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SHAKUNTALA DEVI

THE MATHEMATICAL WIZARD

Figuring MADE EASY

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Shakuntala Devi

Figuring Made Easy

FOR TEENAGERS

Hind Pocket Books

FIGURING MADE EASY

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"What's one and one and one and one and one

and one and one and one and one and one?"

"I don't know," said Alice, "I lost count."

"She can't do addition," said the red queen.

Lewis Carroll

CONTENTS

INTRODUCTION

Numbers originated in the practical needs of humans. The primitive humans bartered articles with one another or divided their prey into portions. And so as to maintain a rough justice in their division or exchange, they were forced to develop some elementary notions of counting.

The human who first realised that the number of fingers on one hand or on two and the number of toes on the feet were a convenient tool for help in counting must have been considered a genius in his or her time.

Since then there has been an immense stride in human thought that has carried us from counting to the study of 'Number' in itself as an object of interest, and it dawned on the human mind that numbers had an interest of their own.

There is hardly any activity in life which does not depend on a certain ability to count. We live in a world that is ruled by numbers. Early in our childhood, the 'number' thought begins in us, when we decide that we would rather have two pieces of chocolate than one or we would rather have more than one toy.

We see numbers and hear them every day at home. Our houses have a number, we all have a birthday that is a number, and those of us who have telephones know that it has also a number—Dad has to be at work at nine o'clock, mum sets her oven temperature at 320°F when she bakes a cake, big sister has two pigtails, baby brother weighed nine pounds when he was born, my favourite TV programme is on channel eight . . . Even family relationships involve a certain amount of counting . . . 3 sisters, 1 brother, 2 uncles and so on.

Besides the practical use numbers have, there is a range of richness to numbers; they come alive, cease to be symbols written on a blackboard, and lead us into a world of arithmetical adventure, where calculating numbers, and getting the right answers, can be exciting and thrilling.

I fell in love with numbers at three. And I found it sheer ecstasy to do sums and get the right answers. Numbers were my toys. I played with them. My interest grew with age and I took delight in working out huge problems mentally—sometimes even faster than electronic calculating machines and computers. And I feel, had I been deprived of my interest in numbers and the ability to calculate, the world would have been a duller place for me. I see the world of numbers as one of magic, full of fantasy and enchantment.

In this book I am taking you on an excursion into the exciting world of numbers, so we can share the adventure of attacking the problems in new ways, discovering different possibilities, make up new problems and explore the strange properties of numbers and the mysteries of arithmetical phenomena.

First of all, let's meet the family of numbers. The family consists of ten numbers and each one of them has a special character. We shall meet them one by one.

THE FAMILY OF NUMBERS

NUMBER 1

Number 1, or the unity, is the basis of any system of notation. Upright and unbending number 1 has some very special characteristics—the first and most important being that whatever figure you multiply by it, or divide by it, it remains unchanged. The first nine places in the 1-times table, therefore, have no secret steps, or at least they are the same as the normal ones, but after that, it becomes more interesting.

$10 \times 1 = 10$	$1 + 0 = 1$
$11 \times 1 = 11$	$1 + 1 = 2$
$12 \times 1 = 12$	$1 + 2 = 3$
$13 \times 1 = 13$	$1 + 3 = 4$
$14 \times 1 = 14$	$1 + 4 = 5$
$15 \times 1 = 15$	$1 + 5 = 6$
$16 \times 1 = 16$	$1 + 6 = 7$

and so on. No matter how far you go, you will find that the Secret Steps always give you the digits from 1 to 9 repeating themselves in sequence, for example:

Straight Steps	*Secret Steps*	
$154 \times 1 = 154$	$1 + 5 + 4 = 10$	$1 + 0 = 1$
$155 \times 1 = 155$	$1 + 5 + 5 = 11$	$1 + 1 = 2$

Straight Steps	*Secret Steps*	
$156 \times 1 = 156$	$1 + 5 + 6 = 12$	$1 + 2 = 3$
$157 \times 1 = 157$	$1 + 5 + 7 = 13$	$1 + 3 = 4$
$158 \times 1 = 158$	$1 + 5 + 8 = 14$	$1 + 4 = 5$
$159 \times 1 = 159$	$1 + 5 + 9 = 15$	$1 + 5 = 6$
$160 \times 1 = 160$	$1 + 6 + 0 =$	$1 + 6 = 7$
$161 \times 1 = 161$	$1 + 6 + 1 =$	$1 + 6 + 1 = 8$

In this case, it is fairly obvious why the Secret Steps work, but nonetheless, it is a curiosity that may not have occurred to you before.

Here is another curiosity about the number 1 showing its talent for creating palindromes (numbers that read the same backwards and forwards)

$$
\begin{aligned}
1 \times 1 &= 1\\
11 \times 11 &= 121\\
111 \times 111 &= 12321\\
1111 \times 1111 &= 1234321\\
11111 \times 11111 &= 123454321\\
111111 \times 111111 &= 12345654321\\
1111111 \times 1111111 &= 1234567654321\\
11111111 \times 11111111 &= 123456787654321\\
111111111 \times 111111111 &= 12345678987654321
\end{aligned}
$$

At that point, it stops but the same thing works briefly with number 11:

$$
\begin{aligned}
11 \times 11 &= 121\\
11 \times 11 \times 11 &= 1331\\
11 \times 11 \times 11 \times 11 &= 14641
\end{aligned}
$$

The fact that 1 is, so to speak, immune to multiplication means that whatever power you raise it to, it remains unchanged:

$$1^{564786} = 1$$

By the same token $\sqrt[8]{1} = 1$

It is the only number all of whose powers are equal to itself.

And any number whatever except zero raised to the zero power is equal to 1.

The powers exceeding 1, of all numbers greater than 1, are greater than the numbers so raised:

$$2^2 = 4$$
$$3^2 = 9$$

But the powers exceeding 1, of all quantities smaller than 1, but greater than 0 are smaller than the quantities:

$$(1/2)^2 = 1/4$$
$$(1/3)^2 = 1/9$$

The product obtained by multiplying together any two numbers, each greater than 1, gives a quantity larger than either of the multipliers and if the quantities are less than 1, but greater than 0, their product will be a smaller quantity than either of them.

Thus, the number 1 stands as a dividing point and quantities greater than 1 present characteristics differing from those of quantities less in value than 1.

NUMBER 2

Number 2 has one very obvious characteristic—to multiply any number by 2 is the same as adding any number to itself. As for the multiplication and its Secret Steps:

$2 \times 1 = 2$	2
$2 \times 2 = 4$	4
$2 \times 3 = 6$	6
$2 \times 4 = 8$	8
$2 \times 5 = 10$	$1 + 0 = 1$
$2 \times 6 = 12$	$1 + 2 = 3$
$2 \times 7 = 14$	$1 + 4 = 5$
$2 \times 8 = 16$	$1 + 6 = 7$

$2 \times 9 = 18$	$1 + 8 = 9$
$2 \times 10 = 20$	$2 + 0 = 2$
$2 \times 11 = 22$	$2 + 2 = 4$
$2 \times 12 = 24$	$2 + 4 = 6$
$2 \times 13 = 26$	$2 + 6 = 8$
$2 \times 14 = 28$	$2 + 8 = 1\ (0)$
$2 \times 15 = 30$	$3 + 0 = 3$
$2 \times 16 = 32$	$3 + 2 = 5$
$2 \times 17 = 34$	$3 + 4 = 7$
$2 \times 18 = 36$	$3 + 6 = 9$

and so on.

The Secret Steps go on working *ad infinitum,* always giving you the same sequence of the four even digits followed by the five odd ones.

Here is an amusing party trick that can be played with number 2.

The game is to express all ten digits, in each case using the number 2 five times and no other numbers. You are allowed to use the symbols for addition, subtraction, multiplication and division and the conventional method of writing fractions.

Here we are:

$$2 + 2 - 2 - 2/2 = 1$$
$$2 + 2 + 2 - 2 - 2 = 2$$
$$2 + 2 - 2 + 2/2 = 3$$
$$2 \times 2 \times 2 - 2 - 2 = 4$$
$$2 + 2 + 2 - 2/2 = 5$$
$$2 + 2 + 2 + 2 - 2 = 6$$
$$22 \div 2 - 2 - 2 = 7$$
$$2 \times 2 \times 2 + 2 - 2 = 8$$
$$2 \times 2 \times 2 + 2/2 = 9$$
$$2 - 2/2 - 2/2 = 0$$

Finally, here is another oddity associated with 2:

$$
\begin{array}{r}
1\,2\,3\,4\,5\,6\,7\,8\,9 \\
+\,1\,2\,3\,4\,5\,6\,7\,8\,9 \\
+\,9\,8\,7\,6\,5\,4\,3\,2\,1 \\
+\,9\,8\,7\,6\,5\,4\,3\,2\,1 \\
+\,2 \\
\hline
2\,2\,2\,2\,2\,2\,2\,2\,2\,2 \\
\hline
\end{array}
$$

NUMBER 3

The most distinguished characteristic of this number is that it is the first triangle number—that is, a number the units of which can be laid out to form a single triangle, like this $_{0}{}^{0}{}_{0}$.

Triangle numbers have importance and peculiarities of their own which we shall encounter later on.

Three is also a prime number. A prime number is one which is indivisible except by 1 and by itself. 1, 2, 3, 5, 7, 11 and 13 are prime numbers. The next prime numbers are 17, 19 and 23 and so on.

Here are some strange things about the first few prime numbers 1, 3, 5 :

$$153 = 1^3 + 5^3 + 3^3$$

And 3 and 5 can also be expressed as the difference between two squares.

$$3 = 2^2 - 1^2$$
$$5 = 3^2 - 2^2$$

Now the Secret Steps in the 3-times table:

Straight Steps	*Secret Steps*
$3 \times 1 = 3$	3
$3 \times 2 = 6$	6

Straight Steps	*Secret Steps*
3 × 3 = 9	9
3 × 4 = 12	1 + 2 = 3
3 × 5 = 15	1 + 5 = 6
3 × 6 = 18	1 + 8 = 9
3 × 7 = 21	2 + 1 = 3
3 × 8 = 24	2 + 4 = 6
3 × 9 = 27	2 + 7 = 9
3 × 10 = 30	3 + 0 = 3
3 × 11 = 33	3 + 3 = 6
3 × 12 = 36	3 + 6 = 9

Very simple really. The pattern of the Secret Steps recurs whatever stage you carry the table upto—why not check for yourself?

NUMBER 4

With number 4, the Secret Steps in the multiplication tables become a little more intricate:

Straight Steps	*Secret Steps*	
4 × 1 = 4		4
4 × 2 = 8		8
4 × 3 = 12		1 + 2 = 3
4 × 4 = 16		1 + 6 = 7
4 × 5 = 20		2 + 0 = 2
4 × 6 = 24		2 + 4 = 6
4 × 7 = 28	2 + 8 = 10	1 + 0 = 1
4 × 8 = 32		3 + 2 = 5
4 × 9 = 36		3 + 6 = 9
4 × 10 = 40		4 + 0 = 4
4 × 11 = 44		4 + 4 = 8
4 × 12 = 48	4 + 8 = 12	1 + 2 = 3
4 × 13 = 52		5 + 2 = 7
4 × 14 = 56	5 + 6 = 11	1 + 1 = 2
4 × 15 = 60		6 + 0 = 6
4 × 16 = 64	6 + 4 = 10	1 + 0 = 1

At first, the sums of the digits look like a jumble of figures, but choose at random any sequence of numbers and multiply them by 4 and you will see that the pattern emerges of two interlinked columns of digits in descending order, for example:

2160 × 4 = 8640	8 + 6 + 4 + 0 = 18	1 + 8 =		9
2161 × 4 = 8644	8 + 6 + 4 + 4 = 22	2 + 2 =	4	
2162 × 4 = 8648	8 + 6 + 4 + 8 = 26	2 + 6 =		8
2163 × 4 = 8652	8 + 6 + 5 + 2 = 21	2 + 1 =	3	
2164 × 4 = 8656	8 + 6 + 5 + 6 = 25	2 + 5 =		7
2165 × 4 = 8660	8 + 6 + 6 + 0 = 20	2 + 0 =	2	
2166 × 4 = 8664	8 + 6 + 6 + 4 = 24	2 + 4 =		6
2167 × 4 = 8668	8 + 6 + 6 + 8 = 28			
	and 2 + 8 = 10	1 + 0 =	1	
2168 × 4 = 8672	8 + 6 + 7 + 2 = 23	2 + 3 =		5

NUMBER 5

The most important thing about number 5 is that it is half of ten. This fact is a key to many shortcuts in calculation.

The Secret Steps in the 5-times table are very similar to those in the 4-times table; the sequences simply go upwards instead of downwards:

5 × 1 = 5				5
5 × 2 = 10	1 + 0 =		1	
5 × 3 = 15	1 + 5 =			6
5 × 4 = 20	2 + 0 =		2	
5 × 5 = 25	2 + 5 =			7
5 × 6 = 30	3 + 0 =		3	
5 × 7 = 35	3 + 5 =			8
5 × 8 = 40	4 + 0 =		4	
5 × 9 = 45	4 + 5 =			9
5 × 10 = 50	5 + 0 =		5	
5 × 11 = 55	5 + 5 = 10	1 + 0 =		1
5 × 12 = 60	6 + 0 =		6	
5 × 13 = 65	6 + 5 = 11	1 + 1 =		2

5 × 14 = 70	7 + 0 =		7
5 × 15 = 75	7 + 5 = 12	1 + 2 =	3
5 × 16 = 80	8 + 0 =		8
5 × 17 = 85	8 + 5 = 13	1 + 3 =	4
5 × 18 = 90	9 + 0 =		9
5 × 19 = 95	9 + 5 = 14	1 + 4 =	5

and so on.

NUMBER 6

This is the second triangle number; and the first perfect number—a perfect number is one which is equal to the sum of all its divisors. Thus 1 + 2 + 3 = 6.

The Secret Steps in the 6-times table are very similar to those in the 3-times table, only the order is slightly different.

Straight Steps	*Secret Steps*	
6 × 1 = 6		=6
6 × 2 = 12		1 + 2 = 3
6 × 3 = 18		1 + 8 = 9
6 × 4 = 24		2 + 4 = 6
6 × 5 = 30		3 + 0 = 3
6 × 6 = 36		3 + 6 = 9
6 × 7 = 42		4 + 2 = 6
6 × 8 = 48	4 + 8 = 12	1 + 2 = 3
6 × 9 = 54		5 + 4 = 9
6 × 10 = 60		6 + 0 = 6
6 × 11 = 66	6 + 6 = 12	1 + 2 = 3
6 × 12 = 72		7 + 2 = 9

and so on.

NUMBER 7

This is the next prime number after 5.

The Secret Steps in the 7-times table almost duplicate those in the 2-times, except that they go up instead of down at each step:

Straight Steps	*Secret Steps*	
$7 \times 1 = 7$		7
$7 \times 2 = 14$		$1 + 4 = 5$
$7 \times 3 = 21$		$2 + 1 = 3$
$7 \times 4 = 28$	$2 + 8 = 10$	$1 + 0 = 1$
$7 \times 5 = 35$		$3 + 5 = 8$
$7 \times 6 = 42$		$4 + 2 = 6$
$7 \times 7 = 49$	$4 + 9 = 13$	$1 + 3 = 4$
$7 \times 8 = 56$	$5 + 6 = 11$	$1 + 1 = 2$
$7 \times 9 = 63$		$6 + 3 = 9$
$7 \times 10 = 70$		$7 + 0 = 7$
$7 \times 11 = 77$	$7 + 7 = 14$	$1 + 4 = 5$
$7 \times 12 = 84$	$8 + 4 = 12$	$1 + 2 = 3$
$7 \times 13 = 91$	$9 + 1 = 10$	$1 + 0 = 1$
$7 \times 14 = 98$	$9 + 8 = 17$	$1 + 7 = 8$
$7 \times 15 = 105$		$1 + 0 + 5 = 6$
$7 \times 16 = 112$		$1 + 1 + 2 = 4$
$7 \times 17 = 119$	$1 + 1 + 9 = 11$	$1 + 1 = 2$
$7 \times 18 = 126$		$1 + 2 + 6 = 9$
$7 \times 19 = 133$		$1 + 3 + 3 = 7$
$7 \times 20 = 140$		$1 + 4 + 0 = 5$

There is a curious relationship between 7 and the number 142857. Watch

$$
\begin{array}{lr}
7 \times 2 = 7 \times 2 = & 14 \\
7 \times 2^2 = 7 \times 4 = & 28 \\
7 \times 2^3 = 7 \times 8 = & 56 \\
7 \times 2^4 = 7 \times 16 = & 112 \\
7 \times 2^5 = 7 \times 32 = & 224 \\
7 \times 2^6 = 7 \times 64 = & 448 \\
7 \times 2^7 = 7 \times 128 = & 896 \\
7 \times 2^8 = 7 \times 256 = & 1792 \\
7 \times 2^9 = 7 \times 512 = & 3584 \\
\hline
 & 142857142857142(784) \\
\hline
\end{array}
$$

No matter how far you take the calculation, the sequence 142857 will repeat itself though the final digits on the right-hand side which I have bracketed will be 'wrong' because they would be affected by the next stage in the addition if you took the calculation further on.

This number 142857 has itself some strange properties; multiply it by any number between 1 and 6 and see what happens:

$$142857 \times 1 = 142857$$
$$142857 \times 2 = 285714$$
$$142857 \times 3 = 428571$$
$$142857 \times 4 = 571428$$
$$142857 \times 5 = 714285$$
$$142857 \times 6 = 857142$$

The same digits recur in each answer, and if the products are written each in the form of a circle, you will see that the order of the digits remains the same. If you then go on to multiply the same number by 7, the answer is 999999. We will come back to some further characteristics of this number in a later chapter.

NUMBER 8

This time the Secret Steps in the multiplication table are the reverse of those in the 1-times table:

Straight Steps	*Secret Steps*	
$8 \times 1 = 8$		8
$8 \times 2 = 16$		$1 + 6 = 7$
$8 \times 3 = 24$		$2 + 4 = 6$
$8 \times 4 = 32$		$3 + 2 = 5$
$8 \times 5 = 40$		$4 + 0 = 4$
$8 \times 6 = 48$	$4 + 8 = 12$	$1 + 2 = 3$
$8 \times 7 = 56$	$5 + 6 = 11$	$1 + 1 = 2$
$8 \times 8 = 64$	$6 + 4 = 10$	$1 + 0 = 1$
$8 \times 9 = 72$		$7 + 2 = 9$

$8 \times 10 = 80$		$8 + 0 = 8$
$8 \times 11 = 88$	$8 + 8 = 16$	$1 + 6 = 7$
$8 \times 12 = 96$	$9 + 6 = 15$	$1 + 5 = 6$
$8 \times 13 = 104$		$1 + 0 + 4 = 5$
$8 \times 14 = 112$		$1 + 1 + 2 = 4$
$8 \times 15 = 120$		$1 + 2 + 0 = 3$
$8 \times 16 = 128$	$1 + 2 + 8 = 11$	$1 + 1 = 2$
$8 \times 17 = 136$	$1 + 3 + 6 = 10$	$1 + 0 = 1$

So it continues.

If this is unexpected, then look at some other peculiarities of the number 8:

$$\begin{array}{r} 888 \\ 88 \\ 8 \\ 8 \\ 8 \\ \hline 1000 \\ \hline \end{array}$$

and

$$\begin{aligned}
88 &= 9 \times 9 + 7 \\
888 &= 98 \times 9 + 6 \\
8888 &= 987 \times 9 + 5 \\
88888 &= 9876 \times 9 + 4 \\
888888 &= 98765 \times 9 + 3 \\
8888888 &= 987654 \times 9 + 2 \\
88888888 &= 9876543 \times 9 + 1
\end{aligned}$$

and lastly

$$123456789 \times 8 = 987654312$$

NUMBER 9

With 9 we come to the most intriguing of the digits, indeed of all numbers. The number 9 exceeds in interesting and practical properties of all other digits:

Straight Steps	*Secret Steps*	
$9 \times 1 =$		9
$9 \times 2 = 18$		$1 + 8 = 9$
$9 \times 3 = 27$		$2 + 7 = 9$
$9 \times 4 = 36$		$3 + 6 = 9$
$9 \times 5 = 45$		$4 + 5 = 9$
$9 \times 6 = 54$		$5 + 4 = 9$
$9 \times 7 = 63$		$6 + 3 = 9$
$9 \times 8 = 72$		$7 + 2 = 9$
$9 \times 9 = 81$		$8 + 1 = 9$
$9 \times 10 = 90$		$9 + 0 = 9$
$9 \times 11 = 99$	$9 + 9 = 18$	$1 + 8 = 9$
$9 \times 12 = 108$		$1 + 0 + 8 = 9$

It is an absolute rule that whatever you multiply by 9, the sum of the digits in the product will always be 9 provided this sum consists of one digit only. If the sum of the digits consists of more than one digit, go on adding the digits of the sum till you get a one-digit number, as shown in the Secret Steps above. You will find that this one-digit number will always be 9. Moreover, not only are there no steps in the hidden part of the 9-times table but, for its first ten places, it has another feature:

$1 \times 9 = 09$	$90 = 9 \times 10$
$2 \times 9 = 18$	$81 = 9 \times 9$
$3 \times 9 = 27$	$72 = 9 \times 8$
$4 \times 9 = 36$	$63 = 9 \times 7$
$5 \times 9 = 45$	$54 = 9 \times 6$

The product in the second half of the table is the reverse of that in the first.

Now, take any number, say, 87594. Reverse the order of the digits, which gives you 49578. Subtract the lesser from the greater

$$\begin{array}{r} 87594 \\ -\ 49578 \\ \hline 38016 \\ \hline \end{array}$$

Now add up the sum of digits in the remainder:

$$3 + 8 + 0 + 1 + 6 = 18 \qquad 1 + 8 = 9$$

The answer will always be 9.

Again, take any number, say 64783. Calculate the sum of its digits: $6 + 4 + 7 + 8 + 3 = 28$. (You can stop there or go on as usual to calculate $2 + 8 = 10$, and again only if you wish, $1 + 0 = 1$.)

Now take the sum of the digits away from the original number, and add up the digits of the remainder. Wherever you choose to stop, and whatever the number you originally select, the answer will be 9 if the sum of the digits of the remainder is a one-digit number, otherwise this sum will be a multiple of 9.

$$\begin{array}{r} 64783 \\ -\ 28 \\ \hline 64755 \end{array} \quad 6 + 4 + 7 + 5 + 5 = 27 \qquad 2 + 7 = 9$$

$$\begin{array}{r} 64783 \\ -\ 10 \\ \hline 64773 \end{array} \quad 6 + 4 + 7 + 7 + 3 = 27 \qquad 2 + 7 = 9$$

$$\begin{array}{r} 64783 \\ -\ 1 \\ \hline 64782 \end{array} \quad 6 + 4 + 7 + 8 + 2 = 27 \qquad 2 + 7 = 9$$

Take the nine digits in order and remove the 8: 12345679.

Then multiply by 9:

$$12345679 \times 9 = 111\ 111\ 111$$

Now try multiplying by the multiples of 9:

$$12345679 \times 18 = 222\ 222\ 222$$
$$12345679 \times 27 = 333\ 333\ 333$$
$$12345679 \times 36 = 444\ 444\ 444$$
$$12345679 \times 45 = 555\ 555\ 555$$
$$12345679 \times 54 = 666\ 666\ 666$$
$$12345679 \times 63 = 777\ 777\ 777$$
$$12345679 \times 72 = 888\ 888\ 888$$
$$12345679 \times 81 = 999\ 999\ 999$$

There is a little mathematical riddle that can be played using 9. What is the largest number that can be written using three digits?

The answer is $9^{9^{9}}$ or 9 raised to the ninth power of 9. The ninth power of nine is 387420489. No one knows the precise number that is represented by $9^{387420489}$, but it begins 428124773...and ends 89. The complete number will contain 369 million digits, would occupy over five hundred miles of paper and take years to read.

ZERO

I have a particular affection for zero because it was some of my countrymen who first gave it the status of a number. Though the symbol for a void or nothingness is thought to have been invented by the Babylonians, but without symbol it was the Hindu mathematicians who first conceived of 0 as a number, the next in the progression 4-3-2-1. The Mayans also invented zero

Now, of course, the zero is a central part of our mathematics, the key to our decimal system of counting. And it signifies something very different from simply 'nothing'—just think of the enormous difference between .001, .01, 1, 10, 100 to remind yourself of the importance of the presence and position of a 0 in a number.

Zero is both negative and positive, at the same time, it is neither negative nor positive. In other words, it has no sign connected with it. But it is the dividing point between the negative and positive numbers.

The operations with zero can be summarized as follows:

(a) Any number plus zero equals the number
(b) Any number minus zero equals the number
(c) Zero minus any number equals the negative of the number
(d) Any number times zero equals zero
(e) The operation of dividing by zero is not permitted.

About zero Alfred North Whitehead said:

'The point about zero is that we do not need to use it in the operation of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all cardinals, and its use is only forced on us by the need of cultivated modes of thought.

2

ADDITION

Addition is the first rule of arithmetical operations and it can be thought of as an extension in counting. Almost all calculations involve addition and yet of all the four basic things in arithmetic of addition, subtraction, multiplication and division, addition is the most troublesome and is one which is most subject to error. Mistakes generally occur in addition when long lists of numbers are being added together, or when numbers are carried from one column to the next. Carrying errors very often occur when you are adding in your head, and have to keep both the digits of the final answer and those you are carrying to add to the next column in your mind at the same time. However, by proper methods and by analysis of its combinations, the process of addition can be facilitated to a greater extent than would seem possible. There are a number of different ways of adding up columns of figures and many individuals have what may be called their own ways of getting at results.

In the multiplication table, there are 144 different multiplications to be remembered. But in the addition table, there are only 45 additions to be memorised and to be known so perfectly that no time or thought is to be given to them when asked or required. The table is very simple really. It consists of addition of the nine digits to each other, taking any two at a time.

The total number of such additions is fortyfive.

Of these additions, twenty give simple numbers such as:

$$3 + 3 = 6, \qquad 5 + 2 = 7$$

Of these additions, twentyfive give double numbers such as:

$$9 + 3 = 12, \qquad 8 + 6 = 14$$

The largest number given by the addition of two digits is $9 + 9 = 18$.

The figure on the left i.e. the one in the tens place, number 1, is the figure to be carried when we add. Therefore the figure to be carried in the addition of two single digits is never more than 1.

Here is the simple table to be memorised:

$1 + 1 = 2$	$2 + 2 = 4$	$3 + 3 = 6$	$4 + 4 = 8$
$1 + 2 = 3$	$2 + 3 = 5$	$3 + 4 = 7$	$4 + 5 = 9$
$1 + 3 = 4$	$2 + 4 = 6$	$3 + 5 = 8$	
$1 + 4 = 5$	$2 + 5 = 7$	$3 + 6 = 9$	
$1 + 5 = 6$	$2 + 6 = 8$		
$1 + 6 = 7$	$2 + 7 = 9$		
$1 + 7 = 8$			
$1 + 8 = 9$			

These are the twenty additions which do not give 'one to carry'. And now comes the table of twentyfive additions which do involve the carrying of 1:

$1+9=10$	$2+9=11$	$3+8=11$	$4+6=10$	$4+8=12$
$2+8=10$	$3+7=10$	$3+9=12$	$4+7=11$	$4+9=13$
$5+5=10$	$5+7=12$	$5+9=14$	$6+7=13$	$6+9=15$
				$8+8=16$
$5+6=11$	$5+8=13$	$6+6=12$	$6+8=14$	$7+9=16$
			$8+9=17$	$9+9=18$

For example, suppose 6, 7, 8 and 9 are to be added together:

6 and 7 are 13.

This gives 1 or 1 'ten' to carry.

Then come:

3 and 8 giving 11.

This gives 1 to carry to which is added the 1 'ten' carried from the first addition, giving 21. And now there are 2 'tens' to carry.

Next, 1 is added to 9 giving 10. This gives 1 to carry, to which is added 2, carried from the previous addition giving 30, which is the sum of the four numbers.

There was first the 1 to carry, and we saw that it could never be more than 1. The next number to be carried, due to the addition this time of three figures, was 2, one more than the first figure carried, which was 1. Then when the addition of the four was completed, the figure in the tens place was one more than the figure carried, it was 3.

This brings us to the following conclusion:

When a column of single digits is added up, the successive lefthand figures in the tens places can never exceed each other by more than 1 at a time. However, the successive figures in the tens places may sometimes be the same, depending on the numbers added.

Given below are some examples of additions, with the numbers to be added given in the first column (on the left), the second column (on the right) containing numbers which are successive sums obtained by addition, starting from the bottommost number:

E.g.	*a*	*E.g.*	*b*	*E.g.*	*c*	*E.g.*	*d*
9	50	1	22	9	32	9	40
8	41	2	21	8	23	8	31
9	33	2	19	7	15	7	23
7	24	8	17	8 ↑		8	16
8	17	9 ↑				8 ↑	
9 ↑							

E.g. a

Shows that it contains figures involving the 'carrying over' of an additional 1 for each adding. The successive sums also increase in the tens place by '1' viz. 1, 2, 3, 4, 5. They can never increase more rapidly than this.

E.g. b

Shows smaller numbers being added and therefore the tens place in the successive sums also increases at a slower pace viz. 1, 1, 2, 2.

E.g. c

As in e.g. a, here too the tens place figures increase with maximum rapidity, viz. 1, 2, 3. In column d, the same 4 figures are taken but an 8 is placed below them. On addition, we again have the regular maximum succession of tens place figures but in this case running upto 4 because of the extra 8 which has been added in.

In adding numbers, it would be of help to add them without naming the numbers at each step of addition. For example, while adding the figures in column *a*, you should say to yourself in quick succession 9, 17, 24, 33, 41, 50 instead of 9 and 8 are 17 and 17 and 7 are 24 and so on.

There are different ways of adding numbers. It can be done column by column taking one digit at a time. This, of course, is the conventional way of doing it. While it is the most obvious and simplest, it is also the slowest. There are two ways of doing it, from above downwards and from bottom upwards.

However, to be a hundred per cent sure of the result, it is well to do it first one way and then the other.

The addition of two columns at once can be done easily by anyone by using the following method:

The bottom or top number of the two columns is added to the tens of the number next to it and then the units of the next number are added, which gives the sum of the two. Then the tens of the third number are added and then the units of the third number giving the sum of the three. This is continued to the end of the columns.

This is the way to proceed

31	Add 70 to 83. This gives 153.
46	
74	To this add the 4 of the 74 giving 157.
83	To 157 add 40 giving 197 and to this add the 6 of the 46 giving 203. To
234	203 add 30 giving 233 and to this

add the 1 of the 31 and the total is 234. This method of adding is known as zigzag addition.

A three-column addition can also be carried out by an extension of the same method. Here is an example:

643	1134 + 600 = 1734
124	1094 + 40 = 1134
967	1091 + 3 = 1094
1734	991 + 100 = 1091
	971 + 20 = 991
	967 + 4 = 971

The addition was started at the bottom of the columns, and the sum was reached at the top.

This is a method well worth acquiring.

Let us see how an addition of 3 numbers of 4 digits each is done.

```
 8642
 9187
 3854
 ----
 1513
2017
-----
21683
```

Steps:

1. The first column from the right is added up $2 + 7 + 4 = 13$ with giving 3 under the first column and 1 under the second column, the figures are written down. If the sum gives a 3-digit figure, the third digit is written under the third column.

2. The third column $6 + 1 + 8$ is added and the sum 15 is written down alongside the first sum 13, with 5 under the third column and 1 under the fourth column.

3. The second column $4 + 8 + 5$ is then added and the sum 17 written down so that 7 comes under the second column and 1 under the third column.

4. Finally, the fourth column $8 + 9 + 3$ is added and the sum is written down alongside 17.
   A final addition of two rows of figures 1513 and 2017 gives the total 21683, as given in the worked out example.

However, when the columns are more than twelve figure high, three figures in a sum can occur. But this can be tackled by setting down the figures as described, but the only effect is that one more row is required. Suppose that the first column of four columns added up to 126, the third 348, the second 547 and the fourth 231. The sum would be put down thus:

```
      126
    348
     547
   231
   ------
Total: 271396
   ------
```

Another interesting method of adding is shown in the following example:

```
 569847      15
 316283       13
 769418        24
-------         14
152413           13
 131418           18
-------     --------
1655548     1655548
```

Starting at the left, the column is added up giving 15, which is put down. The next column is added up which gives 13. This is put down with the 1 under the 5 and the 3 next to the 5 on the same line. This is carried on through the rest of the columns. Their sums are 24, 14, 13 and 18. All are put down obliquely on two lines and the final addition gives the sum of the whole.

Another method that can be used to advantage is to write down the sum of each column separately, one sum under the other and each successive sum set one space to the left. Then the subsidiary addition gives the total.

This is how it is done:

```
 3348     Addition of first column       23
 6897     Addition of second column     17
 8436     Addition of third column     24
 5912     Addition of fourth column   22
 ----                                 -----
24593                                 24593
 ----                                 -----
```

In the lefthand addition, it is added up in the regular way, and in the righthand addition the way just described is carried out.

You can, of course, work from left to right if you wish and set each column total one place further right rather than one place further left:

3348	Addition of the fourth column	22
6897	Addition of the third column	24
8436	Addition of the second column	17
5912	Addition of the first column	23
	Total:	24593

There are even tricks you can play with addition. Ask a friend to write down any five-figure number—say, he writes 18463. You, apparently at random, choose figures to write below his. You put down 81536—these are, in fact, digits each of which, added to each of those they stand below, will total 9. You now ask your friend to add a further five-figure number, and you again write below his digits those each of which would make each of them upto 9. Your friend then adds a final fifth line of five figures and you instantly draw a line and add all five numbers together—the total will always be the last number your friend wrote with two subtracted from the last digit and two inserted before the first one:

```
 18463
 81536
 92843
  7156
 58462
______
258460
```

CHECKING BY CASTING OUT THE NINES

Additions can be checked by a method which is known as 'Casting out the nines'. Though this method is not infallible, it is a useful thing to know.

First add up the digits of each of the numbers you are adding and of the answer you have arrived at. Divide each of these numbers by 9 and set down the remainder. Total the remainders of the numbers you were adding together, and again cast out the nines. If the remainder you are left with is the remainder when the sum of the digits of your answer was divided by 9, you may assume that your answer was correct.

Here is an interesting rhyme describing how zigzag addition is done for two column additions from 'Rapid Arithmetic' of W. Stokes:

By Zigzag Addition, two columns, or three
Can be cast up together, you'll readily see.
For two columns, tens and then units first take,
Then with the next unit a mental sum make.
Then add in its tens; then the units above,
Then its tens will fill in, as a hand fits a glove.
All the figures above you can thus take in turn,
And the accurate total you'll quickly discern!

For three columns, two tens and two units combine,
And add in the hundreds upon the first line,
Then the hundreds above, then from line number three
The tens and the units must with the rest be.
Then hundreds same line must the whole amount swell
Tens and hundreds above, as perhaps you will tell.
Then the hundreds again, in same style as before,
So with figures which reach to the ceiling from floor!

3

SUBTRACTION

Subtraction is the opposite of addition. In arithmetic, subtraction is the taking away of a smaller number from a larger one, whereas in algebra, the reverse may also be done. When the smaller quantity is the minuend, the remainder will be affected by a minus sign. Of all the four elementary processes of arithmetic, subtraction may be taken as the simplest and least complicated.

Counting 0 also as a number or digit, there will be 100 subtractions of number from number. Thus from 1 we may have to subtract any one of the ten digits. The same is to be said for 2 and as there are ten digits, the total number of subtractions is 10×10 or 100.

In some of the subtractions, it is necessary to 'borrow ten' and to 'carry one'. There are fortyfive subtractions of single digit from single digit in which this has to be done. It helps to know these operations perfectly in order to perform the subtraction quickly.

In general ordinary subtraction, the smaller number, the subtrahend, is usually placed below the larger, the minuend, and the subtraction is done digit by digit.

A simpler way of carrying out a subtraction would be based on the following considerations. It is easier to subtract a multiple of ten from another quantity than to subtract any other double-digit number. It is easier to subtract 20 than to subtract 17. This is the main consideration. The other consideration is that if numbers are to be subtracted one from the other, the result will be unchanged if we add the same amount to each or if we subtract the same amount from each. This is particularly a useful trick when subtracting sums of money in decimal currencies. For example, to subtract 46 from 58 add 4 to both numbers—the problem then becomes the easy one of

$$62 - 50 = 12$$

or to deduct £1.72 from £3.64, add 8 pence to each sum:

$$£3.72 - £1.80 = £1.92$$

When you are dealing with three-figure numbers, you will bring the number being subtracted upto the nearest hundred:

246 — 182 becomes 264 — 200 and the answer, 64, is obvious.

However, when working by couples, there will be one to carry. This introduces difficulty at any moment. If there is one to carry, one is subtracted from the difference of the couple next on the left.

For example, in subtracting 4968 from 8254, we do it in couples:

Add 1 to the lefthand couples, and add 7 to the righthand couples. This gives:

8356	84	63
3983	40	90
4373	43	73

In subtracting 90 from 63 we had to borrow 1, so there is one to carry, and this is subtracted from the difference of 84 — 40 giving 43 instead of 44.

Subtraction can also be performed by the method of addition. Suppose we have to subtract 386 from 923, here is an example of how it can be done:

$$\begin{array}{r} 923 \\ -\ 386 \\ \hline 537 \\ \hline \end{array} \qquad \begin{array}{r} 537 \\ +\ 386 \\ \hline 923 \\ \hline \end{array}$$

The above lefthand example shows the regular subtraction and the righthand one the addition method. To do the first, write down 386 the subtrahend. Put a line under it and below this line put down 923, the minuend. Then proceed to write above the two numbers a number which added to 386 will give 923. This number is 537 and this is the answer.

When the sum of several numbers is to be subtracted from the sum of several others, the usual way is to add each set of numbers separately and subtract the sum of one from that of the other set. However, it can also be done directly.

In the following example, the three numbers below the upper line are to have their sum subtracted from the sum of those above the line:

$$\begin{array}{r} 9342 \\ 4564 \\ 4723 \\ \hline 1335 \\ 2662 \\ 3141 \\ \hline 11491 \\ \hline \end{array}$$

In this case, we proceed as follows:

Add 1, 2 and 5 giving 8. Keep this in mind and add 3, 4 and 2 giving 9. Subtract 8 from 9 giving 1. Put down the 1 under the lower line and add the next column above the upper line. This gives 12 from which is to be subtracted the sum of 3, 6 and 4. Then in order to subtract 13 from 12, we borrow 1 from the next line, deducting 1 from that line, giving $22 - 13 = 9$.

The principal point to remember in this process is to keep correct on the carrying. You must carry to the proper place.

There is a simple way of checking errors in subtraction. Add your answer to the number being subtracted, the sum will be the same as the number being subtracted from, if you have made no mistakes.

4

MULTIPLICATION

Multiplication is only a repeated addition. We are all supposed to know, by heart, the multiplication tables and when we are asked to multiply 8 by 7, we at once put down the product as 56. If you analyse the operation, you will find that what we have done actually is we have actually added seven 8's to one another. Putting it down as a formula, we have

$$8 + 8 + 8 + 8 + 8 + 8 + 8 = 8 \times 7 = 56$$

Addition of the seven 8's is the same thing in its result as multiplying 8 by 7.

And the reverse holds good, the addition of the eight 7's is the same in result as multiplying 7 by 8.

The most essential thing in multiplication is to know the multiplication table. As almost universally taught, the table includes as its upper limit, twelve times twelve. The multiplication table upto twelve times involves one hundred forty-four operations to be memorized. At first sight, though it may seem formidable, on analysis, it becomes much simpler.

We may safely omit the one-times table as not to be learned, because it is known to anyone who can count upto twelve.

This leaves us with one hundred and thirtytwo operations. Of these, eleven are one-times — such as 3 times one is 3, 4 times one is 4. Leaving these out, we are left with one hundred and twentyone operations. However, many of these are simple reversals of each other, such as 2 times 4 equals 8 and 4 times 2 equals 8. Counting 2 reversals as one operation, which is perfectly correct, the operations reduce to sixtyseven and many products of these operations are repeated, so that the number of products is only fortynine. They are the following:

4	6	8	9	10	12	14	15	16	18	20
21	22	24	25	27	28	30	32	33	35	36
40	42	44	45	48	50	56	60	64	66	70
72	77	80	81	84	88	90	99	100	108	110
120	121	132	144							

While doing higher multiplications, one can take advantage of the lower known products by two or by 4. For example, sixteen times is eight times multiplied by 2. Fourteen times is seven times multiplied by 2 and so on for other even multipliers.

The squares of higher numbers such as sixteen times sixteen may be taken as four times sixtyfour — which is the square of eight.

However, this easy method does not apply to 'prime numbers' — numbers not divisible by any other number except 1 and itself — for example, eleven, thirteen, seventeen and so on.

It is always easier to multiply by a single number than a double one. So when you want to multiply a two-digit number by another two-digit number, here is a simple way. Suppose 39 is to be multiplied by 16:

If twice of 39 is multiplied by half of 16 the answer will be given.

So

$$39 \times 2 = 78$$
$$16 \div 2 = 8$$
$$\text{and} \quad 78 \times 8 = 624$$

The general principle is to multiply or divide by a number which will make a single number out of one of the two given numbers. However, the trouble with this method arises when one of the numbers may be indivisible without a remainder.

When multiplying a two-digit number by another two-digit number, which has the same figure in the tens, such as 47 and 43, 72 and 79, the operation can be carried out in a simple way:

Multiply the units together.

Multiply the tens digit of one of the numbers by the sum of the units of the original numbers and annex a zero.

Multiply the tens figures together and annex two zeros.

Add the products for the final results — the product of the original numbers.

Example:

Multiply 82 by 87
$2 \times 7 = 14$

This is the first of the three quantities.

Now multiply the tens figure 8 by the sum of the units figures:

$2 + 7 = 9$

The product is $8 \times 9 = 72$
Annex a zero giving 720, the second quantity.
Multiply the tens figures together, giving 64, and annex two zeros for the third figure 6400.
The sum of the three is the product of

82×87
$14 + 720 + 6400 = 7134$

Here is one more example:

$$59 \times 53 = 3127$$

$9 \times 3 = 27$	27
$5 \times 12 = 60$	600
$5 \times 5 = 25$	2500
	3127

A similar method can be used to multiply three-digit numbers or even higher numbers. Here are two multiplications of larger numbers:

$$368 \times 364 = 133952$$

$8 \times 4 =$	32	32
$36 \times 12 =$	432	4320
$36 \times 36 =$	1296	129600
		133952

$$568 \times 563 = 319784$$

$8 \times 3 =$	24	24
$56 \times 11 =$	616	6160
$56 \times 56 =$	3136	313600
		319784

One other way can be adopted in multiplying three-digit numbers. Here is the way:

Supposing you have to multiply 346 by 493. First multiply 346 by 90 and this gives 31140. Now multiply 346 by 403 as follows:

> 3 times 346 is 1038. Write down only the figures 38 and carry the rest in your mind. Then start with 4 times 346. Four times 6 is 24, add to this the 10 and you have 34. Write down 4 and carry 3. Then 4 times 4 is 16 and 16 and 3 are 19. Put down 9 and carry 1. Finally, 4 times 3 is 12 and 12 and 1 to carry are 13. We now have 139438 to which is to be added 31140, giving 170578, the product asked for.

This method is an excellent practice in mental arithmetic. There are other different methods, but this method will be the quickest and work best.

To multiply by 100 you add two zeros.

To multiply by 1000, you add three zeros and so on.

This basic technique can be widely extended.

If you are asked to multiply 36 by 5, if you simply followed the general method, you would write down both numbers, multiply 6 by 5, put down the zero and carry the 3, multiply 3 by 5 and add the 3 you carried, getting the correct result 180. But a moment's thought will show you that 5 is half of 10, so if you multiply by 10, by the simple expedient of adding 0 and then divide by 2 you will get the answer much quicker. Alternatively, you can first halve 36 giving you 18, and then add the zero to get 180.

Here are some extensions of this method:

To multiply 15, remember that 15 is one and a half times 10. So to multiply 48 by 15, first multiply by 10:

$$10 \times 48 = 480$$

Then to multiply by 5, simply halve that figure

$$480 \div 2 = 240$$

Add the two products together to get the answer:

$$480 + 240 = 720$$

To multiply by $7\frac{1}{2}$, $7\frac{1}{2}$ is three-quarters of 10.
For example, $64 \times 7\frac{1}{2}$:

$$64 \times 10 = 640$$

The easiest way of finding three-quarters of 640 is to divide by 4 and multiply by 3, thus:

$$640 \div 4 = 160$$
$$160 \times 3 = 480$$

It is easy to see that this method can work just as well for 75 or 750, and there is no difficulty if the multiplicand is a decimal figure. For example, to take a problem in decimal currency, suppose you are asked to multiply 187.60 by 75. Instead of adding a zero, just move the decimal point:

$$187.60 \times 100 = 18760$$
$$\frac{18760}{4} = 4690$$
$$4690 \times 3 = 14070$$

To multiply by 9, just remember that 9 is one less than 10, so all that is necessary is to add a zero and then subtract the original multiplicand. Take 9×84

$$10 \times 84 = 840$$
$$840 - 84 = 756$$

This can be extended, if asked to multiply by 18. All that is necessary is to multiply by 9 and double the product. For example, 448×18

$$448 \times 10 = 4480$$
$$4480 - 448 = 4032$$
$$4032 \times 2 = 8064$$

Alternatively, you can start from the fact that 18 is 20 less 2, in which case the sequence is:

$$448 \times 2 = 896$$
$$896 \times 10 = 8960$$
$$8960 - 896 = 8064$$

This method can be used for all numbers which are multiples of 9. For example, if asked to multiply 765 by 54, you will realise that 54 is equivalent to 6×9, the calculation then goes:

$$765 \times 6 = 4590$$
$$4590 \times 10 = 45900$$
$$45900 - 4590 = 41310$$

Multiplying by 11 is easily done if you remember that 11 is $10 + 1$. Therefore to multiply any number by 11, all that is necessary is to add a zero and then add on the original number. For example, to multiply 5342 by 11:

$$5342 \times 10 = 53420$$
$$53420 + 5342 = 58762$$

In the case of 11, there is an even shorter method that can be used if the multiplicand is a three-digit number. For example, if asked to multiply 653 by 11, you proceed as follows:

First multiply the last two digits, 53 by 11

$$53 \times 11 = 583$$

To this add the first hundreds digits multiplied by 11.

$$600 \times 11 = 6600$$
$$6600 + 583 = 7183$$

This method is also valid even if the multiplicand is more than a three-digit number.

Another case where this method can be used is with $12\frac{1}{2}$. Consider the product $872 \times 12\frac{1}{2}$. Here you have a choice, you can either work on the basis that $12\frac{1}{2}$ is 10 plus a quarter of 10, in which case

$$872 \times 10 = 8720$$
$$8720 \div 4 = 2180$$
$$8720 + 2180 = 10900$$

Or, you can work from the basis that $12\frac{1}{2}$ is one-eighth of 100, in which case

$$872 \times 100 = 87200$$
$$87200 \div 8 = 10900$$

It's best to check if the multiplicand is divisible by 8, before adopting the second way. In fact, this process of checking pretty well does the calculation for you. If you have to multiply 168 by 12½, a moment's thought shows that 8 goes into 168 exactly 21 times. All you then have to do is add two zeros to get the correct product 2100.

But if the multiplicand had been, say, 146, then clearly the first method is the one to use.

Here are some other relationships that can be exploited to use this basic method:

112½ is 100 plus one-eighth of 100.
125 is 100 plus one-quarter of 100; or 125 is one-eighth of 1000.
45 is 50 minus 5, 50 is half of 100 and 5 is one-tenth of 50.
25 is a quarter of 100.
35 is 25 plus 10.
99 is 100 minus 1.
90 is 100 minus one-tenth of 100

and so on. If you experiment, you will find many more of these useful relationships, all of which can be used to take advantage of the basic shortcut offered by the fact that to multiply by 10, all you do is add a zero. After some practice, you will find that you can spot almost without thinking a case where this method is going to help.

The next method I am going to describe can be used when the multiplier is a relatively small number, but one for which our first method is unsuitable because there is no simple relationship to 10 which can be spotted and exploited.

When, for instance, you multiply by 20 by the simple and obvious means of multiplying by two and then adding a zero, what you are doing is taking out the factors of 20, 2 and 10, and multiplying by them each in turn. This method can be extended to any number which can be broken down into factors. For example, if you are asked to multiply a number by 32 you can break 32 into its factors, 8 and 4, and proceed as follows (here, to start with anyway, I suggest the use of paper and pencil):

To multiply 928 by 32

$$\begin{array}{r} 928 \\ \times\ 4 \\ \hline 3712 \\ \times\ 8 \\ \hline 29696 \\ \hline \end{array}$$

Even if you have to write the calculation out as above it is a great deal quicker than the conventional method, which would not only involve two sequences of multiplication but also one of addition.

In practice, this method is best used where the multiplier is relatively small and where its factors, therefore, can easily and quickly be extracted; if you remember multiplication tables upto 12, you will be able to judge at a glance whether or not this is a suitable method in the case of a two-digit multiplier.

There is another method that could be used where there is no easily discernible means of using the other methods I have explained so far, and where the number of digits involved makes them impracticable.

I am going to start by explaining the method with relatively small numbers, so that you can grasp the essentials.

For our first example, let us take 13×19. First add the unit digit of any one number to the number thus:

$$9 + 13 = 22 \text{ (also } 3 + 19 = 22)$$

You then think of that sum as so many tens, in this case 22 tens. Now multiply the units digits together

$$3 \times 9 = 27$$

Finally, add the product to the tens figure you already have:

$$27 + 220 = 247$$

Here is another example, 17×14:

$$4 + 17 = 21 \text{ (also } 7 + 14)$$
$$7 \times 4 = 28$$
$$28 + 210 = 238$$

The above method is valid only when the two digits in tens places of the two numbers are equal to 1.

There are particular shortcuts for multiplying together two-digit numbers with the same tens figure 1.

When the tens figure is common, you add to one number the units figure of the other, multiply this sum by the common tens figure and add to this product, considered as tens, the product of the two units digits. For example, 49×42:

$$49 + 2 = 51 \text{ (also } 9 + 42)$$
$$51 \times 4 = 204$$

Add to this product (2040, when considered as tens) the product of the units ($9 \times 2 = 18$) to get the final product of 2058.

Another example, 58×53:

$$58 + 3 = 61 \text{ (also } 8 + 53)$$
$$61 \times 5 = 305$$
$$8 \times 3 = 24$$
$$3050 + 24 = 3074$$

If the common tens figure is 9, there is an even simpler method. Subtract each of the numbers from 100. Multiply the remainders together—this gives you the last two digits of the final product. To arrive at the first two digits—take away from one of the numbers the figure by which the other was short of 100; for example, 93×96:

$$100 - 93 = 7$$
$$100 - 96 = 4$$
$$4 \times 7 = 28, \text{ your last two digits.}$$

$93 - 4$ or $96 - 7$ gives you 89, your first two digits.

The product is therefore 8928.

If the units figures of two two-figure digits are the same, you can multiply them by adding the product of the two tens digits (considered as hundreds) to the sum of the tens digits multiplied by the common units digit and the square of the common units digit. For example, consider 96×46:

$$\begin{array}{lr} & 9 \times 4 = 36 \\ 9 + 4 = 13 & 13 \times 6 = 78 \\ & 6 \times 6 = 36 \end{array}$$

$$3600 + 780 + 36 = 4416$$

Or to multiply 62×42:

$$\begin{array}{lr} & 6 \times 4 = 24 \\ 6 + 4 = 10 & 10 \times 2 = 20 \\ & 2 \times 2 = 4 \end{array}$$

$$2400 + 200 + 4 = 2604$$

If the common final digit is 5, it is even simpler—to the product of the two tens figures considered as hundreds, add half the sum of the two tens figures, still considered as hundreds.

Then add 25 to arrive at the final product.

For example, 45×85:

$$4 \times 8 = 32$$

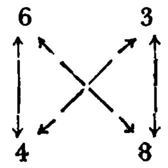
$$\frac{4 + 8}{2} = 6$$

$$3200 + 600 = 3800$$

Add 25 to obtain the final product of 3825.

All the methods I have described so far can be done mentally when you have a little practice. I will now describe others which can be used more generally, but which require pencil and paper. Even with these methods, most of the calculations can be done mentally; you use the paper to keep note of your intermediate results. In each case, you do a sequence of diagonal or vertical multiplications—the pattern of these is shown in the diagrams to the right of the examples.

For example, to multiply 63 by 48, write down the numbers thus:



$3 \times 8 = 24$, so put 4 in the units column and carry the 2, you will add this to the sum of the products of the 'diagonals'—8 and 6, and 4 and 3. Your mental calculation runs thus:

$$6 \times 8 = 48; \; 4 \times 3 = 12: \; 48 + 12 + 2 = 62$$

You write 2 in the tens column and carry 6—this you add to the product of the two tens digits—6 and 4.

The mental calculation $6 \times 4 = 24$, $24 + 6 = 30$ gives the final figures—all you have had to write down is the problem itself and the answer:

$$\begin{array}{r} 63 \\ 48 \\ \hline 3024 \\ \hline \end{array}$$

Set out below are three-digit figures to be multiplied in the same way:

$$\begin{array}{ccc} 4 & 3 & 6 \\ 2 & 5 & 4 \end{array}$$

Again multiply the units digits and write down the units figure of the answer 4 and carry the tens digit 2. Now multiply 3 by 4 and add the 2 you are carrying to make 14. Add this to the product of 5 and 6, i.e. 30, to make a total of 44. You now have two figures of your final answer and are still carrying only one figure—4—in your head. The figures you have written read

$$\begin{array}{r} 436 \\ 254 \\ \hline 44 \\ \hline \end{array}$$

Your next mental steps are to add the 4 you are carrying to the products of 4 and 4, 6 and 2 and 3 and 5, and the calculation will run:

$4 \times 4 = 16$; $16 + 4 = 20$; $6 \times 2 = 12$; $12 + 20 = 32$; $3 \times 5 = 15$; $15 + 32 = 47$.

Set down 7 and carry the 4.

Now multiply the lefthand set of diagonals—4 and 5, and 3 and 2 and add the carried 4 : $4 \times 5 = 20$, $20 + 4 = 24$, $3 \times 2 = 6$, $6 + 24 = 30$.

Set down the zero, carry the 3, and add it to the product of the first hundreds digits, 4 and 2:

$$4 \times 2 = 8, \quad 8 + 3 = 11.$$

Now write 11 next to the 0.

$$\begin{array}{r} 436 \\ 254 \\ \hline 110744 \\ \hline \end{array}$$

Again, all you have had to write down is problem and answer.

If the multiplier has only two figures, you can still use this method by replacing the missing hundreds figure with a **0**. For example, set out 476 × 26 like this:

$$\begin{array}{r} 476 \\ 026 \\ \hline \end{array}$$

6 × 6 = 36, put down the 6, carry 3, and proceed as before.

7 × 6 = 42; plus 3 carried = 45; plus 2 × 6 (= 12) = 57.
Put down the 7 and carry the 5.
4 × 6 = 24; plus the 5 carried = 29;
6 × 0 = 0; 2 × 7 = 14
29 + 14 = 43. Set down 3 and carry 4.
4 × 2 = 8; 8 + 4 = 12; 7 × 0 = 0; 12 + 0 = 12.

So you write and carry 1. The figures you have now written read:

$$\begin{array}{r} 476 \\ 026 \\ \hline 2376 \\ \hline \end{array}$$

4 × 0 = 0, but you are still carrying 1.

So the final answer reads 12376.

We can now extend the method to deal with four-digit numbers. For instance, to multiply 9246 by 2543, set down the problem in the same way as before:

$$\begin{array}{r} 9246 \\ 2543 \\ \hline \end{array}$$

First the units : 3 × 6 = 18, write down the 8 and carry the 1. Next the first pair of diagonals: 4 × 3 = 12; 12 + 1 =13; 4 × 6 = 24; 24 + 13 = 37.

Write down the 7 and carry the 3

$$\begin{array}{r} 9246 \\ 2543 \\ \hline 78 \\ \hline \end{array}$$

Now 2×3, 4×4 and 5×6

$$\begin{array}{lll} 2 \times 3 = 6 & 6 + 3 \text{ (carried)} & = 9 \\ 5 \times 6 = 30 & 30 + 9 & = 39 \\ 4 \times 4 = 16 & 16 + 39 & = 55 \end{array}$$

Set down 5 and carry 5

$$\begin{array}{r} 9246 \\ 2543 \\ \hline 578 \\ \hline \end{array}$$

Now the four sets of diagonals:

$$\begin{array}{lll} 9 \times 3 = 27 & 27 + 5 \text{ (carried)} & = 32 \\ 2 \times 6 = 12 & 12 + 32 & = 44 \\ 2 \times 4 = 8 & 8 + 44 & = 52 \\ 5 \times 4 = 20 & 20 + 52 & = 72 \end{array}$$

Set down 2 and carry 7

$$\begin{array}{r} 9246 \\ 2543 \\ \hline 2578 \\ \hline \end{array}$$

By the same procedures as before:

$9 \times 4 = 36$	$36 + 7$ (carried) $= 43$
$4 \times 2 = 8$	$43 + 8 = 51$
$2 \times 5 = 10$	$51 + 10 = 61$

gives us another digit of the solution, and 6 to carry

$9 \times 5 = 45$	$45 + 6$ (carried) $= 51$
$2 \times 2 = 4$	$51 + 4 = 55$

gives another 5, with 5 to carry

$$\begin{array}{r} 9246 \\ 2543 \\ \hline 512578 \\ \hline \end{array}$$

and we get the final two digits by multiplying 9 by 2 and adding the 5 we are carrying:

$$18 + 5 = 23$$

$$\begin{array}{r} 9246 \\ 2543 \\ \hline 23512578 \\ \hline \end{array}$$

The same method of adding the products of groups of diagonal multiplications can be used for larger numbers, but a little more paper work will make it simpler. For instance, to multiply 637432 by 513124, start by setting the numbers out in the usual way:

$$\begin{array}{r} 637432 \\ 513124 \\ \hline \end{array}$$

Then proceed as follows:

$4 \times 2 =$	8
$(4 \times 3) + (2 \times 2) = 12 + 4 =$	16
$(4 \times 4) + (2 \times 1) + (3 \times 2) = 16 + 2 + 6 =$	24
$(4 \times 7) + (2 \times 3) + (3\times 1)+(4 \times 2) = 28 + 6 + 3 + 8 =$	45
$(4\times 3)+(2\times 1)+(2\times 7)+(3\times 3)+(4\times 1)= 12+2+14+9+4=$	41
$(4\times 6)+(5\times 2)+(2\times 3)+(1\times 3)+(7\times 1)+(3\times 4)= 24+10+6+3+7+12=$	62
$(2\times 6)+(5\times 3)+(1\times 4)+(3\times 1)+(7\times 3)= 12+15+4+3+21=$	55
$(1\times 6)+(5\times 4)+(1\times 7)+(3\times 3)=6+20+7+9=$	42
$(3\times 6)+(5\times 7)+(3\times 1)=18+35+3=$	56
$(1\times 6)+(5\times 3)=6+15=$	21
$(6\times 5)=$	30
	327081657568

All the computing goes on in your head, the addition at the end involves only two digits in each column and the amount of paper work is limited to the figures on the righthand side of the page.

CHECKING BY CASTING OUT THE NINES

Like in addition, multiplication can also be checked by the excess of nines method. If the digits of a number are added together and their sum is divided by 9, the remainder, if there is any, is called the excess of nines. This is the way it is done:

Add up the digits in each factor (the numbers you are multiplying together) and in the product.

Divide each of the sums of digits by 9. Set down the remainder in each case—this is known as the check number.

Multiply the check number of the multiplicand by the check number of the multiplier, and add up the digits in the product. If this gives you the same number as the check number of the original product, you can assume that the product was correct.

For example, to check that $8216 \times 4215 = 34630440$:

$8 + 2 + 1 + 6 = 17$, casting out the nines, leaves 8 as the check number.

$4 + 2 + 1 + 5 = 12$, casting out the nines, leaves 3 as the check number.

$3 + 4 + 6 + 3 + 0 + 4 + 4 + 0 = 24$, casting out the nines, leaves 6 as the check number.

$3 \times 8 = 24$, $2 + 4 = 6$ which was the check number of the original product. Therefore, the multiplication is correct.

However, I must mention here that this system of casting out the nines is not infallible, but if the answer you have obtained checks out as correct when you have used it, the chances of your being wrong are very slim.

SOME CURIOSITIES IN MULTIPLICATION

Some things are curious in the multiplication table, which can make a fascinating study. Here are some of them.

Write down the products in any multiplication table, such as three times or four times, and add up the units figures. You will find that the sum of the figures in question stopping at the 9th place will either be 40 or 45, except in the case of five times. For even number multiplications such as four times or six times, the sum of these units will be 40, for the odd times, three times or seven times, the sum will be 45.

Here are some examples:

Three Times	*Four Times*	*Six Times*	*Seven Times*
3	4	6	7
6	8	12	14
9	12	18	21
12	16	24	28
15	20	30	35
18	24	36	42
21	28	42	49
24	32	48	56
27	36	54	63
45	40	40	45

The righthand columns are the only ones that are added. The lefthand columns are not added up. The five times section is not put down but it gives as the sum of its righthand digits only 25.

If the digits in the units places of odd numbers of the righthand columns, such as three times or seven times are added, their sum will be 25. We shall now do it and see for three times, seven times and nine times:

Three Times	*Seven Times*	*Nine Times*
3	7	9
9	21	27
15	35	45
21	49	63
27	63	81
25	25	25

As before, it is only the righthand numbers which have been added up, and their sum is the same as the sum of all the righthand digits of the five times division of the multiplication table.

Let us now take any of the columns and add, this time horizontally, the component digits of the different products.

Taking the three times column given above, we get:

3, 6, 9, 3, 6, 9, 3, 6, 9

Taking the four times column, we get:

4, 8, 3, 7, 2, 6, 10, 5, 9

The six times column gives us:

6, 3, 9, 6, 3, 9, 6, 3, 9,

Regularity in various degrees can be traced out for other multiplication tables also.

Here is a curious method of writing down the nine times table.

We write down the 8 digits beginning with 1 and ending with 8 in a vertical column. Then to the right of this column we write another column, this time beginning with 9, one space above the 1 of the other column and with 1. The 1 should come by the side of 8 in the other column as shown below:

9
18
27
36
45
54
63
72
81

Don't you now see the full nine times of the multiplication table from nine times one up to nine times nine?

A CURIOUS WAY OF MULTIPLYING

Did you know you can carry out a multiplication of any two numbers together simply by adding, multiplication by 2 and division by 2?

All you have to do is put the numbers down side by side, divide one of them by 2, put the quotient under the same number and divide this quotient by 2. Don't worry about the remainders—leave them. Repeat this until you can go no further or until the quotient 1 is obtained. Multiply the other number by 2, put it alongside the first quotient, multiply this by 2 and put it alongside the second quotient.

Keep this up until you have a multiple for each of the quotients. The quotients can go only a definite distance and are the limiting element. Of the products thus obtained strike out each one, that is opposite an even number quotient.

The sum of the products remaining will give the product of the two original numbers.

For example, let us multiply 92 by 63.

It is immaterial which number is successively divided and which multiplied. We shall do it in both ways here, indicated as 'a' and 'b':

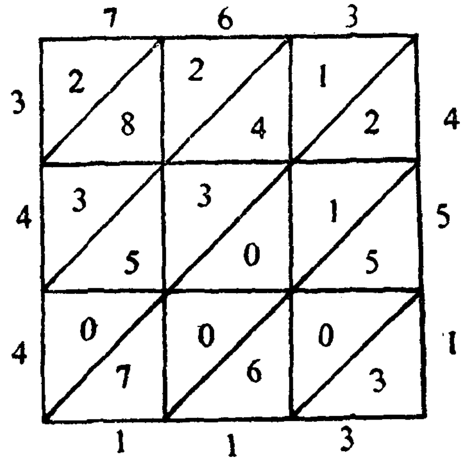
a	b
92→ 63	92←63
46→ 126	184←31
23→ 252	368←15
11→ 504	736← 7
5→1008	1472← 3
2→2016	2944← 1
1→4032	

The quantities opposite the odd number quotients have to be added to give the product of the two original numbers. This is done below in each case below its own calculation:

252	92
	184
504	368
	736
1008	1472
4032	2944
5796	5796

Lattice Method of Multiplication

Here is another curious way of multiplying two numbers. The following is the multiplication of the two numbers 451 and 763 by the Lattice method:



In this multiplication, each cell is the product of the number at the top of each column and to the right of each row, and the sums are added along the diagonals to obtain the final product.

In this case the product is 344113.

Other Curiosities in Multiplication

Write down the nine digits omitting 1, beginning from nine to two and multiply it by 9. This product will be a succession of nine 8's.

$$98765432 \times 9 = 888888888$$

Now let's see what happens when we write down the nine digits including the 1 this time and multiply by 9

$$987654321 \times 9 = 8888888889$$

This gives a product consisting of nine 8's as before with a 9 at the right end.

Let's now multiply it by 18

$$987654321 \times 18 = 17777777778$$

We get a product with the lefthand figure 1 and then come nine 7's and as the righthand figure comes the 8.

If multiplied by 27, the lefthand figure will be a 2, the nine middle figures will be 6's and the righthand figure a 7.

$$\frac{987654321 \times 27}{26666666667}$$

It goes thus through the multiples of 9 used as multipliers, until we finally get nine times nine or 81 as a multiplier, the lefthand figure 8, the righthand figure 1 and zeros to the regular number of 9, as the intermediate digits.

In each product, the left and righthand figures give or repeat the multiplier, and the intermediate figures run in regular order from 8's to zeros. The multiplications are given here:

$$\begin{aligned}
987654321 \times 9 &= 888888889 \\
987654321 \times 18 &= 1777777778 \\
987654321 \times 27 &= 2666666667 \\
987654321 \times 36 &= 3555555556 \\
987654321 \times 45 &= 4444444445 \\
987654321 \times 54 &= 5333333334 \\
987654321 \times 63 &= 6222222223 \\
987654321 \times 72 &= 7111111112 \\
987654321 \times 81 &= 8000000001
\end{aligned}$$

Another interesting multiplication is that of 15873 × 7:

$$15873 \times 7 = 111111$$

Now if you multiply this same number by 9, you get the product 142857. And this number multiplied by 7 gives as product six 9's:

$$15873 \times 9 = 142857 : 142857 \times 7 = 999999$$
$$15873 \times 63 = 999999$$

Here is an odd series of products involving the versatile number, number 9:

$$9 \times 9 = 81 \text{ and } 81 + 7 = 88$$
$$98 \times 9 = 882 \text{ and } 882 + 6 = 888$$
$$987 \times 9 = 8883 \text{ and } 8883 + 5 = 8888$$

The last two in the series are:

$$9 \times 9876543 = 88888887 \text{ and } 88888887 + 1 = 88888888$$
$$9 \times 98765432 = 888888888 \text{ and } 888888888 + 0 = 888888888$$

When these curious multiplications are carried out by further multiplications, they give other interesting results. Here are a few:

$$153846 \times 13 = 1999998$$

If we add one half of itself to the above number, we obtain

$$153846 + 76923 = 230769.$$

This number multiplied by 13 gives:

$$230769 \times 13 = 2999997$$

Adding it again to the last sum, we obtain:

$$230769 + 76923 = 307692$$
$$307692 \times 13 = 3999996$$

Adding it once again to the last sum, we obtain:

$$307692 + 76923 = 384615$$
$$384615 \times 13 = 4999995$$

This process can be carried down to the eighth product or successive additions of 76923. Let us place these products in a column and see what happens:

$$\begin{aligned}
153846 \times 13 &= 1999998 \\
230769 \times 13 &= 2999997 \\
307692 \times 13 &= 3999996 \\
384615 \times 13 &= 4999995 \\
461538 \times 13 &= 5999994 \\
538461 \times 13 &= 6999993 \\
615384 \times 13 &= 7999992 \\
692307 \times 13 &= 8999991
\end{aligned}$$

The lefthand digits run from 1 to 8 and the righthand ones run from 8 to 1. The left and righthand digits together give the product of 9 multiplied by 2, 3, 4 and so on.

Here is a special pattern formed by the multiplication of all 3's:

$$\begin{aligned}
33 \times 33 &= 1089 \\
333 \times 333 &= 110889 \\
3333 \times 3333 &= 11108889 \\
33333 \times 33333 &= 1111088889 \\
333333 \times 333333 &= 111110888889 \\
3333333 \times 3333333 &= 11111108888889 \\
33333333 \times 33333333 &= 1111111088888889 \\
333333333 \times 333333333 &= 111111110888888889
\end{aligned}$$

and so on.

Here are some more odd multiplications:

$$\begin{aligned}
37037037037 \times 9 &= 333333333333 \\
13717421 \times 9 &= 123456789 \\
987654321 \times 9 &= 8888888889
\end{aligned}$$

The pattern formed by the products of the multiplication of all nines is also interesting:

$$99 \times 99 = 9801$$
$$999 \times 999 = 998001$$
$$9999 \times 9999 = 99980001$$
$$99999 \times 99999 = 9999800001$$
$$999999 \times 999999 = 999998000001$$
$$9999999 \times 9999999 = 99999980000001$$
$$99999999 \times 99999999 = 9999999800000001$$
$$999999999 \times 999999999 = 999999998000000001$$

and so on.

5

DIVISION

Division is the reverse of multiplication—just as subtraction is the reverse of addition. To multiply 3 by 4 is to add four threes together, and find the total—12. Dividing 12 by 3 could be said to be subtracting threes until you had nothing left—and you would, of course, do it four times. But division seems more intractable to most people, and particularly difficult to handle mentally. In this chapter, I shall show how division by particular numbers can be handled in particular ways, and, at the end of the chapter, how you can check to see whether a number is or is not divisible by any number from 2 to 11 (or, of course, any multiple of such a number). But first let us consider some special cases.

To divide a number by 5, you take advantage of the fact that 5 is half of 10, multiply the dividend (the number being divided) by 2 and divide by 10 by moving the decimal point one place to the left. For example:

$$165 \text{ divided by } 5 = \frac{330}{10} = 33$$

To divide by 15, multiply the dividend by 2 and divide by 30. For example:

$$165 \text{ divided by } 15 = \frac{210}{30} = 7$$

If you simplify in this way—by doubling divisor and dividend — and are left at the end of the calculation with a remainder, you must remember that this, too, will be doubled. Some divisions can be simplified by halving both divisor and dividend — divisions by 14, 16, 18, 20, 22 and 24 become divisions by 7, 8, 9, 10, 11 and 12, respectively — but this time you must remember to double the remainder. Here are some examples:

$$392 \text{ divided by } 14 = \frac{196}{7} = 28$$

$$464 \text{ divided by } 16 = \frac{232}{8} = 29$$

$$882 \text{ divided by } 18 = \frac{441}{9} = 49$$

$$4960 \text{ divided by } 20 = \frac{2480}{10} = 248$$

$$946 \text{ divided by } 22 = \frac{473}{11} = 43$$

$$1176 \text{ divided by } 24 = \frac{588}{12} = 49$$

Dividing by Factors

If a number can be broken down into factors, it may be simpler to divide by these, successively, than to do a single calculation. A mental division by 8, and then by 4, is simpler than a division by 32. For example, to divide 1088 by 32, first divide by 8:

$$\frac{1088}{8} = 136$$

and then divide the answer, 136, by 4 to get the final result—34.

Numbers easy to handle in this way are the products in the basic multiplication tables—multiples of 11, for instance, go particularly smoothly:

To divide 2695 by 55, first divide by 5:

$$\frac{2695}{5} = 539$$

and then by 11:

$$\frac{539}{11} = 49$$

to get the answer.

To divide by numbers that are powers of 2 (4, 8, 16, and so on), you merely have to go on halving the dividend. 16, for instance, is 2^4, so halving the dividend four times is the same thing as dividing by 16.

To divide 8192 by 16	
halve once to get	4096
and again to get	2048
and a third time to get	1024
and, finally, a fourth time to get the answer	512

This technique makes dividing by high powers of 2 easy, for instance, to divide 32768 by 128 — which is 2^7:

	32768 halved =	16384
	16384 halved =	8192
	8192 halved =	4096
	4096 halved =	2048
	2048 halved =	1024
	1024 halved =	512
and	512 halved =	256, which is the required answer.

The methods I have described so far work only with some numbers — thinking about what sort of numbers are involved in a calculation is often the first step to finding a quick way to do it. The more skilled you get at mental multiplication and division, the more steps in a normal division sum you will be able to do in your head. For instance, in the sum set out below, only the remainders are noted down —partial products are arrived at, and the subtractions done mentally:

```
31 ) 13113 ( 423
       71
        93
       ----
        00
       ----
```

Dividing By Fractions Mentally

It is easier to divide by whole numbers than by fractions — if both divisor and dividend are multiplied by the same factor, the answer to the problem will be the same. (Any remainder, however, will be a multiple or fraction of the correct value.) If, for instance, you are dividing by 7½, it is simpler to multiply both numbers in the calculation by 4 — dividing four times your original dividend by 30. For example, to divide 360 by 7½:

$$360 \times 4 = 1440$$
$$7\tfrac{1}{2} \times 4 = 30$$
$$\frac{1440}{30} = 48$$

By the same principle when dividing by 12½, multiply the dividend by 8 and divide by 100, when dividing by 37½, multiply by 8 and divide by 300, and when dividing by 62½, multiply by 8 and divide by 500.

To divide by 1½, double the dividend and the divisor and divide by 3 (but remember to halve any remainder).

To divide a number by 2½, double the dividend and the divisor and divide by 5 — and go about dividing by 3½ in the same way.

Checking Whether a Number is Exactly Divisible

There are tests that can be made to show whether a number is exactly divisible by another number (or a multiple of it). Here are some of them:

If a number is divisible by two, it will end in an even number or a zero.

If a number is divisible by three, the sum of its digits will be divisible by 3 (for example, $372 = 3 + 7 + 2 = 12$). A corollary of this is that any number made by rearranging the digits of a number divisible by 3 will also be divisible by 3.

If a number is divisible by four, the last two digits are divisible by 4 (or are zeros).

If a number is divisible by five, the last digit will be 5 or 0.

If a number is divisible by six, the last digit will be even and the sum of the digits divisible by 3.

There is no quick test for finding out if a number is divisible by seven.

If a number is divisible by eight, the last three digits are divisible by 8 or are zeros.

If a number is divisible by nine, the sum of its digits is divisible by 9.

If a number is divisible by ten, it ends with a zero.

If a number is divisible by eleven, the difference between the sum of the digits in the even places and the sum of the digits in the odd places is a multiple of 11 or it is 0. For example, 58432 is shown to be divisible by 11 because:

$$\begin{aligned} 5 + 4 + 2 &= 11 \\ 8 + 3 &= 11 \\ 11 - 11 &= 0 \end{aligned}$$

and 25806 is shown to be divisible by 11 because:

$$\begin{aligned} 2 + 8 + 6 &= 16 \\ 5 + 0 &= 5 \\ 16 - 5 &= 11 \end{aligned}$$

Checking By Casting out the Nines

Divisions, like additions and multiplications, can be checked by casting out the nines — although, again, the method is not infallible.

You obtain check numbers as before, by adding up the digits in the numbers in the calculation—in this case the divisor, dividend and quotient — and dividing them by 9.

The check number is the remainder left after each division.

Multiply the check number of the divisor by the check number of the quotient. If the check number of this product is the same as the check number of the dividend, the division may be assumed to be correct. For example, to check:

$$\frac{2426376}{5321} = 456$$

$2 + 4 + 2 + 6 + 3 + 7 + 6 = 30$, casting out the nines leaves 3.

$5 + 3 + 2 + 1 = 11$, casting out the nines leaves 2.

$4 + 5 + 6 = 15$, casting out the nines leaves 6.

$6 \times 2 = 12$ (check number divisor $\times$ check number quotient). Casting out the nines leaves 3 — which is the same as the check number of the dividend.

When there is a remainder, the sum of the digits in the remainder is added to the product of the check number of the quotient and the divisor, and the operation is completed in the usual way. For example, to check:

$$\frac{1481265}{4281} = 346 \text{ remainder } 39.$$

$1 + 4 + 8 + 1 + 2 + 6 + 5 = 27$ casting out the nines leaves 0

$4 + 2 + 8 + 1 = 15$, casting out the nines leaves 6

$3 + 4 + 6 = 13$, casting out the nines leaves 4

$3 + 9 = 12$, casting out the nines leaves 3

$6 \times 4 = 24$, $24 + 3 = 27$, adding these digits and casting out the nines leaves 0, which is the check number of the dividend.

6

SOME SPECIAL NUMBERS

The Revolving Number — 142857

While discussing number 7 in Chapter 1, I described some oddities of the number 142857. For instance, its habit of repeating itself in the total when the powers of 2 are multiplied by 7 and added after being set out in a staggered formation, for instance. Here are some more. First set out the digits in a circle:

	1	
7		4
5		2
	8	

Now multiply 142857 by numbers from 1 to 6:

$$\begin{array}{r} 142857 \\ \times\ 1 \\ \hline 142857 \\ \hline \end{array} \qquad \begin{array}{r} 142857 \\ \times\ 2 \\ \hline 285714 \\ \hline \end{array} \qquad \begin{array}{r} 142857 \\ \times\ 3 \\ \hline 428571 \\ \hline \end{array} \qquad \begin{array}{r} 142857 \\ \times\ 4 \\ \hline 571428 \\ \hline \end{array} \qquad \begin{array}{r} 142857 \\ \times\ 5 \\ \hline 714285 \\ \hline \end{array}$$

$$\begin{array}{r} 142857 \\ \times\ 6 \\ \hline 857142 \\ \hline \end{array}$$

You will see that the numbers start revolving — the same digits in different combinations arrived at by starting from a different point on the circle. Multiply 142857 by 7 and things suddenly change

$$\begin{array}{r} 142857 \\ \times\ 7 \\ \hline 999999 \\ \hline \end{array}$$

But there are still oddities in store, for instance, if you multiply 142857 by a really big number, see what happens:

$$\begin{array}{r} 142857 \\ \times\ 32284662474 \\ \hline 4612090027048218 \\ \hline \end{array}$$

No resemblance to 142857 at first sight perhaps, but divide the product up into groups of 6, 6 and 4 and see what happens:

$$\begin{array}{r} 048218 \\ 090027 \\ 4612 \\ \hline 142857 \\ \hline \end{array}$$

The revolving number appears again: Sometimes 142857 hides deeper than that. For instance, in this multiplication, the product seems immune from 142857 extraction:

$$\begin{array}{r} 142857 \\ \times\ 45013648 \\ \hline 6430514712336 \\ \hline \end{array}$$

But divide it into 6, 6 and 1 in the same way as you did last time and you arrive at the following sum:

$$\begin{array}{r} 712336 \\ 430514 \\ 6 \\ \hline 1142856 \\ \hline \end{array}$$

Treat this total in the same way:

$$\begin{array}{r} 142856 \\ 1 \\ \hline 142857 \\ \hline \end{array}$$

And you winkle 142857 out.

You can find out oddities just by looking at the digits themselves. For instance, if you divide them into two groups, 142 and 857, the second figure of the first group, multiplied by the third figure of the first group gives the first figure of the second group:

$$4 \times 2 = 8$$

The sum of the first two figures of the first group gives the second figure of the second group:

$$1 + 4 = 5$$

And the sum of all the three figures of the first group gives the third figure of the second group:

$$1 + 4 + 2 = 7$$

To show, the next property of 142857 you have to draw up a table of the products of the number when multiplied by 1, 2, 3, 4, 5, and 6. Horizontally and vertically, the digits all add up to 27:

1	4	2	8	5	7	=	27
2	8	5	7	1	4	=	27
4	2	8	5	7	1	=	27
5	7	1	4	2	8	=	27
7	1	4	2	8	5	=	27
8	5	7	1	4	2	=	27
27	27	27	27	27	27		

And if you can't remember the number, there is always an easy way to find it; for 1/7, expressed as a decimal is:
142857 142857 142857 142857 and so on to infinity.

The Number 1089

This, as I mentioned earlier, has some peculiar traits. For example, look at the pattern that is formed when it is multiplied by the numbers 1 to 9:

$$1089 \times 1 = 1089 \qquad 9801 = 1089 \times 9$$
$$1089 \times 2 = 2178 \qquad 8712 = 1089 \times 8$$
$$1089 \times 3 = 3267 \qquad 7623 = 1089 \times 7$$
$$1089 \times 4 = 4356 \qquad 6534 = 1089 \times 6$$
$$1089 \times 5 = 5445$$

Or try this trick: write 1089 on a piece of paper and put it in your pocket. Now ask someone to think of a three-digit number, the first and last digits of which differ by at least 2. Let us suppose he chose 517. Now ask him to reverse the digits and subtract the higher number from the lesser (715 — 517 = 198)

Finally, ask him to add this number to itself reversed (198+891=1089.) No matter what number he starts with, you will always have the final answer — 1089 — in your pocket!

Strange Addition

1								
2	6							
3	9	15						
4	12	20	28					
5	15	25	35	45				
6	18	30	42	54	66			
7	21	35	49	63	77	91		
8	24	40	56	72	88	104	120	
9	27	45	63	81	99	117	135	153

In the table above, the horizontal lines are arithmetical progressions. The difference between each number and the one to its right is twice the figure that stands at the beginning of the row. (Row 7, for example, can be worked out by adding 14, first to 7 and then to each successive total.)

But how would you find the sum of all the numbers in any row? There is no need to add them — it is the same as the cube of the number which stands at the beginning of the row. For instance, the total of the numbers in row 6 is $6 \times 6 \times 6 = 216$.

Number 37

Watch the pattern this number forms when multiplied by 3 and its multiples:

$$\begin{aligned}
37 \times 3 &= 111\\
37 \times 6 &= 222\\
37 \times 9 &= 333\\
37 \times 12 &= 444\\
37 \times 15 &= 555\\
37 \times 18 &= 666\\
37 \times 21 &= 777\\
37 \times 24 &= 888\\
37 \times 27 &= 999
\end{aligned}$$

Number 65359477124183

Watch the pattern formed when you multiply this number by 17 and multiples of 17:

$$
\begin{aligned}
65359477124183 \times 17 &= 1111111111111111\\
65359477124183 \times 34 &= 2222222222222222\\
65359477124183 \times 51 &= 3333333333333333\\
65359477124183 \times 68 &= 4444444444444444\\
65359477124183 \times 85 &= 5555555555555555\\
65359477124183 \times 102 &= 6666666666666666\\
65359477124183 \times 119 &= 7777777777777777\\
65359477124183 \times 136 &= 8888888888888888\\
65359477124183 \times 153 &= 9999999999999999
\end{aligned}
$$

Reversals

$9 + 9 = 18$	$81 = 9 \times 9$
$24 + 3 = 27$	$72 = 24 \times 3$
$47 + 2 = 49$	$94 = 47 \times 2$
$497 + 2 = 499$	$994 = 497 \times 2$

The Number 52631578947368421O

This deceptively straightforward number is also circular, in the sense that when it is multiplied by any number from 2 to 200, the product can always be read off from a circle made up of the figures of the multiplicand.

If the multiplier is 18 or less, the break comes after the digit which is the same as the multiplier or, where the multiplier has two figures, the same as the last figure of the multiplier. When the cut has been made and the two parts joined, add a zero to get the final answer.

As all the digits in the number, except 9 and 0, occur twice, you must know which of the pair to make the break after. The rule is this: look at the figures following the two you are choosing between. If the number you are multiplying by is between 2 and 9, make the break before the lower figure, if it lies between 11 and 18 make the break before the higher one. For example:

To multiply 526315789473684210 by 6.

The break must come between 6 and 3 or 6 and 8. The multiplier lies between 2 and 9, so we make the break between 6 and 3.

We write out the answer by taking the figures that follow 6 : 315789473684210, carrying on from the beginning of the number and joining on 526, and adding 0 to get the answer:

3157894736842105260

Or to multiply 526315789473684210 by 14:

The break can be made between 4 and 7 or 4 and 2. 14 lies between 11 and 18, so we choose the higher number, and the answer can be read straight off:

7368421052631578940

To multiply by numbers between 19 and 200 is a little more complicated. If they are the numbers we have already dealt with multiplied by 10—that is 20, 30, 40 and so on upto 180—there is no problem, for we can carry out the procedure set out above and add 0 to the product. Similarly, to multiply by 200, carry on the operatión as if multiplying by 2 and add two zeros to the answer.

19, 38, 57, 76 and 95 give products in which all the digits are nines except the last two and (in all cases except 19) the first:

526315789473684210
× 19

9999999999999999990

526315789473684210
× 38

19999999999999999980

526315789473684210
× 57

29999999999999999970

526315789473684210
× 76

39999999999999999960

526315789473684210
× 95

49999999999999999950

These five numbers are special cases. The rule for numbers between 21 and 29 is this: add 1 to the second digit of the multiplier. Multiply the special number by making the break before the lower of the two possible figures. When you reach the last digit, reduce it by 1. Insert 1 at the beginning and 0 at the end of the number to arrive at the final product. For example:

526315789473684210 × 27

Increase the second digit of the multiplier by 1: $7+1=8$. Now make a cut between 8 and 4 in our special number and write down the figures

421052631578947367

Annex a zero to this number and attach the 1 at the beginning *to get the product*

142105263157894736 70

To multiply the special number by numbers between 31 and 37, use the same method as that set out above, but make the break before the higher of the two possible figures. For example:

526315789473684210 × 34

Increase the second digit of the multiplier by 1 to make 5. Make the break between 5 and 7 to get the number 789473684210526315. Reduce the last digit by 1, put a 1 at the beginning and a 0 at the end to arrive at the final product:

17894736842105263140

For multipliers between 39 and 48, the method is the same as for numbers between 21 and 29, but instead of 1 at the beginning of the product, write 2, instead of deducting 1 from the last figure of the number, deduct 2, and instead of adding 1 to the second digit of the multiplier, add 2. For example:

526315789473684210 × 46

Add 2 to the second figure of the multiplier: 6 + 2 = 8. Make a cut at the lower succeeding figure. You get:

421052631578947368

Reduce the last figure by two units. You get:

421052631578947366

Annex a zero to this number and attach the number 2 at the beginning of the number to obtain the product:

24210526315789473660

When the product lies between 49 and 56, the method is the same as that for multipliers between 39 and 48, but the cut is made before the higher of the two possible figures.

For multipliers between 68 and 75, the method is as that for 58 to 67, but the break is made at the higher figure.

For multipliers between 77 and 85, insert 4 and not 3, and cut at the lower figure and for multipliers between 86 and 94, do the same but cut at the higher one. For multipliers from 96 to 104, insert 5 not 4, and cut at the lower figure. For other numbers, similar methods apply — you can find them by trial and error, right upto 200. You will find though that 114, 133, 152, 171 and 190 give products made up of a series of nines with two varying digits and a zero or zeros.

The Magical Number

76923

This is one of the most curious numbers in arithmetic. Look what happens when you multiply this number by 1, 10, 9, 12, 3 and 4

$$
\begin{aligned}
76923 \times 1 &= 076923 \\
76923 \times 10 &= 769230 \\
76923 \times 9 &= 692307 \\
76923 \times 12 &= 923076 \\
76923 \times 3 &= 230769 \\
76923 \times 4 &= 307692
\end{aligned}
$$

You get the same sequence of digits when read from top to bottom or from left to right.

That's not all. Multiply the same number by 2, 7, 5, 11, 6 and 8 and see what happens!

$$
\begin{aligned}
76923 \times 2 &= 153846 \\
76923 \times 7 &= 538461 \\
76923 \times 5 &= 384615 \\
76923 \times 11 &= 846153 \\
76923 \times 6 &= 461538 \\
76923 \times 8 &= 615384
\end{aligned}
$$

You get another sequence of digits that reads the same from top to bottom and from left to right.

It is noteworthy that in each of the above two tables, all the answers consist of the same digits arranged in different groupings and the sum of the digits in all the answers is 27.

What a strange peculiarity this number has!

Number 37037

This is another curious number. Look at the following table:

$$37037 \times 3 = 111111$$
$$37037 \times 6 = 222222$$
$$37037 \times 9 = 333333$$
$$37037 \times 12 = 444444$$
$$37037 \times 15 = 555555$$
$$37037 \times 18 = 666666$$
$$37037 \times 21 = 777777$$
$$37037 \times 24 = 888888$$
$$37037 \times 27 = 999999$$

A Curious Multiplication—Addition

Here is a series of multiplications—additions that bring out the nine digits in the natural and also in their inverted order:

$$(1 \times 8) + 1 = 9$$
$$(12 \times 8) + 2 = 98$$
$$(123 \times 8) + 3 = 987$$
$$(1234 \times 8) + 4 = 9876$$
$$(12345 \times 8) + 5 = 98765$$
$$(123456 \times 8) + 6 = 987654$$
$$(1234567 \times 8) + 7 = 9876543$$
$$(12345678 \times 8) + 8 = 98765432$$
$$(123456789 \times 8) + 9 = 987654321$$

Yet another:

$$
\begin{aligned}
(1 \times 9) + 2 &= 11 \\
(12 \times 9) + 3 &= 111 \\
(123 \times 9) + 4 &= 1111 \\
(1234 \times 9) + 5 &= 11111 \\
(12345 \times 9) + 6 &= 111111 \\
(123456 \times 9) + 7 &= 1111111 \\
(1234567 \times 9) + 8 &= 11111111 \\
(12345678 \times 9) + 9 &= 111111111 \\
(123456789 \times 9) + 9 &= 1111111111
\end{aligned}
$$

The Unique Number

Here is the smallest number, of which the alternate figures are zeros, and which is divisible by nine and also by 11.

90 90 90 90 90 90 90 90 90 90 9

Dividing it by 9, we get:

101010101010101010101

Dividing it by 11 gives:

82644628099173553719

This again is a curious number in that it contains various sequences of identical numbers in direct and reversed orders —8264 and 4628, 17355 and 55371, 1735 and 5371 and perhaps various others.

The Versatile Number 9

Here is one more interesting characteristic of number 9.

Multiply 9 by 21. We get the product 189.

Multiply 9 by 321. We get the product 2889. If you carry on successive multiplications in this order with inverted digits, one being added to the row each time, the successive multipliers will be 21, 321, 4321, and so on upto the full row of digits inverted, 987654321. Let us now arrange the successive multiplications in the pyramidal form:

$$
\begin{aligned}
21 \times 9 &= 189 \\
321 \times 9 &= 2889 \\
4321 \times 9 &= 38889 \\
54321 \times 9 &= 488889 \\
654321 \times 9 &= 5888889 \\
7654321 \times 9 &= 68888889 \\
87654321 \times 9 &= 788888889 \\
987654321 \times 9 &= 8888888889 \\
10987654321 \times 9 &= 98888888889
\end{aligned}
$$

7

GAMES TO PLAY WITH NUMBERS

To Find the Age of a Person

(1) Ask your friend to multiply his age by 3 and add six to the product, then ask him to divide the last number by three and tell you the result. Subtract two from that result and give him the number. That will be his age.

(2) You can also tell the age of a person who is under 21, by the help of the following table:

A	B	C	D	E
1	2	4	8	16
3	3	5	9	17
5	6	6	10	18
7	7	7	11	19
9	10	12	12	20
11	11	13	13	
13	14	14	14	
15	15	15	15	
17	18	20		
19	19			

Ask the person to tell you in which columns his age occurs and then add together the numbers at the top of those columns and the sum will be his age.

For example, if he says that his age is found in columns C and E, then $4 + 16 = 20$ is his age.

(3) *A Nice Party Trick*

If you wish to impress your friends with a trick of predicting a number, write out the number 37 on a piece of paper and keep it in your pocket. Ask one of your friends to call out any number formed of three identical figures. Ask him to add up these three figures and divide the number by the answer. Then you take out the piece of paper from your pocket on which is written number 37! Watch the reaction on everybody's face! They'll be amazed.

You see, whatever might be the number of the three identical figures, the result is always the same

555 divided by 5+5+5 gives 37.
333 divided by 3+3+3 gives 37.
888 divided by 8+8+8 gives 37.

(4) *Another Party Trick*

To play this trick, you need at least four other persons besides you in the party.

Ask your neighbour to write down a three-digit number—any three-digit number, without reservations. And ask him to write the same number alongside, which gives a six-digit number.

Then ask him to pass the slip to his neighbour, whom you will ask to divide the six-digit number by seven.

Don't worry about the number not dividing exactly by seven—it will.

Now ask him to pass the result to his neighbour, whom you will ask to divide the number by 11.

Again, don't worry about the number not dividing by 11 — it will.

Ask the result to be passed again to his neighbour, whom you ask to divide by 13.

It will divide by 13—don't worry.

Now ask that person to give you back the slip, the paper well folded so that you don't see the result, and pass that slip on, without unfolding it to your neighbour who first wrote down the number, at the start.
It will be the same he wrote on the slip of paper to begin with!
Surprised?
Here's the secret.
Let's assume that the first number that was written was the number 498.
A similar number written alongside it would make it 498498.
In actual fact what has happened to the number is that it has been multiplied by 1000 and then the original number has been added to it.

$$498 \times 1000 + 498 = 498498$$

which is the same as 498×10001

which equals 498498

And

$$1001 = 7 \times 11 \times 13$$

Therefore what has actually been done is, the number was first multiplied by 1001 and then gradually, hand to hand divided by 1001!

(5) Ask your friend to write a multi-digit number — but with one condition—the number should not end with a zero.
Ask him to add up the digits and subtract the total from the original number. Then ask him to cross out any one of the digits and tell you the remaining numbers. You don't need to know the original number nor what he had done with it.
You can 'pop' tell him the exact number he had crossed out.
How?

Here's how you do it.

Very simple! All you have to do is to find the digit which added to the numbers he gave you will form the nearest number divisible by 9. And that will be the missing number.

For example, if he had in mind the number 9241, he adds up the four digits and subtracts the total from the original number. He gets $9241 - 16 = 9225$. Supposing he crosses out the 5 and gives you 9, 2 and 2, adding 9, 2 and 2 you get 13, and the nearest number to 13 which should not be less than 13 and is divisible by 9 is 18. Therefore, you give the missing number as $18 - 13 = 5$.

(6) *Guessing the Birthday*

Would you like to surprise your friend by guessing his birthday? You can tell your friend the month and the date of his birth very easily.

Here is how to do it.

First of all, ask your friend to keep in mind two numbers, the number of the month in which he was born and the number of the date of the month. The months, of course, are numbered from 1 to 12, beginning from January to December. Then you ask him to multiply the number of the month by 5, add to this 6 and multiply it by 4 and again add 9. Once again ask him to multiply the number by 5 and add to it the number of the date on which he was born.

When he finishes the calculations, ask him for the final result. Then mentally you subtract 165 from the final result. After the subtraction, the remaining number gives the answer.

The last two digits of the number give you the date of the month and the first digit or the first two digits give you the number of the month.

For example, if your friend's final result is 1269, when you subtract 165, you get 1104. From this number you know that he was born on November 4th.

I am sure you would like to know the trick behind it.

The directions you give to your friend are a disguised way of adding 165 to the number of the month, multiplied by 100. When the number 165 is taken away from the total, the number of the day and one hundred times the number of the month are left.

(7) Ask your friends to prove that 7 is equal to one-half of 12. They will naturally give up. Then write twelve in Roman numerals, and then draw a horizontal line through the middle, thus:

VII

ΛII

Thus you can see that seven is one-half of twelve, the upper half being VII.

(8) Ask your friends to add five and six in such a way that the sum will be nine. They will give up. Then make the following six strokes 111111. And to them add the following five strokes thus:

I I I I I I NINE

(9) Ask your friends to put down a three-digit number unknown to you. Tell them to reverse the digits and to subtract the smaller number from the greater. Then ask them to give the first digit of the result, whereupon you will be able to give the entire answer, by subtracting the first digit given to you from nine, which will form the last digit and, of course, the middle digit will always be nine.

For example, if your friends have in mind the number 843, they will reverse it and get 348. From 843 they will take out 348, give you 4 as the first digit of the answer. Now that you got the first digit, all that you have to do is subtract 4 from 9 and get 5 as the last digit. And, of course, the middle digit will always be nine. Therefore you give the answer as 495.

This applies only when the digits in the units and hundreds places are not equal.

8

PUZZLES TO PUZZLE YOU

Here are a few puzzles to tickle your imagination and to excite your interest in further arithmetical inquiries. These puzzles are not riddles made to deceive you or just puzzles without purpose made only to tease, but they are straightforward exercises in reason that could give you many hours of fun and pleasure.

(1) *The Gong*

Supposing a clock takes 7 seconds to strike 7, how long will it take for the same clock to strike 10?

(2) *A Problem of Candy*

If 6 men can pack 6 packets of candy in 6 minutes, how many men are required to pack 60 packets in 60 minutes?

(3) *A Question of Distance*

It was a beautiful sunny morning. The air was fresh and a mild wind was blowing against my windscreen. I was driving from Bangalore to Brindavan gardens. It took me 1 hour and 30 minutes to complete the journey.
After lunch, I returned to Bangalore. I drove for 90 minutes. How do you explain it?

(4) Can you write 23 with only 2's, 45 only with 4's and 1000 with only 9's?

(5) Do you notice anything interesting in the following multiplication?

$$138 \times 42 = 5796$$

You will note that there are nine digits in the multiplication and all the nine digits are different. Can you think up other similar numbers?

(6) *Three Fat Men*

In my neighbourhood lives a very fat man who weighs 200 kilos. He has two very fat sons who weigh 100 kilos each. On a festival day, they decided to go across the river on a boat to visit some relations. But the boat could carry a maximum load of only 200 kilos.
Can you tell how they managed to get across the river by boat?

(7) *A Problem of Gooseberries*

When I was a little girl, one day my mother had left a bowl of gooseberries to be shared between my two sisters Lalitha, Vasantha and myself. I went home first. I ate what I thought was my share of gooseberries and left. Then Lalitha arrived. She thought she was the first one to arrive and ate the number of gooseberries she thought was her share and left. Lastly, Vasantha arrived. She again thought she was the first one to arrive and she took what she thought was her share and left 8 gooseberries in the bowl.
When we three sisters met in the evening, we realised what had happened and my mother distributed the remaining 8 gooseberries in a fair share.
How did my mother do it?

(8) Can you write 1, by using all the ten digits?

(9) *Mystery of the Missing Paisa*

Two women were selling marbles in the market place —one at three a paisa and the other at two a paisa. One day they both were obliged to return home when each had thirty marbles unsold. They put together the two lots of marbles and handing them over to a friend asked her to sell them at five for 2 paise. According to their calculation, after all, 3 for one paisa and two for one paisa was exactly the same as 5 for 2 paise.
But when the takings were handed over to them, they were both most surprised, because the entire lot together had fetched only 24 paise! If, however, they had sold their marbles separately, they would have fetched 25 paise.
Now where did the one paisa go?
Can you explain the mystery?

(10) *The Pen and The Pencil*

I bought a pen and a pencil for Rs 1.10 p. If the pen costs 1 rupee more than the pencil, how much does the pencil cost?

(11) Can you write 1000 by using eight 8's?
There are many possibilities.

(12) Here is an epitaph of the celebrated Greek mathematician of 250 AD, Diophantus. Can you calculate his age from this?
Diophantus passed one-sixth of his life in childhood, one-twelfth in youth, and one-seventh more as a bachelor; five years after his marriage, a son was born who died four years before his father at half his final age. How old is Diophantus?

(13) I have a little friend named Rajiv. He bought a used cricket ball for 60 paise only. Having bought it, he somehow did not find it to his liking and so when a friend of his offered him 70 paise, he sold it to him.

However, having sold it, Rajiv felt bad and decided to buy it back from his friend by offering him 80 paise. Having bought it, once again Rajiv felt that he did not really like the ball, and so he sold it again for 90 paise only.
Did Rájiv make at all any profit in this transaction? If so, how much?

(14) Can you write 30 using any other 3 identical digits except 5's?

(15) Take the eight digits 1, 2, 3, 4, 5, 6, 7 and 8. Without the aid of any other digits and using the addition and subtraction signs only, can you make 100?

(16) At this moment, it is 9 p.m. Can you tell me what time it will be 23999999992 hours later?

(17) *On the Way to Market*

One morning, I was on my way to the market and met a man who had 5 wives. Each of the wives had five bags containing five dogs and each dog had 5 puppies.
Taking all things into consideration, how many were going to the market?

(18) *The Multiplying Bacteria*

Bacteria are known to multiply very rapidly. A certain container contains just one bacteria on the first day, and there are twice as many on the next day. In this manner, the number of bacteria in the container double themselves every day.
Assuming that the container would be full of bacteria on the 10th day, on which day would the container be half full?

(19) *Gold for All Occasions*

Which is worth more, a bucketful of half-sovereign gold pieces or an identical bucketful of 1-sovereign gold pieces?

SOLUTIONS

(1) While striking 7, the clock strikes its first gong at 7 o'clock and it strikes 6 more at regular intervals. Those 6 intervals take seven seconds, so that the interval between gongs is 7/6 seconds. However, to strike 10, there are 9 intervals, each taking 7/6 seconds for a total of $7/6 \times 9$ or $10\frac{1}{2}$ seconds.

(2) 6 men pack 6 packets in 6 minutes
6 men pack 1 packet in 1 minute
6 men pack 60 packets in 60 minutes.

(3) There is nothing to explain here. The driving time there and back is absolutely the same because 90 minutes and 1 hour and 30 minutes are one and the same thing. This problem is for inattentive readers who may think that there is some difference between 90 minutes and 1 hour and 30 minutes.

(4) $22 + \frac{2}{2}$, $44 + \frac{4}{4}$, $999 + \frac{9}{9}$

(5) Here is a group of 9 such numbers:

$$12 \times 483 = 5796$$
$$18 \times 297 = 5346$$
$$39 \times 186 = 7254$$
$$48 \times 159 = 7632$$
$$27 \times 198 = 5346$$
$$28 \times 157 = 4396$$
$$4 \times 1738 = 6952$$

(6) First the two sons rowed across the river and one stayed behind, while the other returned in the boat to his father. The son remained behind, while the father crossed the river. Then the other son brought back the boat and the two brothers rowed over together.

(7) My mother gave Lalitha 3 and Vasantha 5. I ate my share of gooseberries which was $\frac{1}{3}$. Therefore, there

were $\frac{2}{3}$ of the gooseberries left in the bowl. Lalitha took $\frac{1}{3}$ of these or $\frac{1}{3}$ of $\frac{2}{3}$ or $\frac{2}{9}$ of them. So when Vasantha arrived, already
$\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$ of the original gooseberries had been eaten.
Therefore only $\frac{4}{9}$ of the original number of gooseberries remained from which Vasantha proceeded to eat her share.
Therefore only Vasantha ate $\frac{1}{3}$ of $\frac{4}{9}$ and there remained $\frac{2}{3}$ of $\frac{4}{9} = \frac{8}{27}$.
But in the evening we saw that eight gooseberries remained in the bowl.
Therefore 8/27 of the original number=8
So there were 27 gooseberries in the bowl when I first took my share of 9.
I was the only one to have had my fair share of the gooseberries.
Lalitha took what she thought was her share from the remaining 18 gooseberries, namely, 6. And from the remaining 12, Vasantha had taken 4 gooseberries, thinking that to be her share.
Now after Lalitha got her 3 and Vasantha her 5 gooseberries, we all had eaten our even share of 9 gooseberries each.

(8) $$\frac{148}{296} + \frac{35}{70} = 1$$

(9) There isn't really any mystery, because the explanation is simple. The two ways of selling are identical when the number of marbles sold at three for a paisa and two for a paisa is in the proportion of two to three.
When the first woman handed over 36 marbles and the second woman 24, they would have fetched 24 paise, immaterial whether sold separately or at five for 2 paise.
But if they each held the same number of marbles, there would be a loss when sold together of 1 paisa in every 60 marbles.
So if they had 60 each, there would be a loss of 2 paise and if there were 90 each (180 altogether), they would lose 3 paise and so on.

In case of 60, the missing one paisa arises from the fact that the 3 a paisa woman gains 2 paise and the 2 a paisa woman loses 3 paise.
The first woman receives $9\frac{1}{2}$ paise and the second woman $14\frac{1}{2}$, so that each loses $\frac{1}{2}$ paisa in the transaction.

(10) If the pen costs Re 1 and the pencil 10 paise, the difference would be 90 paise and therefore the pen must cost more and the pencil less. A little thought indicates that the pen costs Rs 1.05 paise and the pencil 5 paise. So the difference is Re 1.00.

(11) Here are four of them

$$888 + 88 + 8 + 8 + 8 = 1000$$

$$\frac{8+8}{8} (8 \times 8 \times 8 - 8) - 8 = 1000$$

$$\frac{8888 - 888}{8} = 1000$$

$$8 (8 \times 8 + 8 \times 8) - 8 - 8 - 8 = 1000$$

(12) Let us assume a is Diophantus' age.

$$\frac{a}{6} + \frac{a}{12} + \frac{a}{7} + 5 + \frac{a}{2} + 4 = a$$

Diophantus lived to be 84 years old.

(13) Rajiv made altogether 20 paise in the transaction. He made 10 paise when he sold the ball for the first time and another 10 paise when he sold it for the second time.

(14) Here are three solutions:

$6 \times 6 - 6 = 30$; $\quad 3^3 + 3 = 30$

$33 - 3 = 30$

(15) $86 + 2 + 4 + 5 + 7 - 1 - 3 = 100$

(16) 24 billion hours later it would be 9 p.m. And 8 hours before that it would be 1 p.m.

(17) Just myself. Only I was going to the market and I met all the others coming from the opposite direction.

(18) The container would be half full on the 9th day. Since the number of bacteria doubles each day, the container should be half full on the day before it became full.

(19) The bucketful of half-sovereign gold pieces are worth more since the denominations of the gold pieces make no difference. What is most important is, the bucket containing half-sovereign gold pieces is full of gold, whereas the other is only half full.

SHAKUNTALA DEVI lives in Bombay in India, but spends much of her life travelling round the world giving demonstrations of her talents. Incredibly, in addition to travelling Miss Devi has found time to write novels, cookery books and an account of Hindu astrology. The photograph shows her concentrating on working out the twenty-third root of the number on the blackboard, which contains 201 digits — it took her fifty seconds. The problem was set by students at the Southern Methodist University in Dallas, Texas; but in order to check her answer they had to consult a Univac 1108 computer at the National Bureau of Standards in Washington DC. It took the computer a full minute to confirm that she was right — but it had to be given over 13,000 instructions before it started on the problem.

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