely quite comfortable with the use of this technique of PIN, and all you need is to do frequent practice. When you have just learned a concept, it is easy to forget some steps unless you keep working with it. However, after a period of constant practice the concept becomes so easy that it joins the list of concepts that you learned ages ago. You need to practice this PIN technique until it comes to mind fast when there is a problem that might be solved with its application.

The SAT and ACT are some of the exams with questions that can be conveniently solved by way of PIN.

1) Test all the answers provided when using PIN

Why do you have to test each and every one of the answer choices provided, especially if you have already found one that works with your chosen plug-in number? There is a very good reason. When using the PIN technique, you can find more than one answer option that works well with your chosen number, and if you only test one answer option, it means you can end up with the wrong answer by choosing one that gives you just one option.

Example:

For each positive number with 2 digits, such as x, and a tens digit such as t, and a unit digit such as u, make y that number with 2 digits that is formed when you reverse the digits that belong to x. After doing this, find out which answer option has an expression that is equivalent to x – y.

Answer choices:

9 (t – u)

9 (u – t)

9t – u

9u – t

0

For this question, we are going to randomly pick 95 as our number to plug in. It is our chosen 2-digit number for variable x.

Supposing x = 95, then the tens digit, call it ‘t,’ is 9, and the units digit, call it ‘u,’ is 5

How are we going to find y? We need to reverse the digits of our chosen number.

In short, y = 59 because x = 95

So now we can proceed to try and find out what x – y is.

We are going to use the numbers 95 and 59 to fill in for x and y respectively.

x – y = 95 – 59

x – y = 36

We can now test our answer options by replacing the given variables with the values we now know, those of x and y, and also of the individual digits.

Which of the answer options matches 36?

F. 9 (t – u)

9 (t – u)

9 (9 – 5) = 81 – 45 = 36

It is great news when you are testing to see which answer option satisfies the given expression and first option works. As you can see, answer option F has worked with the digits we have.

With a bit of logic, we can go ahead and eliminate answer option K from contention, because there is no way 36 is equal to zero.

For G. 9 (u – t),

9 (5 – 9)

45 – 81 = -36

We must also eliminate answer option G. as it has turned out negative and is not what we are looking for.

Next in line is answer option H. 9t – u

9t – u

9 (9) - 5 = 81 – 5 = 76

Option H is also out of contention because it has given 76, which is not what we are looking for.

Let us find out if answer option J will work.

J. 9u – t

9 (5) – 9 = 45 – 9 = 36

Here is another 36, and so we have two answer options to credit with meeting the required condition.

There are two answer options whose expressions are equivalent to x – y—F and J. However, we want one option that consistently satisfies the given condition that all relationships remaining unaltered. Otherwise, it is possible to have an option that satisfies the given expression only with some chosen numbers and not with others. When using the PIN technique, consistency is of fundamental importance.

Some doubts that might plague students' mind are: If we go for a different set of substitute numbers, do we have to test every answer option all over again? The answer is no, because there are some answer options that were found to be out of contention, and because consistency is crucial, even if they worked with our new set of numbers, we still would not call them fitting for our expression.

In our case here, answer options G, H and K are already out of contention, courtesy of our first testing. As such, we are only going to test answer options F and J once more with different numbers, which we shall get after working with our new PIN.

Let our new PIN be 43 instead of 95, and we will use it in place of variable x. Just as in the case of 95, we have no reason whatsoever for choosing 43 as opposed to any other number with a double digit. Our choice is entirely random.

When x = 43, t can only be 4 and u can only be 3, according to the stipulations of the question we are working on.

At the same time, when x = 43, y can only be 34, also according to the stipulations contained in this question.

x – y = 43 – 34 = 9

x – y = 9

We have now quickly arrived at the stage where we want to find the answer choices matching the value, 9. In the interest of time, it is important to remember that we only need to test two answer options, F and J.

Answer option F. has 9 (t – u)

9 (t – u) = 9 (4 – 3) = 9 (1) = 9

Answer option F has satisfied the relevant expression.

It is now time to test answer option J.

Answer option J. has 9u – t

9u – t = 9 (3) – 4 = 27 – 4 = 23

Answer option J has failed to satisfy the given expression, and so we have to eliminate it.

Consequently, the correct answer option for the question is F. 9 (t – u).

2) Stay clear of 1 and 0 as potential plug-in numbers

It is important to avoid the use of either zero or one as plug-in numbers because they are not reliable. They are the kind of numbers which, when substituted for variables, can give you multiple correct answers until you wonder what is right and what is wrong. They can also lead you to very ridiculous outcomes. In short, whatever they produce is not something that can give a candidate the confidence they need during exam time.

3) 10 and 100 are great for PIN

Among the best numbers to use for the PIN technique are 10 and 100. This is especially great when the question you are trying to solve has percentages, in which case smooth round numbers like 10 or 100 are of great help. There are some numbers you can choose which will lead to percentages that are difficult to work with.

4) Not all questions can be solved with PIN

The idea of plugging in any number at random to replace a variable whose value is unknown is enticing. Yet you should not always rush to use PIN without some initial assessment of the question.

For questions whose answer choices have integers, fractions and even decimal numbers, your best bet is not PIN but its counterpart, PIA. PIN does well in questions whose answer options comprise variables. Even when it is the question that is dominated by variables, PIN helps a great deal in solving the problem quickly.

In short, the liberal usage of random numbers in the PIN method of solving equations should not be misused, because using it inappropriately can lead to time wastage. Moreover, you may not solve the question adequately enough to be able to select the correct answer from the choices provided.

Best Times to Use PIN

Just as we pointed out about PIN, it is equally important to highlight the best times to use this method. That will save you from trying out the PIN method only to realize halfway through that you were better off solving the problem algebraically. One thing that is indisputable, though, is that when applied on the right kind of questions, the PIN method is effective, accurate and fast.

(1) Use PIN if you do not know any other method of tackling the question.

(2) Use PIN if you have plenty of time to spare.

As you must have noted, the PIN and PIA methods involve a good degree of trial and error. So if you are constrained for time, these cannot be your methods unless the question you are addressing is the very last one for your exam.

(3) Use PIN to check whether answers you got through other methods are correct.

(4) Use PIN if you are convinced you made a mistake on a certain question.

(5) Use PIN on questions you are prone to missing.

If you are not confident about the topic on which a particular question is based, the PIN method can be very helpful.

Chapter 17: Parentheses And Algebra

Parentheses help us to know which order to undertake different functions, that is, addition, subtraction, multiplication and division.

Example 1

7 + 4 (5 – 2) =

From the above example, you are required to first work out the values inside the p by subtracting 2 from 5. The answer is multiplied by 4 and then, finally, you add the 7 as follows:

5 – 2 = 3

4 (3) = 12

7 + 12 = 19

Therefore, 7 + 4 (5 – 2) = 19

However, the brackets can also be removed by multiplying each number in the brackets by the number outside the bracket. If you choose to use this method, then it is important to pay close attention to the signs used, especially when the –ve sign is used as in the example above.

4 x 5 = 20

4 x (- 2) = - 8

7 + 20 – 8=

7 + 20 = 27

27 – 8 = 19

Hence, 7 + 4 (5 – 2) = 19

To avoid making mistakes when multiplying, especially where the negative sign is involved, always remember the following:

Positive multiplied by positive means the answer is always positive.

5 x 7 = 35

Positive multiplied by negative means the result is always negative.

6 x (- 7) = - 42

Negative multiplied by positive means the result is always negative.

(- 4) x 5 = - 20

Negative multiplied by negative means the result is always positive.

(-3) x (- 7) = 21

The above are basic rules which must always be remembered when multiplying numbers.

Basically, if there is a + sign before a bracket then the signs of the terms remain the same after opening the brackets.

P + 2 (3a - 4c) = P + 6a – 8c

a + 5 (4b + 6c) = a + 20b + 30c

However, if the sign is negative then the resultant must change its sign to that which opposite to the term inside the bracket when the bracket is removed.

7 –a (2b + 3c) = 7 - 2ab – 3ac

Note that the term 2b, which is positive, changes its sign to –ve after opening the bracket due to –a. 3c, which is also positive, changes to – 3ac for the same reason.

y – 3 (6a – 2y) = y – 18a + 6y = y + 6y – 18a = 7y – 18a

We start by opening the brackets and multiplying each term by -3. The sign changes as explained above. The next step is putting the like terms together, y + 6y = 7y, which is then added to 18a.

Exercise 23

Remove the brackets and simplify (put the like terms together.)

1) 4 + 8 (2a – 7)

2) 9 – (-b + 4)

3) 3 (x + 5) -2 (x + 7)

4) y - {3y – (4 + y)}

5) 3 {(2a + b) – (a – b)}

6) 7a + 3a ^{2} + 5a

Terms are called like terms if they have the same letters raised to the same power. Otherwise, they are unlike terms. For example, 5b and 7b ^{2} are unlike terms, while b ^{2} and 6b ^{2} are like terms.

In each of the following, pick the term which is not like the others.

a) 3a, 5a, 13a, 4a ^{2}

b) m ^{4} , 7m ^{3} , 6m ^{3} , 11m ^{3}

We have learned to remove brackets in some expressions like 6 (m + n) = 6m +6n. This process is called expansion. Sometimes the reverse process is required, such as in 6m + 6n. Since 6 is a common factor, we divide each term by the factor as follows:

6m ÷ 6 = m

6n ÷ 6 = n

Hence, 6m + 6n = 6 (m + n)

The above process of including (introducing) parentheses in various algebraic equations is called factorization.

Exercise 24

Complete the following

a) 2a + 2b = 2 (__ + __)

b) 4c – 4d = __ (c – d)

c) 3a + 6b = __ (__ + __)

d) 3a + 6ab – 9a ^{2} b = __ (1 + 2b – 3ab)

Factorize each of the following

2a + 4b + 3a + 6b

3p + 3r + 3q + 3r

Multiply the sum of b and c by a

Multiply h + 4 by s + b

Find the area of the rectangle shown below p + q

r + s

Peter has x books while James has 5 more books than him. Julie has t books and Brian has one less than Julie. How many books do they have altogether?

Factorization By Grouping

In the previous section we dealt with factorizing in expressions such as bc + ba.

It involved us looking for the common factor between terms then dividing the terms by that factor.

In our expression above, the common factor is b. We therefore divide both terms by b as follows:

bc ÷ b = c

ba ÷ b = a

We now introduce the bracket and start by writing the common factor outside, and the resultants inside, as shown below:

b (c + a)

In this section we will factorize expressions that don’t have a common factor for all the terms.

Example 1

Factorize

ax + a + bx + b

There is no common factor in our expression. However, if we take them pairwise, i.e., ax + a and bx + b, then each pair has a common factor.

We can thus factorize the first equation by taking the common factor, which is a, and dividing both terms by it.

ax ÷ a = x

a ÷ a = 1

a (x + 1)

Let’s do the same for the second equation, bx + b, whose common factor is b

bx ÷ b = x

b ÷ b = 1

b (x + 1)

The two equations can be written as one

a (x + 1) + b (x + 1)

x + 1 is also a common factor in both expressions. Thus,

ax + a + bx + b = a (x + 1) + b (x + 1)

= (x + 1) (a + b)

The above method is called factorization by grouping because we paired our terms, then sought the common factors in each pair.

NOTE : We can also pair the terms any other way and still get the same answer.

Consider if the expression was written as follows:

$ax + bx + a + b$

We start by pairing them up, i.e., $ax + bx$ and $a + b$

in the first pair.

x is common, thus:

$ax \div x = a$

$bx \div x = b$

$x (a + b)$

For the second pair, 1 is common, thus:

$a \div 1 = a$

$b \div 1 = b$

$1 (a + b)$

The two equations can be written as one $x (a + b) + 1 (a + b)$

$a + b$ is our common factor in both expressions and thus:

$ax + bx + a + b = x (a + b) + 1 (a + b)$

$= (x + 1) (a + b)$

It is therefore important to note that it does not matter how the expression has been arranged. As long as there is a common factor between each pair then the expression can be factorized.

Interchanging the final brackets e.g., $(a + b) (x + 1)$ does not in any way affect the answer.

Consider $3 \times 2 = 6$ and $2 \times 3 = 6$.

Therefore, it does not matter which term begins the sequence. The answer is always the same.

Exercise 25

$nx + bn + mx + bm$

Factorize

$ab + bc – a – c$

Factorize

$3n – 3w + mn – mw$

Addition And Subtraction Of Algebraic Fractions

In algebra we can still add and subtract fractions just like in arithmetic. The process is the same and involves the following steps:

✓ Start by getting the LCM of the denominators.

✓ Multiply all terms by the LCM.

✓ Put the like terms together, if there are any.

Example 1

-

=

The LCM of 4 and 5 is 20.

Multiply the terms by 20.

x 20 = 5a

x 20 = 4b

The answer is, therefore,

Example 2

Simplify

-

=

We start by getting the LCM of 5 and 3 which is 15.

(

) +($\frac{a}{3}$ x 15

) =

$\frac{3a + 5a}{15}$

We complete the problem by putting the like terms together.

The answer is

.

Example 3

Simplify

2x -

From this question we realize that not all terms are fractions. However, the denominator of a whole number is always 1. Whether it is written or not makes no difference to the value of the whole number. Bearing that in mind, we can now rewrite the equation as

-

=

The LCM of 5 and 1 is 5.

Multiply all the terms by the LCM.

{(

) – (

)}

{2x – (

)}

{2x – 15x}

= - 13x

Example 4

Simplify

-
-

We start by getting the LCM of 2, 4 and 5, which is 20.

Multiply each term in the equation by the LCM.

{(

x 20) + (

x 20) – (

x 20)}

{10 (x – 1) + 5 (x + 2) – 4 (x)}

{10x – 10 + 5x + 10 – 4x}

20

Put the like terms together:

{10x + 5x – 4x + 10}

20

And the answer is

.

Exercise 26

Express each of the given expressions as one fraction and make it as simple as possible.

-
-
-
-

Chapter 18: Simplification Of Algebraic Fractions Involving Multiplication And Division

This is also called cancelling.

If the numerator and denominator of a fraction are divided by a common factor then the ratio of the fraction is unaltered and remains the same. For example:

=

=

However, it is important to note that we only cancel when the sign between the terms is multiplication.

= 14 . This is wrong because:

=

correct answer)

Let’s see how we could have canceled if the sign was multiplication instead of addition.

3

= 33 correct answer

Similarly

=

= bc

Example:

5xy = 5xy = 5x

y ² y.y

Simplification by Factorization

One of the uses of factorization is simplification of various expressions.

Example 1

Simplify:

We will start by factorizing the numerator and the denominator.

We start with the numerator:

ra + rb

The common term in our expression is r. We therefore divide both terms by r:

ra ÷ r =

a

rb ÷

r = b

The answer is r (a + b) for the numerator.

We also work out the denominator in the same manner, i.e., ma + mb.

The common factor here is m. We therefore divide both terms by m.

ma ÷ m = a

mb ÷ m = b

The answer for the denominator is m (a + b).

If we combine both the numerator and the denominator, our expression is as follows:

Since (a + b) is common in both the numerator and the denominator, and the sign between the two terms is multiplication, then we can cancel out as shown below:

r (a + b)

m (a + b)

The answer is, therefore,

.

Example 2

$$\frac{ay - ax}{by - bx}$$

Steps to follow:

✓ Start by working out the numerator by factorizing.

✓ Work out the denominator by factorizing.

✓ Combine the answer for the numerator and the denominator.

✓ Check whether there is a common factor between the numerator and the denominator.

✓ If there is a common factor, cancel them out.

ay – ax

Our common factor in this expression is a. We will therefore divide both terms by a:

ay ÷ a = y

-ax ÷ a = - x

Our numerator can now be written as a (y – x)

Let's work in the same way for the denominator, which is by – bx.

Our common factor in the terms is b.

We then divide both terms by the common factor:

by ÷ b = y

-bx ÷ b = - x

The denominator is now written as b (y – x).

We now combine the numerator and the denominator to get:

We realize there is a common factor between the numerator and the denominator which is (y – x).

We now cancel the common factor:

a (y – x)

b (y – x)

The final answer is, therefore,

.

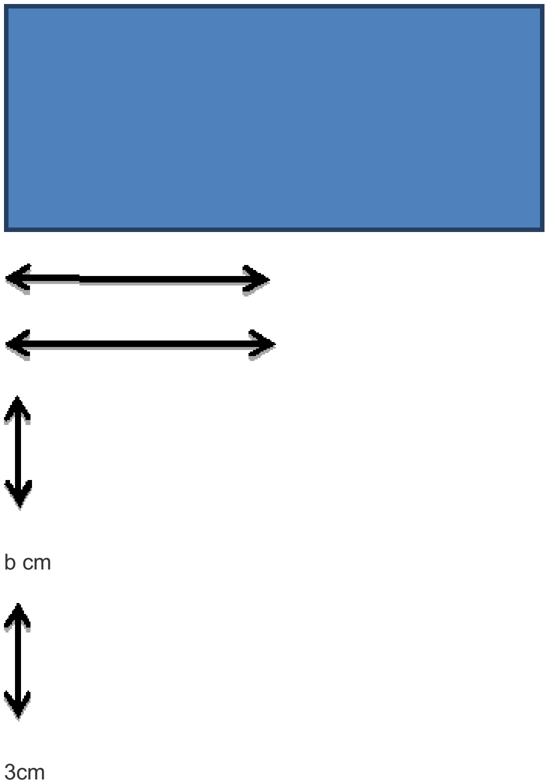
Substitution

This is a process which involves giving variables specific values in an expression.

Example 1

Consider the rectangle shown below. Find the area of the figure.

a cm 4cm



The area of a rectangle is derived by length multiplied by width.

The length for this rectangle is (a + 4) cm while the width is (b + 3) cm.

Its area is, therefore, (a + 4) multiplied by (b + 3)

This can be written as (a + 4) x (b + 3)

However, the multiplication sign can be ignored and the expression written as:

(a + 4) (b + 3)

We are now required to remove the brackets through a process called expansion, as follows.

Multiply the terms in the second bracket by those in the first bracket as shown below:

a (b + 3) + 4 (b + 3)

ab + 3a + 4b + 12

It's always important to check whether there are like terms which can be combined. In this case, there are none. Hence, we proceed and leave the area of our rectangle as

= ab + 3a + 4b + 12.

If we know the values of a and b, we can replace them in our expression and get the actual area of the rectangle.

Let's take, for instance, if a = 4 and b = 2. We can replace the values of a and b in our first expression as follows:

Method 1

(a + 4) (b + 3)

Where there is letter a, we put 4, and where there is letter b, we put 2, such that we will have:

(4 + 4) (2 + 3)

(8) (5)

8cm x 5cm = 40cm ^{2}

The actual area of the rectangle after substitution is $40cm^2$.

Method 2

We can also get the actual area by substituting in the final expression which we had after expansion, i.e.,

ab + 3a + 4b + 12.

Where there is an a, we substitute it with 4. And where there is a b, we substitute it with 2,

such that:

ab + 3a + 4b + 12 = {(4 x 2) + (3 x 4) + (4 x 2) + 12}

= {8 + 12 + 8 + 12}

Answer = $40cm^2$

It does not matter at what juncture you substitute. Whether at the beginning, like in Method 1, or at the end, like in Method 2, the final answer is the same.

Example 2

Evaluate the expression

$r^2 + q^2 + m$, given that r = 3, q = 4 and m = 5.

The same process applies. We check where in our equation there is an r and we substitute it with 3. Where there is a q we replace it with 4, and where letter m appears we replace it with 5, as shown below:

$3^2 + 4^2 + 5$

= {(3 x 3) + (4 x 4) + 5}

= {9 + 16 + 5}

The answer is, therefore, 30.

Example 3

The surface area of an open box of side a, b and c centimeters is given by

A = 2b (a + c) + ac

Find A if a = 40, b = 20 and c = 12.

We replace the values of a, b and c in the expression as follows:

A = 2b (a + c) ac

A = {(2 x 20) (40 + 12) + (40 x 12)}

We proceed and open the brackets.

A = {(40) (40 + 12) + (40 x 12)}

A = {(40) (52) + (40 x 12)}

A = {(40) (52) + (480)}

Note that the sign between the brackets of 40 and 52 is multiplication.

A = {(2080 + 480)}

A = $2560cm^2$

Practice Exercise

If $V = l \times w \times h$.

a) Find V if l = 4cm, w = 5cm and h = 6cm.

$V = l \times w \times h$

$V = 4cm \times 5cm \times 6cm$

$V = 120cm^3$

b) Find l if wh = 8cm and $V = 24cm^3$

$V = l \times w \times h$

$24cm^3 = l \times 8cm$

Our interest is in l. We therefore should eliminate 8 on the right-hand side (RHS) by dividing all through by 8:

$24 \div 8 = l \times 8 \div 8$

$3 = l$

If $A = (R^2 – r^2)$, find A when:

a) R = 15 and r = 11

$A = (15^2 – 11^2)$

A = {(15 x 15) – (11 x 11)}

A = {(225) – (11 x 11)}

A = {(225) – (121)}

A = {225 – 121}

A = 104

Don't write any units because the substitution values have not been indicated in specific units. If the values you are substituting with have units, it is important to write the units at the end of the question.

Linear Equations With One Unknown

This is a simple algebraic expression with only one variable.

Example 1

$3x + 4 = 10$

Make sure the unknown remains on one side of the equation by taking the other values of the expression on one side.

$3x + 4 = 10$

Take 4 to the RHS by subtracting it from both sides.

$3x + 4 - 4 = 10 – 4$

$3x = 6$

To get the value of x, we divide both sides by 3:

=

$x = 2$

Example 2

– 2 = 4

We make sure that the unknown is one side of the equation by taking -2 on the RHS.

– 2 + 2 = 4 + 2

= 6

The next step involves removing the denominator by multiplying both sides by 3.

x 3 = 6 x 3

x = 18

Example 3

=

The first step here is removing the denominator by getting the LCM of 3 and 4, which is 12.

x 12 =

x 12

4 (p + 5) = 3 (5)

4p + 20 = 15

Make sure the unknown is on one side of the equation by subtracting -20 from both sides.

4p + 20 – 20 = 15 – 20

4p = - 5

We remove 4 from the left-hand side (LHS) by dividing both sides by 4.

=

The final answer is, therefore, p =

.

P can also be written as a decimal as p = - 1.25.

It can also be written as a mixed fraction as p =

.

Practice Exercise

1)

-

=

Start by removing the denominators by getting the LCM of 2, 3 and 8, then multiplying each term in the expression by it.

The LCM of 2, 3 and 8 is 24.

{(

x 24) – (

x 24) = (

x 24)}

{12 (x + 1) – (

x 24) = (

x 24)}

{12 (x + 1) – 8 (x – 2) = (

x 24)}

{12 (x + 1) – 8 (x – 2) = (3)}

{12x + 12 – 8 (x – 2) = (3)}

{12x + 12 – 8x + 16 = 3}

Combine the like terms:

12x – 8x + 12 + 16 = 3

4x + 28 = 3

Make sure the unknown is on one side of the equation by subtracting -28 from both sides.

4x + 28 – 28 = 3 – 28

4x = - 25

Divide both sides by 4 to be left with x on one side.

=

x =

The answer can also be written as a decimal as x = - 6.25

or as a mixed fraction as x = $-6\frac{1}{4}$

.

NOTE: If any operation is performed on one side of an equation, it must also be performed on the other side.

2) A piece of wire 200cm long is bent to form a rectangular shape. One side of the rectangle is 4cm longer than the other. Find the dimensions of the rectangle.

In such word problems, it is always advisable to start by reading and rereading the question so as to interpret it well.

After fully understanding the question, it is important to draw a sketch when possible. This helps in interpreting the question as per the examiner wishes.

If the question has any measurements, put them where they should be on the sketch.

Let's start by rereading the question to ensure our understanding.

Points to note:

If the length of the wire is 200cm and the whole of it is used to form a rectangular shape, then the perimeter of the rectangle is 200cm.

One side of the rectangle is 4cm longer than the other. That statement simply means that if one side of the rectangle is xcm, then the other, which is 4cm longer, is (x + 4) cm.

Let's start by drawing our sketch:

A(x + 4) cm

x cm

____

We then ask ourselves what perimeter is. Perimeter is the distance all around an object. Let's consider the rectangle below, say, PQSR.

____

The perimeter of the rectangle above involves adding the distance from

(P to Q) + (Q to R) + (R to S) + (S to P)

It is also important to note that opposite sides of a rectangle are equal.

In our question we are required to get the perimeter from the top shape, presumably, ABCD.

(A to B) + (B to C) + (C to D) + (D to A)

(x + 4) + (x) + (x + 4) + (x) = 200

x + 4 + x + x + 4 + x = 200

Put the like terms together:

x + x + x + x + 4 + 4 = 200

4x + 8 = 200

Make sure the term with the unknown is on one side of the expression, alone, by subtracting 8 from both sides:

4x + 8 – 8 = 200 – 8

4x = 192

Divide both sides by 4 to get the value of x:

=

x = 48

Having gotten the value of x as 48, we can now replace 48 in both terms so as to have the dimensions of the rectangle as needed.

The expression for the length is (x + 4) and x = 48. We substitute 48 in our expression to have

(48 + 4) = 52. The length of the triangle is, therefore, 52cm

The expression for the width was simply (x) cm, therefore, the width of the rectangle is 48cm.

Measurements for the rectangle are as follows:

Length = 52cm

Width = 48cm

3) A man earns x shillings while his wife earns

of this. After spending a third of their combined income, they have 2400 shillings left. How much money does the man earn?

Reread the problem, to ensure you've fully understood it. Then:

Man's earnings = x

Wife earnings =

x

Combined income = x +

x =

x

After spending a third of their combined earnings, they are left with 2,400 shillings.

Linear Equations with Two Unknowns

If x + 3 = 8, then x can only be 5 so as to satisfy the expression given. However, in an expression with two unknowns there are a number of pairs of values of the unknowns which satisfy it.

e.g., if x + y = 8, then x could be any number depending on the value of y.

if y = 1 x = 7; if y = 2, x = 6; if y = 3, x = 5.

Consider the following pairs of equations:

$x + y = 8$

$x – y = 6$

Each pair has two unknowns of the first degree. Such equations have only one pair of values which satisfy both of them.

Complete the table below for $x + y = 8$:

Copy and complete the table for $x – y = 6$

Check your answers. Note that there is only one pair of values that appears in both tables: When $x = 7$, $y = 1$, which is the solution of the equation $x + y = 8$ and $x – y = 6$ simultaneously.

In the last section we dealt with equations with one unknown. We shall now deal with equations with two unknowns. Equations with two unknowns are also called simultaneous equations . Consider the following situation:

Example 1

The cost of two skirts and three blouses is 600 shillings. If the cost of one skirt and two blouses of the same quality is 350 shillings, find the cost of each item

Let the cost of one skirt be x shillings and the cost of one blouse be y shillings.

We can now form equations representing the above information. Let's start with the first part of the question, which states:

The cost of two skirts and three blouses is 600 shilling.

The cost of one skirt is x. Therefore, the cost of two skirts is $(x * 2) = 2x$

The cost of one blouse is y. Therefore, the cost of 3 blouses is $(y \times 3) = 3y$

$2x + 3y = 600$......................... equation one

We also form another equation to represent the other part of the question, i.e.,

if the cost of one skirt and two blouses of the same quality is 350 shillings,

the cost for one skirt remains as x while the cost of the blouses is $(y \times 2) = 2y$.

$x + 2y = 350$ .... equation two

So, our two equations are as follows:

$2x + 3y = 600$

$x + 2y = 350$

We can solve these two equations by eliminating either x or y. This will help to reduce it from two unknowns to one unknown. This is called the elimination method of solving simultaneous equations.

Let's see how it is done:

Method 1

We will eliminate x.

Steps to follow:

Make sure that the coefficients (the value in front of x in an equation) of x are the same in both equations.

If they are not, make them similar by multiplying equation one with the coefficient of x in equation two and multiplying equation two with the coefficient of x in equation one.

Subtract equation two from equation one.

You are now left with one unknown, which is y, in a single variable equation.

Make sure the unknown is on one side of the equation and no other value is on the same side.

It is now time to divide each side using the unknown's coefficient.

2x + 3y = 600

x + 2y = 350

The coefficients of x are not similar. We make them common by multiplying equation one by 1 and equation two by 2.

NOTE: When multiplying, multiply all the terms of the equation.

1 (2x + 3y = 600) = 2x + 3y = 600

2 (x + 2y = 350) = 2x + 4y = 700

Subtract the two equations:

2x + 3y = 600

2x + 4y = 700

0 – y = - 100

-y = - 100

Since we do not want the negative value of y, we divide both sides of the equation by -1 to eliminate the negative sign before y, as follows:

–

=

y = 100

Having gotten the value of y, we can now substitute it on either of the equations so as to get the value of x.

Let's substitute it on our equation 1:

2x + 3y = 600

2x + (3 x 100) = 600

2x + 300 = 600

2x + 300 – 300 = 600 – 300

2x = 300

=

x = 150

The cost of a skirt is 100 shillings while the cost of a blouse is 150 shillings.

We can confirm our answer by taking any of the two equations and substituting with the actual values of x and y. Let's use equation two:

x + 2y = 350

150 + (2 x 100)

150 + 200 = 350

We have successfully confirmed and proved our answers.

Example 2

Solve the following simultaneous equations by elimination method, then check your Answers :

m + n = 8

m – n = 4

Since the coefficients of m are similar, we can subtract directly.

m + n = 8

m – n = 4

0 + 2n = 4

2n = 4

=

n = 2

We now substitute n with 2 in any of our equations, such that:

m + 2 = 8

m + 2 – 2 = 8 – 2

m = 6

We check our answer by replacing n with 2 and m with 6 in any of the equations.

m + n = 8

6 + 2 = 8

The value of m = 6 and the value of n = 2.

Example 3

Solve the following simultaneous equations by elimination method, then check your answers:

2m + n = 7

3m – 2n = 10

Multiply the first equation with the coefficient of m in the second equation and the second equation with the coefficient of m in the first equation, as follows:

3 (2m + n = 7)

6m + 3n = 21

(3m – 2n = 10)

6m – 4n = 20

We subtract the two equations:

6m + 3n = 21

6m – 4n = 20

0 + 7n = 1

7n = 1

Divide all through by n:

=

n =

Substitute n with

in any of the equations:

2m + n = 7

2m +

= 7

2m +

-

= 7 -

2m = 6.85714285715

=

m = 3.428571428575

We now confirm our answer by substituting both m and n in any of the two equations:

2m + n = 7

{(2 x 3.428571428575) +

)} =

{6.85714285715 +

} = 7

We have successfully found the values of m and n and also confirmed our answers.

Practice Exercise

1 3a + 5b = 20

6a + 5b = 12

Since the coefficients of b are similar, we can just subtract the two equations as they are and eliminate b as follows:

3a + 5b = 20

6a + 5b = 12

-3a + 0 = 8

-3a = 8

=

a =

Substitute b in any of the two equations:

{(3 x

) + 5b = 20}

-8 + 5b = 20

-8 + 8 + 5b = 20 + 8

5b = 28

=

b =

We check our answers by substituting the values in our equations:

3a + 5b = 20

{(3 x

) + (5 x

)

(- 8) + (28) = 20

We have successfully confirmed our answer. Thus, the value of a =

and the value of b =

.

2 The price of four handkerchiefs and three pairs of socks is 340 shillings. If three handkerchiefs and four pairs of socks cost 395 shillings, find the price of each item.

Let the price of the handkerchief be x and the price of a pair of socks be y.

Equation one.......4x + 3y = 340

Equation two.......3x + 4y = 395

3 (4x + 3y = 340)

12x + 9y = 1020

4 (3x + 4y = 395)

12x + 16y = 1580

12x + 9y = 1020

12x + 16y = 1580

- 7y = - 560

=

y = 80

4x + 3y = 340

{(4x + (3 x 80)} = 340

4x + 240 = 340

4x + 240 – 240 = 340 – 240

4x = 100

=

x = 25

Substitute the values of x and y in any of the two equations to prove your answer.

4x + 3y = 340

{(4 x 25) + (3 x 80) = 340

100 + 240 = 340

340 = 340

Since the values on the LHS and those on the RHS are equal, then it proves that x = 25 and y = 80.

Biscuits are sold in two types of packets, A and B. 6 packets of A and 7 packets of B contain 84 biscuits, while 3 packets of A and 2 packets of B contain 33 biscuits. Find the number of biscuits in each type of packet.

Let packet A be represented by letter a and packet B by letter b.

6a + 7b = 84

3a + 2b = 33

3 (6a + 7b = 84)

18a + 21b = 252

6 (3a + 2b = 33)

18a + 12b = 198

Subtract the two equations.

18a + 21b = 252

18a + 12b = 198

- 9b = 54

=

b = 6

6a + 7b = 84

6a + (7 x 6) = 84

6a + 42 = 84

6a + 42 – 42 = 84 – 42

6a = 42

=

a = 7

6a + 7b = 84

{(6 x 7) + (7 x 6)}

42 + 42 = 84

Substitution Method Of Solving Simultaneous Equations

Example 1

Solve the following equation using the substitution method.

x + 3y = 8............................equation 1

5x + 7y = 24........................equation 2

Taking equation 1 alone, we can make x the subject, then substitute the value of x in equation 1 to equation 2.

You realize that we are now left with one unknown.

Proceed and put the like terms together, then make sure that the terms with the unknown are on one side of the equation as follows:

x + 3y = 8

x + 3y – 3y = 8 – 3y

x = 8 – 3y

Substitute the value of x in equation 2:

{5 (8 – 3y) + 7y = 24}

{40 – 15y + 7y = 24}

{40 – 8 y = 24}

40 – 40 – 8y = 24 – 40

-8y = - 16

Divide both sides by -8:

=

y = 2

Having gotten the value of y, we can now replace it in either of the two equations.

Let’s use equation one:

x + 3y = 8

x + 3 (2) = 8

x + 6 = 8

x + 6 – 6 = 8 – 6

x = 2

Always check and confirm your answer by substituting the values of x and y. You can use any of the equations. For our case, we will use equation 2:

5x + 7y = 24

{(5 x 2) + (7 x 2)} = 24

{10 + 14} = 24

24 = 24

Since the values on the LHS and the RHS are equal, then our values for x and y are correct.

EXAMPLE 2

Solve the following equation using the substitution method

x + y = 7

3x + y = 15

In this example we are going to take equation 2 and make y the subject as follows:

3x + y = 15

3x – 3x + y = 15 – 3x

y = 15 – 3x

We substitute the value of y in the first equation:

x + 15 – 3x = 7

Put the like terms together:

x – 3x + 15 = 7

-2x + 15 = 7

Let the term with the unknown be left on one side of the equation by subtracting 15 on both sides:

-2x + 15 – 15 = 7 – 15

-2x = -8

Divide both sides by -2:

x = 4

Having gotten the value of x, we can replace it in either the first or the second equation. We replace the value of x in the second equation.

3x + y = 15

{(3 x 4) + y = 15

12 + y = 15

Subtract 12 from both sides, so as to be left with y on one side, as follows:

12 – 12 + y = 15 – 12

y = 3

We now replace the values of x and y in either equation, to confirm your answer.

3x + y = 15

{(3 x 4) + 3 = 15

12 + 3 = 15

15 = 15

Since the values on the LHS and the RHS are equal, then our values for x are correct.

Exercise

Solve the following simultaneous equations using the substitution method:

9x – y + 7 = 0...equation one

13x – 4y + 5 =0..equation two

Making y the subject in equation one,

take all other values on the RHS to be left with y alone on the LHS, as follows:

9x – 9x – y + 7 = 0 – 9x

-y + 7 = - 9x

-y + 7 – 7 = -9x -7

-y = -9x - 7

Since we got the negative value of x, we divide all terms by negative one to make the value of y positive.

=

-

y = 9x – 7

Substitute y in the second equation:

13x – 4y + 5 =0

13x – 4 (9x – 7) + 5 =0

13x – 36x + 28 + 5 = 0

-23x + 28 + 5 = 0

-23x + 33 = 0

-23x + 33 – 33 = -33

-23x = -33

Replace x with its value in the second equation so as to get the value for y:

13x – 4y + 5 =0

{(13 x

) – 4y + 5 = 0}

– 4y + 5 = 0

Add 4y to both sides so as to make sure it is on one side of the equation and alone:

– 4y + 4y + 5 = 4y

- 5 = 4y

= 4y

Divide both sides by 4. But since on the LHS there is a fraction, we multiply by:

x

= 4y x

= y

The cost of 3 sandwiches and 2 cups of tea is 60 shillings. If 2 sandwiches and 3 cups of tea cost 65 shillings. Find the cost of:

A) a sandwich

B) a cup of tea

Let the cost of a sandwich be x shillings and that of tea be y shillings.

Form the two equations as follows:

3x + 2y = 60

2x + 3y = 65

Quadratic Expressions and Equations

Expansion

Previously, we removed brackets in expressions of the form a (x + y) to have ax + ay.

The same idea can be used to remove brackets in expressions such as the following example:

Example 1

(x + 5) (2x + 3)

Let (x + 5) be a

Thus, a (2x + 3) = 2xa + 3a

2xa + 3a = 2x (x + 5) + 3 (x + 5)

= $2x^2 + 10x + 3x + 15$

Put the like terms together:

$2x^2 + 13x + 15$

When the expression (x + 5) (2x + 3) is written in the form of $2x^2 + 13x + 15$, it is said to have been expanded. This expression is said to have been written in the form of $ax^2 + bx + c$ where a, b and c are constants and it's called a quadratic expression. In such equations, a is called the coefficient of x^2 , b is the coefficient of x and c is the constant term.

Example 2

Expand the expression:

(m + n)(m – n)

Let (m – n) be a

(m + n) a = ma + na

m (m - n) + n (m – n)

$m^2 – mn + nm – n^2$

Put the like terms together. Note that nm and mn are similar.

Therefore, $m^2 – n^2$ is the expanded form of (m + n)(m – n).

Practice Exercise

(6x + 2) (4x + 3)

Let (4x + 3) be a

a (6x + 2) = 6xa + 2a

6x (4x + 3) + 2 (4x + 3)

$24x^2 + 18x + 8x + 6$

Ensure the like terms are together:

$24x^2 + 26x + 6$

(x – 1) (x + 2)

Let (x – 1) be a

a (x + 2)

xa + 2a

x (x – 1) + 2 (x – 1)

$x^2 - x + 2x - 2$

$x^2 - x - 2$

The expanded form of $(x - 1)(x + 2)$ is, therefore, $x^2 - x - 2$

$(x + 2)^2$

When an expression is raised to a certain power, you are required to multiply the power by itself the number of times it has been raised. e.g.,

$(a + b)^3 = (a + b)(a + b)(a + b)$

In our case $(x + 2)^2 = (x + 2)(x + 2)$

$x(x + 2) + 2(x + 2)$

$x^2 + 2x + 2x + 4$

$x^2 + 4x + 4$

Factorizing Quadratic Equations

Factorizing involves introducing brackets where there are none.

We explained the general quadratic expression as an expression written in the form of $ax^2 + bx + c$ where a, b and c are constants. In such equations, a is called the coefficient of x^2, b is the coefficient of x and c is the constant term.

Factorizing Expressions

Example 1

$x^2 + 6x + 5$

In a situation where the coefficient of x^2 is 1 then the factors will be in the form of

$(x + a)(x + b)$

Look for two whole numbers such that if you add them, they will give you 6 and if you multiply them, they will give you 5.

In this case, the two numbers are 5 and 1.

We substitute them to the coefficient of x such that our equation will be as follows:

$x^2 + 5x + x + 5$

We pair our expression to have $x^2 + 5x$ and $x + 5$.

We start by factorizing the first pair $x^2 + 5x$ and look for a common factor which is x.

Divide both terms by x.

$x^2 \div x = x$

$5x \div x = 5$

$x(x + 5)$

For the second pair, $x + 5$, the common factor is 1:

$x \div 1 = x$

$5 \div 1 = 5$

$(x + 5)$

Since $(x + 5)$ is a common factor, our factorized equation is $(x + 1)(x + 5)$.

Example 2

Factorize the expression:

$8x^2 + 10x + 3$

Multiply the coefficient of x^2 and the constant term:

$8 \times 3 = 24$

Get the coefficient of x, which is 10.

Check which two numbers when multiplied will give 24 but, when added, give 10.

The two numbers in question are 6 and 4.

Rewrite the term 10x as 6x + 4x.

Thus, the whole expression can be written as

$8x^2 + 6x + 4x + 3$

We pick our terms pair-wise, i.e., $8x^2 + 6x$ and $4x + 3$

Check for the common factor in the first pair, i.e., $8x^2 + 6x$

2x is the common factor. Hence, we divide all terms by the common factor as follows:

$8x^2 \div 2x = 4x$

$6x \div 2x = 3$

$2x (4x + 3)$

Look for the common factor in the second pair, i.e., 4x + 3.

1 is our common factor here, so we divide all terms by it.

$4x \div 1 = 4x$

$\div 1 = 3$

$(4x + 3)$

(4x + 3) is also common. Hence, our factorized expression can be written as

$2x (4x + 3) + 1 (4x + 3)$

Our final answer is (2x + 1) (4x + 3).

Calculating Percentages

Are you good at percentages ? How about addition? Just know that there are faster ways of doing the same. As has been previously discussed, 10% is the easiest percentage to figure out. Example, 100 becomes 10 and 80 becomes 0.8 by simply moving the decimal point one place to the left. The GMAT test will obviously not ask for 10% in calculations but it remains the core from which all other percents are derived. Hence, it is important to keep in mind.

A great example to show percentage at work is when tipping. Most, if not all people, especially in the United States and other first world countries, follow the “double the tax” method of tipping. The “double the tax” method works favorably well in the US because the sales tax is about 8%.

Let us work out a mathematical representation of the double the tax method where y will represent the total cost of the meal.

16% * y = 2 * 8% * y. In this case, we are using 16%, since this is double the tax. That is 8% tax multiplied by 2.

When we use 20% instead of 16%, the figure will look like this:

20% * y = 2 * 10% * y

When manually calculating this percentage in our heads, the calculation looks like this: The tab is $65 which means that ten percent of $65 is $6.50. Using the “double the tax” method, we take the tax, which is $6.50, and multiply it by two. That is $6.50 * 2. The result is $13.00. This, however, is the simple kind of percent calculation but it paves the way to more complex percents that one may come across in the GMAT test.

Calculating the Tens, Fives and Ones in Percents

It is fairly easy to figure out 5% and 1%, just as it is easy to get 20% when you know 10%. If have the knowledge that 10% = $6.50, then you are only a step away from figuring out that 5% = $3.25 and that 1% = $0.65. With this in mind, if you wish to calculate 16%, but there is no indication of the 8% sales tax, you can simply add the three figures. That is:

$6.50 + $3.25 + $0.65 = $10.40.

This is, however, not the sole way to get to 16%. You could start by finding 20% (with the double 10% in mind) and then subtract 4% (in other words, 1 percent times 4). Here is the calculation:

($6.50)(2) � ($0.65)(4) = $13.00 - $2.60 = $10.40

Any number can be expressed as a combination of tens and ones but you may find it easier to use fives too. Once you know that 45 = 10(4) + 5, or that 28 = 10(3) � 1(2), you are only a few simple steps away from deriving a percentage.

Summary

Growing your mental ability to calculate figures in your mind takes practice. You might find that you are rather slow at first, but as you frequently and consciously practice mental calculations, you will get better. Remember, you can use mental math anywhere you go and it will not only improve your GMAT Quant test score but will also help in you in your day-to-day activities.

Chapter 19: How Best to Improve on Your GMAT Quant Test Results

There are some people who consider themselves good at math, but as they prepare for GMAT, they realize it is a different ballgame. There could be two reasons for this:

1) They never attended a class of pure math, and if they did, it was ages ago.

2) They were never great at mental calculations.

Many GMAT test takers learn that they did better at past tests than they did on the GMAT. Some take up advanced math classes from high school to college and even go on to careers that rely heavily on math such as analytical scientists, data analysts and actuarial scientists. All the while, they believe that they are good in math but the truth is that most of us can hardly do any mental calculations without the help of a device.

A good example is when we take a visit to the grocery store. It becomes very challenging to do a simple mental calculation of how much change one should receive back. This can become very embarrassing and leaves one questioning the authenticity of our so-called math skills.

GMAT test takers often find themselves between a rock and a hard place when they do well in other areas of the GMAT test and perform less impressively on the Quant test. Most highly rated MBA programs require that one pass both the math in the Quant test and the verbal test with at least an 80% mark. Failure to meet this requirement due to a low Quant score could lock one out of their desired programs. This book will therefore address the challenges GMAT Quant test takers likely face and how to go about each hurdle for higher scores.

There are two main strategies that have been proven to produce better Quant test scores. The information shared here is a contribution from GMAT Quant tutors as well as successful GMAT test takers who have used said strategies on their tests.

These two strategies are:

(1) Have a core understanding of all Quant concepts.

(2) Work on your mental math skills.

There will also be a small task to carry out at the end of every strategy that will help you evaluate yourself. Let us now expound on the above strategies.

Core Understanding Of All Quant Concepts

All concepts in the Quant test must be fully understood for one to successfully tackle Quant questions. One comes across new concepts that they have not been exposed to before such as the topic of combinatorics. Other concepts might be familiar, as one comes across them in high school such as calculus, trigonometry and geometry, among others. There are also those that are relatively easy to grasp while others have varying levels of difficulty. Failure to understand even one simple concept could cost you dearly. It is therefore imperative to put some effort and time into each, learning, relearning and refreshing, regardless of how you feel about them.

A student who had taken the GMAT quant test gave this account to his tutor on a topic that cost him greatly. Combinatorics was fairly new to him since he had not come across it in his previous studies in high school and college. He said that all along he would ‘study’ the topic by following the explanation given on sample questions. When he failed a particular problem, he would review the answer and, with the help of the explanation, he would work on the problem again. He knew instinctively that he did not quite understand the concept, despite getting correct answers on the questions he tackled. The pattern he used to solve the problems was not foolproof since it could only work when the questions were set in a specific way.

His fear of not solving the problems correctly was confirmed when he took the final test. The questions were framed in a way that he was not familiar with. All the practice he had done with sample questions proved futile as he helplessly tried to maneuver through the questions. This is a classic example of how failure to deeply comprehend a concept can cost you greatly in the end. He was able to understand the concept years later, but he wished that he had done so before because it would have surely saved him a great amount of trouble. It is therefore crucial to learn new concepts from the start and understand each term, function, etc.

There are certain parameters that you can use to gauge your understanding of topics on the Quant test. You learn of your lack of mastery on a topic with the five signs below:

(i) You find it difficult to explain a particular concept clearly.

(ii) You repeatedly fail questions on a given topic.

(iii) It is a new concept that was never introduced to you until the GMAT test.

(iv) You are comfortable with a question when it is worded in a specific way.

(v) You are not able to solve a problem when it is put in a different format.

Learning From Intuition When You Have Not Grasped the Concept

It is okay to have a few problem areas and this should not get you down. It’s of great importance, however, to acknowledge this lack of understanding as this is the first step toward improvement. It is dangerous to assume that you will figure out the concept while in the exam room. The learning should then start as soon as you identify your problem areas so that you can create enough time to both learn and practice the concept ahead of the test.

There are several topics on the GMAT Quant test that have proven to be of higher difficulty than others. According to tutors and test takers, these are the four main topics that are challenging to the students: rates, probability, number theory and combinatorics. Let us briefly define each of the said topics.

(i) Probability: This is the study of the likelihood that a particular event will occur. (What is the chance that x will occur, given Y and Z?)

(ii) Number theory: This is the branch of math that focuses on properties of numbers and their relationship, such as squares and primes.

(iii) Rates: This is a topic that answers problems on distance, movement and time.

(iv) Combinatorics: Here we look at the branch of math that deals with objects combinations, counting objects that belong to a finite set guided by certain constraints.

As stated earlier, there is a small task that all test takers need to take up in a bid to evaluate themselves. This will only require minimum tools and honest reflection.

Take a pen and piece of paper.

Write down all the topics covered on the GMAT Quant test.

Highlight those that give you some level of difficulty. (As assessed with the help of the parameters we have used above.)

Gather all relevant material that focuses on the topics you have highlighted.

Use your GMAT prep time to dive deep into the topics, refreshing, learning and relearning. (Take notes as you go along.)

Practice as much as you can with the help of sample problems.

Believe in yourself and all the effort you have put in.

Use your mental ability to solve problems.

Most of us hardly use our mental ability to solve even the simplest of math problems because we have become heavily dependent on calculators that effortlessly display answers for us. On the GMAT, no one is allowed to have a calculator in their possession during the test. This ensures that the test takers use their mental skills to deduce answers.

This is a commendable practice that enhances mental math skills. Many have tried to push for the removal of calculators in the school system, especially in the exam room, but their efforts have remained futile. On the GMAT test, however it is an offense to have one.

It helps greatly to know the figures that one is frequently likely to see. A good example is the case of a pizza delivery guy. He is likely to deliver similar orders and get the same amount of money on most trips to certain delivery locations. Let’s say that one large pizza costs $12.40. When the delivery guy receives $20, he automatically knows that he is expected to give back $7.60. When he gets two $20 for two large pizzas, he knows that he should give twice the balance of one large pizza.

In the GMAT test there are also figures that one is likely to come across frequently. A crucial tip that most GMAT tutors offer is for students to know the multiplication table, especially the 12*12, grid through and through. Knowing this saves a lot of time in the exam room since you only need to apply the concepts on the already mastered figures. Most of the figures that come up are those that one learned in grade school and thus should not be hard to grasp.

However, there are different numbers that one finds on the GMAT test that do not involve the 12*12 grid. It is also important to learn numbers that are in the multiple line of 2^x and 3^x. The powers along this line, up to the power of 5, are also frequent and so it would be wise to learn them. Figures along the power of 2, for example, are very useful when tackling coin flip probability and binary decisions.

Sample GMAT Question

If a and b are integers and $(a*b)^5 = 96y$, y could be:

5; 9; 27; 81; 125;

In this problem, we need to look at the given figures and think logically. The product (answer) will need to be an integer as both a and b are also integers. You need to be sharp as you are expected to figure out the problem in about two minutes.

On the GMAT, you are likely to work with the fifth power that is 2 and 3, with 2^5 being 32 and 3^5 being 243. When you look at 96y, the number 32 is what comes to mind. If either a or b were 2, then 32 could be easily factored out of 96y, leaving 3y. If 3y had to be equivalent to a number (either a or b to the power of 5) it is then clearly 3. Assuming b is leftover, then $b^5=3y$*. Replacing b with 3, we get* $3^5=3^1$ y, so y must be 3^4.

The answer choice D is exactly 3^4 (or 81) Therefore the best choice answer is D.

There is an advantage that comes with not using calculators on the GMAT test. Despite not having the luxury of calculating figures in a few seconds, you are not required to calculate real numbers, which generally means that having a good understanding of the concepts and the tables is good enough.

There are a few tried and tested ways to improve one’s mental ability. Choose a few, if not all, of the ideas listed below and apply them in your practice.

Helpful GMAT Tips

Start calculating tips, change and simple everyday math problems using your mind.

Prime factor each and every number you come across.

Memorize the following facts:

Prime factoring:

Under the number theory topic, the prime numbers that divide a positive integer are called prime factors. This process is known as prime factorization or integer factorization. The easiest way to go about prime factorization is to break down a number into several factors then work the factors down to primes.

Prime powers:

A simple definition of a prime number is that it is a positive integer which is only divisible by 1 and itself. It is also greater than 1. e.g., 2, 3, 5, 7, 11, 13, 17, 19 (infinite) An integer is prime to any integer but not to itself. It is also important to note that the least common multiple of two relatively prime integers is their product and the greatest common factor (greatest common divisor) of two relatively prime integers is 1.

Squares:

This is inevitable on the GMAT test. It is wise to memorize squares from 2 up to 25 for easier calculations.

Cubes:

Try and memorize the cubes from 2 to 11.

Divisibility Rules:

A number is divisible by 2 if it ends in 0, 2, 4, 6 or 8. This means that the number must be divisible by 2.

A number is divisible by 3 if the sum of its digits is divisible by 3. For example, a number like negative 18 is divisible by 3. – 1 + 8 = 9

A number is divisible by 4 if it is twice divisible by 2 and the last two digits form a number that is divisible by 4.

A number is divisible by 8 if it is divisible by 2 three times and the last three digits form a number that is divisible by 8.

The GMAT test mostly deals with rational numbers such as integers, percents, fractions, etc. However, once in a while, a question with irrational numbers can be found. This means that one needs to know a few approximations that might save the day.

Chapter 20: Exercises And Their Solutions

How to Multiply Big Numbers Mentally

Exercise 1

(a) 99 x 23

100 x 30 = 3,000

(1 x 30) + (7 x 100) = 730

3,000 – 730 = 2,270

(1 x 7) + 2,670 =7 + 2,670

Answer is 2,670

(b) 86 x 37

90 x 40 = 3,600

3,600 - (4 x 40) + (3 x 90) = 3600 - (160 + 270) = 3,600 – 430 = 3,170

3,170 + (4 x 3) = 3,170 + 12 = 3,182

Answer is 3,182

(c) 119 x 99

120 x 100 = 12,000

(1 x 100) + (1 x 120) = 100 + 120 = 220

12,000 – 220 = 11,780

11,780 + (1 x 1) = 11,680 + 1 = 11,781

Answer is 11,781

(d) 255 x 19

300 x 20 = 6,000

(45 x 20) + (1 x 300) = 900 + 300 = 1200

6,000 - 1,200=4,800

45 x 1 = 45

4,800 + 45 = 4,845

Answer is 4,845

(e) 1,098 x 99

1,100 x 100 = 11,000

(2 x 100) + (1 x 1100) = 200 + 1100 = 1300

110,000 – 1,300 = 108,700

(2 x 1) = 2

108,700 + 2 = 108,702

Answer is 108,702

(f) 78 x 39

80 x 40 = 3,200

(2 x 40) + (1 x 80) = 80+80=160

3200-160=3040

(2 x 1) = 2

3040 + 2 = 3042

Easiest Way to Multiply a 2-digit number by 11

Exercise 2

(a) 81 x 11 = **891**___

(b) 72 x 11 = ***792***_

(c) 15 x 11 = **165**___

(d) 22 x 11 = ***242***_

(e) 26 x 11 =***286***__

(f) 33 x 11 = **363**

(g) 18 x 11 = ***198***__

(h) 34 x 11 =**374**____

(i) 24 x 11 =***264***__

(j) 36 x 11 =**396**

(k) 41 x 11 =**451**

(l) 52 x 11 =____**572**

(m) 63 x 11 =***693***__

(n) 30 x 11 =**330**____

(o) 45 x 11 =**495**____

(p) 71 x 11 =__***781***

(q) 70 x 11 =**770**

(r) 62 x 11 =**682**

(s) 61 x 11 =***671***__

(t) 80 x 11 =**880**

Easiest Way To Multiply A Big 2-Digit Number By 11

Exercise 3

(a) 39 x 11 = 429

(b) 46 x 11 = 506

(c) 38 x 11 = 418

(d) 29 x 11 = 319

(e) 28 x 11 = 308

(f) 48 x 11 = 528

(g) 49 x 11 = 539

(h) 56 x 11 = 616

(i) 65 x 11 = 715

(j) 66 x 11 = 726

(k) 73 x 11 = 803

(l) 67 x 11 = 737

(m) 78 x 11 = 858

(n) 69 x 11 = 759

(o) 74 x 11 = 814

(p) 76 x 11 = 836

(q) 68 x 11 = 748

(r) 83 x 11 = 913

(s) 93 x 11 = 1023

(t) 87 x 11 = 957

Easiest Way To Multiply A 3-Digit Number By 11

Exercise 4

(a) 263 x 11 = 2893

(b) 114 x 11 = 1254

(c) 621 x 11 = 6831

(d) 261 x 11 = 2871

(e) 101 x 11 = 1111

(f) 144 x 11 = 1584

(g) 218 x 11 = 2398

(h) 416 x 11 = 4576

(i) 209 x 11 = 2299

(j) 812 x 11 = 8932

(k) 804 x 11 = 8844

(l) 723 x 11 = 7953

(m) 727 x 11 = 7997

(n) 711 x 11 = 7821

(o) 909 x 11 = 9999

(p) 634 x 11 = 6974

(q) 544 x 11 = 5984

(r) 271 x 11 = 2981

(s) 362 x 11 = 3982

(t) 111 x 11 = 1221

When You Need to Carry in 11 Times

Exercise 5

(a) 174 x 11 = 1914

(b) 173 x 11 = 1903

(c) 148 x 11 = 1628

(d) 177 x 11 = 1947

(e) 267 x 11 = 2937

(f) 264 x 11 = 2904

(g) 357 x 11 = 3927

(h) 175 x 11 = 1925

(i) 358 x 11 = 3938

(j) 349 x 11 = 3839

(k) 347 x 11 = 3817

(l) 329 x 11 = 3619

(m) 269 x 11 = 2959

(n) 419 x 11 = 4609

(o) 429 x 11 = 4719

(p) 438 x 11 = 4818

(q) 447 x 11 = 4917

(r) 439 x 11 = 4829

(s) 519 x 11 = 5709

(t) 719 x 11 = 7909

When The Carrying Affects The First Digit

Exercise 6

(a) 190 x 11 = 2090

(b) 392 x 11 = 4312

(c) 381 x 11 = 4191

(d) 475 x 11 = 5225

(e) 483 x 11 = 5313

(f) 477 x 11 = 5247

(g) 582 x 11 = 6402

(h) 732 x 11 =8052

(i) 831 x 11 = 9141

(j) 851 x 11 = 9361

(k) 883 x 11 = 9713

(l) 828 x 11 = 9108

(m) 568 x 11 = 6248

(n) 662 x 11 = 7282

(o) 670 x 11 = 7370

(p) 191 x 11 = 2101

(q) 292 x 11 = 3212

(r) 391 x 11 = 4301

(s) 731 x 11 = 8041

(t) 733 x 11 = 8063

Math Strategies That Anyone Can Master

Exercise 7

(a) 8 + 7 = 15

(b) 7 + 6 = 13

(c) 19 + 12 = 31

(d) 17 + 18 = 35

(e) 22 + 31 = 53

(f) 257 + 13 = 270

(g) 2,998 + 100 = 3,098

(h) 10,098 + 198 = 10,296

(i) 100,299 + 16 = 100,315

(j) 200,998 + 999 = 201,997

Exercise 8

(a) 8 + 9 = 17

(b) 6 + 7 = 13

(c) 30 + 31 = 61

(d) 200 + 201 =401

(e) 300 + 301 = 601

(f) 1,000 + 1,001 = 2,001

(g) 900 + 901 = 1,801

(h) 10,000 + 10,001 = 20,001

(i) 7,000 + 7,001 = 14,001

(j) 100,000 + 100,001 = 200,001

Exercise 9

(a) 30 + 20 = 50

(b) 400 + 300 = 700

(c) 405 + 24 = 429

(d) 64 + 25 = 99

(e) 116 + 105 = 221

(f) 500 + 405 = 905

(g) 7,000 + 8,000 = 15,000

(h) 2,500 + 2,400 = 4,900

(i) 57 +26 = 83

(j) 25 + 14 = 39

(k) 44 + 23 = 67

(l) 99 + 11 = 110

(m) 107 + 26 = 133

(n) 134 + 13 = 147

(o) 267 + 25 = 292

(p) 1,111 + 1,112 = 2223

(q) 83 + 12 = 95

(r) 833 + 122 = 955

(s) 105 + 104 = 209

(t) 17 + 16 = 33

Exercise 10

(a) 20 – 6 = 14

(b) 20 – 13 = 7

(c) 17 – 4 = 13

(d) 18 – 12 = 6

(e) 19 – 2 = 7

(f) 12 – 9 = 7

(g) 29 – 20 = 9

(h) 68 – 18 = 50

(i) 688 – 608 = 80

(j) 510 – 400 = 110

(k) 475 – 100 = 375

(l) 888 – 800 = 88

(m) 919 – 19 = 900

(n) 120 – 90 = 30

(o) 10,000 – 8,000 = 2,000

(p) 12,000 – 5,000 = 7,000

(q) 27 – 5 = 22

(r) 32 – 30 = 2

(s) 680 – 180 = 500

(t) 100 – 30 = 70

Exercise 11

(a) 85 x 5 = 425

(b) 27 x 5 = 135

(c) 35 x 5 = 175

(d) 17 x 5 = 85

(e) 250 x 5 = 1250

(f) 112 x 5 = 560

(g) 42 x 5 = 210

(h) 30 x 5 = 150

(i) 19 x 5 = 95

(j) 90 x 5 =450

Exercise 12

(a) 5 x 4 = 20

(b) 6 x 4 = 24

(c) 12 x 4 = 48

(d) 21 x 4 = _84

(e) 35 x 4 = 140

(f) 105 x 4 = 420

(g) 115 x 4 = 260

(h) 410 x 4 = 1,640

(i) 531 x 4 = 2,124

(j) 331 x 4 = 1,324

Exercise 13

(a) 25 x 8 = 200

(b) 31 x 8 = 248

(c) 13 x 8 = 104

(d) 45 x 8 = 360

(e) 102 x 8 = 816

(f) 71 x 8 = 568

(g) 21 x 8 = 168

(h) 222 x 8 = 1,776

(i) 300 x 8 = 2,400

(j) 112 x 8 = 896

Exercise 14

(a) 35 x 8 = 70 x 4 = 280

(b) 60 x 8 = 120 x 4 = 480

(c) 17 x 20 = 34 x 10 = 340

(d) 92 x 20 = 184 x 10 = 1,840

(e) 24 x 20 = 48 x 10 = 480

(f) 55 x 4 = 110 x 2 = 220

(g) 110 x 4 = 220 x 2 = 440

(h) 15 x 8 = 120

(i) 202 x 20 = 4,040

(j) 22 x 20 =440

Exercise 15

(k) 76 x 2 = (70 x 2) + (6 x 2) = 140 + 12 = 152

(l) 83 x 3 = (80 x 3) + (3 x 3) = 240+9 = 249

(m) 123 x 5 = (100 x 5) + (20 x 5) + (3 x 5) = 500 + 100 + 15 = 615

(n) 304 x 6 = (300 x 6) + (4 x 6) = 1,800 + 24 = 1,824

(o) 661 x 3 = (600 x 3) + (60 x 3) + (1 x 3) = 1,800 + 180 + 3 = 1,983

(p) 252 x 4 = (200 x 4) + (50 x 4) + (2 x 4) = 800 + 200 + 8 = 1,008

(q) 117 x 7 = (100 x 7) + (10 x 7) + (7 x 7) = 700 + 70 + 49 = 819

(r) 91 x 8 = (90 x 8) + (1 x 8) = 720 + 8 = 728

(s) 82 x 4 = (80 x 4) + (2 x 4) = 320 + 8 = 328

(t) 344 x 5 = (300 x 5) + (40 x 5) + (4 x 5) = 1,500 + 200 + 20 = 1,720

Exercise 16

(7) Solve the expression below.

-2x (-5 + 6y)

Answer choices:

-10x – 12xy

10x – 12xy

-10x + 12xy

10x – 12y

Solution:

Remember, a negative multiplied with a negative becomes a positive and a negative multiplied by a positive becomes a negative.

-2x (-5 + 6y) = -2x * -5 + -2x * 6y

The negative before 2, matched against the negative before 5, produces a positive. Hence, 10x below is written without a sign. When a number does not have a plus or minus sign before it, the implication is that it is a positive number.

The negative before 2, matched with the positive before 6, produces a negative, and that is the reason we have ended up with a -12xy, as shown below:

-2x * -5 + -2x * 6y = 10x + -12xy

10x + -12xy = 10x – 12xy

Hence, the answer is B. 10x – 12xy

(8) Solve the expression 5,327 x 7 and do not use a calculator

Answer choices:

35,928

35,289

35,298

37,289

Solution:

You can solve this by distributing the values that make up the big number, 5,327, according to their significance. You will, therefore, have:

(5,000 + 300 + 20 + 7) x 7

With this manner of distribution, solving the problem becomes much easier.

With (5,000 + 300 + 20 + 7) x 7, you are expected to multiply each of the terms inside the parenthesis by the 7 that is outside. You will end up with:

35,000 (from 5,000 x 7) + 2,100 (from 300 x 7) + 140 (from 20 x 7) + 49 (7 x 7)

That expression is 35,000 + 2,100 + 140 + 7 = 37,289

Hence, the answer is D. 37,289.

(9) By using distribution, simplify the expression, 5 (6y + -1).

Answer choices:

30y + - 5

30y + 5

30y - 1

6y - 5

Solution :

5 (6y + -1)

By using the distributive property, you will multiply each term inside the bracket with the 5 that is outside the bracket before proceeding with your calculations.

5 * 6y + 5 * -1

As usual, a plus multiplied by a plus remains a plus, but a plus multiplied by a minus turns into a minus.

So, 5 * 6y + 5 * -1 = 30y + - 5

Hence, the answer is A. 30y + - 5

(10) Using the distributive property, simplify the expression -3 (2z – 4).

Answer choices:

-6z – 12

-6z + 12

-3z - 4

6z -12

Solution:

Here you need to begin by multiplying each term within the brackets with the -3 which is the multiplier outside the bracket. As such, you will have:

-3 * 2z - -3 * 4 = -6z + 12

Whereas the minus before 3 has combined with the implicit plus before 2z to form a minus, the minus before 3 has combined with the minus before 4 to form a plus.

Hence, the answer is B. -6z + 12.

(11) You have been shown a rectangular plot whose size is 12ft by 15ft, and you have been asked to increase the measurement of each side by x. What will the new plot measure in terms of area?

Answer choices:

180 + 2x

x^2 + 27x + 180

12x + 15x + 27

27 x^2 + 180

Solution

The length of the plot was originally 15ft.

The new length is (15 + x).

The width of the plot was originally 12ft.

The new width is (12 + x).

The new area of the rectangular plot is, therefore, (15 + x) + (12 + x).

$(15 + x)(12 + x) = 180 + 15x + 12x + x^2$

$180 + 15x + 12x + x^2 = 180 + 27x + x^2$

Hence, the answer is B. $x^2 + 27x + 180$

(12) Solve the expression given below.

$-3y(x + 6y)(3x – 4y)$

Answer choices:

$-9yx^2 – 42yx^2 + 72y^3$

$-9yx^2 – 54yx^2 + 18y^3$

$-9yx^2 – 54xy^2 + 18y^3$

$-9yx^2 – 42xy^2 + 72y^3$

Solution:

It is important to remember how different signs relate with one another, e.g., a plus sign changing to minus when multiplied by minus, and a minus changing to plus when multiplied by another minus.

$-3y(x + 6y)(3x – 4y)$

$(-3y * x + -3y * 6y)(3x – 4y)$

$(-3yx + -18y^2)(3x – 4y) = -9yx^2 + 12xy^2 – 54xy^2 + 72y^3$

$-9yx^2 + 12xy^2 – 54xy^2 + 72y^3 = -9yx^2 – 42xy^2 + 72y^3$

Hence, the answer is D. $-9yx^2 – 42xy^2 + 72y^3$

Exercise 17

Ten equations are given below, followed by some statements. For each statement, choose the equation that best represents it.

$n / 6 = 3$

$n – 6 = 18$

$6n = 36$

$6 + 18 – 9 = 15$

$n – 18 = 6$

$¼n = 6$

$^6 / n + 12 = 18$

$6n + 2 = 18$

$n + 6 = 18$

$6 + 15 – 9 = 12$

Statements and their corresponding equations

(11) An unknown number plus 6 is equal to 18.

Answer : The matching equation is I, which is: $n + 6 = 18$

(12) 6 is a quarter of a certain number.

Answer : The matching equation is F, which is $\frac{1}{4}n = 6$.

(13) The sum of 6 and 18 less 9 is equal to 15.

Answer : The matching equation is D, which is $6 + 18 - 9 = 15$.

(14) The sum of 6 and 15 less 9 is equal to 12.

Answer : The matching equation is J, which is $6 + 15 - 9 = 12$.

(15) An unknown number divided by 6 gives you 3.

Answer : The matching equation is A, which is $n / 6 = 3$.

(16) An unknown number times 6 is equal to 36.

Answer : The matching equation is C, which is $6n = 36$.

(17) 6 is the difference between an unknown number and 18.

Answer : The matching equation is E, which is $n - 18 = 6$.

(18) A unknown number less 6 is equal to 18.

Answer : The matching equation is B, which is $n - 6 = 18$.

(19) An unknown number times 6 plus 2 is equal to 18.

Answer : The matching equation is H, which is $6n + 2 = 18$.

(20) 6 divided by an unknown number plus 12 is equal to 18.

Answer : The matching equation is G, which is $^{6} / n + 12 = 18$.

Exercise 18

(1) What are the three consecutive numbers which, when added together, result in 156?

Let the first number among the three consecutive numbers be x.

The second number must be $x + 1$.

The third of the three consecutive numbers must be $(x + 1) + 1$ or $x + 2$.

You can now form an equation to show that the first number, plus the second consecutive one, plus the third one is equal to 156.

$x + (x + 1) + (x + 2) = 156$

$x + x + 1 + x + 2 = 156$

$3x + 3 = 156$

$3x + 3 - 3 = 156 - 3$

$3x = 153$

$3x \div 3 = 153 \div 3$

$x = 51$

You only need to substitute x for 51 in the operations that represent the three consecutive numbers.

The first number is x. Therefore, it is 51.

The second number is x + 1, therefore it is 51 + 1 = 52.

The third number is x + 2, therefore it is 51 + 2 = 53.

Essentially the answer is 51, 52 and 53.

Conclusion

Thank you for reading this book. I hope you are now a mental math genius. If you benefitted from this book, would you kindly take a few seconds to leave a positive review?

Good luck!