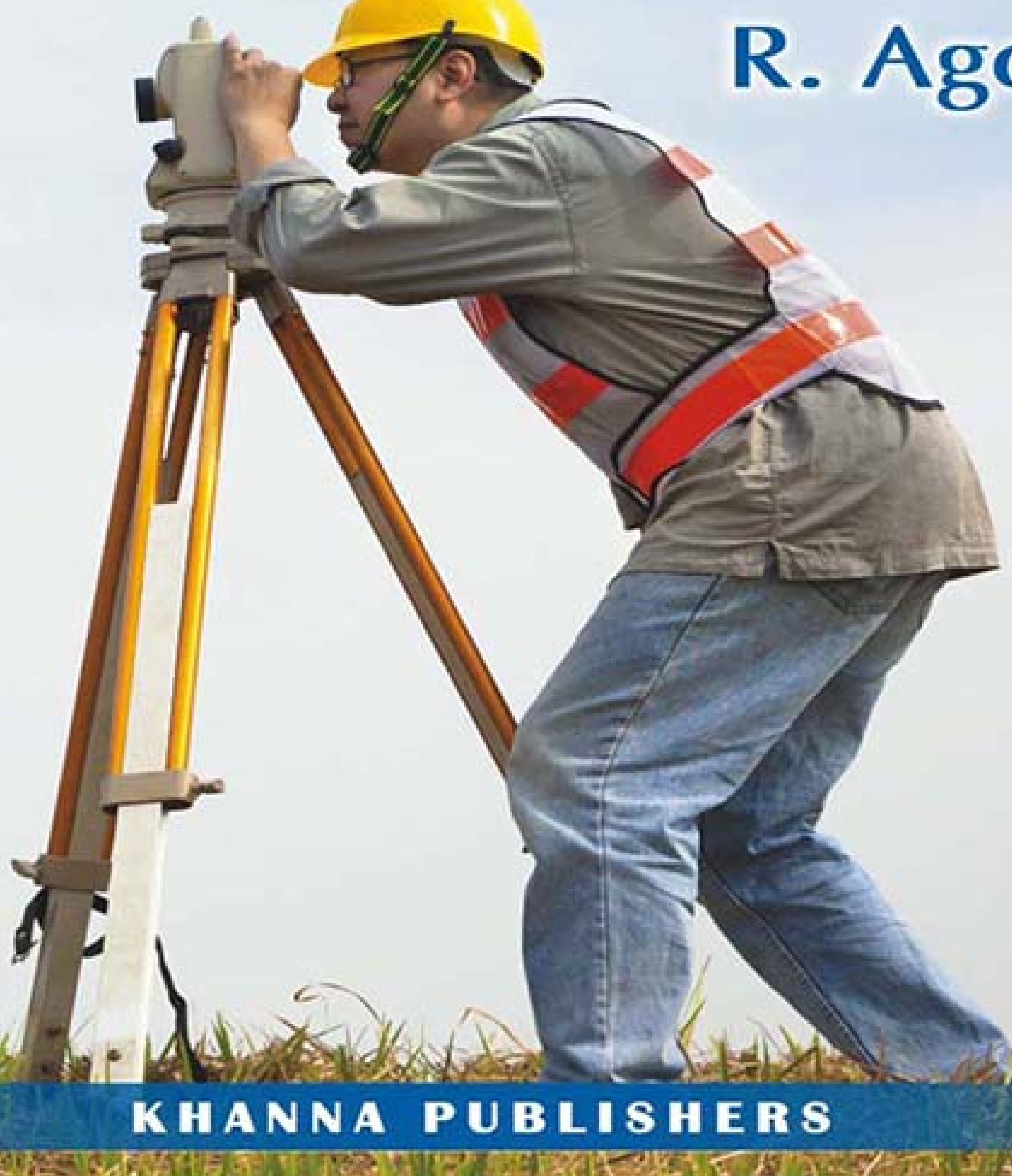
A Textbook of SURVEYING and LEVELLING

R. Agor



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A TEXT BOOK OF
SURVEYING AND LEVELLING

[*For Degree, Diploma Students; Practicing Engineers and Surveyors*]

By

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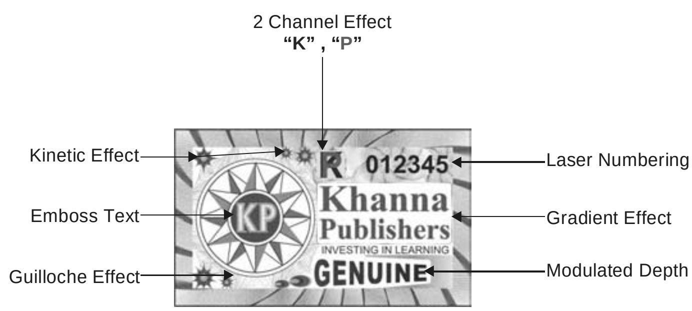

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PREFACE TO THE ELEVENTH EDITION

This edition of the treatise is thoroughly revised, edited and recomposed. Latest questions of various competitive examinations and other universities have been added to respective chapters. Besides this new chapters on **Remote Sensing System** and **Geographical Information System** have also been added at the end, to meet the requirements of students. Besides, this many Objective Type and Multi Choice Questions (M.C.Q.) have been added throughout the text.

It is hoped that the book will be further useful to B.Sc. Engg., Degree, Diploma students of various Indian Universities and Board of Technical Education as well as it will serve as a reference for field engineers and surveyors.

The author is thankful to the readers who had sent their valuable suggestions/comments for the improvement of this edition.

The author shall be grateful if any short comings in the text or contents are brought to his notice. Any suggestions/criticism for the improvement of the edition will be gratefully acknowledged.

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PREFACE TO THE FIRST EDITION

The author has made an attempt to write this textbook after twenty-five years experience in the field of 'Surveying and Teaching'. This book is primarily written for the students of A.M.I.E. (India), Degree and Diploma classes, but it will also be useful as a reference book for practicing engineers and surveyors.

While writing this book, the requirements of all the students regarding the latest trend of examinations, have been kept in view. The subject matter has been divided into sixteen chapters which are systematically arranged and discussed in detail with elaborate use of illustrations. Important questions from examination papers of A.M.I.E. (India), Universities and State Technical Boards, are given as solved examples in a logical sequence. At the end of each chapter, large number of objective type questions, essay type questions and numerical problems, have been added for the students to solve them independently and then to compare their results with those given in the book.

An attempt has been made to explain the method of contouring with an Indian tangent clinometer and the method of tacheometric planetabling and a solution of three points problem by making tacheometric observations in the respective chapters.

Though, every effort has been made to avoid composing mistakes, a few of them might have occurred due to over-sight. The author will feel high obliged if such errors and omissions are brought to his notice. Suggestions and criticisms for the improvement of this textbook shall be gratefully acknowledged and incorporated in the revised edition.

The author expresses his sincere thanks to Shri R.N. Saxena, Principal, Third Boys, Polytechnic, Delhi for encouraging him to write this book.

The author expresses his thanks to M/s. Wild Heerbrugg Ltd., M/s. Tama Sokki, Co. Ltd, and the National Instrument Factory Ltd., for providing photographs of surveying instruments for illustrations in the book.

The author also expresses his sincere gratitude to the Surveyor General of India, under whose guidance, the author took comprehensive training in different branches of surveying and attained practical experience of supervision and execution of various projects in different climatic conditions all over India.

In the end, the author expresses his sincere thanks to Sh. R. C. Khanna for rendering his valuable suggestions.

November, 1979 — R. Agor

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1

Introduction

1.1. DEFINITION

Surveying is the art of determining the relative positions of distinctive features on the surface of the earth or beneath the surface of the earth, by means of measurements of distances, directions and elevations. The branch of surveying which deals with the measurements of relative heights of different points on the surface of the earth, is known as *levelling*.

1.2. OBJECT OF SURVEYING

The objet of surveying is the preparation of plans and maps of the areas. The science of surveying has been developing since the very initial stage of human civilization according to the requirements. The art of surveying and preparation of maps has been practised from the ancient times. As soon as the man developed the sense of land property, he evolved methods for demarcating its boundaries. Hence, the earliest surveys were performed only for the purpose of recording the boundaries of plots of land. Due to advancement in technology, the science of surveying has also attained its due importance. The practical importance of surveying cannot be over-estimated. In the absence of accurate maps, it is impossible to lay out the alignments of roads, railways, canals, tunnels, transmission power lines, and microwave or television relaying towers. Detailed maps of the sites of engineering projects are necessary for the precision establishment of sophisticated instruments. Surveying is the first step for the execution of any project. As the success of any engineering project is based upon the accurate and complete survey work, an engineer must, therefore, be thoroughly familiar with the principles and different methods of surveying and mapping. It is for this reason, the subject of surveying has been made compulsory to all the disciplines of engineering at diploma and degree courses.

1.3. PRIMARY DIVISIONS OF SURVEYING

The surveying may primarily be divided into two divisions:

1. Plane surveying, 2. Geodetic surveying

1. Plane Surveying. The surveys in which earth surface is assumed as a plane and the curvature of the earth is ignored, are known as *Plane surveys.* As the plane survey extends only over small areas, the lines connecting any two points on the surface of the earth, are treated as straight lines and the angles between these lines are taken as plane angles. Hence, in dealing with plane surveys, plane geometry and trigonometry are only required. Surveys covering an area up to 260 sq. km may be treated as plane surveys because the difference in length between the arc and its subtended chord on the earth surface for a distance of 18.2 km, is only 0.1 m.

Scope and Use of Plane Surveying. Plane surveys which generally cover areas up to 260 sq. km, are carried out for engineering projects on sufficiently large scale to determine relative positions of individual features of the earth surface.

Plane surveys are used for the lay-out of highways, railways, canals, fixing boundary pillars, construction of bridges, factories etc. The scope and use of plane surveys is very wide. For majority of engineering projects, plane surveying is the first step to execute them. For proper, economical and accurate planning of projects, plane surveys are basically needed and their practical significance cannot be over-estimated.

2. Goodetic Surveying. The surveys in which curvature of the earth is taken into account and higher degree of accuracy in linear as well as in angular observations is achieved, are known as *'Geodetic Surveying'.* In geodetic surveying, curvature of the earth's surface is taken into account while making measurements on the earth's surface. As the surveys extend over large areas, lines connecting any two points on the surface of the earth, are treated as arcs. For calculating their projected plan distances for the plotting on the maps, the curvature correction is applied to the measured distances. The angles between the curved lines are treated as spherical angles. A knowledge of spherical trigonometry is necessary for making measurements for the geodetic surveys.

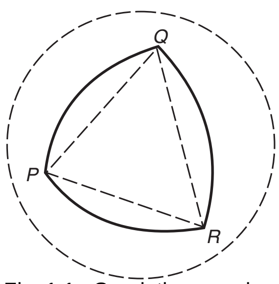


Fig. 1.1. Geodetic surveying.

Scope and use of Geodetic Surveying. Geodetic surveys are conducted with highest degree of accuracy to provide widely spaced control points on the earth's surface for subsequent plane surveys. Provision of such control points, is based on the principle of surveying from the whole to the part and not from the part to the whole. Geodetic surveys require the use of sophisticated instruments, accurate methods of observations and their computation with accurate adjustment. These surveys are generally carried out to provide plan control. To eliminate the errors in observations due to atmospheric refraction, angular observations are generally restricted to nights and arc lamps are used as signals on the survey stations.

Geodetic surveys are usually carried out by the department of National Surveys. In India, geodetic surveys are conducted by the department of the Survey of India under the direction of the Surveyor General of India.

1.4. CLASSIFICATION OF SURVEYS

According to the use and the purpose of the final maps, surveys may be classified, under the following different heads:

1.4.1. Classification based upon the nature of the field

Land Surveys. These include the following:

(*i*) **Topographic surveys.** The surveys which are carried out to depict the topography of the mountaineous terrain, rivers, water bodies, wooded areas and other cultural details such as roads, railways, townships etc., are called *topographical surveys.*

(*ii*) **Cadastral surveys.** The surveys which are generally plotted to a larger scale than topographical surveys and arc carried out for fixing the property lines, calculation of area of landed properties and preparation of revenue maps of states, are called *cadastral survey.* These are also sometimes used for surveying the boundaries of municipalities, corporations and cantonments.

(*iii*) **City surveys.** The surveys which are carried out for the construction of roads, parks, water supply system, sewer and other constructional work for any developing township, are called *City surveys.* The city maps which are prepared for the tourists are known as *Guide Maps.* Guide maps for every important city of India, are available from the offices of the department of Tourism.

2. Hydrographic Surveys. The surveys which deal with the mapping of large water bodies for the purpose of navigation, construction of harbour works, prediction of tides and determination of mean sea-level, are called *Hydrographic surveys.* Hydrographic surveys consist of preparation of topographical maps of the shores and banks, by taking soundings and determining the depth of water at a number of places and ultimately surveying bathymetric contours under water.

3. Astronomical Surveys. The surveys which are carried out for determining the absolute locations *i.e.*, latitudes of different places on the earth surface and the direction of any line on the surface of the earth by making observations to heavenly bodies, *i.e.,* stars and sun, are called *astronomical surveys.* In nothern hemisphere, when night observations are preferred to, observations are usually made to the Polaris, *i.e.,* the pole star.

1.4.2. Classification based on the purpose of the survey

1. Engineering Surveys. The surveys which are carried out for determination of quantities or to afford sufficient data for designing engineering works, such as roads, reservoirs, sewage disposal, water supply, etc., are called *Engineering Surveys.*

2. Military or Defence Surveys. The surveys which are carried out for preparation of maps of the areas of Military importance, are called *military surveys.*

3. Mine Surveys. The surveys which are carried out for exploration of mineral wealths beneath the surface of the ground, *i.e.,* coal, copper, gold, iron ores etc., are called *Mine surveys.*

4. Geological Surveys. The surveys which are carried out to ascertain the composition of the earth crust *i.e.*, different stratas of rocks of the earth crust, are called *Geological surveys.*

5. Archaelogical Surveys. The surveys which are carried out to prepare maps of ancient culture *i.e.*, antiquities, are called *Archaelogical surveys.*

1.4.3. Classification based on instruments used

According to the instruments used and method of surveying, the surveys may also be classified as under :

1. Chain surveying
2. Compass surveying
3. Plane table surveying
4. Theodolite surveying
5. Tacheometric surveying
6. Triangulation surveying
7. Aerial surveying
8. Photogrammetric surveying

1.5. GEOGRAPHICAL SURVEY

The following technical terms are generally used in surveying:

1. Plan. A plan is the graphical representation of the features on the earth surface or below the earth surface as projected on a horizontal plane. This may not necessarily show its geographical position on the globe. On a plan, horizontal distances and directions are generally shown.

2. Map. The representation of the earth surface on a small scale, is called a *map*. The map must show its geographical position on the globe. On a map the topography of the terrain, is depicted generally by contours, hachures and spot levels.

3. Topographical map. The maps which are on sufficiently large scale to enable the individual features shown on the map to be identified on the ground by their shapes and positions, are called *topographical maps.*

4. Geographical maps. The maps which are on such a small scale that the features shown on the map are suitably generalized and the map gives a picture of the country as a whole and not a strict representation of its individual features, are called *Geographical maps.*

1.6. PRINCIPLE OF SURVEYING

The fundamental principles upon which different methods of surveying are based, are very simple. These are stated as under:

1. Working from the whole to the part. The main principle of surveying whether plane or geodetic is to work from the whole to the part. To achieve this in actual practice, a sufficient number of primary control points, are established with higher precision in and around the area to be detail-surveyed. Minor control points in between the primary control points, are then established with less precise method. Further details are surveyed with the help of these minor control points by adopting any one of the survey methods. The main idea of working from the whole to the part is to prevent accumulation of errors and to localise minor errors within the frame work of the control points. On the other hand, if survey is carried out from the part to the whole, the errors would expand to greater magnitudes and the scale of the survey will be distorted beyond control.

In general practice the area is divided into a number of large triangles and the positions of their vertices are surveyed with greater accuracy, using sophisticated instruments. These triangles are further divided into smaller triangles and their vertices are surveyed with lesser accuracy.

2. Location of a point by measurement from two control points. The control points are selected in the area and the distance between them, is measured accurately. The line is then plotted to a convenient scale on a drawing sheet. In case, the control points are co-ordinated, their locations may be plotted with the system of coordinates, *i.e.,* cartesian or spherical. The location of the required point may then be plotted by making two measurements from the given control points as explained below.

Let P and Q be two given control points. Any other point, say, R can be located with reference to these points, by any one of the following methods (Fig.1.2).

(*a*) *By measuring the distances PR and QR.* The distances PR and QR may be measured and the location of R may be plotted by drawing arcs to the same scale to which line PQ has been drawn [Fig.1.2 (*a*)].

(*b*) *By Dropping a perpendicular from R on PQ.* A perpendicular RT may be dropped on the line PQ. Distances PT, TQ and RT are measured and the location of R may be plotted by drawing the perpendicular RT to the same scale to which line PQ has been drawn [Fig. 1.2 (*b*)].

Principles (*a*) and (*b*) are generally used in the method of 'Chain surveying'.

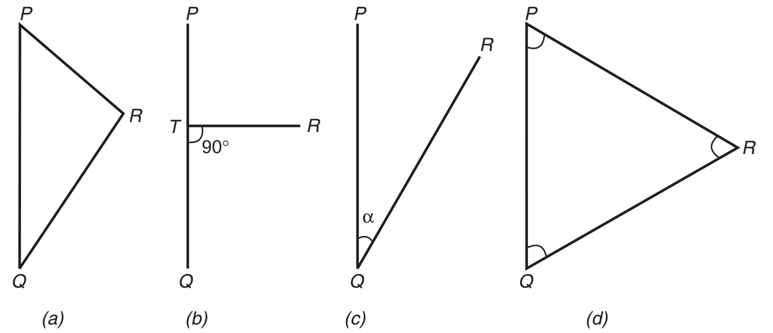


Fig.1.2. Location of a point.

(*c*) *By measuring the distance QR and the angle PQR.* The distance QR and the angle PQR equal to α are measured and the location of R may be plotted either by means of a protractor or trigonometrically [Fig. 1.2 (*c*)].

This principle is used in the method of '*Theodolite Traversing*'.

(*d*) *By measuring the interior anles of the triangle PQR*. The interior angles P, Q and R of the triangle PQR are measured with an angle-measuring instrument such as theodolites. The lenghts of the sides PR and QR are calculated by solving the triangle PQR and the coordinates of R are calculated in the same terms as those of P and Q. Even without calculating the coordinates, or sides the location of R can be obtained by plotting the angles PQR and QPR [Fig. 1.2 (*d*)].

This principle is used in the method of 'Triangulation'.

(*e*) *By measuring the sides of the triangle PQR.* The interior angles P, Q and R are calculated from the measured sides of the triangle PQR by applying cosine rule.

This principle is used in the method of Trilateration.

1.7. UNITS OF MEASUREMENTS

There are two kinds of measurements used in plane surveying;

1. Linear measure, *i.e.*, horizontal or vertical distances.

2. Angular measure, *i.e.,* horizontal or vertical angles.

1. Linear Measures. According to the standards of Weight and Measure Act (India) 1956, the metric system has been introduced in

India. The units of measurement of distances, have been recommended as metre and centimetre for the execution of surveys.

(*a*) *Basic units of length in metric system:*

10 millimetres= 1 centimetre

10 centimetres = 1 decimetre

10 decimetres = 1 metre

10 metres = 1 dekametre

10 dekametres = 1 hectametre

10 hectametres = 1 kilometre

1.852 kilometres = 1 nautical mile.

(*b*) Basic units of area in metric system:

100 sq. metres = 1 are

10 ares = 1 *deka–are*

10 deka ares = 1 hecta–are

(*c*) *Basic units of volume in metric system:*

1000 cub. milimetres = 1 cub. centimetre

1000 cub. centimetres = 1 cub. decimetre

1000 cub. decimetres = 1 cub. metre.

Before 1956, F.P.S. (Foot, pound, second) system was used for the measurement of lenghts, areas and volumes. These units which are known as British units, are:

(*a*) *Basic units of length in F.P.S. System:*

12 inches = 1 foot

3 feet = 1 *yard*

5.5. yards = 1 *rod*, *pole or* 1 *sq*. *perch*

4 poles = 1 *chain*(66 feet)

10 chains = 1 *furlong*

8 furlongs = 1 *mile*

6 feet = 1 *fathom*

120 fathoms = 1 *cable length*

6080 feet = 1 *nautical mile*

(*b*) *Basic units of area in F.P.S. System :*

144 sq. inch = 1sq. foot

9 sq. feet = 1sq. yard

30.25 sq. yard = 1sq. rod or pole

40 sq. rods = 1 rood

4 roods = 1 acre

640 acres = 1 sq. mile

484 sq. yards = 1 sq. chain

10 sq. chains = 1 acre.

(c) Basic units of volume in F.P.S. System :

1728 cu. inches = 1 cu. foot.

27 cu. feet = 1 cu. yard.

Conversion Factors for Lengths

(Metres, yards, feet and inches)

Metres	*Yards*	*Feet*	*Inches*
1	1.0936	3.2808	39.37
0.9144	1	3	36
0.3048	0.3333	1	12
0.0254	0.0278	0.0833	1

Conversion Factors for Areas

(Sq. metres, sq. yards, sq. feet and sq. inches)

Sq. metres	*Sq. yards*	*Sq. feet*	*Sq. inches*
1	1.196	10.7639	1550
0.8361	1	9	1296
0.0929	0.1111	1	144
0.00065	0.00077	0.0069	1

Conversion Factors for Areas

(Ares, acres and sq. yards)

Ares	*Acres*	*Sq. metres*	*Sq. yards*
1	0.0247	100	119.6
40.469	1	4046.9	4840
0.01	0.000247	1	1.196
0.0084	0.00021	0.8361	1

Conversion Factors for Volumes

(Cub. metres, cub. yards, gallons)

Cub. metres	*Cub. yards*	*Gallons (Imps)*
1	1.308	219.969
0.7645	1	168.178
0.00455	0.00595	1

2. **Angular Measures.** An angle may be defined as the difference in directions of two intersecting lines, or it is the inclination of two straight lines. The unit of a plane angle is 'radian'. Radian is defined as the measure of the angle between two radii of a circle which contain an arc equal to the radius of the circle [Fig. 1.3].

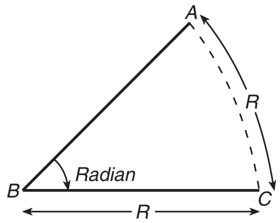


Fig. 1.3. A radian.

The popular systems of angular measurements, are:

(*i*) *Sexagesimal System of Angular Measurements*

In this system the circumference of a circle, is divided into 360 equal parts, each part is known as *one degree.* 1/60th part of a degree is called *a minute* and 1/60th part of a minute, is called a *second. i.e.*

1circumference = 360 degrees of arc

1° = 60 minutes of arc

1 minute = 60 seconds of arc.

(*ii*) *Centesimal System of Angular Measurements*

In this system, the circumference of a circle, is divided into 400 equal parts, each part is known as *one grad.* One hundredth part of a grad is known *as centigrad* and one hundredth part of a centigrad is known as *centi-centigrad i.e.,*

1 circumference = 400 grads

1 gard = 100 centigrads

1 centigrad = 100 centi–centigrads.

From the ancient times, sexagesimal system is widely used in different countries of the world. Most complete mathematical tables are available in this system and most of surveying instruments *i.e.,* theodolites, sextants etc., are graduated according to this system. However, due to facility in computation and interpolation, the centesimal system for angular measurements is gaining popularity in the western countries these days.

Conversion Factors from one System to other

Degrees	*Grads*	*Minutes*	*Centigrads*	*Seconds*	*Centi-centigrads*
1	1.1111	60	111.11	3600	11111
0.9	1	54	100	3240	10000
0.0167	0.01852	1	1.8518	60	185.18
0.0090	0.0100	0.5405	1	32.4	100
0.00027	0.0003	0.0167	0.0309	1	3.0864
0.00009	0.0001	0.0054	0.0100	0.324	1

1.8. MAP SCALES

Considering the actual surface dimensions, drawings are made to smaller scale of the area. It is never possible to make its drawing to full size. This operation is generally known as *'drawing to scale'*.

The scale of a map may be defined as the fixed proportion which every distance between the locations of the points on the map, bears to the corresponding distances between their positions on the ground. For an example, if 1 cm on a map represents a distance of 5 metres on the ground, the scale of the map is said to be 1 cm = 5 m. The scale of a map is also sometimes expressed by a fraction generally called, *'Representative Fraction'* (R.F.). Scales of the maps are represented by the following two methods:

(*i*) Numerical scales. (*ii*) Graphical scales.

1. Numerical scales. Numerical scales are further divided into two types, *i.e.*, (*a*) Engineer's scale (*b*) Fraction scale.

(*a*) **Engineer's scale.** The scale on which one cm on the plan represents some whole number of metres on the ground, is known as *Engineer's scale.* For example, 1 cm = 5 m ; 1 cm = 10 m, etc.

(*b*) **Fraction scale.** The scale on which an unit of length on the plan represents some number of the same unit of length on the ground is known as *Fraction Scale.* For example, 1 : 500; 1 : 1000; 1 : 5,000, etc.

To convert an engineer's scale into fraction scale, multiply the whole number of metres by 100. Similarly, a fraction scale may be converted into engineer's scale by dividing the denominator by 100 and equating the quotient to 1 cm.

Example 1.1. *The engineer's scale of a drawing, is stated to be 1 cm = 4 m. Convert this to fraction scale.*

Solution.

Engineer's scale is 1 cm = 4 m

$\therefore$ Fraction scale is 4×100 *or* 1 : 400. **Ans.**

Example 1.2. *The fraction scale of a map is stated to be 1 : 50,000. Convert this to Engineer's scale.*

Solution.

1 unit on plan = 50,000 units on the ground

$\therefore$ 1 cm on plan = 50,000 cm on the ground

or 1 cm on plan = 500 m on the ground

Hence, Engineer's scale is 1 cm = 500 m. **Ans.**

2. Graphical scales. A graphical scale is a line subdivided into plan distances corresponding to some convenient units of length on the surface of the earth.

1.9. NECESSITY OF DRAWING SCALES ON MAPS

When a map is used after a considerable time or in different climatic conditions, the dimensions of the paper usually get distorted. Due to distortion in the paper the numerical scales will not give accurate results. On the other hand, if a graphical scale is drawn on the map, there will also be a proportional distortion in the length of the scale and the distances from the distorted map will be accurately scaled off. This is why scales are always drawn on the maps and charts whcih are maintained for future reference.

1.10. REQUIREMENTS OF A USEFUL SCALE

A useful map scale should possess the following essential requirements.

1. It should be sufficiently long and should not be less than 18 cm and more than 32 cm.
2. Inter-divisions should be accurately done and correctly numbered.
3. The zero must always be placed between unit and its subdivisions.
4. The name of scale and its R.F. should always be written on the plan.
5. It should be easily readable without making any arithmetical calculations for measuring the distances on a map. The main divisions should, therefore, represent one, ten, hundred or thousand units.

1.11. CLASSIFICATION OF SCALES

The scales drawn on the maps or plans, may be classified as under :

(*i*) Plain scale
(*ii*) Diagonal scale
(*iii*) Scale of chords
(*iv*) Vernier scale.

1. **Plain Scales.** A plain scale is one on which it is possible to measure only two dimensions, *i.e.*, metres and decimetres; kilometres and hectametres; miles and furlongs, etc.

Plain Scales as Recommended by IS : 1491 – 1959

Full Size	1 : 1
50 cm to a metre	1 : 2
40 cm to a metre	1 : 2.5
20 cm to a metre	1 : 5
10 cm to a metre	1 : 10

5 cm to a metre	1 : 20
2 cm to a metre	1 : 50
1 cm to a metre	1 : 100
5 mm to a metre	1 : 200
2 mm to a metre	1 : 500
1 mm to a metre	1 : 1000
0.5 mm to a metre	1 : 200

Example 1.3. *Construct a plain scale whose R.F. is 1 : 50,000, to measure miles and furlongs.*

Solution.

$$\therefore \quad 50{,}000 \text{ yds} = 1 \text{ yd} = 36''$$

$$= 36 \times 2.54 \text{ cm}$$

$$\therefore \quad 1 \text{ mile or } 1760 \text{ yds} = \frac{36 \times 2.54}{50{,}000} \times 1760 = 3.219 \text{ cm.}$$

To have the length of the scale more than 18 cm, multiply 3.219 by 6.

∴ The length representing 6 miles

$$= 3.219 \times 6 = 19.314 \text{ cm.}$$

Take a length of 19.314 cm and divide it into 6 equal parts, each part representing one mile. Subdivide the left hand division into 8 equal parts, each part representing one furlong. Place zero of the scale between the undivided part and divided part and mark the readings on the scale as shown in Fig. 1.4.

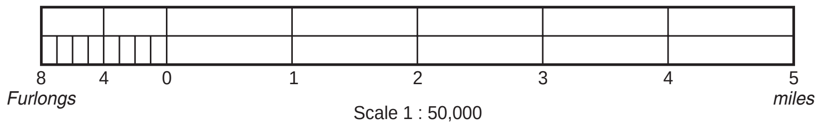


Fig. 1.4. Plain scale.

Example 1.4. *Construct a plain scale 1 cm = 250 m and show 3 kilometres and 7 hectametres thereon.*

Solution. (Fig. 1.5).

$$\therefore \quad 250\, m = 1 \text{ cm}$$

$$1000\, m = \frac{1}{250} \times 1000 = 4 \text{ cm}$$

Take a 24 cm length and divide it into 6 equal parts, each part representing 1 kilometre. Subdivide the left hand part into 10 divisions, each representing one hectametre. Place the zero of the scale between the sub divided part and undivided part and mark the scale as shown in Fig. 1.5.

To measure a distance of 3 kilometres and 7 hectametres, place one leg of the divider at 3 kilometres and the other at 7 hectametres, as shown in Fig.1.5.

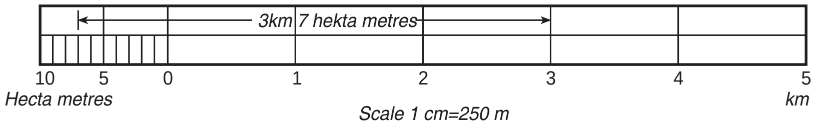


Fig. 1.5. Plain Scale.

2. Diagonal Scales. On a diagonal scale, it is possible to measure three dimensions such as kilometres, hectametres and decametres; or yards, feet and inches, etc.

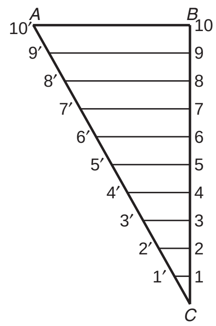


Fig. 1.6. Principle of diagonal scale

Principle of 'a Diagonal Scale.' The construction of a diagonal scale is based on the principle of similar triangles in which corresponding sides are proportional.

Take a line *BC*. Erect a perpendicular *BA* at *B*. Divide length *BC* into ten (or as required) equal parts. Draw lines parallel to *AB* from each point on *BC*, so that they cut the diagonal *AC* at points 1′, 2′, etc. In this way diagonal *AC* is also divided into 10 equal parts (Fig. 1.6).

It may be noted that the distance

$$1 - 1' = \frac{1}{10} \text{ th of } AB$$

$$2 - 2' = \frac{2}{10} \text{ th of } AB$$

....

....

$$\text{distance } 9 - 9' = \frac{9}{10} \text{th of } AB.$$

Example 1.5. *Construct a diagonal scale 1 cm = 5 metres to read metres and decimetres.*

Solution. (Fig. 1.7)

$\therefore$ 1 cm= 5 metres

$\therefore$ 22 cm = 5 × 22 = 110 m.

Construction. Take a length *AB* of 22 cm and divide it into 11 equal parts, each part represents 10 metres. Subdivide the left hand part into 10 equal parts, each part represents one metre. Draw ten lines equidistant and parallel to *AB*. Erect perpendiculars at *A, B* and other division

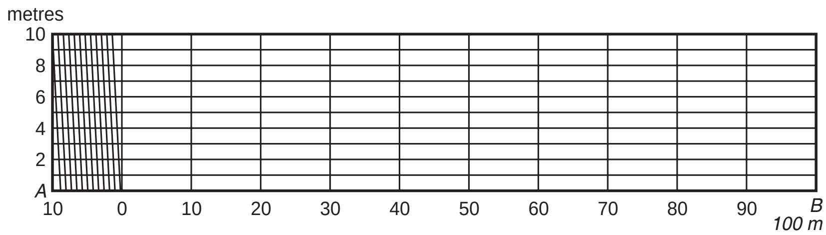


Fig. 1.7. Diagonal scale.

points. Subdivide the left-hand division of the topmost line, *i.e.*, the tenth line and join these diagonally to the corresponding subdivisions on the bottom line *AB*.

Example 1.6. *The area of a field is 45,000 sq.m. The length and breadth of a field on the map are 9 cm and 8 cm. Construct a diagonal scale which can be read up to one metre. Find out the representative fraction of the scale.*

Solution.

The area of the field on paper = 8 × 9= 72 sq. cm.

The area of the field on ground = 45,000 sq. m (given)

$$\therefore \quad 1 \text{ sq. cm} = \frac{45,000}{72} = 625 \text{ sq m}$$

or

$$1 \text{ cm} = \sqrt{625} = 25 \text{ m.}$$

∴ Representative fraction = 1:2500. **Ans.**

To read, up to one metre, we must have a length

$$= 1 \times 10 \times 10 = 100 \text{ metres}$$

Now 25 m = 1 cm

∴ 100 m = 4 cm.

Take a 24 cm length and divide it into 6 equal parts, each representng 100 metres. Divide the left hand division into 10 equal parts and finally draw 10 parallel lines. Construct the diagonal scale as shown in Fig. 1.8.

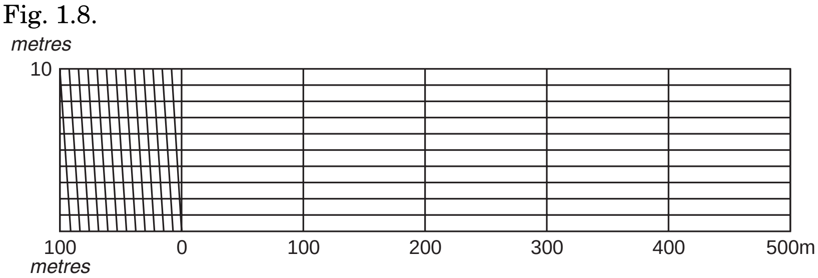


Fig.1.8. Diagonal scale.

Example 1.7. *An aeroplane at an altitude of 1000 metres covers a horizontal distance of 5 kilometres in one minute. Draw a scale of* $R.F. = \frac{1}{200{,}000}$ *to measure minutes and seconds. Show on the scale a distance the aeroplane covers in 4 minutes and 25 seconds.*

Solution.

$$\therefore \quad 200{,}000 \text{ metres} = 1 \text{ metre}$$

$$\therefore \quad 5{,}000 \text{ metres} = \frac{1 \times 5{,}000}{200{,}000} \times 100 \text{ cm}$$

$$= 2.5 \text{ cm.}$$

A 2.5 cm distance on the scale represents a distance covered in one minute.

$\therefore$ 25 cm distance on the scale represents

$$= \frac{1 \times 25}{2.5} = 10 \text{ minutes.}$$

Take a 25 cm length and divide it into 10 equal parts, each representing one minute. Divide the left hand division into six equal parts, each representing 10 seconds. Finally draw 10 parallel lines and construct the diagonal scale as shown in Fig. 1.9.

The distance covered in 4 minutes and 25 seconds is marked by *AB* in Fig. 1.9.

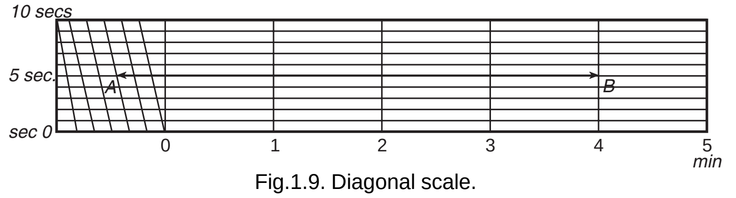


Fig.1.9. Diagonal scale.

3. Scale of Chords. A scale of chords is used to measure or to set off angles. It is marked either on a rectangular protractor or on an ordinary box wooden scale.

1. Construction of a Chord Scale. The following steps are followed:

1. Draw a quadrant *ABC*, making *AB* = *AC*. Prolong *BA* to *D*, making *BD* = *BC*.

2. Divide the arc *BC* into eighteen equal parts, each part representing 5°.

3. With *B* as a centre describe the arcs from each of the division, cutting *BAD* at points marked 5°, 10°, 15°, 20°, 25°, 30°,....

4. Subdivide these parts if required by first subdividing each division of the arc *BC*, and then drawing arcs with *B* as centre, as in step (3).

5. Complete the scale as shown in Fig. 1.10. It should be noted that the arc through the 60° division will always pass through the point *A* (since the chord of 60° is always equal to radius *BA*). The distance from *B* to any mark on the scale is equal to the chord of the angle of that mark. For example, the distance between *B* to 50° mark on the scale is equal to the chord of 50°.

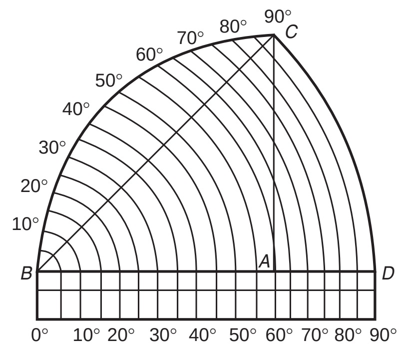


Fig. 1.10. A scale of chords.

2. Construction of Angles of 40° and 70° with a Scale of Chords (Fig. 1.11). The following steps are followed :

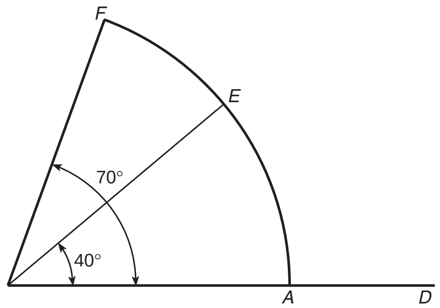


Fig. 1.11.

1. Draw a line *BD* and mark *BA* = chord of 40° from the scale of chords.
2. With *B* as centre and *BA* as radius, draw an arc.
3. With *A* as centre and radius equal to chord of 40° (*i.e.*, distance from 0° to 40° on to the scale of chords) draw an arc to cut the previous arc at *E*. Join *BE*. Then the angle *FBA* = 40°.
4. Similarly, with *B* as centre and radius equal to the chord of 70° (*i.e.* distance from 0° to 70° on the scale of chords), draw an arc to cut the previous arc at *F*. Join *BF*. Then, the angle *FBA* = 70°.

3. Measurement of an Angle EBD with the Scale of Chords (Fig. 1.12). The following steps are followed:

1. On BD, measure BA = chord of 60°.
2. With B as centre and BA as radius, draw an arc to cut line BE at F.
3. With the help of a pair of dividers, take the chord distance AF and measure it on the scale of chords to get the value of the angle θ, which is found to be 34°.

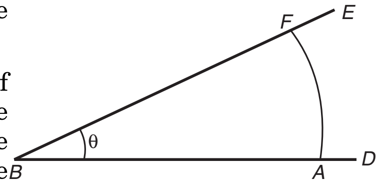


Fig. 1.12.

4. Vernier Scales. In 1631, Pierre Vernier invented a device for the purpose of measuring a fractional part of a graduated scale. It consists of two approximating scales, one of them is fixed and is called the *primary scale*, the other is movable and is called the *vernier.* The fineness of reading or the least count of a vernier is equal to the difference between the smallest division on the main scale and the smallest division on the vernier. The principle of vernier is based on the fact that the eye can perceive without strain and with considerable precision when two graduations coincide to form one continuous straight line. Depending upon the graduations of the main scale, the vernier may be called either single vernier or double vernier.

(*a*) **Single Vernier.** If the graduations of the main scale are numbered in one direction only, and the vernier extends also in one direction, is called a *single vernier*.

(*b*) **Double Vernier.** If the graduations of the main scale are numbered in both directions and the vernier also extends in both directions, having its index mark in the middle, then, the vernier is called a *double vernier.*

5. Classification of Verniers. Verniers are further classified primarily into two types :

(*a*) Direct verniers, (*b*) Retrograde verniers.

(*a*) **Direct Vernier. (Fig. 1.13).** The verniers which extend or increase in the same direction in which the graduations of their main scale increase and in which the smallest division is shorter than the smallest division of the main scales, are called as *Direct Verniers.* In such verniers, n divisions of the main scale equal in length to $(n + 1)$ divisions of the vernier scale.

If p = value of the smallest division of the primary scale

v = value of the smallest division of the vernier scale

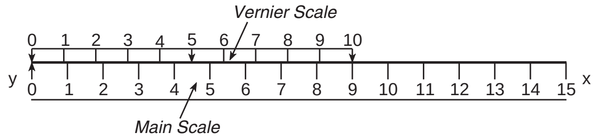


Fig. 1.13. A vernier scale.

n = number of divisions of the primary scale of a specified length.

$n + 1$ = number of divisions of the vernier scale of the same length.

As the length occupied by $(n + 1)$ divisions of the vernier scale is the same as occupied by n divisions of the primary scale.

$\therefore \quad (n + 1) \cdot v = n \cdot p$

or $$v = \frac{n \cdot p}{n+1}$$

$\therefore$ Difference between the value of a primary scale division and that of a vernier scale *i.e.*, least count.

Least count (L.C.) $= p - v$

$$= p - \frac{np}{n+1} = \frac{p}{n+1} \qquad \text{...(1.1)}$$

$$= \frac{\text{Value of one division of primary scale}}{\text{Number of divisions of vernier scale}}$$

Hence, the least count (L.C.) of a vernier can be obtained by dividing the value of one primary scale division by the total number of the divisions of the vernier scale.

Example 1.8. *Calculate the least count of a vernier whose 60 divisions coincide with 59 divisions of its primary scale and if each degree on the primary scale is subdivided into10′intervals.*

Solution.

Here

n = 59, the number of divisions of primary scale

$n + 1$ = 60, the number of divisions of vernier scale

p = 10′, the value of one division of primary scale

Substituting the values in eqn. (1.1) we get

$$\therefore \quad L.C. = \frac{p}{n+1} = \frac{10}{60} \text{ minutes}$$

or $= \mathbf{10''}$. **Ans.**

Example 1.9. *The primary scale of a box sextant is graduated to read 30 minutes. Construct a direct vernier to read to one minute and also show thereon a reading of 35° 14′.*

Solution.

We know that the least count of the vernier

$$\text{L.C.} = \frac{p}{n+1}$$

p = value of one division of primary scale = 30′

Required L.C. = 1 ′.

Substituting the values in eqn. (1.1) we get

$$p = 30'$$

$$\therefore \quad 1 = \frac{30}{n+1}$$

$$n + 1 = 30$$

or $$n = 29$$

∴ Length of vernier scale

$$= 29 \times 30 = 870' = 14° \, 30'$$

Take an arc of 14° 30′ and divide it into 30 equal parts.

The vernier scale is shown in Fig.1.14.

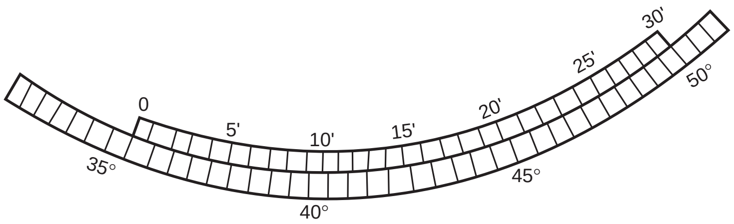


Fig.1.14.

Example 1.10. *Construct a direct vernier reading to 1 mm for a scale graduated to 5 mm.*

Solution. We know from eqn. (1.1) that

$$L.C. = \frac{p}{n+1}$$

where L.C. = least count of vernier

p = value of one division of primary scale

$n + 1$ = number of divisions of vernier scale

Substituting the values in eqn. (1.1) we get

Here L.C. = 1 mm

$$\therefore \quad 1 = \frac{5}{n+1}$$

or $\quad n = 4$

$\therefore$ Length of vernier scale

$= 4 \times 5 = 20$ mm.

Take a 20 mm length and divide it into 5 equal parts.

The vernier scale is shown in Fig. 1.15.

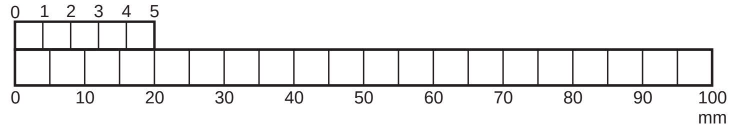


Fig 1.15. Vernier scale.

(*b*) **Retrograde Verniers.** The verniers which extend or increase in opposite direction of their main scales and also in which the smallest division of the vernier is longer than the smallest division of their main scales, are called *'retrograde verniers'*.

In such verniers, $(n + 1)$ divisions of the primary scale are equal, in length, to n divisions of the vernier scale.

Let p = value of smallest division of the primary scale

v = value of smallest division of the vernier scale

n = number of divisions of vernier scale

n + 1 = number of divisions of primary scale

Then, $n \,.\, v = (n + 1) \,.\, p$

$$\text{or} \quad v = \frac{(n+1) \,.\, p}{n}$$

$\therefore$ Least count $= v - p$

$$= \frac{(n+1) \,.\, p}{n} - p = \frac{p}{n}$$

$$\text{or} \quad \text{L.C.} = \frac{p}{n} \qquad \text{...(1.2)}$$

$$\text{or Least count} = \frac{\text{Value of one division of primary scale}}{\text{Total number of divisions of vernier scale}}$$

Hence, the least count of a retrograde vernier can be obtained by dividing the value of one primary scale division by the total number of divisions of the vernier scale.

A retrograde vernier in which 11 divisons of the main scale coincide with 10 divisions of the vernier scale, is illustrated in Fig. 1.16.

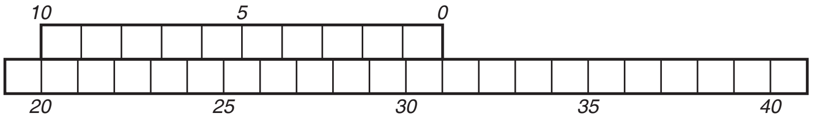


Fig. 1.16. A retrograde vernier.

Example 1.11. *Design a retrograde vernier for a theodolite circle divided into degrees and one-third degree to read up to 20".*

Solution.

From eqn. (1.2) We know that L.C. $= \frac{p}{n}$

where p = measure of one division of primary scale

n = number of divisions of vernier scale.

$$\text{L.C.} = 20'' = \frac{20}{60} \text{ minutes} \qquad \text{(Given)}$$

Substituting the values in eqn. (1.2) we get

$$\frac{20}{60} = \frac{20}{n}$$

or $n = 60$.

i.e., sixty-one divisions of the theodolite circle should be taken for the vernier scale and divided into 60 parts for a retrograde vernier.

6. Reading a Vernier Scale. The following steps are followed :

(*i*) Bring the zero of the vernier scale against a full division mark of the primary scale.

(*ii*) Ascertain the number of primary scale divisions equivalent in the length of the vernier scale.

(*iii*) Ascertain the value of the smallest division of the primary scale.

(*iv*) Calculate the least count of the direct vernier by the formula *i.e.*, L.C. $= \frac{p}{n+1}$ where p is the value of the smallest division of the primary scale and $(n+1)$ is the total number of divisions of the vrenier scale.

or

Calculate the least count of the retrograde vernier by the formula $L.C. = \frac{P}{n}$ where p is the value of the smallest division of the primary scale and n is the total number of divisions of vernier scale.

(*v*) Note the exact coincidence of the vernier division with the primary scale division.

(*vi*) Multiply the value of least count by the number of divisions of vernier scale coincident with the main scale division.

(*vii*) Note the reading of the main scale just before the vernier index.

(*viii*) Add the value obtained in step (*vi*) to the reading obtained in step (*vii*).

Example 1.12. *The least count of a theodolite vernier is 10″ and 53rd division of the vernier is coincident with a division of the graduated circle. If the vernier index is between 27° 30′* and 27° *40′*, find the reading of the theodolite.

Solution.

The reading of the main scale = 27° 30′

Least count of the vernier = 10″

∴ Reading of the vernier scale = 53 × 10″ = 530″ = 8′ 50″

∴ The theodolite reading = 27° 30′ + 8′ 50″

= 27° 38′ 50″ **Ans.**

Example 1.13. *A theodolite circle is divided into degrees and one-third degree, 59 divisions of the main scale coincide with 60 divisions of the vernier. If, for a particular setting of the instrument, the vernier index is between 30° 20′ and* 30° 40′ *and 35th division of the vernier coincides with a division of main scale, calculate the reading of the theodolite for the setting.*

Solution. We know

$$\text{L.C.} = \frac{p}{n+1}$$

Here $p = \frac{1}{3}$ rd degree = 20′

$$n + 1 = 59 + 1 = 60$$

∴ Substituting the values in eqn. (1.1) we get

$$\text{L.C.} = \frac{20}{60} \text{ minutes} = \frac{1}{3} \times 60 = 20''$$

Reading of the main scale = 30° 20′

Reading of the vernier scale = 35 × 20″ = 0° 11′ 40″

∴ The theodolite reading = 30° 31′ 40″. **Ans.**

Example 1.14. *The value of the smallest division of a theodolite graduated circle is 10 minutes. Design a suitable vernier to read up to 10″.*

Solution.

We know $\text{L.C.} = \dfrac{p}{n+1}$

Hence, $\text{L.C.} = \dfrac{10}{60}$ minutes where $p = 10'$

$\therefore \quad \dfrac{10}{60} = \dfrac{10}{n+1}$

or $n + 1 = 60$

$\therefore \quad n = 59.$

Take a length of 59 primary scale divisions and divide it into 60 equal parts to have the required vernier.

1.12. THE MICROMETER MICROSCOPE

To achieve a finer degree of accuracy, micrometer microscopes are used. A micrometer miroscope generally consists of a small low-powered microscope with an eyepiece, a diaphragm and an object glass. The diaphgram can be moved at right angles to the longitudinal axis of the tube. A typical micrometer is shown in Fig. 1.17 (*a*) and the field of view while taking a reading, is shown in Fig. 1.17 (*b*).

In the present case, the circle of the horizontal plate is divided into 10 minute divisions. The objective of micrometer is placed close to the circle graduations and thus it enlarges the image. Two vertical wires mounted on a movable frame placed in the plane of the image can be moved left or right by a micrometer screw drum. One complete revolution of the graduated drum moves the vertical wire across one division of the circle (10′) division. Graduated drum is divided into 10 large divisions and each large division is further divided into 6 small divisions. The fractional part of a division on the horizontal circle may be read on the graduated drum against an index mark fitted on one side.

The approximate reading is determined with the help of a specially marked *V* notch at the mid point of the lower side of the field of view. In Fig 1.17 (*a*) the circle reading is between 52° 10′ and 52° 20′ and the double wire index is in between the readings. With the help of the drum, move the index till the nearest reading seems to be mid way between

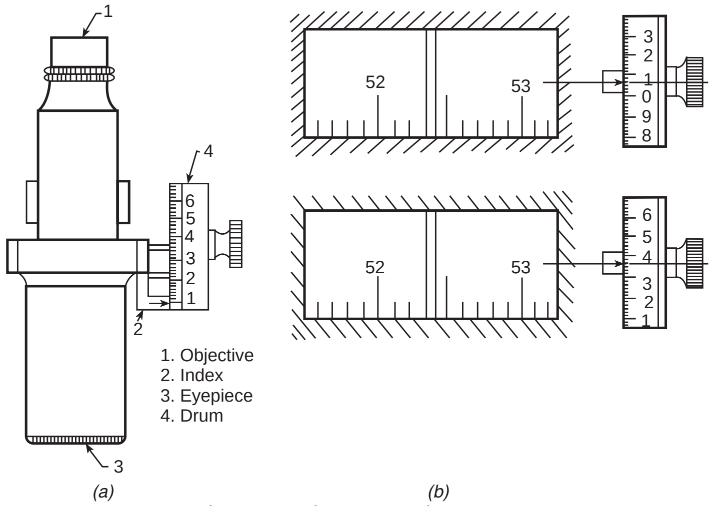


Fig. 1.17. Micrometer microscope.

the vertical wires. Note the reading on the drum. The complete reading is 52° 16′ 10″. Two vertical wires are used to increase the precision of observations.

Example 1.15. *The horizontal circle of a theodolite is graduated to read to 15 minutes. Design a suitable micrometer drum to read to 10 seconds.*

Solution.

The value of one division of the circle = 15′

Divide the circumference of the drum into 15 equal parts, each representing one minute.

Divide each one minute division into 6 equal parts, each representing 10 seconds. **Ans.**

1.13. MEASURING A CORRECT LENGTH WITH A WRONG SCALE

To obtain true measurements of lines and areas on a map using a wrong scale, the following formulae are used :

(1) Correct length

$$= \frac{\text{R.F. of wrong scale}}{\text{R.F. of correct scale}} \times \text{measured length.} \qquad \text{...(1.3)}$$

(2) Correct area

$$= \left(\frac{\text{R.F. of wrong scale}}{\text{R.F. of correct scale}}\right)^2 \times \text{measured area.} \qquad \text{...(1.4)}$$

Example 1.16. *A surveyor measured the distance between two points on a plan and calculated the length to be equal to 650 m assuming the scale of plan to be 1 cm = 50 m. Later, it was discovered that the scale of plan was 1 cm = 40 m. Find the true distance between the points.*

Solution.

Measured length = 650 m (Given)

$$\text{R.F. of wrong scale} = \frac{1}{50 \times 100} = \frac{1}{5000}$$

$$\text{R.F. of correct scale} = \frac{1}{40 \times 100} = \frac{1}{4000}$$

$$\therefore \quad \text{Correct length} = \frac{\text{R.F. of wrong scale}}{\text{R.F. of correct scale}} \times \text{measured length}$$

$$= \frac{\frac{1}{5000}}{\frac{1}{4000}} \times 650$$

$$= 520 \text{ m.} \quad \textbf{Ans.}$$

Example 1.17. *A plot of land acquired for a factory site measures* 25 cm × 20 cm *on village map drawn on a scale 1 cm = 100 m. What is its area in hectares ? What will be its area on a toposheet on 1 : 50,000 scale ?*

Solution.

Scale of village map

1 cm = 100 m

$\therefore$ 1 cm^2 on the village map = 100 × 100

= 10,000 sq. m. on ground

The area of the plot on map = 25 × 20 = 500 sq. cm.

$\therefore$ Area of the plot on the ground = 500 × 10,000

= 50,00,000 sq. m. = 500 hectares. **Ans.**

On the topo sheet

50,000 m = 1 m = 100 cm

$$\therefore \quad 1000 \text{ m} = \frac{100}{50,000} \times 1000 = 2 \, cm$$

$\therefore$ 1 sq. km = 4 sq. cm.

$\therefore$ Area on the topo sheet

$$= \frac{50,00,000}{1000 \times 1000} \times 4$$

= **20 sq. cm. Ans.**

1.14. DISTORTED OR SHRUNK SCALES

Due to change in climatic conditions, the plans and maps generally get distorted. If no graphical scale is drawn on the plan, correct scale of the distorted plan (or map), may be calculated by the following method:

(1) Measure a distance between any two well defined points on the plan and calculate its corresponding ground distance from the scale *i.e.,*

1 cm = x metres. Let it be l metres.

(2) Measure the horizontal distance between the same points on the ground by chaining. Calculate the distance on plan with the scale. Let it be y cm.

(3) Calculate the shrinkage ratio or shrinkage factor which is equal to shrunk length / the actual length.

(4) Shrunk scale of plan = Shrinkage factor × Original scale.

Example 1.18. *The area of a plot on a map is found, by planimeter, to be 10.22 cm^2. The scale of the map was 1 : 25000, but at present it is shrunk such that a line originally 5 cm on the map is now 4.8 cm. What is the correct field area in hectares ?*

Solution.

The area of the plot measured by the planimeter on the shrunk map

$= 10.22 \text{ cm}^2$ (Given)

The ratio of shrinkage $= \frac{4.8}{5.0} = 0.96$

The ratio of shrinkage of the area

$= (0.96)^2 = 0.9216$

The area of the plot on unshrunk map on 1 : 25,000 scale

$$= \frac{10.22}{0.9216} = 11.08941 \text{ cm}^2$$

Area of 1 cm^2 on scale 1 : 25000

$= 250 \times 250 = 62500 \text{ m}^2$

∴ Area of 11.08941 cm on scale 1 : 25000

$= 62500 \times 11.08941 = 693088.12 \text{ m}^2$

= 69.3088 hectares. **Ans.**

Example 1.19. *A rectangular plot of land measures* 30 *cm* × *40 cm on a cadastral map drawn on scale : 1 : 5000. Calculate its area in hectares. If a topographical sheet of the area is compiled on scale 1 : 50,000, what will be its area on the toposheet ?*

Solution.

(*i*) ***Cadastral map***

$\because$ 1 m on map = 5000 m on the ground.

$\therefore$ 1 cm on map = 50 m on the ground.

$\therefore$ $1 cm^2$ on map = $(50)^2$ m^2 on the ground.

The plot measures 30 cm × 40 cm *i.e.*, 1200 sq. cm on the map.

$\therefore$ Area of the plot = 1200 × 50 × 50 sq. metres.

= 300 hectares **Ans.**

(*ii*) ***Topo sheet***

50,000 m is represented by 1 m.

or $(50{,}000)^2$ m^2 is represented by 1 m^2,

$$\therefore \quad 300{,}0000 \text{ m}^2 \text{ is represented} = \frac{1 \times 30{,}00{,}000 \times 100 \times 100}{50{,}000 \times 50{,}000}$$

$\therefore$ The area on the topo sheet = 12 cm^2 **Ans.**

1.15 STAGES OF SURVEY OPERATIONS

The entire work of a survey operation may be divided into three distinct stages :

(*i*) Field work (*ii*) Office work

(*iii*) Care and adjustment of the instruments.

1. Field work. The field work consists of measurement of distances and angles required for plotting to scale and also keeping a systematic record of what has been done in the form of a field book or measurement book. Field work is further divided into three stages.

(*a*) Reconnaissance, (*b*) Observations, (*c*) Field Record.

(*a*) **Reconnaissance.** During reconnaissance, the surveyor goes over the area to fix a number of stations, ensuring necessary inter-visibility between survey stations, to establish a system of horizontal control. A few permanent stations are also selected for an extension of the survey in future.

(*b*) **Observations.** The surveyor makes necessary observations with survey instruments for linear and angular measurements. The observations also include determination of differences in elevations between the stations, establishment of points at given elevations and surveying contours of land areas and bathymetric contours (fathoms) of water bodies. Method of observation depends upon the nature of the terrain, type of the instruments and the method of surveying.

(*c*) **Field records.** All the measurements are recorded in a field book. Every care is made to ensure correct entries of all the observations otherwise the survey records may be useless. The competency of a surveyor is judged by his field records.

Some of the operations which a field surveyor is required to do in the field, are as follow:

1. Selection of the sites and establishment of stations and bench marks in the area.
2. Measuring the horizontal distances between stations either by chaining on the surface of the earth or by trigonometrical computation.
3. Locating the detail points with respect to survey lines such as in chain surveying or by methods of planetabling.
4. Determination of elevations of stations and bench marks either by spirit levelling or by trigonometrical levelling.
5. Surveying contours of land areas and bathymetric contours for water bodies.
6. Determination of latitude, longitude or local time by making astronomical observations to either the sun or stars.

Important rules for note keeping. These include the following:

1. As soon as observations are made, readings should be recorded in the field book. Nothing should be kept in mind for recording later.
2. Only one field book should be maintained.
3. Entries should be made by a sharp 2H or 3H pencil and not by a soft pencil. This keeps the field book neat and clean.
4. Style of writing should be consistent and numericals should be bold and legibly written.
5. Neat sketches should invariably be drawn to explain relative positions and directions.
6. Never erase wrong readings. If readings are to be scored out, rule one line through the incorrect value and record the correct reading above it. All cuttings must always be initialled.
7. Each day's work and field notes must be signed daily.

2. Office work. The field notes are brought to the office and necessary drafting, computing and designing work, are done by draftsmen and computors.

1. Drafting. This process consists of preparation of plans and sections (longitudinal or cross section) by plotting the field measurements to the desired scale.

2. Computing. This process consists of calculating data necessary for plotting and also includes determining the areas and volumes for the earth work.

3. Designing. This process consists of selection of best alignments of roads, railways, canals etc. on the plotting plans.

3. Care and adjustments of instruments. A great care is required to handle survey instruments. A beginner should always be made familiar with care and adjustment of the instruments and its limitations. Precision instruments such as theodolite, level, prismatic compass need more care than the equipment such as chains, arrows, ranging rods, etc. Following precautions must be taken:

1. While removing a theodolite or a level from its box, do not lift it by its telescope. It should be lifted by its standards by placing hands under the levelling head or the foot plate.
2. While carrying an instrument from one place to the other, it should be carried on the shoulder if the distance is short, otherwise it should be carried in its box.
3. Do not set an instrument on smooth surfaces, to avoid spreading its tripod legs and ultimately falling of the instrument. In unavoidable circumstances, tripod legs should be inserted in the joints or cracks.
4. The instruments must be kept clean and frequently dusted with a small brush. Lenses should be dusted lightly with a brush.
5. Keep the hands off the vertical circles and other exposed graduations to avoid tarnishing.
6. Do not expose the instrument to dust, dampness and scorching sun. An umbrella may be conveniently used to protect it from these.
7. Do not leave the instrument on the road, foot path or in an unguarded posture.
8. Do not force the foot-screws and tangent screws too hard.
9. Whenever observations are interrupted, the cap of the objective should be placed.
10. In case of a compass, its needle should never be left to swing unnecessarily. When not in use, it should be lifted off the pivot.
11. After day's work the steel tapes should be wiped clean and dried with a dry cloth slightly oily.
12. The theodolite should be turned a few revolutions in altitude and azimuth before starting actual work.

It may be remembered that if you respect an instrument, the instrument will respect you by giving good results. It is an old proverb.

1.16. PRECISION IN SURVEYING

The degree of precision required in surveying mainly depends upon the purpose and scale of the map. Larger the scale, better the precision required and *vice versa.* If a map is required to be on scale 1 cm = 1 km and the plotted permissible error on a map is 0.25 mm. It is therefore necessary to have a precision in linear measurements to 25 metres *i.e.,* an error of 25 metres in linear measurement of a line does not affect the accuracy of the map. On the other hand, if the scale is 1 cm = 5 m, the plotted permissible error on the map is given by $0.5 \times 0.25 = 0.125$ *m i.e.*, an error of 12.5 cm can hardly be tolerated.

Similarly, the degree of precision required in a topographical map is not the same as that required of a cadastral map. Since the value of land in the city is more than the value of land in the rural areas, greater precision in the measurements is required in cities and hence nearest fraction of a centimetre is observed. The cost of survey is directly proportional to the accuracy of the map. Before commencing a survey work, the surveyor must, therefore, consider the following factors to decide the method to be adopted and instruments best suited to the particular case.

1. The purpose of surveying
2. The degree of precision required
3. The scale of the map
4. The extent of the area
5. The nature of the country
6. The time available
7. The fund available for the survey

EXERCISE 1

1. Fill in the blanks with suitable word(s).

(*i*) Surveying is the art of determining.........positions of different features on the surface of the earth.

(*ii*) The object of surveying is the preparation of ....of the area.

(*iii*) In the absence of accurate ....it is difficult to layout the alignment of roads, railways and canals.

(*iv*) Surveying is the first...............for the execution of any engineering project.

(*v*) Surveys in which curvature of the earth is ignored, are known as..........surveys whereas surveys in which curvature of the earth is taken into account, are known as.......surveys.

(*vi*) The branch of surveying which deals with the measurements in vertical planes, is known as...........

(*vii*) Surveys which are carried out to depict the general topography of the terrain, are known as ....surveys.

(*viii*) The small scale maps on which features are suitably generalised so that a picture of the country as a whole can only be visualised, are known as........maps.

(*ix*) The main principle of surveying is to work from the........to the........

(*x*) Location of a point can be fixed with respect to given two points by measuring............between the known point and the point.

(*xi*) The smallest basic unit of length in metric system, is ....

(*xii*) The measure of the angle between two radii of a circle which contain an arc equal to the radius on the circumference of the circle, is known as...........

(*xiii*) In sexagesimal system the circumference in divided into.............equal part whereas in centesimal system, it is divided into..........equal parts.

(*xiv*) π radians =.............rt angles = ...degrees = ...grads.

(*xv*) Diagonal scales are............accurate than plane scales.

(*xvi*) The difference between the smallest division on the main scale and the smallest division on the vernier is called........ of the vernier.

(*xvii*) Least count of a vernier can be found out by dividing the value of one primary scale division by.........number of divisions of the vernier.

(*xviii*) If the smallest division of a vernier is longer than the smallest division of the main scale, the vernier is called a........

2. Define surveying. Explain its importance for Civil Engineers.

3. Explain the fundamental principles on which the art of surveying is based.

4. (*i*) What are the objects of plane surveying ?

(*ii*) Give a classification of surveys based on the instruments used.

5. What are the different kinds of verniers ? Explain the object of vernier and the principle of its working.

6. What is the difference between direct vernier and retrograde vernier? Construct a dircet vernier reading to 1 mm to a scale graduated to 5 mm.

7. Explain the main characteristics of 'plain' and 'diagonal' scales and also show the utility of a vernier scale.

8. A boat in the Ganges is timed to move down a distance of 60 metres in one minute. Draw a scale of R.F. $\frac{1}{5500}$ to meature minutes and diagonally spaces of 5 seconds. Show on the scale the distance the boat moves down in 7 minutes and 35 seconds.

9. The horizontal circle of a theodolite is graduated to read to 10 minutes. Design a suitable vernier to read to 10″.

10. The least count of a vernier theodolite is 20′ and the smallest division of the main scale is 20′. If the zero of the vernier is in between 120° 40′ and 121° 0′ and 27th division of the vernier is in exact coincidence with the division of the main scale, calculate the exact reading on the theodolite. Draw a neat diagram.

ANSWERS

1. (*i*) relative; (*ii*) a map; (*iii*) maps; (*iv*) step; (*v*) plain, geodetic; (*vi*) levelling; (*vii*) topographical; (*viii*) geographical; (*ix*) whole, part; (*x*) distances; (*xi*) centimetre; (*xii*) radian; (*xiii*) 360, 400 (*xiv*) 2, 180°, 200; (*xv*) more; (*xvi*) least count; (*xvii*) total; (*xviii*) retrograde.

10. 120° 49′.

2

Linear Measurements

2.1. INTRODUCTION

There are two main methods of determining the distances between points on the surface of the earth : (*i*) Direct method, and (*ii*) Computative method.

1. Direct Measurement. In this method, distances are actually measured on the surface of the earth by means of chains, tapes, etc.

2. Computative Measurement. In this method distances are determined by calculation as in tacheometry and triangulation.

Several methods are available for measuring the distances directly on the earth surface. Each methods has its own limitation of the degree of accuracy (precision).

2.2. INSTRUMENTS FOR MEASURING DISTANCES

The instruments used for measuring distances are as discussed below:

1. Tapes. Depending upon the material, tapes are classified as under:

(*i*) Cloth or linen tape (*iii*) Steel tape

(*ii*) Metallic tape (*iv*) Invar tape

(1) Cloth or Linen Tape (Fig. 2.1)

Linen tapes are closely woven linen and varnished to resist moisture. They are generally 10 metres to 30 metres in length and 12 mm to 15 mm in width. One end of the tape is provided with a ring whose length is included in the total length of the tape. Cloth tapes are generally used for measuring offset measurements due to the following reasons:

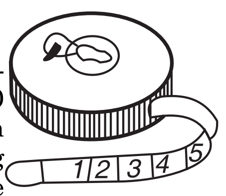


Fig. 2.1. A cloth or linen tape.

(*i*) It is easily affected by moisture and thus gets shrunk.

(*ii*) Its length gets altered by stretching.

(*iii*) It is likely to twist and tangle.

(*iv*) It is not strong as a chain or steel tape.

(*v*) It is light and flexible and it does not remain straight in strong wind.

(*vi*) Due to continuous use, its figures get in distinct.

2. Metallic Tape. A linen tape reinforced with brass or copper wires to prevent stretching or twisting of fibers is called a *metallic tape.* As the wires are interwoven and the tape is varnished, these wires are not visible to naked eyes. These tapes are also available in different lengths but tapes of 20 m and 30 m lengths are more common. These are supplied in leather cases, with a winding device. Each metre is divided into decimetres, and each decimetre is further subdivided into centimetres.

Fig. 2.2. A metallic tape.

Fig. 2.3 A steel tape.

A metallic tape can be used for measuring accurate distances. It is commonly used for taking offset distances in chain surveying.

3. Steel Tape. Steel tapes are available with different accuracy of graduations. A steel tape of lowest degree of accuracy is generally superior to a metallic or cloth tape for near measurements. Steel tapes which consist of a light strip of width 6 mm to 10 mm are accurately graduated. Steel tapes are available in different lengths but 10 m, 20 m, 30 m, and 50 metre steel tapes are usually used for survey measurements. At the end of the tape, a brass ring is provided. The length of the metal ring is included in the length of the tape. It is wound in a leather metal case, having a suitable winding device. (Fig. 2.3).

As steel tapes are delicate, they are generally not used in terrain with vegetations or rocky grounds.

4. Invar Tape. Invar tapes are made of an alloy of nickel (36%) and steel (64%) having very low coefficient of thermal expansion (0.000000122 per 1°C). These are 6 mm wide and are available in length of 30 m, 50 m and 100 m. Invar tapes are used mainly for high degree of precision required for base measurements.

Disadvantages of Invar Tapes. Following are the disadvantages of invar tapes:

(*i*) These are more expensive, softer and get deformed more easily than steel tapes.

(*ii*) The invar tape develops creep with time.

(*iii*) Their co-efficient of thermal expansion goes on changing.

(*iv*) A number of assistants are required to stretch and handle them.

(*v*) They need the greatest care to handle them to avoid bending and kinking.

(*vi*) They cannot be used for ordinary work.

2. Steel Bands. (Fig. 2.4). The steel band consists of ribbon of steel with a brass swivel handle at each end. It is 20 m or 30 m long and 16 mm wide. The graduations are marked in the following manner :

1. The band is divided by brass studs at every 0.2 m and numbered at every one metre. The first and last links are subdivided into centimetres and millimetres.

2. The graduations are etched as metres, decimetres, centimetres on one side and 0.2 m links on the other side.

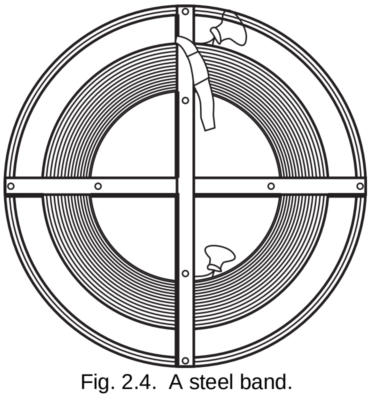


Fig. 2.4. A steel band.

The band is wound on an open steel cross or metal reel in a closed case.

Suitability of Steel Bands. The steel bands are suitable for the following:

(*i*) Whenever accurate measurements are required, a steel band is used.

(*ii*) It is lighter and easier to handle than a chain.

(*iii*) Its length does not alter as compared to a chain due to usage.

(*iv*) Proper care is required to handle it as it breaks easily.

(*v*) In case it is broken it cannot be repaired in the field.

(*vi*) It must be protected from rust by frequent cleaning and oiling.

(*vii*) It cannot be read so easily as a chain.

3. Chains. The chains used in surveying are generally of the following types.

(1) Gunter's Chain. It is 66 ft long and it is divided into 100 links. Each link measures 0.66 ft.

(2) Engineer's Chain. It is 100 ft long and it is divided into 100 links. Each link measures 1 ft.

(3) Metric Chain. It is 20 m or 30 m long and it is divided into 100 or 150 links respectively. Each link measures 20 cm.

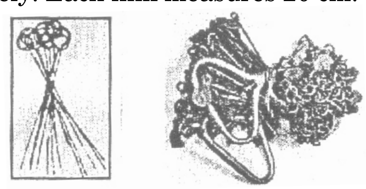


Fig. 2.5.

A metric chain (Fig. 2.5) is generally divided into 100 or 150 links. The links are composed of pieces of galvanised mild steel wire 4 mm in diameter. The ends of each link are bent into loops and connected together by means of three oval shaped rings which afford the flexibility to the chain. The joints of the links are usually open but in good quality chains, these are welded so that true length of the chain, does not alter due to stretching. The ends of the chain are provided with brass handles with swivel joints so that the chain can be turned round without twisting. The outside of the handle is the zero point or the end point of the chain. The length of a chain is measured from the outside of one handle to the outside of the other. The length of a link is the distance between the centres of the two consecutive middle rings (Fig. 2.6). The end link also includes the length of the handle. Metallic tags of different patterns are fixed at various important points of a chain, *i.e.*, 5 m, 10 m, 15 m, 20 m for quick and easy reading of the chain.

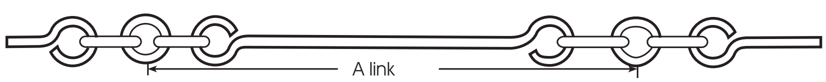


Fig. 2.6. A link of a chain

According to I.S. 1492–1956 the surveying chains are calibrated into metres and its further smaller subdivisions. Metric surveying chains are available in lengths of 20 metres and 30 metres. Small brass rings are provided at every metre length except at 5 m, 10 m, 15 m, 20 m, 25 m, where tallies are fixed, to facilitate the reading of the fractions of the chain.

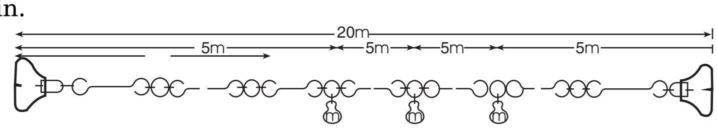


Fig. 2.7. A 20 metre chain

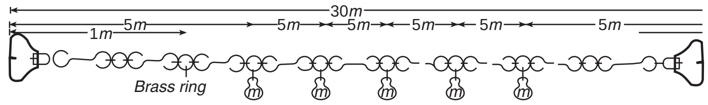


Fig. 2.8. A 30 metre chain.

The handles of the chain are provided with grooves so that the arrow can be held at the correct position. The tallies used for a marking 5 m, 10 m, etc. are marked with letter M to distinguish the metric chain from a non-metric chain. The length of the chain whether 20 m or 30 m, is indicated on the handle, for easy indetification.

Suitability of Chains. The chains are suitable for the following :

(*i*) Its length alters due to continued use. Hence, it is suitable only for ordinary work.

(*ii*) Its length gets shortened due to bending of the links and gets lengthened by flattening of the rings.

(*iii*) Being heavier, a chain sags considerably when suspended at its ends.

(*iv*) It is only suitable for rough usage.

(*v*) It can be easily repaired in the field.

(*vi*) It can be read easily.

Testing a Chain. During its use, links of a chain get bent and consequently the length of the chain decreases. On the other hand, the length of a chain increases due to the following factors :

(*i*) Wearing and tearing of 600 wearing surfaces.

(*ii*) Stretching of the links and the joints.

(*iii*) Opening out of small rings due to prolonged usage.

(*iv*) Rough handling through hedges, fences, etc.

Because of the above reasons, it is necessary to check the length of the chain before commencing the day's work and at frequent intervals afterwards. If the chain is not tested, the measurements will become unreliable. Before checking a chain, the surveyor should ensure that its links, are not bent, openings are not too wide and there is no mud attached to them. The connecting rings should be circular.

When a tension of 8 kg (78.5 N) force is applied at the ends of a chain and compared against a certified steel band standardised at 20°C, every metre length should be accurate within ± 2 mm and the overall length of the chain should be accurate within the following limits :

20 metre chain ± 5 mm.

30 metre chain ± 8 mm.

Testing and adjusting a chain in the field. The length of a chain may be tested and adjusted as detail below.

Two pegs are driven at requisite distance apart, *i.e.*, 20 m or 30 m and nails are inserted into their tops to mark their exact points as shown in Fig. 2.9.

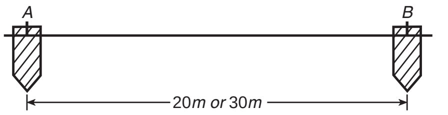


Fig. 2.9. Adjustment of a chain length.

The overall length of the chain should be compared against the fixed points and the difference, if any, should be noted.

On comparison, if a chain is found to be longer than its standard length, it may be adjusted by :

(*i*) Closing the opened joints of the rings.

(*ii*) Reshaping the elongated rings.

(*iii*) Removing one or more small circular rings.

(*iv*) Replacing the worn-out rings.

If, on the other hand, a chain is found to be short, it may be adjusted by :

(*i*) Straightening the bent links.

(*ii*) Flattening the circular rings.

(*iii*) Replacing one or more small circular rings by bigger ones.

(*iv*) Inserting additional circular rings.

However, in both the cases, adjustment must be done symmetrically so that the measurements made by different portions of the chain, do not differ considerably.

Chain Pins (arrows). An arrow is made of steel wire 4 mm (8 S.W.G.) in diameter. The length of the arrow may vary from 25 cm to 50 cm but the length in common use is 40 cm. One end of the arrow is bent into a loop of a circle of 50 cm diameter and the other end is made pointed sharp. 10 arrows generally accompany a chain.

2.3. INSTRUMENT FOR MARKING STATIONS

In soft grounds pegs 40 cm to 60 cm in length and 4 cm to 5 cm square, are found more suitable. These should be driven firmly in ground projecting about 5 cm above the surface of the ground.

Survey stations may be marked as under :

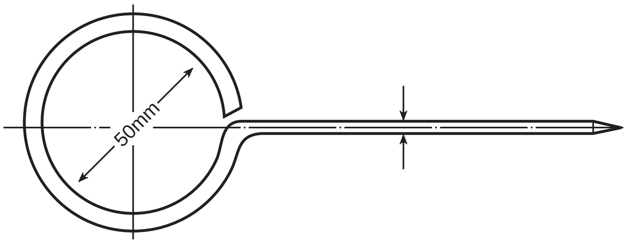


Fig. 2.10. An arrow.

1. Pegs. Wooden pegs usually 2.5 cm square and 15 cm deep are used to mark the position of the survey stations.

2. Iron Pegs. When surveying in a comparatively harder ground, iron pegs or loop wire nails are generally used in place of wooden pegs.

3. Ranging Rods. Ranging rods are used for marking the positions of stations while ranging a line. It is made of well seasoned straight grained timber of teak or deodar and is generally available in 2 m or 3 m length having a 3 cm nominal diameter. It is divided into equal parts, each part measuring 0.2 m. Its lower end is provided with a cross shoe of 15 cm length. It is generally painted alternatively black and white throughout its length.

4. Ranging Poles. The ranging poles are similar to ranging rods excepting that these are of heavier section of length 4 m to 6 m. These are used for ranging very long lines in undulating ground.

5. Offset Rods. The offset rod is also similar to a ranging rod. It is generally 3 m long and is divided into equal parts of 0.2 m. The top is provided with an open hook for pulling or pushing a chain through obstruction *i.e.,* bushes etc. Two narrow vertical slots passing through the centre of the section at right angles to one another are provided at the eye level. It is used for aligning the offset line and measuring short offsets.

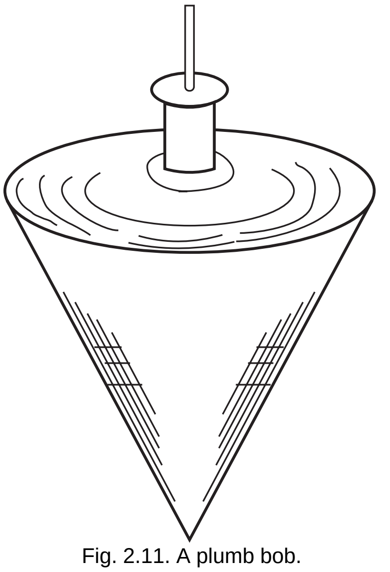


Fig. 2.11. A plumb bob.

6. Plumb Bob (Fig. 2.11). It is used to transfer the end points of the chain onto ground while measuring distances in a hilly terrain. It is also used for testing the verticality of ranging poles, ranging rods or levelling staves.

2.4. RANGING A LINE

The process of marking a number of intermediate points on a survey line joining two stations in the field so that the length between them may be measured correctly, is called *ranging.* When the line is short or its end station is clearly visible, the chain may be laid in true alignment. If the line is long or its end station is not visible due to undulating ground, it is required to mark a number of points with ranging rods such as *a, b, c, d,* etc. (Fig. 2.12) along the chain line prior to chaining the distance between *A* and *B*.

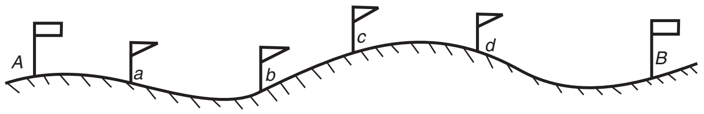


Fig. 2.12. Ranging a line.

Ranging may be done either by eye estimation or by using a line ranger or a theodolite. Theodolites are generally used only for important work.

Classification of Ranging

Ranging may be classified as :

(*i*) Direct ranging (*ii*) Indirect ranging.

1. Direct Ranging. When intermediate ranging rods are fixed along the chain line, by direct observation from either end station, the process is known as *'Direct Ranging'.*

Following steps are taken in direct ranging:

1. Erect ranging rods or poles vertically behind each end of the line.
2. Stand about 2 m behind the ranging rod at the beginning of the line.
3. Direct the assistant to hold a ranging rod vertically at arm's length at the point where the intermediate station is to be established.
4. Direct the assistant to move the rod to the right or left, until the three ranging rods appear to be exactly in a straight line.
5. Stoop down and check the position of the rod by sighting over their lower ends in order to avoid error due to non-verticality of the ranging rods.
6. After ascertaining that the three ranging rods are in a straight line, signal the assistant to fix the ranging rod.

Code of signals. The following codes of signals are used to direct the assistant while ranging a line.

	Code	Meaning
1.	Rapid sweeps with right hand	Move considerably to the right
2.	Rapid sweeps with left hand	Move considerably to the left
3.	Slow sweeps with right hand	Move slowly to the right
4.	Slow sweeps with left hand	Move slowly to the left
5.	Right arm extended	Continue to move to the right
6.	Left arm extended	Continue to move to the left
7.	Right arm up and moved to the right	Plump the rod to the right
8.	Left arm up and moved to the left	Plump the rod to the left
9.	Both arms above head and then brought down	Correct
10.	Both arms extended forward and depressed briskly	Fix

Note: The following points may be noted:

(*i*) The signals should be made clearly without any confusion.

(*ii*) When the assistant is at a great distance, the surveyor should use his handker-chief for signalling.

Line Ranger. It is a small reflecting instrument used for fixing intermediate points on a chain line. It consists of two *right angled isosceles triangular prisms* placed one above the other. (Fig. 2.13.)

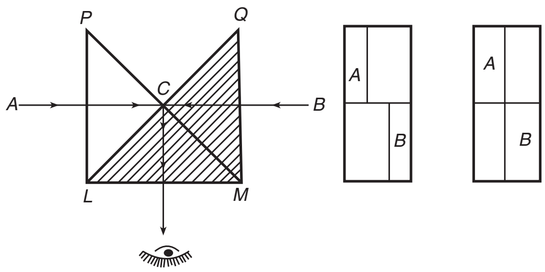


Fig. 2.13. A line ranger.

Suppose A and B are the ends of a line and C is an intermediate point to be fixed on this line. Following steps are followed for locating the intermediate point C.

Steps. The following steps are taken:

(1) Stand approximately in line near C and hold the line ranger at eye level.

(2) Observe the ray of light from *A*, which enters the upper prism, gets reflected from the hypotenuse *LQ* and enters the eye at right angles to *AB*.

(3) Similarly, observe the ray of light from *B*, which enters the lower prism, gets reflected from the hypotenuse *PM* and enters the eye at right angles to *BA*.

(4) Observe the images of the ranging rods *A* and *B* in upper and lower prisms simultaneously.

(5) If the point *C* is not in line with *AB*, two images will appear to be separated (Fig. 2.13. *a*).

(6) Move the instrument backward and forward at right angles to the line-until two images appear one above the other exactly in same vertical line. (Fig. 2.13 *b*).

(7) The centre of the instrument defines the location of *C* on line *AB*.

Adjustment of a Line Ranger. Proceed as under:

Test.(*i*) Fix three ranging rods exactly in a straight line.

(*ii*) Hold the line ranger over the intermediate point.

(*iii*) Observe if the images of the end ranging rods appear in exact coincidence.

Adjustment. If the two images do not coincide, the instrument is out of adjustment. Adjust the movable prism by means of its adjusting screw so that the images appear exactly coincident.

Repeat the observations by reversing the end points.

2. Indirect Ranging. When end stations are not intervisible and the intermediate ranging rods are placed in line by interpolation or by reciprocal ranging or by running an auxiliary line (or random line), the process is known as *Indirect Ranging.*

Indirect ranging is resorted to the following situations:

(1) When the end stations of a line are not intervisible due to interveiwing raised ground.

(2) When the end stations of a line are not distinctly visible due to a large distance.

Case I: Intervening a raised ground

Let *A* and *B* be two end stations intervened by raised ground. The ranging on intermediate points, may be done as discussed below. (Fig. 2.14.)

Steps. The following steps are followed:

(*i*) Fix two ranging rods *A* and *B* at the ends of the chain line.

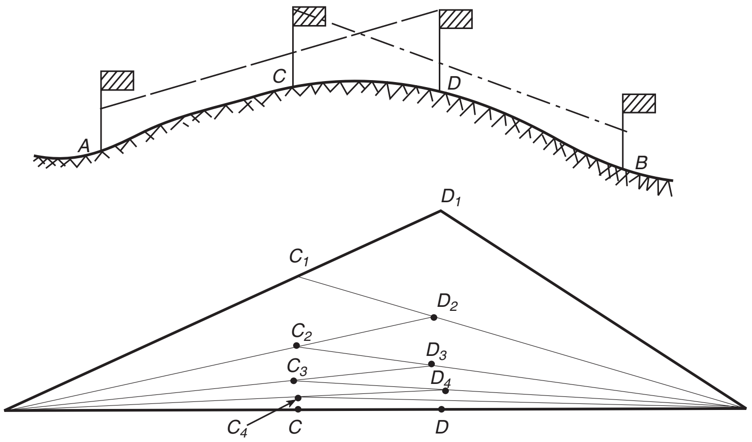


Fig. 2.14. Indirect ranging.

(*ii*) Send two assistants with ranging rods to take positions at *C* and *D* as nearly on the line as can be judged.

(*iii*) Ensure that the assistant at *C* can see the ranging rods held at *B* and *D* and the assistant at *D* can see the ranging rods held *A* and *C*.

(*iv*) Direct the assistants to proceed to line themselves alternately.

(*v*) Assistant at *C* should direct the assistant at *D* to be in line with *B*, and then the assistant at *D* should direct the assistant at *C* to be in line with *A*.

(*vi*) By successively directing each other, the two assistants go on changing their positions until both are exactly on the line *AB*.

(*vii*) Erect the ranging rods at *C* and *D* which serve as intermediate stations for ranging other points.

The above method may also be used for ranging a line across a valley.

Case II: Non-intervisibility due to large distance

When end stations are not visible from any intermediate point, the random line method is used. This method is specially suitable for ranging a line across forests and wooded areas where vision is obstructed.

Let *AB* be a line to be ranged which is intervened by vegetation. (Fig. 2.15.)

Steps. The following steps are followed:

(*i*) Estimate the approximate direction of the station *B* from *A*.

(*ii*) Run a random line *AB′* in the estimated direction and measure it. Fix intermediate points such as *C′* and *D′*.

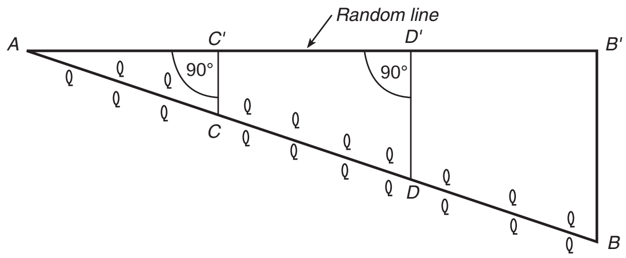


Fig 2.15. A random line.

(*iii*) Continue ranging until the station B is visible.

(*iv*) Measure a perpendicular offset $B'B$ *on* AB'.

(*v*) Calculate the length of $AB = \sqrt{(AB')^2 + (BB')^2}$

(*vi*) Calculate the perpendicular offsets $C'C$ and $D'D$ which are evidently equal to $\frac{AC'}{AB'} BB'$ and $\frac{AD'}{AB'} \times BB'$ respectively.

(*vii*) Lay out perpendicular offsets equal to their calculated values from points C' and D'.

(*viii*) Locate a number of points on the line AB. Clear the line from points vegetation for accurate ranging.

2.5. CHAINING A LINE

For a chaining operation, two chainmen are required. The chainman at the forward end of the chain is called a *leader* while the other chainman at the rear end, is called a *follower*.

The duties of the *leader* and *follower* are tabulated under:

	Leader	*Follower*
s1.	To drag the chain forward	To direct the leader to be in line with the ranging rod at the end station.
2.	To insert an arrow at the end of every chain.	To carry the rear end of the chain ensuring that it is dragged above the ground.
3.	To obey the instructions of the follower.	To pick up the arrows inserted by the leader.

2.6. UNFOLDING A CHAIN

Unfolding a chain must be done with great care. After removing the leather strap, both the handles should be held in the left hand and the chain should be thrown well forward with the right hand. The leader, then should take one handle of the chain and move forward until the chain is extended to its full length. The chain is then examined to see if there are any kinks or bent links. This operation is called *unfolding the chain*.

2.7. METHOD OF CHAINING

To chain a line, the follower holds the handle of the chain in contact with the peg at the beginning of the line and direct the leader to be in line with the ranging rod fixed at the end of the chain line. The leader, taking 10 arrows in one hand and the other handle of the chain in other hand, walks along the line dragging the chain. At the end of the chain, the leader holds a ranging rod vertically in contact with the outside of the handle at arm's length and faces the follower. Using the code of signals, the follower directs the leader to move the rod to the right or left as required by the follower until it is exactly in line. The leader then holds the handle in both hands, stands in the line and straightens the chain by jerking it and stretches over the mark. He then holds the arrow against the end of the handle and inserts it vertically into the ground to mark the end of a chain length. If the ground is hard, the end of chain length may be marked by a cross (×) scratched with an arrow or with a chalk. An arrow is laid at the cross.

The leader, then holding the ranging rod and the remaining nine arrows, starts off dragging the chain a little off the line so that the arrow placed in position, is not disturbed.

The follower, holding the rear handle comes to the last fixed arrow and calls 'chain' to give a warning to the leader that he has already reached a chain length and that he should stop moving forward. The process as explained in earlier paragraphs is again repeated. When the tenth chain length is measured, the follower is left with no arrow. The follower then asks him to wait. He hands over all the ten arrows to the leader. The surveyor records the transfer of arrows in his field work. To measure a fractional length of a chain, the leader should drag the chain beyond the station and the follower holds the chain handle against the last arrow. The leader after stretching the chain comes to the station mark and counts the odd links.

2.8. FOLDING THE CHAIN

After the day's work the chain should be folded into a bundle and fastened with a leather strap. To do this, the handles of the chain should be brought together by pulling the chain at the middle. Commencing from the middle, take two pairs of links at a time with the right hand and place them obliquely across the other in the left hand. When the chain is collected in a bundle which somewhat resembles a sheaf of corn, it is tied with a leather strap. This operation is called *folding the chain.*

2.9. ERROR IN MEASUREMENT DUE TO INCORRECT CHAIN LENGTH

Due to usage of a chain over rough ground, its oval shaped rings get elongated and thus the length of the chain gets increased. On the other hand, sometimes, some of the links get bent and consequently the length of the chain gets decreased. It may therefore, be noted that lengths obtained by chaining with a faulty chain, are either too long or too short than the length which would be obtained by chaining with a chain of standard length. The general rule applicable may be stated :

"If the chain is too long, the measured distance will be less and' if the chain is too short, the measured distance will be more."

Let L be true length of chain

L' be faulty length of the chain

1. The true length of a line

$$= \frac{L'}{L} \times \text{measured length of the line.}$$

2. The true area of plot of land

$$= \left(\frac{L'}{L}\right)^2 \times \text{measured area of the plot.}$$

3. The true volume of an excavation

$$= \left(\frac{L'}{L}\right)^3 \times \text{measured volume of the excavation.}$$

Remember. Product of the correct length and correct chain length = Product of the incorrect length and incorrect chain length *i.e.,*

"Birds of the same feather, flock together."

Example 2.1. *The length of a survey line measured with a 30 m chain was found to be 631.5 m. When the chain was compared with a standard chain, it was found to be 0.10 m too long. Find the true length of the survey line.*

Solution.

True length of the line

$$= \frac{L'}{L} \times \text{measured length of the line.}$$

Here $L' = 30.10$ m, L = 30 m

Measured length of survey line

$$= 631.5 \text{ m}$$

∴ True length of the survey line

$$= \frac{30.10}{30} \times 631.5$$

$= 633.603$ m **Ans.**

Example 2.2 *A 20 m chain was found to be 4 cm too long after chaining 1400 m. It was 8 cm too long at the end of day's work after chaining a total distance of 2420 m. If the chain was correct before commencement of the work, find the true distance.*

Solution.

The correct length of the chain at commencement

$= 20$ m

The length of the chain after chaining 1400 m = 20.04 m.

$\therefore$ The mean length of the chain while measuring

$$= \frac{20 + 20.4}{2} = 20.02 \text{ m}$$

True distance for the wrong chainage of 1400 m.

$$= \frac{20.02}{20} \times 1400 = 1401.400 \text{ m.}$$

The remaining distance

$= 2420 - 1400 = 1020$ m.

Mean length of the chain while measuring the remaining distance

$$= \frac{20.08 + 20.04}{2} = 20.06 \text{ m}$$

$\therefore$ True length of remaining 1020 m

$$= \frac{20.06}{20} \times 1020 = 1023.06 \text{ m}$$

Hence, the total true distance = 1401.40 + 1023.06

$= 2424.46$ m. **Ans.**

Example 2.3. *A distance of 2000 metres was measured by a 30 metre chain. Later on, it was detected that the chain was 0.1 metre too long. Another 500 metre (i.e., total 2500 m) was measured and it was detected that the chain was 0.15 metre too long. If the length of the chain in the initial stage, was quite correct, determine the exact length that was measured.*

Solution.

The length of the chain at the commencement

$= 30$ m

The length of the chain at the end of 2000 m

$= 30.1$ m

$\therefore$ The mean length of the chain while measuring 2000 m.

$$= \frac{30.0 + 30.1}{2} = 30.05 \text{ m.}$$

$\therefore$ True distance for 2000 m

$$= \frac{30.05}{30} \times 2000 = 2003.33 \text{ m}$$

The remaining distance = 2500 – 2000 = 500 m

The length of the chain at the end of 2500 m = 30.15 m

The mean length of the chain while measuring 500 m

$$= \frac{30.10 + 30.15}{2} = 30.125 \text{ m}$$

$\therefore$ True distance for 500 m

$$= \frac{30.125}{30} \times 500 \text{ m} = 502.08 \text{ m}$$

The exact length that was measured

$= 2003.33 + 502.08 = 2505.41$ m. **Ans**.

Example 2.4. *A line was measured by 20 m and 100 ft. chain respectively and was 12 chains in length in each case. If the 30 m chain was 0.2 m too long, find the correct length of the 100 ft. chain upto three decimal places. Take 1 m = 3.28 ft.*

Solution.

True length $= \frac{L'}{L} \times$ measured length

Here $L' = 30.2$ m, L = 30 m

Measured length $= 12 \times 30 = 360$ m

$\therefore$ True length of the line

$$= \frac{30.2}{30} \times 360 \text{ m} = 362.4 \text{ m}$$

$= 362.4 \times 3.28$ ft.

$= 1188.672$ ft.

Correct length of 100 ft. chain

$$= \frac{1188.672}{12}$$

$= 99.056$ ft. **Ans.**

Example 2.5. *The area of a certain field was measured with a 30 m chain and found to be 5000 sq. m. It was afterwards detected that the chain used was 10 cm too short. What is the true area of the field ?*

Solution.

$$\text{True area} = \left(\frac{L'}{L^2}\right) \times \text{Measured area}$$

Here, $L' = 29.9$ m ; $L = 30$ m; measured area = 5000 sq. m

$$\text{True area} = \left(\frac{29.9}{30}\right)^2 \times 5000 \text{ sq. m}$$

$= 4966.72$ sq. m. **Ans.**

Example 2.6. *At the end of survey of a field a 30 m chain was found to be 10 cm too long. The area of the plan drawn with the measurements taken with this chain is found to be 125 cm^2. If the scale of the plan is 1 cm = 10 m, what is the true area of the field. Assume that the chain was exact 30 m at the commencement of the work.*

Solution.

Area of plan $= 125$ cm^2

Scale of plan, 1cm =10 m

$\therefore$ Area of the field $= 125 \times (10)^2 = 12{,}500$ sq. m.

$$\text{True area} = \left(\frac{L'}{L}\right)^2 \times \text{measured area}$$

Here $L = 30$ m

$$L' = \frac{30.0 + 30.10}{2} = 30.05 \text{ m}$$

Measured area $= 12{,}500$ sq. m.

$$\therefore \text{True area} = \left(\frac{30.05}{30}\right)^2 \times 12{,}500$$

$= 12541.7$ sq. m. **Ans.**

Example 2.7. *The area of the plan of an old map plotted to a scale of 10 metres to 1 cm measures now 100.2 sq^2. cm. when measured by a plani meter. The plan is found to have shrunk so that a line originally 10 cm long now measures 9.7 cm only. Further, the 20 m chain used is 8 cm too short. Find the true area of the survey.*

Solution.

Original area plotted on map

$$= \left(\frac{10}{9.7}\right)^2 \times 100.2 = 106.494 \text{ cm}^2$$

The scale of the plan, 1 cm = 10 m

$\therefore$ $\quad 1 \text{ cm}^2 = 10 \times 10 = 100\ m^2$

$\therefore$ The area of the plot

$$= 106.496 \times 100 \text{ sq}^2\text{. m} = 10649.4 \text{ sq}^2\text{. m}$$

Again, true area $= \left(\frac{L'}{L}\right)^2 \times$ Measured area

Here $L' = 19.92$ m, $\quad L = 20$ m

Measured area = 10649.4 sq. m.

$\therefore$ True area of survey $= \left(\frac{19.92}{20}\right)^2 \times 10649.4$

$$= 10{,}564.38 \text{ sq. m.}$$ **Ans.**

Example 2.8. *The volume of an excavation was computed from the measurements taken by a 20 m chain and found to be 58,75,000 cu.m. On the close of the work it was detected that the chain used was 5 cm too long, whereas it was correct at the commencement of the work. Calculate the correct volume of the excavation.*

Solution.

The mean length of the chain

$$= \frac{20.05 + 20.00}{2} = 20.025 \text{ m}$$

Correct volume $= \left(\frac{L'}{L}\right)^3 \times$ Measured volume

$L' = 20.025$ m ; $L = 20$ m

Measured volume = 58,575,000 cu. m

$\therefore$ Correct volume of the excavation

$$= \left(\frac{20.025}{20}\right)^3 \times 58{,}75{,}000 \text{ cu. m.}$$

$$= 5897058.2 \text{ cu. m.} \qquad \text{Ans.}$$

2.10 . CHAINING ON SLOPING GROUNDS

Chaining on the surface of a sloping ground gives the sloping distance. For plotting the surveys, horizontal distances are required. It is therefore, necessary either to reduce the sloping distances to horizontal equivalents or to measure the horizontal distances between the stations directly. There are two methods for getting the horizontal distance between two stations on a sloping ground *i.e.,*

(*i*) Direct method, (*ii*) Indirect method.

1. Direct method (Fig. 2.16). In direct method horizontal distances are measured on the ground in short horizontal lengths. Full length of a chain or tape is not generally used. Depending upon the steepness of the slope, a portion of the chain or tape is used. Direct method is also, sometimes known as '*stepping method*'.

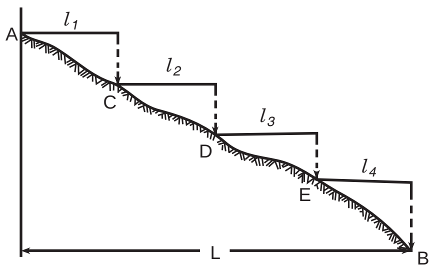


Fig. 2.16. Stepping method.

Procedure. The follower holds the zero end of the tape at *A* while the leader selects any suitable length l_1 of the tape and moves forward along the chain line duly ranged. The follower directs the leader for correct ranging. The leader pulles the tape, stretches it horizontally above the ground at a convenient height (less than 1.8 m). The point *C* vertically below a definite chainage, is accurately located on the ground by suspending a plumb bob or *drop arrow.* For less accurate and small scale surveys where small errors in lengths are not plottable, the end of the suspended length of the chain, may be transferred on to the ground by dropping a pebble. The follower then reaches the point *C* and the process is continued until the entire length of the sloping line, is measured. If $l_1, l_2, l_3, l_4 \ldots l_n$ are the tape lengths used in stepping between stations *A* and B, the total horizontal distance between the two stations, *A* and *B* is equal to the sum of the lengths of the steps *i.e.,* $l_1 + l_2 + l_3 + l_4 \ldots\ldots + l_n$.

Precautions. Following precautions are taken;

(*i*) The tape should be stretched horizontally and should be checked by the surveyor himself, standing clear to one side, directing the leader.

(*ii*) As the error due to sag is proportional to $\left(\frac{\text{Weight}}{\text{Tension}}\right)^2$, the chain or tape lighter in weight, should be used with a maximum pull.

(*iii*) It is not necessary to keep the length of steps uniform throughout and may vary inversely proportional to the steepnes of the slope and the weight of the chain or tape. Short steps may be taken if the chain is heavy or the slope is steep.

(*iv*) It is always convenient to chain the slopes down the hill than to chain up the hill.

2. Indirect Method. The horizontal distance between two stations on a sloping ground may be obtained by any one of the following methods:

(*i*) By measuring along the slope and the angle of slope of the ground.

(*ii*) By applying the Hypotenusal Allowance to each chain length laid along the slope.

When the ground slopes uniformly for a long distance, the distances may be measured more quickly and accurately along the surface of the ground as compared to by the method of stepping. Angle of slope may be measured by a clinometer, or Abney level directly. Angle of slope may also be computed from the difference in elevations obtained by differential or trigonometrical levelling. For details of levelling, please refer to chapter 6 *Levelling*.

Case 1. Measuring the Slope Angle With a Clinometer

The type of clinometer shown in Fig. 2.17, is most commonly used for measuring the angles of slope. It consists of:

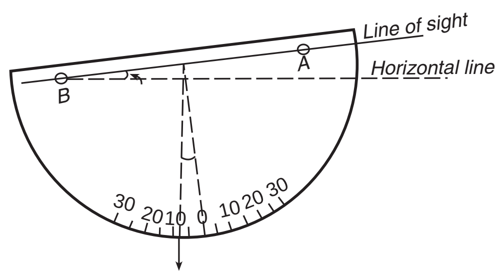


Fig. 2.17. A clinometer.

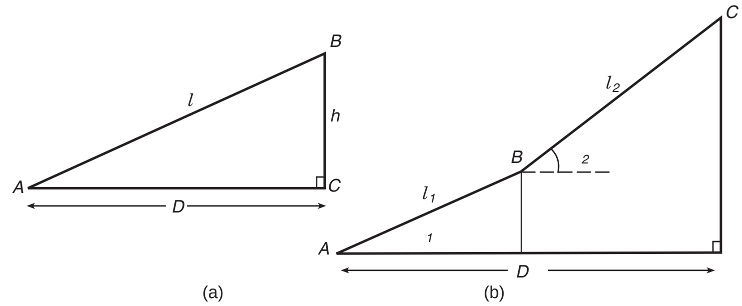
(*i*) A graduated semi-circle similar to a protractor.

(*ii*) Two pins *A* and *B* for sighting.

(*iii*) A light plumb-bob with a long thread suspended from the centre of the graduated semi-circular.

Procedure: The clinometer is fixed to a ranging rod at the height of the observer's eye. The surveyor turns the instrument till the line joining the pins *A* and *B* becomes parallel to the ground slope by sighting a ranging rod of equal height at the other station. The plumb bob gives the required angle of slope, on the graduated arc which may be verified geometrically.

Case 2. Measuring the Slope Angle From the Difference in Levels (Fig. 2.18.)



(a) (*b*).
Fig. 2.18. Measuring the slope angle.

The reduced levels of the two stations are obtained by differential levelling*. Let the difference in levels, of stations *A* and *B* be '*h*'. (Fig. 2.18 a).

If *l* is the inclined distance measured along the slope and '*h*' is the difference in levels, the angle of slope

$$\theta = \sin^{-1} \frac{h}{l}$$

Knowing the angle of slope, the horizontal distance *D* between stations *A* and *B* $= l \cos \theta$.

When the slope of the ground is not regular and it consists of varying slopes the line may be divided into a number of sections of uniform slopes. The length and slope of each section are measured separately. (Fig. 2.18 b).

Let l_1, l_2, l_3, etc. be the measured lengths of the sections

$\theta_1, \theta_2, \theta_3$, etc., be the angles of slope of the sections.

The required horizontal distance *D* between end stations.

$D = l_1 \cos \theta_1 + l_2 \cos \theta_2 + l_3 \cos \theta_3 + ... + l_n \cos \theta_n = \Sigma l \cos \theta$

Case 3. Hypotenusal Allowance Correction Method

This method is commonly used for locating a number of intermediate points by applying a correction to every chain length. The hypotenusal

* Differential levelling is discussed in chapter 6 Levelling.

allowance correction for each chain length, may be computed as under:

Let θ be the angle of slope of the ground.

$$AC = AB = 30 \text{ m} = 150 \text{ links}$$

$$AE = 150 \sec\theta \text{ links}$$

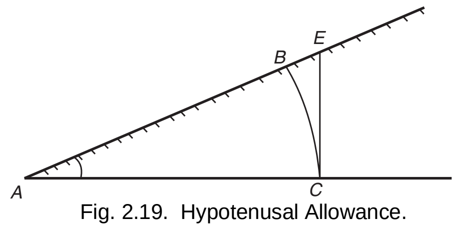


Fig. 2.19. Hypotenusal Allowance.

∴ Hypotenusal allowance

$$BE = AE - AB = 100 (\sec\theta - 1) \text{ links for a 20 m chain}$$

$$= 150 (sec\,\theta - 1) \text{ links for a 30 m chain.}$$

Procedure : While chaining along the slope, the chain should be laid on the surface of the ground. The arrow should be placed forward in the line of *AB* at *E* instead of at *B* by an amount equal to 100 (sec. θ – 1) *links* or 150 (sec θ – 1). The hypotensual allowance can be easily computed from a table of natural, secants. The follower, then holds the zero end of the chain at *E* and the process is continued until the end of the line is reached. The required horizontal distance *AL* is equal to the number of full chains measured plus a part of the chain if any (Fig. 2.20).

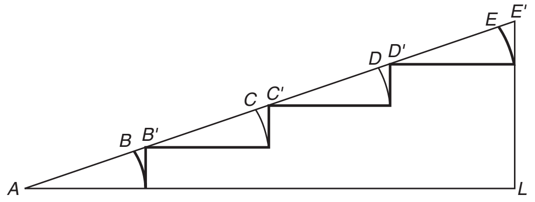


Fig. 2.20. Measurement by hypotenusal allowance.

Case 4. Computation of Horizontal Distances

When the difference h in reduced levels between the end stations of a sloping ground, is determined by levelling, the horizontal distance

$$D = \sqrt{l^2 - h^2}$$

where l = distance measured along the slope

h = difference in elevations *i.e.* the vertical distance between the end stations of the slope.

Comparison between the Direct and Indirect Methods

	Direct Method	*Indirect Method*
1.	It is more convenient, quicker and is preferred to if the ground is undulating and slopes are short and of varying degree.	1. It is more convenient, quicker and is perferred to if the ground is gently sloping with regular and log slopes.
2.	It is unsuitable when the ground is gently sloping and the steps are long.	2. The method of hpotenusal allowance is generally employed in routesurveys where through chainage, is required.
3.	The errors are likely to be introduced due to the sag of the chain and transferring the forward end of the chain vertically below on the ground.	3. No accumulation of errors due to sag.

2.11. ERROR IN CHAINING

The errors that generally occur in chaining, are classified under two categories *i.e.*,

(*i*) Cumulative errors

(*ii*) Compensating errors

1. Cumulative Errors. The errors which occur in the same direction and tend to accumulate, or to add up, are called *Cumulative errors.* Such an error makes the apparent measurements always either too long or too short.

A. Positive Cumulative Errors. The error which makes the measured length more than the actual, is known as *positive cumulative error.* Positive errors are caused in the following situations:

1. The length of the chain or tape is shorter than its standard length due to :

(*i*) Bending of the links.

(*ii*) Removal of too many rings from the chain during adjustment of its length.

(*iii*) Knots in the connecting links.

(*iv*) The field temperature being lower than that at which the tape was calibrated.

(*v*) Shrinkage of the tape when moist.

(*vi*) Clogging of rings with mud.

2. The slope correction ignored while measuring along the sloping ground.

3. The sag correction, if not applied, when the chain or tape is suspended at its ends.

4. Incorrect alignment.

5. Working in windy weather, when the tape bellys out.

B. Negative Cumulative Errors. The error which makes the measured length less than the actual, is known as *Negative Cumulative error.* Negative errors are caused in the following situations :

1. The length of the chain or tape is shorter than its standard length due to :

(*i*) Flattening of the connecting rings.

(*ii*) Opening of the ring joints.

(*iii*) The field temperature being higher than that at which the tape was caliberated.

Note. The following points may be noted :

(*i*) Cumulative errors are always proportional to the length of the line.

(*ii*) Cumulative errors, though large, can be corrected by applying the required correction.

2. Compensating Errors. The errors which are liable to occur in either direction and tend to compensate, are called *compensating errors*. These are caused in the following situations :

1. Incorrect holding of the chain.
2. The chain is not uniformly calibrated throughout its length.
3. Refinement is not made in plumbing during stepping method.
4. Chain angles are set out with a chain which is not uniformly adjusted.

Note. The following points may be noted :

(*i*) Compensating errors are proportional to the length of the line.

(*ii*) Though compensating errors are small as compared to cumulative errors, these can not be corrected as the nature of correction cannot be ascertained.

2.12. COMMON MISTAKES IN CHAINING

While chaining a line, an inexperienced chainman generally commits the following mistakes

1. Displacement of the arrows
2. Failure to observe the zero point of the tape
3. Adding or omitting a full chain length
4. Reading from the wrong end of the chain

5. Reading numbers wrongly
6. Reading wrong metre marks
7. Calling numbers wrongly
8. Wrong recording in the field book

2.13. CORRECTIONS FOR LINEAR MEASUREMENTS

For precise measurements, the following corrections are made :

(*i*) Correction for standard length
(*ii*) Correction for alignment
(*iii*) Correction for slope
(*iv*) Correction for tension
(*v*) Correction for temperature
(*vi*) Correction for sag
(*vii*) Reduction to M.S.L.

1. Correction for Standard Length. Before using a tape, its actual length is ascertained by comparing it with a standard tape of known length. The *designated nominal length* of a tape is its designated length *e.g.*, 30 m, or 100 m. The *absolute length* of a tape is its actual length under specified conditions. The absolute length of a tape is seldom equal to the nominal length of the tape.

Let L = measured length of a line.

C_a = correction for absolute length.

l = nominal or designated length of the tape.

C = correction be applied the tape.

then $$C_a = \frac{\text{L.C.}}{\text{l}} \qquad \text{...(2.1)}$$

The sign of the correction C_a will be the same as that of C. Before applying the Eqn (2.1), L and l must be expressed in the same units and also the units of C_a and C should be the same.

2. Correction for Alignment. Generally a survey line is set out in a continuous straight line. Sometimes, it becomes necessary, due to obstruction, to follow a bent line which may be composed of two or more straight portions subtending an angle other than 180°. (Fig. 2.21)

Let $AC = l_1;\ CB = l_2$

Angle $BAC = \theta_1$; Angle $ABC = \theta_2$

Length $\boldsymbol{AB = l_1 \cos \theta_1 + l_2 \cos \theta_2}$.

∴ The required correction

Fig. 2.21. Correction for alignment.

$$= (l_1 + l_2) - (l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$= l_1 (l - \cos \theta_1) + l_2 (l - \cos \theta_2) \qquad ...(2.2)$$

In case, stations *A* and *B* are not intervisible, the angle *ACB* (say) may be measured accurately with a theodolite* and the distance *AB* may be computed with the cosine formula :

$$AB = \sqrt{AC^2 + BC^2 - 2AC.\, BC.\, \cos \alpha} \qquad ...(2.3)$$

Note. The correction for alignment is always subtracted from the measured length of the line.

3. Correction for Slope. The distance measured along the slope between two stations, is always greater than the horizontal distance between them. The difference in slope distance and horizontal distance, is known as *slope correction* which is always *subtractive.* Angle of slope is measured with a theodolite.

The formulae for the slope correction are derived as follows: (Fig. 2.22).

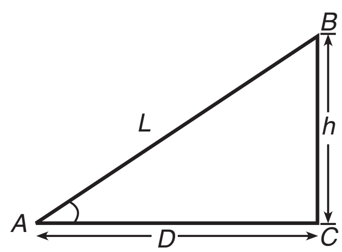


Fig. 2.22. Slope Correction.

Case 1. Let L = slope distance AB

$= D$ horizontal distance AC

$= h$ difference in reduced levels of A and B

$\therefore \quad = D \sqrt{L^2 - h^2}$

$\therefore$ Slope correction

$$= L - D$$

$$= L - (L^2 - h^{2)^{1/2}}$$

$$= L - L\left(1 - \frac{h^2}{2L^2} - \frac{h^4}{8L^4} \ldots\ldots\right)$$

* Theodolite discussed later.

$$= \frac{h^2}{2L} + \frac{h^4}{8L^3}$$

or slope correction $= \dfrac{h^2}{2L}$ ...(2.4)

If difference in elevation h is less than 3 m in a length of 20 metres, the quantity $\dfrac{h^4}{8L^3}$ equals to 0.1266×10^{-2} and hence may be neglected.

Case 2. Let θ = angle of slope

L = measured distance along the slope.

D = horizontal distance between A and $B = L \cos \theta$

Slope correction $= L - L \cos \theta$

$= L\,(1 - \cos \textit{theta}) = L$ versine θ ...(2.5)

Case 3. Let θ = angle of slope in degrees.

L = slope distance of AB

h = difference in elevation of A and B

Now $h = L \sin \theta$

$= L\,.\,\theta$ where θ is in radians.

$= L \times 0.01745 \times \theta$ where θ is in degrees.

Substituting the value of h in slope correction eqn. (2.4), we get

$$\text{Slope correction} = \frac{h^2}{2L} = \frac{(0.01745 \times L\theta)^2}{2L}$$

$$= 0.00015\, L\, \theta^2$$

or $\therefore$ Slope correction $= \dfrac{1.5 \times L\theta^2}{10{,}000}$ (approximately) ...(2.6)

4. Correction for Tension. If the pull applied to the tape during measurements, is more than the pull at which it was standardised, its length increases and hence the measured distances become less than actual. Correction for tension is therefore positive. On the other hand, if the applied pull is less, its length decreases and consequently the measured distances become more. The correction for tension is negative.

Remember. *If applied pull (or tension) is more, tension correction is positive, and if it is less, the correction is negative.*

Derivation of the Formula. When a tape is subjected to a tension of P newton weight, it gets elongated by a small amount within the elastic limits. The ratio of stress and strain which is known as *Young's Modulus of the Elasticity* of the material, is a constant.

Let P = Pull or tension in Newtons (N)

A = Cross–sectional area of the tape in square cm.

L = Length of the measured line.

l = Elongation of the tape.

Stress $= \dfrac{P}{A}$; strain $= \dfrac{l}{L}$

or $$E = \frac{\text{stress}}{\text{strain}} = \frac{P}{A} / \frac{l}{L} = \frac{PL}{Al}$$

or $$l = \frac{PL}{AE}$$

If Po = standard pull

P = pull applied during measurements

Then, correction for tension

$$= \frac{PL}{AE} - \frac{PoL}{AE} = \frac{(P - Po)}{AE} L \qquad ...(2.7)$$

The value of E for steel may be taken as 2.1×10^5 N/mm^2 and that for invar as 1.5×10^5 N/ mm^2.

5. Correction for Temperature. The length of a tape increases if its temperature is raised and decreases if its temperature is lowered. If the temperature of a tape is above normal, the correction is positive and if it is below normal, the correction is negative.

Let L = measured length of the line

T_m = mean temperature during measurements

T_o = normal temperature at standardisation

α = coefficient of thermal expansion of the tape material

Temperature correction

$$\alpha = (T_m \sim T_o)\, L. \qquad ...(2.8)$$

The value of α for invar is 0.000000122 per 1°C or less. For precise measurements, accurate value of the coefficient of expansion of the tape material must be carefully determined.

6. Sag Correction. When a tape is suspended from two supports in air, it assumes the shape of a catenary. The difference between the curved length of the tape and the horizontal distance between the supports, is known as *'sag correction.'* The apparent length of the tape is too long and as such sag correction is always *negative.*

Derivation of the Formula. (Fig. 2.23).

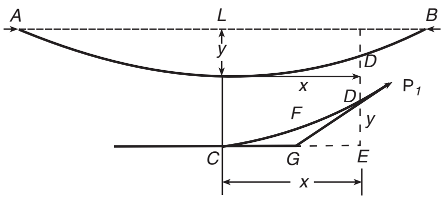


Fig. 2.23. Sag correction.

Suppose A and B are two supports at the same level.

L = horizontal distance between supports

Y = total dip at the centre of the tape.

Consider the equilibrium of a small portion CD of the tape *measured from the centre. Let its horizontal projection* CE be x metres.

The portion CD is subjected to three forces, namely,

H, a horizontal pull at C.

P_1, a pull tangential at D.

W, the weight of the tape portion between C and D.

As the sag is very small in comparison with the span, the length CD may be taken equal to x. The weight of the portion CD is wx where w is the weight per metre length of the tape.

For an equilibrium, three forces must pass through a point *i.e.*, H and P_1 must meet at G *a point* vertically below F.

Construction. Draw DE parallel to FG to meet CG produced at E.

Δ GED is apparently a triangle of forces

$$\therefore \qquad \frac{\omega x}{DE} = \frac{P_1}{\sqrt{GE^2 + DE^2}} = \frac{H}{GE}$$

or
$$\frac{\omega x}{y} = \frac{P_1}{\sqrt{\left(\frac{x}{2}\right)^2 + y^2}} = \frac{H}{\frac{x}{2}} \qquad ...(2.9)$$

where $DE = y$

When $x = \dfrac{L}{2}$ *i.e.*, at the support, $y = Y$ and $P_1 = P$, the pull applied with a spring balance.

or
$$\frac{\omega L}{2y} = \frac{P}{\sqrt{y^2 + \left(\frac{L}{4}\right)^2}} = \frac{4H}{L}$$

i.e., $$H = \frac{\omega L^2}{8Y} \qquad ...(2.10)$$

Substituting the value of H in Eqn. (2.9), we get

$$\frac{\omega x}{y} = \frac{2\omega L^2}{8xY}$$

$$\therefore \quad x^2 = \frac{L^2}{4Y} \cdot y$$

or $$x^2 = C.y \qquad ...(2.11)$$

where C is a constant

i.e., $$C = \frac{L^2}{4Y} \qquad ...(2.11a)$$

$x^2 = C.y$ is an equation of a parabola whose constant is C.

The length of the tape from the centre for a horizontal distance x, is calculated from the formula.

$$L_1 = \int_o^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$\therefore$ Total length of the tape

$$L_1 = 2\int_0^{L/2} \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2} dx \qquad ...(2.12)$$

Differentiating Eqn. (2.11) and substituting the value of $\frac{dy}{dx}$ in Eqn.(2.12), we get

$$L_1 = 2\int_0^{L/2} \left\{\left(1 + \frac{4x^2}{C^2}\right)\right\}^{1/2} dx$$

$$= 2\int_0^{L/2} \left[1 + \frac{1}{2} \cdot \frac{4x^2}{C^2} - \frac{1}{8}\left(\frac{4x^2}{C^2}\right)^2 + ...\right] dx$$

$$= 2\left[x + \frac{2(x)^3}{3C^2} + ...\right]_0^{L/2}$$ ignoring higher powers of x

$$= 2\left[\frac{L}{2} + \frac{2L^3}{24C^2}\right]$$

$$\therefore \qquad L_1 = L + \frac{L^3}{6C^2} \text{ approximately}$$

Substituting the value of C from Eqn. (2.11 a), we get,

$$L_1 = L + \frac{L^3(4Y)^2}{6(L^2)^2} = L + \frac{L^3 \times 16Y^2}{6L^4}$$

$$L_1 = L + \frac{8Y^2}{3L}$$

The parabola being comparatively flat, the pull P may be assumed equal to H.

From Eqn. (2.10), $H = \frac{\omega L^2}{8Y} = P$

Substituting the value of $Y = \frac{\omega L^2}{8P}$, we get

$$L_1 = L + \frac{\omega^2 L^3}{24P^2}$$

i.e.,

$$L_1 = L + \frac{L}{24} \cdot \left(\frac{\omega L}{P}\right)^2$$

$$\therefore \text{ Sag correction } = \frac{L}{24.}\left(\frac{W}{P}\right)^2 \text{ approximately} \qquad ...(2.13)$$

where W is total weight of the tape.

7. Reduction to M.S.L. (Fig. 2.24). The measured length of a line at an altitude of h metres above mean sea-level will be more as compared with the corresponding line on the mean sea-level surface. The difference in the length of the measured line and its equivalent length at sea-level, is known as an *error due to reduction to M.S.L.*

Derivation of the formula :

Let

L= distance AB measured at an altitude h metres above M.S.L.
L'= distance $A' B'$ reduced as M.S.L.

R = radius of the earth

θ = angle AOB subtended by the line AB at the centre of the earth.

Then, $L = (R + h)\,\theta$ ...(2.14)

and $L' = R\,\theta$ ...(2.15)

By dividing Eqn. (2.14) by Eqn. (2.15) and transposing, we get,

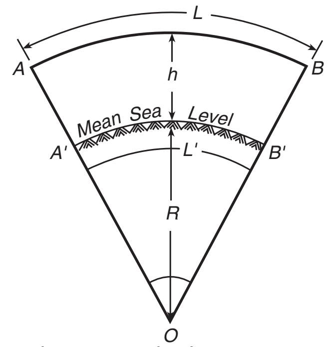


Fig. 2.24. Reduction to M.S.L.

$$\frac{L}{R+h} = \frac{L'}{R}$$

or $$L' = \frac{R}{R+h} \cdot L$$

∴ Correction due to reduction to M.S.L.

$$= L - L' = L - \frac{R}{R+h} L = L\left(1 - \frac{R}{R+h}\right)$$

$$= L \frac{h}{R+h} \qquad \text{...(2.16)}$$

or Correction M.S.L. $$= L \cdot \frac{h}{R} \text{ approximately.} \qquad \text{...(2.16}a\text{)}$$

2.14 NORMAL TENSION

The pull or tension which when applied to a tape supported in air over two ends, equalises the correction due to pull and the correction due to sag is known as *Normal Tension*. As the pull increases the length of a tape and the sag decreases its length, the normal tension neutralizes both the corrections and hence no correction is necessary.

The correction for pull

$$C_1 = \frac{(P - P_0)L}{AE} \qquad (+\text{ ve}) \qquad ..(i)$$

The correction for sag

$$C_2 = \frac{L.W^2}{24P^2} \qquad (-\text{ ve}) \qquad ...(ii)$$

Equating numerically the equations, (*i*) and (*ii*) we get

$$\frac{(P - P_0)L}{AE} = \frac{L \cdot W^2}{24P^2}$$

or $$P = \frac{0.204\, W\sqrt{AE}}{\sqrt{P - P_0}} \qquad ...(2.17)$$

where W = weight of the tape supported

P = applied normal tension

P_0 = tension at which tape was standardised.

The value of the normal tension P, is determined by trial and error with the help of equation (2.17).

Example 2.9. *A line was measured with a steel tape which was exactly 30 metres at 20° C at a pull of 100 N (or 10 kgf), the measured length being 1650.00 metres. The temperature during measurement was 30°C and the pull applied was 150 N (or 15 kgf). Find the length of the line, if the cross-sectional area of the tape was 0.025 sq. cm. The coefficient of expansion of the material of the tape per 1° C =* 3.5×10^{-6} *and the modulus of elasticity of the material of the tape =* 2.1×10^5 *N/mm*^{2} *(*2.1×10^6 *kg/cm*^{2}*).*

Solution.

(*i*) Correction of temperature per tape length

$$= \alpha\,(T_m - T_o)\,L = 0.0000035\,(30 - 20) \times 30$$

$$= 0.00105 \text{ m } (+\text{ve})$$

(*ii*) Correction for pull per tape length

$$= \frac{(P_m - P_o)\,L}{AE} = \frac{(150 - 100) \times 30}{2.5 \times 2.1 \times 10^5}$$

$$= 0.00286 \text{ m } (+\text{ve})$$

Combined correction = 0.00105 + 0.00286 = 0.00391 m

$\therefore$ True length of the tape = 30 + 0.0039 = 30.0039 m

$\therefore$ True length of the line

$$= \frac{30.0039 \times 1650.00}{30}$$

= 1650.21 m. **Ans.**

Example 2.10. *To measure a base line, a steel tape 30m long standardised at* 15°C *with a pull of 100 N (or 10 kgf) was used. Find the correction per tape length, if the temperature at the time of measurement was 20° C and the pull exerted was 160 N (or 16 kgf.) Weight of of 1 cubic* *cm of steel is 0.0786 N (or 0.00786 kgf). Weight of the tape = 8N (or 0.8 kgf). $E = 2.1 \times 10^5$ kg/sq. cm. Coefficient of expansion of the tape per 1°C = 7.1×10^{-7}.*

Solution.

Let A = area of cross–section of the tape in sq. cm.

Then, $A \times 30 \times 100 \times 0.0786 = 8$

$$A = \frac{8}{30 \times 100 \times 0.0786}$$

$$= 0.034 \text{sq. cm} = 3.39 \text{ mm}^2.$$

(*i*) Correction for pull

$$= \frac{(P_m - P_o)\, L}{AE} = \frac{(160 - 100) \times 30}{3.39 \times 2.1 \times 10^5}$$

$$= \frac{1800}{33{,}927 \times 21} = 0.0025 \text{ m}$$

(*ii*) Correction for temperature

$$= \alpha\,(T_m - T_o) \times L = 7.1 \times 10^{-7} \times (20 - 15) \times 30$$

$$= \frac{5 \times 30 \times 7.1}{10^7}$$

$$= \mathbf{0.0001065\ m.}$$

$\therefore$ Total correction per tape length = 0.0025 + 0.0001

= 0.0026 m. **Ans.**

Example 2.11. *A tape 100 m long, 6.35 mm wide, 0.5 mm thick was used to measure a line, the apparent length of which was found to be 1986.96 m. The tape was standardised under a pull of 67.5 N (or 6.75 kgf), but after the line was measured, it was found that the pull actually used during the measurement was 77.5 N (or 7.75 kgf). What was the true length of the line if the tape was standardised and used on the flat?*

Take Young's modulus, E = 200000 N/mm^2 *(or 20,00,000* kg/cm^2*).*

Solution.

Correction for pull per tape length

$$C_p = \frac{(P - P_o)\, L}{AE}$$

$$= \frac{(77.5 - 67.5)\,100}{(6.35 \times 0.5) \times 200000}$$

$$= \frac{1000}{3.175 \times 200000} = 0.001575 \text{ m } (+\text{ ve})$$

∴ True length of the tape

$$= 100 + 0.001575 = 100.001575 \text{ m}$$

∴ True length of the line

$$= \frac{100.001575}{100} \times 1986.96 = 1986.991 \text{ m} \qquad \textbf{Ans.}$$

Example 2.12. *A base line AC was measured in two parts along two straight drains AB and BC of length 1650 m and 1819.5 m with a steel tape which was exactly 30 metres at 25°C at a pull of 9 N (or 10 kgf). The applied pull during measurement of both parts was 200 N (0.20 kgf) whereas respective temperatures were 45°C and 25°C. The slopes of drains AB and BC were 3° and 3°30′ and the deflection angle of BC was 10° right. Find the correct length of the base line if the cross-section area of the tape was 2.5 mm²*. The coefficient of expansion and modulus of elasticity of tape material were 3.5×10^{-6} *per or* 1°*C and* 21×10^5 *N/mm² respectively*.

Solution.

(*i*) Correction for temperature per tape length

For length $AB = \alpha\,(T_m - T_0)\,L = 0.0000035\,(45 - 25) \times 30$

$= 0.00210$ m (+ ve)

For length $BC = 0.000035\,(40 - 25) \times 30$

$= 0.001575$ m (+ ve)

(*ii*) Correction for pull per tape length

$$= \frac{(P_m - T_o)\,L}{AE} = \frac{(200 - 100) \times 30}{2.5 \times 21 \times 10^5}$$

$= 0.0057$ m (+ ve)

∴ Combined correction for the length *AB*

$= 0.0021 + 0.0057 = 0.0078$ m

and Combined correction for the length *BC*

$= 0.001575 + 0.0057 = 0.007275$ m

∴ True length of the tape for measuring *AB* = 30.0078 m and True length of the tape for measuring BC = 30.007275 m

$\therefore$ True length of AB

$$= \frac{30.0078}{30} \times 1650$$

$$= 1650.429 \text{ m}$$

and True length of BC

$$= \frac{30.007275}{30} \times 1819.5$$

$$= 1819.941 \text{ m}$$

Length of AB corrected for slope

$$= 1650.429 \cos 3°$$

$$= 1648.168 \text{ m}$$

Length of BC corrected for slope

$$= 1819.941 \cos 3° \, 30'$$

$$= 1816.547 \text{ m}$$

$\therefore$ Length of the base AC

$$= \sqrt{(1648.168)^2 + (1816.547)^2 - 2 \times 1648.168 \times 1816.547 \cos 170°}$$

$$= 3451.562 \text{ m.}$$ **Ans.**

Example 2.13. *A survey line AB is running along different slopes as detailed below. There is a downward slope of 1 in 10 from station A to chain age 238 m. The ground has an angle of elevation of 8° 15′from chainage 238 m to chainage 465 m. There is a rise of 25 m from chainage 465 m to station B having a change of 665 m. All the measurements of chainages have actually been taken along the ground. It was also found that the 20 m chain used for chaining was 5cm too long throughout the work.*

Calculate the correct horizontal distance from station A to station B in this case.

Solution. (Fig. 2.25)

Let AC, CD and DB be the three slopes between A and B

α be the angle of depression of slope AC.

$$\therefore \qquad \alpha = \tan^{(-1)} \frac{1}{10} = 5° \, 42' \, 38''$$

Correct horizontal distance $AC = 238. \cos \alpha$

$$= 236.82\text{m} \qquad \ldots(i)$$

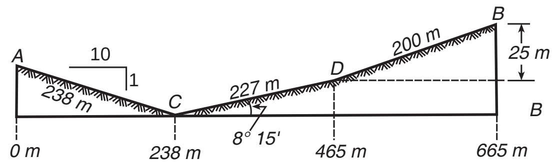


Fig. 2.25.

Correct horizontal distance $CD = 227.568 \cos 8° 15'$

$= 224.65$ m ...(*ii*)

Correct horizontal distance $DB = \sqrt{(200)^2 - (25)^2}$

$= 198.43$ m ...(*iii*)

Correct horizontal distance *AB*

$(i) + (ii) + (iii) = 236.82 + 224.65 + 198.43$

$= 659.9$ m. *Ans.*

Example 2.14. *Show that for a chain of 3 mm² cross-sectional area and 0.48 kg weight (material $E = 2 \times 10^6$ kg/cm²) standardised at 8 kg tension, the normal pull is 12 kg.*

Solution.

We know that

$$P = \frac{0.204\, W \sqrt{AE}}{\sqrt{P - P_o}} \quad ...(i)$$

where P = normal pull

P_o = Pull at standardisation = 8 kg

A = Area of cross-section of the tape

$= 3 \text{ mm}^2 = 0.03 \text{ cm}^2$

W = Weight of the tape = 0.48 kg

A = Youngs modulus of elasticity

$= 2 \times 10^4 \text{ kg/cm}^2$

Substituting the values in equation (*i*) we get

$$P = \frac{0.204 \times 0.48 \sqrt{0.03 \times 2 \times 10^6}}{\sqrt{P - 8}}$$

$\therefore \quad P\sqrt{P - 8} = 23.9854 = 24$

By trial and error $P = 12$ kg Proved.

EXERCISE 2

1. Fill in the blanks with suitable word(s).

(*i*) A 30 m chain is generally divided into ....links and each link measures ....cm.

(*ii*) Links are connected together by means of ....oval rings.

(*iii*) The ends of a chain are provided with ....handles with ....joints.

(*iv*) In.............ranging, the end stations are not intervisible.

(*v*) Three types of chains are (i)............(ii).........(iii)......

(*vi*) A linen tape reinforced with brass or copper wires, to prevent stretching or twisting of fibers, is called a ....tape.

(*vii*) Invar tape is made of an alloy of steel and ....

(*viii*) ....is the most accurate method of measuring distances.

(*ix*) The length of a standard arrow is ....cm.

(*x*) Generally.......arrows accompany a chain.

(*xi*) The process of marking intermediate points on a survey line, is known as............

(*xii*) The method of measuring horizontal distances along the slope directly, is known as ......method.

(*xiii*) The hypotenusal allowance for a chain is ....where θ is the angle of slope.

(*xiv*) The errors which occur in either direction and tend to accumulate or add up, are known as ......errors.

(*xv*) Incorrect length of a chain is a source of ......error.

(*xvi*) The errors which occur is either direction, are known as ......errors.

(*xvii*) Correction for slope is equal to.......where h is the difference in heights and L is the sloping distance.

(*xviii*) Correction for slope is always......

(*xix*) Sag correction is given by.......where L = length of tape; W = weight of tape and P = the pull.

(*xx*) Correction due to reduction to M.S.L. is equal to........where L is the length of a line h metres above M.S.L. and R is the radius of the earth.

(*xxi*) The pull which equalises the corrections due to sag and pull, is known as........

(*xxii*) Correction for sag and slope is given by......where P is the pull applied, P_o is the standard pull, A is the area of its cross-section and E is the young's modulus of elasticity.

2. Use suitable word(s) from the brackets to fill in the blanks:

(*i*) ....is used for accurate measurement.

(chain/invar tape/metallic tape)

(*ii*) Each metre length of a metric chain is divided into.......links. (5, 10, 12)

(*iii*) For the measurement of short offsets.....are used. (offset rods/ranging rods/both)

(*iv*) Links of a chain are connected together with......rings. (one, two, three)

(*v*) End link.......the length of the brass handle. (does not include, includes)

(*vi*) The length of a chain is measured from the........of one handle to the ....of the other. (outside, inside)

(*vii*) The length of a link of a metre chain is........cm. (10, 20, 30)

(*viii*) The over-all length of an adjusted 20 m chain should be within......mm. (± 2, ± 3, ± 8)

(*ix*) A steel tape is........to a metallic tape. (superior, inferior)

(*x*) The length of the metal ring........included in the overall length of a steel tape. (is, is not)

(*xi*) Invar tape is made of an alloy of......and steel. (copper, brass, nickel)

(*xii*) In indirect ranging........is obstructed. (vision, chaining or both)

(*xiii*) A line ranger consists of two......... (plane mirrors, prisms, lenses)

(*xiv*) Random line method of ranging, is adopted when the end stations are........ (visible, not visible)

(*xv*) The accuracy of chaining depends upon the alertness of...... (leader, follower or both)

(*xvi*) The correction for slope is given by..... $\left(\frac{h^3}{2L}, \frac{h^2}{3L}, \frac{h^2}{2L}\right)$

(*xvii*) Hypotenusal correction for 20 m chain for measuring along slopes, is given by ....links. [$100\,(1 - \cos\theta)$, $100\,(\sec\theta - 1)$; $100\,(1 - \sec\theta)$]

(*xviii*) Sag correction can be neutralised by applying.......... (normal tension, tension correction)

3. State whether following statements are true or false:

(*i*) A metallic tape is made of an alloy of nickel and steel.

(*ii*) Error due to faulty length of a chain is cumulative.

(*iii*) Line ranger is used for ranging when end stations are not intervisible.

(*iv*) It is better to move up than step down while measuring the distance along sloping ground.

(*v*) Cumulative errors though large, as compared to compensating errors, can be corrected but not the compensating errors.

4. Describe different types of chains commonly used in surveying, stating the special advantages of each.

5. Describe different types of tapes commonly used in surveying stating the advantages of each.

6. Give a complete list of instruments and equipments required for measuring the distance between two points on the surface of the earth.

7. Describe, in brief, the method of chaining mentioning the duties of the leader and the follower.

8. Explain the method of chaining on sloping ground in detail mentioning the precautions to be taken to get direct horizontal distances.

9. Explain, in detail, how a chain is tested and adjusted in the field.

10. Describe how you would range a survey line between two points which are not intervisible due to an intervening raised ground.

11. Describe the common mistakes which are likely to be made in chaining. What precautions would you take to guard against them ?

12. Give a list of corrections to be applied to measurements made with a tape and say whether they are additive or subtractive.

13. What are the different kinds of ranging ? Describe with sketches the method used for ranging across a high ground.

14. Give a list of sources of errors in linear measurements and say which of them are cumulative and which are compensating.

15. Enumerate the cumulative and compensating errors in chaining and state how these are eliminated.

16. What are the sources of cumulative errors in chaining survey times?

17. Explain the following ;

(*i*) The leader and the follower. (*ii*) Hypotenusal allowance.

(*iii*) Drop arrow. (*iv*) Invar tape.

(*v*) Offset rod.

(*vi*) Nominal and designated lengths of a chain.

(*vii*) Normal tension

(*viii*) Reciprocal ranging

(*ix*) A line ranger.

(*x*) A mallat.

18. How will you use a line ranger in the field ? Draw a neat sketch.

19. The length of a line measured with a 20-metre chain was found to be 375 metres. The true length of the line was known to be 374.5 metres. Find the error in the chain.

20. The length of a line found to be 600 metres when measured with a 20-metres chain. If the chain is 15 cm too short, find out the correct length of the line.

21. A survey line was measured with a 30 metre chain on a falling gradient of 1 in 8 and found to be 294.6 metres. Later, however it was noted that the chain used was 0.6 link too short. Find out the correct length of the line.

22. A 30 m chain was found to be 15 cm too long after changing 1524 m. The same chain was found to be 30.5 cm too long after chaining the total distance of 3048 m. Find the correct length of the total distance chained assuming that the chain was correct at the commencement of chaining.

23. The length of a line measured on a slope of 150 was recorded as 1500 metres. It was subsequently found that 20-metre chain was 0.5 link too long. Calculate the true horizontal length of the line.

24. A line measured on a rising gradient of 1 in 12 was found to be 450.5 m. It was afterwards found that the 30 m chain used for the purpose was 5 cm too short. Find the correct horizontal length of the line.

25. A 20 metre tape is held 40 cm out of line. Find the resulting error per tape length.

26. A chain was tested before starting a survey and was found to be exactly 30 m. At the end of the survey it was tested again and found to measure 30.10 m. The area of the plan of the field drawn to a scale 1 cm = 20 m was 160 sq. centimeters. Find the true area of field in square metres.

27. (a) How is a chain standardised ? Describe how you would adjust it if it is found too long.

(*b*) A 20 metre chain was found to be 20.05 m at the beginning of a survey and 20.15 m long at the end of the survey. The area of the plan drawn to scale of 1 cm = 10 m, was measured with the help of a planimeter and was found to be 32 sq. cm. Find the correct area of the field.

28. (*a*) Explain the different types of chains commonly used in surveying stating the special advantages of each. Explain how a chain is tested and adjusted.

(*b*) A chain was tested before starting a survey and was found to be exactly be 20 metres. At the end of the survey, it was tested again and found to measure 20.05 metres. The area of the plan of the field drawn to a scale of 1 cm = 16 metres was 130.7 sq. cm. Find the true area of the field in hectares.

29. A 30 m tape is suspended between its ends under a pull of 100 N (10 kgf). If the weight of the tape is 7 N (or 0.7 kgf), find the correct length of the distance between the ends of the tape.

30. A line 3.2 km long is measured with a steel tape which is 20 m under no pull at 30° C. The tape in section is 1/8 cm wide and 1/20 cm thick. If one-half of the line is measured at a temperature of 40° C and the other half at 50° C and the tape is attached to a pull of 200 N (or 20 kgf), find the corrected total length of the line given the coefficient of expansion is 11.5×10^{-6} per degree C, weight of tape per cu cm. of steel = 0.077504 N (or 7.7504 kgf), $E = 2.11 \times 10^5$ kg/cm^2.

ANSWERS

1. (*i*) 150 ; 20; (*ii*) three; (*iii*) brass, swivel; (*vi*) Indirect; (*v*) Engineer, Metric, Gunter; (*vi*) Metallic; (*vii*) nickel; (*viii*) Chaining; (*ix*) 40; (*x*) 10; (*xi*) Ranging; (*xii*) Stepping; (*xiii*) (*sec* θ – 1); (*xiv*) Cumulating; (*xv*) Cumulative; (*xvi*) Compensating; (*xvii*) $\frac{h^2}{2L}$; (*xviii*) Negative; (*xix*) $\frac{LW^2}{24P^2}$; (*xx*) $L\frac{h}{R+h}$ (*xxi*) normal tension; (*xxii*) Negative; (xxiii) $\frac{(P_m - P_0)\,L}{AE}$.

2. (*i*) Invar tape; (*ii*) 5; (*iii*) Offset rods; (*iv*) three; (*v*) includes; (*vi*) outside; (*vii*) 20; (*viii*) ± 5; (*ix*) superior; (*x*) is; (*xi*) nickel; (*xii*) vision; (*xiii*) prisms; (*xiv*) not visible; (*xv*) follower; (*xvi*) $\frac{h^2}{2L}$; (*xvii*) 100 (*sec* (θ – 1); (*xviii*) normal tension.

3. (*i*) True; (*ii*) True; (*iii*) False; (*iv*) False; (*v*) True.

19. 2.7 cm	20. 595.5 m.
21. 291.16 m.	22. 3063.37 m.
23. 1456.13 m.	24. 448.20 m
25. 0.004 m.	26. 64213.50 sq. m.
27. 3232.08 sq. m.	28. 3.3543 hectares.
29. 29.94 m.	30. 3199.418 m.

3

Chain Surveying

3.1. INTRODUCTION

Chain surveying is one of the methods of land surveying.

It is the system of surveying in which sides of various triangles are measured directly in the field and no angular measurements are taken. It is the simplest but accurate method of land surveying.

3.2. PURPOSE OF LAND SURVEYING

The land surveys are generally carried out for one of the following purposes:

1. To secure necessary data for exact description of the boundaries of a plot of land.
2. To determine the area of a plot of land.
3. To prepare an accurate plan of a plot of land.
4. To demarcate the boundaries of a plot of land in a previously surveyed area.
5. To divide a plot of land into a number of smaller units.
6. To secure data for executing engineering projects, *i.e.* alignment of roads, railway lines, canals, etc.

3.3. SUITABILITY OF CHAIN SURVEYING

Chain surveying is most suitable in the following cases:

1. When the ground is fairly level and open with simple details.
2. When large scale plans are required such as those for a factory site.
3. When the area is comparatively small in extent.

3.4. UNSUITABILITY OF CHAIN SURVEYING

Chain surveying is unsuitable in the following cases:

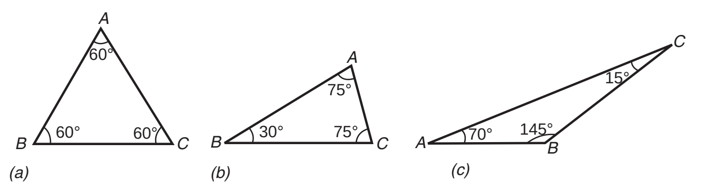
1. For large areas.
2. For areas crowded with many details.
3. For wooded countries.
4. For undulating areas.

3.5. PRINCIPLE OF CHAIN SURVEYING

The principle of chain surveying is to divide the area into a number of triangles of suitable sides. As a triangle is the only simple plane geometrical figure which can be plotted with its sides alone, a network of triangles is preferred to in chain surveying.

3.6. SHAPE, SIZE AND ARRANGEMENT OF TRIANGLES

When the position of a point, is fixed by the intersection of two arcs, its displacement due to errors in their radii is minimum, if the arcs intersect at 90°. In chain surveying all the three sides of a triangle are liable to error. Hence, all the sides of the triangle, should preferably be equal, having each angle nearly 60°. An equilateral triangle is also more accurately plottable than an obtuse angled triangle. Hence, to ensure minimum distortion due to errors in measurement and plotting, the best shaped triangle is an equilateral triangle. Due to configuration of the ground, it is not always possible to have equilateral triangles. Attempts should, therefore, be made to have triangles which are very nearly equilateral. Such triangles are known as *well conditioned or well shaped triangles*. A well conditioned triangle should not contain any angle smaller than 30° and greater than 120° (Fig. 3.1).



(a) An ideal triangle. (b) A well conditioned triangle (c) An ill conditoned triangle
Fig. 3.1.

The triangles having angles smaller than 30° or greater than 120°, are known as *ill-conditioned triangles*. Ill conditioned triangles should always be avoided. In case, ill-conditioned triangles are unavoidable, greater care must be taken while their chaining and plotting of their sides.

The size of a triangle depends upon the nature of the details and the terrain. If the terrain is open with lesser details, large triangles may be adopted. On the other hand, if the terrain is crowded with details, small sized triangles may be suitable.

The exact arrangement of the triangles to be adopted, in the field, depends upon the shape, topography of the ground and the natural or artificial obstacles met with.

In the layout of chain survey (Fig. 3.2)

1. Base line; *AC*
2. Main surveying lines : *AB, BC, CD* and
3. Subsidiary or tie lines : *BE* and *FD*
4. Check lines : *HG* and *MN*

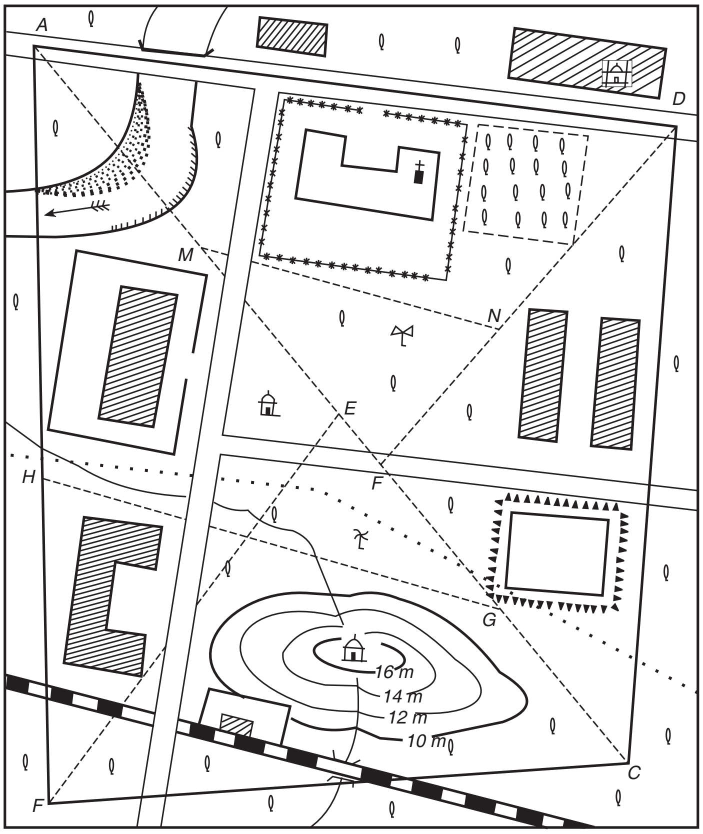


Fig. 3.2. Layout of a chain survey.

5. Main surveying stations : *A, B, C* and *D*
6. Subsidiary stations : *E* and *F*.

3.7. TECHNICAL TERMS AND THEIR DEFINITIONS

The important technical terms used in chain susveying are :

1. Main Survey Station. The point where two sides of a main triangle meet is called, *a main survey station.* Main survey station is a point at either end of a chain line.

2. Subsidiary Survey Station. (or tie station). The stations which are selected on the main survey lines for running auxiliary lines, are called *subsidiary stations.*

3. Main Survey Lines. The chain line joining the two main survey stations, is known as the *main survey line*.

4. Auxiliary, Subsidiary, or Tie Lines. The chain line joining two subsidiary survey stations, is known as *auxiliary, subsidiary* or more commonly as *tie line.* Auxiliary lines are provided to locate the interior details which are far away from the main lines.

5. Base Line. The longest of the main survey lines, is called a *base line.* Various survey stations are plotted with reference to the base line.

6. Check Lines. The line which is run in the field to check the accuracy of the field work, is called the *check line.* If the measured length of a check line agrees with the length scaled off the plan, the survey is accurate.

Each triangle is generally provided with a check line. The check lines may be laid in such a way that maximum number of details are intersected by it. Check lines may also be laid by joining the apex of the main triangle to any point on the opposite side or by joining two points on any two sides of the triangle.

3.8. SELECTION OF STATIONS

The following points should be kept in mind while selecting survey stations:

1. Main survey stations at the ends of chain lines, should be intervisible.
2. Survey lines should be minimum possible.
3. The main principle of surveying *viz.,working from the whole to the part and not from the part to the whole,* should be strictly observed.
4. Survey stations should form well conditioned triangles.
5. Every triangle should be provided with a check line.
6. Tie lines should be provided to avoid too long offsets.
7. Obstacles to ranging and chaining, if any, should be avoided.
8. The larger side of the triangle should be placed parallel to boundaries, roads, buildings, etc. to have short offsets.
9. To avoid tresspassing, main survey lines should remain within the boundaries of the property to be surveyed.
10. Chain lines should lie preferably over level ground.
11. Lines should be laid on one side of the road to avoid interruption to chaining by moving traffic.

3.9. SELECTION AND MEASUREMENT OF THE BASE LINE

In chain surveying, the base line is the most important line as it fixes the directions of all other chain lines. The following points are kept in view while selecting and measureing a base line.

1. It should be laid preferably on a level ground.
2. It should be run through the centre of the length of the area.
3. It should be correctly measured horizontally.

4. It should be measured twice or thrice and the mean value accepted as its correct length.
5. Great care should be taken, to ensure straightness of the base line while measuring.
6. If convenient, two base lines perpendicularly bisecting each other should be laid out.

3.10. OFFSETS

In chain surveying, the positions of details *i.e.*, boundaries, culverts, roads stream bends, etc., are located with respect to the chain line by measuring their distances right or left of the chain line. Such lateral measurements are called *offsets.* There are two types of offsets *i.e.,*

(*i*) Perpendicular offsets, (*ii*) Oblique offsets.

1. Perpendicular offsets. When the lateral measurements for fixing detail points, are made perpendicular to a chain line, the offsets are known as *perpendicular or right angled offsets. EN* is a perpendicular offset on the right side of the chain line *AB*. (Fig. 3.3 *a*)

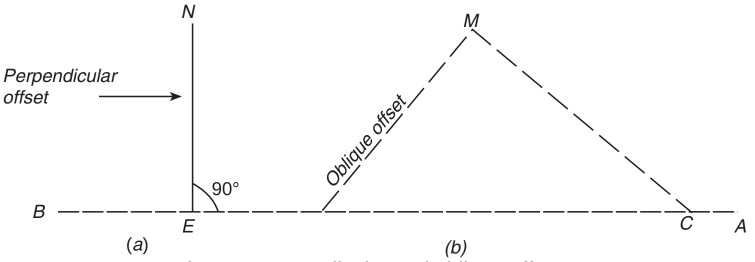


Fig. 3.3. Perpendicular and oblique offsets.

2. Oblique Offsets. When the lateral measurements for fixing detail points, are made at any angle to the chain line, the offsets are known as *oblique offsets*. *CM* and *DM* are oblique offsets on the right side of the chain line *AB*. (Fig. 3.3 b)

3. Short Offsets. The offsets having their length less than 15 m are called *short offsets*.

4. Long Offsets. The offsets having their length more than 15 m, are called *long offsets.*

3.11. MEASUREMENT OF PERPENDICULAR OFFSETS

The offsets are generally measured either with a metallic or steel tape depending upon the accuracy aimed at in surveying.

For every offset, following two measurements, are involved:

1. The distance along the chain line. The total distance of the foot of an offset from the starting station of a chain line, is called *chainage.*

2. The length of the offset. The distance of the detail point from the foot of its offset is called *length of the offset.*

In Fig. 3.4 *AB* is a chain line and *CP* is a perpendicular offset on leftside. The distance *AC* from *A* is known as the *chainage of the offset* and distance *CP* from *C* is the length of the offset.

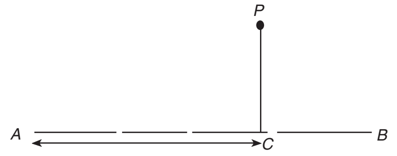


Fig. 3.4. Measurement of perpendicular offset.

For short offsets, perpendicular directions are generally judged by eye only. When offsets are long and better accuracy is required, right angles are set out with an optical square* or a cross-staff.*

Long offsets should be avoided to minimise the error due to incorrect length of the tape or incorrect direction. Moreover, short offsets are measured more quickly and accurately as compared to long offsets.

3.12. MEASUREMENT OF OBLIQUE OFFSETS

Oblique offsets are taken to locate details at a great distance from the chain line or for important details such as the corners of a building or boundary pillars of adjoining properties. These are also sometimes used for checking the accuracy of the perpendicular offsets. Two oblique offsets are taken for locating the position of a detail point by drawing two arcs. For an example, oblique offsets are taken from *C* and *D* on chain line *AB* to survey the corner H of a building. Similarly, oblique offsets are taken from *E* and *G* to check the accuracy of the perpendicular offset taken from *F* which was measured to survey the location T of a tree (Fig. 3.5).

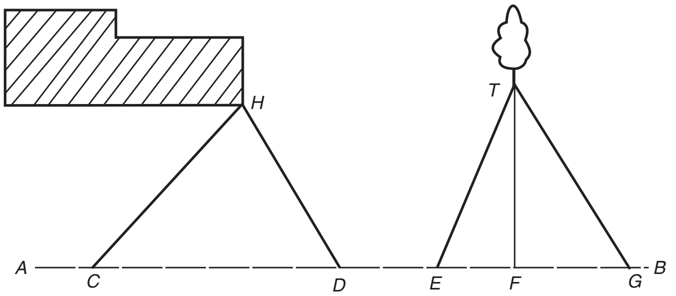


Fig. 3.5. Measurement of oblique offsets

3.13. TAKING OFFSETS

Offset measurements are taken and noted in the field and the complete operation is known as *'taking offsets'.* In the absence of an optical square or a cross-staff, an offset is taken as follows :

* Optical square, discussed later
** Cross staff, discussed later.

The leader holds the zero end of the tape at *P* for which offset is taken and the follower swings off the chain in a short arc about the point *P* as its centre. He finds the minimum reading on the tape which gives the position of the foot of the perpendicular from *P* on *AB*. (Fig. 3.6) Such an offset is called a *swing offset*.

The follower then fixes an arrow at *C* so found and reads the chainage and the length of the offset. The surveyor, after checking, records the readings in a field book.

The leader holds the zero end of the tape at *P* and follower swings an arc to cut chain at *E* and *G* to intersect the chain line at two points, (Fig.3.6). He finds the mid-point of *E* and *G* at *F* which the foot of the offset.

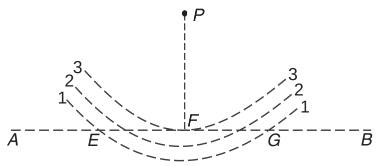


Fig. 3.6. Swing offset.

3.14. AVOIDING LONG OFFSETS

Where a better accuracy is required, long offsets as far as possible, should be avoided. To avoid long offsets, the chain lines must be arranged carefully. As already explained in article 3.9 chain lines must be laid parallel to boundaries and roads where possible. When there is a considerable bend in the outline of a road, fence or nalla, it is always advisable to run a subsidiary triangle such as *OPQ* on the main line *AB*. The subsidiary triangle should be well proportioned and the subsidiary lines *OP* and *QP* run sufficiently close to the detail lines. (Fig. 3.7.)

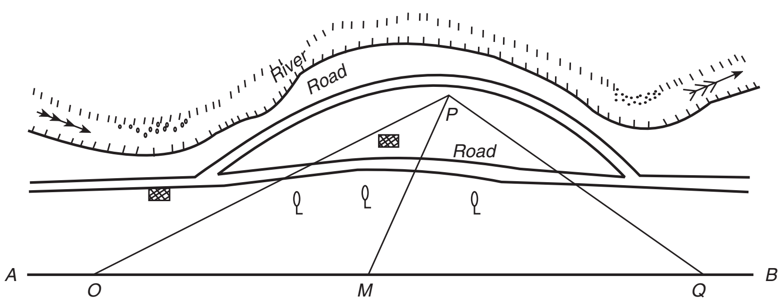


Fig. 3.7. Avoiding long offsets.

The offsets to the road and nalla, are then taken from the subsidiary lines *OP* and *PQ* thus avoiding long offsets from the main survey line *AB*. The accuracy of the subsidiary triangle *OPQ* should be checked by measuring an extra check line such as *MP*. It may be noted that running a few extra lines, expedites the survey to locate the details with short offsets than to locate them with long offsets.

Number of offsets. Number of offsets depend upon the nature of the detail points and the accuracy aimed at. The main rule to be kept in mind is :

"Take as many offsets as are sufficient to define the out-line of the object clearly and accurately". Offsets are taken to every change in the alignment of the detail. The following points must be kept in mind while taking offsets.

1. Circular details. If a detail is round, an offset should be taken to its centre and its radius is measured. (Fig. 3.8.)

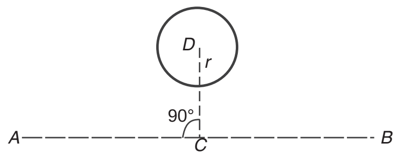


Fig. 3.8. Offset to ground details.

2. Polygonal details. If a detail is hexagonal or octagonal, the ends of the side nearer to the chain line should be located by taking offsets and the length of the sides measured. (Fig. 3.9).

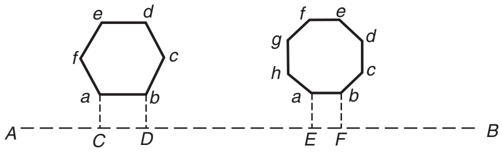


Fig. 3.9. Offsets to hexagonal or octagonal details.

3. Straight details. For straight details such as a road, a boundary, etc., offsets to its each end or change point need only be taken. In case of long straight details, a few additional offsets at intervals, may be taken to provide a check on the accuracy of the work. (Fig. 3.10.)

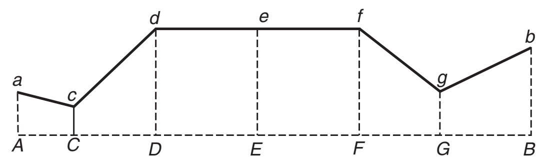


Fig. 3.10. Offset to straight details.

4. Irregular details. While surveying an irregular boundary or detail, it may be divided into a series of lengths which could be assumed as straight. Sufficient number of offsets are taken to locate their positions.

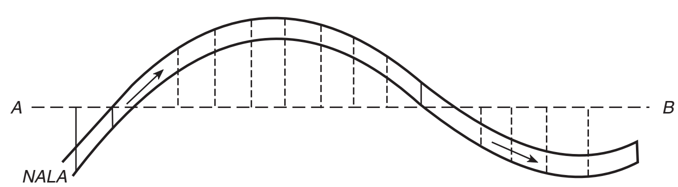


Fig. 3.11. Offsets to irregular boundaries.

5. Curved details. For surveying curved foot paths or roads, offsets may be taken to its beginning, middle, and the end of the curve and also a few points in between. (Fig. 3.12.)

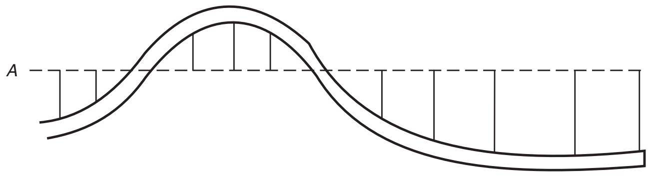


Fig. 3.12. Offset to curved details.

6. Intersecting details. When straight details such as roads, fences, hedges, etc. cross a chain line, the chainage of their points of intersection may be noted. To determine the direction of the alignment of the detail, offsets may be taken to some other points on each detail of the chain line (Fig. 3.13.)

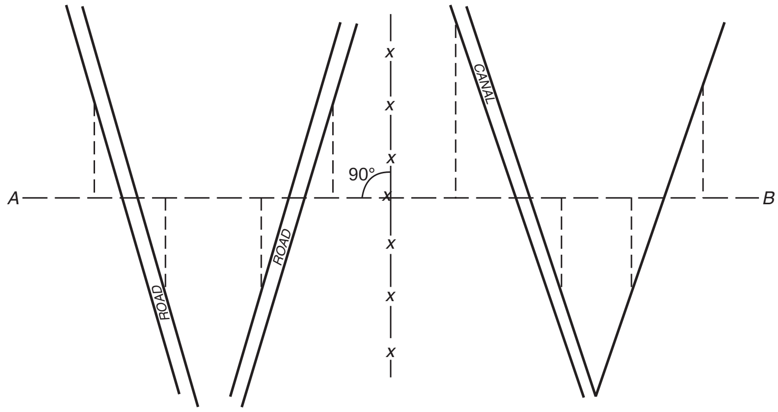


Fig. 3.13. Offsets to straight details intersection the chain line.

3.15. LOCATING BUILDING CORNERS AND POINT OF INTERSECTIONS

(1) The corners of a field are located with perpendicular offsets and their accuracy is checked by providing check ties. Similarly, every point

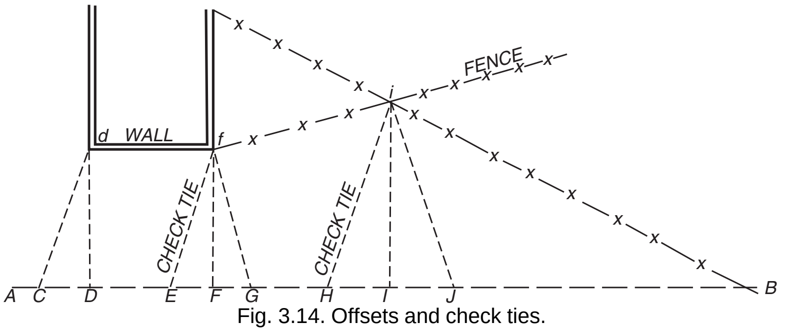


Fig. 3.14. Offsets and check ties.

of intersection must also be provided with a set of check ties in addition to the perpendicular offset (Fig. 3.14).

(2) For locating buildings, their corners must be accurately surveyed by taking normal offsets and check ties. In addition measurements of the periphery of the building, may also be recorded. Layouts of the offsets and check ties for locating building corners, are illustrated in Fig. 3.15.

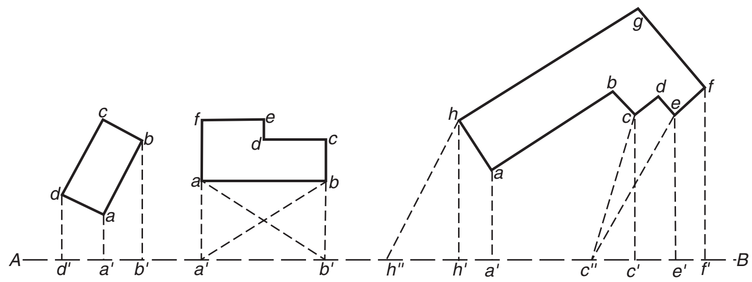


Fig. 3.15. Offsets to building corners.

3.16. DEGREE OF ACCURACY OF OFFSETS

Before commencing the field measurements, degree of precision to be maintained in measuring offsets must be decided. Accuracy mainly depends on the scale of the survey. Normally, limit of precision in plotting is 0.25 mm and the accuracy of taking offsets, depends upon the length of the offset and the importance of the detail.

Number of offsets required is also directly proportional to scale of the plan *i.e.,* larger the scale, greater number of offsets and smaller the scale, lesser number of offsets are required. The length of offsets is inversely proportional to the scale *i.e.*, larger the scale, lesser the length and smaller the scale, longer the length is required.

Example 3.1. *Determine the accuracy required in laying down perpendicular offsets for preparing a plan on scale 1 cm to 5 m.*

Solution.

$\because$ 1 cm on paper = 5 m = 500 cm on the ground

or 10 mm on paper = 500 cm of the ground

$\therefore$ 0.25 mm on paper

$$= \frac{500}{10} \times .25 = 12.5 \text{ cm on the ground}$$

Hence, offsets should be measured correct to 12.5 cm or roughly half a link.

3.17. ERROR DUE TO INCORRECT RANGING

Due to incorrect ranging, the distance between two stations is obviously increased. The error due to incorrect ranging may be deduced as under :

Let the length between two stations A and B is required to be measured. (Fig. 3.16).

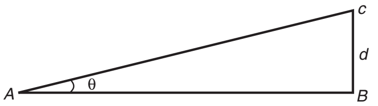


Fig. 3.16. Incorrect ranging.

Due to incorrect ranging, the chain is laid along AC where perpendicular distance BC is equal to d.

Error in the distance $\delta l = AC - AB$ ...(3.1)

Suppose AC = 20 m, a metric chain length $BC = d$

then $AB = \sqrt{20^2 - d^2}$ m

$= 20\left(1 - \dfrac{d^2}{800}\right)$ expanding binomially and ignoring higher power of d.

Substituting the values of AC and AB in Eqn. (3.1), we get

$$\delta l = 20 - 20\left(1 - \frac{d^2}{800}\right)$$

or $$\delta l = \frac{d^2}{40} \text{ m.} \quad ...(3.2)$$

If the total length of the line is 100 m and displacement per 20 m chain is 1 m, then total error in the length

$$= \frac{1 \times 5}{40} = 12.5 \text{ cm.}$$

Hence, very refined ranging is unnecessary if the distance between two stations is required only for plotting on a plan.

3.18. LIMITING LENGTH OF OFFSETS

The allowable length of the offsets, depends upon the following factors :

(*i*) accuracy desired

(*ii*) scale of the plotting

(*iii*) maximum error in laying off the direction of the offsets

(*iv*) nature of the ground.

Consider the offset CP to be laid out from a point C on the chain line AB to an object P. (Fig. 3.17)

Let $\alpha°$ be the angle of deviation.

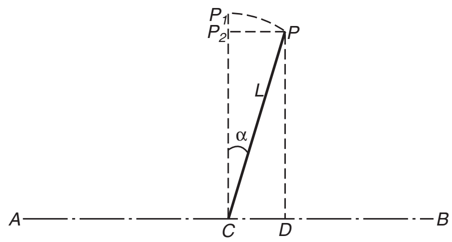


Fig. 3.17. Limiting length of offsets.

D be the foot of the perpendicular from P on the chain line AB.

l be the correct length of the offset PC.

Angle $PCD = 90° - \alpha$

The plotted location of P gets displaced to P_1 and the amount of displacement PP_1 equals to $l \sin \alpha°$ approximately

The displacement should never exceed the limit of precision of plotting *i.e.*, 0.025 cm.

If the scale of the plan is 1 cm = n metres

$$\frac{l \sin \alpha°}{n} = 0.025$$

or $$l = 0.025\ n \operatorname{cosec}\ \alpha° \quad ...(3.3)$$

Also, displacement of the point perpendicular to chain line

$$P_1 P_2 = CP_1 - CP_2$$

or $$= \frac{l - l \cos \alpha°}{n} \quad ..(3.4)$$

3.19. COMBINED ERROR IN LENGTH AND DIRECTION OF OFFSETS

Let CP be the true length of the offset in metres

CP_1 be the measured length l of the offset in metres

CP_2 be the perpendicular offset as plotted in metres

α be the angular error in the direction.

1 in n be the accuracy with which length of the offset is measured.

PP_1 = displacement due to incorrect measurement.

$P_1 P_2$ = displacement due to incorrect direction.

PP_2 = total resultant displacement.

Then, $$P_1 P_2 = l \sin \alpha \text{ and } PP_1 = \frac{l}{n}$$

Fig. 3.17

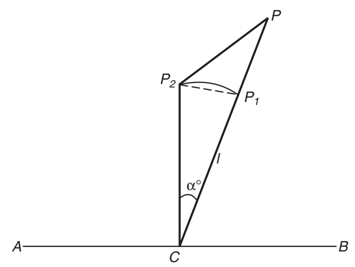


Fig. 3.18. Combined error in length and direction.

To have equal displacement of the point due to errors in measurement and direction of the offset, two errors must be equal.

$$\therefore \quad l \sin \alpha = \frac{l}{n}$$

or $$n = \text{cosec } \alpha \qquad ..(3.5)$$

Since the two errors are assumed to be equal, $P_1P_2 = PP_1$.

$$\therefore \quad PP_2 = \sqrt{(P_1 P_2)^2 + (PP_1)^2}$$
$$= \sqrt{2} \times P_1P_2 = \sqrt{2} \times l \sin \alpha$$

or $$= \sqrt{2} \times PP_1 = \sqrt{2}\frac{1}{n} \qquad ...(3.6)$$

assuming the angle PP_1P_2 very nearly equal to 90°.

If the scale of the plan is 1 cm equal to p metres, then

$$PP_2 = \frac{\sqrt{2} \times l \sin \alpha}{p} = \frac{\sqrt{2} \times l}{np} \text{ cm.}$$

But, displacement should be restricted to 0.025 cm.

$$\therefore \quad \frac{\sqrt{2} \times l \sin \alpha}{p} = 0.025 = \frac{\sqrt{2} \times l}{np} \text{ (from Eqn. 3.6)}$$

or $$l = \frac{P \text{ cosec } \alpha}{40\sqrt{2}} = \frac{pn}{40\sqrt{2}} \text{ (from Eqn. 3.5)} \qquad ...(3.7)$$

If the error PP_1 in the length of the offset is x metres, the total displacement$PP_2 = \sqrt{l^2 \sin a^2 + x^2}$

$$\therefore \quad PP_2 \text{ on paper} = \frac{\sqrt{l^2 \sin^2 \alpha + x^2}}{p} = 0.025 \text{ cm.}$$

$$\therefore \qquad \text{Sin}^2 \alpha = \frac{1}{l^2}\left(\frac{p^2}{40^2} - x^2\right)$$

or

$$\text{Sin}\,\alpha = \left\{\frac{1}{l^2}\left(\frac{P^2}{40^2} - x^2\right)\right\}^{1/2} \qquad ...(3.8)$$

Example 3.2. *Find the maximum length of an offset so that displacement of a point on plan on scale 1 cm = 10 m should not exceed 0.025 cm, if the offset was laid out 5° from its true perpendicular direction.*

Solution.

Let l = limiting length of the offset in metres.

α^0 = angular error in direction.

The displacement of the point on ground $= l \sin \alpha^o = l \sin 5^o$

But, the scale of the plan is 1 cm = 10 m.

∴ The displacement of the point on paper

$$= \frac{l \sin 5°}{10} \text{ cm}$$

$$\therefore \qquad \frac{l \sin 5°}{10} = 0.025$$

or

$$l = \frac{0.025 \times 10}{\sin 5°} = 0.25 \text{ cosec } 5°$$

$$= \mathbf{2.868 \text{ m. Ans.}}$$

Example 3.3. *If an offset is laid out 5° from its true direction in the field, find the resulting displacement of the plotted point on paper: (a) in a direction parallel to the chain line and (b) in a direction perpendicular to the chain line. Given : the length of the offset is 20 m and scale of plotting is 1 cm = 5 m.*

Solution.

Let l = length of the offset in metres

α = angular error in direction

(*a*) Displacement of the point on the ground

$$= l \sin \alpha = 20 \sin 5^o$$

The scale of the plan is 1 cm = 5 m

∴ Displacement of the point parallel to chain line

$$= \frac{20 \times \sin 5^0}{5} = \frac{20 \times 0.0872}{5}$$

$= \mathbf{0.35\ cm.}$ **Ans.**

(*b*) Displacement of the point perpendicular to chain line.

$$= l\,(1 - \cos \alpha) = 20\,(1 - \cos 5)^o = 20\,(1 - 0.9962)$$

Displacement of the point perpendicular to the chain line on paper.

$$= \frac{20 \times\ .0038}{5}$$

$= \mathbf{0.0152\ cm.}$ **Ans.**

Example 3.4. *With what accuracy an offset should be measured if the angular error in laying off the perpendicular direction is 5°, so that the maximum displacement of the point on paper is the same due to two sources of error.*

Solution.

Let l = length of the offset in metres.

α = angular error in direction

1 in n = accuracy of measurements

Displacement of the point due to the angular error in direction

$$= l \sin \alpha° = l \sin 5°$$

Displacement of the point due to error in measurement $= \dfrac{1}{n}$

$$\therefore \quad l \sin 5° = \frac{l}{n} \text{ or } n = cosec\ 5° = 11.47$$

Hence, the accuracy in measurement should be 1 in 11.5. **Ans.**

Example 3.5. *Find the maximum length of an offset so that displacement on paper from both sources of error should not exceed 0.025 cm ; given that the offset is measured with an accuracy of 1 in 25, and the scale is 1 cm = 30 m.*

Solution.

Let l be the maximum length of the offsets in metres

1 in n be the accuracy in its measurement.

The displacement of the point on the ground from both the sources of error from Eqn. (3.6)

$$= \sqrt{2} \times \frac{l}{n} = \sqrt{2} \times \frac{l}{25} \text{ m}$$

The scale of plotting is 1 cm = 30 m

∴ The displacement on paper

$$= \frac{\sqrt{2} \times l}{25 \times 30}$$

But, $\frac{\sqrt{2} \times l}{25 \times 30}$ should not exceed 0.025 cm

$$= \frac{\sqrt{2} \times l}{25 \times 30} = 0.025 \quad or \quad l = 0.025 \times 25 \times \frac{30}{\sqrt{2}}$$

∴ l = **13.26 m. Ans.**

Example 3.6. *Find the maximum permissible error in laying off the direction of an offset so that maximum displacement may not exceed 0.025 cm on paper given that length of the offset is 15 m, the scale is 1 cm to 50 cm and the maximum error in length of the offset is 0.5 m.*

Solution.

Let $\alpha°$ = maximum permissible angular error

l = length of the offset = 15 m

Displacement of the point due to incorrect direction

$$= l \sin \alpha°$$

$$= 15 \sin \alpha°$$

Maximum error in length of offset = 0.5 m (given)

∴ Displacement of the point due to both errors.

$$= \frac{\sqrt{(15 \sin \alpha)^2 + (0.5)^2}}{50}$$

But, maximum displacement is not to exceed 0.025 cm. *i.e.*,

$$\frac{\sqrt{(15 \sin \alpha)^2 + (0.5)^2}}{50} = 0.025$$

$$\sqrt{(15 \sin \alpha)^2 + (0.5)^2} = 0.025 \times 50$$

$$(15 \sin \alpha^2) = (0.025 \times 50)^2 - (0.5)^2$$

$$225 \sin^2 \alpha = 1.5625 - 0.25 = 1.3125$$

$$\sin \alpha^\circ = \frac{\sqrt{1.3125}}{225}$$

$$= 0.07638$$

$$\alpha^\circ = \mathbf{4^\circ\ 23'}. \quad \textbf{Ans.}$$

3.20. FIELD BOOK

The notebook in which chainages, offset measurements and sketches of detail points are recorded, is generally called a *field book.* It is an oblong book of size about 20 cm × 12 cm and opens lengthwise. Two types of field books are in general use *i.e.*, single line field book and double line field book. In single line field books, a red line is ruled down the middle of each page which represents the chain line or survey line. In double line field books, two blue lines about 1.5 cm to 2 cm apart are ruled down in the middle of each page to represent the chain line. The chainages are written on the line in single line field books and between the two lines in double line field books. Offsets are written opposite their chainages to the right or left according to their positions whether on the right or on the left of the chain line. The single line field book is generally used for large scale survey with detailed dimension work. Double line book is commonly used for ordinary work. The space on either side of the single line book or the column of double line book is utilized for drawing sketches and symbols of the detail points located from the chain line.

3.21. BOOKING FIELD NOTES

In a field book, the field notes are entered from the bottom of the page upward. At the beginning of a chain line, the following information is recorded in the field book :

(*i*) The name or number of the chain lin.

(*ii*) The name or number of the survey statio.

(*iii*) The symbol Δ denoting the station mark

(*iv*) The direction of survey lines starting from or ending at the station.

(*v*) The initial chainage which is generally zero, is enclosed in the symbol Δ.

All distances along the chain line *i.e.*, chainages are entered on the centre line or in the central column and offsets are written opposite them on the right or left of the column according to their ground positions with respect to the chain line. Close to the offsets, their sketches are drawn to guide the draughtsman to draw them correctly. When any linear detail such as a road, a foot path, a fence, a boundary line, etc. intersects the chain line, chainages of its points of intersection are entered in the column and the direction of the detail sketched. At the end of the chain line, the chainage should be enclosed in the symbol Δ and the name or number of the station and chain line, should be neatly written.

Tie or subsidiary stations along a chain line should be indicated by a circle or an oval round their chainages. At the commencement of a tie line in the field book, the position of the tie station should be described *e.g.*, Tie station T_1 on *AB* at 36.0 m from *A*. Similarly at the end of the tie line, it double line should be described *e.g.*, Tie station T_2 on *BC* at 86.5 m from *B*. A specimen field book for chain survey, is shown on page 93.

3.22. INSTRUCTIONS FOR BOOKING THE FIELD NOTES

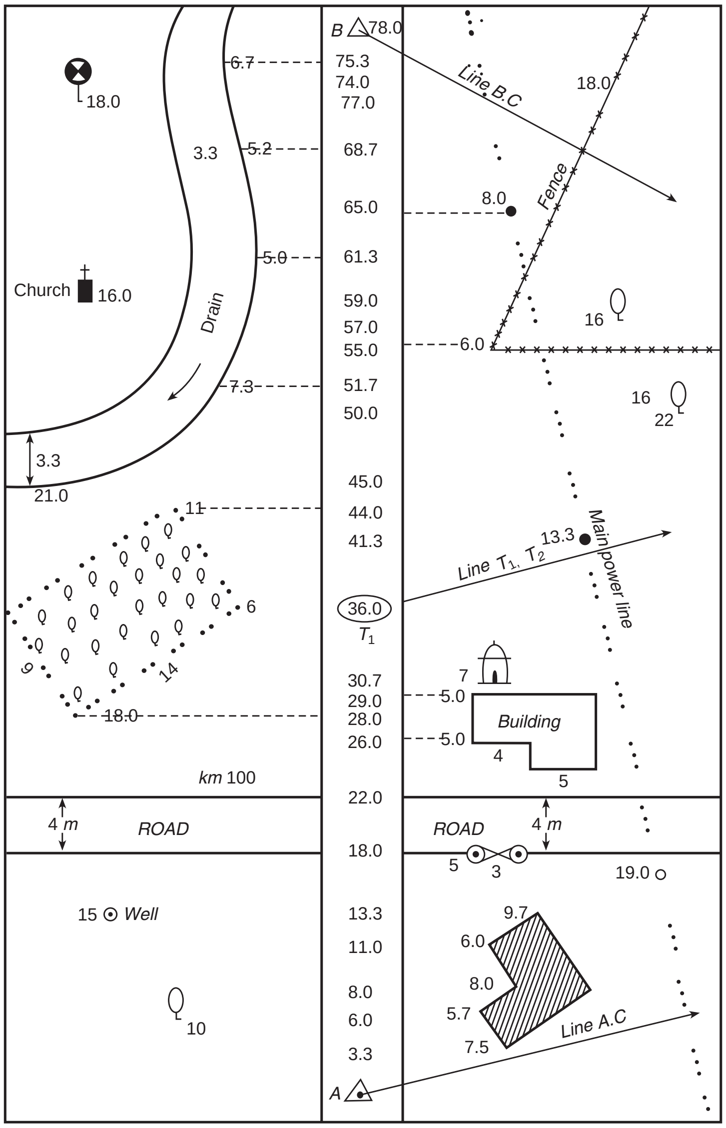
The following points should be kept in mind for booking the chain survey measurements in a field book :

1. Each chain line or a tie line, should be recorded on a separate page.
2. The recorder should always face the direction of chaining while booking the field notes.
3. All the measurements should be recorded as soon as these are taken and nothing should be left to memory.
4. The notes should be complete, neat and accurate with all informations necessary for plotting the survey by a draughtsman in office.
5. Numerals should be neatly and legibly written without any overwriting.
6. Sketches of the details, should be neat and clear.
7. A good quality pencil should be used for recording the entries.
8. The field book is an important document and should be kept clean. Wrong entries should be scored out and correct ones written over the wrong ones. If an entire page is to be discarded, it should be crossed, and marked "cancelled". A reference to the page on which correct readings are recorded, is made on this page.
9. A complete record of the chain survey should include:
   (*i*) A general lay-out plan of the lines.
   (*ii*) The details of the lines.
   (*iii*) The date of the survey.
   (*iv*) A page index of the lines.
   (*v*) Names of the surveyor and his assistants.

3.23. EQUIPMENT

The list of equipments required for chain surveying, should include the following :

1. A 20 m or 30 m chain with 10 arrows.
2. A 20 m metallic tape.



3. Ranging rods and offset rods.
4. An optical square or a cross staff.
5. A plumb bob.
6. Wooden pegs.
7. A hammer.

8. A field book.
9. Two good quality pencils.
10. A pen knife.

3.24. FIELD WORK

Field work of chain surveying is carried out in the following steps.

(*i*) Reconnaissance.

(*ii*) Marking stations.

(*iii*) Running survey lines.

1. Reconnaissance. It is always useful and often absolutely necessary for the surveyor to make a preliminary inspection of the area before commencing his actual detail survey, for the purpose of fixing the survey stations and forming a general plan for the net work of the chainlines. Such preliminary inspection of the area, is generally known as *reconnaissance* or *reconnoitre*. On arriving at the survey site, the surveyor should, therefore, walk over the entire area to examine the ground to decide upon the best layout of the chain lines. During reconnaissance, the surveyor should ensure that the survey stations are intervisible, there is no difficulty in chaining and the angles of the chain triangle are not acute. A fairly accurate key plan should be prepared to show the boundaries, main features, positions of chainlines. and stations duly lettered and numbered. Directions in which chainlines are to be measured, are marked with arrow heads.

2. Marking stations. On completion of successful reconnaissance, all survey stations should be marked in such a way that these are easily discovered during the progress of survey even after sometime, if necessary, to revise a faulty work. In soft ground, wooden pegs are driven, leaving a small portion projected above the ground. In case of roads or hard surface ground, nails of spikes may be driven flush with the payment. Sometimes, in hard ground, a portion may be dug and filled with cement mortar, etc. For marking a permanent station, a stone of any standard shape may be embedded in the ground and fixed with cement mortar.

A brief description of each survey station is given and reference sketches are drawn in the field book. The sketch, showing measure-

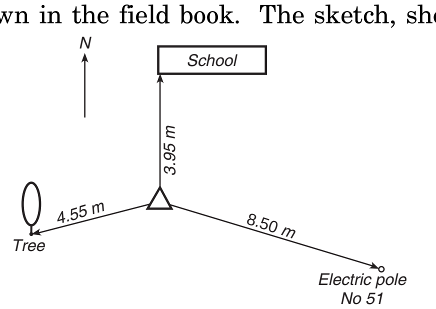


Fig. 3.19. Reference sketch.

ments at least to three permanent and definite points such as gates, pillars, light posts, corners of buildings, etc.., is known as a *reference or location sketch* (Fig. 3.19.)

Reference sketches are very useful to locate the positions of stations, in case their marks are displaced or points are lost or if required, at a later date. Measurement to the reference points must be taken correct to 5 mm. Reference sketches should be drawn by facing the north direction and a north line should be drawn. Measurements to references are written along arrows drawn to indicate their positions, which are technically known as *'ties'*.

3. Running survey lines. On completion of preliminary work, survey lines are run as detailed below:

1. Ranging is done between the end stations of the base line.
2. A chain is stretched in true alignment keeping one end of the chain at the starting station.
3. An arrow is fixed at the other end of the chain while it is kept laying on the ground.
4. The surveyor walks along the chain line and takes offsets and adjacent detail points on the right and left sides of the chain line.
5. Chainages and offsets are recorded in the field book.
6. Process of chaining and offsetting, is repeated until the end of the base line is reached.
7. Other lines are similarly completed.

3.25 INSTRUMENTS FOR SETTING-OUT RIGHTANGLES

For setting out right angles in chain surveying, the following instruments are generally used.

(*i*) Cross staffs

(*ii*) Optical squares

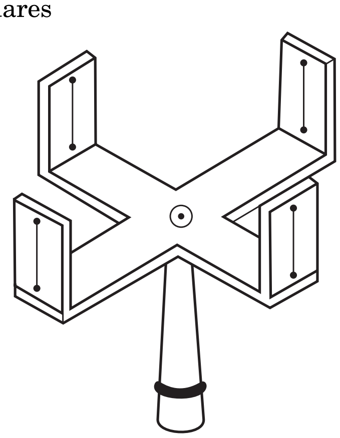


Fig. 3.20. An open cross staff.

(*iii*) Prism squares

1. Cross staffs. Cross staffs are of the following three types:

(*i*) Open cross staffs

(*ii*) French cross staffs

(*iii*) Adjustable cross staffs

1. Open cross staff (Fig. 3.20). It is the simplest type of cross staff which is commonly used. It consists of a head and a leg. The head is a wooden block octagonal or round, about 15 cm side or 20 cm diameter and 4 cm deep. The wooden block is provided with two cuts, 1 cm deep at right angles to each other, establishing two lines of sight. A better form of the open cross staff consists of four metal arms with vertical slits for sighting through at right angles to each other. The head block is fixed to the top of an iron shoed wooden staff (about 2.5 cm in diameter and 1.2 m to 1.5 m long) which is driven into the ground.

Uses of an open cross staff. Open cross staves are used for the following purposes:

A. Finding the foot of a perpendicular offset. For finding the foot of a perpendicular offset, proceed as under :

(*i*) The cross staff is held vertically on the chain line where the perpendicular from an object is expected to meet.

(*ii*) Turn the cross staff until one pair of opposite slits, is directed to a ranging rod fixed at the forward end of the chain line.

(*iii*) Look through the other pair of slits and see if the point to which the offset is taken, is bisected.

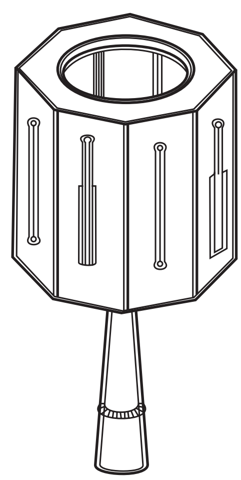


Fig. 3.21. A French cross staff.

(*iv*) If not, the cross staff is moved forward or backward on the chain line until the line of sight through the pair of slits at right angles to the chain line, bisects the desired point.

(*v*) Care should be taken to hold the cross staff vertically while viewing through the slits.

B. Setting out a right angle at a point on a chain line. For setting out a right angle at a point on a chain line, proceed as under :

(*i*) Hold the cross staff vertically over the given point, on the chain line.

(*ii*) Turn it until the ranging rod fixed at either end of the chain line, is bisected by the line of sight through one pair of slits.

(*iii*) Fix a ranging rod in the line of sight through the other pair of slits at a convenient distance.

2. French cross staff (Fig. 3.21). It consists of an octagonal brass tube with slits on all the eight faces. It has alternate vertical sight slits and on opposite vertical faces, windows are provided. Vertical fine wires are stretched on each of the four sides. These are used for setting out right angles. Vertical sight slits at the centres of opposite slits, make angles of 45° with each other. With this arrangement, it is, sometime, possible to set out 45° angles also.

The base of French cross staff is fitted on a pointed staff with a socket. It is inferior to the open cross staff because the sights are too close *i.e.,* 8 cm. apart.

3. Adjustable cross staff (Fig. 3.22). It consists of a brass cylindrical tube about 8 cm in diameter and 10 cm deep, divided at the middle. The lower portion remains fixed and the upper portion can be rotated

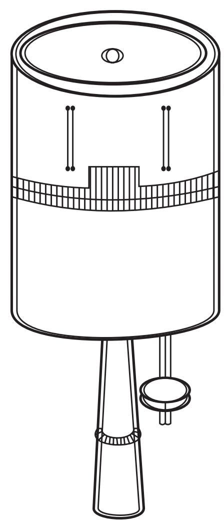


Fig. 3.22. An adjustable cross staff.

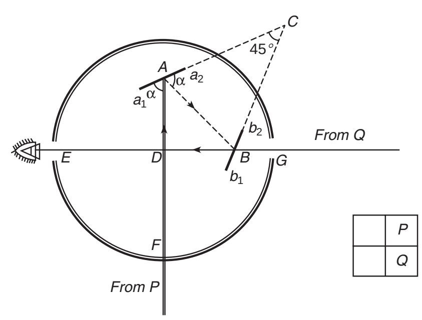


Fig. 3.23. An optical square.

relative to the lower one by a circular rack and pinion arrangement actuated by a milled headed screw. Sighting slits are provided in both the parts. The lower part is graduated into degrees and their sub-divisions, while the upper one carries a vernier. The adjustable cross staff may be used for setting out angles of any magnitude. On the top of the instrument, a magnetic compass is provided to take bearings of lines. It is also an inferior type and is only used for approximate work.

All the three types of cross staffs whether adjustable or non-adjustable, do not provide high accuracy. For more accurate work, optical squares, are generally used.

3.26. OPTICAL SQUARES

There are following two types of optical squares :

(1) Optical Squares (2) Indian Optical Squares.

1. Optical Square (Fig. 3.23). It consists of a circular metal box about 5 cm in diameter and 1.25 cm deep. The periphery is formed of two cylinders, one capable of sliding over the other so that the eye and object openings can be closed to protect the mirrors from dust. Two plain mirrors *A* and *B* are placed inclined at an angle of 45°. Upper half depth of mirror *B*, known as *horizon glass* is a plain glass and lower half is silvered. Mirror *A* known as *Index glass* is completely silvered. To an eye placed at *E*, the ranging rod placed at Q is visible through the transparent half of horizon mirror *B*, and at the same time the image of the ranging rod placed at *P* at right angles to the line *EQ,* is seen after reflection at *B* in its silvered part.

Three openings are cut alike in the rims of box and cover :

(*i*) A pin hole for the eye (*or sight hole*)

(*ii*) A small rectangular, slot for *horizon sight,* diametrically opposite to the sight hole.

(*iii*) A large rectangular slot for object sight, at right angles to the line joining the pin hole and horizon sight hole.

Principle of an optical square : The principle on which construction of an optical square, is based, may be stated as follows :

"If there are two plain mirrors whose reflecting surfaces are inclined at a given angle, with each other and if a ray of light in a plane perpendicular to the planes of both the mirrors is reflected successively from both, it under-goes a deviation of twice the angle between the reflecting surfaces."

In order to have a ray of light reflected through a right angle, the angle between the reflecting surfaces of the optical square, is therefore kept 45°.

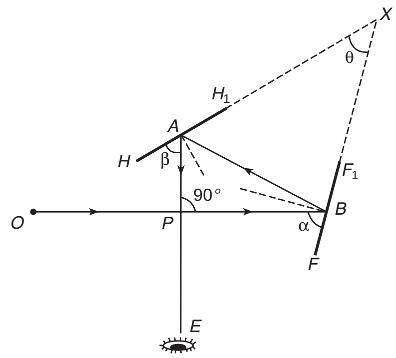


Fig. 3.24. Principle of an optical square.

Proof (Fig. 3.24)

Let a ray of light from *O* an object strike the mirror *B*. The ray gets reflected along *BA*, making an angle of reflection equal to the angle of incidence. When the reflected ray *BA* strikes the horizon mirror *A,* it gets reflected along *AE,* following the rules of reflection.

Let $\theta = \angle AXB$, the angle between two mirrors.

$\alpha = \angle OBF$; $\beta = \angle EAH$

The angle of incidence = the angle of reflection

or $$\angle OBF = \angle ABX = \alpha$$

Similarly, $\angle XAB \times = \angle HAE = \beta$

In the right angled ΔABP,

$$\angle APB = 90°; \angle PAB = 180° - 2\beta \text{ and } \angle PBA = 180° - 2\alpha$$

$\therefore$ $90° + 180° - 2\beta + 180° - 2\alpha = 180°$

or $$\alpha + \beta = 135° \qquad ...(3.9)$$

In ΔABX, $\angle ABX = \alpha$, $\angle BAX = \beta$, and $\angle AXB = \theta$

$\therefore$ $$\alpha + \beta + \theta = 180° \qquad ...(3.10)$$

Substituting the value of $\alpha + \beta = 135°$ from Eqn. (3.9) in Eqn. (3.10), we get

or $$135° + \theta = 180°$$

$$\theta = 180° - 135° = 45^o$$

i.e., the angle *AXB* between two mirrors = 45°

Uses of an optical square. The optical squares are used for the following purposes :

A. To find the foot of a perpendicular to the chain line AB from any given point D. The following procedure may be followed :

(*i*) Let *AB* represent a chain line, *B* being the end station.

(*ii*) Hold the instrument close to the eye horizontally and walk along the chain line towards *B*.

(*iii*) Sight the ranging rod at *B* through unsilvered portion of the horizon glass and the image of the ranging rod at the given point *D* through silvered portion.

(*iv*) Keep on walking along the chain forward or backward until the ranging rod at *B* is seen through unsilvered portion and the image of the ranging rod at *D* appears exactly coincident.

(*v*) Mark the point *C* vertically beneath the instrument to get the position of the foot of the required perpendicular.

Note. The following points may be noted:

(*i*) It is recommended that two forward poles must be erected marking the survey line, to maintain the instrument in the line without any trouble.

(*ii*) If the offset point lies on the right hand side of the chain line, the instrument should be held in left hand.

(*iii*) When the offset point lies on the left hand side of the chain line, the instrument should be held in right hand up side down.

B. To set out a perpendicular to a chain line AB at a given point C.

The following procedure may be followed:

(*i*) Stand on the chain line holding the instrument over the point *C* and sight the ranging rod at *B* through the unsilvered portion of the horizon glass.

(*ii*) Direct the staff man to move his ranging rod *D* to the right or left as found necessary until its image is seen in the silvered portion of the horizon glass exactly in coincidence with the ranging rod at *B*.

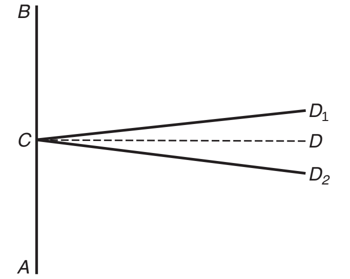


Fig. 3.25. Adjustment of an optical square.

(*iii*) The line *CD* is the required perpendicular to the given line *AB*.

Precautions: Following precautions must be followed:

(*i*) The optical square should be held horizontally.

(*ii*) When the ground is sloping, a number of ranging rods at close intervals say 10 m to 15 m, are fixed in the chain line from the point at which the right angle is to be set out.

(*iii*) Similarly, a rod may be fixed at a shorter distance for fixing a perpendicular direction.

(*iv*) In uneven ground, the use of the instrument should be discouraged.

Adjustment of an Optical Square (Fig. 3.25) To test the adjustment of an optical square, the following steps are taken:

(*i*) Range out a straight line *AB* upon a fairly level ground and fix three points *A*, *B* and *C* on it.

(*ii*) Hold the instrument over the point *C* and sight the ranging rod at *B* and set-out a right angle BCD_1.

(*iii*) Hold the instrument up side down, turn round and sight a ranging rod at *A*. Set-out a right angle ACD_2.

(*iv*) If the point D_2 coincides with D_1, the instrument is in perfect adjustment.

(*v*) If not, fix a ranging rod at *D* exactly midway between D_1 and D_2 to mark the true perpendicular direction *CD*.

(*vi*) Rotate the index mirror by means of a key until the image of the ranging rod *D*, coincides with that of *A*.

(*vii*) Turn round and sight the ranging rod at *B* and observe whether the image of *D* appears coincident with that of *B*.

(*viii*) If not, repeat the adjustment, until correct.

2. Indian Optical Square. (Fig. 3.26). It is a brass wedge shaped hollow box of 5 cm sides and 3 cm deep with a handle 7.2 cm long. Two mirrors m_1 and m_2 are fixed inclined at an angle of 45° to the inner side of the box. Two rectangular openings *ab* and *cd*, are provided in the sides above these mirrors for sighting. *MNOP* is the open face of the square which is kept towards the object to which an offset is to be taken.

Use of an Indian optical square. An optical square may be used for the following purposes:

A. Taking offsets to objects from the Chain line. The following steps are involved.

(*i*) Hold the instrument in a hand.

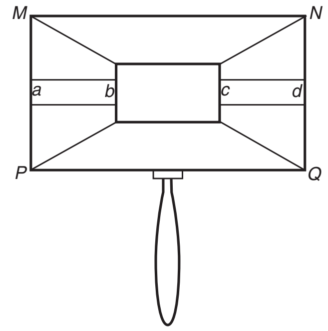


Fig. 3.26. An Indian optical square.

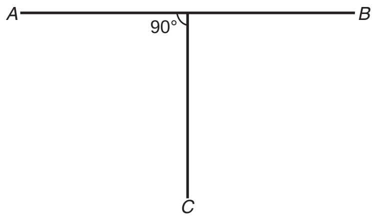


Fig. 3.27.

(*ii*) Stand on the chain line *AB* and turn its open face *MNOP* towards the object *C*.

(*iii*) Sight the ranging rod at the forward station *B* looking through the opening in the direction of *a c* or *d b* according as the object *C* is on the right or on the left.

(*iv*) Walk along the chain line *AB* forward and backward until the image of the object *C*, appears exactly in coincidence with the ranging rod at *B*.

(*v*) The plumb line suspended from the handle gives the required point on the chain line.

B. Setting out right angles at given points on a chain line. The procedure is similar to that of an optical square explained earlier.

3.27. Field Problems and their Solutions

It is never practicable to arrange layouts and surveys so that all the chain lines can be run in straight forward manner described in previous paragraphs. Obstacles generally create difficulties in surveying. Solution of simple difficulties with essential equipments used in chain surveying, may be made as under:

Problem 1. To erect perpendiculars to a chain line

For erecting a perpendicular to a chain line, one of the following methods may be adopted.

Method 1. Select two points *B* and *C* equidistant from the given point *G* on the chain line. Pindown the ends of a 30 m tape, *i.e.*, zero end and 30 m mark at *B* and *C* respectively. Hold the 15 m mark and move it away from the line till two halves are fully stretched. Then the point *G′* at 15 m mark is on the required perpendicular *G′ G*. (Fig. 3.28).

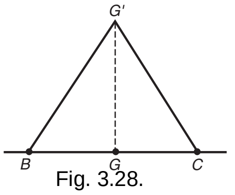


Fig. 3.28.

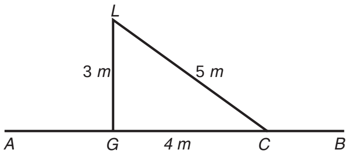


Fig. 3.29.

Method 2. On the chain line *AB*., select a point *C,* 4 m away from *G*. With *O* and *G* as centres and 5 m and 3 m as radii, sweep two arcs to intersect at *L* to get the required perpendicular *GL*. (Fig. 3.29).

Method 3. Select any convenient point *D* so that *GD* is less than the length of the tape. With *D* as centre and *DG* as radius, swing an arc cutting the chain line at *A* and also *AD* produced at *L*. *GL* is the required perpendicular. (Fig. 3.30).

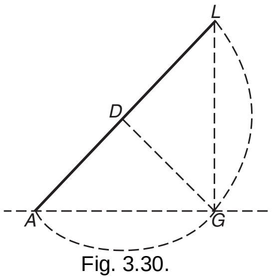


Fig. 3.30.

This solution is based on the geometrical property of a circle *i.e.*, The angle subtended by a diameter on the circumstance, is always a right angle.

Problem 2. To drop perpendiculars to a chain line from outside points

Two cases may arise:

(*a*) **When the point is accessible**. The following method may the adopted:

Method 1. With the given point *G* as centre, swing an arc to cut the chain line at *A* and *B*. Find the mid-point *L* of *AB* which is the foot of the required perpendicular *GL*. (Fig. 3.31).

Method 2. Select any point *B* on the given chain line and measure a convenient length *BG*. Set off *BC* on the chain line equal to *BG*. Measure *CG*. (Fig. 3.32).

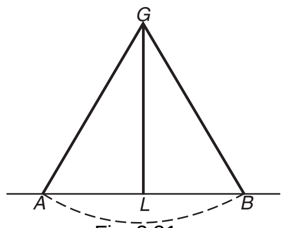


Fig. 3.31.

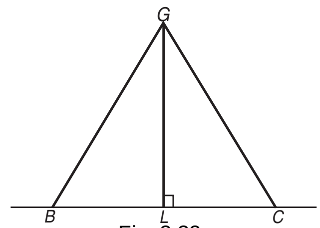


Fig. 3.32.

Obtain the point L on BC by measuring CL equal to $\frac{(CG)^2}{2BC}$.

LG is the required perpendicular.

Proof: From Δ GLC, we get

$$CG^2 = GL^2 + LC^2 = BG^2 - BL^2 + LC^2$$
$$= BG^2 + (LC + BL)(LC - BL) = BC^2 + BC(LC - BL)$$
$$= BC[BC + LC - BL] = BC(BC - BL + LC)$$
$$= BC \times 2LC$$

or $$CL = \frac{(CG)^2}{2BC} \qquad ...(3.11)$$

(*b*) **When the point is in-accessible.** (Fig. 3.33).

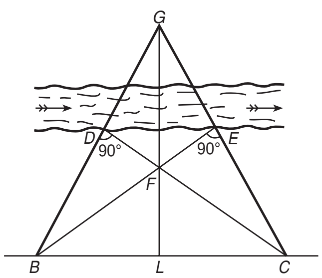


Fig. 3.33.

Method. Select two points B and C on the chain line. Drop *perpendiculars BE to CG* and CD to BG. Locate their point of intersection F. Produce GF to L which is the required perpendicular to BC.

This solution is based on the geometrical property of a triangle *i.e.,* perpendiculars drawn from the vertices on opposite sides, intersect at a point.

Problem 3. To run a Parallel line to an inaccessible line through a given point.

Method. Let AB be an inaccessible line and G is the given point. Fix a point E in line with A and G. Fix any point F in a convenient position, so that *EF* is approximately parallel to *AB*. Through *G* run a line *GC* parallel to *AF* by any one of the methods. Through *C*, run a line *CD* parallel to *FB* intersecting *EB* at *D*. Then *GD* is the required line parallel to *AB*.

Proof. In Δs *ABF* and *GDC*,

GC is parallel to *AF*

CD is parallel to *FB*

∴ Third side *GD* is parallel to *AB*.

3.28. OBSTACLES IN CHAINING

Various types of obstacles generally met during chaining, may be overcome by any one of the following methods.

Obstacles to chaining are of the following types:

1. Obstacles which obstruct ranging but not chaining.
2. Obstacles which obstruct chaining but not ranging.
3. Obstacles which obstruct both ranging and chaining.

1. Obstacles which Obstruct Ranging but not Chaining. In this type of obstacles, the ends of the chain line are not intervisible. Such obstacles are generally met in undulating terrain where area consists of rising grounds, intervening hills or undulations. Two cases may occur:

(1) When ends are visible from intermediate points on the chain line.

(2) When ends are not visible from any intermediate point on the chain line.

Difficulties faced in both the cases may be over-come either by reciprocal ranging or by the random line method, as already described in Chapter 2 Linear measurements.

2. Obstacles which Obstruct Chaining but not ranging. The typical types of obstructions under this category are generally large water bodies, *i.e.*, lakes, ponds, rivers, etc. where distances between two convenient points on the survey line on either side of the obstacles, are required to be determined.

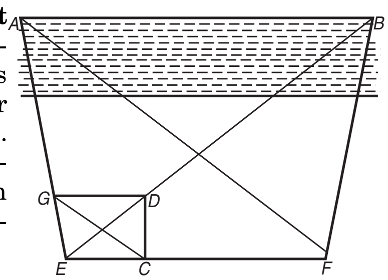


Fig. 3.34.

Two situations may rise:

(*i*) It is not possible to chain round the obstacle.

(*ii*) It is possible to chain round the obstacle.

Case I. Chaining round the obstacle possible. Depending upon the layout of the obstacle and the nature of the terrain around the obstacle a number of methods may be employed. However, a few typical methods generally used, are described here under:

Method 1. By Constructing a Rectangle. (Fig. 3.35).

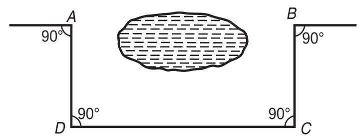


Fig. 3.35.

On the chain line at A and B, erect two perpendiculars AD and BC of equal lengths on the same side on either side of the obstruction. Join D and C and measure DC which is equal to the desired length AB.

Method 2. By Constructing a Right Angled Triangle ABC having 90˚ Angle at either A or B. (Fig. 3.36).

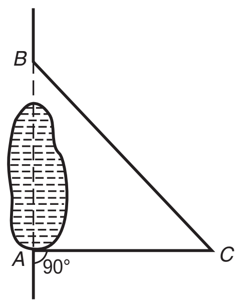


Fig. 3.36.

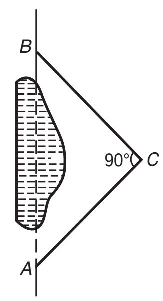


Fig. 3.37

Erect a perpendicular AC of such a length that the line CB is clear of the obstacle. Measure both AC and CB accurately. The line

The required length $AB = \sqrt{BC^2 - AC^2}$

or $$AB = \sqrt{(BC + AC)\,(CB - AC)} \qquad ...(3.12)$$

Method 3. By Constructing a Right Angled Triangle ACB having a 90˚ angle at C. (Fig. 3.37).

Fix two ranging rods at A and B. With the help of an optical square or a cross staff, find a convenient point C where angle ACB is a right angle, ensuring that AC and BC are clear of the obstacle. Measure distances AC and BC.

$$AB = \sqrt{AC^2 + BC^2} \qquad ...(3.13)$$

Method 4. By Constructing a Triangle Enclosing the Obstacle. (Fig. 3.38).

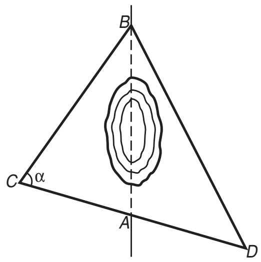


Fig. 3.38.

Set out a straight line *CAD* of a convenient length such that *CB* and *DB* are clear of the obstacle. Measure *AC, AD, BC* and *BD*. The length of *AB* is calculated as follows:

Let angle *BCD* be α.

Applying cosine formula

$$BD^2 = CB^2 + CD^2 - 2CB \cdot CD \cdot \cos\alpha$$

$$\therefore \quad \cos\alpha = \frac{CB^2 + CD^2 - BD^2}{2CB \cdot CD} \quad \text{...(3.14)}$$

Similarly from $\Delta\, ACB$,

$$AB^2 = CB^2 + CA^2 - 2CB \cdot CA \cdot \cos\alpha$$

or $$\cos\alpha = \frac{CB^2 + CA^2 - AB^2}{2CB \cdot CA} \quad \text{...(3.15)}$$

Equating the values of cos α, from Eqns. 3.14 and 3.15, we get

$$\frac{CB^2 + CD^2 - BD^2}{2CB \,.CD} = \frac{CB^2 + CA^2 - AB^2}{2CB.CA}$$

or $$CA\,(CB^2 + CD^2 - BD^2) = CD\,(CB^2 + AC^2 - AB^2)$$

or $$AB^2 = \frac{CD.BC^2 + CD.AC^2 - AC.BC^2 - AC.CD^2 + AC.BD^2}{CD}$$

$$= BC^2 + AC^2 - AC.CD - \frac{AC.BC^2}{CD} + \frac{AC.\,BD^2}{CD}$$

$$= \frac{AC.BD^2 - BC^2.\,AC}{CD} + BC^2 + AC^2 - AC.CD$$

or $$AB^2 = \frac{AC.BD^2 + BC^2\,(CD - AC)}{CD} + AC\,(AC - CD)$$

$$= \frac{AC.BD^2 + BC^2.\,AD}{CD} - AC\,(CD - AC)$$

$$= \frac{AC.BD^2 + BC^2.AD}{CD} - AC \times AD$$

or $$AB = \sqrt{\frac{BD^2 \times AC + BC^2 \times AD}{CD} - AC \times AD} \quad ...(3.16)$$

Method 5. By Constructing Similar Triangles. (Fig. 3.39).

Fix a point C clear of the obstacle. Range a point D in line of AC such that $AC = CD$. Range another point E in line of BC such that $BC = CE$. Measure ED which is equal to AB.

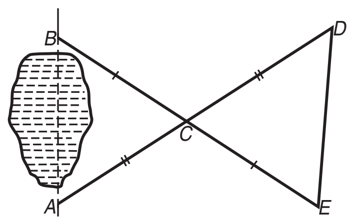


Fig. 3.39.

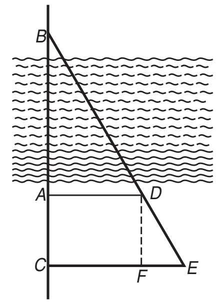


Fig. 3.40.

Case II. Chaining round the obstacle not possible. Few typical methods for overcoming the obstacles are described below:

Method 1. By construction of similar triangles

(*a*) From two points A and C on the chain line CB, erect perpendiculars AD and CE such that points B, D and E are in a straight line. (Fig. 3.40).

Measure AC, AD and CE; then from similar Δs BAD and DFE, we get

$$AB = \frac{AC \cdot AD}{CE - AD}$$

Proof. Drop DF ⊥ CE

Δs BAD and DFE are similar.

$$\therefore \quad \frac{AB}{DF} = \frac{AD}{FE}$$

or $$AB = \frac{AD.AC}{CE - AD} \quad ...(3.17)$$

(*b*) Set out a perpendicular AC. Mark the mid-point E of AC. Set out perpendicular CD such that points B, E and D lie in one straight line.

Then, $AB = CD$. (Fig. 3.41).

Method 2. By constructing right angled triangles

* Theodolite discussed later

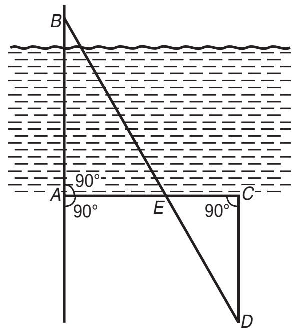


Fig. 3.41.

Erect a perpendicular AC to the given chain line. Find a point D on the chain line such that $\angle DCB$ is 90°. Measure AC and AD. (Fig. 3.42).

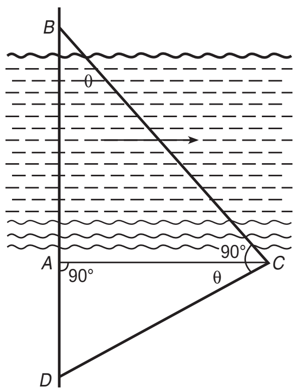


Fig. 3.42.

Now $$AB = \frac{AC^2}{AD} \qquad ...(3.18)$$

Proof. From the similar right angled triangles BCD and DAC, we get

$$\frac{DC}{BD} = \frac{AD}{DC}$$

$$DC^2 = AD \,.\, BD$$

$$DC^2 = AD \,.\, (AD + AB)$$

$$DC^2 = AD^2 + AD \,.\, AB$$

$$AB = \frac{DC^2 - AD^2}{AD}$$

or $$AB = \frac{AC^2}{AD}$$ Proved

Method 3. By constructing a parallelogram (Fig. 3.43).

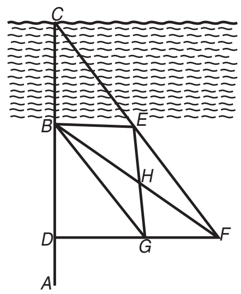


Fig. 3.43.

Let ABC be the chain line.

Set out a line BE roughly parallel to the river.

Extend CE to F and establish the mid-point H of BF.

Extend EH to G such that $EH = HG$.

Extend FG to cut the chain line AC at D.

Measure GD and BD.

$$BC = \frac{BD \times BE}{DG}$$

Proof. From similar Δ *s* CBE and CDF, we get

$$\frac{BC}{CD} = \frac{BE}{DF}$$

$$\frac{BC}{BC + BD} = \frac{BE}{BE + DG}$$

$$\frac{BC + BD}{BC} = \frac{BE + DG}{BE}$$

$$\frac{BC}{BC} + \frac{BD}{BC} = \frac{BE}{BE} + \frac{DG}{BE}$$

$$\frac{BD}{BC} = \frac{DG}{BE}$$

or $$BC = \frac{BD \times BE}{DG}$$

3. Obstacles which obstruct-both Ranging and Chaining. In such cases prolonging the line beyond the obstacle and finding the distance across it, may be overcome by any one of the following methods.

Method 1. From two points A and B on the chain line, erect equal perpendiculars AC and BD. Range G and H in line with C and D. Set out GE and HF perpendiculars to CH equal to AC. E and F are on the chain line and $BF = DG$. (Fig. 3.44).

Method 2. Erect a perpendicular AC. Mark a point B on the chain line such that $AC = AB$. Produce BC to D. Set out a right angle BDF. Measure DE equal to CD and EF equal to BC.

From E and F swing two arcs of radii equal to AC to get the location of G. Points G and F are along the chain line. Now CE may be measured to get the length AG. (Fig. 3.45).

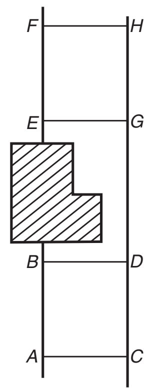


Fig. 3.44.

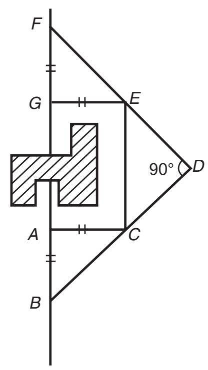


Fig. 3.45.

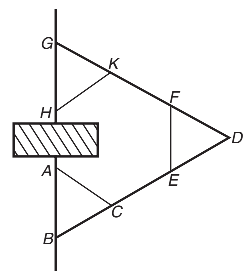


Fig. 3.46.

Method 3. By swinging a tape, construct an equilateral triangle ABC. Produce BC to D such that $CD = 2BC$ and mark the mid point of CD at E. An equilateral triangle DEF is constructed and the side DF is produced so that $FG = BE$ and mid-point K of FG located. By intersection of two arcs of radius AB with K and G as centres get H. Then $AH = AB$ and HG is along the chain line (Fig. 3.46).

Method 4. Select two points C and D on both sides of A in any straight line and a point B on the chain line. (Fig. 3.47).

Fix a point E in line of BC and F in line of BD making $BE = n \times BD$. Divide EF so that $EG = n \times CA$, where $n = CA/AD$.

Similarly, fix points H in line of BCE and J in line of BDF, making $BH = n \times BE$ and $BJ = n \times BF$ respectively. Divide HJ so that $HK = n \times EG$. Points G and K are on the chain line BA.

Proof: $\dfrac{CA}{AD} = \dfrac{EG}{GF} = \dfrac{HK}{KJ} = n$

Measure BC, CE and EH, then

$$\frac{AB}{BC} = \frac{BG}{BE}$$

or
$$\frac{AB}{BC} = \frac{AB + AG}{BE}$$

$$\frac{AB}{BC} - \frac{AB}{BE} = \frac{AG}{BE}$$

$$\frac{AB\,(BE - BC)}{BE\,.\,BC} = \frac{AG}{BE}$$

$$\therefore \quad AG = \frac{AB \times EC}{BC}$$

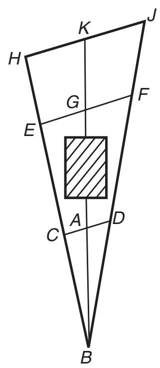


Fig. 3.47.

It may be noted that the above methods for prolongation of a chain line, do not ensure much accuracy. For accurate work, a theodolite* should preferably be used.

Example 3.7. *A survey line BAC crosses a river; A and C being the near and far banks respectively. A perpendicular AD, 40 metres long is set out at A. If the bearings of AD and DC are 38° 45′ and 278° 45′ respectively, find the width of the river.*

Solution. (Fig. 3.48).

The bearing of a line may be defined as the horizontal angle between the north direction and the line measured in a clockwise direction.

Bearing of $AD = 38° \, 45'$ (given)

$\therefore$ Bearing of $DA = 38° \, 45' + 180°$

$= 218° \, 45'$

Bearing of $DC = 278° \, 45'$ (given)

$\angle CDA$ = Bearing of DC – Bearing of DA

$= 278° \, 45' - 218° \, 45' = 60°$

From ΔACD, we get

$$AC = AD \tan 60°$$

* Theodolite disscussed later

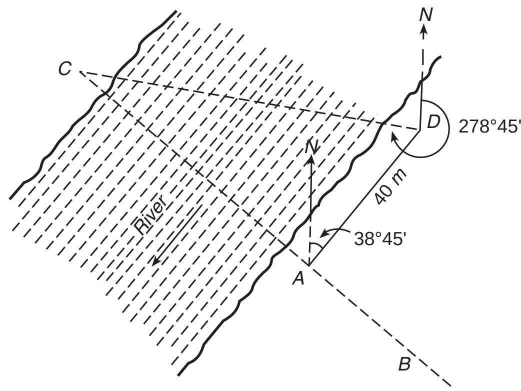


Fig. 3.48.

$$= 40 \tan 60° = 40\sqrt{3}$$

∴ Width of the river = **69.28 m. Ans.**

Example 3.8. *A survey line CDE crosses a river, D being on the near bank, and E on the opposite bank. A perpendicular DF = 150 metres is ranged at D on the left. From F bearings of E and C are observed to be 25° and 115° respectively. If the chainage of C is 1250 metres and that of D is 1620 metres, find the chainage of E.*

Solution. (Fig. 3.49).

Chainage of $D = 1620$ m

Chainage of $C = 1250$ m

∴ The length $CD = 1620 - 1250 = 370$ m

Bearing of $FC = 115°$

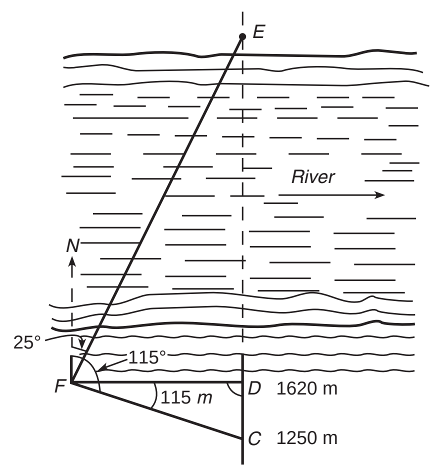


Fig. 3.49.

Bearing of $FE = 25°$

$\therefore \quad \angle EFC$ = Bearing of FC – Bearing of FE

$= 115° - 25° = 90°$

From similar Δs EFD and Δ FDC, we get

$$\frac{ED}{FD} = \frac{FD}{DC}$$

or
$$ED = \frac{FD^2}{DC} = \frac{150^2}{370} = 60.81 \text{ m}$$

∴ Chainage at E = Chainage at $D + ED$

= 1620 + 60.81 = **1680.81 m Ans .**

Example 3.9. *A chain line PQR crosses a stream, Q and R being the near and far off banks respectively. A line QM of length 60 m is set out at right angles to the chain line at Q. If the bearings of QM and MR are 282° 45′ and 42° 45′ respectively, find the width of the stream.*

Solution. (Fig. 3.50).

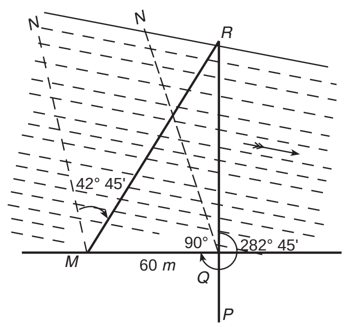


Fig. 3.50.

Bearing of QM = 282° 45′

Bearing of MQ = 282° 45′ – 180°

= 102° 45′

Angle RMQ = Bearing of MQ – Bearing of MR

= 102° 45′ – 42° 45′ = 60°

From the right angled trianlge RQM we get

∴ Width of river QR

= MQ tan 60°

= 60 × 1.732050 = 103.92 m. **Ans.**

Example 3.10 *A chain line PQR crosses a river, Q and R being on the near and distant banks respectively. A perpendicular QS, 90 m long, is set out at Q on the left of the chain line. The respective bearings of R and P taken at S are 77° 30′ 20″ and 167° 30′ 20″. Find the chainage of R given that PQ is 45 m and the chainage of Q is 650 m.*

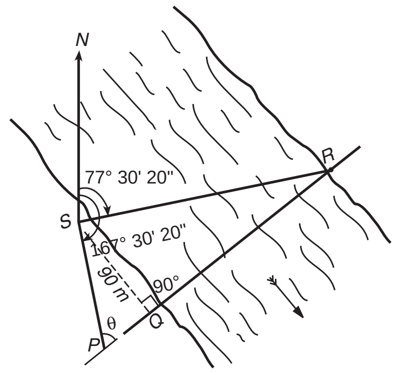


Fig. 3.51.

Solution.

Angle RSP = Bearing of SP − Bearing of SR

= 167° 30′ 20″ − 77° 30′ 20″

= 90°

∴ ΔRSQ is a right angled triangle at S.

Let $\angle SPQ$ be θ

∴ $\angle PSQ = 90° - \theta$

∴ $\angle RSQ = 90° - (90° - \theta) = \theta$

Similarly, $\angle QRS = \angle PSQ$.

∴ Δs PSQ and RQS are similar

∴ $$\frac{PQ}{QS} = \frac{QS}{RQ}$$

∴ $$RQ = \frac{QS^2}{PQ} = \frac{90^2}{45} = 180 \text{ m}$$

Chainage of R = Chainage of $Q + RQ$

= 650 + 180 = 830 m **Ans.**

Example 3.11. *A chain line ABC crosses a river, B and C being on the near and distant banks respectively. A line BD of length 100 m is set out at right angles to the chain line at B. If the bearings of BD and DC are 287° 15′ and 62° 15′ respectively, find the width of the river.*

Solution. (Fig. 3.52.)

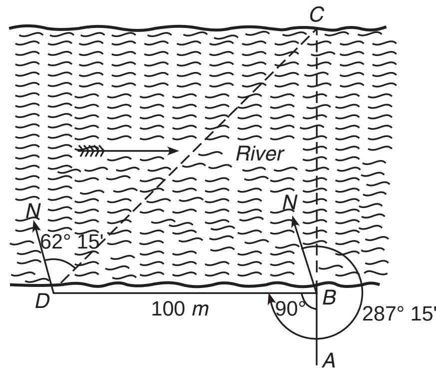


Fig. 3.52.

Bearing of $BD = 287° 15'$

Bearing of $DB = 287° 15' - 180° 0' = 107° 15'$

Bearing of $DC = 62° 15'$ (given)

$\angle CDB$ = Bearing of DB − Bearing of DC

$= 107° 15' - 62° 15' = 45°$

In ΔBCD, angle $DBC = 90°$ and angle $CDB = 45°$

$\therefore$ Angle $DCB = 90° - 45° = 45°$

Hence $BC = BD =$ **100 m** **Ans.**

Example 3.12. *A big pond obstructs the chain line ab. A line al was measured on the left of line ab for circumventing the obstacle. The lengths al was 901 m. Similarly, another line am was measured on the right of line ab whose length was 1100 m. Points, m, b, and l are on the same straight line. Lengths of lines bl and bm are 502 m and 548 m respectively. Find the distance ab.*

Solution. (Fig. 3.53.)

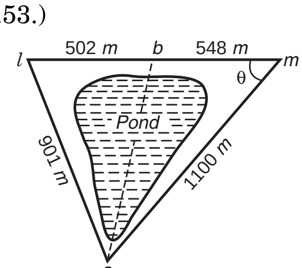


Fig. 3.53.

Let angle *aml* be θ, then

From Δ *alm*.

$$\cos\theta = \frac{(am)^2 + (ml)^2 - (al)^2}{2 \times am \times ml}$$

or $$\cos\theta = \frac{(1100)^2 + (1050)^2 - (901)^2}{2 \times 1100 \times 1050} \qquad ...(i)$$

From Δ *amb*, $$\cos\theta = \frac{(am)^2 + (mb)^2 - (ab)^2}{2 \times am \times mb}$$

or $$\cos\theta = \frac{(1100)^2 + (548)^2 - (ab)^2}{2 \times 1100 \times 548} \qquad ...(ii)$$

Comparing the values of cos θ, from Eq. (*i*) and (*ii*) we get

$$\frac{(1100)^2 + (1050)^2 - (901)^2}{2 \times 1100 \times 1050} = \frac{(1100)^2 + (548)^2 - (ab)^2}{2 \times 1100 \times 548}$$

or $$\frac{15,00,699 \times 548}{1050} = 15,10,304 - (ab)^2$$

or $$(ab)^2 = 15,10,304 - 7,83,221.95$$
$$= 72,7082.05$$
$$= \sqrt{7,27,082.05}$$
$$ab = \mathbf{852.69\ m.} \quad \textbf{Ans.}$$

Example 3.13. *A river is flowing from west to east. For determining the width of the river two points A and B are selected on southern bank such that distance AB = 75 m. Point A is westwards. The bearings of a tree C on the northern bank are observed to be 38° and 338° respectively from A and B. Calculate the width of the river.*

Solution. (Fig. 3.54.)

In triangle *ABC*

$\angle CAB = 90° - 38° = 52°$

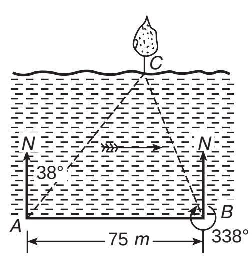


Fig. 3.54.

$\angle CBA = 338° - 270° = 68°$

$\angle ACB = 180° - (\angle CAB + \angle CBA)$

$= 180° - (52° + 68°) = 60°$

Applying sine rule to ΔABC

$$\frac{AC}{\sin ABC} = \frac{BC}{\sin CAB} = \frac{AB}{\sin ACB}$$

$$AC = \frac{AB \sin ABC}{\sin ACB} = \frac{75 \times \sin 68°}{\sin 60°}$$

$$= \frac{75 \times 0.92718}{0.866025} = 80.30 \text{ m}$$

and $$BC = \frac{AB \sin CAB}{\sin ACB} = \frac{75 \times \sin 52°}{\sin 60°}$$

$$= \frac{75 \times 0.78801}{0.866025} = 68.24 \text{ m}$$

Width of river $= AC \sin 52°$

$= 80.30 \times 0.78801 = 63.27$ m

or Width of river $= BC \sin 68° = 68.24 \times 0.92718 = 63.27$ m.

$\therefore$ The width of the river **63.27 m. Ans.**

Example 3.14. *A survey line ABC crossing a river at right angles, cuts its banks at B and C. To determine width BC of the river, the following operation was carried out :*

A line BE 60 m long was set out roughly parallel to the river. Line CE was extended to D and mid point F of DB was established. Then EF was extended to G such that FG = EF. And DG was extended to cut the survey line ABC at H. GH and HB were measured and found to be 40 m and 80 m respectively.

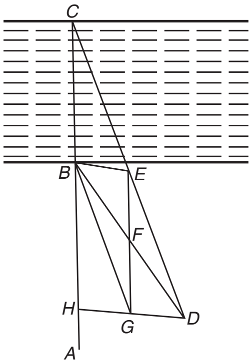


Fig. 3.55.

Find the width of the river.

Solution. (Fig. 3.55)

In the quadrilateral *BEDG*, diagonals *BD* and *EG* bisect at *F*.

∴ *BEDG* is a parallelogram and hence, $BE = GD = 60$ m.

Let the width of the river $BC = x$.

From similar Δs *CBE* and *CHD*,

$$\frac{BC}{BE} = \frac{CH}{HD}$$

or $$\frac{x}{60} = \frac{x + 80}{40 + 60}$$

or $$100x = 60x + 4800$$

or $$x = 120 \text{ m.} \quad \textbf{Ans.}$$

Example 3.15. *Two ranging rods, one of 3.00 m and the other of 1.50 m length, were used in the effort to find the height of an inaccessible tower. In the first setting the rods were so placed that their tops were in line with the top of the tower. The distance between the rods was 15 m. In the second setting the rods were ranged on the same line as before. This time the distance between the rods was 30 m. If the distance between the two longer rods was 90 m, find the height of the tower.*

Solution.

Let $BE = x$

$$\tan \alpha = \frac{3.0 - 1.5}{30} = 0.05$$

$$\tan \beta = \frac{3.0 - 1.5}{15} = 0.10$$

$$FT = (x + 15) \tan \beta = (x + 90 + 30) \tan \alpha$$

∴ $$(x + 15) \times 0.1 = (x + 120) \times 0.05$$

$$2(x + 15) = x + 120$$

$$x = 90 \text{ m.}$$

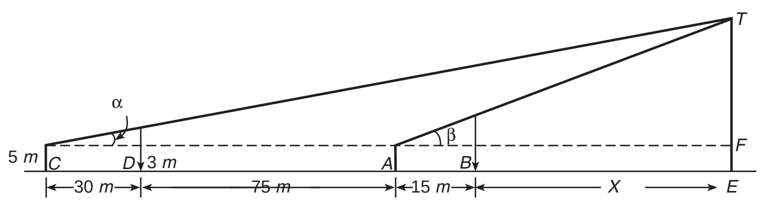


Fig. 3.56.

$\therefore \qquad FT = (90 + 15) \tan \beta$

$= 105 \times 0.1 = 10.5$ m

Ht. of tower $= EF + FT$

$= 1.5 + 10.5 = 12$ m **Ans.**

3.29. CROSS STAFF SURVEY

The survey which is carried out to prepare cadastral (or revenue) maps for locating the boundaries of each and every field and also to determine their areas, is called *cross staff surveying.*

3.30. METHOD OF CROSS STAFF SURVEY

A chain line is run through the centre of the field and the area is divided into a number of right angled triangles and trapezoids. Offsets for each turning point of the field boundary, are taken with a cross staff and their chainages are noted in a field book as already explained.

3.31. INSTRUMENTS REQUIRED FOR CROSS STAFF SURVEY

The following instruments are required for cross-staff survey.

1. A cross staff to divide the area into right angled triangles and trapezoids.
2. Two chains to measure the lengths of the base line and perpendicular offsets.
3. Arrows and ranging rods.
4. A plumb bob.

3.32. CALCULATION OF THE AREA OF A CROSS STAFF SURVEY

(*a*) **Triangle**. The area of a right angled triangle is calculated from the formula *i.e., the area of a right angled triangle is equal to its base multiplied by half the perpendicular.*

(*b*) **Trapezoid**. The area of a trapezoid is calculated from the formula *i.e., the area of a trapezoid is equal to the base multiplied by half the sum of two perpendiculars.*

3.33. PLOTTING A CROSS STAFF SURVEY

On completion of field observations and measurements, the survey is plotted to a convenient scale as explained in the following solved examples.

Example 3.16. *Plot the following details of a field and calculate its area in hectares, all measurements being in metres.*

	U	
T *80*	Δ *100*	
	74	
	54	42 S
R 50	26	
	14	30 Q
	Δ	
	P	

Solution. Fig. (3.57).

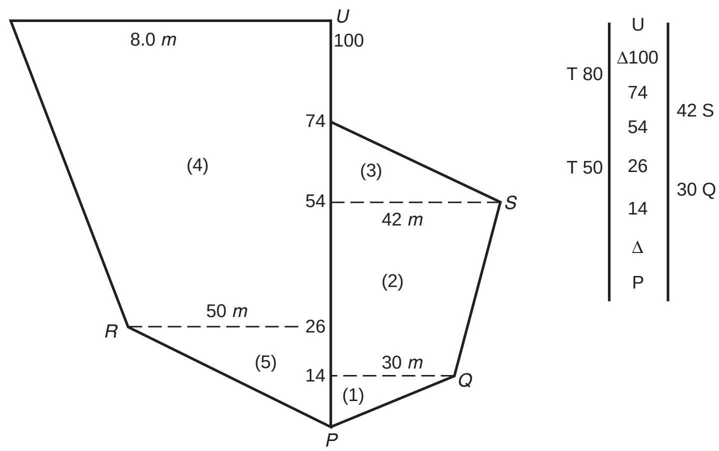


Fig. 3.57.

Area of figure (1) $= \frac{1}{2}(14 \times 30) = 210$ sq. m.

Area of figure (2) $= \frac{(54 - 14)\ (30 + 42)}{2} = 1440$ sq. m.

Area of figure (3) $= \frac{1}{2}(20 \times 42) = 420$ sq. m

Area of figure (4) $= \frac{1}{2}[(100 - 26)\ (50 + 80)] = 4810$ sq. m

Area of figure (5) $= \frac{1}{2}(26 \times 50) = 650$ sq. m

Total Area = 7530 sq. m

∴ The area of the field $= \frac{7530}{10,000}$ = **0.753 hectare. Ans.**

Example 3.17. *Draw a rough sketch of the following cross-staff survey of a field ABCDEFG and calculate its area in hectares. All distances are given in metres.*

		750		*D*
		650	*105*	*E*
C	90	*490*		
		300	*125*	*F*
B	*80*	*180*		
		100	*25*	*G*
		0		*A*

Solution. (Fig. 3.57.)

Area of figure (1) $= \frac{1}{2} \times 100 \times 25 = 1{,}250$ sq. m.

Area of figure (2) $= \frac{1}{2}(200 \times 150) = 15{,}000$ sq. m.

Area of figure (3) $= \frac{1}{2}(350 \times 230) = 40{,}250$ sq. m.

Area of figure (4) $= \frac{1}{2}(100 \times 105) = 5{,}250$ sq. m.

Area of figure (5) $= \frac{1}{2}(260 \times 90) = 11{,}700$ sq. m.

Area of figure (6) $= \frac{1}{2}(310 \times 170) = 26{,}350$ sq. m.

Area of figure (7) $= \frac{1}{2}(180 \times 80) = 7{,}200$ sq. m.

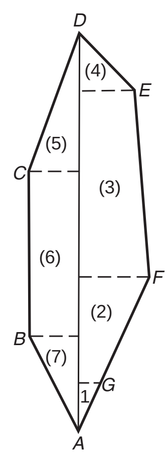


Fig. 3.57a.

$$= 1{,}07{,}000 \text{ sq. m.}$$

∴ Total area = **10.7 hectares** **Ans.**

Example 3.18. *Sketch the following cross-staff survey of a field ABCDEF and calculate the area in hectares, assuming all measurements in metres.*

	300	*Y*
D 20	*270*	
	260	*80 C*
	220	*60 B*
E 80	*180*	
F 100	*120*	
	70	
	50	*40 A*
	0	*X*

Solution. (Fig. 3.58.)

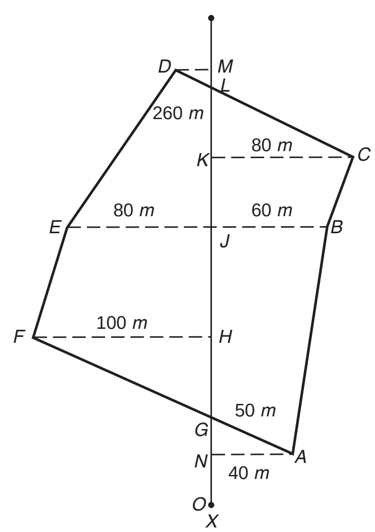


Fig. 3.58.

Area of figure $ANG = \frac{1}{2}(20 \times 40) = (-)\ 400$ sq. m.

Area of figure

$$ANJB = \frac{1}{2}(130 \times 100) = 6500 \text{ sq. m.}$$

Area of figure $BJKC = \frac{1}{2}(40 \times 140) = 2800$ sq. m.

Area of figure $CKL = \frac{1}{2}(40 \times 80) = 1600$ sq. m.

Area of figure $DML = \frac{1}{2}(10 \times 20) = (-)\ 100$ sq. m.

Area of figure $DMJE = \frac{1}{2}(90 \times 100) = 4500$ sq. m.

Area of figure $EJHF = \frac{1}{2}(60 \times 180) = 5400$ sq. m.

Area of figure $FHG = \frac{1}{2}(50 \times 100) = 2500$ sq. m.

Area of the field $= 6500 + 2800 + 1600 + 4500 + 5400 + 2500 - (400 + 100)$

$= 22800$ sq.m.

Hence area = **2.28 hectares. Ans.**

3.34. CONVENTIONAL SIGNS

A map is a graphical representation of the earth's surface on a plane paper. As the earth surface contains varieties of natural and cultural features, their depiction graphically will not be possible unless their descriptions are typed, which consequently make a map overcrowded. Such crowded maps are of little utility to map readers and field engineers. To overcome this difficulty, standard symbols have been adopted for each type of details.

Let us suppose that there are ten temples falling in the body of a map. There are following two alternatives to depict their locations on the map :

(*i*) Drawing a hut symbol for each temple and typing the word temple close to it such as temple

(*ii*) Drawing a special symbol for the temple acceptable by all map producers and map readers such as.

The symbols which are drawn to depict natural or cultural details on a map, are known as '*Conventional Signs*'.

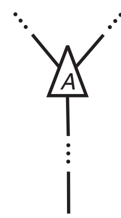
Utility of conventional signs can be realised from the fact that ten words would have been typed for ten temples. Similarly, if there are other details like mosque, church, tomb, Idgah etc., typing of equal number of descriptions of these details, will make the map overcrowded and the purpose of the map will be defeated due to lack of clarity. Conventional signs have been designed so as to closely resemble the details either in elevation or in plan. For an example, the conventional sign of a mosque in profile may be drawn as . O the other hand the conventional sign of triangulation stations may be drawn in plan like Δ. It may be noted that conventional signs drawn in profile should be drawn clearly so that their actual surveyed positions fall at the centre of the base of symbols. For the symbols drawn in plan, surveyed positions should be at the centre of the symbols. As far as possible, typing should be avoided in the body of a map for explaining the conventional signs.

Some of the conventional signs in common use, are shown here under:

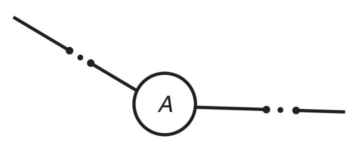
Sl. No. *Name* *Conventional Sign*

1. Chain line

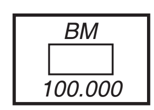
2. Triangulation station



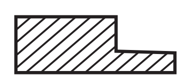
3. Traverse station



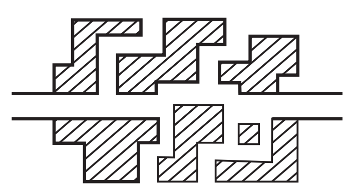
4. Bench mark



5. Building

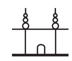


6. Township



7. Temple

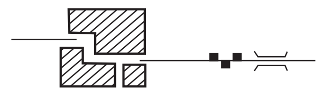
8. Mosque



9. Well lined

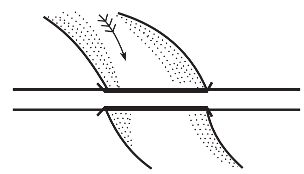
10. Foot path

11. Cart track with a bridge

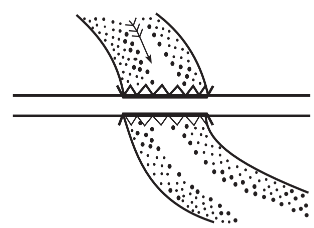


12. Unmetalled road

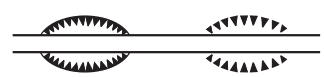
13. Metalled road with a bridge



14. Metalled road with boat bridge

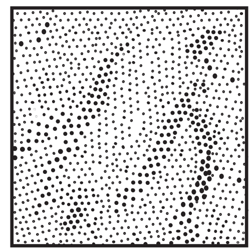


15. Road in cuttings



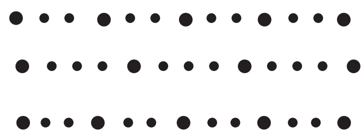
16. Road on embankments

17. Sand dunes



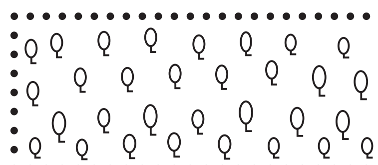
18. Pipe line

19. Telephone/telegraph line



20. Main Power line

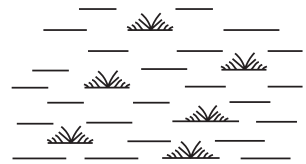
21. Power line



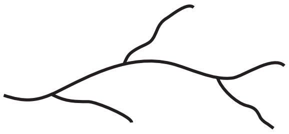
22. Trees

23. Orchard

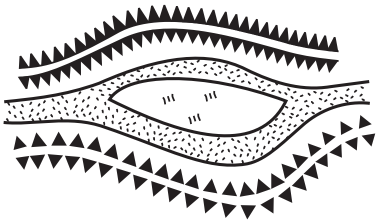
24. Swamp or Marsh



25. Stream single line



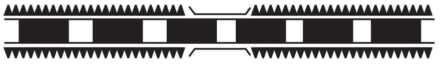
26. River double line with embankments



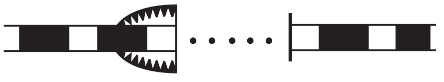
27. Railway single line with station

28. Railways, other gauge

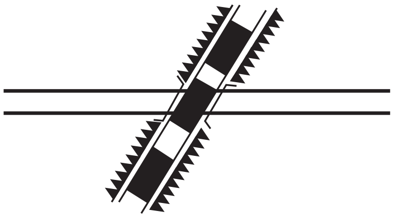
29. Railway bridge



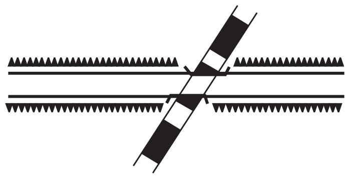
30. Railway tunnel with or without cuttings



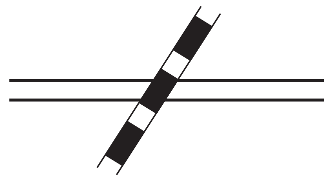
31. Railway over road



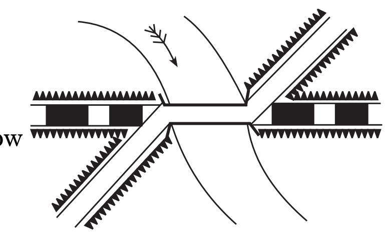
32. Road over railway



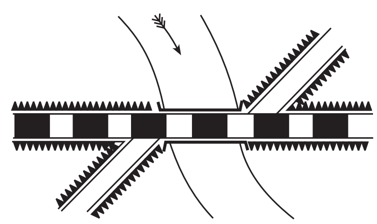
33. Level crossing



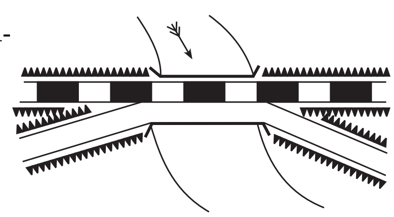
34. Bridge carrying railway below road



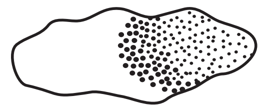
35. Bridge carrying railway over road



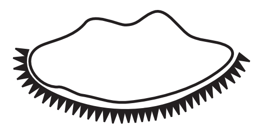
36. Bridge carrying road and railway



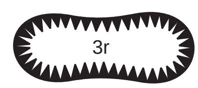
37. Ropeway with terminus



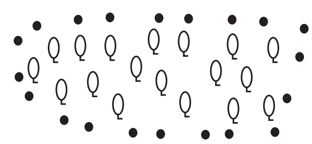
38. Lake as surveyed



39. Lake as surveyed with embankment



40. Quarry

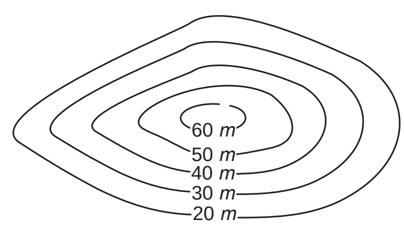


41. Reserved protected forest

42. State boundary demarcated

43. State boundary undemarcated

44. District boundary



45. Contours

3.35. PLOTTING A CHAIN SURVEY

On successful completion of field observations and measurements, plotting of a chain survey, is done in the office by draftsmen. Plotting includes preparation of plans and computation of their areas with the help of field notes. The plotting of a chain line may be done in the following steps:

1. Selection of scale. Scale of a map is very important. It should be decided before the commencement of the field work. The choice of the scale is governed by the size of sheet and the extent and purpose of the survey.

2. Plotting of Survey Lines. The longest line generally known as *base line* is first drawn in the appropriate position on the sheet. The positions of intermediate survey stations are carefully scaled and marked with fine pencil dots. Other chain lines forming triangles with base line are plotted by describing short intersecting arcs with the lengths of their sides as radii. The accuracy of plotting of these triangles can be checked by fitting in the check lines. The whole framework must be plotted and checked before plotting of the details of chain lines, is commenced.

3. Plotting of Details. Plotting of offsets may be done in two ways:

(*i*) In the first method, chainages of the offsets, are marked along the survey lines and their lengths are plotted at right angles. In plotting short offsets, perpendicularity of offsets may be estimated by eye. But, for long offsets, pencil lines are drawn perpendicular to survey lines by set squares.

(*ii*) In the second method, a short scale, called an *offset scale* is used. An ordinary scale is laid parallel to the chain line such that zero of the offset scale coincides with the chain line. The chainages can be read on an ordinary scale. The lengths of offsets are read on offset scale. The offset scale is slided along the ordinary scale which is held by weights. The various offset lengths are pricked off rapidly. If the offset scale is graduated such that its zero division is at the centre of its length, the ordinary scale is laid down parallel to the chain line and at a distance equal to half the length of the offset scale so that zero of the offset scale coincides with the chain line.

The offset scale may then be slided to various chainages. The offsets are marked on both the sides of the chain line.

The plotted points are joined by straight or curved lines as the case may be. It should be remembered that changes in direction of the boundary occur only at the ends of offsets.

Common mistakes in plotting details. The following mistakes are generally committed in plotting details:

1. Plotting offsets from wrong points
2. Plotting offsets on wrong side of survey line
3. Omitting offsets
4. Scaling chainages from wrong end of the chain line
5. Joining wrong detail points.

3.36. COMPLETION OF DETAILS

The details on a drawing sheet are first drawn in pencil and subsequently inked up in colours. It is always convenient to ink up the details from top of the plan downwards and from left to right. Curved lines should be inked up with the help of French curves. Straight lines should be ruled with a straight scale. The junction of curved line with a straight should be inked up neatly. The chain lines are drawn with dash and dot in blue with 6 mm circles in red at the end stations. The standard conventional signs are used for important details.

3.37. COMPLETION OF A SHEET

The title of the plan is printed with vertical or sloping letters in south east corner. It should invariably include the name of area, owner's name, etc.

A north line pointing upwards must be drawn on the sheet in any convenient blank space on the paper.

The scale of the plan should be drawn under the title or just inside the border at the bottom of the plotted area. The scale should be drawn before offsets are plotted.

EXERCISE 3

1. Use suitable words given in brackets to fill in the blanks:

(*i*) The simplest type of surveying is.....surveying.
(Plane table, compass, chain)

(*ii*) Chain surveying is most suitable when the ground is fairly level with .....details. (simple, crowded)

(*iii*) A system of surveying in which sides of various triangles are measured directly in the field with a chain and no angular measurement taken, is known as .....surveying.
(compass, plane table, chain)

(*iv*) The principle of chain surveying is to divide the area into .....
(rectangles, triangles, squares)

(*v*)

A chain triangle is said be well-conditioned if none of its angle is less than.... (100, 200, 300, 400)

(*vi*) The length of the handle of a chain.....the part of the end link. (forms, does not form)

(*vii*) Each metric chain is accompanied by ....arrows. (1, 5, 10)

(*viii*) A 30-metre chain contains....links and each link measures .....cm. (100, 150, 20, 30)

(*ix*) A ranging rod is generally.....metres long. (2 to 3, 3 to 4, 4 to 5)

(*x*) An offset rod is provided at its top with a ...... (hook, flag)

(*xi*) A straight line joining a station on a main survey line and another station on another survey line, is called a .....line. (subsidiary, tie, check)

(*xii*) In a chain survey, subsidiary lines are provided to.....the interior details. (check, locate)

(*xiii*) The longest chain line passing through the centre of the area, is known as.....line. (survey, chain, base)

(*xiv*) The survey line provided to check the accuracy of the frame work is known as....line. (check, tie, subsidiary)

(*xv*) To range a line across a mound with ends not visible, the method of....ranging is used. (direct, indirect)

(*xiv*) The method of stepping is used for measuring horizontal distances in the case of ....surface. (level, undulating, sloping)

(*xvii*) Measurements and sketches of chain survey are recorded in a.....book. (field, measurement, exercise)

(*xviii*) Walking over the area and observing its main features and boundaries, is known as.... (inspection, reconnaissance, observation)

(*xix*) The principle of working of an optical square is based upon... (double reflection, double refraction)

(*xx*) The angle between two plane mirrors of an optical square is.... $(45^o, 30^o, 60^o)$

(*xxi*) Incorrect length of a chain is a source of....error. (Compensating, cumulative)

(*xxii*) For accurate measurement of distances,....are used. (chains, metallic tapes, invar tapes)

(*xxiii*) The length of a line measured with an erroneous chain is ...where D and D′ are true and erroneous distances of the line and L and L′ are true and erroneous chain lengths.

$$\left(\frac{L'}{L} \times D', \frac{L}{D'} \times D, \frac{D}{D'} \times L\right)$$

(*xxiv*) In case of chaining across a building....is obstructed. (ranging, vision, ranging and vision both)

(*xxv*) A.....field book is convenient for large scale and detail dimensions.

(single line, double line)

2. Write 'Correct' or 'Wrong' against the following statements:

(*i*) Correction for temperature is applied in chaining.

(*ii*) Correction for slope is applied in chaining as well as in taping.

(*iii*) No correction for sag is applied while taping or changing.

(*iv*) Measurement of distances by taping is more accurate than that obtained by chaining.

(*v*) Accuracy of chaining depends upon the alertness of the follower and not the leader.

(*vi*) It is not necessary to measure offsets with the same accuracy as the chain line from which offsets are taken.

(*vii*) A line ranger is used for ranging a line when end stations are not visible.

(*viii*) Offsets should be as long as possible.

(*ix*) The length of a chain should be checked against a standard at frequent intervals.

(*x*) Two links of a chain, are connected to each other by three rings.

(*xi*) Principle of chain survey is to work from the part to the whole.

(*xii*) The field book of chain survey, is commenced at the bottom of a page and worked upwards.

(*xiii*) Metal arm cross-staff, is a modified form of wooden cross-staff.

(*xiv*) In case of a river, chaining is free but vision is obstructed.

(*xv*) Base line is the most important line of a chain survey.

(*xvi*) It is always better to work from the part to the whole.

(*xvii*) It is better to move up than step down while measuring a distance along sloping ground.

(*xviii*) Cumulative errors though large, as compared to compensating errors, can be corrected but not the compensating errors, though small.

3. Explain the principle used in Chain Surveying. What are the limitation of chain surveying ? Explain briefly the situations where it can be suilably employed.

4. Explain the principle of Chain Survey. When does it become in-covenient ?

5. Which type of area is best suited for Chain Survey ? Give reasons.

6. What are the instruments used in Chain Surveying ? How is the Chain Survey executed in the field ?

7. What is Reconnaissance ? State its importance in Chain Surveying.

8. Draw a neat sketch of an optical square and give in details its principle, construction and working.

9. Attempt the following :

(*a*) Advantages of working from the whole to the part.

(*b*) Continuing a chain line when it crosses a river at an oblique angle.

(*c*) What do you understand by well-conditioned triangles and why are they used ?

10. (*a*) What survey equipments will be required for chain survey of a field?

(*b*) Describe with the help of neat sketches

(*i*) An optical square

(*ii*) An Engineer's chain

(*iii*) A Cross-staff.

11. Describe in detail how you would range and chain a line between two points which are not intervisible because of an intervening hillock.

12. (*a*) Describe the principle, construction and working of an optical square.

(*b*) What are the different kinds of ranging across a high ground.

13. (*a*) Define surveying. Explain the principle of Chain Surveying.

(*b*) Give a list of sources of error in chain survey and say which of these are cumulative and which are compensating.

14. (*i*) Enumerate the instruments required for making a chain survey.

(*ii*) Describe a field book and show how the field measurements are entered in it.

15. Define the following terms :

(*i*) Swing offset, (*ii*) Oblique offset, (*iii*) Random line, (*iv*) Reference sketch, (*v*) Key plan, (*vi*) Base line, (*vii*) Check line, (*viii*) Tie line, (*ix*) Well-conditioned triangle, (*x*) Tie station.

16. During the process of chaining, you come across (i) a pond (ii) a tall building. Describe how you would continue the line with chain only.

17. Describe the method of determining the width of a river when a chain line crosses the river: (i) normally, (ii) obliquely.

18. (*a*) What are the conventional Signs ?

(*b*) Give the conventional signs for the following :

(*i*) Metalled road in cutting, a culvert, Bridge, Road on embankments.

(*ii*) Railway double line, Railway bridge, a single line railway track.

(*iii*) Compound wall, Hedge, compound wall with gate, Grassy land, Marshy land, and lake.

(*iv*) Canal, Pond, Canal with lock.

19. What are offsets ? How are they taken and recorded ? Why is it desirable to take short offsets.

20. Find the maximum length of an offset so that displacement of a point on a map, does not exceed 0.025 cm, given that the offset was laid out 3° from its true direction and the scale of the map was 1 cm to 10 m.

21. Find the maximum permissible error in laying off the direction of the offset so that maximum displacement may not exceed 0.025 cm, given that length of the offset is 12 m, the scale of the plan is 1 cm = 50 m and maximum error in the length of the offset, is 50 cm.

22. Find the maximum length of an offset so that displacement on paper from both the sources of error, does not exceed 0.025 cm, given that the offset is measured with an accuracy of 1 in 25 and the scale is 20 m to 1 cm.

23. A survey line ABC crosses normally a river flowing EW. Points B and C being on the near and far banks respectively. A perpendicular BD 50 m long is set out at B. If the bearings of DC and DA are 35° 25′ and 125° 25′ respectively, the chainages of A and B are 862 m, and 900 m respectively, find the chainage of C.

24. A survey line ABC crosses normally a river flowing EW. Points B and C being on the near and far banks respectively. A perpendicular BD 36.44 m long is set out at B. The bearings of AD and DC are 39° and 309° respectively. If distance AB is 45 m, find the width of the river.

25. A chain line CDE crosses a river, D and E being on the near and distant banks respectively. A perpendicular DF 54.865 m long is set out at D on the left of the chain line. The respective bearings of E and C taken at F are 67° 30′ and 157° 30′. Find the chainage of E, given that CD is 27.630 m and the chainage of D is 382.525 m.

26. A chain line ABC is obstructed by a pond. To prolong the chainage a line EBF is set out such that EC and FC are clear of obstruction. If EF = 140 m, BE = 50 m, FC = 115 m and EC = 125 m, calculate the length BC.

27. A line ABC crosses a river, B and C being on the near and distant banks respectively. Perpendiculars BD and AE 30.5 m and 50.5 m long respectively are drawn such that C, D and E are in a straight line. If the chainage of A and B are 505.5 m and 555.5 m respectively, calculate the chainage of C.

28. A survey line ABC crossing a river at right angles cuts its banks at B and C. To determine the width BC of the river, the following operation was carried out.

A point E was established on the perpendicular BE such that angle CEF is a right angle where F is a point on the survey line. If the chainage of F and B are respectively 1200 m and 1320 m, and the distance EB is 90 m, calculate the width of the river and also the chainage of C.

29. A, B and C are three points on a chain line running along the magnetic meridian from south to north. Points B and C are on the south and north banks of the river respectively. To determine the width of the river a point D, 200 m from B is fixed perpendicular to the chain line and the following magnetic bearings were taken:

Line	*F.B.*	*B.B.*
BC	357°	180°
BD	87°	270°
CD	150°	330°

Calculate the chainage of C, if that of B is 573.6 m.

30. Magnetic bearing of a chain line which crosses a river is N 45° W. A and B are two stations on the chain line on either bank of river, from where magnetic bearings of an electric pole are observed and found 330° and 355° respectively. If the perpendicular offset of the electric pole is 78.73 m, calculate the width of the river.

ANSWERS

1. (*i*) Chain; (*ii*) simple; (*iii*) chain; (*iv*) triangles; (*v*) 30° (*vi*) forms; (*vii*) 10; (*viii*) 150, 20; (*ix*) 2 to 3; (*x*) hook; (*xi*) tie; (*xii*) locate; (*xiii*) base; (*xiv*) check; (*xv*) indirect; (*xvi*) sloping; (*xvii*) field; (*xviii*) reconnaissance; (*xix*) double reflection; (*xx*) 45° ; (*xxi*) cumulative; (*xxii*) invar tape; (*xxiii*) $\frac{L'}{L} \times D'$; (*xxiv*) ranging and vision both; (*xxv*) single line.

2. (*ii*), (*iii*), (*iv*), (*v*), (*vi*), (*ix*), (*x*), (*xii*), (*xiii*), (*xv*), (*xvi*), (*xviii*), Correct.

(*i*), (*vii*), (*viii*), (*xi*), (*xiv*), (*xvii*) Wrong.

20. 4.78 m21. 5o29′.

22. 8.84 m.23. 965.8 m.

24. 29.5 m.

25. 491.471 m.

26. 101.33 m.

27. 631.75 m.

28. 67.5 m; 1387.5 m.

29. 920 m.

30. 200 m.

4

Compass Surveying

4.1. INTRODUCTION

The branch of surveying in which directions of survey lines are determined by a compass and their lengths by chaining or taping directly on the surface of the earth, is called *compass surveying*. Method of chain surveying is preferred to, if the area to be surveyed is small in extent and higher accuracy is aimed at. On the other hand, if the area is comparatively large, with undulations, compass surveying is adopted. Before recommending the compass survey for any area, it must be ascertained that area is not magnetically disturbed.

4.2. TRAVERSE

A series of connected straight lines each joining two ground stations on the ground, is called a *traverse*. End points are known as *traverse stations* and straight lines between two consecutive stations, are called *traverse legs*.

Traverses may be either a closed traverse or an open traverse.

1. Closed traverse. The traverse which either originates from a station and returns to the same station completing a circuit or runs between two known stations, is closed called a *closed traverse*. The closed circuit of the traverse legs, is known as *traverse circuit*.

In Fig. 4.1 (*a*), the traverse originates from station *A* and follows the route as indicated by arrows, connecting stations *B, C, D, E,...,*etc. and finally closes on the originating station *A*.

In Fig. 4.1 (*b*), the traverse originates from a known station *A* and follows the route as indicated by arrows, connecting stations *B, C, D, E,* etc. and finally closes on another known station *G*.

2. Open traverse. The traverse which neither returns to its starting station nor closes on any other known station, is called an *open traverse*. In this case, a series of connected lines extends in the same general direction.

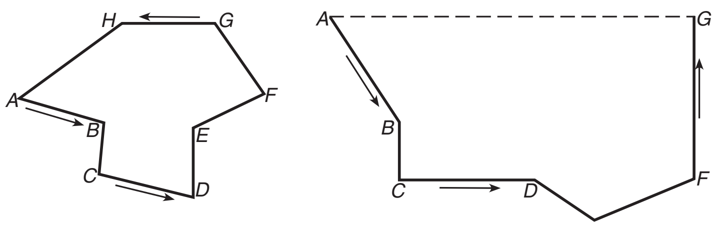


Fig. 4.1. Closed traverses.

In Fig. 4.2 the traverse originates from a station *A* and follows the route as indicated by arrows, connecting stations *B, C, D,* and finally closes on *E* whose position is not predetermined.

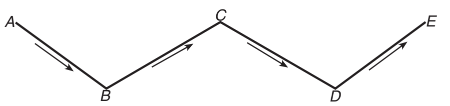


Fig. 4.2. An open or unclosed traverse.

4.3. CLASSIFICATION OF TRAVERSES BASED ON INSTRUMENTS USED

The classification of traverses based upon the instruments used, is as under:

(*i*) Chain traversing
(*ii*) Compass traversing
(*iii*) Plane table traversing
(*iv*) Theodolite traversing
(*v*) Tacheometric traversing.

1. Chain traversing or chain angles method. In chain traversing the entire work is done by a chain or tape and no angle measuring instrument is needed. The angle computed by the measurements, is known as a *chain angle*.

Derivation of the formulae for chain angles (Fig. 4.3).

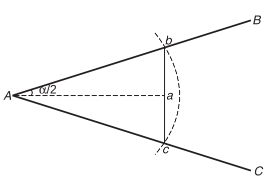


Fig. 4.3. A Chain angle

First method

Consider the angle *BAC* equal to α. With *A* as centre and radius equal to a chain length *i.e.* 30 m or 20 m, draw an arc intersecting *AB* and *AC* at *b* and *c* respectively. Bisect *bc* at *a*.

then, $$\sin\frac{\alpha}{2} = \frac{ab}{Ab} = \frac{bc}{2\,Ab} \text{ where } bc \text{ is the chord.}$$

$$= \frac{bc}{40} \text{ for a 20 m chain length}$$

or $$\frac{\alpha}{2} = \sin^{-1}\frac{bc}{40}$$

∴ $$\alpha = 2\sin^{-1}\frac{bc}{40} \text{ for a 20 m chain.}$$

Similarly $$\alpha = 2\sin^{-1}\frac{bc}{60} \text{ for a 30 m chain.}$$

It may be noted that chain angles are generally liable to an error as the principle of surveying *i.e.* working from the whole to the part is not adopted. The accuracy in the measurement of the angle is proportional to the accuracy achieved in measuring the distances *Ab, Ac and bc*. The chain angle method of traversing is more or less obsolete these days.

If the chain angle is an obtuse angle, its supplementary angle obtained by prolongation of one of the traverse lines, is measured. The chain angle is computed by subtracting the supplementary angle from 180°.

Second method

Measurement of included angles of compass traverse. When a compass is not available, the included angles may be measured as detailed below;

Let *ABCDE* be a traverse whose stations *A*, *B*, *C*, *D* and *E* are marked on the ground after ascertaining the intervisibility of adjacent stations (Fig. 4.3(a))

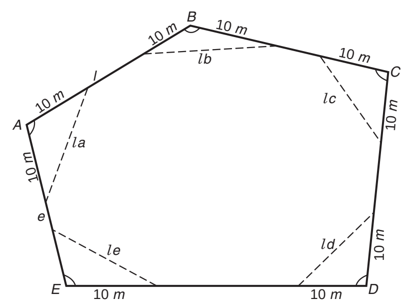


Fig. 4.3(a)

Let us measure angle *BAE* with a tape only.

The following steps are taken:

(*i*) Fix the position of traverse station *A* with a pencil mark on the top of a wooden peg driven into the ground.

(*ii*) Align the direction AB and AE with ranging rods held at B and E respectively.

(*iii*) Measure the distance Ab on AB and AC on AE equal to 10 metres and mark the locations of b and e with chain pins.

(*iv*) Measure the distance *be* with a tape correct to a millimetre.

Let be '*la*'

By applying the Cosine formula, $\cos \alpha = \dfrac{(Ab^2) + (Ae^2) - (be^2)}{2 \,.\, Ab \,.\, Ae}$, we get

$$\cos \alpha = \frac{10^2 + 10^2 - (la^2)}{2 \times 10 \times 10} = \frac{200 - (la^2)}{200}$$

$$\alpha = \cos^{-1} \frac{200 - (la^2)}{200} \text{ in degrees.}$$

(*v*) The included angles B, C, D and E may be measured in a similar manner.

(*vi*) Apply the geometrical check on the observed angle *i.e.*

Sum of included angles = $(2\,n - 4)$ *rt* angles where n is the number of sides of the closed traverse.

Example. *ABCD* is a closed transverse whose interior angles are measured as detailed below:

Station	*Side*	*Distance (m)*
A	*la*	11.90
B	*lb*	15.97
C	*lc*	11.76
D	*ld*	16.38

Determine the value of each angle and provide a geometrical check.

Solution

Angle $$A = \alpha = \cos^{-1} \frac{200 - (la)^2}{200}$$

$$= \cos^{-1} \frac{200 - (11.90)^2}{200} = 0.29195$$

$$\alpha = 73° \; 1' 31''$$

Angle $$B = \beta = \cos^{-1} \frac{200 - (lb^2)}{200}$$

$$= \cos^{-1} \frac{200 - (15.97)^2}{200} = 0.27520$$

$$\beta = 105° \; 58' \; 26''$$

Angle $$C = \gamma = \cos^{-1} \frac{200 - (lb^2)}{200}$$

$$= \cos^{-1} \frac{200 - (11.76)^2}{200} = 0.30851$$

$$\gamma = 72° \; 01' \; 50''$$

Angle $D = \delta = \cos^{-1} \dfrac{200 - (l6.38)^2}{200} = -\,0.34152$

$$\delta = 109° \; 58' \; 10''$$

Results:

Angle $A = 73° \; 01' \; 31''$

$B = 105° \; 58' \; 26''$

$C = 72° \; 01' \; 50''$

$D = 109° \; 58' \; 10''$

Sum $= 360° \; 59' \; 57''$

Interior angles of a quadrilateral $= (2n - 4)rt$ angles $= (2\times 4 - 4)$ *right* angles = 4 *right* angles ∴ Error = 360° 59′ 57″ – 360° = 59′ 57″

Correction to each angle, $\dfrac{59'57''}{4} = 14' \; 59''$ Say15′

The traverse angles, nowadays, are generally measured either with a compass, or a theodolite for better accuracy.

Setting out a of 60° angle with a chain and tape only (Fig. 4.4)

Let *AX* be one of the given lines.

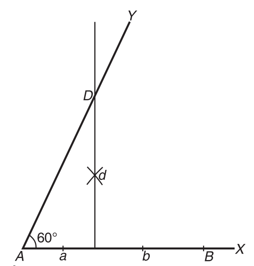


Fig. 4.4.

Following steps are followed:

1. Measure *AB* = 30 m along *AX* and fix a ranging rod at *B*.
2. Fix the mid point **C** at 15 m either from *A* or *B*:
3. Fix two points *a* and *b* equidistant from *C*.

4. With a and b as centres, draw two arcs of equal radii (greater than aC or bC) to intersect at d.
5. Join Cd and prolong it.
6. With A as centre, draw an arc of 30 m radius to cut Cd (or produced) at D.
7. Prolong AD to Y.
8. YAX is the required angle of 60°.

To check the accuracy of setting out the angle, procced as under :

(*i*) Measure CD which should be $= \sqrt{30^2 - 15^2} = 25.98$ m.

(*ii*) Measure BD which should evidently be 30 m.

Note: The problem may also be solved by other methods. Students may try these methods.

Setting out a 30° angle with chain and tape only (Fig. 4.5)

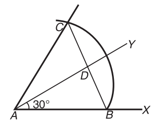


Fig. 4.5.

Let AX be one of the given lines.

Following steps are followed:

1. Measure AB= 30 m along AX and fix a chain pin at B.

2. With A as centre and radius equal to AB, draw an arc.

3. With B as centre and radius equal to AB, draw another arc to cut the earlier one at C.

4. Join BC and bisect it at D.

5. Join AD and prolong it to Y.

6. YAX is the required angle of 30°.

To check the accuracy of setting out the angle, draw arcs of equal radii with centres B and C. Their point of intersection must lie on AY, if the work is accurate.

2. Compass traversing. The traverse in which angular measurements are made with a surveying compass, is known as *compass traversing*. The traverse angle between two consecutive legs is computed by observing the magnetic bearings of the sides. This method is explained in detail in this chapter.

3. Plane table traversing. The traverse in which angular measurements between the traverse sides are plotted graphically on a plane table with the help of an alidade, is known as *plane table traversing*. For detailed description of the method, refer to chapter 5. **"Plane Table Surveying".**

4. Theodolite traversing. The traverse in which angular measurements between traverse sides are made with a theodolite, is known as *theodolite traversing*. For detailed description of the method, refer to chapter 12 **"Theodolite Traversing"**

5. Tacheometric traversing. The traverse in which direct measurements of traverse sides by chaining is dispensed with and these are obtained by making observations with a tacheometer, is known as *tacheometric traversing*. For detailed description of the method, refer to chapter 13 **"Tacheometry"**

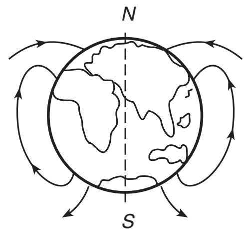


Fig. 4.6.

4.4. THEORY OF MAGNETISM (FIG. 4.6)

When a magnetic needle is balanced at its centroid on a hard steel pivot, it swings freely in a horizontal plane and ultimately rests along N-S direction of the earth magnetic field. The north pole of the earth magnet is assumed to be located near the south geographical pole and south pole near the north geographical pole. Hence, lines of force of the earth magnet travel from the north magnetic pole (the pole near the south geographical pole) to its south magnetic pole (the pole near the north geographical pole). It is also a well established fact that like poles of two magnets repel each other and opposite poles attract each other. Hence, the end of a magnetic needle pointing towards the geographical north is conventionally accepted as north pole of the magnetic needle.

The poles of earth magnet do not coincide with geographical poles and hence the magnetic meridian *i.e.* N-S direction as defined by the axis of the magnetic needle at any place is not true meridian *i.e.* N-S direction of geographical poles. South pole of the earth magnet which is called the *North magnetic pole*, is located near 70° North Latitude and 96° West Longitude, in the northern hemisphere. A similar area having north pole of the earth magnet, is called *south magnetic pole*.

Dip of magnetic needle. If a needle is perfectly balanced at its centre of gravity before magnetization, it does not remain so after magnetisation, due to the magnetic influence of the earth. The lines of the force of earth magnetic field run from geographical south pole to geographical north pole and near the equator these are parallel to the earth surface. In northern and southern hemispheres, the magnetic needle does not remain in a horizontal plane. In northern hemisphere, the north end and in southern hemisphere, the south end of the needle, deflects downwards (Fig. 4.7).

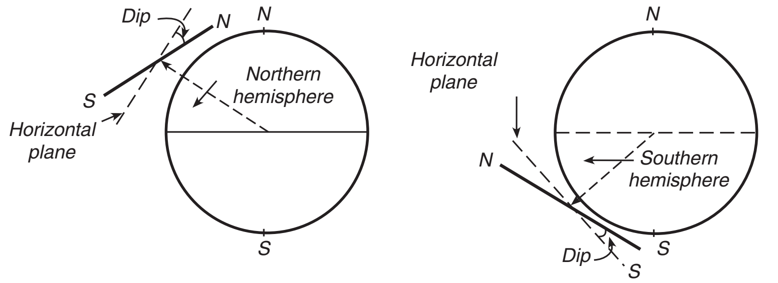


Fig. 4.7. Dip of a magnetic needle.

The angle of inclination between the longitudinal axis of a magnetic needle and the horizontal plane through its pivot is known as *Dip* or inclination of the needle measured in vertical plane. The amount of dip is not uniform. It varies differently in different parts of the earth. It is zero at the equator and 90° at the north and south magnetic poles.

The imaginary line joining the points having same dip on the surface of the earth, is known as *Isoclinic line.* The line along which there is no dip, which lies in the vicinity of the geographical equator on the earth surface, is generally known as *magnetic equator.*

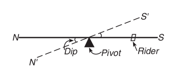


Fig. 4.8. A rider.

To keep the magnetic needle in horizontal plane at any place, a small sliding weight-known as *rider,* is attached to the higher portion of the needle (Fig. 4.8). By moving the rider towards or away from the pivot, the balance of the magnetic needle may be adjusted at any place.

4.5. SURVEYING COMPASSES

Surveying compasses are of the following two types :

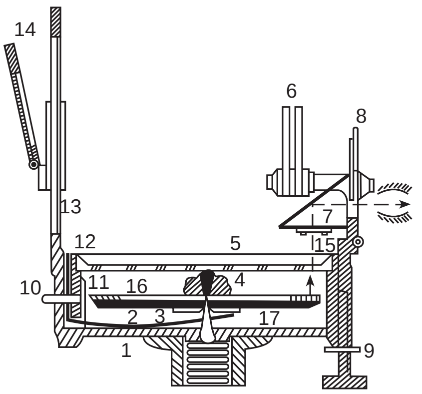


Fig. 4.9. A Prismatic Compass.

1. Prismatic Compass 2. Surveyor's Compass.

1. Prismatic Compass. It is most suitable type of surveying compass which consists of a circular box about 100 mm in diameter. Prismatic compass can be used as a hand instrument or on a tripod. Prismatic compass can be accurately centered over the ground station mark.

The main parts of a prismatic compass, are shown in FIg. 4.9. A broad magnetic needle attached to an aluminium circular ring is balanced on a hard steel pointed pivot. The circular ring is graduated to degrees and half degrees. The graduations run clockwise, having zero at South, 90° at West, 180° at North and 270° at East. The gradations are written inverted. When the needle is balanced on the pivot, it orients itself in the magnetic meridian. The North and South ends of the aluminium ring lie in N-S direction (Fig. 4.10).

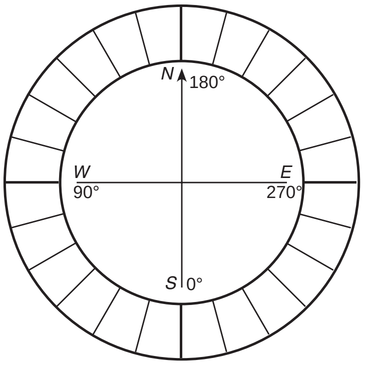


Fig. 4.10. Graduated ring.

1. Compass box	2. Lifting lever	3. Needle
4. Agate cap	5. Glass cover	6. Sun glass
7. Prism	8. Eye vane	9. Focussing stud
10. Brake pin	11. Spring brake	12. Lifting pin
13. Object vane	14. Adjustable mirror	15. Prism cap
16. Graduated ring	17. Pivot	

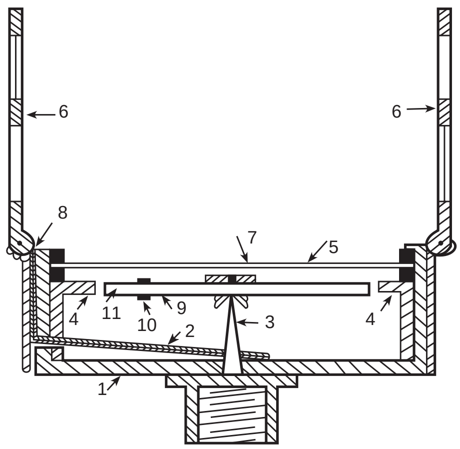
The object vane carries a vertical hair attached to a suitable frame. The object vane is sometimes provided with a hinged mirror which can be moved upwards and downwards by a screw. The mirror can be inclined at any desired angle so that objects too high or too low, can be sighted.

Sight vane or eye slit consists of vertical slit cut into the upper assembly of a prism. The object vane and sight vane, are hinged to the box diagonally opposite at the top.

When an object is sighted out of magnetic meridian, sight vane rotates with respect to N-S ends of ring through the angle which the line makes with the magnetic meridian. When the line of sight falls along the magnetic meridian, the South end of the ring comes vertically below the horizontal face of the sighting prism and the 0° or 360° reading is seen through the prism. The inverted figures of the graduations are reflected by the hypotenusal side of the eye prism. This arrangement makes the eye to read figures erect and magnified manifold. A glass lid is fitted over the box to protect the needle from dust.

When not in use, the object vane may be folded on the glass lid. It presses against a bent lever which lifts the needle off the pivot and holds it against the glass lid. Oscillations of the needle can be dampen to facilitate the reading of the graduated ring by a braking pin placed at the base of the object vane. When bright objects are sighted, a dark glass is fitted in the eye vane to diminish the intensity of light.

2. Surveyor's Compass. (Fig. 4.11).



1. Box
2. Lifting lever
3. Pivot
4. Circular graduated arc
5. Glass top
6. Sight vanes
7. Jewelbearing
8. Lifting pin
9. Magnetic needle
10. Rider
11. Metal pin

Fig. 4.11. Surveyor's Compass.

Surveyor's compass is similar in construction to the prismatic compass with a few modifications stated below :

1. The graduated ring is directly attached to the circular box and not with the magnetic needle.

2. The edge bar magnetic needle floats freely over the pivot.

3. The eye vane consists of a simple metal vane with a fine sight hole.

4. No mirror is attached to the object vane for sighting object at higher elevation or depression.

5. Readings are taken against the north end of the needle.

6. The ring is graduated in quadrental system having 0° at North and South ends ; 90° at East and West ends.

4.6. ADJUSTMENTS OF SURVEYING COMPASS

The following are the adjustments of a surveying compass are :

(*a*) **Temporary Adjustments.** The adjustments which are required to be made at every set up of the instrument, are known as *temporary* or *Station Adjustments.* Temporary adjustments include the following operations:

(*i*) Centering, (*ii*) Levelling,

(*iii*) Focusing the prism (only in the case of prismatic compass).

(*b*) **Permanent Adjustments.** The adjustments which are made only if the fundamental relations between the various parts of a compass, are disturbed due to careless handling or otherwise, are called *permanent adjustments.*

1. Temporary Adjustments. Temporary adjustments of a compass, are made at every station, as discussed below :

(*i*) Centering. The process of centering the instrument *i.e.* making the pivot exactly vertically over the ground station mark, is called *centering.* The compass is fixed on the top of a tripod. By adjusting the legs of the tripod, centering is achieved. A plumb bob may be hung from the centre of the circular box, to check the centering of the compass. If no plumb bob is provided, the centering may be judged by dropping a small pebble freely from the centre of the bottom of the circular box. If the compass is centred perfectly, the pebble will fall exactly over the ground station mark.

(*ii*) Levelling. The process of holding the compass in such a way that its graduated ring swings freely, is called *levelling.* The levelling is done by eye judgement. Generally the compass is provided with a ball and socket arrangement attached to the tripod for achieving quick levelling of the instrument. In surveyor's compass two plate levels at right angles to each other, are sometimes provided. The ball and socket arrangement is adjusted till the two bubbles remain central in both the plate levels.

(*iii*) Focussing the prism. (Only applicable to prismatic compass). The process of moving up or down the prism for obtaining the figures and graduations sharp and clear, is called *focussing the prism.*

2. Permanent Adjustments. The following fundamental relationships between different parts of a compass, are established by making permanent adjustments :

(*i*) When plate bubbles, if provided, are at the centres of their run, the vertical axis of the compass should be truly vertical.

(*ii*) When the instrument is perfectly levelled, sight vanes should be vertical.

(*iii*) The ends of the needle and the centre of the pivot should lie in the same vertical plane.

(*iv*) The centre of the pivot should coincide with the geometrical centre of the graduated ring.

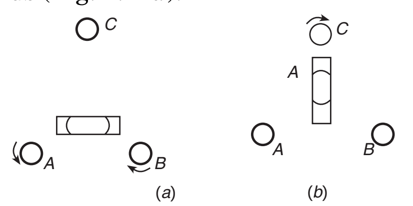
I. Adjustment of the plate levels

Object. To make the vertical axis truly vertical when plate bubbles are at the centres of their run.

Test. Proceed as under.

1. Set up the compass with a tripod on a firm ground.

2. Bring one of the levels parallel to a pair of levelling screws. Bring the bubble central by turning both the screws simultaneously either inwards or outwards (Fig. 4.12*a*).



(a) (b)

Fig. 4.12. Plate level adjustments.

3. Rotate the instrument through 90° about its vertical axis till the level tube lies parallel to the line joining the third screw and the mid-point of the first two screws. Centre the bubble with the help of third screw. (Fig. 4.12*b*).

4. Rotate the instrument through 180°. If the bubble remains central, the instrument is in adjustment.

Adjustments : If the bubble does not remain central, correct half the error by capstan's screw and the remaining half by the levelling screws.

Repeat the adjustment until the bubble remains central throughout a complete rotation of the compass.

Note. The following points may be noted :

(*i*) In compasses, if levelling screws are not provided, the ball and socket arrangement may be used for making this adjustment.

(*ii*) Considering the limitations of a compass, plate level adjustment is an unnecessary refinement.

(*iii*) Level tubes are generally not provided, in surveying compasses.

II. Adjustment of sight vanes

Object. To make sight vane and object vane vertical when the compass is level.

Test. Proceed as under :

1. Level the compass carefully.

2. Suspend a plumb bob roughly 25 metres away.

3. Move the eye from the top to the bottom of the eye vane slit. Note whether the hair of object vane remains in coincidence with the vertical plumb bob. In case the sight vane is provided with a simple hole, one should check the line of sight from the top to the bottom along the hair of object vane.

4. Swing the compass through 180° and test the sight vane in the same way, sighting through the hole in the object vane.

Adjustment : If the hair of object vane does not remain coincident either file one side of the bottom of the vane where it rests on the plate or insert a paper packing.

Repeat the test and adjustment until the error is completely eliminated.

III. Adjustment of the magnetic needle

Object. To straighten the magnetic needle

1. Test for Vertical Bending. Lower the needle on its pivot and observe if there is any rocking at its upper surface at the ends.

Adjustment. If so, remove the needle and bend it in a vertical plane till no rocking is noted on replacement.

2. Test for Horizontal Bending. Proceed as under.

(*i*) Level the compass and read both the ends of the magnetic needle in any position.

(*ii*) Rotate the compass until the graduation originally opposite the north end of the needle faces the south end.

(*iii*) If the reading at the north end is now the same as that was originally at the south end, the needle is straight. It not, half the discrepancy in reading, represents the deviation from the straightness.

Adjustment. Remove the needle and bend the north end horizontally through half the angle in a direction which would carry it from the north end reading on reversing to south end reading. Replace and repeat *till* adjusted correctly.

IV. Adjustment of the pivot

Object. To set the pivot point at the geometrical centre of the graduated ring.

Test. Read both the ends of the magnetic needle at various portions of the graduated ring. If the readings differ by 180° assuming the needle to be straight, the pivot is central.

Adjustment. If the readings do not differ by 180°, proceed as under:

1. If the difference of the readings is variable, ascertain the position at which the discrepancy is maximum.

2. Remove the needle and bend the pivot at right angles to this position of the needle and towards the greater segment of the graduated ring formed by the needle.

3. Repeat the test and adjustments until end readings agree in all the positions.

4.7. COMPARISON BETWEEN A SURVEYOR'S COMPASS AND PRISMATIC COMPASS

Sl. No.	*Item.*	*Surveyor's Compass*	*Prismatic Compass*
1.	Magnetic needle	The needle is of edge bar type and also acts as an index.	The needle is broad needle type but does not act as an index.
2.	Graduated ring	(*i.*) The graduated ring is attached to the box and not to the needle. This rotates along with the line of sight. (*ii*) The graduations are in Q.B. system, having 0° at North and South and 90° at East and West. East and West are interchangeable. (*iii*) The graduations are engraved erect.	(*i*) The graduated ring is attahced with the needle. This does not rotate along with the line of sight. (*ii*) The graduations are in W.C.B. system having 0_o at South
3.	Sighting vanes	(*i*) The object vane consists of a metal vane with a vetical hair	(*i*) The object vane consists of a metal vane with a vertical hair.
		(*i*) The eye vane consists of a small vane with fine slit.	(*ii*) The eye vane consists of a metal vane with a slit.
4.	Reading system	(*i*) The readings are taken directly be seeing through the top of the glass (*ii*) Sighting and reading cannot be done simultaneously from one position of the observer.	(*i*) Th readings are taken with the help of a prism provided at the eye slit. (*ii*) Sighting and reading can be done simultaneously from one position of the observer.
5.	Tripod	The instrument cannot be used without a tripod.	Tripod may or may not be provided. The instrument may be used even by holding in hand.

4.8. MERIDIANS AND BEARINGS

The directions of survey lines may be defined in two ways :

(*i*) Relatively to each other

(*ii*) Relatively to some reference direction

In the first case, directions are expressed in terms of the angles between two consecutive lines. In second case, these are expressed in terms of bearings.

Meridian. The fixed direction on the surface of the earth, with reference to which, bearings of survey lines are expressed, is called a *meridian.*

Bearing. The horizontal angle between the reference meridian and the survey line measured in a clockwise direction, is called *bearing.*

The meridians of reference directions employed in surveying may be one of the following.

1. True meridian 2. Magnetic meridian
3. Grid meridian 4. Arbitrary direction

1. True Meridian. The line of intersection of the earth surface by a plane containing north pole, south pole and the observer's position is called *true meridian* or *geographical meridian.* It represents true north-south direction at the place. Geographical meridian at different places are not parallel to each other. These converge to a point in northern and southern hemispheres as shown in Fig. 4.13.

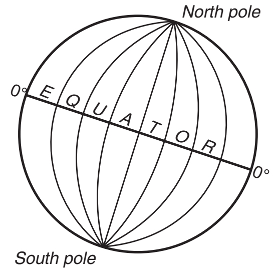


Fig. 4.13

Equatorial circumference of the earth surface is divided into 360°. The true meridian of Greenwich has been assumed internationally as 0°. The meridians on its eastern sides are known as *East of Greenwich* and on the western side of zero meridian as *West of Greenwich.* 180° meridian on the globe is just opposite to Greenwich meridian.

Determination of the true meridian at any place precisely, is made by making astronomical observations to heavenly bodies, *i.e.* sun and stars.

Detailed description of the method is described in Author's book ***Advanced Surveying.***

The true meridian at any place is not variable. In engineering surveys it is very useful to save time in laying the surveying lines during construction. Due to convergence of true meridians, it is adopted for large scale surveys only in the area of limited extent.

It may be mentioned here that the maps prepared by the National Survey Department of any country, such as Survey of India are based on true meridians.

Convergency of true meridians. True meridians of two places other than those which lie on the same meridian, do not run parallel to each other. Convergency of the meridians varies according to the latitude of the place. It is more in higher latitudes than near the equator. The convergency of meridians at latitude λ is the angle between their tangents at latitudes λ. It may be computed from the formula :

Convergency is seconds = Diff. in longitudes in seconds $\times \sin \lambda$

where λ is the latitude of the place.

Determination of true meridian (Fig. 4.14). The direction of the true meridian at a place, may be determined as under:

On a fairly horizontal ground, draw a circle with any point *O* as centre and having a radius sufficiently large. Fix a straight rod AO vertically at *O*. Note the directions *OB* and *OC* of the shadow of the rod when it just toaches the circumference of the drawn circle. When the sun is low in the east, the shadow falls far away from the circumference. The length of the shadow decreases as the sun's altitude increases. When the end of shadow just appears to cross the circumference, mark its position *B*. When the sun crosses the meridian of the place *O*, the shadow starts increasing. Again, when the shadow appears to approach the circumference, mark the position *C*. The sun's altitudes and azimuths are equal when the shadows are equal, *AOB* and *AOC* are the vertical planes passing through the sun at these instants, and these are equally inclined to the meridian *ON*. The bisector of the angle *BOC*, is the direction of the true meridian at *O*.

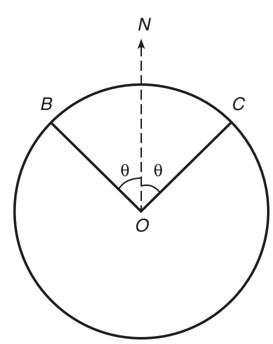


Fig. 4.14. Determine of true merdian.

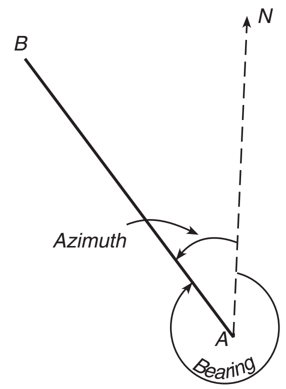


Fig. 4.15. Azimuth and bearing.

True Bearing. The horizontal angle between the true meridian and the survey line measured in a clockwise direction, is called *true bearing* of the line.

Azimuth. The smaller angle which a survey line makes with the true meridian, is called *'azimuth'*. Azimuth does not specify the direction of measuring the angle between the true meridian and the survey line. Azimuth and true bearing of the survey line *AB*, are shown in Fig. 4.15.

If *AN* represents the true meridian and *AB* is any given survey line, true bearing of the line *AB* is the angle *NAB* measured in clockwise direction whereas its azimuth is the acute angle NAB, measured in anti-clockwise direction.

Calculation of azimuth. The azimuth of lines may be calculated as under:

1. If true bearing of any line is more than 180°, then the azimuth of the line may be calculated by subtracting it from 360°.

2. If true bearing of any line is less than 180°, then, the azimuth of the line will be equal to the true bearing.

Note. In some countries azimuth of survey lines, is reckoned from south.

Example 4.1. *Calculate the azimuth of a line if its true bearing is 275° 45′.*

Solution.

True bearing of the line = 275° 45′

Azimuth of the line = 360° – 275° 45′ = 84° 15′. **Ans.**

Example 4.2. *Calculate the azimuth of a line if its true bearing is* 30° 45′.

Solution. True bearing is less than 180°.

Azimuth of the line = True bearing of the line

= 30° 45′. **Ans.**

2. Magnetic Meridian. The longitudinal axis of a freely suspended and properly balanced magnetic needle, unaffected by local attractive forces, defines the magnetic north-south line which is called the *magnetic meridian.* It does not coincide with the true meridian except in certain localities during the year.

Magnetic Bearing. The horizontal angle which a survey line makes with the magnetic meridian, is called *magnetic bearing.* It is not constant at a point but varies with laps of time. (Fig. 4.16.)

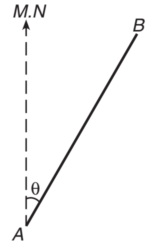


Fig. 4.16. Magnetic Bearing.

3. Grid Meridian. In every country, state survey maps are based on one or more true meridians of the places so chosen that they are centrally placed in definite belts bounded by some geographical meridians and parallels (Longitudes and latitudes). Such maps generally have a rectangular 'grid' plotted on them. The northsouth lines of the grid, are parallel to the line representing the central meridian. The direction of the grid lines along the north-south direction, is generally known *Grid meridian*.

Grid Bearing. Bearings of survey lines referred to and reckoned from grid lines are called *grid bearings.* As the grid meridian and true meridian at any place other than the central point, are not parallel, the former is inclined at a small angle to the later. The angle between the true meridian and the grid meridian at any place, is known as *grid convergence*. When the grid line is inclined towards east, the convergence is east and if it is towards west, it is west.

4. Arbitrary Meridian. The convenient direction which is assumed as a meridian for measuring bearings of survey lines, is known as *arbitrary meridian.* Arbitrary meridians are generally assumed for survey of small plots of land. An arbitrary meridian has the merit of being invariable and its direction can be easily recovered at a future date, if and when required. If the angle between the true meridian and the assumed arbitrary meridian, is established later, the arbitrary meridian may also be converted to true meridian by applying the desired correction.

Example 4.3. *The bearing of a line with reference to an arbitrary meridian is 85° 30′. At a later date, it was established that the angle between the arbitrary meridian and the true meridian, is 15° 10′ W. Calculate the true bearing of the line.*

Solution. (Fig. 4.17.).

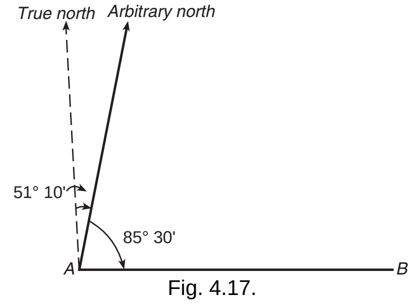


Fig. 4.17.

Arbitrary bearing of the line = 85° 30′ Given

Angle between two meridians = 15° 10′

∴ True bearing of the line = 85° 30′ + 15° 10′

= 100° 40′. **Ans.**

4.9. DESIGNATION OF BEARINGS

Bearings of survey lines are designated in the following systems:

(*i*) The whole circle bearing system (W.C.B.)

(*ii*) The quadrental bearing system (Q.B.)

1. The Whole Circle Bearing System. The whole circle bearing system is also sometimes known as *Azimuthal system,* In this system bearing of a line is measured from the true north or magnetic north in clockwise direction. The value of a bearing may vary from 0° to 360°, utilising the whole circle of graduation. Prismatic compass is graduated on the whole circle bearing system. The system of measuring bearings from the north direction, is adopted in India and United Kingdom.

Referring to Fig. 4.18, W.C.B. of lines *OA, OB, OC* and *OD* are θ_1, θ_2, θ_3 and θ_4, respectively.

Note. In some countries, W.C.B. of survey lines are reckoned from the South. Those bearings differ by 180° in magnitude as compared to those expressed from the North.

2. The quadrental bearing system. In quadrantal bearing system, bearings of survey lines are measured eastward or westward from

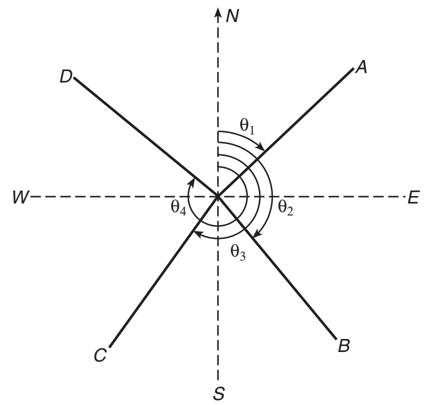


Fig. 4.18. Whole circle bearings of lines.

North and South whichever is nearer. In this system, both north and south directions are used as reference meridians and bearings are reckoned either clockwise or anticlockwise, depending upon the position of the line. The quadrant in which a line lies is mentioned, to specify the location of the line. Surveyor's compass is graduated in quadrantal bearing system.

Bearings designated by quadrantal bearing system, are sometime called *Reduced Bearings.*

Referring to Fig. 4.19, Q.B. of lines *OA, OB, OC* and *OD* are designated as: $N\ \alpha°\ E$, $S\ \beta°\ E$, $\lambda°\ W$, $N\ \delta°\ W$, respectively.

Thus, in quadrantal bearing system, reference meridian is prefixed and the direction of measurement whether eastward or westward, is affixed to the numerical value of the bearing. The numerical value of a quadrantal bearing, may vary from 0° to 90°.

Conversion of bearings from one system to the other. The conversion of bearings from one system to another may be *easily done by drawing a diagram.*

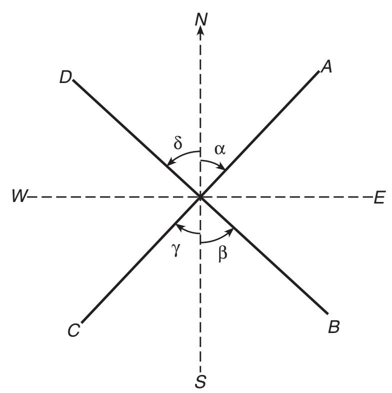


Fig. 4.19. Quadrantal bearnings.

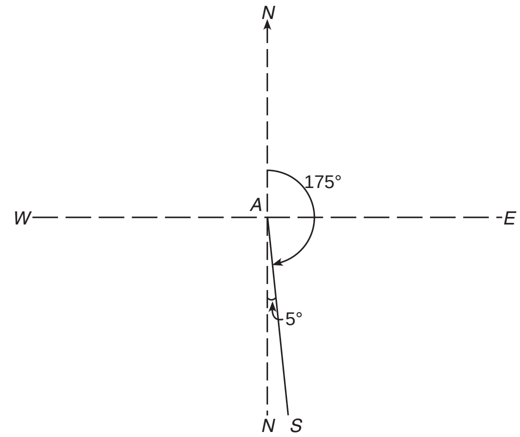


Fig. 4.20.

Suppose W.C.B. *of any line is* 175°.

The Q.B. of the line = 180° – 175° = 5°. The line is lying in *SE* quadrant. It is also nearer to the South direction.

Hence, the Q.B. of the line is designated as *S* 5° *E*.

For converting W.C. bearings into reduced bearings or Q.B., the rules are stated in Table 4.1.

Table 4.1. Conversion of W.C.B. into Q.B.

Case	*W.C.B. between*	*Rule of Q.B.*	*Quad-rant*
I	0° and 90°	W.C.B.	N.E.
II	90° and 180°	180° –– *W.C.B.*	S.E.
III	180° and 270°	*W.C.B.* — 180°	S.W.
IV	270° and 360°	360° — *W.C.B.*	N.W.

Note. When a line lies exactly either along North, South, East or West, the W.C.B. of the line is converted in the quadrantal system as follows:

If W.C.B. of a line = 0° then, Q.B. of the line is *N*.

W.C.B. of a line = 90° then, Q.B. of the line is *E* 90°.

W.C.B. of a line = 180° then, Q.B. of the line is *S*

W.C.B. of a line = 270° then, Q.B. of the line is *W* 90°.

The rules for the conversion of Q.B. into W.C.B. are stated in Table 4.2.

Table 4.2. Conversion of Q.B. into W.C.B.

Case	R.B.	Rule for W.C.B.	W.C.B. between
I	$N \alpha E$	R.B.	0° and 90°
II	$S \beta E$	180° — *R.B.*	90° and 180°
III	$S \gamma W$	180° + *R.B.*	180° and 270°
IV	$N \delta W$	360° — *R.B.*	270° and 360°

4.10. FORE AND BACK BEARINGS

A line may be defined by two bearings, one observed at either end of the line. Both the bearings expressed in W.C.B. system differ each other by 180°. The bearing of a line in the direction of the progress of survey, is called *Fore or Forward Bearing* (*F.B.*) while the bearing in the opposite direction of the progress of survey, is known as *Reverse or Back bearing* (*B.B.*).

In Fig. 4.21 the bearing of the line *AB* in the direction from *A* to *B* is a fore bearing (α) whereas the bearing of the line *AB* in the direction from *B* to *A*, is a back bearing (β).

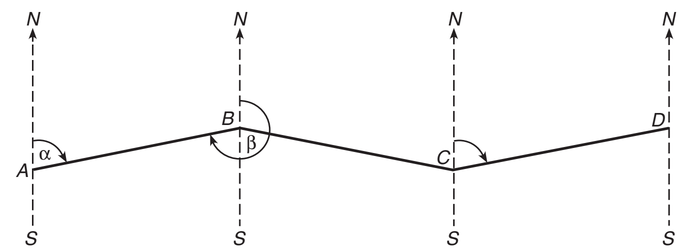


Fig. 4.21. Direction of survey is West to East.

Relationship between fore and back bearings

W.C.B. system: (Fig. 4.22)

Let the fore bearing of a line $AB = \alpha°$

Back bearing of $BA = \beta°$

or $$\beta = 180° + \angle SBA = 180° + \angle BAN' = \alpha + 180°$$

$\therefore$ Back bearing

$$= \text{Fore bearing} + 180° \quad ...(i)$$

Now, consider the bearing of *BA* as a fore bearing $= \beta$

Then, $$\alpha = 180° - \angle S'AB = 180° - \angle ABN = 180° - (360° - \beta)$$

$$= \beta - 180°$$

or Back bearing = Fore bearing – 180° ...(*ii*)

Equations (*i*) and (*ii*) may be combined into one equation *i.e.*

Back bearing = Fore bearing ± 180°, using +ve sign if the fore bearing is less than 180° and – ve sign if it is more than 180°.

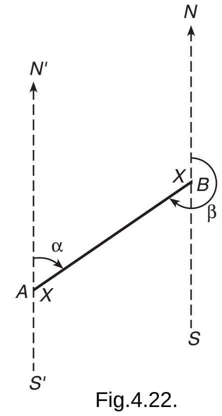


Fig.4.22.

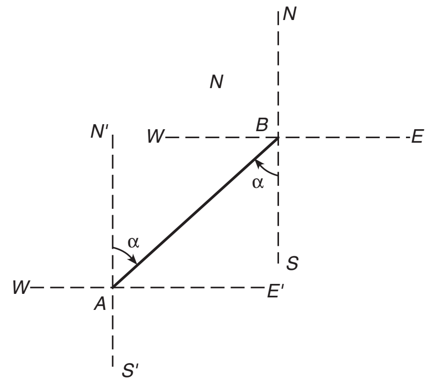


Fig.4.23.

Q.B. system: (Fig. 4.23)

Let the fore bearing of a line $AB = N\alpha°\ E$

or Back bearing of line $AB = \angle SBA = \angle BAN' = \alpha°$

To convert the fore bearing of a line into its back bearing in Q.B. system, replace *N* by *S, S* by *N, E* by *W* and *W* by *E*, without changing the numerical value of the bearing.

4.11. CALCULATION OF INCLUDED ANGLES FROM BEARINGS

Knowing the bearings of two adjacent lines, their included angle may be easily calculated as under:

I. Given W.C.B. of two lines

Let W.C.B. of the line $AB = \alpha°$

W.C.B. of the line $AC = \beta°$ (Fig. 4.24.)

∴ The included angle *BAC*

$= \angle NAC - \angle NAB = \beta - \alpha$

= Bearing of *AC* – Bearing of *AB*

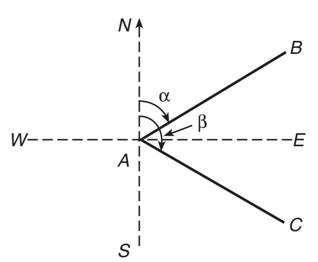


Fig. 4.24.

Note. The difference of bearings of two adjacent lines, is the included angle measured clockwise from the line whose bearing is less.

A diagram may be drawn and bearings of the lines plotted in their respective quadrants. The included angle is calculated from one of the undermentioned formulae.

II. Given Q.B. of lines.

1. If the bearings have been measured to the same side of the common meridian, the included angle $\alpha = \theta_2 - \theta_1$ *i.e.* the difference of the bearings. This is true for all the quadrants. [Fig. 4.25 *a*].

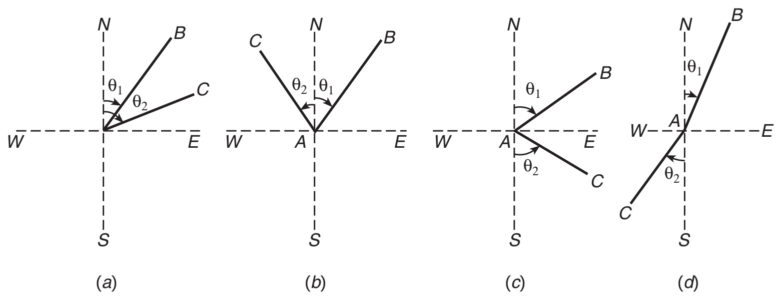


Fig. 4.25.

2. If the bearings have been measured to the opposite side of the common meridian, the included angle $\alpha = \theta_1 + \theta_2$ *i.e.* the sum of the bearings [Fig. 4.25 *b*].

3. If the bearings have been measured to the same side of different meridians, the included angle $\alpha = 180° - (\theta_1 + \theta_2)$ *i.e.* the difference of 180° and the sum of the bearings. [Fig. 4.25 *c*].

4. If the bearings have been measured to the opposite side of different meridians, the included angle $\alpha = 180° - (\theta_2 - \theta_1)$ *i.e.* the difference of 180° and the difference of the bearings. [Fig. 4.25 *d*].

4.12. CALCULATION OF BEARINGS FROM INCLUDED ANGLES

Knowing the bearing of a line and included angles between the successive lines, the bearings of remaining lines, may be calculated as under:

Let the observed bearing of the line *AB* be θ_1 (given)

$\alpha, \beta, \gamma, \delta, \varphi$...etc., the included angles measured clockwise between adjacent lines.

$\theta_2, \theta_3\ \theta_4, \theta_5$, etc., the bearings of successive lines.

The bearing of $BC = \theta_2 = \theta_1 + \alpha - 180°$ ...(*i*)

The bearing of $CD = \theta_3 = \theta_2 + \beta - 180°$ ...(*ii*)

The bearing of $DE = \theta_4 = \theta_3 + \lambda - 180°$ ...(*iii*)

The bearing of $EF = \theta_5 = \theta_4 + \delta - 180°$ ...(*iv*)

The bearing of $FG = \theta_6 = \theta_5 + \varphi - 180°$ ...(*v*)

From Fig. 4.26, it is evident that each of $(\theta_1 + \alpha)$, $(\theta_2 + \beta)$ and $(\theta_3 + \gamma)$ is more than 180°; $(\theta_4 + \delta)$ is less than 180° and $(\theta_6 + \phi)$ is greater than 540°. Hence, in order to calculate the bearing of the next line, the following statements may be made:

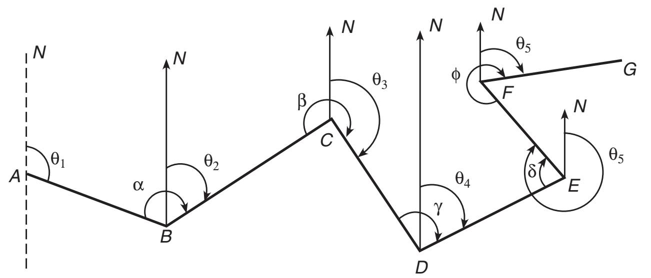


Fig. 4.26.

Add the included angle measured clockwise to the bearing of the previous line. If the sum is :

more than 180°, deduct *180°,*

more than 540°, deduct 540°,

less than 180°, add 180°, to get the bearing of the next line".

Note. The following points may be noted:

(*i*) In a closed traverse run in anticlockwise direction, the observed included angles are interior angles.

(*ii*) In a closed traverse run in clockwise direction, the observed included angles, are exterior angles.

Example 4.4. (*a*) *Convert the following whole circle bearings to quadrantal bearings:*

(*i*)*12° 45′* (*ii*) *160° 10′* (*iii*) *210° 30′* (*iv*) *285° 50′*.

(*b*) *Convert the following quadrantal bearings to whole circle bearings:*

(*i*) *N 30° 30′ E* (*ii*) *S 70° 42′ E* (*iii*) *S 36° 35′ W* (*iv*) *N 85° 10′ W.*

Solution.

(*a*) Refer to Fig. 4.19 and Table 4.1.

(*i*) Reduced Bearing = W.C.B. = 12° 45′ = N 12° 45′ E

(*ii*) Reduced Bearing = 180° − W.C.B. = 180° − 160° 10′

= 19° 50′ = S 19° 50′ E

(*iii*) Reduced Bearing = W.C.B. – 180° = 210° 30′ – 180°

= 30° 30′ = S 30° 30′ W

(*iv*) Reduced Bearing = 360° – W.C.B. = 360° – 285° 50′

= 74° 10′ = N 74° 10′ W

(*b*) Refer to Fig. 4.18 and Table 4.2

(*i*) W.C.B. = R.B. = 30° 30′

(*ii*) W.C.B. = 180° – R.B. = 180° – 70° 42′ = 109° 18′

(*iii*) W.C.B. = 180° + R.B. = 180° + 36° 35′ = 216° 35′

(*iv*) W.C.B. = 360° – R.B. = 360° – 85° 10′ = 274° 50′.

Example 4.5. *The whole-circle- bearing of a line is* (*i*) 180° (*ii*) 270°. *What will be its reduced bearing in each case* ?

Solution. (Fig. 4.27)

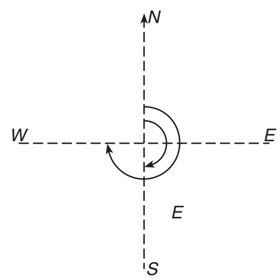


Fig.4.27.

(*i*) R.B. = 180° – W.C.B. = 180° – 180°

= 0° = S **Ans.**

(*ii*) R.B. = 360° – W.C.B. = 360° – 270°

= 90° = W 90° **Ans.**

Example 4.6. *The fore bearings of traverse sides are as follow: AB 85° 10′*; *BC 155° 30′*; *CD 265° 5′* and *DE 355° 30′*. *Find their back bearings.*

Solution. We know that

Back bearing of a line = fore bearing of the line ± 180°.

∴ Back bearing of *AB* = 85° 10′ + 180° = 265° 10′ . **Ans.**

Back bearing of *BC* = 155° 30′ + 180°

= 335° 30′. **Ans.**

Back bearing of $CD = 265° 05' - 180°$

$= 85° 05'$ **Ans.**

Back bearing of $DE = 355° 30' - 180°$

$= 175° 30'$. **Ans.**

Example 4.7. *The fore bearings of AB and BC are respectively N 30° 10′ E* and *S 20° 20′ W. Find their back bearings.*

Solution.

We know that the back bearing of a line in Q.B. is obtained by simply replacing N by S and E by W and *vice versa*.

∴ Back bearing of $AB = S\ 30° 10'$ W. **Ans.**

and Back bearing of BC = N 20° 20′ E. **Ans.**

Example 4.8. *Find the included angles between lines AB and AC if their whole circle bearings are :*

(*i*) *AB* 75° 30′ *AC* 108° 50′

(*ii*) *AB* 185° 50′ *AC* 269° 25′

(*iii*) *AB* 60° 10′ *AC* 245° 10′

(*iv*) *AB* 70° 20′ *AC* 285° 40′.

Solution.

(*i*) Bearing of $AB = 75° 30'$

Bearing of $AC = 108° 50'$

∵ Included angle BAC = Bearing of AC – Bearing of AB

$= 108° 50' - 75° 30' = 33° 20'$. **Ans.**

(*ii*) Bearing of $AB = 185° 50'$

Bearing of $AC = 269° 25'$

∴ Included angle $BAC = 269° 25' - 185° 50' = 83° 35'$. **Ans.**

(*iii*) Bearing of $AB = 60° 10'$

Bearing of $AC = 245° 10'$

Difference in bearings $= 245° 10' - 60° 10' = 185° 0'$

As it is more than 180°, deduct it from 360°

∴ Included angle $BAC = 360° - 185° 0' = 175° 0'$. **Ans.**

(*iv*) Bearing of $AB = 70° 20'$

Bearing of $AC = 285° 40'$

∴ Difference in bearings = 285° 40′ – 70° 20′. = 215° 20′

As it is more than 180°, deduct it from 360°

∴ Included angle *BAC* = 360° – 215° 20′ = 144° 40′. **Ans.**

Example 4.9. *Find the included angle between lines AB and AC, if their reduced bearings are*:

(*i*) *AB N*40° 10′ *E* *AC N* 89° 45′ *E*

(*ii*) *AB N*10° 50′ *E* *AC S* 40° 40′ *E*

(*iii*) *AB S* 35° 45′ *W* *AC N* 45° 20′ *E*

(*iv*) *AB N* 30° 25′ *E* *AC N* 30° 25′ *W*.

Solution.

Refer to the formulae stated in Table 4.1.

(*i*) (Fig. 4.28).

Bearings of *AB* = *N* 40° 10′ E; Bearing of *AC* = N 89° 45′ E

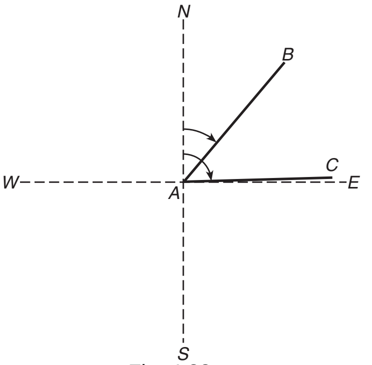


Fig. 4.28.

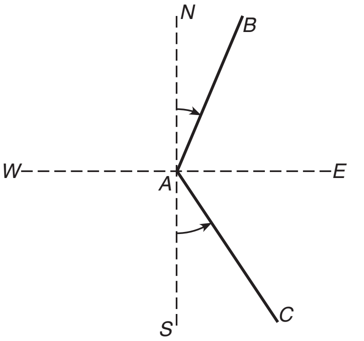


Fig. 4.29.

∵ Bearings are measured on the same side of the north meridian, and both lie in *NE* quadrant.

∴ Included angle *BAC* = difference in the bearings

= 89° 45′ – 40° 10′ = 49° 35′. **Ans.**

(*ii*) (Fig. 4.29)

Bearing of *AB* = N 10° 50′ E; Bearing of *AC* = S 40° 40′ E

The bearings are measured on the same side of N-S meridian, and lie in adjacent quadrants.

∴ Included angle *BAC* = 180° – sum of the bearings

$= 180° – (10° 50' + 40° 40)' = 128° 30'$. **Ans.**

(*iii*) (Fig. 4.30)

Bearing of *AB* = S 35° 45′ *W*

Bearing of *AC* = N 45° 20′ E

The bearings are measured on opposite sides of the meridian and lie in opposite quadrants,

∴ Included angle *CAB* = 180° – (difference in bearings)

= 180° – (45° 20′ – 35° 45′)

= **170° 25′. Ans.**

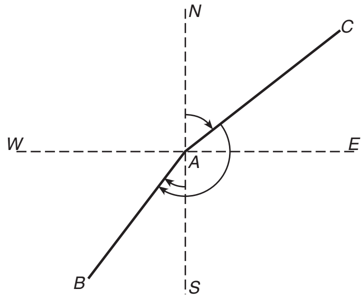


Fig. 4.30.

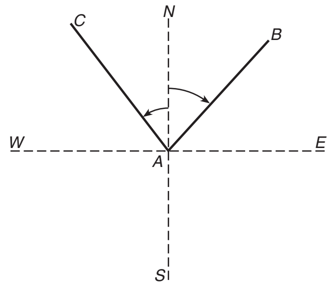


Fig. 4.31.

(*iv*) (Fig. 4.31)

Bearing of *AB* = N 30° 25′ *E*

Bearing of *AC* = N 30° 25′ *W*

The bearings are measured on the opposite side of common meridian and lie in adjacent quadrants.

∴ The included angle *CAB* = sum of the bearings

= 30° 25′ + 30° 25′ = 60° 50′ . **Ans.**

Example 4.10. *The bearings of the sides of a closed transverse ABCDEA are as follow :*

Side	*F.B.*	*B.B.*
AB	*107° 15′*	*287° 15′*
BC	*22° 00′*	*202° 00′*
CD	*281° 30′*	*101° 30′*
DE	*181° 15′*	*1° 15′*
EA	*124° 45′*	*304° 45′*

Compute the interior angles of the traverse and exercise necessary checks.

Solution. (Fig. 4.32).

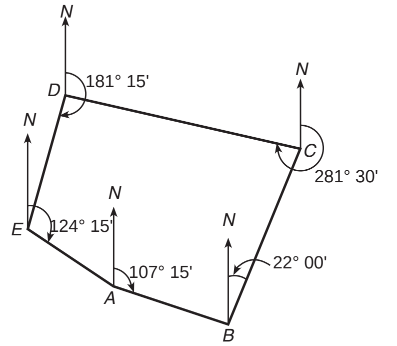


Fig. 4.32.

(*i*) The included angle A = The difference in bearings of AB and AE.

As the bearing of AB is less than that of AE, add 360°. to the bearing AB.

$\therefore$ Included angle A

$= (107° 15' + 360°) - 304° 45'$

$= 162° 30'$. **Ans.**

(*ii*) The included angle at B:

The difference in bearings of BC and BA

$$= (22° 00' + 360°) - 287° 15'$$

$\therefore$ Included angle $B = 94° 45'$. **Ans.**

(*iii*) The included angle at C:

The difference in bearings of CD and CB

$$= 281° 30' - 202° 00' = 79° 30'$$

$\therefore$ Included angle $C = 79° 30'$. **Ans.**

(*iv*) The included angle at D:

The difference in bearings of DE and $DC = 181° 15' - 101° 30'$

$$= 79° 45'$$

$\therefore$ Included angle $D = 79° 45'$. **Ans.**

(*v*) The included angle at E:

The difference in bearings of EA and $ED = 124° 45' - 1° 15'$

$$= 123° 30'$$

$\therefore$ Included angle $E = 123° 30'$. **Ans.**

Check:

Sum of the included angles of a pentagon

$$= (2 \times 5 - 4) = 6 \text{ right angles.}$$

Sum of the included angles $A + B + C + D + E$

$$= 162° 30' + 94° 45' + 79° 30' + 79° 45' + 123° 30'$$

$$= 540° 00' \text{ or } 6 \text{ right angles} \quad \text{O.K.}$$

Example 4.11. *The following bearings were taken in a closed traverse ABCD*:

Line	*Fore bearing*	*Back bearing*
AB	45° 15′	225° 15′
BC	123° 15′	303° 15′
CD	181° 00′	1° 00′
DA	289° 30′	109° 30′

Calculate the interior angles of the traverse.

Solution.

Calculation of the interior angles of the closed traverse *ABCD*.

Angle A = Bearing of AD – Bearing of AB

= 109° 30′ – 45° 15′ = 64° 15′

Angle B = Bearing of BA – Bearing of BC

= 225° 15′ – 123° 15′ = 102° 00′

Angle C = Bearing of CB – Bearing of CD

= 303° 15′ – 181° 00′ = 122° 15′

Angle D = 360° – [Bearing of DA – Bearing of DC]

= 360° – (289° 30′ – 1° 00′) = 71° 30′

Check : $\angle A + \angle B + \angle C + \angle D$

= 64° 15′ + 102° 00′ + 122° 15′ + 71° 30′

= 360° O. K.

Angle A = 64° 15′

Angle B = 102° 00′

Angle C = 122° 15′

Angle D = 71° 30′ **Ans.**

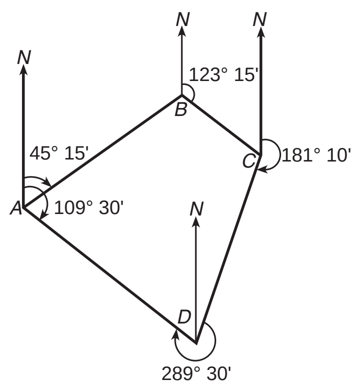


Fig. 4.33.

4.13. LOCAL ATTRACTION

North end of a freely suspended magnetic needle always points to the magnetic north, if it is not influenced by any other external forces except the earth's magnetic field. It is a common experience that the magnetic needle gets deflected from its normal position, if placed near magnetic rocks, iron ores, cables carrying current or iron electric poles. Such a disturbing force is known as '*Local attraction*'. Magnetic bearings are, therefore, not reliable unless these are checked against the presence of local attraction at each station and their elimination.

Detection of Local Attraction. The presence of local attraction at any station may be detected by observing the fore and back bearings of the line. If the difference between fore and back bearings is 180°, both end stations are free from local attraction. If not, the discrepancy may be due to:

1. An error in observation of either fore or back bearings or both.

2. Presence of local attraction at either station.

3. Presence of local attraction at both the stations.

It may be noted that local attraction at any station affects all the magnetic bearings by an equal amount and hence, the included angles deduced from the affected bearings are always correct. In case, the fore and back bearings of neither line of a traverse differ by the permissible error of reading, the mean value of the bearings of the line least affected, may be accepted. The correction to other stations, may be made according to the following methods.

1. By calculating the included angles at the affected stations.

2. By calculating the local attraction of each station and then applying the required corrections, starting from the unaffected bearing.

1. Method of elimination of local attraction by included angles.

The following steps are followed :

(*i*) Compute the included angle at each station from the observed bearings, in case of a closed traverse.

(*ii*) Starting from the unaffected line, run down the correct bearings of the successive sides.

The complete procedure is explained in the following solved examples.

Example 4.12. *The following fore and back bearings were observed in traversing with a compass where local straction was suspected :*

AB	65° 30′	245° 30′
CD	43° 45′	226° 30′

BC	104° 15′	283° 0′
DE	326° 15′	144° 45′

Determine the corrected *FB* and *BB* and true bearing of the lines assuming magnetic declination to be 5° 20′ *W*.

Solution. (Fig. 4.34)

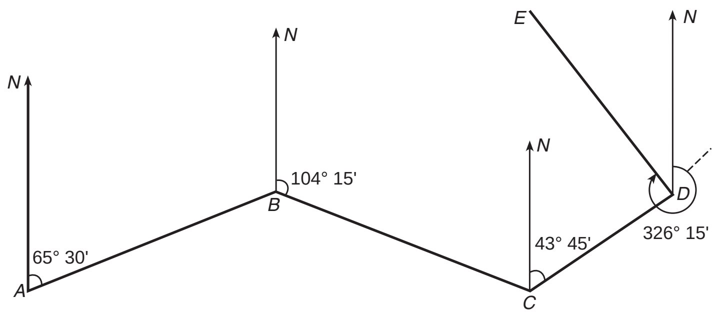


Fig. 4.34.

Fore Bearing of *AB* = 65° 30′

Back Bearing of *AB* = 245° 30′ (Given)

∴ Difference of bearings = 245° 30′ – 65° 30′ = 180° 0′

∴ Stations *A* and *B* are free from local straction.

and F. *B*. of *BC* = 104° 15′ is correct.

Correct *B.B* of *BC* = 104° 15′ + 180° = 284° 15′

Observed *B.B* of = 283° 0′

∴ Error = Observed Bearing – true *BC*

= 283° 0′ – 284° 15′ = – 1° 15′

∴ Correction at *C* = + 1 15′

∴ Correct *FB* of *CD* = Observed Bearing of *CD* + correction

= 43° 45′ + 1° 15′ = 45° 0′

∴ Correct *BB* of *DC* = 45^o 0′ + 180° 0′ = 225^0 0′

Observed *BB* of *DC* = 226° 30′

∴ Error at *D* = Observed bearing – correct bearing

= 226° 30′ – 225° 0′ = + 1° 30′

∴ Correction at *D* = – 1^o 30′

∴ Correct FB of DE = Observed bearing – Correction

$$= 326° 15' – 1° 30' = 324° 45'$$

The corrected FB, BB and true bearing of the lines are tabulated here under.

Line	*F.B.*	*B.B.*	*Declination*	*True Fore Bearing*	*True B.B.*	
AB	65° 30′	245° 30′	5° 20′W	60° 10′	240° 10′	
BC	104° 15′	284° 15′	5° 20′W	98° 55′	278° 55′	**Ans.**
CD	45° 0′	225° 0′	5° 20′W	39° 40′	219° 40′	
DE	324° 45′	144° 45′	5° 20′W	319° 25′	139° 25′	

Example 4.13. *A closed compass traverse ABCD was conducted round a lake and the following bearings were obtained. Determine which of the stations are suffering from local attraction and give the values of the corrected bearings :*

AB	74° 20′	256° 0′
BC	107° 20′	286° 20′
CD	224° 50′	44° 50′
DA	306° 40′	126° 00′

Solution. (Fig. 4.35)

An examination we find that fore and back bearings of CD differ exactly by 180°. Hence, stations C and D are free from local attraction. Stations affected by local attraction are A and B.

Calculation of included angles :

Interior angle at A = bearing of AD – bearing of AB

$$= 126° 00' – .74° 20' = 51° 40'$$

Exterior angle A = 360° – 51° 40′ = 308′ 20′

Interior angle at B = bearing of BA – bearing of BC

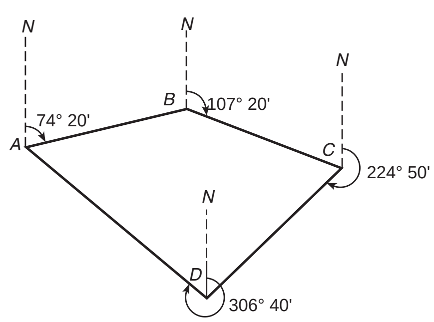


Fig. 4.35.

$= 256° 0' - 107° 20' = 148° 40'$

$\therefore$ Exterior angle at $B = 360° 0' - 148° 40' = 211° 20'$

Interior angle at C = bearing of CB – bearing of CD

$\therefore$ Exterior angle at $C = 360° 00' - 61° 30' = 298° 30'$

Exterior angle D = bearing of DA – bearing of DC

$= 306° 40' - 44° 50' = 261° 50'$

Check : Sum of exterior angles of the quadrilateral $ABCD$ $(2 \times 4 + 4) = 12$ right angles. **O.K.**

Total sum of exterior angles

$= 308° 20' + 211° 20' + 298° 30' + 261° 50'$

$= 1080° = 12$ right angles. **O.K.**

Calculation of bearings :

Bearing of CD $= 224° 50'$ (given)

Add traverse angle at $D = 261° 50'$

Sum $= 486° 40'$

Sum is more than 180°, subtract $= 180° 00'$

$\therefore$ Bearing of $DA = 306° 40'$

Add traverse angle at $A = 308° 20'$

$= 615° 00'$

Sum is more than 540°, subtract $= 540° 00'$

$\therefore$ Bearing of $AB = 75° 00'$

Add traverse angle at $B = 211° 20'$

Sum $= 286° 20'$

Sum is more than 180°, subtract 180° 00′

$\therefore$ Bearing of $BC = 106° 20'$

Add traverse angle at C $= 298° 30'$

Sum $= 404° 50'$

Sum is more than 180° , subtract $= 180° 00'$

$\therefore$ Bearing of $CD = 224° 50'$ checked

Result : Corrected bearings of the lines are :

Side	FB	BB
AB	75° 00′	225° 0′
BC	106° 20′	286° 20′
CD	224° 50′	44° 50′
DA	306° 40′	126° 40′

2. Method of elimination of local attraction by applying Corrections to Bearings.

Following steps are followed :

(*i*) Calculate the magnitude and direction of the error due to local attraction at each affected station.

(*ii*) Run down the bearings, starting from the bearing unaffected by local attraction.

Complete procedure is explained in the following solved Examples.

Example 4.14. *Solve the example 4.13 by applying corrections to the bearings.*

Solution. On inspection we find that the fore and back bearings of line *CD* differ exactly by 180°. Hence, both stations *C* and *D* are free from local attraction. Bearing of *CD* may therefore be accepted as correct.

Back bearing of *BC* = 286° 20′ (Correct)

Subtract 180° = – 180° 00′

∴ Correct fore bearing of *BC* = 106° 20′

Observed bearing of *BC* = 107° 20′

∴ Error due to local attraction at

B = Observed bearing – correct bearing

= 107° 20′ – 106° 20′ = + 1° 0′

Correction at *B* = – 1° 0′

∴ Back bearing of *AB* = Observed bearing + correction

= 256° 0′ – 1° 0′ = 255° 0′

Subtract 180° – 180° 0′

Correct fore bearing of *AB* = 75° 0′

Error due to local attraction at *A* = Observed bearing – correct bearing

= 74° 20′ – 75° 0′ = – 0° 40′

Correction at *A* = + 0° 40′

∴ Back bearing of *AD* = Observed bearing + correction

= 126° 0′ + 0° 40′ = 126° 40′.

∴ Fore bearing of line *AD* = 126° 40′ + 180° = 306° 40′

Result :

Line	*F.B.*	*B.B.*
AB	75° 0′	255° 0′
BC	106° 20′	286° 20′
CD	224° 50′	44° 50′
DA	306° 40′	126° 40′

Example 4.15. *The following fore and back bearings were observed in traversing with a compass. Correct for local attraction :*

Line	*Fore bearing*	*Back bearing*
AB	44° 30′	226° 30′
BC	124° 30′	303° 15′
CD	181° 0′	1° 0′
DA	289° 30′	108° 45′

Solution. (Fig. 4.36.)

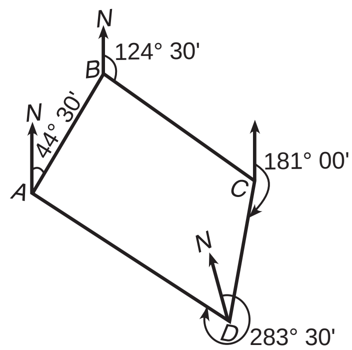


Fig. 4.36.

First method. We shall first calculate the including angles and thereafter, we will calculate the bearing of each line.

Angle *A* = Bearing of *AD* – Bearing of *AB*

= 108° 45′ – 44° 30′ = 64° 15′

Angle *B* = Bearing of *BA* – Bearing of *BC*

= 226° 30′ – 124° 30′ = 102° 00′

Angle *C* = Bearing of *CB* – Bearing of *CD*

= 303° 15′ – 181° 0′ = 122° 15′

Angle *D* = Bearing of *DC* – Bearing of *DA*

= (1° + 360°) – 289° 30′ = 71° 30′

Check :

$\angle A + \angle B + \angle C + \angle D = 64° 15' + 102° 0' + 122° 15' + 71° 30' = 360°$

Considering the traverse in anticlock wise and starting from station D, we calculate bearing of the remaining side.

As the F.B. and BB of CD differ exactly by 180°, we assume that the bearing of DC is correct.

Bearing of DC	$= 1° 0'$
Add $\angle C$	$+ 122° 15'$
Sum	$= 123° 15'$
Sum is less than 180°, add 180°	$+ 180° 00'$
$\therefore$ Bearing of CB	$= 303° 15'$
Add $\angle B$	$+ 102° 00'$
Sum	$= 405° 15'$
Sum is more than m 180°, subtract	$- 180° 0'$
$\therefore$ Bearing BA	$= 225° 15'$
Add $\angle A$	$+ 64° 15'$
Sum	$= 289° 30'$
Substract 180°	$- 180° 00'$
$\therefore$ Bearing of AD	$= 109° 30'$
Add $\angle D$	$+ 71° 0'$
Sum	$= 181° 0'$
	$180° 0'$
Bearing of DC	$= 1° 0'$

Corrected bearings are as under :

Side	F.B.	BB.
AB	45° 15′	225° 15′
BC	123° 15′	303° 15′
CD	181° 0′	1° 00′
DA	289° 30′	109° 30′

Second method. We shall correct the bearings, by applying correction at each station.

Correct bearing of $CB = 303° 15'$

$\therefore$ Correct bearing of $BC = 303° 15' - 180° 0' = 123° 15'$

Observed bearing of $BC = 124° 30'$

$\therefore$ Error at station B = Observed bearing – true bearing

$= 124° 30' - 123° 15' = 1° 15'$

Correction at stations $B = -1° 15'$

Correct bearing of $BA = 226° 30' - 1° 15' = 225° 15'$

Correct bearing of $AB = 225° 15' - 180° 0' = 45° 15'$

Observed bearing of $AB = 44° 30'$

$\therefore$ Error at station A = Observed bearing – true bearing

$= 44° 30' - 45° 15' = -0° 45'$

Correction at station $A = +0' 45'$

Correct bearing of $AD = 108° 45' + 0° 45' = 109° 30'$

Correct bearing of $DA = 109° 30' + 180° = 289° 30'$

The corrected *FB* and *BB* of the lines are as already computed by the first method.

Example 4.16. *Following is the data regarding a closed compass traverse PQRS taken in clockwise direction.*

(*i*) *Fore-bearing and back bearing at station P = 55° and 135° respectively*

(*ii*) *Fore-bearing and back bearing of line RS = 211° and 31° respectively*

(*iii*) *Included angles :* $\angle Q = 100°$, $\angle R = 105°$

(*iv*) *Local attraction at station R = 2° W*

(*v*) *All the observations were free from all the errors except local attraction.*

From the above data :

(*i) Calculate the local attraction at stations P and S.*

(*ii*) *Calculate the Corrected bearings of all the lines and tabulate the same.*

Solution. (Fig. 4.36*a*)

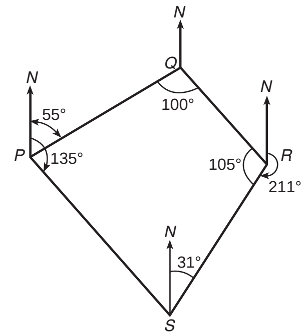


Fig. 4.36*a*.

From the observations it is noticed that the difference between fore bearing and back bearing of line *RS* is 180°. It means that either stations *R* and *S* are either free from local attraction or both are affected equally. As local attraction at station *R* is 2° W. Station *S* is also having local attraction equal to 2° W.

Angle *P* = Bearing of *PS* – Bearing of *PQ* = 135° – 55° = 80°.

Angle *S* = 360° – (80° + 100° + 105°) = 75°

Corrected bearing of line *RS*	
= 211° – 2°	= 209°
Bearing of line *RS*	= 209°
Add exterior angle *S*	+ 285°
Sum	= 494°
Subtract 180°	– 180°
Bearing of line *SP*	= 314°
Add exterior angle *P*	+ 280°
Sum	= 594°
Sum is more than 540°, subtract	– 540°
Bearing of line *PQ*	= 54°
Add exterior angle *Q*	+ 260°
Sum	= 314°
Subtract 180°	– 180°
Bearing of line QR	= 134°
Add exterior angle *R*	+ 255°

Sum	= 389°
Subtract 180°	= – 180°
Bearing of line *RS*	= 209° O.K.
Observed bearing of line *PQ*	= 55°
True bearing of line *PQ*	= 54°
∴ Local attraction at *P*	= True bearing – Observed bearing
	= 54° – 55° = – 1°

As the sign is negative, the local attraction is 1° W

Corrected bearings of the lines are tabulated below :

Line	*F.B.*	*B.B.*
PQ	54°	234°
QR	134°	314°
RS	209°	29°
SP	314°	134°

3. Practical Hints for Locating Local Attraction and its Correction

The following steps may be followed for calculation of corrected bearings:

(*i*) Observe the line whose fore and back bearings differ exactly by 180°.

(*ii*) Accept the bearings taken from the end stations of the line having no error, as correct.

(*iii*) Calculate the back or fore bearings of the next line and find the error between the observed bearing and its correct bearing.

(*iv*) If, at a station, observed bearing of a line is *more* than that of its correct value, *the error at the station is positive* and the correction to be applied to other bearings, *is negative and vice versa.*

Example 4.17. *The following bearings were observed in case of a closed traverse. At what stations, local attraction is suspected. Also compute the corrected bearings,*

Line	*F.B.*	*B.B.*
AB	*S 40° 30′ W*	*N 41° 15′ E*
BC	*S 80° 45′ W*	*N 79° 30′ E*
CD	*N 19° 30′ E*	*S 20° 00′ W*
DA	*S 80° 00′ E*	*N 80° 00′ W*

Solution. (Fig. 4.37).

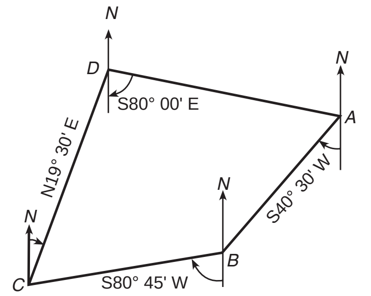


Fig. 4.37.

Fore and back bearings of DA in Q.B. system differ exactly by 0°, stations A and D are free from local attraction.

Correct fore bearing of AB

= S 40° 30′ W

∴ Correct back bearing of AB

= N 40° 30′ E

Observed back bearing of AB

= N 41° 15′ E

∴ Error at station B = 41° 15′ – 40° 30′ = + 0° 45′

∴ Correction at B = – 0° 45′

∴ Correct fore bearing of BC = Observed bearing + Correction

= S 80° 45′ W – 0° 45′

∴ = S 80° 0′ W

Now, correct back bearing of BC = N 80° 0′ E

observed back bearing of BC = N 79° 30′ E

∴ Error at station C = 79° 30′ – 80° 0′ = – 0° 30′

∴ Correction at station C = + 0° 30′

∴ Correct fore bearing of CD = Observed bearing + Correction

= N 19° 30′ E + 0° 30′

= N 20° 0 ′E

∴ Correct back bearing of CD = S 20° 0′ W **Checked**.

Line	*F.B.*	*BB.*
AB	*S* 40° 30′ *W*	*N* 40° 30′ *E*
BC	*S* 80° 00′ *W*	*N* 80° 00′ *W*
CD	*N* 20° 00′ *E*	*N* 20° 00′ *W*
DA	*S* 80° 00′ *E*	*N* 80° 00′ *W*

Ans.

4. Method of elimination of local attraction when none of the lines has a difference of 180° in its fore and back bearings

Sometimes it may so happen that in a closed traverse, no line has a difference of 180° in its fore and back bearings. In such cases, local attraction exists at every station and the affected bearings are corrected as follows :

1. Calculate the interior angles of the traverse and check their sum against ($2n \pm 4$) right angles.
2. Distribute the error, if any, equally to all the angles.
3. Locate the line whose fore and back bearings differ the least from 180° in W.C. bearing system and from 0° in Q.B. system.
4. Find out the correct bearings of the line by distributing half the error to each of the bearings.
5. Calculate correct bearings of other lines as already explained.

Example 4.18. *Find which stations are affected by local attraction and work out correct bearings of the lines of a closed traverse ABCDEA.*

Line	*F.B.*	*B.B.*
AB	191° *30′*	*13° 0′*
BC	*69° 30′*	*246° 30′*
CD	*32° 15′*	*210° 30′*
DE	*262° 45′*	*80° 45′*
EA	*230° 15′*	*53° 00′*

Solution. (Fig. 4.38)

On examination the fore and back bearings of the lines of the traverse, we find that no line has a difference of 180° in its fore and back bearings.

(*i*) **Calculation of included angles :**

Angle A = Bearing of AB – Bearing of AE

= 191° 30′ – 53° 00′ = 138° 30′

Angle B = Bearing of BC – Bearing of BA

= 69° 30′ – 13° 0′ = 56° 30′

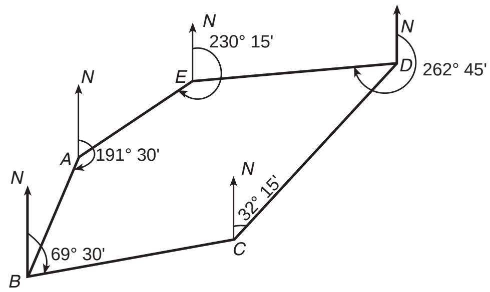


Fig. 4.38.

Angle C = Bearing of CB – Bearing of CD

= 246° 30′ – 32° 15′ = 214° 15′ being an exterior angle

= 360° – 214° 15′ = 145° 45′

Angle D = Bearing of DE – Bearing of DC

= 262° 45′ – 210° 30′ = 52° 15′

Angle E = Bearing of EA – Bearing of ED

= 230° 15′ – 80° 45′ = 149° 30′

Sum of the interior angles

= 138° 30′ + 56° 30′ + 145° 45′ + 52° 15′ + 149° 30′

= 542° 30′

The required sum of the interior angles = (2 × 5 – 4) 90°

= 540° 00′

∴ Total error in the five angles = 542° 30′ – 540° 00′

= + 2° 30′

∴ Correction for the five angles = + 2° 30′

∴ Correction for each angles = – 30′

Corrected angle A =138° 30′ – 30′ = 138° 00′

Corrected angle B = 56° 30′ – 30′ = 56° 00′

Corrected angle C = 145° 45′ – 30′ = 145° 15′

Corrected angle D = 52° 15′ – 30′ = 51° 45′

Corrected angle E = 149° 30′ – 30′ = 149° 00′

Sum = 540° 00′

(*ii*) Comparison of the fore and back bearings of the lines.

Bearing of AB – Bearing of BA = 191° 30′ – 13° 0′ = 178° 30″

Bearing of CB – Bearing of BC = 246° 30′ – 69° 30′ = 177° 00′

Bearing of DC – Bearing of CD = 210° 30′ – 32° 15′ = 178° 15′

Bearing of DE – Bearing of ED = 262° 45′ – 80° 45′ = 182° 00′

Bearing of EA – Bearing of AE = 230° 15′ – 53° 00′ = 177° 15′

It is seen that the fore and back bearings of the side AB differ the least from 180°.

∴ The difference in bearings = 180° – 178° 30′ = 1° 30′

∴ Correct fore bearing of AB $= 191° 30′ + \frac{1° 30′}{2}$

= 192° 15′

∴ Add angle B	= + 56° 00′
Sum	= 248° 15′
Subtract 180°	– 180° 00′
∴ Correct bearing of BC	= 68° 15′
Add angle C	+ 145° 15′
Sum	= 213° 30′
Subtract 180°	– 180° 00′
∴ Correct bearing of CD	33° 30′
Add angle D	+ 51° 45′
Sum	= 85° 15′
Add 180°	+ 180° 00′
∴ Correct bearing of DE	265° 15′
Add angle E	+ 149° 00′
Sum	= 414° 15′
Subtract 180°	– 180° 00′

∴ Correct bearing of *EA* = 234° 15′

Add angle *A* + 138° 00′

Sum = 372° 15′

Subtract 180° – 180° 00′

192° 15′ **Checked.**

(*iii*) The result may be tabulated as under.

Line	*Observed bearings*		*Included Agnles*			*Correct*		*Remarks*
	FA	*BB*		*Observed*	*Corrected*	*F.B.*	*B.B.*	
AB	191° 30′	13° 0′	*A*	138° 30′	138° 00′	192° 15′	12° 15′	No station is free from local attraction
BC	69° 30′	246° 30′	*B*	56° 30′	56° 00′	68° 15′	248° 15′	
CD	32° 15′	210° 30′	*C*	145° 15′	145° 15′	33° 30′	213° 30′	
DE	262° 30′	80° 45′	*D*	52° 45′	51° 45′	265° 15′	85° 15′	
EA	230° 15′	53° 00′	*E*	149° 30′	149° 00′	234° 15′	54° 15′	

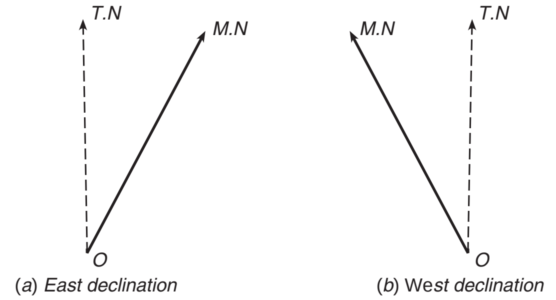
4.14. MAGNETIC DECLINATION

The horizontal angle between true north and magnetic north at a place at the time of observation, is called *magnetic declination.* The angle of convergence between the true north and magnetic north at any place does not remain constant. It depends upon the direction of the magnetic meridian at the time of observation. If the magnetic meridian is on eastern side of true meridian, the angle of declination is said to be *eastern declination* or *positive declination.* On the other hand if the magnetic meridian is on western side, the declination is said to be *western declination* or *negative declination.* When both true and magnetic meridians coincide, magnetie declination is zero.

The imaginary lines joining the places of equal declination either positive or negative, on the surface of the earth, are called *"Isogonic lines"*. As the earth magnetism is not regular and the intensity of its magnetic field also varies, the isogonic lines do not form complete circles but these follow irregular paths. The isogonic lines having zero declination, are known as *'Agonic lines'*

Mariners generally call magnetic declination as *'variation'*.

1. Determination of Magnetic Declination. True meridians at a number of places in the area, are determined by making astronomical observations (specially to stars). Compass observations are made by sighting the true meridians at the places. The angle of inclination between true meridian and magnetic meridian given by a compass reading, is the desired magnetic declination at the place.



(a) East declination. (b) West declination.

Fig. 4.39. Magnetic Declination.

It may be remembered that "Magnetic declination = True bearing -Magnetic bearing".*

Geographical positions of the places where declinations are observed, may be plotted. Interpolation of the isogonic lines** at constant interval, may be done as shown in Fig. 4.40.

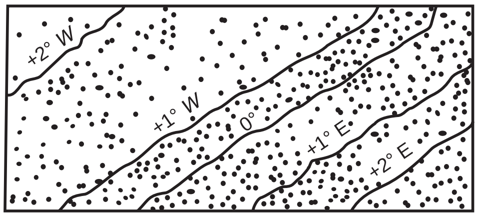


Fig. 4.40. Isogonic chart.

2. Calculation of True Bearing. If we know magnetic bearing of a line and magnetic declination at that place, then, true bearing of the line, may be calculated from the formula.

True bearing = Magnetic bearing ± magnetic declination, use +ve sign if declination is east and -ve sign, if it is west.

3. Calculation of Magnetic Bearing. If we know true bearing of a line and magnetic declination of that place, then magnetic bearing of the line, may be calculated from the formula :

Magnetic bearing = True bearing ± magnetic declination, use –ve sign for eastern declination and + ve sign for western declination.

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* If the sign of the difference is + ve the declination is east and if negative, it is to the west of the true meridian.

** The isogonic lines are generally surveyed by the department of the National Survey. The isogonic charts, showing the lines of equal magnetic declinations ar pulished at an interval of 5 years for reference by the mariners.

4.15. VARIATION OF DECLINATION

Declination at any place does not remain constant but keeps on changing from time to time. These variations may be classified under four heads *viz*.

1. Secular variation
2. Annual variation
3. Diurnal variation
4. Irregular variation

1. Secular Variation. The earth magnetic poles are continually changing their positions relatively to the geographical poles. Earth magnetic meridian also changes and affects the declination of places. Secular variation is a slow continuous change in declination of places. It alters in a more and less regular manner from year to year. Due to its magnitude, secular variation is the most important for land surveyors. It appears to be of periodic character and follows a sine curve. The swing of declination at a place over a period of centuries, may be compared to a simple harmonic motion. A secular change from year to year is also not uniform for any given place. It is also different for different places. To convert magnetic bearings into true bearings, an accurate amount of declination is essentially required. As such it is very important for a surveyor to know the exact amount of declination. When observations for the declination are made in different years of a century, it is revealed that magnetic meridian moves from one side of true meridian to the other. The change produced annually by secular variation at different places amounts from 0.02 minute to 12 minutes. The variation depends upon the geographical position of different places. The annual secular change is greatest near the middle point of a complete cycle and least at it extreme limits.

2. Annual Variation. Change in declination at a place over a period of one year, is known as *annual variation.* From the observations made at different places over a period of 12 months, it is found that annual variation is about 1 minute to 2 minutes, depending upon their geographical positions.

3. Diurnal Variation. The departure of declination from its mean value during a period of 24 hours at any place is called *diurnal variation.* The duirnal variation depends upon the following factors :

(*a*) **The geographical position of the place.** It is greatest for the places in higher latitudes and lesser near the equator.

(*b*) **Season of the year.** It is comparatively more in summer than in winter at the same place.

(*c*) **The time at the place.** It is more in day and less at night.

(*d*) **The year of the cycle.** It is different for different years in the complete cycle of secular variation.

4. Irregular Variation. Abrupt changes of declinations at places due to magnetic storms, earthquakes and other solar influences, are called *irregular variations.* These disturbances may occur at any time at any place and cannot be predicted. The displacement of a needle, may vary in extent from 1° to 2°.

Example 4.19. *The true and magnetic bearings of a line are 78° 45′ and 75° 30′ respectively. Calculate the magnetic declination at the place.*

Solution. (Fig. 4.41).

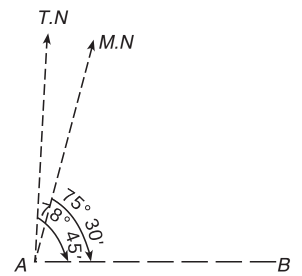

Fig. 4.41.

Magnetic declination = True bearing – Magnetic bearing

$$= 78° \ 45' - 75° \ 30' = 3° \ 15'$$

As the sign is +ve, declination is east of true meridian.

$\therefore$ Magnetic declination = 3° 15′ East. **Ans.**

Example 4.20. *The true and magnetic bearings of a line AB are 120° 45′ and 123° 15′ respectively. Calculate the magnetic declination at the station A.*

Solution. (Fig. 4.42).

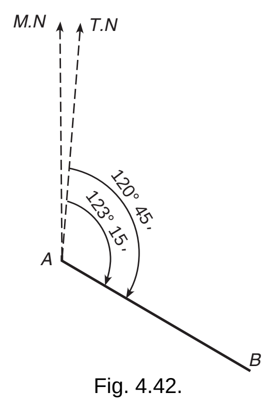

Fig. 4.42.

Magnetic declination

= True bearing – Magnetic bearing

= 120° 45′ – 123° 15′

= – 2° 30′

As the sign is negative, magnetic declination is west.

∴ Magnetic declination = 2° 30′ west. **Ans.**

Example 4.21. *Calculate the true bearing of a line CD if its magnetic bearing is S 50° 45′ W and the declination is 3° 45′ E.*

Solution. (Fig. 4.43).

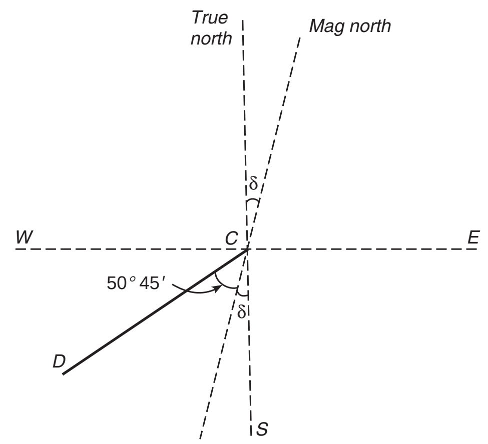

Fig. 4.43.

As the declination is eastern, magnetic meridian will be east of true meridian.

∴ True bearing = Magnetic bearing + Declination

= S 50° 45′ W + 3° 45′

= S 54° 30′ W. **Ans.**

Example 4.22. *In an old map a survey line was drawn with a magnetic bearing of 202° when the declination was 2° W. Find the magnetic bearing of the line at a time when magnetic declination was* 2° *.E*

Solution. The magnetic declination is W

∴ True bearing = Magnetic bearing -Declination

= 202° – 2° = 200°

Present declination = 2° E

$\therefore$ Magnetic bearing of the line = True bearing – Declination

= 200° – 2° = 198° **Ans.**

Example 4.23. *In 1935, a certain line had a magnetic bearing of S67° 30′ E and then the magnetic declination at that place was 8° E. In 1977, the magnetic declination was* 4° *W. Find the magnetic bearing of the line in 1977.*

Solution. True bearing = Magnetic bearing – Declination

= S 67° 30′ E – 8° = S 59° 30′ E

Magnetic bearing of the line in 1977

= True bearing – Declination in 1977

= S 59° 30′ E – 4°

= S 55° 30′ E. **Ans.**

Example 4.24. *The magnetic bearing of the sun at noon is 175°. Show with a sketch the true bearing of sun and the magnetic declination.*

Solution. (Fig. 4.44).

True bearing of sun at noon = 180°

Magnetic bearing of sun at noon = 175°

Magnetic declination = True bearing – magnetic bearing

= 180° – 175° = 5°

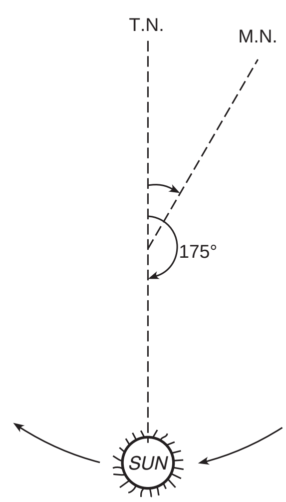

Fig. 4.44.

As the sign is + ve, the declination is east

∴ Magnetic declination is 5° east. **Ans.**

Example 4.25. *The declination at a place 50 years back was 2° 15′ W. The present declination is 1° 30′ E. If the old magnetic bearing of a line in such an area, is recorded as 153°, find its present magnetic bearing.*

Solution.

True bearing of the line

= Magnetic bearing – Magnetic declination

= 153° 0′ – 2° 15′ = 150° 45′

Present Magnetic bearing = True bearing – Magnetic declination

= 150° 45′ – 1° 30′ = 149° 15′. **Ans.**

Example 4.26. *The angles of a triangular net ABC, named clockwise are found by compass as 58°.25,* 62°.50 *and 60°* .00 *If the corrected magnetic bearing of CB be N 15.75* W *and declination* 2° *E, tabulate the geographic fore bearings of the sides.*

Solution. (Fig. 4.45).

Angle A = 58.25

Angle B = 62° .50

Angle C = 60°.00

Sum = 180° .75

Correction per angle = $\frac{0.75}{3}$ = – 0° .25

Angle A = 58° .25 – 0° .25 = 58°.00

Angle B = 62° .50 – 0° .25 = 62° .25

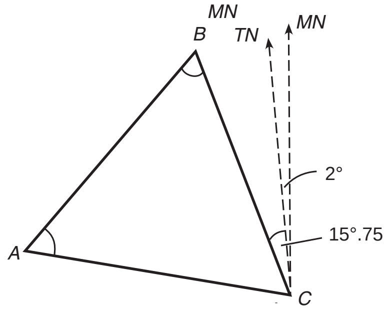

Fig. 4.45.

Angle $C = 60° .00 - 0° .25 = 59° .75$

Correct magnetic reduced bearing of $CB = N$ 15°.75 W (Given)

∴ Correct magnetic W.C. bearing of CB = 360° − 15°.75
= 344°.25

Correct geographic W.C. bearing of CB = 344° .25 − 2°
= 342°.25

Add angle B + 62.25

Sum = 404° .50

Subtract 180° − 180°.00

Correct geographic W.C. bearing of BA = 224°50

Add angle A + 58°.00

Sum = 282°.50

Subtract 180° − 180°.00

Correct geographic W.C. bearing of AC = 102°.50

Add angle C + 59.75

Sum = 162° .25

Add 180° + 180° .00

Correct geographic W.C. bearing of CB = 342° .25 O.K.

Side	*Back bearing*	*Fore bearing*		
		(*W.C.*)	(*R.B.*)	
AB	224.50	44° 50	*N* 44° 50 *E*	**Ans.**
BC	342.25	162.25	*S* 17.75 *E*	
CA	102° 50	282° 50	*N* 77°.50 *W*	

Example 4.27. *In an anticlockwise closed traverse ABCA all the sides were equal. Magnetic fore bearing of BC was obtained to be 15° 30′. The bearing of sun was observed to be 184° 30′ at local noon with a prismatic compass. Calculate the magnetic bearings and true bearings of all the sides of the traverse.*

Tabulate the results and draw a neat sketch to show true bearings.

Solution. (Fig. 4.46)

As the traverse legs are the sides of an equilateral Δ ABC,

Angle A = Angle B = Angle C = 60°.

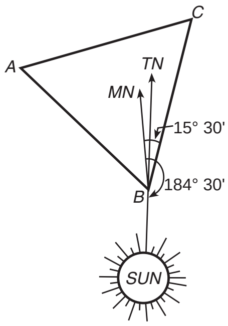

Fig. 4.46.

Calculation of bearings :

Bearing of *BC* = 15° 30′ (given)
Add Angle *C* = 60° 00′

Sum = 75° 30′
Add 180° 180° 00′

Bearing of *CA* = 255° 30′
Add angle *A* = 60° 00′

Sum = 315° 30′
Subtract – 00′ = 180°

Bearing of *AB* = 135° 30′
Add angle *B* = 60° 00′

Sum = 195° 30′
Subtract 180° 00′

Bearing of *BC* = 15° 30′ OK

Calculation of magnetic declination :

True bearing of the sun at local noon

$= 180° \ 00'$

Magnetic bearing of the sun at local noon

$= 184° \ 30'$

Magnetic declination $= \text{True bearing} - \text{Magnetic bearing}$

$= 180° \ 00' - 184° \ 30'$

$= -(4° \ 30')$

As the sign is negative the declination is west.

Corrected bearings of the sides are tabulated under :

Side	*Magnetic bearing*	*Declination*	*True bearing*
AB	135° 30′	4° 30′ W	131° 00′
BC	15° 30′	4° 30′ W	11° 00′
CA	255° 30′	4° 30′ W	251° 30′

Example 4.28. *Three aeroplanes A, B and C left station O at 6.00 AM, 6.10 AM and 6.20 AM, at speeds 300, 360 and 450 kmph in three directions having bearings N 70° E, S 66° E* and *S 10° E respectively. Determine the bearing and distance of A and C as observed from B at 7.00 AM, that is one hour after A left the station O.*

Solution. (Fig. 4.47).

Distance *OA* traversed by aeroplane *A,* at 7 AM

$= 300 \text{ km}$

Distance *OB* traversed by aeroplane *B* at 7 AM

$$= \frac{360}{60} \times 50 = 300 \text{ km}$$

Distance OC traversed by aeroplane *C* at 7 AM

$$= \frac{450}{60} \times 40 = 300 \text{ km.}$$

Angle $BOC = 66° - 10° = 56°$

Angle $AOB = 180° - (70 + 66) = 44°$

$$\text{Angle } OBA = \frac{180° - 44°}{2} = 68°$$

$$\text{Angle } OBC = \frac{180° - 56°}{2} = 62°$$

Reduced bearing of $BO = N \ 66° \text{ W}$

$\therefore$ WCB of $BO = 360° - 66° = 294°$

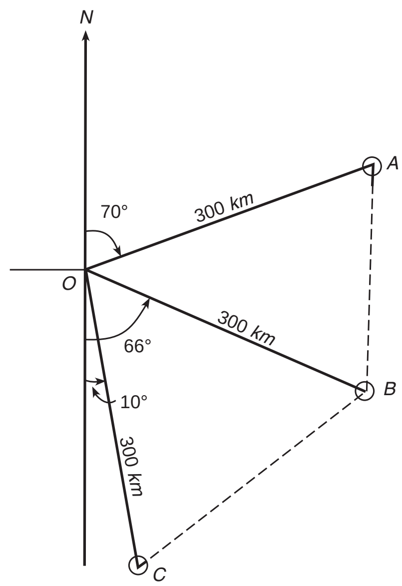

Fig. 4.47.

$\therefore$ Bearing of BC

$= 294° - \angle OBC$

$= 294° - 62° = 232°$

or QB of $BC = 232° - 180° = S\ 52°$ W. **Ans.**

Bearing of $BA = 294° + \angle OBA$

$= 294° + 68° = 362°$

or QB of $BA = 2° = N\ 2°\ E$. **Ans.**

Distance $BC = 2 \times BO \cos 62°$

$= 2 \times 300 \times 0.469472$

$= 281.683$ m. **Ans.**

Distance $BA = 2 \times BO \cos 68° = 2 \times 300 \times 0.374607$

$= 224.764$ m. **Ans.**

Example 4.29. *A line was drawn to a magnetic bearing of S 32° W, when the magnetic declination was 4° W. To what bearing should it be set now if the magnetic declination is 8° E.*

Solution. (Fig. 4.48).

Let AB be the given line.

Magnetic bearing of AB S 32° W

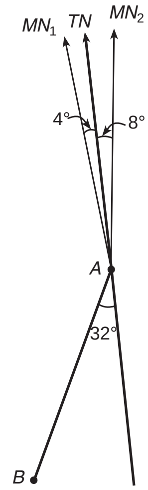

Fig. 4.48.

W.C.B. = 180° + 32° = 212°

Magnetic declination at the time of drawing the line AB

= 4° *W*.

∴ True Bearing of *AB*

= Magnetic bearing – Declination

= 212° – 4° = 208°

Present Magnetic declination = 8° *E*.

∴ Present Magnetic bearing of *AB*

= True bearing of *AB* – Magnetic declination

= 208° – 8° = 200°

∴ The present magnetic bearing of *AB* = 200° – 180° = 20°

= *S* 20° *W*. **Ans.**

4.16. TRAVERSING WITH A CHAIN AND COMPASS

To survey an area bounded by stations *A, B, C, D, E* etc. Fig. 4.49. with a chain and compass, the following steps are involved :

1. Reconnaissance of the area
2. Determination of the directions of chain lines
3. Measurement of the traverse legs (chain lines) and taking off-sets.

1. Reconnaissance of the area. The entire area is gone over to ascertain the following points in respect of the survey stations :

(*i*) The adjacent stations are intervisible.

(*ii*) Chaining between the stations is easy and without any obstruction.

(*iii*) Chain lines are as near the detail points as possible.

(*iv*) Chain lines connecting consecutive stations are as long as possible.

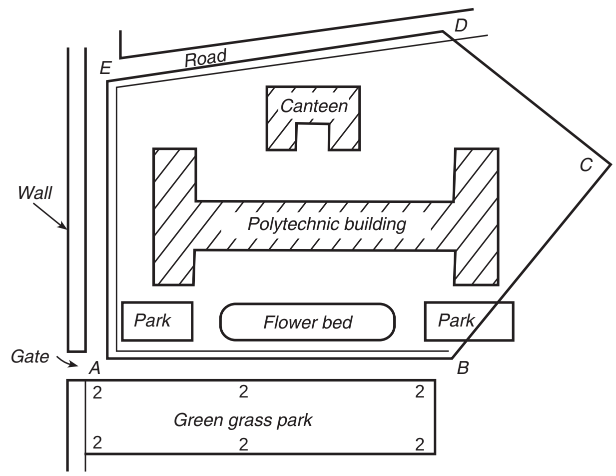

Fig. 4.49. Layout of the area.

2. Determination of direction of chain lines. In traversing with a chain and compass, directions of chain lines are determined by free or loose needle method. The compass is set up at each successive station. The fore and back bearings of chain lines are observed and recorded in a field book. As each bearing is observed independently of the other, errors do not accumulate but tend to compensate. If error between the fore and back bearings of a line exceeds the permissible limit *i.e.* 15′, the fore and back bearings of the chain line, are reobserved. Even if, on checking, the error remains it may be assumed that one or both the stations of the line are affected by local attraction. The fore and back bearings should be corrected for the local attraction, if any, before these are used in plotting.

3. Measurement of Traverse Legs and Taking Off-sets. A compass is centered over station *A* and is levelled properly. The fore bearing of *AB* and back bearing of *EA* are taken by sighting the ranging rods held at *B* and *E* respectively. The line *AB* is chained and the off-sets to the detail points, on either side of the line *AB* are taken as usual. The operation is repeated at other stations *B, C, D* and *E*.

It may be noted that in a compass traverse, the accuracy of field work mainly depends upon the accuracy with which bearings of survey lines, are observed. Fore and back bearings of each line provide a check only if they agree within the limit of permissible error of reading. This is why traverse legs should be as long as possible, to have less number of lines and consequently less number of bearings.

The traverse may be run in either direction *i.e.* clockwise or anticlockwise. For rough and speed work, compass may be set up at alternate stations.

4.17. METHODS OF PLOTTING OF TRAVERSES

Before plotting a traverse survey, it should be checked whether observed bearings are correct. If not the required correction to each bearing, may be made so as to have a perfect geometrical figure based on the field notes. It is advisable to draw a rough sketch of the chain lines to decide a proper layout of the plan on the sheet.

The traverse survey may be plotted by any one of the following methods:

1. By Parallel Meridians. After deciding the layout of the traverse a line representing the magnetic meridian, through the location of the starting station, is drawn on the paper. The bearing of the line *AB* is plotted with an ordinary protractor and its length duly reduced to scale, is marked off to get the location of station *A*, is drawn. The bearing of *BC* is plotted and length *BC* is plotted to scale. The process is continued till last station is plotted. In a closed traverse, last line should end on the starting station *A*, incase of a closed circuit or at any other known station in case of linear closed traverses. If it does not, the distance between two locations of the same station, is termed as *closing error*. (Fig. 4.50).

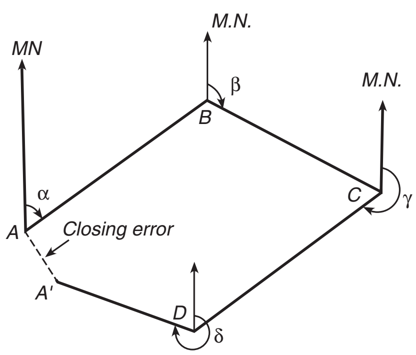

Fig. 4.50.

2. By Included Angles. After deciding the location of the starting station *A* on the paper, draw a line to represent the magnetic meridian passing through *A*. Plot the magnetic bearing of first chain line *AB* and plot *AB* duly reduced to scale. Now, plot the included angle *ABC* by a protractor and plot the location of station *C*. The process is continued till all the stations are plotted. It may be noted that for a closed traverse if linear measurements between stations are correct and plotting is errorless, the closing station will coincide with the starting station *A*. If not, the dis-

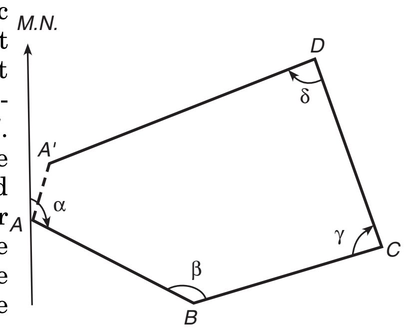

Fig. 4.51.

tance between two locations of the starting station, is known as *closing error* (Fig. 4.51).

3. Plotting by tangents. Deflection angles of the chain lines are plotted by geometrical construction with the help of their natural tangents. The traverse may be plotted as explained under :

From the location of the starting station *A,* draw a line passing through *A* to represent its magnetic meridian. To draw the bearing of traverse leg *AB,* cut a length of 10 cm on the magnetic meridian of station *A,* at B_1. At B_1 erect a perpendicular $B_1 B_2$ on the proper side of the meridian. Take $B_1 B_2$ equal to 10 × tangent of the reduced bearing *i.e.* angle of deflection of the line *AB*, in centimetres.

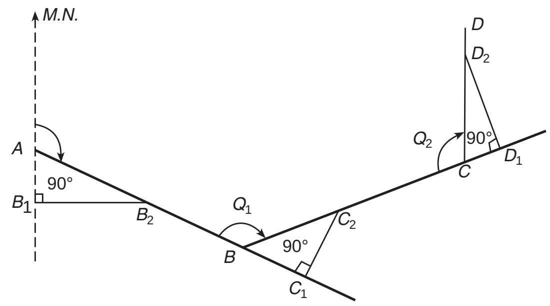

Fig. 4.52. Plotting by tangents.

Join AB_2 and produce it to get the direction of traverse line *AB*. Plot the length of *AB* on the line AB_2, (or produced) to the desired scale.

Deflection angle at station *B* may be plotted as under :

Take BC_1 equal to 10 cm on the line *AB* produced. Erect a perpendicular at C_1 on the side of line *BC.* Plot C_1C_2 equal to 10 × tangent of the deflection angle $C_2\,BC_1$.

The deflection angles of the successive chain lines for the purpose of plotting are obtained by the following formulae :

1. If the included angle between adjacent lines is between 0° and 90°, deflection angle is equal to the included angle.
2. If the included angle is between 90° and 180°, subtract the given included angle from 180° to get the deflection angle.
3. If the included angle is between 180° and 270°, subtract 180° from the given included angle.
4. If the included angle is between 270° and 360°, subtract the included angle from 360°.

The process is continued till all the traverse legs are plotted.

4.18. ADJUSTMENT OF CLOSING ERROR

When a closed traverse is plotted from the field measurements, the end station of a traverse generally does not coincide exactly with its starting station. This discrepancy is due to the errors in the field observations *i.e.* magnetic bearings and linear distances. Such an error of the traverse, is known as *closing error* or *error of closure.*

When the angular and linear measurements are of equal precision, graphical adjustment of the traverse may be made. This method is based on the Bowditch's rule. Corrections are applied to lengths as well as to bearings of the lines in proportion to their lengths. Graphical method is also sometimes known as *proportionate method* of adjustment.

Method. The adjustment of a compass traverse graphically, may be made as under :

Let *ABCDEA′* be a closed traverse as plotted from the observed magnetic bearings and linear measurements of the traverse legs. *A* is the starting station and *A′* is the location of the station *A* as plotted. Hence, *A′ A* is the closing error.

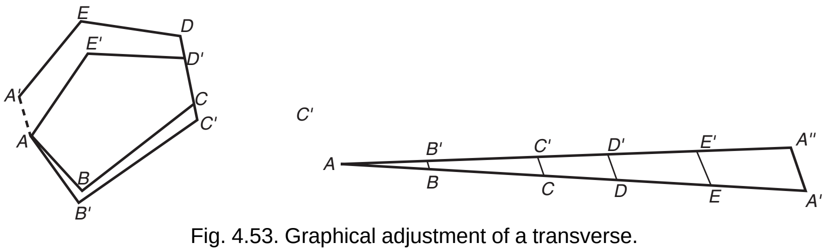

Fig. 4.53. Graphical adjustment of a transverse.

Adjustment. Proceed as under :

1. Draw a straight line *AA′* equal to the perimeter of the traverse to any suitable scale. Set off along it the distances *AB*, *BC*, *CD*, *DE* and *EA′* equal to the lengths of the sides of the traverse.

2. Draw *A′ A″* parallel and equal to the closing error *A′ A*.

3. Draw parallel lines through points *B*, *C*, *D*, and *E* to meet *AA″* at *B′*, *C′*, *D′* and *E′*.

4. Draw parallel lines through the plotted stations *B, C, D, E* and plot the errors equal to *BB′*,*CC′*, *DD′* in the direction of *A′A′*.

5. Join the points *AB′C′D′E′ A* to get the adjusted traverse.

4.19. SOURCES OF ERROR IN COMPASS TRAVERSING

The errors in compass traversing, may be broadly classified as under:

1. Instrumental Errors 2. Observational Errors

1. Instrumental Errors. The instrumental errors which are caused by defective parts of the instrument, are :

(*a*) Sluggish magnetic needle. The needle of the compass gets sluggissh due to dullness of the pivot. The pivot gets dull when the magnetic needle is un-necessarily allowed to swing even when not in use.

(*b*) The eccentricity of the pivot. If the pivot is not at the centre of the graduated ring, readings will be erroneous.

(*c*) Non-verticality of the sight vanes. The sight vane, object vane and the pivot may not be in the same vertical plane.

(*d*) Non-horizontality of the graduated ring

(*e*) Un-equal divisions of the graduated ring.

(*f*) The line of sight may not be passing through the centre of the graduated circle.

(*g*) Non-coincidence of the magnetic and geometrical axes of the needle.

2. Observational Errors. The observational errors include the following:

1. Incorrect bisection of the ranging rods.
2. Incorrect reading of compass.
3. Incorrect recording of readings.
4. Presence of magnetic substances in the vicinity of stations.
5. Magnetic changes in the area.

4.20. PRECAUTIONS TO BE TAKEN IN COMPASS SURVEY

The instrumental and observational errors during a compass survey, may be minimized by taking the following precautions :

1. Set up and level the compass carefully.

2. Stop the vibrations of the needle by gently pressing the brake-pin so that it may come to rest soon.

3. Always look along the needle and not across it, to avoid parallax.

4. When the instrument is not in use, its magnetic needle should be kept off the pivot. If it is not done, the pivot is subjected to unnecessary wear which may cause sluggishness of the magnetic needle.

5. Before taking a reading, the compass box should be gently tapped to ensure that the magnetic needle is freely swinging and has not come to rest due to friction of the pivot.

6. Stations should be selected such that these are away from the sources of local attraction.

7. Surveyor should never carry iron articles, such as a bunch of keys which may cause local attraction.

8. Fore and back bearings of each line should be taken to guard against the local attraction. If the compass cannot be set at the end of a line, the bearings may be taken from any intermediate point along that line.

9. Two sets of readings should be taken at each station for important details by displacing the magnetic needle after taking one reading.

10. Avoid taking a reading in wrong direction *viz.* 25° to 20° instead 20° to 25° and so.

11. If the glass cover has been dusted with a handkerchief, the glass gets charged with electrostatic current and the needle adheres to the glass cover. This may be obviated by applying a moist finger to the glass.

12. Object vane and eye vane must be straightened before making observations.

EXERCISE 4

1. Pick up the correct word(s) from the brackets to fill in the blanks :

(*i*) Compass survey is suitable where .... is the main consideration. (accuracy, speed)

(*ii*) The box of the compass is made of .... (brass, iron, aluminium)

(*iii*) The north end of a magnetic needle deflects .... in the northern hemisphere. (upwards, downwards)

(*iv*) The sum of the interior angles of a closed traverse is equal to .... right angles, where n is the number of its sides. ($4n - 2$, $2n - 4$)

(*v*) Fore and back bearings of a line whose end stations are free from local attraction, should differ by .... (180°, 90°, 360°)

(*vi*) .... meridian of a place changes its position with time. (Arbitrary, Magnetic, True)

(*vii*) Magnetic declination at a place is said to be .... when magnetic north is east of the true north. (east, west)

(*viii*) Angle of inclination between the longitudinal axis of a magnetic needle and the horizontal plane at its pivot, is known as .... (dip, declination, bearing)

(*ix*) Imaginary lines joining the points having same dip on the surface of the earth, are known as .... lines. (agonic, isoclinic) (*x*) Magnetic equator is an imaginary line along which dip, is............... (least, greatest, zero)

(*xi*) The zero of the graduations of a prismatic compass is at .... end of the graduated circle. (north, east, south, west)

(*xii*) The ring of a surveyor's compass is graduated into....... bearing system. (quadrantal, whole circle)

(*xiii*) The graduated ring of a surveyor's compass contains two zeros one at .... end and other at .... end. (East, North, West, South)

(*xiv*) Meridian is a fixed direction on the surface of the earth, with reference to which .... of survey lines are expressed. (bearings, directions, angles)

(*xv*) True meridian at any place is .... (variable, not variable)

(*xvi*) The smaller angle which a line makes with true meridian, is called............ (bearing, declination, azimuth)

(*xvii*) .... is the horizontal angle between true north and magnetic north at any place. (Bearing, Dip, Declination)

(*xviii*) An imaginary line joining the points of equal declination either positive or negative, on the surface of the earth, is called .... line. (isoclinic, dip, isogonic, agonic)

(*xix*) At magnetic poles, dip of a magnetic needle is .... (0°, 45°, 90°, 180°)

(*xx*) The whole circle bearing of a line........................ on the quadrant in which it lies. (depends, does not depend)

2. Fill in the blanks with suitable word(s).

(*i*) The compass surveying is one of the methods of surveying in which .... of survey lines are determined by .... and their lengths by ....directly on the surface of the ground.

(*ii*) Compass survey is more suitable for ....areas with undulating ground than chain survey.

(*iii*) Angles computed by the measurements, are known as .... angles.

(*iv*) The least count of a prismatic compass, is..........

(*v*) The reduced bearing of a line whose whole-circle bearing is 90° is....

(*vi*) The sum of the interior angles of a regular pentagon is.......... where *n* is the number of sides.

(*vii*) A series of connected survey lines originating from a station and closing on the same station, whose directions and lengths are known, is known as...........

(*viii*) The angle between the true north and magnetic north at a place, is known as.........

(*ix*) The lines joining the places having equal declination, are known as..........

(*x*) If true bearing of a line is $125^\circ\ 30'$ and declination is $2^\circ\ 30'\ W$, magnetic bearing equals to..........

3. State whether the following statements are correct or not :

(*i*) At local noon, the sun is exactly on the geographical meridian.

(*ii*) The magnetic needle deflects downwards in north as well as in south hemispheres.

(*iii*) The dip of a magnetic needle at equator is zero.

(*iv*) The fore and back bearings of a line differ by 90°.

(*v*) The zero graduation of the prismatic compass, is marked at the south end of the rim.

(*vi*) The local attraction does not affect the horizontal angles between the lines joining at a point.

(*vii*) $S\ 45^\circ\ E$ is the bearing of a line expressed in quadrantal system.

(*viii*) The compass box is made of brass.

(*ix*) If the fore bearing of a line is S 60° W then, its back bearing is N 60° E.

(*x*) If the whole circle bearing of a line is 270°, then its bearing in quadrantal system is 90° W.

4. What is meant by traverse surveying ? How does it differ from chain surveying ? Distinguish between a closed and an open traverse.

5. Explain, with the help of neat sketches the graduations of a prismatic compass and a surveyor's compass.

6. (*a*) Draw a neat sketch of a prismatic compass and name the parts thereon.

(*b*) Explain the significance of any two of its important components.

7. Tabulate the differences of a prismatic compass and a surveyor's compass.

8. Draw a neat sectional elevation of a prismatic compass and name the different parts of the instrument thereon.

9. What are the sources of errors in compass survey and what precautions are taken to eliminate them.

10. Define a traverse. Give its classifications based on the instrument used. Explain each briefly.

11. Define the following.

(*i*) Dip (*ii*) Isoclinic lines (*iii*) Magnetic equator.

12. what are the tests for the adjustment of a prismatic compass ?

How will you adjust a prismatic compass ?

13. Explain clearly the points of difference between the prismatic compass and the surveyor's compass ?

14. Define the following terms :

(*i*) Meridian (*ii*) True meridian

(*iii*) Magnetic Meridian (*iv*) Convergency of meridians.

15. Without resorting to astronomical observations with a theodolite, how will you determine the true meridian at a place ?

16. Define the following terms :

(*i*) Bearing (*ii*) True bearing

(*iii*) Magnetic bearing (*iv*) Azimuth

(*v*) Grid bearing (*vi*) Fore and back bearings.

17. Explain a bearing. What are different systems of designation of bearings. Explain each system with neat sketches.

18. What is local attraction ? How is it detected and removed ?

19. Write short notes on :

(*i*) Fore and back bearings.

(*ii*) Reduced bearings and whole circle bearings.

(*iii*) True bearing and magnetic bearing.

(*iv*) Magnetic declination and convergence of meridians.

(*v*) Local attraction and dip of needle.

(*vi*) Isogonic lines and Agonic lines.

(*vii*) Secular and irregular variations.

20. Describe in brief, different methods of plotting a compass traverse.

21. What do you understand by 'closing error' of a compass traverse. Show can how can it be adjusted by graphical method.

22. Describe the method of traversing with a chain and compass.

23. What are the advantages and disadvantages of compass surveys ? Describe the limits of precision of compass surveying. Where is the compass survey normally used ?

24. Convert the following whole circle bearings to quadrantal bearings;

(*a*) 87° 30′ (*b*) 120° 05′

(*c*) 210° 10′ (*d*) 266° 36′

(*e*) 310° 10′ (*f*) 359° 15′.

25. Convert the following quadrantal bearings to the whole circle earnings:

(*a*) N 30° 30′ E (*b*) S 20° 45′ E

(*c*) S 10° 45′ W (*d*) N 50° 45′ W

26. (*i*) Write the back bearings of the following fore-bearings :

(*a*) 30° 05′ (*b*) 120° 25′

(*c*) 225° 15′ (*d*) 310° 36′.

(*ii*) Write the fore bearings of the following back bearings :

(*a*) 67° 15′ (*b*) 136° 36′

(*c*) 189° 20′ (*d*) 7° 07′.

27. The bearings of the sides of a triangle ABC are as under :

AB = 45° 15′

BC = 150° 50′

CA = 270° 00′

Calculate the interior angles of the triangle.

28. The bearings of lines AB, and AC are 37° 45′ and 127° 35′ respectively. Calculate the acute angle BAC.

29. The bearing of a diagonal AC of a left handed square is 36° 30′. Find the bearing of the diagonal BD.

30. The following fore bearings were observed in running a closed compass traverse :

AB = 80° 35′

BC = 170° 35′

CD = 260° 35′

DA = 350° 35′

Calculate the interior angles of the closed traverse.

31. The bearings of the sides of a traverse *ABCDEA* are as follows :

Side	F.B.	B.B.
AB	150° 10′	285° 10′
BC	20° 20′	200° 20′
CD	275° 35′	95° 35′
DE	179° 45′	359° 45′
EA	120° 50′	300° 50′

Compute the interior angles of the traverse and exercise the geometric checks.

32. The magnetic bearing of a line : AB is 125° 25′. Find its true bearing if the magnetic declination at A is (*a*) 9° 15′ *W* (*b*) 5° 30′ *E*.

33. The true bearing of a line CD is 135° 35′. Find its magnetic bearing if the magnetic declination at C is (*a*) 5° 25′ *W* ; (*b*) 3° 10′ E.

34. A line has a true bearing of 235°. The declination is 3° 30′ *E*. Calculate the magnetic bearing on whole circle as well as reduced bearing systems.

35. Find the magnetic declination, if magnetic bearings of the sun at noon are : (a) 182° 30′(*b*) 177° 30′ (*c*) 359° 10′.

36. The true bearing of a line is *N* 30° 45′ *E*. Compute the magnetic bearing of the line if the magnetic declination is : (a) 3° 10′ *E* and (*b*) 5° 25′ *W*.

37. In an old survey made when the magnetic declination was 3° 39′ *W*, the magnetic bearing of a line *AB* was *N* 56° 12′ *E*. If the present magnetic declination in the same locality is 3° 26′ *E*, calculate the true bearing and magnetic bearing of *AB*.

38. The following bearings were observed with a compass on a traverse:

Line	*F.B.*	*B.B.*	
AB	72° 0′	*BA*	252° 0′
BC	93° 0′	*CB*	273° 0′
CD	168° 0′	*DC*	344° 0′
DE	176° 0′	*ED*	0° 0′
EF	187° 0′	*FE*	7° 0′

At what station(s) do you suspect local attraction ? Find the correct bearings.

39. The following bearings were observed on a traverse :

Line	*F.B.*	*B.B.*
AB	80° 45′	260° 00′
BC	130° 30′	311° 35′
CD	240° 15′	60° 15′
DA	290° 30′	110° 10′

Make corrections for local attraction and declination of 1° 30′ *W* and calculate true fore bearings.

40. The following fore and back bearings were observed in an open compass traverse :

Line	*F.B.*	*B.B.*
AB	223° 00′	42° 45′
BC	166° 30′	346° 45′
CD	02° 15′	182° 15′
DE	174° 15′	354° 00′

Which stations are affected by local attraction and how much ? Determine the true fore and back bearings if area is known to have declination of 2° 15′ east.

41. The following bearings were taken in running an open traverse with a compass in a place where local attraction was suspected :

Line	*F.B.*	*B.B.*
AB	44° 40′	225° 20′
BC	96° 20′	274° 18′
CD	30° 40′	212° 02′
DE	320° 12′	140° 12′

At what stations do you expect local attraction. Find the corrected bearings.

42. Give the corrected bearings of the following traverse taken from a compass survey :

Line	*F.B.*	*B.B.*
AB	N 55° 00′ E	S 54° 00′ W
BC	S 68° 30′ E	N 66° 30′ W
CD	S 24° 00′ W	N 24° 00′ E
DE	S 77° 00′ W	N 75° 30′ E
EA	N 64° 00′ W	S 63° 30′ E

43. A compass was set on the station A and the bearing of AB was observed 309° 15′. Then, the same instrument was shifted to station B and the bearing of BA was found to be 129° *15′*.

Is there any local attraction at the station A, or at station B ? Can you give a precise answer ?

State your comments and support it with rational argument.

44. Calculate the true bearing of a line if there is a local attraction of 2° E and declination is 3° W. The observed magnetic bearing of the line is 222°.

45. Following observations were taken with a compass in case of a closed traverse. Calculate the angles and correct the bearings for local attraction, if any. Calculate the true bearings if the declination is 1° 30′ west.

Line	*F.F.*	*B.B.*	*Declination*
AB	51° 30′	230° 00′	
BC	182° 40′	356° 00′	1° 30′ W
CD	104° 15′	284° 55′	
DE	165° 15′	345° 15′	
EA	251° 30′	79° 00′	

46. The following bearings were observed with a compass.

Line	*F.B.*	*B.B.*
AB	165° 58′	346° 00′
BC	78° 43′	258° 40′
CD	26° 32′	206° 30′
DE	355° 47′	155° 50′
EF	264° 35′	84° 32′
FG	224° 52′	44° 55′
GH	179° 07′	359° 05′
HA	107° 10′	287° 12′
AE	27° 15′	207° 15′

At which stations you suspect local attraction ? Find the correct bearings of the lines.

47. To determine the angle *ACB*, magnetic bearings of *AB* and *BC* as observed from *A* and *B* were *N* 25° 30′ *E* and N 18° 15′ W respectively. Stations A and B were suspected to have local attraction. A point P free of local attraction was selected. The respective bearings of AP and BP were observed as S 15° 30′ E and S 50° 45′ W, while their back bearings were found to be N 16° 15′ W and N 50° 15′ E. Find the correct angle *ACB*.

48. A surveyor equipped with the following materials has to set out two lines making an angle of 60°, with each other.

Chain	= 30 m
Metallic tape	= 30 m
Ranging rods	= 4 Nos.
Arrows	= 10 Nos.

Describe the procedure for setting the same at the desired angle.

49. The magnetic bearing of a line AB as observed from A was N 79° 30′ W. Station A is suspected to have local attraction. In order to determine the correct bearing of the line, a point C which was free from local attraction was selected. The bearing of AC was observed at S 53° 45′ E, while the bearing of CA was found to be N 57° 45′ W. Find the correct bearing of AB.

50. The following bearings were taken in running a closed compass traverse :

Line	*F.B.*	*B.B.*
AB	48° 25′	230° 00′
BC	177° 45′	356° 00′
CD	104° 15′	284° 55′
DE	165° 15′	345° 15′
EA	259° 30′	79° 90′

(*i*) State what stations were affected by local attraction and by how much.

(*ii*) Determine the corrected bearings.

(*iii*) Calculate the true bearings if the declination was 1° 30′ W.

Answers

1. (*i*) Speed (*ii*) Brass (*iii*) Downwards (*iv*) 2n - 4 (*v*) 180° (*vi*) Magnetic (*vii*) East (*viii*) Dip (*ix*) Isoclinic (*x*) Zero (*xi*) South (*xii*) Quadrantal (*xiii*) North, South (*xiv*) Bearings (*xv*) Not variable (*xvi*) azimuth (*xvii*) Declination (*xviii*) Isogonic (*xix*) 90° (*xx*) does not depend.

2. (*i*) Bearings compass, chaining (*ii*) Large (*iii*) Tie (*iv*) 15′ (*v*) *E* 90° (*vi*) (*2n - 4*) (*vii*) Closed circuit (*viii*) Declination (*ix*) Isogonic (*x*) 128°.

3. (*i*) Correct (*ii*) Correct (*iii*) Correct (*iv*) Incorrect (*v*) Correct (*vi*) Correct (*vii*) Correct (*viii*) Correct (*ix*) Correct (*x*) Correct.

24. (*a*) N 87° 30′ E (*b*) S 59° 55′ E (*c*) S 30° 10′ W (*d*) S 86° 36′(*e*)

(*e*) N 49° 50′ W (*f*) N 0° 45′ W.

25. (*a*) 30° 30′ (*b*) 159° 15′ (*c*) 190° 45′ (*d*) 309° 15′.

26. (*i*) (*a*) 210° 05′ (*b*) 300° 25′ (*c*) 45° 45′ (*d*) 130° 36′.

(*ii*) (*a*) 247° 15′ (*b*) 316° 36′ (*c*) 9° 20′ (*d*) 187° 07′.

27. A = 44° 45′ ; *B* = 74° 25′ ; *C* = 60° 50′.

28. Angle BAC = 89° 50′.

29. 306° 30′

30. Angle A = B = C = D = 90° .

31. A = 164° 20′ ; B = 95° 10′ ; C = 75° 15′ , D = 84° 10′ ; E = 121° 05′.

32. (*a*) 116° 10′ (*b*) 132° 55′.

33. (*a*) 141° (*b*) 132° 25′.

34. 231° 30′, S 51° 30′ W.

35. (*a*) 2° 30′ W (*b*) 2° 30′ E (*c*) 0° 50′ E.

36. (*a*) N 27° 35′ E (*b*) N 36° 10′ E.

37. N 52° 33′ E, N 49° 07′ E.

38. Station D is affected by local attraction.

Bearing of DC = 348° 0′ ; *DE* = 180° 0′.

39. AB = 79° 35′ ; BC = 130° 05′ ; CD = 238° 45′ ; DA = 289° 00′

40. AB = 225° 15′ ; BC = 169° 00′ ; CD = 04° 30′ ; DE = 176° 30′.

Stations B and E are affected by local attraction.

41. AB = 44° 40′ ; BC = 275° 40′ ; CD = 32° 02′ ; DE = 320° 12′.

42. AB = N 56° 00′ E ; BC = S 66° 30′ E ; CD = S 24° 00′. *W*;

DE = S 77° 00′ W ; EA = N 77° 30′ W.

43. Though Fore and Back bearings exactly differ by 180° ; it can not be precisely said that both are free from attraction, because both stations might be affected equally.

44. 221° 00′.

45. AB = 42° 30′ ; BC = 175° 10′ : CD = 103° 25′ ; .

DE = 163° 45′ EA = 250° 00′

46. AB = 165° 58′ ; BC = 78° 41′ ; CD = 26° 33′ ; DE = 335° 50′ ;

EF = 264° 35′ ; FG = 224° 55′ ; GH = 179° 07′ ; HA = 107° 12′.

47. 43° 30′.

48. By construction.

49. N 83° 30′ *W*.

50. (*i*) Stations A, B and C.

(*ii*)Corrected bearings ; AB = 48° 55′, BC = 176° 40′ ; CD = 104° 55′.

DE = 165° 15′ , EA = 259° 30′.

(*iii*) True bearings : AB = 47° 25′, BC = 175° 10′, CD = 103° 25′ .

DE = 163° 45′, EA = 258° 00′.

5

Plane Table Surveying

5.1. INTRODUCTION

Plane Table Surveying is one of the methods of surveying in which field observations and plotting proceed simultaneously. For correct representation of various features on the surface of the earth by planetabling, surveyor must be a good artist.

5.2. PRINCIPLE OF PLANETABLING

The principle of planetabling is based on the fact that the lines joining the points on the planetable, are made to lie parallel to their corresponding lines joining the ground points while working at each station. The principle can be best understood by considering the graphical reduction of a triangle to the given dimensions. The base of the triangle is plotted on the desired scale and the base angles are plotted directly by turning the alidade at each end. The intersection of the rays gives the desired location of the triangle vertex. *The planetabling, may be defined as graphical construction of straight lines, angles and triangles for plotting the ground detail points.*

5.3. INSTRUMENTS USED IN PLANETABLING

The instruments required for planetabling are :

1. Plane table with stand
2. Alidade or sight rule
3. Spirit level
4. Magnetic compass
5. Plumbing fork (for large scale surveys only)
6. Drawing paper.

1. The Plane Table. It consists of a wooden table mounted on a light wooden tripod in such a way that the table top (board) may be rotated about its vertical axis and can be clamped in any position. The table top is levelled by adjusting the legs of the tripod. The table measures 750 mm × 600 mm and the legs of the tripod are usually 1200 mm long. The instrument is made entirely of well-seasoned wood except for the metal plate, bolts, nuts and screws, which are of brass and the shoes of the legs, which are of iron. (Fig. 5.1).

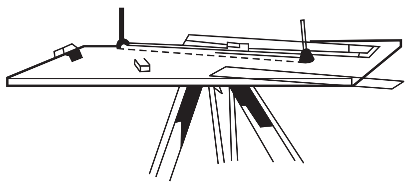

Fig. 5.1. A plane table with an alidade.

Qualities of a good planetable. A good plane table should possess the following qualities :

1. The table-top should be truly flat and free from knots.
2. The butterfly nuts which clamp the legs to the clamping head, should not be free.
3. The clamping assembly should fit the plate at the bottom of the plane table.
4. Annular ring should be properly fixed with the plane table.
5. There should not be any movement of the table top when properly clamped.

2. The Alidade. A plane table alidade is a straight edge with some form of sighting device. Two types of alidades are generally used :

(*i*) Plain alidade (*ii*) Telescopic alidade :

(1) Plain Alidade. It generally consists of a metal or wooden rule with two vanes at the ends. Vanes are hinged and can be folded on the rule when the alidade is not in use. (Fig. 5.2)

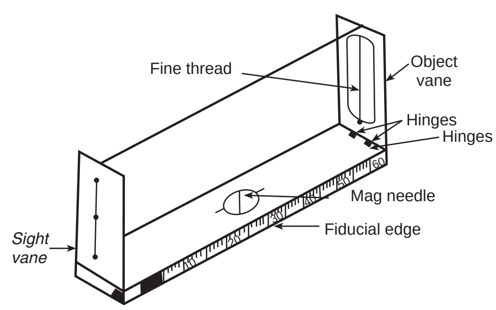

Fig. 5.2. A Plain Alidade.

One of the vanes known as *sight vane* is provided with a narrow slit with three holes, one at the top, one at the bottom and one in the middle. The other vane which is known as *object vane,* is open and carries a hair or a fine thread or a thin wire stretched between the top and bottom of the slit. With the help of the slit, a definite line of sight may be established parallel to the ruling edge of the alidade. The alidade can be rotated about the point which represents the location of the instrument station on the sheet so that line of sight passes through the station sighted. The length of the ruling edge of the alidade should not be shorter than the longest side of the plane table. The two vanes should be perpendicular to the surface of the table. The working edge of the alidade is known as *fiducial edge.* A plain alidade can be used only when the elevations or depressions of the objects are low. If elevations of the objects are more than what can be accommodated by the line of sight, the alidade can be used by stretching a thin thread tightly between the tops of the sight and object vanes.

(2) Telescopic Alidade (Fig. 5.3). The alidade which is fitted with a telescope is known as *telescopic alidade.* It is generally used when it is required to take inclined sights. Telescope increases the range and accuracy of the sights. It consists of a small telescope with a level tube. A graduated scale is mounted on a horizontal axis. The horizontal axis rests on an A-frame which is supported on a heavy metal ruler. One side of the metal ruler is used as the working edge along which lines are drawn. The angles of elevation or depression can be read on the vertical circle. The horizontal distance between the instrument station and the detail point can be deduced by taking stadia readings on a levelling staff held at the point, using tacheometric formulae. The difference in elevations can be computed by multiplying the horizontal distance and the tangent of the angle of elevation or depression. Sometimes tacheometric tables can also be referred to, for finding the horizontal and vertical distances between the stations. For tacheometric observations and calculations, refer to Chapter 13 "Tacheometric Surveying".

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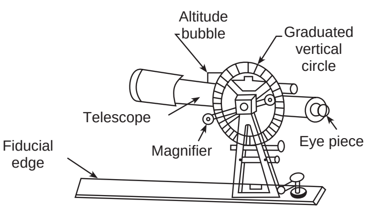


Fig. 5.3. A telescopic alidade.

3. Spirit Level (Fig. 5.4). It consists of a small metal tube which contains a small bubble. The spirit level may also be circular but its base must be flat so that it can be laid on the table. The table is truly level when the bubble remains central all over the table.

4. The Magnetic Compass (Fig. 5.5). A box compass consists of a magnetic needle pivoted at its centre freely. It is used for orienting the plane table to magnetic north. The edges of the box compass are

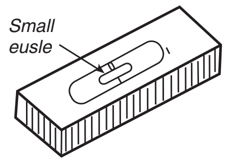


Fig. 5.4. A spirit level.

straight and the bottom is perfectly flat. The dip can be temporarily adjusted by tying a rider around the elevated end. The magnetic needle should be fairly sensitive and play freely. When a compass works unsatisfactorily due to worn out *agate,* a new agate should be replaced. At the first plane table station, the longer edges of the compass are placed parallel to the sides of the plane table. The plane table is then rotated till the needle points N-S direction. A line drawn along the longer edge represents the magnetic north. In case ground control points are already plotted on the sheet, the table is set with respect to the ground control points by any one of the methods described in article 5.4. The box compass is rotated till its needle rests in N-S direction. A line along the edge of the compass is drawn, which defines the magnetic north.

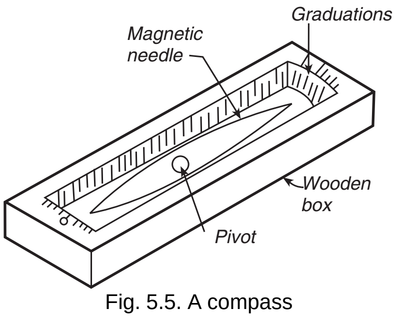


Fig. 5.5. A compass

5. Plumbing Fork (Fig. 5.6). The plumbing fork consists of a hair pin-shaped brass frame, having two equal arms of equal lengths. One end has a pointer while a plumb bob is attached to the other end. It is used in large scale survey for accurate centering of the station location on the table over its ground position. It is also used for transferring the location of the instrument station on the sheet on to the ground.

The fork is placed with its upper arm lying on the top of the table and the lower arm below it. The table is said to be centered when the

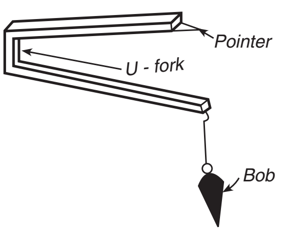


Fig. 5.6. A Plumbing form.

plumb bob hangs freely over ground mark. The pointed end of the fork points the required location on the plane table. On small scale survey exact centering of the point is not required. The centre of the table may be used as the location of the ground station.

6. Drawing Paper. The drawing paper used for planetabling must of be superior quality so that it can stand erasing. It may experience minimum effect of distortion due to climatic changes. Changes in humidity of the atmosphere alter the dimensions of paper in different directions differently. Sometimes drawing paper is mounted on a zinc sheet to avoid shrinkage and expansion due to atmospheric humidity.

5.4. WORKING OPERATIONS

Following three operations are carried out at each plane table station.

(*i*) Fixing the planetable on the tripod.

(*ii*) Setting up the planetable.

(*iii*) Sighting the ground stations and intersected points.

1. Fixing the plane table on the tripod. In this operation, leather strap of the tripod, is unfolded and legs of the tripod are well spread. The tripod is held so that its top height is roughly 1.2 m above the ground level. The bolt is removed from the brass annular ring and table top is placed on the top of the tripod so that it fits well with the clamping assembly of the tripod. The bolt with a washer is then tightened.

2. Setting up the plane table. The setting up operation consists of the following :

(*i*) Levelling the plane table

(*ii*) Centering the plane table

(*iii*) Orienting the plane table.

1. Levelling. In this operation, the table top is made truly horizontal. For rough and small scale work, levelling can be done by eye estimation whereas for accurate and large scale work, levelling is achieved with an ordinary spirit level. The levelling is specially important in hilly terrain where some of the control points are situated at higher level and some other at lower level. The dislevelment of the plane table, throws the location of the point considerably out of its true location.

Procedure : Following steps are involved :

(*i*) Set up the planetable at the convenient height (nearly 1.2 metres) by spreading the legs to keep the table approximately levelled, ensuring that location of the occupied station, is also roughly centered over its ground position.

(*ii*) Rotate the plane table about its vertical axis till its longer edge is parallel to the line joining the shoes of any two legs of the tripod. Place the step third leg pointing towards the observer in between his / her legs.

(*iii*) Place a spirit level on the plane table such that its longitudinal axis is parallel to longer edge of the table. With the help of the third leg, by moving it right or left, bring the bubble of the spirit level central.

(*iv*) Next place the spirit level perpendicular to its previous position. With the help of the third leg, by moving it forward or backward, bring the bubble of the spirit level central.

(*v*) Rotate the table top through 180° . Check if the bubble remains central in all positions.

(*vi*) Repeat the above procedure if found, necessary.

2. Centering. In this operation, the location of the plane table station on the paper, is brought exactly vertical above the ground station position. For rough and small scale work, exact centering of the station, is not necessary and only centre of the table may be centered over the ground position.

Procedure. Place one end of the U-fork touching the plotted location and the plumb bob hanging from the other end below the table, points towards the ground point. In case it does not, shift the plane table bodily such that the plumb bob is exactly over the ground station without disturbing levelling. Before centering is done, the table should be roughly oriented otherwise centering might be disturbed when orientation is done.

3. Orientation. In this operation, the plane table is set at a station such that its edges make a fixed angle with a fixed direction. The fixed direction is known as the *meridian*. In case, the table is not correctly oriented at each station, the locations of detail points obtained by any one of the methods of planetabling *i.e.* Radiation, Resection or Intersection described in article No. 5.5., will not represent their correct relative positions. *The main principle of planetabling is based on the fact that the lines joining the locations of the ground stations on the sheet, are made parallel to their respective ground lines.* This is achieved by the process of orientation which involves rotation of the table about its vertical axis in azimuth. The operation of orientation is sometimes called "*Setting the plane table"*. As already discussed, the process of orientation disturbs the centering and *vice versa*. For accurate and large scale work, centering must be checked before orientation. Sometimes, both the processes of centering and orientation, are repeated till the two required conditions are statisfied.

Orientation of a planetable may be done by the following methods :

1. Orientation with a magnetic compass

2. Orientation with a back ray.

Method 1. Orientation with a Magnetic Compass. In case true north is not known at the plane table station, a magnetic north is sometimes used as reference *i.e., meridian.* At the starting station, the table is set such that the entire area falls on it. Place a box magnetic compass such that its magnetic needle rests in N-S direction. Draw a pencil line along the longer edge of the box. On subsequent stations after levelling and centering the table over the ground mark, the magnetic compass is laid along the drawn magnetic north. The table is then rotated until the needle rests in N-S direction. Clamp the table. The table is correctly oriented in magnetic meridian if the plane table station is free from local attraction.

Method 2. Orientation with a Back Ray. In this method, a ray is drawn from the plotted location of the instrument station to the next forward station. Its extremities are marked on both the ends of the alidade. On arrival at the forward station, the alidade is laid along the ray drawn from the previous station. The table is rotated until the line of sight intersects the previous station. This operation is termed *"setting by the back ray"*. This method is independent of the defects of magnetic compass and local attraction. It is essential that the same edge of the alidade is used for drawing lines. It may also be ensured that the line *i.e.*, back ray remains vertically above the ground position of the forward station.

3. Sighting the ground Station. In this operation, the table is accurately centered and levelled, over the ground station. The fiducial edge or working edge of the alidade is kept touching the plotted location of the instrument station. The ground control point is sighted through its sight vane so that the station, the thread of the object vane and the slit hole of the sight vane, all are in a straight line. The sighting operation is required for sighting all the stations or details whose locations are either known or are to be surveyed on the plane table.

5.5. METHODS OF PLANE TABLE SURVEYING

Plane table surveying may be carried out by one of the following methods:

(*i*) Radiation Method.

(*ii*) Intersection Method.

(*iii*) Traversing Method.

(*iv*) Resection Method

Before describing each method in detail, the following technical terms may be clearly understood.

Fore sight. When a station is sighted and a ray is drawn through the plotted location of the instrument station towards that station, the sight is called a "*fore sight*".

Back sight. When the alidade is placed along a previously plotted line passing through the plotted location of the station occupied and that of another station and then the table is rotated until the line of sight bisects the station, the sight is called a *back sight.*

Resector. When a known station is sighted and a line is drawn through the plotted location of that station towards the instrument station, the sight is called a '*resector*'.

1. Radiation Method. In this method, a plane table is set up at any commanding station. Detail points are plotted on their radiating lines drawn from the location of the instrument station, after reducing their respective ground distances on the desired scale of survey.

This method is suitable for the survey of small areas which can be commanded from a single station. Particularly, preparation of plans on large scales, is conveniently done by this method. This method is rarely used to survey a complete project. It is generally combined with other methods for surveying details within a chain length from the instrument station.

Principle of the Method. (Fig. 5.7).

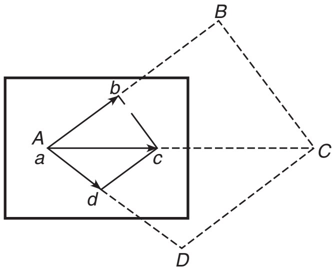


Fig. 5.7. Principle of Radiation.

Let *A* represent the instrument station and *B, C* and *D* represent other ground stations whose locations are required to be plotted. Lines *ab, ac, ad* etc. are drawn from the location *a* of the station *A*. Distances *AB, AC* and *AD* are reduced to the same ratio. Let the ratio $\frac{AB}{ab} = \frac{AC}{ac} = \frac{AD}{ad} = K$. From a well known geometrical property, *b* and *c* are the points on the sides *AB* and *AC* of the triangle *ABC* such that $\frac{AB}{ab} = \frac{AC}{ac} = K$. The line *bc* will therefore be parallel to *BC*.

$\Delta\, abc$ and $\Delta\, ABC$ are therefore similar

i.e. $$\frac{bc}{BC} = \frac{ab}{AB} = \frac{ac}{AC} = \frac{1}{K}$$

Hence, the line $bc = \dfrac{BC}{K}$ has been plotted on the same scale to which lines AB and AC have been plotted.

Procedure. The following steps are followed to locate the points from the instrument station. (Fig. 5.8.)

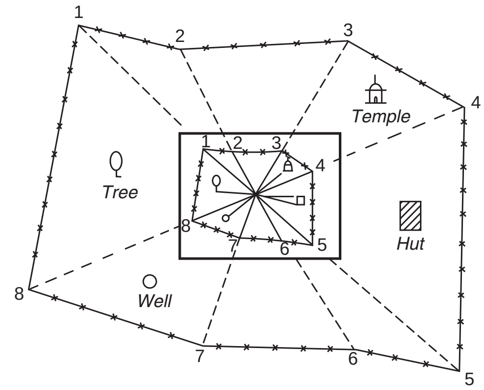


Fig. 5.8. Radiation Method.

(*i*) Set up the plane table at the station, centre and level it accurately.

(*ii*) Choose a location of the station *A* on the drawing paper at a convenient position, considering the general lay out of the area.

(*iii*) Transfer the point on to the ground by means of a plumbing U-fork for setting up the table on subsequent days.

(*iv*) Clamp the plane table tightly and draw the magnetic north with the help of a magnetic compass.

(*v*) Pivoting the alidade about *a,* the location of instrument station, sight the detail points *B, C, D* etc. in turn and draw rays along the fiducial edge of the alidade.

(*vi*) Measure the ground distances by direct chaining and plot them on their respective lines drawn on the desired scale. If the ground is sloping, slope correction is applied and equivalent horizontal distances are plotted.

(*vii*) Conventional symbols are drawn for different details and inked up.

2. Intersection Method. In this method, either the coordinates of at least two accessible and intervisible points must be known or the distance between them is measured directly in the field. These points are plotted on the required convenient scale. The locations of other detail points are determined by drawing rays from each end station after proper orientation of the table. The intersection of rays gives the location of detail point. It is thus evident that it is very essential to have at least two points whose locations are plotted before the survey may be started.

The line joining the locations of the given stations is known as the base line. In this method, no other linear measurement is required except that of the base line. The point of intersection of the rays drawn from the ends of the base line, forms the vertex of the triangle and two rays represent the remaining two sides. the position of the vertex is determined by completing the triangle graphically. This is why the method is also known as '*Graphic triangulation*'.

Principle of the intersecting Method Let A and B represent two ground points d metres apart. a and b represent their plotted locations on the table. The table is oriented first at the station A. Placing the edge of the alidade along ab, rotate the table until B is sighted. Clamp the table. From a, draw a ray towards C. The table is then shifted to station B and after centering carefully over the ground mark, the table is oriented with station A. Pivoting the alidade about b, a ray is drawn towards C to intersect the previous ray drawn from A. The intersection of the two rays, gives the desired location c of the point C on the same scale to which AB was plotted. (Fig. 5.9)

Proof :

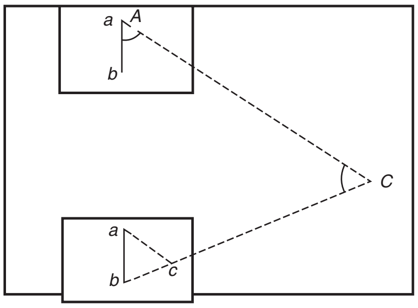


Fig. 5.9. Principle of intersection method.

In Δs s abc and ABC

$$\angle bac = \angle BAC \quad \text{(By construction)}$$

$$\angle abc = \angle ABC \quad \text{(By construction)}$$

$$\therefore \quad \angle acb = \angle ACB$$

Hence, the triangles abc and ABC are similar.

or $$\frac{ab}{AB} = \frac{ac}{AC} = \frac{bc}{BC} = k, \text{ a constant}$$

$$\therefore \quad ab = k.\,AB \;;\; ac = k.\,AC \text{ and } bc = k.\,BC.$$

i.e. the sides AC and BC of Δ ABC have been reduced to the same ratio to which the side AB was initially plotted.

Procedure : Following steps are followed to locate the points by the method of intersection.

(*i*) Select two points A and B on a fairly levelled ground at sufficiently large distance apart. Measure the distance AB directly. In case their independent coordinates are available, these can be plotted to get the locations of the two stations.

(*ii*) Plot the base line AB on the plane table on the desired scale in a convenient position, keeping in view the general layout of the area to be surveyed.

(*iii*) Set up the instrument on the station A such that its plotted location is centered over the ground point. The line ab is also kept approximately coincident with the ground line AB.

(*iv*) Level the table and place the fiducial edge of the alidade along the line ab.

(*v*) Rotate the plane table until the point B is sighted.

(*vi*) Check up whether the location of A is vertically over the ground point. If not, centre the table and repeat the step No. 5.

(*vii*) With the help of a magnetic compass, mark the north direction on the drawing sheet.

(*viii*) Pivoting the alidade about a, sight the points 1, 2, 3, etc. in turn and draw their corresponding rays, a_1, a_2 , a_3, etc.

(*ix*) Shift the plane table to the station B and centre it over the ground mark.

(*x*) Placing the alidade along line ba, rotate the table till the station A is sighted. Clamp it.

(*xi*) Pivoting the alidade about b, sight the points 1, 2, 3, etc. Draw their corresponding rays b_1, b_2, b_3, *etc. to intersect the rays previously drawn from the station A*. The intersections of the corresponding rays, give the required locations of the points, 1, 2, 3, etc. (Fig. 5.10).

Suitability. The method of intersection is suitable when distances between detail points are either two large or can not be measured accurately due to undulations. The method is generally used for surveying the detail points. Whenever this method is used for locating other points to be used at subsequent plane table stations, the points should be got by way of intersection of at least three rays. It may be noted that the angles of intersection of differnt rays should not be acute to obtain accurate locations of the points. Triangles should be well conditioned. The angle of intersections of rays, should not preferably be less than 30° and not more than 120°. As no linear measurements are required in this method it can be suitably employed for surveying mountaineous regions. It may not be out of place to mention here that mapping of large areas is mostly done by the method of intersection by the department of the Survey of India. As accumulation of error is limited only to the scale of

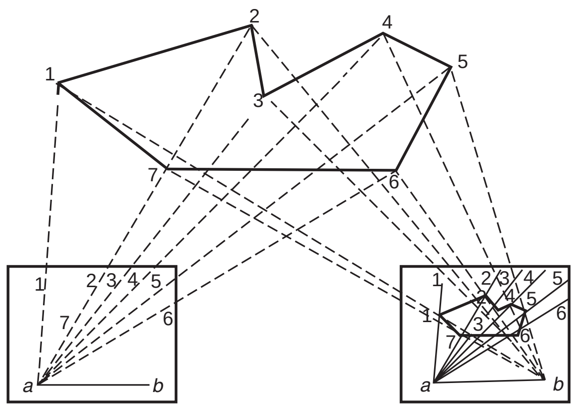


Fig. 5.10. Intersection method.

plotting of the base line, graphic triangulation can be extended to cover a large area without introducing any appreciable error.

3. Traversing Method. The method of traversing by a plane table is similar to that of compass and theodolite traverses. In plane table traverse, the table is set at each successive station, a foresight is taken to the next station and its location is plotted on the foresight, by measuring the distance directly between the two stations. Sometimes a resector may be drawn from the location of a well fixed point, to intersect the foresight at a good angle. The location of the following station is obtained without resorting to the measurement of the distance between stations.

Principle of the Traversing Method. The principle of traversing is similar to that of radiation method. The only difference is that in the case of radiation method, the observations are made for locating all the points in the neighborhood of the station whereas in traversing, observations are made only to points which will subsequently be occupied by the surveyor for locating the details. Plane table traversing is always carried out in a closed circuit or it may originate from and close on known points or other resected points. (Fig. 5.11)

Procedure. The plane table traverse is carried out as follows:

(*i*) Reconnoitre the area to be surveyed for selecting a number of stations, sufficiently far apart, ascertaining their intervisibility and feasibility of chaining.

(*ii*) Set up the plane table over the starting station *A*. Transfer the ground point with a *U* fork on the sheet.

(*iii*) Orient the plane table approximately so that the area to be surveyed, falls on the table.

(*iv*) Draw a magnetic north line with a box compass.

(*v*) With the alidade pivoted at *a*, the assumed location of *A*, sight the next station *B*. Draw a ray along the fiducial edge of the

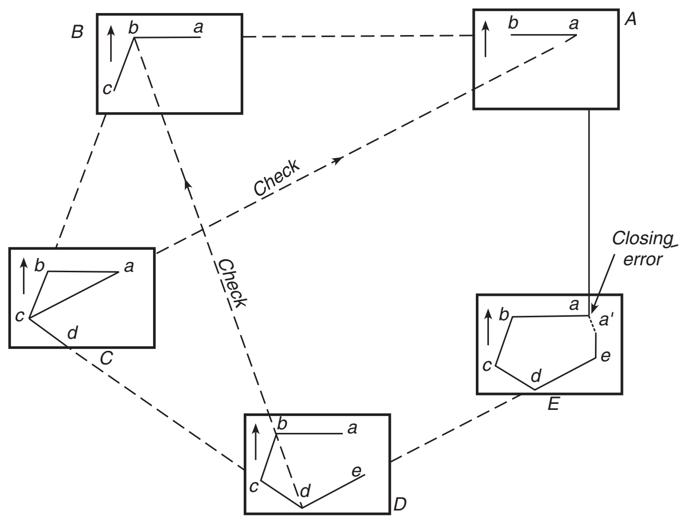


Fig. 5.11. A plane table transverse

alidade and also its extremities are marked on both the edges of the plane table.

(*vi*) Measure the distance *AB* accurately and plot a line *ab* on the scale of survey.

(*vii*) Shift the table to station *B*. Centre the plane table such that the ray *ab* passes vertically above the ground point *B*.

(*viii*) Place the alidade along the ray *ba*. Rotate the table till ground station *A* is sighted. Clamp it.

(*ix*) Pivating the alidade about *b*, sight next station *C*. Measure *BC* and plot *bc* on the ray drawn towards the station *C*.

The planetable is set up on other succeeding stations till the last station is plotted. The plotted position of the last station should normally coincide with the location of the first station, in case of a closed circuit. The distance between the two locations of the starting station, if any, is known as, '*closing error*'.

Suitability. When surveying a township or forested areas where clearings are rare and distant views seldom obtainable, it is evident that the normal methods of plane tabling (*i.e.* radiation, intersection or resection) are not possible. To survey such areas, planetable traverse can be suitably employed. This method is suitable only for large scale surveys as orientation of the planetable is done with back ray method in which longer rays are required. In case of a small scale plotting say 1 : 5000, a distance of 25 metres will be represented by 5 mm on the sheet while on scale 1 : 1000, the same distance will be represented by a distance of 25 mm. Planetable traversing can also be suitably employed for surveying an area magnetically disturbed.

Adjustment of a planetable traverse. The closing error of a plane table traverse if any, may be adjusted graphically as explained below:

Let *abcdea′* denote the traverse whose starting and closing station is *A* (Fig. 5.12). *a* and *a′* denote the two locations of starting station *A* as obtained by traversing. In case of an errorless traverse the location *a′* would have coincided with its plotted position *a*. The distance *a′a* is known as *closing error*. The traverse should be adjusted such that the error is distributed equally among all the stations without disturbing the shape of the traverse.

Procedure. Lay out a straight line parallel to longer edge of the table and mark *a, b, c, d, e,* and *a′* at their respective distances from each other either on the scale of the survey or on any other convenient scale.

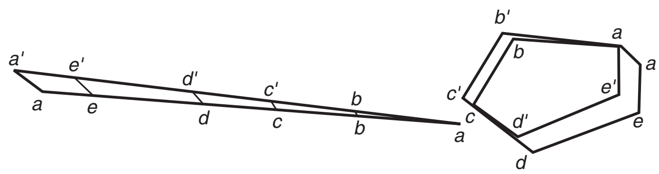


Fig. 5.12. Graphical adjustment of traverse.

At *a′*, draw *a′ a″* parallel and equal to the length of the closing error. Join *a a″* and draw parallel lines through points *b, c, d* and *e* to meet the line *a a″* at *b′*, *c′*, *d′*, and *e′*. Join *a a′* on the traverse and draw lines parallel to *a′ a″* through traverse points *b, c, d,* and *e*. On these parallels mark off *b b′*, *c c′*, *d d′*, etc. equal to the respective corrections *b b′*, *c c′*, *d d′*, etc. Join *a*, *b′*, *c′*, *d′*, *e′*, *a*, to represent the adjusted traverse.

4. Resection Method. The process of determining the location of the station occupied by the plane table, by means of drawing rays from stations whose locations have already been plotted on the sheet, is called *resection*. This method which is also generally known as *Interpolation Method* or *Fixing Method* consists of drawing rays from known points whose locations are already available on the sheet. The intersection of these rays will be at a point if the orientation of the table was correct before rays are drawn. It is seldom possible to get an accurate orientation even with a magnetic compass. The problem, therefore, lies in orienting the table at the unknown occupied station. It may be solved by one of the following methods:

(*i*) Back ray method (*ii*) Three points method

(*iii*) Two points method (*iv*) A box compass method.

1. Back ray method. In this method the planetable is orientated by laying the alidade along the line drawn from the previous station. The location of the unknown occupied station is determined by drawing a ray from another station or point whose locations is already plotted on the sheet, ensuring that two rays intersect at a good angle.

Procedure. The following steps are followed :

(*i*) Let A and B be two ground stations whose location, a and b are plotted on the sheet.

(*ii*) Set up, level and centre the table over the point B. Placing the alidade along ba, rotate the table till station A is sighted. Clamp it.

(*iii*) Pivoting the alidade about b, sight the station C and draw a ray bc. Draw rays also at the extremities of the alidade edge.

(*iv*) Shift the instrument and set it up at C. Centre and level it over the ground point.

(*v*) Place the alidade along cb and rotate the table till the station B is sighted, ensuring that the line bc remains vertically above the ground point C.

(*vi*) With the alidade pivoted at a, sight the station A and draw a ray to intersect the ray previously drawn from station B for C. It gives the required location of the station C.

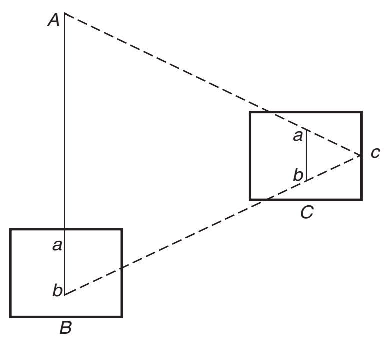


Fig. 5.13. Resection by 'back ray'.

Suitability. This method can be suitably employed for the plane table survey when a prominent point such as a temple spire, chimney, etc. is available in the centre of the area. After setting the plane table at succeeding stations, their locations can be obtained by drawing rays from it. This method is also suitable for large scale surveys where long rays are possible. The accuracy of the survey depends upon the accuracy with which the initial setting of the table was made.

Precautions. In orienting the table by the back ray method, the following precautions must be taken.

(*i*) The actual plotted location of the station on the sheet (not the centre of the table) must be centred over the ground station mark. An error of 30 cm in centering, in the case of a short ray of 20 metres introduces an error of 1° in orientation. This error is accumulative.

(*ii*) The forward station must be carefully selected and marked by a wooden peg before a forward ray is drawn. The back station must also similarly be marked with a wooden peg by transferring its position vertically below the plotted position with a U-fork before leaving the station.

(*iii*) The forward ray should be marked on both the edges of the alidade so that an accurate setting of the table can be made by using the full length of the alidade, at the next station.

(*iv*) The same side of the alidade, should be used for drawing rays.

2. Three Point Method of Resection *"Finding the location of the station occupied by a planetable on the sheet, by means of sighting to three well defined points whose locations have previously been plotted on the sheet, is known as three point resection".*

In this method, the plane table is set up with the help of three known points without visiting them. Let *a, b, c,* represent the locations of *A, B, C,* three ground stations and *P* represents the instrument position, the location of which is to be determined. The table is said to be oriented when rays drawn from three points *A, B* and *C* intersect at a point, and they do not form any triangle. The point of intersection of three rays is the required location of the instrument station *P*.

The orientation of the plane table by three point method can be achieved by one of the following methods.

(*i*) Mechanical (or Tracing Paper) Method.

(*ii*) Graphical Method.

(*iii*) Trial and Error Method or Lehmann's Method.

A. Mechanical (or Tracing Paper) Method Let *A, B,* and *C* be three known points whose locations on the sheet are respectively, *a, b,* and *c*. The instrument station *P* is represented by *p*. (Fig. 5.14)

Procedure. Following steps are followed :

(*i*) Set up the plane table on the station *P*. Orient it roughly with the help of a magnetic compass or by eye judgement.

(*ii*) Fix a tracing paper large enough to include the locations of all the four points on the sheet. Mark a point p' on the tracing paper to represent the instrument position.

(*iii*) Pivoting the alidade about p', sight *A, B, C* in turn and draw rays, $p'A$, $p'B$ and $p'C$ on the tracing paper.

(*iv*) Now, remove the tracing paper. Move it on the sheet in such a way that lines $p'A$, $p'B$ and $p'C$ are made to pass through the plotted locations *a, b* and *c* of the ground stations respectively.

(*v*) Prick through the point P' to get the location *p* of station *P* on the sheet.

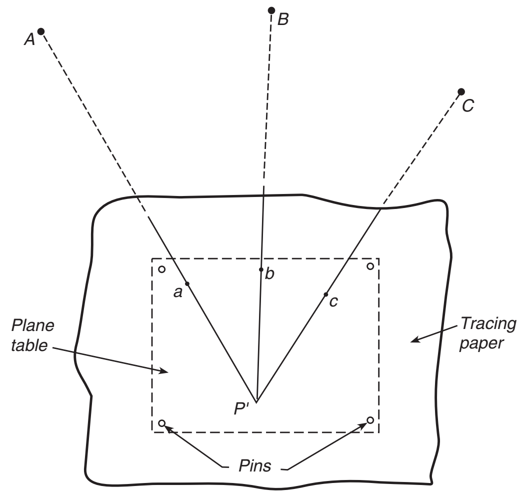


Fig. 5.14. Tracing paper method.

(*vi*) Align the alidade along the longest ray *pa* (assuming *A* to be the farthest point). Rotate the table until the point *A* is sighted.

(*vii*) Pivoting the alidade about the locations *b* and *c*, draw rays from stations *B* and *C*. These rays should also pass through the point *p*. This provides a check on the orientation of the table.

It may be noted that accuracy of the work depends upon the accuracy with which lines are drawn from the assumed position of the instrument station on the tracing paper and also upon the fineness of the lines drawn. This method may be used only to survey detail points. It should not be used for providing control points.

B. Graphical Methods There are various methods for solving a three point problem graphically. Only important ones have been described here:

(*i*) Bessel's Method (*ii*) Perpendicular Method.

(*iii*) Method of arcs.

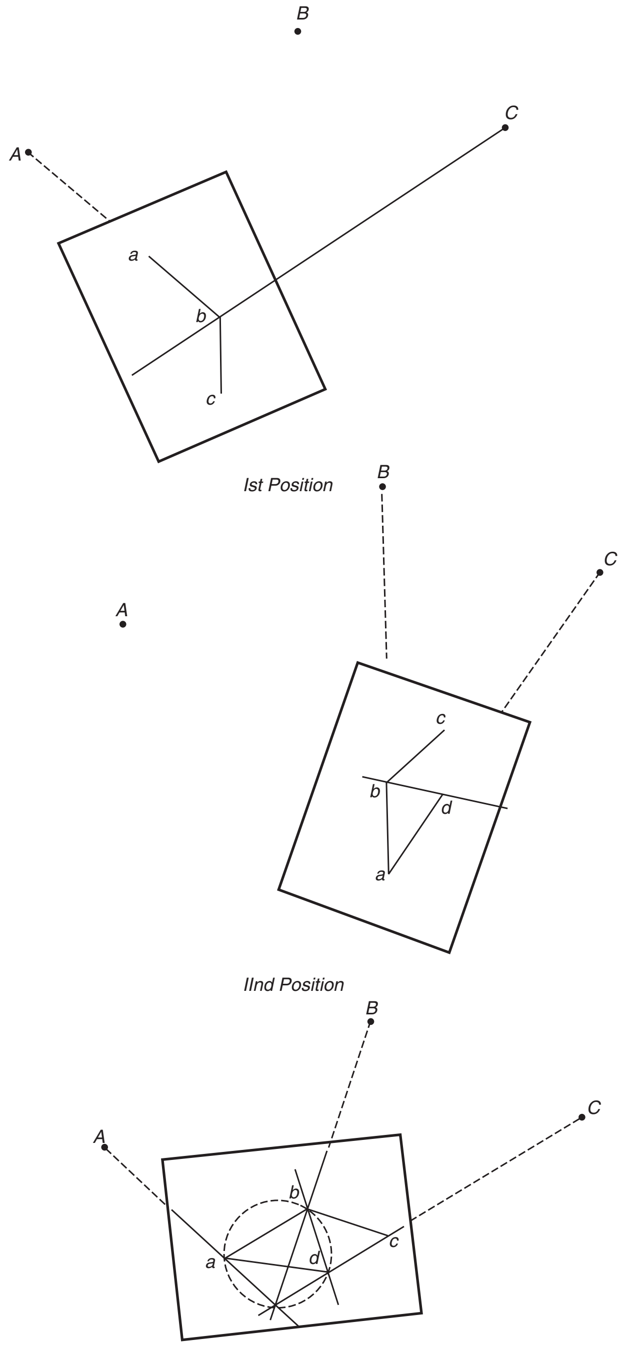
(*a*) **Bessel's Method.** Let *A, B* and *C* be the ground stations whose locations on the sheet are represented by *a, b* and *c* respectively. Let *p* represent the instrument station *P*. (Fig. 5.15).

Procedure. Following steps are followed:

(*i*) Set up the instrument on the station *P*.

(*ii*) Align the alidade along *ba* and rotate the table till station *A* is sighted. Clamp the table.

(*iii*) Pivoting the alidade about *b*, sight the third station *C* and draw a ray *bc*.



3rd Position (c)

5.15. Bessel's Method

(*iv*) Align the alidade along *a b* and rotate the table till station *B* is sighted. Clamp the table.

(*v*) Pivoting the alidade about *a*, sight the third Station *C* and draw a ray to intersect the ray *bc* at *d*.

(*vi*) Join *dc*. Keeping the alidade along *dc* rotate the table till station *C* is sighted. Clamp the table which is now correctly oriented.

(*vii*) Pivoting the alidade about *a* and *b*, sight *A* and *B* in turn and draw rays to intersect on the line *dc* (or produced), if the work is accurate.

This method is sometimes called as *Bessel's method of described quadrilateral* as the points *a, b, c,* and *p* lie on the circle passing through them.

Note : If the instrument station *P* lies on the circumference of the circle passing through *A, B* and *C*, Bessel's method is not suitable.

(*b*) **Perpendicular Method.** Let *A, B* and *C* be the ground stations. *a, b* and *c* be their locations on the sheet. The location of the instrument station may be obtained by sighting to the given three stations. (Fig. 5.16)

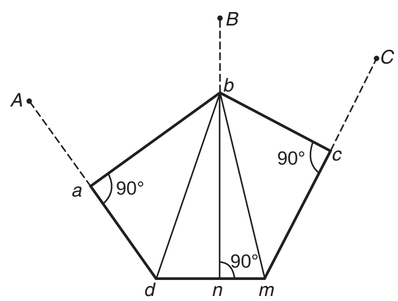


Fig. 5.16. Perpendicular Method.

Procedure. The following steps are followed :

(*i*) Draw a line *ad* perpendicular to *ba* at *a*

(*ii*) Align the alidade along *da* and rotate the table till station *A* is sighted. Clamp the table.

(*iii*) Pivoting the alidade about *b*, sight *B* and draw a ray through *b* to intersect *ad* at *d*.

(*iv*) Draw a line *cm* perpendicular to *bc* at *c*.

(*v*) Align the alidade along *mc* and rotate the table till station *C* is sighted. Clamp the table.

(*vi*) Pivoting the alidade about *b*, sight *B*. Draw a ray through *b* to intersect *cm* at *m*.

(*vii*) Join *dm* and drop a perpendicular *bn* from *b* on *dm.*

(*viii*) *n* is the required location of the instrument station.

(*ix*) Align the alidade along *na* (assuming *A* to be the farthest station) and rotate the table till *A* is sighted. Draw rays from other stations *B* and *C* which will pass through *n* if the orientation is accurate.

(*c*) **Method of arcs :** The three point problem may also be solved by the method of arcs as described below :

Let *A, B* and *C* be the ground stations whose locations on the sheet are *a, b* and *c* respectively.

Let *P* be the plane table position whose location *p* is desired.

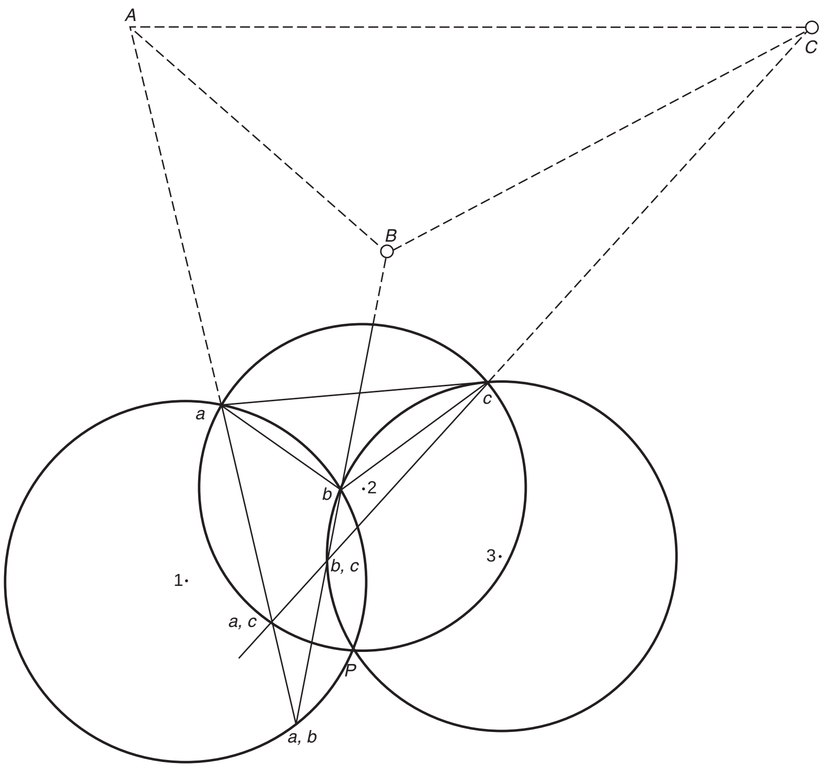


Fig. 5.17. Method of arcs.

Procedure : The following steps are followed :

(*i*) Set up the plane table at *P* and level it carefully.

(*ii*) By eye estimation, orient the plane table such that the locations of points towards their respective ground stations *A, B* and *C*.

(*iii*) Join *a, b, c* to form a triangle *abc*.

(*iv*) Pivoting the alidade about *a, b* and *c* in turn, sight respective ground stations *A, B, C* and draw rays.

(*v*) Locate the points of intersections of the rays from *A* and *B* as *ab,* from *B* and *C* as *bc*, and from *A* and *C* as *ac*.

(*vi*) Draw a circle to pass through *a, b* and *ab*. Draw another circle to pass through *b, c* and *bc.* Similarly, draw a circle to pass through *a, c* and *ac.*

(*vii*) The point *p* at which these circles intersect, is the required location of the plane table station.

(*viii*) Align the alidade along *pa*, (assuming *A* to be farthest station) and rotate the table to sight the ground station *A.*

(*ix*) Draw rays from ground stations *B* and *C*. These rays shall pass through *p*, if the orientation is correct.

Final orientation of the Plane Table by method of arcs is shown in Fig. 5.18.

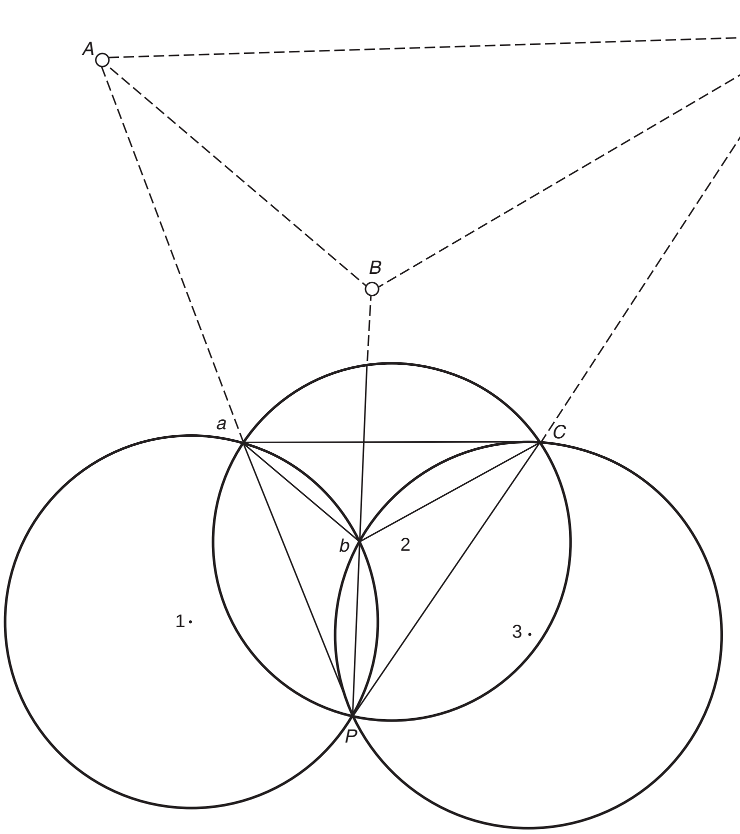
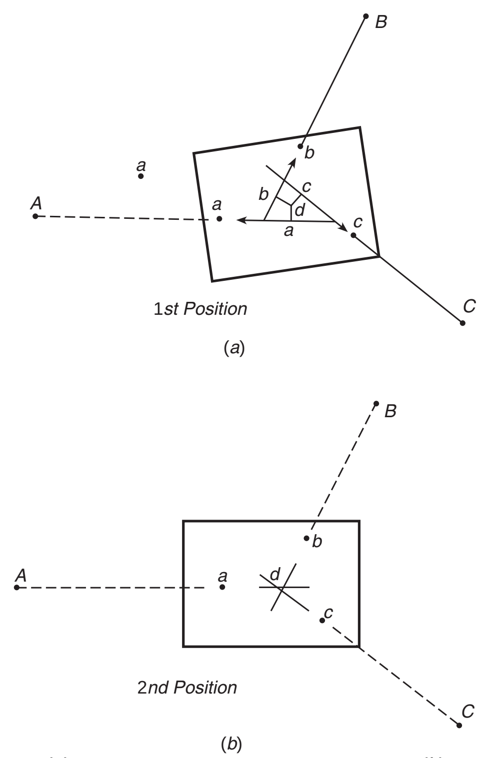


Fig. 5.18. Method of arcs.

C. Trial and Error Method (or Lehmann's Method).

In this method the location of the station occupied, is determined by trial and error. This method was invented by a well-known mathematician, Lehmann and hence this method is sometimes known as *Lehmann's method*. After orientation by eye judgement or with the help of a magnetic compass, rays are drawn to *A, B, C* pivoting the alidade about *a, b, c*. If the orientation is correct, the rays will intersect at one point which gives the required location of the instrument position. This is seldom possible to get an intersection of three rays at a point. Generally three rays intersect forming a small triangle known as *triangle of error*. The size of the triangle of error depends upon the angular error in the orientation. There are three cases which are to be considered while solving the triangle of error, graphically'.

Case I. When the observer's position is inside the triangle formed by joining the locations of the known stations. (Fig. 5.19).



(*a*) (*b*)
Fig. 5.19. Trial and error method (inside fixing).

In this case the location of the instrument station will be within the small triangle of error formed by the intersections of the rays. Its position will be such that its perpendicular distances from each rays will be in proportion to the distances of the plane table position from the respective fixed stations.

Case II. When the observer's position lies outside the triangle formed by joining the locations of the known stations. (Fig. 5.20).

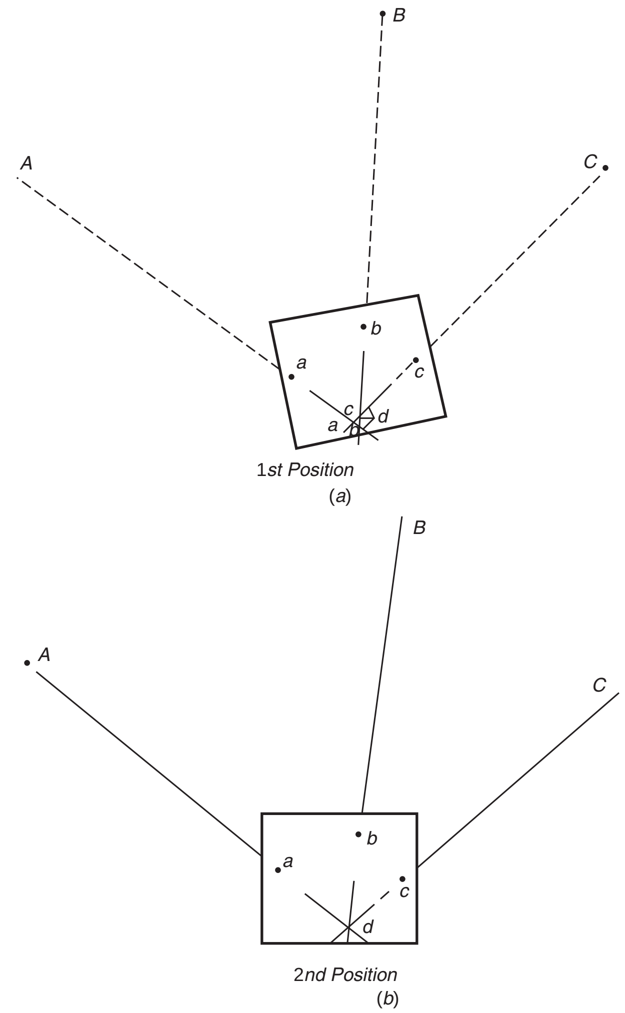


Fig. 5.20. Trial and error method (out side fixing).

In this case, the location of instrument station will be outside the triangle of error formed by the intersection of rays. Its position will also be such that its perpendicular distances from each ray will be in proportion to the distance of instrument position from the respective fixed stations with the condition that all the rays have to move in the same direction round their fixed station in order to reach it when the table is rotated for orientation.

Case III. When the observer's position lies on the circumference of the circle passing through or nearly through the ground stations. (Fig. 5.21).

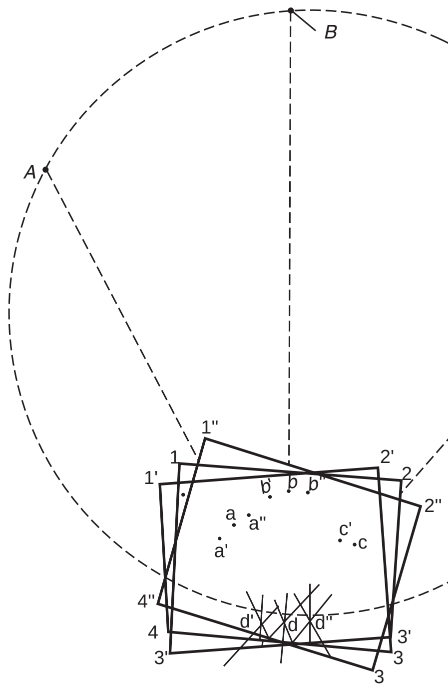


Fig. 5.21. Plane table on Danger circle.

In this case, no accurate determination of the instrument position is possible as the three rays always intersect at a point within a certain limit of error in orientation. It may be noted that interpolation by solving the three point problem should be limited to make inside resection to avoid any possibility of case III arising without being noticed by the observer.

Procedure. Following steps are followed.

Let *A, B* and *C* represent the ground stations and *a, b* and *c* their locations on the sheet. Let *D* be the plane table station whose location is at *d*. (Fig. 5.19).

(*i*) Set up the table over *D*. Orient it approximately such that the locations of the given points lie towards their respective ground stations and the lines *ab, bc* become parallel to *AB* and *BC* respectively.

(*ii*) Pivoting the alidade about *a, b* and *c*, sight ground stations *A, B* and *C* in turn and draw rays. If the orientation of the table is correct, the three rays will meet at one point. If not, they will form a small triangle of error.

(*iii*) Choose a point *d′* such that its perpendicular distances from each ray is in proportion to their distances of instrument position from the respective stations. The point *d′* will lie inside the triangle of error (case I) if the instrument position is within the great triangle. The point *d′* will be outside the triangle of error (case II) if the instrument position is outside the great triangle.

(*iv*) Align the alidade along *d′ a* (assuming *A* to be farthest station) and rotate the table till *A* is sighted. Clamp the table.

(*v*) Pivoting the alidade about *a, b* and *c* in turn, sight *A, B* and *C* and draw rays. If the estimation was correct, the three rays will intersect at a point otherwise the size of the triangle of error will be further reduced. Another estimation may again be made and the steps (*iv*) and (*v*) repeated.

(*vi*) The procedure is repeated till all the three rays intersect at a point. The point of intersection gives the required location of the instrument position.

The great triangle. The triangle formed by joining the ground points *A, B, C* or their locations *a, b, c* on the sheet, is known as the *great triangle.* (Fig. 5.22).

The great circle. The circle passing through the ground points *A, B, C* or their locations *a, b, c* on the sheet, is known as the *great circle.* (Fig. 5.22).

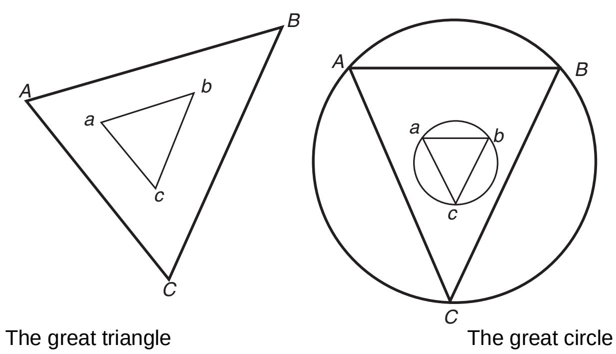


Fig. 5.22.

Lehmann's Rules. Mr. Lehmann, a well known mathematician suggested the following rules for solving a three point problem.

Rule 1. If the instrument position *P* lies inside the great triangle *ABC*, the triangle of error will also fall inside the great triangle and the point *p′* should always be chosen inside the triangle of error. Similarly, if the instrument position *P* lies outside the great triangle *ABC,* the triangle of error will also fall outside the great triangle and the point p' should be chosen outside the triangle of error. (Fig. 5.23).

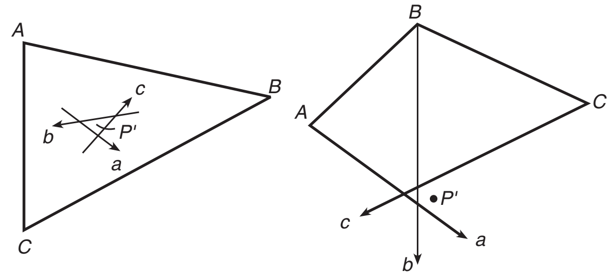


Fig. 5.23. Solution of triangles of error.

Rule 2. The point p' should be chosen such that its distances from the rays *Aa, Bb* and *Cc* are propotional to the distances of *P* from *A, B* and *C* respectively.

Rule 3. In case, the triangle of error falls outside the great triangle, point p' should be so chosen that it is on the same side of the rays *Aa, Bb, Cc. i.e.* if point p' is chosen on the right side of *Aa,* then it should also be on the right side of the other two rays.

Rule 4. When the point *P* lies outside the great circle, the point *p* is chosen on the same side of the ray drawn to the most distant point as the intersection of the other the two rays.

Rule 5. When the point *P* lies out side the great triangle *ABC* but inside the great circle *i.e.* within one of the three segments of the great circle formed by the sides of the great triangle, the ray drawn towards the middle point lies between the point p' and the intersection of the other two rays.

3. The Two Point Problem Statement. *"Finding the location of the station occupied by the table on the sheet, by means of sighting to two well defined points whose locations have previously been plotted on the sheet, is known as the Two point problem".*

Let there be two points *A* and *B* whose locations have been plotted as *a* and *b* on the sheet. Let *C* be the instrument position whose location is required. (Fig. 5.24).

Procedure. Following steps are followed :

(*i*) Choose an auxiliary point *D* such that *CD* is approximately parallel and roughly equal to *AB* by eye judgement.

(*ii*) Orient the table over the point *C* such that locations *a*, and *b* lie parallel to their ground positions *A* and *B*. This can be done by eye judgement. Clamp the table.

(*iii*) Pivoting the alidade about *a* and *b* draw rays to intersect at *c* (1st Position). The degree of accuracy of the location *c* thus obtained, depends upon the approximation that has been made

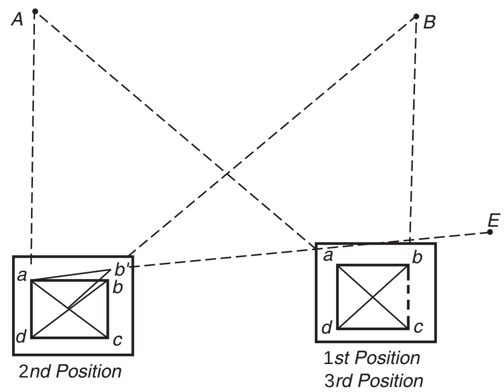


Fig. 5.24. Two point problem.

in the orientation. Transfer the point c on the ground with a U-fork and fix a wooden peg.

(*iv*) Pivoting the alidade about c, sight the station D and draw a ray. Also, draw rays at the extremities of the alidade.

(*v*) Shift the table to station D and orient it accurately with the back ray method, ensuring that the ray cd passes through a point vertically above the ground mark D. (2nd Position).

(*vi*) Pivoting the alidade about a and b, draw resectors which will intersect on the line drawn from C if the orientation was correct.

(*vii*) If not, pivoting the alidade about the point of intersection of the rays drawn from C and obtained from A, sight the station B. Draw a ray to cut cb (or cb produced) at b'.

(*viii*) Align the alidade along the line ab' and fix a point E in the line of sight, at a great distance.

(*ix*) Align the alidade along ab and rotate the table until the point E is again sighted. Clamp the table.

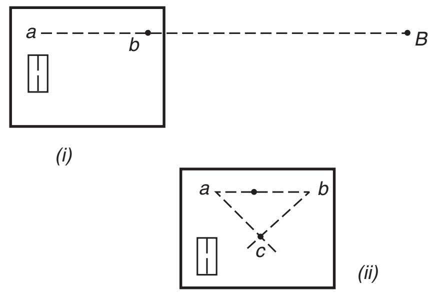
(*x*) Pivoting the alidade about a and b, draw resectors to intersect at d. Pivoting the alidade about d draw a ray towards C.

(*xi*) Shift the table to C (3rd Position). Orient it with a back ray method. Pivoting the alidade about a and b, draw rays. The rays intersect on the line drawn from D, to give the correct location of the station C.

Suitability. The accuracy of the interpolation by two point problem depends upon the selection of the station D which should lie on a line approximately parallel to AB. CD should also be equal to distance as AB. The resection by two point problem does not give much reliable result. Moreover, labour and time are wasted to set the table at two stations to achieve the orientation.

4. Orientation by Compass

Procedure. Following steps are followed. (Fig. 5.25)



(*i*) (*ii*)

Fig. 5.25. Resection by compass.

(*i*) Select a base line *AB* as in the case of intersection method. Measure it accurately and plot *a* and *b* on the sheet in a convenient position.

(*ii*) Set up the table at station *A* and centre the location *'a'* over the ground mark *A*. Level the table.

(*iii*) Place the alidade along *ab*, orient the table by turning it until the station *B* is sighted. Clamp the table.

(*iv*) Place the magnetic compass on the table and rotate the compass till magnetic needle rests in north-south direction. Draw a line along the longer side of the compass.

(*v*) Shift the table to the next instrument station *C* and level it.

(*vi*) Place the compass in its marked position and rotate the table till the magnetic needle rests in north south direction . Clamp the table.

(*vii*) Pivoting the alidade about the points *a* and *b*, draw resectors to intersect at *c* which is the required instrument position.

Suitability. This method is suitable only in areas which are not magnetically disturbed and when the scale of survey is comparatively small. The magnetic compass should be in perfect working order to achieve accurate results.

5.6. ADVANTAGES AND DISADVANTAGES OF PLANE TABLING

Advantages. Following are the advantages :

1. Less number of control points are required as extension of planimmetric control is provided by planetabling itself while the survey proceeds.
2. Depiction of irregular details and contours can be done accurately as the entire area remains in view during survey.
3. As the numerical values of angles as well as linear measurements are not observed, the errors and mistakes due to reading, recording the plotting, are eliminated.

4. As plotting is done in the field itself, chances of omissions of important measurements, are avoided.
5. The principles of intersection and resection are conveniently used to avoid computation.
6. Checking of plotted details can be done easily.
7. The amount of office work is practically reduced to nil.
8. It is less costly as compared to other methods of surveying.

Disadvantages. Following are the disadvantages.

1. The plane table and its accessories are cumbersome which are required to be carried in the field.
2. Considerable time is required for a surveyor, to gain proficiency in planetabling.
3. The time required to survey the area in the field, is comparatively more.
4. The method can only be used in open country with clear visibility.
5. The rainy season and cold wind affect the progress of survey considerably.

5.7. ERRORS IN PLANE TABLING

The various errors is plane tabling are as under :

1. Instrumental Errors. These include the following :

(a) The top surface of the plane table may not be perfectly plane. It may be warped.
(b) The fiducial edge of the alidade may not be straight.
(c) Fittings of the table and tripod may be loose.
(d) The magnetic compass may be defective.

2. Errors of Plotting. Error of plotting is a common source of inaccuracy and can only be minimised by drawing fine lines. The errors are accumulative if the survey is extended from a small base to larger area. Hence, errors in plane tabling are brought within limits if the survey is done for filling in details between control points.

3. Errors of Manipulation and Sighting. The errors which are caused due to carelessness of the surveyor, include :

(*a*) **Non-horizontality of the board.** If the table top is not horizontal in a direction perpendicular to the edges, both the vanes of the alidade will be inclined to the vertical. The point of intersection obtained by inclined rays, does not give correct location.

(*b*) **Errors of Centering.** If the point on the sheet does not lie exactly vertical over the ground mark, an error in orientation is introduced. The error due to inaccurate centering is appreciable on large scale surveys.

(c) **Defective Sighting.** The ground station should remain bisected when an eye is moved from the top to the bottom along the eye vane. If not, an error in intersection of the detail point is introduced.

(d) **Movement of the board during sighting.** Once the table is clamped for drawing rays, the movement of the table introduces an error in intersections.

4. Error due to Inaccurate Centering. To avoid unnecessary wastage of time to achieve accurate centering of the plane table at every station, the surveyor should have a proper conception of the amount of error likely to be introduced, due to inaccurate centering. (Fig. 5.26).

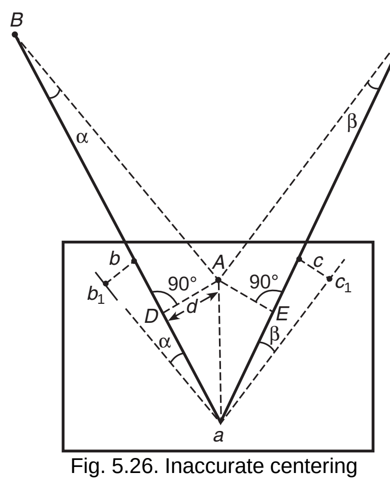


Fig. 5.26. Inaccurate centering

Let *A, B* and *C* represent the positions of the ground stations and *a, b* and *c* their plotted locations on the table. Let *A* and *a* represent the exact instrument station and plotted location on the table respectively. The angular error ($\angle BAC - \angle bac$) depends upon the distances *BA* and *CA* as well as upon the error of centering.

Let the angle $aBA = \alpha$

and angle $aCA = \beta$.

From Δs *ABa* and *ACa*

we get

Angle BAC = Angle ABa + Angle ACa + Angle BaC

or angle BAC − Angle BaC. = Angle ABa + Angle $ACa = \alpha + \beta$

Construction. Drop *AD* and *AE* perpendiculars to *aB* and *aC* respectively.

Now $$\alpha + \beta = \sin^{-1} \frac{AD}{AB} + \sin^{-1} \frac{AE}{AC}$$

In actual practice, the maximum length of the perpendiculars AD and AE does not exceed 30 cm. The extent of the resulting error for equal lengths of sight, may be judged by the following :

Distance in metres	*Angular Error*		
30	1°	09′	00′
100	0°	20′	40″
200	0°	10′	20″
1000	0°	02′	04″

But, the more significant is the positional errors of b and c. To obtain these errors, draw rays ab_1 and ac_1 parallel to AB and AC respectively. Evidently, the true position of c will be c_1 to the right of c and that of b at b_1 to the left of b.

$$\angle b_1 ab = \alpha \; ; \angle c_1 ac = \beta$$

Displacement of $C = cc_1 = ac \times \beta$

Displacement of $B = bb_1 = ab \times \alpha$

The displacements are generally too small to be plotted on the paper to affect the accuracy of the map.

Let the scale of the map be 1 cm = n metres ; $AD = AE = d$ metres.

$\therefore$ Plotted length $ab = \dfrac{AB}{n}$ cm

Plotted length $ac = \dfrac{AC}{n}$ cm

Actual displacement $bb_1 = \dfrac{AB}{n} \times \alpha$ centimeters

Actual displacement $cc_1 = \dfrac{AC}{n} \times \beta$ Centimetres

But $bb_1 = cc_1 = \dfrac{d}{n}$ cm

Assuming 0.025 cm as the limit of precision in plotting on paper, we get

$$\frac{d}{n} = 0.025$$

or $$d = \frac{n}{40}$$

i.e., d must not exceed $\dfrac{n}{40}$.

It may be noted that very accurate centering is only needed for short sights and when the scale of plotting is large. On small scale surveys having longer sights, accurate centering is not necessary. The centre of the table may be centred over the ground position. For very small scale, the table may be set up within a radius of reasonable distance. For example, the plottable error on scale 1 : 50,000 is 17 metres. Setting up the table within a distance of 5 metres does not affect the accuracy of the map.

Example 5.1. *In setting up a plane table at station X, it was noticed that the point representing ground station X on the paper is 30 cm in a direction at right angles to ray to Z (XZ). Calculate the corresponding displacement of z from its true position, assuming the following data.*

(*i*) *Scale of the map : 20,000; distance XZ = 2000 m*

(*ii*) *Scale of the map 1 : 500; distance XZ = 2000 m*

(*iii*) *Scale of the map 1 cm = 2 m; distance XZ = 20 m.*

Solution.

(*i*) Here, scale 1 : 20,000 means 1 cm = 200 m *i.e.,*

$$n = 200$$

Displacement of z from its true position.

$$= xz \times \text{error in direction}$$

Now, $$xz = \frac{\text{Distance } XZ}{n} = \frac{2000}{200} \text{ m}$$

and error in direction $= \dfrac{d}{XZ} = \dfrac{0.30}{2000}$ radian

$\therefore$ Displacement of z from its true position.

$$= \frac{2000}{200} \times \frac{0.3}{2000} = 0.0015 \text{ cm.} \quad \textbf{Ans.}$$

(*ii*) Displacement of z from its position

$$= \frac{2000}{5} \times \frac{0.3}{2000} = 0.06 \text{ cm.} \quad \textbf{Ans.}$$

(*iii*) Displacement of z from its position

$$= \frac{20}{2} \times \frac{0.3}{20} = 0.15 \text{ cm.} \quad \textbf{Ans.}$$

Example 5.2. *For the purpose of up-dating an old map on ground station P has been chosen. The old map has already three points plotted on it which are A, B and C. The distance AB is 120 m and BC = 90 m. The angle ABC = 140°. Further from the ground observation by a prismatic compass, it is seen that* $\angle APB = 45°$ and $\angle BPC = 60°$. *Plot* *the point P on the old map by graphical method and scale off the distances PA, PB and PC.*

Adopt the following scale :

1 cm = 10 m

Solution. (Fig. 5.27).

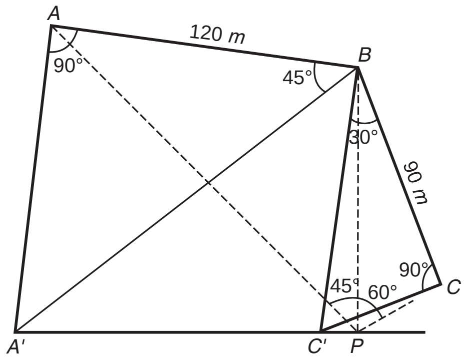


Fig. 5.27.

To solve the problem graphically, the following steps may be followed :

1. Join *AB* and *BC* on the existing map.
2. Erect perpendiculars *AA′* and *CC′* at *A′* and *C* to the sides *AB* and *BC* respectively.
3. Plot angle *ABA′* = 45° to intersect the perpendicular *AA′* meet at *A*.
4. Plot angle *CBC′* = 30° to intersect the perpendicular *CC′* at *C′*.
5. Join *A′ C′*.
6. Drop a perpendicular from *B* to *A′ C′* (or produced) to at *P*.
7. Join *PA, PB* and *PC* which are the desired distances.

PA = 260 m

PB = 106 m

PC = 54 m **Ans.**

Note :- Quadriltrals *PC′ BC*, and *PBAA′* being concyclic, angle *BPC* = angle *BC′ C* = 60^o and Angle *BPA* = Angle *BA′ A* = 45′.

EXERCISE 5

1. Pick up the correct words from brackets to complete the sentences.

(*i*) Topography is the graphical representation of the surface of the earth in a (horizontal/vertical) plane.

(*ii*) Plane tabling is the method in which field observations and plotting (do not proceed/proceed) simultaneously.

(*iii*) The principle of plane tabling is based on the fact that the lines on the sheet (do not lie/lie) parallel to their corresponding ground lines simultaneously.

(*iv*) In plane tabling, the straight lines, angles and triangles are constructed with a (protactor/divider/graphically).

(*v*) The top of a plane table is made of (iron/steel/brass/wood).

(*vi*) The length of an alidade should be equal to (length/breadth) of the plane table.

(*vii*) Spirit level is used for obtaining (centering/levelling) of the plane table.

(*viii*) The trough compass is used for obtaining (centering/levelling orientation) quickly.

(*xi*) Plumbing fork is used for accurate (levelling/centering /orientation).

(*x*) The levelling of a plane table top is done for making it truly (horizontal/vertical).

(*xi*) The levelling of a plane table is not important while working in (hilly/plain) areas.

(*xii*) Centering of a table means to bring (the centre of the table/plotted location of the instrument station) exactly over the ground mark.

(*xiii*) Accurate centering is done by a (spirit level/plumbing fork alidade).

(*xiv*) The process of orientation means setting up the table (over the ground mark/parallel to the meridian).

(*xv*) When a station is sighted and a ray is drawn through the plotted location of the instrument station towards that station, the sight is called a (resector/foresight/back sight).

(*xvi*) The process of determining the location of the instrument station by drawing resector from the locations of known stations, is called (radiation/resection /intersection).

(*xvii*) To locate the position of an inaccessible point, the method of (resection/radiation/intersection) is used.

(*xviii*) The method of graphical adjustment of a plane table traverse and a compass traverse is (same/not same).

(*xix*) When the instrument station is inside the triangle formed by joining the locations of the known points, the approximate position of the instrument location will be (inside/outside) the triangle of error.

(*xx*) The triangle formed by joining the positions of the known points or their locations on a plane table, is known as (triangle of error/great triangle).

(*xxi*) The circle passing through the positions of the known points or their locations, is known as (zero circle/great circle).

(*xxii*) To get an accurate orientation, the instrument station (should lie/should not lie) on the great circle.

(*xxiii*) Two point problem can be solved by setting the table at (one station/two stations).

(*xxiv*) In plane tabling (less/more) control is required as, compared to chain surveying.

(*xxv*) Depiction of irregular details can be done (accurately/less accurately) by plane tabling.

(*xxvi*) Measurement of angles and sides in plane table surveying are (not required/required).

(*xxvii*) The inaccurate centering of a plane table, throws the (orientation/levelling) of the table.

(*xxviii*) (One side/both sides) of an alidade may be used for accurate plane tabling.

(*xxix*) The purpose of levelling a plane table top is to make its top (parallel/perpendicular) to the vertical line.

(*xxx*) Bisection of a detail point should be checked by viewing through (central/lower/upper/all the three holes) of the sight vane of the alidade.

(*xxxi*) It is not necessary to do an accurate centering of a plane table for (large/small) scale surveys.

(*xxxii*) Plane tabling is (more/less) accurate than chain surveying.

2. Fill in the blanks with suitable word(s).

(*i*) The instruments required for plane tabling are :

(*a*) .... (*b*).......................

(*c*) .... (*d*)...................

(*ii*) The working edge of the alidade, is known as........

(*iii*) A telescopic alidade is superior to a plain alidade because the former is fitted with a telescope which facilitates sighting of distant objects also to compute horizontal, vertical or both the distances, by using............. formulae.

(*iv*) Three operations needed at every plane table station, are :

(*a*) .... (*b*)....................................

(*c*) ....

(*v*) The setting up a plane table means

(*a*) .... (*b*)..............................

(*c*) ....

(*vi*) The orientation of a plane table may be made by using

(*a*) .... (*b*)..

(*viii)* The method of intersection is sometimes called..........................

(*vii*) The methods of plane tabling are :

(*a*) .... (*b*)...............

(*c*) .... (*d*)......

(*ix*) Finding the location of the station occupied by the table, on the sheet by means of sighting to three well defined points whose locations have previously been plotted on the sheet, is known as....................................

(*x*) Three resectors drawn from three known points may form a triangle known as...............

3. State whether the following statements are true or false. If false, rewrite their corrected ones.

(*i*) The orientation of a plane table may not be necessary when the method of radiation is adopted and only one station is occupied to survey the area.

(*ii*) Quickest method for obtaining an accurate orientation is with the help of a compass.

(*iii*) It is not necessary to do exact centering of a plane table on small scale surveys.

(*iv*) Radiation method of plane tabling has a wider scope if telescopic alidade is used for determining the horizontal distances.

(*v*) In a plane table traversing intermediate checks are not possible as in the case of theodolite surveying.

(*vi*) If orientation of a plane table at each station during traversing is made with the help of a compass, the accuracy of the survey is the same as obtained by compass traverse.

(*vii*) A small and light form of table with a trough compass recessed into the board is known as a traverse plane table which is used when rapid plane table traverse is required by setting up the table at alternate stations.

(*viii)* Extension of survey by plane tabling may be made without any appreciable error on small scales.

(*ix*) To survey an area by intersection method, the surveyor needs to measure the distances between the plane table station and every detail point.

(*x*) In the absence of a measured base the scale of any graphic triangulation survey may be ascertained by including two points whose distance apart has already been previously determined trignometrically or by direct measurement.

(*xi*) If an extensive area is required to be surveyed by graphictriangulation uncontrolled by theodolite points, there is every chance of propagation of compensating error.

(*xii*) Error of centering does not affect the accuracy of the survey when the method of resection, is used for small scale work.

(*xiii*) The back ray method of orienting a table does not necessitate a ray being drawn from the preceding station to that being occupied and this does not involve the previous selection of the instrument station.

(*xiv*) In solving a three point problem by Bessel's method, any two points be used and rays drawn towards the third point.

(*xv*) Necessary elevations required for contouring in conjunction with plane tabling may be obtained by using a clinometre or telescopic alidade.

(*xvi*) Tacheometric levelling is generally preferred to where good contour delineation is required, particularly in flat country.

(*xvii*) In graphic triangulation, intersections should be obtained without actually drawing the second rays from second station, by marking the points at which the fiducial edge of the alidade cuts the appropriate first rays drawn from the first station.

(*xviii*) It is better to obtain locations of detail points from longer rays rather than shorter rays.

(*xix*) The plain alidade with open sights is much superior to the telescopic alidade in the definition of the line of sight.

(*xx*) The effect of dislevelment of the table is most marked when there is a considerable difference of level between the points sighted.

4. (*a*) Enumerate the instruments used in planetabling.

(*b*) Describe the construction and requirement of each instrument used in plane tabling with neat diagrams.

(*c*) Describe the qualities of a good plane table.

(*d*) How will you check the accuracy of a plane alidade.

5. (*a*) Narrate the working operations of plane tabling at each station and describe each briefly.

(*b*) Describe the method of orientation with a back ray.

6.(*a*) Define the following terms : (*i*) Radiation (*ii*) Intersection (*iii*) Resection.

(*b*) What is the basic difference in the above three methods.

7.(*a*) Describe the method of plane table surveying.

(*b*) Explain the method of adjustment of a planetable traverse graphically.

8. State the three point problem in plane table surveying and describe how it is solved by Bessel's method.

9. (*a*) State three point problem in plane table surveying and describe its solution by trial and error method.

(*b*) What are the Lehman's rules which are followed in estimating the position of the point sought.

10. (*a*) Discuss the advantages and disadvantages of plane table surveying over other methods of surveying.

(*b*) What is three point problem and how it is solved by the tracing paper method.

11. (*a*) Describe two methods of orienting a plane table.

(*b*) State the various sources of error in plane table surveying.

12. A plane table survey is to be plotted to a scale of 1 : 5000. How much error can be tolerated in the exact centering of the table over the ground station if the surveyor can plot to no finer than 0.25 mm ?

13. P and Q the two corner points of a field are plotted on the plan. Further survey is to commence from the third corner R of the said field with a plane table. Both P and Q are visible from R. Describe in detail with sketches as to how you would locate R on the plan with reference to the plotted positions of P and Q.

14. (*i*) Explain with neat sketches, 'failure of fix', 'bad fix' and 'good fix' in plane table surveying, particularly in a three point problem.

(*ii*) Cite 'two point and three point problems in plane table surveying and describe any one method of solving a three point problem.

15. (*a*) Enunciate (*i*) two point and (*ii*) three point problems in plane table surveying.

(*b*) Give Bessel's graphical method to solve 'three point' problem.

(*c*) Discuss the nature and extent of the error caused by inaccurate centering of plane table.

16. (*a*) What is meant by two-point problem ? Explain by sketches how you would it in the field. (No compass is available)

(*b*) When does the three problem fail ? Give reason .

(*c*) The reduced level of a plane table station is 60 metres and the height of the alidade above the ground is 1.5 m. find the reduced level of a station A, when the reading upon the Indian clinometre scale is +0.03. The distance to A scaled from the plan is 3000 m and the vane 3 m above the ground at A, was sighted.

17. Explain the procedure for solving two point problem in plane table survey.

18. You are required to conduct a plane table survey of your college campus. Explain stepwise how you would proceed to conduct the survey to prepare a dimensional plan.

19. Enumerate the various sources of error in plane tabling. How will you guard against them ?

20. A plane table survey is to be carried out on scale 1 : 5000. Show that at this scale, accurate centering of the plane table over the survey station is not necessary. What error would be caused in position on the map if the map point is 45.7 cm out of vertical through the station point ?

21. (*a*) Enumerate the different methods of plane tabling and highlight the typographical conditions under which each one is preferred to.

(*b*) Explain, with neat sketches, any one method of plane tabling for locating details.

Answers

1. (*i*) horizontal (*ii*) proceed (*iii*) lie (*iv*) graphically (*v*) wood (*vi*) length (*vii*) levelling (*viii*) orientation (*xi*) centering (*x*) horizontal (*xi*) plain (*xii*) plotted location of the instrument station (*xiii*) plumbing fork (*xiv*) parallel to meridian (*xv*) fore sight (*xvi*) resection (*xvii*) intersection (*xviii*) same (*xix*) inside (*xx*) great triangle (*xxi*) great circle (*xxii*) should not lie (*xxiii*) two stations (*xxiv*) less (*xxv*) less accurately (*xxvi*) are not required (*xxvii*) orientation (*xxviii*) one side (*xxix*) perpendicular (*xxx*) all the three holes (*xxxi*) small (*xxxii*) less.

2. (*i*) (*a*) plane table with stand (*b*) An alidade. (*c*) plumbing fork, (*d*) spirit level (*e*) compass.

(*ii*) Fiducial edge

(*iii*) Tacheometric

(*iv*) Centering, levelling, orientation

(*v*) Levelling, centering and orientation

(*iv*) Magnetic compass, back ray

(*vii*) Graphical triangulation

(*viii*) Three point problem

(*ix*) Triangle of error.

3. (*i*) True, (*ii*) False, (*iii*) True, (*iv*) True, (*v*) False, (*vi*) True, (*vii*) True, (*viii*) False, (*ix*) False, (*x*) True, (*xi*) True, (*xii*) True, (*xiii*) False, (*xiv*) True, (*xv*) True, (*xvi*) True (*xvii*) True (*xviii*) False, (*xix*) False (*xx*) True.

12. 1.25 m

16. 148.0 m

17. 0.0914 cm.

6

Levelling

6.1 INTRODUCTION

The art of determining relative altitudes of points on the surface of the earth or beneath the surface of the earth, is called *levelling*. This branch of surveying deals with measurements in vertical planes.

For the execution of engineering projects, such as railways, highways, canals, dams, water supply and sanitary schemes, it is very necessary to determine elevations of different points along the alignments of the proposed projects. Success of such projects, depends upon accurate determination of elevations. Levelling is employed to provide an accurate network of heights, covering the entire area of the project. Levelling is of prime importance to the engineers, both in acquiring necessary data for the design of the project and also during its execution.

6.2. THE LEVEL

The instrument which is used for levelling, is known as *a level*. It consists essentially of the following parts :

(*i*) A telescope to provide a line of sight.

(*ii*) A level tube to make the line of sight horizontal.

(*iii*) A levelling head to bring the bubble of the tube level at the centre of its run.

A tripod to support the above three parts of the level.

1. Telescope. It is an optical instrument used for magnifying and viewing the images of distant objects. The telescopes which are fitted in levels, are generally of two types :

1. External focussing telescope
2. Internal focussing telescope.

The external focussing telescopes were generally used in olden type levels whereas internal focussing telescopes, are being used in modern survey instruments.

A surveying telescope is similar to Keplar's telescope. It consists of two convex lenses fitted in a tube. The lens fitted near the eye, is called the *eye piece* and the other fitted at the end nearer to the object, is called *the objective.*

Principle of a Telescope. (Fig. 6.1). The principle of the surveying telescope, is based on the following optical phenomenon.

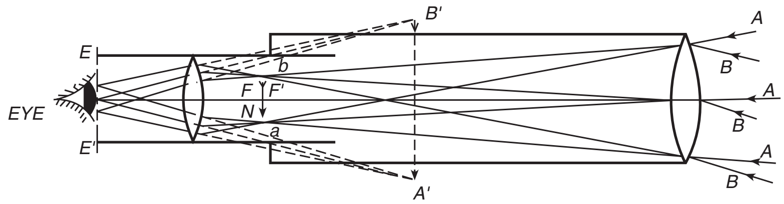


Fig. 6.1. An optical diagram of a telescope.

"All parallel rays of light incident on a convex lens, get bent due to refraction and they leave the lens in such a manner that they intersect at a common point, generally known as the focus and all other rays passing through the optical centre of the lens, leave the lens without bending".

The objective provides a real inverted image in front of the eye piece at a distance lesser than its focal length. The eye piece in turn produces a magnified and vertical image of the object on the same side of the eye piece.

Cross Hairs of the Eye Piece. The cross hairs are placed in front of the eye piece within its focal distance, where an inverted image of the object is produced by the objective. The cross hairs also get magnified by the eye piece along with the inverted image of the object.

Relationship between the Distances of the Object and Image, and Focal Length of the Objective.

If u = distance of the object from the optical centre of the objective.

v = distance of the image from the optical centre of the objective.

f = focal length of the objective

then $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

As the focal length of the objective remains constant, the distance of the image of an object, depends upon the distance of the object from the objective.

The following two essential conditions are involved :

(*i*) The real image of the object, must be formed in front of the eye-piece in its focal distance.

(*ii*) The plane of the image must coincide with that of the cross hairs.

Focussing of telescope. The operation of obtaining a clear image of the object in the plane of cross hairs, is known as *focussing*.

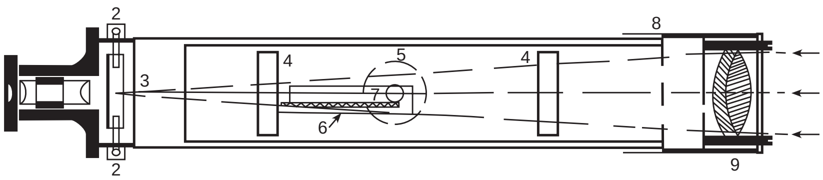
Focussing of a telescope is achieved in two steps :

(1) Focussing the eye piece. In this operation, the cross hairs are made to appear clearly visible, with the help of the eye piece unit which may be moved in or out. By doing so the cross hairs are brought in the plane of distant vision. The focussing of the eye piece, depends on the observer's eye sight.

Focussing the objective. In this operation, the image of the object is brought in the plane of the cross hairs which are clearly visible. The focussing of a telescope can be done externally or internally. Accordingly the telescopes are also classified as external or internal focussing telescopes.

External Focussing Telescope. The telescope in which focussing is achieved by the external movement of either objective or eye-piece, is known as *an external focussing telescope.*

In an external focussing telescope, the body is formed by two tubes at the ends of each, objective and eye piece are fitted. One of the tubes is made to slide axially within the other by means of a rack and pinion arrangement attached to the focussing screw of the telescope. (Fig. 6.2).

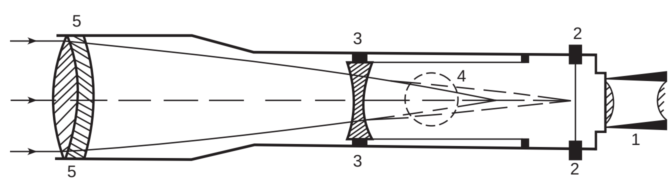


1. *Eye piece* 2. *Diaphragm screws* 3. *Diaphragm*
4. *Stops* 5. *Focussing screws* 6. *Rack*
7. *Pinion* 8. *Ray shade* 9. *Objectives glass*

Fig. 6.2. An external focussing telescope.

Internal Focussing Telescope. The telescope in which focussing is achieved internally with a concave lens, is known as *internal focussing telescope.*

In an internal focussing telescope, the objective and eye piece are kept at a fixed distance, and focussing is achieved by a double concave lens mounted in a short tube capable of sliding axially to and from



1. *Eye piece* 2. *Diaphragm screws* 3. *Focussing lens*
4. *Focussing screws* 5. *Object glass*

Fig. 6.3. An internal focussing telescope.

between the eye piece and the objective with a rack and pinion arrangement attached to the focussing screw. (Fig. 6.3).

Parallax. When the image of an object formed by the objective does not lie in the plane of the cross hairs, any movement of the eye causes an apparent movement of the image with respect to cross hairs. This shift of the image, is called 'parallax'.

Objective (Fig. 6.4). An objective is a lens on which rays from an object are incident. It is invariably a compound lens consisting of :

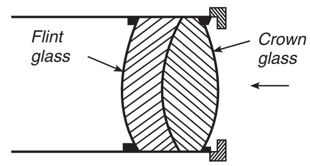


Fig. 6.4. An objective.

(*i*) The front double convex lens made of crown glass.

(*ii*) The back convex lens made of flint glass.

The two lenses when commented together with balsam at their common surface, are generally known as *achromatic lens*. In such lenses, the spherical and chromatic aberations known as optical serious defects, are practically eliminated.

Eye-piece. It is an assembly through which image of the object formed by the objective and magnified by itself, is viewed. It consists of two plano-convex lenses of equal focal length placed in a small tube such that their spherical surfaces face each other and are separated by a distance equal to the focal length of either (Fig. 6.5).

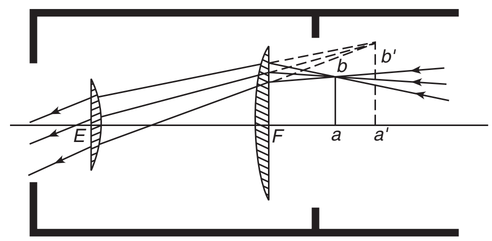


Fig. 6.5. An eye-piece.

Ramsden's eye-piece is used in most of the surveying instruments. It is also sometimes called as *positive* or *non-erecting eye-piece* as it does not change the inverted image formed by the objective. This eye-piece does not satisfy the conditions of minimum spherical aberration but the curvature of the lenses are arranged so as to remedy the defects as far as possible.

Let E and F represent the eye lens and field lens respectively of a Ramsden eye-piece. Rays coming from the objective, converge to a focus at b in front of field lens F and after passing through E the rays seem to diverge from b'. In order that the ray may emerge parallel from E, the point b' should be in the focus of that lens. In other words, the distance Ea' should be equal to $-f$, where f is the focal length of either lens. Here, the distances measured against the direction of rays, are assumed negative.

We know $$EF = -\frac{2}{3}f$$

$\therefore$ $$Fa' = -f - \left(\frac{2}{3}f\right) = -\frac{f}{3}$$

To determine the position of ab, apply the lens equation,

i.e. $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{Fa'} + \frac{1}{Fa'} = \frac{1}{f}$$

or $$-\frac{3}{f} - \frac{1}{Fa} = \frac{1}{f}$$

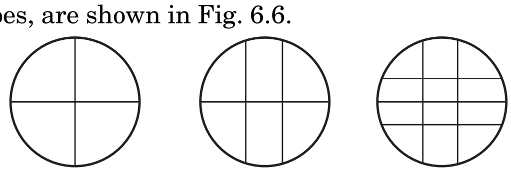
or $$\frac{1}{Fa} = -\frac{3}{f} - \frac{1}{f} = -\frac{4}{f}$$

or $$Fa = -\frac{f}{4} \quad \text{...(6.1)}$$

Thus, the real image formed by the objective should fall in front of field lens at a distance from it numerically equal to one fourth of its focal length. The is the correct position for the frame carrying the cross hairs.

Diaphragm. A frame carrying cross hairs usually made of either silk threads, spider threads or platinum wires and placed at the plane at which vertical image of the object is formed by the objective, is known as a *diphragm*.

A few typical arrangements of cross hairs used in diaphragms of level telescopes, are shown in Fig. 6.6.



(*a*) (*b*) (*c*)

Fig. 6.6. Types of Diaghragms.

The vertical hair of the diaphragm enables the surveyor to check the verticality of the levelling staff along sideways whereas the horizontal hairs are used to read the staff graduations. With the diaphragms shown in Fig 6.6 (*a*) and 6.6. (*b*) only one reading of the staff is possible but with the third one [Fig. 6.6 (*c*)] three readings of a staff for each setting of the instrument are possible. The mean of the three readings should generally agree with the middle wire reading. When the telescope is fitted with more than one horizontal wire called *stadia hairs,* the horizontal distance between the staff position and the level may be calculated by stadia tacheometer. For details refer to Chapter 13 'Tacheometric Surveying'.

2. Level Tube. A level tube also known as *bubble tube*, consists of a glass tube placed in a brass tube which is sealed with plaster of Paris. The whole of the interior surface or the upper half is accurately ground so that its longitudinal section, is an arc of a circle. The level tube is nearly filled with either ether or alcohol or a mixture of both. The remaining space is occupied by an air bubble. The centre of the air bubble always rests at the highest point of the tube. (Fig. 6.7).

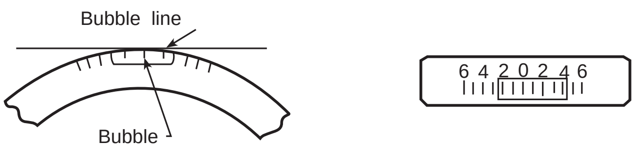


Fig. 6.7. A level tube.

The outer surface of the level tube is graduated in both the directions from the centre. The exact centering of the bubble can be ascertained by observing the number of divisions of its ends from the centre. *The line tangential to the circular arc at its highest point, i.e., the middle of the tube or the zero of the graduations, is called the axis of the level tube.* When the bubble is central, the axis of the bubble becomes horizontal. The length of the bubble changes with a change in temperature. With a rise in temperature the liquid expands and thus the bubble shortens and consequently, its sensitiveness is reduced.

The level tube is attached on the top of the telescope by means of capstan-headed nuts.

3. Levelling Head. The levelling head generally consists of two parallel plates with three or four foot screws. The upper plate is known as *tribarch* and the lower plate as *trivet* which can be screwed on to the tripod. A levelling head has to perform the following three distinct functions.

(*i*) To support the telescope.

(*ii*) To attach the level to the tripod.

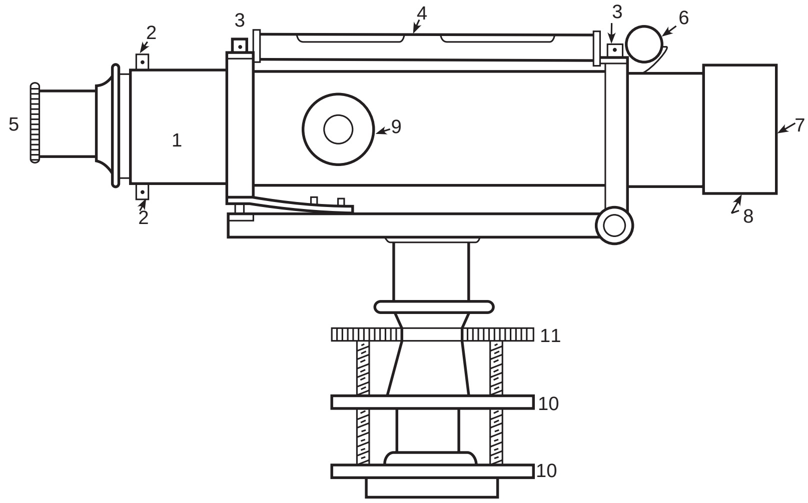
(*iii*) To provide a means for level.

4. Tripod. When in use, the level is supported on a tripod which consists of three solid or framed legs. At the lower ends, the legs are provided with pointed iron shoes. The tripod head carries at its upper surface, an external screw to which the foot-plate (trivet) of the levelling head can be screwed.

6.3. TYPES OF LEVELS

According to the construction, the levels may be classified and discussed as under :

1. The Dumpy Level. The dumpy level designed by Gravatt, consists of a telescope rigidly fixed to its support. It can neither be rotated about its longitudinal axis, nor it can be removed from its supports. A long bubble tube is attached to the top of the telescope. (Fig. 6.8).



1. *Telescope.*
2. *Diaphragm adjusting screws.*
3. *Bubble tube adjusting screws.*
4. *Longitudinal bubble*
6. *Transverse bubble tube*
7. *Objective end.*
8. *Ray shade.*
9. *Focussing screw.*
10. *Levelling head.*
11. *Foot screw.*

Fig. 6.8. A Dumpy level

The fact that a dumpy level in its original design, was comparatively shorter than a Wye Level of the same magnifying power, originated the name dumpy level. Dumpy literally means short and thick. Its levelling head generally consists of two parallel plates with either three or four foot screws. The upper plate is known as *tribarch* and the lower plate known as *trivet* is screwed on to the tripod before setting up.

2. The Wye Level. A wye level consists of a telescope held in two vertical wye supports. The name 'Y-Level' is originated from the fact that the telescope is supported in Y supports and is not rigidly fixed to the supports. The Y-supports are two curved clips which may be raised to enable the telescope to be rotated about its longitudinal axis. It may be removed and turned end to end. (Fig. 6.9).

When the clips are tightened, the telescope is held in supports rigidly. A level tube is attached to the stage carrying the wyes. In some levels, the level tube of reversible type, is attached on the telescope as in the case of a dumpy level. The levelling head is similar to that of a dumpy level. In some levels, a clamp and fine motion tangent screw, are also provided to control the movement in the horizontal plane.

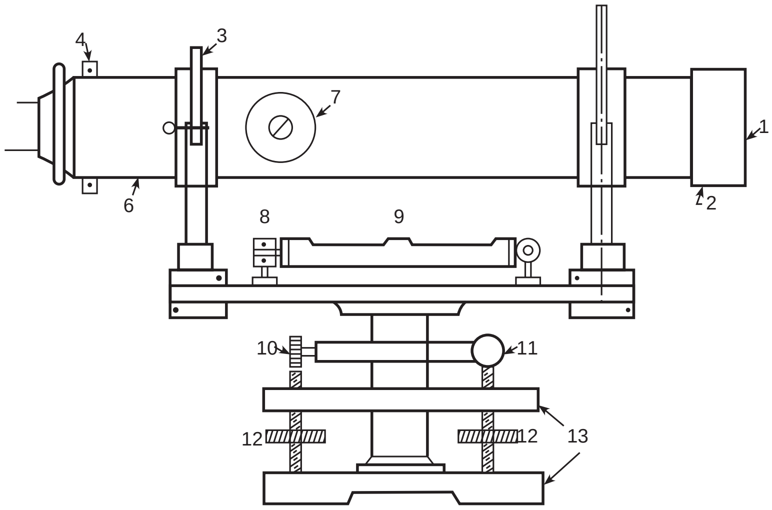


Fig. 6.9. A Wye Level.

3. The reversible level. In a reversible level, the features of both dumpy and wye levels are combined so that the advantages of each could be utilised. The telescope is similar to that of a wye level and is supported by two rigid sockets into which it is introduced from either end and fastened by screws. The sockets are rigidly connected to the cage which is fixed to the spindle. Once, the telescope is introduced into sockets and screws tightened, it acts as a dumpy level.

For testing and adjusting the level, the screws of the sockets are loosened, the telescope is removed and reversed end to end.

4. The tilting level. It consists of a telescope attached with a level tube which can be tilted within few degrees in vertical plane by a tilting screw. The line of collimation and vertical axis of a tilting level need not be at right angles as required in the case of a dumpy level or wye level. The tilting level is designed for precise work. (Fig. 6.10).

How to use a tilting level. The level is approximately travelled with the help of levelling screws with reference to a circular level provided forthe purpose. The staff is sighted and focussed properly. Before reading the staff, the bubble of the main level tube is centered accurately by means of the tilting screw. The movement of the telescope in vertical plane, enables the centering of the bubble. The fact that the axis of the level tube is not generally perpendicular to the vertical axis, demands that the bubble of the main level tube is accurately centered before taking each observation. It is however very essential that the observer should have the view of the bubble tube while sighting the staff.

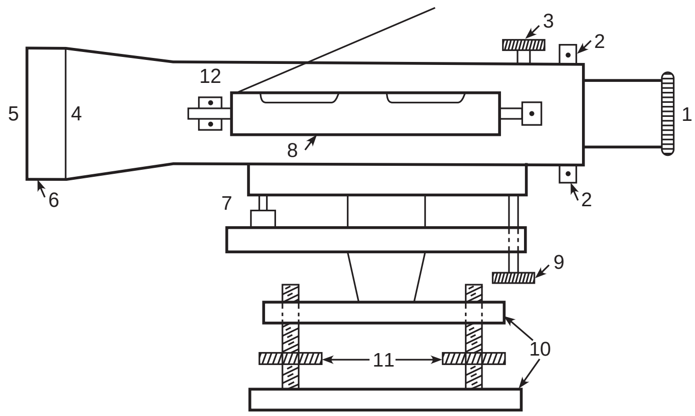


Fig. 6.10. A Tilting level.

6.4. ADVANTAGES AND DISADVANTAGES OF DIFFERENT TYPES OF LEVELS

1. Dumpy level. These include :

(*i*) It is simple in construction with a few movable parts.

(*ii*) It requires fewer permanent adjustments.

(*iii*) Adjustments once carried out remain for a longer period.

2. Wye level. These include :

(*i*) The adjustments are tested with greater rapidity and ease.

(*ii*) Adjustments need be checked so often as they get disturbed due to large number of movable parts.

3. Tilting level. These include :

(*i*) The exact centering of the level tube, is possible at the time of observation by the tilting screw.

(*ii*) It saves time in levelling the instrument as only rough levelling is required. Accurate centering of the bubble is done with the help of tilting screw before taking each reading.

6.5. LEVELLING STAFF

A straight, rectangular wooden rod graduated into metres/feet and further smaller divisions, is called *a levelling staff*. The bottom of the levelling staff represents the zero reading. The reading given by the line of sight on a levelling staff held vertically is the height of the line of collimation above the point on which staff is held. The levelling staves may be divided into two classes :

1. Self reading staff.
2. Target staff.

(a) Self reading staff. A staff on which readings are directly read by the observer through the telescope, is known as *self-reading staff*. Self reading staffs are of three types as discussed below :

(b) Solid staff. These are usually 3 m long in one length. Due to the absence of a hinge or socket on these staffs, greater accuracy in reading is achievable but on the other hand it is inconvenient to carry them in the field. Use of a solid staff is generally restricted to only precise levelling work.

(c) Folding or hinged staff (Fig. 6.11). A folding staff is made of well-seasoned timber. It is 4 m long and consists of two portions, each being 2 m hinged together. The width and thickness of the staff, is kept 75 mm and 18 mm respectively.

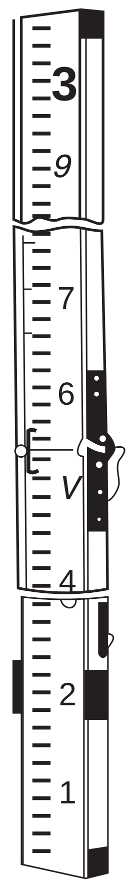


Fig. 6.11.

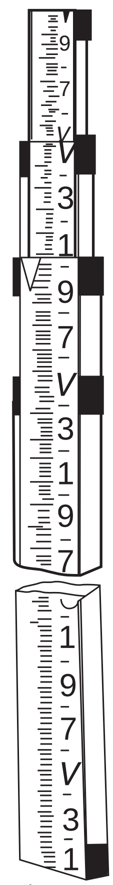


Fig. 6.12.

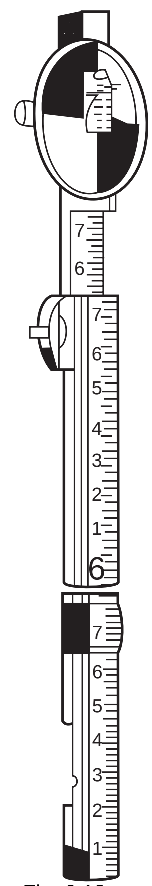


Fig. 6.13.

The foot of the staff is provided with a brass cap to avoid wear and tear due to usage. Sometimes, a plummet is also provided to test the verticality of the staff by the staffman.

Each metre length is sub-divided into decimetres and each decimetre is further divided into 20 equal divisions of 5 mm width. Decimetre numerals 1 to 9 of each metre length, are marked in black and metre numerals in red. The graduations are marked inverted so that they appear erect when viewed through the telescope. In modern lavelling staff, graduations are marked erect.

The staff may be folded together so that :

(*i*) One 2 m piece is capable of folding on the other when not in use.

(*ii*) Two pieces are detachable from one another so that one half may be used while working in plain areas.

(*iii*) When the two portions are locked together, the entire length behaves as a rigid rod.

3. Telescopic or Sopwith type staff (Fig. 6.12). It consists of three pieces. Top piece is solid 1.25 m long whereas central piece 1.25 m and lower piece 1.5 m are hollow. The top portion slides into the central portion telescopically. When fully extended, total length of the staff is 4 m. The upper two pieces are held by brass spring catches.

The smallest division of this type of levelling staff is also 5 mm. The metre numerals which are shown on the left are marked in red. The decimetre numerals 1 to 9 are shown on the right and marked in black. The decimetre numeral 10 of each metre length is omitted and letter *M* is marked to indicate the end of the metre length. Graduations are marked erect and when viewed through the telescope these appear inverted. While using a telescopic staff it may be ensured that the three parts are fully extended in length when using the full length *i.e.* 4 m.

4. Target staff (Fig. 6.13). It consists of two ordinary rods, the upper rod 6 ft. in length slides into lower one which is 7 ft. in length. A target which can be moved up and down is attached to the staff. The rod is graduated in feet, and its tenths and hundreds. For taking readings the level man directs the staff man to raise or lower the target till it is bisected by the line of sight. The staff man clamps the target and takes the reading.

6.6. RELATIVE MERITS OF SELF READING STAFF AND TARGET STAFF

1. It is easier and quicker to read a self reading staff than a target staff.
2. It is tedious to adjust the target so that the line of sight bisects it accurately.
3. The readings on a self reading staff are taken by the leveller himself but in case of a target staff, the staff man is responsible for noting down the readings.

4. For holding a target staff, an experienced staff man is required whereas in the case of a self reading staff even an inexperienced staff man may be employed.
5. Though the fineness of reading is greater in the case of a target staff than with a self-reading staff, the accuracy of reading a target staff much depends upon the accuracy with which the bisection of the target by the line of sight, is estimated.

Generally a self-reading staff is preferred to a target staff so that the instrument man is made responsible for the accuracy of the whole work.

6.7. TECHNICAL TERMS USED IN LEVELLING

1. Level surface. The surface which is parallel to the mean spheroidal surface of the earth, is known as *level surface*. Every point on this surface is equidistant from the centre of the earth. It is also normal to the plumb line at every point. The surface of still water in a lake represents a level surface.

2. Level line. A line lying on the level surface, is known as a *level line*. Every point of a level line, is equidistant from the centre of the earth. The cross section of still water of a lake, represents a level line.

3. Horizontal surface. A surface tangential to the level surface at any point, is known as a *horizontal surface*. It is perpendicular to the plumb line at the tangent point.

4. Horizontal line. A line lying on the horizontal surface, is known as a *horizontal line*. It is a straight line tangential to the level line.

5. Vertical line. A line perpendicular to the level line is called a *vertical line*. The plumb line at any place, is called the *vertical line*.

6. Vertical plane. The plane which contains the vertical line at a place is, called a *vertical plane*.

7. Vertical angle. The angle between an inclined line and a horizontal line at a place, in vertical plane, is called *vertical angle*.

8. Detum Surface. The imaginary level surface with reference to which vertical distances of the points (above or below) are measured, is called *Datum surface*.

9. Mean sea level datum. The mean sea level datum obtained by making hourly observations of the tides at any place over a period of 19 years, is known as *Mean Sea level*. The M.S.L. datum adopted by the Survey of India for determining the elevations of different points in India is that of Mumbai.

10. Reduced Level (R.L.) The height or depth of a point above or below the assumed datum, is called *reduced level* (*R.L.*). It is also known as elevation of the point. Elevations of the points below the datum surface, are known as *negative elevations*.

11. Line of sight. The line passing through the optical centre of the objective, traversing the eye-piece and entering the eye, is known as a *line of sight*.

12. Line of collimation. The line passing through the optical centre of the objective and the point of intersection of the cross hairs stretched in front of the eye piece and its continuation, is called *line of collimation.*

13. Plane of collination. The horizontal plane in which the telescope of a duly adjusted and corrected level is rotated about its vertical axis, is known as *plane of collimation.*

14. Optical centre of a lens. The point in a lens through which rays pass without any lateral displacement, is called *optical centre*. It is so situated in the lens that its distances from the curved surfaces, are directly proportional to their radii.

15. Axis of the telescope. The line joining the optical centre of the objective and the centre of the eye piece, is called *axis of the telescope*.

16. Bench mark. (B.M.) A relatively permanent and fixed reference point of known elevation above the assumed datum, is called a *bench mark*.

6.8. PRINCIPLE OF LEVELLING

The principle of levelling lies in furnishing a horizontal line of sight and finding the vertical distances of the points above or below the line of sight. The line of sight is provided with a level and a graduated levelling staff is used for measuring the height of the line of sight above the staff positions.

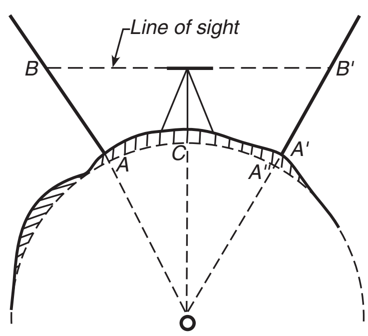


Fig. 6.14.

In Fig. 6.14 let O represents the centre of the earth. A and A' are the points whose difference in elevation is required. C is the position of the level. The line CO is the direction of plumb line. BB' represents the line of sight which is perpendicular to CO. AB and $A'B'$ are the readings on a staff vertically held at A and A' respectively.

$$OA + AB = OA'' + A''A' + A'B'$$

or $$AB - A'B' = A''A'' = \delta h \qquad (\because OA = OA'')$$

i.e. δh = the difference of the staff readings.

It may be noted that the distances of both the staff positions from the instrument station have been kept equal to cancel the effect of the curvature of the earth surface, discussed later.

6.9. SPECIAL TERMS AND THEIR ABBREVIATIONS USED IN LEVELLING

1. Instrument station. The point where instrument is set up for observations, is called *instrument station*.

2. Station. The point where levelling staff is held, is called *station*. It is the point whose elevation is to be determined or the point that is to be established at a given elevation.

3. Height of instrument. (H.I.) The elevation of the line of sight with respect to the assumed datum, is known as *height of instrument*. In levelling it does not mean the height of the telescope above the ground level where the level is set up.

4. Back sight. (B.S.) The first sight taken on a levelling staff held at point of known elevation, is called *back sight*. It ascertains the amount by which the line of sight is above or below the elevation of the point. Back sight enables the surveyor to obtain the height of the instrument.

5. Fore sight. (F.S.) The sight taken on a levelling staff held at a point of unknown elevation to ascertain the amount by which the point is above or below the line of sight, is called a *fore sight*. Fore sight enables the surveyor to obtain the elevation of the point. It is also generally known as *minus sight* as the foresight reading is always subtracted from the height of the instrument (except when the staff is held inverted) to obtain the elevation.

6. Change point. The point on which both the fore sight and back sight, are taken during the operation of levelling, is called a *change point*. Two sights are taken from two different instrument stations, a fore sight to ascertain the elevation of the point while a back sight is taken on the same point to establish the height of the instrument of the new setting of the level. The change point is always selected on a relatively permanent point.

7. Intermediate sight. The fore sight taken on a levelling staff held at a point between two turning points, to determine the elevation of that point, is known as *intermediate sight*. It may be noted that for one setting of a level, there will be only a back sight and a fore sight but there can be a number of intermediate sights.

6.10. ADJUSTMENT OF A LEVEL

A level needs two types of adjustments *i.e*.

1. Temporary adjustments.

2. Permanent adjustment.

6.11. TEMPORARY ADJUSTMENTS

The adjustments which are made for every setting of a level, are called *Temporary Adjustments*. These include:

1. Setting up the level. 2. Levelling up.
3. Elimination of parallax.

1. Setting up the Level. This operation includes fixing the instrument on the tripod and also levelling the instrument approximately by leg adjustment. To achieve this, release the clamp, hold the instrument in the right hand and fix it on the tripod by turning round the levelling head with the left hand. The tripod legs are so adjusted that the telescope is at a convenient height and the tribarch is approximately levelled. Modern levels are provided with a small circular bubble on the tribarch for achieving approximate levelling of the instrument.

2. Leveling up. After setting up the level, accurate levelling is done with the help of foot screws, and by using plate levels. The object of levelling up the instrument is to make its vertical axis truly vertical. Depending upon the number of foot screws provided in the levelling head, there are two methods of levelling.

Method 1. Levelling with a Three screw head: (Fig. 6.15).

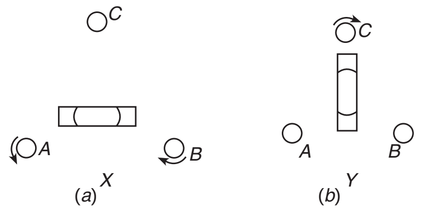


Fig. 6.15. Levelling up with three screws.

The following steps are followed :

1. Loosen the clamp and turn the instrument until the longitudinal axis of the plate level is parallel to a line joining any two levelling screws. Such as *A* and *B* in Fig. 6.15 (*a*).

2. Holding these two foot screws with the thumb and first finger of each hand, turn them uniformly so that the thumbs move either towards each other or away from each other until the plate bubble is central. The bubble moves in the same direction as left thumb.

3. Rotate the upper plate through 90°, *i.e.*, until the axis of the plate level coincides a line joining the third foot screw *C* and the mid point of the first two screws *A* and *B* Fig. 6.14 (*b*).

4. Hold the third screw with the thumb and first finger of the right hand and turn it until the plate bubble is central.

5. Rotate the upper plate through 90° to its original position [Fig. 6.15 (*a*)] and repeat step 2 till the bubble is central.

6. Rotate again through 90° and repeat step (4).

7. Repeat the steps (2) and (4) till the bubble remains central in both the positions.

8. Rotate the instrument through 180°. The bubble should remain central if the instrument is in adjustment. The vertical axis of the level will now be truly vertical. If not, the instrument needs permanent adjustment. For permanent adjustments, refer to article 6.33 of this chapter.

Note. It is very essential to keep the same quarter of the circle for changing directions and not to swing through the remaining three quarters of the circle for repeating steps 2 and 4.

Method 2. Levelling with a Four screw head (Fig. 6.16).

Fig. 6.16. Levelling up with four foot screws.

The following steps are involved :

1. Loosen the clamp and turn the upper plate till the longitudinal axis of the plate level, is approximately parallel to the line joining any two diagonally opposite foot screws such as *B* and *D* Fig. 6.16 (*a*).

2. Bring the bubble central exactly in the same manner as described in step (2) of levelling up with three foot screws.

3. Turn the upper plate through 90° until the plate level axis is parallel to the other two diagonally opposite screws *i.e. A* and *C* Fig. 6.16 (*b*).

4. Make the bubble central as explained earlier.

5. Repeat the above steps till the bubble remains central in both the positions.

6. Turn the instrument through 180° to check the adjustment in the same way as in the case of three screw method.

Note. The following points may be noted.

(*i*) In case the instrument is provided with two plate levels placed at right angles to each other, the step (4) is not necessary.

(*ii*) If the tribarch is not approximately levelled, jamming of a pair of screws may occur.

(*iii*) Screws should be left firmly upon the lower parallel plate so that there is no tendency to rock.

(*iv*) Never force the screw to achieve levelling.

(*v*) As far as possible, the foot screws should be at the centre of their run.

3. Elimination of parallax. If the image formed by the objective does not lie in the plane of the cross hairs, there will be a shift in the image due to shift of the eye. Such a displacement of the image is termed as *parallax*. For an accurate sighting, the parallax is eliminated in two steps : (*i*) focussing the eye-piece for distinct vision of the cross hairs (*ii*) focussing the objective so that the image is formed in the plane of cross hairs.

1. Focussing the eye-piece. Following steps are involved :

(*i*) Direct the telescope either towards the sky or hold a sheet of white paper in front of the objective.

(*ii*) Move the eye piece in or out till the cross hairs appear distinct.

In some levels, the eye-piece is graduated and numbered. Once the eye-piece is focussed, the observer may note this position to save much of his time at other settings.

2. Focussing the objective. Following steps are involved :

(*i*) Direct the telescope towards the levelling staff.

(*ii*) Turn the focussing screw till the image appears clear and sharp.

(*iii*) The image so formed must be in the plane of cross hairs.

6.12. TEMPORARY ADJUSTMENTS OF A TILTING LEVEL

Following steps are involved :

1. Set up the level on firm ground.
2. Bring the bubble of the circular level at its centre by adjusting the legs, so that the vertical axis is brought approximately vertical.
3. Set the micrometer screw (or tilting screw) to read zero so that two halves of the main bubble appear in the prism.
4. Direct the telescope towards the staff and focuss it.
5. Turn the micrometer screw until the end of the bubble coincides accurately with its index line to achieve exact centering of the bubble. In modern sophisticated levels, the two halves of the bubble appear in the prism. When the bubble is central, the two halves appear coincident to form a complete bubble.

Note. The following points may be noted :

(*i*)

The instrument is approximately levelled when the end of the main bubble is brought in view of the prism and the micrometer reads zero.

(*ii*) Final levelling *i.e.*, centering the main bubble by means of micrometer screw is made before every staff reading is taken.

6.13. BENCH MARKS

Depending upon the permanency and precision, bench, marks may be divided into the following types.

1. G.T.S Bench Marks
2. Permanent Bench Marks
3. Arbitrary Bench Marks
4. Temporary Bench Marks

1. G.T.S. Bench Marks. These bench marks are established by the Survey of India with greatest precision at an interval of about 100 km. all over the country. Their elevations refer to the mean sea level datum obtained by hourly observations of the tides over a period of 19 years, at Mumbai port. G.T.S. bench marks falling in the belts of the area bounded by 1° Latitude and 1° Longitude, are published in levelling pamphlets. These are also depicted on topo sheets published by the Survey of India and their elevations correct to two places of decimal of a metre, are entered.

2. Permanent Bench Marks. These bench marks are established between G.T.S. bench marks by the Survey of India or other government agencies such as P.W.D., on clearly defined and permanent natural or cultural detail points such as isolated rocks culverts, kilometre stones, railway platforms, gate pillars of inspections houses, etc. The permanent bench marks established by the Survey of India contain the inscripts. $\overset{\text{G.T.S.}}{\underset{\text{B.M.}}{\text{O}}}$ Their elevations are also published in the levelling pamphlet of the area. P.W.D. bench marks are marked on a plane surface by a rectangle. Below or above the rectangle, the letters B.M. along with R.L. of the bench mark are also cut and filled in Japan black. Such Bench Marks are used for reference and checking purpose. For irrigation projects, G.T.S. or other permanent bench marks are referred to, to decide the required slope of the bed of canals so that water flows freely under gravity.

3. Arbitrary Bench Marks. These are the reference points whose elevations are arbitrarily assumed for small levelling operations. Their elevations do not refer to any fixed datum as in the case of G.T.S. or permanent bench marks.

4. Temporary Bench Marks. These are the reference points on which a day's work is closed and from where levelling is continued next day in the absence of a permanent B.M. Their elevations are referred to as the reduced levels. Such bench marks should be carefully established on permanent detail points such as km stones, parapets, floor of verandahs, roots of old trees, etc. Their correct descriptions should invariably be written in level books.

6.14. CLASSIFICATION OF LEVELLING

Levellings may be classified into two main types *i.e.*

1. Simple Levelling. 2. Differential Levelling.

1. Simple Levelling. The operation of levelling for determining the difference in elevation, if not too great, between two points visible from a single position of the level, is known as *Simple levelling*.

Suppose *A* and *C* are two points whose difference in elevations, is required with a level set up at *B*. To eliminate the effect of the earth curvature and instrumental errors, it is always advisable to ensure that their distances from the level are kept equal but not necessarily on the line joining them. (Fig. 6.17).

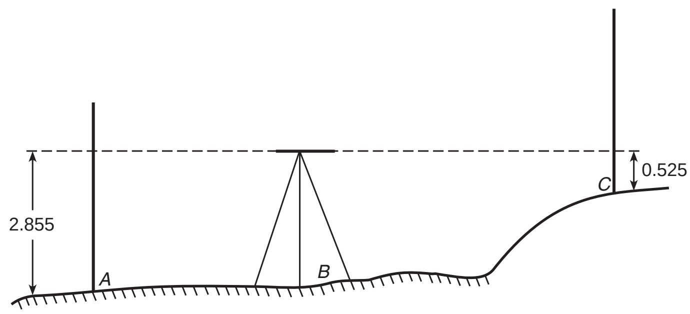


Fig. 6.17. Simple Levelling.

Procedure : Following steps are involved :

(1) Level the instrument correctly.

(2) Direct the telescope towards the staff held vertically on *A*. Focuss it carefully to obtain clear graduations.

(3) Take the reading of the central horizontal hair of the diaphragm where it appears to cut the staff, ensuring that the bubble is central.

(4) Send the staff to the next point *C*.

(5) Direct the telescope towards *C* and focuss it again.

(6) Check up the bubble if central. If not, bring it to the central position by the foot screw *nearest to the telescope* or the micrometer screw in case of a tilting level.

(7) Take the reading of the central horizontal cross hair.

Illustration I. Let the respective readings on staff *A* and staff *C* be 2.855 m and 0.525 m respectively. The difference of level between *A* and *C*

$$= 2.855 - 0.525 = 2.330 \text{ m}$$

If R.L. of A is 500.000, the R.L. of B, may be calculated as under:

R.L. of the point A = 500.000 m.

R.L. of the line of sight = 500.000 + 2.855 = 502.855 m.

R.L. of the point C = 502.855 − 0.525 = 502.330 m.

Illustration II. If one of the points is on the floor and the other on the ceiling such as in tunnels or buildings, the staff at the elevated point, may be held vertically inverted (Fig. 6.18).

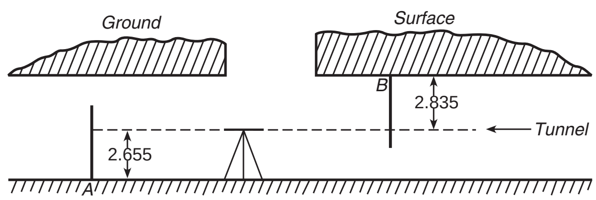


Fig. 6.18. Simple levelling with inverted staff.

If the elevation of A = 200.000 m

back sight reading on A = 2.655 m

fore sight reading on B = 2.835 m.

R.L. of the line of sight = 200.000 + 2.655 = 202.655 m

∴ R.L. of the point B = 202.655 + 2.835 = 205.490 m

2. Differential Levelling (Fig. 6.19). The method of levelling for determining the difference in elevation of two points either too far apart or obstructed by an intervening ground, is known as *Differential levelling*. In this method, the level is set up at a number of points and the difference in elevation of successive points, is determined as in the case of a levelling. This levelling process is also known as *Fly, Compound* or *Continuous levelling*.

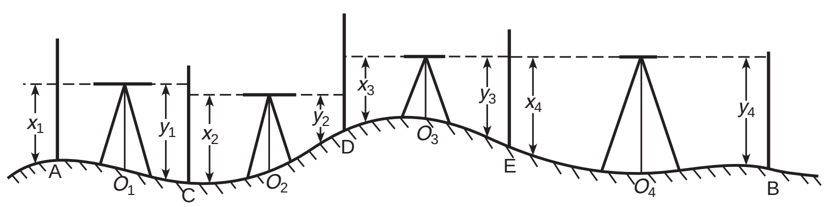


Fig. 6.19. Differential Levelling.

Let us suppose that A and B are two points which are far apart and the difference in their elevations, is to be determined by differential levelling.

Procedure : Following steps are involved :

1. Set up the level at O_1 ensuring that the line of sight intersects the staff held at A. Level it correctly.

2. With the bubble central, take the back staff reading on the staff held vertically at A.
3. Select a point C equidistant from the instrument position O_1 and take the fore staff reading on the staff held vertically at C.
4. Shift the instrument to O_2, set up and level it correctly.
5. With the bubble central, take the back staff reading on the staff held vertically at C again.
6. Select a point D equidistant from the instrument position O_2 and take the fore staff reading on the staff held vertically at D.
7. Repeat the process until the fore staff reading is taken on the staff held on the point B.

Notes : The following points may be noted.

(*i*) The points where two readings are taken at the successive points *C, D, E* etc. are called *change points.*

(*ii*) The level must be set up on firm ground otherwise it may sink during the interval of reading the back and fore sights.

(*iii*) The bubble must always be brought to the centre of its run before staff reading is taken.

(*iv*) The staff from the change point must not be removed till a back sight is taken from the next instrument station by simply turning round to face the telescope.

Illustration III. Let $x_1, x_2, x_3, x_4, \ldots x_n$ be the back sights and $y_1, y_2, y_3 \ldots y_n$ be the fore sights taken on the staff held vertically at *A, C, D,* ...etc.

Here

Difference of level between A and $C = x_1 - y_1$

Difference of level between C and $D = x_2 - y_2$

Difference of level between N and $E = x_3 - y_3$ and so on.

The difference of level of the points A and B is equal to the difference of algebraic sum of the back sights and algebraic sum of the fore sights, *i.e.* $\Sigma B.S. - \Sigma F.S.$

If the difference in level is positive, the closing point B is higher than the starting point A whereas if negative, the point B is lower than the point A.

$\therefore$ R.L. of the point B = R.L. of point $A \pm (\Sigma B.S. - \Sigma F.S.)$

6.15. BOOKING AND REDUCTION OF THE LEVELS

Booking and reduction of the levels may be done by following two methods :

1. Rise and fall method.

2. Height of collimation method.

1. Rise and Fall Method. In this method, the difference of level between two consecutive points for each setting of the instrument, is obtained by comparing their staff readings. The difference between their staff readings indicates *a rise* if the back staff reading is more than the fore sight and *a fall* if it is less than the fore sight. The rise and fall worked out for all the points give the vertical distance of each point relative to the preceding one. If the R.L. of the back staff point is known, then R.L. of the following point may be obtained by adding its rise or subtracting its fall from the R.L. of preceding point as the case may be.

The specimen page of a level book illustrating the method of booking staff readings and calculating R.Ls. of stations by the rise and fall method is shown under.

Rise and fall method of reduction of levels

Stn.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1.	0.585					100.000	B.M.
2.	1.855		2.955		2.370	97.630	C.P.
3.		1.265		0.590		98.220	
4.		2.925			1.660	96.560	
5.	2.350		0.350	2.575		99.135	C.P.
6.		2.855			0.505	98.630	
7.	2.685		1.655	1.200		99.830	C.P.
8.			2.435	0.250		100.080	B.M.
Totals	7.475		7.395	4.615	4.535		

Arithmetic checks. The difference between the sum of the back sights and the sum of the fore sights should be equal to the difference of the sum of rises and the sum of falls and should also be equal to the difference between the R.L. of the last point and that of the first point *i.e.*

$$\Sigma \text{ B. S.} - \Sigma \text{ F. S.} = \Sigma \text{ Rise} - \Sigma \text{ Fall} = \text{Last R. L.} - \text{First R. L.}$$

i.e. $7.475 - 7.395 = 4.615 - 4.535 = 100.080 - 100.000 = 0.080.$

In this method of reduction a complete check on intermediate sights also is provided because these are included for calculating the rises and falls.

2. Height of Collimation Method. In this method, height of the instrument (H.I.) is calculated for each setting of the instrument by adding the back sight (B.S.) to the elevation of the B.M. The reduced level of the first station is obtained by subtracting its fore sight from the instrument height (H.I.). For the second setting of the instrument, the height of the instrument is calculated by adding the back sight taken on the first station to its reduced level. The reduced level of the last point is obtained by subtracting the fore sight of the last point from the height of instrument at the last setting.

If an intermediate sight is observed to an intermediate station, its reduced level is obtained by subtracting its foresight from the height of the instrument for its setting.

The specimen page of a level field book illustrating the method of booking the staff readings and calculating R.Ls. of the stations by the height of collimation method, is shown under.

Hight of Instrument method of reduction of levels.

Stn.	*B.S.*	*I.S.*	*F.S.*	*H.I.*	*R.L.*	*Remarks*
1.	0.585			100.585	100.000	B.M.
2.	1.855		2.955	99.485	97.630	C.P.
3.		1.265			98.220	
4.		2.925			96.560	
5.	2.350		0.350	101.485	99.135	C.P.
6.		2.855			98.630	
7.	2.685		1.655	102.515	99.830	C.P.
8.			2.435		100.080	B.M.
Totals	7.475		7.395			

Arithmetic checks. The difference between the sum of the back sights and the sum of the fore sights should equate to the difference between the R.L. of last station and the R.L. of the first station *i.e.*

Σ B. S. – Σ F. S. = Last R. L. – First R. L.

i.e. 7.475 – 7.395 = 100.080 – 100.000 = 0.080.

In this method there is no check on intermediate sights.

6.16. COMPARISON OF LINE OF COLLMATION METHOD WITH RISE AND FALL METHOD

	Hight of collimation method		*Rise and fall method*
1.	It is more rapid and saves a considerable time and labour.	1.	It is laborious as the staff reading of each station is compared to get a rise or fall.
2.	It is well adopted for reduction of levels for construction work such as longitudinal or cross-section levelling operation.	2.	It is well adopted for determining the difference in levels of two points where precision is required.

3.	There is no check on reduction of R.Ls. of intermediate stations.	3.	There is a complete check on the reduction of R.Ls of intermediate stations.
4.	There are only two arithmetical checks *i.e.* the difference between the sum of the fore sights must be equal to be the difference in R.L. of the last station and first station.	4.	There are three arithmetical checks *i.e.* the difference between the sum of the back sights and the sum of fore sights must be equal to the difference between the sum of the rises and the sum of falls as well as it must also be equal to the difference in R.Ls of the last station and first station.
5.	Errors if any in intermediate sights are not detected	5.	Errors in intermediate sights are noticed as these are used for finding out rises and falls.

Example 6.1. *The following readings were successively taken with an instrument in levelling work :*

0.32, 0.53, 0.62, 1.78, 1.91, 2.35, 1.75, 0.35, 0.69, 1.24 and 0.98 m.

The in position of the instrument was changed after 3rd, 7th and 9th readings. Draw out the form of a level book and enter the above readings properly. Assume the R.L. of the 1st point as 81.53m. Calculate R.L. of all points and apply usual checks.

Solution.

B.S.	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
0.32					81.53	
	0.53			0.21	81.32	
1.78		0.62		0.09	81.23	T.P.
	1.91			0.13	81.10	
	2.35			0.44	80.66	
0.35		1.75	0.60		81.26	T.P.
1.24		0.69		0.34	80.92	T.P.
		0.98	0.26		81.18	
Σ 3.69		4.04	0.86	1.21		

Arithmetical checks :

$\Sigma\, B.S - \Sigma\, F.S. = \Sigma$ Rise $- \Sigma$ Fall = Last $R.S.$ − first $R.S.$

$= 3.69 - 4.04 = 0.86 - 1.21 = 81.18 - 81.53 = 0.35$ O.K.

Example 6.2. *The following consecutive readings were taken with a level and a 4 m staff on a continuously sloping ground at a common interval of 20 metres :*

0.855 (on Q), 1.545, 2.335, 3.115, 3.825, 0.455, 1.380, 2.055, 2.855, 3.455, 0.585, 1.015, 1.850, 1.850, 2.755, and *3.845 (on R).*

Enter the readings as on a field book page, reduce the levels, apply the checks and determine the gradient of line QR.

Solution.

Field Book Page

Chainage	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
0 m	0.855					100.000	B.M. on *Q*
20		1.545			0.790	99.210	
40		2.335			0.790	99.420	
60		3.115			0.780	97.640	
80	0.455		3.825		0.710	96.930	
100		1.380			0.930	96.000	
120		2.055			0.670	95.330	
140		2.855			0.800	94.530	
160	0.585		3.455		0.600	93.930	
180		1.015			0.430	93.500	
200		1.850			0.735	92.765	
220		1.850			0.000	92.765	
240		2.755			0.905	91.860	
260 m			3.845		1.090	90.770	BM on R
Σ	1.895		11.125	0.000	9.230		

Checks :

Σ B.S. – Σ F.S. = Σ Rise – Σ Fall

= Last R.L. – First R.L.

1.895 – 11.125 = 0.000 – 9.230 = 90.770 – 100.00

= – 9.230 M. O.K.

Gradient : The total distance between the starting point and the last point.

$= 260$ m

Difference in level $= -9.230$ m.

$\therefore$ Down gradient $= \dfrac{260 \text{ m}}{9.23 \text{ m}} = 28.169$

i.e., 1 in 28.169. **Ans.**

6.17. GRADIENT OF A LINE

The gradient of a line joining two points, is calculated as under :

Procedure : Following steps are followed.

1. Calculate the R.L. of each station.

2. Apply usual arithmetical checks to the calculations.

3. Calculate the difference in level of the given two points *i.e.* R.L. of the last point –R.L. of the first point.

4. If R.L. of the last point is more as compared to that of the first point, the gradient is positive *i.e. rising gradient.* On the other hand, if R.L. of the last point is less than R.L. of the first point, it is a negative gradient, *i.e., down gradient.*

5. Calculate the distance between the end points. It is equal to nd where n is total number of fore sights and intermediate sight readings on the straight line joining the end points and d is the constant distance between consecutive stations.

6. Divide the distance obtained is step 5 by the difference in level obtained in step (3), to obtain the desired gradient.

Example 6.3. *The following consecutive readings were taken with a dumpy level and a 4 m levelling staff on a continuously sloping ground at 30 m intervals :*

0.680, 1.455, 1.855, 2.330, 2.885, 3.380, 1.055, 1.860, 2.265, 3.540, 0.835, 0.945, 1.530 and *2.250.*

The R.L. of the starting point was 80.750 m.

(*a*) *Rule out a page of level book and enter the above readings.*

(*b*) *Carry out reduction of heights by collimation method.*

(*c*) *Apply the arithmetic checks including the checks on I.S.*

(*d*) *Determine the gradient of the line joining the first and last point.*

Solution. In this question reduction of heights, is asked by collimation method and also checks on Intermediate sights. A modified level book is therefore used as under :

Stn.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*Ht. of collimation*	*R.L.*	*Remarks*
1	0.680			-		81.430	80.750	B.M.
2		1.455		-	0.775		79.975	
3		1.855		-	0.400		79.575	
4		2.330		-	0.475		79.100	
5		2.885		-	0.555		78.545	
6	1.055		3.380	-	0.495	79.105	78.050	C.P.
7		1.860		-	0.805		77.245	
8		2.265		-	0.405		76.840	
9	0.835		3.540	-	1.275	76.400	75.565	C.P.
10		0.945		-	0.110		75.455	
11		1.530		-	0.585		74.870	
12			2.250	-	0.720		74.150	
Sum	2.570		9.170	-	6.600			

Arithmetic checks :

$$\Sigma \text{ B.S.} - \text{F.S.} = 2.570 - 9.170 = -6.600$$

$$\Sigma \text{ Rise} - \Sigma \text{ Fall} = 0.000 - 6.600 = -6.600.$$

R.L. of last –R.L. of first = 74.150 – 80.750 = – 6.600.

$$\text{Gradient } = \frac{\text{Horizontal distance}}{\text{Difference in level}} = \frac{11 \times 30}{6.6} = 50$$

i.e. Gradient = 1 in 50 fall. **Ans.**

6.18. PEGGING STATIONS AT GIVEN GRADIENT

Pegs may be established at the required gradient along the line joining the points A and B as under :

Procedure : Following steps are followed:

1. Find R.L. of the point A say 'x'.

2. Calculate the difference in level for the given distance between two consecutive pegs. If the given gradient is 1 in r and the distance between the consecutive pegs is d meters, then, the difference in level between the pegs is = d/r metres.

3. Calculate the reduced levels of the pegs as under :

R.L. of 1st peg = $x + d/r$

R.L. of 2nd peg = $x + 2\ d/r$

R.L. of 3rd peg = $x + 3\ d/r$

R.L. of 4th peg = $x + 4\ d/r$

....

....

....

R.L. of 5th peg = $x + nd/r$

4. Calculate the staff readings on the pegs as under :

Reading on 1st peg = Ht. of collimation – R.L. of 1st peg.

Reading on 2nd peg = Ht. of collimation – R.L. of 2nd peg.

Reading on 3rd peg = Ht. of collimation – R.L. of 3rd peg.

....

....

....

Reading on nth peg = Ht. of collimation – R.L. of nth peg.

5. Proceed in a similar manner till all the pegs are established.

Note. The following points may be noted.

(*i*) In case all the pegs cannot be fixed from one instrument station, establish a forward station and fix the remaining stations after calculating the height of collimation of the new instrument station.

(*ii*) If the gradient is downward, the reduced levels of the points are determined by substracting d/r,$2d/r$,$3d/r$ etc.

(*iii*) Pegs either are driven or raised till the calculated readings for each peg agree.

Example 6.4. *In running fly levels from a B.M. of R.L. = 250.000 m, the following readings were obtained :*

Back sights = 1.315 ; 2.035 ; 1.980 ; 2.625 ;

Fore sights = 1.150 ; 3.450 ; 2.255.

From the last position of the instrument, five pegs at 20 metre intervals are to be set out on a uniform rising gradient of 1 in 40. The first peg is to have a R.L. of 247.245. Work out the staff readings required for setting the tops of the pegs on the given gradient.

Solution.

St.	*B.S.*	*I.S.*	*F.S.*	*Ht. of collimation*	*R.L.*	*Remarks*
1.	1.315			251.315	250.000	B.M.
2.	2.035		1.150	252.200	250.165	
3.	1.980		3.450	250.730	248.750	
4.	2.625		2.255	251.100	248.475	C.P.
		3.855			247.245	1st peg
		3.355			247.745	2nd peg
		2.855			248.245	3rd peg
		2.355			248.745	4th peg
			1.855		249.245	5th peg
Total	7.955		8.710			

Arithmetic checks :

Σ B.S. – Σ F.S. = 7.955 – 8.710 = – 0.755 m

R.L. of last station – R.L. of first station.

= 2.49.245 – 250.000 = – 0.755 m

Explanation. Difference in level between two consecutive stations.

$$\frac{d}{r} = \frac{20}{40} = 0.5 \text{ m}$$

R.L. of 1st peg = 247.245 m

R.L. 2nd peg = 247.245 + 0.500 = 247.745

R.L. 3rd peg = 247.745 + 0.500 = 248.245

R.L. 4th peg = 248.245 + 0.500 = 248.245

R.L. 5th peg = 248.745 + 0.500 = 249.245

Staff readings on the pegs are obtained by subtracting the height of collimation from the reduced levels of the pegs.

The first four staff readings of the pegs, are written in the intermediate column and the last reading is entered in F.S. column.

Example 6.5. *During fly levelling, the following note is made :*

B.S. = 0.62, 2.05, 1.42 ; 2.63 and 2.42 metres

F.S. = 2.44, 1.35, 0.53 and 2.41 metres.

The first B.S. was taken on a B.M. R.L. 100.00 metres. From the last B.S., it is required to set 4 pegs each at distance of 30 metres on a rising *gradient of 1 in 200. Enter these notes in a form of level book (in your answer book) and calculate the R.L. of the top of each peg by rise and fall method. Also, calculate the staff readings on each peg and apply the usual checks.*

Solution.

Stn.	*B.S.*	*F.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1.	0.62					100.00	B.M.
2.	2.05		2.44		1.82	98.18	C.P.
3.	1.42		1.35	0.70		98.88	C.P.
4.	2.63		0.53	0.89		99.77	C.P.
5.	2.42		2.41	0.22		99.99	C.P.
6.		2.27		0.15		100.14	1st peg
7.		2.12		0.15		100.29	2nd peg
8.		1.97		0.15		100.44	3th peg
9.			1.82	0.15		100.59	4th peg
Total	9.14		8.55	2.41	1.82		

Arithmetic checks :

Σ B.S. – Σ F.S. = 9.14 – 8.55 = 0.59

Σ Rise – Σ Fall = 2.41 – 1.82 = 0.59

Last R.L. - First R.L. = 100.59 – 100.00 = 0.59.

Explanation. The difference in level between two consecutive pegs $= \frac{d}{r} = \frac{30}{200} = 0.15$ m

Staff reading of the 1st peg = 99.99 + 0.15 = 100.14 m

Staff reading of the 2nd peg = 100.14 + 0.15 = 100.29 m.

Staff reading of the 3rd peg = 100.29 + 0.15 = 100.44 m.

staff reading of the 4th peg = 100.44 + 0.15 = 100.59 m.

6.19. CALCULATION OF MISSING READINGS OF A LEVEL BOOK

Readings recorded in a level book, may be classified into two categories:

1. Basic readings. 2. Derived readings.

1. Basic readings. The readings which are observed by a level on a Levelling staff held vertically at the starting B.M. and other intermediate stations, are known as *basic readings*. Such as back sights, intermediate sights, fore sights and the R.Ls. of the given bench marks.

2. Derived readings. The readings which are derived with the help of the basic readings, are known as the *derived readings*. Such as rises, falls, the height of collimation and R.Ls. of unknown points.

For the first setting of the level, there must be at least three basic readings *i.e.*, B.S., F.S. and R.L. of the starting B.M. For other settings at least two basic readings, are needed to run down the levels by any one of the methods.

In a level book every line may have two basic readings and two derived readings. There may be two cases of missing readings, as discussed below :

Case 1. One basic reading missing in a line.

(*a*) Fore sight is missing

1. Fore sight reading = Back sight − Rise

OR

Back sight + Fall

2. Intermediate reading = Back sight or previous I.S. − Rise

OR

Back sight or previous I.S. + Fall

(*b*) Back sight is missing

1. Back sight = Fore sight − *Fall*

OR

Fore sight + Rise

2. Previous intermediate reading = Fore sight (or next I.S.) − Fall

OR

Fore sight (or next I.S.) + Rise

Case 2. Both back and fore sights missing.

1. Calculate the rise or fall by subtracting R.L. of the previous point from R.L. of the point.

2. Sum up all the known back sights and fore sights and calculate missing B.S. or S.F.

3. Similarly, add up rises and falls to get any missing rise or fall.

The method of approach for reproducing missing readings is explained in Example 6.6.

Example 6.6. *Determine the missing data :*

Station	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*HI*	*RL (m)*
1.	*?*					*23.18*	*20.00*
2.		*1.59*		*?*			*?*
3.	*0.28*		*?*		*1.08*	*?*	*?*
4.	*?*		*4.00*		*?*	*18.33*	*?*
5.		*?*			*2.19*		*?*
6.	*?*			*?*			*15.72*
7.			*2.95*		*?*		*?*

Solution.

Station	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*HI*	*RL (m)*
1.	3.18					23.18	20.00
2.		1.59		1.59			21.59
3.	0.28		2.67		1.08	20.79	20.51
4.	1.54		4.00		3.72	18.33	16.79
5.		3.73			2.19		14.60
6.		2.61		1.12			15.72
7.			2.95		0.34		15.38
Σ	5.00		9.62	2.71	7.33		

Checks : Σ B.S. − Σ F.S. = Σ Rise − Σ Fall = Last R.L. − First R.L.

5.00 − 9.64 = 2.71 − 7.33 = 15.38 − 20.00 = 4.62 O.K.

Example 6.7. *The following is a page of a level field book. Fill in the missing readings and calculate the reduced levels of all the points. Also, carry out the necessary checks :*

Sl. No.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*RL*	*Remarks*
1.	*3.250*					...	
2.	*1.880*				*0.600*		
3.		*2.250*			...	...	
4.	...		*1.920*			...	
5.		*2.540*			*0.015*	...	
6.	...		...	*1.000*		...	
7.	*1.175*		*2.115*		...	*225.305*	
8.		*1.625*			...		
9.	...		*1.895*		*0.270*	...	
10.			*1.255*		*0.750*	...	
Sum	*11.450*		...	...	...	...	

Solution. The missing readings and calculation of reduced levels is done in the following table.

Sl. No.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*RL*	*Remarks*
1.	3.250					224.960	
2.	1.880		3.850		0.600	224.360	
3.		2.250			0.370	223.990	
4.	2.525		1.920	0.330		224.320	
5.		2.540			0.015	...	
6.	2.115		1.540	1.000		225.305	
7.	1.175		2.115		0.000	225.305	
8.		1.625			0.450	224.855	
9.	0.505		1.895		0.270	224.585	
10.			1.255		0.750	223.835	
Sum	11.450		12.575	1.330	2.455		

Arithmetical Checks :

Σ B.S.– Σ F.S. = Σ Rise – Σ Fall = Last R.L. – First R.L.

11.450 – 12.575 = 1.330 – 2.455 = 223.835 – 224.960

– 1.125 = – 1.125 = – 1.125. O.K.

Example 6.8. *A gradient of 1 in 400 falling from elevation 67.45 m was set out by driving pegs at 100 metres intervals with the top of the pegs on the required gradient. After a time it was suspected that some of the pegs had been disturbed and the following observations were taken for checking their levels. Draw up a list of errors of the pegs.*

Stn.	*Back sight*	*Inter sight*	*Fore sight*	*R.L.*	*Remarks*
1.	1.76			64.13	B.M.
2.	2.64		0.72		
3.	1.96		1.42		
4.		0.93			Peg No. 1
5.		1.20			Peg No. 2
6.		1.50			Peg No. 3
7.		1.76			Peg No. 4
8.		2.03			Peg No. 5

9.		2.30			Peg No. 6
10.	0.69		2.59		Peg No. 7
11.		0.95			Peg No. 8
12.		1.23			Peg No. 9
13.		1.52			Peg No. 10
14.	0.61		1.21		
15.			1.72	64.13	B.M.

Solution.

Stn.	*BS.*	*IS*	*Rise*	*Fall*	*R.L.*	*Gradient level*	*Error*	*Remarks*	
1	1.76					64.13			B.M.
2	2.64		0.72	1.04		65.17			
3	1.96		1.42	1.22		66.39			
4		0.93		1.03		67.42	67.45	0.03	Peg 1
5		1.20			0.27	67.15	67.20	0.05	Peg 2
6		1.50			0.30	66.85	66.95	0.10	Peg 3
7		1.76			0.26	66.59	66.70	0.11	Peg 4
8		2.03			0.27	66.32	66.45	0.13	Peg 5
9		2.30			0.27	66.05	66.20	0.15	Peg 6
10	0.69		2.59		0.29	65.76	65.95	0.19	Peg 7
11		0.95			0.26	65.50	65.70	0.20	Peg 8
12		1.23			0.28	65.22	65.45	0.23	Peg 9
13		1.52			0.29	64.93	65.20	0.27	Peg 10
14	0.61		1.21	0.31		65.24			
15			1.72		1.11	64.13			B.M.
Totals	7.66		7.66	3.60	3.60				

Arithmethical checks :

Σ B.S. – Σ F.S. = Σ Rise – Σ Fall = 0

R.L. of last Station – R.L. of first station = 0

Explanation :

1. Rule out a page of level book.
2. Enter the given data.
3. Calculate R.L. of the points.
4. Calculate R.Ls. of the points on the given gradient.

5. Enter R.Ls. in the level book.

6. Difference of the R.Ls. of the points equals the error of the pegs.

Example 6.9. *Reproduced below is the page in a level book. Fill in the missing data. Apply usual checks.*

Stn.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1	2.150					450.000	B.M. 1
2	1.645		?	0.500			
3		2.345			?		
4	?		1.965	?			
5	2.050		1.825		0.400		
6		?		?		451.730	B.M. Staff held against ceiling
7	–1.690		?	0.120			
8	?		2.100		?		
9			?	?		499.100	B.M. 3
	8.445						

Solution.

Stn.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1	2.150					450.00	B.M. 1
2	1.645		1.650	0.500		450.50	
3		2.345			0.700	449.800	
4	1.425		1.965	0.380		450.180	
5	2.050		1.825		0.400	449.780	
6		0.100		1.950		451.730	
7	–1.690		–0.020	0.120		451.850	B.M. 2
8	2.865		2.100		3.790	448.060	
9			1.825	1.040		449.100	B.M. 3
Sum	8.445		9.345	3.990	4.890		

Arithmetical checks :

ΣB.S. – ΣF.S. = 8.445 – 9.345 = – 0.900 m

Σ Rise – Σ Fall = 3.990 – 4.890 = – 0.900

R.L. of last point – R.L. of first point

$$= 449.100 - 450.000 = -0.900 \text{ m.}$$

6.20. SPIRIT LEVELLING

Spirit levelling may be classified as under :

1. Differential levelling
2. Check levelling
3. Profile levelling
4. Cross-sectioning levelling
5. Reciprocal levelling
6. Precision levelling.

1. Differential levelling. The operation of spirit levelling which is employed to determine the elevations of a number of points some distance apart to establish Bench Marks in the area, is called *differential levelling*. It is done regardless of the horizontal positions of the points with respect to each other. It is also sometimes known as *taking fly levels or simply fly levelling*.

2. Check levelling. The operation of running level line to check the accuracy of the bench marks previously fixed, is called *check levelling*. The stability of the existing bench marks can only be ascertained if the difference in levels of the sections between three bench marks agree with the observed differences obtained by check levelling.

3. Profile levelling. The operation of levelling carried out to determine the elevations of the points at known distances apart, and also other salient features, along a given straight line is called *profile levelling*. If we plot the elevations of different points as ordinates and the horizontal distances between the points as abscissa, then the line joining the ends of the ordinates, gives the profile of the surface of the earth. Profile levelling is also called *Longitudinal levelling*.

4. Cross-sectioning levelling. The operation of levelling which is carried out to provide levels on either side of the main line at right angles, in order to determine the vertical section of the earth surface on the ground, is called *cross-sectioning*.

5. Reciprocal levelling. The operation of levelling in which difference in elevations between the points is accurately determined by two sets of reciprocal observations, is called *reciprocal levelling*. Reciprocal levelling is employed when it is not possible to set up the level between two points due to an intervening obstruction such as large water bodies.

6. Precision levelling. The operation of levelling in which further refinement of field technique, is made by using precise staves and levels, and special precautions are taken during observation, to eliminate all errors, is known as *precision levelling.* G.T.S. bench marks are established by carrying out precision levelling.

6.21. METHOD OF PROFILE LEVELLING

A longitudinal section is generally carried out along the centre line of the proposed alignments *e.g.*, highways, railways, pipe lines or canals. The alignment may consist of a straight line or a series of straight lines connected by curves. With the help of the profile of the surface of the earth, the designer studies the relationship between the existing ground surface and the levels of the proposed construction work in the direction of the progress of the work. A single profile does not enable the engineer to study the exact topography of the belt of proposed alignment as the character of the ground on either side of the alignment may not be same.

A profile levelling is generally carried out in conjunction with the cross-sectioning for any project except in the case of a pipe line. The profile is usually plotted with the vertical scale much larger than horizontal scale. For proportionate grades and correct estimates of the earth work with least expenditure, the engineer has to study the profiles of the work carefully.

1. Field book for profile levelling (Fig. 6.20). Before actual operation of the levelling, a number of points at equal distances along the centre line of the proposed alignment, are marked on the ground.

The level is set up at a convenient station say I_1 *and a back sight is taken on a levelling staff held vertically on the B.M.*

The levelling staff is then held successively on the points along the profile line and their readings are entered as intermediate staff readings.

When it is not possible to read the staff clearly at a great distance, a fore sight is taken on a relatively permanent point CP_1 not necessarily on the line of the profile. Shift the level to another convenient station say I_2 and take a back sight reading on the change point CP_1.

Proceed in a similar manner till the readings on all the points are observed. In case, it is not possible to locate the points at equal distances

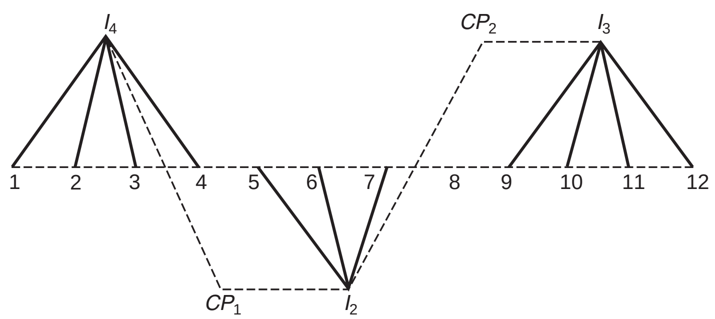


Fig. 6.20. Location of the instrument.

due to irregular undulations, the chaining is done simultaneously to locate the positions of the intermediate points on the profile. The chaining of each point is noted from the point of commencement. The distance between the points along the profile line depends upon the undulations and unevenness of the surface of the ground.

It may be noted that it is always preferred to set up level a bit away from the central line and successively on either side of the profile.

Table 6.1. shows a specimen field book for profile levelling.

Table 6.1. Specimen field book for profile levelling

Stn.	*Distance*	*B.S.*	*I.S.*	*F.S.*	*H.I.*	*R.L.*	*Remarks*
B.M.	--	1.555			501.555	500.000	
1	0		0.525		"	501.030	
2	10		0.155		"	501.400	
3	20		1.655		"	499.900	
4	30		2.050		"	499.505	
CP_1		1.520		3.855	499.220	497.700	CP
5	40		2.560			496.660	
6	60		2.785			496.435	
CP_2		1.860		2.850	498.230	496.370	CP
7	75		1.505		"	496.725	
8	90		1.950		"	496.280	
9	105		0.855		"	497.375	
10	120		3.550		"	494.680	
CP_3	--			2.525	"	495.705	CP
Totals		4.935		9.230			

Checks: Σ B. S. − Σ F.S. =R. L. of last point − R.S. of first point

$$4.935 - 9.230 = 495.705 - 500.00 = 4.295 \text{ m}$$

2. Plotting the profile (Fig. 6.21). The following procedure is adopted.

Draw a straight line *AB* to represent the total horizontal distance between the end stations to a convenient scale. The distances between the consecutive points are marked thereon. Verticals are drawn at each point and their elevations plotted along these verticals. Each ground point is thus plotted by the cartesian co-ordinates *i.e.,* horizontal dis-

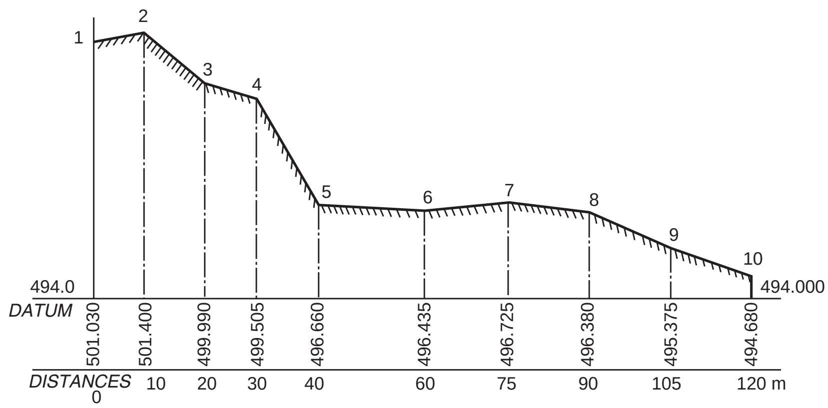


Fig. 6.21. Profile planning.

tance as x-ordinate and elevation as y-ordinates. The end points of all verticals are joined by straight lines, to show the profile of the ground.

The elevations of the datum line, may be assumed as the elevation of the first point. But, for easy calculation of the difference between the elevation of the datum line and the elevation of different points, it is always preferred to, to assume an elevation having full metres.

Generally, horizontal scale is adopted as 1 cm = 10 m and vertical scale is kept 10 times the horizontal scale *i.e.*, 1 cm = 1 m so that inequalities of the ground, may be shown clearly.

6.22 CROSS-SECTIONING.

The method of running sections at right angles to the centre line and on either side of it, for determining the lateral outline of the ground surface, is known as *Cross-Sectioning*. Cross-sections are taken at each 20 m or 30 m station on the centre line. The length of the cross sections depends upon the general terrain and nature of work. *e.g.,* for highway, it may be 30 m or 60 m on each side of the centre line whereas for rail ways 200 m to 300 m or even more on either side of the centre line.

Setting out of the Cross-Sections. Short cross sections are normally setout by eye such as for road/highway. For long cross sections, either an optical square or a box extant or even a theodolite may be used, Cross-sections are serially numbered from the begining of the centre line simultaneously with the progressing the longitudinal section. The reduced levels of equidistant point on the cross-section may be obtained by either with a level, a hand level, an Abney lever or a theodolite.

Cross-Sectioning with a level. Initially, a line perpendicular to and on either isde of the centre line at the desired station on the centre line in section, is setout.

The levelling staff is held at each 10 m point and other points of salilent features such as change in slope which is generallly marked

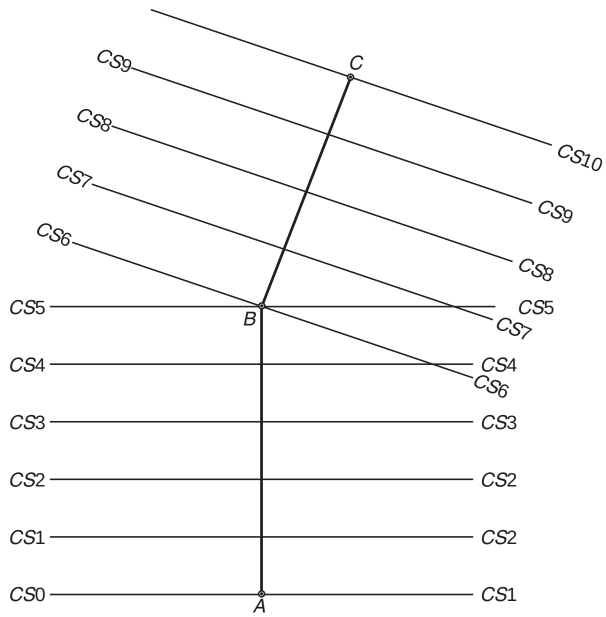


Fig. 6.22. Cross sectioning.

with white paint during the recconnaissance. The staff readings are taken with a dumpy level and the distances of the staff points are measured with a tape, left and right of the centre station on the centre line. The staff readings and distances are recorded as shown in following page of level.

Station	*Distance (m)*	*Side*	*Staff reading*						*Remarks*
			BS.	*I.S.*	*F.S.*	*Rise (+)*	*Fall –*	*Reduced level*	
A			0.125					500.000	B.M
	0			0.255			0.130	499.870	
	10	L		1.300			1.045	498.825	
	20	L		1.380			0.080	498.745	
	30	L		1.389			0.009	498.736	
	10	R		2.365			0.976	497.760	
	20	R		1.685		0.680		498.440	CP
	30	R		2.310			0.625	497.815	

B	30	–	2.170		0.855	1.455		499.270	CP
	10	L		1.250		0.920		500.190	
	20	L		2.690			1.440	498.750	
	30	L		3.150			0.460	498.290	
	10	R		2.830		0.320		498.610	
	20	R		2.980			0.150	498.460	
	30	R		3.250			0.270	498.198	
C	60	-	1.265		2.815	0.435		498.625	
	10	L		2.655			1.390	498.235	
	20	L		3.250			0.595	496.640	
	30	L		3.825			0.575	496.065	
	10	R		1.050		2.775		498.840	
	20	R		2.225			1.175	497.665	
	30	R		2.900			0.675	496.990	
D	90	-	2.710		3.285		0.385	496.605	
	10	L		1.258		1.425		698.030	
	20	L		2.315			1.030	497.000	
	30	L		2.905			0.590	496.410	
	10	R		1.610		1.295		497.705	
	20	R		2.215			0.605	497.705	497.100
	30	R		2.890			0.675	496.425	
E	120	-			1.565	1.325		497.750	
Σ			6.270		8.520	10.630	12.880		

$\Sigma.B.S. - \Sigma.F.S. = \Sigma \text{ rise} - \Sigma \text{ Fall} = \text{Last } RL - \text{First} RL$

$6.270 - 8.520 = 10.630 - 12.880 = 497.750 - 500.000$

$- 2.250 = 2.250 = - 2.250$ O.K.

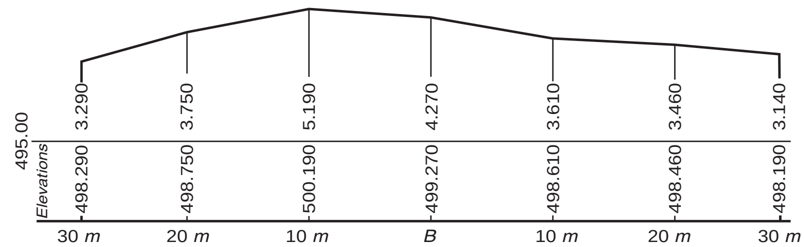


Fig. 6.23.

Plottings Cross-sections. The Cross-sections are plotted in the same manner as the longitudinal sections. In this case, the horizontal and vertical measurements are plotted to the same scale. The most commonly adopted scales are 1 : 100 and 1 : 200. The elevation of the datum lines for each cross section may be kept different to have the ordinates fairly short.

The cross section at chainage 30 m is shown in Fig. 6.23.

6.23. METHOD OF RECIPROCAL LEVELLING

The principle of differential levelling is based on the fact that when the instrument is kept equidistant from the back and forward staff stations, the difference in elevation of the two stations, is equal to the difference of the staff readings.

By setting the level mid way, the errors due to curvature and refraction, and also the collimation error* if any, are eliminated.

When it is not possible to set up the level midway between two points as in the case of levelling across large water bodies, the reciprocal levelling is employed to carry forward the levels on the other side of the obstruction.

Procedure. Let *A* and *B* be two points on opposite banks of an intervening lake. The difference of level of *A* and *B* may be determined, as follows :

1. Set up the level very near to *A*.

2. Keeping the bubble of the level tube central, take readings on the staff held at *A* and at *B*.

3. Let the staff readings on *A* and *B* be a_1 and b_1 respectively. Reading on *A* is usually taken through the objective. As the field of view is very small, a pencil point may be moved up and down the staff till it is seen through the telescope. The correct reading is thus obtained.

4. Transfer the instrument to *B* and set it up very near to *B*. (Fig. 6.24*b*).

5. With the bubble at the centre of its run, read the staffs held at *A* and at *B*.

*Collimation-error discussed later.

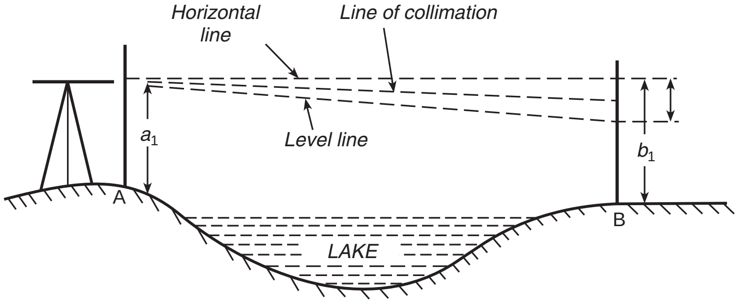


Fig. 6.24(a). Reciprocal levelling 1st position.

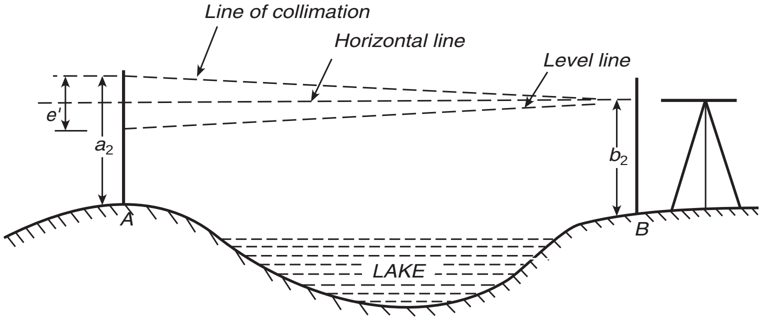


Fig. 6.24(b). Reciprocal levelling 2nd position.

6. Let the staff readings at A and B be a_2 and b_2 respectively.

Computation :

Let h = true difference of level between A and B

e = combined error due to refraction, curvature and imperfect adjustment of the line of collimation.

First Position of the level :

The correct reading on staff $B = b_1 - e$

The correct reading on staff $A = a_1$.

Assuming A to be higher than B, the true difference of level

$$h = (b_1 - e) - a_1.$$

or $$h = (b_1 - a_1) - e \qquad ...(6.2)$$

Second position of the level :

The correct reading on staff $B = b_2$

The correct reading on staff $A = a_2 - e$

The true difference in level

$$h = b_2 - (a_2 - e)$$

or $$h = (b_2 - a_2) + e \qquad ...(6.3)$$

Adding Eqns. (6.2) and (6.3) and dividing by 2, we get

$$h = \frac{(b_1 - a_1) + (b_2 - a_2)}{2} \quad ...(6.4)$$

i.e., the true difference of level between A and B is equal to the mean of the two apparent differences of level.

The combined error can be obtained by equating the Eqns. (6.2) and (6.3).

i.e. $(b_1 - a_1) - e = (b_2 - a_2) + e$

$$e = \frac{(b_1 - a_1) - (b_2 - a_2)}{2} \quad ...(6.5)$$

i.e. the combined error is equal to the half of the difference of the apparent differences of level.

Note. The following points may be noted :

(*i*) In reciprocal levelling, the collimation error and the error due to curvature are completely eliminated.

(*ii*) The elimination of the error due to refraction depends upon the change in the climatic conditions during the transfer of the instrument.

(*iii*) A set of observations at different times of the day, may be made to obtain the accurate difference of levels by taking the mean of all the differences.

Example 6.10. *Reciprocal levels were taken with a dumpy level and following observations were recorded :*

Inst. near Station	*Staff reading at station*	
	A	*B*
A	1.225	1.375
B	0.850	0.500

R.L. of station A is known to be 626.155. Calculate the R.L. of station B.

Also, calculate the error in line of collimation and state clearly whether it is inclined upwards or downwards

Solution.

Instrument near station A : Apparent difference in level between *A* and *B*

= 1.375 – 1.225 = 0.150 m

A being higher

Instrument near station B : Apparent difference in level between *A* and *B*

= 0.500 – 0.850 = (–) 0.350 m

A being higher

The difference in level between station A and B

$$= \frac{0.150 - 0.350}{2} = -0.100 \text{ m}$$

As the difference in level is of negative sign, station A is at lower level than B by an amount 0.100 m.

R.L. of station B = R.L. of station A + true difference

$= 626.155 + 0.100 = 626.255$ m

Instrument at B: Correct reading on stn. A = Reading on Stn. B + true difference in level

$= 0.500 + 0.100 = 0.600$ m

Observed reading on stn. A = 0.850 m

∴ Error of collimation = Observed reading on Stn. A – True reading on station A

$= 0.850 - 0.600 = 0.250$ m

As observed reading on station A is greater than its true reading, the line of collimation is inclined *upwards*.

Example 6.11. *In levelling across a river, two pegs A and B were fixed on opposite banks. The following readings were taken.*

Position of Level	*Staff reading at*	
	A	*B*
Level at A	1.871	1.469
Level at B	1.664	0.706

If R.L. of A is 50.865, find the R.L. of the point B.

Solution.

Instrument near A: The apparent difference in elevation.

$= 1.871 - 1.469$

$= 0.402,$ B being higher

Insurement near B: The apparent difference in elevation.

$= 1.664 - 0.706$

$= 0.958,$ B being higher.

Hence, the true difference in elevation

$$= \frac{0.402 + 0.958}{2} = 0.680 \text{ m}$$

The R.L. of B = R.L. of A + Difference in elevation

$= 50.865 + 0.680 = 51.545$ m. **Ans.**

Example 6.12. *A dumpy level was set up with its eye-piece vertically over a peg C. The height from the top of peg C to the centre of eyepiece was measured and found to be 1.578 m. The reading on the staff held on peg D was 1.008 m. The level was then moved and set up likewise at peg D. The height of eyepiece above D was 1.258 m and the reading on the staff held on peg C was 163.378.*

Solution.

Instrument at C : The apparent difference in elevation of C and D.

$$= 1.578 - 1.008$$

$$= 0.570, \qquad D \text{ being higher.}$$

Instrument at D : The apparent difference in elevation of C and D

$$= 1.812 - 1.258$$

$$= 0.554, \qquad D \text{ being higher.}$$

Hence, the true difference in elevation

$$= \frac{0.570 + 0.554}{2} = 0.562 \text{ m}$$

$\therefore$ R.L. of the point D.

$$= \text{R. L. of the point } C + \text{Difference in elevation}$$

$$= 163.378 + 0.562 = 163.940 \text{ m.}$$ **Ans.**

6.24. PRECISE LEVELLING

Levelling of the highest precision, is used for establishing a network of permanent bench marks in any country for future reference. The precise levelling is mostly done by National Surveying Organisation such as the Survey of India in India. The field method adopted for precise levelling in principle in the same as that of ordinary levelling but using special instruments and by applying the corrections. The levelling staves used for precise levelling are graduated to 0.1 mm or even finer whereas those used in ordinary levelling are graduated to 0.05 cm.

Definition of Precise levelling. Precise levelling may be defined as levelling done with instruments designed so that a finer precision of reading is taken. The most commonly used in the parallel-sided plate micrometer fitted to a telescope of magnification about 30X which is provided with a tubular bubble of high sensitivity.

Classification of Precise levelling. Depending upon the degree of accuracy, the precise levellings are classified as under:

1. Primary Levelling (First levelling). It is the most accurate levelling in which maximum permissible error is $\pm 4\sqrt{K}$ mm, when the level line is a loop of first order precise levelling and K is the total distance of the level line in kilometre.

The maximum distance of the staff station from the instrument is limited to 50 m to 60 m.

2. Secondary Levelling (Second order). The permissible error of closure of precise levelling of second order of running level line from any first order bench mark or begining from and ending on such a bench mark is $\pm 8\sqrt{K}$ mm where K is the length of the level circuit in kilometres.

The maximum distances of the staff stations from instrument is limited to 60 m to 70 m.

3. Tertiary Levelling (Third order). The permissible error of closure of precise levelling of third order by running the level line from any first order/second order bench marks and closing on some other first/second order bench mark is $12 \pm \sqrt{K}$ mm where K is the total length of the level circuit.

The maximum distances of the staff station from the instrument is limited to 90 m.

Precise Levelling staves. For precision levelling one piece staves with graduations marked on strips of invar, are employed. The commonly used length of precision levelling staff is 3 metres. The precision staff is either made of wooden or metal frame with grooves in which 20 or 30 mm wide invar strip is easily fitted. The strip is fixed only at the bottom end of the frame. The invar strip is 2 or 3 mm thick and is capable to stand up rigidly in its grooves or sometimes the strip is a thin tape kept out by a spring at its upper head. The graduations on the invar strip are either painted or the paint is filled into shallow recesses out the metal and smoothened off flush with the surrounding metal other than graduations. The numbers and other marks are written on the frame. The short graduation lines are 1 mm thick and spaced at 10 mm intervals. There are two sets of staggered graduations.

Collimation Correction When the sum of back sight distances and that of foresight distances in a level line are not equal and the line of collimation is inclined upward or downward, the elevations of the points obtained will not be correct. To obtain correct elevations, a collimation correction is applied to the observed elevations.

The collimation factor or C-factor of the instrument may be determined as explained under:

Procedure. The following steps are involved:

(i) Select two staff stations A and B on a fairly level ground about 80 m apart.

Set up the level at station M at a distance x_1 from station A and x_2 from station B. The distance x_1 is taken about 6 m. Take B.S. reading on station A and F.S. reading on station B.

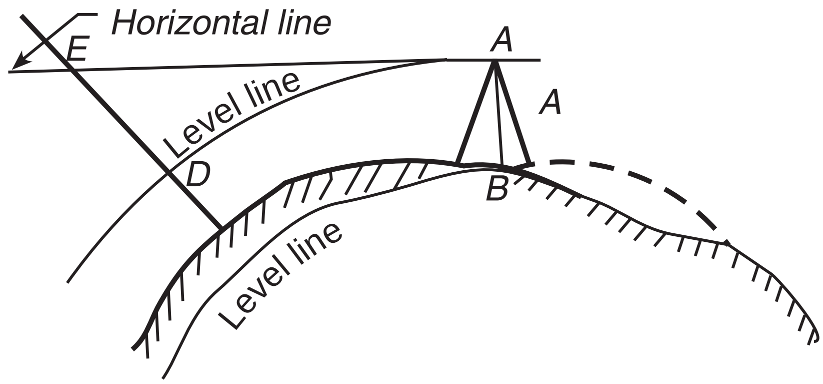


Fig. 6.25. Line of collimation correction.

(*ii*) Shift the level to station *N* at a distance x_3 from station *A* and x_4 from station *B*. The distance x_4 is taken 6 m.

Take B.S. reading on staff at station *A* and F.S. reading on staff at station *B*.

Calculation of the value of C-factor. Let the collimation factor be *C i.e.*, the vertical intercept per unit length of line of collimation. Let the line of collimation be inclined downwards and the elevation of *A* is more than that of section *B*.

In the first settling, let *a* and *b* the observed staff readings. Then,

The correct staff readings are:

$(a + Cx_1)$ and $(b + Cx_2)$ at *A* and *B* respectively

∴ Correct difference in elevations

$$h = (b + Cx_2) - (a + Cx_1) \qquad \ldots(i)$$

In the seocond setting, let *c* and *d* be the staff readings, the correct staff readings are

$(C + Cx_3)$ and $(d + Cx_4)$ at *A* and *B* respectively

Correct difference in elevations

$$h = (d + Cx_4) - (C + Cx_3) \qquad \ldots(ii)$$

Comparing eqns. (*i*) and (*ii*) we get

$$(b + Cx_2) - (a + Cx_1) = (d + Cx_4) - (C + Cx_3)$$

or $\quad b + Cx_2 - a - Cx_1 = d + Cx_4 - C - Cx_3$

$C\,(x_2 - x_1) + b - a = C(x_4 - x_3) + d - C$

$$C[(x_2 - x_1) - (x_4 - x_3)] = (d - C) - (b - a)$$

or
$$C = \frac{d - c - b + a}{x_2 - x_1 - x_3 + x_4} = \frac{(d + a) - (c + b)}{(x_2 + x_4) - (x_1 + x_3)}$$

$$C = \frac{\text{Difference of sum of near reading and far reading}}{\text{Difference of sum far distance and near distance}}$$

In case of value of C is found negative, the assumed line of collination is wrong. It should have been inclined upwards.

i.e. The correct elevation = Calculated elevation + C (ΣBS distance – ΣFS distance)

Standards for levelling of high precision were fixed by International Geodetic Association in 1912.

The specifications of precision levelling are :

Probable accidental error not $> \pm 1\sqrt{k}$, mm

Probable systematic error not $> \pm 0.2\sqrt{k}$, m

where K, is in killometres

Instruments. Levels manufactured by Carl Zeiss or Wild Heerbrug are mostly used for precise levelling. Invar precision levelling staves are in used place of ordinary levelling staves. With the parallel plate micrometers, readings upto $\frac{1}{1000}$ of a metre can be taken accurately.

Precautions for Precise Levelling. The following precautions must be taken for precision levelling:

1. The adjustment of the instrument is carefully tested and perfectly done.
2. A constant watch is made to ensure that the bubble remains central while observations are made.
3. The level is protected from the sun, wind, or rain by an umbrella.
4. The levelling staff is provided with a level to check its vertically.
5. The level is set up on hard and firm ground exactly mid way between the forward staff and back staff.
6. Lengths of the sights are kept within 50 metres.
7. The change points are taken on a steel plate firmly fixed.
8. The fore sight and back sight are taken in quick succession to eliminate the chances of any settlement of the instrument or the staff.
9. The levelling is carried out either in the morning or in late evenings, to avoid errors due to refraction.

10. Check levelling is done in opposite directions by two surveyors on different days and with different change points.

6.25. CURVATURE CORRECTION

As already defined earlier, the liner of sight is a straight line assumed to be free from the effect of refraction. The level line is a curved line having its concavity towards the earth. Due to curvature of the earth, readings taken on a levelling staff held vertically, are always more than what these would have been if the earth had a plain surface. (Fig 6.26)

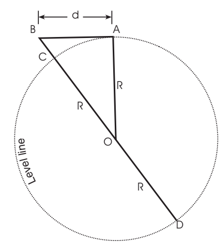


Fig. 6.26. Effect of earth Curvature.

Effect of curvature. *The effect of the curvature on a staff reading, is equal to the distance between the points where the line of sight and level line through the level, intersects the staff.*

In Fig. 6.25, *A E* is the horizontal line which gets defected upward from the level line *AD* by an amount *DE*. To find the difference in elevation between points B and *C*, the staff reading should have been taken at *D* where the level line intersects the staff. But the line of sight being a straight line, in the absence of refraction, the staff reading is taken at the point *E*. It may be seen that the apparent staff reading *CE* is more than the actual reading *CD*, the point *C* therefore, appears to be lower than what it really is.

The correction for curvature is, therefore, negative and is always subtracted from the staff reading.

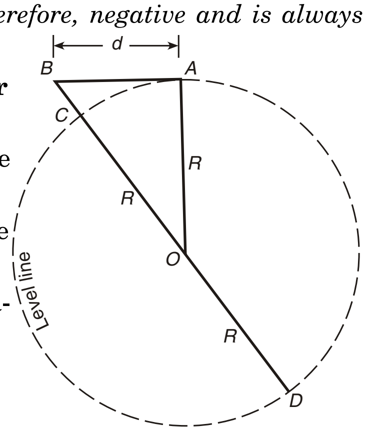


Fig. 6.27

Derivation of the formula for curvature correction (Fig. 6.27)

Let $AB = d$, the horizontal distance between *A* and *C*.

$OC = OA = R$ the radius of the earth.

$BC \quad C_c$, the correction for the curvature.

From the rt. angled ? *BAO*, we get

$$BO^2 \quad AO^2 \; ? \; AB^2$$

$$\therefore \quad (R + C_c^2) = R^2 + d^2$$

or $$R^2 + 2RC_c + C_c^2 = R^2 + d^2$$

or $$C_c(2R + C_c) = d^2$$

or $$C_c = \frac{d^2}{2R + C_c} \quad \text{(Exact)} \qquad ...(6.6)$$

But, C_c is negligible as compared to the diameter of the earth

$$\therefore \quad C_c = \frac{d^2}{2R} \quad \text{(Approximate)} \qquad ...(6.7)$$

If distance d is in kilometers and the radius of earth is assumed as 6370 km, the correction of curvature.

$$C_c = \frac{d^2}{12740} \times 1000$$

or $$= 0.0785\, d^2 \quad \text{metres} \qquad ...(6.8)$$

6.26. REFRACTION CORRECTION

The line of sight provided by a level does not remain straight. It gets bent towards the earth due to refraction as it passes through layers of air of different densities. The effect of refraction is therefore opposite to that of curvature and the points appear higher than what they really are. (Fig 6.28) *The correction of refraction is always added to the staff readings.*

Let the line of sight affected by refraction be represented by AE. The staff held vertically at C, actually reads CE. Under normal atmospheric conditions the arc AE may be assumed circular whose radius is seven times that of the earth. *The effect of refraction is therefore 1/7th that of the curvature but of opposite sign.*

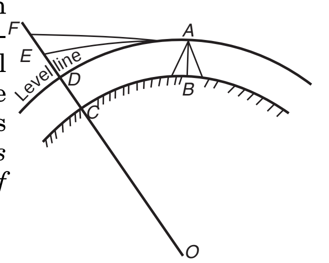


Fig. 6.28.

$\therefore$ The correction for refraction

$$C_r = \frac{1}{7} \times \frac{d^2}{2R} = 0.0112 d^2 \qquad ...(6.9)$$

where d is in kilometres.

(1) Correction due to curvature and refraction

From Eqns. (6.7) and (6.9) we know,

correction for curvature $= \dfrac{l^2}{2R}(-ve)$

correction for refraction $= \frac{1}{7} \times \frac{d^2}{2R}$ (+ *ve*)

$\therefore$ The combined correction due to curvature and refraction

$$= \frac{d^2}{2R} - \frac{1}{7} \times \frac{d^2}{2R}$$

$$= \frac{6d^2}{7 \times 2R} (-ve)$$

$$= \frac{6 \times 1000 \times d^2}{7 \times 2 \times 6370}$$

$$= 0.0673\, d^2 \text{ metres} \quad \text{...(6.10)}$$

where d is in km.

Note. The following points may be noted :

(*i*) For ordinary lengths of sight, the error due to curvature and refraction, is very small and is generally ignored.

(*ii*) The error due to curvature and refraction can be eliminated by equalising back sight and fore sight distances *i.e.*, by *balancing the sights*.

(*iii*) The error due to curvature and refraction can also be eliminated by the method of *reciprocal levelling*.

(2) Distance to the visible horizon. (Fig. 6.29)

Let C be the staff station; $BC = h$ be the staff reading; d be the distance of visible horizon.

Let the horizon meets the earth surface at A, the point where the level line CA meets the horizon of the point B.

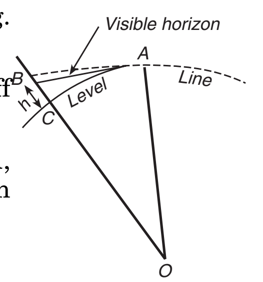


Fig. 6.29. Visicale horizon.

From Eqn. (6.10), we know,

$h = 0.0673\, d^2$ metres where d is in km.

$$d = \sqrt{\frac{h}{0.0673}} \text{ km} \quad \text{...(6.11)}$$

or $\quad d = 3.855 \sqrt{h}$ km. ...(6.12)

In Eqn. (6.11) both the curvature and refraction corrections are taken into account.

6.27 DIP OF THE HORIZON

Let A be the point of observation at an altitude 'h' metres above B which lies on the level line BC. Let AE be a horizontal line through the point of observation A and perpendicular to plumb line AO, O being the centre of the earth. (Fig. 6.30).

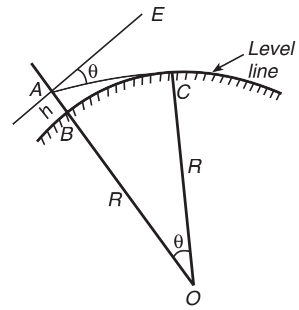


Fig. 6.30.

The angle EAC between the horizontal line AE and the tangent AC to the level line, is known as the *dip* of the horizon.

The dip of horizon is equal to the angle (θ) subtended at the centre of the earth by the arc BC. *i.e.,* the dip of horizon.

$$= \angle EAC = \theta = \angle BOC$$

$$= \frac{\text{arc BC}}{\text{radius of the earth}} \text{ in radian.}$$

The radius of the earth being very large, the arc BC may be approximated to chord BC = d. Hence, the dip of the horizon (in radian)

$= \dfrac{d}{R}$ where d and R are expressed in the same units.

Example 6.13. *A level was set up at a point O and the distances to two staff stations A and B were 150 m and 250 m respectively. The observed staff readings on stations A and B were 2.725 and 1.855. Find the correct difference of levels between stations A and B.*

Solution. Combined correction of curvature and refraction for the staff reading on A

$$= 0.0673\, d^2 = 0.0673 \times \left(\frac{150}{1000}\right)^2 = 0.0015 \text{ m}$$

Combined correction of curvature and refraction for the staff reading on B

$$= 0.0673\, d^2 = 0.0673 \times \left(\frac{250}{1000}\right)^2 = 0.0042 \text{ m.}$$

∴ Correct staff reading on station A

$$= 2.7250 - 0.0015 = 2.7235 \text{ m}$$

and Correct staff reading on station B

$$= 1.8550 - 0.0042 = 1.8508 \text{ m}$$

∴ Correct difference of level between A and B

$$= 2.7235 - 1.8508 = 0.8727 \text{ m } \textbf{Ans.}$$

Example 6.14. *A lamp on the top of a light house is visible just above the horizon at a certain station at the sea level. The distance of the top of the light house from the station of observation is 50 km. Find the height of the lamp above sea level.*

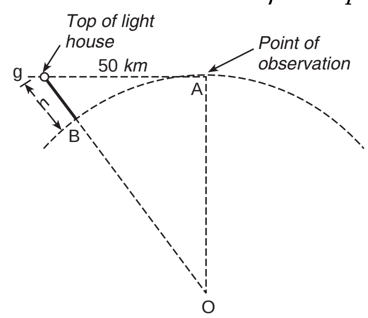


Fig. 6.31.

Solution. (Fig. 6.31).

Let A be point of observation and B be the bottom of the light house of height 'h'.

Due to the combined effect of curvature and refraction the top of the light house just appears above the horizon.

i.e. $h = 0.0673\, d^2$ where d is in km.

$\therefore$ $h = 0.0673 \times 50^2 = 168.25$ metres.

The top of the light house above sea level = 168.25 m. **Ans.**

Example 6.15. *From the deck of a ship, the light at the top of a light house is visible just above the horizon. The heights of the top of light house and the eye of the viewer from the ship above mean sea level may be assumed as 85 m and 6 m respectively. Assuming the radius of the earth as 6370 km and the usual correction refraction, determine the distance between the ship and light house.*

Solution. (Fig. 6.32)

Fig. 6.30.

Let A be the position of ship

B be the position of light house

O be the point where ray touches the sea

d_1 be distance between ship and C

d_2 be distance between ship and B.

We know that combined correction due to curvature and refraction.

$$= \frac{6}{7} \times \frac{(\text{distance})^2}{\text{diameter of the earth}}$$

i.e.

$$h = \frac{6}{7} \times \frac{1000\, d^2}{2 \times 6370} \text{ metres}$$

$$h = 0.06728\, d^2$$

Substituting the value of h in Eqn. (i) we get

$$d = \sqrt{\frac{h}{0.06728}}$$

$$= 3.8553 \sqrt{h}$$

$$\therefore \quad d_1 = 3.8553 \times \sqrt{6}$$

$$= 9.44 \text{ km}$$

$$d_2 = 3.8553 \times \sqrt{85} = 35.54 \text{ km}$$

$\therefore$ Total distance $AB = d_1 + d_2 = 9.44 + 35.54 =$ **44.98 km Ans.**

6.27. THREE WIRE LEVELLING

To establish a network of accurate bench marks, the method of three wire levelling is generally employed.

For this method the diaphragm of the level should be provided with three horizontal cross hairs instead of one. The two extra cross hairs are placed at exactly equal distances above and below the central cross hair. (Fig. 6.33).

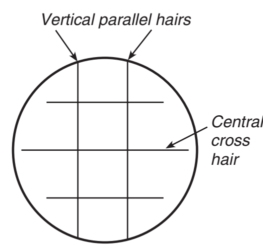


Fig. 6.33.

Procedure : Following steps are followed.

1. Observe the readings on a levelling staff for each cross hair. The average of the three wire readings should agree with the central cross hair reading within a permissible limit (0.003 m).
2. From the average readings of back sights and fore sights at each station, calculate the rise and fall or height of line of collimation.
3. Record the three-wire levelling observations, as illustrated below:

Upper wire	= 2.655 m
Middle wire	= 2.525 m
Lower wire	= 2.400 m
Sum	= 7.580
Average	= 2.527 m.

Specimen field level book is shown in Table 6.3.

Table 6.3. Specimen field book of three wire levelling.

Stn.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*
1	2.525					100.000
	2.425					
	2.320					
Sum	7.270					
	2.423					
2	3.855					
	3.525		1.625			
	3.200		1.205			
Sum	10.580		0.790			
	3.527		3.620			
3	1.265		1.207	1.216		101.216
	1.250		2.655			
	1.240		2.450			
	3.755		7.970			
	1.252		2.657			
4			2.650	0.870		102.086
			2.425			
			2.200			
			7.275			
			2.425		1.173	100.913
Sum	7.202		6.289	2.086	1.173	

Arithmetical Checks :

$$\Sigma \text{ B. S.} - \Sigma \text{ F. S.} = 7.202 - 6.289 = 0.913$$

$$\Sigma \text{ Rise} - \Sigma \text{ Fall} = 2.086 - 1.173 = 0.913$$

R.L. of last station – R.L. of first station

$$= 100.913 - 100.000 = 0.913$$

6.28. DIFFICULTIES IN LEVELLING

The following are some of the common difficulties faced in levelling:

1. Levelling in undulating terrain. While carrying out levelling in undulating terrain, the level should never be set up on the top of the summits or bottoms so shallows. If the level is set up on the summits a large number of stations will be required as illustrated in Fig. 6.34 and the progress of the work will be considerably slow.

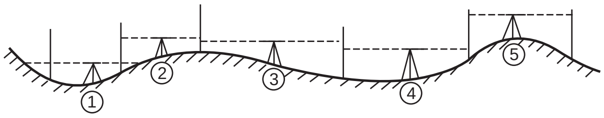


Fig. 6.34. Incorrect positioning of the instrument stations.

In order to avoid short sights and to equalise fore and back sight readings, the instrument may first be set up on the slope as illustrated in Fig. 6.33. In the first case the number of stations are 5 whereas in the second case (Fig. 6.35). The number of stations has been reduced to 3, for the same length.

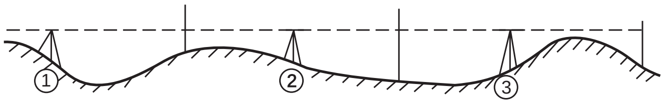


Fig. 6.35. Correct positioning of the instrument.

2. Levelling across a lake. When a staff cannot be read clearly due to great distance, the difficulty may be overcome as explained under:

Procedure: Proceed as under.

Drive in two pegs *A* and *B* flushed with the water surface on the opposite banks of the lake as shown in Fig. 6.36.

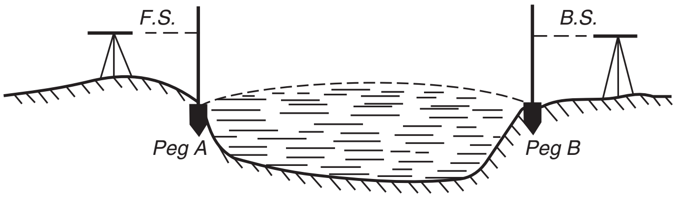


Fig. 6.36. Levelling across a pond.

As per definition of level surface, the surface of water of a still lake or a pond represents a level surface. Hence, the level of the tops of pegs *A* and *B* is the same. Reduced level of the top of peg *A* is determined by taking a foresight from the instrument station *X*. The level is then shifted to the other bank and back sight is taken on the top of peg *B*. In this case the two pegs *A* and *B* together have been considered as a single change point for the purpose of running down the levels.

3. Levelling across a river. The method suggested in para 2 is not suitable in case of a wide river as water flows in rivers with appreciable velocity. This difficulty may be overcome by the method of reciprocal levelling as already explained.

In case the river is too wide to adopt even reciprocal levelling, transfer of level is done by driving two pegs *C* and *D* on opposite banks separated by the shortest distance at right angles to the flow of water as shown in Fig. 6.37.

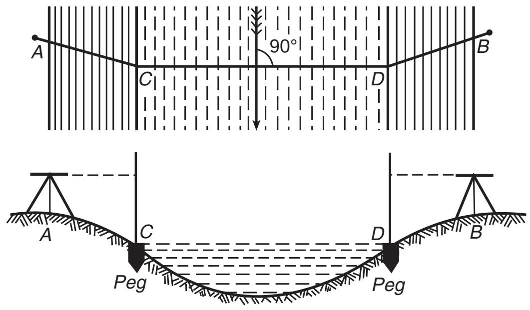


Fig. 6.37. Cross-sectioning of the river at C-D.

Reduced level of the top of peg C is determined by taking a foresight from the instrument set up on its bank. The level is then shifted to the other bank and a back sight is taken on the top of peg D. In this case the level of both pegs C and D is assumed the same.

4. Levelling across an intervening high wall. (Fig. 6.38)

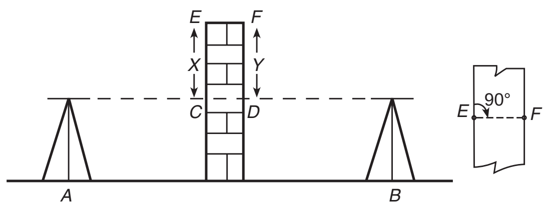


Fig. 6.38. Levelling across a wall.

When a high wall is intervened, levelling may be carried out as explained under :

Mark a point C on the wall in the line of sight. Measure the distance of point E vertically above the point C. Let it be x. Establish another point F on the other face of the wall such that EF is perpendicular to the first face containing the point E. Suspend a plumb bob from F.

Transfer the level on the other side of the wall and adjust its height conveniently and make observation. When the line of sight intersects the suspended chord, mark a point D. Measure the distance FD accurately. Let it be y. The reduced level of D may be calculated as under:

$$\text{R. L. of } D = \text{R. L. of } C + CE - FD$$

$$= \text{Ht. of collimation at } A + x - y.$$

$\therefore$ The hight of collimation at B

$$= \text{Ht. of collimation at } A + x - y. \quad \text{...(6.12)}$$

5. When a B.M. is above the line of collimation. This case occurs when the B.M. is on the underside of a beam. It generally remains much

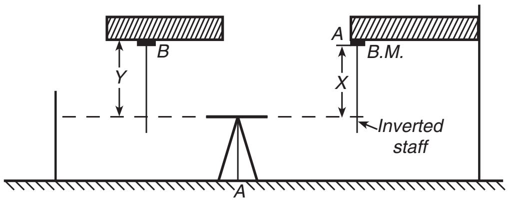


Fig. 6.39. Levelling with inverted staff.

above the line of sight of a level set up on the floor. This difficulty may be overcome as explained under :

Hold the staff inverted on the overhead point keeping the foot of the staff touching the point. The height of line of collimation is equal to R.L. of the B.M. minus the back sight reading. In the level book, inverted staff readings are entered with a negative sign.

In case, a fore sight is taken on an inverted staff, R.L. of the sight is obtained by adding the fore sight.

Illustration. Suppose R.L. of a B.M. is 100.000

Back sight reading on B.M. x = 2.855 m

and fore sight reading on B.M., y = 2.925 m

i.e. R.L. of line of collimation

$$= 100.000 - 2.855 = 97.145 \text{ m}$$

$$\text{R. L. of } B = 97.145 + 2.925 = 100.070 \text{ m}$$

Observations are recorded as under

B.S.	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*
(–) 2.855					100.000
		(–) 2.925	0.070		100.070

Note : It may be noted that –2.925 is less than –2.855 and hence the difference of the two readings is a rise.

6. Levelling on Steep slopes. When levelling is done on steep slopes, back sights and fore sights may not be equal unless short lengths are adopted. It is therefore more convenient to set up the instrument away from the line joining the staff positions and preferably at the same level as the higher one (Fig. 6.40). If I_1, I_2, I_3 etc. be the instrument stations and 1, 2, 3, 4, 5, etc. the staff positions, the distances I_1 and I_2 are kept approximately equal, to eliminate the effect of curvature and non-adjustment of the level. (Fig. 6.40)

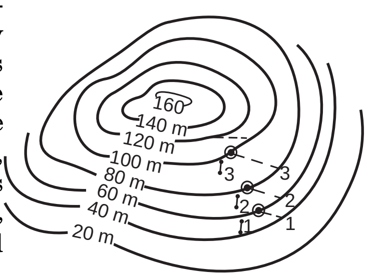


Fig. 6.40. Levelling on steep ground.

6.29. ERRORS IN LEVELLING

Errors in levelling may be categoriesed into following three heads.

(*i*) Personal errors.

(*ii*) Errors due to natural causes.

(*iii*) Instrumental errors.

1. Personal errors. Personal errors include the following :

(*i*) Error in sighting.

(*ii*) Error in manipulation.

(*iii*) Error in reading the staff.

(*iv*) Error in recording and computation.

1. Error in sighting. This error is caused when it is difficult to see the exact coincidence of the cross hairs and the staff graduation. This may be either due to long sights or due to coarseness of the cross hairs and the staff. Sometimes, atmospheric conditions also cause an error in sighting. This error is accidental and may be classified as compensative.

2. Error in manipulation. These include the errors due to the following reasons :

(*i*) **Carelessly setting up the level**. The instrument should be set up on firm ground and carefully levelled. Neighter the telescope nor the triped should be touched while taking readings.

(*ii*) **Imperfect focussing of eyepiece and objective.** To eliminate this errors, the eyepiece must be moved in or out till the cross hairs are distinctly visible against a white paper. The parallax should also be completely removed by properly focussing the object glass before taking every reading.

(*iii*) **The bubble not being central at the time of taking readings.** When the bubble is exactly central, the vertical axis is truly vertical and the horizontal axis of the telescope becomes horizontal. If the bubble is not central, the horizontal axis of the telescope gets inclined which affects the staff reading. The error is more for long sights and less for short sights. To avoid this error, the observer should develop a habit to check the bubble before and after taking each reading.

(*iv*) **Non-verticality of the staff.** In the absence of a plumb bob attached to a staff, it is difficult to judge the verticality of the staff in the line of sight. On an inclined staff, the readings are always too great. For the same angle of the tilt, the error is more for large readings of the staff and less for small readings. When the readings are great, the verticality of the staff should be carefully checked before readings are observed. In case of ordinary levelling, staff man may be asked to move the staff slowly towards or away from the level and the minimum staff reading taken. The non-verticality of the staff in the direction transverse to the line of sight may be checked by two vertical cross hair. Moreover, the horizontal cross hair will not be parallel to the graduations unless the staff is vertical transverse to the line of sight.

Derivation of the formula. (Fig. 6.41.)

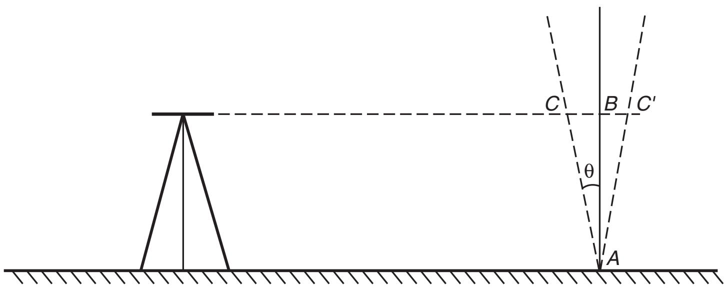


Fig. 6.41. Error due to non-verticality of staff.

Let A be the foot of the staff. The line of sight intersects the vertically held staff at B and the inclined staff at C or C' where angle of tilt is θ.

From the right angled triangle ABC, we get

$$AC^2 = AB^2 + BC^2$$
$$= AB^2 + (AB \tan \theta)^2 = AB^2 (1 + \tan^2 \theta)$$
$$AC = AB\sqrt{1 + \tan^2 \theta}$$

∴ Error due to non-verticality of the staff

$$= AC - AB$$
$$= AB(\sqrt{1 + \tan^2 \theta}) - AB = AB\,(\sqrt{1 + \tan^2 \theta} - 1)$$
$$= AB(\sec \theta - 1). \qquad ...(6.13)$$

3. Errors in reading the staff. These errors are generally committed by a beginner and are :

(*i*) Reading the staff upwards, instead of downwards.
(*ii*) Reading against the top or bottom hair instead of the central hair.
(*iii*) Concentrating the attention on the decimal part of the reading and reading the whole metre wrongly.
(*iv*) Reading the inverted staff as a vertically held staff.

4. Errors in recording and computation. The common errors in recording include :

(*i*) Entering a reading in the wrong column *i.e.* B.S. reading in the I.S. or F.S. columns or *vice versa*.
(*ii*) Recording the readings with digits interchanged *i.e.* 2.654 instead 2.456.
(*iii*) Omitting an entry.

(*iv*) Mistaking the numerical value of a reading called by the level man.
(*v*) Entering the inverted staff reading without a minus sign.
(*vi*) Adding the foresight reading instead of subtraction it and or subtracting a backsight reading instead of adding.

2. Errors due to natural causes. These include the following errors :

(*i*) **Errors due to curvature.** The curvature of the earth surface lowers the elevation of the station and its amount is directly proportional to the square of the horizontal distance between the staff position and the point of the observation. The correction of the curvature has to be subtracted from the observed staff reading to get correct reading. In case of ordinary levelling, error due to curvature is a negligible quantity (only 0.003 m for a sight of 300 m length). For calculation of the curvature correction, refer to article 6.25 of this chapter.

(*ii*) **Errors due to refraction**. The effect of refraction on the observed readings, is opposite to that of the curvature. Refraction raises the elevation of the station and the error is also directly proportional to the square of the horizontal distance of the station from the level. This is negligible for short sights and is generally ignored in ordinary levelling. For calculation of the correction due to refraction, refer to article 6.25 of this chapter.

(*iii*) **Errors due to wind and sun.** Due to strong wind it is always difficult to hold the staff vertical. Due to non-verticality of the staff, the observed readings are erroneous. Similarly the wind also causes vibrations in the instrument and the bubble of the level tube does not remain central. In strong wind it is always advisable to suspend the work, to protect the level by an umbrella and also the staff readings may be kept small.

The sun causes a considerable trouble if it shines on the object glass. It is recommended always to protect the objective by an umberalla. The effect of sun is also to change the length of staff due to change in temperature. But, in ordinary levelling, the change in length is negligible.

3. Instrument errors. Instrumental errors are caused due to :

(*i*) **Imperfect adjustment of the level.** In a perfectly adjusted level, the line of collimation remains horizontal when the bubble of the level tube occupies the central position. The non-adjustment of the level, makes the line of collimation either inclined upward or downward and the observed readings are either more or less. Such errors get compensated if the back sight and fore sight distances are kept equal as in the case of fly-levelling. But, in the case of intermediate sights, the distances considerably differ and the readings are thrown into error by different amounts. In case of levelling on steep slopes, the back sights are either longer or shorter than fore sights, the error becomes cumulative. It may be noted that error due to non-adjustment of the level, is very common and of serious nature. The level must always be carefully tested and adjusted before it is used. Care should also be taken to ensure that back sight and fore sight distances are equal.

(*ii*) ***Defective level tube***. If the bubble of the level tube is sluggish, it will remain central even though the bubble axis is not horizontal. On the other hand, if it is too sensitive, considerable time is spent to bring the bubble central. Irregularity of coverature of the tube is also a serious defect. The effect of defective level tube also gets neutralized if the sights are equal.

(*iii*) **Shaky tripod.** A shaky tripod causes an instability of the instrument. It wastes a considerable time to make accurate observations. Every bolt and nut of the tripod and the screws of the foot shoes should be tightened before observations are made. Best check to test the stability of the tripod is to twist one of its legs after reading a staff and release it. Observe if the reading remains the same as before, if not, the tripod is not stable.

(*iv*) **Incorrect graduations of the staff.** If the graduations of a staff are not perfect, this error is caused. But in ordinary levelling the error may be negligible, because the readings are generally made only correct to 0.005 m. In case of precise levelling, the graduations should be compared against invar tape under magnification.

6.30. SENSITIVENESS OF A LEVEL TUBE

Definition. *The capability of a level tube to exhibit small deviation from the horizontal, is termed as the sensitiveness or the sensivity of the level tube.* Sensitiveness of a level tube is either designated in terms of the radius of creature of its upper portion or by the angle through which the axis is tilted to cause the bubble to move the smallest division of the engraved scale.

The sensitiveness of a level tube depends upon :

1. The radius of the curvature of the internal surface *i.e.*, larger the radius, greater the sensitiveness of the tube.
2. The diameter of the level tube *i.e.*, larger the diameter greater is the sensitiveness of the tube.
3. The length of the vapour bubble *i.e.*, greater the length bubble more is the sensitiveness of the tube.
4. The viscosity and surface tension of the bubble more.
5. Smoothness of the finish of the internal surface of the tube.

The sensitiveness of the tube should accord with that of the instrument on which it is mounted. One should not waste time in levelling the instrument unnecessarily.

6.31. MEASUREMENT OF THE SENSITIVENESS

To determine the curvature of the internal surface or the value of the angle for one division of the graduated scale, the following procedure is followed.

Procedure : Preceed as under :

1. Measure a 100 m base line on a level ground by a steel tape.
2. Set up the instrument at one end of the base line and hold the staff at the other end *i.e.*, at *P*.
3. Observe the staff reading with the bubble near one end of its run by means of the levelling screw beneath the telescope. Let the reading be *PA*.
4. Observe the staff reading with the bubble near the other end of its run by means of the same levelling screw beneath the telescope. Let the reading be *PB*
5. Determine the total run of the bubble *i.e.*, total number of graduations of the scale.

Let,

D = length of the base.

S = average length of the staff intercepted between the upper and lower lines of sight *i.e.* the difference of $PB - PA$.

n = number of divisions through which bubble is moved.

d = length of one division on the tube expressed in the same units as D and S.

R = radius of curvature of the tube.

The length through which the centre of the bubble moves $EF = n.d$.

From the segment *OEF,* we get

$$R\ \alpha = \text{arc } EF$$

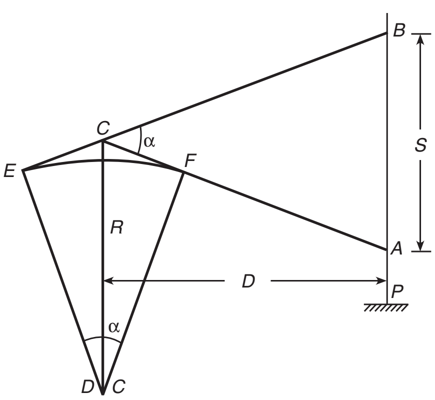


Fig. 6.43. Measurement of sensitiveness of level tube.

But $\text{arc } EF = EF$

$$\alpha = \frac{EF}{R} \qquad \text{...(6.14)}$$

From similar Δs *OEF* and *ACB*

$$\therefore \quad \frac{EF}{AB} = \frac{R}{D}$$

or

$$\frac{EF}{R} = \frac{AB}{D} = \frac{S}{D} \qquad \text{...(6.15)}$$

Combining Eqs. (6.14) and (6.15) we get

$$\alpha = \frac{EF}{R} = \frac{S}{D} \qquad \because \quad \alpha = \frac{nd}{R} = \frac{S}{D}$$

or

$$R = \frac{nd.\,D}{S} \qquad \text{...(6.16)}$$

Angular value for one division in radians

$$= \frac{\alpha}{n} = \frac{d}{R} = \frac{S}{Dn}$$

But, 1 radian = 206,265 seconds

∴ The angular value of one division

$$= \frac{S}{Dn} \times 206{,}265 \text{ seconds}$$

$$= \frac{d}{R} \times 206{,}265 \text{ seconds}$$

$$= \frac{d}{R \sin 1''} = \frac{S}{DN \sin 1''}$$

where 1 radian = 206,265 sec. and $\sin 1'' = \frac{1}{206{,}265}$

Example 6.17. *Bringing the bubble central, the reading taken on a staff 200 m from the instrument was 2,856 m. The bubble was then moved 5 divisions out of the centre and the staff reading observed was 2.806 m. If the length of one division of the bubble is 2 mm, calculate the radius of curvature of the bubble tube and also the angular value of one division of the bubble.*

Solution. Staff intercept *S* for a deviations of 5 division of the bubble. = 2.856 – 2.806 = 0.050 m.

(*i*) The radius of curvature $R = \frac{ndD}{S}$

Here $n = 5$; $d = 2$ mm and $D = 200$ m

$$R = \frac{5 \times 2 \times 200}{1000 \times 0.050} \text{ m.} = 40 \text{ metres.} \quad \textbf{Ans.}$$

(*ii*) The sensitivity of the bubble α' is given by

$$\alpha' = \frac{S}{nD} \times 206{,}265 \text{ seconds}$$

$$= \frac{0.05 \times 206,\,265}{5 \times 200} \text{ seconds} = 10.31 \textbf{ seconds Ans.}$$

Example 6.18. *If the bubble tube of a level has a sensitiveness of 30′ per 2 mm division. find the error in staff reading on a vertically held staff at a distance of 150 m, caused by the bubble 2.5 divisions out of centre.*

Solution. The angular value of one division of 2 mm length

$$\alpha' = \frac{S}{nD} \times 206,\,265 \text{ seconds} \qquad ...(i)$$

Here S = staff intercept = ?

$n = 2.5$ and $D = 150$ m

Substituting the values in equation (*i*), we get

$$S = \frac{\alpha' \, n \, D}{206{,}265} = \frac{30 \times 2.5 \times 150}{206{,}265} = 0.055 \textbf{ m. Ans.}$$

Example 6.19. *The reading taken on a staff held 100 m from the instrument with the bubble central is 1.786. The bubble is then moved 3 divisions out of centre and the staff reading is observed to be 1.817 m. Find the angular value of one division of bubble tube and the radius of curvature of the bubble tube, the length of one division being 2 mm.*

Solution. Staff intercept S for 3-division deviation of the bubble

$$= 1.817 - 1.786 = 0.031 \text{ m}$$

(*i*) The radius of curvature,

$$R = \frac{ndD}{S}$$

Here $n = 3$, $d = 2$ mm, $D = 100$ m

Substituting the values in eqn. (*i*) we get

$$R = \frac{3 \times 2 \times 100}{1000 \times 0.031} = \textbf{19.35 m. Ans.}$$

(*ii*) The angular value of one division

$$\alpha' = \frac{S}{nD} \times 206{,}265 \text{ sec} = \frac{0.031 \times 206{,}265}{3 \times 100} \text{ secs.}$$

= **21.31 seconds. Ans.**

Example 6.20. *For the measurement of the sensitivess of a bubble tube, the following data are observed.*

Bubble Reading		*Staff Readings*	*Remarks*
L.H.	*R.H.*		
14.0	6.0	1.587	Distance from inst. staff = 50 m
7.0	13.0	1.663	Length of one division = 2 mm.

Calculate the angular value of one division.

Solution. The deviation of the bubble from the centre

$n = 14 - 7 = 7$ divisions

Again, $n = 13 - 6 = 7$ divistions.

The staff intercept $= 1.663 - 1.587 = 0.076$

$$\therefore \quad \alpha' = \frac{S}{nD} \times 206{,}265 = \frac{0.076 \times 206{,}265}{7 \times 50}$$

= **44.79 seconds. Ans.**

6.32. PRINCIPLE OF REVERSAL

The testing of instruments is commonly based on the principle of reversal which states that *if there exists any error in a certain part, it gets doubled by reversing the position of that part i.e. revolving through* 180°, the apparent error on reversal also gets doubled as it is evident from Fig. 6.44.

Let ABC be a set square whose right angle ABC is $e°$ less than 90°. Reversing the set square such that A occupies A', C occupies C' and B remains unchanged, the angle of error becomes CBC' which is evidently equal to 2e°.

Proof. $\angle ABC = 90° - e°$

$\angle ABC = \angle A'BC'$ being the same angle

$\therefore \quad \angle A'BC' = 90° - e°$

$\therefore \quad \angle CBC' = 180° - 2\,(90° - e°), = 2e°.$

assuming ABA' to be in a straight line

The principle of reversal also suggests that even though the instrument is out of adjustment, accurate results may be obtained by reversing and taking the mean of the two observations.

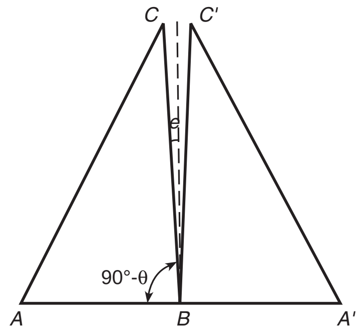


Fig. 6.44. Principle of reversal.

6.33. PERMANENT ADJUSTMENTS OF A LEVEL

Though, all the instruments are properly adjusted by the manufacturers, their fundamental lines get disturbed due to mishandling of the instruments in the field. To establish the fixed relationships between the fundamental lines of a level, is the main object of the permanent adjustments.

Fundamental lines of level. The fundamental lines of a level are:

1. The axis of the bubble tube.
2. The vertical axis.
3. The axis of the telescope.
4. The line of collimation.

Desired relationship of the fundamental lines : The following relationship must be obtained.

1. The line of collimation should be parallel to the axis of the bubble tube. This is applicable to every type of levelling instruments.
2. The axis of the telescope and line of collimation should coincide.
3. The axis of the bubble tube should be perpendicular to the vertical axis of the level.

Methods of adjustments of Dumpy level, Y-level and Tilting level.

1. Adjustment of a Dumpy Level. The following permanent adjustments are made :

(*a*) To make the axis of the bubble tube perpendicular to vertical axis of the level.

(*b*) To make the line of collimation parallel to the axis of the bubble tube.

Adjustment 1. To make the axis of the bubble tube perpendicular to the vertical axis of the level

Test. Proceed as under.

1. Set up the level on firm ground and level it. The bubble now remains central only in two positions *i.e.*, parallel to a pair of foot screws and over the third foot screw.
2. Turn the telescope through 180° in azimuth so that the ends of the telescope are reversed.
3. If the bubble still remains central, the instrument is in perfect adjustment.

Adjustment may be done as explained under :

(*i*) If the bubble does not remain central, note down the deviation of the centre of the bubble.

(*ii*) Bring the bubble half way-back by means of the capstan nut and the other half with the foot-screw beneath the telescope.

(*iii*) Turn the telescope through 90° so that it becomes parallel to the pair of the foot screws. Bring the bubble central by means of this pair of foot screws.

(*iv*) Rotate the telescope and observed if the bubble remains central. If not, repeat the whole process until the adjustment is correct.

Adjustment 2. To make the line of collimation parallel to the axis of the bubble tube

This adjustment may be made by the following two peg method.

Method (A) Let the line of collimation of level be inclined by $e°$.

Procedure. Proceed as under.

(*i*) Fix two pegs *A* and *B* on a fairly level ground, at a distance 2 *D* approximately 100 metres apart.

(*ii*) Set up the level at *C*, the mid point of the line *AB*. (Fig. 6.45*a*.)

(*iii*) Observe the readings *x* and *y* on the staff held vertically at *A* and *B* respectively, ensuring that the bubble of the tube remains central while taking the readings.

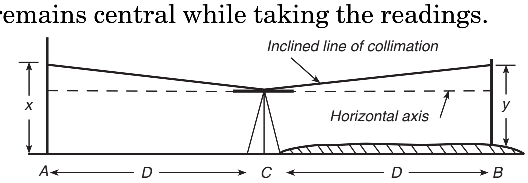


Fig - 6.45 (a).Ist position

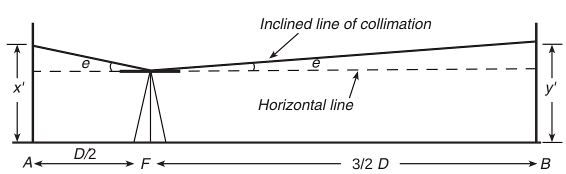


Fig - 6.45 (b).IIst position

Fig. 6.45(a). 1st position.

(*iv*) Shift the level to E, at a distance $D / 2$ from A and $3\,D / 2$ from B. [Fig. 6.45 (*b*)].

In this position, the error in reading the staff

$$\text{at } A = \frac{D}{2} \tan e^\circ \text{ ; at } B = \frac{3D}{2} \tan e^\circ$$

(*v*) Observe the readings x' and y' on the staffs A and B respectively.

(*vi*) Calculate the true and apparent differences in level from the observations at two positions. Let these be a and b respectively.

$$\therefore \quad a - b = D \tan e^\circ \qquad \text{...(6.17)}$$

or

$$\tan e^\circ = \frac{a-b}{D}$$

or

$$e^\circ = \tan^{-1} \frac{a-b}{D} \qquad \text{...(6.18)}$$

(*vii*) In second position, the error in the staff reading at B

$$= \frac{3D}{2} \tan e^\circ = \frac{3}{2}(a-b) \qquad \text{from eqn. (6.17)}$$

(*viii*) Multiply the value of $D \tan e^\circ$, or $(a - b)$ by 3/2 and depress / raise the telescope by this amount. Now the bubble goes out of central position.

(*ix*) Adjust the bubble with the help of the capstan screw and repeat the process for check.

Method B. (Fig. 6.46)

Procedure. Proceed as under,

1. Choose two points A and B on a fairly level ground say 100 m apart.
2. Set up the level near the point A on the line AB such that its eye-piece almost touches the staff held at A.

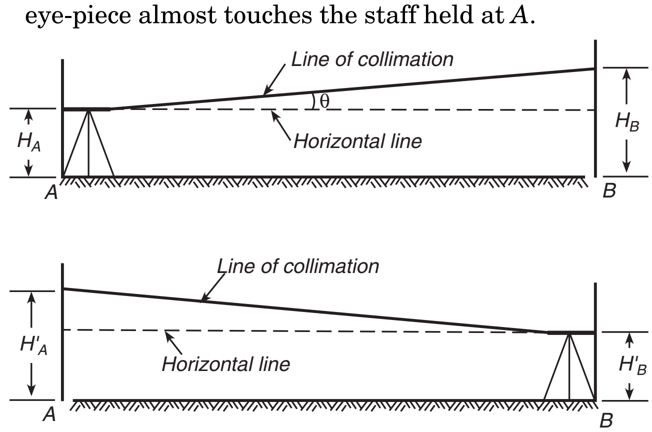


Fig. 6.46. Two peg method

3. Take the reading on staff A through the objective by sliding a pointed pencil along the staff. This reading H_A is taken as a correct reading on staff A.
4. Take a reading H_B on the staff held at peg B.
5. Calculate the apparent difference in level between A and B *i.e.*, $H_A - H_B$, assuming B to be higher.
6. Shift the level near the point B and set it in a similar manner as in step 2.
7. Take reading H'_B on the staff B through the objective and reading H'_A on A through the eye piece.
8. Calculate the apparent difference is level between A and B *i.e.*, $H'_A - H'_B$.
9. If the two apparent differences in level are the same, the instrument is in perfect adjustment. If not, calculate the correct difference in level as in the case of reciprocal levelling *i.e.*, correct difference in elevation.

$$H = \frac{(H_A - H_B) + (H_A' - H'_B)}{2}$$

10. Keeping the instrument near B, calculate the correct staff reading A *i.e.*, Correct reading on staff A = Reading on staff $B + H = H'_B + H$.
11. Loosen the capstan screw so that the central horizontal wire reads the calculated reading as obtained in step (10).
12. Repeat the test for checking.

Note. The following points may be noted.

(*i*) If the computed value of H is positive, the point of staff B is higher than A. If it is negative, B is lower than A.
(*ii*) Use proper sign while calculating the correct reading in step (10).
(*iii*) Readings taken through the objective are taken as correct.

Method (C) (Fig. 6.47)

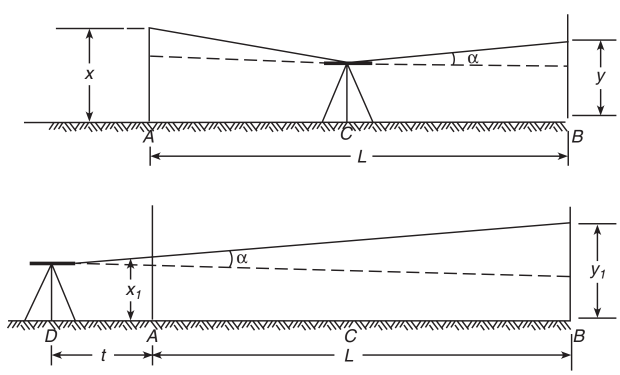


Fig. 6.47. Two-peg method.

Procedure : Proceed as under :

1. Fix two pegs A and B at a distance L on a fairly level ground.
2. Set up the level exactly at the mid point C of AB. Take the staff readings on A and B, keeping the bubble central of its run. Let the staff readings be X and Y respectively.
3. Shift the level to a point D at a distance l from A on the line BA produced.
4. Read the staff readings on A and B. Let the staff readings be X_1 and Y_1 respectively.
5. Calculate the true difference of level by subtracting the reading X from the reading Y when the level was at C.
6. Calculate the apparent difference of level by subtracting the reading X_1 from the reading Y_1.

   If the two differences are equal, the instrument is in adjustment.
7. If not, find whether there is a rise or a fall from peg A to B. If X is greater than Y, B is higher than A and *vice versa*.
8. Calculate the reading Y_2 on the peg B at the same level as of X_1 *i.e.*.,

   Reading on the peg

   $$B = \text{Reading on } A \pm \text{True difference}$$

   i.e. $\quad Y_2 = X_1 \mp$ True difference [+ for fall and – for rise]
9. If Y_1 is greater than Y_2, the line of collimation is inclined upwards. If Y_1 is smaller than Y_2, the line of collimation is inclined downwards.
10. Calculate the net collimation error in the distance L *i.e.*

    Collimation error in distance L

    $$= Y_1 - Y_2$$

    Collimation error per unit distance

    $$= \frac{Y_1 - Y_2.}{L}$$
11. Calculate the required correction for the readings on pegs A and B *i.e.*,

    Collimation correction to the reading on the peg A

    $$= \frac{I}{L}(Y_1 - Y_2)$$

    Collimation correction to the reading on the peg B

    $$= \frac{I + L}{L}(Y_1 - Y_2)$$

12. Add the collimation correction to the staff readings if the line of sight is inclined downward and subtract it, in case it is inclined upwards.
13. Loosen the capstan headed screw and shift the horizontal wire to read the corrected reading of B. Check the reading on A whether it agrees with the calculated reading. Repeat the adjustment, if found necessary.

2. Adjustment of a Y-level. As the telescope of a Wye level can be lifted from the collars during adjustment, one more geometrical condition is to be fulfilled in addition to those required for a dumpy level. The fundamental relationship between different axes to be established, are :

(*i*) The line of collimation should coincide with the axis of collars. It is the manufacturer's responsibility to ensure that the axis of the collars, is also coincident with the optical axis.
(*ii*) The line of collimation of the telescope should be parallel to the axis of the bubble tube.
(*iii*) The axis of the bubble tube and the vertical axis of the level should be mutually perpendicular.

1. Adjustment of the line of collimation. Aim. *To make the line of collimation coincide with the axis of the collars.*

Necessity. This adjustment is very important. The telescope should be clipped in one position. If not, the rotation of the line of collimation about its axis would generate a cone. The fundamental requirement of parallelism between the axis of the bubble tube and the line of collimation is obtained only for a particular position of the telescope in the collars.

Test and Adjustment

(*i*) Set up the level firmly on the ground.
(*ii*) Sight a definite small mark on a wall at a distance approximately 50 to 100 metres.
(*iii*) Fix both the clamps.
(*iv*) Loosen the clips and rotate the telescope through 180° about its longitudinal axis.
(*v*) Sight the point again, if the line of sight still strikes the same point, the instrument is in perfect adjustment. If not, it requires adjustment.
(*vi*) By means of the four capstan screws, make the intersection of hairs half-way towards the point.
(*vii*) Repeat the test and adjustment until correct.

Principle of the test and adjustment. (Fig. 6.48).

Let, the line of the collimation be inclined by $e°$ upward. The mark sighted will be at P_1. After giving the 180° rotation, the line of collimation inclined upwards will now be inclined downwords by the same amount and the point sighted by the line sight of will be at P_2.

The apparent error is therefore twice the actual. Hence, the necessary correction to be applied is only half the apparent error.

2. Adjustment of the level tube

Aim. *To make the axis of level tube parallel to the line of collimation.*

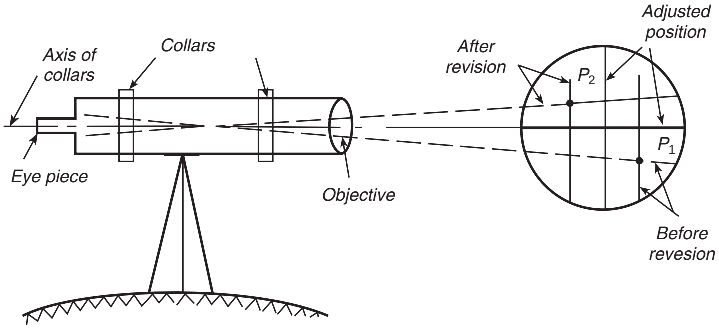


Fig. 6.48. Adjustment of the line of collination.

The main function of a level is to furnish a horizontal line of collimation. It is obtained only if the axis of the bubble is parallel to the line of collimation, when the bubble of the level tube is at the centre of its run.

Test and Adjustment. This adjustment is carried out is under :

First step. *To bring the axis of the bubble tube in the plane of the collimation.*

(*i*) Level up the instrument carefully.
(*ii*) Rotate the telescope in the collars through a small angle. If the bubble remains central, the adjustment is correct.
(*iii*) If not, bring the bubble central by adjusting the screw and controlling the level tube laterally.
(*iv*) Repeat the test and adjustment till the required condition is obtained.

Second step : *To make the axis of the bubble parallel to the line of collimation.*

(*i*) Set the telescope parallel to any pair of levelling screws.
(*ii*) Open the collars and level the instrument carefully.
(*iii*) Lift the telescope from the collars and replace it end to end.
(*iv*) If the level remains central, the adjustment is correct.
(*v*) If not, move the bubble half-way towards the centre by the screw provided at one end of the level tube.
(*vi*) Repeat the test till the required condition is achieved.

3. Adjustment of perpendicularity of the vertical axis and axis of the level tube

Aim. To ensure that the instrument once levelled, the bubble remains central for all positions of the telescope.

Necessity. The adjustment is not very important. It is carried out only for the surveyor's convinience in doing the level as under :

(*i*) Set up the instrument on firm ground.

(*ii*) Bring the level tube parallel to a pair of levelling screws.

(*iii*) Turning both the screws at the same time, bring the bubble central.

(*iv*) Rotate the telescope through 90°. With the help of the third levelling screw, bring the bubble central.

(*v*) Rotate the telescope through 180°. If the bubble remains central, the adjustment is correct.

(*vi*) If not, bring the bubble half-way back by the capstan screw.

(*vii*) Level up and repeat the test and adjustment till the required condition is obtained.

3. Adjustment of a Tilting Level. In tilting levels only one adjustment is of prime importance *i.e.*, the line of collimation is horizontal when the bubble is central at its run.

Test and adjustment. Proceed as in the case of a dampy level.

(*i*) Calculate the correct staff reading.

(*ii*) Raise or lower the line of sight with the help of the tilting screw to read the calculated reading.

(*iii*) Now, the bubble becomes out of the centre.

(*iv*) Bring the bubble central with the help of the capstan screw provided at one end of the level tube.

(*v*)

Repeat the test and adjustment till the instrument is perfectly adjusted.

Example 6.21. (*a*) *In a two peg test of a dumpy level, the following readings were taken:*

(*i*) *Instrument at C, mid-way*	*Staff reading on A* = 1.585
between A and B, AB = 100 m	*Staff reading on B* = 1.225
(*ii*) *Instrument near A*	*Staff reading on A* = 1.425
	Staff reading on B = 1.150

Is the line of collimation inclined upwards or downwards and by how much ? With the instrument at A what should be the staff reading on B in order to place the line of collimation truly horizontal.

Solution. (Fig. 6.48)

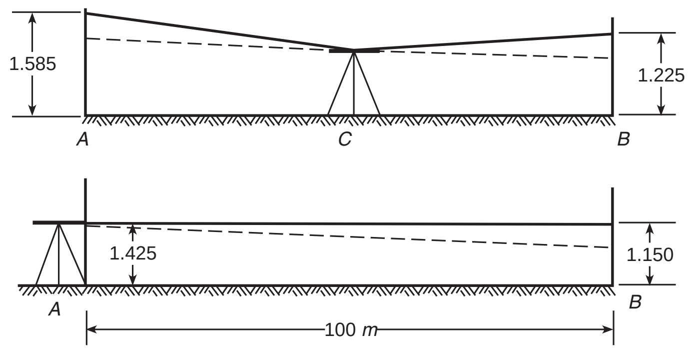


Fig. 6.48.

(*b*) **Level at mid-point C**

Staff reading on A = 1.585 m

Staff reading on B = 1.225 m

True difference of level = 0.360 m, a size from A to B

Level at A :

Staff reading on A = 1.425 m correct reading unaffected by error of collimation

Subtract true rise = **0.360 m**

True staff reading on B = 1.065 m

Observed staff reading on B = 1.150 m

As the observed staff reading is more than the required true staff reading, line of collimation is inclined upwards.

The collimation error = 1.150 – 1.065 = **0.085 m** **Ans.**

Staff reading on B which makes the line of collimation truly horizontal

= 1.150 – 0.085 = **1.065 m.** **Ans.**

Example 6.22. *Following observations were taken for testing of a dumpy level :*

(*i*) *Instrument exactly at the mid-point of line AB*

Staff reading at station A = 1.855

Staff reading at station B = 1.605

(*ii*) *Instrument very near to station B*

Staff reading at station A = 0.675

Staff reading at station B = 0.925

Find out from the above observations whether the line of collimation is in adjustment or not. If it is not in adjustment what is the nature and amount of the error in distance AB ? What will be the correct readings on staff at A and B from station B when the line of collimation is adjusted.

Solution. Instrument at mid point. True difference in level between A and B = 1.855 – 1.605

= 0.250 B being higher.

Instrument an station B.

Correct reading on B = 0.925

Correct reading on A = 0.925 + 0.250 = 0.675

Observed reading on A = 0.675

Collimation error = 1.175 – 0.675 = 0.500

As the observed reading is less than true reading, the line of collimation is inclined downwards.

Example 6.23. *While carrying out the permanent adjustment of a Dumpy level by two peg-method, the following observations were made :*

Instrument at E, midway between points C and D, 100 m apart
Reading at point C = 2.00 m.
Reading at point D = 3.00 m.
Instrument at peg F in line of CD such that CF = 120 m and DF = 20 m.
Reading at point C = 1.50 m
Reading at point D = 2.75 m
Check whether the instrument needs permanent adjustment or not and whether the line of sights is inclined upwards. What should be the correct reading at C if the instrument is to be adjusted ?

Solution. (Fig. 6.49).
Level at mid-point E (Fig. 6.49*a*)

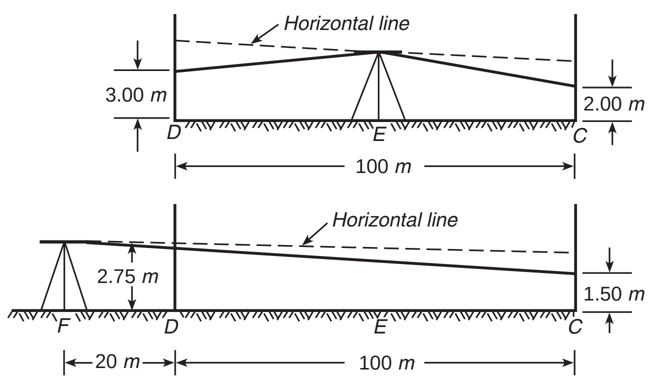


Fig. 6.49.

Staff reading on $D = 3.00$ m

Staff reading on $C = 2.00$ m

$\therefore$ True difference of level

$= 1.00$ m, C being higher

Level at point F (Fig. 6.49*b*)

Staff reading on $D = 2.75$ m

Staff reading on $C = 1.50$ m

$\therefore$ Apparent difference in level

$= 1.25$ m, C being higher.

The staff reading on the point C at the level of D

= Reading on

D – True difference in level

$= 2.75 - 1.00 = 1.75$ m

As the observed reading 1.50 m is less than the calculated value 1.75m the line of collimation is inclined downwards.

The net collimation error in 100 m

$= 1.75 - 1.50 = 0.25$ m

$\therefore$ Correction to the reading on point C

$$= \frac{l + L}{L}\ (y_1 - y_2) = \frac{120}{100} \times (1.75 - 1.50)$$

$= 0.30$ m

$\therefore$ Correct staff reading on the point C

= Observed reading + correction

$= 1.50 + 0.30 =$ **1.80 m . Ans.**

Example 6.24. *The distance between two bench marks. A and B was 40 m. A dumpy level was placed at C on an extension of AB such that AC = 60 m. The following data was recorded :*

Staff on B.M. at A. (R.L. 10.750) 0.750

Staff on B.M. at B. (R.L. 11.750) 1.750

(*i*) *Was the line of collimation inclined upwards or down words and by how much ?*

(*ii*) *Calculate the readings that should be obtained on A and B to have a horizontal line of sight.*

(*iii*) *State in what direction and how the diaphragm has to be moved for adjustment.*

Solution.

(*i*) The true difference in R.L. of Bench marks

= 11.750–10.750 = 1.000 m, *B* being higher

The true reading on B.M. *A* at the reading of *B*

= 1.750 + 1.000 = 2.750 m

The observed reading on B.M. *A* = 0.750 m

∴ Difference in readings = 0.750–2.750 = – 2.000 m

As the difference is negative, the collimation is inclined downwards.

The correction for staff at $A = \frac{60}{40} \times 2.000 = 3.000$ m

The correction for staff $B = \frac{20 \times 2.000}{40} = 1.000$ m

The corrected reading for $A = 0.750 + 3.000 = 3.750$ m

The corrected readings for $B = 1.750 + 1.000 = 2.750$ m **Ans.**

Example.6.25. *A level is set up at O on a line AB, 45 m from A and 1250 m from B. The back sight on A is 0.458 m and the fore sight on B is 3.125 m. Determine the true differences of level between A and B.*

Solution. As the length of line of collination is 1250, we need to apply the correction for curvature and refraction to the observed fore sight reading (*i.e.* 3.125 m) whereas no correction is required for the back sight reading (*i.e.* 0.458), since the distance to staff position of *A* is 45 m. (Fig. 6.50.)

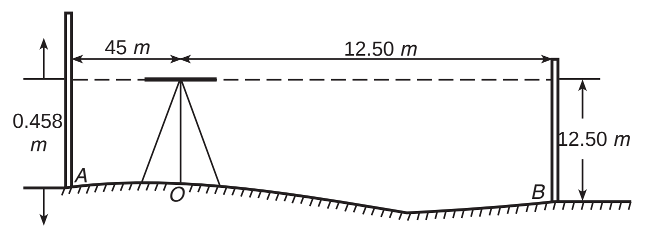


Fig. 6.50.

Combined correction for 1250 m $= 0.673\left(\frac{1250}{1000}\right)^2 = 0.105$ m

Corrected reading on B. = observed reading – Correction

= 3.125 – 0.105 = 3.020 m

Hence, true difference of level between *A* and *B*

= 3.020 – 0.458 = 2.562 m, a fall from *A* to *B*.

Example. 6.26. *A level is set up at O, on line CD. The reading on staff held at C, 450 m from O, is 2.315 and on D 580 m from O, is 3.875*m. *Find the true difference of level between C and D.*

Solution. Correction for curvature and refraction for 450 m

$$= 0.0673\left(\frac{450}{1000}\right)^2 = 0.014 \text{ m}$$

Correction for curvature and refracton for 580 m

$$= 0.0673\left(\frac{580}{1000}\right)^2 = 0.023 \text{ m}$$

Corrected staff reading on $C = 2.315 - 0.014 = 2.301$ m

Corrected staff reading on $D = 3.875 - 0.023 = 3.852$ m

$\therefore$ True difference in level between C and D.

$$= 3.852 - 2.301 = 1.551 \text{ m}$$

i.e., Point D is 1.551 m lower than point C. **Ans.**

Example. 6.27. *Find the distance to the visible horizon from the top of a light house 60 m high. Also, find the dip of the horizon if the radius of the earth is 6371 m.*

Solution.

(i) Distance of the visible horizon

$$D = \sqrt{\frac{h}{0.0673}} \qquad ...(i)$$

Substituting the value of $h = 60$ m, in eqn (i) we get

$$D = \sqrt{\frac{60}{0.0673}} = 29.86 \text{ km}$$ **Ans.**.

(ii) The dip of the horizon $= \dfrac{D}{R}$ radian

$$= \frac{29.86}{6371} \times \frac{180}{\pi} \times 60 = 16.15 \text{ minutes.}$$ **Ans.**

Example. 6.28. *An obsever at a height of 50 m above mean sea level just sees the flash light on the top of a light house. The distance between the observer's station and the light house is 60 Km. What is the height of the flash light?*

Solution. Given: $A' B' = D = 60$ *km*.

Let the line of sight touch the mean sea level at C such that distance $A'C = D_1$ Km and distance $B'C = D_2$ Km.

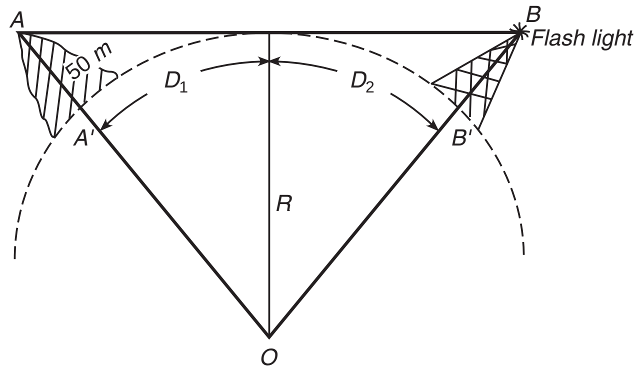


Fig. 6.49.

Let the height of the flash light above MSL be 'h' m. Applying the formula, $h = 0.0673D^2$ we get

$$50 = 0.0673\, D_1^2$$

or üØ

Now, $D_2 = 60 - D_1 = 60 - 27.26 = 32.74$ km

$\therefore$ Height of flash light above MSL

$$= 0.0673\, (32.74)^2 = 72.14 \text{ m} \quad \textbf{Ans.}$$

6.34. BAROMETRIC LEVELLING

Barometric levelling is based on the fact that atmospheric pressure varies inversely with elevation. The pressure at sea level is maximum and it decreases as the height increases. The decrease in pressure for a rise of every 900 ft, is approximately one inch of mercury. (108 m is approximately one cm of mercury) The instrument which is used to measure atmospheric pressure, is known as a *barometer*.

There are two types of barometers :

(1) Mercurial barometer. (2) Aneroid barometer.

1. The Mercurial Barometer (Fig. 6.50)

It consists of a glass tube about one metre long, sealed at the top and immersed in a cistern containing mercury. The mercury rises in the tube due to atmospheric pressure and a vacuum is created in its upper portion. The tube is graduated in centimetres and its decimal parts.

In some mercurial barometers, verniers are used to read the height of the mercury column with a finer accuracy.

A mercurial barometer is a cumbersome apparatus. It can not be easily transported in the field and hence, its use in limited to fixed stations only. It is not much used for survey work.

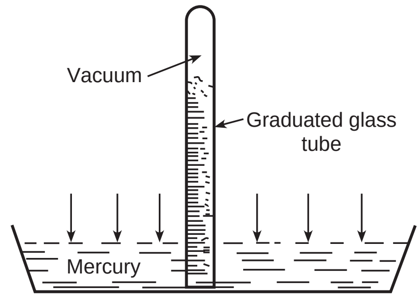


Fig. 6.50. A Mercurial Barometer.

2. The Aneroid Barometer. (Fig. 6.50)

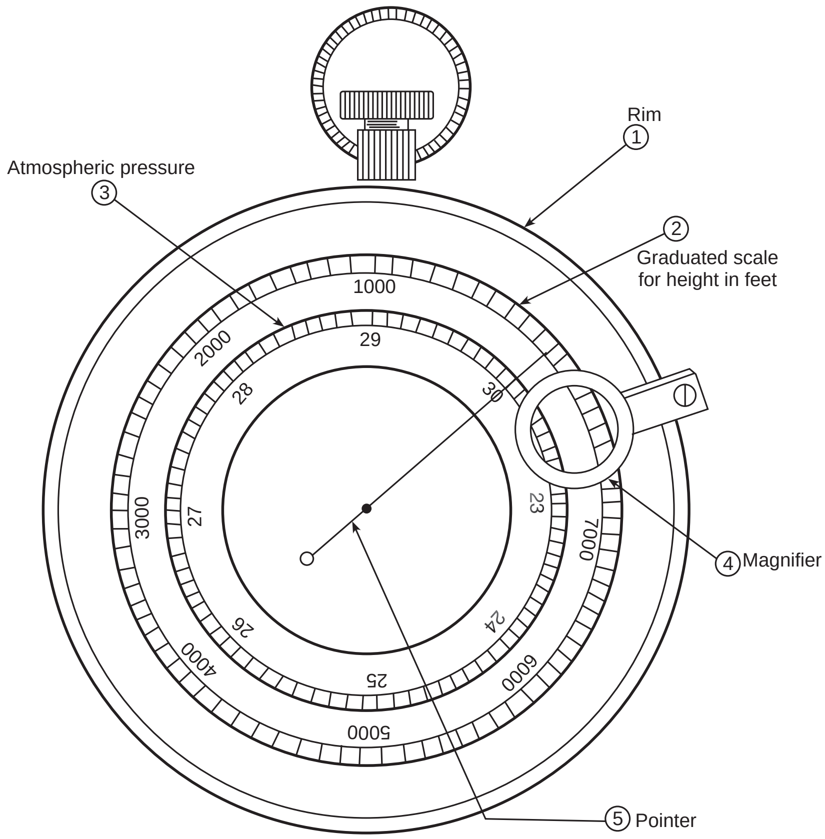


Fig. 6.51. Section of an aneroid barometer.
1. Circular box 2. Altitude graduations
3. Pressure graduations 4. Magnifiers 5. Pointer.

It consists of a thin cylindrical box about 7.5 cm to 12.5 cm in diameter hermetically sealed and from which air is partially exhausted. The ends of the box are corrugated in circular corrungation. These are sensitive to the variations of the atmospheric pressure. The movement of the thin surface of the box, caused due to changes of atmospheric pressure, is transmitted through a system of levers and a spring to a pointer which moves over a graduated dial. The essential parts of an aneroid barometer are shown in Fig. 6.50.

An aneroid barometer though less accurate than a mercurial barometer, is generally used for survey work. It is portable, convenient and light in weight to carry from place to place.

6.35. METHODS OF BAROMETRIC LEVELLING

There are two methods of levelling with a barometer as discussed below.

1. Single observation method. The procedure involved for barometric levelling by single observation method is as follows :

(*i*) Take the reading of a barometer under shade at a station of known elevation.
(*ii*) Take the atmospheric temperature also under shade at the station.
(*iii*) Shift the barometer to the next station for observation and record the pressure and temperature under shade.
(*iv*) If the stations are situated on a circular route, it is better to reobserve the readings of the starting point at regular intervals to find out the rate of change of atmospheric pressure and to correct the intermediate readings against atmospheric variations.

This method is not accurate. The readings involve errors due to the changes in the atmosphere, which take place during the interval between the observations.

2. Simultaneous observations method. In this method two barometers are used at the same time at both the stations. The barometer which is used in the field, is called *field battery* and the other one which is used at the base station, is called a *base battery*.

The following procedure is followed.

(*i*) Both the barometers are read at the base station *i.e.* the starting B.M. to find the difference of readings, if any.
(*ii*) Time of observation and atmospheric temperature under shade, are recorded.
(*iii*) Continuous observations of pressure and temperature are made by the base party at regular interval of time. Time, temperature and pressure are recorded for each set of readings.
(*iv*) Observations are taken by the field party at other stations whose elevations are to be determined either simultaneously with base battery or at any convenient time.

(*v*) In case the observations are not simultaneous, the time of observations is also recorded so that proportionate pressure may be computed later.

Note. The accuracy may be improved by reading two or more barometers at the base as well as in the field and the mean observation accepted for computation.

Advantages of simultaneous observations method. Method of simultaneous observation is more accurate as the effects of atmospheric conditions, are eliminated.

Precautions. The following precautions must be taken while reading an aneroid barometer.

(*i*) It should always be carried in the same position.
(*ii*) It should be read in the same position generally horizontal.
(*iii*) Five to ten minutes are to be taken to attain the pressure before recording a reading.
(*iv*) Before taking a reading, tap the glass plate.
(*v*) Readings should always be taken in shade.
(*vi*) Parallax should be removed perfectly.
(*vii*) Reading should be recorded after proper estimation.

6.36 BAROMETRIC GRADIENT

The inclination of the plane passing through the places of equal pressure to the horizontal, is called *barometric gradient.*

6.37 CORRECTION TO BAROMETRIC LEVELLING

Following corrections are made.

(1) Index error of the barometer. (2) The relative index error of field battery and base battery is calculated. If the difference is small, the mean may be accepted otherwise according to time of observations, the value is interpolated.

Index Error = field battery reading -base battery reading.

Specimen field book for barometric observations

Station	*Barometer*						*Temp.*		*Remarks of atomospheric conditions*
	Time of obsn.	*1050*	*IS*	*1048*	*IC*	*Mean reading*	*Dry F°*	*Wet F°*	
A	0830	26.41	–02	26.55	+02	26.48	70°	57°	Calm and clear weather
B	0900	26.71	–01	26.87	+1	26.79	67°	58°	"
C	0925	26.81	–02	26.99	-00	26.79	71°	60°	"

D	0942	26.91	–01	27.09	-01	27.00	68°	59°	"
E	0905	27.10	–10	27.19	-02	27.13	70°	63°	"
F	1025	27.14	–01	27.30	-01	27.21	79°	63°	"
G	1047	27.27	–01	27.44	-02	27.34	70°	63°	"
H	1105	27.33	–01	27.50	-02	27.40	72°	67°	"
I	1525	27.40	–02	27.58	+02	27.48	78°	68°	"
J	1550	27.35	–03	27.52	+00	27.42	74°	65°	"
K	1608	27.35	–03	27.52	+00	27.42	77°	65°	"
L	1640	27.26	–04	27.42	-00	27.32	75°	64°	"
M	1730	27.04	–04	27.20	+02	27.11	67°	64°	

6.38 BAROMETRIC HEIGHT COMPUTATIONS

Principle of a barometer. Consider two points B and C on the surface of mercury in a dish, point B being inside the glass (Fig. 6.50.)

The pressure at B is due to mercury column of height h.

$= h.\ \rho g$. where ρ is the density of mercury.

The pressure at C is the atmospheric pressure

Since B and C being at the same horizontal level of the mercury,

The atmospheric pressure $= h\rho g$.

As the value of ρ and g remain constant at a place, the atmospheric pressure is proportional to the height of the mercury column.

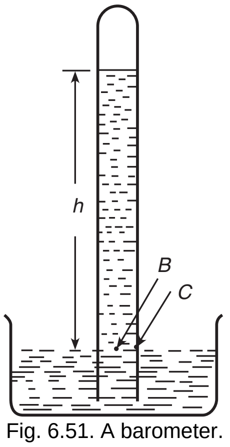


Fig. 6.51. A barometer.

Derivation of barometric height formula

Let ρ_a = density of air at point A.

ρ = density of air at any point say B.

h_a = height of mercury column of the barometer at any point.

L = height of the homogeneous atmosphere (assuming air density to be) constant throughout having a value ρ_a).

P = pressure at A in absolute units.

g = acceleration due to gravity.

h_1 = barometric reading in centimetres at the lower station A.

h_2 = barometric reading in centimetres at the higher station B.

where $$p = H\,\rho_a\,g = h\rho g$$

or $$H\rho a\,g = h.\rho g \qquad ...(6.19)$$

or $$H = \frac{\rho h}{\rho a.} \qquad ...(6.20)$$

If g is taken constant, $\frac{\rho}{\rho_a}$ is also constant by Boyel's law.

Hence the height of the atmosphere is also constant.

Let δH = a small difference in altitude at B above A.

and δh = correspoinding change in the harmonic reading at B

Substituting the values in Eqn. (6.19), we get

$$(h - \delta h)\,\rho.g = (H - \delta H)\rho_a g \qquad [\because h\rho_g = H\rho_a\,.\,g]$$

or $$\delta h\rho = \delta H.\rho a$$

$$\delta H = \frac{\delta h.\rho}{\rho_a}$$

$$\delta H = \frac{\delta h.H}{h}$$

$$\left[\because \frac{\rho}{\rho a} = \frac{H}{h} \text{ from Eqn. (6.19)}\right]$$

Integrating both the sides of the eqn.

$$H = H\int_{h_2}^{h_1} \frac{dh}{h} = H[\,\log_e h_1 - \log_e h_2] \qquad ...(6.20)$$

Reducing this expression to common logarithm and substituting the value of H from Eqn. (6.20) we get

$$H = 60158.6\,(\log_{10} h_1 - \log_{10} h_2) \text{ ft at } 32°\text{ F and } 45°\text{ Lat.} \qquad ...(6.21)$$

But, the density and pressure of air vary with the temperature, and hence, the formula after temperature correction reduces to

$$H = 60158.6\,(\log_{10} h_1 - \log_{10} h_2)\left(1 + \frac{t_1 + t_2 - 64°}{900}\right)\text{ft.}$$

Where t_1 = temperature of atmosphere at A in degrees Fahrenheit.

t_2 = temperature of atmosphere at B in degrees Fahrenheit.

When metric units are used and temperatures recorded in degree centigrades,

$$H = 18336.6\ (\log_{10}h_1 - \log_{10}h_2)\left(1 + \frac{T_1 + T_2}{500}\right) \text{metres}$$

Laplace's formulae of barometric heights

$$H = 60158.6\ (10h_1 - \log_{10}h_2)\left(1 + \frac{t_1 + t_2 - 64°}{900}\right) \times$$

$$(1 + 0.002695 \cos 2\Psi)\ \left(1 + \frac{h_1 + h_2}{R}\right) \text{ft.}$$

or
$$H = 18336.6\ (\log_{10}h_1 - \log_{10}h_2)\left(1 + \frac{T_1 + T_2}{500}\right) \times$$

$$(1 + 0.002695 \cos 2\phi)\left(1 + \frac{h_1 + h_2}{R}\right) \quad \text{metres}$$

where letters have their usual meanings.

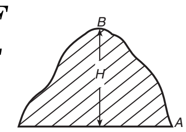
As the corrections $(1 + 0.002695 \cos 2\ \phi)$ and $\left(1 + \frac{h_1 + h_2}{R}\right)$ are unnecessary refinements, these corrections are generally ignored.

Example 6.29. *Find the reduced level of a point B from the following simultaneous observations.*

Barometric reading at A = 27.55 *inches*, *temp*. 75° *F*

Barometric reading at B = 26.50 *inches temp*. 76° *F*

Reduced level of A = 1885 *ft*.



Solution.

From the F.P.S. formula for barometric heights, we get

$$H = 60158.6\ (\log_{10}h_1 - \log_{10}h_2)\left(1 + \frac{t_1 + t_2 - 64°}{900}\right) \text{ft.}$$

$$= 60158.6\ (\log_{10} 27.55 - \log_{10} 26.50)\left(1 + \frac{75° + 76° - 64°}{900}\right)$$

$$= 60158.6 \times 0.0169 \times 1.0967 \text{ ft.} = 1115 \text{ ft.}$$

$\therefore$ Reduced level of B

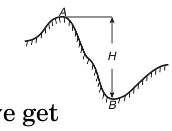
= Reduced level of $A + H$ = 1885 + 1115 = 3000 ft. **Ans.**

Example 6.30. *Find the reduced level of a point B in metres from the following simultaneous observations.*

Barometric reading at A = 69.98 cm, temp. 23.9° C

Barometric reading at B = 67.31 cm, temp. 24.4° C

Reduced level of A = 1254.25 m.



Solution. Given: h_1 = 69.98 cm; h_2 = 67.31 cm

From the metric formula for barometric height, we get

$$H = 18336.6\ (\log_{10} h_1 - \log_{10} h_2)\left(1 + \frac{T_1 + T_2}{500}\right) \text{ metres.}$$

$$= 18336.6\ (1.8450 - 1.8281)\left(1 + \frac{23.9 + 24.4}{500}\right)$$

$$= 18336.6 \times 0.0169 \times 1.0966 = 339.82 \text{ m}$$

R.L. of B = R.L. of $A - H$ = 1254.25 + 339.82

= 914.43 metres. **Ans.**

6.39 HYPSOMETRIC LEVELLING.

The reduced levels of various points at far distances in hilly terrain may be determined by using a thermo-barometer also known as a *hypsometer*. This method is based on the following principle:

"*A liquid boils when its vapours pressure equals the atmospheric pressure*."

The boiling point of a liquid is the temperature at which it starts to boil *i.e.*, starts to bubble up and turn into vapour. The boiling point of water depends upon the pressure to which it is subjected. The boiling point of water decreases as the altitude of a place increases *i.e.*, the atmospheric pressure decreases there. In this method, boiling temperatures are noted at the places where reduced levels are to be determined.

A hyposometer. It is a thermometer graduated to 1/5 degree of Fareinhiet or 1/10 degree of Centrigrade. At the place of observation, it is held vertically by a telescopic tube and suspended over a small boiler filled with rain water which is boiled by a spirit lamp. The lower end of the thermobarometer is so adjusted that it is just above the water surface, and records the temperature of the steam. The temperature of the air in immediate surrounding is measured with another thermometer.

The boiling point of pure water at sea level is 100°C (212°F) when the atmospheric pressure is 76 cm (29.92 inches) and the atmospheric air temperature is 0°C (32°F).

Barometric height or pressure in cm of mercury

$$= 76.00 \pm 2.679\ t_1$$

where t_1 = the difference of boiling point from 100°C.

From these barometric heights, the difference in levels between the points may be determined by using the following barometric formula:

$$H = 18336.3\ (\log h_1 - \log h_2)\left(\frac{T_1 + T_2}{500}\right) \qquad ...(i)$$

The empirical formula. The following empirical formula may be used to find the height of the station above the datum (i.e. the point at which the water boils at 100°C).

$$H = (285.9\ t^2 + 0.74\ t)a$$

where, H = height of the station above the datum.

t = the number of degrees (°C) below 100

a = the air temperature constant

$$= \left(1 + \frac{T_1 + T_2}{500}\right) \qquad ...(i)$$

Example. 6.31. Determine the difference in altitude between Jharipani survey Camp near Mussoorie hill and Rajpur Head Quarter of survey camp along Dehradun Rajpur highway, from the following observed data at 11-30 AM. on 1.2.1956.

Boiling point at Rajpur HQ. survey camp = 99.1°C

Temperature of air at Rajpur HQ. survey camp = 15°C

Boiling point at Jharipani survey camp = 96.4°C

Temperature of air at Jharipani survey camp = 11°C

Solution.

We know that height above MSL

$$H = (285.9\ t + 0.74\ t^2)a \qquad ...(i)$$

(i) Here, $t = 100° - 99.1° = 0.9°$ C

$$a = \left(1 + \frac{T_1 + T_2}{500}\right)\left(1 + \frac{15 + 11}{500}\right) 1 + \frac{26}{500} = 1.052$$

Substituting the values of t and a in eqn. (i) we get:

$$H = [(285.9 \times 0.9 + 0.74\ (0.9)^2)] \times 1.052$$

$$= (257.31 + 0.5994) \times 1.052$$

$$= 257.9094 \times 1.052 = 271.32 \text{ m}$$

(ii) Here, $t = 100° - 96.4° = 3.6°$ C

$$H = [(285.9 \times 3.6 + 0.74\ (3.6)^2)] \times 1.052$$

$$= (1029.24 + 9.59) \times 1.052$$

$= 1038.83 \times 1.052 = 1092.85$ m

$\therefore$ Difference in altitudes

$= 1092.85 - 271.32 = 821.53$ m

i.e. Jharipani camp is 821.5 m higher than that of Rajpur camp. **Ans.**

Alternative method. The barometer height at Rajpur camp and Jharipani survey camp may be determined from the following formula:

$$h = 76.00 - 2.679\, t \qquad ...(i)$$

The difference in altitudes in metres between two camps may then be determined by the application of the following formula;

$$H = 18336.3\ (\log h_1 - \log h_2)\left(1 + \frac{T_1 + T_2}{500}\right) \qquad ...(ii)$$

Here, h_1 at Rajpur camp $= 76.00 - 2.679 \times 0.9$

$= 76.00 - 2.41 = 73.59$ (cm)

h_2 at Jharipani camp $= 76.00 - 2.679 \times 3.6$

$= 76.00 - 9.64 = 66.36$ (cm)

Difference in altitudes

$= 18336.6\ (\log 73.59 - \log 66.36) \times 1.052$

$= 18336.3\ (1.866819 - 1.821906) \times 1.052$

$= 18336.3 \times 0.044913 \times 1.052 = 866.4$ m

i.e. Jharipani survey camp is 866.4 m above the Rajpur survey camp.

EXERCISE 6

1. Fill up the blanks with suitable word(s).

(*i*) Levelling is the branch of surveying in which measurements are made in..............plane.

(*ii*) A level surface is a curved surface which is..............to the vertical at each point.

(*iii*) A line which is normal to the plumb line at all points, is known as................line.

(*iv*) Verizontal line is...............to the vertical line at the point.

(*v*) A vertical line at any point is defined by the ....line.

(*vi*) A level surface to which elevations of different points are referred to, is known as.........

(*vii*) Mean sea level at any place is the average datum of hourly tidal heights over a period of............years.

(*viii*) A relatively permanent point of reference whose elevation with respect to any assumed datum, is known as........

(*ix*) A level essentially consists of the following four parts :

(*a*) .... (*b*)..................

(*c*) .... (*d*)...............

(*x*) In a dumpy level, the telescope is firmly fixed in two..................

(*xi*) In a wye level, the telescope is carried in two...............supports.

(*xii*) In a tilting level, the line of sight may not remain exactly perpendicular to..........axis at each setting.

(*xiii*) Self reading staves are of usually three forms :

(*a*) .... (*b*)..................

(*c*) ....

(*xiv*) A telescopic levelling staff when fully extended is.............metres long. The upper and central parts are............metres each and the lower part is...........metres.

(*xv*) A metric levelling staff is divided into metres, decimeters and its smallest division is...........cm.

(*xvi*) According to IS 1779-1961, a 4 metre............type levelling staff is recommended for surveying.

(*xvii*) A line which passes through the optical centre of the objective and also through the intersection of the cross hair, is called..........

(*xviii*) A first sight taken on a levelling staff held at a point of known elevation, is termed as...........sight.

(*xix*) The point on which both back-sight and fore-sight are taken, is known as..........

(*xx*) When the bubble of a level tube is at the centre of its run, the bubble line is..........

(*xxi*) Levelling should always commence from a................

(*xxii*) The amount of correction of refraction is.............that of the curvature correction.

(*xxiii*) Sensitiveness of a level tube is designated by...........of level tube.

(*xxiv*) Axis of telescope is the line joining the centre of the eye piece and the optical centre of...........

(*xxv*) The arithmetical checks on reduction of levels by rise and fall method, are :

.................=...................=................

(*xxvi*) The method of reduction of levels which provides a full check on calculations of all sights, is known as.............method.

(*xxvii*) The correction for the earth curvature is...............as applied to the staff reading.

(*xxviii*) The correction for atmospheric refraction is.............as applied to the staff reading.

(*xxix*) The longitudinal section of inner surface of a bubble tube by a vertical plane through its axis is ....

(*xxx*) The readings on inverted staffs are entered with a............sign in level books.

2. Pick up the correct word(s) given in brackets to fill in the blanks.

(*i*) A level line at each point is................to the direction of gravity at that point. (perpendicular, parallel)

(*ii*) The still water surface of a pond represents a.............surface. (level, horizontal, vertical)

(*iii*) A reference surface above which elevations of points are determined, is called a...........surface. (datum, M.S.L., level)

(*iv*) The geometrical axis of the telescope, is known as............. (line of collimation, axis of telescope)

(*v*) In an adjusted telescope, the line of collimation............with the axis of the telescope. (coincides, does not coincide)

(*vi*) In a dumpy level, the telescope............from the supports while adjusting. (can be removed, can not be removed)

(*vii*) The reading on a metric levelling staff, can be made accurately to..........m and by estimation to..........m. (0.0005, 0.05, 0.001)

(*viii*) In an internal focusing telescope, focusing is achieved by the movement of a............lens inside the telescope. (convex, concave, plano-convex)

(*ix*) Modern levels are generally fitted with............focussing telescope, (internal, external)

(*x*) Height of instrument of a level is the elevation of the line of collimation above the ....level. (ground, datum)

(*xi*) At every turning point........sight(s) are taken. (fore, back, both)

(*xii*) If the reading of the back sight is more than that of foresight, then the forward station is...........than the back station. (higher, lower)

(*xiii*) If R.L. of a B.M. is 200.000 m, back sight is 1.525 m and foresight is 3.285 m, R.L. of the forward station, is............. (201.760 m, 198.240 m, 201.525)

(*xiv*) In rise and fall method a complete check is provided on............ sights. (back, fore intermediate, all)

(*xv*) The effect of.................can be neutralised by setting the level equidistant from the back staff and forward staff. (curvature, refraction, both) (*xvi*) The curvature of the earth surface.............the elevation of the points. (raises, lowers)

(*xvii*) By making reciproal observations in levelling, the effect of.........can be neutralised. (curvature, refraction, both)

(*xviii*) The combined effect of curvature and atmospheric refraction, is..........where d is in kilometres.
($0.06735\ d^2$ $0.6735\ d^2$, $0.006735\ d^2$)

(*xix*) The observed reading on a levelling staff will be..........when it is held vertically. (least, greatest)

(*xx*) The aneroid barometer is.........accurate than the mercurial barometer. (less, more)

(*xxi*) Observations with a level are......... (angular, linear)

(*xxii*) Reduced level of a point is the elevation with reference to......
(ground level, datum)

3. State whether the following statements are true. If not, rewrite their corrected statements.

(*i*) Height of instrument is the height of the centre of the telescope above the datum.

(*ii*) In levelling, station is a point where instrument is set up.

(*iii*) The error due to refraction is opposite to that of curvature.

(*iv*) Smallest division of a levelling staff is 5 mm.

(*v*) Sensitiveness of a level tube is increased by the increase in its length.

(*vi*) When viewed through the telescope, the staff is seen inverted.

(*vii*) Smallest graduation on an invar tape is 1 mm.

(*viii*) The inner surface of a bubble tube is an arc a of a circle.

(*ix*) The last reading in levelling is always a foresight.

(*x*) For each sight, the focussing of the eye piece is made before reading.

(*xi*) In rise and fall method there is a complete check on all sights.

(*xii*) Tilting levels are commonly used for accurate work.

(*xiii*) The correction for refraction alone in levelling, is always added to the staff reading.

(*xiv*) Error due to refraction may not be completely eliminated by reciprocal levelling.

(*xv*) The inner surface of the bubble tube will be elliptical in cross-section.

4. Draw a neat diagrammatic sketch of a Dumpy level and describe its different parts thereon.

5. (*i*) What are the different types of levels used in levelling ?

(*ii*) Explain the essential differences between them. Which instrument would you prefer and why ?

6. Define the following : Level line, Level surface, Horizontal line, Horizontal surface, Line of collimation, Axis of telescope. Foresight, Back sight, Intermediate sight, Bench mark, Mean sea level, Height of instrument and Reduced level.

7. Differentiate between the following with neat sketches :

(*i*) Foresight and back sight

(*ii*) Curvature and refraction correction

(*iii*) G.T.S. Bench mark and temporary bench mark

(*iv*) Level surface and horizontal surface

(*v*) Line of collimation and height of instrument

(*vi*) Line of collimation and optical axis of a surveying telescope.

8. Describe the temporary adjustments of a level.

9. Describe fully the methods of reduction of levels and discuss their merits and demerits.

10. Describe a level field book for rise and fall method and explain how the field notes are booked and the accuracy of the reduction of levels checked.

11. What are the common difficulties generally faced in levelling ? How will you overcome each of them ?

12. What are the sources of error in levelling ? What precautions you will take to avoid them ?

13. Explain fully the process of reciprocal levelling and state its advantages.

14. Discuss the effects of Curvature and Refraction in levelling. Find the correction due to each and the combined correction. Why are these effects ignored in ordinary levelling ?

15. What is meant by sensitiveness of a bubble tube ? Explain how this could be determined in the field ?

16. Compare 'line of collimation' method with the 'rise and fall' method for reducing levels.

17. Explain (*i*) reciprocal levelling (*ii*) fly levelling (*iii*) differential levelling (*iv*) simple levelling and state where each is used.

18. If it is known that permanent adjustment of an instrument are faulty, what precautions are necessary so that fairly accurate levelling may be done without adjustment ?

19. (*a*) Name the permanent adjustments of a Dumpy level in the proper order to performance.

(*b*) Describe briefly how the line of collimation can be adjusted in a Dumpy level.

20. Write short notes on :

(*i*) Fly level

(*ii*) Cross-section levelling

(*iii*) Compound levelling

(*iv*) Reciprocal levelling

(*v*) Sensitiveness of bubble tube

(*vi*) Profile levelling

21. The back sight reading at A is 3.562 m and the fore sight reading at B is 2.863 m. Find the difference in level of A and B.

22. The back sight reading on a levelling staff held vertically on a bench mark whose R.L. 100.000 m was 2.965 m and the fore sight on the staff held vertical on a rail was 0.895 m. Find the reduced level of the rail.

23. The back sight reading on a staff held vertical on a bench mark whose R.L. is 501.000 m is 1.585 m and the fore sight on a staff held vertically inverted against a beam is 3.585 m. Find the reduced level of the beam.

24. Find the error of reading of a levelling staff if the observed reading is 3.555 m and the point sighted at the staff is 10 cm away from the vertical through the bottom.

25. A 4 m levelling staff was found to be 20 cm out of plumb line at one of its ends when held inverted. If the reading on the staff is 3.850 m and reading on a staff held on a B.M. whose R.L. is 425.000 m was 2.555 m. Find out the correct reduced level of the point.

26. R.L. of a factory floor is 520.000 m. Staff reading on the floor is 1.255 m and reading on a staff held inverted with its bottom touching the beam at the roof truss is 3.785 m. Find the R.L. of the beam.

27. The following readings are successively taken with an instrument in a levelling work : 0.359, 0.489, 0.622, 1.758, 1.895, 2.350, 1.780, 0.345, 0.687, 1.230.

The position of the instrument was changed after taking 4th and 7th readings.

Draw out the form of a level field book and enter the above readings properly. Assume R.L. of the first point as 85.000 m. Calculate R.L. of all the points and apply usual checks.

28. The following consecutive readings were taken with a dumpy level and a 4 m staff on a continuously sloping ground on a straight line at a common interval of 30 m. 0.680, 1.455, 1.855, 2.330, 2.885, 3.380, 1.055, 1.860, 2.265, 3.540, 0.835, 0.945, 1.530, and 2.445.

The reduced level of the first point was 80.750. Rule out a page of a level field book and enter above readings. Calculate the reduced levels of the points by the rise and fall method, and also the gradient of the line joining the first and last points.

29. The following consecutive readings were taken with a level and a 4 metre levelling staff on a continuously sloping ground at common interval of 30 metres.

0.855, (on A). 1.545, 2.335, 3.115, 3.825, 0.455, 1.380, 2.055, 2.855, 3.455, 0.585, 1.015, 1.850, 2.755, 3.845 (on B).

The R.L. of A was 380.500. Make a level book and apply usual checks. Determine the gradient of the line AB.

30. The following consecutive readings were taken with a level and a 3 metre levelling staff on a continuously sloping ground at a common interval of 20 metres.

0.602, 1.234, 1.860, 2.574, 0.238, 0.914, 1.936, 2.872, 0.568, 1.824, 2.722.

The reduced level of the first point was 200.000. Rule out a page of a level field book and enter the above readings. Calculate the reduced levels of the points and also the gradient of the line joining the first and last points.

31. Given the following data :

Distance (m)	*Station*	*B.S.*	*I.S.*	*F.S.*	*R.L.*	*Remarks*
	B.M. 1	2.91		-	103.67	
0	1	-	3.15			
30	2	-	1.06			
60	3	3.43	-	0.23		
90	4	-	3.17			
120	5	3.72	-	3.56		
150	6	-	-	2.39		

Obtain the reduced levels of 1, 2, 3, 4, 5 and 6. If an even gradient of 1 in 100 starts at 1, at a level of 100 m above datum, calculate the height of filling or depth of cutting at the points 1, 2, 3, 4, 5 and 6.

32. It was required to ascertain elevations of A and B. A line of level was taken from A to B and then continued to a bench mark of elevation 130.300 m. The observations are record below. Obtain the R.Ls. of A and B.

If the distance between A to B is 3.980 km. find the gradient between A and B.

B.S.	*I.S.*	*F.S.*	*R.L.*	*Remarks*
0.92				A
1.46		2.78		
2.05		3.27		
	2.36			
2.81		0.85		B
2.63		2.97		
1.02		3.19		
		2.28	130.300	B.M.

33. The following is the page of a level book. Fill in the missing readings and calculate levels of the stations and apply usual checks :

	B.S.	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1	3.202						B.M. 1
2	1.883				0.550		
3	2.204		2.853				
4			1.153				
5		0.420		1.606		653.908	B.M. 2
6	1.245				1.092		
7	1.793		0.716				
8	1.557		0.690				
9				1.065			B.M. 3

34. The following is the page of a level field book. Fill in the missing readings and calculate R.Ls. of all points. Check th accuracy of calculations:

	Staff Readings						
Sl. No.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1	3.250					...	
2	1.880		...		0.600	...	
3		2.250			...	...	
4	...		1.920	...		...	
5		2.540			0.015	...	
6	...		...	1.000		...	
7	1.175		2.115		...	225.305	
8		1.625			...	...	
9	...		1.895		0.270	...,	
10			1.255		0.750	,	
Sum	11.450					,	

35. The following is the page of a level field book. Fill in the missing readings and calculate the reduced levels of all the points. Apply the usual checks.

Stn.	*B.S.*	*I.S.*	*F.S.*	*Rise*	*Fall*	*R.L.*	*Remarks*
1	2.150					450.000	B.M.1
2	1.645		?	0.500			
3		2.345		?	?		
4	?		1965	?			
5	2.050		1.825		0.400		
6	?		?	?		451.500	B.M. 2
7	1.690		1.570	0.120			
8	2.865		21.00		?		
9			?	?		451.250	B.M. 3

36. Given the following data, obtain the R.L. of the stations 1, 2, 3, 4, 5, and 6. If an even upward gradient of 1 in 20 starts at station 1, at a level 100 above datum, calculate the height of embankment and depth of cutting at the points 1, 2, 3, 4, 5 and 6.

Distance	*Station*	*B.S.*	*I.S.*	*F.S.*	*R.L.*
--	B.M.	9.71	--	--	103.62
0	1	--	3.15		
100	2		1.06		
200	3	7.43		0.23	
300	4	-	4.17		
400	5	11.72	-	3.56	
500	6	-	-	6.39	

37. It is required to determine the accurate difference of level between points C and D on opposite banks of a wide river. The level was set up very near to C and the staff readings on C and D were 2.705 and 1.970. The instrument was then set up at D and observed staff readings on C and D were 2.850 and 2.050 respectively. What is the true difference of level between the two points.

38. A dumpy level was set up very close to a peg A. The height from the top of peg A to the centre of eye-piece was measured and found 1.467 m. The reading on the staff held on peg B was 0.897 m. The level was shifted and set up likewise at peg B. The height of eye-piece above B was 1.147 m and the reading on the staff A was 1.701 m. Determine the true R.L. of peg B if that of peg A was 101.555 m.

39. Two pegs C and D are fixed in the ground at a distance of 100 m. A dumpy level is set up near C and levelled. The staff readings at C andD are 2.165 and 2.195 respectively. The level is then shifted to D and levelled. The staff readings at C and D are 2.135 and 2.105 respectively. State, if the instrument is in adjustment. If R.L. of C is 501.015 m, calculate R.L. of D.

40. Reciprocal levelling was done to determine the difference in level between two points A and B fixed on the opposite banks of a river. The following readings were taken :

Position of level	*Staff Readings (m)*	
	A	B
Level near A	2.570	2.168
Level near B	2.363	1.405

If R.L. of A is 300.585 m, find the R.L. of B.

41. A luminous object on the top of a light house constructed on a hill is just visible above the horizon at a certain point A at the sea level. The distance of the luminous object from the point A is 50 km. Find the elevation of the luminous object above sea level, assuming the radius of the earth to be 6370 km.

42. A passenger who is travelling in a ship stands on the deck of a ship and sees a light house which is 50 m above sea level at a port. If the height of the passenger's eye above sea level is 4.5 m, find the distance of the passenger from the light house.

43. An observer at a height of 27 m above sea level just sees a luminous lamp on a hill top. The distance between the observer's station and the hill is 100 km. Calculate the height of the hill top above M.S.L.

44. A level with perfect adjustment was set up at a station P. The reading on the staff held at Q 500 m away, from P was 2.865 m and the staff reading at R 800 m away from P was 3.495 m, find the true difference of level between Q and R.

45. The top of a receiving television tower at A is visible from the top of a transmitting television tower at B, across a sea. If the distance between the tops of towers is 220 km and the elevation of the top of transmitting tower above M.S.L. is 450 m, calculate the elevation of the top of the receiving television tower. Assume the radius of the earth as 6370 km.

46. While carrying out permanent adjustments of a Dumpy level by the two peg method, the following observations were made. Instrument was at E midway between points C and D 100 m apart.

Reading at point $C = 2.00$ m

Reading at point $D = 3.00$ m

Instrument at peg F in line of CD such that

$$CF = 120 \text{ m and } DF = 20 \text{ m}$$

Reading at point $C = 1.50$ m

Reading at point $D = 2.75$ m

Check whether the instrument needs permanent adjustment or not and whether the line of sight is inclined upwards or downwards. What should be the correct reading at C if the instrument is to be adjusted ?

47. (*a*) What is the object of permanent adjustments of a level ? State the desired relationships and their necessities between the fundamental axes of a levelling instrument for it to be in permanent adjustment.

(*b*) Fly levels were run from a bench mark P, R.L. 325.120 to determine the reduced level of a point Q on the top of a hill. The average lengths of the 12 back sights and 12 foresights were 18 m and 10 m respectively. The observed difference in level between P and Q was found to 31.854 m. The instrument was suspected to be out of adjustment. It was therefore tested and the following results obtained :

	Reading on	Reading on
Level at *A*	*A*	*B*
Midway	1.632	1.014
Near *B*	1.694	1.166

Distance between A and B = 80 metres. Work out the true reduced level of Q.

48. While checking a dumpy level, the following readings were obtained:

Level set up midway between two staff stations A and B 100 m apart : Staff readings on A 1.90 m and on B 1.40 m. Level set up 10 m behind B and in line AB : Staff readings on B 1.10 m and on A 1.35 m. Determine the amount of instrument error and its inclination.

49. The following observations were taken during the testing of dumpy level :

Instrument at	*Staff readings on*	
A	1.275	2.005
B	1.040	1.660

Is the instrument in adjustment ? To what reading should the line of collimation be adjusted when the instrument was at B.

50. The following notes refer to reciprocal levels taken with a level :

Instrument	*Staff Reading on*		
Near	*P*	*Q*	
P	1.540	2.500	PQ = 1055
Q	0.810	1.620	R.L. of P = 125.885 m

Determine :

(*i*) The true R.L. of Q.

(*ii*) The combined correction for curvature and refraction.

(*iii*) The angular error in collimation adjustment of the instrument if any.

51. The reading taken on a staff held 100 m from the instrument with the bubble central is 1.565 m. The bubble is then moved 2 divisions out of centre, and the staff reading is observed to be 1.585 m. Find the angular value of one division of bubble and the radius of curvature of the bubble tube, the length of one division being 2 mm.

52. Find the radius of curvature of a bubble tube whose length of one division 2 mm and if the angular value of its one division is (*a*) 20 seconds (*b*) 30 seconds.

53. A staff is held at a distance 100 m from the level. If the bubble tube of the level has a sensitiveness of 30 seconds per division, find the error in the staff reading when the bubble is 3 divisions out of the centre.

54. Find the radius of curvature of bubble tube and angular value of its one division from the following data :

Bubble	*Readings*	*Staff readings*	*Remarks*
L.H.	R.H.		
12.5	7.5	2.895	Distance from the instrument to the staff = 100 m.
5.5	14.5	2.980	

55. Compute the reduced level of a point B in metres from the following simultaneous barometric observations :

Barometric reading at A = 65.5 cm., temp. 20° C

Barometric reading at B = 62.5 cm.,temp. 18° C

Reduced level of A = 1500 m.

ANSWERS

1. (*i*) Vertical (*ii*) perpendicular (*iii*) level (*iv*) perpendicular (*v*) plumb (*vi*) datum (*vii*) 19 (*viii*) bench mark (*ix*) telescope, level tube, levelling head, and tripod (*x*) supports (*xi*) Y (*xii*) vertical (*xiii*) solid, folding, telescopic (*xiv*) 4, 1.25, 1.50 (*xv*) 0.5 (*xvi*) telescopic (*xvii*) line of mark (*xviii*) back (*xix*) change point (*xx*) horizontal (*xxi*) Bench mark (*xxii*) $\frac{1}{7}$th (*xxiii*) radius (*xxiv*) objective (*xxv*) $\Sigma BS - \Sigma SF = \Sigma$ rise $- \Sigma$ Fall = –R.L. of last point - R.L. of first point (xxvi) rise and fall (*xxvii*) negative (*xxviii*) positive (*xxix*) circular. (*xxx*) negative.

2. (*i*) Perpendicular (*ii*) level (*iii*) datum (*iv*) axis of the telescope (*v*) coincides (*vi*) can not be removed (*vii*) 0.005, 0.001 (*viii*) concave (*ix*) internal (*x*) datum (*xi*) both (*xii*) higher (*xiii*) 198.240 (*xiv*) all (*xv*) both (*xvi*) lowers (*xvii*) both (*xviii*) $0.06735d^2$ (*xix*) least (*xx*) less (*xxi*) linear (*xxii*) datum.

3. (*i*) true (*ii*) false (*iii*) true (*iv*) true (*v*) true (*vi*) true (*vii*) true (*viii*) true (*ix*) true (*xi*) true (*xii*) true (*xiii*) true (*xiv*) true (*xv*) false.

21. 0.699 m

22. 102.070 m

23. 506.170 m

24. 0.001 m

25. 431.100 m

26. 525.040 m

27. Σ BS – Σ FS = Σ Rise – Σ Fall = 2.169, R.L. of last point = 82.831 m.

28. R.L. of last point 73.955 m; Gradient 1 in 48.56

29. Gradient 1 : 39

30. R.L. of last point = 193.240 m ; gradient 1 in 23.67

31. R.Ls: 103.43, 105.52, 106.35, 106.61, 106.22, 107.55 m

32. R.L. of A = 134.75, R.L. of B = 130.77, Gradient 1 : 1000

33. Σ B. S. – Σ F.S. = Σ Rise Σ Fall

= R.L. of Last point – R.L. of first point = 2.742 m.

34. R.L. of first point = 224.960; R.L.of last point = 223.835

35. B.S. :1.425, 1.690 ; F.S. 1.650, 0.330, 2.825.

36.

Point	*Cutting*	*Filling*
1	10.18	...
2	7.27	...
3	3.10	...
4	1.36	...
5	...	3.03
6	...	2.70

37. 0.7675 m

38. R.L. of B = 102.117 m

39. Not in adjustment ; R.L. of D = 501.015 m

40. 301.265 m

41. 168.25 m

42. 35.437 km

43. 430.40 m

44. 0.604 m

45. 1285.94 m

46. Inclined downward ; 1.800 m

47. 356.824 m

48. 0.25 m in 100 m ; 8″.56 downwards

49. Not in adjustment ; reading on A = 0.985 m

50. 125.000 m, 0.75 m, 14″. 66

51. 20″. 63; 20 m,

52. (a) 20.63; m, (b) 13.75 m

53. 0.044 m

54. 25″, 16.5 m

55. 1991.7 m

7

Contouring

7.1. INTRODUCTION

Relief on a map enhances its utility. A map without relief representation is simply a plan on which relative positions of details are only shown in horizontal plane. Relative heights of various points on the map, may be represented by any one of the following methods :

1. Spot heights.
2. Altitude Tints or layers.
3. Shading.
4. Hachuring.
5. Contours.

1. 'Relief represented by spot heights. Whenever ground is flat, there is hardly any difference in elevation of different points in the area, *e.g.,* the plains of West Bengal. On the other hand, if contours are drawn at larger vertical difference, slight variation in heights cannot be depicted. In such cases, the relief is indicated by showing spot levels of a number of points. Spot levels are generally obtained by spirit levelling for accurate maps and with ordinary instruments for less accurate maps. (Fig. 7.1).

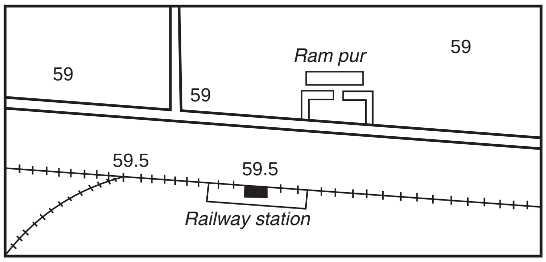


Fig. 7.1. Relief with spot heights.

2. Relief represented by altitude tints or layers. Whenever ground is steep, the relief may be represented by layers of different heights and coloured with different colours. One layer usually contains several contours. The tints generally range from green to red, brown, blue and white representing the natural colours. Layers are generally used for depiction of relief on geographical maps on which only general topography of the ground, is shown.

3. Relief represented by shading. In case the area consists of moderately low hills, the relief can be indicated by varying intensity of shading with a stump or by a colour wash. (Fig. 7.2).

Fig. 7.2. Shading with vertical light.

There are three systems of hill shading :

(*a*) Shade formed by a vertical light.

(*b*) Shade formed by a horizontal light from a definite direction.

(*c*) Shade formed by a combined vertical and horizontal light.

(*d*) With vertical light, all flat ground and the crests of the ridges appear white, whilst the slopes appear shaded. The intensity of darkness of the shade, decreases from the steep slope to a gentle slope.

Note. The following points may be noted.

1. A vertical light actually casts on shadow, but on the slopes, the intensity diminishes as the slope gets steeper and the shadow thus may be assumed darker.
2. With horizontal light the convention has been to assume a light varying from the North-West direction to North-East direction to suit the layout of the grounds. (*i.e.* general layout of the ridge lines). In this method the lighted portion of the ridges are kept white, while the other sides are shaded black. Steeper the slope, darker the shading is used.
3. Whenever a combined light is used, the slopes of ridges are shaded. Slopes on the illuminated sides are lightly shaded than those on the unilluminated side. This system of combined light is, probably, most suitable for typographical maps on small scales.

4. Relief represented by hachuring. In this method of representing relief, short lines called hachures are used. Hachures are of two types :

1. Vertical Hachures 2. Horizontal Hachures.

1. Vertical Hachures. Vertical hachuring is a method of indicating relief by a set of parallel lines closely drawn in the direction in which water would flow on the ground surface. Generally, there may be the same number of lines to a centimetre, but where the slope is steeper, lines are drawn closer and thicker. On flat surface, the lines are drawn finer and widely spaced. Hachures are broken at contours. (Fig. 7.3)

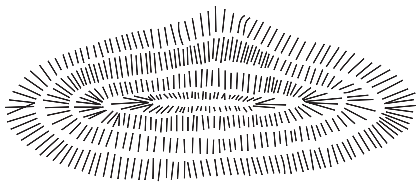


Fig. 7.3. Relief represented by vertical hachures.

2. Horizontal Hachures. In this method, short lines are drawn parallel to contours. Steeper the slopes, thicker the lines are drawn. Horizontal hachures are generally used to depict the rocky features in higher altitudes. (Fig. 7.4).

Fig. 7.4. Relief represented by horizontal hachures.

7.2. CONTOURS, CONTOUR INTERNAL AND HORIZONTAL EQUIPMENT

1. Contour. An imaginary line, on the ground, joining the points of equal elevation above the assumed datum, is called a *contour.* It is a plan projection of the plane passing through the points of equal height on the surface of the earth. Concept of a contour can be made clear by surveying the boundary of still water in a pond. If the level of the water surface is 100 m, then the periphery of water represents a contour of 100 metres. Now imagine that water level is lowered by 5 metres, the new periphery of water will then represent a contour of 95 m. (Fig. 7.5).

2. Contour Interval. The vertical distance between any two consecutive contours, is called *contour interval.* It is kept the same on a map to depict correct topography of the terrain.

3. Horizontal Equivalent. The least-horizontal distance between two consecutive contours, is called *horizontal equivalent.* It is different for different contours and is dependent on the slope of the ground surface. It is comparatively less in hills than in plains.

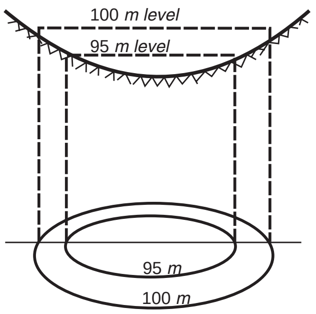


Fig. 7.5. Contours

7.3. FACTORS FOR DECIDING CONTOUR INTERVAL

Contour interval on a map is decided on the following considerations:

1. Scale of the map. The contour interval is kept inversely proportional to the scale of the map. Smaller the scale of the map, larger the contour interval. On the other hand, if the scale of the map is large, the contour interval should be small. If, on a small scale map, a small contour interval is adopted the horizontal distance between two consecutive contours *i.e. horizontal equivalent,* is also small and when plotted on the scale of the map, the two contours might unite together. It necessitates to increase the contour interval on small scale maps.

2. Purpose of the map. The contour interval on a map also depends upon the purpose for which the map is to be utilised. If the map is prepared for setting out a high-way on hill slopes, a large contour interval might suffice. But, if the map is required for the construction of an university campus, a small contour interval will be required for accurate work.

3. Nature of the ground. The contour interval depends upon the general topography of the terrian. In flat ground, contours at small intervals are surveyed to depict the general slope of the ground whereas high hills can only be depicted with contours at larger contour interval. In other words, we may say that the contour interval is inversely proportional to the flatness of the ground *i.e.*, steeper the terrain, larger the contour interval.

4. Availability of time and funds. If the time available is less, greater contour interval is adopted to complete the project in the specified time. On the other hand, if sufficient time is at the disposal, a smaller contour interval might be decided, keeping in view all the other factors already described.

It may be noted that contour interval should be such so that, depending upon the scale of the map, purpose of the map, availability of time and the nature of the ground, correct topography of the terrain may be depicted clearly and without any confusion.

For general topographical maps, the contour interval may be decided from the following rule :

Contour interval

$$= \frac{20}{\text{No. of centimetres per kilometre}} \text{ Metres.}$$

$$= \frac{50}{\text{No. of inches to a mile}} \text{ Feet.}$$

Example 7.1. *Find a suitable contour interval on a map on scale 1 : 50,000.*

Solution :

On a 1 : 50,000 scale

50,000 m = 1 m = 100 cm.

$$\therefore \qquad 1000 \quad \text{m} = 1 \text{ km} = \frac{100 \times 1000}{50{,}000} = 2 \text{ cm}$$

Contour interval

$$= \frac{20}{\text{No. of cms per km.}} = \frac{20}{2} = 10 \text{ metres.} \quad \textbf{Ans.}$$

7.4. COMPARATIVE ADVANTAGES AND DISADVANTAGES OF THE METHODS OF RELIEF REPRESENTATION

A. **Contour system**

Advantages : The following are the advantages :

1. The height of every point between the contours can be read reasonably accurate directly from the maps.
2. Angle of slope of the hills can be easily determined.
3. Intervisibility between any two points can be ascertained.
4. Contours can also be combined with other methods of relief representation from a contour map.

Disadvantages. The untrained map reader cannot read the slope of the ground from a contour map.

B. **Hachuring system**

Advantages. The following are the advantages :

1. The formation of hill features can be visualised even by an untrained map reader.
2. It shows the slope conditions of the hill quite well.

Disadvantages. The following are the advantages :

1. Elevation is not indicated on maps.

2. Angle of slope cannot be determined.
3. It requires a great deal of skill and practice for drawing hachures.
4. It is not satisfactorily adoptable to very small scales.
5. It obscures other details and typing.

Note. As disadvantages are many and of serious nature while advantages are few, the use of hachuring, has therefore declined considerably.

C. **Shading system**

Advantages. The following are the advantages :

1. It can be combined with contours.
2. It allows overwriting of letterings, because shading is generally done in brown or grey colours.

Disadvantages : The following are the disadvantages

1. It does not indicate the elevations.
2. It represents generalised features on small scale maps.

D. **Layers system**

Advantages : The following is the advantage.

1. It gives the general topography of the country.

Disadvantages : The following is the disadvantage.

1. It is not suitable for large scale maps with small contour interval.

Considering the utility of the map, the following combinations of the methods of relief representation, are commonly used :

1. Contours with shading.
2. Contours with hachures.
3. Contours with altitudes tints.

7.5. CHARACTERISTICS OF CONTOURS

The following characteristics of contours are kept in view while preparing or reading a contour map.

1. Two contours of different elevations do not cross each other except in the case of an overhanging cliff.
2. Contours of different elevations do not unite to form one contour except in the case of a vertical cliff.
3. Contours drawn closer depict a steep slope and, if drawn far apart, represent a gentle slope.
4. Contours equally spaced depict a uniform slope. When contours are parallel, equidistant and straight, these represent an inclined plane surface.
5. Contour at any point is perpendicular to the line of the steepest slope at the point.

6. A contour line must close itself but need not be necessarily within the limits of the map itself.
7. A set of ring contours with higher values inside, depict a hill whereas a set of ring contours with lower values inside depict a pond or a depression without an outlet.
8. When contours cross a ridge or V-shaped valley, they form sharp V-shapes across them. Contours represent a ridge line, if the concavity of higher value contour lies towards the next lower value contour and on the other hand these represent a valley if the concavity of the lower value contour, lies towards the higher value contour.
9. The same contour must appear on both the sides of a ridge or a valley.
10. Contours do not have sharp turnings.

7.6. CONTOURS OF NATURAL FEATURES

Keeping in view, the characteristics of contours enumerated above, different natural features may be shown by contours

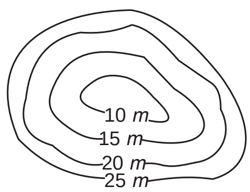


Fig. 7.6 (*a*). A pond or depression without an outlet

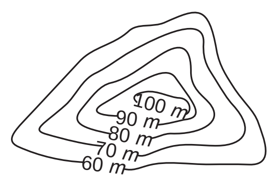


Fig. 7.6 (*b*). A hill or a mound.

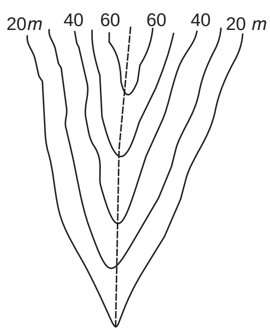


Fig. 7.6 (c). A ridge line.

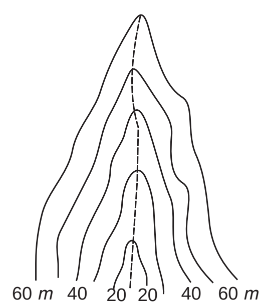


Fig. 7.6 (d). A valley.

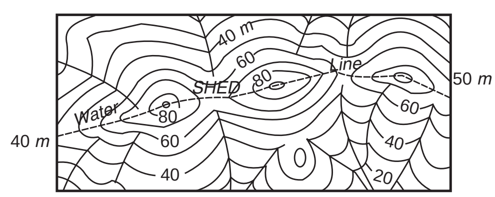


Fig. 7.6 (e). A water shed line.

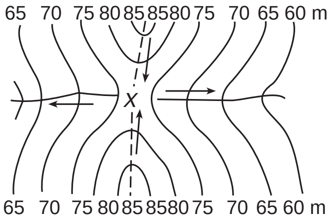


Fig. 7.6 (f). A pass or a saddle.

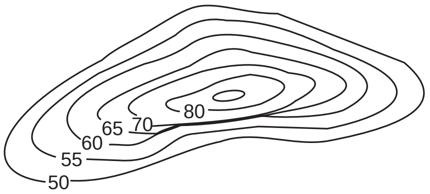


Fig. 7.6 (g). A vertical cliff.

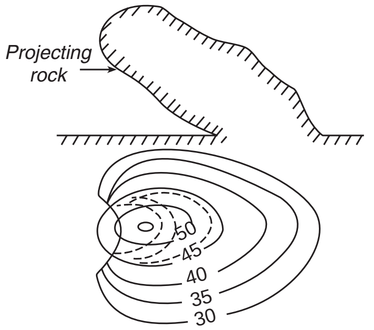


Fig. 7.6 (h). A hanging cliff.

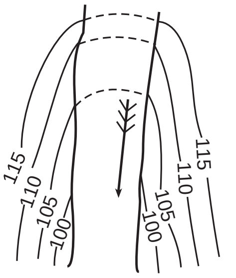


Fig. 7.6 (i). Contour in river bed.

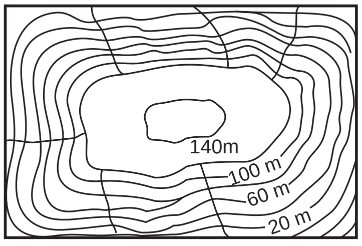


Fig. 7.6 (j). A plateau.

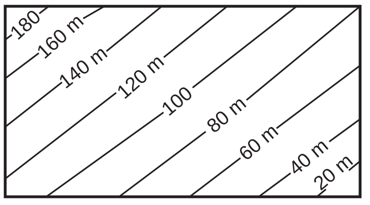


Fig. 7.6 (k). An inclined plane.

7.7. METHODS OF CONTOURING

Field work for locating contours may be executed in various ways according to the instruments used. The various methods of locating contours, may be divided into two main classes :

(*i*) Direct method. (*ii*) Indirect method.

Direct method No. 1. In this method, contours to be plotted are actually traced out in the field by locating and marking a number of points on each. These points are surveyed on a plane table section and the appropriate contours are drawn through them. This method is comparatively slow and is generally not adopted on large surveys unless a superior accuracy is demanded. It is suitable for contouring of small areas where better accuracy is required.

The whole field work may be divided into two steps :

(*a*) The location of the points on the contours *i.e.*, vertical control.

(*b*) Plotting of the points on the plane table section *i.e.,* horizontal control.

The two operations may be carried out simultaneously if one surveyor is employed on levelling for locating the points and the other for surveying their locations, by plane tabling.

1. **Vertical Control.** Instruments required : level, levelling staff, plane table with its accessories.

Procedure : Proceed as under.

(*i*) Establish a permanent bench mark of known elevation above mean sea level, in the area of the project or accept any suitable point as datum level with an arbitrary value say 500 metres.

(*ii*) Set up the level on any commanding position *A* from where a reading on the staff placed on the bench mark can be made easily.

(*iii*) Read the level staff held on the bench mark. Assume that its middle wire reading is 1.523 m.

(*iv*) Calculate the height of the line of collimation of the level *i.e.,* 500 + 1.523 = 501.523 m. Refer to Fig. (7.7 *a*)

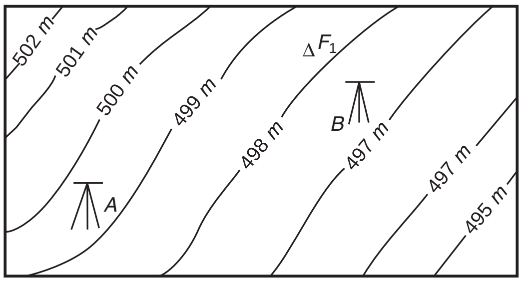


Fig. 7.7 (*a*). Contouring with a level.

(*v*) To locate the 500 metre contour on the ground, direct the staff man to the points where staff reading is 501.523 – 500.00 = 1.523 m, and locate their positions by inserting wooden pegs. For locating the 501 metre contour, the staff man should be directed to the points where the staff reading is 501.523–501.0 = 0.523 m. It may be noted that from this setting of the level, 502 metre contour can not be located as the elevation of the line of collimation is only 501.523 m as shown in the Fig. (7.7 *b*) but the lower value contours *i.e.* 499 m, 498 m, etc. can be located depending upon the length of the levelling staff *i.e.* whether it is 4 m, 4.5 m or 5 m. It is necessary that the line of collimation should intersect some portion of the staff for locating the contour.

(*vi*) Establish a forward station *F* on a pakka mark and read the levelling staff held on it. Let its value be 3.426 m. The reduced level of *F* is 501.523 – 3.426 = 498.097 m.

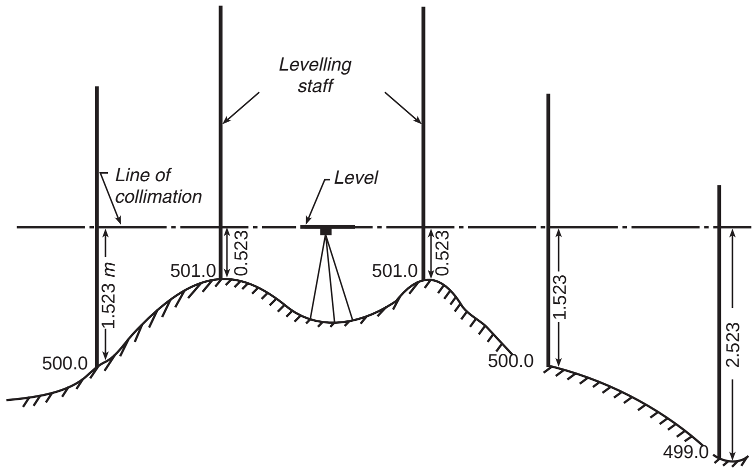


Fig. 7.7 (*b*). Locating Contours.

(*vii*) Shift the level to another commanding station say *B* and read the back staff on the point *F*. Let the reading be 0.852 m.

(*viii*) Calculate the height of the line of collimation of the level at *B* *i.e.* 498.097 + 0.852 = 498.949 m.

(*ix*) The points on the contours of lower values *i.e.*, 497, 496, 495 m, etc. can be further located from this setting of the level.

(*x*) Proceed in a similar manner till entire area is contoured.

2. **Horizontal Control.** Instruments required : a planetable with its accessories, chains, etc.

Procedure (Fig. 7.8.) In case of small areas, the locations of the points on each contour are surveyed on the plane table section by radiation method. If the area is large and all the contours can not be plotted from one setting, the plane table is shifted to another commanding station *B* to which a long radial line *AB* is drawn.

On reaching the station *B,* the planetable is set by 'Back Ray Method' and the measured distance between the two stations is plotted on the scale of the planetable section. The points on different contours

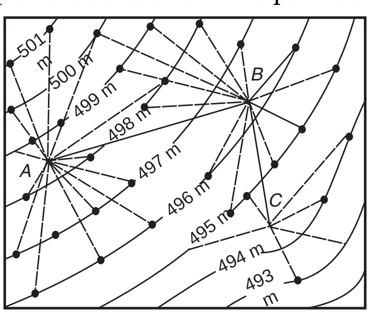
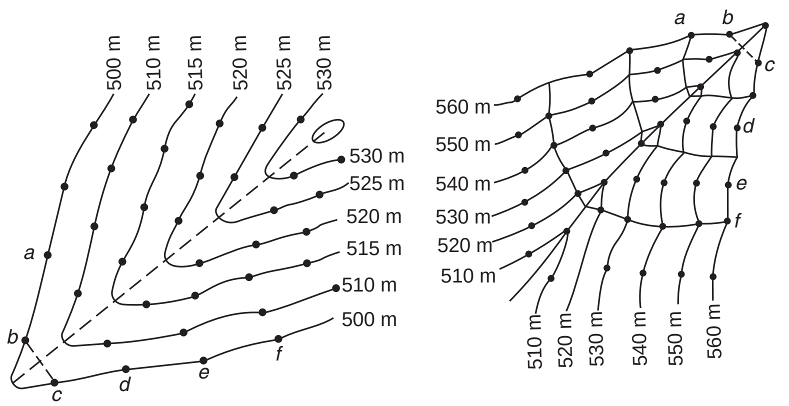


Fig. 7.8. Surveying locations of Contours.

are surveyed by radial line method and the process continued till all the contours are plotted. It may be appreciated that the contours are drawn through the points located on the plane table. Hence, sufficient number of the points at close interval are required to be located for accurate contouring.

While locating contours with this method, the following points should be kept in mind.

The plotted positions of the consecutive points on any contour are simply joined by straight lines on the plane table section and as such locating some points on the changes of slope should also be made. (Fig. 7.9).



(*a*) Ridge contours (*b*) Valley contours
Fig. 7.9. Locating highest and lowest points.

Suppose *a, b, c, d, e* and *f* are the points on a 500 metre contour located on the ground. The points *a* and *b* are on one side of a ridge and the points *c, d, e* and *f* on the other side whereas water shed line is shown by a dotted line. It should be ensured in such cases that a point on the contour must be located on the ridge line otherwise by simply joining the points *b* and *c* the contour will be seriously in error as shown by a dotted line joining *b* and *c*. This is equally applicable to locate contours in stream beds and valleys.

Direct Method No. 2. Instruments : Indian tangent clinometer, a ranging rod, a planetable and a chain.

In this method, contours are also actually traced by locating a number of points on each. For the vertical control, a clinometer is used instead of a level. The distances are measured by direct chaining from the planetable station to the points.

Procedure. A number of bench marks are established in the area by normal method of levelling. The planetable section is set upon a commanding position and levelled accurately. In case, few coordinated control points are available, they may be plotted on the P.T. Section.

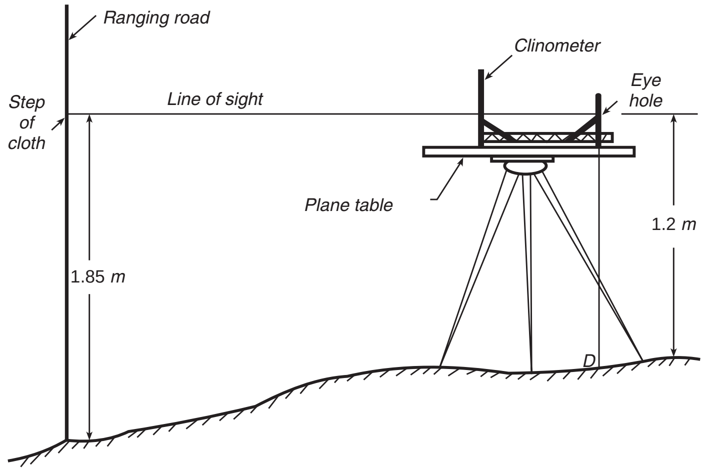


Fig. 7.10. Contouring with an Indian tangent clinometer.

Otherwise assume any suitable location on the planetable for the station occupied by the planetable, keeping in view the general layout of the area.

Place an Indian tangent clinometer on the top of the planetable section and level it carefully by its levelling screw. Sight through the eye hole of the clinometer, a ranging rod held vertically on the nearby bench mark.

The man holding the ranging rod should be asked to tie a 5 cm white cloth strip round the rod and slide it up and down till the clinometer reading becomes zero for the upper edge of the cloth strip. The exact height of the cloth strip above the bench mark is measured with a tape.

Let the ground elevation of the bench mark be 405.62 m correct upto second place of decimal and the height of the cloth strip above ground level be 1.85 m. The height of the eye hole above the datum is therefore equal to 405.62 + 1.85 = 407.47 m.

Now to locate the points on a 405 metre contour, tie the white cloth at a height of 407.47 – 405.00 = 2.47 m.

The ranging rod man may then be asked to hold the rod on the points where the clinometer reads zero. The distances of the points are measured directly by chaining and plotted to scale, on the rays drawn to them from the plane table station.

To locate other contours of 404 m and 403 m the white cloth strip may be tied at 3.47 m and 4.47 m height respectively. Sufficient number of points on each contour may be located as explained in the previous paragraph.

This method can be suitably employed when a level is not available during plane tabling or only one surveyor performs both the operations. It may be appreciated that horizontal and vertical controls are done simultaneously from the same station. The measurement of the distances are made from the point vertically below the eye vane *i.e.*, the point *D* as shown in Fig. (7.10).

Direct Method No. 3. Instruments. Photogrammetric plotting machine, a pair of vertical aerial. The method of direct contouring with the help of aerial photographs, is the latest technique of surveying. In this method, a pair of vertical serial photographs overlapping 60%, are exposed keeping the camera axis vertical. The diapositives or negatives of these exposures are mounted on the picture carriers of photogrammetric machines such as wild *A7, A8, B8,* etc.

Relative orientation of the three dimensioned model is achieved by a systematric procedure of applying rotation and translation to the diapositives/negatives mounted on the picture carriers until corresponding images of two photographic exposures are made to coincide over the entire model area. This process is known as '*Relative Orientation'* or *Removal of Y-Parallax'* over the entire model.

After relative orientation, the correct scale of the model is obtained by altering the distance between the perspective centres of photographs and the entire model is rotated and displaced along three different directions *i.e. X, Y, Z* axes until the heights and distances of known points in the model, agree.

To achieve correct and absolute orientation of a model, at least four points of known plan and height coordinates in four corners of the model are required. Three points are required for bringing the model in correct spatial position and the fourth one provides a check.

The floating mark is then set on a desired elevation. Keeping the *Z*-column fixed, the floating mark is moved in *X* and *Y* directions and the desired contour is plotted on a plan with a coordinatograph attached to the photogrammetric machine.

By changing the *Z* column readings, the required number of contours may be surveyed.

It may be noted that in this method, every point on a contour is plotted and as such, it is the most accurate method. This method is employed when higher degree of accuracy is demanded.

7.8. INDIRECT METHOD

In this method sufficient number of points are given spot levels. The locations of such points can be conveniently plotted on a plane table section as these generally form the corners of well, shaped geometrical figures *i.e.*, squares, rectangles, triangles, etc. It is seldom possible to

Fig. 7.11. A plotting machine for contouring by photographic method
(Photo by the courtesy of M/S Wild Herburg Ltd.)

have exact spot level of any point on exact value of the contour. The spot levels of important features which represent hill tops, ridge lines, beds of streams and lowest points of the depression are also taken, to depict their correct features while drawing contour lines. The contours in between spot levels are interpolated and drawn. This method of contouring is sometimes known as *Contouring by spot levels.*

Indirect method of contouring is commonly employed in small scale surveys of extensive areas. This method is cheaper, quicker and less tedious as compared with direct method of contouring.

Indirect method of contouring can be employed in three different ways detailed below :

(*i*) By squares method. (*ii*) By cross sections method.
(*iii*) By tacheometric method.

1. **By Square Method.** In this method, the entire area is divided into a number of squares, the sides of which may vary from 5 m to 25 m, depending upon the nature of the ground, the contour interval and the scale of the plan. The squares may not be of the same size throughout but may vary according to the requirements of the map. The corners of the squares are marked on the ground and spot levels of these points, are given with a level by normal method of levelling. Special care is to be taken to give spot levels to the salient features of the ground such as hill tops, deepest point of the depressions, etc. and their measurements from respective corners of the squares noted. (Fig. 7.12).

The squares are plotted on the desired scale of the plan and reduced levels of the corners as well as that of the salient features are entered. The contours of desired values are then interpolated.

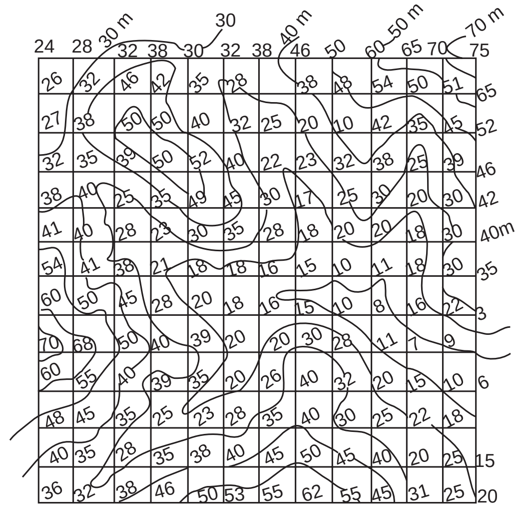


Fig. 7.12. Locating contours by method of squares.

Suitability. This method is suitable in low undulations without any vegetative covers.

2. **By Cross Section Method.** In this method, cross sections perpendicular to the centre line of the area are set out. The spacing of the cross-section depends upon the contour interval, scale of the plan and the characteristics of the ground. In general, spacing of cross-sections at 20 m in hilly country and 100 m in flat country are adopted. Points of salient features along the centre line and on cross-sections are also located. The layout of the cross-sections need not necessarily be at right angles to the centre line. These may be inclined at suitable angles to the centre if found necessary. First plot the centre line and cross-sections on the desired scale and enter their reduced levels. The contours are then interpolated with respect to these reduced levels. (Fig. 7.13).

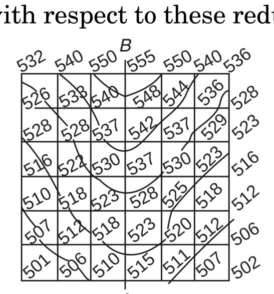


Fig. 7.13. Locating contours by method of cross-section.

Suitability. This method is suitable for preparing a contour plan of a road, railway or canal alignment.

3. **By Tacheometric Method.** In this method a number of radial lines at known angular interval, are drawn on the ground and the

positions of the points at equal distances are marked. Salient points of the ground are also located in the field by observing the vertical angles and the staff readings of the bottom, middle and top wires. Calculations of the reduced levels and the horizontal distances of the points from the instrument position, are done using the tacheometric formulae. For further details refer to chapter 13 'Tacheometric Surveying'. The radial lines, and the positions of the points on each line, are plotted on the desired scale and their spot levels entered. Now interpolation of required contours can be done with respect to the spot levels. (Fig. 7.14).

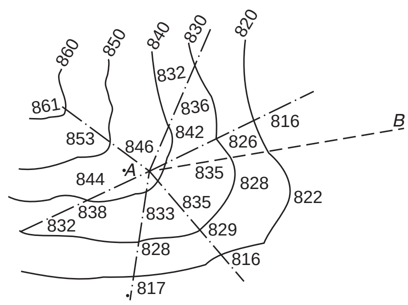


Fig. 7.14. Locating contours by tacheometric method.

Similarly, the instrument is set up at other commanding tacheometric stations such as *B, C, D,* etc. and the entire area is covered.

Suitability. This method is suitable for contouring the area of long strips with mountaneous/undulations where direct chaining is difficult.

7.9. INTERPOLATION OF CONTOURS

The process of drawing contours proportionately between the plotted ground points or in between plotted contours, is known as *interpolation of contours.* Interpolation of contours between points is done assuming that the slope of the ground between any two points is uniform. It may be done by one of the following methods :

(*i*) Estimation

(*ii*) Arithmetical calculation

(*iii*) Graphical method.

1. **By Estimation.** The positions of the contour points between ground points are estimated and the contours are then drawn through them. This method is rough and the accuracy depends upon the skill and experience of the surveyor. It is usually used for contouring small scale maps. The method of contouring of topographical maps on scale 1 : 50,000 in the Survey of India, is generally based by method of estimation.

Procedure. Heights of the selected ground points are calculated by reading clinometric readings and multiplying by their scaled distances from the plane table position. Plan positions of the heighted points are surveyed by normal method of plane tabling. In between these points, contours are interpolated, keeping in view the nature of the ground. An experienced surveyor draws contours without any serious error.

2. **By Arithmetical Calculation.** In this method, positions of contours between two known points, are located by making accurate calculations. Hence, the method, though very accurate is time consuming and laborious. It is generally adopted when higher accuracy is demanded for a limited area. (Fig. 7.15).

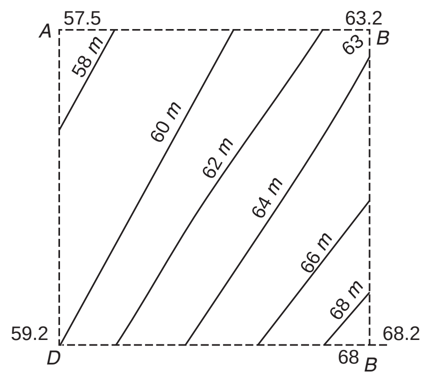


Fig. 7.15. Interpolation by arithmetical calculations.

Suppose *A, B, C* and *D* are four plotted points at 2 cm apart and their ground reduced levels are 57.5, 63.2, 68.2, 59.2 m respectively. It is required to draw contours at 2 m vertical interval.

(*a*) **Interpolation along AD.** The total difference in elevation between *A* and *D* is 59.2 – 57.5 = 1.7 m.

The difference of level between *A* and 58 m contour is 58.0 – 57.5 = 0.5 m.

Hence, the distance of the 58 m contour from *A* $= \dfrac{0.5}{1.7} \times 2$ cm = 5.9 mm.

(*b*) **Interpolation along AB.** The total difference in elevation between points *A* and *B* is 63.2 – 57.5 = 5.7 m.

The difference of level between point *A* and the 58 m contour is 58.0 – 57.5 = 0.5 m.

Hence, the distance of the 58 m contour from the point *A* along *AB* is $\dfrac{0.5}{5.7} \times 2 = 1.8$ mm.

Similarly the distances of the 60 m and 62 m contours from the point *A* can be calculated. These will be $\dfrac{2.5}{5.7} \times 2 = 8.8$ mm and $\dfrac{4.5}{5.7} \times 2 = 15.8$ mm respectively.

(*c*) **Interpolation along BC.** The total difference in elevation between points *B* and *C* is 68.2 – 63.2 = 5 m.

The difference of level between point *B* and the 64 m contour is 64.0 – 63.2 = 0.8 m.

Hence, the distance of the 64 m contour from the point *B* along *BC* is $\frac{0.8}{5.0} \times 2 = 3.2$ mm.

Similarly the distances of the 66 m and 68 m contours from the point *C* will be $\frac{2.8}{5.0} \times 2 = 11.2$ mm and $\frac{4.8}{5.0} \times 2 = 19.2$ mm respectively.

(*d*) **Interpolation along CD.** The total difference is elevation between points *D* and *C* is 68.2 – 59.2 = 9.0 m.

The difference of level between point *D* and the 60 m contour is 60.0 – 59.2 = 0.8 m.

Hence, the distance of the 60 m contour from the point *D* along *DC* is $\frac{0.8}{9} \times 2 = 1.8$ mm.

Similarly, the distances of 62 m, 64 m, 66 m, and 68 m, contours from the point *D* can be calculated. These will be

$\frac{2.8}{9} \times 2 = 6.2$ mm $\frac{4.8}{9} \times 2 = 10.7$ mm, $\frac{6.8}{9} \times 2 = 15.1$ mm and $\frac{8.8}{9} \times 2$ = 19.6 mm respectively.

The points having equal elevations are joined and the required contours of 58 m, 60 m, 62 m, 64 m, 66 m and 68 m may be drawn.

To achieve better accuracy, intrepolation along the diagonals *AC* and *BD* may also be done. Then the contours are drawn through the points of equal elevation.

It may be appreciated that instead of squares, the rectangles whose sides are in the ratio of 3 : 4 are preferred to. The diagonals of the rectangles will be in the ratio of 3 : 4 : 5. Calculation of the diagonls will be easier and their lengths will also be in full metres so long as the ratio 3 : 4 for the sides of the rectangle, is kept in full metres.

3. **Graphical Method of Interpolation.** In this method, actual calculation for interpolation of contours between known heights, is not done but the locations of the contours are obtained graphically with the help of a tracing paper or tracing cloth. There are two methods.

(*i*) y drawing radiating lines. (*ii*) By drawing parallel lines.

First Method. In this method, several lines are drawn parallel to each other on a piece of tracing paper. Let the interdistance between the lines be 2 mm. Every fifth line may be drawn heavier to facilitate easy counting. (Fig. 7.16).

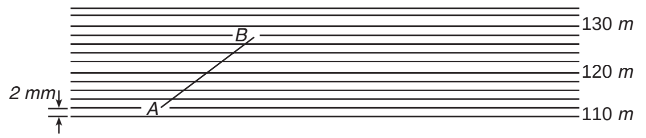


Fig. 7.16. First method of graphical interpolation.

Suppose we have to interpolate contours at 2 metre vertical interval, between two points *A* and *B* whose elevations are 112.5 m and 127.5 m. Let the bottom line on the tracing paper represent 110 m whereas other lines represent every two metres. Place the tracing paper on the plan on which heighted points have been plotted in such a way that the line representing 112.5 m (in this case a point in between the two lines 112 m and 114 m and at 0.5 mm from 112 m line) passes through the point *A*. Now rotate the tracing paper about the point *A* till the line representing 127.5 m (in this case a point in between the 126 m and 128 m lines and at 0.5 mm from 128 m line) is made to pass through the point *B*.

Prick through the points of intersection of the ruled lines on the line joining the points *A* and *B* to get the required positions of the desired contours.

It may be noted that the same set of ruled parallel lines may be utilised to interpolate contours between other points so long as the distance between them is greater than the perpendicular distance between lines representing 112.5 m and 127.5 m. In case the distance is less, then another set of values are to be given. Assume that each line represents 3 m so that the bottom line and every fifth line thereafter represents 110, 125, 140 m etc. Now the distance between points, which was less in earlier case is more than the perpendicular distance between the lines representing 112.5 m and 127.5 m. Rotate the tracing paper so that 127.5 m line is made to pass through the point of 127.5 m.

Similarly interpolation between other points of different heights may be made by assigning different values to the ruled lines.

After interpolation between the points has been done, the contour lines are drawn through the points of equal elevations.

Second Method. In this method, a line *XY* of any convenient length is drawn and divided into small equal parts, say each 2 mm width on a piece of a tracing paper. At its mid-point, a perpendicular is erected and a pole *O* is so chosen on the perpendicular that the angle *YOX* is approximately equal to 45°. Draw radial lines through pole *O* and division marks on the line *XY*. Rule out few lines parallel to *XY* across the radial lines. Such lines which serve as a guide are generally known as *guide lines.* It can be easily verified that these guide lines are equally divided by the radial lines. The first radial line and every fifth line thereafter are drawn a little heavier or bolder to facilitate quick reading of the diagram. (Fig. 7.17).

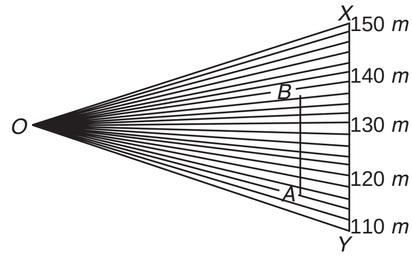


Fig. 7.17. Graphical interpolation.

Let us assume that we have to interpolate contours at 2 m vertical interval between points *A* and *B* whose elevations are 114.5 m and 137.5 m. Assume *Y* to read 110 m and every fifth line 10 m apart. Place the tracing paper over the plan such that the 114.5 m and 137.5 m radial lines simultaneously pass through the points *A* and *B* below the tracing paper.

The points at which radial lines of 116, 118, 120, 122, 124, 126, 128, 130, 132, 134 and 136 m intersect the line *AB*, may then be pricked through to get the positions of required contours on the plan. In a similar way interpolation between other points may be done and contour lines drawn through the points of equal elevation.

7.10. COMPARISON OF DIRECT AND INDIRECT METHODS OF CON TOURING

	Direct method	Indirect method
1.	The method is most accurate but very slow and troublesome.	1. The method is not very accurate but it is quicker and less troublesome.
2.	It is very expensive.	2. It is very cheap.
3.	It is used for small projects where greater accuracy is desired *i.e.* layout of a factory building structural foundations etc.	3. It is used for large projects where greater accuracy is not desired. i.e., layout of a road canal railway line etc.
4.	It is unsuitale for hilly terrain.	4. It can be suitably used for preparing contour plan by tacheometric method. The route surveys of canals roads etc. can be preapared by the method of cross-sections.
5.	Calculation of reduced levels is to be done in the field and once the area is contoured, the calculations can not be checked.	5. The calculation of the reduced levels is not done in the field and hence these can be checked as and when needed.

7.11. CONTOUR GRADIENT

The imaginary line lying throughout on the surface of the earth and preserving a constant inclination to the horizontal, is known as *contour gradient*. The inclination of a contour gradient is generally given either as rising gradient or falling gradient and is expressed as the ratio of the vertical height in a specified horizontal distance. Suppose the bed of a canal is lowered by one metre in a length of 100 metres, then, the gradient is 1 in 100. Similarly, when a railway track rises 1 metre in every 250 metres of its length, the inclination of the contour gradient is 1 in 250.

If we know the inclination of the contour gradient, its direction may be easily located on the ground by using one of the following surveying instruments.

(*i*) Indian tangent clinometer (*ii*) Theodolite

(*iii*) Level (*iv*) Ghat tracer.

1. Locating a contour gradient with an Indian tangent Clinometer. For the description and working of an Indian tangent clinometer, refer to chapter 10, 'Minor Instruments'.

The clinometer is placed on a plane table which is centred over the point *A* from where the direction of the contour gradient is required to be located, The natural tangent of the angle of elevation for a rising gradient or the angle of depression of a down gradient, is calculated, *i.e.*

$$\tan\ \theta = \frac{\text{Vertical height difference of the points}}{\text{Horizontal distance between the points}}$$

The clinometer is levelled and the line of sight is clamped on the natural tangent reading for the given gradient. For an example, the natural tangents for 1 : 100, 1 : 200 and 1 : 250 are 0.01, 0.005, 0.004 respectively. Another person holding a ranging rod with a target fixed at the height of the eye hole of the clinometer, is directed to move up and down the slope till the target is bisected by the line of the sight. (Fig. 7.18).

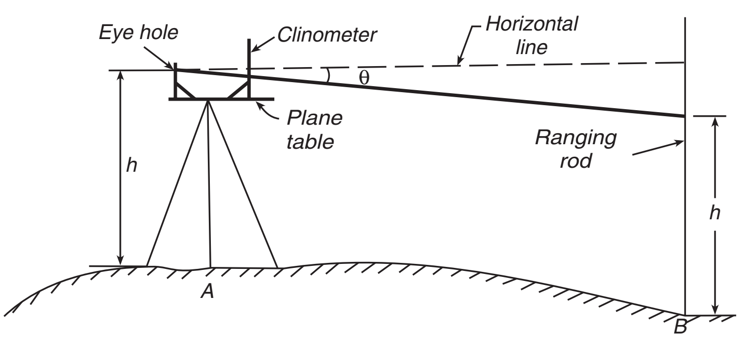


Fig. 7.18. Locating a contour gradient with an Indian tangent clinometer.

The point B is pegged on the ground. The instrument is then transferred to the point B. Other points C, D, etc. on the contour gradient are located in a similar manner.

It may be noted that the line of sight at points A, B, C, D, etc. remains parallel to the lines joining the points AB, BC, CD, etc.

2. Locating a contour gradient with a theodolite (Fig. 7.19). For the description and working of a Theodolite refer to chapter 11, "Theodolite".

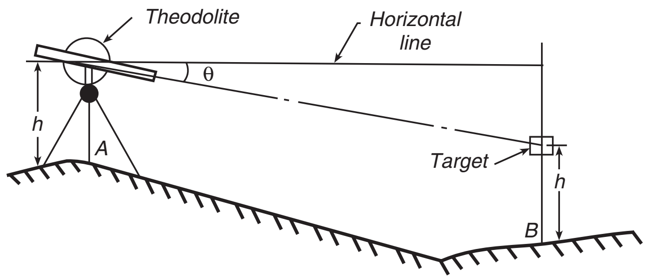


Fig. 7.19. Locating contour gradient with a theodolite.

The theodolite is centered and levelled over the starting point of the contour gradient. The reading of the vertical circle of theodolite is set to the required angle of slope. Now the telescope points in the required direction of slope. Telescope is then swung in the direction of the proposed alignment of the canal or road. The height h of the trunnion axis of the theodolite is measured accurately and a target fixed to a rod at the same height, is sighted. When the target is bisected by the horizontal cross hair of the telescope, the position of the target rod is the required point on the contour gradient. For accurate work, a levelling staff is sighted instead of a target. The instrument is then shifted to the point so obtained on the contour gradient and the line of slope is extended further.

3. Locating contour gradient with a level. The level is set up over any commanding position but not necessarily on the desired contour gradient. The height of the line of collimation for this setting of the instrument is determined by reading a levelling staff held on a nearby bench mark. The reduced level of the first point on the contour gradient is then obtained by reading a levelling staff. If the reduced level of the line of collimation is 501.655 and the reading on the staff held at the starting point A is 1.655 then, the reduced level of the point A is 501.655 − 1.655 = 500.000 metres.

Now to locate another point B on a falling contour gradient 1 : 100, direct the staff man to walk along the proposed alignment so that it is 20 m (say) away from A. The reading on this point should therefore be 1.655 + 0.20 = 1.855 m. Fix a wooden peg into the ground such that the reading on the staff when held on the top of the peg is 1.855 m. The top of the wooden peg is on the contour gradient. The staffman is again directed to move to a point *C*, 20 m away from *B* and in the same straight line. Now, the reading on staff must be 1.655 + 0.400 = 2.055 m. Let us suppose that the staff reading at *C* is 1.755 m. The staffman will be directed to dig a pit such that the reading on the staff when held vertically becomes exactly 2.055 m. In this way a number of points on the contour gradient are located before actual construction of the road or railway may be started.

4. Locating a contour gradient with a Ghat Tracer. For the description of a ghat tracer, refer to chapter 10, 'Minor Instruments'.

The wooden stand on which the ghat tracer is suspended, is held on the starting point of the desired contour gradient. The height of the sighting tube above the top of the peg driven at *A* is measured accurately with a tape. A target is then fixed on a rod at the same height as the height of the sighting tube above the ground level at *A*.

The target man is then directed to walk along the line joining the starting and closing points on the contour gradient. The bevelled edge of the weight is then kept at the falling gradient 1 in 100 so that the line of sight of the instrument becomes parallel to the gradient. Sighting through the eye hole, direct the target man to move up and down along the line till the target is bisected accurately by the intersection of the cross wires. The staff man marks the position of *B* at the foot of the target rod. The instrument is then shifted to *B* and the procedure is repeated to fix another point *C*.

In a similar manner, other points are fixed enroute till the last point is located.

Note. When locating points on a contour gradient with a ghat tracer, Abney's level, theodolite or Indian tangent clinometer inter-distances between the points fixed may not be same as required in the case of a level.

7.12. CONTOURING WITH AN INDIAN TANGENT CLINOMETER

Contouring may be carried out with an Indian Tangent Clinometer and a plane table *pari passu* with the survey of the other details as under:

At first setting of the plane table at station *A* (its location and height being known) the surveyor orients his plane table with respect to another point *B* whose location and height are also predetermined (Fig. 7.20) Pivoting the alidade about *A*, rays are drawn to a number of points on the changes of slope, intersection of streams, isolated tops, etc. Keeping the bubble of the clinometer central, clinometric readings to the same points are then observed and noted (in pencil) against their respective rays on the plane table. The plane table is then shifted to

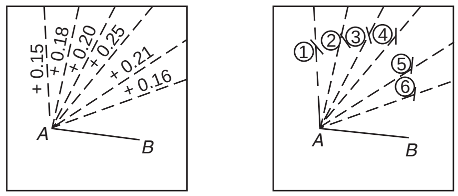


Fig. 7.20.

station B. After centering and levelling, orientation is done with respect to station A by back ray method.

Pivoting the alidade about B, rays are drawn to intersect the respective rays and their points of intersection represent their plan positions. The clinometric readings to the same points are again observed and noted against each.

The reduced levels of these points are computed as under :

Let Ha be the R.L. of station A.

D_1 be the distance of 1st point from A (scaled from plan)

α_1 be the clinometric reading for first point.

h_a be the height of the eye hole above ground point A.

then

Difference in levels, $h_1 = D_1 \times \alpha_1$ metres

$\therefore$ R.L. of first point $=$ R.L. of $A + h_a + h_1$ for angle of elevation

$=$ R.L. of $A + h_a - h_1$ for angle of depression

In this way reduced levels of all the points are computed from clinometric readings taken from station A and also from B. Mean values of the reduced levels of each, is accepted as the correct value of the point.

Keeping in view general terrain, the surveyor will then commence to sketch lightly the shape of the ground. The degree of accuracy of such sketches will depend upon the ability of the surveyor to identify different degrees of slope with the corresponding horizontal equivalents between contours, (Fig. 7.21).

Let the reduced levels of the points computed and entered against each, are as under :

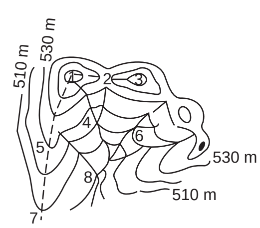


Fig. 7.21.

Point	*R.L.*
1	568 m
2	542 m
3	562 m
4	518 m
5	530m
6	522 m
7	508 m
8	502 m

Knowing that points 1, 5 and 7 are on a ridge line, the contours of 560m, 550 m, 540 m, 530 m at 10 m vertical interval, are drawn across the ridge in between points 1 and 5. Again, contours of 520 m and 510 m are drawn across the ridge in between points 5 and 7.

Knowing that points 1 and 3 are on the isolated hill tops and point 2 is in the saddle, contours of 550 m and 560 m are drawn across the water shed line defined by lines joining points 1, 2 and 3. Knowing that point 4 is at the junction of streams originating from points 1 and 2, contoure of 520 m, 530 m, 540 m, 550 m, and 560 m are drawn, across the stream originating from top 1. Similarly contours of 520, 530 and 540 m are drawn across the stream originating from the saddle 2. The surveyor should ensure that the horizontal equivalents along ridge line, water shed line or stream should be in agreement with the steepness of their slopes. Small portions of contours thus drawn, may then be joined to represent the topography of the hill slopes.

In a similar manner other contours are interpolated between the clinometric heights and area is contoured. The extension of the height control may be made during the course of survey. When any height is used for the extension of the height control, its computation should be checked at least from three stations.

Height indicator. The difference in height may also be obtained without computation by a special scale known as the *'Height Indicator'*. On such scale the horizontal equivalents for given vertical intervals are arranged. Such scales are only useful for small scale mapping. For engineering surveys on large scales, the distances are always scaled off from plan and multiplied by the clinometric reading, to obtain the difference in level for the points.

Precautions. While contouring with an Indian tangent clinometer the surveyor should guard against the following :

(*i*) **Economy of heights.** The accuracy of the contours directly depend upon the number of spot heights which have been used. Hence, the surveyor should provide maximum number of heights before interpolation of the contours is done.

(*ii*) **Direction of sun rays.** A surveyor should contour the area when the sun rays fall obliquely on the hill slope so that minor spurs get exposed prominently.

(*iii*) **Interpolation of contours in camp.** A surveyor should never shirk his responsibility of drawing contours in the field. He should never interpolate contours in his camp.

7.13. USES OF CONTOUR MAPS (FIG. 7.22)

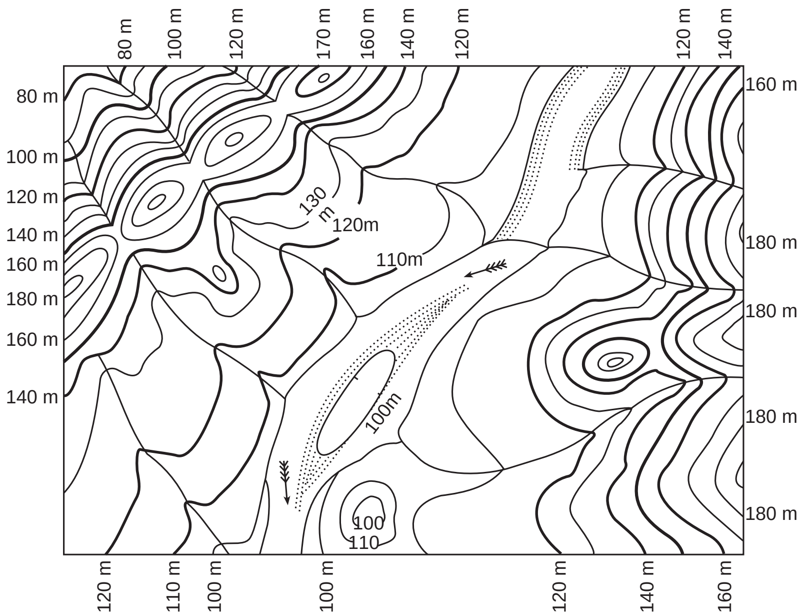


Fig. 7.22. A part of a Topographical map.

Contour maps are used for the following :

1. To study the general character of the tract of the country without visiting the ground. With the knowledge of the characteristics of the contours, it is easier to visualise whether the country is flat, undulating or mountaineous.

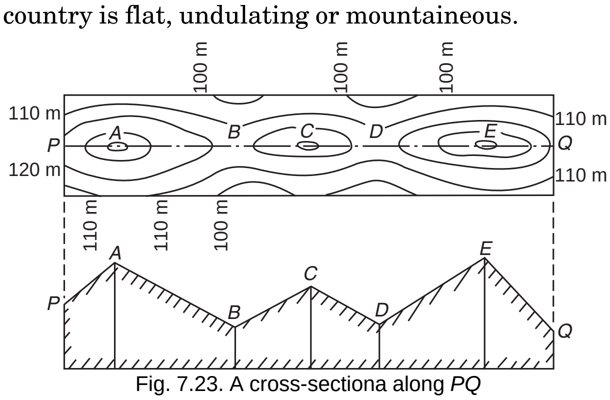


Fig. 7.23. A cross-sectiona along *PQ*

2. To decide the most economical and suitable sites for engineering works such as canals, sewers, reservoirs, roads, railways etc.
3. To determine the catchment area of the drainage basin and hence the capacity of the proposed reservoir.
4. To compute the earth work required for filling or cutting along the linear alignment of projects such as canals, roads, etc.
5. To ascertain the intervisibility of points.
6. To trace a contour gradient for the road alignment.
7. To draw longitudinal sections and cross-sections to ascertain the nature of the ground.
8. To calculate the water capacities of reservoirs.
9. To decide the best positions of the guns, the line of march and camping grounds by the army commanders during wars.

I. Drawing of sections. On a contour map, sections in any direction may be drawn to study the general shape of the ground. Such sections are also required for calculating the earth work needed for road, railway or canal projects.

Let it be required to draw a section along the line *PQ*. Plot the points at which various contours intersect the line *PQ* along X-axis and their corresponding heights are plotted along Y-axis to any convenient scale. The ends of the perpendiculars representing the contour heights may then be joined by smooth straight lines, to give the configuration of the ground surface. (Fig. 7.23).

II. Selection of a Canal Alignment. Suppose a reservoir is to be constructed at *R* across a river. An irrigation canal *RL* is required to be constructed to irrigate the areas of two villages Rampur and Shyampur located on opposite slopes of the ridge.

We know that water in canals flows under gravity. From the contour map (Fig. 7.24) it may be noted that the elevation of the reservoir

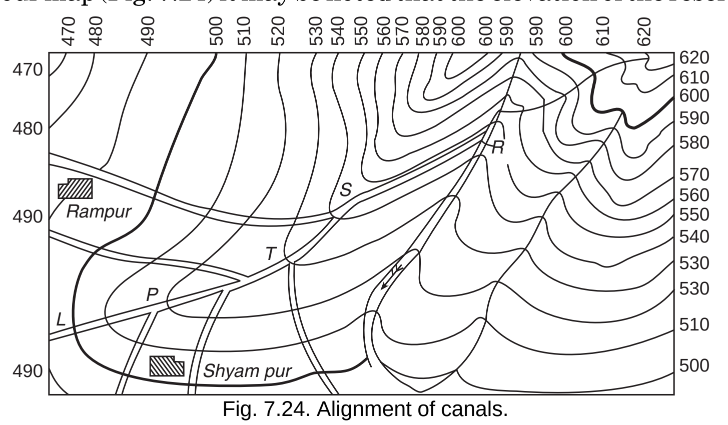


Fig. 7.24. Alignment of canals.

site R is 547 m approximately. The canal alignment should therefore be kept below this level. The water shed or ridge line is first marked by a dotted line. From R, the canal is taken along RS where S is a point on the water shed line and its height is approximately 542 m. Points may be chosen by considering the other factors to avoid construction of aquaducts over the intercepting drainages enroute. From S onward the canal may be aligned along the water shed line *S-T-P-L.,* ensuring proper gradient. Branch canals may then be constructed on either slope so that the areas of both the villages may be irrigated.

It may be noted that in the absence of a contour map, the layout of the canals cannot be decided economically.

III. Determination of Intervisibility. (*a*) Intervisibility between two points situated on different elevations with no intervening raised ground may be ascertained by inspection from the map.

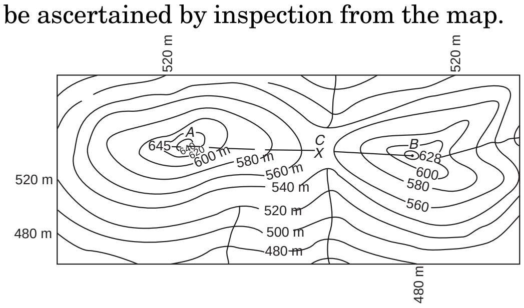


Fig. 7.25. Intervisiblity of points.

Let A and B be two given points whose elevations as obtained by interpolations of contours are 645 and 628 metres respectively. To ascertain the interversibility of the points A and B join A and B by a dotted pencil line. By inspection of the contours intercepted by the line AB, it is seen that contour values decrease up to C and then increase up to point B. As no higher value contour appears in between, the points A and B are intervisible.

(*b*) When it is not quite evident from the inspection of the map whether two points are intervisible or not, intervisibility may be ascertained as follows:

Join the given points by a straight line and note down the values of the intervening contours on this line. (Fig. 7.26)

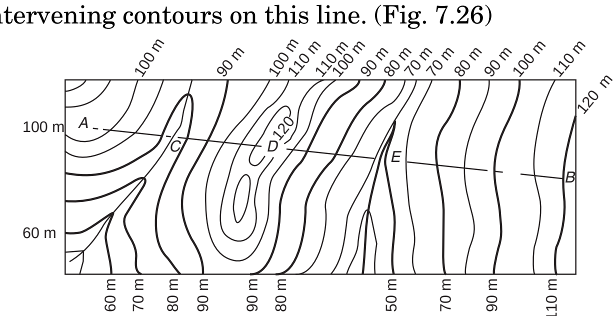


Fig. 7.26. Intervisibility of points.

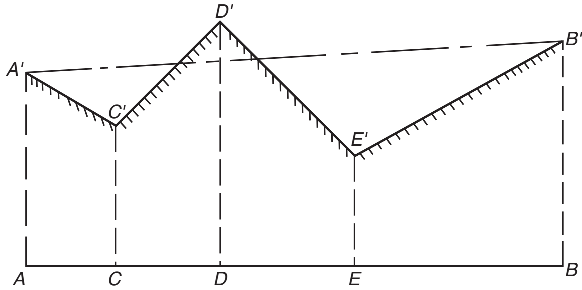


Fig. 7.27. Section *AB*.

Suppose *A* and *B* are two given points marked on a contour map. Draw a straight line *AB* preferably equal to the map distance *AB*. (Fig. 7.27).

At the respective distance points *C, D* and *E* plot their positions on the straight line *AB*. By interpolation of the contours estimate the heights of the points *A, C, D, E* and *B*. Let their heights be 105, 75, 125, 57 and 120 metres respectively.

At points *A, C, D, E* and *B* on the line *AB,* erect perpendiculars, *AA', CC', DD', EE'* and *BB'* and plot their elevations to some suitable scale say 1 mm = 5 m. Join the ends ordinates *A′* and *D′* by a straight line *A′B′*. As the ordinate *DD′* projects above this line, the line of sight is obstructed at *D* whereas the line of sight is higher at points *C* and *E*. Hence, the points *A* and *B* are not intervisible.

IV. Tracing of a Contour Gradient (Fig. 7.28). On a contour map, the alignment of a proposed road or railway, may be decided and located by drawing a contour gradient.

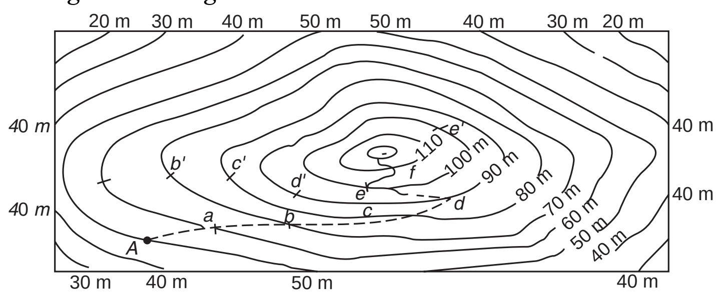


Fig. 7.28. Tracing a contour gradient.

Let the contour interval on the supplied contour map be 10 metres and an upward gradient 1 in 30 is required to be drawn from *A* to *B*.

Ground Horizontal equivalent

$$= \text{Contour interval} \times \text{gradient} = 10 \times 30 = 300 \text{ m.}$$

With *A* as centre and a radius representing 300 metres on the scale of the map, draw an arc to cut the 60 m contour at *a*. Similarly with *a* as centre and with the same radius, draw another arc to cut 70 m contour at *b*. In a similar manner points *c* and *d* are obtained. When an arc with *d* as centre is drawn, it cuts 100 m contour at *e e′*. Point *e′* though in the continuance of the route, is rejected because *d* and *e′* when joined cuts the 100 m contour again and thus a deep cutting is involved. At *e,* the arc of the same radius if drawn cuts 110 m contour nowhere. To overcome this difficulty, the 105 m contour is interpolated and half the radius is used to obtain the point *f*. Locations of a number of points are made till the last point is reached.

During the process of drawing arcs, it may be noted that it cuts next higher contour at two points. Only the point suitable for the proposed alignment, considering the general layout and geological features, is accepted and the other point ignored.

During construction, the route of the road is made to follow the contour gradient line so obtained as closely as possible.

V. Measurement of Catchment Area (Fig. 7.29). The area of the tract of a land which contributes water which flows over the surface of the earth into a stream at any point A, is called *catchment area.* The limit of such an area may be marked sufficiently accurate on a contour map. The line that marks the limits of drainage area, possesses the following characteristics :

1. The line follows the highest ridge line and divides the drainage area from other area.
2. It follows the ridges.
3. It is everywhere perpendicular to the direction of the contours.

An imaginary line possessing the above characteristics, is generally known as *water-shed line.*

A line is drawn on the map with dashes and dots to mark the bounding limits of the catchment area. The plan area is then determined with the help of a planimeter. For description and working of a planimeter, refer to chapter 8, 'Area'.

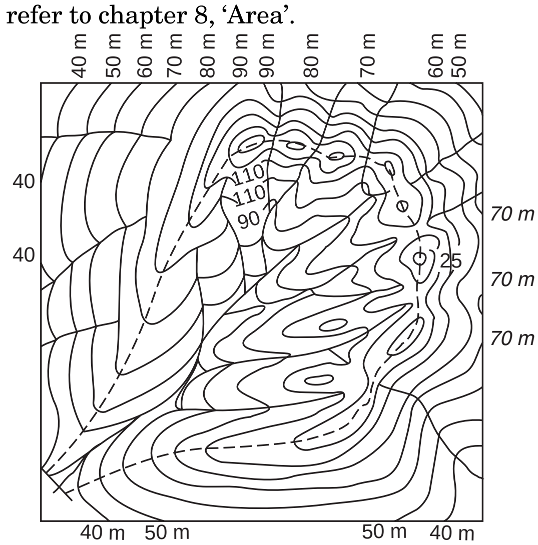


Fig. 7.29. A catchment area.

Knowing the maximum water level of the reservoir, the extent of the area to be submerged, will be the area enclosed by the contour of the maximum water level and its area may also be calculated by a planimeter. The tract of the area to be submerged under water $= \frac{a}{A} \times 100\%$ where a is the area of water body of the reservoir and A is the area of its catchment basin.

VI. Calculation of Storage Capacity of Reservoirs

The capacity of a reservoir at its dam site may easily be calculated with the help of a contour map. Knowing the maximum water level of the dam and contour interval, the area enclosed at respective elevations may be found out by a planimeter. The capacity of the reservoir may then be computed by using Trapezoidal or Prismoidal formulae.

Trapezoidal formula :

$$\text{Volume } (V) = h\left[\frac{A_1 + A_n}{2} + A_2 + A_3 ..... + A_{n-1}\right] \qquad ...(7.1)$$

Prismoidal formula :

$$V = \frac{h}{3}\left[A_1 + A_n + 4\,(A_2 + A_4 + ...) + 2\,(A_3 + A_5 + ...)\right] \qquad ...(7.2)$$

where $A_1, A_2, A_3 ... A_n$ are the areas enclosed between successive contours and h is the vertical contour interval.

Example 7.1. *Following data refers to a site of a reservoir. The areas are the ones which will be contained by a proposed dam and contour lines as given below :*

Contour in meteres	*Area enclosed in hectares*
610	22
615	110
620	410
625	890
630	1158

Calculate the total volume of water impounding.

Solution.

Applying prismoidal formula, we get

$$V = \frac{h}{3}\left[A_1 + A_n + 2 \times \text{ Odd ordinates} + 4 \times \text{ Even ordinates}\right]$$

$$= \frac{5}{3} [22 + 1158 + 2 \times 410 + 4 \times (110 + 890)] \times 100 \times 100$$

$$= \frac{5}{3} [22 + 1158 + 820 + 4000] \times 10000$$

$$= \frac{5}{3} \times 6000 \times 10000$$

= **10,00,00,000 cu. m. Ans.**

Example 7.2. *From a topographical map, the areas enclosed within the contour lines and along the face of a proposed dam, are as given below:*

Contour	*Area (square metres)*
300	29,750
295	26,850
290	21,050
285	18,500
280	13,440
275	8,750
270	5,180
265	735
260(Bottom)	30

Calculate the volume of water in the reservoir formed, when the water level is at an elevation of 30 m, using :

(*i*) *Trapezoidal formula* (*ii*) *Prismoidal formula*

Solution. Here contour interval = 5 m

(*i*) **Trapezoidal formula**

$$V = h\left[\frac{A_1 + A_n}{2} + A_2 + A_3 ...... + A_{n-1}\right]$$

$$= 5\left[\frac{29.75 + 0.03}{2} + 26.85 + 21.05 + 18.50\right.$$

$$\left. + 13.44 + 8.75 + 5.18 + 0.735\right] \times 1000$$

$$= 5 [14.890 + 94.505] \times 1000 = 5 \times 109.395 \times 1000$$

= 5,46,975 cu, m. **Ans.**

(*ii*) **Prismoidal formula**

$$V = \frac{h}{3} [A_1 + 2 \text{ times odd} + 4 \text{ times even} + A_n]$$

$$= \frac{5}{3} [29.75 + 2(21.05 + 13.44 + 5.18) + 4(26.85 + 18.50 + 8.75 + 0.735) + 0.030] \times 1000$$

$$= \frac{5}{3} [29.75 + 2 \times 39.67 + 4 \times 54.835 + 0.030] \times 1000$$

$$= \frac{5}{3} [29.75 + 79.34 + 219.34 + 0.030] \times 1000$$

$$= \frac{5}{3} \times 328.46 \times 1000$$

V = 547,433 cu. m. **Ans.**

Example 7.3. *In a proposed hydro-electric project a storage reservoir was required to provide a storage of 4.50 million cu. m. between lowest draw down (L.D.D.) and top water level (T.W.L.). The areas contained within the stated contours and up stream face of the dam were as follows:*

Contour	100	95	90	85	80	75	70	65
Area in hectares	30	25	23	17	15	13	7	2

If L.D.D. was to be 68 m, calculate the T.W.L. for :

(*a*) *Full storage capacity* (*b*) *60% full storage capacity*

(Use end area method for calculating volumes).

Solution. It is assumd that area of cross-sections increases with height.

(*a*) The area contained in 68 m contour

$$= 2 + \frac{68 - 65}{5} \times 5 = 5 \text{ hectares.}$$

Volume of water contained between 68 m and 70 m

$$= 2 \times \frac{(5 + 7)}{2} \times 10{,}000 = 120{,}000 \text{ m}^3 \qquad ...(i)$$

Let the T.W.L. for full capacity be above 95 m.

∴ Volume of water contained between 70 m and 95 m

$$= 5 \left[\frac{7 + 25}{2} + 13 + 15 + 17 + 23 \right] 10{,}000$$

$$= 5 \times 84 \times 10{,}000$$

$$= 42{,}00{,}000 \text{ m}^3 \qquad ...(ii)$$

Total volume between 68 m and 95 m = 42,00,000 + 1,20,000

$= 43{,}20{,}000 \text{ m}^3$

Volume short of target = 45,00,000 – 43,20,000

$= 1{,}80{,}000 \text{ m}^3$

Let the height of water level above 95 m contour for full capacity be x m, then, $x\left(\dfrac{25+25+x}{2}\right) \times 10{,}000 = 18{,}000$

or $\quad x^2 + 50x - 36 = 0$

or $\quad x = 0.71$ m

$\therefore$ Water level for full capacity = 95 + 0.71 = 95.71 m **Ans.**

(*b*) 60% of full capacity $= 45{,}00{,}000 \times \dfrac{60}{100} = 27{,}00{,}000 \text{ m}^3$

Let us assume that water level coincides the 90 m contour.

Volume of water contained between 68 m and 90 m contours

$$= 120{,}000 + 5\left[\frac{7+23}{2} + 13 + 15 + 17\right] \times 10000$$

$$= 1{,}20{,}000 + 30{,}00{,}000 = 31{,}20{,}000 \text{ m}^3$$

which is more than what is required.

Again, volume contained between 68 m and 85 m contours

$$= 120{,}000 + 5\left[\frac{7+17}{2} + 13 + 15\right] 10{,}000$$

$$= 1{,}20{,}000 + 20{,}00{,}000$$

$$= 21{,}20{,}000 \text{ m}^3$$

which is less than what is required.

Hence, it is evident that water level for 60% capacity is above 85 m but below 90 m

Difference in volume = 27,00,000–21,20,000

$= 5{,}80{,}000 \text{ m}^2$

Let the height of water level above 85 m contour for 60% capacity be 'x' m

$$\therefore \quad x\left(\frac{17 + 17 + \frac{6}{5}x}{2}\right) \times 10{,}000 = 580{,}000$$

or $\quad 34x + 1.2x^2 = 116$

or $\quad 1.2x^2 + 34x - 116 = 0$

or $\quad x = 3.08$ m

$\therefore$ Top level for 60% storage capacity = 85 + 3.08 + 88.08 m **Ans.**

Example 7.4. *From the following spot RL's, draw contours of 10 m, 20 m, 30 m on a plan (scale 1 : 1000) :*

North (m)	50	50	50	50	100	100	100	150	150	150
East (m)	0	50	100	150	0	50	150	0	100	150
R.L. (m)	5	13	22	28	12	18	34	21	33	27

Solution. The required contours are shwon in Fig. 7.30.

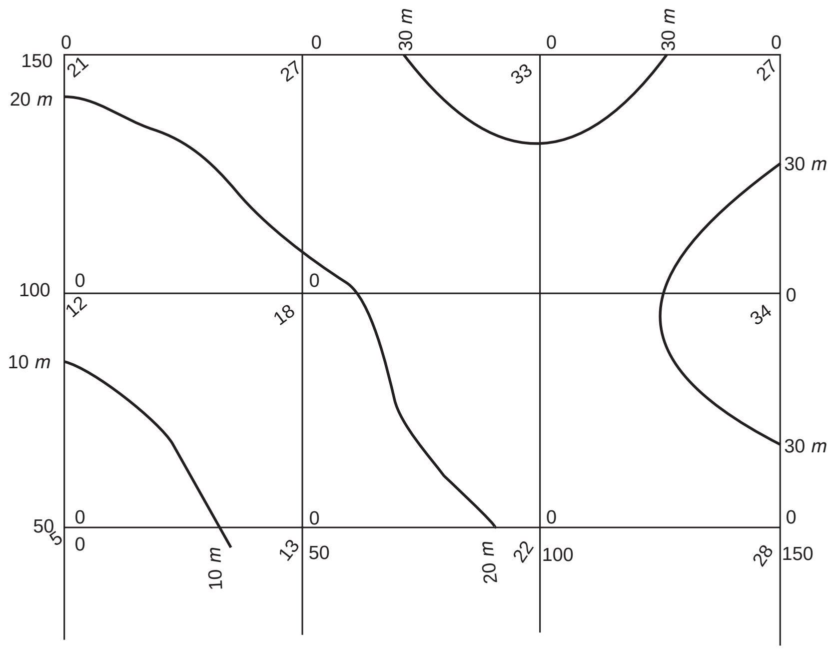


Fig. 7.30

EXERCISE 7

1. Fill in the blanks with suitable word(s).

(*i*) An imaginary line joining the points of same elevation, is known as.....................

(*ii*) The vertical distance between any two consecutive contours, is called.....................

(*iii*) The shortest horizontal distance between two consecutive contours, is known as....................

(*iv*) The contour interval of a map depends upon :

(*a*) .... (*b*) ....

(*c*) .... (*d*)....................

(*v*) The general formula for deciding a contour interval, is :

$$\text{Contour interval} = \frac{\text{...........}}{\text{No. of cm per km}} \text{ metres.}$$

(*vi*) Contour interval for a map on scale 1 : 100,000 is .... metres.

(*vii*) Contours equally spaced depict....................

(*viii*) Two contours of different elevations do not cross each other except in the case of....................

(*ix*) The method of contouring in which contours are actully traced out in the field, is known as ....method of contouring.

(*x*) An imaginary line lying throughout on the surface of the ground and preserving a constant inclination to the horizontal, is known as....................

2. Fill in the blanks with suitable word(s) from the brackets.

(*i*) Lines on a map which are at equal vertical distances are known as .... (gradients, contour gradients, contours)

(*ii*) Contours when unite together form a ....
(cliff, overhanging cliff, ridge)

(*iii*) Equidistant and parallel contours represent ....areas
(flat, steep slope, gentle slope, slope)

(*iv*) With the help of contours, angle of slope of a hill.....................be determined. (can, cannot)

(*v*) Contour at any point is ....to the line of the steepest slope at that point. (perpendicular, parallel)

(*vi*) Ring contours of higher values inward represent a ....
(hill, lake, depression)

(*vii*) Square method of indirect contouring is commonly used in ....areas. (hill, flat, nearly flat)

(*viii*) Direct method of contouring is ....accurate than indirect method of contouring. (more, less)

(*ix*) An Indian tangent clinometer reads the ....of the vertical angles. (sine, tangent, cotangent)

(*x*) Distances computed by observations with an Indian tangent clinometer, ... slope correction. (require, do not require)

(*xi*) In case of square method of contouring, the size of squres depends upon the....................

(Contour interval, scale of the plan, nature of the ground, all) (*xii*) For contouring a hilly terrain ....method is most suitable. (tacheometric, direct, squares)

(*xiii*) If contour interval is 5 metres, and the lowest point in an area is at 57m above datum, the lowest contour to be surveyed, is..................... (57 m, 60 m, 62 m)

(*xiv*) Angle of slope of any two consecutive contours at any place on a map, is .... (same, not same)

(*xv*) On a contour map intervisibility of points ....ascertained. (can be, cannot be)

3. State whether following statements are 'True' or 'False'. If false, rewrite their corrected statements :

(*i*) Contour interval on any map is kept constant.

(*ii*) Contours cannot intersect but may unite to form a single contour.

(*iii*) Direct method of contouring is more accurate than an indirect method of contouring.

(*iv*) The shortest horizontal distance between two consecutive contours, is known as horizontal equivalent which is kept constant throughout a map.

(*v*) The square method of contouring is one of the indirect methods of contouring.

(*vi*) The contour interval on a map sheet is changed, where area consists of fairly flat area.

(*vii*) Three dimensioned configuration of the earth surface, can only be accurately represented by contours on a map.

(*viii*) Contour lines are the intersections of the surface of the earth and the level surfaces separated by a distance equal to the contour interval.

(*ix*) The perimeter of a lower value contour is always less than the perimeter of the next higher value contour.

(*x*) Equispaced and parallel contours show that the ground is flat.

(*xi*) Contour interval is inversely proportional to the scale of the contour plan.

(*xii*) In hilly areas, level will be easier to work with than a tachnometer for contouring.

(*xiii*) The accurate limits of the catchment area of a river, may be ascertained with the help of a contoured map.

(*xiv*) More the accuracy required, larger should be the contour interval.

(*xv*) Horizontal equivalent is more if the slope is steeper.

(*xvi*) The horizontal equivalent will be kept constant throughout the contour map.

4. Define contour. What do you understand by contour interval and on what factors does it depend ?

5. What is meant by "Contour interval" ? Name the factors that govern the selection of the contour interval and describe how these affect the choice.

6. Show with neat sketches, the characteristic features of contour lines of the following : (*i*) An overhanging cliff; (*ii*) A pond; (*iii*) A depression; (*iv*) A ridge line; (*v*) Area having flat slope; (*vi*) A valley; (*vii*) A saddle or pass; (*viii*) A vertical cliff; (*ix*) A plateau; (*x*) A plain with a knoll;

7. (*a*) Define contours and give characteristics of contours.

(*b*) Describe the method of plotting contours by taking spot levels in the field.

8. What are the uses of a map ? How will you determine the inter-visibility of points if the contour map is given to you. Explain by giving an example.

9. What are the different methods of contouring ? Under what conditions these are used. Discuss their merits and demerits.

10. (*a*) Explain the use of a contour map in finding the volume of water of a proposed dam at a particular place.

(*b*) What do you understand by interpolation of contours ? Explain in detail.

11. (*a*) Wht do you understand by interpolation of contours ? Explain their importance in location of a hill road.

(*b*) Explain the method of locating the alignment of a road with a rising gradient 1 in 40 on a contour map, having 25 m contour interval and scale 1 : 100,000.

12. (*a*) What is the difference between direct and indirect methods of contouring ?

(*b*) What do you understand by interpolation of contours ?

13. R.L.'s. of the corners of 20 m side squares have been worked out and tabulated below. Prepare a contour map with 5 m vertical interval.

513.0	518.0	527.0	535.0	543.0	550.0
522.0	519.0	526.0	537.0	547.0	548.0
532.0	533.0	528.0	530.0	537.0	537.0
538.0	543.0	533.0	521.0	531.0	530.0
546.0	542.0	532.0	515.0	522.0	523.0
544.0	535.0	527.0	515.0	510.0	513.0

14. The following table gives areas contained within the stated contours on the upstream face of a dam in a proposed hydroelectric project. If the lowest draw down level is to be 234.00 m, calculate the total contents at R.L. 300.00 m (Use the Prismodial formula).

R.L. in (m)	*Area in sq. m.*
300	312600
290	265900

280	247400
270	220800
260	192200
250	175900
240	127600
230	52000

15. From a topographical map, the areas enclosed within the contour lines and along the face of a proposed dam are as given under :

Contour (m)	*Area (square metres)*
300	29750
295	26850
290	21050
285	18500
280	13440
275	8750
270	5180
265	735
260 (Bottom level of reservoir)	30

Compute the volume of water in the reservoir formed, when the water level is at elevation 300 m using :

(*a*) Trapezoidal formula (*b*) Prismoidal formula

16. Show valleys, ridge lines, saddle points, stream and water shed line on the contour map shown in Fig. 7.31.

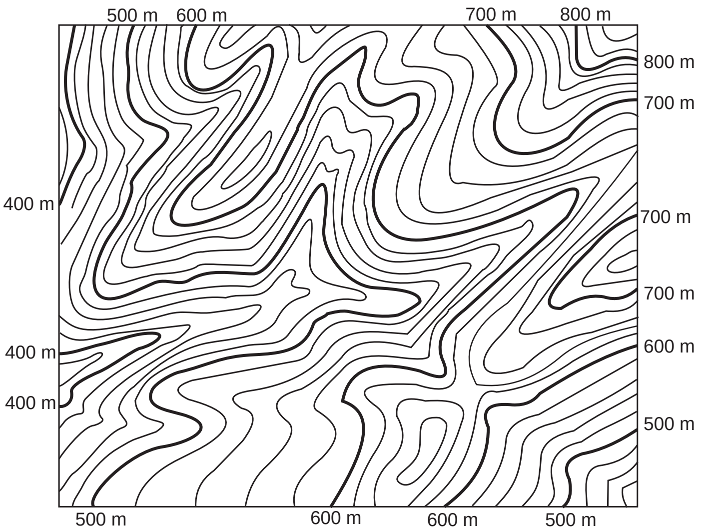
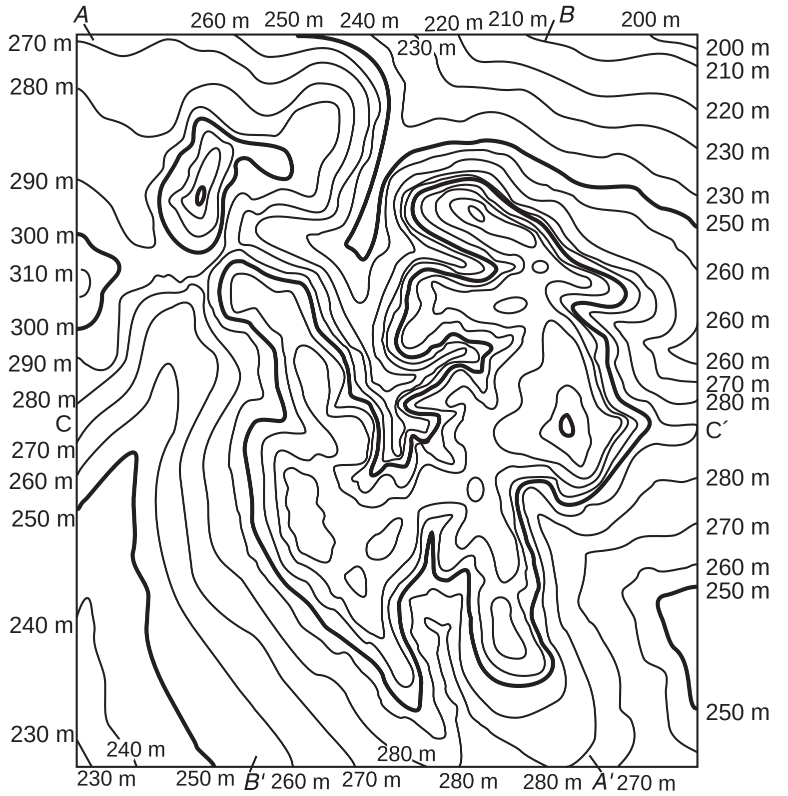


Fig. 7.31.

17. Draw sections along AA', BB' and CC' on the contour map shown in Fig. 7.32.



Scale 1 cm = 10 m

Fig. 7.32.

18. (*a*) What are the characteristics of contour lines ?

(*b*) At the site of a reservoir, areas enclosed between the dam and the contours are as follows :

Contour level (metres)	Enclosed Area (square metres)
95	552
97	6350
99	52600
101	78000
103	368000

Calculate the capacity of the reservoir by "Traperzoidal formula".

ANSWERS

1. (*i*) Contour (*ii*) Contour interval (*iii*) Horizontal equivalent (*iv*) Scale, available time, purpose of map, nature of the ground (*v*) 20 (*vi*) 20 (*vii*) inclined plane (*viii*) hanging cliff (*ix*) direct (*x*) contour gradient.

2. (*i*) Contours (*ii*) cliff (*iii*) slope (*iv*) can (*v*) perpendicular (*vi*) hill (*vii*) nearly flat (*viii*) more (*ix*) tangent (*x*) do not require (*xi*) all the three (*xii*) tacheometric (*xiii*) 60 m (*xiv*) not the same (*xv*) can be.

3. (*i*) True (*ii*) True (*iii*) True (*iv*) False (*v*) True (*vi*) False (*vii*) True (*viii*) True (*ix*) False (*x*) False (*xi*) True (*xii*) False (*xiii*) True (*xiv*) False (*xv*) False (*xvi*) False.

14. 13,862,186 m^3.

15. (*a*) 546,975 m^3. (*b*) 547,433 m^3

18. 540768 m^3.

8

Areas

8.1. INTRODUCTION

Area is defined as the area of a tract of land as projected upon a horizontal plane and not the actural area of the surface of the land.

The units of area in metric system, commonly used, are :

(*i*) The square metre *i.e.* area of a square whose each side is one metre.

(*ii*) The 'are' *i.e.* area of a square whose each side is 10 metres.

(*iii*) The hectare *i.e.* area of a land containing 100 square arcs or 10,000 sq. metres.

The area of agricultural land is measured in hectares whereas area of urban properties is measured in square metres.

The units of area in F.P.S. system or English units, are :

(*i*) The square foot *i.e.* the area of a square whose each side is one foot.

(*ii*) The acre *i.e.* the area of a land containing 4840 sq. yards or 43,560 sq. feet.

Table 8.1. Conversion of British Units to Metric Equivalents

Acre	*Sq. Yards*	*Sq. Feet*	*Metric Equivalent*
1	4840	43,560	0.40467 hactare
—	1	9	0.836 m^2
—	—	1	0.929 m^2

Table 8.2. Conversion of Metric Units to British Equivalents

Hectare(ha)	*Areas (a)*	*Sq. metre (m^2)*	*British Equivalent*
1	100	10,000	2.4710 acres
—	1	100	1076.4 sq.ft.
—	—	1	10.764 sq. ft

8.2. DETERMINATION OF AREAS

Depending upon the topography of the terrain and the required accuracy, the area of land portion, may be determined by the following methods.

(*i*) From the field notes

(*ii*) From the plotted plan or map.

1. Computation of areas from field notes. Whenever the area of a plot of land is to be determined directly from the field notes, it should be ensured that survey lines include the whole area and the land is divided into geometrical figures such as triangles, squares, rectangles, etc. In case of triangles, altitudes of the triangles, should also be measured in the field to simplify calculations.

The area of geometrical figures is calculated from one of the following formulae :

1. The area of a triangle

$$= \sqrt{s\,(s-a)\,(s-b)\,(s-c)} \text{ where } a, b \text{ and } c$$

are its sides and $s = \frac{1}{2}(a + b + c)$.

OR

The area of a triangle $= \frac{1}{2} \times b \times h$ where b is the base and h is the altitude of the triangle.

2. The area of a square $= a^2$ where a is the side of the square.
3. The area of a rectangle $= a \times b$ where a and b are the sides of the rectangle.
4. The area of a trapezium $= \frac{1}{2}(a + b) \times d$ where a and b are its parallel sides and d is the perpendicular distance between them.

2. Areas between the survey lines and boundaries. The area between boundaries and the survey lines may be calculated as under :

A number of offsets are measured from the survey line to the nearest boundary line (Fig. 8.1). The area of the belts bounded by adjacent offsets, the boundary line and the base line may be assumed as trapeziums and their area may be computed as under:

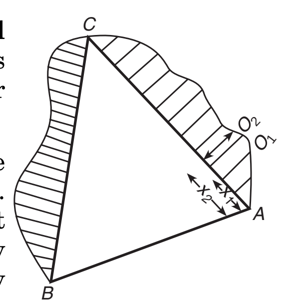


Fig. 8.1.

The area is calculated by multiplying the mean of each successive pair of adjacent offsets, by the distance between them.

Let O_1 = offset length at a change x_1 from A along AC.

O_2 = offset length at a chainage x_2 from A along AC.

Then, the mean offset length = $\frac{O_1 + O_2}{2}$

The distance between offsets = $x_2 - x_1$

$\therefore$ Area of the trapezium = $\frac{O_1 + O_2}{2} \times (x_2 - x_1)$

8.3. COMPUTATION OF AREAS FROM PLANS

The methods of determination of area from plans, may be further sub-divided into two categories :

(*i*) Graphical method; (*ii*) Instrumental method.

1. Graphical method. In this method, area of plots of land is obtained from scaled measurements on the plan. The method consists of dividing the whole area into a number of geometrical figures and calculating their areas and also the area of irregular strips along the boundaries.

2. Entire area of geometrical figures. The whole area may be divided into convenient geometrical figures *i.e.,* triangles, squares, trapeziums, etc.

(*i*) **By division into triangles**. The whole area may be divided into a number of traingles of convenient bases and altitudes. The area of each triangle is obtained by multiplying its half base by its altitude.

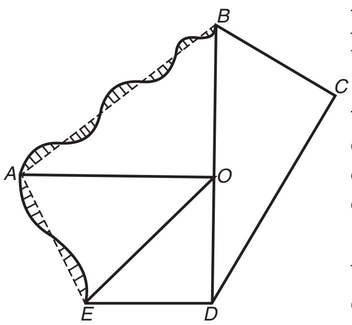


Fig. 8.2. A net work of triangles.

In case, boundaries are irregular, these may be replaced by straight lines drawn in such a way that the areas included and excluded are approximately equal.

Suppose the area of a plot is bounded by figure *ABCDEA* (Fig. 8.2). Sides *BC, CD* and *DE* are straight and the sides *AB* and *AE* are along zig zag lines. For calculating the area, the plot is divided into four triangles. Irregular boundaries *AB* and *AE* have been replaced by straight sides by drawing straight lines *AB* and *AE* as shown in Figure by dotted lines.

(*ii*) **By division into squares** (Fig. 8.3). In this method, squares of known sides and area, are ruled on a tracing paper which is placed on the plan. The number of complete squares is counted. The fraction of a square is judged by eye and

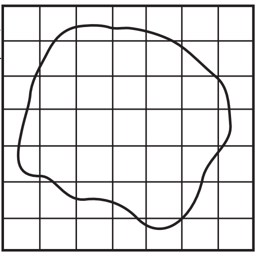


Fig. 8.3. A net work of square.

taken as full square if it is more than half and ignored if less than half.

The required area of the plot = Total number of these squares multiplied by the area of one square.

(*iii*) **By division into trapezoids** (Fig. 8.4). In this method, equidistant lines are drawn on a transparent paper. The constant distance should represent either some full metres or centimetres depending upon the scale of the plan. A ruled tracing paper is then placed on the plan in such a way that entire area is brought between any two parallel lines. The area is then divided into a number of strips, the curved ends are replaced by perpendicular *give and take lines* as shown in Fig. 8.4.

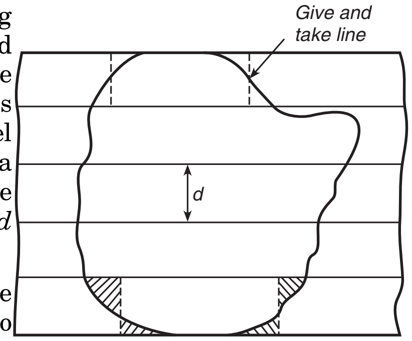


Fig. 8.4. A network of trapezoids.

"The total area of the plan = the sum of the lengths of all rectangles so formed × common breadth *i.e.,* the distance between the parallel lines".

The length of the rectangles may be scaled by means of an ordinary plotting scale or a computing scale.

The computing scale. It consists of an ordinary scale and a crusor with a fine wire stretched at right angles to the graduated scale. (Fig. 8.5).

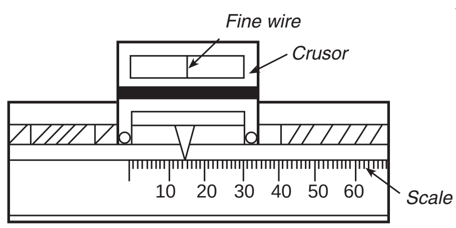


Fig. 8.5. A computing scale.

The computing scale is used for measuring the length of the rectangles without drawing the give and take lines on transparent paper.

Use. The ruled line paper is placed on the plan so that entire area of the plan is brought between two parallel lines. Transparent paper is then fixed on the plan. Place the computing scale over it such that the edge of the graduated scale is kept parallel to the lines. With the crusor wire at zero, the scale is adjusted so that crusor wire balances the boundary at the left end of the upper most line. The crusor is then slided to the right end till it balances the boundary. The final reading on the scale is the required length. In a similar way the scale is placed parallel to the next parallel line and with the crusor wire at the fixed reading. Adjust the wire so that it balances the boundary. The crusor is shifted to right side. In this way, the accumulated length of the rectangles is the last reading of the scale for the lowest line, with the crusor at the right side.

8.4. AREA BETWEEN A STRAIGHT LINE AND IRREGULAR BOUNDARY

The area enclosed between a straight line drawn on the plan and an irregular boundary line, may be determined as under :

(*i*) Divide the base line into suitable number of equal parts.

(*ii*) Draw ordinates at each of the points of division and scale off their lengths.

The required area between the base line and the boundary line may be calculated by one of the following methods :

1. The mid-ordinate rule.
2. The average ordinate rule.
3. The trapezoidal rule.
4. The Simpson's rule.

In the first three rules, it is assumed that the portions of the boundary between the ends of the ordinates, are straight lines. In Simpson's rule, it is assumed that the portions are parabolic arcs and as such the Simpson's rule, is some times known as the *Parabolic Rule.*

1. The mid-ordinate rule. In this method a base line *AB* is divided into a number of equal parts and ordinates are drawn at the mid-points of each division. The length of each ordinate, is then scaled off. (Fig. 8.6).

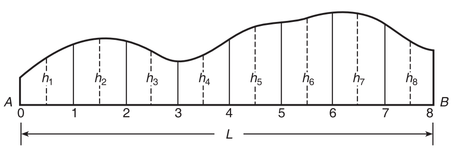


Fig. 8.6. The mid-ordinate rule.

If L = scaled length of the base line *AB*

n = number of divisions of the base line.

d = common distance between the ordinates.

$h_1, h_2, \ldots h_n$ etc. = scaled lengths of the ordinates at the mid points of each divisions.

The area of the plot

$$= h_1 \times d + h_2 \times d + h_3 \times d + \ldots\ldots + h_n \times d$$

$$= (h_1 + h_2 + h_3 + \ldots + h_n)\, d \qquad \ldots(8.1.)$$

or Area = sum of the mid ordinates multiplied by the common distance d.

2. The average ordinate rule. In this method, a base line is drawn on the plan dividing the area in two approximately equal parts and is divided into a number of equal divisions. Ordinates are drawn at the points of division and their lengths are scaled off. (Fig. 8.7).

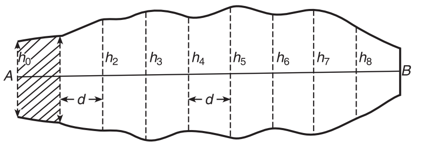


Fig. 8.7. The average ordinate method.

Let $h_0, h_1, h_2, h_3 \ldots h_n$ etc. be the lengths of the ordinates

d = distance between adjacent ordinates

L = length of the base line AB.

n = number of equal parts in which base line AB is divided.

The area of the plot

= Average ordinate × Length of the base

$$= \frac{h_0 + h_1 + h_2 + \ldots + h_n}{n+1} \times L \quad \ldots(8.2)$$

Note. It is assumed that the entire area is made equal to the rectangle whose length is equal to the base line and breadth is equal to the average ordingate. The accuracy of the area obtained by this method, will depend upon the number of divisions.

3. The trapezoidal rule. In this method, a base line AB is drawn and is divided into equal parts. The ordinates at each point of division are drawn and their lengths scaled off. This method assumes that the area between adjacent ordinates is of the shape of a trapezoid.

Statement. "*To the sum of the first and the last ordinates, add twice the sum of the remaining ordinates. Multiply the total sum by the common distance between the ordinates. Half of the product equals to the required area.*"

Let $h_1, h_2, h_3 \ldots\ldots h_n$ etc. be the lengths of the ordinates

n = number of the divisions.

L = length of the base line.

d = distance between adjacent ordinates.

$$\text{Area} = \frac{1}{2}\frac{L}{n}\left[\frac{h_1 + 2h_2 + \ldots + 2h_{n-1} + h_n}{2}\right]$$

$$= \frac{1}{2} d\ [h_1 + 2h_2 + 2h_3 + \ldots + 2h_{n-1} + h_n]$$

or $$= \frac{d}{2} [h_1 + 2h_2 + 2h_3 + ... + 2h_{n-1} + h_n]$$

$$= d\left[\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + \dots + h_{n-1}\right] \quad ...(8.3)$$

Note. When there is an apex at one or both the ends of the base, resulting h_1 or/and h_n, though being equal to zero, is also included in the trapezoidal formula.

Derivation of the trapezoidal formula (Fig. 8.8)

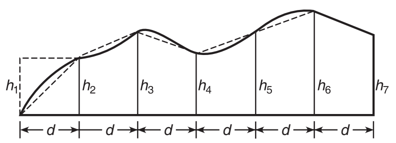


Fig. 8.8. Trapezoidal formula.

Assumption. It is assumed that the ends of ordinates are joined by straight lines. Thus, the area between the base line and the irregular boundary is divided into a series of trapezoids whose parallel sides (*i.e.* ordinates) are equidistant.

Let h_1, h_2, h_3.....h_n be the lengths of ordinates at equal interval.

and d = distance between adjacent ordinates

We know that the area of a trapezoid

= half the sum of parallel sides × perpendicular distance between them.

∴ Area of the first trapezoid

$$= \frac{h_1 + h_2}{2} \times d$$

Area of the second trapezoid

$$= \frac{h_2 + h_3}{2} \times d$$

Area of the third trapezoid

$$= \frac{h_3 + h_4}{2} \times d$$

....

....

Area of the last trapezoid

$$= \frac{h_{n-1} + h_n}{d} \times d$$

Hence, total area

$$A = \frac{d}{2}\,[h_1 + h_2 + h_2 + h_3 + h_3 + h_4 + h_{n-1} + h_n]$$

$$= d\,[\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + ...h_{n-1}] \qquad ...(8.4)$$

or

$$= \frac{d}{2}\left[h_1 + 2h_2 + 2h_3 + 2h_4 + \ldots + 2h_{n-1} + h_n\right] \qquad ...(8.4a)$$

4. The Simpson's Rule. In this method, a base line is drawn and divided into portions of equal length. At each point of divisions, ordinates are drawn and their lengths are scaled off. The area of the portion bounded by the base line and the irregular boundary, is calculated from the Simpson's rule which is stated below.

Statement. "*To the sum of the first and last ordinates, add twice the sum of the remaining odd ordinates and four times the sum of the even ordinates. Multiply the total sum by one third the common distance between the ordinates, the result gives the required area*" *i.e.*

$$\text{Area} = \frac{d}{3}\,[h_1 + 2\,(h_3 + h_5 + h_7 + ...) + 4\,(h_2 + h_4 + ...) + h_n] \qquad ...(8.5)$$

Note. The following points may be noted :

1. The Simpson's rule is applicable only if number of ordinates is odd. If the number of the ordinates is even, the area of the last trapezoid may be calculated separately and added to the result obtained by applying the Simpson's rule to remaining trapezoids.

2. Even if first or / and last ordinate happens to be zero, these are not omitted from the Simpson's formula.

Derivation of the Simpson's Formula

Assumption. In this method, it is assumed that the portions of irregular boundary between the ordinates, are the arcs of parabolas. (Fig. 8.9).

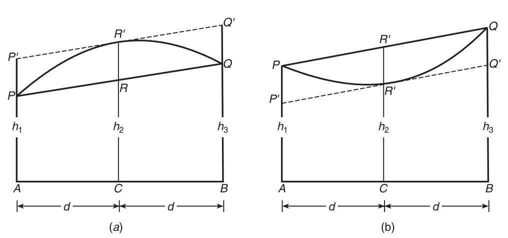


Fig. 8.9. The Simpson's formula

Let h_1, h_2 and h_3 be three consecutive ordinates

d = distance between the ordinates

$\therefore$ Area = Area $APR'QB$ bounded by the base line AB, ordinates, h_1 and h_3 and the parabolic arc $PR'Q$.

Construction. Join P and Q. Draw a parallel line $P'R'Q'$ through R' to meet AP and BQ produced at P' and Q'.

The area of the portion $APR'QB$ = Area of the trapezoid $APQB$ $\pm$ area of the segment $PR'QRP$ between the parabolic arc $PR'Q$ and the chord PQ.

The area of trapezoid $APQB$

$$= \frac{h_1 + h_3}{2} \times 2d. \qquad ...(8.6)$$

The area of the segment $PR'QRP$

$$= \frac{2}{3} \times \text{Area of the enclosing parallelogram } PP'Q'Q.$$

$$= RR' \times AB \qquad ...(8.7)$$

where $AB = 2d$

But, $RR' = CR' - CR$

$$= h_2 - \left(\frac{h_1 + h_3}{2}\right) \quad \text{for Fig. 8.9 } (a) \qquad ...(8.8)$$

$$= \left(\frac{h_1 + h_3}{2}\right) - h_2 \quad \text{for Fig. 8.9}(b)$$

From Eqns. (8.6), and (8.7) (Fig. 8.9 a), we get

$$\text{Area} = \left[\frac{h_1 + h_3}{2} \times 2d\right] + \frac{2}{3}\left\{h_2 - \frac{h_1 + h_3}{2}\right\} \times 2d.$$

$$= d(h_1 + h_3) + \frac{2}{3}(2h_2 - h_1 - h_3)d$$

$$= d\left(h_1 + h_3 + \frac{4h_2}{3} - \frac{2h_1}{3} - \frac{2h_3}{3}\right)$$

$$= \frac{d}{3}(3h_1 + 3h_3 + 4h_2 - 2h_1 - 2h_3) = \frac{d}{3}(h_1 + 4h_2 + h_3)$$

Similarly the area of 2nd trapezoid

$$= \frac{d}{3}(h_3 + 4h_4 + h_5)$$

Area of 3rd trapezoid

$$= \frac{d}{3}(h_5 + 4h_6 + h_7)$$

∴ Total required area

$$A = \frac{d}{3}(h_1 + 4h_2 + 2h_3 + 4h_4 + \ldots\ldots + 2h_{n-2} + 4h_{n-1} + h_n)$$

or
$$A = \frac{d}{3}[h_1 + h_n + 2\,(\text{Sum of remaining odd ordinates}) + 4\,(\text{Sum of remaining even ordinates})$$

8.5. COMPARISON OF ACCURACIES ACHIEVED BY SIMPSON'S RULE AND TRAPEZOIDAL RULE.

The results obtained by using the Simpson's rule are more accurate as compared to those obtained by trapezoidal rule. Hence, Simpson's rule is invariably used when better accuracy is required.

Results obtained by using the Simpson's rule are greater or lesser than those obtained by using the trapezoidal rule according as the curve of the boundary is concave or convex towards the base line.

Note. For application of both the rules, the interval between the consecutive ordinates *must* be uniform through out the length of the base. If the interval is not the same, the base line may be divided into different sections, each having the same interval. The areas may be calculated separately and the total area may be obtained by adding the areas of each section.

Example 8.1. *The following perpendicular offsets were taken at 10 m intervals from a survey line AB to an irregular boundary line, 2.30, 3.80, 4.55, 6.75, 5.25, 7.30, 8.95, 8.25 and 5.50 metres. Calculate the area in square metres, enclosed between the survey line, the irregular boundary, the first and last offsets by the application of* (*i*) *the Simpson's rule,* (*ii*) *the trapezoidal rule, and* (*iii*) *the average ordinate rule.*

Solution. (Fig. 8.10)

Given : Common distance = 10 m.

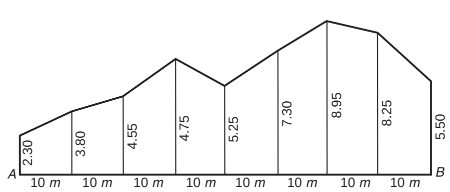


Fig. 8.10

(*i*) **By Simpson's rule**

The area $A = \frac{d}{3}$ [First offset + Last offset + Twice the sum of remaining odd offsets + Four times the sum of remaining even offsets]. ...(*i*)

Substituting the values in Eqn. (*i*) we get,

$$A = \frac{10}{3} [2.30 + 5.50 + 2\ (4.55 + 5.25 + 8.95) + 4\ (3.80 + 6.75 + 7.30 + 8.25)]$$

$$= \frac{10}{3} [2.30 + 5.50 + 37.50 + 104.40]$$

$$= \frac{10}{3} \times 149.70 = \textbf{449 sq. m. Ans.}$$

(*ii*) **By the trapezoidal rule**

The area

$$A = \frac{d}{2} [\text{First offset} + \text{Last offset} + 2 \times \text{Sum of remaining offsets}]$$

$$A = \frac{10}{2} [2.30 + 5.50 + 2\ (3.80 + 4.55 + 6.75 + 5.25 + 7.30 + 8.95 + 8.25)\ ]$$

$$= \frac{10}{2} [2.30 + 5.50 + 2 \times 44.85] = 5 \times 97.5$$

$$= \textbf{487.5 sq. m. Ans.}$$

(*iii*) **By the average ordinate rule**

The area $A = \frac{\text{sum of ordinates}}{\text{No. of ordinates}} \times \text{length of base}$

$$= \frac{2.30 + 3.80 + 4.55 + 6.75 + 5.25 + 7.30 + 8.95 + 8.25 + 5.50}{9} \times 80$$

$$= \frac{4212}{9} = \textbf{468.0 sq. m. Ans.}$$

Example 8.2. *The following offsets were taken from a chain line to an irregular boundary :*

Distance in m	0	6	12	18	24	36	48	60	72	81	90
Offset in m	3.6	3.0	2.4	1.8	1.2	1.3	2.1	2.4	3.0	3.3	3.9

Calculate the area enclosed between the chain line, the irregular boundary and the end offsets by Simpson's rule and trapezoidal rule.

Solution. (Fig. 8.11).

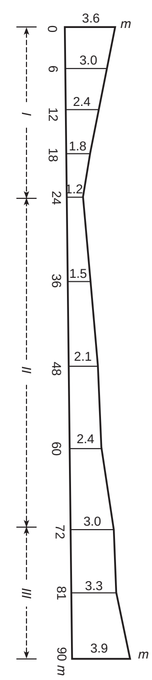


Fig. 8.11.

(*a*) **Simpson's rule.** The entire chain line may be divided into three portions as shown Fig. 8.11.

The area of the portion

$$I = \frac{6}{3}\,[\,3.6 + 1.2 + 4\,(3.0 + 1.8) + 2\,(2.4)]$$

$$= 2\,[4.8 + 19.2 + 4.8] = 57.6 \text{ m}^2$$

$$II = \frac{12}{3}\,[1.2 + 3.0 + 4\,(1.5 + 2.4) + 2 \times 2.1] = 4\,[4.2 + 15.6 + 4.2]$$

$= 96.0 \text{ m}^2$

$$III = \frac{9}{3} \; [3.0 + 3.9 + 4\,(3.3) + 2 \times 0] = 3\;[6.9 + 13.2] = 60.3 \text{ m}^2$$

∴ Total area enclosed

$= I + II + III = 57.6 + 86.0 + 60.3 = 213.9 \text{ m}^2$. **Ans.**

(*b*) **Trapezoidal rule**

The area of the portion

$$I = d\left[\frac{x_1 + x_n}{2} + (x_2 + x_3 \ldots)\right] = 6\left[\frac{3.6 + 1.2}{2} + (3.0 + 2.4 + 1.8)\right]$$

$$= 6\left[\frac{4.8}{2} + 7.2\right] = 6\;[2.4 + 7.2] = 6 \times 9.6 = 57.6 \text{ m}^2$$

$$II = 12\left[\frac{1.2 + 3.0}{2} + (1.5 + 2.1 + 2.4)\right]$$

$$= 12\left[\frac{4.2}{2} + 6.0\right] = 12 \times 8.1 = 97.2 \text{ m}^2$$

$$III = 9\left[\frac{3.0 + 3.9}{2} + 3.3\right]$$

$$= 9\left[\frac{6.9}{2} + 3.3\right] = 9\;[3.45 + 3.3\,] = 9 \times 6.75 = 60.75 \text{ m}^2$$

∴ Total area enclosed $= I + II + III$

$= 57.6 + 97.2 + 60.75 = 215.55 \text{ m}^2$ **Ans.**

Example 8.3. *A series of offsets were taken from a chain line to a curved boundary line at an interval of 10 m in the following order :*

0, 2.85, 3.95, 6.45, 8.60, 8.90, 5.25, 0, metres. Calculate the area between the chain line and the curved boundary line by the Simpson's rule.

Solution. (Fig. 8.12).

Note. In order to the apply Simpson's rule, the number of offsets should be odd. In this problem, number of offsets is even. Hence, the last offset is ignored for calculating the area of the remaining portion and the area between last two offsets is calculated by trapezoidal rule and added, to get the required area.

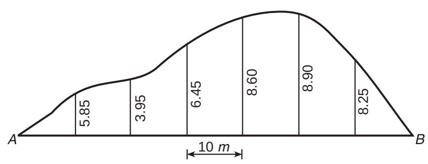


Fig. 8.12.

$$\therefore \quad \text{Area} = \frac{10}{3}\,[0 + 5.25 + 2\,(3.95 + 8.60) + 4\,(2.85 + 6.45 + 8.90)\,]$$

$$+ \frac{10}{2}\,(0 + 5.25)$$

$$= \frac{10}{3}\,[5.25 + 25.10 + 72.80] + 26.25 = 343.83 + 26.25$$

$$= \mathbf{370.08\ sq.\ m.} \quad \mathbf{Ans.}$$

Example 8.4. (a) *What is Simpson's Rule in the computation of areas of figures ? Derive an expression for it.*

The following offsets were taken from a chain line to a hedge.

Distance in metres	0	30	60	90	120	150	180
Offsets in metres	9.4	10.8	12.5	10.5	14.5	13.0	7.5

Compute the area included between the chain line, the hedge and end offsets by the Simpson's rule.

Solution. (a) For statement and derivation of the Simpson's rule, refer to article 8.4.

Area

$A = \frac{d}{3}$ [sum of the first and last offsets + Twice the sum of the remaining odd offsets + Four times the sum of remaining even offsets

$$= \frac{30}{3}\,[19.4 + 7.5 + 2\,(12.5 + 10.5) + 4\,(10.8 + 10.5 + 13.0)]$$

$$= 10\,(16.9 + 54.0 + 137.2) = \mathbf{2081\ sq.\ m.} \quad \mathbf{Ans.}$$

Example 8.5. *A plot of land ABCDA has four sides. The sides AB and BC are straight and the sides CD and DA are irregular. The above plot was surveyed by chain and tape by the method of chain surveying, fixing four stations at A, B, C and D and was moved and measured in* *the clockwise direction. The straight distances measured from one station to the other, were as follows :*

AB = 150 m ; BC = 165 m; CD = 155 m; DA = 162 m; AC = 230 m; The offsets measured from chain lines CD and DA to the irregular boundaries are as under :

Distance from C in (m)	*Offsets in (m)*
0	*0.00 left*
30	*150 "*
60	*2.00 "*
90	*2.25 "*
120	*1.75 "*
155	*0.00 "*
Distance from D in (m)	*Offsets in (m)*
0	*0.00 right*
30	*1.62 "*
60	*2.45 "*
90	*2.30 "*
120	*1.22 "*
162	*0.00 "*

Calculate the area of the plot ABCDA.

Solution. (Fig. 8.13).

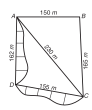


Fig. 8.13.

We know that the area of a triangle whose sides are a, b and c is given by the equation :

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad ...(i)$$

where $2s = a + b + c$.

Area of Δ ABC :

$$a = 165 \; ; b = 230 \; ; c = 150 \; ; 2s = 545 \; ; s = 272.5$$

$$\therefore \; A_1 = \sqrt{272.5\,(272.5 - 165)\,(272.5 - 230)\,(272.5 - 150)}$$

$= \sqrt{272.5 \times 107.5 \times 42.5 \times 122.5}$

$= 12349.49$ sq.m.

Area of Δ ACD

$a = 155, c = 162, d = 230, 2s = 547, s = 273.5$

$\therefore\ A_2 = \sqrt{273.5\ (273.5 - 155)\ (273.5 - 162)\ (273.5 - 230)}$

$= \sqrt{273.5 \times 118.5 \times 111.5 \times 43.5}$

$= 12537.75$ sq. m.

Area between irregular boundary and side *CD*

$$A_3 = \frac{30}{3}\left[\ 0 + 1.75 + 4\ (1.50 + 2.25) + 2 \times 2.0 + \frac{0 + 1.75}{2} \times 35\right]$$

$= 10\ [\ 0 - 1.75 + 15.0 + 4.0] + 0.875 \times 35 = 207.5 + 30.625$

$= 238.125$ sq. m.

Area between irregular boundary and side *DA*

$$A_4 = \frac{30}{3}\ [0 + 1.22 + 4\ (1.62 + 2.30) + 2 \times 2.45\ ] + \frac{0 + 1.22}{2} \times 42$$

$A_4 = 10\ [\ 0 + 1.22 + 15.68 + 4.9\ ] + 0.61 \times 42$

$= 218.0 + 25.62 = 243.62$ sq. m.

Area of plot of land *ABCDA*

$=$ Area of Δ *ABC* + Area of Δ *CDA* $+ A_3 - A_4$

$= 12349.49 + 12537.75 + 238.125 - 243.62$

$= 24881.745$ sq. m. **Ans.**

Example 8.6. *Calculate using Simpson's rule the area enclosed between the boundaries of the field, offsets to which have been taken from a chain line at intervals of 25 m to the right and left.*

Offset left	*Distance*	*Offset right*
33.6 metres	*0 metres*	31.9 metres
29.8 "	*25* "	*27.5* "
55.3 "	*50* "	*48.9* "
47.2 "	*75* "	*56.6* "
24.6 "	*100* "	*18.3* "
21.7 "	*125* "	*25.4* "
18.1 "	*150* "	*41.6* "

Solution. (Fig. 8.14).

We shall calculate the area in two parts :

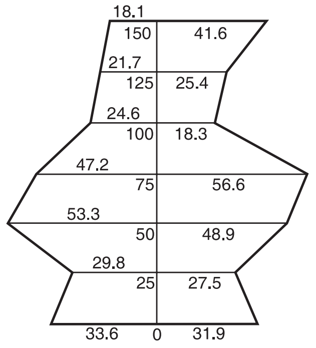


Fig. 8.14.

The area enclosed by the chain line and ends of the offsets on the right.

$$A = \frac{d}{3} \; [\; h_1 + h_n + 2\,(h_3 + h_5 + \ldots + 4\,(h_2 + h_4 + \ldots\,)]$$

$$A = \frac{25}{3}\,[\;(31.9 + 41.6) + 2\,(48.9 + 18.3) + 4\,(27.5 + 56.6 + 25.4\;]$$

$$= \frac{25 \times 645.9}{3}$$

$$= 5382.5 \text{ sq. m.}$$

The area enclosed by the chain line and the ends of the offsets on left.

$$A_2 = \frac{d}{3}\;[\;h_1 + h_n + 2\,(h_3 + h_5 + \ldots\,) + 4\,(h_2 + h_4 + \ldots..\,)]$$

$$= \frac{25}{3}\;[\;33.6 + 18.1 + 2\,(55.3 + 24.6)\;4\,(29.8 + 47.2 + 21.7)]$$

$$= \frac{25}{3} \times 606.3 = 5052.5 \text{ sq. m.}$$

Total area $= A_1 + A_2 = 53825 + 5052.5 = 10435\ \text{m}^2$ **Ans.**

8.6. CALCULATION OF AREAS OF A CLOSED TRAVERSE FROM CO-ORDINATES

The area of a closed traverse from field notes, may be calculated by one of the following methods :

1. Areas from coordinates (x and y)
2. Areas from Latitudes and Double Meridian Distances
3. Areas from Departures and Total Latitudes

1. Areas from Co-ordinates (Fig. 8.15).

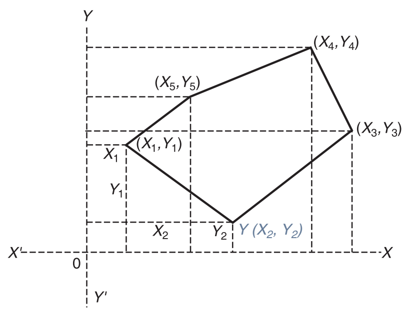


Fig. 8.15.

If we know the coordinates of a traverse stations, then the area enclosed by the closed traverse, may be calculated as under :

Rules. The following rules are followed.

1. Number the stations of traverse in a serial order.
2. Multiply each ordinate / abscissa by the difference between the following and the preceding abscissa/ordinate. It may be ensured that the same order is followed *i.e.* either subtracting the preceding one from the following or subtracting the following from the preceding one.
3. Find the sum of the products.
4. Half of the sum gives the area enclosed by the traverse.

i.e. Areas $= \frac{1}{2} [y_1 (x_2 - x_n) + y_2 (x_3 - x_1) + y_3 (x_4 - x_2) + ... + y_n (x_1 - x_{n-1})]$

where x_1, x_2, x_3 ,......., x_n = abscissa and y_1, y_2, y_3 ,........y_n = ordinates.

OR

1. Arrange the coordinates in the order shown in Fig. 8.16.
2. Find the sum of the products of the coordinates joined by the full lines, and the sum of the products of the coordinates joined by broken lines.

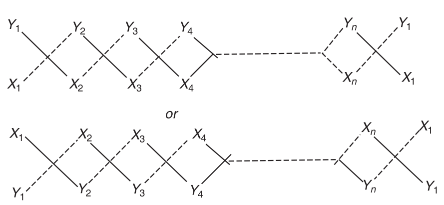


Fig. 8.16. Areas from Coordinates.

3. Find the difference of these two sums, which gives twice the area of the traverse.
4. Half of this difference gives the required area of the traverse.

2. Areas from latitudes and double meridian distance (D.M.D.)

The following definitions may be clearly understood before discussing the method.

1. **The meridian distance of a line or longitude.** It is the perpendicular distance of the mid-point of the line from the reference meridian. In Fig. 8.17, the meridian distance of line *BC* is marked by the arrow M.D.
2. **The double meridian distance (D.M.D.) or double longitude of a line.** It is the sum of meridian distances of the two ends of the lines.
3. **The departure of a line.** It is the abscissa of the consecutive coordinates.

Rules for finding out the D.M.D. or double longitude.

1. The D.M.D. of the first line is equal to the departure of that line.
2. The D.M.D. of each succeeding line = D.M.D. and departure of the preceding line + departure of the line itself.
3. The D.M.D. of the last line = departure of the last line with the opposite sign.

Explanation of the rules (Fig. 8.17).

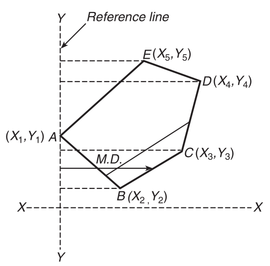


Fig. 8.17.

D.M.D. of the line AB = departure of the line $= x_2$

D.M.D. of the line BC $= x_2 + x_2 + (x_3 - x_2) = (x_2 + x_3)$

D.M.D. of the line CD $= (x_2 + x_3) + (x_3 - x_2) + (x_4 - x_3) = x_3 + x_4$

D.M.D. of the line DE $= x_3 + x_4 + (x_4 - x_3) + (x_5 - x_4) = x_4 + x_5$

D.M.D. of the line EA $= x_4 + x_5 + (x_5 - x_4) - x_5 = x_5$.

Hence, the area of a closed traverse = Half the algebraic sum of the products of the latitude of each line by its D.M.D.

Note. The following points may be noted :

1. The reference meridian should be assumed to pass through the most westernly station.
2. The most westernly station is the station at which the latitude changes from south to north or the departure changes from west to east.
3. While calculating the D.M.D. of the lines, due regards should be paid to the signs of the departure.

3. Area from departure and total latitudes (Fig. 8.18)

Assume any one of the stations as a reference station from which the total latitudes of the other stations may be calculated.

Rules :

1. Calculate the total departure of each station.
2. Calculate the algebraic sum of the departures of the lines meeting at that station.
3. Multiply total latitudes of each station by corresponding algebraic sum of the departures.
4. Calculate the algebraic sum of the products.
5. Half the sum gives the required area of the closed traverse.

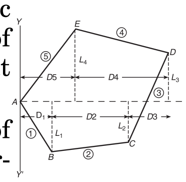


Fig. 8.18. Area from departures and total latitudes.

Or

"Twice the area within the lines of a closed traverse, is equal to the algebraic sum of products of the total latitudes of each station by the algebraic sum of the departures of the two lines meeting at the station."

Example 8.7. *Calculate the area enclosed by a closed traverse with the following data :*

Side	Latitudes (m)		*Departures (m), +*	
	N	*S*	*E*	*W*
AB	*220.5*	-	*120.0*	-
BC	-	*240.2*	*200.5*	-
CD	-	*160.0*	-	*100.5*

DA	*179.7*	-	-	*220.0*

Solution. The area of the enclosed traverse may be calculated by three different methods as discussed below :

(*a*) **Area from coordinates**

(*i*) Assuming the coordinates of *A* as (N 200, E 100), the independent coordinates of each station, are calculated and tabulated as under :

Side	*Lat.*	*Dep.*	*Station*	*Independent coordinates*	
AB	+ 220.5	+ 120.0	A	200.0	100.0
BC	–240.2	+ 200.5	B	420.5	220.0
CD	–160.0	-100.5	C	180.3	420.5
DA	+179.7	-220.0	D	20.3	320.0
			A	200.0	100.0

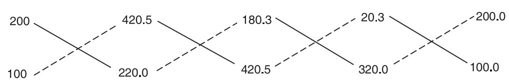


Fig. 8.19.

(*ii*) Arrange the independent coordinates in the form of a determinant. (Fig. 8.19).

$\Sigma P = 200 \times 220.0 + 420.5 \times 420.5 + 180.3 \times 320.0 + 20.3 \times 100.0$

$= 280546.25$

$\Sigma Q = 100 \times 420.5 + 220.0 \times 180.3 + 420.5 \times 20.3 + 320.0 \times 200.0$

$= 154252.15$

$\therefore$ Twice the area

$= \Sigma P - \Sigma Q = 28054.625 - 15452.15 = 126294.10$

$\therefore$ Area of the closed traverse *ABCDA*

$= 63147.05$ sq. m. **Ans.**

(*b*) **Area from latitudes and double meridian distances**

Assuming *A* to be most westernly station, calculate the double meridian distances of the sides *AB, BC, CD* and *DA*.

Calculation of D.M.D.

D.M.D. of *AB* = 120.0

D.M.D. of *BC* = 120.0 + 120.0 + 200.5 = 440.5

D.M.D. of *CD* = 440.5 + 200.5 − 100.5 = 540.5

D.M.D. of *DA* = 540.5 − 100.5 − 220.0 = 220.0

The results are tabulated as under :

Side	*Lat.*	*Dep.*	*D.M.D.*	*Double area = Col. 2 × Col. 4*	
1	2	3	4	5	6
AB	+ 220.5	+ 120.0	120.0	26460.0	–
BC	– 240.2	+200.5	440.5	–	–105808.10
CD	–160.0	-100.5	540.5	–	–86480.00
DA	+ 179.7	-220.0	220.0	+39534.0	
			Total	+65994.0	–192288.10

Algebraic sum = 192288.10 – 65294.0 = 126294.10

∴ Twice the area enclosed = 126294.10

or Area = 63147.05 sq.metre. **Ans.**

(*c*) **Area from the departures and total latitudes**

Assuming station *A* as the reference point, calculate the total latitudes of the station *B, C* and *D* and tabulate :

Total latitude of $B = \Sigma L = 220.5$

Total latitude $C = \Sigma L = 220.5 - 240.2 = -19.7$

Total latitude $D = \Sigma L = -19.7 - 160.0 = -179.7$

Total latitude of $A = \Sigma L = -179.7 + 179.7 = 0.0$

Side	*Lat.*	*Dep.*	*Station*	*Total Lat.*	*Algebraic sum of the adjoining departures*	*Double area = Col 5 × Col 6*	
						+	–
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
AB	+ 220.5	+ 120.0	B	+ 220.5	+ 320.5	70670.25	–
BC	– 240.2	+ 200.5	C	– 19.7	+ 100.0	–	1970.0
CD	– 160.0	– 100.5	D	– 179.7	– 320.5	57593.85	–
DA	+ 179.7	– 220.0	A	0.0	– 100.0	–	0.0
					Total	128264.10	1970.0

Algebraic sum = 128264.10 – 1970.0 = 12694.10

∴ Area of the enclosed traverse

$$= \frac{126294.10}{2} = 63147.05 \text{ sq.metre.} \quad \textbf{Ans.}$$

Example 8.8. *The latitudes and departures of the lines of a closed traverse are given below. Calculate the area of the traverse.*

Line	*Latitude (m)*		*Departure (m)*	
	Northing	*Southing*	*Easting*	*Westing*
AB	...	*157.2*	*158.4*	-
BC	*210.5*	...	*52.5*	-
CD	*175.4*	...	...	*98.3*
DA	...	*228.7*	...	*109.0*

Solution. Assuming the station *A* as western most, calculate total latitudes of the stations *B, C* and *D* and tabulate the result.

Side	*Lat.*	*Dep.*	*Station*	*Total Lat.*	*Algebraic sum of the adjoining departures*	*Double area = Col 5 × Col 6*	
						+	–
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
AB	+ 157.2	+ 158.4	B	– 157.2	+ 210.5		33150.48
BC	+ 210.5	+ 52.5	C	+ 53.3	– 45.8		2441.14
CD	+ 175.4	– 98.3	D	+ 228.7	– 207.3		47409.51
DA	- 228.7	– 109.0	A	0.0	+ 49.4		0.0
						Total	83004.13

∴ Double the area enclosed by the traverse = 83004.13

Area = 41502.07 sq. m. **Ans.**

Example 8.9. *The following are the latitudes and departures of the sides of 4 closed treaverse ABCD :*

Line	*Latitude (m)*	*Departure (m)*
AB	–116.1	–44.4
BC	+6.8	+58.2
CD	+ 80.5	+17.2
DA	+28.8	–31.0

Compute the area of 5.4 traverse by the departures and total method.

Solution.

We shall assume station *B* as reference point.

Total latitude of *C* = 6.8

Total latitude of *D* = 6.8 + 80.5 = 87.3

Total latitude of *A* 87.3 + 28.8 = 116° 1

Total latitude of B 116.1 – 116.1 = 0

Side	Lat.	Dep.	Stn.	Total latitude	Depature		
BC	+6.8	+58.2	C	6.8	75.4	512.72	–
CD	+80.5	+17.2	D	87.3	–13.8	–	1204.74
DA	+28.8	–31.0	A	116.1	–75.4	–	8753.94
AD	–116.1	–44.4	B	0.0	+13.8	–	
						–	9958.68
							512.72

Twice the area = 9445.96

∴ Area of the traverse = 4722.98

Note : The total departure of a station is the algebraic sum of the departure of the adjacent sides of traverse.

For example, the total departure of static C

= departure of the side BC and that of CD.

= 58.2 + 17.2 = 75.4

8.7. AREA WITH A PLANIMETER

Fig. 8.20. A planimeter

The instrument which is employed for determining the areas of plotted maps, is known as *Planimetre.* Areas of plotted maps, obtained by a planimetre are more accurate than those obtained by any other methods discussed in previous articles.

Planimeters are of the following two types ;

(*i*) The rolling planimeter. (*ii*) Amslar Polar planimeter.

The Amslar planimeter which is generally used these days, is shown in Fig. 8.20.

Amslar Planimeter. It consists of two arms hinged at a point. One of the two arms, is called the *anchor arm* which carries a needle for fixing in the paper with a small weight placed on the needle for holding the end in position. The other arm, called the *tracing arm* is of an adjustable length. It carries a tracing point which is moved along the boundary of the area. A wheel whose axis of rotation is parallel to the tracing arm, measures the normal displacement of the tracing point. The measuring wheel may be placed either between the pivot point and the tracing point or beyond the pivot point away from the tracing point. The wheel drum is graduated into 100 parts. A vernier is used to read tenths of a part of the wheel. The wheel is geared to a counting disc which gives the complete revolutions of the wheel. The counting disc is divided into 10 equal parts. Ten turns of the wheel are required for one revolution of the counting disc. Each complete reading of a planimeter, therefore, contains a number of 4 digits. The units are given by the counting disc, the tenths and hundredths are read on the wheel and the thousandths part is read on the vernier.

The planimeter rests on three points *i.e.,* the tracing point, the anchor point and the periphery of the wheel drum.

When the tracing point is moved along the boundary of the area, the wheel partly rotates and partly slides. The normal component of the motion causes the rotation of the wheel and the axial component of the motion causes a slip which does not affect the reading of the graduated wheel. The total displacement normal to the tracing arm, measures the area of the enclosed boundary.

8.8. USE OF A PLANIMETER

The area of a plan may be determined with a planimeter as explained below :

1. Set the index mark on the bevelled edge of the slide to the scale to which the plan is drawn.
2. Fix the anchor point firmly in the paper outside or inside the figure, ensuring that the tracing point reaches every point on the boundary line without any difficulty. It is always preferable to keep the anchor point outside the figure. However, for larger areas, the anchor point may be kept inside the figure.

3. Set the tracing point on a marked point on the boundary of the figure.
4. Observe the readings on the wheel, the counting disc and the vernier. Record the reading which is called the *initial reading (I.R.)*.
5. Move the tracing point along the periphery of the area in a clockwise direction until it reaches the starting point.
6. Observe the readings on the wheel, the counting disc and the vernier. Record the reading which is called the *final reading (F.R.)*.
7. Keep on watching the number of times the zero mark of the dial passes the index mark in a clockwise or anticlockwise directions
8. The area of the figure may be calculated from the formula.

Area $A = M\,(FR - IR \pm 10\,N + C)$ ...(8.9)

where M = a multiple whose value is marked on the tracing arm.

N = number of times of zero mark of the dial passes the fixed mark.

C = a constant marked on the tracing arm just above scale divisions and is added only when the anchor point is inside the figure.

A + ve sign of N is to be accepted if the zero of the dial passes the fixed index in the clockwise direction *i.e.*

$A = M\,(F.R. - I.R. + 10\,N + C)$, when the anchor point is inside.

and $A = M\,(F.R. - I.R. + 10\,N)$, when the anchor point is outside.

A negative sign of N is to be accepted if the zero of the dial passes the fixed index in an anti-clockwise direction *i.e.*

$A = M\,(F.R. - I.R. - 10\,N + C)$, when the anchor point is inside.

and $A = M\,(F.R. - I.R. - 10\,N)$, when the anchor point is outside.

8.9. ZERO CIRCLE OF A PLANIMETER

The circle along which, if the tracing point is moved, no rotation of the wheel takes place and it only slides on the paper without changing the reading, is called a *zero circle*. It is also sometimes called a *circle of correction.*

The zero circle is obtained by moving the tracing point in such a way that the line joining the tracing point to the wheel, is kept at right angles to the line joining the anchor point to the wheel. (Fig. 8.21).

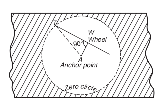


Fig. 8.21. Circle of correction.

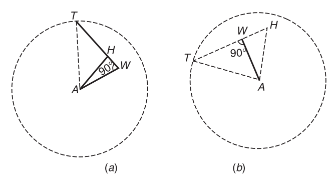
The line joining the anchor point and the tracing point is known as the *radius of the zero circle* whose centre remains at the anchor point A.

When the anchor point is kept inside the figure, the area of the zero circle is always added to the recorded area of the planimetre because the planimetre records only the area of the annular space between the figure and the circumference of the zero circle.

8.10. THE AREA OF THE ZERO CIRCLE

Depending upon the position of the anchor point, the area of the zero circle may be calculated, as under :

(*a*) **When the wheel is outside the pivot and tracing point** (Fig. 8.22 *a*).



(*a*) (*b*)

Fig. 8.22. Area of zero circle.

Let A represent the anchor point

T represent the tracing point

H represent the pivot point

W represent the wheel.

Then, angle AWT must be a right angle,

$$\therefore \quad AT^2 = AW^2 + TW^2.$$

But, from $\Delta\, AWH$, $AW^2 = AH^2 - HW^2$

$$\therefore \quad AT^2 = AH^2 - HW^2 + TW^2 \qquad \text{...(8.10)}$$

Again, Let L = length of tracing arm *i.e.*, the distance between the tracing point and the pivot.

L_1 = distance between the pivot and the wheel.

R_1 = length of the anchor arm i.e., the distance between the pivot and the anchor point.

R = radius of the zero circle.

Substituting the values in Eqn. (8.10), we get

$$R^2 = R_1^2 - L_1^2 + (L_1 + L_2)^2$$

$$= L^2 + 2L\,L_1 + R_1^2$$

or $$R = \sqrt{L^2 + 2L\,L_1 + R_1^2} \quad \text{...(8.11)}$$

∴ The area of the zero circle

$$= \pi\, R^2$$

$$= \pi\,(L^2 + 2L\,L_1 + R_1^2) \quad \text{...(8.12)}$$

(*b*) **When the wheel is between the pivot and tracing point** (Fig. 8.22 *b*).

Proceeding as in (*a*) above it may be proved that the area of the zero circle

$$A = \pi\,(L^2 - 2L\,L_1 + R_1^2) \quad \text{...(8.13)}$$

8.11. METHODS OF FINDING THE AREA OF THE ZERO CIRCLE

It may be noted that the area of the zero circle changes according to the setting of the instrument. It may be determined by one of the following methods :

1. By using formulae. The area of zero circle may be computed from the following formulae.

(*a*) Area of the zero circle $= \pi\,(L^2 \pm 2L\,L_1 + R_1^2)$, the letters having their usual meanings. Use a plus sign when wheel is placed outside the pivot and the tracing point.

(*b*) Area of the zero circle $= M \times C$ ...(8.14)

M = multiplier, the value of which is marked next to the scale division.

C = additive constant, the value of which is engraved on the top of the tracing arm just above the scale division.

2. By Measuring the radius of the zero circle. (Fig. 8.23). Draw two lines AW and TW at right angles to each other intersecting at W on a drawing paper. Place the wheel at W and the anchor point on the line WA at A'. Now move the tracing point till it rests on the line WT at T'.

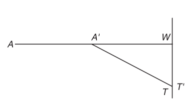


Fig. 8.23. Radius of zero circle.

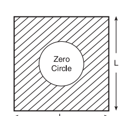


Fig. 8.24. Zero circle.

Measure the distance $A' T'$ which is the required radius (R) of the zero circle.

$\therefore$ Area of the zero circle $= \pi R^2$. ...(8.15)

3. By using the Planimetre (Fig. 8.24). There are two methods, *i.e.,*

(*a*) Find out the area of a known geometrical figure by a planimetre, keeping its anchor point inside.

The area of the zero circle = Actual area of the figure 196 Area recorded by the planimetre

(*b*) Find out the area of the geometrical figure by keeping the anchor point outside and again by keeping the anchor point inside.

The area of the zero circle = the difference of the two areas.

Precautions. The following precautions must be taken while using a planimetre.

1. The figure of the area must be kept on a horizontal table.
2. To avoid additive constant, the anchor point must be kept outside the figure.
3. If the area of the figure is too large, it may be divided into parts of convenient sizes and the area of each part is measured with the anchor point outside the part in each case and the results are added.
4. The area of the figure should be determined twice with different positions of the anchor point and the mean value to be accepted.
5. Time should not be wasted to set the vernier to read zero. Whatever the reading it indicates, may be accepted as the initial reading (I.R.).
6. The surface of the paper on which the wheel rolls must by smooth. If the plan is drawn on a rough paper, a koda trace may be placed over it.
7. The folds in the drawing paper should be avoided as it affects the results.
8. The tracing point should be moved exactly on the line. To facilitate this, a straight edge or a French curve may be used where possible.
9. The area should also be checked roughly by calculation to provide a check against gross error.

8.12. PRACTICAL METHOD OF USING A PLANIMETRE

The area of any figure plotted on paper may be found out using a planimetre as follows :

1. Draw a square of suitable size whose side represents some whole metres.
2. Set the tracing point on one of the corners of the square and read the vernier, the drum and the counting disc. Let it be the initial reading.
3. Move the tracing point along the sides of the square in a clockwise direction until it reaches the starting point.
4. Observe and record the final reading.
5. Calculate the constant x of the planimetre by finding out the difference of the two readings.
6. Similarly, with the same anchor point, find out the initial and final readings for the figure and let the difference of readings be y.
7. Area of the figure $= \frac{y}{x} \times$ area of the square ...(18.15)

Example 8.9. *With the anchor point outside, the difference of readings of a planimetre for a square* 2 *cm* × 2 *cm is 1.655 and when it is moved round the perimeter of an irregular area is 8.275. Find the area of the figure if the scale of the map is 1 cm = 5 metres.*

Solution. The area of the square = 2 × 2 = 4 sq. cm

Ground area of the square = 4 × 25 = 100 sq. m.

∴ Area of the figure

$$= \frac{y}{x} \times \text{Area of the square} = \frac{8.275}{1.655} \times 100$$

= 500 sq. metres. **Ans.**

Example 8.10. *Calculate the area of a figure from the following readings recorded by a planimetre with the anchor point inside the figure.*

Initial reading 8.277 ; final reading 2.256

M = 100 sq cm ; C = 23.521

It was observed that the zero mark on the dial passed the index once in the anticlockwise direction.

Solution. From Eqn. (8.9) we get

$$A = M\,(\text{F.R.} - \text{I.R.} \pm 10N + C) \qquad ...(i)$$

Here M = 100 sq cm ; I.R. = 8.277 ; F.R. = 2.256 ; C = 23.521 N = –1

Substituting the values in Eqn. (i) we get

$$\therefore \quad A = 100\,(2.256 - 8.277 - 10 + 23.521)$$

= 750 sq. cm. **Ans.**

Example 8.11. *The following readings were obtained when the perimetre of a rectangle 15″ × 10″ was traversed clockwise with the anchor point inside the rectangle and the tracing arm set to the natural scale (M = 10 sq inches). Initial reading = 0.586 ; final reading = 9.876. The zero of the counting disc passed the fixed index mark twice in the reverse direction. Find the area of the zero circle.*

Solution. Area of the rectangle

$- 15 \times 10 = 150$ sq. inch. ...(*i*)

Measured area of the rectangel

$= M$ (F.R. – I.R. – $10\,N + C$)

$= 10\,(9.876 - 0.586 - 10 \times 2 + C)$

$= 10\,(-10.710 + C)$ ...(*ii*)

Equating the Eqns. (*i*) and (*ii*), we get

$10\,(-10.710 + C) = 150$

$\therefore \quad C = 15 + 10.710 = 25.710$

Hence, the area of the zero circle $= MC = 10 \times 25.710$

$= 257.10$ sq. in. **Ans.**

Example 8.12. *The area of a figure drawn to a scale of 10 metres to 1 cm was measured by a planimetre with the anchor point inside. The tracing arm being set to the natural scale. The initial and final readings were 6.582 and 4.698 respectively. The zero of the dial passed the fixed index mark once in the reverse direction. Calculate the area of the figure given that C = 20.*

Solution.

Here $I.R. = 6.582$; $F.R. = 4.698$; $M = 100$; $C = 20$; $N = -1$

$\therefore$ Area of the figure

$= M\,(F.R. - I.R. - 10\,N + C)$

$= 100\,(4.698 - 6.582 - 10 \times 1 + 20) = 811.6$ sq. cm.

$\therefore$ Area of the figure $= 811.6 \times 100 = 811.6$ sq. m. **Ans.**

Example 8.13. *The area of a square, 10 cm side was measured by a planimetre with the anchor point outside the figure and the initial and final readings were found to be 6.852 and 8.764 respectively. With the same setting of the tracing arm and the anchor point outside, another irregular figure was traversed clockwise and the initial and final readings were found to be 2.378 and 8.656 respectively. What is the area of the figure.*

If the plan scale is 1 cm = 10 m, calculate the area in sq metres.

Solution. Area of the figure

$$= M\,(F.R. - I.R. -+ 10\,N + C)$$

In this problem, $N = 0$, and also $C = 0$ as the anchor point is outside the figure.

1st case :

$$\therefore \quad 10 \times 10 = M\,(8.764 - 6.852)$$

or
$$M = \frac{100}{1.912}$$

Second case :

$$\text{Area} = M\,(8.656 - 2.378)$$

or
$$= \frac{100}{1.912} \times 6.278 = 328.347 \text{ sq. cm.}$$

Again, 1 sq cm = 100 sq. m.

∴ Area of the figure 328.347 × 100 = 32834.7 sq. m. **Ans.**

Example 8.14. *Calculate the area of the zero circle with the following data :*

I.R.	*F.R.*	*Position of anchor point*	*Remarks*
7.520	*3.826*	*Outside the figure*	*The zero of the counting disc crossed the fixed index mark once in the clockwise direction.*
2.223	*8.853*	*Inside the figure*	*The zero of the counting disc crossed the fixed index mark twice in the anticlock direction.*

Assume that tracing arm of the planimetre was so set that one revolution of the measuring wheel measures 100 sq cm on the paper.

Solution.

(*i*) Area of the figure in the first case

$$= M\,(F.R. - I.R. + 10\,N), \text{ C being zero}$$

$$100\,(3.826 - 7.520 + 10 \times 1), \text{ being 1}$$

$$= 100 \times 6.306 = \mathbf{630.6} \quad \textbf{sq. cm.}$$

(*ii*) Area of the figure in the second case

$$= M\,(\text{F.R.} - \text{I.R.} - 10\,N + C)$$

$= 100\,(8.853 - 2.223 - 10 \times 2 + C) = 100\,(-13.370 + C)$

Equating the two values, we get

$$100\,(-13.370 + C) = 630.6$$

or $\qquad C = 6.306 + 12.370 = 19.676$

Hence, the area of the zero circle

$$= M \times C = 100 \times 19.676$$

$= 1967.6$ sq. cm. **Ans.**

Example 8.15. *The length of tracing arm of a planimetre is 15.92 cm. the distance from the hinge to the anchor point is 16.0 cm. The diametre of the rim of the wheel is 2 cm. The wheel is placed outside (beyond the hinge from the tracing point) at a distance of 3.00 cm from the hinge. Calculate the area corresponding to one revolution of the wheel and the area of the zero circle.*

Solution. (*i*) We know that the value of M is the area of the plan corresponding one revolution of the wheel = $L \times$ circumference of the rim where L is the distance from the hinge to the tracing point.

Here, $\quad L = 15.92$ cm; $d = 2$ cm

$\therefore \qquad M = 15.92 \times \pi \times 2 = 100.03$ sq. cm. **Ans.**

(*ii*) We know that the area of the zero circle

$= \pi\,(L^2 + 2LL_1 + R^2)$ wheel being outside

Here $\quad L = 15.92$ cm ; $L = 3.0$, $R = 16.0$ cm

$\therefore$ Area of the zero circle

$$= \pi\,[(15.92)^2 + 2 \times 15.92 \times 3.00 + (16.0)^2]$$

$$= \pi\,(253.446 + 95.52 + 256.0) = \pi \times 604.966$$

$= 1900.555$ sq. cm. **Ans.**

Example 8.16. *Calculate the area of a figure from the following data:*

Initial reading = 1.785 ; Final reading = 9.615

The distance from the hinge to the tracing point = 16 cm

The distance from the hinge to the anchor point = 15.92 cm

The distance from the hinge to the wheel = 1.6 cm

The wheel is placed inside (i.e. between hinge and tracing point).

The diametre of the wheel = 2 cm.

The zero of the counting dial crossed the index mark twice in the reverse direction.

Solution.

We know that the area of a figure

$$M = (\text{F.R.} - \text{I.R.} + 10\,N + C) \quad ...(i)$$

where M is a multiplying constant

$$= L \times \pi \times d$$

N is the number of zero passed the dial

C is the constant of the planimetre

Here, $M = 16 \times \pi \times 2 = 100.53$; $N = 2$

$$C = \frac{\text{Area of zero circle}}{N}$$

$$= \frac{\pi\,(16^2 - 2 \times 16 \times 1.6 + 15.92^2)}{100.53} = \frac{1439.62}{100.53} = 14.32$$

Substituting the value in equation (*i*) we get

Area $= 100.53\,(9.615 - 1.785 - 10 \times 2 + 14.32)$

$= 100.53\,(7.83 - 20 + 14.32) = 100.53\,(22.15 - 20.0)$

$= 100.53 \times 2.15$

or $= 216.14$ sq. cm. **Ans.**

Example 8.17. *Calculate the length of the tracing arm and anchor arm so that the multiplying constant and the area of the zero circle equal to 100 sq cm and 2500 sq cm respectively, with the following data :*

Diametre of the wheel = 2 cm, wheel placed 3 cm away from the pivot.

Solution. Multiplying constant

$$M = L\,\pi d$$

$\therefore \quad 100 = l \times \pi \times 2$

or $\quad L = 15.92$ cm.

Area of the zero circle

$$= \pi\,(L^2 + 2LL_1 + R^2)$$

$\therefore \quad 2500 = \pi\,[(15.92)^2 + 2 \times 15.92 \times 3 + R^2)]$

or $\quad R = 21.14$ cm. **Ans.**

EXERCISE 8

1. (*a*) Fill in the blanks with word(s) given in the brackets.

(*i*) Smallest unit of area of urban properties is............(sq. metre, ace)

(*ii*) The accuracy of the area obtained by the average ordinate rule depends upon the ....of divisions of the base line. (number, length)

(*iii*) In trapezoidal formula of areas, the line joining the ends of the ordinates is assumed................ (straight, parabolic, circular)

(*iv*) Simpson's rule of areas can only be applied if the number of ordinates is............ (odd, even)

(*v*) Planimetre is used for measuring......... (volume, area, length)

(*vi*) Areas calculated from plotted plans are ....accurate by a planimetre than by any other method. (less, more)

2. State whether the following statements are true or false.

(*i*) The area of a piece of land is equal to its actual area of the surface of land.

(*ii*) When an end ordinate is zero, it is ignored in the computation by the trapezoidal formula.

(*iii*) If the first and last ordinates are each zero, these are omitted while applying the Simpson's formula.

(*iv*) The planimetre can only be used by placing its anchor point outside the area.

(*v*) The measuring wheel of a planimetre is divided into 1000 parts of its circumference.

(*vi*) Circle of correction and zero circle of a planimetre are the same.

(*vii*) When the anchor point is kept inside the figure, the area of the zero circle is always subtracted.

(*viii*) In ordinary land surveying, the area of a piece of land is taken as its projection upon a horizontal plane, and not the actual area of the surface of the land.

(*ix*) For precise determination of the area of a large tract, the area is taken as the projection of the tract upon the earth's spheroidal surface at mean sea level.

(*x*) Results obtained by using Simpson's Rule are always inferior to those obtained by the Trapezoidal Rule if the boundary curve is convex towards the base line.

(*xi*) The results obtained by the use of Simpson's Rule are always inferior to those obtained by the Trapezoidal Rule.

(*xii*) For small areas, the anchor point of the planimetre is always placed outside the figure.

(*xiii*)

For counter clockwise traversing of the tracing point, the rotation of the roller is in the opposite direction that for clockwise traversing.

(*xiv*) When the tracing arm is held in such a position relative to the anchor arm that the plane of roller passes through the anchor point, the tracing point traverses along the circumference of the zero circle without there being any revolution to the roller.

(*xv*) The net rotation of the roller of a planimetre will always be backward if the area of the figure is greater than that of the zero circle.

(*xvi*) The planimetre can be used for finding the area of all shapes with highest degree of accuracy when measuring areas from plotted plans.

(*xvii*) In any closed traverse if all sides having east departures have accidental (random) errors with minus sign and all sides having west departures have accidental errors with a plus sign, the errors accumulate in departures like systematic errors but compensate in latitudes.

(*xviii*) The area of the zero circle for a planimetre is determined by measuring an area first with the anchor point inside and again with the anchor point outside same figure. The difference between the two results is equal to the area of zero circle.

(*xix*) The accuracy of a planimetre varies with the size of the measured area.

(*xx*) Polar planimetres are more accurate than rolling planimetres.

3. What is Simpson's Rule in the computation of areas of figures. Derive an expression for it.

4. What do you understand by the zero circle of a planimetre ? How can its area be determined. Give two methods only.

5. Describe the planimetre. Explain how you would use it in finding the area of a given figure. What precautions would you take in its manipulation.

6. Describe with the help of neat sketches, the construction and working of a planimetre.

7. Write short notes on :

(*i*) Determination of areas of irregular field without the help of planimetre.

(*ii*) Zero circle.

(*iii*) Simpson's rule.

(*iv*) A planimetre.

8. The offsets taken at 5 m intervals from a chain line to a curved boundary are : 0, 4.6, 6.5, 6.8, 5.2, 3.5, 2.2 metres. Calculate the area between the chain line, the curved boundary line and the end offsets using by Simpson's rule.

9. Calculate the area between the chain line and an irregular boundary and the first and last offsets by Trapezoidal Rule, if the observed data are as follows :

Distance (m)	0	8	16	24	32	10	40
Offset (m)	2.5	4.8	4.8	5.6	4.2	3.8	2.2

10. The following offsets were taken from a chain line to a hedge :

Distance (m)	0	5	10	15	20	25	30	35	40
Offset (m)	0	2.5	5.0	7.5	8.8	7.5	6.5	3.5	0

Calculate the area enclosed between the chain line and hedge by:

(*a*) the Simpon's Rule and (*b*) the Trapezoidal Rule.

11. A rectangular plot of land *ABCD* has the following dimensions:

AB = 50 m ; *BC* = 80 m ; *CD* = 50 m side *DA* is a curved line and to calculate the area of the plot, the following left offsets to the curved boundary while chaining along DA, were taken.

Distance (m)	0	10	20	30	40	50	60	70	80
Offset (m)	0	2.0	3.0	4.0	5.0	4.0	3.0	2.0	0

Calculate the area of the plot of land using Simpson's rule.

12. A series of offsets were taken at 3 m intervals in the following order from a chain line to a curved hedge.

0, 2.26, 1.62, 2.80, 2.04, 2.22, 2.46, 0 m.

Calculate the area between the chain line, the hedge and the end points of the chain line by (*a*) Simpson's Rule and (*b*) Trapezoidal rule.

13. To calculate the area enclosed by a closed traverse the following data was obtained :

Side	*Latitude*		*Departure*	
	N	*S*	*E*	*W*
AB	150.8	–	200.5	–
BC	–	250.5	155.6	–
CD	–	120.5	–	200.8
DA	220.5	–	–	150.3

Calculate the area by using coordinates and check the same by latitudes and double meridian distances.

14. Determine the area enclosed by the traverse legs of a closed traverse ABCDEA whose data is given below :

Traverse leg	*AB*	*BC*	*CD*	*DE*	*EA*
Latitude	+218	–340	–109	–207	+375
Departure	+ 351	+202	+80	–332	–301

(*a*) by independent coordinates method

(*b*) by method of total latitudes and adjacent departures

(*c*) by D.M.D. method.

15. If the multiplying and additive constants of a planimetre are 100 sq. cm and 20.0 respectively, calculate the area of a figure from the following data.

Initial reading = 2.586, final reading = 9.476. The zero of the dial passed the index mark once in the reverse direction.

16. Following observations were taken with a planimetre for finding out the area of a plan :

I.R. = 6.129, F.R. 1.673, multiplying constant = 100 cm^2 ; zero circle constant = 21.372. Zero crosses the pointer once in the anticlockwise direction.

Calculate the area of the plan. Is the anchor point inside or outside the area ?

17. Find the lengths of the tracing arm and the anchor arm to obtain the value of the multiplying constant equal to 100 sq. cm and the area of the zero circle equal 2000 sq. cm units from the following data.

Diametre of the wheel 2 cm.

Wheel placed away from tracing point at a distance = 3 cm.

18. For calculating the area from tracing point outside. The multiplying constant = 100 sq. and the additive constant of planimetre = 200 sq. m.

	Initial reading	*Final reading*
Shyam pur	1.526	3.256
Ram pur	2.866	4.826
Ban pur	4.847	6.027

Calculate the total area of the plan in sq. cm. If the scale of the plan is 1 cm = 100 m, claculate the total area in hectares.

19. The circumference of a circle 20 cm in diametre is traversed clockwise with the anchor point of the planimetre out side the circle. The initial reading is 6.526 and the final reading is 9.668. What is the planimetre constant.

20. The tracing arm of a planimetre is set so that the roller reads 0.625 revolution for 62.5 sq. cm. The perimetre of an area is traversed clockwise first with anchor point outside and then anchor point inside. The corresponding differences in readings are 4.655 and 2.325. What is the planimetre constant ? What is area of the zero circle?

21. A plot of land ABCD is bounded by the straight lines AB and CD of 280 m length. Sides BC and AD are bounded by hedge lines. The length of BC, DA and AC are respectively 240, 320 and 350 m. The perpendicular offsets from AC and AD to the hedge lines are :

Distances from Line *BC*	0	40	80	120	160	200	240	
Left offsets	0	3.5	4.8	5.6	4.6	3.2	0	
Distances from D	0	50	100	150	200	250	300	320
Left offsets Line *DA*	0	4.2	6.5	7.2	4.8	3.2	4.5	0

Calculate the area in hectares of the plot.

ANSWERS

1. (*a*) (*i*) Square metre (*ii*) number (*iii*) straight (*iv*) odd (*v*) area (*vi*) more.

2. (*i*) False (*ii*) True (*iii*) False (*iv*) False (*v*) False (*vi*) True (*vii*) False (*viii*) True (*ix*) True (*x*) False (*xi*) False (*xii*) True (*xiii*) True (*xiv*) True (*xv*) True (*xvi*) True (*xvii*) True (*xviii*) False (*xix*) True (*xx*) False.

8. 42.0 m^2

9. 204.4 m^2

10. (*a*) 207.67 m^2 (*b*) 206.5 m^2

11. 4233.3 m^2

12. (*a*) 42.59 m^2 (*b*) 40.20 m^2

13. 9665.84 m^2

14. 209.233 m^2

15. 1689 m^2

16. 691.6 sq. cm. : inside

17. 15.92 cm., 16.96 cm.

18. 196 sq. cm.; 4.87 hectares

19. 100

20. 100 ; 233 sq. cm.

21. 79, 169.56 sq. m.

9

Volumes

9.1. INTRODUCTION

Measurement of volumes for all types of projects for their designing and estimation of earth work, is commonly required during construction works such as highways, railways, canals, etc. Similarly, capacities of reservoirs are required to be estimated for proper designs of dams, water supplies, hydro-electric and irrigation schemes.

Calculation of the earth work can be made by providing a sufficient number of spot levels by spirit levelling. Accuracy of calculation of the earth work, depends upon the layout of the level network and the density of the level points.

9.2. METHODS OF COMPUTATION

Measurement of volumes may be made by one of the following methods:

1. From cross-sections
2. From spot levels
3. From contours.

The first two methods are generally used for calculation of earth work either in cuttings or in fillings. The third method is used for calculation of reservoir capacities.

The basic unit of the volume of earth work is *cubic metre*.

9.3. MEASUREMENT FROM CROSS SECTIONS

In this method, the total volume is divided into a series of solids bounded by the plane cross-sections. The spacing of the sections depends upon the general characteristics of the ground and the desired accuracy of the earth work. Additional sections may also be taken at the points of change of slope along the centre line.

The various cross-sections likely to occur on the ground surface may be classified in five groups *i.e.*

1. Level sections
2. Two level sections
3. Three-level sections
4. Side-hill two level sections
5. Multi-level sections.

9.4. FORMULAE FOR CALCULATION OF AREAS OF CROSS SECTIONS

(Fig. 9.1)

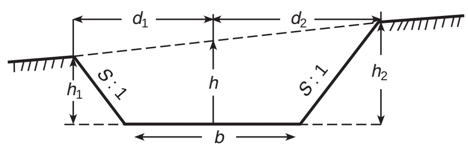


Fig. 9.1.

The notations used in deriving various formulae for volume will be as under :

b = formation or sub-grade width (width at formation level)

h = centre cut or fill, cut being denoted by a plus (+) sign and fill by a minus (–) sign

S to 1 = side slope *i.e.* S horizontals to 1 vertical

n to 1 = lateral or transverse slope of the natural ground *i.e.* n horizontals to 1 vertical.

d_1 and d_2 = side widths of half breadths *i.e.* the horizontal distance from the centre line to the intersections of the side slopes with the natural ground surface

h_1 and h_2 = depths of cuttings or heights of the banks at the edge points of cuttings or banks.

A = Area of the cross section.

1. Level Section. (Fig. 9.2). In this type of cross section, the ground is assumed to be level transversely *i.e.* the value of n approaches to infinity.

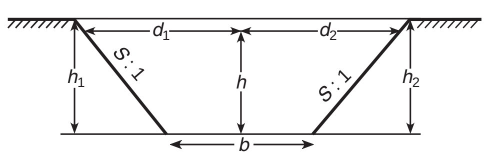


Fig. 9.2. Level Section.

From elementary knowledge of plane geometry, we know

$$h_1 = h = h_2$$

or $$d_1 = d_2 = \left\{\frac{b}{2} + sh\right\}$$

$\therefore$ Area of the trapezium = half the sum of parallel sides × perpendicular distance between them.

i.e. $$A = \frac{\left\{b + 2\left(\frac{b}{2} + sh\right)\right\}}{2} \times h$$

or $$= (b + sh)\,h \qquad ...(9.1)$$

$$= bh + sh^2 \qquad ...(9.1a)$$

2. Two-level section. (Fig. 9.3). Let us assume that AB represents the formation level in a cutting.

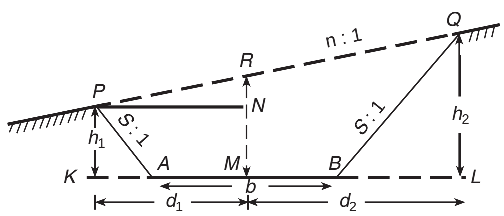


Fig. 9.3. A two level section

P and Q are the points where sloping sides AP and BQ intersect the natural ground surface PQ.

Construction : Drop PN perpendicular to RM where RM is the central line. Drop PK and QL perpendiculars to AB produced on either side to meet at K and L respectively.

Now $$d_1 = AM + AK$$

$$= \frac{b}{2} + sh_1 \qquad ...(9.2)$$

Also $$d_1 = PN = n.RN$$

$$= n\,(h - h_1) \qquad ...(9.3)$$

Substituting the value of h_1 from Eqn. (9.3) in Eqn. (9.2), we get

$$d_1 = \frac{b}{2} + s\left(h - \frac{d_1}{n}\right)$$

or $$d_1 = \frac{b}{2} + sh - \frac{sd_1}{n}$$

or $$d_1\left(1 + \frac{s}{n}\right) = \frac{b}{2} + sh = s\left(h + \frac{b}{2s}\right)$$

or $$d_1 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n + s}\right) \qquad ...(9.4)$$

Similarly, we may get

$$d_2 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n-s}\right) \qquad ...(9.5)$$

Area of $ABQP$ = area of $PQLK$ -(Area PKA + Area BQL)

$$A = \frac{1}{2}(h_1 + h_2)(d_1 + d_2) - \frac{1}{2}\left[\left(d_1 - \frac{b}{2}\right)h_1 + \left(d_2 - \frac{b}{2}\right)h_2\right]$$

...(9.6)

Substituting the values of h_1 and h_2 from Eqn. (9.2) in Eqn. (9.6), we get

$$A = \frac{1}{2}\left(\frac{d_1 - \frac{b}{2}}{s} + \frac{d_2 - \frac{b}{2}}{s}\right)(d_1 + d_2)$$

$$-\frac{1}{2}\left[\left(d_1 - \frac{b}{2}\right)\left(\frac{d_1 - \frac{b}{2}}{s}\right) + \left(d_2 - \frac{b}{2}\right)\left(\frac{d_2 - \frac{b}{2}}{s}\right)\right]$$

$$= \frac{1}{2}\left(\frac{d_1 + d_2 - b}{s}\right)(d_1 + d_2) - \frac{1}{2s}\left(d_1 - \frac{b}{2}\right)^2 - \frac{1}{2s}\left(d_2 - \frac{b}{2}\right)^2.$$

or $$A = \frac{d_1 d_2}{s} - \frac{b^2}{4s} \qquad ...(9.7)$$

Again, substituting the values of d_1 and d_2 from Eqn. (9.2) in Eqn. (9.6) we get

$$A = \frac{1}{2}(h_1 + h_2)\left(\frac{b}{2} + sh_1 + \frac{b}{2} + sh_2\right)$$

$$-\frac{1}{2}\left(\frac{b}{2} + sh_1 - \frac{b}{2}\right)h_1 - \frac{1}{2}\left(\frac{b}{2} + sh_2 - \frac{b}{2}\right)h_2$$

$$= \frac{1}{2}(h_1 + h_2)(b + sh_1 + sh_2) - \frac{1}{2}sh_1^2 - \frac{1}{2}sh_2^2.$$

or $$A = sh_1 h_2 + \frac{b}{2}(h_1 + h_2) \qquad ...(9.8)$$

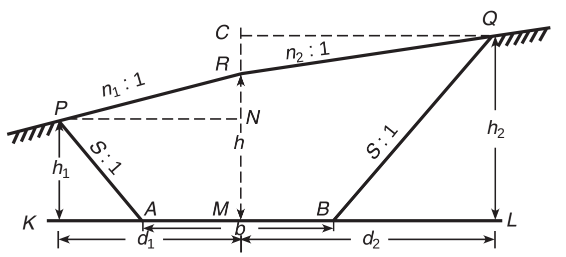


Fig. 9.4. Three level section.

3. Three level section. Let us assume that transverse slope of the natural ground is not uniform. Let it be n_1 to 1 and n_2 to 1 on either side of the central line. (Fig. 9.4)

Construction : Drop *PN* and *QC* perpendiculars to *MR* and *MR* produced respectively. Drop *PK* and *QL* perpendiculars to *AB* produced on either side.

Here $$d_1 = AM + AK$$
$$= \frac{b}{2} + sh_1 \quad \text{...(9.9)}$$

Again, $$d_1 = n_1 \times RN$$
$$= n_1(h - h_1) \quad \text{...(9.10)}$$

Eliminating h_1 from Eqns. (9.9) and (9.10), we get

$$d_1 = (h + \frac{b}{2s})\left[\frac{n_1 s}{n_1 + s}\right] \quad \text{Refer to Eqn. (9.4)}$$

Similarly, may we get

$$d_2 = [h + \frac{b}{2s}]\left[\frac{n_2 s}{n_2 - s}\right] \quad \text{Refer to Eqn. (9.5)}$$

where n_1 and n_2 are both rising gradients

Area of the cross-section *PRQBAP*

= Area of trapezium *PRMK* + Area of trapezium *RQLM* – Area of Δ *PAK* – Area of Δ *QLB*

or $$A = \frac{1}{2}(h_1 + h) \cdot d_1 + \frac{1}{2}(h_2 + h) \cdot d_2 - \frac{1}{2}(d_1 - \frac{b}{2})\, h_1 - \frac{1}{2}(d_2 - \frac{b}{2})h_2 \quad \text{...(9.11)}$$

$$A = \frac{h}{2}(d_1 + d_2) + \frac{b}{4}(h_1 + h_2) \quad \text{...(9.12)}$$

Substituting the values of h_1 and h_2 as $\dfrac{d_1 - \frac{b}{2}}{s}$ and $\dfrac{d_2 - \frac{b}{2}}{s}$ in Eqn. (9.11), we get

$$A = \frac{1}{2}\left(\frac{d_1 - \frac{b}{2}}{s} + h\right) d_1 + \frac{1}{2}\left(\frac{d_2 - \frac{b}{2}}{s} + h\right) d_2$$
$$- \frac{1}{2}\left(d_1 - \frac{b}{2}\right)\left(\frac{d_1 - \frac{b}{2}}{s}\right) - \frac{1}{2}\left(\frac{d_2 - \frac{b}{2}}{s}\right)$$

or $$A = \frac{1}{2s}\Big[(d_1 - \frac{b}{2} + shd_1 + \left(d_2 - \frac{b}{2} + sh\right) d_2$$
$$- \left(d_1 - \frac{b}{2}\right)^2 - \left(d_2 - \frac{b}{2}\right)^2\Big]$$

$$= \frac{1}{2s}\left[\frac{bd_1}{2} + \frac{bd_2}{2} - \frac{b^2}{2} + shd_1 + shd_2\right]$$

$$= \frac{b(d_1 + d_2)}{4s} - \frac{b^2}{4s} + \frac{h(d_1 + d_2)}{2}$$

or $$A = (d_1 + d_2)\left[\frac{b}{4s} + \frac{h}{2}\right] - \frac{b^2}{4s} = \frac{D}{2}[h + \frac{b}{2s}] - \frac{b^2}{4s} \quad ...(9.13)$$

where D is total top width.

4. Side hill two level section. In such cases, the ground slopes transversely and the slope of the ground surface cuts the formation level in such a way that one portion of the area is in cutting and the other portion is in embankment *i.e.* the section consists of two parts one in cutting and the other in filling. In general, two cases may arise :

(*i*) When the centre line of the formation is in excavation.

(*ii*) When the centre line of the formation is in embankment.

Case I. When centre line of the formation is in excavation (Fig. 9.5).

Let the transverse slope of the ground surface be n to 1 and the side slopes in the cutting and embankment be s to 1. Let AB be the width of the formation level of which CB is in the excavation and CA is in embankment. Let h be the depth of the cutting at the centre line.

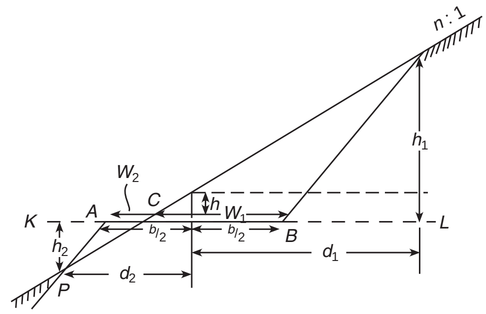


Fig. 9.5. A side hill two level section.

$BC = w_1$ and $AC = w_2$

then ∴ $$w_1 = \frac{b}{2} + nh$$

and $$d_1 = \frac{b}{2} + sh_1 \quad ...(9.14)$$

Again $$d_1 = n(h_1 - h) \quad ...(9.15)$$

Substituting the value of h_1 from Eqn. (9.15) in Eqn. (9.14), we get

$$d_1 = \frac{b}{2} + s\left[h + \frac{d_1}{n}\right]$$

$$= \frac{b}{2} + sh + \frac{sd_1}{n}$$

or $$d_1\left[1 - \frac{s}{n}\right] = \frac{b}{2} + sh = s\left(\frac{b}{2s} + h\right)$$

or $$d_1 = \left[\frac{b}{2s} + h\right]\frac{ns}{n - s}$$

(*i*) **For the excavation :**

$$w_1 = CL - BL$$
$$= nh_1 - sh_1$$
$$= h_1(n - s)$$

or $$h_1 = \frac{w_1}{n - s}$$

∴ The area in excavation = triangular area BCQ

$$A = \frac{1}{2} \times BC \times QL$$
$$= \frac{1}{2} \times w_1 \times h_1$$
$$= \frac{1}{2} \times w_1 \times \frac{w_1}{(n - s)}$$

∴ $$A = \frac{w_1^2}{2(n - s)} \qquad ...(9.16)$$

or $$A = \frac{1}{2}\left[\left(\frac{\frac{b}{2} + nh}{n - s}\right)^2\right] \qquad ...(9.16a)$$

(*ii*) **For the embankment**

$$w_2 = AB - BC$$
$$= b - w_1$$
$$= b - \left[\frac{b}{2} + nh\right]$$
$$= \frac{b}{2} - nh$$

and $$d_2 = \left[\frac{b}{2s} - h\right]\left(\frac{ns}{n - s}\right)$$

∴ Area in embankment = triangular area ACP

$$= \frac{1}{2} \times w_2 \times h_2$$

$$= \frac{1}{2} \times w_2 \times \frac{w_2}{n - s}$$

$$\therefore \qquad A = \frac{w_2^2}{2\,(n - s)} \qquad ...(9.17)$$

$$= \frac{1}{2}\left[\frac{\left(\frac{b}{2} - nh\right)^2}{n - s}\right] \qquad ...(9.17\text{ a})$$

Case II. When centre line of the formation is in embankment. (Fig. 9.6).

Let AB be the formation width = b

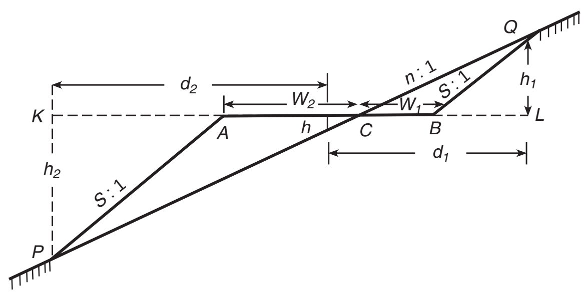


Fig. 9.6. A side hill two level section.

w_1 be the formation width in cutting

w_2 be the formation width in embankment

(*i*) **For excavation**

$$w_1 = nh_1 - sh_1$$

or
$$= (n - s)\, h_1$$

$$d_1 = nh_1 + nh \qquad ...(9.18)$$
$$= n\,(h + h_1)$$

$$\text{Area of the section} = \frac{1}{2} \times BC \times LQ$$

$$= \frac{1}{2} \times w_1 \times h_1$$

$$= \frac{1}{2} \times w_1 \times \frac{(w_1)}{(n - s)} \text{ from Eqn. 9.18}$$

$$A = \frac{w_1^2}{2\,(n - s)} \qquad ...(9.19)$$

(*ii*) **For embankment**

$$w_2 = b - w_1$$

or
$$w_2 = \frac{b}{2} + nh$$

$$d_2 = \frac{b}{2} + sh_2$$

Also
$$d_2 = nh_2 - nh = n\,(h_2 - h) \qquad ...(9.20)$$

∴
$$d_2 = \frac{b}{2} + s\left(h + \frac{d_2}{n}\right)$$

or
$$d_2 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n - s}\right)$$

$$\text{Area of the section} = \frac{1}{2} \times w_2 \times h_2$$

But,
$$h_2 = \frac{w_2}{n - s}$$

∴
$$A = \frac{1}{2} \times \frac{w_2}{n - s} \times w_2$$

$$A = \frac{w_2^2}{2\,(n - s)} \qquad ...(9.21)$$

5. Multi-Level Section (Fig. 9.7). In this case, apot levels and their distances from the central line, are usually recorded as shown in Table 9.1.

Table 9.1

Left		Centre	Right	
$\frac{\pm H_2}{W_2}$	$\frac{\pm H_1}{W_1}$	$\frac{\pm h}{0}$	$\frac{\pm h_1}{W_1}$	$\frac{\pm h_2}{W_2}$

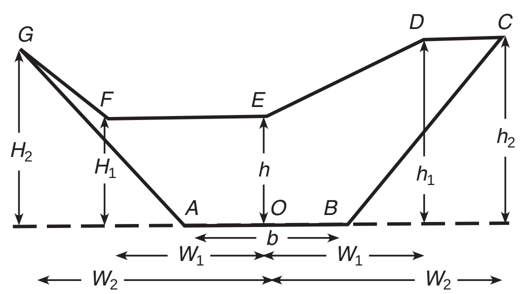


Fig. 9.7. Multi-level Section.

In Table 9.1, numerators of the fractions denote the amount of cutting (+ ve) and filling (– ve) at the various points whereas the denominators denote their horizontal distances from the centre line of the section.

Assuming the formation level *AB* as *X* axis and *OE*, the perpendicular bisector of *AB* as the *Y* axis, these notes may be considered as *x* and *y* coordinates of each point of the section. The area of the cross-section may then be computed by the method of coordinates. The points *A, B, C, D, E, F, G* may be written irrespective of their signs in the form of a determinant. (Fig. 9.8)

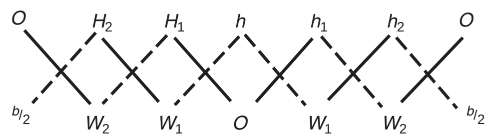


Fig. 9.8. A determinant of spot levels.

For preparation of the determinant, start from the centre and proceed to the right and to the left.

For calculation of the area, proceed as under :

1. Find the sum of the products of the co-ordinates joined by full lines and denote it by Σ *F i.e.*

$$\Sigma F = OH_1 + W_1H_2 + W_2O + Oh_1 + w_1h_2 + w_2O.$$

2. Find the sum of the products of the coordinates joined by dotted lines and denote it by Σ*D i.e.*

$$\Sigma D = h\ W_1 + H_1\ W_2 + H_2 \frac{b}{2} + hw_1 + h_1w_2 + h_2 \frac{b}{2}$$

3. The area of the section is equal to half the difference of the sums. *i.e.*

Area $$A = \frac{1}{2}\,[\Sigma F - \Sigma D]$$

Example 9.1. *Compute the volume of the earth work in a road cutting 50 metres long from the following data :*

The formation width 10 metres; side slopes 1 to 1 ; average depth of the cutting along the centre line 5 m ; slope of the ground transverse to cross-section 10 to 1.

Solution. (Fig. 9.9)

The cross-sectional area in terms of d_1, d_2 and s is given by equation (9.7)

$$A = \frac{d_1d_2}{s} - \frac{b^2}{4s}$$

where $$d_1 = \left\{h + \frac{b}{2s}\right\}\left\{\frac{ns}{n+s}\right\}$$

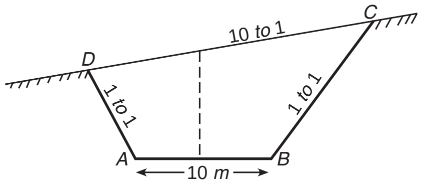


Fig. 9.9.

and $$d_2 = \left[h + \frac{b}{2s}\right]\left[\frac{s}{n - s}\right]$$

Here $h = 5\ m; b = 10\ m;$

$s = 1; n = 10.$

$$\therefore \quad d_1 = \left[5 + \frac{10}{2 \times 1}\right]\left[\frac{10 \times 1}{10 + 1}\right]$$

$$= \frac{10 \times 10}{11} = \frac{100}{11}\ \text{m}$$

$$d_2 = \left[5 + \frac{10}{2 \times 1}\right]\left[\frac{10 \times 1}{10 - 1}\right]$$

$$= \frac{10 \times 10}{9} = \frac{100}{9}\ \text{m}$$

$$\therefore \quad A = \frac{100}{11} \times \frac{100}{9} - \frac{10^2}{4 \times 1}$$

$$= \frac{100{,}00}{99} - 25$$

$= \mathbf{76.01\ sq.\ m.}$

$\therefore$ The required volume of the cutting

$V = A \times L = 76.0 \times 50.$

$=$ **3800.5 cubic metres. Ans.**

Example 9.2. *Compute the volume of the earth work in a road cutting 100 metres in length from the following data :*

Formation width 8 metres ; sides 2 to 1 ; average depth of cutting along the centre line = 0.6 m ; Transverse cross-section of the ground 8 to 1.

Solution. (Fig. 9.10)

Given data :

Formation width (b) = 8 m

Side slope (s) = 2 to 1

Transverse slope (n) = 8 to 1

Average depth of cutting

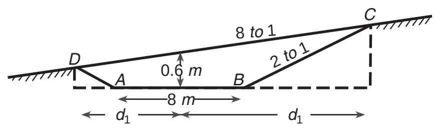


Fig. 9.10.

$$h = 0.6 \text{ m.}$$

$$\therefore \quad d_1 = \left[h + \frac{b}{2s}\right]\frac{ns}{n+s}$$

$$= \left(0.6 + \frac{8}{2 \times 2}\right)\left(\frac{8 \times 2}{8+2}\right) = \frac{2.6 \times 8}{5} \text{ m}$$

$$d_2 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n-s}\right)$$

$$= \left(0.6 + \frac{8}{2 \times 2}\right)\left(\frac{8 \times 2}{8-2}\right)$$

$$= 2.6 \times \frac{8}{3} \text{ m}$$

Substituting the values of d_1, d_2, b and s in Eqn (9.7), we get,

Area of cross-section

$$= \frac{d_1 d_2}{s} - \frac{b^2}{4s}$$

$$= \frac{2.6 \times 8}{5} \times \frac{2.6 \times 8}{3} \times \frac{1}{2} - \frac{8 \times 8}{4 \times 2}$$

$$= 14.42 - 8.00 = 6.42 \text{ sq. m.}$$

$\therefore$ The required volume of the earth work

$$= A \times L = 6.42 \times 100$$

$$= 642 \text{ cubic metres.}$$ **Ans.**

Example 9.3. *Compute the volume of the earth work in a road embankment 100 metres long from the following given data* :

The formation width 6 metres ; side slope of banking 2 to 1

Transverse slope of the ground 5 to 1 ; the mean height of the embankment 2 metres.

Solution. (Fig. 9.11)

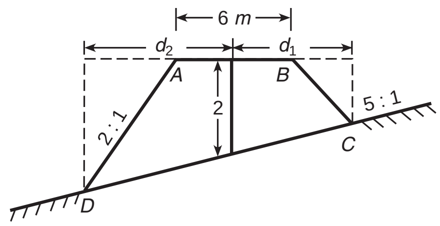


Fig. 9.11.

While calculating the earth work, the embankment may be considered as a cutting upside down.

Given data :

Formation width $b = 6$ m

Side slope $s = 2$

Transverse slope $n = 5$

Average height of embankment

$$h = 2.00 \text{ m}$$

Using equations (9.4) and (9.5) we get

$$d_1 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n+s}\right)$$

$$= \left(2 + \frac{6}{2 \times 2}\right)\left(\frac{5 \times 2}{5+2}\right)$$

$$= 3.5 \times \frac{10}{7} \text{ m}$$

and

$$d_2 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n-s}\right)$$

$$\left(2 + \frac{6}{2 \times 2}\right)\left(\frac{5 \times 2}{5-2}\right)$$

$$= 3.5 \times \frac{10}{3} \text{ m}$$

Substituting the values of d_1, d_2, b and s in equation (9.7) we get

Area of cross-section

$$= \frac{d_1 d_2}{s} - \frac{b^2}{4s}$$

$$= \frac{3.5 \times 10}{7} \times \frac{3.5 \times 10}{3} \times \frac{1}{2} - \frac{36}{4 \times 2}$$

$$= \frac{35 \times 35}{42} - 4.5 = 24.67 \text{ sq. m.}$$

∴ The required volume of the earth work

$$V = A \times L$$

$$= 24.67 \times 100 = 2467 \text{ cubic metres.} \quad \textbf{Ans.}$$

Example 9.4. *The width of a certain road at the formation level is 20 m, side slopes 1 in 1 for cutting and 1 in 2 for filling. The original ground transverse to the centre of the road is 1 in 5. If the depth of the excavation at the centre line of three sections 25 metres apart are 0.4, 0.8 and 1.2 m respectively, calculate the volume of the cutting and the filling over the length of the road.*

Solution. (Fig. 9.5)

Section I. Depth of excavation,

$$h = 0.4 \text{ m}, n = 5$$

$$s = 1 \text{ for cutting and 2 for filling}$$

Substituting the values of h, n, s and b in equation (9.16*a*) we get area of the cutting

$$A_1 = \frac{1}{2}\left[\frac{\left(\frac{b}{2} + nh\right)^2}{n - s}\right] = \frac{1}{2}\left[\frac{(10 + 5 \times 0.4)^2}{5 - 1}\right]$$

$$= 18 \text{ sq m}$$

Area of the filling

$$A_1' = \frac{1}{2}\left[\frac{\left(\frac{b}{2} - nh\right)^2}{n - s}\right] = \frac{1}{2}\left[\frac{(10 - 5 \times 0.4)^2}{5 - 2}\right]$$

$$= 10.67 \text{ sq m}$$

Section II

$$h = 0.8$$

∴ Area of cutting

$$A_2 = \frac{1}{2}\left[\frac{(10 + 5 \times 0.8)^2}{5 - 1}\right] = 24.5 \text{ sq.m}$$

Area of filling

$$A_2' = \frac{1}{2}\left[\frac{(10 - 5 \times 0.8)^2}{5 - 2}\right]$$

$$= 6 \text{ sq. m.}$$

Section III. $h = 1.2$

Area of cutting

$$A_3 = \frac{1}{2}\left[\frac{(10 + 5 \times 1.2)^2}{5 - 1}\right] = 24.5 \text{ sq.m} = 32 \text{ sq m.}$$

Area of filling

$$A_3' = \frac{1}{2}\left[\frac{(10 - 5 \times 1.2)^2}{5 - 2}\right] = 2.67 \text{ sq. m.}$$

∴ Volume of the cutting by the prismoidal rule

$$= \frac{25}{3}\,[18 + 32 + 4 \times 24.5]$$

$$= 1233.33 \text{ m}^{3.}$$ **Ans.**

and volume of the filling by the prismoidal rule

$$= \frac{25}{3}\,[10.67 + 2.67 + 4 \times 6] = 311.17 \text{ m}^3 \text{ Ans.}$$

It may be noted that the volume of the cutting is more than the volume of the filling by an amount

$$= 1233.33 - 311.17 = \mathbf{922.16\ m^{3.}}$$ **Ans.**

Example 9.5. *A straight road is to be formed along hill side, having a uniform lateral slope of 10 horizontals to 1 vertical. The formation width is 30 metres with side slope 1 : 1 in cutting and 2 : 1 in filling. Calculate the total volume of earth work in a length of 450 metres, if the areas of cut and fill in each cross-section are equal.*

Solution. (Fig. 9.5)

We know that if

w_1 = formation width in cutting

n = transverse slope

s = slope of cutting

Referring to equation (9.16) we get

The area of cutting

$$A = \frac{w_1^2}{2\,(n - s)} \qquad ...(i)$$

Similarly, with the same notations,

Area of filling

$$A = \frac{w_2^2}{2\,(n - s)} \qquad ...(ii)$$

Comparing the equations (i) and (ii), we get

$$\frac{w^{2_1}}{2\,(n - s_1)} = \frac{w_2^2}{2\,(n - s_2)}$$

or $$\frac{w^{2_1}}{(10 - 1)} = \frac{w_2^2}{(10 - 2)}$$

or $$\left(\frac{w_1}{w_2}\right)^2 = \frac{9}{8} = 1.125$$

$\therefore$ $$\frac{w_1}{w_2} = 1.06$$

But $w_1 + w_2 = 30$ m

$\therefore$ $1.06w_2 + w_2 = 30$

$2.06w_2 = 30$

or $w_2 = 30/2.06$

$= 14.5631$

and $w_1 = 30 - 14.5631 = 15.4369$

$\therefore$ Area of the cross-section in cutting

$$= \frac{w_1}{2\,(n - s)} = \frac{(15.4369)^2}{2\,(10 - 1)} = 13.239 \text{ m}^2$$

Area of the cross-section in filling

$$= \frac{w_2}{2\,(n - s)} = \frac{(14.5631)^2}{2\,(10 - 2)} = \mathbf{13.255\ m^2}$$

$\therefore$ Mean area of the cutting or filling

$$= \frac{1}{2}\,[13.239 + 13.255] = 13.247 \text{ m}^2$$

$\therefore$ Total volume of the earth work

$$V = 13.247 \times 450 = \mathbf{5961.15\ m^3}$$ **Ans.**

9.5. CALCULATION OF VOLUMES

The volume of the earth work between-cross sections taken along a route, may be calculated by one of the following methods :

1. Prismoidal formula 2. End area formula,

1. Prismoidal formula. A Prismoid is a solid bounded by planes of which two, called end faces, are parallel. The end faces may be both polygons, not necessarily similar or with the same number of sides, one of them may even be a point. The other faces, called the longitudinal faces are planes extending between the end planes.

Statement of the formula: "*The volume of a prismoid is equal to the sum of the areas of end parallel faces plus four times the area of the central section, and multiplied by ⅙th of the perpendicular distance between the end sections.*"

Let $A_1, A_2, A_3, \ldots\ldots A_n$ be areas of cross-sections and D is the distance between consecutive sections.

Volume $$V = \frac{D}{3}[A_1 + 4A_2 + 2A_3 + 4A_4 + \ldots + 2A_{n-2} + 4A_{n-1} + A_n]$$

or $$V = \frac{D}{3}[A_1 + A_n + 2\,\Sigma\text{ odds} + 4\,\Sigma\text{ evens}] \qquad \ldots(9.22)$$

The prismoidal formula for volume, is sometimes known as the *Simpson's rule.* for volume.

Note : The following points may be noted.

(1) In order to apply the prismoidal formula for volume, number of cross-sections should always be odd.

(2) If there are even number of cross-sections, one of them (preferably the last one) may be treated separately and the volume of the remaining length of the route may be computed by the prismoidal formula. The volume of the last section may be calculated by the trapezoidal rule or prismoidal formula.

Derivation of the prismoidal formula (Fig. 9.12)

Let *ABCDEFGH* be a prismoid whose end faces *ABCD* and *EFGH* are parallel.

Construction. Draw a plane parallel to the base and containing *BC* to cut the given solid by the lines *CK, KJ* and *JB*.

The prismoid may now be considered to be made of the following three solids :

1. The prism *ABCD HJKG*.
2. The pyramid *CKFEJ*.C.
3. The pyramid *EBCJ*.

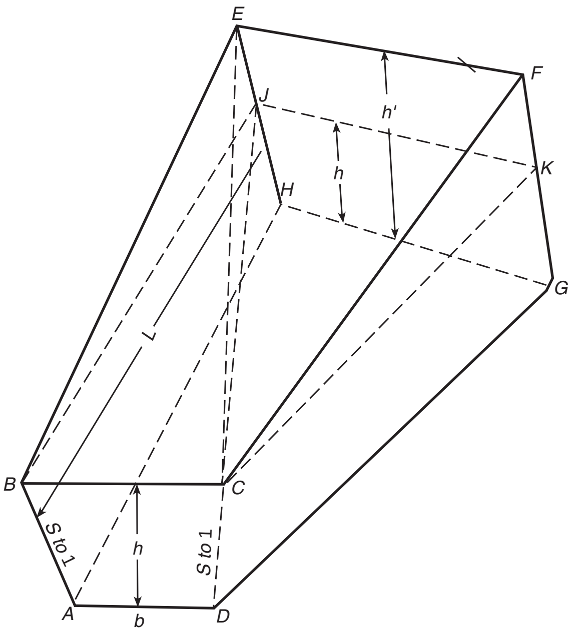


Fig. 9.12. Prismoidal formula.

Let $AD = HG$ = formation width.

h = central depth of the cutting at AD

h' = central depth of the cutting at HG

L = length of the prismoid

s to 1 = side slopes of the cutting *i.e.s.* horizontals to 1 vertical

d = distance between the sections.

Proof :

The area of the cross-section $ABCD$

$$A_1 = (b + sh) \times h$$

The area of the cross-section $EFGH$

$$A_3 = (b + sh') \times h'$$

The area of the mid-section

$$A_2 = \left[b + s\left(\frac{h + h'}{2}\right)\right]\left(\frac{h + h'}{2}\right)$$

The volume of the prism $ABCDHJKG$

V_1 = area of the end face × distance

$$= (b + sh)\, h \times L$$

The volume of the pyramid *CKFEJ*

$$V_2 = \text{area of } KFEJ \times \frac{1}{3}$$

$$= [(b + sh')\, h' - (b + sh)\, h] \times \frac{1}{3}$$

The volume of pyramid *EBCJ*

$$V_3 = \frac{1}{2} BC \times CJ \times \left(\frac{h' - h}{3}\right)$$

$$= \frac{1}{2}(b + 2sh)\, l \left(\frac{h' - h}{3}\right)$$

$$= (b + 2sh)(h' - h)\frac{L}{6}$$

$\therefore$ Volume of the prismoid $= V_1 + V_2 + V_3$

$$= (b + sh)\, h \times L + [(s + sh')\, h' - (b + sh)\, h]\,\frac{1}{3} + (b + 2sh)(h' - h) \times \frac{1}{6}$$

$$= \frac{L}{6}[6\,(b + sh)\, h + 2\,(b + sh')\, h' - 2\,(b + sh)\, h + bh' + 2shh' - bh - 2sh^2]$$

$$= \frac{L}{6}[4\,(b + sh)\, h + (b + sh')\, h' + bh' + 2shh' - bh - 2sh^2 + (b + sh')\, h']$$

$$= \frac{L}{6}[(b + sh)\, h + 3bh + 3sh^2 + bh' + sh'^2 + bh + 2shh' - bh - 2sh^2 + (b + sh')\, h']$$

$$= \frac{L}{6}[(b + sh)\, h + 2bh + 2bh' + sh^2 + sh'^2 + (b + sh')\, h']$$

$$= \frac{L}{6}[(b + sh)\, h + 2bh' + 2bh + 2shh' + sh^2 + sh'^2 + (b + sh')\, h']$$

$$= \frac{L}{6}[(b + sh)\, h + 2b\,(h + h') + s\,(h^2 + h'^2 + 2hh') + (b + sh')\, h']$$

$$= \frac{L}{6}\left[(b + sh)\, h + 4\left[\frac{b(h + h')}{2} + \frac{s(h + h')^2}{4}\right] + (b + sh')\, h'\right]$$

$$= \frac{L}{6}\left[(b + sh)\, h + 4\left\{b + s\,\frac{(h + h')}{2}\right\}\frac{(h + h')}{2} + (b + sh')\, h'\right]$$

$$= \frac{L}{6}[A_1 + 4A_2 + A_3]$$

Adding the volume of the next prismoid, we get

$$= \frac{L}{6} [A_1 + 4A_2 + A_3 + A_3 + 4A_4 + A_5]$$

$$= \frac{L}{6} [A_1 + 4 (A_2 + A_4) + 2 (A_3) + A_5]$$

But, $L = 2d$

$$\therefore \quad V = \frac{d}{3} [A_1 + 4 (A_2 + A_4 + ...) + 2 (A_3 + A_5 + ...) + A_n]$$

or $V = \frac{d}{3}$ [area of the first section + 4 times area of remaining even sections + 2 times area of remaining odd sections + area of the last section].

2. End area (or Trapezoidal) formula. While calculating volumes by the end area formula, it is assumed that volume of a prismoid, is equal to the product of the length of the prismoid by the average of the end areas

i.e.

$$V = L \times \frac{A_1 + A_2}{2} \qquad ...(9.23)$$

where V = Volume of the prismoid

A_1 = Area of one end section

A_2 = Area of other end section

L = distance between the sections.

For a series of cross sections, the Eqn. (9.23) may be simplified as

$$V = \frac{L}{2} (A_1 + 2A_2 + 2A_3 + ... + 2A_{n-1} + A_n)$$

or

$$= L \left(\frac{A_1 + A_n}{2} + A_2 + A_3 ...... + A_{n-1} \right) \qquad ...(9.24)$$

9.6. PRISMOIDAL CORRECTION

The volume obtained by the end area formula is not as accurate as obtained by the prismoidal formula. The accuracy of the result obtained by the end area formula may be increased by applying a correction called, *the prismoidal correction*. As the volume of any prismoid calculated by the end area formula is somewhat larger than that obtained by the prismoidal formula, prismoidal correction is always negative.

Prismoidal correction (P.C.) = volume by the end area formula − volume by the prismoidal formula

i.e. $$\text{P.C.} = \frac{L}{2}(A_1 + A_2) - \frac{L}{6}(A_1 + 4A_m + A_2)$$
$$= \frac{L}{3}(A_1 + A_2 - 2A_m) \qquad ...(9.25)$$

Derivation of the formula for prismoidal correction for Level Section :

Let L = distance between two adjacent sections

b = constant formation width

s to 1 = side slopes, s horizontal to 1 vertical

h_1, h_2 = central heights of the two sections

A_1, A_2 = areas of the two sections

From Eqn. (9.1), we get

$$A_1 = (b + sh_1)\, h_1 \quad ; \quad A_2 = (b + sh_2)\, h_2$$

Averaging the dimensions of the end sections for obtaining the dimensions of the mid-section, we get

The central height of the mid section $= \dfrac{h_1 + h_2}{2}$

$$\therefore \quad A_m = \left[b + \frac{s\,(h_1 + h_2)}{2}\right]\left(\frac{h_1 + h_2}{2}\right)$$

Substituting the values of central height and area of the midsection in equation (9.25), we get

$$\text{P.C.} = \frac{L}{3}(b + sh_1)\, h_1 + (b + sh_2)\, h_2 - \frac{2\,(h_1 + h_2)}{2}\left(b + s\,\frac{h_1 + h_2}{2}\right)$$
$$= \frac{L}{3}\left[bh_1 + sh_1^2 + bh_2 + sh_2^2 - (h_1 + h_2)\, b - \frac{(h_1 + h_2)^2}{2}\, .\, s\right]$$
$$= \frac{L}{6}\,[2bh_1 + 2sh_1^2 + 2bh_2 + 2sh_2^2 - 2\,(h_1 + h_2)\, b - (h_1 + h_2)2\, s]$$
$$= \frac{L}{6}\,[h_1^2 + h_2^2 - 2h_1h_2]\, s$$
$$= \frac{L}{6}\, s(h_1 - h_2)^2\, . \qquad ...(9.26)$$

9.7. FORMULAE FOR OBTAINING PRISMOIDAL CORRECTIONS FOR DIFFERENT SECTIONS.

(*a*) **Level section :**

$$\text{P.C.} = \frac{Ls}{6}\,(h_1 - h_2)^2 \qquad ...(9.27)$$

(*b*) **Two level section :**

$$\text{P.C.} = \frac{L}{6s}(d - d')(d_1 - d_1') \quad ...(9.28)$$

(*c*) **Three level section :**

$$\text{P.C.} = \frac{L}{12}(D - D')(h_1 - h_2) \quad ...(9.29)$$

(*d*) **Side hill two level section :**

(*i*) The centre line being in excavation

$$\text{P.C.} = \frac{L}{12s}(w - w')(d - d') \quad ...(9.30)$$

(*ii*) The centre line being in embankment

$$\text{P.C.} = \frac{L}{12s}(w_1 - w_1')(d_1 - d_1') \quad ...(9.31)$$

where letters carry their usual meanings.

9.8. CURVATURE CORRECTION FOR VOLUMES

The formulae obtained for the earth work calculation in the previous Article 9.7 are based on the assumptions that the sections are parallel to each other and normal to the centre line. But, on curves the cross-sections are run in radial lines and consequently the earth solids between them do not have parallel faces. Therefore, the volumes computed by the usual methods, assuming the end faces parallel, require a correction for the curvature of the central line. The calculation of the volumes along the curved line, is made by the Pappu's Theorem.

Pappu's Theorem. *It states that the volume swept by a constant area rotating about a fixed axis is equal to the product of that area and the length of the path traced by the centroid of the area.*"

Let e_1 = distance of centroid of area A_1 from the centre line.

e_2 = distance of centroid of area A_2 from the centre line.

$\frac{e_1 + e_2}{2}$ = mean distance of the areas A_1 and A_2 from the centre line.

L = distance between two curved centre lines.

R = radius of the curved central line.

The length of the path traversed by the centroid, may be calculated with reference to (Fig. 9.13).

$$\frac{L}{l} = \frac{R \pm e}{R}$$

or $$L = \frac{l}{R}(R \pm e)$$

or $$L = I\left(1 \pm \frac{e}{R}\right)$$

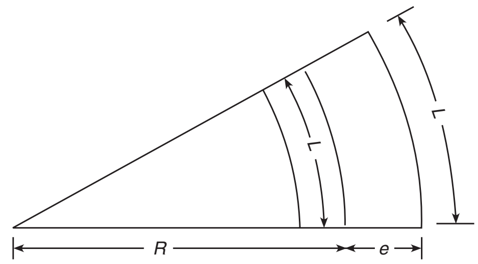


Fig. 9.13. A curature correction.

Again if A_1 and A_2 are the areas of end sections, then,

$$\text{Mean area} = \frac{A_1 + A_2}{2}$$

$$\therefore \quad \text{Volume} = \frac{A_1 + A_2}{2} \times l \left(1 \pm \frac{e}{R}\right) \quad \text{...(9.32)}$$

$$= \frac{A_1 + A_2}{2} \times \frac{l}{R} (R \pm e) \quad \text{...(9.32}a\text{)}$$

(+ ve) sign is accepted when the centroid of the area is on the other side of the centre line. Negative sign is accepted when the centroid is on the same side as the centre of the curve.

Derivation of the formula for the curvature correction. (Fig. 9.14)

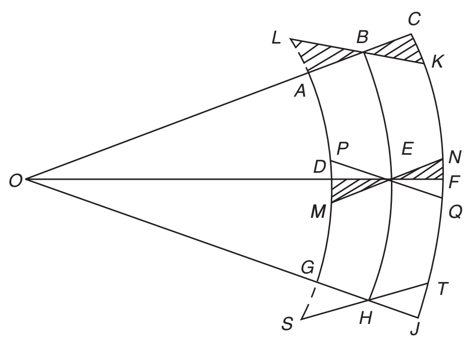


Fig. 9.14. Curvature correction.

Let *B, E* and *H* be the successive pegs on the central line of a curve. *AC, DF* and *GJ* are the radial lines of cross-sections respectively.

Construction. Draw *LK* and *ST* normal to *BE* and *EH* respectively. The portion of the solid from *E* to *B* is shown in plan by *LKNM* whereas the end cross-sections are normal to the centre line. The volume calculated by usual formula is too small by the volume *BCK* and *ENF* and too large by those volumes *ABL* and *EDM*. If the cross-section area is symmetrical about the centre line, these wedge shaped masses practically balance and no curvature correction is required. The curvature correction is usually computed for each peg station. For example the required correction at station *E* is the volume *NEQ* – volume *MEP*.

Let us represent the cross-section at E by *DEFNL* (Fig. 9.15).

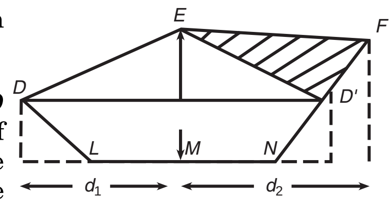


Fig. 9.15.

Draw ED' symmetrical to ED with respect to the centre line of depth h. The volume swept out by the area EFD' turning through the angle NEQ, (Fig. 9.14) is the required curvature correction at station E.

Area $\quad EFD'$ = Area of $DLNFED$ − Area $DLND'E$

or Area $\quad EFD' = \left[\left(\frac{d_1 + d_2}{2}\right)\left(h + \frac{b}{2s}\right) - \frac{b^2}{4s}\right] - \left[\left(\frac{d_1 + d_1}{2}\right)\left(h + \frac{b}{2s}\right) - \frac{b^2}{2s}\right]$

$$= \left(\frac{d_2 - d_1}{2}\right)\left(a + \frac{b}{2s}\right) \qquad ...(9.33)$$

and the horizontal distance of the centroid of the area from the vertical through $E = \frac{d_2 + d_1}{3}$

The length of the path traversed by the centroid

$$= \frac{L}{3R}(d_1 + d_2)$$

where $\frac{L}{R}$ is the circular measure of angle NEQ.

Applying Pappu's theorem of volume, we get

Curvature correction

$$= \left(\frac{d_2 - d_1}{2}\right)\left(h + \frac{b}{2s}\right)(d_2 + d_1)\frac{L}{3R}$$

or
$$= (d_2^2 - d_1^2)\left(h + \frac{b}{2s}\right)\frac{L}{6R} \qquad ...(9.34)$$

The correction is positive or negative according to whether greater half breadth is on the convex or concave side of the curve.

The following points may be noted :

Note. 1. The Eqn. (9.34) for the curvature correction is applicable on the station on the curve and for each tangent point with the station, half the station correction is applied.

2. The formula for a three level section is equally applicable to a two level section.
3. The curvature correction for a level section is zero.
4. The curvature correction for the cutting in a side hill two level section is

$$\frac{Lwh}{6R}(d + b - w) \qquad ...(9.35)$$

9.9. MEASUREMENT OF VOLUMES FROM SPOT LEVELS

Whenever earth work is required for large excavations, the site is divided into triangles, squares or rectangles of equal area of convenient size. The depths of cuttings at the corners of these geometrical figures are obtained by finding the difference in levels between the original and proposed ground surfaces. These differences in level may be regarded as the length of the sides of a number of vertical truncated prisms of which areas of horizontal base are known.

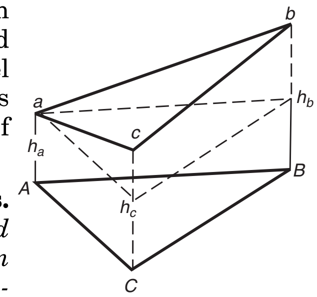


Fig. 9.16. Volume by spot levels.

The volume formula by spot levels. *The volume of each prism may be obtained by the product of the area of the right section multiplied by the average height of the vertical edges (Fig. 9.16).*

Let A be area of the horizontal base of the prism. h_a, h_b, h_c be depths of the excavation at each corner.

The volume of the truncated triangular prism.

$$= A\left(\frac{h_a + h_b + h_c}{3}\right) \qquad ...(9.36)$$

and similarly the volume of a truncated rectangular prism

$$A = \left(\frac{h_a + h_b + h_c + h_d}{4}\right) \qquad ...(9.37)$$

Total volume of any excavation which may consist of a number of prisms, having the same cross-section, may be computed as under :

1. Multiply each corner height by the number of times it is used *i.e.* the number of prisms in which it occurs.
2. Add the products and divide the sum by 4. Denote the result by H.

i.e.
$$H = \left(\frac{\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4}{4}\right) \qquad ...(9.38)$$

where Σh_1 = sum of the heights used once.

Σh_2 = sum of the heights used twice.

Σh_3 = sum of the heights used thrice.

Σh_4 = sum of the heights used four times.

3. The total volume V = horizontal area of the cross-section of one prism $\times H$.

i.e. $$V = A \times H \quad ...(9.39)$$

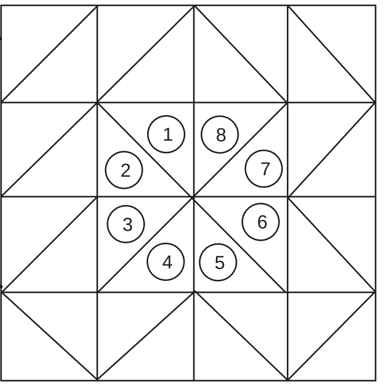


Fig. 9.17. Volume by spot levels.

Alternative Method. If the entire area of the excavation is divided into a number of equal triangles, the height of any corner will be used in calculation once, twice, thrice etc. upto eight times according to the number of truncated prisms to which it may belong (Fig. 9.17).

Let

A = horizontal area of each triangle in sq. m.

h_1 = total sum of the heights used once.

h_2 = total sum of the heights used twice.

h_3 = total sum of the heights used thrice.

h_4 = total sum of the heights used four times.

h_5 = total sum of the heights used five times.

h_6 = total sum of the heights used six times.

h_7 = total sum of the heights used seven times.

h_8 = total sum of the heights used eight times.

$$\text{Volume} = A\left(\frac{h_1 + 2h_2 + 3h_3 + 4h_4 + 5h_5 + 6h_6 + 7h_7 + 8h_8}{3}\right) \text{ cu. m.} \quad ...(9.40)$$

If the entire area is divided into squares or rectangles of equal area, the required volume V may be calculated by the following formula.

$$\text{Volume} = 4\left(\frac{h_1 + 2h_2 + 3h_3 + 4h_4 + 5h_5 + 6h_6 + 7h_7 + 8h_8}{4}\right) \text{ cu. m.} \quad ...(9.41)$$

9.10. MEASUREMENT OF RESERVOIR CAPACITIES

The capacity of a reservoir may be easily determined with the help of a contour map. The area enclosed by each contour line is measured by a planimeter. (Fig. 9.18).

When the finished surface of the ground is horizontal, it becomes parallel to the surface defined by the contour lines. (It may be noted that the plane containing any contour represents a horizontal plane). In case of a reservoir, the data may be compared exactly as in the area cross-sections method. The area bounded by each contour will be treated

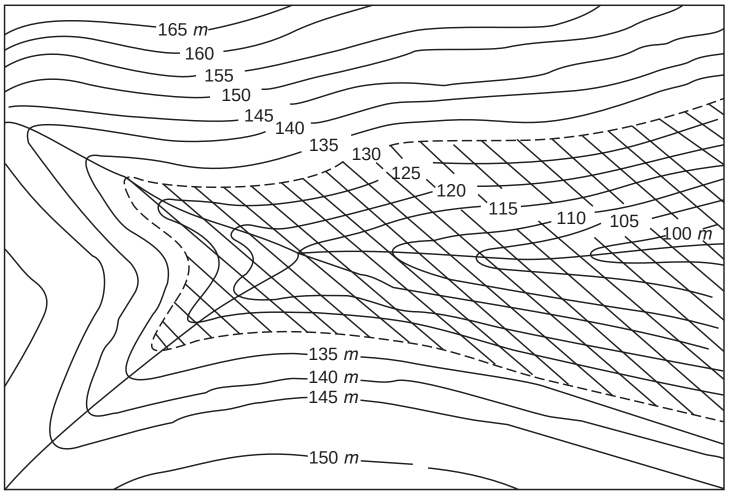


Fig. 9.18. The capacity of a reservoir.

as the area of the cross-sections and the vertical contour interval will be taken as the distance between the adjacent cross-sections.

Calculation of the volume may be done either by the Prismodial formula or by the End area formula. Cubic contents between successive contours when added together gives the required capacity of the reservoir. While applying the prismoidal formula, every second contour area is treated as the area of mid-section, *i.e.*

Trapezoidal formula :

$$V = \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \ldots + A_{n-1}\right]$$

Prismoidal formula :

$$V = \frac{h}{3}\,[A_1 + A_n + 4\,(A_2 + A_4 + \ldots) + 2\,(A_3 + A_5 + \ldots)]$$

For numerical problems on calculation of reservoir capacities, refer to chapter 7 "contouring."

Example 9.6. *A road embankment is 30 metre wide at the top with side slopes of 2 to 1. The ground levels at 100 metre intervals along line PQ are as under :*

P 153.0 ; 151.8 ; 151.2 ; 150.6 ; 149.2 Q

The formation level at P is 161.4 m with a uniform falling gradient of 1 in 50 from P to Q. Calculate by prismoidal formula the volume of earth work in cubic metres, assuming the ground to be level in cross-section.

(*U.P.S.C. Engg. Services, Exam. 1969*)

Solution. (Fig. 9.19)

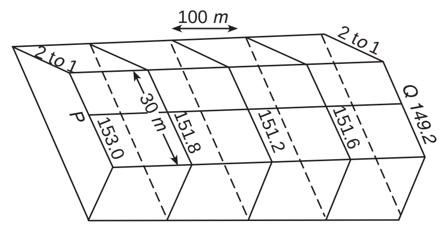


Fig. 9.19

'The formation level at p = 161.4 m.

Uniform falling gradient is 1 in 50 from P to Q.

∴ The formation levels at successive cross-sections are obtained by deducting $\frac{1}{50} \times 100 = 2$ m from the level of preceding section.

The formation level at P, 0 m = 161.4 m

The formation level at 100 m = 159.4 m

The formation level at 200 m = 157.4 m

The formation level at 300 m = 155.4 m

The formation level at 400 m = 153.4 m

The depths of the embankment at various sections

= Formation level – Ground level, *i.e.*

The depth at P, 0 m 161.4 – 153.0 = 8.4 m.

The depth at 100 m = 159.4 – 151.8 = 7.6 m

The depth at 200 m = 157.4 – 151.2 = 6.2 m

The depth at 300 m = 155.4 – 150.6 = 4.8 m

The depth at Q, 400 m = 153.4 – 149.2 = 4.2 m

(i) Area of cross-section at P, *0* m. (Fig. 9.20)

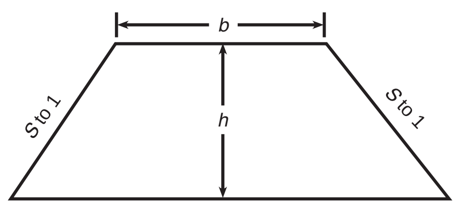


Fig. 9.20.

$$A_0 = (b + sh)\,h$$
$$= (30 + 2 \times 8.4) \times 8.4$$
$$= 46.8 \times 8.4$$
$$= 393.12 \text{ sq. m}$$

(ii) Area of cross-section at 100 m

$A_{100} = (b + sh)h \quad = (30 + 2 \times 7.6) \times 7.6 = 45.2 \times 7.6$

$= 343.52$ sq. m

(iii) Area of cross-section at 200 m

$A_{200} = (b + sh)\ h = (30 + 2 \times 6.2) \times 6.2 = 42.4 \times 6.2$

$= 262.88$ sq. m

(iv) Area of cross-sections at 300 m

$A_{300} = (b + sh)\ h = (30 + 2 \times 4.8) \times 4.8 = 39.6 \times 4.8$

$= 190.08$ sq. m

(v) Area of cross-section at 400 m

$A_{400} = (b + sh)\ h = (30 + 2 \times 4.2) \times 4.2 = 38.4 \times 4.2$

$= 161.28$ sq. m

Applying the Prismoidal formula, we get

$V = \frac{d}{3}$ [Area of first section + 4 times area of even sections.

+ 2 times area of odd sections + Area of last section]

$= \frac{100}{3}\ [393.12 + 2 \times 262.88 + 4\ (343.52 + 190.08) + 161.28]$

$= \frac{100}{3}\ [393.12 + 525.76 + 2134.4 + 161.28]$

= 107152 cubic metres **Ans.**

Example 9.7. *A road embankment 35 m wide at formation level with side slopes 1 : 1 and with an average height of 12 m is constructed with an average gradient 1 in 30 from contour 140 m to 580 m. The ground has an average slope of 12 to 1 in direction transverse to centre line. Calculate (i)* length of the road ; (ii) volume of the embankment in cm^3

Solution.

(i) Difference in elevation = 580 – 140 = 440 m.

For 1 m rise, the length = 30 m

∴ 440 m rise, the length = 30 × 440 = 13200 m

∴ The road length = 13.2 km. **Ans.**

Formation width = 35 m

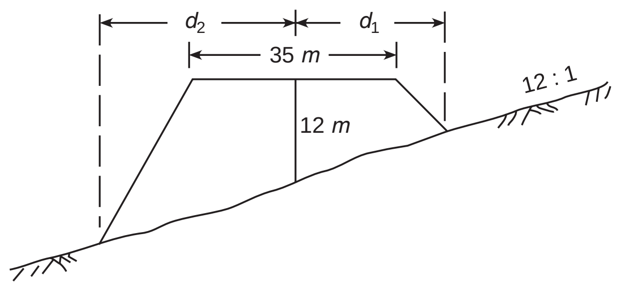


Fig. 9.21.

Side slope $s = 1$

Transverse slope $n = 12$

Average height of embankment = 12 m.

$$d_1, = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n+s}\right) = \left(12 + \frac{35}{2 \times 1}\right)\left(\frac{12+1}{12 \times 1}\right)$$

$$= \left(\frac{59}{2}\right)\left(\frac{12}{13}\right) = 29.5 \times \frac{12}{13}$$

$$d_2 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n-s}\right) = \left(12 + \frac{35}{2 \times 1}\right)\left(\frac{12 \times 1}{12 - 1}\right) = 29.5 \times \frac{12}{11}$$

$\therefore$ Average of cross-section $= \dfrac{d_1\, d_2}{S} - \dfrac{b_2}{4s}$

$$= 29.5 \times \frac{12}{13} \times 29.5 \times \frac{12}{11} \times \frac{l}{1} - \frac{35^2}{4 \times 1}$$

$$= \frac{29.5^2 \times 12^2}{143} - \frac{35^2}{4} = 876.33566 - 306.25$$

$$= 570.08566 \text{ m}^2$$

$\therefore$ The volume of embankment

$$= 570.08566 \times 13200 = 7525130.7 \text{ m}^3$$ **Ans.**

Example 9.8. *An embankment of formation width 10 m with side slopes 2 to 1 is to be constructed on a curve of 300 m radius. The surface of the ground slopes at 5 to 1 downwards, the centre of the curve and the height of bank at the centre is 4 m. Calculate the curvature correction per 20 m length.*

Solution.

Here $b = 10$ m, $h = 4$ m, $s = 2$, $n = 5$

$$d_1 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n+s}\right) = \left(4 + \frac{10}{2 \times 2}\right)\left(\frac{5 \times 2}{5+2}\right)$$

$$= 6.50 \times \frac{10}{7} = \frac{65}{7}\ m$$

$$d_2 = \left(4 + \frac{10}{2 \times 2}\right)\left(\frac{5 \times 2}{5-2}\right)$$

$$= 6.5 \times \frac{10}{3} = \frac{65}{3}\ \text{m}$$

But, from Eqn. (9.7), the area of cross-section

$$= \frac{d_1\, d_2}{s} - \frac{b^2}{4s} \qquad \text{...}(i)$$

Substituting the values of d_1, d_2, s and b in Eqn. (i)

$$A = \frac{65}{7} \times \frac{65}{3} \times \frac{1}{2} - \frac{10^2}{4 \times 2}$$

$$= \frac{65^2}{42} - \frac{100}{8}$$

$$= 100.6 - 12.5 = 88.1\ \text{m}^2$$

$\therefore$ The total volume $V = 88.1 \times 20 = 1762$ cu. m.

From Eqn. (9.34) we get

$$\text{Curvature correction} = \frac{L}{6R}(d_2^2 - d_1^2)\left(h + \frac{b}{2s}\right)$$

$$= \frac{20}{6 \times 300}\left[\left(\frac{65^2}{3}\right) - \left(\frac{65^2}{7}\right)\right]\left(4 + \frac{10}{2 \times 2}\right)$$

$$= \frac{1}{90}\left(\frac{4225}{5} - \frac{4225}{49}\right)(4 + 2.5)$$

$$= \frac{1}{90} \times 4225 \times \frac{40}{441} \times 6.5$$

$$= \frac{4225 \times 260}{90 \times 441}$$

$= 27.68$ cu. m. **Ans.**

Example 9.9. *A swimming pool is to be made by excavating earth on flat ground. The top edges are rectangular 40 m × 30 m Side slop is kept 1 : 1 in short edges ; 1 V : 2 H in long edges. If the depth is 5 m, determine the bottom area and the volume of cutting.*

Solution. (Fig. 9.22).

depth h = 5m

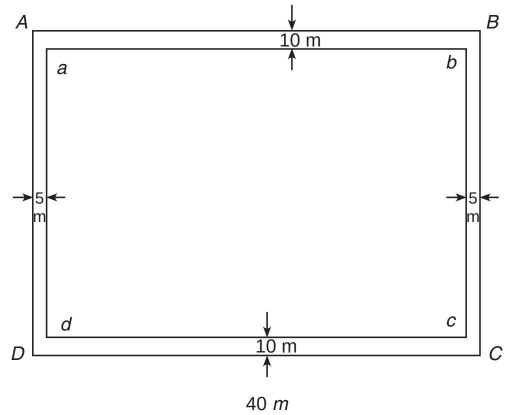


Fig. 9.22.

Bottom long edge $40 - (5 + 5) = 30$m

Bottom short edge $= 30 - (10 + 10) = 10$m

Area of bottom $A_1 = 30 \times 10 = 300 \text{ m}^2$. **Ans.**

Area of top $A_2 = 40 \times 30 = 1200 \text{ m}^2$

Volume of cutting $= \dfrac{A_1 + A_2}{2} \times h$

$= \dfrac{300 + 1200}{2} \times 5 = 3750 \text{ m}^2$. **Ans.**

Example 9.10. *A rectangle ABCD (Fig. 9.23) which represents the plan of the part of an excavation, is 30 m × 20 m, E being the point of intersection of its diagonals. The depth of excavations at the points A, B, C, D and E are 2.50 m, 3.75 m, 4.25, m 5.85 m and 5.50 m respectively. Calculate the volume of the excavation within ABCD.*

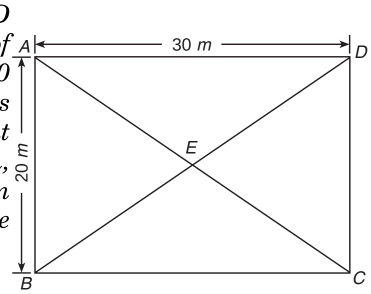


Fig. 9.23.

Solution.

Area of the four triangular prisms

$= \dfrac{1}{2}(20 \times 15)$

= 150 sq. m.

Average depth of $\Delta ABE = \frac{1}{3}(2.50 + 3.75 + 5.50)$

$$= \frac{11.75}{3}$$

Volume $V_1 = \frac{11.75}{3} \times 150$ cu. m

Average depth of $\Delta EBC = \frac{1}{3}(3.75 + 4.25 + 5.50)$

$$= \frac{13.50}{3}$$

Volume $V_2 = \frac{13.50}{3} \times 150$ cu. m

Average depth of

$$\Delta CDE = \frac{1}{3}(4.25 + 5.85 + 5.50) = \frac{15.60}{3}$$

Volume $V_3 = \frac{15.60}{3} \times 150$ cu. m

Average depth of

$$\Delta ADE = \frac{1}{3}(2.50 + 5.85 + 5.50) = \frac{13.85}{3}$$

Volume $V_n = \frac{13.85}{3} \times 150$ cu. m

$\therefore$ The required volume

$$V = 150\left(\frac{11.75}{3} + \frac{13.50}{3} + \frac{15.60}{3} + \frac{13.85}{3}\right)$$

$$= 50 \times 54.70 = 2735 \text{ cu.m.} \quad \textbf{Ans.}$$

Example 9.11. *A railway embankment is 9 m wide at formation level, with side slope of 2 to 1. Assuming the ground to be level transversely, calculate the volume of the embankment in cubic metres in a length of 180 m, the centre heights at 30 m intervals being 0.6, 0.8, 1.5, 1.8, 0.75, 0.3 and 0.67 m respectively. Use Traperzoidal method.*

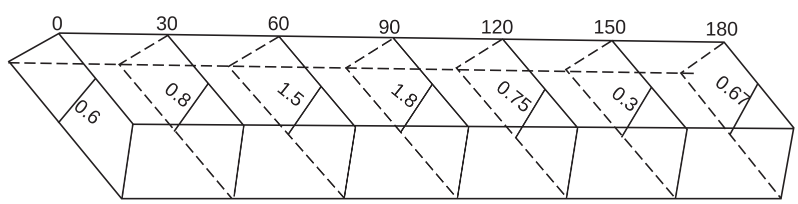


Fig. 9.24.

1. Area of cross section at 0 m $= (b + sh)\, h$

$= (9 + 2 \times 0.6)\ 0.6 = 6.12\ \text{m}^2$

2. Area of Cross section at 30 $= (9 + 2 \times 0.8)\ 0.8 = 8.48\ \text{m}^2$

3. Area of Cross section at 60 m $= (9 + 2 \times 1.5)\ 1.5 = 18.0\ \text{m}^2$

4. Area of Cross section at 90 m $= (9 + 2 \times 1.8)1.8 = 22.68\ \text{m}^2$

5. Area of Cross section at 120 m $= (9 + 2 \times 0.75)\ 0.75 = 7.875\ \text{m}^2$

6. Area of Cross section at 150 $= (9 + 2 \times 0.3)\ 0.3 = 2.88\ \text{m}^2$

7. Area of Cross section at 180 m

$= (9 + 2 \times 0.67)0.67 = 6.928\ \text{m}^2$

∴ Volume of the embankment by Traperzoidal method.

$$V = h\left[\frac{A_1 + A_n}{2} + A_2 + A_3 + A_4 + \ldots.. + A_{n-1}\right]$$

$$= 30\left[\frac{6.12 + 6.928}{2} + 8.48 + 18.0 + 22.68 + 7.88 + 2.88\right]$$

$$= 1993.35\ \text{m}^3 \quad \textbf{Ans.}$$

Example 9.12. *The dimensions of two embankment sections are :*

$\frac{-2.07}{7.14}$	$\frac{-2.34}{0.0}$	$\frac{-3.15}{9.30}$
	and	
$\frac{-2.16}{8.82}$	$\frac{-3.45}{0.0}$	$\frac{-3.81}{12.12}$

The distance between them being 30 m. The formation width increases uniformly from 6 m at the first section to 9 m at the second. Calculate the volume contained between these sections by trapezoidal formula.

Solution.

Area of the first section (Fig. 9.25)

Area of the first section

= area of trapezium *EFGH* + area of trapezium *HEDC* – area of Δ *FGA* – area of Δ *BCD*.

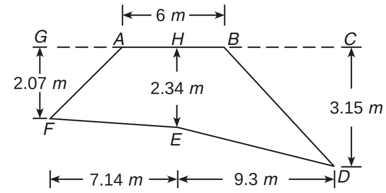


Fig. 9.25.

Area of trapezium *EFHG* $= \frac{1}{2}(2.07 + 2.34) \times 7.14 = 15.7437$

Area of trapezium $ECDH$ $= \frac{1}{2}(2.34 + 3.15) \times 9.30 = 25.5285$

Area of $\Delta\, FGA = \frac{1}{2} \times 4.14 \times 2.07 = 4.2849$

Area of $\Delta\, BCD = \frac{1}{2} \times 6.30 \times 3.15 = 9.9225$

$\therefore$ Area $(A_1) = 15.7437 + 25.5285 - (4.2849 + 9.9225)$

$= 27.0648$ sq. m.

Area of the second section (Fig. 9.26)

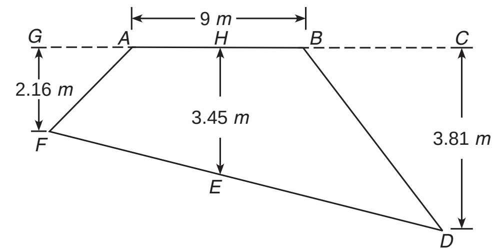


Fig. 9.26.

Area of the second section = area of trapezium EFGH + area of trapezium EDCH – area of $\Delta\, AFG$ – area of $\Delta\, BCD$

Area of trapezium $EFGH = \frac{1}{2}(2.16 + 3.45) \times 8.82 = 27.7401$

Area of trapezium $ECDH = \frac{1}{2}(3.45 + 3.81) \times 12.12$

$= 43.9956$

Area of $\Delta\, AFG = \frac{1}{2}(8.82 - 4.5) \times 2.16$

$= \frac{1}{2} \times 4.32 \times 2.16 = 4.6656$

Area of $\Delta\, BCD = \frac{1}{2}(12.12 - 4.5) \times 3.81$

$= \frac{1}{2} \times 7.62 \times 3.81 = 14.5161$

$\therefore$ Area $(A_2) = 24.7401 + 43.9956 - (4.6656 + 14.5161)$

$= 68.7357 - 19.1817 = 49.5540 \text{ m}^{2.}$

$\therefore$ Total volume (V) of the embankment

$$= \frac{1}{2}(A_1 + A_2) \times d$$

$$= \frac{1}{2}(27.0648 + 49.5540) \times 30$$

$$= 1149.282 \text{ cu. m.} \quad \textbf{Ans.}$$

Example 9.13. *In a certain road cutting the width at the formation level is 20 metres, the sides of the cutting slope are 1.5 horizontals to 1 vertical and the surface of the ground has a uniform transverse slope of 1 in 10. Compute the volume of the excavation contained in a length of 200 metres. The depth of cutting at 100 metre intervals along the centre of formation level are 25, 26 and 30 metres respectively at the three consecutive sections.*

Solution. (Fig. 9.27)

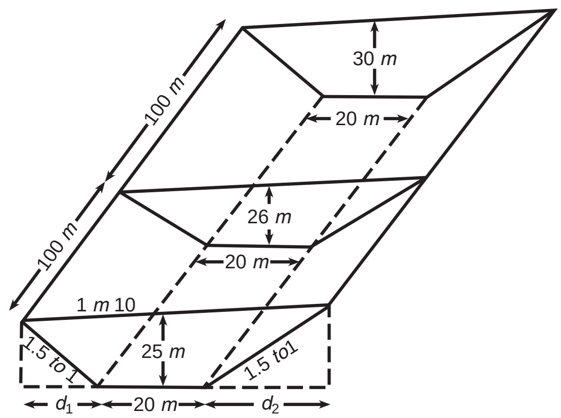


Fig. 9.27.

Section 1.

From Eqn. (9.4)

$$d_1 = (h + \frac{b}{2s})\left(\frac{ns}{n+s}\right)$$

$$= \left(25 + \frac{20}{2 \times 1.5}\right)\left(\frac{10 \times \frac{3}{2}}{10 + \frac{3}{2}}\right)$$

$$= \frac{950}{23} \text{ m}$$

From Eqn. (9.5)

$$d_2 = \left(h + \frac{b}{2s}\right)\left(\frac{ns}{n-s}\right)$$

$$= \left(25 + \frac{20}{2 \times 1.5}\right)\left(\frac{10 \times \frac{3}{2}}{10 - \frac{3}{2}}\right) = \frac{950}{17} \text{ m}$$

$\therefore$ Area of the first section from Eqn. (9.7)

$$A_1 = \frac{d_1 d_2}{s} - \frac{b^2}{4 \times s}$$

$$\frac{\frac{950}{23} \times \frac{950}{17}}{\frac{3}{2}} - \frac{20^2}{4 \times \frac{3}{2}} = 1472.12 \text{ sq. m.}$$

Section 2.

$$d_1 = \left(26 + \frac{20}{3}\right)\left(\frac{10 \times \frac{3}{2}}{10 + \frac{3}{2}}\right) = \frac{980}{23} \text{ m}$$

$$d_2 = \left(26 + \frac{20}{3}\right)\left(\frac{10 \times \frac{3}{2}}{10 - \frac{3}{2}}\right) = \frac{980}{17} \text{ m}$$

$$A_2 = \frac{\frac{980}{23} \times \frac{980}{17}}{\frac{3}{2}} - \frac{(20)^2}{4 \times \frac{3}{2}} = 1570.84 \text{ sq. m}$$

Section 3.

$$d_1 = \left(30 + \frac{20}{3}\right)\left(\frac{10 \times \frac{3}{2}}{10 \times \frac{3}{2}}\right) = \frac{1100}{23} \text{ m}$$

$$d_2 = \left(30 + \frac{20}{3}\right)\left(\frac{10 \times \frac{3}{2}}{10 - \frac{3}{2}}\right) = \frac{1100}{17} \text{ m}$$

$$\therefore \quad A_3 = \frac{\frac{1100}{23} \times \frac{1100}{17}}{\frac{3}{2}} - \frac{(20)^2}{4 \times \frac{3}{2}} = 1996.42 \text{ m}$$

Total volume by end area formula

$$= 100\left(\frac{1472.12 + 1996.42}{2} + 1570.84\right)$$

$$= 100\ (1734.27 + 1570.84)$$

$$= 330511 \text{ cu. m.}$$ **Ans.**

Example 9.14. *The centre line of an existing canal cutting is on a curve of 400 metre radius, the original surface of the ground is approximately level. The cutting be widened by increasing the canal bed width from 20 metres to 30 metres, the excavation to be entirely on the inside of the curve and to retain the existing side slope of* $1\frac{1}{2}$ *horizontals to 1 vertical. If the depth of the canal bed increases uniformly from 8 metres at chainage 1400 metres to 17 metres at chainage 1700 metres, compute the volume of the earth work to be removed in this 300 metre length of the canal.*

Solution. (Fig. 9.28).

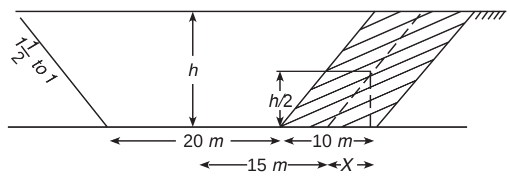


Fig. 9.28.

The area of the proposed excavation $= 10\ h$

The eccentricity of the centroid of excavation from the existing centre line

$$= 15 + \frac{3}{2} \times \frac{h}{2}$$

$$= 15 + \frac{3h}{4}$$

Calculations may be made in a tabular form as shown as under :

Chainage (*m*)	*h* (*m*)	*Area* (*m*)	*Eccentricity*(*m*)	*Average eccentricity between sections* (*m*)
1400	8	80	15 + 6 = 21.00 ,	
1500	11	110	$15 + \frac{33}{4} = 23.25$	22.125
1600	14	140	$15 + \frac{42}{4} = 25.50$	24.375
1700	17	170	$15 + \frac{51}{4} = 27.75$	26.625

Using Eqn. (9.32*a*) compute the volume as under.

(*i*) Volume of excavation between 1400 m and 1500 m

$$= \frac{1}{2}(80 + 110) \times \frac{100}{400}(400 - 22.125)$$

$$= 8975 \text{ m}^{3.}$$

(*ii*) Volume of excavation between 1500 m and 1600 m

$$= \frac{1}{2}(110 + 140) \times \frac{100}{400}(400 - 24.375)$$

$$= 11{,}738 \text{ m}^{3.}$$

(*iii*) Volume of excavation between 1600 m and 1700 m

$$= (140 + 170)\,\frac{100}{400}(400 - 26.625) = 14468 \text{ m}^3.$$

∴ Total volume of excavation

$$= 8975 + 11{,}738 + 14{,}468 = 35{,}181 \text{ m}^3 \textbf{ Ans.}$$

Example 9.15. *Fig. 9.29 shows the reduced levels of a rectangular field which is to be excavated to a uniform reduced level of 100 metres above datum. Assuming the sides to be vertical, calculate the volume of the earth work involved.*

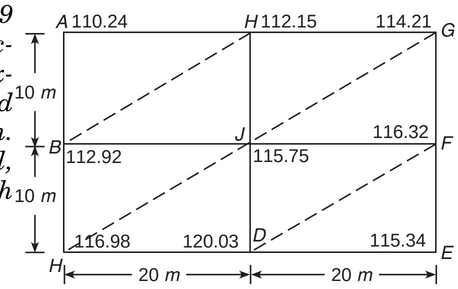


Fig. 9.29.

Solution.

Let us suppose that the area is subdivided into four rectangles. The number of times the depths at the corners of all the rectangles are used, are tabulated here under :

Corner	*Depth of Excavation* (*h*)	*No. of rectangles in which it occurs* (*n*)	*Product* $h \times n$
A	10.24	1	10.24
B	12.92	2	25.84
C	16.98	1	16.98
D	20.03	2	40.06
E	15.34	1	15.34
F	16.32	2	32.64
G	14.21	1	14.21
H	12.15	2	24.30
J	15.75	4	63.00

$$\Sigma h \times n = 242.61$$

Area of each rectangle

$= 10 \times 20 = 200$ sq m

$\therefore$ Volume (V) = Area of one rectangle $\times \dfrac{\Sigma h \times n}{4}$

$= 200 \times \dfrac{242.61}{4}$

$= 12130.5$ cu. m. **Ans.**

(b) Let us suppose that the area is divided into triangles as shown by the dotted lines. The number of times the depths of the triangles used at the corners are tabulated as under :

Corner	*Depth of Excavation* (h)	*No. of triangles in which it occurs* (n)	*Product* $h \times n$
A	10.24	1	10.24
B	12.92	3	38.76
C	16.98	2	33.96
D	20.03	3	60.09
E	15.34	1	15.34
F	16.32	3	48.96
G	14.21	2	28.42
H	12.15	3	36.45
J	15.75	6	94.50

$\Sigma\, h \times n = 366.72$

Area of triangle $= \dfrac{1}{2} \times 20 \times 10 = 100$ sq m.

Volume = Area of triangle $\times \dfrac{\Sigma\,(h \times n)}{3}$

$= 100 \times \dfrac{366.72}{3}$

$= 12224$ cu. m. **Ans.**

Example 9.16. *The areas enclosed by contours in a lake and a hill situated side by side in a plot of land are as under* :

Lake	Contours (m)	200	190	180	170	160	150
	Area (m)^2	5000	3500	2000	1250	500	0.0
Hill	Contours (m)	200	210	220	230	240	250
	Area (m)^2	4500	3500	2250	1500	760	0.0

If the lake is to be filled up to 200 m level with the excavated material from the hill, ascertain whether excavated material is just sufficient or in excess.

Solution.

(*i*) The volume of the lake cavity :

$$= \frac{10}{3}\,[5000 + 500 + 4\,(3500 + 1250) + 2 \times 2000] + 10 \times \frac{(500 + 0)}{2}$$

$$= \frac{10}{3}\,[5500 + 19000 + 4000] + 2500$$

$$= 95000 + 2500 = 97500 \text{ m}^3$$

(*ii*) The earth volume of the hill.

$$= \frac{10}{3}\,[4500 + 760 + 4\,(3500 + 1500) + 2 \times 2250] + 10 \times \frac{(0 + 760)}{2}$$

$$= \frac{10}{3}\,[5260 + 20,000 + 4500] + 3800$$

$$= 99200 + 3800 = 103,000 \text{ m}^3$$

The volume of the earth work is more *i.e.*

$= 103,000 - 97500 = 5500 \text{ m}^3$ **Ans.**

9.11. MASS DIAGRAM

For ascertaining in advance, proper distribution of excavated material and the amount of waste and borrow required for the estimation of cost, a *mass diagram* is commonly used. It is a curve plotted on a distance base, the ordinate at any point of which represents the algebraic sum of the volumes of cuttings and fillings from the starting point of the earth work to that point. In plotting a mass diagram, cuttings are assumed as positive and the fillings as negative. If cuttings and fillings are on the same length of the longitudinal section, as in hill side roads, their difference only is used in the algebraic summation, the sign of the greater volume is accepted in the computation.

It may also be noted that while computing volumes of the earth work actually to be moved, due allowance is made for the 'swelling' of the excavated material and for the settlement and shrinkage (compaction) of the filled material.

Definitions. The definitions of important terms are listed below.

1. **Haul distance.** It is the distance at any time between the working face of an excavation and the tip end of the embankment formed from the hauled material.
2. **Average haul distance.** It is the distance between the centre to gravity of a cutting and centre of gravity of filling.
3. **Haul.** It is the sum of the products of the each volume by its haul distance i.e. $\Sigma v.\, d = VD$, where V is the total volume of an excavation and D is the average haul distance.

4. **Free haul distance.** It is the specified distance in terms of contracts, up to which the excavated material, is transported regardless to the haul distance.
5. **Over haul distance.** If the excavated material from a cutting has to be moved to a greater distance than the free haul distance, the extra distance is known as *over haul distance.*
6. **Balancing line.** Any horizontal line drawn on the curve balances the volumes of cutting and filling because, there is no difference in aggregate volume between the two points, such a line is known as, a *balancing line,* such as *MN* in Fig. (9.30).

Construction of a Mass Diagram (Fig. 9.30)

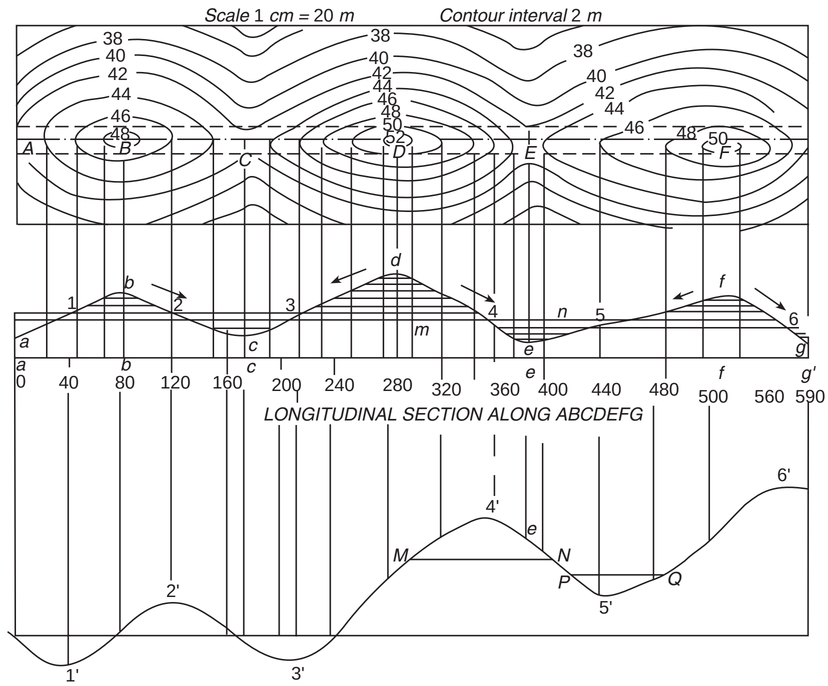


Fig. 9.30. A mass diagram.

The following steps are involved for the construction of a Mass Diagram:

(*i*) Divide the length of the road or railway in separate sections of convenient distances.

(*ii*) Calculate the volumes of the earth work for each section.

(*iii*) Plot a longitudinal section of each section of the road on a convenient scale.

(*iv*) Plot the volumes as ordinates and distances between the sections which are kept same, as the longitudinal section.

(*v*) Plot the positive volumes above the base line and the negative volumes below the base line.

(*vi*) Join the ends of the adjacent ordinates by a smooth curve to obtain the required mass diagram.

Example 9.17. *Draw a mass diagram for a road 590 metres in length. The details of the earth work involved is tabulated here under.*

	Volume (cu. m.)		
Distance (metres)	*Cutting (+)*	*Filling (–)*	*Total Volume (cubic metres)*
0		140	0
40	140		-140
80	130		0.0
120			+130
160		100	+30
200		140	-110
240	80		-30
280	230		+200
320	170		+370
360	180		+450
400		150	+300
440		140	+160
480	50		+210
520	130		+340
560	210		+550
590		20	+530

Solution.

For the solution of this example, see Fig. 9.30 on previous page.

9.12. CHARACTERISTICS OF A MASS DIAGRAM.

Assuming the volumes of cuttings as positive and fillings as negatives, characteristics of a mass diagram may be studied as under :

1. If the slope of the curve in the direction of the increasing abscissa is upward, it indicates an excavated section.

2. If the slope of the curve in the direction of the increasing abscissa is downward it indicates an embankment.
3. The vertical distance between a maximum ordinate and the next forward minimum ordinate represents the whole volume of a filling.
4. The vertical distance between a minimum ordinate and the next forward maximum ordinate, represents the whole volume of a cutting.
5. The vertical distance between two points on the curve which has neither a maximum nor a minimum point, represents the volume of earth work between their abscissa *i.e.* the chainages.
6. If the mass diagram curve cuts the base line at any two points in succession then, the volume of cutting equals that of filling between these points, since the algebraic sum of the quantity between such points, is zero.
7. Any horizontal line *MN* drawn parallel to the base line and intersecting the curve at two points, indicates a length over which the volumes of cutting and filling will be equalized.
8. When the loop of the curve cut off by a balancing lines above that line, the excavated material must be hauled forward in the direction of the increasing abscissa. And, if the loop lies below the balancing line, the direction of haul must be backward.
9. The length of a balancing line intercepted by a loop of the curve is equal to the maximum distance involved in disposing off the excavation. In Fig. 9.30, the haul distance for the loop *MN* is *mn*, where *MN* is the balancing line. The haul distance increases from zero at *m* to a maximum at point *n*.
10. The area enclosed by a loop of the curve and a balancing line, measures the haul in that direction.
11. The haul over any length is a maximum when the balancing line is so situated that the sum of all areas cut off by it, ignoring the sign, is a minimum.

Drawing a balancing line. In general, a balancing line must be drawn in such a manner that the balancing of the material is economical. The base line itself, on which the mass diagram is drawn may some times be used as the balancing line. Alternatively, a number of horizontal lines may be drawn on the curve to find suitable balancing lines or line which may enable the work to be done in the most economical manner. Such lines may not always necessarily be continuous. If different balancing lines are not connected together, then the volume of the earth work between those points on the profile, not included by the line, will not be balanced. If the mass curve between such points rises, the material will be carried to the tip as it is surplus and if the curve falls, it will mean that material will be borrowed as it indicates a 'fill'.

It may be summerised that a number of balancing lines may be drawn, not necessarily all continuous so that a series of balanced earthwork may be obtained to avoid unnecessary wastage of power. The project should be planned in such a way that the excavated material is hauled down the hill.

Use of a mass diagram. In cases, where more than one alternatives of the utilisation of the excavated material is possible, the mass diagram may be used to compare the projects as regards the economy of the haulage. The exact interpretation of any mass diagram depends upon the correct positioning of the balancing line. The mass diagram may be used for the following purposes.

1. To find an economical scheme for distributing the excavated material by comparing a number of balancing lines drawn on the curve.
2. To avoid the wastage of the material at one place and borrowing at another place.
3. To overcome the difficulty of estimation of the proposed wastage, the designer may advise widening an embankment or a cutting where necessary.
4. To ascertain the cost of excavation and balancing one cubic metre as compared to balancing to waste one cubic metre plus that of excavating and hauling one cubic metre from the borrow pits.

9.13. LEAD AND LIFT

Lead. The horizontal distance through which excavated material from a cutting or borrow pit is transported to the nearby embankment is known as *Lead.* This is the distance on which normally earthwork is estimated. A 30 metre lead is generally used for normal rate for earthwork.

Lift. It is the vertical distance through which excavated material from a cutting or borrow pit is transported. A 1.5 metre lift is generally used for earth work.

When either the lead or the lift or both are greater than normal, the rates of estimation will be higher for every unit of 30 m lead and for every unit of 1.5 m lift.

EXERCISE 9

1. State whether following statements are True of False

(*i*) The method of calculation of volumes of earthwork based on the field measurements are of precise nature as these involve assumptions of geometrical forms of the solids.

(*ii*) A solid having its end faces in parallel planes and consisting of any two polygonws, not necessarily of the same number of sides, the longitudinal faces being plane surfaces extending between the end planes, is known as a wedge.

(*iii*) A prismoid whose end polygons are equal and side faces are parallelograms, is generally known as prism.

(*iv*) In the prismoidal formula $\left[V = \frac{L}{6}(A_1 + A_2 + 4M)\right]$

A_1, A_2 are the areas of end surfaces. L is the distance between these faces and M is the average of end areas A_1 and A_2.

(*v*) A prismoidal formula can only be used provided end faces are in parallel planes.

(*vi*) Volumes of earthwork computed by applying Simpson's rules are not so accurate as individual treatment of the solids between adjacent cross section.

(*vii*) The volume of earthwork computed by prismoidal formula is always more than the volume computed by the method of End Areas.

(*viii*) It is less troublesome first to compute the approximate volume by the method of End Areas and then to apply prismoidal correction to the result, rather that to obtain the volume directly from the prismoidal formula.

(*ix*) The formula for the curvature correction for a three level section is $\frac{3}{2}$ of the correction for a two level section.

(*x*) The distance from the centre of gravity of a cutting to that of the tipped material, is generally known as Haul Distance.

(*xi*) The points at which the curve of a longitudinal section crosses the base line from positive to negative, gives the locations of the minimum on the mass diagram.

(*xii*) The vertical distance between a maximum point and the next forward minimum point on a mass diagram represents the whole volume of an embankment.

2. Write short notes on the following :

(*a*) Straight volume.

(*b*) Curved volume.

(*c*) Simpson's rule.

(*d*) Prismoidal correction.

(*e*) Curvature correction.

(*f*) Mass diagram

(*g*) Lift and lead.

3. Enumerate different methods of determination of volume of earthwork. Describe their merits and demerits.

4. What curvature correction is done while determining the volume. How can the volume be determined from contours.

5. Derive an expression for the prismoidal formula as applicable to volume.

6. Derive an expression for the prismoidal correction for a level section.

7. Establish the formula $V = \frac{1}{2}(A_1 + A_2) \times 1 \left[1 \pm \frac{e}{R}\right]$

where A_1, A_2, l, e and R have their usual meanings.

8. Describe the various characteristics or mass diagram with the help of a neat sketch.

9. What do you understand by mass diagram ? For what purposes this can be used ? Explain by an example.

10. Write a short note on use of mass diagram in earthwork.

11. The formation width of a certain road embankment was 20 m. Bank slopes are two horizontals to one vertical. Formation level at the starting point was 161.0 m. Formation was on a uniform falling gradient of 2.0 %. The ground levels along the centre line were as follows :

Distance (m)	0	100	200	300	400
G.L.	155.0	154.0	153.0	152.5	152.0

Assuming the ground to be level transversely, calculate the volume of earth work in cubic metres by :

(*a*) Prismoidal formula

(*b*) Trapezoidal formula.

12. (*a*) How will you determine the capacity of a reservoir with the help of contours ?

(*b*) The following cross section notes show the height of slope stakes formation and corresponding slope stake distances from the centre line. The formation width is 24 m and the slopes 1 : 1.

Station	*Chainage (m)*	*Left*	*Centre*	*Height*
14	1400	$+\frac{5.7}{17.7}$	$+\frac{5.20}{0}$	$+\frac{4.9}{16.9}$
13	1300	$+\frac{5.0}{17}$	$+\frac{4.4}{0}$	$+\frac{4.2}{16.2}$
12	1200	$+\frac{5.5}{17.4}$	$+\frac{4.9}{0}$	$+\frac{4.4}{16.4}$
11	1100	$+\frac{5.1}{17.1}$	$+\frac{4.4}{4}$	$+\frac{3.9}{15.9}$
10	1000	$+\frac{3.7}{15.7}$	$+\frac{3.8}{8}$	$+\frac{3.4}{15.4}$

Using the prismoidal formula, calculate the volume of cutting in cu. metre between stations 10 and 14.

13. A straight railway embankment is made on a ground having a transverse slope of 1 in 8. The formation width of the embankment is 30 m and the side slopes are 1½ horizontals to 1 vertical. At three sections 50 m apart, the heights of the bank , at the centre of the formation level, are 10 m, 15 m and 18 m.

Compute the volume of the earth work involved in the embankment.

14. Cross-sections at 100 m interval along the centre line of a proposed straight cutting are levelled at 20 m intervals 60 m either side and the following data obtained.

Distance (m)		*Left*		*Centre*		*Right*	
	60	40	20	0	20	40	60
0	4.0	1.0	0.0	0.0	0.0	1.0	2.8
100	12.9	8.6	5.0	3.0	2.0	3.0	6.0
200	17.5	14.1	10.9	8.0	6.0	6.0	9.6
300	21.8	17.7	14.4	11.3	9.7	9.7	11.0
400	25.0	21.2	18.0	15.2	12.8	12.0	13.2

If the formation width and level are 20 m and zero m respectively and side slopes 2 horizontals to 1 vertical, calculate the volume of excavation in cu. metre between the chainages 0 to 400 m.

15. In a road cutting the width at the formation level is 20 m with sides 1½ horizontals to 1 vertical and the ground transversely has a uniform slope of 1 in 10. Compute the volume of the excavation in cubic metres between two points 200 metres apart on the centre line if the depths of the cutting at the first point, second point and at a point half way between are 25 m, 30 m and 26 m respectively.

16. The areas enclosed by the contours in a lake are as under :

Contour (m)	270	275	280	285	290
Area (m^2)	2050	8400	16300	24600	31500

Calculate the volume of water in the lake between 270 m and 290 m contours.

17. A straight road embankment is made on ground having a transverse slope of 1 in 8. The formation width of the embankment is 30 m and the side slopes are 1½ horizontals to 1 vertical. At three sections spaced 50 metres apart the heights of the bank, at the centre of the formation level are 10 m, 15 m and 18 m. Calculate the volume of the embankment assuming ends to be vertical.

18. A road embankment is 30 metre wide with side slopes 1.5 horizontals to 1 vertical. If the ground in a direction transverse to the centre line is level, calculate by the end area method the volume contained in a length of 600 metres, the centre heights of the embankment at 100 metre intervals being in metres as under :

2, 4, 4.5, 6, 5.5, 4, 0.5.

19. A straight level road is to be constructed along hill side, having lateral slope of 1 in 8. The formation width is 25 m with side slopes 1 : 1 in cutting and 2 : 1 in filling. Calculate the total volume of the earth work in a length of 257 m, if the area of cutting and fill in each cross section are equal.

20. A level book page given below shows the readings taken on the centre line of a highway alignment.

The designed crest of the proposed embankment of reduced distance (R.D.) 1500 m is 262.155 with a falling gradient of 1 in 100. The designed crest width of the said embankment is 10 m with side slopes 1 : 1.

R.D.	*B.S.*	*I.S.*	*F.S.*	*R.L.*	*Remarks*
	2.725			255.000	
	2.810		0.290		
1500	1.425		0.115		
1530		1.530			
1560		1.685			
1590	2.030		1.775		
1620		2.525			
1650		2.880			
1680	1.895		3.380		
1710		2.145			
1740			2.370		

Assuming the ground to be level across the alignment, calculate the earth work in filling from R.D. 1560 to R.D. 1650 m.

ANSWERS

1. (*i*) False (*ii*) False (*iii*) True (*iv*) False (*v*) True (*vi*) True (*vii*) False (*viii*) True (*ix*) False (*x*) False (*xi*) True (*xii*) True.

11. (*a*) 42933 m^3 (*b*) 43150 m^3 **12.** 56731 m^3

13. **79002** m^3 **14.** 137700 m^3

15. 325350 m^3

16. 330375 m^3 by Trapozoidal formula

330250 m^3 by Prismoidal formula

17. 79002 m^3 **18.** 61938 m^3

19. 35805 m^3 **20.** 1664.55 by Prismoidal formula.

10

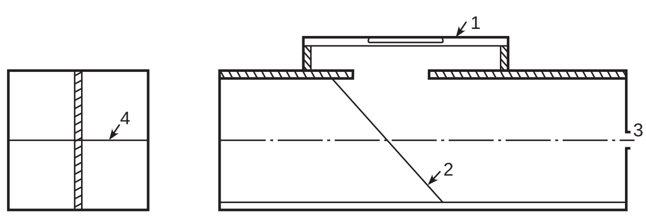
Minor Instruments

10.1. INTRODUCTION

There are a number of minor instruments which are used in exploratory surveys. Important ones are discussed in this chapter.

10.2. THE HAND LEVEL

As the name of the instrument suggests it is a simple and compact instrument which may be held by hand while making observations. It consists of a sighting tube about 10 to 15 cm long having a rectangular or circular cross-section. On one end of the tube *i.e.* the sighting vane, a pin hole is provided and on the other end, cross hairs are provided. A small bubble tube is fitted on the top of the main tube, having an opening through which the reflected image of the bubble may be seen in a mirror fixed inside the tube at an angle of 45° to its axis and exactly under the bubble tube. The mirror occupies only half the width of the tube and objects are viewed through the other half. The line joining the pin hole and the intersection of the cross hairs fixed at other end of the tube, defines the line of sight of the instrument. (Fig. 10.1).



1. Bubble tube 2. Reflecting Mirror
3. Eye hole 4. Cross hair.
Fig. 10.1. The hand Level.

Observations with a Hand Level. The following steps are taken for making observations with a hand level :

1. Hold the level in hand at a known height (1.2 m to 1.5 m) above the ground level and sight the levelling staff.

2. By raising or lowering the forward end, bring the image of the bubble at the intersection of the cross wires.
3. Read the levelling staff and note the reading at which horizontal hair appears to cut the levelling staff (Fig. 10.2).

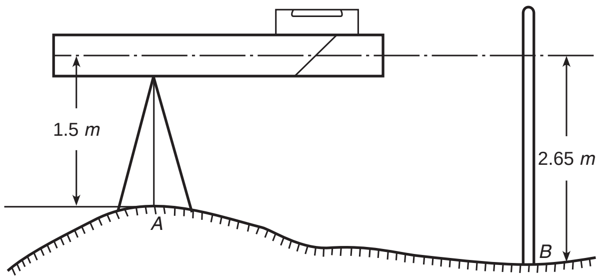


Fig. 10.2. Observations with a hand level.

Let the height of the line of sight at *A* and *B* be 1.50 m and 2.65 m respectively.

$\therefore$ Difference in level = 2.65 – 1.50 = 1.15 m

As the reading on the staff held at *B* is more than the height of line of sight at *A*, the point B is lower than the point *A*.

Uses. A hand level is generally used for the following.

(*i*) Rough work such as reconnaissance and exploratory surveys.

(*ii*) Locating the contours on small scale topographical maps.

(*iii*) Taking short cross sections where longitudinal section has already been surveyed.

Adjustment of the Hand Level (Fig. 10.3).

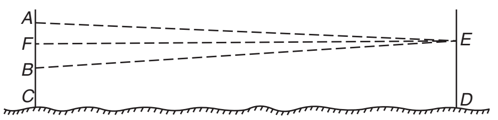


Fig. 10.3. Adjustment of a hand level.

Procedure : The following steps are followed.

1. Fix two ranging rods at *C* and *D* about 25 m apart.
2. Hold the level at *A* about 1.5 m above *C* and sight a point *E* on the ranging rod fixed at *D*, ensuring that the bubble is central.
3. Shift the instrument to *D* and hold it against the point *E*. Centre the bubble and sight a point *B* on the ranging rod held at *C*. If the points *A* and *B* on the ranging rod held at *C* coincide, the instrument is in adjustment.

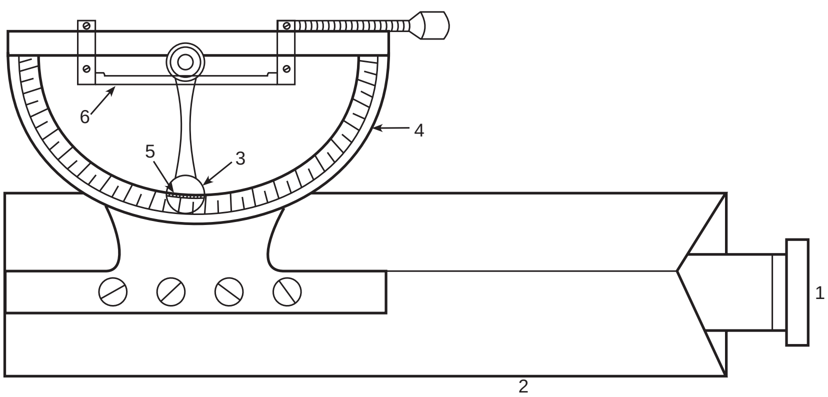
If not, adjust it as under :

1. Mark a point *F* mid-way between *A* and *B*.
2. Raise or lower the horizontal cross wire till the line of sight bisects F.

3. Repeat the procedure till the instrument is adjusted.

10.3. ABNEY'S LEVEL

It is one of the types of clinometers, which is commonly used for rapid work. Abney's level consists of a square sighting tube having a pin hole or an eye piece on one end and a cross-wire at the other end. Near the objective end a mirror is placed at an angle of 45° to the axis of the tube. The mirror occupies only half the width of the sighting tube and the other half is used for sighting objects. Immediately above the opening a small bubble tube is fixed so that the reflected image of the bubble may be seen in the mirror. A vernier arm is attached to the centre of the bubble, which may be rotated either by means of a milled headed screw or by a rack and pinion arrangement. The line joining the pin hole and the intersection of cross wires defines the line of sight of the instrument. When the line of sight is at any inclination, the exact bisection of the bubble and cross wire is done by means of milled headed screw. By doing so the vernier index is moved from its zero-position. The angular movement of the vernier index is equal to the angle of inclination of the line of sight. (Fig. 10.4).



(1) Eye-piece (2) Tube (3) Magnifier (4) Vernier (5) Level tube

Fig. 10.4. An Abney's level.

A semi-circular arc graduated into degrees and minutes, is attached to the sighting tube such that the zero mark of its graduations coincides with the zero mark of the index vernier. The readings on the graduated arc increases from 0° to 90° in both the directions. If the index mark is towards the pin hole, the reading on the graduated arc is a depression and if on the other side, the reading gives an elevation. An extended type of vernier having a least count of 5′ to 10′ is provided.

Uses. Abney's level is very useful for the following work :

(*i*) Measuring vertical angles.

(*ii*) Measuring the slopes of the ground surface.

(*iii*) Tracing grade contours.

(*iv*) Taking cross sections in hilly ground.

Note : It may also be used as a hand level by setting its vernier index to the zero of the graduated scale.

1. Measuring a Vertical Angle with Abney's Level

Vertical angles whether elevation or depression may be measured with an Alney's level as under :

1. Hold the instrument at eye level firmly in a hand and direct it towards the object such that the line of sight bisects it.
2. As the axis of the bubble tube is parallel to the line of sight, the bubble will go out of the centre. Bring the bubble to the centre with the help of the milled headed screw, ensuring that the line of sight passes through the object.
3. Read the angle on the graduated arc by means of the index vernier.
4. If the index vernier is towards the pin hole, the angle is a depression and if on the opposite side, it is an elevation.

2. Measuring a Slope with Abney's Level

The slope of the line joining two points on the surface of the earth may be measured as under :

1. Tie a red or white cloth at the height of observer's eye on a ranging rod and fix it at the other end of the given line.
2. Hold the instrument at the eye level and direct it towards the ranging rod till the line of sight coincides with the strip of the cloth tied round the ranging rod.
3. Bring the bubble central of its run by means of the milled headed screw.
4. Read the angle on the arc by means of the index vernier.

3. Tracing a Grade Contour with Abney's Level

The grade contour of a given inclination may be traced with an Abney's level as under :

Suppose a gradient of an inclination 1 in 50 is required to be traced, starting from any point *A*.

Procedure : Following steps are followed :

(*i*) Calculate the vertical angle corresponding to the inclination 1 in 50.

(*ii*) Hold the instrument at observer's level and fix a target at the height of observer's eye on a ranging rod.

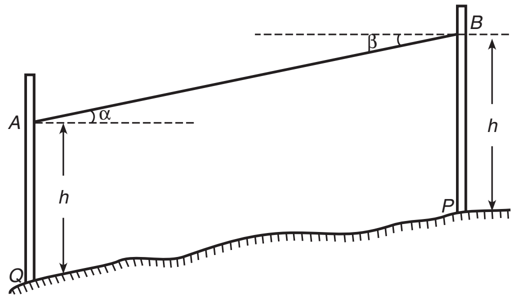


Fig. 10.5. Adjustment of an Abney level.

(*iii*) Set the vernier index to read the angle computed in step (*i*). Bring the bubble central of its run by raising or lowering the object end.

(*iv*) Ask the target man to move with the ranging rod up and down along the line of sight till the target is bisected by the cross wire.

(*v*) The line joining the instrument station and the ranging rod is on the given grade. Fix a peg at the foot of the ranging rod.

(*vi*) Shift the instrument to the peg fixed in step (*v*) and hold it vertically over the peg.

(*vii*) Repeat the steps (*iii*) and (*iv*), and fix another peg.

(*viii*) Continue the process until the last point is located.

4. Adjustment of an Abney's Level.

The test and adjustment of an Abneys's level are done as explained under:

1. Fix two ranging rods at *P* and *Q* about 50 metres apart. Fix targets *A* and *B* one on each rod at equal height approximately equal to the height of observer's eye.
2. Hold the Abney's level at *A* on the ranging rod at *Q*. Measure the angle of elevation α the line *AB* makes with the horizontal.
3. Shift the level to *P*. Hold it at *B* on the ranging rod at *P*. Measure the angle of depression β the line *AB* makes with the horizontal.
4. If the angle of elevation α is equal to the angle of depression β, the instrument is in perfect adjustment.
5. If not, make the vernier to read the mean reading *i.e.* $\frac{\alpha + \beta}{2}$. Now the bubble will be out of its central position.
6. By means of the adjusting screw, bring the bubble to the centre of its run.
7. Repeat the test and adjustment till it is adjusted.

10.4. AN INDIAN TANGENT CLINOMETER

A tangent clinometer which is also known as the Survey of India clinometer, is used for determining the difference in elevations of points. It is specially used in plane table surveys for contouring simultaneously. (Fig. 10.6)

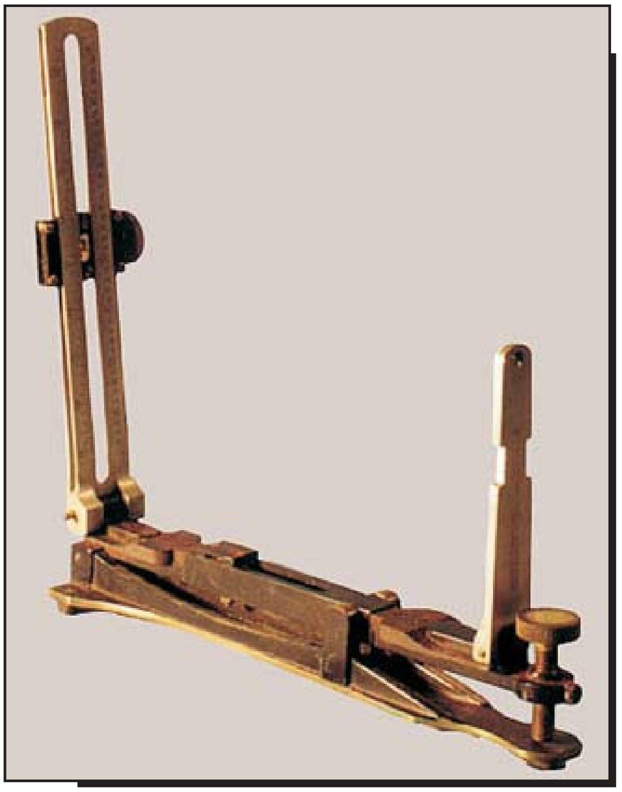


Fig. 10.6. An Indian Tangent Clinometer.

The instrument stands on three buttons and is placed on the surface of the plane table. The base plate carries two vanes *i.e.* an eye vane and an object vane. A small level is attached at the centre of the base plate. When the bubble is central at its run, the line joining the hole of the eye vane and zero of the graduations of the object vane, becomes horizontal. The object vane has an object slit and a scale of natural tangents is engraved on both sides. The distance between the two vanes is 20.30 cm. The distance from the eye hole to the extreme graduation, + 0.40 above or –0.40 below the zero on the tangent scale, is kept 21.88 cm.

1. Observations with a Tangent Clinometer.

The observations with a tangant clinometer are made as meter :

1. The clinometer is placed on a plane table roughly levelled.

2. Sight the object through the pin hole and the objec slit. Bring the bubble central by turning the milled headed screw.
3. Read the tangent value on the scale against the top of the object. (In some clinometers a horizontal wire is stretched across the object slit, which may be moved up and down till the object is bisected. The reading against the wire is noted).

2. Precautions to be Taken while Using a Clinometer. The following precautions are taken while making observations with a tangent clinometer.

(*i*) The clinometer should never be lifted on or off the plane table by either of its vanes to avoid bending.

(*ii*) Both the vanes should be upright and parallel. These should not be bent.

(*iii*) The bubble of the level tube should be brought central before taking readings.

(*iv*) The eye should not be too close to the eye-hole and should preferably be about 5 to 10 cm away from it.

(*v*) Diagonal braces should be pressed right down on their studs.

3. Calculation of Heights. The tangent of the vertical angle (either elevation or depression) when multiplied by the horizontal distance in metres from the plane table position to the object, gives the difference of height in metres. The horizontal distance may be either measured directly, or scaled off from the plane table section. When the horizontal distance exceeds three kilometres, the resulting difference of height should also be corrected for curvature and refraction as discussed under :

The curvature of the earth lowers the distant objects and on the other hand the refraction of the atmosphere raises them. The effect of refraction is always less than the effect of curvature. The amount of correction varies as the square of the horizontal distance. Special scales for the combined correction of curvature and refraction are available for use. In case such a special scale is not available, the amount of correction in metres may be taken one fifteenth (correction factor is 0.0674) of the square of the distance in kilometres. The sign of the correction of curvature and refraction is always positive.

If the observations are made to the top of a signal, the height of the signal of subtracted for obtaining the required ground height.

4. Adjustment of a Tangent Clinometer. There are several methods of testing and adjusting a tangent clinometer. Three of them are described below:

I. Adjustment of a Tangent clinometer with the Help of a Theodolite

First method : The following steps are involved.

(*i*) Place the clinometer on the edge of a plane table.

(*ii*) Set up a theodolite along side such that the horizontal axis of the theodolite is at the level of the sight vane of the clinometer.

(*iii*) Level the theodolite carefully and clamp the telescope to read zero on the vertical circle. Vertical collimation should be ascertained carefully before making observations.

(*iv*) Fix a small piece of paper about 5 cm square on a wall at a distance of about 100 metres where the horizontal wire of the theodolite intersects.

(*v*) By means of the milled headed screw of the clinometer, make the scale reading as zero for the given point.

(*vi*) Now the bubble will be out of the centre. Bring the bubble of the level at the centre of its run by adjusting the screw attached to the level.

(*vii*) Repeat the procedure till adjusted.

Second method : The following steps are involved :

1. Set up a plane table and place the clinometer near its edge.
2. Observe the natural tangent to a distinct object about two kilometres away.
3. Set up a theodolite near the plane table such that its horizontal axis is at the same level as the clinometer. Observe the vertical angle to the same object.
4. Find the natural tangent corresponding to the vertical angle and compare it with that observed with the clinometer.
5. If the two values are the same, the clinometer is in adjustment, if not.
6. Make the clinometer to read the correct natural tangent by means of the milled headed screw.
7. Bring the bubble to the centre of its run by the adjusting crew.
8. Check the natural tangents for vertical angles of other distinct elevated objects.
9. Repeat the procedure till adjusted.

II. Adjustment of a Tangent Clinometer Without a Theodolite

Procedure : Following steps are involved :

1. Place the clinometer on a plane table set up at A.
2. Tie a strip of white cloth round a ranging rod at the same height as the eye hole of the clinometer above the peg A.
3. Sendan assistant with the ranging rod to a peg B about 100 metres away.
4. Read the clinometric reading of the cloth tied on the ranging rod.

5. Shift the plane table and clinometer to *B*. Set the eye hole at the same height above the peg *B* as it was at *A*.
6. Send the assistant to *A* and again observe the reading to the cloth tied on the ranging rod.
7. If the two readings are the same but with opposite sign, the clinometer is in adjustment, if not.
8. Find the difference of two readings. Correct the reading by applying half of the difference. Make the clinometer to read and corrected reading.
9. Bring the bubble to the centre of its run by the adjusting screw.
10. Repeat the above procedure till the clinometer is perfectly adjusted.

Example 10.1. *The reduced level of a plane table station is 100 m and height of the clinometer above the ground level is 1.2 m. Find the reduced level of the hill top when the reading upon an Indian clinometer scale is + 0.025. The horizontal distance to the hill top, scaled off the plane table is 2000 m.*

Solution. (Fig. 10.7)

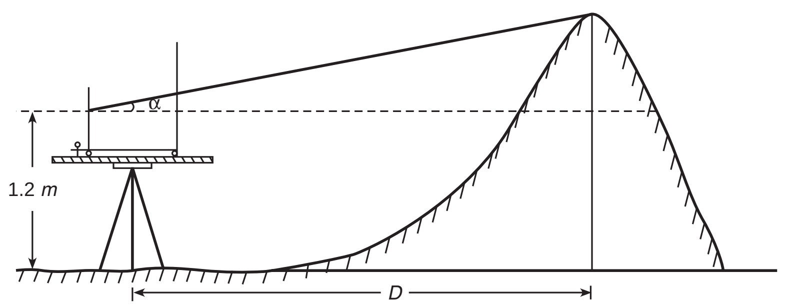


Fig. 10.7.

Let the angle of elevation be α

$\therefore$ $\tan \alpha = 0.025$

The horizontal distance $D = 2000$ m (Given)

$\therefore$ Difference in elevation $= D \tan \alpha$

$= 2000 \times 0.025 = 50$ m

$\therefore$ Reduced level of the hill top = R.L. of the station + Ht. of the clinometer above ground + difference in elevation.

$= 100 + 1.2 + 50 = 151.2$ m **Ans.**

Example 10.2. *The clinometric reading at a station A to a station B was + 0.038. The horizontal distance AB as scaled off the plan was*

1775 m. The reduced level of the plane table station A was 250.50 m and the height of the clinometer above the ground was 1.30 m. Find the reduced level of the station B.

Solution.

Let the angle of elevation be α,

$$\tan\ \alpha = 0.038$$

The horizontal distance $D = 1775$ m (Given)

$\therefore$ Difference in height $= D \tan \alpha$,

$$= 1775 \times 0.038 = 67.45 \text{ m.}$$

$\therefore$ R.L. of B = R.L. of A + height of the clinometer above ground + difference in heights = 250.50 + 1.30 + 67.45.

$$= 319.25 \text{ m.}$$ **Ans.**

Note. As the horizontal distance in both the above examples is less than three kilometres, the correction for curvature and refraction has been ignored.

Example 10.3. *The reduced level of a plane table station is 60 metres and the height of the alidade above the ground 1.5 m. Find the reduced level of staff station A, when the reading upon the Indian clinometre scale is 0.03. The distance to A sealed from the plan is 3,000 m and the vane 3 m above the ground at A was sighted.*

Solution. Here tangent of the angle of elevation = 0.03

Distance of A = 3000 m

$\therefore$ Difference in level $= 3000 \times 0.03 = 90$ m

Correction of curvature and refraction

$$= 0.0676 \times 3^2 = 0.6084 \text{ m}$$

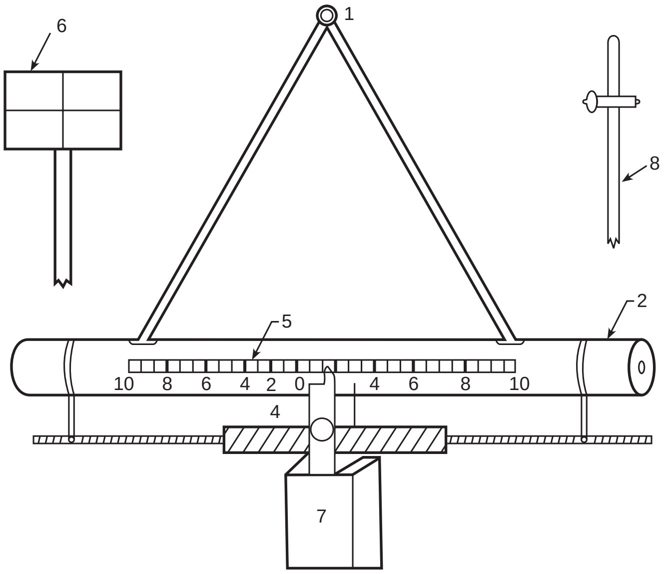
$\therefore$ R.L. of A = R.L. of station +*Ht.* of clinometer above ground level + difference in level + Correction of curvature and refraction - Height of the signal.

$$= 60.0 + 1.5 + 90.0 + 0.6 - 3.0$$

$$= 149.1 \text{ m}$$ **Ans.**

10.5. GHAT TRACER (OR CYLONE GHAT TRACER) (FIG. 10.8)

It is a very simple instrument extensively used for locating a number of points on a given contour gradient during preliminary route surveys of a hill road.



1. Supporting hole 2. Tube
3. A-frame 4. Rod
5. Graduated Scale 6. Target
7. Sliding weight 8. Stand

Fig. 10.8. A Ghat tracer.

Construction : It consists of a hollow tube having an eye hole on one end and a cross wire fitted on the other end. The tube is attached to a bracket which may be suspended from wooden rod. With a suitable rack and pinion arrangement, a heavy weight may be slided along the tube. The weight at the top contains one bevelled edge which slides along the graduations of the scale attached to the tube and serves as an index. The line of sight of a Ghat tracer may be defined as an imaginary line joining the eye hole to the intersection of the cross wire and its prolongation. When the bevelled edge of the sliding weight is against the zero reading of the scale, the line of sight becomes horizontal. If the weight is out of the centre and is towards the observer, the line of sight is elevated and if the weight is away from the observer, the line of sight is depressed. The scale attached to the tube gives the readings of different gradients.

Preparation of a Graduated Scale of a Ghat Tracer

Following steps are involved :

1. Hold the tube against the wooden rod of the Ghat tracer at a point C on a levelled ground.
2. With the help of a spirit level, make the tube horizontal.
3. Engrave a mark at the mid-point of the scale to represent zero.
4. Measure a distance *CD* approximately 100 m from *C*.

5. Fix a mark A on the rod at the same height as the height of the tube.
6. Fix another mark B on the ranging rod at 5 metres above the mark A. Hold the rod truly vertical at D.
7. Slide the weight along the tube towards the observer till the mark B is accurately bisected by the cross-wire.
8. Mark a point on the scale to give a gradient of 1 in 20. Another mark at equal distance is marked on the other side of the zero to give a down gradient of 1 to 20.
9. Similarly the gradient readings, may be engraved on the scale.

1. Measuring the Slope with a Ghat Tracer. The slope between two points A and B on the ground may be measured as follows :

1. Fix the instrument on the stand so that ground mid-point of the tube lies vertically above the point A.
2. Keep the height of the target equal to the height of the centre of the tube and hold it at the other end.
3. Looking through the eye hole, move the sliding weight till the target is bisected by the line of sight.
4. If the weight is near the observer, the slope is along an elevation and if away from the observer, it is along a depression.

2. Tracing the Gradient with a Ghat Tracer. Any gradient (say 1 in 25) may be set out with a Ghat travel as follows :

1. Suspend the instrument from the wooden rod held vertically at the starting point A.
2. Fix a target at the same height as the tube above ground level.
3. Slide the weight along the graduated scale by means of milled headed screw until its bevelled edge reads 25 on the scale.
4. Send the target to a point along the line of sight.
5. Looking through the eye hole, direct the target man to move up or down the slope until the line of sight bisects the target accurately.
6. Fix a peg at the foot of the target rod.
7. Shift the instrument to the peg fixed in step 6.. Repeat the steps 1. to 5. for fixing another point on the gradient.
8. Continue the process until the last point is established.

Note. Target sighted is generally a T-shaped staff. The centre of the target represents height of the tube. If it is not available, an ordinary ranging rod may be used. A target at the height of the tube of the instrument may be fixed for the purpose of sighting.

10.6. SEXTANT

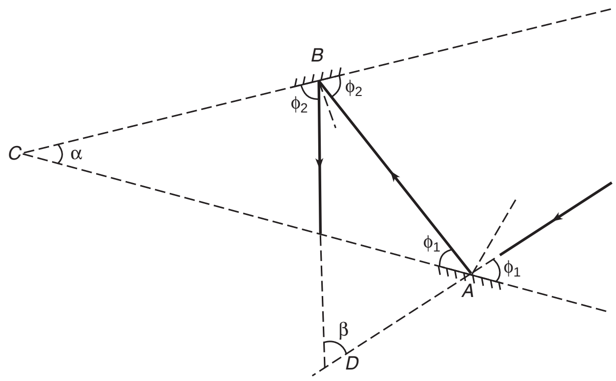


Fig. 10.9. Principle of the sextant.

The sextant is an instrument employed for measuring the angle between two distant objects as seen from the observer's position. The salient feature of the instrument is the arrangement of mirrors which enable the observer to sight the two objects simultaneously. A sextent can be used to measure horizontal as well as vertical angles.

Principle of the Sextant. *The principle of sextant is "when a ray of light is reflected successively from two plane mirrors, the angle between the first incident ray and the last reflected ray, is twice the angle between the planes of the two mirrors."*

Proof : Let *A* and *B* represent two plane mirrors inclined at an angle α represented by *ACB*. Let incident ray *PA* after reflection at *A* travels along *AB* and again after reflection at *B* travels along *BD*. Had there been no mirror at *A*, the ray *PA* would have travelled along *PAD*. Hence the angle of deviation caused due to reflection at two mirrors *A* and *B* is represented by the angle *ADB*. Let ϕ_1 and ϕ_2 denote the angles which the incident and reflected rays make with the reflecting surfaces of the mirrors *A* and *B* respectively.

From triangle *ABC*

$$\phi_2 = \alpha + \phi_1 \quad ...(10.1)$$

Again,

$$\alpha + 180° - \phi_2 = 180° - \phi_1 \quad ...(10.2)$$

From triangle *ABD*

$$\beta + 180° - 2\phi_2 = 180° - 2\phi_1 \quad ...(10.3)$$

From equations (10.2) and (10.3) we get

$$\alpha + 180° - \phi_2 = \beta + 180° - 2\phi_2 + \phi_1$$

$$\alpha = \beta - (\phi_2 - \phi_1)$$

Substituting the values of $(\phi_2 - \phi_1)$ from Eqn. (10.1)

$$\alpha = \beta - \alpha$$

or

$$\beta = 2\alpha \quad ...(10.4)$$

Hence, the angle of deviation of the reflected ray is twice the angle between the reflecting surfaces.

1. Construction. A sextant consists of the following components shown in Fig. 10.10.

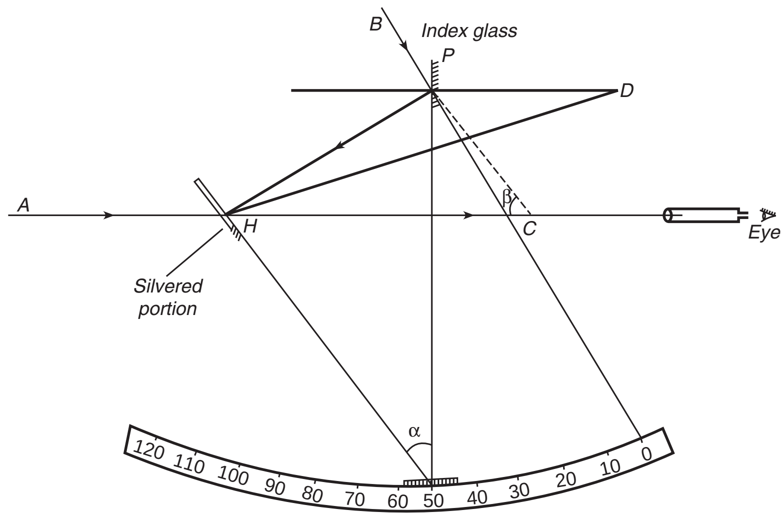


Fig. 10.10. A sextant.

1. A fixed glass (*H*) which is silvered in its lower half and the other upper half is plain.
2. An index glass (*P*) attached to a movable arm which can be moved by means of a milled headed screw.
3. A vernier attached to the other end of the movable arm.
4. A graduated arc against which the vernier moves.

2. Observations with a Sextant. To find out the angle subtended by two points *A* and *B* at the point of observation *C*, the following steps are taken.

1. Hold the instrument such that the point *A* is visible through the upper unsilvered portion of the mirror *H* and read the graduated scale with the vernier.
2. Rotate the movable arm with the milled headed screw until the ray from *B*, after reflection at *P* is seen in the silvered portion of the mirror *H*. Read the graduated scale again.
3. The difference of the two readings is half the angle subtended by the two points *A* and *B*.

To read the angles directly, the scale of the sextant is graduated in values equal to twice the actual angles.

Note. The following points may be noted :

(*i*) The reading of a sextant should be zero when the two mirrors *H* and *P* are placed parallel.

(*ii*) The two mirrors should be perpendicular to the plane of the graduated arc.

(*iii*) The optical axis of the instrument should be parallel to the plane of the graduated arc.

10.7. TYPES OF SEXTANT.

There are three types of sextants in common use :

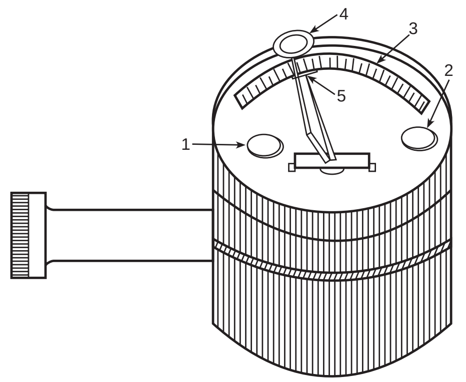
1. Nautical Sextant 2. Sounding Sextant 3. Box Sextant.

1. Nautical Sextant. A sextant used for measuring the angles of elevation of the stars by navigators, is known as a *nautical sextant*.

The graduated arc is replaced by a silver one of large radius (15 to 20 cm) fixed into the gun metal casting carrying the main parts. The least count of the vernier of a nautical sextant is 10 seconds. The horizon glass is placed opposite the telescope.

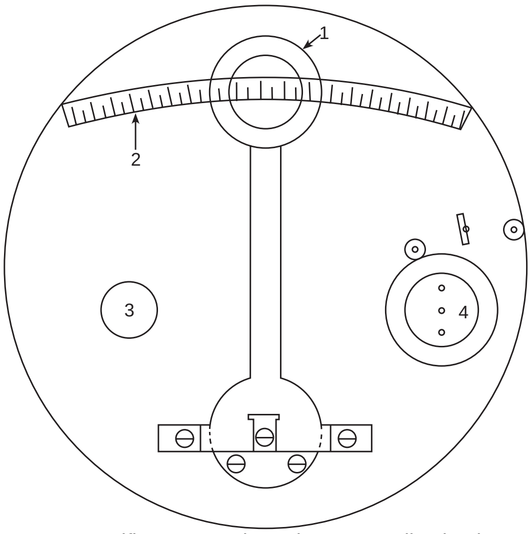
2. Sounding Sextant. It is used for locating the soundings in hydrogracal surveys.

3. Box Sextant. (Fig. 10.11). A box sextant is very handy and compact instrument.



1. Adjusting key. 2. A screw to move index arm and glass simultaneously.
3. A graduated arc. 4. A magnifier. 5. A vernier.
Fig. 10.11. External view of a box sextant.

It is used for the measurement of angles. It consists of a cylindrical box of about 8 cm diameter. The lid may be removed and fitted to its bottom so that it can be used as a handle. A telescope is attached in position by means of a screw on the face of the box. In some box sextants, the telescope is completely eliminated and a pin hole is provided in the movable slide. Opposite to the pin hole, a horizon glass having its upper half silvered and the lower half plain, is provided. (Fig. 10.12)



1. A magnifier 2. A graduated arc 3. A adjusting key
4. A milled headed screw for turning index glass.
Fig. 10.12. Internal view of a box sextant.

The index mirror is attached to the lever system which is moved by means of a large milled headed screw on the top of the instrument. On the other end of the lever, a vernier is attached which moves against a scale graduated to about 140°. The least count of the vernier is one minute. A pair of coloured glasses are generally provided for making observations to the sun or other bright objects.

1. Observations of horizontal angles with a box sextant. The instrument may be used for the measurement of the horizontal angles in the following manner :

1. Hold the sextant in the right hand directly over the point *A* where the angle is to be measured.
2. Sight the left hand object B through the lower or clear portion of the horizon glass.
3. By means of the milled headed screw, the index mirror is rotated until the image of the right hand object *C* is seen in the silvered portion of the horizontal glass.
4. Read the graduated scale against the vernier when the two objects *B* and *C* apparently coincide.

2. Observations of vertical angles with a box sextant. The instrument may be used for measuring the vertical angles as detailed under:

1. Hold the instrument such that its graduated arc lies in a vertical plane.

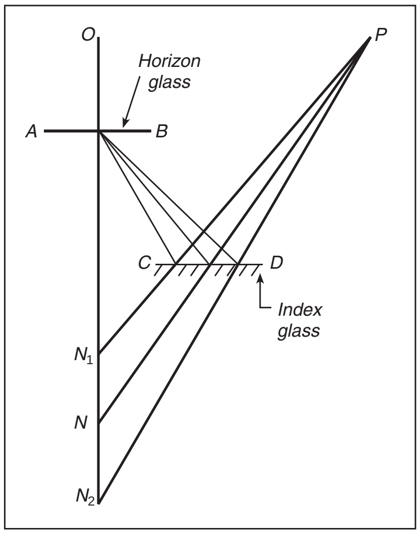


Fig. 10.13. Parallax of box sextant

2. Sight the lower object directly through the plain portion of the horizon glass.
3. Turn the index mirror till the higher object is seen through the silvered portion of the horizon mirror.
4. Reading given by the vernier is the required vertical angle subtended by the object at the instrument station.

10.8. PARALLAX OF THE BOX SEXTANT

The optical principle of a box sextant assumes that the position of the observer's eye while making observations, remains at the point of intersection of the direct ray and the incident ray. In case the eye is kept at some other point, an error is introduced in the observed angles. This error is known as the *parallax* of the box sextant.

Let *AB* be the horizon glass
CD be the index glass
ON be the direct ray
PN be the incident ray

Apparently, the error (the angle NPN_1, or NPN_2 subtended at the reflected station *P* by NN_1 or NN_2) varies inversely as the distance of the reflected station *P*. The amount of error is

$$\alpha = \frac{NN_1}{PN} \text{ or } \frac{NN_2}{PN}$$

radians. If the error in the position of eye is 2.5 cm. and the distance of the reflected station is 90 m, the error due to parallax of box sextant approximately equals to one minute. The error decreases as the distance of the reflected station increases and it amounts only 3 seconds when the distance is 1700 m.

Eccentric Error of the Box Sextant. In case the point N does not coincide with the station of observation, an error due to *eccentricity* is also introduced. The error mainly depends upon the angle subtended at the reflected station P by the displacement of the station of observation from the reflected ray PN. To minimise the error, the nearer station should be sighted directly and more distance station should be viewed by reflection.

Note. The resulting error due to parallel and eccentricity is minimum if the far distant station is viewed by reflection.

10.9. REDUCTION OF OBLIQUE ANGLE TO ITS HORIZONTAL EQUIVALENT

The angle measured by a sextant lies in the plane containing the station of observation and the stations sighted. Unless these stations have equal elevations, the measured angles are always oblique angles. If the elevations of two stations differ considerable from each other, the oblique angle observed by the box sextant considerably differs from the horizontal angle measured by a theodolite. The observed oblique angles may be reduced to their horizontal equivalents by the following formulae.

Observed data : The following data must be carefully observed. (Fig. 10.14).

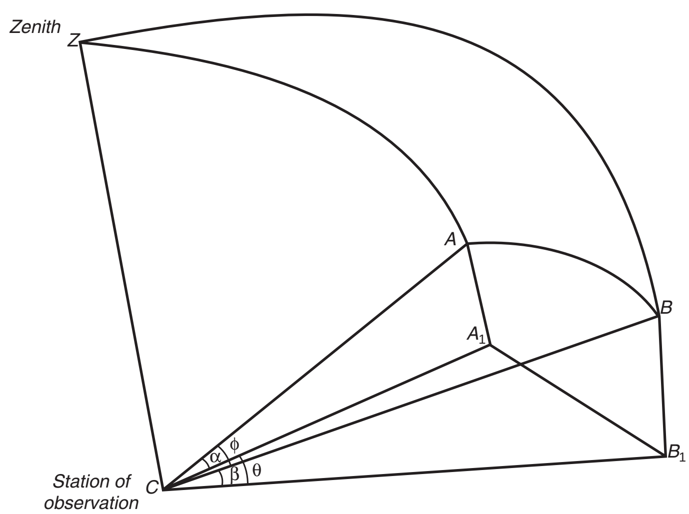


Fig. 10.14.

1. The vertical angle of each station.
2. The oblique angle between two stations.

Let, C be the station of observation

A and B be the stations sighted

Z be the zenith of the observer.

$\angle ACA_1 = \alpha$ = vertical angle of A at C

$\angle BCB_1 = \beta$ = vertical angle of B at C

$\angle ACB = \phi$ = observed oblique angle

$\angle A_1 CB_1 = \phi$ = horizontal equivalent of the observed angle.

The spherical angle $ABZ = A_1CB_1 = \theta$

In spherical triangle ABZ, we have

Fig. 10.14.

$ZA = z_1 =$ zenith distance of station $A = 90^o - \alpha$

$ZB = z_2 =$ zenith distance of station $B = 90^o - \beta$

$AB = \phi$ = observed oblique angle.

The half sum of the sides of sphecical triangle ABZ

$$S = \frac{1}{2}(AZ + BZ + AB) = \frac{1}{2}(z_1 + z_2 + \phi)$$

$$= \frac{1}{2}[(90° - \alpha) + (90° - \beta) + \phi]$$

Then $$\sin\frac{\theta}{2} = \sqrt{\frac{\sin(s - z_1)\sin(s - z_2)}{\sin z_1 \sin z_2}} \quad ...(10.5)$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{\sin(s - z_1)\sin(s - z_2)}{\sin s \sin(s - \phi)}} \quad ...(10.6)$$

$$\cos\theta = \frac{\cos\phi - \cos z_1 \cos z_2}{\sin z_1 \sin z_2} \quad ...(10.7)$$

Example 10.4. *The oblique angle measured with a sextant at a station C between stations A and B is B is* 68° 30′. *The angle of elevation of A is* 30° 30′ *and that of B is* 12° 30′. *Calculate the true horizontal angle ACB.*

Solution. (Fig. 10.12)

Here $\alpha = 30° 30'$; $\beta = 12^o 30'$; $\phi = 68° 30'$

In spherical triangle ABZ,

$AZ = z_1 = 90° - \alpha = 90° - 30° 30' = 59° 30'$

$BZ = z_2 = 90° - \beta = 90° - 12° 30' = 77° 30'$

AB = observed oblique angle $\phi = 68^o 30'$

$$2s = 205° \ 30'$$

$$s = 102° \ 45'$$

$$s - z_1 = 102° \ 45' - 59° \ 30' = 43° \ 15'$$

$$s - z_2 = 102° \ 45' - 77° \ 30' = 25° \ 15'$$

$$s - \phi = 102° \ 45' - 68° \ 30' = 34° \ 15'$$

$$\text{sum} = 102° \ 45'$$

(*i*) Substituting the values in Eqn. 10.6, we get

$$\tan \frac{\theta}{2} = \left[\frac{\sin 43° \ 15' \sin 25° \ 15'}{\sin 102° \ 45' \sin 34° \ 15'}\right]^{1/2}$$

$$= \left[\frac{0.685183 \times 0.426569}{0.97534 \times 0.562805}\right]^{1/2}$$

$$= 0.72969351$$

$$\frac{\theta}{2} = 36° \ 07' \ 5''$$

$$\theta = 72° \ 14' \ 10'' \quad \textbf{Ans.}$$

(*ii*) Substituting the values in Eqn. (10.7) we get

$$\cos \theta = \frac{\cos 68° \ 30' - \cos 59° \ 30' \cos 77° \ 30}{\sin 59° \ 30' \sin 77° \ 30'}$$

$$= \frac{0.366501 - 0.507539 \times 0.21644}{0.861629 \times 0.976296}$$

$$= 0.30509718$$

$$\theta = 72° \ 14' \ 10'' \quad \textbf{Ans.}$$

Example 10.5. *The oblique angle measured with a sextent at a station C between two stations A and B was found to be* 67° 15′. *The angle of elevation of A was* 25° 25′ *and the angle of depression of B was* 8° 20′. *Compute the true horizontal angle ACB.*

Solution. (Fig. 10.12)

$$\alpha = 25° \ 25' \ ; \ \beta = -8° \ 20' \ ; \ \phi = 67° \ 15'$$

In the spherical triangle *ABC*

$$ZA = z_1 = 90° - \alpha = 90° - 25° \ 25' = 64° \ 35'$$

$$ZB = z_1 = 90° \ \beta = 90° - (-8° \ 20') = 98° \ 20'$$

$$AB = \text{the observed oblique angle } \phi = 67° \ 15'$$

$$2s = 230° \; 10''$$

$$s = 115° \; 05'$$

$$s - z_1 = 115° \; 05' - 64° \; 35' = 50° \; 30'$$

$$s - z_2 = 115° \; 05' - 98° \; 20' = 16° \; 45'$$

$$s - \phi = 115° \; 05' - 67° \; 15' = 47° \; 50'$$

$$\text{Sum} = 115° \; 05'$$

(*i*) Substituting the values in Eqn. (10.6), we get

$$\tan\frac{\theta}{2} = \left[\frac{\sin 50° \; 30' \sin 16° \; 45'}{\sin 115° \; 05' \sin 47° \; 50'}\right]^{\frac{1}{2}}$$

$$= \left[\frac{0.771625 \times 0.288196}{0.905692 \times 0.741195}\right]^{\frac{1}{2}}$$

$$= 0.57555998$$

$$\theta/2 = 29° \; 55' \; 23''$$

$$\theta = 59° \; 50' \; 46'' \quad \textbf{Ans.}$$

(*ii*) Substituting the values in Eqn. (10.7), we get

$$\cos \theta = \frac{\cos 67° \; 15' - \cos 64° \; 35' \cos 98° \; 20'}{\sin 64° \; 35' \sin 98° \; 20'}$$

$$= \frac{0.386711 - 0.429198 \times (-0.144932)}{0.903210 \times 0.989442}$$

$$= 0.50232585$$

$$\theta = 59° \; 50' \; 46'' \quad \textbf{Ans.}$$

Example 10.6. *The inclined angle between two stations A and B from O was measured to be 74° 25′. It was later determined that the elevation of A from O was 15° 22′ and the elevation of B from O was 4° 06′.*

Calculate the horizontal angle AOB.

Solution.

Here $\alpha = 15° \; 22'$; $\beta = 4° \; 06'$: $\phi = 74° \; 25'$

In spherical triangle *ABC*

$$ZA = z_1 = 90° - \alpha = 90° - 15° \; 22' = 74° \; 38'$$

$$ZB = z_2 = 90° - \beta = 90° - 4° \; 06' = 85° \; 54'$$

$$AB = \text{the observed oblique angle } \phi = 74° \; 25'$$

$$2s = 234° \ 57'$$

$$s = 117° \ 28' \ 30'$$

$$s - z_1 = 117° \ 28' \ 30'' - 74° \ 38' = 42° \ 50' \ 30''$$

$$s - z_2 = 117° \ 30' - 85° \ 54' = 31° \ 34' \ 30''$$

$$s - \phi = 117° \ 28' \ 30'' - 74° \ 25' = 43° \ 03' \ 30''$$

$$\text{sum} = 117° \ 28' \ 30''$$

Substituting the values in Eqn. (10.6), we get

$$\tan \frac{\theta}{2} = \sqrt{\frac{\sin 42° \ 50' \ 30'' \sin 31^{\circ} \ 34' \ 30'}{\sin 117° \ 28' \ 30'' \sin 43° \ 03' \ 30''}}$$

$$= \sqrt{\frac{0.679975 \times 0.523614}{0.887212 \times 0.682743}}$$

$$= 0.6322025$$

$$\frac{\theta}{2} = 32° \ 18' \ 04''$$

$$= 64° \ 36' \ 08'' \quad \textbf{Ans.}$$

10.10. PANTAGRAPH (FIG. 10.15)

A pantagraph is an instrument used for enlarging, reducing or reproducing the plans.

Construction : It consists of a light brass frame of tubular construction and hinged at the joints. A heavy fulcrum (*G*) is used to fix the frame and about which the instrument moves. Two tracing points *H* and *J* are provided. *J* is fixed to the arm *AC* near *C* and *H* is movable. Fulcrum *G* is also movable and may be fixed such that *G*, *H* and *J* are in one straight line. The frame is supported on small rollers which allow the instrument to move freely on the paper in all directions.

The points *A, D, F* and *E* always make a parallelogram in any position.

A thread is fitted over the hinges so that the pencil may be lifted whenever required.

Principle of Pantagraph Let *AB* and *AC* be two straight arms hinged at *A*. *D* and *E* are two hinge points equidistant from *A*. Arms *DE* and *EF* are kept equal to *AD* and *AE* respectively. Hence, *ADEF* is a parallelogram. If the arm *AB* is hinged at *G* and the end *C* of the arm *AC* is moved, then movements of the point *H* on *DF* and *J* on *EC* will be in the ratio of their distances from *D* and *E* respectively. (Fig. 10.16)

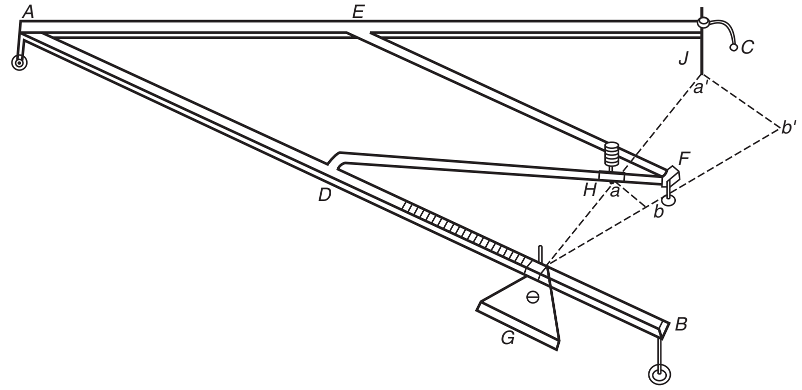


Fig. 10.15. A pantagraph.

Proof. *ADFE* remains always a parallelogram, having equal sides in every position. So *DH* is parallel to *AC*.

Now $\angle DHG = \angle CAG$

$\therefore$ Δs *HDG* and *JAG* are similar

$$\frac{HD}{GD} = \frac{JA}{GA}$$

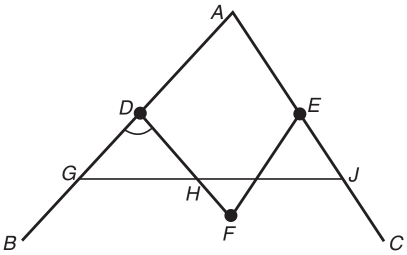


Fig. 10.16. Principle of a pantagraph.

when *G, H* and *J* are in the same straight line.

Now *J* is kept fixed on *AC* and the point *H* is selected on the arm *DF* such that *G, H* and *J* are in one straight line.

From similar Δ *GDH* and Δ *GAJ*, we get

$$\frac{HG}{JG} = \frac{GD}{GA}$$

Any displacement of *J* through a distance *X* will give a corresponding displacement of *H* through $\frac{GD}{GA}$ and hence the plan placed at J will be reduced. Similarly it may be proved that for enlargement, the plan is placed under *H* and the enlargement is obtained at *J*.

Procedure to enlarge or to reduce a plan.

The plan is fixed on a smooth surface table. The fulcrum and tracing point *H* are set to the required ratio of enlargement. A paper is placed under the tracing point *J*. The tracing point *H* is moved along the details of the plan. The enlargement of the same is obtained on the paper. (Fig. 10.15)

When a reduction of a plan is required, the tracing point and tracing pencil are interchanged.

EXERCISE 10

1. State whether following statements are true or false.

(*i*) In a hand level, the bubble is seen in a mirror fixed inside the tube at an angle 45° with horizontal axis.

(*ii*) If the index mark of an Abney's level is towards the observer, the reading on the graduated arc is an elevation.

(*iii*) If the index mark of an Abney's level is towards the cross hair, the reading on the graduated arc is an elevation.

(*iv*) An Abney's level may be used as a hand level by setting the vernier index to zero.

(*v*) The distance between two vanes of an Indian clinometer is kept equal to 21.88 cm.

(*vi*) The distance between the eye hole and the +0.4 mark on the object vane of an Indian clinometer is kept equal to 20.32 cm.

(*vii*) The distance between the graduations of +0.4 and –0.4 of an Indian clinometer is kept to 8.13 cm.

(*viii*) A Ghat tracer is used to find out the natural tangent of vertical angle between two stations.

(*ix*) Scale of a sextant is graduated in values equal to twice the actual angles.

(*x*) A box sextant is used only to measure the horizontal angles.

(*xi*) With the help of a Ghat tracer, the angles of elevations as well as depressions, can be measured.

(*xii*) Pantagraph is an instrument which can be used to enlarge or to reduce the plans.

2. (*a*) Describe the construction and uses of a hand level with a neat diagram. (*b*) Explain how you would make observations with a hand level.

3. (*a*) Describe an Abney's level with a neat sketch. (*b*) How will you measure the angles of slope of the ground with an Abney's level.

4. How will you adjust an Abney's level ? Describe different steps which are taken to trace a grade contour on the ground with the help of an Abney's level.

5. Sketch and describe the Indian tangent clinometer. How is it used?

6. What are the different methods of adjusting an Indian tangent clinometer. Describe any one of them in detail.

7. Describe the construction of a Ghat tracer with a neat sketch. Explain the method of setting out a gradient contour along a hill slope with the help of a Ghat tracer.

8. Explain the principle of sextant and prove mathematically that the angle of deviation of the reflected ray is twice the angle between the reflecting surfaces of the sextant.

9. Sketch and describe a box sextant. Explain how the index error of a box sextant is determined and eliminated.

10. Describe with a neat sketch, the construction and use of a pantagraph.

11. Write short notes on the following :

(*a*) Pantagraph (*b*) Hand level

(*c*) Tangent clinometer (*d*) Ghat tracer

(*e*) Box sextant (*f*) Abney's level.

12. The clinometric reading from a station A to a station B was 0.045. The distance AB as scaled from the plan was 1525 m. The reduced level of the station peg A was 152.85 m, and the height of the eye hole of the clinometer above the top of peg A was 1.25 m. Calculate the reduced level of the station B.

13. The clinometer reading from a station A to a station B was –0.057. The distance AB as scaled from the plan was 2850 m and the correction for curvature and refraction added to the height of the plane table as 3.10 m. Calculate the reduced level of the station B if the reduced level of A was 250.35 m.

ANSWERS

1. (*i*) True (*ii*) False (*iii*) True (*iv*) True (*v*) False (*vi*) False (*vii*) True (*viii*) False (*ix*) True (*x*) False (*xi*) False (*xii*) False (*xiii*) True.

12. 222.88 m

13. 4001 m.

11

Theodolite

11.1. INTRODUCTION

A theodolite is a precise instrument for measuring horizontal angles, angles of elevation and depression *i.e.*, vertical angles, bearing and azimuth of a line. Theodolite is also used for prolongation of survey lines, finding difference in elevations and setting out engineering works requiring higher precision *i.e.* ranging the highway and railway curves, aligning tunnels, etc.

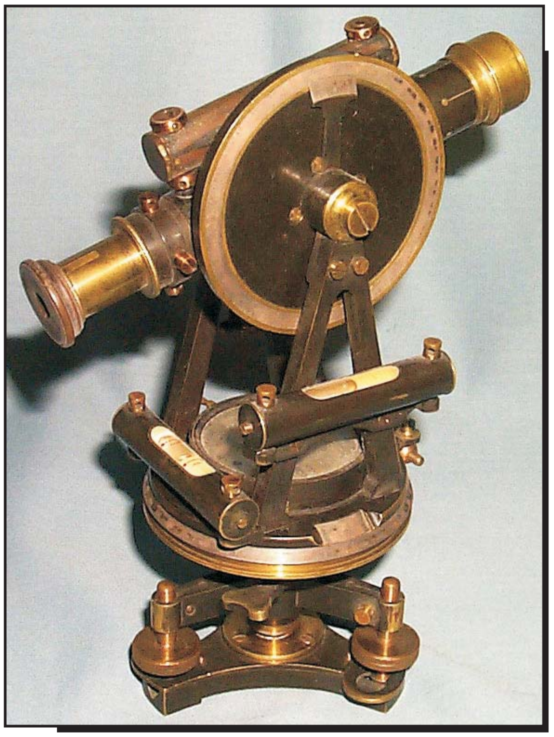


Fig. 11.1. A transit theodolite (By courtesy key bee and company).

11.2. CLASSIFICATION OF THEODOLITES

Theodolites are primarily classified as (*i*) Transit theodolite (*ii*) Non-transit theodolite.

1. **Transit Theodolite.** The theodolite whose telescope can be transited, is called a *transit theodolite.* A transit telescope can be revolved through a complete revolution about its horizontal axis in the vertical plane.
2. **Non -Transit Theodolite.** The theodolite whose telescope can not be transited, is called a *non-transit theodolite.* A non-transit telescope cannot be revolved through a complete revolution about its horizontal axis in the vertical plane.

Various types of theodolites are as shown here under:

Non-transit theodolites are much inferior as compared to transit theodolites. The non-transit theordolites have become almost obsolete now a days.

Various types of theodolites are :

1. Vernier Transit
2. Double arcs
3. Watts micrometer
4. Tavistock
5. Wild T_2
6. Wild T_4

Vernier theodolites. It is the most simple type of theodolite. It is called vernier transit theodolite because its telescope can be transited or rotated around the horizontal or transverse axis through a complete revolution and the readings are taken with the help of vermiers provided on horizontal plate.

The diameter of the main scale plate defines the size of the theodolite *i.e.* 15 cm theodolite or so.

The main plate is graduated in degrees and each degree is subdivided into 3 equal parts; each small division equals to 20 minutes.

Reading with the Verniers. 59 small divisions of main scale plate coincide with 60 divisions of the vernier scale.

So, the least count of the vernier.

$$= \frac{\text{Value of smallest division of main scale}}{\text{Number of divisions on the vernier scale}}$$

$$= \frac{20'}{60} = \frac{20 \times 60''}{60} = 20'' \text{ (seconds of arc)}$$

Hence, the readings on the main scale can be read to 20 seconds

11.3. PARTS OF A TRANSIT THEODOLITE (FIG. 11.2).

A transit theodolite consists of the following essential parts :

1.Levelling Head. It consists of two parts *i.e.* upper tribarch and lower tribarch.

1. **The upper tribarch.** It has three arms. Each arm carries a levelling screw. Levelling screws are provided for supporting and levelling the instrument. The boss of the upper tribarch is pierced with a female axis in which lower male vertical axis operates.
2. **The lower tribarch.** It has a circular hole through which a plumb bob may be suspended for centering the instrument

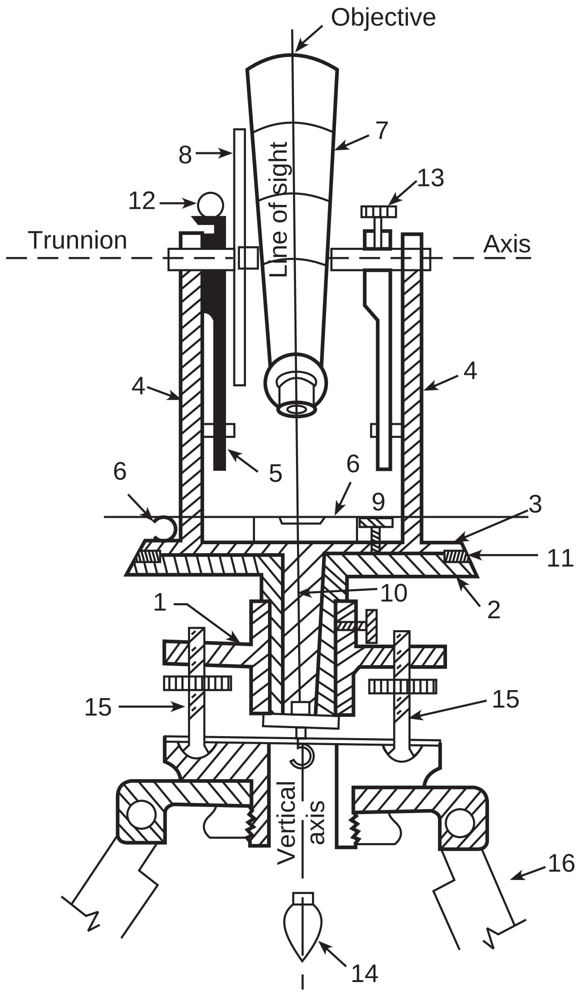


Fig. 11.2. Parts of a theodolite.

quickly and accurately on the ground station.

The three distinct functions of a levelling head are :

(*i*) To support the main part of the instrument.

(*ii*) To attach the theodolite to the tripod.

(*iii*) To provide a means for levelling the theodolite.

2. Lower plate (or scale plate). The lower plate which is attached to the outer spindle, carries a horizontal graduated circle at its bevelled edge. It is therefore sometimes known as the *scale plate.* It is divided into 360° . Each degree is further divided into ten minutes or twenty minutes arc intervals. Scale plate can be clamped at any position by a clamping screw and a corresponding slow motion can be made with a tangential screw or slow motion screw.

When the lower clamp is tightened, the lower plate is fixed to the upper tribarch of the levelling head. The size of the theodolite is determined by the size of the diameter of the lower plate.

3. Upper plate (or vernier plate). The upper plate or vernier plate is attached to the inner spindle axis. Two verniers are screwed to the upper plate diametrically opposite. This plate is so constructed that it overlaps and protects the lower plate containing the horizontal circle completely except at the parts exposed just below the verniers. The verniers are fitted with magnifiers. The upper plate supports the Y_S or A_S which provide the bearings to the pivots of the telescope. It carries an upper clamp screw and a corresponding tangent screw for accurately fixing it to the lower plate. On clamping the upper clamp and unclamping the lower clamp, the instrument may be rotated on its outer spindle without any relative motion between the two plates. On the other hand, if the lower clamp screw is tightened and upper clamp screw is unclamped, the instrument may be rotated about its inner spindle with a relative motion between the vernier and the graduated scale of the lower plate. This property is utilised for measuring the angles between two settings of the instrument. It may be ensured that the clamping screws are properly tightened before using the tangent screws for a finer setting.

4. The Standards (or A Frame). Two standards resembling the English letter *A* are firmly attached to the upper plate. The tops of these standards form the bearings of the pivots of the telescope. The standards are made sufficiently high to allow the rotation of the telescope on its horizontal axis in vertical plane. The T-frame the and arm of vertical circle clamp, are also attached to the standards.

5. T-frame or Index -Bar. It is T-shaped and is centered on the horizontal axis of the telescope in the frame of the vertical circle. The two verniers *C* and *D* are provided on it at the ends of the horizontal arm, called the *'Index Arm'*. A vertical leg known as clipping arm is provided with a fork and two clipping screw at its lower extremity. The index and clipping arms together, are known as *T-frame.* At the top of this frame, is attached a bubble tube which is called the *altitude bubble tube.*

6. Plate levels. The upper plate carries two plate levels placed at right angles to each other. One of the plate bubbles is kept parallel to the trunnion axis. The plate levels can be centered with the help of the foot screws. In some theodolites only one plate level is provided.

7. Telescope. The telescopes may be classified as

(*i*) The external focussing telescope.

(*ii*) The internal focussing telescope.

1. The external focussing Telescope. This type of telescope consists of an outer tube and an inner tube. The outer tube is attached to the pivot by a thick metal band and the inner tube slides in the outer tube by means of a rack and pinion turned by a large milled-head screw. With this arrangement, the telescope can be focussed to varying distances. The object glass is usually fixed at the end of the inner tube.

2. The Internal Focussing Telescope. In this type of telescope, an internal focussing lens (double concave) is introduced between the objective and eye piece both of which are mounted at the ends of the tube. Focussing may be achieved by moving the internal focussing lens with a focussing screw. The interior of the telescope tube is painted dull black to prevent reflection of light from the internal surface.

Principle of focussing internal focussing telescope. The image of the object is formed by the objective glass O in the absence of the concave lens C is at B. Due to the presence of the lens C, the image is brought to focus at the plane of cross hairs of the eye-piece at a distance D from the objective O. (Fig. 11.3).

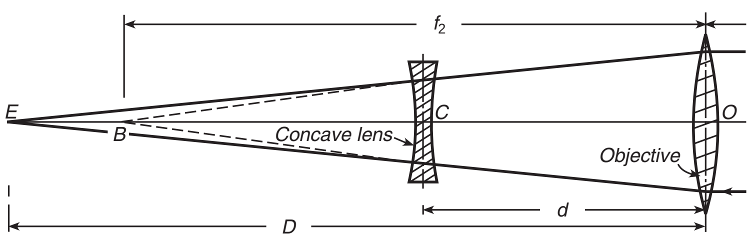


Fig. 11.3. Principle of focussing an internal focussing telescope.

With an elementary knowledge of the optics of a convex lens having f as its focal length, we know

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad ...(11.1)$$

For the concave lens C having f' as its focal length

$$\frac{1}{f'} = \frac{1}{(f_2 - d)} + \frac{1}{(D - d)} \qquad ...(11.2)$$

where d is the distance between the lenses O and C, *and* D is the distance between O and E.

From the above two conjugate focal equations, the distance d may be calculated for a particular value of f_1. When the object sighted is at infinity, f_2 equals f and d attains its minimum value.

Advantages of an internal focussing telescope. The advantages of an internal focussing telescope as compared to an external focussing telescope are as under :

1. The overall length of the telescope is not altered during focussing and hence the balance of the telescope is not affected.
2. There is no risk of breaking the parallel plate bubble or glass cover of the compass underneath while transiting the telescope.
3. Wear of the rack and pinion is less due to lesser movement of the concave lens.
4. The line of collimation is least affected by focussing.
5. The telescope remains perfectly free and dust moisture.
6. When making tacheometric observations, the additive constant is generally eliminated and computations are thus simplified. For Tacheometric Surveying, refer to chapter 13 Tacheometric Surveying.
7. The combined focal length of the lenses, increases the power of the telescope.
8. By fitting a concave lens, the diameter of the objective can be increased without aberrative effects.

8. Vertical Circle. A vertical circle is attached to the telescope and is graduated in various ways by the manufacturers. The following

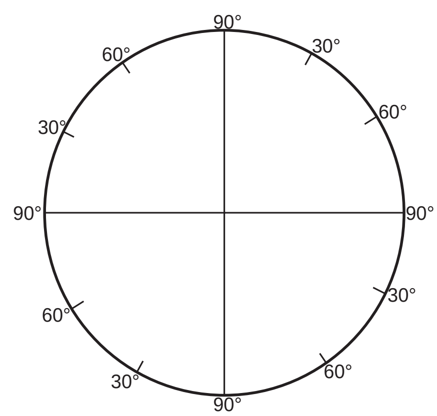


Fig. 11.4. Graduations of a Vertical circle.

graduations of vertical circles are in common use :

1. The vertical circle is divided into four quadrants each reading from 0° to 90° in the same direction. (Fig. 11.4).

The theodolites which are provided with such vertical circles read elevations on one face and depressions on the other face.

2. The vertical circle is divided into four quadrants from 0° at the eye-piece and objective end to 90° in both the directions. (Fig. 11.5).

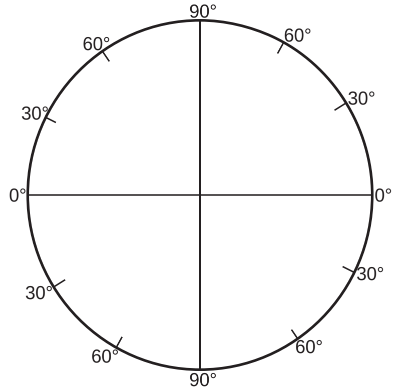


Fig. 11.5. Graduations of a Vertical circle.

The theodolites which are provided with such vertical circles, read elevations and depressions on both the faces.

3. The vertical circle is divided from 0° to 360° and placing of zero does not matter. (Fig. 11.6).

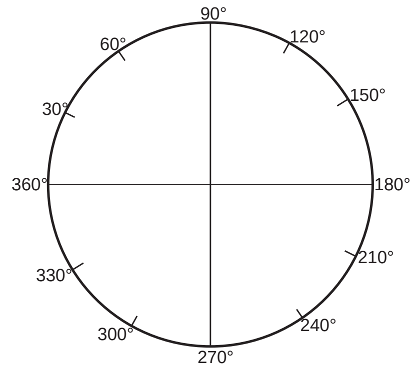


Fig. 11.6. Graduations of a vertical circle.

The true elevations and depressions on such vertical circles may be obtained by applying the following rules :

Subtract the lesser reading from the greater,

(*a*) If the result is greater than 180°, the angle is an elevation ; to obtain this, subtract it from 180° and divide by 2, the result will be the true elevation.

(*b*) If the result is less than 180°, the angle is depression ; to obtain this, subtract it from 180° and divide by 2, the result will be the true depression.

Note. It may be noted that the same vernier should be read first on both faces to read full degrees correctly.

9. Tripod. While working, the theodolite is supported on a tripod which consists of three solid or framed legs. The lower ends of each leg are provided with pointed iron shoes. The tripod head carries at its upper surface an external screw to which foot plate of the levelling head may be screwed.

10. The Plumb bob. A plumb bob is suspended from a hook fitted to the bottom of the main vertical axis to centre the theodolite exactly over the ground station mark.

11.4. DEFINITIONS AND OTHER TECHNICAL TERMS

Following terms are used while making observations with a theodolite.

1. **Vertical axis.** The axis about which the theodolite, may be rotated in horizontal plane, is called *vertical axis.* Both upper and lower plates may be rotated about vertical axis.

2. **Horizontal axis.** The axis about which the telescope along with the vertical circle of a theodolite, may be rotated in vertical plane, is called *horizontal axis.* It is also sometimes called *trunnion axis, or transverse axis.*

3. **Line of Collimation.** The line which passes through the point of intersection of the cross hairs of the eye-piece and optical centre of the objective and its continuation, is called *line of collimation.* The angle between the line of collimation and the line perpendicular to the horizontal axis, is called *error of collimation.*

The line passing through the eye piece and any point on the objective, is called *line of sight.*

4. **Axis of telescope.** The axis about which the telescope may be rotated, is called *axis of telescope.*

5. **Axis of the level tube.** The straight line which is tangential to longitudinal curve of the level tube at its centre is called *axis of the level*

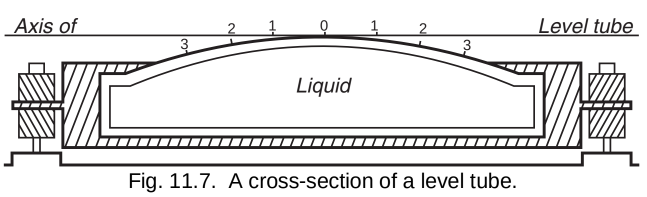


Fig. 11.7. A cross-section of a level tube.

tube. When the bubble of the level tube is at central position, the axis of the level tube becomes horizontal (Fig. 11.7).

6. **Centering.** The process of setting up a theodolite exactly over the ground station mark, is known as *centering*. It is achieved when the vertical axis of the theodolite is made to pass through the ground station mark.

7. **Transiting.** The process of turning the telescope in vertical plane through 180° about its horizontal axis, is known as *transiting*. The process is also sometimes known as *reversing* or *plunging*.

8. **Swing.** A continuous motion of the telescope about the vertical axis in horizontal plane, is called *swing*. The swing may be in either direction, *i.e.,* right or left. When the telescope is rotated in clockwise (right) direction, it is known as *right swing*. When it is rotated in the anticlockwise (left) direction, it is known as *left swing*.

9. **Face left observations.** When the vertical circle is on the left of the telescope at the time of observations, the observations of the angles are known as *'face left observations'*.

10. **Face right observations.** When the vertical circle is on the right of the telescope at the time of observations, the observations of the angles, are known as *"face right observations*.

11. **Changing face.** The operation of changing the face of the telescope from left to right and *vice versa,* is called changing face.

12. **A 'measure'.** It is the determination of the number of degrees, minutes and seconds, or grades contained in an angle.

13. **A 'set'.** A 'set' of horizontal observation of any angle consists of two horizontal measures, one on the face left and the other on the face right.

14. **Telescope normal.** A telescope is said to be normal when its vertical circle is to its left and the bubble of the telescope is up.

15. **Telescope inverted.** A telescope is said to be inverted or reversed when its vertical circle is to its right and the bubble of the telescope is down.

11.5. FUNDAMENTAL LINES OF A TRANSIT

The fundamental lines of a transit are :

1. The vertical axis.
2. The axis of plate bubble.
3. The line of collimation which is also sometimes called line of sight.
4. The horizontal axis, transverse axis or trunnion axis.
5. The bubble line of telescope bubble or altitude bubble.

11.6. GEOMETRY OF THE TRANSIT. (FIG. 11.8.)

The following geometrical requirements must be maintained in a transit.

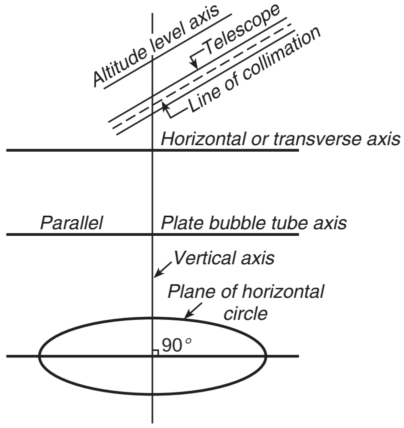


Fig. 11.8. Fundamental lines of transit.

1. The inner and outer spindles and the bearings of the horizontal axis (trunnion axis) must be made in such a way that the instrument rotates about its geometric lines and not cones or cylinders.
2. The inside and outside tapered surfaces of the outer spindle must be concentric. If this condition is not met, the instrument will not remain level when its horizontal circle is turned.
3. The vertical axis, horizontal axis and line of collination must meet in a point known as *instrument centre*.
4. The vertical axis must be perpendicular to the horizontal axis.
5. The line of collimation must be perpendicular to the horizontal axis. The movement of the telescope lens must be so guided that it does not alter the line of sight.
6. When the vertical axis is truly vertical, the plate bubbles must occupy their central positions.
7. When the line of sight is horizontal, the altitude bubble must be at the centre of its run.
8. The horizontal graduated circle must be concentric with and perpendicular to the vertical axis.
9. The graduations on the horizontal circle must be concentric with the vertical axis.
10. The graduations on the vertical circle must be concentric with the horizontal axis or trunnion axis.
11. When the line of collination is perpendicular to the horizontal axis, the vertical circle must read zero.

11.7 ADJUSTMENTS OF A THEODOLITE

The adjustments of a theodolite are of two kinds :

1. Temporary Adjustments. 2. Permanent Adjustments

1. Temporary Adjustments. The adjustments which are required to be made at every instrument station before making observations, are known as *temporary adjustments.*

The temporary adjustments of a theodolite include the following :

(*i*) Setting up and centering the theodolite over the station.

(*ii*) Levelling of the theodolite.

(*iii*) Elimination of the parallax.

1. Setting up and centering. The operation of setting up a theodolitde includes the centering of the theodolite over the ground mark and also approximate levelling with the help of tripod legs.

The operation with which vertical axis of the theodolite, represented by a plumb line, is made to pass through the ground station mark, is called *centering.* The operation of centering is carried out in following steps :

(*i*) Suspend the plumb bob with a string attached to the hook fitted to the bottom of the instrument to define the vertical axis.

(*ii*) Place the theodolite over the station mark by spreading the legs well apart so that telescope is at a convenient height.

(*iii*) The centering may be done by moving the legs radially and circumferentially till the plumb bob hangs within 1 cm horizontally of the station mark.

By unclamping the centre-shifting arrangement, the finer centering may now be made.

Approximate levelling with the help of the tripod. It is very necessary to ensure that the level of the tripod head is approximately level before centering is done. In case there is a considerable dislevelment, the centering will be disturbed when levelling is done. The approximate levelling may be done either with reference to a small circular bubble provided on the tribarch or by eye judgement.

2. Levelling of a theodolite. The operation of making the vertical axis of a theodolite truly vertical, is known as *levelling of the theodolite.* After having levelled approximately and centred accurately, accurate levelling is done with the help of plate levels. Two methods of levelling are adopted to the theodolites, depending upon the number of levelling screws.

***(a)* Levelling with a three Screw head.** The following steps are involved. (Fig. 11.9).

1. Turn the horizontal plate until the longitudinal axis of the plate level is approximately parallel to a line joining any two levelling screws [Fig. 11.9(*a*)].

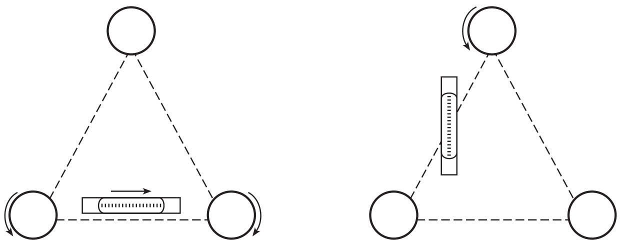


Fig. 11.9. Levelling of a theodolite with a three-screw head.

2. Bring the bubble to the centre of its run by turning both foot-screws simultaneously in opposite directions either inwards or outwards. The movement of the left thumb indicates the direction of movement of the bubble.
3. Turn the instrument through 180° in azimuth.
4. Note the position of the bubble. If it occupies a different position, move it by means of the same foot-screws to the approximate mean of the two positions.
5. Turn the theodolite through 90° in azimuth so that the plate level becomes perpendicular to the previous position [Fig. 11.9(*b*)].
6. With the help of the third foot-screw, move the bubble to the approximate mean position already indicated.
7. Repeat the process until the bubble retains the same position for every setting of the instrument, in azimuth.

The mean position of the bubble, is called the **zero** of the level tube.

If the theodolite is provided with two plate levels placed perpendicular to each other, the instrument is not required to be turned through 90° . In this case, the longer plate level is kept parallel to any two foot-screws and the bubble is brought to central position by turning both the foot screws simultaneously. Now, with the help of the third foot screw, bring the bubble of the second plate level central. Repeat the process till both the plate bubbles occupy the central positions of their run for all the positions of the instrument.

(*b*) Levelling with a Four Screw Head. The following steps are involved. (Fig. 11.10)

1. Turn the upper plate until the longitudinal axis of the plate level is roughly parallel to any two *diagonally opposite* foot screws such as D and B (Fig. 11.10 *a*).
2. Bring the bubble central of its run by turning both the foot-screws simultaneously in opposite directions.
3. Turn the upper plate through 90° until the plate level is parallel to the other two diagonally opposite screws such as A and C (Fig. 11.10 *b*).

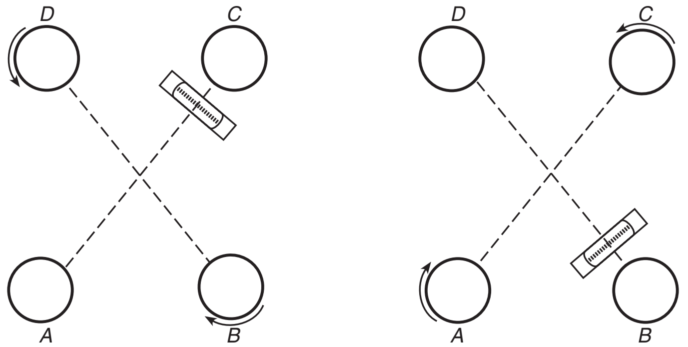


Fig. 11.10. Levelling of a theodolite with a four-screw head.

4. Bring the bubble to the centre of its run as before.
5. Repeat the process till the bubble remains central in all the positions of the instrument.

Elimination of Parallax. An apparant change in the position of the object caused by the change in the position of the observer's eye, is known as *parallax.*

In a telescope, parallax is caused when the image formed by the objective is not situated in the plane of the cross-hairs. Unless parallax is removed, accurate bisection and sighting of objects become difficult.

Elimination of parallax may be done by focussing the eyepiece for distinct vision of cross hairs and focussing the objective to bring the image of the object in the plane of the cross-hairs as discussed below.

Focussing the eye-piece. To focus the eye-piece for distinct vision of cross hairs, either hold a white paper in front of the objective or sight the telescope towards the sky. Move the eye piece in or out till the cross-hairs are seen sharp and distinct.

Focussing the objective. After cross-hairs have been properly focussed, direct the telescope on a well defined distant object and intersect it with vertical wire. Focus the objective till a sharp image is seen. Removal of the parallax may be checked by moving the eye slowly to one side. If the object still appears intersected, there is no parallax.

If, on moving the eye laterally, the image of the object appears to move in the same direction as the eye, the observer's eye and the image of the object are on the opposite sides of the vertical wire. The image of the object and the eye are brought nearer to eliminate the parallax. This parallax is called *far parallax.*

If, on the other hand, the image appears to move in reverse direction to the movement of the eye, the observer's eye and the image of the object are on the same side of the vertical wire and the parallax is then called *near parallax.* It may be removed by increasing the distance between the image and the eye.

11.8. PERMANENT ADJUSTMENTS OF A THEODOLITE

The permanent adjustments of a transit theodolite are :

1. Adjustment of the horizontal plate level.
2. Adjustment of the horizontal axis (or trunnion axis).
3. Adjustment of the telescope.
4. Adjustment of the telescope level.
5. Adjustment of the vertical circle index.

1. Adjustment of the horizontal plate level. With this adjustment the axis of the plate levels, is made perpendicular to the vertical axis of the theodolite.

Object. When the plate levels are in perfect adjustment, their bubbles must remain at the centre of their run during a complete revolution of the theodolite in azimuth.

Necessity. For accurate measurement of horizontal and vertical angles, the vertical axis should remain truly vertical.

Test. The following steps are involved.

1. Set up the theodolite on a firm ground. Clamp the lower plate and turn the upper plate until the plate level becomes parallel to any pair of foot screws. Bring the bubble to the centre of its run by means of foot screws as explained in Para 11.7.
2. Rotate the instrument about the vertical axis through 180°. The plate level is now again parallel to the same pair of foot screws but with the ends reversed in direction. If the bubble remains central, the vertical axis of the theodolite is perpendicular to the axis of the plate level (Fig. 11.11).

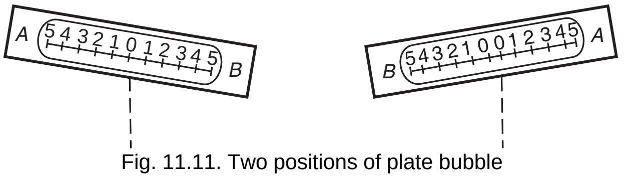


Fig. 11.11. Two positions of plate bubble

Adjustment. 1. If the bubble does not remain central, note down the reading of the bubble. It is the apparent error and is twice the actual error of the axis of the plate bubble.

2. Bring the bubble to the mean position with the help of the same pair of foot-screws.
3. Remaining half of the correction is corrected by means of the Capstan headed screw provided at the end of the level tube.
4. Repeat the test and adjustment until the bubble remains central at its run during a full rotation of the instrument in azimuth.

2. Adjustment of the horizontal axis. With this adjustment the horizontal axis (or trunnion axis) is made perpendicular to the vertical axis.

Object. The object of this adjustment is to ensure that the line of collimation revolves in a vertical plane, perpendicular to the horizontal axis.

Necessity. This adjustment is very necessary for prolonging straight lines, by making observations on one face only.

This adjustment is made by the *spire test* (Fig. 11.12).

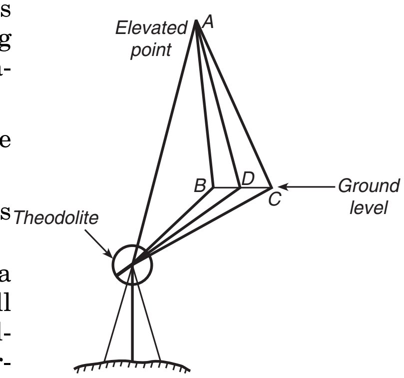


Fig. 11.12. Spire test.

Spire Test. The following steps are involved :

1. Set up the theodolite near a tall building on which a well defined point say *A*, is available at about 60° to 70° vertical angle.
2. Level the instrument carefully so that its vertical axis becomes truly vertical.
3. Sight *A*. Keeping both the horizontal plates clamped, depress the telescope and fix a point *B* on the ground.
4. Transit the telescope and swing the theodolite through 180° about its vertical axis until positions of the two supports are reversed.
5. Bisect *A* again and depress the telescope as in step 3. above. Note if the line of collination passes through *B*. If so, the horizontal axis is perpendicular to the vertical axis.

Adjustment. Proceed as under :

1. If the line of collination does not pass through *B*, fix a point *C*′ in the line of sight close to *B* on right or left as the case may be.
2. Locate *D* at the mid-point of *B* and *C*. The point *D* will lie in the same vertical plane which contains the elevated point *A* on the building.
3. Sight *D* and elevate the telescope until *A* appears in the field of view.
4. Loosen the screw of the bearing cap of the horizontal axis. Raise or lower the end of the horizontal axis until line of collination passes through *A*.

Principle of the adjustment. It may be seen that the conditions of the telescope and the horizontal axis are the same for both observations except that the positions of the supports have been reversed for the second observation. Hence, the existing discrepancy is due to difference in elevation of the supports of the trunnion axis.

3. Adjustment of the telescope. This adjustment includes :

(*i*) Adjustment of the horizontal hair.
(*ii*) Adjustment of the vertical hair.

1. Adjustment of the horizontal hair.

Object. The object of this adjustment is to bring the horizontal hair of the eye piece into the horizontal plane through the optical axis.

Necessity. The direction of the line of sight also changes when the objective of the instrument is moved in and out for focussing. If the horizontal hair does not lie in the horizontal plane through the optical axis, vertical angles will be in error. It is particularly required when the instrument is used for levelling operations. It has no effect in the measurement of horizontal angles. (Fig. 11.13).

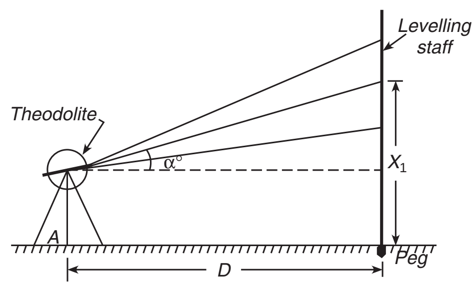


Fig. 11.13. Adjustment of horizontal hair

Test : Following steps are involved.

1. Set up the instrument at a convenient station *A* and level it carefully.
2. Hold a levelling staff vertically on a peg *B* at a distance *D* about 100 m. Read the staff and the vertical angle for the central hair. Let the staff reading and vertical angle be x_1 and α° respectively on face left.
3. Change the face of the theodolite to right. Set the vernier of vertical circle to the former reading α° and read the levelling staff again. If this staff reading is also x_1, the instrument is in perfect adjustment.

Adjustment. If the reading differs, let it be x_2.

1. Move the horizontal hair by means of the vertical diaphragm screws until mean reading on the levelling staff, is sighted
2. Repeat the adjustment, if necessary.

2. Adjustment of the vertical hair

Object. The object of this adjustment is to make the line of collimation perpendicular to the horizontal axis (or trunnion axis)

Necessity. If this adjustment exists, the line of collimation generates a plane when the telescope is revolved in vertical plane. In case it does not exist it generates a cone about the horizontal axis. This adjustment is necessary for measuring horizontal angles between points at different elevations and also for prolonging the lines by making observations on one face only. (Fig. 11.14)

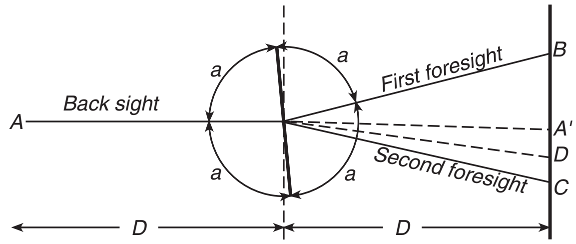


Fig. 11.14. Adjustment of vertical hair

Test. Following steps are involved.

1. Level the instrument carefully and sight *A* about 100 metres away, with the telescope normal *i.e.* keeping vertical circle on the face left. The point so chosen should be at about the same elevation as the instrument axis.
2. Clamp both the plates. Transit the telescope and fix a point *B* on the line of sight at the same distance on the opposite side of the instrument as *A*.
3. Unclamp the upper plate, swing the telescope and again sight *A*. The telescope now is inverted.
4. Clamp the upper plate and transit the telescope. If the line of sight passes through *B* previously fixed, the line of sight is perpendicular to the horizontal axis and no adjustment is therefore needed.

Adjustment. If the line of sight does not pass through *B*, the adjustment may be done as follows :

1. Fix a point *C* very close to *B* along the line of sight.
2. Mark a point *D* at one fourth the distance of *CB* from *C*.
3. Adjust the vertical hair by means of two opposite horizontal screws of the diaphragm so that the line of sight passes through *D*.
4. Repeat the adjustment, if needed.

Principle of the adjustment. The principle of the adjustment is based on the double application of the principle of reversal. Transiting the telescope once doubles the error. Transiting it second time after changing the face doubles the error on the opposite side. Hence, the total apparent error, is four times the true error. (Fig. 11.15)

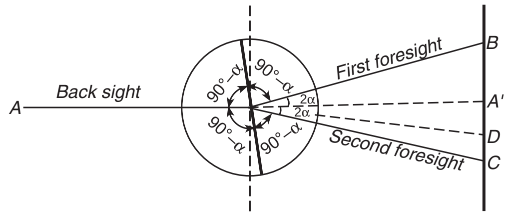


Fig. 11.15. Principle of the adjustment of vertical hair.

4. Adjustment of the telescope

Object. The object of this adjustment is that the line of collimation should remain horizontal when the bubble of the level tube fitted on telescope is brought at the centre of its run. (Fig. 11.16)

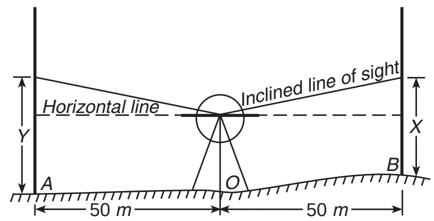


Fig. 11.16. Adjustment of the level of the telescope.

Necessity. This adjustment is essential when a theodolite is used as a *level* and also when vertical angles are observed.

Test. Following steps are involved.

1. Fix two pegs A and B on a fairly levelled ground about 100 m apart.
2. Set up the theodolite at O, exactly midway between A and B.
3. Clamp the vertical circle and bring the bubble central by means of vertical tangent screw. Let the readings be y and x on a levelling staff held at A and B respectively.
4. Calculate the true difference in level between the points A and B. Let it be X_1.
5. Shift the instrument to O, on the prolongation of line BA, about 25 m from A (Fig. 11.17.)
6. With the bubble central, read the levelling staff first on A and then on B. Let the readings be y_1 and x_1 respectively. Find the

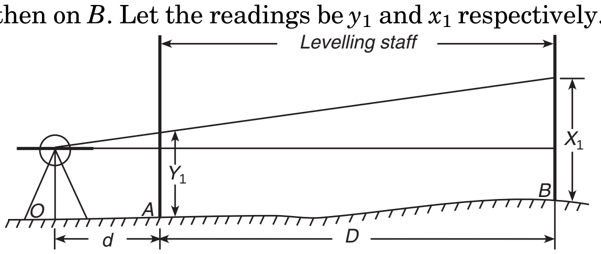


Fig. 11.17. Adjustment of the level of the telescope.

difference in elevation again. Let it be X_2. If this difference in level equals the first difference, the instrument is in adjustment.

Adjustment. 1. If the difference in level is not the same *i.e.* $X_1 \neq X_2$, calculate the true difference from first observations and also ascertain whether the difference is a rise or a fall from peg A to peg B. In first setting, the errors due to refraction and curvature are balanced because the theodolite was kept equidistant from A and B.

2. Calculate the correct staff reading for B with the following formula :

$$\text{Reading on } B = \text{Reading on } A \pm \text{ true difference} \begin{Bmatrix} + \text{ sign for fall} \\ - \text{ sign for rise} \end{Bmatrix}$$

i.e. $x_2 = y_1 \pm$ true difference in elevation.

3. Calculate the collimation error for a distance D, *i.e.* $x_1 - x_2$

4. Calculate the correction to the staff readings as under :

Correction to the reading on A.

$$C_A = \frac{d}{D}(x_1 - x_2).$$

Correction to the reading on B

$$C_B = \frac{(d + D)}{D}(x_1 - d_2).$$

5. These corrections are additive or subtractive if the line of collimation is inclined *downwards or upwards*. The inclination of the line of collimation can be ascertained as under :

If $x_1 > x_2$, the line of collimation is inclined *upwards* and if, $x_1 < x_2$, it is inclined *downwards.*

11.9. ORDER OF CARRYING OUT THE PERMANENT ADJUSTMENTS OF A THEODOLITE.

It is evident that while carrying out some adjustment, other adjustments get disturbed. To avoid this possibility, it is recommended that permanent adjustments are carried out in the following order :

1. Make the vertical cross hair to lie in a plane perpendicular to the horizontal axis.
2. Make the plate bubbles central to their run when the vertical axis is truly vertical.
3. Make the line of collimation perpendicular to the horizontal axis.
4. Make the horizontal axis perpendicular to the vertical axis.
5. Make the telescope level bubble central when the line of sight is horizontal.
6. Make the vertical circle to indicate zero when the line of sight is perpendicular to the vertical axis.

11.10. USES OF THEODOLITES.

Theodolites are commonly used for the following operations :

(*i*) Measurement of horizontal angles.
(*ii*) Measurement of vertical angles.
(*iii*) Measurement of magnetic bearing of a line.
(*iv*) Measurement of direct angles.
(*v*) Measurement of deflection angles.
(*vi*) Prolongation of straight a line.
(*vii*) Running a straight line between two points.
(*viii*) Laying off an angle by repetition method.

1. Measurement of horizontal angles

1. Direct method of measuring the angle :

Procedure. To measure a horizontal angle *ABC* between *BA* and *BC*, the following procedure is followed :

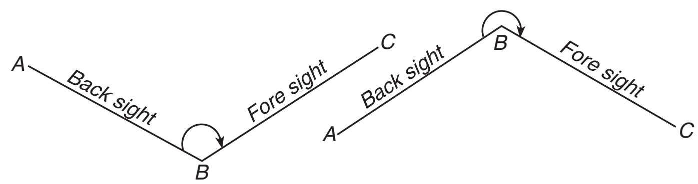


Fig. 11.18. Measurement of horizontal angles.

(*i*) Set up, centre and level the theodolite over the ground point *B*. (Fig. 11.18)
(*ii*) Loosen the upper plate, set the vernier to read zero and clamp the upper plate.
(*iii*) Loosen the lower plate and swing the telescope until the left point *A* is sighted. Tighten the lower clamp. Accurate bisection of the arrow held on station *A* is done by using the lower tangent screw. Read both the verniers and take the mean of the readings.
(*iv*) Unclamp the upper plate and swing the telescope in clockwise direction until point *C* is brought in the field of view. Tighten the upper clamp and bisect the arrow on station *C* accurately, using the upper tangent screw.
(*v*) Read both the verniers and take the mean of the readings. The difference of the means of the readings to stations *C* and *A*, is the required angle *ABC*.
(*vi*) Change the face of the instrument and repeat the whole procedure. The measure of the angle is again obtained by taking the difference of the means of the readings to *C* and *A* on face right.
(*vii*) The mean of two measures of the angle *ABC* on two faces, is the required value of the angle *ABC*.

Note. The following points may be noted :

(*i*) To eliminate the error due to imperfect adjustment, observations on both the faces should be made for precise work.

(*ii*) To eliminate the error of eccentricity of the graduated circle and verniers, both verniers should be read.

2. To measure the angle by method of repetition (Fig. 11.19)

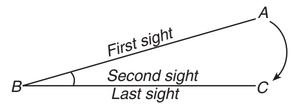


Fig. 11.19. Repition method.

Let *ABC* be the required angle between sides *BA* and *BC*, to be measured by the repetition method. When the measure of an angle is small, slight error in its sine value introduces a considerable error in the computed sides as the sine value of the angle changes rapidly. Therefore, for accurate and precise work, the method of repetition is generally used. In this method, *the value of the angle is added several times mechanically and the accurate value of the angular measure is determined by dividing the accumulated reading by the number of repetitions.*

Procedure. To measure a small horizontal angel *ABC*, proceed as follows:

(*i*) Keeping the face of the instrument left, centre and level it accurately over the ground point *B*.

(*ii*) Set the vernier to read zero. Loosen the lower plate and swing the telescope in azimuth to sight the left hand point *A*. Using lower tangent screw, bisect the point *A* accurately.

(*iii*) Read both the verniers and take the mean of two readings.

(*iv*) Loosen the upper plate and swing the telescope in clockwise direction until point *C* is brought in the field of view. Using the upper tangent screw, bisect the mark *C* accurately.

(*v*) Read both the verniers and take the mean of the readings. The difference of the mean readings of points *C* and *A*, gives the approximate value of the angle. Let it be $3° \, 45' \, 20''$.

(*vi*) Unclamp the lower plate and turn the telescope in clockwise direction until point *A* is again sighted. Clamp it and bisect the station mark accurately with the help of the lower tangent screw, ensuring that the vernier readings do not change.

(*vii*) Loosen the upper plate and swing the telescope in clockwise direction and again bisect *C* exactly by using upper tangent screw. The verniers will now read double the value of the angle *ABC*.

(*viii*) Repeat the process until the angle *ABC* is repeated by the required number of times, say 5.

(*ix*) Read both the verniers. The final reading after 5 repetitions should be approximately equal to 5 times the approximate value of the angle. In this case, say $5 \times (3° \, 45' \, 20'') +$ index error if any. Take the mean of the two vernier readings.

Table 11.1. Specimen field book for recording observations by the method of repetition (Fig. 11.19)

Observation at 'A' Theodolite no. 5286

Instru-ment at	*Sighted to*	*Face : Left*												*Swing : Right*	*Face : Right*														*Swing : Left*
		A			*B*		*Mean*			*No. of Rep.*	*Horizontal Angle*			*A*			*B*		*Mean*			*No. of Rep*	*Horizontal Angle*			*Average Horizontal Angle*		
		°	′	″	′	″	°	′	″		°	′	″	°	′	″	′	″	°	′	″		°	′	″	°	′	″
B	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	C	20	21	20	21	20	20	21	20	1	20	21	20	20	21	40	21	40	20	21	40	1	20	21	40	20	21	40
	C	101	47	20	47	20	101	47	20	5	20	21	28	101	46	40	46	20	101	46	30	5	101	46	30	20	21	18

Date of observation : 1-10-1979 Signature of observer (*x*) The accumulated angle is obtained by taking the difference of the two mean readings to stations *C* and *A*.

(*xi*) Divide the accumulated angle by the number of repetitions (in this case 5) to get the correct value of the angle *ABC*.

(*xii*) Change the face of the instrument and repeat the whole procedure to get the value of the angle *ABC* on face right.

(*xiii*) The mean of the two values of the angle obtained on face left and on face right, gives the required value of the angle *ABC*.

Specimen field book for recording the observations by the method of repetition, is shown in Table 11.1. on page 525.

Note. The following points may be noted :

(*i*) The angles measured by the method of repetition are free from errors caused due to :

(*a*) *Eccentricity of the centres.* This is eliminated by reading both verniers and taking the mean reading.

(*b*) *The imperfect adjustment of the line of collimation and the trunnion axis.* This error is eliminated by making observations on both the faces and taking the mean value.

(*c*) *The pointing of the telescope.* This is eliminated by compensating the error in each pointing and the resulting error is further minimised by dividing it by the number of repetitions.

(*d*) *The graduations of the horizontal circle.* This error may be eliminated by measuring the angle on different parts of the circle, *i.e.*, on different zeros and the mean value of the angle accepted.

(*ii*) By the method of repetition, angles may be measured to a finer degree of accuracy than that obtainable with the least count of the vernier.

(*iii*) Measurement of the angles by the method of repetitions is generally made for the substance bar* measurement and also for the extension of triangulation base line. It may be appreciated that the subtense bar and base line measurements generally subtend very small angels (varying from 3Ž° to 8°). Slight error in the angle throws the horizontal distances computed from the known base considerably.

3. To measure horizontal angles by reiteration method. When several angles having a common vertex, are to be measured, the *reiteration method* is generally adopted. In this method angles are measured successively, starting from a reference station and finally closing on the same station. The operation of making last observation on the starting station, is known as *closing horizon.* Making observations on the starting station twice provides a check on the sum of all angles around a station. The sum should invariably be equal to 360°, provided the instrument is not disturbed during observations. As the angles are measured by sighting the stations in turn, this method is sometimes known as *direction method* of observation of the horizontal angles.

Procedure. Let the instrument station be *O* whereas *A, B, C, D* and *E* are the stations sighted for measuring the angles *AOB, BOC, COD, DOE* and *EOA*. To measure the angles by reiteration method, the following steps are involved. (Fig. 11.20)

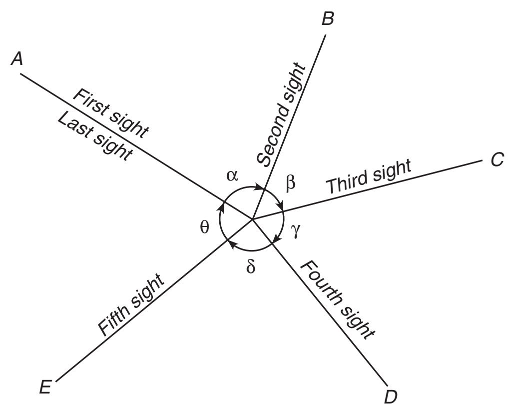


Fig. 11.20. Reiteration method.

(*i*) Centre the theodolite accurately over the ground station mark *O* and level it.

(*ii*) Bisect a well defined distant station (say *A*) using the lower clamp and make the vernier to read zero degree, few minutes and few seconds. Clamp the upper and lower plates. Read both the verniers and take the mean of the readings.

(*iii*) Unclamp the upper plate, swing the theodolite clockwise and bisect *B* accurately, using the upper tangent screw.

(*iv*) Read both the verniers and take the mean of the readings.

(*v*) Similarly, bisect stations *C, D, E,* etc. successively and finally the starting station *A*. In each case, read both the verniers and take the mean of the readings.

(*vi*) Calculate the included angles by taking the differences between two consecutive readings, *i.e.*, mean reading of *B* minus mean reading of *A*; mean reading of *C* minus mean reading of *B*, etc.

(*vii*) Transit the telescope, swing the instrument in an anticlockwise direction and make observations on the face right to get the measure of each angle.

(*viii*) The mean of two measures of each angle, is accepted as the correct value of the angle.

Specimen filed book for recording the observations by reiteration method, is shown in Table 11.2 on page 528.

Table 11.2. Specimen field book for recording the observation by the reiteration method (Fig. 11.20)

Observation at '0'

Theodolite no. 5265

Station sighted	*Face*	*Horizontal Angles*												
		Vernier A			*Vernier B*		*Mean*		*General mean*			*Angles*		
A	L R	0° 180	05′ 05	20″ 40	05 05	40″ 20	05′ 05	30″ 30	0°	05′	30″			
B	L R	50 230	55 55	00 40	55 55	20 20	55 55	10 30	50	55	20	50	49	50
C	L R	120 300	34 34	20 00	34 34	40 20	34 34	30 10	120	34	20	69	39	00
D	L R	210 30	24 24	40 20	24 24	40 00	24 24	40 10	210	24	25	89	50	05
E	L R	276 96	14 14	00 20	14 14	20 00	14 14	10 10	276	14	10	65	49	45
F	L R	0 180	05 05	20 40	05 05	40 20	05 05	30 30	0	05	30	83	51	20
					Check : Total							360	00	00

Note. The following points may be noted.

(*i*) Time should never be spent to set the verniers to read exactly zero degree, zero minute and zero second.

(*ii*) On face left, observations should be made in a clockwise direction and on face right, these are made in an anti-clockwise direction.

(*iii*) While swinging the telescope from station to station the graduated scale remains in a fixed position and only verniers move through-out the entire process.

(*iv*) On the close of the horizon, vernier readings should be the same as that of the original setting or within a permissible limit. If the difference is large, the complete set of observations should be rejected and a fresh set of readings taken.

(*v*) To eliminate the error due to inaccurate graduations, observations of the angles must be made on different parts of the circle.

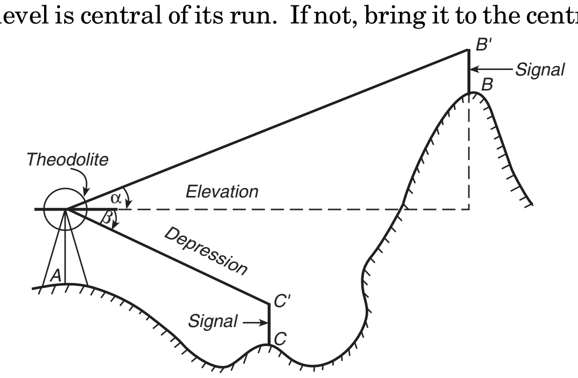
(*vi*) Method of reiteration is commonly used in triangulation surveys.*

2. Measurement of vertical angles A vertical angle may be defined as the angle subtended by the inclined line of sight and the horizontal line of sight at the station in vertical plane. If the point sighted is above the horizontal axis of the theodolite, the vertical angle is known as an *angle of elevation* and if it is below, it is known as an *angle of depression.* (Fig. 11.21).

Procedure : To measure a vertical angle subtended by the station *B* at the instrument station *A*, the following steps are involved :

(*i*) Set up the theodolite over the ground station mark *A*. Level it accurately by using the altitude bubble

(*ii*) Set the zero of the vertical vernier exactly in coincidence with the zero of the vertical scale using vertical clamp and vertical tangent screw. Check up whether the bubble of the altitude level is central of its run. If not, bring it to the centre of its run



Fi.g 11.21. Measurement of vertical angles.

*Refer Chapter 13, Tacheometry Surveying.

by means of the clip *screw*. In this position, the line of collimation of the telescope is horizontal and the verniers read zero.

(*iii*) Loosen the vertical circle clamp and move the telescope in vertical plane until the station *B* is brought in field of view. Use vertical circle tangent screw for accurate bisection.

(*iv*) Read both the verniers of the vertical circle. The mean of two vernier readings gives the value of the vertical angle.

(*v*) Change the face of the instrument and make the observations exactly in similar way as on the face left.

(*vi*) The average of two values of the vertical angle is the required value of the vertical angle.

Measurement of vertical angle between two stations at different elevations (Fig. 11.22 *a, b, c*).

The vertical angle subtended by two points lying in vertical plane containing the trunnion axis of the theodolite, may be made as under:

(*i*) Measure the vertical angle of the higher station than the other as explained earlier. Let it be α.

(*ii*) Measure the vertical angle of the lower station. Let it be β.

(*iii*) The required vertical angle between stations may be calculated by finding the algebraic difference between two readings, assuming angles of elevation as positive and angles of depression as negative.

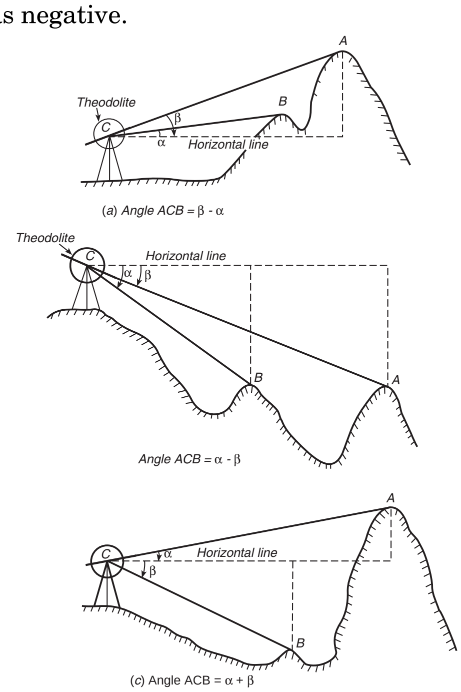


Fig. 11.22 (a, b) Measurement of vertical angles.

Fig. 11.22 (*c*). Measurement of vertical angles.

3. **Measurement of magnetic bearing of a line**. To measure the magnetic bearing of a line *AB*, the theodolite should be provided with either a circular or a trough compass. The following steps are involved:

1. Centre and level the instrument accurately on station *A*.
2. Set the vernier to read zero.
3. Loosen the lower plate and also release the magnetic needle.
4. Swing the telescope about its vertical axis until the magnetic needle points *N-S* graduations of the compass box scale.
5. Clamp the lower plate. Uthe lower tangent screw bring the needle exactly against the zero graduation in exact coincidence with the north end of the needle.
6. In this position, the line of collimation of the telescope lies in the magnetic meridian at the place while vernier still reads a zero. *The setting of the instrument is now said to be oriented on the magnetic meridian.*
7. Loosen the upper plate, swing the instrument and bisect *B* accurately, using the upper tangent screw.
8. Read both the vernier. The mean of two readings is the required magnetic bearing of the line *AB*.
9. Change the face of the instrument and observe the magnetic bearing exactly in a similar way as on the left face.
10. The mean of magnetic bearings observed on both faces, is the accurate value of the magnetic bearing of line *AB*.

Note. The following points may be noted :

(*i*) Before observing the magnetic bearings, it must be ensured that area is free from local attraction.

(*ii*) Theodolite must be set away from power lines, telegraph lines, etc. Iron articles should not be kept near the instrument to avoid attraction of the needle.

4. Measurement of direct angles. The angle measured clockwise from the preceding line to the following line is called a *direct angle*.

These angles are also some times known as *azimuths* from the back line, or *angles to the right* and may vary from 0° to 360°. (Fig. 11.23)

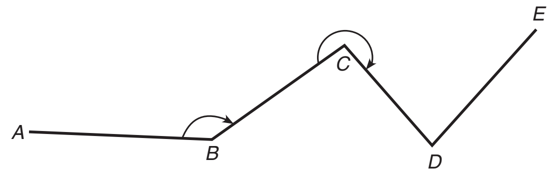


Fig. 11.23. Measurement of direct angles.

Procedure. To measure a direct angle say *BCD,* proceed as under:

(*i*) Set up the theodolite over station *C*, centre and level it carefully.

2. Keeping the face of the instrument left, set the vernier to read zero degree by turning the upper plate.
3. Unclamp lower clamp and bisect the station *B* accurately using the lower tangent screw. Clamp the lower plate.
4. Unclamp the upper plate swing the telescope clockwise and bisect the forward station *D*. Read both the verniers.
5. Plunge the telescope, unclamp the lower clamp and bisect the preceding station *B* without disturbing the vernier readings.
6. Unclamp the upper plate, swing the telescope clockwise and bisect the station *D* accurately. Read both the verniers again.
7. Take the mean of the final vernier readings. Now, the angle is doubled and hence the average value gives the value of the direct angle *BCD*.

Note. The following points may be noted :

(*i*) Measurement of direct angles is generally made on both faces to eliminate instrumental errors.

(*ii*) Two measures of the angle may be obtained by finding the difference between second reading and the first and the other value from the difference between the final reading and the second reading.

(*iii*) The average of the two measures gives the correct value of the direct angle.

(*iv*) Direct angles are generally used in a theodolite traverse.

5. Measurement of deflection angles. The angle which any survey line makes with the prolongation of the preceding line, is called *deflection angle*. Its value may vary from 0° to 180° and is designated as right deflection angle if it is measured in clockwise direction and as left deflection angle if it is measured in an anticlockwise direction. In Fig. 11.24 the deflection angles α and δ at stations *B* and *E* respectively are left deflection angles whereas angles β and γ at stations *C* and *D*, are right deflection angles.

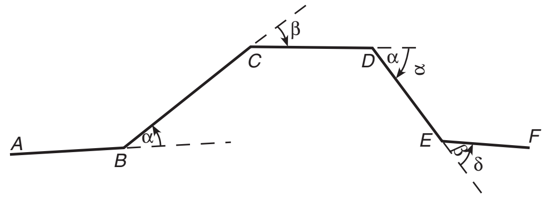


Fig. 11.24. Deflection angles

Procedure. To measure the deflection angle α at station *B*, proceed as under :

1. Set up, centre and level the instrument over station *B* carefully.
2. Make the verniers to read zero. Take a back sight station *A* and clamp both the plates.
3. Transit the telescope so that the line of sight is in the direction of *AB* produced. The verniers should still read zero.
4. Unclamp the upper plate and swing the telescope in an anticlockwise direction to sight station *C*. Read both the verniers.
5. Unclamp the lower plate and swing the telescope to sight *A* again. Ensure that readings of verniers have not changed. Clamp the lower plate and transit the telescope.
6. Loosen the upper clamp and swing the telescope to sight *C* in an anticlockwise direction. Bisect the mark at *C* accurately using the upper tangent screw. Read both the verniers.
7. Deflection angle has been doubled by making observations on both faces. Half of the final reading is the required deflection angle at *B*.

Note. The following points may be noted:

(*i*) If the reading of the vernier after plunging the telescope and sighting the forward station is less than 180°, the deflection angle is left and if more thatn 180° it is right.

(*ii*) Deflection angles are used in theodolite transverses carried out in a linear direction, *i.e.,* traverses for the alignments of highways, railways, canals etc.

6. Prolongation of a straight line. Prolongation of any straight line *AB* to a point *F* may be done by any one of the following methods :

First Method (Fig. 11.25)

Fig. 11.25. Prolongation of a line.

The followiing steps are involved :

1. Set up the theodolite at *A*, centre and level it accurately.
2. Bisect an arrow centred over the mark at *B*.

3. Establish a point C in the line of sight at a convenient distance.
4. Shift the instrument to B.
5. Centre the theodolite over B, level it and sight C. Establish another point D.
6. Proceed in a similar manner until the desired point F is established.

Second Method (Fig. 11.26)

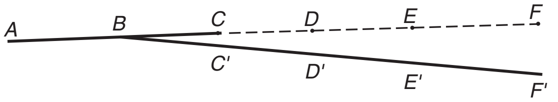


Fig. 11.26. Prolongation of a line.

The following steps are involved :

1. Set up the theodolite at B and centre it carefully.
2. Bisect A accurately and clamp both the plates.
3. Plunge the telescope and establish a point C in the line of sight.
4. Shift the instrument to C and centre it carefully.
5. Bisect B and clamp both the plates.
6. Plunge the telescope and establish a point D in the line of sight.
7. Continue the process till the last point F is established.

Note: The following points may be noted.

(*i*) If the instrument is in perfect adjustment, the points B, C, D, E and F will lie in a straight line.

(*ii*) If the line of collimation is not perpendicular to the horizontal axis, the established point C', D', E' and F' would lie on a curve.

Third Method (Fig. 11.27).

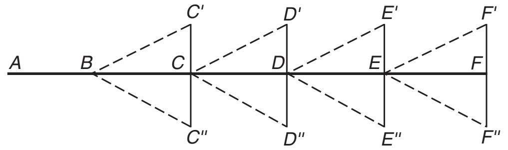


Fig. 11.27. Prolongation of a line.

Following steps are involved.

1. Set up the theodolite at B and centre at carefully.
2. Bisect A on face left and clamp both the plates.
3. Plunge the telescope and establish a point C'.
4. Change the face and bisect A again.
5. Plunge the telescope and establish a point C'' at the same distance as C' from B.
6. If the instrument is in adjustment, the point C' and C'' will coincide.
7. If not, establish a point C midway between C' and C''.

8. Shift the instrument to C and repeat the process to establish a point D.
9. Repeat the process until the required point F is established.

Note. The following points may be noted :

(*i*) This method of prolongation of a line requires two sightings and as such it is known as *double sighting method.*

(*ii*) This method is used only when greater precision is required with a poorly adjusted instrument.

7. Running a straight line between two stations. The following three cases may arise :

1. Both stations intervisible.
2. Both stations not intervisible, but visible from an intermediate station.
3. Both stations not visible from any intermediate station.

First Case. Both stations intervisible (Fig. 11.28)

Fig. 11.28. Ranging a line between two points.

Procedure : Following steps are involved.

1. Set up the instrument at either end A or B.
2. Bisect the other point B or A as the case may be.
3. Establish a number of stations, *C, D, E, F* in the line of sight to define the straight line AB.

Second Case. Both stations not intervisible, but visible from an intermediate station (Fig. 11.29)

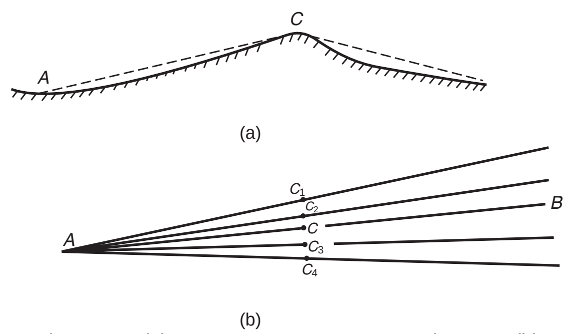


Fig. 11.29. (a) Fig.11.29 (b)

The following steps are involved :

1. Set up the instrument at an intermediate point C as nearly in line with AB as possible by eye judgement.
2. Bisect A and clamp both the plates.
3. Plunge the telescope and sight B.
4. The line of sight does not pass exactly through B.

5. Shift the instrument laterally by estimation.
6. Repeat the process until, on plunging the telescope, the line of sight passes through B.
7. Suspend a plumb bob from the hook and fix a peg C.
8. Check the location of the point C by double sighting.
9. Establish other intermediate points between AB and BC as explained in first case.

Note. The following points may be noted :

(*i*) This method of establishing intermediate points on a straight line between two stations, is sometimes known as *balancing in*.

(*ii*) This case is encountered when two points are on either side of an intervening raised ground.

Third case. Both stations not visible from any intermediate station. (Fig. 11.30)

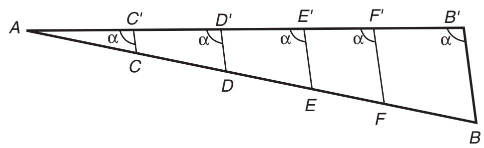


Fig. 11.30. A random line method.

The following steps are involved:

Run down a random line AB' in the expected direction.

1. Set up the instrument at A and centre it carefully.
2. Bisect a point C' in the approximate direction of B.
3. Establish a point C' in the line of sight.
4. Shift the instrument to C' and repeat the process to establish the stations E', F', etc.
5. Finally establish a point B' from where given point B is visible. It should be as near to B as possible.
6. Centre the theodolite over the point B'. Measure the included angle $AB'B$ between $B'A$ and $B'B$. Also, measure horizontal distances AB' and BB'.
7. To locate the station C, D, E and F, proceed as under :

(*i*) Set up the instrument C' and set the angle $AC'C$ equal to angle $AB'B$ so that the line of sight is along $C'C$.

(*ii*) Calculate $C'C = \dfrac{AC'}{AB'} \times BB'$ and measure it along $C'C$ to fix C.

(*iii*) Similarly establish the stations *D, E, F,* etc., on AB.

The following points may be noted.

Note. (*i*) This method is not suitable when station on the random line are also not visible due to intervening undulations.

(*ii*) It is assumed that there is no obstruction to chaining along random line.

(*iii*) For higher precision work, location of the points on the random line must be checked by double sighting method.

(*iv*) Stations *C, D, E* and *F* must be established by making observations on the same face.

8. Laying of an angle by repetition method. The method of repetition is generally used to lay off angles whenever greater precision is required. (Fig. 11.31).

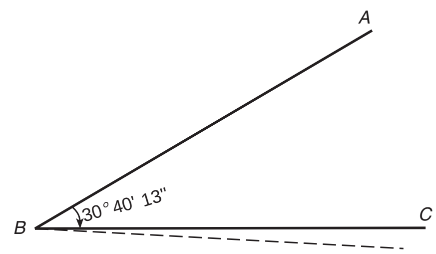


Fig. 11.31. Laying off an angle by repetition method.

Let *BA* be a fixed line and it is required to lay off *BC* making an angle *ABC* equal to 30° 40′ 13″ with a theodolite having a 20″ least count. Proceed as follows :

1. Centre the instrument over *B* and level it carefully.
2. Sight *A* accurately and make the vernier to read zero.
3. Unclamp the upper plate and swing the telescope till reading is approximately equal to the required angle. Use the upper tangent screw to set the angle exactly equal to 30° 40′ 20″.
4. Fix *C*′ in the line of sight at the required distance away from *B*.
5. Measure the angle *ABC*′ by the method of repetition. Let the number of repetition be 5 and the accumulated angle *ABC*′ be153° 22′ 20″. The average value of the angle $ABC = \frac{1}{5}(153° \, 22' \, 20'') = 30° \, 40' \, 28''$.
6. Calculate the required correction to be applied to the angle *ABC*, *i.e*. 30° 40′ 28″ — 30° 40′ 13″ = 15″.
7. Since the correction is very small, it may be applied linearly by making offset *C*′*C* = *BC*′ tan *CBC*′.
8. Measure the length of *BC*′. Let it be 250 m.
9. Calculate the distance *C*′*C* = 250 tan 15″ = 0.018 m assuming tan 1′
   = 0.0003 nearly.
10. Establish *C* by setting a perpendicular *C*′*C* to *BC*′. Measure

$C'C$ equal to 0.018 m.

11. Check the accuracy of the angle ABC by measuring its value by the method of repetition. Apply further correction, if found necessary.

11.11. ACCURACY REQUIRED IN MEASURED ANGLES (FIG. 11.32)

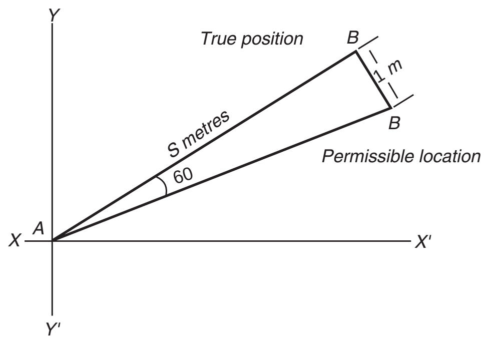


Fig. 11.32.

The accuracy of the angles must be in consistance with the accuracy of the linear measurements. If the required accuracy of linear measurements on a given survey is of the order 1 : S and if the co-ordinates of another point n metres away, are plotted, the point should be located within 1 m of its true location in any direction. Hence, the angle subtended by 1 metre of arc at a distance of S metres must be the maximum permissible closing error per single angle measured for any length of the sight distance.

The angular error $\delta\theta = \frac{1}{S} \times \frac{180}{\pi} \times 3600 = \frac{206265}{S}$ seconds

When there are n angles measured in a traverse, the allowable angular closure should be $\frac{206265}{S}\sqrt{n}$.

Example 11.1. *Calculate the maximum permissible error per angle measured for any length of sight distances, if the accuracy of linear measurements is 1 : 10,000. When the number of traverse angles is 25, find the maximum angular closure.*

Solution. The angular error $= \frac{1}{10,000} \times \frac{180}{\pi} \times 3600 = 20''.6$ **Ans.**

The angular closure $= 20.6\sqrt{25} = 103'' = 1'43''$ **Ans.**

11.12. CARE OF A TRANSIT

As transits, like other surveying instruments, are made with precision, these must be handled with great care. The following precautions and suggestions for their maintenance, must be clearly understood by the transitmen.

1. Before removing the transit from its case, one should observe how it is secured in its case, so that it can be correctly replaced in its case, after use.
2. Ensure that the transit is properly fastened to the tripod without over-tightening.
3. While carrying a transit in a building through doors, and beneath low-hanging trees, hold the transit waist high in horizontal position with the tripod trailing the head. To avoid shock due to impact with any solid object, the clamp screws should be loosened.
4. Do not leave the transit on a tripod unattended.
5. Do not overstress the clamp and levelling screws.
6. Keep a water proof hood available, in case of sudden rain.
7. Always use the sunshade. In case, it is to be removed, or replaced, do so with a clockwise rotary motion to avoid loosening of the objective lens.
8. Clean and lubricate lightly the threads of levelling, clamp and tangent screws at regular intervals.
9. Clean the vertical circle and the vernier, with a chamois or very soft cloth. Do not touch the graduations with the fingers.
10. When placing the transit in its case, replace the dust cap over the objective. Centre the head on the foot plate, and equalise the foot screws.
11. If the case of the transit does not close easily, find out what is wrong. Never use force to close the case.

11.13. PRECAUTIONS TO BE TAKEN IN THEODOLITE OBSERVATIONS

The following precautions must be taken by the observer.

1. Turn the theodolite by the standards and not by the telescope, ensuring even and slow movement.
2. Make a few revolutions of the theodolite in azimuth and that of telescope in altitude before making observations.
3. Keep the theodolite clean. Lenses should be dusted with a brush.
4. Lift the theodolite by its standards.
5. Do not force the foot screws and tangent screws too hard.
6. Ensure that movement of the observer does not affect the levelling of the instrument.
7. Bisect the signal when it is properly fixed.
8. Clamp the vertical axis tightly while observing the horizontal angles.
9. Ensure that the telescope does not overshoot the signal mark. In case it does, rotate the theodolite round till the mark comes again. The final intersection of the mark should be made with

the tangent screw and the last motion of the screw should be against the spring on swing right and in the direction of spring on swing left.

10. Do not force the instrument in its carrying case to avoid damage to its parts.

11.14. SOURCES OF ERROR IN THEODOLITE WORK

The sources of error in theodolite work may be broadly divided into three categories, *i.e.*

1. Instrument errors.
2. Personal errors.
3. Natural errors.

I. **Instrumental Errors.** The theodolites are very delicate and sophisticated surveying instruments. In spite of best efforts during manufacturing, perfect adjustment of fundamental axes of the theodolite, is not possible. The unadjusted errors of the instrument, are called *residual errors.* We shall now discuss how best to avoid the effect of these residual errors while making field observations.

Instrumental errors may also be divided into different types as discussed below :

1. **Error due to imperfect adjustment of plate level.** If the plate bubbles are not adjusted properly, the vertical axis of the instrument does not remain vertical even if plate bubbles remain at the centre of their run. Non-verticality of the vertical axis introduces errors in the measurements of both the horizontal and vertical angles. Due to non-verticality of vertical axis, the horizontal plate gets inclined and it does not remaining in horizontal plane. The error is specially important while measuring the horizontal angles between stations at considerable different elevations.

Elimination of the error. This error can be eliminated only by levelling the instrument carefully, with the help of the altitude or telescope bubble, before starting the observations.

2. **Error due to line of collimation not being perpendicular to the trunnion axis.** If the line of collimation of the telescope is not truly perpendicular to the trunnion axis, it generates a cone when it is rotated about the horizontal axis. The trace of the intersection of the conical surface with the vertical plane containing the station sighted, is hyperbolic. This imperfect adjustment introduces errors in horizontal angles measured between stations at different elevations.

Derivation of the formula. Let M and N be two stations between which horizontal angle MAN is measured, M be the higher station. Let α_m be the vertical angle MAM_1 where M_1 and N are in the plane of the instrument axis.

Bisect the station M on left face and depress the telescope. In cuts the horizontal line NM_1 at M_2 right of M_1, assuming the cross hair intersection to be a little left of the optical axis. Apparently the measured horizontal angle M_2AN is too small than the true horizontal angle M_1AN. Mathematically.

$$e = \beta \sec \alpha_m$$

where e is the error introduced.

β is the error of collimation.

Now, change the face of the theodolite. The cross hair intersection now lies right of the optical axis.

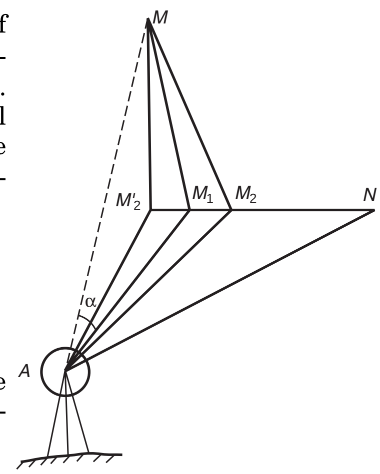


Fig. 11.33.

If the telescope on face right is depressed after bisecting M, it cuts the horizontal line at M_2', left of M_1. On the face right the theodolite measures the angle M_2'AN which is too large than true angle.

In case station N also does not lie in the horizontal plane of the theodolite, a similar error $e' = \beta \sec \alpha_n$ is also introduced in the direction of inclined sight AN, when α_n is the vertical angle of station N.

Total error $E = \beta (\sec \alpha_m - \operatorname{Sec} \alpha_n)$ ...(11.1)

Elimination of the error. This error may be eliminated from the measured angle by taking the average of the two values of the horizontal angle observed on both the faces.

Note. The following points may be noted :

(*i*) The error reduces to zero if $\alpha_m = \alpha_n$ or α_m and α_n are each equal to zero.

(*ii*) Due to non-perpendicularity of line of collimation, vertical angles are very little affected and hence the error may be ignored in ordinary observations.

3. Error due to horizontal axis not being perpendicular to the vertical axis. If the horizontal axis is not perpendicular to the vertical axis, the line of collimation does not revolve in vertical plane, when the telescope is raised or lowered. Due to this imperfect adjustment, the error is introduced in both horizontal and vertical angles. The magnitude of the error depends upon :

(*i*) The angle between the horizontal axis and the vertical axis.

(*ii*) The vertical angle of the station sighted.

(*iii*) Elevations of the stations sighted. It is considerable if the stations sighted are at different elevations.

Derivation of the formula

Let A be the station of observation.

M and N be the stations sighted where M is higher.

α_1 be the observed vertical angle of station M.

β be the angle between the horizontal axis and vertical axis.

In Fig. 11.34 let M_1 be the point vertically below M.

AM_2 is the direction of AM as recorded by the graduated circle.

The angular error in horizontal angle M_1AN is apparently M_1AM_2.

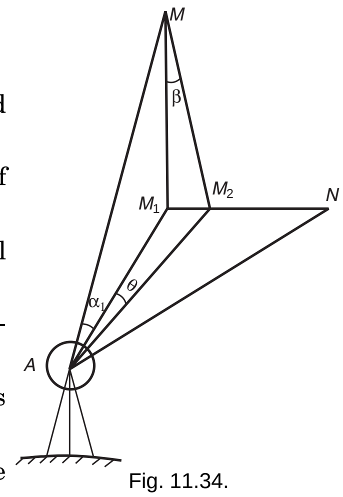


Fig. 11.34.

Let it be denoted by e.

From right angled triangle M_2M_1A, we get

$$\tan e = \frac{M_1M_2}{AM_1} \quad ...(11.2)$$

From right angled triangle MM_1M_2, we get

$$M_1M_2 = MM_1 \tan \beta \quad ...(11.3)$$

From right angled triangle MAM_1, we get

$$MM_1 = AM_1 \tan \alpha_1 \quad ...(11.4)$$

From eqns. (11.2), (11.3) and (11.4) we get

$$\tan e = \frac{AM_1 \tan\alpha_1 \tan\beta}{AM_1} = \tan \alpha_1 \tan \beta$$

or

$$e = \beta \tan \alpha_1 \quad ...(11.5)$$

where e and β are very small.

Assuming vertical angle for station N as α_2, the angular error introduced in direction

$$e' = \beta \tan \alpha_2 \quad ...(11.6)$$

The net error introduced in horizontal angle MAN

$$E = \beta(\tan \alpha_1 - \tan \alpha_2) \quad ...(11.7)$$

using proper sign.

Elimination of the error. As the average of the two values of the horizontal angle observed on both faces is equal to the correct value of the angle, the observations must be made on both the faces.

Note. The following points may be noted :

(*i*) The error becomes zero if the stations between which horizontal angle is measured, are at the same elevation.

(*ii*) The error due to non-perpendicularity of horizontal axis to the vertical axis is more serious as compared to non-perpendicularity of line of collimation.

4. Error due to non-parallelism of the axis of the telescope level and line of collimation. If the axis of the telescope level is not parallel to the line of collimation, an error is introduced in the vertical angle, because zero line of the vertical verniers does not represent true line of reference.

Elimination of the error. The error can be eliminated by taking the mean of the two observed values of the angle, one with the telescope normal and the other with telescope inverted.

5.. Error due to eccentricity of inner and outer vertical axes. If the centre of the graduated circle plate does not coincide with the centre of vernier plate, the angle recorded by either vernier is incorrect.

Deviation of the formula (Fig. 11.35.)

Let C represent the centre of the graduated circle.

C' represent the centre of the vernier plate.

M be the position of either vernier when back sight is taken.

B be the position when theodolite is swung through angle $MC'B$.

Here the arc MB measures the angle $MC'B$ and not MCB.

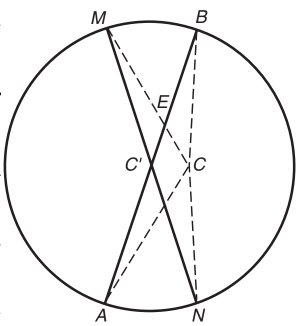


Fig. 11.35.

Mathematically, we get,

$$\text{Angle } MC'B = \text{Angle } MEB - \text{Angle } C'MC$$

$$\text{Angle } MEB = \text{Angle } MCB + \text{Angle } C'BC$$

$$\therefore \quad \text{Angle } MC'B = \text{Angle } MCB + \text{Angle } C'BC - \text{Angle } C'MC \quad ...(i)$$

$$\text{Angle } NC'A = \text{Angle } NCA + \text{Angle } C'NC - \text{Angle } C'AC \quad ...(ii)$$

But $\angle C'MC = \angle C'NC$; $\angle C'BC = \angle C'AC$; $\angle MC'B = \angle NC'A$.

Adding eqns. (*i*) and (*ii*) we get

$$2\angle MC'B = \angle MCB + \angle NCA$$

or $$\angle MC'B = \frac{\angle MCB + \angle NCA}{2}$$

i.e. the true angle is the mean of the values observed by the verniers.

Elimination of the error. To eliminate the error due to this source, observe both the verniers and take the mean value.

6. Error due to eccentricity of verniers. If the line joining the zeros of the herizontal plate verniers does not pass through the centre of vernier plate, an error in the measured horizontal angle is introduced. The eccentricity of the verniers may be easily ascertained by reading both the verniers on different parts of the graduated circle.

Elimination of the error. The error may be eliminated by taking the mean of two values of the angle by reading both verniers.

7.. Error due to unequal graduations. If the graduation of the lower plate are unequal, the observed angle on different portion of the graduated circle, will be apparently different.

Elimination of the error. The error in the observed horizontal angle, may be minimised by measuring the angle on different zeros and taking the mean of all the values of the angle.

2. Personal errors. In this category, following errors are included.

(*i*) Errors of manipulation (*ii*) Errors of sighting and reading.

1. Errors of manipulation. The errors included in this sub-group, are as explained below.

***(a)* In-accurate Centering.** If the centre of the theodolite does not coincide with the ground station mark, the horizontal angles measured at the station are affected with an error, known as *Centreing error.*

The magnitude of the error depends upon the distance between the theodolite centre and ground station mark, the direction and distance of the station sighted.

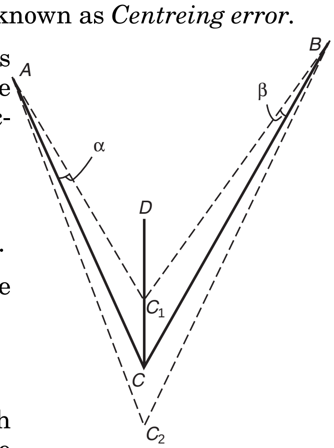


Fig. 11.36.

Derivation of the formula.

Let C be the position of ground Station.

C_1 be the position of theodolite centre

α be the angle CAC_1

β be the angle CBC_1

Apparently, the angle measured with the theodolite is AC_1B and not true angle ACB. Now

$$\angle ACB = \angle ACD + \angle BCD$$

$$= \angle AC_1D - \alpha + \angle BC_1D - \beta$$
$$= \angle AC_1D + \angle BC_1D - (\alpha + \beta)$$
$$= \angle AC_1B - (\alpha + \beta)$$

The value of α will be maximum when C_1C is perpendicular to AC. Similarly β will be maximum when C_1C will be perpendicular to CB. *i.e.*

$$\text{Maximum value of } \alpha = \tan^{-1} \frac{CC_1}{AC}$$

$$\text{Maximum value of } \beta = \tan^{-1} \frac{CC_1}{BC}$$

When the centre of the theodolite lies at C_2, it can similarly be proved that $\angle ACB = AC_2B + (\alpha + \beta)$.

Note. The following points may be noted :

(*i*) It may be noted that the error due to inaccurate centering can not be eliminated unless accurate centering is done.

(*ii*) The error due to defective centering varies inversely as the length of the sights.

(*iii*) The error of one centimetre centering in a length of 34.4 m, introduces an error of one minute arc in the measured angle.

(*b*) **Error due to inaccurate levelling.** Inaccurate levelling introduces a serious error in the horizontal angles when the stations sighted are at considerable height difference. This error is similar to the error due to non-adjustment of the plate levels. If the stations sighted are at the same level, the error is small.

Elimination of the error. Accurate levelling should be made with the help of altitude bubble or telescope bubble which is generally more sensitive.

(*c*) **Error due to manipulation of wrong tangent screw.** An in-experienced surveyor generally commits mistakes of using wrong tangent screws. The observer should bear in mind that manipulation of the upper tangent screw changes the graduated circle reading whereas manipulation of lower tangent screw swings the theodolite without changing the readings.

Elimination of the error. Greatest care must be taken in using the proper tangent screws. The lower tangent screw should be used for taking back sights and upper tangent screw is used for fore sights.

Note. The following points may be noted :

(*i*) The mistake of manipulation of the upper tangent screw for back sights, can be easily deducted by checking the vernier reading before taking a fore sight.

(*ii*) The mistake of manipulation of the lower tangent screw for a fore sight can not be deducted.

2. Errors due to sighting and reading

These errors may arise due to the following sources :

(*a*) **In-accurate bisection of signals.** If the signal erected at the station sighted is not visible clearly, due to vegetative cover or intervening ground, the observer may bisect the signal wrongly. In-accurate bisection of the station mark introduces an error whose magnitude varies inversely with the length of sights. This error is similar to that caused due to inaccurate centering.

Elimination of the error. It may be eliminated by sighting the signal clearly and always at its lowest portion.

(*b*) **Non-verticality of signals.** If the signal is not truly vertical, an error is introduced. This error is inversely proportional to the length of sight *i.e.*

$$\tan \alpha = \frac{\text{error of verticality}}{\text{length of sight}}$$

Elimination ofthe error. The error due to non-verticality and signal, may be eliminated by erecting the signal truly vertical and also bisecting its lowest portion.

(*c*) **Error due to parallax.** If the objective and eye piece of the theodolite are not properly focussed before bisecting the station mark, this error is introduced.

Elimination of the error. The error may be eliminated by properly focussing the eyepiece and objective before bisecting the station mark.

(*3*) **Errors due to natural causes.** In this category the errors included are due to higher temperature, strong wind, scorching sun and unequal settlement of the tripod.

EXERCISE 11

1. State whether following statements are True or False. If false, rewrite the corrected statements :

(*i*) The telescope of a non-transit theodolite can be rotated through 360° in vertical plane.

(*ii*) An external focussing telescope is better than an internal focussing telescope and it is generally provided in modern theodolite.

(*iii*) Glass arc theodolites are provided with improvised verniers.

(*iv*) The size of a theodolite is determined by the length of its telescope.

(*v*) The lower plate which carries a horizontal graduated circle, is some times known as 'Scale Plate'.

(*vi*) The upper plate of the theodolite is provided with two verniers.

(*vii*) After clamping the upper clamp and unclamping the lower plate, the theodolite may be rotated on its outer spindle without any relative motion between the two plates.

(*viii*) The graduations of the vertical circles of all theodolite are done in a similar manner as on horizontal plate.

(*ix*) The vertical axis of a theodolite is the axis about which horizontal plates rotate.

(*x*) The clip screw of a theodolite changes the pointing of the telescope as well as reading of the vertical circle.

(*xi*) The line joining the optical centres of the eye-piece and objective lens, is called the line of sight of surveying telescope.

(*xii*) The cross hair in a surveying telescope, are placed much closer to the eye piece than to objective lens.

(*xiii*) When the upper clamp of a theodolite is tight and its lower plate clamp is loose, turning the theodolite telescope in azimuth alters the horizontal circle readings.

(*xiv*) The operation of turning the telescope in azimuth through 180° is known as 'transiting' the theodolite.

(*xv*) Observations with telescope normal, are sometimes known as 'face left observations'.

(*xvi*) Spire test is used for the adjustment of the line of sight of a theodolite.

(*xvii*) The errors due to imperfect adjustment of a theodolite are eliminated by making observations on both faces.

(*xviii*) To eliminate the errors due to eccentricity of the graduated circle and verniers, both verniers must be read.

(*xix*) Measurement of horizontal angles by the method of reiteration is done in 'traverse surveying.

(*xx*) The angle which any survey line makes with the prolongation of the preceding line, is known as 'direct angle' or azimuth from the back line.

(*xxi*) In an adjusted theodolite, the axis of the telescope should be perpendicular to the line of collimation.

(*xxii*) Stadia markings are made on the eye piece.

(*xxiii*) The upper plate of a theodolite is to be clamped if it is required that there should be no change in readings on moving the telescope in azimuth.

(*xxiv*) The vertical axis, horizontal axis and the line of sight of a transit must meet in a point.

(*xv*) When the line of sight is perpendicular to the horizontal axis, the vertical circle reads zero.

2. Fill up the blanks by using suitable words written in the brackets.

(*i*) There......be index error in the vertical circle. (cannot, can)

(*ii*) Face left and face right observations eliminate......error. (eccentricity, graduation, index)

(*iii*) With the method of repetition, horizontal angles may be measured to....accuracy than the least count of the vernier. (finer, lesser)

(*iv*) Vernier is a device for measuring fractional part of the smallest division on...... (horizontal plate, vertical circle, both)

(*v*) Objective of the telescope is always a ......lens. (compound, convex, convave)

(*vi*) The axis of a telescope and the line of collimation of the telescope are...... (one and the same, different)

(*vii*) The difference of vertical angles measured on both faces, divided by 2 is ......error of the vertical circle vernier. (Index, collimation)

(*viii*) The inclination of the vertical axis of a theodolite through an angle α in the direction of sighting, introduces an error in measured vertical angle equal to...... $\left(\alpha, 2\alpha, \frac{1}{2}\alpha\right)$

(*ix*) If the horizontal axis of a theodolite is inclined due to non-verticality of its vertical axis, the reversal of the telescope......errors in horizontal angles. (does not eliminate, eliminates)

(*x*) If the angle of inclination of the horizontal axis of the theodolite is α, then the error in the horizontal angle whose angle of elevation is β, is......where α is in radians. (α, tan β, α sec β. α sin β)

3. Explain how you will use a theodolite as a level.

4. Describe with a neat sketch how the trunnion axis of a theodolite can be set at right angles to the vertical axis.

5. How will you do the temporary adjustments of a theodolite.

6. Describe temporary adjustments of a theodolite. How will you measure the horizontal angle by it.

7. Describe temporary adjustments of a theodolite. How can these be tested.

8. (*a*) Mention the permanents of a transit theodolite. Explain the object of these adjustments.

(*b*) Describe with the aid of neat sketches, how you would set the plate level at right angles to the vertical axis.

9. (*a*) Name the fundamental axes of a theodolite. State the relationship that must exist between them when the instrument is in adjustment.

(*b*) Describe with the aid of neat sketches.

(*i*) Collimation adjustment.

(*ii*) Horizontal axis adjustment

10. Describe how you would test and, if necessary, adjust the line of collimation of a vernier theodolite.

11. (*a*) What is the difference between temporary and permanent adjustments of a transit theodolite ? Name these.

(*b*) What is spire test ? Describe the test in detail and also the method of adjustment.

12. What are the temporary adjustments of a transit theodolite ? Describe in details you would measure a horizontal angle to finer than 20 seconds if you are provided with a vernier theodolite with a least count of 20 seconds.

13. What is meant by face left and face right of a theodolite ? How would you change face ? What instrumental errors are eliminated by face right and face left observations.

14. Give a list of temporary and permanent adjustments of a transit theodolite.

15. (*a*) State the various permanent adjustments of a transit theodolite in the order in which they are carried out.

(*b*) Explain the object of each of the above adjustments.

(*c*) Describe the procedure of carrying out the adjustment of the horizontal axis perpendicular to the vertical axis.

16. (*a*) Describe with the aid of a sketch, the function of an internal focussing lens in a surveying telescope, and state the advantages of internal focussing as compared to external focussing.

(*b*) State the procedure step by step, how you would carry out the adjustment of collimation perpendicular to the horizontal axis in case of a theodolite.

17. A horizontal angle was measured by repetition with a transit 8 times. The initial reading, the reading of horizontal circle after first and final measurement of the angle, are as shown below.

Initial	*first*	*final*
0° 00′00	30° 25′20	243° 22′10″

Find the angle measured.

18. The location survey for a railway line was run. If the prescribed accuracy of linear measurement is 1 : 3000 and there are 20 angles measured between the starting and closing traverse sides, determine :

(*a*) the allowable angular closure

(*b*) maximum permissible correction per angle.

19. While making the layout survey of a large stadium, a 90° angle was set with a single measurement with an optical transit. The preliminary angle was then measured by repetition and found to be 89° 59′ 44″. What offset (nearest mm) should be made at a distance of 200 m from the instrument in order to establish the true line.

20. Observations were made at the centre of the top of Qutab Minar to check its verticality. With the transit telescope normal, top centre was sighted, the telescope plunged about the horizontal axis, and a point was set on the ground 40 cm to the left of the point on the base of the minar. The same procedure was repeated with the telescope in the inverted position and a point was set 70 cm to the right. How much is the top of Qutab Minar out of plumb and in which direction whether left or right ?

ANSWERS

1.(*i*) false	(*ii*) false	(*iii*) false	(*iv*) false
(*v*) true	(*vi*) true	(*vi*) true	(*viii*) false
(*ix*) true	(*x*) true	(*vi*) false	(*xii*) true
(*xiii*) false	(*xiv*) false	(*vi*) true	(*xvi*) true
(*xvii*) true	(*xviii*) true	(*vi*) false	(*xx*) false
(*xxi*) false	(*xxii*) false	(*vi*) true	(*xxiv*) true
(*xxv*) true.			

2. (*i*) Can

(*v*) compound

(*ix*) does not eliminate

17. 30′ 25′ 16″

18. (*a*) 5′ 8″

(*b*) 1′ 8′

19. 15.5 mm

20. 15 cm right.

12

Theodolite Traversing

12.1. INTRODUCTION

A traverse may be defined as the course taken when measuring a connected series of straight lines, each line joining two points on the ground. These points are called *traverse stations*. The straight line between two consecutive traverse stations, is called *a traverse leg*. The angle at any station between two consecutive traverse legs, is known as *traverse angle*.

The traversing in which traverse legs are measured by direct chaining on the ground and the traverse angle at every traverse station is measured with a theodolite, is known as *theodolite traversing*.

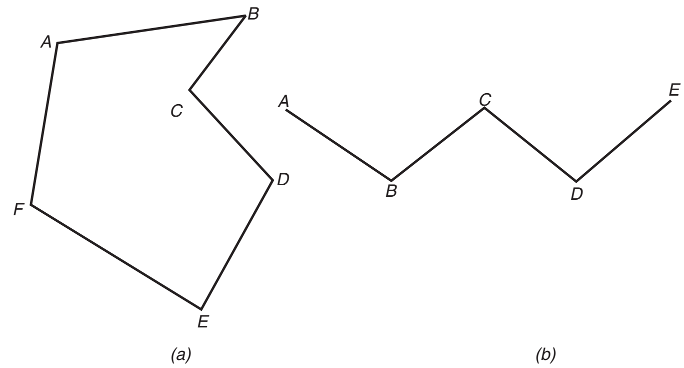
According to the nature of the starting or closing stations, the traverses are classified as under :

(*i*) Closed traverse (*ii*) Open traverse.

1. Closed traverse. A traverse which either emanates from a station and closes on the same station, or runs between two stations whose co-ordinates are known in terms of a common system of co-ordinates, is known as *closed traverse*. A traverse which closes on the starting station is also, some times known as *traverse circuit*. In a closed traverse accuracy of linear as well as angular measurements may be ascertained.

2. Open traverse. A traverse which neither returns to its starting station nor ends on another known station, is known as an *Open traverse* or *'unclosed traverse'*. In open traverses, accuracy of linear as well as angular measurements may not be checked.

In Fig. 12.1 (*a*) the traverse *ABCDEFA* which originates and closes on station *A*, is a traverse circuit and is known as a *closed traverse*. In Fig. 12.1 (*b*) the traverse *ABCDE* which originates from *A* and ends at station *E*, is known as *Open traverse*. In this case, neither the co-ordinates of *A* nor the co-ordinates of *E* are known. Thus the accuracy of the linear and angular measurements of an open traverse cannot be checked. For large scale survey where better accuracy is aimed at, open traverses should never be carried out.



(*a*) Closed traverse Fig. 12.1. (*b*) Open traverse

12.2. PURPOSES OF A THEODOLITE TRAVERSE

As the principle of surveying is to work from the whole to the part, precision control points are fixed by triangulation at distances 5 km to 10 Km apart. Ground control points at closer distances are usually required for plane tabling and chain surveying. The theodolite traverse is, therefore, carried out for the following purposes :

1. To provide control points for chain surveying, plane tabling and photo-grammetric surveys, in flat country.
2. To fix the alignment of roads, canals, rivers, boundaries, etc. when better accuracy is required as compared to plane tabling.
3. To ascertain the co-ordinates of boundary pillars in numerical terms that can be preserved for future reference such as cantonment boundary pillars, forest boundary pillars, international boundary pillars, etc. In case the pillars get disturbed, their positions can be relaid with the help of their co-ordinates.

12.3. GENERAL PRINCIPLE OF THEODOLITE SURVEY

According to the accuracy aimed at and the nature of the ground, the lengths of traverse legs are measured directly on the ground either by chaining or taping. The traverse angles, the angles between consecutive traverse legs are measured with a theodolite by setting up the instrument at each station in turn. If the co-ordinates of one station and the true bearing of the traverse leg connected to it, are known, the coordinates of the other traverse station, may be calculated with the following formulae :

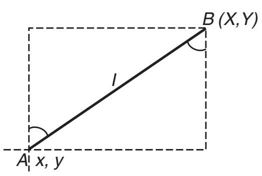


Fig. 12.2.

1. X co-ordinate of $B = X$ coordinate of $A + l \sin \theta$
2. Y co-ordinate of $B = Y$ coordinate of $A + l \cos \theta$

where l is the length and θ is the reduced bearing of the traverse leg AB.

12.4. METHODS OF THEODOLITE TRAVERSING

On the basis of measuring relative directions of traverse legs, the methods of theodolite traversing, may be divided into two groups :

1. By measuring the direct angles between two consecutive traverse legs.
2. By measuring the direct bearings of the traverse legs.

The first method is generally adopted for long traverses when higher degree of accuracy is required. In this method, bearings of the starting and closing traverse legs are generally determined by making astronomical observations to either pole star or sun.

The second method is used when it is not possible or desirable to make astronomical observations for obtaining true bearings of the starting and closing traverse legs and also when higher accuracy is not required. This method can be conveniently adopted for small projects where a few traverse legs might cover the required area of the project.

1. Theodolite traversing by direct measurement of the angles. In this method angles between relative directions of traverse legs, are directly measured with a theodolite. The bearing of initial line being either observed from astronomical observations, or assumed arbitrarily, the bearings of remaining traverse legs may be easily calculated from the bearing of the initial traverse line and the measured traverse angles. Direct angles measured at different traverse stations may be either included angles or deflection angles. For measuring the direct angles, the following two methods are adopted :

A. Theodolite traversing by observing included angles

In this method, bearings of the initial traverse leg and other traverse legs at frequent intervals as well as that of the last traverse leg, are generally observed from astronomical observations. The included (or direct) angles are those angles which are measured on the left side of the direction of the traverse. In closed traverses included angles may either be exterior angles or interior angles. It is customary to run a closed traverse in an anti-clockwise direction in which only interior angles are measured. In closed circuits, the accuracy of angular measurements is easily checked by summing up all the included angles as their sum total should be equal to ($2n \pm 4$) right angles, where n is the number of traverse legs, the positive sign is used for exterior angles and negative sign for interior angles.

In Fig. 12.3, the included angles of a linear traverse are measured in clockwise direction. The traverse does not close on the starting station.

In Fig. 12.4, the interior angles of a closed circuit are measured in anticlock wise. It may be noted that the direction of the traverse, is anti-clockwise as shown by the arrows.

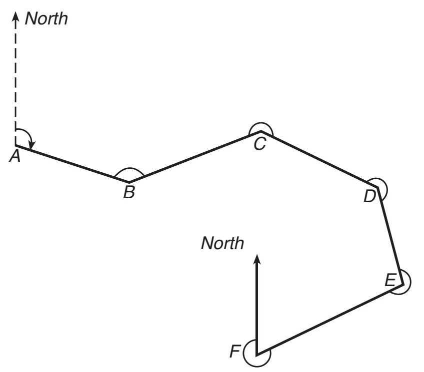


Fig. 12.3. A traverse in a clockwise direction.

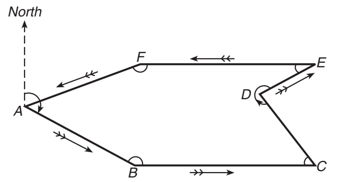


Fig. 12.4. A closed traverse with interior angles.

In Fig. 12.5, exterior angles of a closed circuit are measured. It may be noted that the direction of the traverse, is clockwise as shown by the arrows.

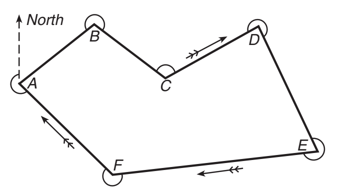


Fig. 12.5. A closed traverse exterior angles.

Procedure. Let us assume that a closed traverse *A B C D E F* is run between two pre-determined stations *A* and *F*. (Fig. 12.3), or two closed circuits *ABCDEF A* (Fig. 12.4, 12.5) are run starting from and closing on the same station *A*. In all the three cases, the bearing of the initial traverse line, *i.e.* the angle θ between the north direction and traverse leg *AB* is required to be determined. In first case (Fig. 12.3) the bearing of the last traverse leg *EF* must also be predetermined to have a check on the run-down bearing of the traverse leg *EF*. For measuring the included angles of a traverse, the following steps are followed:

(*i*) Set up a theodolite on the face left at station *B*. Centre it over the ground station mark and level it accurately with levellling screws.

(*ii*) Sight the telescope towards station *A*, clamp the upper and lower plates and sight the signal at *A* accurately, using either upper or lower tangent screw.

(*iii*) Read both the verniers and calculate the mean value.

(*iv*) Unclamp the upper plate, swing the theodolite clockwise and sight the signal at *C*. Accurate bisection of the signal, is made by using *upper tangent screw only*.

(*v*) Read both the verniers and find the average reading. The difference of the mean readings of the forward station *C* and back station *A* is the included angle at *B*. To get better accuracy, observations on both the faces are made and the average value of the angle is accepted for computation.

(*vi*) Shift the theodolite to the next traverse stations in turn and repeat the steps (*i*) to (*v*) at every traverse station in the same sequential order, for measuring the included angles at *C, D, E,* etc.

(*vii*) Measure the length of each traverse leg with a chain or a tape directly on the ground, depending upon the accuracy of the traverse aimed at.

B. Theodolite traversing by observing deflection angles

A deflection angle may be defined as the angle between the prolongation of the preceding leg and forward leg. The deflection angle will either be right hand deflection or left hand deflection angle, according to the layout of the forward leg with respect to the preceding leg.

This method is used for open traverses and is most suitable when the traverse legs make small deflection angles with each other. Hence, the method can very conveniently be employed while running traverses along straight alignments of roads, railway lines, canals, transmission lines, etc.

Procedure. Suppose a traverse is run from station *A* to station *G* connecting station *B, C, D, E* and *F*. (Fig. 12.6).

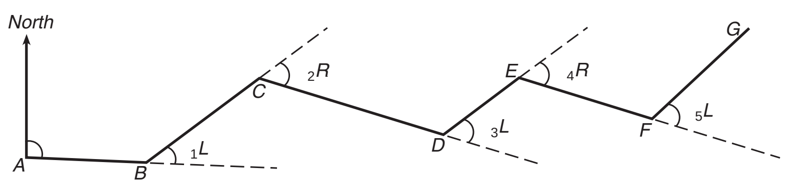


Fig. 12.6. Layout of a traverse with deflection angles.

Following steps are involved for measuring the deflection angles.

(*i*) Observe the bearing of the initial traverse leg *AB* by making astronomical observations or assuming its value arbitrarily.

(*ii*) Set up the theodolite at station *B*, centre it over the ground mark and level it carefully.

(*iii*) Set the vernier *A* to read zero unclamp the lower clamp.

(*iv*) Take a back sight on the station *A* and bisect the signal accurately with *lower* tangent screw.

(*v*) Transit the telescope. The reading of vernier remains the same as there is no movement of the upper plate.

(*vi*) Unclamp the upper plate and swing the theodolite until station *C* is sighted. For accurate bisection of the signal, use *upper* tangent screw.

(*vii*) Read both the verniers. The mean of the two vernier readings gives the deflection angle of the traverse leg *BC* with respect to traverse leg *AB*.

(*viii*) Note the direction of deflection angle in the field book whether the deflection is right or left depending upon the movement of the theodolite whether it is clockwise or anti-clockwise direction with respect to the prolongation of the preceding traverse leg. Similarly, the theodolite is set up at other stations *C, D, E,* etc. in succession and deflection angle at each station is observed. It may be noted that the error in recording the direction of the deflection angles cannot be detected after the instrument is shifted from the station of observation. Hence, utmost care need be taken to note down the direction of the deflection angle correctly in the field book before shifting the instrument to next station.

2. Theodolite traversing by direct observation of bearings of traverse legs

In this method, the bearing of each traverse leg is observed directly by a theodolite in the field and no calculations are necessary for deducing the same as in direct angle method. The method of observing direct bearings of traverse legs, is suitable only for short traverses when better accuracy is not required. In other words, this method can be conveniently used for open traverses in which no checks are possible and closing error cannot be ascertained.

Direct bearings of traverse legs can be observed in the field by the following three methods :

(*i*) Direct method in which telescope is transited.

(*ii*) Direct method in which telescope is not transited.

(*iii*) Back bearing method.

I. Direct method in which telescope is transited

This method of observation of direct bearings of traverse lines is only suitable when the theodolite is fitted with a transit telescope and having its line of collimation in perfect adjustment.

Suppose *A B C D E F* is the required route of an open traverse. (Fig. 12.7).

Procedure : The following steps are taken :

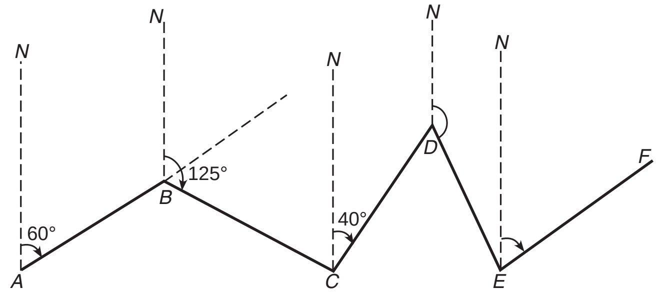


Fig. 12.7. Direct bearings with a transit theodolite.

(*i*) Set up the theodolite over the starting station *A*. Centre it over the ground station mark and level it carefully. Make the vernier *A* to read zero degree, zero minute and zero second.

(*ii*) Bring the telescope in line of magnetic meridian whose direction is determined with a magnetic compass attachable with a theodolite by using the lower clamp and lower tangent screw. In this position the line of collimation, the axis of the telescope and the magnetic meridian of the station of observation, all coincide each other.

(*iii*) Unclamp the upper plate, swing the theodolite right and bisect the mark at station *B* accurately by upper tangent screw.

(*iv*) Read the vernier *A* which gives the direct magnetic bearing of the traverse line *AB*. Let its value be 60° as shown in Fig. 12.7.

(*v*) Shift the theodolite to station *B*. After centering and levelling the theodolite over the ground mark *B*, ensure that the reading of the vernier *A* has not changed while shifting the instrument from station *A* to *B*. If, due to slip the reading has changed, set the original reading, *i.e.* 60° using the upper tangent screw.

(*vi*) Make a back sight to the station *A* and bisect the mark with lower clamp and lower tangent screw.

(*vii*) Transit the telescope so that the line of sight is directed towards *AB* produced and the vernier *A* still records the magnetic bearing of *AB*. Release the upper plate, swing the telescope to sight the forward station *C* and bisect the ground mark using upper tangent screw.

(*viii*) The reading of vernier *A* will give direct magnetic bearing of *BC*. Let its value be 125°.

(*ix*) With the vernier *A* clamped at 125°, shift the theodolite to station *C* and repeat the process to get the magnetic bearing of traverse leg *CD*.

(*x*) In a similar manner, magnetic bearings of remaining traverse legs may be observed. It may be noted that the error, if any, in the magnetic bearing of the starting line exists, it affects all other bearings with the same amount of error.

2. Direct method in which telescope is not transited

This method is employed when the telescope of the theodolite cannot be transited and the line of collimation of the telescope, is not in perfect adjustment.

Procedure. Let us assume that the traverse *A B C D E F* is to be run, with a non-transit theodolite. (Fig. 12.7).

Following steps are involved :

(*i*) Set up the theodolite and centre it over the ground mark of station *A*. Set the vernier *A* to read zero, using upper clamp and upper tangent screw.

(*ii*) Release the lower plate, bring the telescope along the magnetic meridian as already explained in the previous method. Levelling of the instrument should not be ignored.

(*iii*) Release the upper plate, swing the theodolite in right direction and bisect the mark at the station *B* accurately by using upper tangent screw.

(*iv*) Note down the reading of the vernier *A*, which is the direct magnetic bearing of the traverse line *AB*.

(*v*) Shift the instrument to station *B*. After careful centering and levelling direct the telescope to sight the back station *A*, ensuring that vernier *A* still reads the magnetic bearing of traverse line *AB*. In case the vernier *A* is disturbed, set it to read the original reading with the upper tangent screw.

(*vi*) Swing the telescope in clockwise direction to sight the signal at station *C* by releasing the upper plate. In this case, the reading of vernier *A* does not give the direct magnetic bearing of *BC* but it differs by $180°$. *To get the correct value of the magnetic bearing of BC, add* $180°$ *if the reading is less than 180° and subtract* $180°$ *if the reading is more than 180°*.

(*vii*) Shift the instrument to station *C* and centre it over the ground mark *C* by releasing the lower plate and sight the telescope to station *B*. Bisect the signal at station *B* accurately by lower tangent screw, ensuring that the reading of vernier *A* is not changed.

(*viii*) Swing the telescope clockwise and sight the signal at station *D*. In this case the vernier *A* will read $180°$ more than what it read in earlier position at station *B*. It means a complete revolution of the theodolite and thus the vernier *A* reads direct magnetic bearing of *CD*.

Illustration. **Let us suppose that magnetic bearing of the first, second and third traverse legs are** $60°, 125°$ and $40°$ **respectively.**

After setting the telescope in the magnetic meridian, when it sights *B*, the vernier *A* reads $60°$.

When the instrument is at B and telescope sights A with vernier reading 60°, the theodolite is swung, the vernier moves from the reading of 60° to 305° in the following stages :

$$60° + 120° + 125° = 305°$$

where 60° is the original reading of the vernier, 120° is the angle between the line BA and the magnetic north at B and 125° is the magnetic bearing of BC.

To get correct bearing of BC, subtract 180° from 305° *i.e*.

$$305° - 180° = 125°$$

The instrument is now set up at station C. The telescope sights B and vernier A reads 305°. When the theodolite is rotated about its vertical axis to sight D, the vernier A moves from 305° to 40° in the following stages.

$$305° + 55° + 40° = 400°$$

where 305° is the original reading of the vernier A, 55° is the angle between CB and the magnetic north at C and 40° is the magnetic bearing at CD. As the sum is more than 360°, the vernier will read only (400° − 360°) = 40° which is a desired value. *It may be noted that the correction of* 180° *is only applied to readings at even station, i.e. 2nd, 4th, 6th etc.*

Though correction of 180° may be avoided, by reading alternate verniers at alternate stations, it is always preferred to read the same vernier at every station and correction applied to the stations where needed, to avoid risk of confusion.

3. Back bearing method. This method is based on the principle that by setting the back bearing of preceding line, the telescope is brought in correct orientation at every station. Without transiting the telescope and by simply giving a rotation, the direct magnetic bearing of the next line is obtained on the same vernier which was made to read zero at the first station after setting the telescope along the magnetic meridian. (Fig. 12.8).

To bring the telescope along the magnetic meridian at B, the theodolite is only required to be rotated through 360° − θ, where θ is the back bearing of the preceding line. With an angular rotation of 360° − θ, the vernier again reads zero. Further rotation in clockwise direction to sight the forward station will be equal to direct magnetic bearing of the forward line. It may be noted that back bearing of each succeeding line is calculated and vernier A is set to read the calculated value of the bearing accurately at every forward station.

Procedure. Following steps are involved :

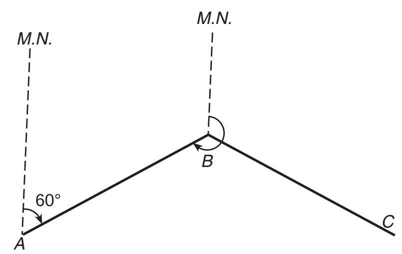


Fig. 12.8. Back bearing method.

(*i*) Set up the theodolite at the starting station *A* and observe the fore or direct bearing of the traverse side *AB*.

(*ii*) Shift the instrument to station *B*.

(*iii*) Calculate the back bearing of *BA*, *i.e.* fore bearing ± 180° and set the vernier to read this. Bisect the mark at station *A*.

(*iv*) Swing the telescope clockwise to sight station *C*.

(*v*) The reading of the vernier will be the direct bearing of the side *BC*.

(*vi*) Repeat the process till bearings of all traverse legs are observed.

12.5. FIELD WORK OF THEODOLITE TRAVERSING

In theodolite traversing, the field work is carried out in the following stages :

1. Reconnaissance.
2. Selection and marking of stations.
3. Measurement of traverse legs.
4. Measurement of traverse angles.
5. Measurement of angles for intersected points.
6. Booking of field notes.

1. Reconnaissance. It is always useful, and often absolutely necessary, for a traverser to make reconnaissance before commencing his actual traverse lines for the purpose of selecting suitable routes along which traverse lines are to be carried out. On reaching the project site, the traverser should hurriedly go over the entire area and select the most suitable routes along which measurements on the ground may be easily made. It should also be kept in mind that the number of traverse stations should be least possible to reduce the accumulated errors in angular and linear measurement. While walking along the proposed alignment of the traverse line, he should select some well-defined points to be fixed from traverse stations by the method of intersection. As the ground control points in profile are more useful for a planetabler than in plan, sufficient number of intersected points should, therefore, invariably be provided. The well-marked points, fixed by good intersections from two or three consecutive traverse stations used with same confidence as traverse stations on a planetable section, are called *intersected points*. Fixing more intersected points from less number of stations, indicates the ability and the efficiency of the traverser.

2. Selection and marking of stations. Every traverse station is selected, keeping in view that consecutive stations are intervisible. The traverse legs, as far as possible, are kept of same length to have systematic error in angular measurements. The closing error in angular measurement is, therefore, divided equally to all traverse angles assuming all angles of equal weights. Every traverse station is marked as conspicuously as possible for easy identification of plane tabler later. As far as possible traverse stations are marked on pakka points, *i.e.* distance stones, culverts, road crossing, etc. Precise and exact description of each station should be entered in the field book giving distances of the marks on easily recognizable points close-by. Description of traverse stations neatly written in the field book enables the plane tabler to find them at later date.

When the length of the traverse circuit is great extending over few kilometres it is advisable that a *traverse chart* on suitable scale may be maintained, to show the locations of the proposed traverse stations and intersected points. The traverse chart is also very useful for the computers. Asured results may be easily verified from plotted positions of intersected points on the traverse chart. A specimen traverse chart is shown in Fig. 12.9.

3. Measurement of traverse legs. Distances between traverse stations are measured directly by chaining which is a more reliable method except in rough ground. Each distance must be measured independently by a 30 metre chain and 20 metre chain separately. Both chains are tested regularly against standard tapes. When better accuracy is required, steel tapes are used for measuring the traverse legs. In case, measurements by two chains differ by more than 1 in 1000 in between two stations, the line must be remeasured by both the chains.

The distance given by a 20 metre chain serves only a check on measurement. The distance measured by a 30 metre chain is only used in computation. *The mean of distances measured by long and short chains should never be accepted for computation.* The undermentioned points are kept in mind while measuring a distance.

(*i*) Measurements start and close at the centres of the station marks.

(*ii*) Chains are always laid straight and pulled out their full lengths.

(*iii*) Arrows (chain pins) are put into the ground vertically.

(*iv*) In rocky grounds or on cemented/tar surfaces, the full chain lengths are marked by chalk or paint.

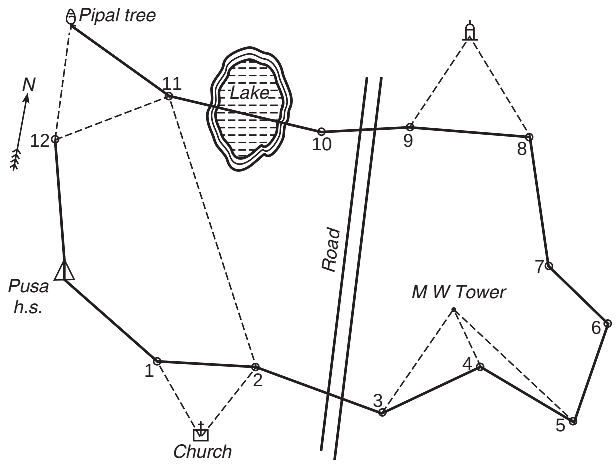


Fig. 12.9. A traverse chart.

(*v*) Measurements are taken from the point where the chain pin enters the ground.

To get a horizontal distance between two traverse stations, corrections are applied to the measured lengths. In ordinary chaining however, all corrections may not be necessary but in important and precise work, the following corrections are applied :

(*i*) Correction for standardization.

(*ii*) Correction for slope.

(*iii*) Correction for temperature.

(*iv*) Correction for tension.

(*v*) Correction for sag.

For the amount of each correction and their effects on the measured lengths, refer to Chapter 2 'Linear Measurements.

4. Measurement of traverse angles. The measurement of traverse angles may be made by either repetition method or direct method as already explained in chapter 11 'Theodolite'.

Practical Method. The following procedure should be carefully carried out at each traverse station.

1. Centre the theodolite over the station mark and level it carefully.
2. Set the instrument roughly to the north with the help of a magnetic needle or a map and clamp its lower plate.
3. Swing the telescope on to the back station, and clamp its upper plate. Bisect the signal at back station using either tangent screw. Record the first reading.
4. Loosen the upper plate and intersect the forward station using the upper tangent screw. Record the second reading.

5. Keeping the upper plate clamped, release the lower plate, turn the telescope to the back station. Final bisection of the back station mark should be made with lower tangent screw. Vernier reading remains unchanged.
6. Release the upper plate, swing the theodolite to bisect the forward station again with upper tangent screw. Record the third reading.
7. The two measures of the traverse angle (to the left of the observer when he faces his forward station) may be calculated as under :

(*i*) Subtract the first reading from the second reading to get the first measure of the angle.

(*ii*) Subtract the second reading from the third reading to get the second measure of the angle.

Enter these two measures in the column of deduced angles. The two measures should invariably agree within 1′ of arc. The mean value is accepted for computation.

Traverse field Book. The observations of the traverse are recorded in the field book beginning at the bottom and working upwards. Specimen field book is as under.

Station observed	*Observed angle*	*Deduced angle*
Forward station	25° 08′ 40″	174° 43′ 00″
Forward station	210° 25′ 20″	174° 43′ 20″
Back station	35° 42′ 40″	174° 42′ 40″

5. Measurement of angles for intersected points. The observations of the horizontal angles for the intersected points should be taken on a separate round, starting from the back station.

6. Booking of field notes. It should be appreciated that utmost care taken in making field observations goes waste unless observations are neatly and systematically recorded in the field books, to derive correct data during computation. Specimen field book, depending upon the method of observation of traverse angles, have been given on pages 564 and 565.

12.6. TRAVERSE COMPUTATIONS

For calculation of coordinates from the observed field data, the following computations are made in their squential order.

1. Checking the field observations.
2. Setting up traverse angles and distances of traverse legs.
3. Ascertaining the bearing of first traverse leg.
4. Running down the bearing of remaining traverse legs.
5. Computation of reduced bearings of traverse legs.

Table 12.1. Specimen field book for recording Traverse Angles by repetition method

Observation at 'B' Theodolite no. 5265

Station	Sighted to	Face : Left														Face : Right														
		A			B		Mean			No. of Rep	Horizontal Angle			A			B		Mean			No. of Rep	Horizontal Angle			Average Horizontal Angle		
		°	′	″	′	″	°	′	″		°	′	″	°	′	″	′	″	°	′	″		°	′	″	°	′	″
B	A	0	0	0	0	0	0	0	0					0	0	0	0	0	0	0	0	0						
	C	80	32	40	32	20	80	32	30	1				80	32	40	32	40	80	32	40	1						
	C	241	38	20	38	40	241	38	30	3	80	32	50	241	38	40	38	20	241	38	30	3	80	32	50	80	32	50

Recorded by : Sd. 5-10-1979 Observed by : Sd. 5-10-1979

Table 12.2. Specimen field of recording of observation on traverse angles by practical method

Theodolite no. 5438

Stn. No.	*Readings of horizontal circle*					*Mean*			*Deduced Angles*			*Horizontal distance (m)*
	A			*B*								
	°	′	″	°	′	°	′	″	°	′	″	
									121	20	05	
C	243 122 01	55 35 15	40 40 20	55 35 15	40 40 40	243 122 01	55 35 15	40 40 30	121 121	20 20	00 10	137.25
									82	21	35	
B	168 85 03	08 47 25	40 00 25	08 47 25	40 20 40	168 85 3	08 47 25	40 10 30	82 82	21 21	30 40	150.85
									109	42	00	
A	225 115 05	07 25 43	40 20 40	07 25 43	20 40 20	225 115 05	07 25 43	30 30 30	109 109	42 42	00 00	270.52

Recorded by : ....

Observed by : ....

6. Calculation of consecutive co-ordinates of traverse stations.
7. Calculation of the closing error.
8. Balancing of consecutive co-ordinates.
9. Calculation of independent co-ordinates.

1. Checking the field observations. Before proceeding for computation, the deductions of observations and their means are checked independently in the office.

2. Set-up of traverse angles and distances. The traverse computation may be systematically done in a tabular form as suggested by Gale. Such a table is known as *Gale's table*. One must study Gale's table carefully before attempting the traverse computations. Refer to Table 12.3 on page 567.

The columns of the Gale's traverse table are filled in as explained below:

1. Enter the traverse station *A, B, C, D,* .... in anti-clockwise direction.
2. Enter the distance between stations *A* and *B* against *B*, the distance between stations *B* and *C* against *C* and so on.
3. Enter the traverse angle *B* against *B*, traverse angle *C* against *C* and so on.
4. Provide a geometrical check for the observed traverse angles.

In case of a close traverse in anti-clockwise direction.

The sum of all angles must be ($2n - 4$) right angles. If not, apply corrections to all angles.

5. Enter the correct bearing of traverse leg *AB* against *A* and calculate the bearings of remaining traverse legs.
6. Enter the reduced bearing of each leg.
7. Calculate the consecutive co-ordinates of each leg.
8. Find the algebraic sum of Eastings and Northings and ensure that for a closed traverse, each equals to zero.
9. If not, balance the consecutive co-ordinates by either Bowditch's rule or Transit rule.
10. Calculate the independent co-ordinates of each traverse station.

A completed Gale's traverse is shown in table 12.3.

3. Bearing of traverse legs. The bearings entered in Gale's table are always from the back station to the forward station. Bearing is entered in the set up (Gale's table) on the same line as the forward station.

The bearings of few intermediate traverse legs at suitable intervals and the last leg are also observed independently and entered in the set up on the lines of their stations of observations. In case of a closed

Table 12.3. Gale's Traverse Table or Traverse set up

Stn	Length (m)	*Included Angle*	*Corn.*	*W.C.B*	*R.B.*	Cosecutive Co-Ordinates				*Independent Co-ordinates*		
						E +	*W* –	*N* +	*S* –	*E*	*N*	*Remarks*
A	...	...	...	140° 11′ 40″	*S* 39° 48′ 20″ *E*	...	...	...	...	1000.0	1000.0	
B	156.2	80° 59′ 34″	– 5″	41° 11′ 09″	*N* 41° 11′ 09″ *E*	100.0	...	...	120.0	1100.0	880.0	
C	318.9	91° 31′ 29″	– 5″	312° 42′ 33″	*N* 47° 17′ 27″ *W*	210.0	...	240.0	...	1310.0	1120.0	
D	88.4	100° 59′ 15″	– 5″	233° 41′ 43″	*S* 53° 41′ 43″ *W*	...	65.0	60.0	...	1245.0	1180.0	
A	304.0	86° 30′ 02″	– 5″	140° 11′ 40″	...	...	245.0	...	180.0	1000.0	1000.0	
	Sum	360° 00′ 20″			Total	310.0	310.0	300.0	300.0			

circuit, observations of bearings of intermediate stations is generally not necessary as the accuracy of angular measurements may be checked by summing up the traverse angles. The sum should equal to $2n \pm 4$ right angles, where n is the number of sides of the traverse circuit. Positive sign in used for exterior angles and negative sign for interior angles.

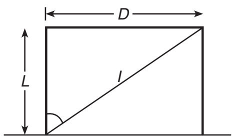


Fig. 12.10. Latitude and departure.

To prove the bearing of each circuit, sum up all the observed traverse angles and divide the sum total of each compartment by 360° . The remainder should equal to the difference between the opening and closing bearings of the circuit.

Discrepancy in bearings is equally distributed to all traverse angles of the circuit.

4. Calculation of whole circle bearings. The bearings of individual traverse legs are determined as discussed below :

(*a*) **From included angles :** Apply the following formulae.

To the bearing of the preceding leg, add the corrected traverse angle.

1. If the sum is more than 180° subtract 180°
2. If the sum is less than 180° add 180°.
3. If the sum is more than 540°, subtract 540°,

In this way, bearings of all traverse legs may be calculated. The calculated bearing of the closing line should agree with its observed bearing.

(*b*) **From deflection angles.** Knowing the whole circle bearing of the initial traverse leg and deflection angle the whole circle bearings of the remaining traverse legs, may be calculated by the following rules :

Rule 1 :

W.C.B. of a traverse leg = W.C. bearing of the preceding leg ± Deflection angle.

Use + sign if the deflection angle is right (clockwise)

Use – sign if the deflection angle is left (anti-clockwise)

If the result is greater than 360°, subtract 360°

If the result is negative add 360°

Rule 2 :

Bearing of the last leg

= Bearing of the initial leg + (Sum of the positive deflection angles)

– (Sum of the negative deflection angles)

where right deflection angles are treated positive and left deflection angles negative.

5. Calculation of reduced bearings of lines. The whole circle bearings of traverse legs should be converted into quadrantal system of bearings. The reduced bearing of a line decides the quadrant in which it falls. The bearing of a leg may be easily converted from the whole circle bearing to quatrantal bearing with the aid of a diagram. For conversion of W.C.B. into R.B. refer to Chapter 4 Compass Surveying.

6. Consecutive co-ordinates (Latitude and Departure). The *latitude* (northing or southing) of a survey line is defined as its co-ordinate measured parallel to the assumed meridian. Sometimes latitudes are also called *meridians*.

The *Departure* (easting or westing) of a survey line is defined as its co-ordinate measured at right angles to the assumed meridian. Sometimes departures are also called *perpendiculars*.

Sign Conventions. The latitudes measured north-ward are termed positive and those measured south-ward are termed as negative. Similarly departures measured eastward are termed positive and those measured westward as negative.

It is very important to enter the latitudes and departures in their correct columns, *i.e.* northings, southings, eastings or westings.

If l is the length and θ is the reduced bearing of a traverse leg AB (Fig. 12.10)

Latitude $L = l \cos \theta$

Departure $D = l \sin \theta$

The sign of latitudes and departures depends upon the reduced bearing of the line. For proper signs of the latitudes and departures, the diagram shown in Fig. 12.11 may be referred to.

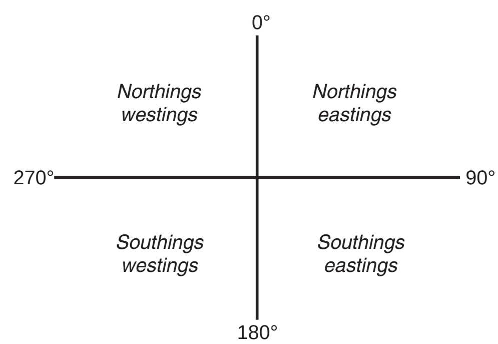


Fig. 12.11. Sign of Latitudes and Departure.

The latitude and departure of any station with respect to the preceding station, are known as *consecutive co-ordinates* or "*dependent co-ordinates*".

The latitude of a line is obtained by multiplying its length by the cosine of its reduced bearing. The departure of the line is obtained by multiplying its length by the sine of its reduced bearing.

Example 12.1. *Calculate the consecutive co-ordinates of the following traverse legs whose reduced bearings are shown against each.*

Line	*Length (metres)*	*Reduced bearings*
AB	305	*N* 30° 30′ *E*
BC	550	*S* 42° 42′ *E*
CD	830	*S* 48° 48′ *W*
DE	630	*N* 60° 0′ *W*

Solution (Fig. 12.12)

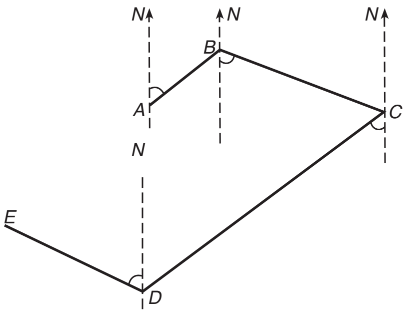


Fig. 12.12

(*i*) **Consecutive co-ordinates of the traverse leg AB :**

Latitude = 305 cos 30° 30′

= 305 × 0.8616292

= 262.80 m.

Departure = 305 sin 30° 30′ = 305 × 0.5075383 = 154.80 m.

Both are positive because the traverse leg lies in the first quadrant.

(*ii*) **Consecutive co-ordinates of traverse leg BC :**

Latitude = 550 cos 42° 42′ = 550 × 0.7349146 = 404.20 m.

Departure = 550 sin 42° 42′ = 550 × 0.6781596 = 372.99 m

Latitude is negative (southing) and departure is positive (easting) as the leg *BC* lies in fourth quadrant.

(*iii*) **Consecutive co-ordinates of traverse leg CD** :

Latitude = 830 cos 48° 48′ = 830 × 0.6586894

= 546.71 m

Departure = 830 sin 48° 48′ = 830 × 0.7524149

= 624.50 m.

Both are negative as the leg lies in third quadrant.

(*iv*) **Consecutive co-ordinates of the traverse leg DE**

Latitude = 630 sin 60° = 600 × 0.5000 = 315.00 m

Departure = 630 sin 60° = 630 × 0.8660254 = 545.60 m.

Latitude is positive (northing) and departure is negative (westing) as the leg *DE* lies in the second quadrant.

7. Calculation of the closing error. In a complete transverse circuit the sum of north latitudes must be equal to that of south latitudes, the sum of eastings must be equal to that of westings, if linear as well as angular measurements of the traverse alongwith their computation are correct.

If not, the distance between the starting station and the position obtained by calculation, is known as a *closing error*.

Let us assume that the sum of the northings of a traverse exceeds the sum of the southings by 1.5 metres and that of eastings exceeds the sum of the westings by 1.8 metres. So the resultant closing error = $\sqrt{(1.8^2) + (1.5^2)}$ = 2.34 m. In Fig. 12.13, *A* is the position of the starting station and *A′* is the position as plotted with its co-ordinates.

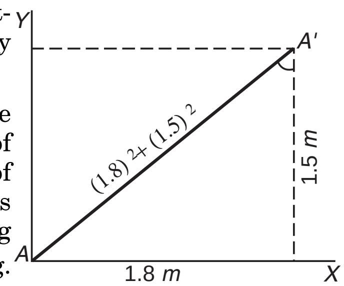


Fig. 12.13. Closing error.

The reduced bearing θ of the closing line *A′A* is evidently equal to $\tan^{-1}\dfrac{1.8}{1.5}$ or S 50° 11′40″ W and its W.C. bearing

$$= 50° \, 11' \, 40'' + 180° = 230° \, 11' \, 40''$$

The closing error is generally expressed as a fraction *i.e.*

$$\frac{\text{closing error}}{\text{perimeter of the traverse}} .$$

Let the perimeter of a traverse be 5388.9 m and closing error be 2.343 m.

$\therefore$ The closing error $= \dfrac{2.343}{5388.9} = \dfrac{1}{2300}$ *i.e.*1 in 2300.

Note. The length of a chain does not affect the closing error of a closed circuit whereas the cumulative and compensating errors of the chain length affect the traverse between stations whose positions were previously determined.

8. Balancing the consecutive coordinates. The process of adjusting consecutive coordinates of each line by applying corrections to them in such a way that each algebraic sum of the latitudes and departures of a closed circuit should be equal to zero *i.e.* the sum of the northings should be exactly equal to the sum of the southings, and the sum of westings should be exactly equal to the eastings, is called *balancing the consecutive coordinates.*

The closing error, however small is distributed throughout the whole traverse circuit such that its effect is not apparent on the plotted locations of the stations.

The operation of balancing the consecutive coordinates of a theodolite traverse, is usually accomplished by one of the following methods detailed below.

1. Bowditch's Method. Independent traverse lines between predetermined stations may be adjusted by distributing the misclosures in proportion to the accumulated distance (*i.e.* perimeter of traverse line in case of closed circuits or the total length of the traverse closing on another point) from the starting station, in accordance with C.F. Bowditch rule which states :

'As the sum of all the distances is to each particular distance so is the total error for latitude (or departure) to the required correction for latitude (or departure)'. i.e. correction to latitude (or departure) of a line = closing error in latitude (or departure)

$$\times \frac{\textit{Length of that traverse leg}}{\textit{Total length of the traverse}}.$$

Angular errors, if any in the observed bearings, are not adjusted before applying the Bowditch rule.

Let l = length of any leg

$\Sigma\, l$ = total length of the traverse

$\Sigma\, L$ = total error in latitude

$\Sigma\, D$ = total error in departure

δL = correction to the latitude of the leg

$\delta\, D$ = correction to the departure of the leg

then
$$\delta\, L = \Sigma\, L \times \frac{l}{\Sigma\, l} \qquad ...(12.1)$$

and
$$\delta\, D = \Sigma\, D \times \frac{l}{\Sigma\, l} \qquad ...(12.2)$$

Note. Bowditch rule is employed when both linear and angular measurements of the traverse are of equal accuracy.

2. Transit Rule. This method of adjusting the consecutive coordinates of theodolite traverses, may be conveniently employed *where* *angular measurements are more accurate than the linear measurements.* Generally, traverse angles of a theodolite traverse are measured more accurately than traverse legs and a well known empirical transit rule for such traverses states, "*The correction to latitude (or departure) of any traverse leg should be proportional to the Latitude (or departure) instead of the length of the traverse leg itself.*

According to the Transit rule, correction to latitude of a traverse leg

$$= \text{total error in latitude} \times \frac{\text{latitude of that traverse leg}}{\text{total sum of latitudes}}$$

Let δL = correction to the latitude of the leg

δD = correction to the departure of the leg

l = latitude of any traverse leg

d = departure of the same traverse leg

L = arithmetic sum (total sum ignoring signs) of the latitudes

D = arithmetic sum (total sum) of the departures (ignoring signs)

SUM L = total error (algebraic sum) of latitude

SUM D = total error (algebraic sum) of departure.

then $$\delta L = SUM\ L \times \frac{l}{K} \qquad ...(12.3)$$

and $$\delta D = \Sigma D \times \frac{d}{D} \qquad ...(12.4)$$

Example 12.2. *An abstract from a traverse sheet for a closed traverse is given below :*

Line	*Length*	*Latitude*	*Departure*
AB	200 m	–173.20	+100.00
BC	130 m	00.0	+130.00
CD	100 m	+86.60	+50.00
DE	250 m	+250.00	0.00
EA	320 m	–154.90	– 280.00

Balance the traverse by Bowditch's method.

Solution.

Total error in latitude

$= -173.20 + 0.00 + 86.60 + 250.00 - 154.90$

$= 8.50$ m

Total error in departure

$= 100.00 + 130.00 + 50.00 + 0.00 - 280.00$

$= 0.00$ m

Perimeter of the traverse

$= 200 + 130 + 100 + 250 + 320$

$= 1000$ m

Corrections to latitudes :

$$AB = 8.50 \times \frac{200}{1000} = 1.700 \text{ m}$$

$$BC = 8.50 \times \frac{130}{1000} = 1.105 \text{ m}$$

$$CD = 8.50 \times \frac{100}{1000} = 0.850 \text{ m}$$

$$DE = 8.50 \times \frac{250}{1000} = 2.125 \text{ m}$$

$$EA = 8.50 \times \frac{320}{1000} = 2.720 \text{ m}$$

Total $= 8.500$ m

As the error is possitive, the sign of the corrections is negative

Corrected latitude of $AB = -173.20 - 1.700 = -174.900$

Corrected latitude of $BC = 0.00 - 1.105 \quad = -1.105$

Corrected latitude of $CD = 86.60 - 0.850 \quad = +85.750$

Corrected latitude of $DE = 250.00 - 2.125 \quad = +247.875$

Corrected latitude of $EA = -154.90 - 2.720 \quad = -157.620$

Total $= 0.000$

There is no error in departure and hence no correction.

Example 12.3. *Balance the traverse of example 12.2 by transit rule*.

Total error in departure is zero, hence no correction for departures

Solution.

Total error in latitudes 8.50m

Total sum of latitudes

$= 173.20 + 0.00 + 86.60 + 250.00 + 154.90 = 664.7$m

Corrections in latitudes :

$$AB = \frac{8.50 \times 173.20}{664.7} = 2.215 \text{ m}$$

$$BC = \frac{8.50 \times 0.00}{664.7} = 0.000 \text{ m}$$

$$CD = \frac{8.50 \times 86.60}{664.7} = 1.107 \text{ m}$$

$$DE = \frac{8.50 \times 250.00}{664.7} = 3.197 \text{ m}$$

$$EA = \frac{8.50 \times 154.90}{664.7} = 1.981 \text{ m}$$

$$\text{Total} = 8.500 \text{ m}$$

As the sign of error is positive, hence sign of corrections is negative

Corrected latitude of $AB = -173.20 - 2.215 = -175.415$

$BC = +0.00 - 0.00 = 0.000$

$CD = +86.60 - 1.107 = 85.493$

$DE = +250.00 - 3.197 = 246.803$

$EA = -154.90 - 1.981 = -156.881$

Total = 0.000

Example 12.4. *Calculate latitudes, departures, and closing error for the following traverse, and adjust using Bowditch's rule*

Line	*Length*	*R.B.*
AB	89.31	45° 10′
BC	219.76	72° 05′
CD	151.18	161° 52′
DE	159.10	228° 43′
EA	232.26	59° 18′

Solution. (Fig. 12.14)

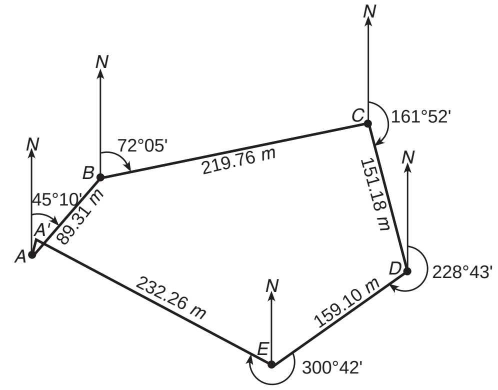


Fig. 12.14

Let $ABCDEA'$ be the given traverse.

Line	*Length*	*R.B.*
AB	89.31	*N* 45° 10′ *E*
BC	219.76	*N* 72° 05′ *E*
CD	151.18	*S* 18° 08′ *E*
DE	159.10	*S* 48° 43′ *W*
EA	232.26	*N* 59° 18′*W*

Calculation of Latitudes :

Latitude of AB = 89.31 cos 45° cos 45° 10′ = 62.97 (+ve)

Latitude of BC = 219.76 cos 72° 05′ = 67.61 (+ve)

Latitude of CD = 151.18 cos 18° 08′ = 143.67 (– ve)

Latitude of DE = 159.10 cos 48° 43′ = 104.97 (–ve)

Latitude of EA = 232.26 cos 59° 18′ = 118.58 (+ve)

Algebraic sum = 0.52

Calculation of Departures :

Departure of AB = 89.31 sin 45° 10′ = 63.34 (+ ve)

Departure of BC = 219.76 sin 72° 05′ = 209.10 (+ ve)

Departure of CD = 151.18 sin 18° 08′ = 47.05 (+ ve)

Departure of DE = 159.10 sin 48° 43′ = 119.56 (– ve)

Departure of EA = 232.26 sin 59° 18′ = 199.71 (– ve)

Algebraic sum = + 0.22

Let θ be the reduced bearing of closing line $A' A$.

$$\tan\theta = \frac{0.22}{0.52} = 0.4230769$$

$$\theta = 22° \, 55' \, 56''$$

Length of the closing error $A' A$

$$= \sqrt{(0.22)^2 + (0.52)^2} = 0.565 \text{ m.}$$ **Ans.**

Perimeter of the traverse

$$= 89.31 + 219.76 + 151.18 + 159.10 + 232.26$$
$$= 851.61 \text{ m}$$

Calculation of Corrections to Latitudes

Correction for $AB = \dfrac{89.31}{851.61} \times 0.52 = 0.06$

Correction for $BC = \dfrac{219.76}{851.18} \times 0.52 = 0.13$

Correction for $CD = \dfrac{151.18}{851.61} \times 0.52 = 0.09$

Correction for $DE = \dfrac{159.10}{851.61} \times 0.52 = 0.10$

Correction for $EA = \dfrac{232.26}{851.61} \times 0.52 = 0.14$

Total = 0.52

Calculation of Corrections to Departures

Correction for $AB = \dfrac{89.31}{851.61} \times 0.22 = 0.02$

Correction for $BC = \dfrac{219.76}{851.61} \times 0.22 = 0.06$

Correction for $CD = \dfrac{151.18}{851.61} \times 0.22 = 0.04$

Correction for $DE = \dfrac{159.10}{851.61} \times 0.22 = 0.04$

Correction for $EA = \dfrac{232.26}{851.61} \times 0.22 = 0.06$

Total = 0.22

As errors of latitudes and departures are positive, their corrections are of negative sign.

Calculation of Corrected Latitudes

Side	*Latitude*	*Correction*	*Corrected latitude*
AB	62.97	-0.06	+62.91
BC	67.61	-0.13	67.48
CD	-143.67	-0.09	-143.76
DE	-104.97	-0.10	-105.07
EA	118.58	-0.14	+118.44
		Algebraic sum = 0.00	

Calculation of Corrected Departures

Side	*Latitude*	*Correction*	*Corrected latitude*
AB	63.34	-0.02	63.32
BC	209.10	-0.06	209.04

CD	47.05	-0.04	47.01
DE	-119.56	-0.04	-119.60
EA	199.71	-0.06	-199.77
		Algebraic sum = 0.00	

3. Graphical method for adjusting a traverse. (Fig. 12.15). For rough traverses where angular measurements are of inferior degree of accuracy, the Bowditch's rule of adjusting the consecutive coordinates may be applied graphically without doing numerical calculations. In the graphical method of adjustment, the traverse is directly plotted from field observations on sufficiently large scale after adjusting the traverse angles or the bearings of traverse legs to satisfy the geometric conditions of the traverse.

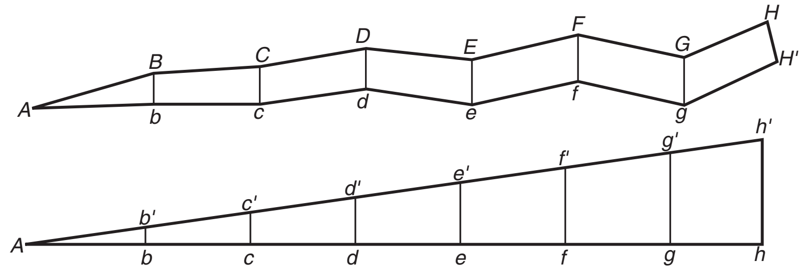


Fig. 12.15. Graphical adjustment of a linear traverse.

Let *A, b, c, d, e, f, g, H′* denote the succeeding traverse stations of a closed traverse between stations *A* and *H* whose positions are fixed and known. *H′* denotes the position of *H*, as given by the traverse which should have been identical with *H* if the traverse had no error. *H′ H* is the closing error which is required to be adjusted.

Procedure. Lay out a straight line *Ah* and mark on it *A, b, c, d, e, f, g,* and *h* at their respective distances from other, on the same scale that of the plotted traverse or on another convenient reduced scale. At *h* erect a perpendicular *hh′* equal to the length of the closing error. Join *A* and *h′* and erect perpendiculars at points *b, c, d, e, f,* and *g* to meet the line *Ah′*at *b′*, *c′*, *d′*, *e′*, *f′*, and *g′*. Join *HH′* and draw parallels to *HH′* through points *b, c, d, e, f* and *g*. On these parallels, mark off *B, C, D, E, F,* and *G*, such that *bB, cC, dD,* etc. are equal to the perpendiculars *bb′*, *cc′*, *dd′*, etc.

Join *A, B, C, D, E, F, G* and *H*. *ABCDEFG* is the adjusted traverse.

In case of closed circuits, the procedure is the same except that *A″* will concide with *A* as shown in Fig. 12.16.

4. The Axis method of adjusting a traverse. When the accuracy of the angular measurements is more than that of linear measurements, the adjustment of the closing error of the traverse is done in such a way that the direction of each line remains unchanged and the

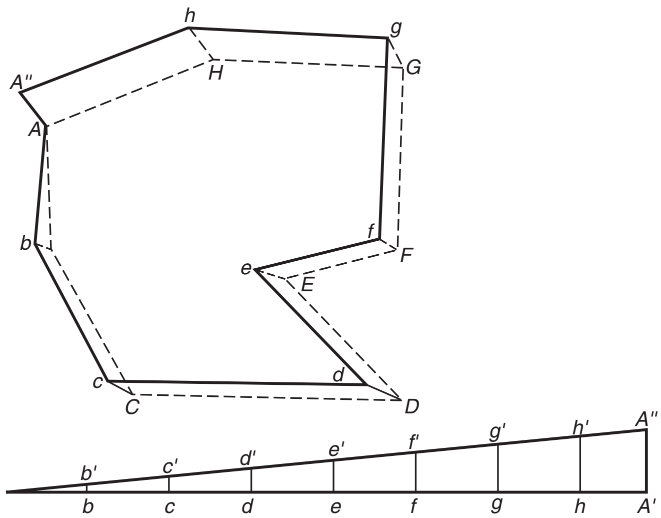


Fig. 12.16.

general shape of the diagram is preserved. The method employed for adjusting such traverses, is known as the '*Axis Method*'.

Procedure. Let *ABCDEFA′* be the uncorrected plan of a closed traverse when plotted on any desired scale.

Apparently *A′ A* is the closing error which may be distributed as under :

(*i*) Join *A′ A* and produce it to cut the side *CD* at *M*. The line *A′ AM* which is known as the *axis of adjustment* should be drawn such that it divides the main traverse in roughly two equal parts *MD EFA′* and *MCBA*.

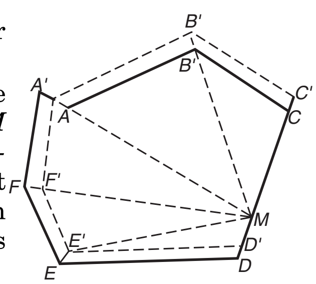


Fig. 12.17. Axis method of adjustment of a traverse.

(*ii*) Divide *A′ A* in the ratio of the perimeters on either side of the axis of adjustment.

(*iii*) Select *N* such that $\dfrac{A'N}{AN} = \dfrac{A'F + FE + ED + DM}{AB + BC + CM}$

(*iv*) Through *N*, draw a line *NB′* parallel to *AB* and cutting *MB* produced at *B′*. Through *B′* draw a line *B′ C′* parallel to *BC* and cutting *MC* produced at *C′*.

(*v*) Similarly, through *N* draw a line *NF′* parallel to *A′F* cutting *MF* in *F′*. Through *F′* draw *F′ E′* parallel to *EF* cutting *ME* in *E′*. Finally draw *E′ D′* parallel to *ED* to cut *MD* in *D′*.

(*vi*) *NB′ C′ D′ E′ F′N* is the adjusted plan of the traverse.

Principle of the method

Δs $NB'M$ and ABM are similar triangles

$$\therefore \quad \frac{NB'}{NM} = \frac{AB}{AM}$$

or

$$NB' = \frac{NM}{AM} \times AB$$

Correction to side

$$AB = NB' - AB = \frac{NM \times AB}{AM} - AB$$

$$= \frac{AB\,(NM - AM)}{AM} = \frac{AN}{AM} \times AB$$

$$= \frac{\frac{1}{2} AA'}{AM} \times AB \text{ (assuming } AN = A'N)$$

$$= \frac{\frac{1}{2}\text{ closing error}}{AM} \times AB.$$

Correction to

$$A'F = \frac{\frac{1}{2}\text{ closing error}}{A'M} \times A'F$$

Assuming $A'M$ very nearly equal to AM,

$$\text{Correction } = \frac{1}{2} \times \frac{\text{side length}}{\text{length of Axis}} \times \text{closing error.}$$

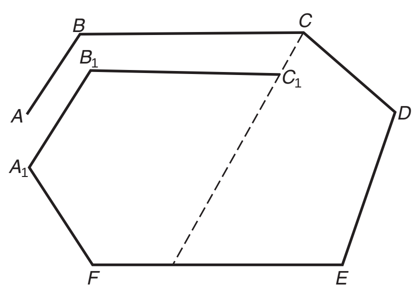


Fig. 12.18. The axis method of adjusting traverse.

Accuracy of the adjustment by the Axis Method depends upon the location of the axis of adjustment *i.e.*, the axis of adjustment divides the figure roughly in two equal portions. However, in some cases, the closing error A_1A when produced may not cut the traverse or may cut it in two very unequal parts. In such cases, the error is transferred to some other point say C, by drawing A_1B_1 parallel and equal to AB, and B_1C_1 parallel and equal to BC, etc. (Fig. 12.18)

The figure $CDEF\ A_1B_1C_1$ is now the unadjusted traverse and C_1C is the closing error. The line CC_1 when produced divides the figure into two roughly equal portions.

Note. The Axis Method of adjusting traverses is not suitable for linear traverse *i.e.*, when the traverse does not end on the starting point.

9. Independent co-ordinates. The total latitude and total departure of any station with respect to a common origin of coordinates, are called *total coordinates* or *independent coordinates.*

After the traverse has been properly balanced *i.e.*, the algebraic sum of the northings is equal to algebraic sum of the southings and the algebraic sum of the eastings is equal to algebraic sum of the westings, the independent coordinates may be calculated as under:

Two reference axes are chosen such that the whole traverse falls in the first quadrant and the total latitude and departure of each stations get positive sign.

The independent coordinates of any station are obtained by adding algebraically the latitudes and departures of the traverse legs between that station and the origin. It may be stated mathematically. "*Total latitude (or departure) of a point=Algebraic sum of all the latitudes (or departures) upto the point.*

In the case of a closed circuit, the coordinates of the closing station should agree with those of the starting station.

A specimen Gale's Table duly completed, for the calculation of independent coordinates of points, is shown on page 582.

Advantages of Independent Coordinates: The accuracy of a survey depends upon the accuracy with which its control points are plotted. When plotting of the traverse is done with consecutive coordinates, the accuracy of the location of each point upon the plan evidently depends upon the accuracy with which previous points are plotted. Due to a slight error in plotting of each point due to scaling, the error gets accumulated and the position of the last point may be displaced to a considerable extent from its true position.

This difficulty is overcome by plotting the points with independent coordinates and thus error does not accumulate.

Knowing independent coordinates of two points, the distance between them and their reduced bearing, may easily by calculated by distance formula:

$$\text{Distance} \qquad AB = \sqrt{(\Delta E)^2 + (\Delta N)^2} \qquad \text{...(12.5)}$$

$$\text{Reduced bearing} \quad = \tan^{-1} \frac{\text{departure}}{\text{latitude}} \qquad \text{...(12.6)}$$

12.7 OMITTED MEASUREMENTS IN TRAVERSING

In case of a closed traverse, the length and bearing of each traverse leg are generally measured in the field. Sometimes, due to obstacles, or

Table 12.4. Gale's Traverse Table

Traverse Legs	*Angles*	*Correc-tions*	*Corrected Angles*	*W.C.B.*	*Reduced Bearing*	*Lengths*	*Stn.*	Consecutive Co-ordinates				Corrections			
								Lat.		Dep.		Lat.		Dep.	
								N.	S.	E.	W.	N.	S.	E	W.
AB	94° 10′ 05″	5″	94° 10′ 00″	187° 20′ 20″	*S* 7° 20′ 20″ *W*	31.02	B	...	30.77	...	3.96	...	0.003	...	00.08
BC	178° 19′ 05″	5″	178° 19′ 00″	185° 39′ 20″	*S* 5° 39′ 20″ *W*	47.18	*C*	...	46.95		4.65	...	0.012	...	0.011
CD	118° 21′ 45″	5″	118° 21′ 40″	124° 01′ 00″	*S* 55° 59′ 00″ *E*	43.01	*D*	...	24.06	35.65	...	...	0.002	0.013	...
DE	94° 42′ 25″	5″	94° 42′ 20″	38° 43′ 20″	*N* 38° 43′ 20″ *E*	50.72	*E*	39.57	...	31.73	...	+0.005	...	0.013	...
EF	158° 07′ 30″	5″	158° 07′ 25″	16° 50′ 45″	*N* 16° 50′ 45″ *E*	53.02	*F*	50.74	...	15.37	...	+0.005	...	0.014	...
FG	89° 03′ 55″	5″	89° 03′ 50″	285° 54′ 35″	*N* 74° 05′ 25″ *W*	33.18	*G*	9.10	...	...	31.91	+0.003	...	...	0.009
GA	167° 15′ 50″	5″	167° 15′ 45″	273° 10′ 20″	*S* 86° 49′ 40″ *W*	42.25	*A*	2.34	...	...	42.19	+0.004	...	...	0.012
Total	900° 00′ 35″	35″				300.38		101.75	101.78	82.75	82.71	+0.017	−0.017	+0.040	−0.040

Closing error = $\sqrt{(0.03)^2 + (0.04)^2} = 0.05$ m

Δ Lat. = 101.78 − 101.75 = − 0.03

Reduced bearing of the closing error = $\tan^{-1} \dfrac{0.04}{0.03} = S\ 53°0.7'\ 48''\ E$

Δ Dep. = 82.75 − 82.71 = − 0.04

Perimeter of the traverse = 300.38 m ; Accuracy of the traverse $= \dfrac{0.05}{300.38} = \dfrac{1}{6008}$

due to oversight it is not possible to make all measurements. Such omitted measurements or missing quantities may be calculated only in case of a closed traverse, provided the required quantities are not more than two. Of course, in such cases no check on the field work may be made and the error propagated throughout the traverse, is brought into the computed values of the missing quantities.

Calculation of the missing data is based upon the fact that in case of a closed traverse starting from and closing on the same station, the algebraic sum of the latitudes and departures is zero and the sum of the interior angles and exterior angles of a polygon are $(2n - 4)$ and $(2n + 4)$ right angles, respectively. Whereas in case of a closed traverse starting from a known station and closing on another known station, the algebraic sum of the latitudes and the departures are equal to the difference of latitudes and departures of the starting and closing stations.

If $l_1, l_2, l_n ....l_n$ are the lengths of a traverse legs and $\theta_1, \theta_2, \theta_3 ....\theta_n$ their reduced bearings, then,

$$\Sigma L = l_1 \cos \theta_1 + l_2 \cos \theta_2 + ... + l_n \cos \theta_n = 0 \quad ...(12.7)$$

$$\Sigma D = l_1 \sin \theta_1 + l_2 \sin \theta_2 + ... + l_n \sin \theta_n = 0 \quad ...(12.8)$$

Solving Eqs. (12.7) and (12.8) any two missing quantities may be calculated.

In Fig. (12.19)

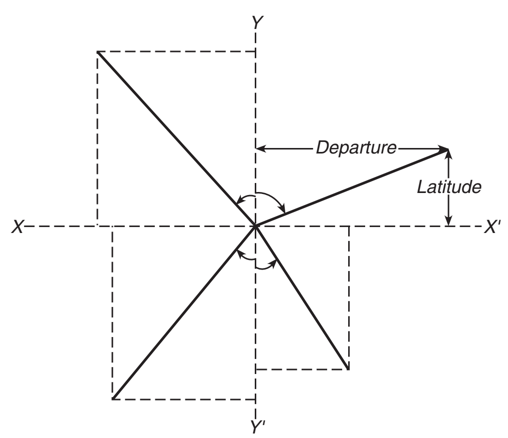


Fig. 12.19. Calculation of omitted measurements.

Let l = length of a traverse leg.

θ = reduced bearing of the traverse leg.

L = latitude of the traverse leg.

D = departure of the traverse leg.

then, $l = \sqrt{D^2 + L^2}$ ...(12.9)

$L = l \cos \theta$ ...(12.10)

$D = l \sin \theta$ ...(12.11)

$\theta = \tan^{-1} D/L$ ...(12.12)

If any two quantities of the above equations are known, the remaining quantities may be easily calculated.

12.9 TYPES OF OMITTED MEASUREMENTS

The omitted measurement in a theodolite traverse, may be classified in four general cases.

I. (*a*) When length of one traverse leg is omitted.
   (*b*) When bearing of one traverse leg is omitted.
   (*c*) When length and bearing of one traverse leg are omitted.

II. When length of one traverse leg and bearing of another adjacent leg are omitted.

III. When lengths of two legs are omitted.

IV. When bearings of two legs are omitted.

In case I only one traverse leg is affected. In remaining cases two legs are affected, both of which may be either adjacent or may not be adjacent.

Case I. Either Length or Bearing or both of one Traverse Leg are Omitted (Fig. 12.20).

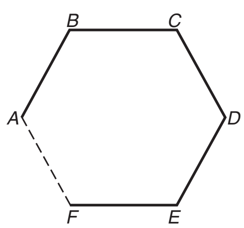


Fig. 12.20.

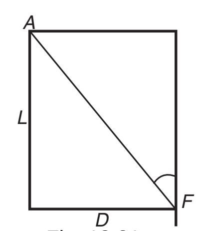


Fig. 12.21.

Let *ABCDEFA* be a closed traverse. Assume that either the length or bearing or both of the traverse leg *FA* is/are omitted from field measurements.

Proceed as under:

1. Calculate the algebraic sum of the latitudes from *A* to *F*. Let it be $\Sigma L'$.
2. Calculate the algebraic sum of the departures from *A* to *F*. Let it be $\Sigma D'$.

We know that for a closed traverse the algebraic sum of all latitudes and departures should each be equal to zero.

If L and D are the latitude and the departure of the traverse leg FA, then

$$L \pm \Sigma L' = 0$$

or $$\text{latitude of } FA = \pm\Sigma L' \qquad ...(12.13)$$

Similarly,

$$\text{Departure of } FA = \pm\Sigma D' \qquad ...(12.14)$$

Knowing the latitude and departure of FA, (Fig. 12.21) its length and bearing may be calculated by equation (12.9) and (12.12) respectively.

Case II. Length of one leg and bearing of an adjacent leg omitted (Fig. 12.22).

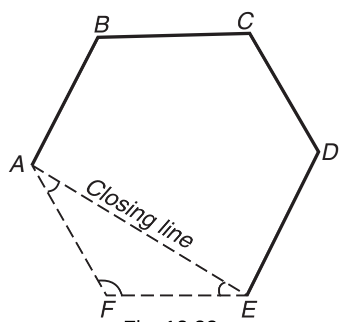


Fig. 12.22

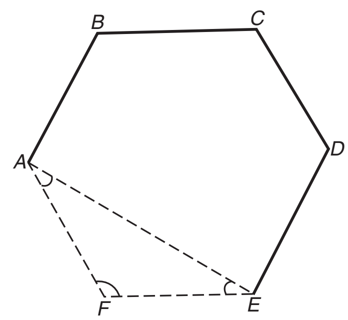


Fig. 12.23

Let $ABCDEF\,A$ be a closed traverse in which length of the leg EF and bearing of the leg FA are omitted.

Construction. Join EA which becomes the closing line of the traverse $ABCDEA$. Now all quantities of traverse $ABCDEA$ being known, the length and bearing of EA may be calculated as explained in case I.

In $\Delta\, AEF$ the sides AE and AF are known.

Angle FEA = bearing of EA bearing of $EF = \beta$

Applying the sine formula, we get

$$\frac{AE}{\sin\theta} = \frac{AF}{\sin\beta}$$

or $$\sin\theta = \frac{AE}{AF}\sin\beta$$

or $$\theta = \sin^{-1}\left(\frac{AE}{AF}\sin\beta\right) \qquad ...(12.15)$$

$\therefore$ $$\alpha = 180° - (\theta + \beta)$$

Again $\quad \dfrac{EF}{\sin\alpha} = \dfrac{AE}{\sin\theta}$

or $$EF = \frac{AE}{\sin\theta}\sin\alpha \qquad \text{...(12.16)}$$

Also, knowing α, the bearing of *AF* can be calculated *i.e.* Bearing of *AF* = Bearing of *AE* + α.

Case III. Length of two adjacent legs omitted (Fig. 12.23)

Let *ABCDEFA* be a closed traverse in which lengths of both *EF* and *FA* are omitted.

Construction. Join *EA* which becomes the closing line of traverse *ABCDEA*. The length and bearing of *AE* may now be calculated as explained in case I.

In Δ *AEF*,

Angle *FEA* = bearing of *EA* – bearing of *EF* = β

Angle *FAE* = bearing of *AF* – bearing of *AE* = α

$\therefore$ Angle *AFE* = $180° - (\alpha + \beta) = \theta$.

Applying the sine formula, we get

$$\frac{AF}{\sin\beta} = \frac{FE}{\sin\alpha} = \frac{AE}{\sin\theta}$$

$\therefore$ $$AF = \frac{AE \times \sin\beta}{\sin\theta}$$

and $$FE = \frac{AE \times \sin\alpha}{\sin\theta}$$

Case IV. Bearings of two adjacent legs omitted (Fig. 12.24)

Let *ABCDEFA* be a closed traverse in which bearings of *EF* and *FA,* both are omitted.

Area of $\Delta\, AEF = \sqrt{s(s-e)\,(s-f)\,(s-a)}$ ...(12.7)

where s = half the sum of the sides, e, f and a

Also, area of $\Delta\, AEF = \dfrac{1}{2} AE \times AE \sin\theta$

$$= \frac{1}{2} FE \times AE \sin\beta$$

$$= \frac{1}{2} AF \times AE \sin\alpha \qquad \text{...(12.8)}$$

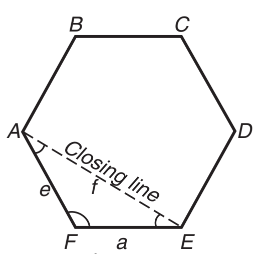


Fig. 12.24

By comparing Eqns. (12.7) and (12.8) we may calculate the values of α, β and θ.

Knowing the bearing of *EA* and angles α, β and θ, bearings of legs *FE* and *FA* may be easily calculated.

Cases II, III and IV. When affected legs are not adjacent (Fig. 12.25)

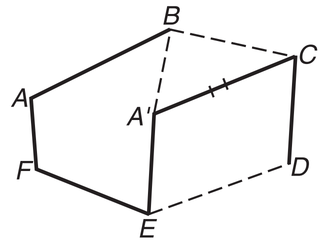


Fig. 12.25

If the affected sides are not adjacent, one of these may be shifted and brought adjacent to the other by drawing lines parallel to the given lines.

Let *ABCDEFA* be a closed traverse in which either the length or the bearing of legs *BC* and *DE* are omitted. To bring the affected sides adjacent to each other, draw *CA′* parallel and equal to *DE*. Join *A′ E* which will be parallel and equal to *CD*. The line *A′B* becomes the closing line of the traverse *BAFEA′B*. The length and bearing of the closing line *A′B* may be calculated. After knowing the length and bearing of the side *A′B* of the triangle *BCA′*, the omitted measurement of *BC* and *CA′* (or *DE*) may be obtained as explained in earlier cases.

Example 12.5. *The coordinates of two points A and B are as follow:*

Point	*Co-ordinates*	
	N	*E*
A	481.5	324.2
B	607.6	75.4

Find the length and bearing of line AB.

Solution. (Fig. 12.26)

Difference in eastings of the stations *A* and *B*

$$\Delta E = 324.2 - 75.4 = 248.8 \text{ m}$$

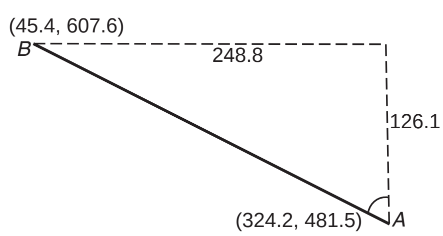


Fig. 12.26

Difference in northings of stations A and B

$$\Delta N = 607.6 - 481.5 = 126.1 \text{ m.}$$

Let θ be reduced bearing of AB

$$\tan\theta = \frac{248.8}{126.1} = 1.97304$$

$$\theta = 63°07' 21''$$

Whole circle bearing of AB

$$= 360° - 63° 07' 21''$$

$$= 296° 52' 39'' \quad \textbf{Ans.}$$

$$\text{Distance } AB = \sqrt{\Delta E^2 + \Delta N^2} = \sqrt{(248.8)^2 + (126.1)^2}$$

$$= 278.93 \text{ m.} \quad \textbf{Ans.}$$

Example 12.6 *A traverse is run to set out a line MQ = 1900 m at right angles to a given line MN. The Lengths and bearings of traverse legs are observed as follows :*

Line	*Length*	*Bearings*
MN	—	360°00′
MO	850	120°00′
OP	1000	86°30′
PQ	—	––

Compute the length and bearing of PQ.

Solution. (Fig. 12.27)

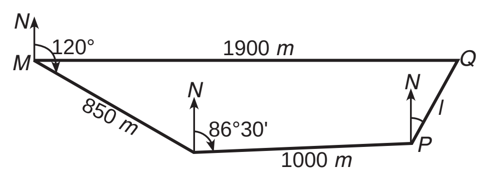


Fig. 12.27.

The line MQ being perpendicular to MN whose bearing is 360°.

The bearing of MQ is 90° and that of QM = 270°.

Latitude of QM = 1900 cos 90° = 0.00 m

Departure of QM = 1900 sin 90° = – 1900.00 m

Latitude of MO = 850 cos 60° = – 425.00 m

Departure of MO = 850 sin 60° = + 736.12 m

Latitude of OP = 1000 cos 86° 30′ = + 61.05 m

Departure of OP = 1000 sin 86° 30′ = + 998.14 m

Let l be the length and θ be the reduced bearing of PQ

Latitude of PQ $= l \cos \theta$

Departure of PQ $= l \sin \theta$

As $QMOP$ is a closed circuit,

Σ Latitude = 0 and Σ Departure = 0

i.e. $0.00 - 425.00 + 61.05 + l \cos \theta = 0$

$l \cos \theta = 425.00 - 61.05 = +363.95$ m ...(*i*)

Similarly $-1900 + 736.12 + 998.14 + l \sin \theta = 0$

$l \sin \theta = 1900 - 736.12 - 998.14 = +165.74$ m ...(*ii*)

Dividing Eq. (*ii*) by Eq. (*i*) we get

$$\therefore \quad \tan \theta = \frac{165.74}{363.95} = 0.45539222$$

$$\theta = 24° \, 29' \, 03''$$

As the latitude and departure of line PQ are both positive, the line PQ lies in the first quadrant

W.C.B. of PQ $\theta = 24° \, 29' \, 03''$ **Ans.**

$\therefore$ Length of $PQ = \sqrt{(165.74)^2 + (363.95)^2} = 399.91$ m,

$= 333.91$ m **Ans.**

Example 12.7. *The lengths and bearings of a traverse ABCD are as follows :*

Line	*Lengths (in metres)*	*Bearings*
AB	250.5	30° 15′
BC	310.4	145° 30′
CD	190.2	222° 15′

Calculate the length and bearing of the line DA.

Solution. (Fig. 12.28)

Reduced bearing of

$AB = 30° \, 15' =$ N30′ 15′ E

Reduced bearing of

$BC = 180° - 145° \, 30'$

$=$ S34° 30′ *E*

Reduced bearing of

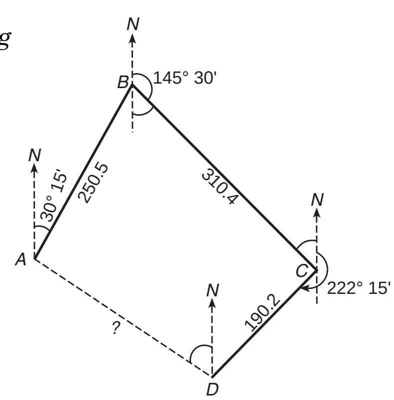


Fig. 12.28.

$CD = 222° 15' - 180°$

$= S42° 15' W$

Calculation of latitudes and departures :

Latitude of

$AB = 250.5 \times \cos 30° 15' = 250.5 \times 0.863836 = +216.39$ m

Departure of

$AB = 250.5 \times \sin 30° 15' = 250.5 \times 0.503774 = +126.20$ m

Latitude of $BC = 310.4 \times \cos 34° 30' = 310.4 \times 0.824126$

$= -255.81$ m

Departure of $BC = 310.4 \times \sin 34°, 30' = 310.4 \times 0.566406$

$= +175.81$ m

Latitude of $CD = 190.2 \times \cos 42° 15' = 190.2 \times 0.740218$

$= -140.79$ m

Departure of $CD = 190.2 \times \sin 42° 15' = 190.2 \times 0.672367$

$= -127.88$ m.

Total Latitude of $D = 216.39 - 255.81 - 140.79 = -180.21$ m

Total departure of $D = 126.20 + 175.81 - 127.88 = +174.13$ m

Let l = length ofthe line DA

θ = reduced bearing of the line Da.

Since the traverse is a closed circuit, the algebraic sum of the latitudes and that of the departures should be each equal to zero.

i.e. $l \cos \theta - 180.21 = 0$

or $l \cos \theta = 180.21$

and $l \sin \theta + 174.13 = 0$

or $l \sin \theta = -174.13$

$$= \tan \theta = \frac{-174.13}{180.21} = -0.96626158$$

$$\theta = 44° 01' 01''$$

$\therefore$ W.C. bearing of $DA = 360° - 44° 01'01''$

$= 315° 58' 59''$

Length of $DA = \sqrt{(\text{Lat.})^2 + (\text{Dep.})^2}$

$= 250.59$ m. **Ans.**

Example 12.8. *It is not possible to measure the length and fix the direction of line AB directly on account of an obstruction between the stations A and B. A traverse ACDB was, therefore, run and following data was obtained :*

Line	*Length in m*	*Reduced bearing*
AC	45	*N* 50° *E*
CD	66	*S* 70° *E*
DB	60	*S* 30° *E*

Find the length and direction of line BA. It was also required to fix a station E on line BA such that line DE will be perpendicular to BA. If there is no obstruction between stations B and E, calculate the data required for fixing the station as required.

Graphical solution will not be accepted.

Solution. (Fig. 12.29)

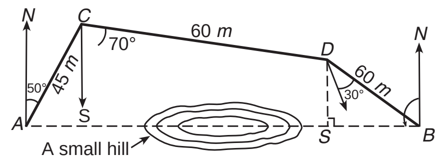


Fig. 12.29.

(*i*) Let the coordinates of *A* be (0, 0).

Calculation of Consecutive Coordinates :

Lat. of *AC* = 45 cos 50° = + 28.92 m

Dep. of *AC* = 45 sin 50° = + 34.47 m

Lat. of *CD* = 66 cos 70° = − 22.57 m

Dep. of *CD* = 66 sin 70° = − 62.02 m

Lat.of *DB* = 60 sin 30° = − 51.96 m

Dep. of *DB* = 60 sin 30° = − 30.00 m

Total latitude of *B* = 28.92 − 22.57 − 51.96 = − 45.61 m

Total departure of *B* = 34.47 + 62.02 + 30.00 = + 126.49 m

Let θ be the reduced bearing of *BA*

$$\tan \theta = \frac{126.49}{45.61} = 2.773295$$

$$\theta = 70° \; 10' \; 18''$$

Reduced bearing of *BA* = *N* 70° 10′ 18″ *W*. **Ans.**

Distance $BA = \sqrt{(126.49)^2 + (45.61)^2}$

= 134.46 m **Ans.**

(*ii*) Drop *DE* perpendicular to *AB*.

Bearing of $DB = S\ 30° E$ (Given)

Bearing of $BD = N\ 30° W$

Bearing of *BA* or $BE = N\ 70° 10' 18'' W$.

$\therefore$ Angle $EBD = 70° 10' 18'' - 30° = 40° 10' 18''$

From right angled triangle *BED* we get

$$BE = BD \cos 40° 10' 18'' = 60 \times 0.764115$$

$$= 45.85 \text{ m.}$$

To locate point *E* on *BA*, proceed as under :

1. Set out an angle $DBA = 40° 10'18''$
2. Measure a length of 45.85 m along the line of sight to get the location of *E*.

Example 12.9. *The following observations were made to provide planimetric control in hilly terrain where direct linear measurements were not possible.*

RD	= 75.0 *m*
Angle DRH	= 125° 19′ 20″
Angle RHS	= 118° 07′ 10″
Angle HST	= 89° 46′ 20″
Angle STH	= 38° 51′ 40″
Angle SDH	= 27° 03′ 20″
Angle DHR	= 42° 40′ 50″
Bearing of RD	= 90° 00′ 00″

Calculate the independent co-ordinates of the stations R, H, S and T if the co-ordinates of the station D are (E 5000, N 4000) metres.

Also, calculate the distance and bearing of the side DT

Solution. (Fig. 12.30)

From $\Delta\, RHD$, we get

Angle $RHD = 180° - (\angle HRD + \angle RDH)$

$$= 180° - (125° 19' 20'' + 42° 40' 50'') = 11° 59' 50''$$

From $\Delta\, HST$, we get

Angle $SHT\ 180° - (\angle HST + \angle STH)$

$$= 180° - (89° 46' 20'' + 38° 51' 40'')$$

$$= 51° 22' 00''$$

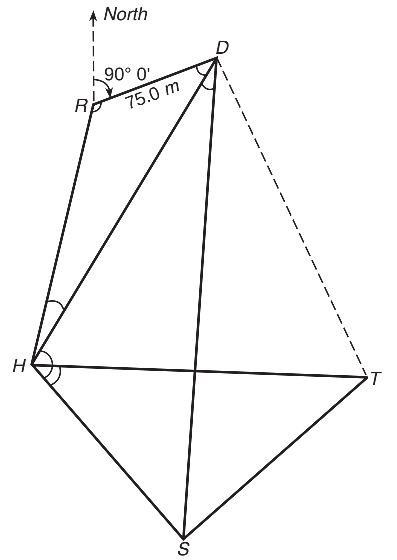


Fig. 12.30

$\therefore$ Angle $DHT = \angle RHS - (\angle RHD + \angle THS)$

$$= 118° 07' 10'' - (11° 59' 50'' + 51° 22' 00'')$$

$$= 54° 45' 20''$$

Applying sine rule to $\Delta\, RHD$, we get

$$\frac{RH}{\sin 42° 40' 50''} = \frac{HD}{\sin 125° 19' 20''} = \frac{75.0}{\sin 11° 59' 50''}$$

$$RH = \frac{75.0 \times \sin 42° 40' 50''}{\sin 11° 59' 50''} = \frac{75.0 \times 0.67791}{0.207864}$$

$$= 244.60 \text{ m.}$$

$$HD = \frac{75.0 \times \sin 125° 19' 20''}{\sin 11° 59' 50''} = \frac{75.0 \times 0.815914}{0.207864}$$

$$= 294.39 \text{ m}$$

Applying sine rule to $\Delta\, DHS$, we get

$$\frac{HS}{\sin 27° 03' 20''} = \frac{SD}{\sin 106° 07' 20''} = \frac{HD}{\sin 46° 49' 20''}$$

$$HS = \frac{294.39 \sin 27° 03' 20''}{\sin 46° 49' 20'}$$

$$= \frac{294.39 \times 0.454854}{0.729234} = 183.62 \text{ m}$$

$$SD = \frac{294.39 \sin 106° \, 07' \, 20''}{\sin 46° \, 49' \, 20''}$$

$$= \frac{294.39 \times 096072}{0.729230} = 387.82 \text{ m}$$

Applying sin rule to Δ *HST*, we get

$$\frac{ST}{\sin 51° \, 22' \, 00''} = \frac{HT}{\sin 89° \, 46' \, 20''} = \frac{HS}{\sin 38° \, 51' \, 40''}$$

$$ST = \frac{183.62 \times \sin 51° \, 22' \, 00''}{\sin 38° \, 51' \, 40''}$$

$$= \frac{183.62 \times 0.781157}{0.627435} = 228.61 \text{ m}$$

$$HT = \frac{183.62 \times \sin 89° \, 46' \, 20''}{\sin 38° \, 51' \, 40''}$$

$$= \frac{183.62 \times 0.999992}{0.627435} = 292.65 \text{ m}$$

Bearing of *RD* = 90° 00′ 00″

Add angle *DRH* + 125° 19′ 20″

Bearing of *RH* = 215° 19′ 20″

Add angle *RHS* + 118° 07′ 10″

Sum = 333° 26′ 30″

Subtract 180° – 180° 00′ 00″

Bearing of *HS* = 153° 26′ 30″

Add angle *HST* + 89° 46′ 20″

Sum = 243° 12′ 50″

Subtract 180° – 180° 00′ 00″

Bearing of *ST* = 63° 12′ 50″

Calculation of consecutive co-ordinates

Departure of *DR* = – 75.00 m

Latitude of *DR* = 0.00 m

Departure of *RH* = 244.60 sin 35° 19′ 20″ = 244.6 × 0.578174

= – 141.42 m

Latitude of *RH* = 244.60 cos 35° 19′ 20″ = 244.6 × 0.815914

= – 199.57 m

Departure of $HS = 183.62 \sin 26° \, 33' \, 30'' = 183.62 \times 0.447109$
$= + 82.10$ m

Latitude of $HS = 183.62 \cos 26° \, 33' \, 30'' = 183.62 \times 0.89448$
$= - 164.24$ m

Departure of $ST = 228.61 \sin 63° \, 12' \, 50'' = 228.7 \times 0.892695$
$= + 204.08$ m

Latitude of $ST = 228.61 \cos 63° \, 12' \, 50'' = 228.7 \times 0.450661$
$= 103.03$ m

Co-ordinates of D	= 5000.00	4000.00 m
Subtract	= – 75.00	0.00
Co-ordinates of R	= 49.25.00	4000.00
Subtract	= – 141.42	199.57
Co-ordinates of H	= 4783.58	3800.43
Add	+ 82.10	– 164.24
Co-ordinates of T	= 5069.68	3636.19
Add	+ 204.08	+ 103.03
Co-ordinates of T	= 5069.76	3739.22

Δ E = 5069.76 – 5000 = 69.76 m

Δ N = 3739.22 – 4000 = – 260.78 m

$\therefore \quad DT = \sqrt{(69.76)^2 + (260.78)^2} = 269.95$ m. **Ans.**

Let θ be the reduced bearing of TD

$$\tan \theta = \frac{\Delta E}{\Delta N} = \frac{69.76}{260.78} = 0.26750517$$

$$\theta = 14° \, 58' \, 35''$$

$\therefore$ Bearing of $DT = 180° - 14° \, 58' \, 35'' = 165° \, 01' \, 25''$. **Ans.**

Example 12.10. *A closed traverse ABCD has the following lengths and bearings.*

Line	*Length*	*Bearing*
AB	*200.0 m*	*Roughly east*
BC	*98.0 m*	178°
CD	*Not obtained*	270°
DA	*86.4 m*	1°

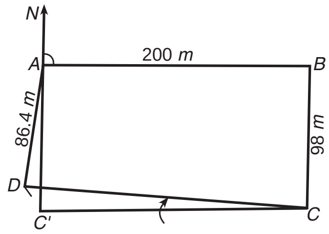


Fig. 12.31.

The length CD could not be measured due to some obstruction to chaining. The bearing of AB could not be taken as the station A is badly affected by local attraction. Find the exact bearing of the side AB and calculate the length CD.

Solution. (Fig. 12.31).

The affected side *AB* and *CD* are not adjacent. To make them adjacent, draw *AC* parallel and equal to *BC*. Line *CC′* is thus equal and parallel to *AB*. The line *C′D* becomes the closing line of the traverse *DAC′*.

The Latitude and departure of *C′D* are calculated as under :

Line	*Length*	*Bearing*	*Reduced Bearing*
DA	86.4 m	1°	*N* 1° *E*
AC′	98.0 m	178°	*S* 2° *E*

Latitude of *DA* = 86.4 cos 1°

= 86.4 × 0.999848 = + 86.39 m

Departure of *DA* = 86.4 sin 1°

= 86.4 × 0.0174525 = + 1.51 m

Latitude of *AC′* (or *BC*) = 98.0 sin 2°

= 98.0 × 0.99939 = – 97.94 m

Departure of *AC′* (*or BC*) = 98.0 sin 2°

= 98.0 × 0.0348995 = + 3.42 m

∴ Total latitude of *C′* = 86.39 – 97.94 = – 11.55 m

and Total departure of *C′* = 1.51 + 3.42 = 4.93 m

For R.B. of *C′D*, tan θ $= \dfrac{\text{Departure}}{\text{Latitude}} = \dfrac{4.93}{11.55}$

∴ = 0.42683982

θ = 23° 06′ 53″

∴ Reduced bearing of *C′D* = **N** 23° 06′ 53″ **W**

and W.C.B. of *C′D* = 360° – 23° 06′ 53″ = 336° 53′ 07″

Angle *CDC′* = β = 90° – 23° 06′ 53″ = 66° 53′ 07″

Length *C prime D* $= \sqrt{(11.55)^2 + (4.93)^2}$

$= \sqrt{133.6336 + 243049}$

= 12.56 m

By applying sine rule to Δ CDC', we get

$$\frac{CC'}{\sin\beta} = \frac{DC'}{\sin\alpha}$$

or $$\sin\alpha = \frac{DC'}{CC'} \times \sin\beta$$

$$= \frac{12.56}{200.00} \times \sin 66° 53' 07''$$

$$= 0.057758475$$

or $$\alpha = 3° 18' 40''$$

Bearing of CC' $= 270° - 3° 18' 40'' = 266° 41' 20''$

$\therefore$ Bearing of AB $= 266° 41' 20'' - 180° = 86° 41' 20''$

In Δ $CC'D$, Angle $CC'D = 180° - (66° 53' 07'' + 3° 18' 40'')$

$$= 109° 48' 13''$$

$$\frac{CC'}{\sin\beta} = \frac{CD}{\sin 109° 48' 13''}$$

or $$CD = \frac{CC' \times \sin 109° 48' 13''}{\sin 66° 53' 07''}$$

$$= \frac{200 \times 0.9408590}{0.9197721} = 204.60$$

Length of CD = 204.60 m

Bearing of AB = 86° 41′ 20″ **Ans.**

Example 12.11. *A and B are two stations of a location traverse, their independent co-ordinates in metres are :*

Independent co-ordinates	*Latitude*	*Departure*
A	*27456.8*	*6007.2*
B	*26936.0*	*7721.6*

A straight reach of railway is to run from C, roughly south of A to D, invisible from C and roughly north of B, the perpendicular offsets to the railway are : AC = 104 m and BD = 57.6 m.

Calculate the bearing of CD.

Solution. (Fig. 12.32)

Let CD and AB intersect at M; $\angle CAM = \angle MBD = \theta$

From Δ s ACM and BDM, we get

$$AM \cos\theta + MB \cos\theta = 104 + 57.6$$

or $$AB \cos\theta = 161.6 \text{ m} \quad ...(i)$$

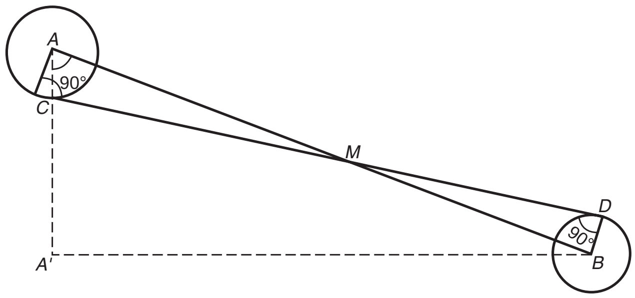


Fig. 12.32

Δ Lat. = 27456.8 – 26936.0 = 520.8 m

Δ Dep. = 7721.6 – 6007.2 = 1714.4 m

$$AB = \sqrt{\Delta\ \text{Dep.}^2 + \Delta\ \text{Lat.}^2}$$

$$= \sqrt{(1714.4)^2 + (520.8)^2}$$

$$= 1791.76 \text{ m}$$

Substituting the value of AB in Eqn. (i)

$$\cos\theta = \frac{161.6}{1791.76} = 0.09019065$$

$$\therefore \qquad \theta = 84°\ 49'\ 31''$$

Let the reduced bearing of AB be α

$$\therefore \qquad \tan\alpha = \frac{\Delta\ \text{Dep.}}{\Delta\ \text{Lat.}} = \frac{1714.4}{520.8} = 3.2918586$$

$$\alpha = 73°\ 06'\ 08''$$

$$\therefore \qquad \angle A'AC = 84°\ 49'\ 31'' - 73°\ 06'\ 08'' = 11°\ 43'\ 23''$$

Hence, Bearing of $CD = 90° + 11°\ 43'\ 23'' = 101°\ 43'\ 23''$ **Ans.**

Example 12.12. *The details of a part of a theodolite traverse survey are as under.*

Line	*Length*	*Bearing*
AB	*200*	300° 20′
BC	*500*	25° 30′
CD	*300*	145° 30′

Calculate the distance between a point P on AB 60 m from A and a point Q on CD 250 m from C and also determine the bearing of line PQ.

Solution. (Fig. 12.33)

PBCQ may be assumed as a closed traverse.

Assume co-ordinates of P as (0, 0)

PB = 200 – 60 = 140 m

1. Consecutive co-ordinates of B.

Latitude = 140 cos 59° 40′

= 140 × 0.505030 = 70.70(+)

Departure = 140 sin 59° 40′

= 140 × 0.863102 = 120.83(–)

2. Consecutive co-ordinates of C.

Latitude = 500 cos 25° 30′

= 500 × 0.902585 = 451.29(+)

Departure = 500 sin 25° 30′

= 500 × 0.430511 = 215.26(+)

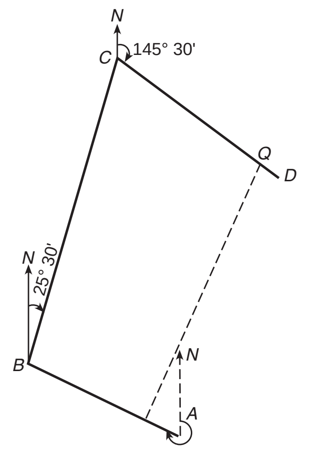


Fig. 12.33.

3. **Consecutive co-ordinates of Q.**

Latitude = 250 cos 34° 30′

= 250 × 0.824126 = 206.03(–)

Departure = 250 sin 34° 30′ = 250 × 0.566406 = 141.60(+)

Independent co-ordinates of Q

Latitude = 70.70 + 451.29 – 206.03 = 315.96 m

Departure = – 120.83 + 215.26 + 141.60 = 236.03 m

Let bearing of *PQ* be θ

$$\tan\theta = \frac{\Delta \text{ Dep.}}{\Delta \text{ Lat.}} = \frac{236.03}{315.96} = 074702493$$

or $$\theta = 36°\ 45'\ 38''$$

∴ W.C.B of $PQ = 36°\ 45'\ 38''$

Distance $PQ = \sqrt{(315.96)^2 + (236.03)^2}$

$= 394.39$ m **Ans.**

Example 12.13. *ABCD is a closed traverse in which the bearing of AD has not been observed and the length of BC has been missed to be recorded. The rest of the field record is as follows :*

Line	*Bearing*	*Length (m)*
AB	181° 18′	*335*
BC	90° 00′	*?*
CD	357° 36′	*408*
DA	*?*	*828*

Calculate the bearing of AD and the length of BC.

Solution. (Fig. 12.34)

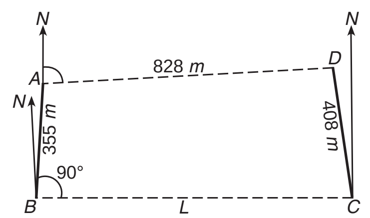


Fig. 12.34.

Let the bearing of *AD* be θ and length of *BC* be *L*

Consecutive co-ordinates of B :

Latitude = 335 cos 1° 18′ = 335 × 0.999743 = – 334.91

Departure = 335 sin 1° 18′ = 335 × 0.0226874 = – 7.60

Consecutive co-ordinates of C :

Latitude = *L* cos 90° = *L* × 0.000 = 0

Departure = *L* sin 90° = *L* × 1.000 = *L*

Consecutive co-ordinates of D :

Latitude = 408 cos 2° 24′ = 408 × 0.999123 = + 407.64

Departure = 408 sin 2° 24′ = 4 + 0.0419 = – 17.09

Consecutive co-ordinates of A :

Latitude = 828 cos θ = 828 cos θ = 828 sin θ

Departure = 828 sin θ = 828 sin θ = – 828 sin θ

The traverse being a closed traverse

Σ Dep. = – 7.60 + *L* – 17.09 + 828 sin θ = 0 ...(*i*)

Σ Lat. = – 334.91 + 0 + 407.64 – 828 cos θ = 0 ...(*ii*)

or 828 cos θ = 72.73

$$\cos\theta = \frac{72.73}{828} = 0.087838164$$

or θ = 84° 57′ 39″ **Ans.**

Substituting the value of θ in Eqn. (*i*)

– 7.60 + *L* – 17.09 – 828 × sin 84° 57′ 39″ = 0

– 7.60 + *L* – 17.09 – 828 × 0.996135 = 0

∴ *L* = 7.60 + 17.09 + 824.80

or *BC* = **848.69 m** **Ans.**

Example 12.14. *A straight highway is to be constructed between two points A and B which are not intervisible due to intervening undulations. Two points M and N, approximately in line with AB are established on the intervening undulations by triangulation. The total co-ordinates of the points are tabulated below :*

Pont	*Easting (m)*	*Northing (m)*
A	*24,723.0*	*15857.9*
B	*20,235.2*	*9025.8*
M	*23,926.4*	*14512.2*
N	*20,996.2*	*10119.2*

Calculate the perpendicular offsets MC and ND. Find the angles AMC and BND and also suggest the method to locate the points on the ground.

Solution. (Fig. 12.35)

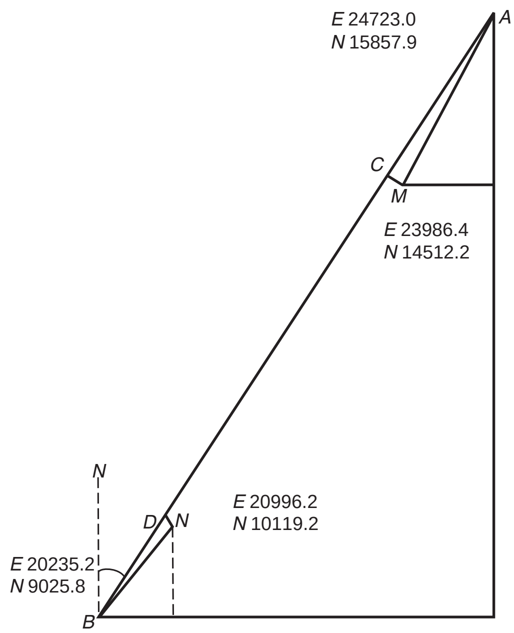


Fig. 12.35.

Calculation of the bearing of AB

Difference in eastings = 24723.0 – 20235.2 = 4487.8 m

Difference in northings = 15857.9 – 9025.8 = 6832.1 m

If θ° is the bearing of the line *AB,* then

$$\tan \theta = \frac{\Delta E}{\Delta N} = \frac{4487.8}{6832.1} = 0.65686977$$

$$\theta = 33° 17' 59''$$

$\therefore$ Bearing of $AB = 33° 17' 59'' + 180° 0' 00''$

$= 213° 17' 59''$

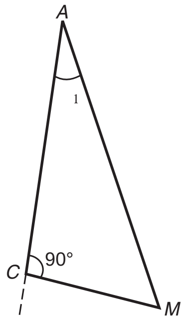


Fig. 12.36.

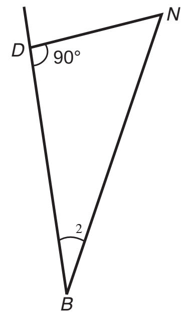


Fig. 12.37.

Calculation of the bearing of AM

Difference in eastings = 24723.0 – 23926.4 = 796.6 m

Difference in northings = 15857.9 – 14512.2 = 1345.7 m

If α is the bearing of line *MA*, then

$$\tan\alpha = \frac{\Delta E}{\Delta N} = \frac{796.6}{1345.7} = 0.59195957\ \alpha$$

$$= 30° 37' 26''$$

$\therefore$ Bearing of the line *AM*

$$= 30° 37' 26'' + 180° 00' 00''$$

$$= 210° 37' 26''$$

In the right angled triangle *ACM*, (Fig. 12.38)

Angle *CAM* (α_1) = bearing of *AC* – bearing of *AM*

$$= 213° 17'59'' - 210° 37' 26''$$

$$\alpha_1 = 2° 40' 33''$$

$$AM = \sqrt{(796.6)^2 + (1345.7)^2}$$

$$= 1563.8 \text{ m}$$

$\therefore$ Perpendicular offset $MC = AM \sin \alpha_1\ 2° 40' 33''$

$$= 1563.8 \times 0.0466852$$

$= 73.01$ m. **Ans.**

Angle $CMA = 90° - 2° 40' 33'' = 87° 19' 27''$

Calculation of the bearing of BN

Difference in eastings = 20996.2 – 20235.2 = 761.0 m

Difference in northings = 10119.2 – 9025.8 = 1093.4 m

If β is the bearing of BN, then

$$\tan \beta = \frac{\Delta E}{\Delta N} = \frac{761.0}{1093.4} = 0.69599414$$

$$\beta = 34° \, 50' \, 16''$$

$\therefore$ Bearing of $BN = 34° \, 50' \, 16''$

In a right angled triangle *BDN,* (Fig. 12.39)

Angle DBN (α_2) = Bearing of BN – Bearing of BD

$$= 34° \, 50' \, 16'' - 33° \, 17' \, 59'' = 1° \, 32' \, 17''$$

$$BN = \sqrt{(761.0)^2 + (1093.4)^2}$$

$$= 1332.16 \text{ m}$$

$\therefore$ Perpendicular offset ND

$$= BN \sin 1° \, 32' \, 17''$$

$$= 1332.16 \times 0.026841$$

$$= 35.76 \text{ m.}$$ **Ans.**

Locations of C and D may be obtained as under :

Set up the theodolite over station *M,* centre and level it carefully. Intersect the arrow held vertically over the station mark at *A* and make the horizontal circle reading 0° 00′ 00″. Loosen the upper plate and swing the theodolite till the reading on the horizontal circle becomes 360° – (87° 19′ 27″) = 272° 40′ 33″. Ask a staffman to walk along the line of sight defined by the telescope and measure a distance 73.01 m along it from station *M*. Point *C* thus fixed will lie on the straight line *AB*.

Similarly, the other point *D* may be located on the ground by making observations from station *N*.

Example 12.15. *A helicopter flies in sky from A to E as per following conditions :*

(*a*) *5 km along a* 5° *up gradient in east direction upto B*

(*b*) *3 km along a* 3° *up gradient in north direction upto C*

(*c*) *4 km along a* 4° *up gradient in N.W. direction upto D*

(*d*) *4 km along a* 4° *down gradient in south direction upto E.*

Calculate its distance, bearing and gradient to reach starting point A. If the co-ordinates of the starting point A are : E 2000 m, N 1000 m and its height above datum 100 m, calculate the total co-ordinates and elevations of the points B, C, D and E.

Soltuion. (Fig. 12.38)

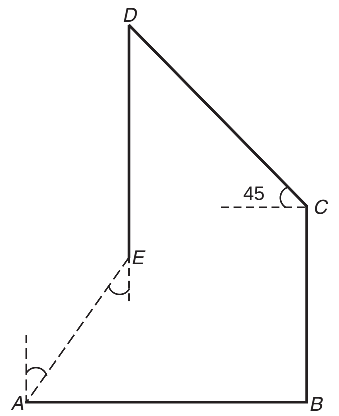


Fig. 12.38

Horizontal distance AB $= 5000 \cos 5°$

$= 5000 \times 0.996195 = 4980.98$ m

Horizontal distance BC $= 3000 \cos 3°$

$= 3000 \times 0.99863 = 2995.89$ m

Horizontal distance CD $= 4000 \cos 4°$

$= 4000 \times 0.997564 = 3990.26$ m

Horizontal distance DE $= 4000 \cos 4°$

$= 4000 \times 0.997564 = 3990.25$ m

Consecutive co-ordinates of B

$\Delta E = 4980.98$ m ; $\Delta N = 0.00$ m

Consecutive co-ordinates of C

$\Delta E = 0.00$ m ; $\Delta N = 2995.89$ m

Consecutive co-ordinates of D

$\Delta E = 3990.26 \times \cos 45° = 3990.26 \times 0.707107 = -2821.54$

$\Delta N = 3990.26 \times \sin 45° = 3990.26 \times 0.707107 = +2821.54$

Consecutive co-ordinates of E

$\Delta E = 0.00$ m ; $EN = -3990.26$ m

$\Sigma \Delta E = 4980.98 - 0.000 - 2821.54 + 0.000 = 2159.44$ m

$\Sigma \Delta N = 0.00 + 2995.89 + 2821.54 - 3990.26 = 1827.17$ m

$\therefore$ Horizontal distance $AE = \sqrt{\Delta E^2 + \Delta N^2}$

$= \sqrt{(2159.44)^2 + (1827.27)^2}$

$= 2828.73$ m **Ans.**

Let θ be the reduced bearing of EA

$$\tan \theta = \frac{\Delta E}{\Delta N} = \frac{2159.44}{1827.17} = 1.1818495$$

$$\theta = 49° \; 45' \; 52''$$

$\therefore$ R.B. Bearing of $EA = S \; 49° \; 45' \; 52''$ W

Height of point B = height of point $A + AB \tan 5°$

$= 100.00 + 4980.98 \times 0.087488$

$= 535.78$ m

Height of point C = height of point $B + BC \tan 3°$

$= 535.78 + 2995.89 \times 0.0524078$

$= 692.79$m

Height of point D = height of point $C + CD \tan 4°$

$= 692.79 + 3990.26 \times 0.0699269$

$= 971.82$ m

Height of point F = height of point $D - DE \tan 4°$

$= 971.82 - 3990.26 \times 0.0699269$

$= 692.79$ m

$\therefore$ Difference in heights of A and E

$= 692.79 - 100.00$

$= 592.79$ m

Gradient of EA $= \dfrac{2828.73}{592.79} = 4.77$

i.e., = **1 in 4.77 downward. Ans.**

Calculation of total co-ordinates :

Co-ordinates of A	E 2000.00	N 1000.00
	+ 4980.98	0.00
Co-ordinates of B =	6980.98	1000.00

	0.0	+2995.89
Co-ordinates of C =	6980.98	3995.89
	– 2821.54	+2821.54
Co-ordinates of D =	4159.44	6817.43
	+ 0.00	-3990.26
Co-ordinates of E =	4159.44	2827.17

Point	*Easting*	*Northing*	
A	2000.00	1000.00	
B	6980.98	1000.00	
C	6980.98	3995.89	**Ans.**
D	4159.44	6817.43	
E	4159.44	2827.17	

Example 12.16. (*a*) *A man travels from a point A due to west and reaches the point B. The distance between A and B = 139.6 m. Calculate the latitude and departure of the line AB.*

(*b*) *What is closing error in a 'theodolite traverse' ? How would you distribute the closing error graphically ?*

(*c*) *In a closed traverse, 'Latitudes and Departures' of the sides were calculated and it was observed that :*

Σ *Latitude* = *1.39 m*

Σ *Departure* = *– 2.17 m*

Calculate the length and bearing of the closing line.

Solution. (Fig. 12.39).

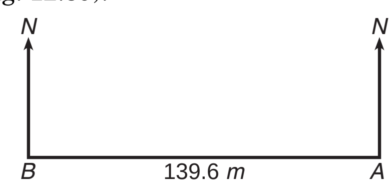


Fig. 12.39.

(*a*) As the point *B* lies exactly due west, the bearing of the line AB = 270° or its reduced bearing = 90° *W*

$\therefore$ Latitude of the line $AB = AB \cos 90°$

$= 139.6 \times 0 = 0.000$ m

Departure of the line $AB = AB \sin 90°$

$= 139.6 \times 1 = 139.6$ m

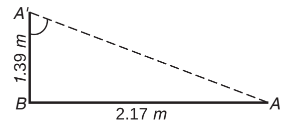


Fig. 12.40

Latitude of AB = 0.000 m

Departure of AB = – 139.6 m **Ans.**

(b) For the procedure of distributing the closing error in a theodolite traverse graphically, refer to page 608.

Let A be the starting station and A' be the closing location of A respectively. $A'A$ is the required closing error.

Σ Latitude = + 1.39 m; Σ Departure = 2.17 m

$\therefore$ Distance $AA' = \sqrt{(1.39)^2 + (2.17^2)} = 2.58$ m

Let θ be the reduced bearing of the closing line $A'A$

i.e. $\tan\theta = \dfrac{2.17}{1.39} = 1.561151$

$\theta = 57° 21' 30''$

W.C.B. of the closing line $A'A = 180° - 57° 21' 30''$

$= 122° 38' 30''$ **Ans.**

Example 12.17. *Uncorrected lengths and bearings of the legs of a closed traverse, are tabulated under* :

Leg	*AB*	*BC*	*CD*	*DE*	*EA*
Length(m)	316.0	650.5	189.0	442.0	334.5
Bearing	20° 31′ 30″	357° 16′ 00″	120° 04′ 00″	188° 27′ 30″	213° 31′ 00′

It is the suspected that one of the values of the lengths is erroneous. Find the error.

Solution. (Fig. 12.41)

Leg	*Length*	*R.B.*	*Latitude*		*Departure*	
			+	–	+	–
AB	316.0	*N* 20° 31′ 30″ *E*	295.94		110.79	
BC	650.5	*N* 2° 44′ 00″ *W*	649.76			31.02
CD	189.0	*S* 59° 56′ 00″ *E*		94.69	163.57	
DE	442.0	*S* 8° 27′ 30″ *W*	437.19		65.01	
EA	334.5	*S* 33° 31′ 00″ *W*	278.88		184.70	
		Sum	945.70	810.76	274.36	280.73
			810.76			274.36
			+134.94			– 6.37

$$\tan \alpha = \frac{6.37}{134.94} = 0.0472062$$

$$\alpha = 2° \ 42' \ 10''$$

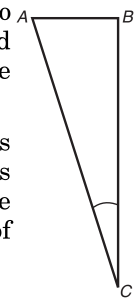


Fig. 12.41

As the bearing of line *BC* is very approximately equal to α, line *BC* is in error. As seen from the table of latitudes and departures the length is to be reduced so as to diminish the excessive latitude by equating Σ Latitudes to zero.

Note. If there is a large error in linear measurements due to failure to book one full chain length, then so long as only one such error has been made, it will be seen that the bearing of the closing error, will be similar to the bearing of the erroneous line.

Latitude of *BC* $= +(810.76 - 295.94)$

$= +514.82$ m

Departure of *BC*

$= +274.36 - (65.01 + 184.70)$

$= +(274.36 - 249.71) = +24.65$ m.

$\therefore$ Length of *BC* $= \dfrac{514.82}{\cos 2° \ 44'} = 515.40$ m

The length of *BC* should be 515.40 m. **Ans.**

Example 12.18. *Due to certain reasons it was not possible to measure the length and bearing of AB. Following lengths and bearings were, therefore, taken from X and Y.*

Line	*Bearing*	*Length*
XA	200° 0′	202 m
XY	338° 36′	579 m
YB	227° 36′	183 m

Find the angles XAY and YBA, and, length and bearing of AB.

Solution. (Fig. 12.42)

Assume the traverse *AXYB* originating from *A* and closing at *B*, having closing line *BA*.

Reduced bearing of

$XA = 200° - 180° = S\ 20°\ W$

$\therefore$ Reduced bearing of

$AX = N\ 20°\ E$

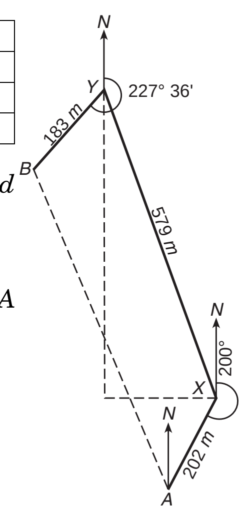


Fig. 12.42.

Reduced bearing of XY = 360° – 338° 36′ = N 21° 24′ W

Reduced bearing of YB = 227° 36′ – 180° = S 47° 36′ W

(i) Latitude of AX = AX cos 20°

= 202 × 0.939693

= 189.82 (+)

Departure of AX = AX sin 20°

= 202 × 0.342020

= 69.09 (+)

(ii) Latitude of XY = XY cos 21° 24′ = 579 × 0.931056

= 539.08 (+)

Departure of XY = XY sin 21° 24′ = 579 × 0.364877

= 211.26 (–)

(iii) Latitude of YB = YB cos 47° 36′ = 183 × 0.674302

= 123.40 (–)

Departure of YB = YB sin 47° 36′ = 183 × 0.738455

= 135.14 (–)

Total Latitude of B = 189.82 + 539.08 – 123.40 = 605.5 (+)

Total Departure of B = 69.09 – 211.26 – 135.14 = 277.31 (–)

If θ is the reduced bearing of AB, then

$$\tan\theta = \frac{\text{Total departure of } B}{\text{Total latitude of } B}$$

$$= \frac{277.31}{605.5} = 0.45798513$$

$$\therefore \quad \theta = 24° \ 36' \ 26''$$

W.C. Bearing of AB = 360° – 24° 36′ 26″ = 335° 23′ 34″

$$\text{Distance} \quad AB = \sqrt{(277.31)^2 + (605.5)^2}$$

$$= \sqrt{443531.08} = 665.98 \text{ m}$$ **Ans.**

If α is the R.B. of AY, then

$$\tan\alpha = \frac{\text{Total departure of Y}}{\text{Total latitude of Y}}$$

$$= \frac{69.09 - 211.26}{189.82 + 539.08}$$

$$= \frac{-142.17}{728.90} = 0.19404733$$

or $\alpha = 11° \ 02' \ 12''$

$\therefore$ Bearing of AY

$= 360° - 11° \ 02' \ 12'' = 348° \ 57' \ 48''$

Exterior angle XAY = Bearing of AY – Bearing of AX

$= 348° \ 57' \ 48'' - 20° \ 00' = 328° \ 57' \ 48''$

Angle $XAY = 360° - 328° \ 57' \ 48'' = 31° \ 02' \ 12''$ **Ans.**

Angle YBA = Bearing of BA – Bearing of BY

$= (335° \ 23' \ 34'' - 180° \ 0') - (227° \ 36' - 180° \ 0')$

$= 155° \ 23' \ 34'' - 47° \ 36' \ 00''$

$= 107° \ 47' \ 34''$ **Ans.**

Angle $XAY = 31° \ 02' \ 12''$

Angle $YBA = 107° \ 47' \ 34''$ **Ans.**

Length of AB = 665.98 m

Bearing of $AB = 335° \ 23' \ 34''$

Example 12.19. *From two points A and B on either side of a creek, two traverses were conducted which are as follows* :

Traverse 1.

Line	*Length(m)*	*Bearing*
BA	?	*270° 00′ 00″*
AM	*1000*	*323° 07′ 48″*
MT_1	*1140.2*	*344° 44′ 41″*

Traverse (*2*)

AB	?	*90° 00′ 00″*
BN	*1655.3*	*25° 01′ 00″*
NT_2	*1612.4*	*352° 52′ 30″*

Calculate the width of the creek if coordinates of towers T_1 and T_2 are :

Tower	*Easting(m)*	*Northing(m)*
T_1	*2100*	*2900*
T_2	*5000*	*4100*

Solution. (Fig. 12.43)

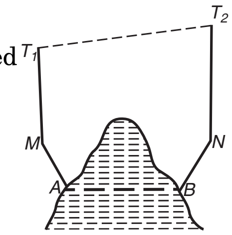


Fig. 12.43.

$ABNT_2T_1MA$ may be assumed as a closed traverse whose side AB is unknown.

Difference in eastings of T_1 and

$$= 5000 - 2100 = 2900 \text{ m}$$

Difference in northings of T_1 and T_2

$$= 4100 - 2900 = 1200 \text{ m}$$

Let bearing of $T_1T_2 = \theta$,

where $$\tan \theta = \frac{\Delta E}{\Delta N} = \frac{2900}{1200} = 2.416667$$

or $$\theta = 67° \, 31' \, 14''$$

Distance $T_1T_2 = \sqrt{(2900)^2 + (1200)^2}$

$$= 3138.47 \text{ m}$$

The lengths and bearings of the traverse legs are tabulated here.

Side	*Length(m)*	*W.C.B.*	*R.B.*
BN	1655.3	25° 01′ 00″	N 25° 01′ 00″ E
NT_2	1612.4	352° 52′ 30″	N 7° 07′ 30″ W
T_2T_1	3138.47	247° 31′ 14″	S 67° 31′ 14″ W
T_2M	1140.2	164° 44′ 41″	S 15° 15′ 19″ E
MA	1000.0	143° 07′ 48″	S 36° 52′ 12″ E

Calculation of consecutive co-ordinates :

Latitude of

$BN = 1655.3 \cos 25° \, 01' \, 00'' = 1655.3 \times 0.906185 = +1500.00$ m

Latitude of

$NT_2 = 1612.4 \cos 7° \, 07' \, 30'' = 1612.4 \times 0.992278 = +1600.00$ m

Latitude of

$T_2T_1 = 3138.47 \cos 67° \, 31' \, 14'' = 3138.47 \times 0.382352 = -1200.00$ m

Latitude of

$T_1M = 1140.2 \cos 15° \, 15' \, 19'' = 1140.2 \times 0.964763 = -1100.0$ m

Latitude of

$MA = 1000.0 \cos 36° \, 52' \, 12'' = 1000 \times 0.799999 = -800.0$ m

∴ Total Latitude of

$A = 1500 + 1600 - 1200 - 1100 - 800 = 0$

Departure of

$BN = 1655.3 \sin 25° \, 01' \, 00'' = 1655.3 \times 0.422882 = +700.0$ m.

Departure of

$NT_2 = 1612.4 \sin 7° \, 07' \, 30'' = 1612.4 \times 0.124034 = -200.0$ m

Departure of

$T_2T_1 = 3138.47 \sin 67° \, 31' \, 14'' = 3138.47 \times 0.924017 = -2900.00$ m

Departure of

$T_1M = 1140.2 \sin 15° \, 15' \, 19'' = 1140.2 \times 0.263120 = +300.0$ m

Departure of

$MA = 1000 \sin 36° \, 52' \, 12'' = 1000 \times 0.600001 = +600.0$ m

$\therefore$ Total departure of

$A = 700 - 200 - 2900 + 300 + 600 = 1500$

$\therefore$ Side $AB = \sqrt{0^2 + (1500)^2}$

or width of creek = 1500 m. **Ans.**

Example 12.20. *Surveyor X is standing 1 km north of surveyor Y on a road which runs along the meridian of both the surveyors.*

Surveyor X moves to A, 511.40 m, N 50° E; then to B, 58.0 m S 25° E and finally to C, 394 m, S 30° W.

Surveyor Y moves to M, 366.0 m s 45° E: then to N 520.00 m N 30° E and finally to O, 436.24 m N 45° W.

Ascertain whether C, O and P are in a straight line where P is the middle point of the initial positions of X and Y. Also, prove that the distance between their final positions is 251.57 m.

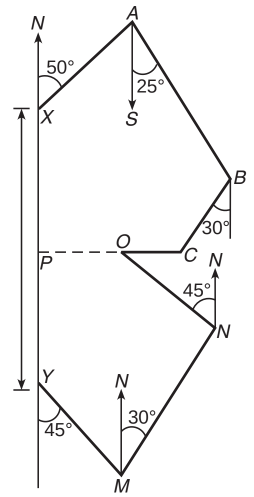


Fig. 12.44

Solution. (12.44)

PXABCP may be considered as a closed traverse No. 1.

PYMNOP may be considered as a closed traverse No. 2.

Consecutive co-ordinates of both the traverses may be computed and tabulated as under :

Traverse 1

Line	*Length (m)*	*R.B.*	*Latitude*		*Departure*	
			N (+)	*S*(–)	*E*(–)	*W*(–)
PX	500.00	*N*	500.00	––	––	––
XA	511.40	*N*50° *E*	328.72	––	391.75	––
AB	580.00	*S* 25° *E*	––	525.66	254.12	––
BC	394.94	*S* 30° *W*	––	303.06	––	174.97
CP	461.90	90° *W*	––	––	––	461.90
		Total	828.72	828.72	636.87	636.87

After calculating the consecutive co-ordinates, algebraic sum of the latitudes must be equal to zero.

∴ The Departure of line *CP* = 636.87 – 174.97 = 461.90 m *i.e.* surveyor *X* has to move 461.90 m westward to reach the middle point *P*.

Traverse 2

Line	*Length (m)*	*R.B.*	*Latitude*		*Departure*	
			N (+)	*S*(–)	*E*(–)	*W*(–)
PY	500.00	*S*	––	500.00	––	––
YM	366.00	*S* 45° *E*	––	258.80	258.80	––
MN	520.00	*N* 30° *E*	450.33	––	260.00	––
NO	436.24	*N* 45° *W*	308.47	––	––	308.47
OP	210.33	––	––	––	––	210.33
		Total	758.80	758.80	518.80	518.80

After calculating the consecutive co-ordinates, the algebraic sum of northings must be equal to zero.

∴ The departure of line *OP* = 518.80 – 308.47 = 210.33 m. *i.e.* surveyor *Y* also has to move westward by 210.33 m. to reach the middle point *P*.

Because the algebraic sum of latitudes of both the traverses are each the zero. Points *P, O* and *C* lie along east-west direction.

Distance between the final positions *C* and *O*

= 461.90 – 210.33

= 251.57 m. **Ans.**

Example 12.21. *From a common point A, traverses are conducted on either side of a harbour as follows* :

Traverse 1

Line	*Length* (*m*)	*Bearing*
AB	200	85° 26′ 20″
BC	100	125° 10′ 40″
	Traverse (2)	
AD	225	175° 50′ 00″
DE	500	85° 06′ 40″

Calculate the distance from C to a point F on DE due south of C and the distance EF.

Solution. Fig. (12.45)

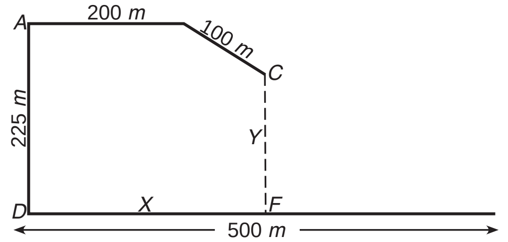


Fig. 12.45.

CBADFC may be considered as a closed traverse. The lengths and bearings of sides are as under :

Side	*Length* (*m*)	*W.C.B.*	*R.B.*
CB	000	305° 10′ 40″	*N* 54° 49′ 20″ *W*
BA	200	265° 26′ 20″	*S* 85° 26′ 20″ *W*
AD	225	175° 50′ 00″	*S* 04° 00′ 00″*E*
DF	?	85° 06′ 40″	*N* 85° 06′ 40″ *E*
FC	?	0° 00′ 00″	*N*.

Calculation of Departures

Dep. of $B = 100 \sin 54° \, 49' \, 20'' = 100 \times 0.8173684 = -81.737$ m

Dep. of $A = 200 \sin 85° \, 26' \, 20'' = 200 \times 0.996833 = -199.367$ m

Dep. of $D = 225 \sin 4° \, 10' \, 00'' = 225 \times 0.0726579 = +16.348$ m

Dep. of $F = X \sin 85° \, 06' \, 40'' = X \times 0.9963618 = 0.9963618X$

Dep. of $C = Y \sin 0° \, 00' \, 00'' = Y \times 00 = 0.000$

For a closed traverse algebraic sum of departures must be zero

i.e. $0.9963618X + 16.348 - 199.367 - 81.737 = 0$

or $0.9963618X = 264.756$

or $X = \dfrac{264.756}{0.9963618} = 265.723$ m.

∴ Distance $EF = 500 - 265.723 = 234.277$ m. **Ans.**

Calculation of Latitudes :

Lat. of $B = 100 \cos 54° 49' 10'' = 100 \times 0.5761153 = +57.612$ m

Lat. of $A = 200 \cos 85° 26' 20'' = 200 \times 0.0795223 = -15.904$ m

Lat. of $D = 225 \cos 4° 10' 00'' = 225 \times 0.9973569 = -224.405$ m

Lat. of

$F = 265.723 \cos 85° 06' 40'' = 265.723 \times 0.0852236 = +22.646$ m

Lat. of $C = Y \cos \theta° = +Y$

For a closed travese algebraic sum of latitudes must be zero

i.e. $57.612 - 15.904 - 224.405 + 22.646 + Y = 0$

or $Y = -57.612 + 15.904 + 224.405 - 22.646 = 160.05$ m

or Distance of $CF = 160.05$ m. **Ans.**

Example. 12.22. *In order to fix a point F, exactly between A and E, following traverse was run.*

Side	*Length*	*Bearing*
AB	*400*	*330° 0′*
BC	*350*	*0° 0′*
CD	*350*	*31° 11′*
DE	*400*	*319° 30′*

Assuming the co-ordinates of A as (500, 500), calculate :

(*a*) *The independent co-ordinates of C, E and F.*

(*b*) *The length and bearing of CF.*

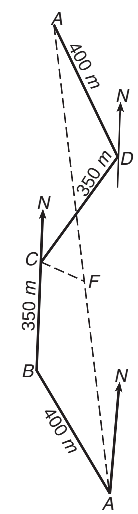


Fig. 12.46.

Solution. (Fig. 12.46)

R.B. of $AB = 360° - 330° 0' = N 30°$ W

R.B. of $BC = 0° 0' = N$

R.B. of $CD = 31° 11' = N 31° 11' E$

R.B. of $DE = 360° - 319° 30' = N 40° 30' W$

Calculation of consecutive co-ordinates :

Latitude of

$AB = AB \cos 30° = 400 \times 0.86603$

$= +346.41$ m

Departure of

$AB = AB \sin 30° = 400 \times 0.5 = -200.00$ m

Latitude of

$$BC = BC \cos 0° = 350 \times 1.0 = +350.00 \text{ m}$$

Departure of

$$BC = BC \sin 0° = 350 \times 0.0 = 0.0 \text{ m}$$

Latitude of

$$CD = CD \cos 31° 11'$$

$$= 350 \times 0.85552 = +299.43 \text{ m}$$

Departure of

$$CD = CD \sin 31° 11' = 350 \times 0.511778 = +181.22 \text{ m}$$

Latitude of

$$DE = DE \cos 40° 30'' = 400 \times 0.76041 = +304.16 \text{ m}$$

Departure of

$$DE = DE \sin 40° 30' = 300 \times 0.64945 = -259.78 \text{ m}$$

Total Latitude of

$$E = 346.41 + 350.00 + 299.43 + 304.16 = 1300.00 \text{ m}$$

Total Departure of

$$E = -200.0 + 0.0 + 181.22 - 259.78 = -278.56 \text{ m}$$

Easting of E = Easting of A + total departure = 500.00 − 278.56 = 221.44 m

Northing of E = Northing of A + total latitude = 500.0 + 1300 = 1800 m

Easting of $F = \frac{1}{2}$ (Easting of A + Easting of E)

$$= \frac{1}{2}(500.0 + 221.44)$$

$$= 360.72 \text{ m}$$

Northing of $F = \frac{1}{2}$ (Northing of A + Northing of E)

$$= (500 + 1800)$$

$$= 1150 \text{ m.}$$

Total Latitude of $C = 346.41 + 350.0 = +696.41$ m

Total departure of $C = -200.0 + 0.0 = -200.0$ m

Easting of C = Easting of A + total Departure of C

$$= 500 - 200 = 300 \text{ m}$$

Northing of C = Northing of A + total latitude of C

$$= 500.0 + 696.41 = 1196.41 \text{ m}$$

Difference of Eastings of C and F

$$300.0 - 360.72 = -60.72 \text{ m}$$

Difference of Northing of C and F

$$= 1196.41 - 1150.0 = +46.41 \text{ m}$$

Let the reduced bearing of CF be θ, then

$$\tan\theta = \frac{\Delta E}{\Delta F} = \frac{60.72}{46.41} = 1.3083387$$

or $\theta = 52° 36' 30''$

W.C.B. of $CF = 180° - 52° 36' 30''$

$= 127° 23' 30''$ **Ans.**

Distance $CF = \sqrt{(60.72)^2 + (46.41)^2}$

$= 76.43$ m. **Ans.**

Example 12.23. *The co-ordinates of three points A, B and C, which are equidistant from a point O, are as tabulated here under :*

Point	*Easting (m)*	*Northing (m)*
A	*5685.00*	*2550.00*
B	*3934.77*	*3364.34*
C	*3055.00*	*1680.00*

To establish a fourth point D such that ABCD is a perfect square, a traverse OEFD is carried out from a point O, equidistant from A, B and C, as under :

Side	*Length (m)*	*Bearing*
OE	925	140°
EF	460	210°
FD	?	?

Calculate the length and bearing of FD.

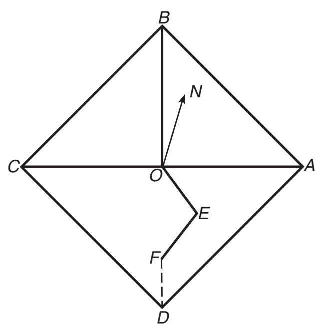


Fig. 12.47.

Solution. (Fig. 12.47).

Co-ordinates of O, the mid-point of AC are :

$$\text{Easting} = \frac{\text{Easting of } A + \text{Easting of } C}{2}$$

$$= \frac{5685.0 + 3055.0}{2} = 4370 \text{ m}$$

Northing $= \dfrac{\text{Nothing of } A + \text{Northing of } C}{2}$

$= \dfrac{255 + 1680}{2} = 2115 \text{ m}$

BD being the other diagonal let the co-ordinates of *D* be (*X, Y*).

$\therefore \quad \dfrac{X + 3934.77}{2} = 4370$

$X = 8740 - 3934.77 = 4805.23 \text{ m}$

and $\quad \dfrac{Y + 3364.34}{2} = 2115$

or $\quad Y = 4230 - 3364.34 = 865.66 \text{ m}$

Co-ordinates of *D* are : Easting 4805.23 ; Northing 865.66 m

Difference in Northings of points *D* and *O* $= 4805.23 - 4370.0$

$= 435.23 \text{ m}$

Difference in Northing of point *D* and *O* $= 865.66 - 2115$

$= -1249.34.$

Consecutive Co-ordinates of E :

Latitude $= 925 \cos 40° = 925 \times 0.766045 = -708.59 \text{ m}$

Departure $= 925 \sin 40° = 925 \times 0.642788 = +594.58 \text{ m}$

Consecutive Co-ordinates of F :

Latitude $= 460 \cos 30° = 460 \times 0.866026 = -398.37 \text{ m}$

Departure $= 460 \sin 30° \; 460 \times 0.50000 = -230.00 \text{ m}$

Let ΔE and ΔN be the latitude and departure of *FD*

$594.58 - 230.37 + \Delta E = 435.23$

or $\quad \Delta E = 435.23 - 594.58 + 230.00 = 70.65 \text{ m}$

and $\quad -708.59 - 396.37 + \Delta N = -1249.34$

or $\quad \Delta N = -149.34 + 708.59 + 398.37$

$= -142.38 \text{ m}$

$FD = \sqrt{(70.65)^2 + (142.38)^2} = 158.94 \text{ m.}$ **Ans.**

Let θ be the reduced bearing of *FD*, then

$$\tan\theta = \frac{\Delta E}{\Delta N} = \frac{70.65}{142.38} = 0.49620733$$

$$\theta = S\ 26°\ 23'\ 27''\ E$$

and Bearing of $\quad FD = 180° - 26°\ 23'\ 27''$

$$= 153°\ 36'\ 33'' \qquad \textbf{Ans.}$$

Example 12.24. *A, B, C and D are four pylones whose coordinates are as under :*

Pylone	*Easting (m)*	*Northing (m)*
A	1000.00	1000.00
B	1180.94	1075.18
C	1021.98	1215.62
D	939.70	1102.36

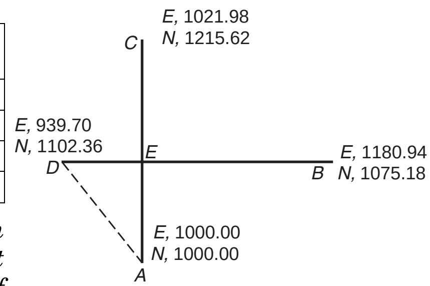


Fig. 12.48.

A ranging rod E is fixed such that ABC and BED are in straight lines. Calculate the coordinates of the ranging rod E.

Solution. (Fig. 12.48). As *AEC* and *BED* are straight lines *E* is situated on the intersection of *AC* and *BD*.

Difference in Eastings of *C* and *A*.

$$= 1021.98 - 1000.00 = +21.98 \text{ m}$$

Difference in Northings of *C* and *A*

$$= 1215.62 - 1000.00 = +215.62 \text{ m}$$

$$\therefore \qquad AC = \sqrt{(21.98)^2 + (215.62)^2} = 216.74 \text{ m}$$

Let α be the reduced bearing of *AC*

$$\therefore \qquad \tan\alpha = \frac{\Delta E}{\Delta N} = \frac{21.98}{215.62} = 0.101943859$$

or $\qquad \alpha = N\ 5°\ 49'\ 14''\ E$

W.C.B. of $\quad AC = S°\ 49''\ 14''$

Difference in Easting of *B* and *D*

$$= 1180.94 - 939.70 = 241.24 \text{ m}$$

Difference in Northings of *B* and *D*

$$= 1075.18 - 1102.36 = -27.18 \text{ m}$$

$$\therefore \qquad BD = \sqrt{(241.24)^2 + (27.18)^2} = 242.77 \text{ m}$$

Let β be the reduced bearing of BD,

$$\tan \beta = \frac{\Delta E}{\Delta N} = \frac{241.24}{27.18} = 8.8756438$$

or $$\beta = N\ 83^\circ\ 34'\ 18'\ W$$

W.C.B. of $BD = 360^\circ - 83^\circ\ 34'\ 18'' = 276^\circ\ 25'\ 42''$

Difference in Eastings of A and D

$$= 1000.00 - 939.70 = 60.30 \text{ m}$$

Difference in Northings of A and D

$$= 1102.36 - 1000.00 = 102.36 \text{ m}$$

$$AD = \sqrt{(60.30)^2 + (102.36)^2} = 118.80 \text{ m}$$

Let γ be the reduced bearing of AD

$$\tan \gamma = \frac{\Delta E}{\Delta N} = \frac{60.30}{102.36} = 0.5890973$$

$$\gamma = N\ 30^\circ\ 30'\ 08''\ W$$

W.C.B. of $AD = 360^\circ - 30^\circ\ 30'\ 08' = 329^\circ\ 29'\ 52''$

In $\Delta\ ADE$,

Angle DAE = Bearing of AC – Bearing of AD

$$= 360^\circ + 5^\circ\ 49'\ 14'' - 329^\circ\ 29'\ 52''$$

$$= 36^\circ\ 19'\ 22''$$

Angle EDA = Bearing of DA – Bearing of DE

$$= (329^\circ\ 30'\ 08'' - 180^\circ\ 00') - (276^\circ\ 25'\ 42'' - 180^\circ\ 00')$$

$$= 53^\circ\ 04'\ 26''$$

$\therefore$ Angle $AED = 180^\circ - (36^\circ\ 19'\ 22'' + 53^\circ\ 04'\ 26'')$

Applying sine rule to $\Delta\ AED$,

$$DE = \frac{AD \sin DAE}{\sin AED} = \frac{118.80 \sin 36^\circ\ 19'\ 22''}{\sin 90^\circ\ 36'\ 12''}$$

$$= \frac{118.80 \times 0.592333}{0.999945} = 70.37 \text{ m}$$

Latitude of $DE = DE \cos 83^\circ\ 34'\ 18''$

$$= 70.37 \times 0.11196 = 7.88 \text{ m} \quad (-\text{ve})$$

Departure of $DE = 70.37 \sin 83^\circ\ 34'\ 18''$

$$= 70.37 \times 0.993713 = 69.93 \text{ m} \quad (+\text{ve})$$

Co-ordinates of E :

Easting = Easting of D + departure

= 939.70 + 69.93 = 1009.63 m

Northing = Northing of D + Latitude

Co-ordinate of ranging rod : = 1102.36 − 7.88 = 1094.48 m

Easting = 1009.63 m

Northing = 1094.48 m **Ans.**

Example 12.25. *A theodolite traverse was conducted from a point A, south of a temple T_1 (E2550, N 1250) to a point H, north of another temple T_2 (E4000, N 1500) to determine the width of a river at a bridge site DE and the following observations were made :*

Side	Length	Angle
T_1A	?	T_1AB = 53° 07′ 48″
AB	250.00	ABT_1 = 63° 26′ 06″
T_1B	?	T_1BC = 153° 26′ 06″
BC	250.00	BCD = 143° 07′ 40″
CD	250.00	CDE = 216° 52′ 12″
DE	?	DEF = 116° 33′ 55″
EF	223.61	EFG = 187° 07′ 30″
FG	180.28	FGH = 222° 16′ 25″
GT_2	?	FGT_2 = 273° 10′ 47″
GH	206.16	GHT_2 = 284° 02′ 10″

Determine the width of the river.

Solution. (Fig. 12.49).

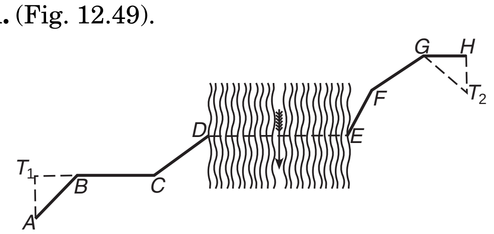


Fig. 12.49.

In Δ T_1AB,

Angle T_1AB = 53° 07′ 48″ ; Angle ABT_1 = 63° 26′ 06″

∴ Angle AT_1B = 180° − (53° 07′ 48″ + 63° 26′ 06″)

= 63° 26′ 06″

Applying sine rule, we get

$$T_1A = \frac{250 \sin 63° 26'' 06'}{\sin 63° 26' 06''} = 250 \text{ m}$$

Calculation of independent coordinates

Stn.	Length	Included Angle	W.C.B.	R.B.	Consecutive Co-ordinates				Independent Co-ordinates	
					E +	W –	N +	S –	E	N
T_1	—	—	180° 00′ 00″	S	...	...	...	...	2550	1250
A	250.00	53° 07′ 48″	53° 07′ 48″	N 53° 07′ 48″ E	...	...	...	250	2550	1000
B	250.00	216° 52′ 12″	90° 00′ 00″	E 90°	200	...	150	...	2750	1150
C	250.00	143° 07′ 48″	53° 07′ 48″	N 53° 07′ 48″ E	250	...	...	...	3000	1150
D	250.00	216° 52′ 12″	90° 00′ 00″	E 90°	200	...	150	...	3200	1300
E	?	206° 33′ 55″	26° 33′ 55″	N 26° 33′ 55″ E	ΔX	...	ΔY	...	3600	1300
F	223.61	187° 07′ 30″	33° 41′ 25″	N 33° 41′ 25″ E	100	...	200	...	3700	1500
G	180.28	222° 16′ 25″	75° 57′ 50″	N 75° 57′ 50″ E	100	...	150	...	3800	1650
H	206.16	284° 05′ 10″	180° 00′ 00″	S	200	...	50	...	4000	1700
T_2	200.00	—	—	—	...	...	...	200	4000	1500
				Sum	$1050 + \Delta Y$	...	$700 + \Delta Y$	450	...	...

In $\Delta\ GHT_2$,

Angle HGT_2 = Angle FGT_2 – Angle FGH

$= 273°\ 10'\ 47'' - 222°\ 16'\ 25'' = 50°\ 54'\ 22''$

Angle $GHT_2 = 360° - 284°\ 02'\ 10'' = 75°\ 57'\ 50''$

$\therefore$ Angle $GT_2H = 180°\ (50°\ 54'\ 22'' + 75°\ 57'\ 50'') = 53°\ 07'\ 48''$

Applying sine rule, we get

$$HT_2 = \frac{206.16 \sin 50°\ 54'\ 22''}{\sin 53°\ 07'\ 48''}$$

$$= \frac{206.16 \times 0.776114}{0.799999} = 200.0 \text{ m}$$

Calculations of independent co-ordinates of various points of the closed traverse $T_1 ABCDEFGHT_2$, are done in a tabular form on page 558.

Difference in Easting of T_2 and T_1

$= 4000 - 2550 = 1450$ m

Difference in Northings of T_2 and T_1

$= 1500 - 1250 = 250$ m

Let ΔX and ΔY be the consecutive co-ordinates of E.

$\therefore \quad 1050 + \Delta X = 1450$

or $\quad \Delta X = 1450 - 1050 = 400$ m

Again, $700 + \Delta Y - 450 = 250m$

or $\quad \Delta Y = 450 + 250 - 700 = 0.0$ m

$\therefore$ The width of the river at bridge site DE is 400 m . **Ans.**

Example 12.26. *Part of data and calculations in respect of a closed theodolite traverse ABCDA are as under :*

Line	*Length (m)*	*R.B.*	*Northing*	*Southing*	*Easting*	*Westing*
AB		*S* 60° *E*		30.00		
BC		*N* 45° *E*			49.50	
CD						
DA				51.65		63.15

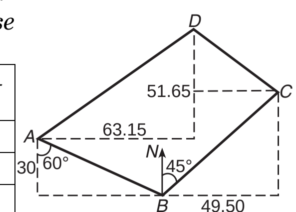


Fig. 12.50.

Complete the above table in all respects if there is no closing error for the traverse.

Solution. (Fig. 12.50).

Let L_{AB} be the length of side AB.

$$L_{AB} \cos 60° = 30°$$

or
$$L_{AB} = \frac{30}{\cos 60°} = 60 \text{ m.}$$

Departure of $AB = 60 \sin 60° = 51.96$ m

Bearing of $BC = 45°$

Latitude of BC = Departure of BC = 49.50 m

Let L_{BC} be the length of BC

∴
$$L_{BC} = \sqrt{(49.5)^2 + (49.5)^2} = 70.00 \text{ m}$$

Let L_{AD} be the length of side AD

∴
$$L_{AD} = \sqrt{(63.15)^2 + (51.65)^2} = 81.58 \text{ m}$$

Let θ be the reduced bearing of DA

$$\tan \theta = \frac{63.15}{51.65} = 1.22265$$

or
$$\theta = 50° \, 43' \, 14''$$

Let ΔE be the departure and ΔN be the latitude of CD *i.e.*

$$-63.15 + 51.96 + 49.50 + \Delta E = 0$$

or
$$\Delta E = 63.15 - 51.96 - 49.50 = -38.31 \text{ m}$$

Similarly, $-51.65 - 30.00 + 49.50 + \Delta N = 0$

or
$$\Delta N = 81.65 - 49.50 = 32.15 \text{ m.}$$

Let L_{CD} be the length of CD.

∴
$$L_{CD} = \sqrt{(32.15)^2 + (38.31)^2} = 50.01 \text{ m}$$

Let θ be the reduced bearing of CD

$$\tan \theta = \frac{32.15}{38.31} = 0.839206$$

or
$$\theta = 40°$$

∴ Reduced bearing of $CD = N40° \, W$.

The missing readings are shown under.

Line	*Length*	*R.B.*	*Northing*	*Southing*	*Easting*	*Westing*
AB	60.00	*S* 60° *E*		30.00	51.96	
BC	70.00	*N* 45° *E*	49.50		49.50	
CD	50.01	*N* 40° *W*	32.15			38.31
DA	81.58	*S* 50° 43′ 14″ *W*		51.65		63.15

Example 12.27. *Following table gives data of consecutive coordinates in respect of a closed theodolite traverse ABCDA.*

Stn.	*N*	*S*	*E*	*W*
A	300.75			200.50
B	200.25		299.25	
C		299.00	199.75	
D		200.00		300.50

From the above data calculate :

(*i*) *Magnitude and direction of closing error.*

(*ii*) *Corrected consecutive coordinates of station B, using transit rule ;*

(*iii*) *Independent coordinates of station B if those of A are (100, 100)*

Solution. (Fig. 12.51).

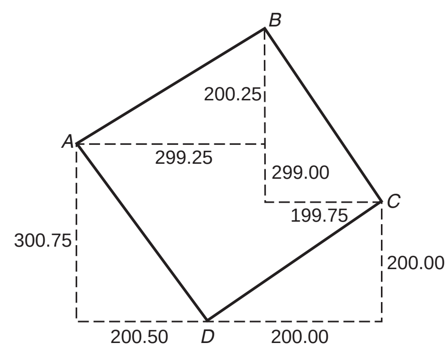


Fig. 12.51.

(*i*) Total error in latitude

$= 300.75 + 200.25 - 299.00 - 200.00$

$= 501.00 - 499.00 = +2.0$ m

Total error in departure

$= -200.50 + 299.25 + 199.75 - 300.50$

$= -501 + 499 = -2.0$ m.

$\therefore$ Magnitude of closing error $= \sqrt{2^2 + 2^2} = 2.828$ **Ans.**

Let θ be the reduced bearing of the closing line

$$\tan\theta = \frac{2}{2} = 1 \text{ or } 4\ \theta = 45°$$

$\therefore$ Bearing of the closing line $= 180° + 45° = 225°$ **Ans.**

(*ii*) Total latitude $= 300.75 + 200.25 + 299.00 + 200.00 = 1000.00$

Total departure $= 200.50 + 299.25 + 199.75 + 300.50 = 1000$ m

$\therefore$ Correction of latitude for $B = \dfrac{2.00 \times 200.25}{1000} = 0.4005$ m

Correction of departure for $B = \dfrac{2.00 \times 299.25}{1000} = 0.60$ m

As the error in latitudes is positive, the necessary correction is of negative sign.

$\therefore$ Corrected latitude of stn. $B = 200.25 - 0.40$

$= 199.85$ m

Again, the error in departure being negative, the correction is positive.

$\therefore$ Corrected departure of

$B = 299.25 + 0.60$

$= 299.85$ m.

Corrected consecutive coordinates of station B

Easting = 299.85 m

Northing = 199.85 m **Ans.**

(*iii*) Independent coordinates of station B

Easting = 100 + 299.85 = 399.85 m

Northing = 100 + 199.85 = 299.85 m **Ans.**

Example 12.28. *The following theodolite observations were made to two light points A [Easting 5720, Northing 3845], and B [Easting 5765, Northing 3785], on the roofs of tall buildings, and a third point C on the ground, from where points A and B appear to be in one straight line.*

Station of obsn.	*Point sighted*	*Horizontal circle reading*
D	*A*	5° 35′ 36″
	B	13° 49′ 35″
	C	69° 17′11″

If the observed bearing of DB is 34° 32′ 24″, calculate the co-ordinates of the points C and D.

Solution. (Fig. 12.52).

Difference of eastings of A and B

$= 5765 - 5720 = 45$ m

Difference of northings A and B

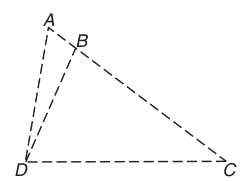


Fig. 12.52.

$= 3845 - 3785 = 60$ m

$\therefore \quad AB = \sqrt{(45)^2 + (60)^2} = 75$ m

Reduced bearing of $BA = \dfrac{45}{60} = 0.75$

$= N\ 36°\ 52'\ 11''\ W$

or $\quad W.C.B.$ of $BA = 323°\ 07'49''$

Bearing of $\quad BD = 34°\ 32'\ 24'' + 180° = 214°\ 32'\ 24''$

In $\Delta\, ABC$,

Angle ABD = Bearing of AB – Bearing of BD

$= 323°\ 07'\ 49'' - 214°\ 32'\ 24''$

$= 108°\ 35'\ 25''$

Angle ADB = Reading to B – Reading to A

$= 13°\ 49'\ 35'' - 5°\ 35'\ 36''$

$= 8°\ 13'59''$

Angle DAB $= 180° - [8°\ 13'\ 59'' + 108°\ 35'\ 25'']$

$= 63°\ 10'\ 36''$

Solving $\Delta\, ABD$, we get

$$BD = \frac{AB \sin DAB}{\sin ADB}$$

$$= \frac{75 \times \sin 63°\ 10'\ 36''}{\sin 8°\ 13'\ 59''}$$

$$= \frac{75 \times 0.8924021}{0.143999} = 467.39 \text{ m}$$

In $\Delta\, BCD$,

Angle BDC = Reading to C – Reading to B

$= 69°\ 17'\ 11'' - 13°\ 49'\ 35''$

$= 55°\ 27'\ 36''$

Angle DBC = Bearing of AB – Bearing of BC

$= 214°\ 32'\ 24'' - 143°\ 07'\ 49''$

$= 71°\ 24'\ 35''$

Angle DCB $= 180° - [55°\ 27'\ 36'' + 71°\ 24'\ 35'']$

$= 53°\ 07'\ 49''$

Solving $\Delta\, BCD$, we get

$$CD = \frac{BD \sin DBC}{\sin DCB}$$

$$= \frac{467.39 \sin 71° \, 24' \, 35''}{\sin 50° \, 07' \, 49''}$$

or $$CD = \frac{467.39 + 0.9478225}{0.8000018} = 553.75 \text{ m}$$

Departure of D from B

$= BD \sin 34° \, 32' \, 24'' = 467.39 \times 0.5669814$

$= 265.00$ m $\quad (-ve)$

Latitude of D from B

$= BD \cos 34° \, 32' \, 24'' = 467.39 \times 0.8237306$

$= 385.00$ m $= 385.00$ m $\quad (-ve)$

Co-ordinates of D are :

Easting : 5765 – 265 = 5500m

Northing : 3785 – 385 = 3400 m **Ans.**

Angle BDC = Reducing to C – Reading to$B$

$= 69° \, 17' \, 11'' - 13° \, 49' \, 35''$

$= 55° \, 27' \, 36''$

Bearing of DC = Bearing of DB + Angle BDC

$= 34° \, 32' \, 24'' + 55° \, 27' \, 36'' = 90° \, 00' \, 00''$

Departure of C from D

$= DC \sin 90° = 553.75 \times 1 = 553.75$ m

Latitude of C from D

$= DC \cos 90° = 553.75 \times 0 = 0.0$

Co-ordinates of C are :
Easting : 5500 + 553.75 = 6053.75 m **Ans.**
Northing : 3400 + 0.0 = 3400.00 m

Example 12.29. *The bearings of two inaccessible stations A and B taken from station C were 225° 00′ and 153° 26′ respectively. The co-ordinates of A and B were as under :*

Station	*Easting*	*Northing*
A	*300*	*200*
B	*400*	*150*

Calculate the independent co-ordinate of C.

Solution. (Fig. 12.53)

Difference in Eastings of stations A and B

$$= 400 - 300 = 100 \text{ m}$$

Difference in Northings of stations A and B

$$= 200 - 150 = 50 \text{ m}$$

$$\therefore \quad AB = \sqrt{(50)^2 + (100)^2} = 111.80 \text{ m.}$$

Let θ be the reduced bearing of AB

Fig. 12.53.

$$\theta = \tan^{-1} \frac{100}{50} = \tan^{-1} 2$$

$$= 63° \, 26' \, 06''$$

W.C. bearing of $AB = 180° - 63° \, 26' \, 06''$

$$= 116° \, 33' \, 54''$$

In $\Delta\, ABC$, $\angle CAB$ = Bearing of AB – Bearing of AC

$$= 116° \, 33' \, 54'' - 45° = 71° \, 33' \, 54''$$

{Bearing of} BA

$$= (180° + 153° \, 26' \, 00'') - (180° + 116° \, 33' \, 54'')$$

$$= 36° \, 52' \, 06''$$

$\angle ACB$ = Bearing of CA– Bearing of CB

225° 00 – 153° 26′ = 71° 34′ 00′

Applying sine rule, we get

$$AC = \frac{111.80 \times \sin 36° \, 52' \, 06''}{\sin 71° \, 34'}$$

$$= \frac{111.8 \times 0.599978}{0.948692} = 70.71 \text{ m.}$$

Latitude of $AC = AC \cos 45°$

$$= 70.71 \cos 45° = 50 \text{ m}$$

Departure of $AC = AC \sin 45° = 70.71 \sin 45° = 50$ m

Co-ordinates of C :

Easting = 300 + 50 = 350 **Ans.**

Northing = 200 + 50 = 250

Example 12.30. *The bearings of two lines AC and CB are respectively 55° 24′ 30″ and 170° 36′ 20″. The co-ordinates of stations A and B are as under :*

Station	*Easting (m)*	*Northing*
A	*300*	*400*
B	*750*	*150*

Calculate the co-ordinates of station C.

Solution. (Fig. 12.54)

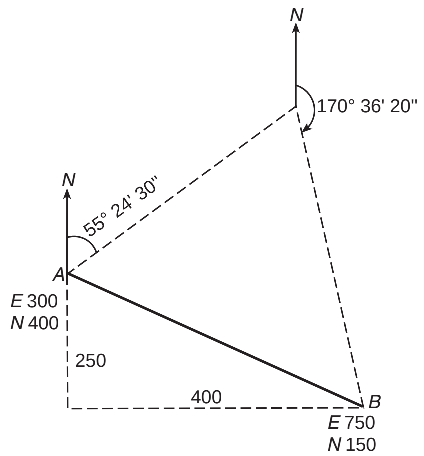


Fig. 12.54.

Bearing of AC $= 55° 24' 30''$

Bearing of CA $= 235° 24' 30''$

Angle ACB = Bearing of CA – Bearing of CB

$= 235° 24' 30'' - 170° 36' 20''$

$= 64° 48' 10''$

Difference of Eastings of A and B

$$\Delta N = 750 - 300 = 450 \text{ m}$$

Difference of Northings of A and B

$$\Delta E = 400 - 150 = 250 \text{ m}$$

Length of AB $= \sqrt{\Delta E^2 + \Delta N^2}$

$= \sqrt{450^2 + 250^2} = 514.78 \text{ m}$

Reduced bearing of AB be θ

$$\tan \theta = \frac{450}{250}$$

$$\theta = 60° 56' 43''$$

$\therefore$ Whole circle bearing of $AB = 180° - 60° 56' 43''$

$= 119° 03' 17''$

Included Angle CAB = Bearing of AB – Bearing of AC

$= 119° 03' 17'' - 55° 24' 30''$

$= 63° 38' 47''$

Angle $ABC = 180° - (64° 48' 10'' + 63° 38' 47'')$

$= 51° 33' 03''$

Applying sine rule to ΔABC, we get

$$\frac{AC}{\sin ABC} = \frac{BC}{\sin CAB} = \frac{AB}{\sin ACB}$$

$$\therefore \quad AC = \frac{AB \sin ABC}{\sin ACB} = \frac{514.78 \sin 51° 33° 03''}{\sin 64° 48' 10''} = 445.55 \text{ m}$$

$$BC = \frac{AB \sin CAB}{\sin ACB} = \frac{514.78 \sin 63° 38' 47'}{\sin 64° 48' 10''} = 509.79 \text{ m}$$

Consecutive coordinates of C with respect to A

Latitude (Northing) $= 445.55 \cos 55° 24' 30''$

$= 252.95$ m

Departure (Easting) $= 445.55 \sin 55° 24' 30''$

$= 366.79$ m

Easting of A = 300 + 366.79 = 666.79 m **Ans.**

Northing of B = 400 + 252.95 = 652.95 m

Example 12.31. *Two points* P_1 and P_2 *are to be located on-line AB such that* $AP_1 = 50$ *m* and $P_1 P_2 = 150$ *m. The coordinates of stations A and B are:*

Station	*Easting*	*Northing*
A	500	800
B	800	1200

Calculate the coordinates of P_1 and P_2.

Solution. (Fig. 12.55).

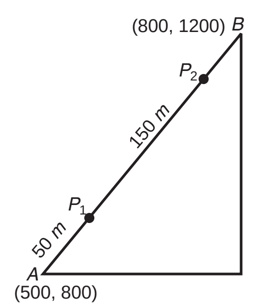


Fig.12.55.

The difference in the Eastings of A and B

$$\Delta E = 800 - 500 = 300$$

The difference in the Northings of A and B

$$\Delta N = 1200 - 800 = 400 \text{ m.}$$

The length of $AB = \sqrt{\Delta E^2 + \Delta N^2}$

$= \sqrt{300^2 + 400^2}$

$= 500$ m

$$\Delta E \text{ for } P_1 = \frac{AP_1}{AB} \times \Delta E = \frac{50}{500} \times 300$$

$$\Delta E = 30 \text{ m}$$

$$\Delta E \text{ for } P_2 = \frac{AP_2}{AB} \times \Delta E = \frac{200}{500} \times 300$$

$$\Delta E = 120 \text{ m}$$

$$\Delta N \text{ for } P_1 = \frac{AP_2}{AB} \times \Delta N = \frac{50}{500} \times 400 = 140 \text{ m}$$

$$\Delta N \text{ for } P_2 = \frac{AP_2}{AB} \times \Delta N = \frac{200}{500} \times 400 = 160 \text{ m}$$

Coordinates of P_1 :

Easting = 500 + 30 = 530 m

Northing = 800 + 40 = 840 m **Ans.**

Coordinates of P_2

Easting = 500 + 120 = 620 m

Northing = 800 + 160 = 960 m **Ans.**

12.10. LAND PARTITIONING (FIG. 12.56)

First Method. To divide the area of a plot in two portions in the given ratio, using a transit, the following steps are involved :

1. Observe the lengths and bearings of all sides of the closed traverse *ABCDEA*.
2. Calculate the independent co-ordinates of all corners of the plot.
3. Calculate the area of the plot by any method.
4. Assume one of the corners as the end of the dividing line say *A*.
5. Adopt the trial line *AD* and calculate its length and bearing.
6. Calculate the area of Δ *ADE*.
7. If the required area is more than the area of Δ *ADE*, the other end of the dividing line *AF* will be along *DC*. And if it is less, then the point *F* will be on line *DE*.
8. From Δ *AFD*, calculate *FD, i.e.*

   Area of $ADF \times \frac{1}{2} DF \times AD \sin. FDA$
9. Measure *DF* along *DC* and locate *F*.

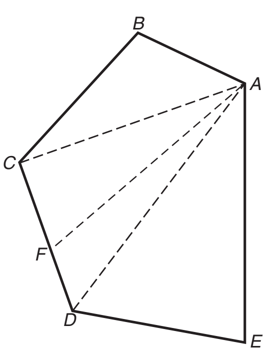


Fig. 12.56.

10. Join *AF* which divides the plot of land in the given ratio.

Second Method. (Fig. 12.56). In case a transit is not available, the area of the given plot may be divided into portions of given ratio, as explained below:

1. Measure all the sides of the plot *AB, BC. CD,* ... etc.
2. Choose one end of the dividing line at the required corner of the plot say *A*.
3. Measure *AC* and *AD* diagonally.
4. Measure the areas of Δ*s ABC, ACD* and *ADE*.
5. Let the area of Δ*ADE* be less than the area of the required portion.
6. Choose a point F on *DC* as under.

(*i*) Calculate the angle *CDA* from the formula

$$\tan \frac{D}{2} = \sqrt{\frac{(S - CD)\,(S - AD)}{S(S - AC)}}$$

(*ii*) $$\text{Area of } \Delta\, ADF = \frac{1}{2} FD.\, AD \sin FDA$$

7. Locate *F* by measuring *DF* along *DC*.

12.11. PRACTICAL PROBLEMS IN THEODOLITE SURVEYING

Solutions of simple difficulties for ranging and chaining distances, with the help of chain and optical square as discussed in chapter 3 'Chain Surveying' may also be employed rapidly and accurately by using a theodolite. To familiarise the students the geometrical methods of solving typical problems generally encountered in the field while setting out engineering works, are given below :

I. Obstacles which obstruct ranging. The following two important cases are generally encountered in the field.

Problem 1. *To measure the distance between two points A and B.*

Procedure. Proceed as follows :

(*i*) Select a point *C* from where both points *A* and *B* are visible and the angle subtended at *C* is approximately 60° . (Fig. 12.57).

(*ii*) Measure sides *AC* and *BC* of the Δ *ABC*. Also measure the angle *C*.

(*iii*) Calculate the values of the angles *A* and *B* by solving triangle *ABC*.

(*iv*) Centre the theodolite over stations *A* and *B* in turn and set out angles *CAB* and *CBA*.

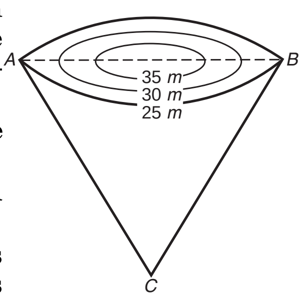


Fig. 12.57.

(*v*) Measure the distance *AB* along the ranged line.

Problem 2. *To determine the distance between two points AB far apart and not intervisible.*

Procedure. Proceed as under :

(*i*) Select two intervisible points *P* and *Q* from where both points *A* and *B* are visible. (Fig. 12.58).

(*ii*) Measure the distance between *P* and *Q*. Also measure

Angle $APB = \alpha_1$;

Angle $BPQ = \alpha_2$;

Angle $AQB = \beta_1$;

Angle $AQP = \beta_2$

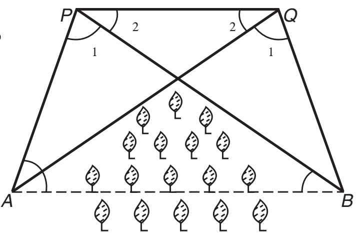


Fig. 12.58.

(*iii*) Calculate the sides *AP* and *AQ* by solving the triangle *APQ*. Similarly calculate the sides *BP* and *BQ* by solving the triangle *BPQ*.

(*iv*) Calculate the angle $PAB = \gamma$ and angle $ABP = \delta$ by solving the triangle *ABP*.

(*v*) Centre the theodolite over station *A* and set out the angle γ with reference to station *P*.

(*vi*) Shift the theodolite to station *B* and measure angle *ABP* which should evidently be equal to angle δ.

(*vii*) Calculate the distance *AB* by solving the trianlge ABP.

It may be noted that the accuracy of this method depends upon the distance between selected points *P* and *Q* and also on the perpendicular distance between *AB* and *PQ*.

II. Obstacles which abstruct chaining. The following four cases are generally met during setting out works in the field.

Problem 1. *To determine the distance between two points A and B, not intervisible, point B also being inaccessible.*

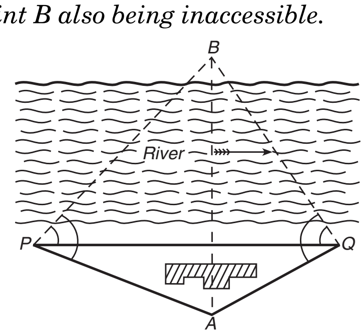


Fig. 12.59.

Procedure. Proceed as under :

(*i*) Select two intervisible points P and Q, such that B is visible from both P and Q. (Fig. 12.59).

(*ii*) Measure the distance AP, PQ and AQ. Also measure angle $BPA = \alpha$ angle $BPQ = \gamma$; angle $BQA = \beta$; angle $BQP = \delta$.

(*iii*) Calculate BP and PQ by solving $\Delta\, BPQ$.

(*iv*) Calculate AB by solving triangle ABP.

(*v*) Check the length of AB by solving the triangle ABQ.

Problem 4. *To determine the distance between points A and B both inaccessible.*

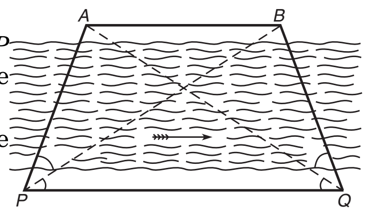


Fig. 12.60.

Procedure. (*i*) Select the points P and Q from where both A and B are visible. (Fig. 12.60).

(*ii*) Measure PQ. Also measure angle $APB = \alpha$:
angle $BPQ = \beta$
angle $AQP = \gamma$:
angle $AQB = \delta$.

(*iii*) Calculate AP and AQ by solving the triangle APQ

(*iv*) Calculate BP and BQ by solving the triangle BPQ.

(*v*) Calculate AB by solving the triangle ABP and check its value by solving the triangle ABQ.

III. Dropping perpendiculars and drawing parallel to given sides.

The following few cases are generally encountered in setting out works in field.

Problem 3. *To drop a perpendicular from the inaccessible point P on line AB.*

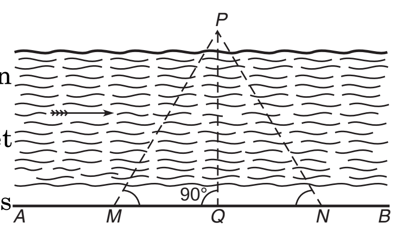


Fig. 12.61.

Procedure. (Fig. 12.61).

(*i*) Select two points M and N on AB.

(*ii*) Measure MN and also let $PMN = \alpha$; angle $PNM = \beta$.

(*iii*) Calculate the value of MQ as under.

Applying sine rule to $\Delta\, PNM$, we get

$$\frac{PM}{\sin\beta} = \frac{MN}{\sin\,[(180^\circ - (\alpha+\beta)]} = \frac{MN}{\sin\,(\alpha+\beta)}$$

or
$$PM = \frac{MN \sin\beta}{\sin\,(\alpha+\beta)}$$

$$MQ = PM \cos \alpha$$

$$= \frac{MN \sin \beta \cos \alpha}{\sin (\alpha + \beta)} \quad ...(1)$$

$$= \frac{MN \sin \beta \cos \alpha}{\sin \alpha \cos \beta + \cos \alpha \sin \beta}$$

or $$MQ = \frac{MN \tan \beta}{\tan \alpha + \tan \beta} \quad ...(2)$$

(*iv*) Measure MQ and locate the foot of the perpendicular.

Problem 4. *To set out a parallel line through a given inaccessible point P to the given line AB*

Procedure.

(*i*) Select two points M and N on given line AB from where point P is visible. (Fig. 12.62).

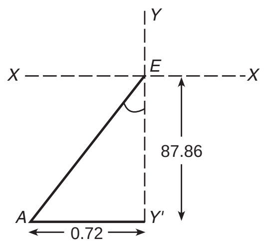


Fig. 12.62.

(*ii*) Measure MN and also angle $PMA = \alpha$; angle $PNA = \beta$

(*iii*) Calculate the lengths of perpendicular ML and NO as under :

Drop PQ, perpendicular to AB.

From ΔPQM, we get

$$PQ = QM \tan \alpha \quad ...(i)$$

From $\Delta\, PQN$, we get

$$PQ = QN \tan \beta \quad ...(ii)$$

Equating (*i*) and (*ii*) we get

$$QM \tan \alpha = QN \tan \beta = (QM + MN) \tan \beta = (QM + MN) \tan \beta$$

or $$QM = \frac{MN \tan \beta}{\tan \alpha - \tan \beta}$$

$\therefore$ $$PQ = \frac{MN \tan \beta \tan \alpha}{\tan \alpha - \tan \beta}$$

or $$PQ = \frac{MN}{\cos \beta - \cos \alpha}$$

(*iv*) Erect perpendiculars at M and N equal to length PQ.

Example 12.32. *The following are the consecutive coordinates of a closed traverse ABCD. Balance the survey traverse by (a) Bowditch rule (b) Transit rule*

Line	*Length*	*N*	*S*	E	*W*
AB	751	750.84	—		15.297
BC	392	177.56	—	349.48	—
CD	561	—	551.0	105.45	—
DA	579.4	—	387.4	—	459.633
Perimeter	2283.4	928.40	938.40	454.93	474.93

Solution. We notice that difference of Northing and Southing = 10 m and the difference of Eastings and Westings = 20 m.

(*a*) *Adjustment by the Bowditch rule*

Correction to latitude of anyside

$$= \text{Total error in latitude} \times \frac{\text{Length of the side}}{\text{Perimeter of the traverse}}$$

$$\text{Correction to } AB = \frac{10 \times 751}{2283.4} = 3.289$$

$$BC = 10 \times \frac{392}{2283.4} = 1.717$$

$$CD = 10 \times \frac{561}{2283.4} = 2.457$$

$$DA = 10 \times \frac{579.4}{2283.4} = 2.537 \text{ m}$$

$$\text{Total} = 10.00 \text{ m}$$

The latitude of AB = 750.84 + 3.289 = 754.129

The latitude BC = 177.56 + 1.717 = 179.277

The latitude CD = 551.00 – 2.457 = 548.843

The latitude DA = 387.4 – 2.537 = 384.863

Check on Latitude: (754.129 + 179.277) – (548.543 + 384.863)

= 933.41 – 933.4 = 0

Correction to departure

$$= \text{Total error in departure} \times \frac{\text{length of the side}}{\text{Perimeter of the traverse}}$$

$$\text{Correction to } AB = 20 \times \frac{751}{2283.4} = 6.578$$

Correction to $BC = 20 \times \dfrac{392}{2283.4} = 3.433$

Correction to $CD = 20 \times \dfrac{561}{2283.4} = 4.913$

Correction to $DA = 20 \times \dfrac{579.4}{2283.4} = 5.075$

$\therefore$ The departure of AB = 15.297 – 6.578 = 8.719

The departure of BC = 349.480 + 3.433 = 352.913

The departure of CD = 105.450 + 4.913 = 110.363

The departure of DA = 459.633 – 5.075 = 454.558

Check on deparature: (352.913 + 110.363) – (8.719 + 454.558)

= 463.28 – 463.28 = 0 O.K.

(*b*) *Adjustment by Transit rule*

Correction to latitude

$$= \text{Total error in latitude} \times \frac{\text{Latitude of the line}}{\text{arithmetic sum of latitudes i.e. (N + S)}}$$

Correction to $AB = 10 \times \dfrac{750.84}{1866.80} = 4.022$

$BC = 10 \times \dfrac{177.56}{1866.80} = 0.951$

$CD = 10 \times \dfrac{551}{1866.00} = 2.952$

$DA = 10 \times \dfrac{387.4}{1866.80} = 2.075$

The latitude of AB = 750.84 + 4.022 = 754.862

The latitude of BC = 177.56 + 0.951 = 178.511

The latitude of CD = 551.0 – 2.92 = 548.048

The latitude of DA = 387.4 – 2.075 = 385.325

Check on latitudes : (754.862 + 178.511) – (548.048 + 385.325)

= 933.374 – 933.373 = 0.001 O.K.

Correction to departure = total error

$$\text{in deparature} \times \frac{\text{deparature of the line}}{\text{arithematic sum of departures i.e., (E + W)}}$$

Correction to $AB = 20 \times \dfrac{15.297}{929.86} = 0.329$

$$\text{Correction to } BC = 20 \times \frac{349.48}{929.86} = 7.517$$

$$\text{Correction to } CD = 20 \times \frac{105.45}{929.86} = 2.267$$

$$\text{Correction to } DA = 20 \times \frac{459.633}{929.86} = 9.886$$

Hence, the departure of AB = 15.297 – 0.329 = 14.968

BC = 349.48 + 7.517 = 356.997

CD = 105.45 + 2.267 = 107.717

DA = 459.633 – 9.886 = 449.747

Check on departures:(356.997 + 107.717)–(14.968 + 449.747) = 0

Example 12.33. *Calculate the independent coordinates of A, B, C and D of a closed traverse ABCD Given:*

Side	*Length (m)*	*W.C.B.*
AB	*751*	*358° 50′*
BC	*392*	*63° 04′*
CA	*561*	*169° 10′*
DA	*579.4*	*239° 22′*

Solution. Traverse table

Consecutive coordinates — Independent coordinates

Line	*Length (m)*	*W.C.B.*	*R.B.*	*Latitudes = l cos RB*		*Departure = l sin RB*		*N*	*S*	*E*	*W*	*Stn*
AB	751	358° 50′	*N* 01° 10 W	750.84	S	E	15.29	750.84			15.29	B
BC	392	63° 04′	*N* 63° 04′ E	177.56		349.48		938.40		334.183		C
CD	561	169° 10′	S 10° 50′ E		551.00	105.44		387.40		439.633		D
DA	579.4	239° 22′	S 59° 22′ W		295.33		498.54	0	0	0	0	A
			Sum	928.40	928.40	454.93	454.23					

Latitude = l cos RB	*Departure = l sin RB*
B = 751 × cos 01° 10′ = 750.84 *N*	751 × sin 01° 10′ = 15.297 *W*
C = 392 × cos 63° 04′ = 177.56 *N*	392 × sin 63° 04′ = 349.48 *E*
D = 561 × cos 10° 50′ = 551.00 *S*	561 × cos 10° 50′ = 105.45 *E*
A = 579.4 × cos 59° 22′ = 377.40 *S*	579.4 × cos 59° 22′ = 489.633 *W*

Independent Coordinates:

B = 750.84 N;	15.297 W
C = 750.84 + 177.56	– 15.597 + 349.480 = 334.183 E

= 928.40 N

D = 928.40 – 551.00 + 334.183 + 105.45 = 439.633 E

= + 377.40 N

A = 377.40 – 377.40 + 439.633 489.633

= 0 = 0

Example 12.33. *The length and bearing of the closing line EA of the following closed traverse ABCDEA are not known. Determine them.*

Line	*Length (m)*	*W.C.B.*	*R.B.*	*Latitudes = l cos R.B.*		*Departures = l sin R.B.*	
				N+	*S –*	*E+*	*W –*
AB	*204*	*87° 30′*	*N 87° 30′ E*	*8.90*	—	*203.80*	—
BC	*226*	*20° 20′*	*N 20° 20′ E*	*211.92*	—	*78.52*	—
CA	*187*	*280° 00′*	*N 80° W*	*32.48*	—		*184.16*
DE	*192*	*210° 30′*	*S 30° 30′ W*	—	*165.44*	—	*97.44*
EA	—	—	—	—	—	—	—
			Sum	*253.30*	*165.44*	*282.32*	*281.60*
			Difference		*= 87.86 S*	= 0.72 W	

The line EA lies in SW quadrant (Fig. 12.62.)

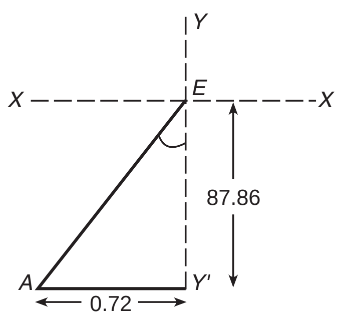


Fig. 12.62

$$\text{Tan reduced bearing } (\theta) = \frac{0.72}{87.86} = 0.0081948$$

$$\theta = 0° \; 28' \; 10''$$

∴ Reduced bearing of E = 0° 28′ W **Ans.**

Length of $EA = \sqrt{(87.86)^2 + (0.72)^2}$ = 87.86 m **Ans.**

Example 12.34. *The following lengths and bearings are recorded in running a theodolite traverse in anti-clock wise direction. The length of CD and bearing of DE could not be obtained. Find them.*

Line	*Length*	*W.C.B.*	*R.B.*
AB	*1970*	110° 49′	*S* 69° 11′*E*
BC	*906*	21° 49′	*N* 21° 49′*E*
CD	—	340° 26′	*N* 19° 34′*W*
DE	*1011*	—	—
EA	*1181*	254° 24′	*S* 74° 24′*W*

Solution. (Fig. 12.63.)

Fig. 12.63

Join *CE*. By using the standard formulae for eastings and northings, compute the length and bearing of *CE* from the closed traverse *ABCEA*.

Line	*N*+	*S* –	*E*+	*W*–
AB	—	700.10	1841.40	—
BC	841.11	—	336.71	—
CE	—	—	—	—
EA	—	317.59	—	1137.49
Sum	841.11	1017.69	2178.11	1137.49
	Diff. = 176.58 N		Diff. = 1040.62 E	

From the difference of northings and southings, and eastings and westing, we find that line *CE* lies in *NW* quadrant

$\therefore$ Length $CE = \sqrt{(176.58)^2 + (1040.52)^2} = 1056$ m

$$\text{Tan reduced bearing } \theta = \frac{\text{Departure}}{\text{Latitude}} = \frac{1040.62}{176.58} = 5.893$$

$\therefore$ $\theta = N\ 80°\ 22'$ W

In ΔCDE, angle ECD =

$$\alpha = (80°\ 22') - [360° - (340°\ 26')] = 60°\ 48'$$

Applying the cosine rule to ΔCDE, we get

$$ED^2 = CE^2 + CD^2 - 2CD \,.\, CE \cos ECD$$

$$(1011)^2 = CD^2 + (1056)^2 - 2 \times CD \times 1056 \cos 60°\ 48'$$

$\therefore$ $CD = 931.10$ m **Ans.**

Applying sine rule to ΔECD we get

$$\frac{\sin \beta}{1056} = \frac{\sin \alpha}{1011} = \frac{\sin 60°\ 48'}{1011}$$

$$\therefore \quad \sin \beta = 1056\, \frac{\sin 60°\ 48'}{1011} = 0.9117762$$

$$\beta = 65°\ 45'$$

$$\gamma = 180° - (60°\ 48' + 65°\ 45') = 53°\ 27'$$

Now, the bearing of $DE = \angle NDE = \angle NDC + \beta$

$= (340°\ 26' - 180°) + (65°\ 45')$

$= 160°\ 26' + 65°\ 45'$

$= 226°\ 11'$ *i.e.* S 46° 11′ W **Ans.**

Example 12.35. *The following traverse notes were taken during a field survey in which the bearings of lines BC and DE could not be taken. (Fig. 12.64). Compute the same.*

Line	*Length (m)*	*R.B.*
AB	*872*	*N 41° 35′ W*
BC	*322*	—
CD	*770*	*S 63° 12′ W*
DE	*406*	—
EF	*1079*	*S 26° 39′ E*
FA	*1489*	*N 53° 30′ E*

Solution. Lines BC and DE whose bearings could not be observed are not adjacent.

Bring CD parallel to itself at *ED′*. Join *BD′*. Now in the closed traverse ABD′EFA line *BD′* is having neither its length nor its bearing.

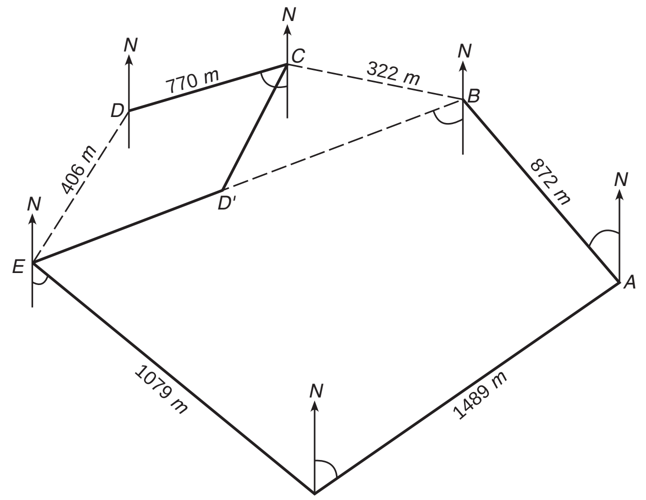


Fig. 12.64

Line	*Length (m)*	*R.B.*	*l cos RB*		*l sin RB*	
			N+	*S –*	*E+*	*W –*
AB	872	N 41° 35′ W	652.25	—	—	578.75
BD′	—	—	—	—	—	—
D′ E or *CD*	770	S 63° 12′W	—	347.18	—	687.29
EF	1079	S 26° 39′ E	—	964.37	483.97	—
FA	1480	N 53° 30′ E	880.34	—	1189.71	—
		Sum	1532.59	1311.55	1673.68	1266.04
			Diff. = 221.04 S		Diff. = 407.64 W	

Apparently, line *BD′* lies in 3rd quadrant (SW).

Length $BD' = \sqrt{(221.04)^2 + (407.64)^2}$

$= 463.71$ m

Tangent of R.B. of line

$$BD' = \frac{407.64}{221.04}$$

$= 1.8442$

or R.B. of line $BD' = S\ 61°\ 31°\ 54'\ W$

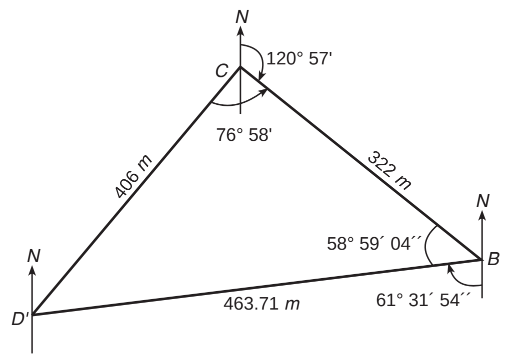


Fig. 12.65

Applying cosine rule to $\Delta BCD'$ (Fig. 12.65), we get

$$(CD')^2 = (BC)^2 + (BD')^2 - 2\,BC \times BD' \times \cos CBD'$$

or $$\cos CBD' = \frac{(322)^2 + (463.71)^2 - (406)}{2 \times 322 \times 463.71}$$

$$= \frac{103684 + 215026.96 - 164836}{298629.24}$$

or $$\cos CBD' = 515271$$

$$\therefore \quad \angle CBD' = 58° \, 59' \, 04''$$

Now bearing of BC = Bearing of $BD' + \angle CBD'$

$= 241° \, 31' \, 54'' + 58° \, 59' \, 04''$

$= 300° \, 30' \, 58''$ **Ans.**

Calculation of bearing of CD' (or DE)

Bearing of CD' = Bearing of CB + $\angle D'CB$

But, Bearing of CB = Bearing of $BC - 180°$

and Bearing of BC = Bearing of $BD' + \angle CBD'$

$= (180° + 61° \, 31' \, 54'') + 58° \, 59' \, 04''$

$= 300° \, 30' \, 58''$

$\therefore$ Bearing of $CB = 300° \, 30' \, 58'' - 180°$

$= 120° \, 30' \, 58''$

By applying the cosine rule to $\Delta BCD'$, (Fig. 12.66)

$$(D'B)^2 = CB^2 + (CD')^2 - 2CB \times CD' \times \cos C$$

$$(463.71)^2 = (322)^2 + (406)^2 - 2 \times 322 \times 406 \times \cos C$$

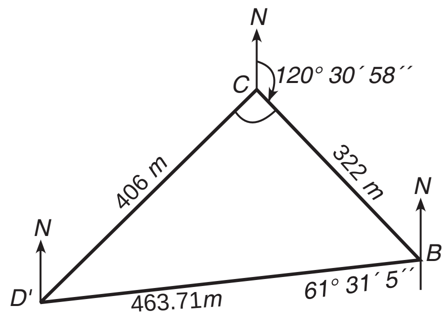


Fig. 12.66

$$= \frac{(322)^2 + (406)^2 - (463.71)^2}{2 \times 322 \times 406}$$

$$= \frac{103684 + 164836 - 2150216.96}{2 \times 322 \times 406} = 0.204590$$

$\therefore \qquad C = 78° + 11' 40''$

$\therefore$ Bearing of CD' = 120° 30′ 58″ + 78° 11′ 40″

= 198′ 42′ 38″

Hence Bearing of DE = 198° 42′ 38″ **Ans.**

EXERCISE 12

1. Pick up correct word(s) from the brackets to complete the sentences.

(*i*) In a traverse ....measurements is/are made either directly or indirectly in the field. (angular, linear, both)

(*ii*) A closed traverse...............close on the starting station, (may, may not)

(*iii*) Theodolite traverse is generally carried out to.........points. (Survey detail, provide control)

(*iv*) Theodolite traverses are generally run in...........direction. (Anti-clockwise, clockwise)

(*v*) The angle between the prolongation of the preceding line and the forward line of a traverse, is called.......angle. (deflection, included, direct)

(*vi*) Measurement of several angles at a common station, is made by.....method. (repetition, reiteration)

(*vii*)

A traverse angle at any station, subtended by adjoining traverse legs is the angle.........of the observer in the direction of the traverse.

(left, right)

(*viii*) To obtain whole circle bearing of the next trverse leg, add the traverse angle to the bearing of the previous leg and if the sum is more than 540° , subtract........ (180° , 360° 540°)

(*ix*) W.C.B. of a line = W.C.B. of the preceding line + deflection angle if the deflection angle is............ (right, left)

(*x*) Departure of a line is obtained by multiplying its length by the........of its reduced bearing. (sine, cosine, tangent)

(*xi*) Σ Latitudes of a traverse is equal to the amount of closing error multiplied by the........of the reduced bearing of the closing line. (tangent, sine, cosine)

(*xii*) Compensating errors of a chain length.............the accuracy of the traverse. (do not affect, affect)

2. State whether following statements are true or false. Rewrite the false statements as true.

(*i*) Theodolite traversing by observing angles between consecutive traverse legs is resorted to when the traverse is long and better accuracy is required.

(*ii*) Included angle or direct angle of a traverse is the angle which is measured on the left side of the direction of the traverse.

(*iii*) It is customary to run a theodolite traverse in clockwise direction when interior angles are only observed.

(*iv*) While measuring the direct angles of a theodolite traverse, final sight is made on the back station and exact bisection is made using lower tangent screw.

(*v*) Deflection angle traverse is generally preferred to when the traverse is a closed circuit and better accuracy is required.

(*vi*) Direction of deflection angles may be conveniently decided at a later date in office.

(*vii*) For correct direct magnetic bearings of traverse legs, the line of collimation and axis of the telescope must coincide the magnetic meridian of the first station of observation.

(*viii*) An error in the bearing of the initial leg affects all other bearings with the same amount of error.

(*ix*) In case a non-transit theodolite the bearing of the next line is obtained by adding 180° if the reading is less than 180° and by subtracting 180° if it is more than 180° .

(*x*) To obtain direct bearings of lines by making observation with a non-transit theodolite, correction of 180° is only applied to readings of odd stations.

(*xi*) To provide a check on the distances of traverse legs, 30 m chain and 20 m chain must be independently used for the measurements and the mean of the distances obtained by both chain is accepted for computation.

(*xii*) The method of repetition is used to measure traverse angles to a finer degree of accuracy than that achievable with the least count of the vernier fitted on the theodolite.

(*xiii*) In precision surveys of higher accuracy, true bearings observed from astronomical observations are not preferred to magnetic bearings.

(*xiv*) In case of closed circuits, it is not necessary to observe bearings of intermediate stations as accuracy of the angular measurements can be checked by summing up the traverse angles, the total of which should be ($2n = 4$) right angles.

(*xv*) Bearing of the closing line is equal to the forward bearing of the initial line plus sum of all right deflection angles minus sum of all left deflection angles.

(*xvi*) Latitude of a traverse leg can be obtained by multiplying its distance by the cosine of the reduced bearing of the line.

(*xvii*) Total latitude and total departure of any point with respect to the origin of co-ordinates, are known as consecutive co-ordinates of the point.

(*xviii*) If the angular measurements of a traverse are more accurate as compared to linear measurements, the misclosures of the traverse can be conveniently adjusted by 'Bowditch's Rule.

(*xix*) Plotting a traverse with the help of independent co-ordinates is more accurate than by lengths and bearings of sides.

(*xx*) The algebraic sum of the total departures of the stations of an adjusted closed traverse should be equal to zero.

3. What is the purpose of a theodolite traverse. Give its main classifications according to its starting and closing stations.

4. Describe the procedure of a theodolite traverse by the method of included angles.

5. When would you suggest a theodolite traversing by the method of deflection angles. Explain this method with neat sketches.

6. What are the different stages of field work during a theodolite traverse. Discuss each briefly.

7. Differentiate :

(*i*) Consecutive co-ordinates and Independent co-ordinates

(*ii*) Open traverse and closed traverse (*iii*) Bowditch's Rule and Transit Rule

(*iv*) Latitudes and Departures

8. What do you understand by a Gale's table. How are its calculations made ?

9. Co-ordinates of *A* and *B* are given below. Third point *C* has been chosen such a way that bearing of line *AC* and *CB* are 29° 30′ and 45° 45′ respectively. Calculate the lengths of lines *AC* and *CB*.

Point	*Northing*	*Easting*
A	150	200
B	1500	1300

10. The data and different values of the lines, are given below. Point *F* is situated in the centre of line joining *A* and *E*. Find out the length and bearing of line *CF* using the principles of latitude and departure.

Line	*Length* (*m*)	*Bearing*
AB	150	*N* 75° 43′ *E*
BC	100	*N* 32° 48′ *E*
CD	300	*S* 28° 54′ *E*
DE	800	*S* 5° 36′ *E*

11. Calculate the missing length and bearing of the line *AB* from the following theodolite traverse data :

Line	*Length*	*Reduced Bearing*
AB	??	??
BC	453.00	*N* 21° 49′ *E*
DC	529.40	*N* 80° 22′ *W*
DA	589.00	*S* 74° 20′ *W*

12. During a theodolite survey the following details were noted :

Line	*Length* (*m*)	*W.C.B.*
AB	550	60°
BC	1200	115°
CD	?	?
DA	1050	310°

Calculate the length and bearing of the line *CD*

13. Following is the data of a closed traverse. Calculate the lengths of the sides *BC* and *CD*.

Line	*Length*	*W.C.B..*	*Lat.*	*Dep.*
AB	34.4	14° 31′	+ 33.3	+ 8.62
BC	?	319° 42′	-	-
CD	?	347° 15′	-	-
DE	30.0	5° 16′	+ 29.88	+ 2.76
EA	195.8	168° 12′	– 191.64	+ 40.04

14. Two points *A* and *D* are connected by a theodolite traverse *ABCD* and the following records are obtained :

AB = 87.6 m; *BC* = 68.2 m; *CD* = 166.0 m

Angle *ABC* = 118° 15′; Angle *BAD* = 108° 40′

Assuming *AB* as meridian, calculate

(*i*) The length of *AD*

(*ii*) The angle *BAC*

(*iii*) The latitude and departure of *D* with respect to *A*.

15. In a closed traverse *ABCDE*, the deflection angles were measured as shown below :

Deflection angle at	0			*Direction*
A	22	15	30	*L*
B	36	42	00	*R*
C	97	05	30	*R*
D	152	17	30	*R*
E	96	13	00	*R*

Calculate the error of closure for the angles and adjust the measured angles.

16. The following data pertains to a theodolite traverse :

Line	*Length*	*Lat.*	*Dep.*
AB	300.00	+ 129.56	+ 4.52
BC	147.60	+ 17.27	+ 299.58
CD	307.20	- 147.53	+ 4.94
DA	129.60	0	- 307.20

Balance the traverse by Transit Rule and compute the independent co-ordinates of stations, given the co-ordinates of station *A* as (400 *N*, 200 *E*).

17. *A* and *B* are two triangulation stations established for location of a tunnel. Independent co-ordinates of the points are :

Point	*Latitude*	*Departure*
A	5530.0	27059.5
B	5005.5	28736.37

A straight tunnel is to be constructed from *C*, a point roughly north of *A* to *D* a point roughly south of *B*. If the perpendicular offsets to the proposed tunnel are *AC* = 110 m and *BD* = 60 m, calculate the bearing of *CD*.

18. The bearings of two in accessible points *A* and *B*, taken at a point *C* are 254° 42′ and 168° 45′ respectively. The co-ordinates of *A* and *B*, taken at a point *C* are 254° 42′ and 168° 45′ respectively. The co-ordinates of *A* and *B* are as under :

Point	*Easting* (*m*)	*Northing* (*m*)
A	2900.5	2142.5
B	4152.5	1685.5

Calculate the independent co-ordinates of the point *C*

19. A straight tunnel is to be bored between the points *A* and *B*, whose co-ordinates are :

Point	*Easting* (*m*)	*Northing* (*m*)
A	2568.3	1235.5
B	3852.4	1855.8

To sink a shaft at *D*, the middle point of *AB*, a point *C* whose coordinates are *E* 3318.8, *N* 1345.5 is fixed on the hill as it was impossible to measure along *AB* directly calculate

(*i*) the co-ordinates of *D*

(*ii*) the length and bearing of *CD*

20. The following are the lengths and bearings in a traverse *ABCD*.

Line	*Length* (*m*)	*Bearing*
AB	124	32^o
BC	168	168^o
CD	97	44^o

Find the length and bearing of *DA*.

ANSWERS

1. (*i*) both (*ii*) may not (*iii*) provide control (*iv*) anti-clockwise (*v*) deflection (*vi*) reiteration (*vii*) left (*viii*) 540^o (*ix*) right (*x*) sine (*xi*) cosine (*xii*) affect.

2. (*i*) True (*ii*) True (*iii*) Wrong (*iv*) Wrong (*v*) Wrong (*vi*) Wrong (*vii*) True (*viii*) True (*ix*) True (*x*) Wrong (*xi*) Wrong (*xii*) True (*xiii*) Wrong (*xiv*) True (*xv*) True (*xvi*) True (*xvii*)(*xviii*) Wrong (*xix*) True (*xx*) True.

9. *AC* = 712.72 m ; *CB* = 1045.70 m

10. 509.06 m; 1° 09″

11. 985.01 m; 69° 10′ 52″

12. 879.18 m; 239° 45′ 35″

13. CD = 94.85 m; BC = 47.14 m

14. AC = 117.18 m;

$\angle BAC$ = 26° 36′ 59″; Lat – 37.50 m; Dep. 111.02 m

15. 2′ 30″ to each.

17. 112° 55′ 17″

18. E. 4001.68 m; N. 2443.75 m

19. E. 3210.35 m; N. 1540.65 m (*ii*) 331° 32′ 57″; 227.63 m

20. Length 168.35 m ; Bearing 266° 23′ 12″.

13

Tacheometric Surveying

13.1. INTRODUCTION

Tacheometry is the branch of surveying in which both horizontal and vertical distances between stations are determined by making instrumental observations. In this method, measurements by a tape or a chain is completly dispensed with. Horizontal distances obtained by tacheometric observations do not require slope correction, tension correction, etc. This method is very rapid and convenient. Though accuracy of tacheometric distances is low as compared to direct chaining on flat ground but the accuracy achievable by tacheometry is better as compared to chaining in broken grounds, deep ravines or across large water bodies. The instrument employed for tacheometric observations, is generally known as a *tacheometer* which is similar to theodolite having diaphragm fitted with two additional horizontal wires, called *stadia hairs.* The accuracy of tacheometric distances under favourable conditions, is such that the error seldom exceeds 1 in 1000.

13.2. PURPOSE OF TACHEOMETRIC SURVEYING

The primary object of tacheometry is the preparation of contoured plans. It is considered to be rapid and accurate in rough country and has thus been widely used by engineers in location surveys for railways, canals, reservoirs, etc. Whenever surveys of higher accuracy are carried out, tacheometry provides a good check on distances measured with a tape or a chain.

13.3. INSTRUMENTS USED FOR TACHEOMETRIC SURVEYING

The following instruments are used :

1. Tacheometer. A tacheometer which is essentially nothing more than a theodolite fitted with stadia hairs, is generally used for tacheometric surveying. The stadia diaphragm consists of one stadia hair above and the other at equal distance below the horizontal cross hair. The stadia hairs are kept in the same vertical plane as the horizontal and vertical cross hairs.

Different forms of stadia diaphragms commonly used in tacheometers, are shown in Fig. 13.1.

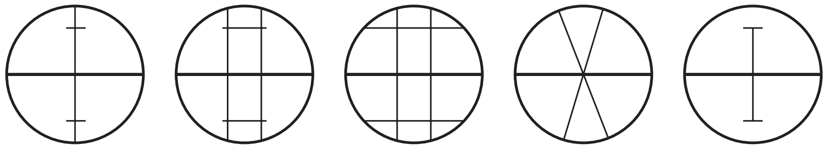


Fig. 13.1. Types of Diaphragms.

2. Stadia rods. For short distances (say up to 100 metres) ordinary levelling staves may be used. For greater distances, the stadia rods 3 to 5 metres in length, are generally used.

13.4. SYSTEMS OF TACHEOMETRIC MEASUREMENTS

Tacheometric measurements, are made by one of the following systems:

(*i*) The stadia hair system. (*ii*) The tangential system.

1. Stadia hair system. The stadia hair system may further be divided into two types :

1. Fixed hair method 2. Movable hair method.

Fixed hair method. In this method, stadia hairs are kept at fixed interval. The intercept on the levelling staff (or stadia rod) varies, depending upon the horizontal distance between the instrument station and the staff. The intercept used in computation is deduced by subtracting the lower stadia reading from the upper stadia reading. When the staff intercept is more than the length of the staff, only half intercept is read, which is equal to the difference between central stadia hair reading and the lower / upper stadia hair reading.

This method can be suitably employed even when horizontal sights are not possible. For inclined sights, readings may be taken by holding the staff either vertical or normal to the line of sight. This is the most common method of tacheometry and the name *stadia hairs method* generally refers to this method.

Movable hair method. In this method, the intercept on the levelling staff is kept constant and the distances between the stadia hairs are variable. Two targets on the staff at a known distance apart are fixed and the stadia hairs are adjusted such that the upper hair bisects the upper target and the lower hair bisects the lower target. In this case, a provision is made for the measurement of the variable interval between the stadia hairs. For inclined sights, readings may be taken by holding the staff either vertical or normal to the line of sight as in the case of fixed hair method.

2. The tangential method. In this method, the stadia hairs are not used. Readings on a staff are taken against the horizontal cross hair.

To measure the staff intercept, two pointings of the telescope are therefore, necessary. Readings to full metre values on the staff are generally taken to avoid the decimal part and also for simplification of computations. This method is generally not adopted as two vertical angles are required to be measured for one single observation.

13.5. PRINCIPLE OF TACHEOMETRY. (Fig. 13.2)

Statement. *"In isosceles triangles, the ratio of the perpendiculars from the vertex on their bases is constant.* This can be proved as under.

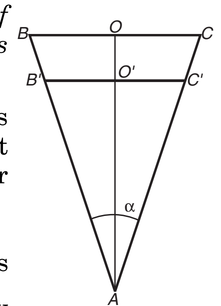


Fig. 13.2. Principle of tacheometry.

Let *ABC* and *AB′C′* be two isosceles triangles whose bases are *BC* and *B′ C′* and their vertex is at *A*. If *AO* and *AO′* are the perpendiculars to their respective bases, then

$\frac{AO}{BC} = \frac{AO'}{B'C'} = 2 \cot \frac{\alpha}{2} = K$, a constant where α is the apex angle. The value of constant *K* entirely depends upon the magnitude of the apex angle α.

For horizontal sights, the difference in elevations of the instrument station and staff position is deduced in a similar way as in the case of differential levelling.

THE STADIA HAIR METHOD

1. **Distance and Elevation Formulae for Horizontal sights by Fixed Hair Method.** (Fig. 13.3)

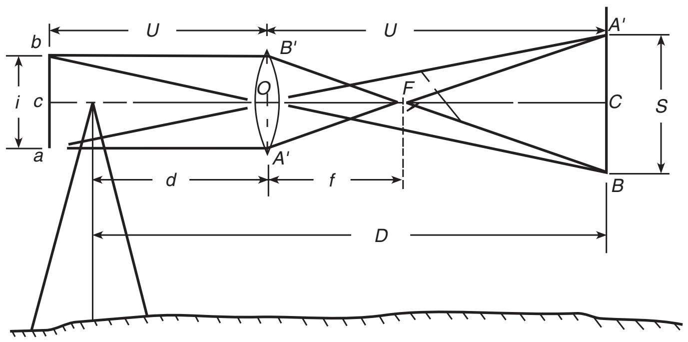


Fig. 13.3. Fixed hair method.

1. Horizontal distance of the staff position. Assume that *O* is the optical centre of the object lens of the external focusing telescope; *a, b, c* represent the three horizontal hairs. *A, B, C*, rule represent respective points on the staff which appear cut by three hairs, *ab* is the length of the image of the staff intercept *AB* as seen in the diaphragm.

Let f = focal length of the object lens

i = stadia hair interval ab,

s = staff intercept AB,

D = horizontal distance from the vertical axis of the tacheometer to the staff.

d = the distance between the optical centre of the object glass and the vertical axis of the theodolite.

With the elementary knowledge of optics, it is clear that the rays from A and B which pass through the exterior principal focus of the objective F, travel parallel to the principal axis after refraction at $A'B'$.

Proof :

$A'B' = ab$ = stadia hair distance.

From similar Δs ABF and $A'B'F$, we get

$$\frac{CF}{OF} = \frac{AB}{A'B'}$$

or

$$CF = \frac{OF \times AB}{A'B'}$$

Substituting the values of AB, OF and $A'B'$, we get

or

$$CF = \frac{f}{i} .s$$

But

$$D = CF + f + d \qquad ...(13.1)$$

Substituting the value of CF in Eq. 13.1), we get

$$D = \frac{f}{i}.s + (f + d) \qquad ...(13.2)$$

Equation (13.2) is known as the *tacheometric distance equation* in which f, i and d are constants for a particular tachometer. The tacheometric distance formula may be stated as :

$$D = AS + B \qquad ...(13.2a)$$

where A and B are generally known as *tacheometric constants* of the tacheometer.

The values of tacheometric constants A and B are determined before making observations by a particular theodolite if these are not given by the manufacturers.

The value of the constant $\frac{f}{i}$ is known as a *multiplying constant.* It is usually kept 100 by the manufacturers. The value of other constant $(f + d)$, known as an *additive constant* is generally kept between 0.3 to 0.5 m.

Alternative Proof. (Fig. 13.3)

Let $OC = u$ and $Oc = v$

From similar Δ s *Oab* and *OAB*, we get

$$\frac{ab}{AB} = \frac{Oc}{OC}$$

Substituting the values $ab = i$; $AB = s$; $Oc = v$; $OC = u$, we get

$$\frac{i}{s} = \frac{v}{u}$$

or $$v = \frac{iu}{s} \qquad ...(13.3)$$

From the lens formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad ...(13.4)$$

Substituting the value of v from Eq. (13.3) in Eq. (13.4), we get

$$\frac{1}{iu/\text{s}} + \frac{1}{u} = \frac{1}{f}$$

or $$\frac{s}{iu} + \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{u}\left(\frac{s}{i} + 1\right) = \frac{1}{f}$$

By cross-multiplication, we get

$$u = \left(\frac{s}{i} + 1\right)f$$

or $$u = \frac{sf}{i} + f$$

But $$D = u + d$$

∴ $$D = \frac{f}{i}s + (f + d)$$

or $$D = \frac{f}{i}s + (f + d) \text{ same as Eq. (13.2)}$$

Alternative Proof. (Fig. 13.3)

From similar Δ*s AOB* and *aOb*, we get

$$\frac{OC}{Oc} = \frac{AB}{ab}$$

or $$\frac{u}{v} = \frac{s}{i} \qquad ...(13.5)$$

where u = horizontal distance of the staff from the optical centre of the objective.

v = horizontal distance of the cross wires from the optical centre of the objective .

u and v being conjugate focal distances of the object glass, we get

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad ...(13.6)$$

where f is the focal length of the objective.

Multiplying Eq. (13.6) throughout by fu, we get

$$u = f + \frac{fu}{v}$$

$$u = \frac{u}{v} f + f$$

Substituting the value of u / v from Eq. (13.5), we get

$$u = \frac{s}{i} . f + f \quad ...(13.7)$$

But $\quad D = u + d$

$\therefore \quad D = \frac{s}{i} . f + f + d$

or $\quad D = \frac{f}{i} s + (f + d)$. same as Eq. (13.2)

Note. The following points may be noted:

(*i*) The distance formula $D = \frac{f}{i} s + (f + d)$ is applicable only if the line of the sight is horizontal and the staff is held truly vertical.

(*ii*) The point F, the vertex of measuring triangle is sometimes known as *anallatic point.*

2. Elevation of the staff station. Since the line of sight is kept horizontal and the staff is held vertical, the elevation of the staff station is obtained exactly in a similar way as in ordinary levelling, by observing the central hair reading and measuring the height of the instrument above the instrument station, *i.e.*

Elevation of the staff station = Elevation of the horizontal instrument axis – Central hair reading.

2. Distance and Elevation formulae for inclined sights by fixed hair method. In this case, the staff may be held either vertical or normal to the line of sight. The line of sight may be either elevated or depressed depending upon the relative positions of the instrument and the staff stations.

1. Distance and Elevation Formulae for inclined sights with Staff Vertical (Fig. 13.4)

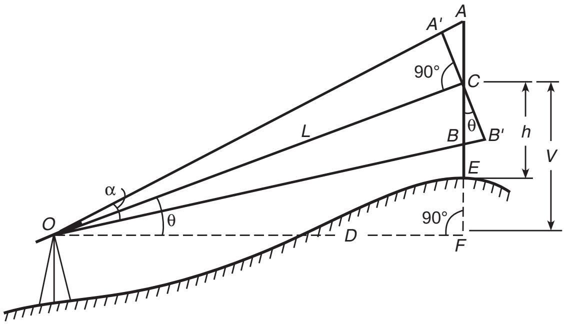


Fig. 13.4. (*a*) Elevated line of sight with staff vertical.

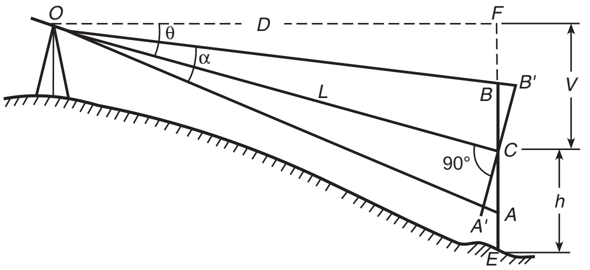


Fig. 13.4 (*b*) Depressed line of sight with staff vertical.

(*i*) **Horizontal distance formula.** Let θ be the angle of elevation or depression of the line of sight from the horizontal. As the staff is held vertical, the staff intercept *AB* is not normal to the line of sight *OC*.

Draw a line *A′B′* passing through *C* and perpendicular to *OC*, cutting *OA* at *A′* and *OB* produced at *B′*.

In Δ*OCF*, angle *OCF* $= 90° - \theta$

But angle *OCB′* = 90°, *OC* being perpendieular to *A′B′*

∴ Angle *BCB′* $= 90° - (90° - \theta) = \theta$

Angle *A′CA* = angle *BCB′* = θ. being opposite angles

Let angle *A′ OB′* subtended by stadia hairs *AB*, be α

i.e. Angle $A'OC = \frac{\alpha}{2}$

or Angle $OA'C = 90° - \frac{\alpha}{2}$

or Angle $AA'C = 180° - \left(90° - \frac{\alpha}{2}\right) = 90° + \frac{\alpha}{2}$

Similarly from $\Delta OCB'$, we get

$$\text{Angle } BB'C = 90° - \frac{\alpha}{2}$$

The value of $\alpha/2$ is generally very small. If the ratio of OC and AB is 100, the value of $\alpha/2$ equals to 17′ 11″. Ignoring $\alpha/2$, both angles $AA'C$ and $BB'C$ may be assumed very nearly equal to right angles.

From Δ s $AA'C$ and $BB'C$

$$A'C = AC \cos\theta;\ B'C = BC \cos\theta$$

$$\therefore \quad A'C + B'C = AC\cos\theta + BC\cos\theta$$

$$A'B' = AB\cos\theta = s\cos\theta$$

The inclined distance OC

$$= \frac{f}{i} \times A'B' + (f + d)$$

i.e.

$$L = \frac{f}{i} s\cos\theta + (f + d) \qquad ...(13.8)$$

Horizontal distance, $D = L\cos\theta$

$$= \left[\frac{f}{i} s\cos\theta + (f + d)\right]\cos\theta$$

or

$$D = \frac{f}{i} s\cos^2\theta + (f + d)\cos\theta$$

$$= A.\ s\cos^2\theta + B.\cos\theta \qquad ...(13.8a)$$

where A and B are the tacheometric constants.

(*ii*) **Elevation formula.**

From $\Delta\ OCF$, $CF = V = L\sin\theta$

$$= \left[\frac{f}{i} s\cos\theta + (f + d)\right]\sin\theta$$

(Refer eqn.13.8)

$$= \frac{f}{i} s\cos\theta\sin\theta + (f + d)\sin\theta$$

$$= A.s.\sin\theta\cos\theta + B\sin\theta$$

or

$$V = A.s.\frac{\sin 2\theta}{2} + B\sin\theta \qquad ...(13.9)$$

Thus equations (13.8a) and (13.9) are the required distance and elevation formulae for inclined line of sights.

(*a*) **Elevation of the staff station for an angle of elevation** [Fig. 13.4 (*a*)]

Let the central hair reading on the staff be h.

The difference in level between O and E

$$FE = V - h.$$

If *H.I.* is the reduced level of the trunnion axis above datum, the reduced leve of

$$E = H.I. + V - h \qquad ...(13.10)$$

(*b*) **Elevation for the staff station for an angle of depression** [Fig. 13.4(*b*)]

Let the central hair reading on the staff be h.

The difference in level between O and E

$$FE = V + h$$

If *H.I.* is the reduced level of the trunnion axis above datum, the reduced level of $E = H.I. - (V + h)$

$$= H.I. - V - h \qquad ...(13.11)$$

Remember. *The reduced level of the staff station = the difference in level of trunnion axis and central hair reading* $\pm V$, *using positive sign for angles of elevation and negative sign for angle of depression.*

2. Distance and elevation formulae for inclined sights with staff normal

The inclination of the staff when held normal to the line of sight will be different for angle of elevation and angle of depression. For angles of elevation the staff will be inclined from the vertical towards the instrument and for angles of depression, it will be inclined from vertical away from the instrument.

Case I. Line of sight at an angle of elevation. (Fig. 13.5)

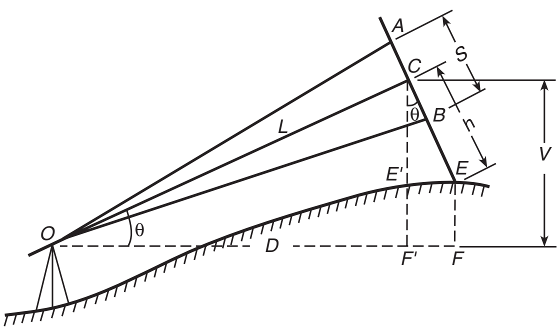


Fig. 13.5. Elevated line of sight with staff normal.

(a) **Horizontal distance formula**

Let $AB = s$, the staff intercept

$CE =$ central hair reading $= h$

$\theta =$ angle of elevation for the central hair

$L =$ inclined distance $= OC$.

Construction : Drop $C F'$ perpendicular to meet the horizontal line OF at F' and drop EE' perpendicular to meet CF' at E'.

From $\Delta\, OCF'$, $OF' = L \cos\theta$

From $\Delta\, ECE'$, $EE' = h \sin\theta = F'F$, angle $E'CE$ being equal to θ.

Applying tacheometric distance formula, we get

$$L = \frac{f}{i}s + (f + d)$$

$$\therefore \quad OF' = \left[\frac{f}{i}s + (f + d)\right]\cos\theta$$

But $D = OF' + F'F$

$$\therefore \quad D = \left[\frac{f}{i}s + (f + d)\right]\cos\theta + h\sin\theta \qquad ...(13.12)$$

(b) **Elevation of the Staff Station**

From $\Delta\, OF'C$, $V = OC \sin\theta$

$$= L\sin\theta$$

$$= \left[\frac{f}{i}s + (f + d)\right]\sin\theta$$

R.L. of E = height of the trunnion axis $+ V - CE'$

$$= \text{H.I.} + \left(\frac{f}{i}s + f + d\right)\sin\theta - h\cos\theta \qquad ...(13.13)$$

Case II. Line of sight at an angle of depression. (Fig. 13.6)

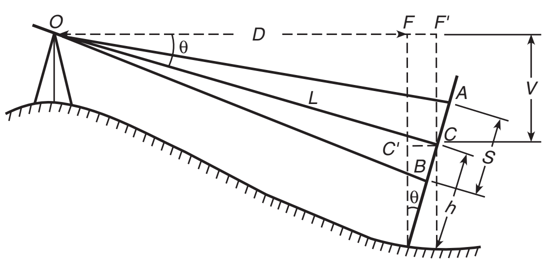


Fig. 13.6. Depressed line of sight with staff normal.

(*a*) **Horizontal distance formula**

Let $AB = s$, the staff intercept

CE = central hair reading = h

θ = angle of depression

L = inclined distance = OC

Construction : Drop CF' and EF perpendiculars to meet the horizontal line through O at F' at F respectively.

Drop CC' perpendicular to EF to meet at C'.

Drop EE' perpendicular to $F'C$ (produced) to meet at E'.

From $\Delta\, ECE'$, $EE' = h \sin\theta$.

Applying tacheometric distance formula,

$$L = \frac{f}{i} s + (f + d) \text{ we get}$$

Now

$$OF' = L \cos\theta$$

$$= \left(\frac{f}{i} s + f + d\right) \cos\theta$$

But

$$D = OF' - FF'$$

or

$$= OF' - EE'.$$

Substituting the values of OF' and EE', we get

$$D = \left(\frac{f}{i} s + f + d\right) \cos\theta - h \sin\theta \qquad \text{...(13.14)}$$

(*b*) **Elevation of the staff station**

From $\Delta\, OCF'$,

$$CF' = OC \sin\theta$$

$$= L \sin\theta$$

or

$$V = \left(\frac{f}{i} s + f + d\right) \sin\theta$$

R.L. of E = height of trunnion axis $- V - CE'$

$$= \text{H.I.} - \left(\frac{fs}{i} + f + d\right) \sin\theta - h \cos\theta \qquad \text{...(13.15)}$$

Remember. *The R.L. of the staff station = the difference of the elevation of the trunnion axis and the vertical component of the central hair reading ± component of the inclined distance, the positive sign is taken for angles of elevation and the negative sign for angles of depression.*

13.7. TYPES OF TELESCOPES FITTED IN STADIA THEODOLITES

The telescopes used in stadia theodolites are of the following three types.

(*i*) Internal focusing telescope.
(*ii*) External focussing telescope.
(*iii*) External focussing telescope fitted with an anallatic lens.

The theodolite fitted with the second type, is known as *stadia theodolite* whereas that fitted with third type is known as a *tacheometer*. In a tacheometer, additive constant is usually kept zero. Before using a stadia theodolite, its additive constant is determined. In internal focussing telescopes, the additive constant varies depending upon the measured distance. However, its value is so small that if it is ignored, it hardly makes any difference in the plotted locations of the points. Some modern theodolites fitted with internal focussing telescopes, may be regarded strictly as anallatic.

A tacheometer must meet the following requirements :

1. The multiplying constant must be 100 and the error in the computed distance should not exceed 1 in 1000.
2. The central hair should be exactly midway between the stadia hair.
3. The telescope must be truly anallatic, *i.e.* the additive constant must be zero.
4. The telescope should be powerful, having a magnification of 20 to 30 diameters. To have a sufficiently bright image, its aperture should be 35 to 45 mm in diameter.

13.8. DETERMINATION OF TACHEOMETRIC CONSTANTS

The tacheometric constants of stadia theodolites may be determined by one of the following methods :

(*i*) Determination of multiplying constant $\frac{f}{i}$ by field measurement and the additive constant $(f + d)$ by direct measurement along the telescope.
(*ii*) Determination of both the constants by field measurements.

First Method : Following steps are followed.

1. Sight any sharp distant object and focus the telescope.
2. Measure the distance along the top of the telescope between the object lens and the plane of the cross hairs accurately with a graduated scale. This is equal to the focal length (f) of the object lens.
3. Measure the distance between the object lens and the vertical axis of the theodolite accurately. This is equal to d.

4. Add the measured value of the focal length f and distance d to get the value of the additive constant, *i.e.* $(f + d)$.
5. Measure three lengths D_1, D_2 and D_3, from the instrument position along a straight line on a fairly flat ground and observe intercepts s_1, s_2 and s_3 on a staff held vertically at each point. The vertical circle vernier should read zero to have a horizontal line of sight.
6. Substitute the value of $(f + d)$ and different staff intercepts in the distance formula, $D = \frac{f}{i} s + (f + d)$ to obtain three independent equations.
7. Solve the three simultaneous equations to get the value of multiplying constant $\frac{f}{i}$.
8. The mean of the three values of $\frac{f}{i}$ is the required value of the multiplying constant $\frac{f}{i}$.

Second method. This method is sometimes called field method.

Following steps are followed :

1. Measure a line AB 100 metres long on a fairly level ground and fix pegs at 25 m intervals.
2. Set up the instrument at A and centre it over the ground point accurately.
3. Obtain the staff intercepts s_1, s_2, s_3 and s_4 by taking stadia readings on a staff held vertically at each peg, keeping the telescope horizontal by setting the vertical circle verniers to read zero.
4. Substitute the different values of D and s in the tacheometric distance formula, *i.e.* $D = \frac{f}{i} s + (f + d)$ to get four quadratic equations in $\frac{f}{i}$ and $(f + d)$.
5. Solve the quadratic equations in pairs to get the values of tacheometric constants.
6. Mean values are the required values of the constants.

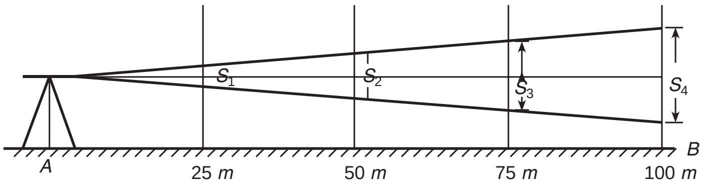


Fig. 13.7. Determination of tacheometric constants.

13.9. ANALLATIC LENS

A concave lens specially provided in a telescope between the object lens and eye piece to eliminate the additive constant $(f + d)$ from the tacheometric distance equations, is known as an *anallatic lens.* It is fitted in internal focussing telescopes only.

1. **Advantages of an anallatic lens.** The main advantage of an anallatic lens is that by its introduction, the calculation of distances and heights, is very much simplified. If the multiplying constant is 100 and additive constant 0, the horizontal distance is obtained by simply multiplying the staff intercept by 100.

2. **Disadvantages of an anallatic lens.** The anallatic lens increases the absorption of light in telescope which consequently results in reduction of the brilliancy of the image. It also adds to the initial cost of manufacturing the telescope.

Theory of the anallatic lens (Fig. 13.8.)

Let AB be the position of the staff

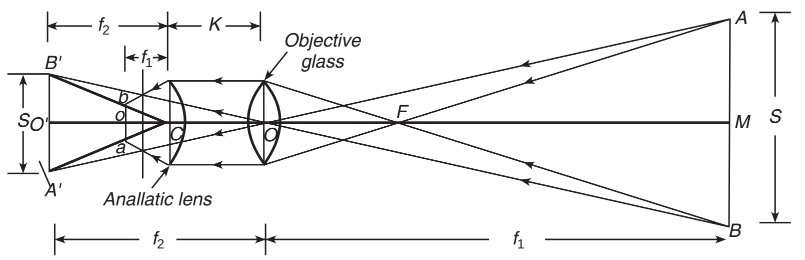


Fig. 13.8. Theory of an anallatic lens.

S = stadia intercept on the staff

O = optical centre of the object glass

$A' B'$ = position of the image at which it would have formed in the absence of the anallatic lens.

K = distance between anallatic lens and object glass.

F_1 and F_2 are the conjugate focal distances of object glass.

In Fig. 13.8 two rays originating from each A and B are only drawn. One of the them passes through the outside principal focus of the object glass and emerges parallel to the axis of the telescope. The other ray travels in a straight line through the optical centre O of the object glass.

The rays after passing through the object glass, get intercepted by the anallatic lens, and hence get refracted. The image is formed at some other position $b\ o\ a$.

Let c be the optical centre of anallatic lens,

$co = f_1$, $co' = f_2$ be the conjugate focal distances of the anallatic lens.

From the laws of lenses, we know

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} \qquad ...(13.16)$$

and
$$\frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2} \qquad ...(13.17)$$

(negative sign is used in Eq. (13.17) since $b\ o\ a$ and $B'O'A'$ are on the same side of the anallatic lens.)

Let $A'B' = i$ and $ab = i'$

From similar Δs ABO and $A'B'O$, we get

$$\frac{S}{i} = \frac{F_1}{F_2} \qquad ...(13.18)$$

From similar Δs $A'B'c$ and $a\ b\ c$, we get

$$\frac{i}{i'} = \frac{f_2}{f_1} \qquad ...(13.19)$$

By cross-multiplication of Eqn. (13.18) and Eqn. (13.19), we get

$$\frac{S}{i} = \frac{F_1}{F_2} \times \frac{f_2}{f_1} \qquad ...(13.20)$$

Substituting the values of $\frac{F_1}{F_2}$ and $\frac{f_2}{f_1}$ from Eqns. (13.16) and (13.17) in Eqn. (13.20), we get

$$\frac{S}{i'} = \frac{F_1 - F}{F} \cdot \frac{f + f_2}{f}$$

or
$$= \frac{F_1 - F}{F} \cdot \frac{(f + F_2 - k)}{f} = \frac{F_1 - F}{F} \cdot \frac{\left(f + \frac{F.\,F_1}{F_1 - F} - k\right)}{f}$$

$$= \frac{FF_1 + (F_1 - F)\,(f - k)}{Ff} = \frac{F_1\,(F + f - k) + F\,(k - f)}{Ff}$$

$$\frac{F_1\,(F + f - k)}{Ff} = \frac{S}{i'} - \frac{F(k - f)}{Ff}$$

$$\therefore \qquad F_1 = \frac{S}{i'} \times \frac{Ff}{(f_1 + f - k)} - \frac{F\,(k - f)}{(F + f - k)}$$

But $\qquad D = F_1 + d$

where d is the distance of the objective from the vertical axis of the theodolite.

or $$D = \frac{S \,.\, Ff}{i' \,(F + f - k)} - \frac{F\,(k - f)}{(F + f - k)} + d \qquad ...(13.21)$$

Now, the condition that distance **D** should be proportional to the intercept S, requires that the factor $\frac{F\,(k - f)}{(F + f - k)} + d$ in Eqn. (13.21) must be zero, *i.e.*

$$-\frac{F\,(k - f)}{(F + f - k)} + d = 0$$

or $$\frac{F\,.\,(k - f)}{(F + f - k)} = d \qquad ...(13.22)$$

or $$k = f + \frac{Fd}{(F + d)} \qquad ...(13.23)$$

Hence, the distance between the anallatic lens and the objective should be made equal to k.

By adopting suitable values of F, f, i' and k in the first term of equation (13.21), $\frac{Ff}{i' \,(F + f - k)}$ is made equal to 100.

or $$D = 100\, S \qquad ...(13.24)$$

From equation (13.24) it is seen that if the theodolite is fitted with an anallatic lens, the horizontal distance between the instrument axis and the staff position is obtained by multiplying the staff intercept by the multiplying constant.

Example 13.1. *A tacheometer has a diaphragm with three cross hairs spaced at distances 1.15 mm. The focal length of the object glass is 23 cm and the distance from the object glass to the trunnion axis is 10 cm. Calculate the tacheometric constants.*

Solution.

Here $f = 23$ cm : $d = 10$ cm

$i = 2 \times 1.15 = 2.30$ mm $= 0.23$ cm

Substituting the values in the standard formulae

$$\frac{f}{i} = \frac{23}{0.23} = 100$$

and $(f + d) = 23 + 10 = 33$ cm

Multiplying constant = 100

Additive constant = 0.33 **Ans.**

Example 13.2. *A staff was held vertically at a distance of 46.2 m and 117.6 m from the centre of a theodolite fitted with stadia hairs and the staff intercepts with the telescope horizontal were 0.45 m and 1.15 m respectively. The instrument was then set over a station P of RL. = 150 m, the height of instrument axis being 1.38 m. The stadia hair readings on a staff held vertically at a station Q were 1.2 m, 1.93 m and 2.65 m respectively, while the vertical angle was –9° 30′. Find the distance PQ and RL. of Q.*

Solution. (Fig. 13.9) By substituting the values in equation

$$D = AS + B, \text{ we get}$$

$$46.2 = A \times 0.45 + B \qquad ...(i)$$

$$117.6 = A \times 1.15 + B \qquad ...(ii)$$

By solving equations (*i*) and (*ii*) we get, $A = 102$ and $B = 0.3$

∴ Tateometric constants are :

Multiplying cost = 102,

Additive constant = 0.3

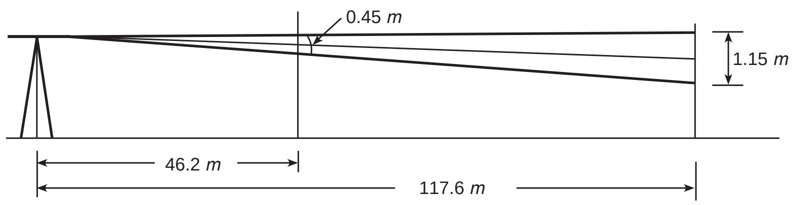


Fig. 13.9 (*a*)

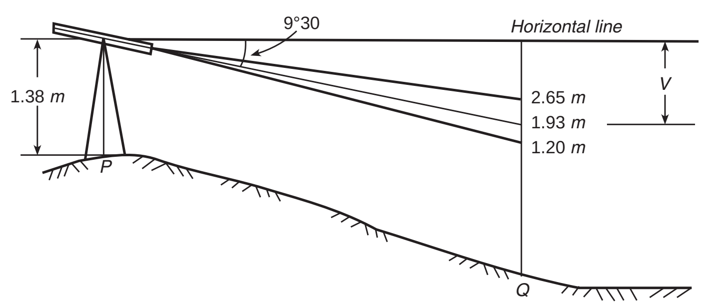


Fig. 13.9 (*b*)

Calculation of horizontal distance

$$S = 2.65 - 1.20 = 1.45 \text{ m}$$

$$\theta = 9°.5$$

$$PQ = \text{As} \cos \theta^2 + B \cos \theta.$$

$$= 102 \times 1.45 \cos^2 9°.5 + 0.3 \cos 9°.5$$

$= 143.87 + 0.30 = 144.17$ m

Vertical Component $V = As\,\frac{\sin 2\theta}{2} + B\sin\theta$

$= 102 \times 1.45\,\frac{\sin 19°}{2} + 0.3 \times \sin 9°.5$

$= 24.08$ m $+ 0.05$ m $= 24.13$ m

$\therefore$ R.L. of Q = R.L. of P + H.I.– V – Central hair reading

$= 150.0 + 1.38 - 24.13 - 1.93$

$= 151.38 - 26.06 = 125.32$ m. **Ans**.

Example 13.3. *The stadia readings with horizontal sight on a vertical staff held 50 m away from a tacheometer were 1.284 and 1.780. The focal length of object glass was 25 cm. The distance between the object glass and trunnion axis of the tacheometer was 15 cm.*

Calculate the stadia interval.

Solution.

From Eqn. (13.2) we know

$$D = \frac{f}{i}s + (f + d) \qquad ...(i)$$

$D = 50$ m, $f = 0.25$ m, $d = 0.15$ m

$S = 1.780 - 1.284 = 0.496$

Substituting the values in Eqn. (i)

$$50 = \frac{0.25 \times 0.496}{i} + (0.25 + 0.15)$$

$$50 = \frac{0.25 \times 0.496}{i} + 0.4$$

or $$i = \frac{0.25 \times 0.496}{50 - 0.4} = \frac{0.25 \times 0.496}{49.6} = 0.0025 \text{ m}$$

$= 2.5$ mm. **Ans.**

Example 13.4. *A tacheometer was set up at an intermediate station C of the line AB and following readings were obtained :*

Staff Station	*Vertical angle*	*Staff readings*		
A	– 6° 20′	0.445	1.675	2.905
B	+ 4° 20′	0.950	1.880	2.810

The instrument was fitted with an anallatic lens and the constant was 100. Find the gradient of the line joining station A and station B.

Solution.

Staff held at A :

$$S_1 = 2.905 - 0.445 = 2.460 \text{ m} ; \theta_1 = -6° 20'$$

Applying tacheometric formulae, we get

Distance $CA = AS_1 \cos^2 \theta_1$

$= 100 \times 2.460 \times \cos^2 6° 20' = 243.0$ m.

Vertical component $V_1 = \dfrac{AS_1 \sin 2\theta_1}{2}$

$= 100 \times 2.46 \times \dfrac{\sin 12° 40'}{2} = 26.971$ m. (– ve)

Staff held at B :

$$S_2 = 2.810 - 0.950 = 1.860 ; \theta_2 = 4° 20'$$

∴ Distance $CB = 100 \times 1.860 \cos^2 4° 20'$

$= 184.94$ m

Vertical component $V_2 = AS_2 \dfrac{\sin 8° 40'}{2}$

$= 100 \times 1.860 \times \dfrac{\sin 8° 40'}{2} = 14.014$ m (+ve)

∴ The distance $AB = 243.0 + 184.94 = 427.94$ m.

Let X be the reduced level of the trunnion axis

R.L. of station $A = X - V_1$ – central hair reading

$= X - 26.971 - 1.675 = X - 28.646$

R.L. of station $B = X + V_2$ – central hair reading

$= X + 14.014 - 1.880 = X + 12.134$

Difference in elevation of A and B

$= X + 12.134 - (X - 28.646) = 40.780$ m

∴ Gradient of line AB

$= \dfrac{427.94}{40.780}$ or 1 in 10.49 upward **Ans.**

Example 13.5. *Determine the gradient from a point A to a point B from the following observations made with a fixed hair tacheometer fitted with an anallatic lens, the constant of the instrument being 100.*

	Bearing	*Reading of stadia hairs*	*Reading of axial hair*	*Vertical angle*
To A	*345°*	*0.750 2.120*	*1.435*	*+15°*
To B	*75°*	*0.625 3.050*	*1.835*	*+10°*

Solution. (Fig. 13.9).

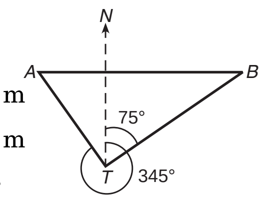


Fig. 13.9

Let the station of observation be T.

$$S_1 = 2.120 - 0.750 = 1.370 \text{ m}$$

$$S_2 = 3.050 - 0.625 = 2.425 \text{ m}$$

Distance TA $= 100 \times 1.370 \times \cos^2 15°$

$= 100 \times 1.370 \times (0.965926)^2$

$= 127.82$ m

Distance $TB = 100 \times 2.425 \cos^2 10° = 100 \times 2.425 \times (0984808)^2$

$= 235.19$ m

Angle ATB = Bearing of TB – Bearing of TA

$= 75° + 360° - 345° = 90°$

Distance $AB = \sqrt{TA^2 + TB^2}$

$= \sqrt{(127.82)^2 + (235.19)^2} = 267.68$ m

Vertical component for A

$$V_1 = 127.82 \tan 15° = 127.82 \times 0.267949 = 34.25 \text{ m}$$

Vertical component for B

$$V_2 = 235.19 \tan 10° = 235.19 \times 0.176327 = 41.47 \text{ m}$$

Difference in elevation between B and A

$= (41.470 - 1.835) - (34.250 - 1.435)$

$= 39.635 - 32.815 = 6.820$ m, B being higher

$\therefore$ Gradient from A to $B = \dfrac{267.68}{6.820}$ *i.e.*, 1 in 39.25 **Ans.**

Example 13.6. *A staff was held vertically at a distance of 100 m and 300 m from the centre of a theodolite fitted with stadia hairs and the staff intercepts with the telescope horizontal were 0.990 and 3.000 respectively. The instrument was then set over a station A of R.L. 950.50 m and the height of instrument was 1.42 m. The stadia hair readings of a staff held vertically at station B were 1.00, 1.83 and 2.67 m while the vertical angle was — 10° 00′. Find the distance AB and R.L. of B.*

Solution.

(*i*) **Calculation of Tacheometric constants :**

Distance	*Intercept*
100 m	*0.990*
300 m	*3.000*

Substituting the above values in the tacheometric equation, *i.e.*

$$D = A.S. + B$$

$$100 = A \times 0.990 + B \quad ...(i)$$

$$300 = A \times 3.000 + B \quad ...(ii)$$

Substracting Eqn. (*i*) from Eqn. (*ii*), we get

$$200 = 2.001\,A$$

or $$A = \frac{200}{2.010} = 99.5$$

Substituting the value of A in Eqn. (*ii*), we get

$$300 = 99.5 \times 3.000 + B$$

or $$B = 300 - 298.5 = 1.5$$

Tacheometric constants are : 99.5 and 1.5. **Ans.**

(*ii*) **Calculation of the Horizontal distance AB** (Fig. 13.10).

Here $S = 2.67 - 1.00 = 1.67$; $\theta = 10°$

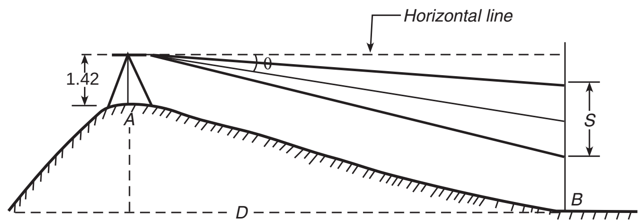


Fig. 13.10.

$$A = 99.5 \text{ and } B = 1.5$$

Substituting these values in equation (13.8 a), we get

$$D = 99.5 \times 1.67 \cos^2 10° + 1.5 \cos 10°$$

$$= 99.5 \times 1.67 \times (0.984808)^2 + 1.5 \times 0.984808$$

$$= 161.15 + 1.48$$

$\therefore$ Distance $AB = 162.63$ m. **Ans.**

(*iii*) **Calculation of R.L. of B**

Vertical component $V = D \tan \theta$

$$= 162.63 \times 0.176367 = 28.68 \text{ m}$$

R.L. of B = R.L. of A + H.I. –– V – central wire reading

$= 950.50 + 1.42 - 28.68 - 1.83$

$= 921.41$ m **Ans.**

Example 13.7. *The following observations pertain to a tacheometric traverse conducted with a tacheometer fitted with an anallatic lens and having a multiplying constant of 100, the levelling staff having been kept vertical :*

Inst. Stn.	*Staff Stn.*	*Hair Reading (m) upper-middle-lower*			*Ht.of Inst. (m)*	Vertical angle
P	Q	0.660	1.750	2.840	1.60	*0° 00′ 00″*
Q	P	0.715	1.810	2.905	1.56	*0° 00′ 00″*
Q	R	1.845	2.520	3.195	1.56	+13° 30′ 00″

The R.L. of P is 587.75 metres. Determine the distance PQ and QR and the reduced levels of Q and R.

Solution.

(*i*) Calculation of the distance PQ with instrument at P

$$S = 2.840 - 0.660 = 2.180 \text{ m}$$

$$\therefore \quad PQ = 100 \times 2.180 = 218 \text{ m.}$$

(*ii*) Calculation of distance PQ with instrument at Q

$$S = 2.905 - 0.715 = 2.190 \text{ m}$$

$$PQ = 100 \times 2.190 = 219 \text{ m.}$$

$$\therefore \quad \text{Mean distance } PQ = \frac{218 + 219}{2}$$

$$= 218.5 \text{ m.} \quad \textbf{Ans.}$$

(*iii*) Horizontal distance QR.

Here $S = 3.195 - 1.845 = 1.350$ m ; $\theta = 13° 30'$

Substituting the values of S and θ in distance equation for inclined sights.

i.e. $$D = AS \cos^2 \theta$$

$$\therefore \quad \text{Distance } QR = 100 \times 1.35 \cos^2 13° 30'$$

$$= 100 \times 1.35 \times (0.97237)^2$$

$$= 127.64 \text{ m} \quad \textbf{Ans.}$$

(*iv*) Vertical component $V = D \tan \theta$

$$= 127.64 \times \tan 13° 30'$$

$$= 127.64 \times 0.240079 = 30.64 \text{ m.}$$

(*v*) R.L. of Q = R.L. of P + H.I. at P – Central wire reading on staff held at Q = 587.75 + 1.60 – 1.750 = 587.60 m

Again, if R.L. of Q is x, then

$$x + 1.56 - 1.81 = 587.75$$

or $$x = 587.75 + 1.81 - 1.56 = 588.00$$

Mean R.L. of

$$Q = \frac{587.60 + 588.00}{2} = 587.80 \text{ m} \quad \textbf{Ans.}$$

(*vi*) R.L. of R = R.L. of Q + H.I. + V – Central wire reading

$$= 587.80 + 1.56 + 30.64 - 2.52$$

$$= 617.48 \text{ m} \quad \textbf{Ans.}$$

Example 13.8. *Following observations were taken with a tacheometre fitted with an anallatic lens having value of constant to be 100.*

Inst. Station	*Staff station*	*R.B.*	*Vertical angle*	*Staff Readings*		
O	P	*N* 37° *W*	4° 12′	0.910	1.510	2.110
O	Q	*N* 23° *E*	5° 42′	1.855	2.705	3.555

Calculate the horizontal distance between P and Q.

Solution. (Fig. 13.11)

Staff at station P

$S_1 = 2.110 - 0.910 = 1.200$ m

$\theta_1 = +4° 12′$; $A = 100$

Substituting the values in Eqn.

$D = AS \cos^2 \theta$, we get

$OP = 100 \times 1.2 \times \cos^2 4° 12′$

$= 119.36$ m.

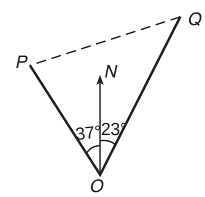


Fig. 13.11.

Staff at station Q

$S_2 = 3.555 - 1.855 = 1.700$

$\theta_2 = +5° 42′$; $A = 100$

Substituting the values again in eqn. $D = AS \cos^2 \theta$, we get

$OQ = 100 \times 1.7 \times \cos^2 5° 42′$

$= 168.32$ m.

From $\Delta\, POQ$ we get

$\angle POQ = 37° + 23° = 60°$

Now applying cosine formula to ΔPOQ we get

$$PQ^2 = OP^2 + OQ^2 - 2\, OP.OQ \cos POQ$$

Substituting the values in Eqn. (*i*) we get

$$PQ = \sqrt{119.36^2 + 168.32^2 - 2 \times 119.36 \times 168.32 \cos 60°}$$

$$= \sqrt{42578.43 - 20090.67}$$

Example 13.9. *Determine the gradient from a point P to another point Q from the following observations made with a tacheometer fitted with an anallatic lens. The constant of the instrument was 100 and the staff was held vertically.*

Inst. Stn.	*Staff. Stn.*	*Bearing*	*Vertical angle*	*Staff readings*
	P	*130°*	+ *10° 32′*	1.255, 1.810 2.365
R				
	Q	*220°*	+5° *06′*	1.300, 2.120 2.940

Solution. (Fig. 13.12).

Let Q and P represent the locations of staff stations and R is the instrument station.

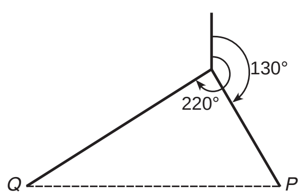


Fig. 13.12.

Angle QRP = Bearing of RQ -- Bearing RP

$$= 220° - 130° = 90°$$

$\therefore$ ΔQRP is a right angled triangle at R

Calculation of Distance RP

Staff intercept $= 2.365 - 1.255 = 1.110$; $\theta = 10° 32'$

$$\therefore \quad RP = 100 \times S \times \cos^2 10° 32'$$

$$= 100 \times 1.11 \times (0.983149)^2 = 111 \times (0.983149)^2$$

$$= 107.29 \text{ m}$$

Vertical component $V_1 = RP \tan 10° 32'$

$$= 107.29 \times \tan 10° 32' = 107.29 \times 0.185941$$

$= 19.95$ m

Let X be the R.L. of the tacheometer axis at R

R.L. of P

= R.L. of the tacheometer axis + V_1 – central wire reading

$= X + 19.95 - 1.81$

$= X + 18.14$ ...(*i*)

Calculation of Distance RQ

Staff intercept $= 2.940 - 1.300 = 1.640$; $\theta = 5° 06'$

$\therefore \quad RQ = 100 \times S \times \cos^2 5° 06'$

$= 100 \times 1.64 \times (0.996041)^2 = 162.70$ m

Vertical component $V_2 = RQ \tan 5° 06'$

$= 162.7 \times \tan 5° 06' = 162.7 \times 0.0892477$

$= 14.52$ m

R.L. of Q = R.L. of tacheometer axis $+V_2$ – Central wire reading

$= X + 14.52 - 2.12 = X + 12.40$

Difference in level between Q and P

$= X + 12.40 - (X + 18.14) = 5.74$ m fall

Distance $\quad PQ = \sqrt{(107.29)^2 + (162.70)^2}$

$= 194.89$ m

$\therefore$ Gradient of $\quad PQ = \dfrac{194.89}{5.74}$

1 in 33.95 fall. **Ans.**

Example 13.10 *The following tacheometric observations were made with an anallactic telescope having a multiplying constant 100 on a vertically held staff*:

Instrument station	*Height of instrument*	*Staff station*	*Vertical angle*	*Station readings*		
A	*1.480*	*BM*	*– 1° 54′*	1.02	*1.72*	*2.42*
A	*1.480*	*P*	*+ 2° 36′*	*1.22*	*1.825*	*2.43*
Q	*1.500*	*P*	*+ 3° 06′*	*0.785*	*1.61*	*2.435*

If R.L. of B.M. is 100.000, find the RLs of stations A, P and Q.

Solution. (Fig. 13.13)

(*i*) **Observation from station A to B.M.**

Vertical component

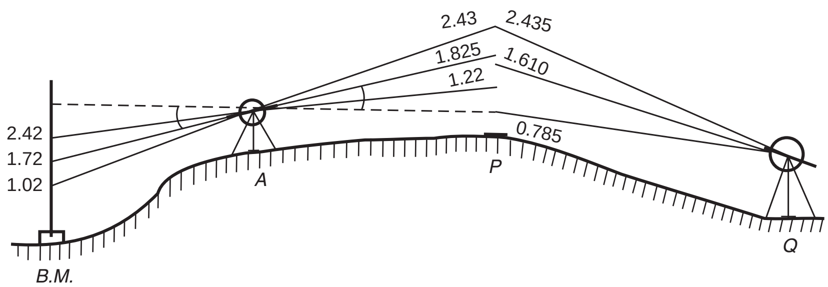


Fig. 13.13

$$V_1 = \frac{AS \sin 2\alpha}{2} \qquad ...(i)$$

Here $A = 100$

$S_1 = 2.42 - 1.02 = 1.40$

$2\alpha_1 = 2 \times (1° \ 54') = 3° \ 48'.$

Substituting the values in Eq. (i), we get

$$V_1 = \frac{100 \times 1.4 \times \sin 3^o \ 48'}{2}$$

$$= \frac{100 \times 1.4 \times 0.0662739}{2} = 4.639.$$

R.L. of station A = R.L. of B.M. + Central hair reading + $V_1 - HI_A$

= 100.000 + 1.720 + 4.639 − 1.480

= 104.879. **Ans.**

(ii) **Observation from station A to station P**

Here $S_2 = 2.43 - 1.22 = 1.21$

$2\alpha_2 = 2 \ (2° \ 36') \ 5^o \ 12'$

Substituting the values is Eq. (i), we get

$$V_2 = \frac{100 \times 1.21 \sin 5° \ 12'}{2}$$

$$= \frac{100 \times 1.21 \times 0.0906325}{2}$$

= 5.483

R.L. of P = R.L. of $A + H.I_a + V_2$ − Central hair reading

= 104.879 + 1.480 + 5.483 − 1.825

= 110.017.

(*iii*) **Observation from station Q**

Here $S_3 = 2.435 - 0.785 = 1.650$

$$2\alpha_3 = 2\,(3^\circ\ 06') = 6^\circ\ 12'$$

Substituting the values in Eq. (*i*), we get

$$V_3 = \frac{100 \times 1.65 \sin 6^\circ\ 12'}{2}$$

$$= \frac{100 \times 1.65 \times 0.1079993}{2}$$

$$= 8.910$$

R.L. of Q = R.L. of P + Central hair reading $- V_3 - H.I.$

= 110.017 + 1.610 − 8.910 − 1.500

= 101.217

Result : R.L. of station A = 104.879

R.L. of station P = 110.017

R.L. of station Q = 101.217 **Ans.**

Example 13.11. *The following readings were taken on a vertical staff with a tacheometer fitted with an anallatic lens and having a constant of 100.*

Staff Station	*Bearing*	*Stadia Readings*			*Vertical Angles*
A	*47° 10′*	*0.940*	*1.500*	*2.060*	*+ 8° 00′*
B	*227° 10′*	*0.847*	*2.000*	*3.153*	*– 5° 00′*

Calculate the relative level of the ground at A and B and the gradient between A and B.

Solution. (Fig. 13.14)

(*i*) **Staff at Station A**

Staff intercept $S_1 = 2.060 - 0.940 = 1.120$ m; $\theta_1 = 8^\circ$

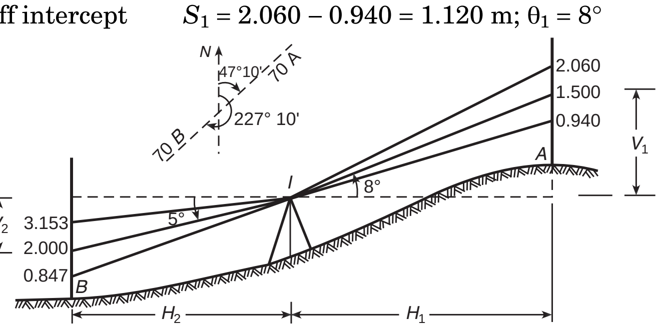


Fig. 13.14

Central wire reading = 1.500 m

We know that horizontal distance $H = 100\ S \cos^2 \theta$ ...(*i*)

and vertical component $V = 100\ S \dfrac{\sin 2\theta}{2}$ ...(*ii*)

Substituting the values of S_1 and θ_1 in Eqs. (*i*) and (*ii*), we get

$$H_1 = 100 \times 1.120 \times \cos^2 8°$$
$$= 100 \times 1.120 \times (0.990268)^2$$
$$= 109.83 \text{ m}$$

and
$$V_1 = 100 \times 1.120 \times \frac{\sin 16°}{2}$$
$$= 100 \times 1.120 \times \frac{0.275637}{2}$$
$$= 15.436 \text{ m.}$$

(*ii*) **Staff at Station B**

$$S_2 = 3.153 - 0.847 = 2.306 \text{ m}; \theta_2 = 5°$$

Central wire reading = 2.000

Substituting the values of S_2 and θ_2 in Eqs. (*i*) and (*ii*), we get

$$H_2 = 100 \times 2.306 \times \cos^2 5°$$
$$= 100 \times 2.306 \times (0.996195)^2$$
$$= 228.85 \text{ m}$$
$$V_2 = 100 \times 2.306\ \frac{\sin 10°}{2}$$
$$= 100 \times 2.306 \times \frac{0.173648}{2} = 20.022 \text{ m}$$

Let x be height of the instrument above the datum.

Level at A $= x + 15.436 - 1.500$

$= x + 13.936$

Level at B $= x - 20.022 - 2.000$

$= x - 22.022$

$\therefore$ Fall from A to B $= x + 13.936 - (x - 22.022)$

$= 35.958$ m

Bearing of B $= 227°\ 10'$

Bearing of A = 47° 10′

∴ Angle between A and B at the instrument station

= 227° 10′ – 47° 10′

= 180°

Hence A, B and the instrument station are on a straight line

$$\therefore \text{ Gradient of } AB = \frac{\text{Difference in level}}{\text{Total distance}}$$

$$= \frac{109.83 + 228.85}{35.958} = \frac{338.68}{35.958} = \frac{1}{9.42}$$

i.e. Gradient of AB is 1 in 9.42. **Ans.**

Example 13.12. *In a tacheometer survey made with an instrument whose constants are 100 and 0.5, the staff was inclined so as to be normal to the line of sight for each reading. Two sets of readings were as given below. Calculate the gradient between the staff stations P and Q and the reduced level of each if that of R is 41.800 m.*

Instrument Station	*Height of Instrument Axis*	*Staff Station*	*Bearing*	*Vertical Angle*	*Stadia Reading*
R	*1.600*	*P*	*85°*	*+ 4° 30′*	*1.000 1.417 1.833*
		Q	*135°*	*– 4° 00′*	*1.000 1.657 2.313*

Solution. (Fig. 13.15)

Given $\frac{f}{i} = 100$, and $(f + d) = 0.5$

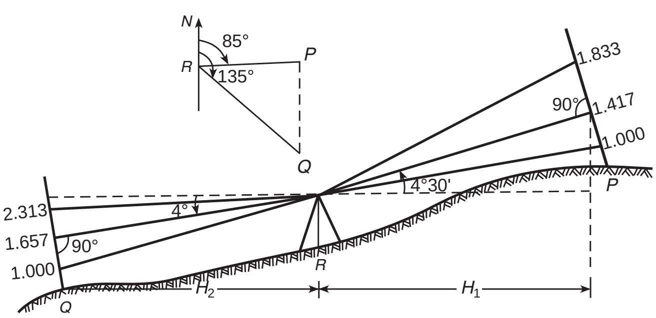


Fig. 13.15

If S = staff interval, h = the central wire reading

and θ = vertical angle, then

$$H = (AS + B) \cos \theta -+ h \sin \theta \qquad ...(i)$$

$V = (AS + B) \sin \theta$

(*a*) **Staff Station P**

Staff interval $S_1 = 1.833 - 1.000 = 0.833$ m

Central wire reading= 1.417 m

Substituting the values of S and θ in Eqns. (*i*) and (*ii*), we get

$$H_1 = (100 \times 0.833 + 0.5) \cos 4° \, 30' + 1.417 \times \sin 4° \, 30'$$
$$= 83.8 \times 0.996917 + 1.417 \times 0.0784592$$
$$= 83.54 + 0.11$$
$$= 83.65 \text{ m}$$

and
$$V_1 = (100 \times 0.833 + 0.5) \sin 4° \, 30'$$
$$= 83.8 \times 0.07846$$
$$= 6.575 \text{ m.} \qquad ...(ii)$$

$\therefore$ Ground level at $P = 41.800 + 1.600 + 6.575 - 1.417 \cos 4° \, 30'$
$$= 41.800 + 1.600 + 6.575 - 1.413$$
$$= 48.562 \text{ m}$$

(*b*) **Staff Station Q**

Staff interval $S_2 = 2.313 - 1.000 = 1.313$

Central wire reading= 1.657

Substiuting the values of S and θ, in Eqs. (*i*) and (*ii*), we get

$$H_2 = (100 \times 1.313 + 0.5) \cos 4° - 1.657 \sin 4°$$
$$= 131.8 \times 0.997564 - 1.657 \times 0.06976$$
$$= 131.48 - 0.12 = 131.36 \text{ m}$$
$$V_2 = 131.8 \sin 4° = 131.8 \times 0.0697566$$
$$= 9.194 \text{ m}$$

$\therefore$ Ground level at

$$Q = 41.800 + 1.600 - 9.194 - 1.657 \times \cos 4°$$
$$= 41.800 + 1.600 - 9.194 - 1.653$$
$$= 32.553 \text{ m}$$

Difference in level of P and Q = 48.562 – 32.553 = 16.009 m

Angle PRQ = Bearing of RQ – Bearing of RP = 135° - 85° = 50°

Applying cosine formula to ΔRPQ , we get

$$PQ^2 = (83.65)^2 + (131.36)^2 - 2 \times 83.65 \times 131.36 \cos 50°$$
$$= 69{,}97.32 + 172{,}55.45 - 2 \times 479.46 \times 0.642788$$
$$= 10126.520$$

or $PQ = 100.63$ m.

∴ The gradient between P and Q

$$= \frac{100.63}{48.562 - 32.553}$$
$$= \frac{100.63}{16.009} = 1 \text{ in } 6.286 \text{ fall.}$$ **Ans.**

Example 13.13. *Two sets of tacheometric readings were taken from an instrument station A (R.L. 100.000) to a staff station B.*

Instrument	*P*	*Q*
Multiplying constant	100	95
Additive constant	30 cm	45 cm
Height of instrument	1.40 m	1.45 cm
Staff held	Vertical	Normal to line of sight
Inst. *at* *to*	*Vertical angle*	*Stadia readings*
P A B	5° 44′	1.090
Q A B	5° 44′	?

Determine

(*a*) *The distance between instrument station and staff station.*

(*b*) *The R.L. of staff station B.*

(*c*) *The stadia readings with instrument Q.*

Solution.

(*a*) Using observations of instrument *P*.

Horizontal distance $AB = AS \cos^2 \theta + B \cos \theta$ ...(*i*)

Here $A = 100$; $\theta = 5° 44'$; $S = 1.795 - 1.090 = 0.705$ m

Substituting the values in equation (*i*), we get

$$AB = 100 \times 0.705 \times (\cos 5° 44')^2 + 0.3 \cos 5° 44$$
$$= 100 \times 0.705 \times (0.995) + 0.3 \times 0.995$$
$$= 69.797 + 0.298$$
$$= 70.095 \text{ m}$$ **Ans.**

(*b*) Using observations of instrument *P*.

Vertical component

$$V = \frac{AS \sin 2\theta}{2} + B \sin \theta$$

$$V = 100 \times 0.705 \times \frac{\sin 11° 28'}{2} + 0.3 \times \sin 5° 44'$$

$$= 100 \times 0.705 \times \frac{0.19880}{2} + 0.3 \times 0.999$$

$$= 7.008 + 0.0300$$

$$= 7.038 \text{ m}$$

$\therefore$ R.L. of staff station = R.L. of A + H.I. + V – central hair reading

= 100.000 + 1.400 + 7.038 – 1.440

= 106.998 m **Ans.**

(c) Using observations of instrument Q

Let S = stadia intercept; h = central hair reading

Horizontal distance

$$AB = (AS + B) \cos \theta + h \sin \theta$$

$$= (95 \times S + 0.45) \cos 5° 44' + h \sin 5° 44'$$

$$= (95\ S + 0.45) \times 0.995 + h \times 0.0999$$

$$= (95\ S + 0.45) \times 0.995 + 0.0999\ h = 70.095$$

or $\quad 946.1962\ S + h = 697.167$ ...(i)

Again, vertical component

$$V = (AS + B) \sin \theta$$

$$= (95\ S + 0.45) \times \sin 5° 44'$$

$$= (95\ S + 0.45) \times 0.0999$$

$\therefore$ R.L. of B = R.L. of A + H.I. + V – $h \cos \theta$

= 100.000 + 1.450 + (95 S + 0.45) × 0.0999 – h × 0.995

100.000 + 1.450 + (95 S + 0.45) × 0.0999 – h × 0.995 = 106.998

$9.4905\ S - 0.995\ h = 5.503$

or $\quad 9.5382\ S - h = 5.5307$ ...(ii)

Adding equations (i) and (ii), we get

$$955.7344\ S = 702.698$$

or $\quad S = 0.735244$ m

and $\quad h = 9.5382 \times 0.735244 - 5.631$

or $h = 1.4819$ m

Stadia upper reading $= 1.4819 + 0.3676 = 1.8495$ m

Stadia central reading $= 1.4819$ m

Stadia lower reading $= 1.4819 - 0.3676$

$= 1.1143$ m

$\therefore$ Stadia readings of instrument Q are:

1.8495, 1.4819, 1.1143. **Ans.**

Example 13.14. *Following observations were taken for determining the R.L. of station A :*

Inst. stn.	*H.I. of Inst.(m)*	*Staff stn.*	*Vertical angle*	*Staff Readings*	*Remarks*
Q	*1.60*	*B.M.*	*+ 6° 12′*	*0.945 1.675 2.405*	*R.L. of B.M. = 421.625*
Q	*1.60*	*P*	*– 4° 12′*	*1.450. 2.380 3.310*	
A	*1.65*	*P*	*7° 0′*	*X, 0.655 1.255*	*Reading X could not be observed*

The instrument was fitted with an anallatic lens and the value of the constant was 100.

Calculate the RL. of station A.

Solution. (Fig. 13.16)

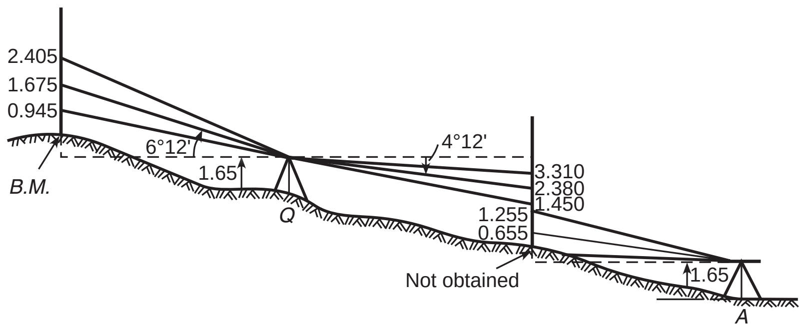


Fig. 13.16

Instrument at Q, staff held on B.M.:

$$S_1 = 2.405 - 0.945 = 1.460 \text{ m}$$

$$\theta = +6° \ 12' : A = 100.$$

Let V_1 be the vertical component, then

$$V_1 = AS \,.\, \frac{\sin 2\,\theta}{2} = 100 \times 1.46 \times \frac{\sin 12° \ 24'}{2}$$

$= +15.676$ m.

$R.D.$ of stn. Q = $R.L.$ of $B.M.$ + central hair reading $- V_1 - H.I.$

$= 421.625 + 1.675 - 15.676 - 1.600 = 406.024$ m

Instrument as station Q, staff held on station P

$S_2 = 3.310 - 1.450 = 1.860$; $\theta = -4° 12'$

Let V_2 be the vertical component

$$V_2 = 100 \times 1.86 \times \frac{\sin 8^\circ 24'}{2} = 13.586 \text{ m.} \quad (-\text{ve})$$

R.L. of station P = R.L. of station Q + H.I. $- V_2 -$ central hair reading

$= 406.024 + 1.600 - 13.586 - 2.380$

$= 391.658$ m

Instrument at station A, staff held on station P.

$$S_3 = 2(1.255 - 0.655) = 1.200; \theta = 7° 0'$$

Let V_3 be the vertical component,

$$V_3 = 100 \times 1.200 \times \frac{\sin 14^\circ}{2}$$

$= 14.515$ m

$\therefore$ R.L. of station A = R.L. of P + central hair reading $- V_3 -$ H.I.

$= 391.658 + 0.655 - 14.515 - 1.650$

$\therefore$ R.L. of $A = 376.148$ m **Ans.**

Example 13.15. *Determine the distance, bearing and gradient of line BC from the following observations which were made with a tacheometer fitted with an anallatic lens, having its constant 100.*

Stn.	*Staff*	*Bearing*	*Vertical angle*	*Stadia readings*
A	*B*	*N 40° W*	*+ 8° 05′*	*2.855 2.350 1.845*
	C	*N 50° E*	*− 8° 41′*	*2.370 1.600 0.835*

Solution. (Fig. 13.17)

Staff held at B

$$S_1 = 2.855 - 1.845 = 1.010 : \theta_1 = 8° 05'$$

$\therefore$ Horizontal distance AB

$= 100 \times 1.010 \times \cos^2 8° 05'$

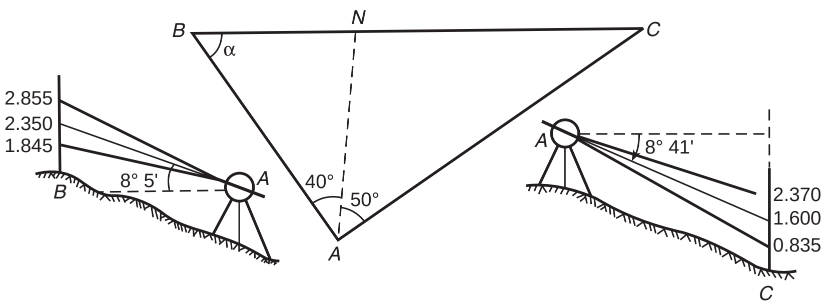


Fig. 13.17

$$= 99 \text{ m}$$

Vertical component

$$V_B = 99 \tan 8° \, 05' = 14.06 \text{ m}$$

Staff held at C

$$S_2 = 2.370 - 0.835 = 1.535 \text{ m}; \theta_2 = 8° \, 41'$$

Horizontal distance AC

$$= 100 \times 1.535 \cos^2 8° \, 41' = 150 \text{ m.}$$

Vertical component

$$V_c = 150 \tan 8° \, 41' = 22.91 \text{ m}$$

Apparently ABC is a right angled triangle at A.

$\therefore$ $$BC = \sqrt{AB^2 + AC^2} = \sqrt{99^2 + 150} = 179.72 \text{ m}$$ **Ans.**

Let angle ABC be α

$\because$ $$\tan \alpha = \frac{150}{99} = 1.5151515$$

or $$\alpha = 56° \, 34' \, 31''$$

Bearing of BA $= 180° - 40° = 140°$

Bearing of $BC = 140° - \alpha = 140° - 56° \, 34' \, 31''$

$= 83° \, 25' \, 29''$ **Ans.**

Let R.L.of trunnion axis of tacheometer be 'x'

R.L. of B = R.L. of trunnion axis + V_B − central hair reading

$$= x + 14.06 - 2.35 \quad \ldots(i)$$

R.L. of C = R.L. of trunnion axis + V_C − central hair reading

$$= x + 22.91 - 1.60 \quad \ldots(ii)$$

$\therefore$ Difference of level of points B and C

$= x + 14.06 - 2.35 - (x + 22.91 - 1.600)$

$= 14.06 - 2.35 - 22.91 + 1.60$

$= 11.71 - 21.31 = -9.60$ m, C being higher

$\because$ Gradient of $BC = \dfrac{179.72}{9.6} =$ **1 in 18.72 Ans.**

Example 13.16. *The following observations were made from stations A and B to points C and D lying northward, with a fixed hair tacheometer fitted with an anallactic lens, the constant of the instrument being 100.*

Stn.	*Ht. of Inst.*	*Staff at*	*Vertical angle*	*Staff intercept*	*Axial line*
A	*1.4 m*	*C*	*8° 36′*	*1.880*	*2.105*
	1.4 m	*D*	*10° 00′*	*2.425*	*1.835*
B	*1.5 m*	*C*	*4° 38′*	*2.075*	*1.675*
	1.5 m	*D*	*8° 14′*	*2.180*	*1.750*

The coordinates of stations of observation are :

Stn.	*Easting(m)*	*Northing(m)*	*R.L.(m)*
A	*5320*	*3580*	*300.85*
B	*5440*	*3670*	*311.41*

Calculate the distance and gradient of CD and coordinates of the points C and D.

Solution. (Fig. 13.18)

Observations at station A

$S_c = 1.880$ m; $\theta = 8° 36'$

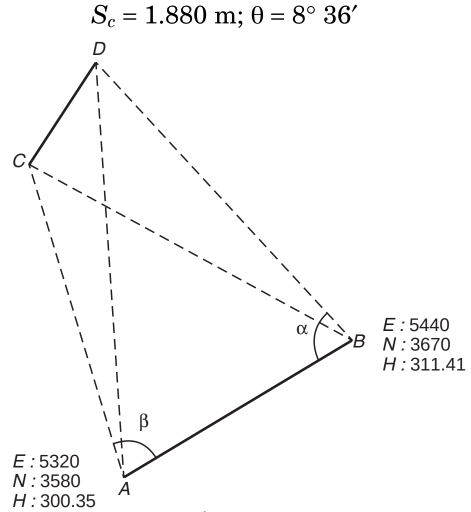


Fig. 13.18

Horizontal distance

$$AC = 100 \times 1.880 \times \cos^2 8° 36' = \mathbf{183.80\ m.}$$

Vertical component

$$V_C = 183.79 \tan 8° 36' = 27.80 \text{ m}$$

Horizontal distance

$$AD = 100 \times 2.425 \times \cos^2 10° 00' = 235.19 \text{ m}$$

Vertical component

$$V_D = 235.19 \tan 10° 00' = 41.47 \text{ m}$$

Observations at station B

Horizontal distance

$$BC = 100 \times 2.075 \times \cos^2 4° 38'$$
$$= 206.15 \text{ m}$$

Vertical component $V_C = 206.15 \tan 4° 38'$

$$= 16.71 \text{ m}$$

Horizontal distance

$$BD = 100 \times 2.180 \cos^2 8° 14'$$
$$= 213.53 \text{ m}$$

Vertical component

$$V_D = 213.53 \tan 8° 14'$$
$$= 30.90 \text{ m}$$

R.L. of point C = R.L. of A + H.I. + V_C – central hair reading

$$= 300.85 + 1.400 + 27.80 - 2.105$$
$$= 327.95 \text{ m}$$

R.L. of point C = R.L. of B + H.I. + V_C – central hair reading

$$= 311.41 + 1.5 + 16.71 - 1.675$$
$$= 327.95 \text{ m}$$

$\therefore$ Mean R.L. of $C = \dfrac{327.95 + 327.95}{2} = 327.95$ m

R.L. of point D = R.L. of A + H.I. + V_D – central hair reading

$$= 300.85 + 1.4 + 41.47 - 1.835$$
$$= 341.89 \text{ m}$$

R.L. of point D = R.L. of B + H.I. + V_D – central hair reading

$$= 311.41 + 1.5 + 30.90 - 1.750 = 342.06 \text{ m}$$

Mean R.L. of $D = \dfrac{341.89 + 342.06}{2} = 341.98$ m

Side $AB = \sqrt{(5440 - 5320)^2 + (3670 - 3580)^2} = 150$ m

From $\Delta\, ABC$ we get

$$\cos \beta = \frac{AC^2 + AB^2 - BC^2}{2 \times AC \times AB}$$

$$\frac{(183.80)^2 + (150)^2 - (206.15)^2}{2 \times 183.80 \times 150} = \frac{56282.44 - 42497.822}{55140}$$

$$= .2499 = 0.24999307$$

or $\beta = 75° \, 31' \, 22''$

From $\Delta\, ABD$, we get

$$\cos \alpha = \frac{AB^2 + \text{BD}^2 - AD^2}{2 \times AB \times BD}$$

$$= \frac{150^2 + (213.53)^2 - (235.19)^2}{2 \times 150 \times 213.53} = 0.19951488$$

$$\alpha = 78° \, 29' \, 29''$$

Let the bearing of line AB be θ

$$\tan \theta = \frac{120}{90} = 1.33333$$

$$\theta = 53° \, 07' \, 48''$$

Bearing of BA = Bearing of AB + 180°

$= 53° \, 07' \, 48'' + 180° = 233° \, 07' \, 48''$

Bearing of AC $= 53° \, 07' \, 48'' + 360° - \beta$

$= 53° \, 07' \, 48'' + 360° - 75° \, 31' \, 22''$

$= 337° \, 36' \, 26''$ or N 22° 23′ 46″ W

Bearing of BD = Bearing of $BA + \alpha$

$= 233° \, 07' \, 48'' + 78° \, 29' \, 29'' = N \, 48° \, 22' \, 34''$ W.

Side	*Length*	*R.B.*	*Latitude* (+)	*Departure* (–)
AC	183.80	*N* 22° 23′ 34″ W	169.94 - ...	...+ 70.02 -
BD	213.53	*N* 48° 22′ 43″ W	141.83 ...	...159.62

	Easting	*Northing*
Coordinate of *C*	5320 – 70.02 = 5249.98	3580 + 169.94 = 3749.94
Coordinater of *D*	5440 – 159.62 = 5280.38	3670 + 141.83 = 3811.83

Distance

$$CD = \sqrt{(5280.38 - 5249.98^{2)} + (3811.83 - 3749.94)^2}$$

$$= \sqrt{30.40)^2 + (61.89)^2} = 68.95 \textbf{ m.} \quad \textbf{Ans.}$$

Difference in level of points *C* and *D*

$$= 341.98 - 327.95 = \mathbf{14.03\ m}$$

$$\therefore \quad \text{Gradient of line } CD = \frac{68.95}{14.03} = 4.91$$

i.e. **1 in 4.91 Ans.**

13.10. MOVABLE HAIR METHOD

In this method the staff intercept is kept constant whereas the distance between stadia hairs is variable. Instruments used in this method are a theodolite with a special type of diaphragm and a staff provided with two targets at a known distance.

1. Diaphragm of the Theodolite [Fig. 13.19]

In this type of diaphragm, the central or axial wire is fixed in the plane of the telescope. The stadia hairs are moved in vertical plane by means of two finely threaded micrometer screws. The distance through which either wire is moved from the fixed central wire, is measured by the number of turns made by the micrometer screw. The full turns are read on the graduated scale seen in the field of view and the fractional part of a turn is read on the graduated drum of the micrometer screw placed one above and one below the eye piece. The total distance through which stadia wires move, is equal to the sum of the micrometer readings.

It may be noted that graduations on the micrometer drums are in opposite directions.

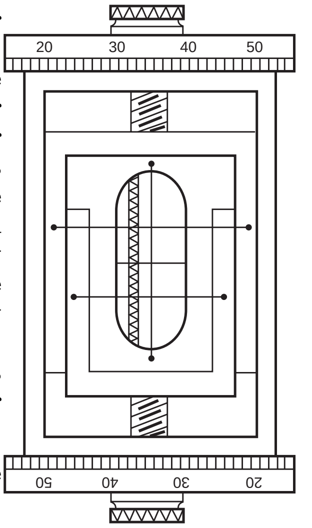


Fig. 13.19. A special type diaphragm of a moving hair theodolite.

2. The staff targets. If the distance between the instrument station and staff position is within 200m, an ordinary levelling staff may be used and a full metre reading used for the purpose of observing a constant intercept. For distances exceeding 200 m, it becomes difficult to read graduations. In such cases two vanes or targets fixed at a known distance apart on a staff, are observed. A third target is fixed at the mid-point of the two targets.

13.11. METHOD OF OBSERVATIONS

While making observations with a theodolite fitted with special type of diaphragm, the central target is first bisected with the axial wire. Micrometers are then turned simultaneously to move the stadia wires in vertical plane. The readings are then noted.

1. Tacheometric formulae for subtense theodolite. A theodolite fitted with a diaphragm in which stadia wires are movable, is known as a *subtense theodolite.* The tacheometric horizontal formula, $D = \frac{f}{i} s + (f + d)$ is equally applicable to the movable hair method. In this case all terms on right hand side are constant except i, the stadia distance. Hence, the multiplying constant is (sf) and the additive constant is $(f + d)$. If m is the total number of the revolutions of the micrometer of pitch p, for a staff intercept then $i = mp$.

Substituting the value of i in tacheometric equation, we get

$$D = \frac{fs}{mp} + (f + d)$$

or $$D = \frac{A}{m} S + B \qquad ...(13.25)$$

However, if e is the index error eqn. (13.25) reduces to

$$D = \frac{A}{m - e} s + B. \qquad ...(13.25a)$$

2. Determination of tacheometric constants. The value of additive constant $B = (f + d)$ may be easily obtained by measuring distances along the telescope of the theodolite. Values of both the constants A and B may be calculated by making field observations as detailed below :

Procedure. The following steps are followed.

1. Measure two distances D_1 and D_2 on a fairly level ground, from the instrument station.
2. Hold the staff carrying two targets S metres apart at the end of distance D_1.
3. Bisect the central target and rotate both the micrometers simultaneously to bisect upper and lower targets. Let the total distance moved be m_1.
4. Now shift the staff to the end of distance D_2.
5. Bisect the central target and rotate both the micrometers simultaneously to bisect upper and lower targets. Let the total distance moved be m_2.

6. Substituting the values in Eq. (13.25). we get

$$D_1 = \frac{A}{m_1} S + B \quad ...(13.26)$$

and

$$D_2 = \frac{A}{m_2} S + B \quad ...(13.27)$$

Subtracting Eq. (13.26) from Eq. (13.27), we get

$$D_2 - D_1 = AS\left(\frac{1}{m_2} - \frac{1}{m_1}\right)$$

$$= AS\,\frac{(m_1 - m_2)}{m_1 m_2}$$

or

$$A = \frac{(D_2 - D_1)\, m_1 m_2}{S\,(m_1 - m_2)} \quad ...(13.28)$$

Substituting the value of A in Eq. (13.27), we get

$$D_2 = \frac{(D_2 - D_1)\, m_1 m_2}{(m_1 - m_2) \times m_2} + B$$

$$B = D_2 - \frac{(D_2 - D_1)\, m_1}{m_1 - m_2}$$

$$= \frac{D_2\,(m_1 - m_2) - (D_2 - D_1)\, m_1}{m_1 - m_2}$$

$$= \frac{D_2 m_1 - D_2 m_2 - D_2 m_1 + D_1 m_1}{m_1 - m_2}$$

or

$$B = \frac{D_1 m_1 - D_2 m_2}{m_1 - m_2} \quad ...(13.29)$$

Note. The following points may be noted.

(*i*) The value of multiplying constant A in the equation $D = \frac{AS}{m} + (f + d)$ varies from 600 to 1000.

(*ii*) For inclined line of sights, the formulae used for fixed hair stadia tacheometry are equally applicable to this method.

Example 13.17. *The constant of an instrument being 600, the value of $(f + d) = 0.5$ m and the distance between targets is 3 m. Calculate the distance from the instrument to the staff position when the micrometer readings are 4.485 and 4.515.*

Solution.

From eqn. (13.25) we know,

$$D = \frac{AS}{m} + B \quad ...(i)$$

where $A = \frac{f}{p}$ = a constant

Here $S = 3.00$ m; $A = 600$ given)

$$m = 4.485 + 4.515 = 9.000;$$

$$B = f + d = 0.5 \text{ m}$$

Substituting the values in eqn. (*i*) we get

$$D = \frac{600 \times 3}{9.00} + 0.5$$

or $D = 200.0 + 0.5 = 200.5$ m **Ans.**

Example 13.18. *The stadia intercept read by means of a fixed hair instrument on a vertically held staff is 1.05 metres, the angle of elevation being 5° 36′. The instrument constants are 100 and 0.3. What would be the total number of turns registered on a movable hair instrument at the same station for a 1.75 metre intercept on a staff held on the same point, the vertical angle in this case being 5° 24′ and the constants 1000 and 0.5?*

Solution.

For the inclined sight and staff vertical.

$$D = A.S.\cos^2\theta + B\cos\theta$$

Here $A = 100$, $B = 0.3$; $S = 1.05$ m; $\theta = 5° 36'$

$$D = 100 \times 1.05 \times \cos^2 5° 36' + 0.3 \times \cos 5° 36'$$

$$= 100 \times 1.05 \times (0.9952)^2 + 0.3 \times 0.9952$$

$$= 103.99 + 0.300$$

or $D = 104.29$ m ...(*i*)

Again, $D = \frac{AS}{m}\cos^2\theta + B\cos\theta$

Here $S = 1.75$; $A = 1000$; $B = 0.5$; $\theta = 5° 24'$

$$\because \quad D = \frac{1.75 \times 1000}{m}(\cos 5° 24')^2 + 0.5 \times \cos 5° 24'$$

$$= \frac{1750 \times (0.9956)^2}{m} + 0.5 \times 0.9956$$

or $D = \frac{1734.63}{m} + 0.4978$ ...(*ii*)

Substituting the value of D from Eq. (i) in Eq. (ii), we get

$$104.29 = \frac{1734.63}{m} + 0.4978$$

or $$103.7922 = \frac{1734.63}{m}$$

$$m = \frac{1734.63}{103.7922}$$

or $$m = 16.7 \text{turns. Ans.}$$

13.12. THE TANGENTIAL METHOD OF TACHEOMETRY

In tangential tacheometry, horizontal and vertical distances from the instrument to the staff position, are computed from the observed vertical angles to two targets fixed at a known distance S on the staff.

Depending upon the vertical angles, three cases may arise :

1. Both vertical angles may be elevation angles.
2. Both vertical angles may be depression angles.
3. One of the angles may be an elevation angle and the other may be a depression angle.

Case I. Distance and Elevation Formulae

Let A and B represent two targets fixed S metres apart on a staff held vertical at D. (Fig. 13.20).

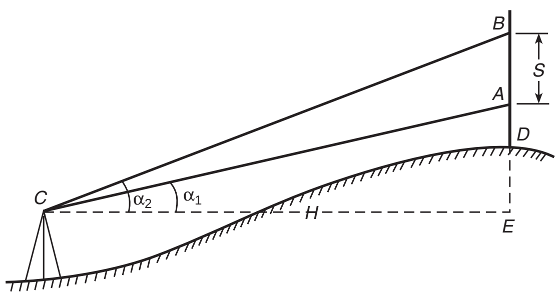


Fig. 13.20. Both vertical angles are elevations.

Let O represent the trunnion axis of the theodolite

Let α_1 and α_2 be the angles of elevation to the targets A and B respectively.

Horizontal distance formula :

From $\Delta\, ACE$, $CE = AE \cot a_1$ ...(13.30)

From ΔBCE, $CE = BE \cot \alpha_2$ ...(13.31)

Comparing Eqs. (13.30) and (13.31), we get

$$AE \cot \alpha_1 = BE \cot \alpha_2$$

$$AE \cot \alpha_1 = (AE + AB) \cot \alpha_2$$

$$AE \cot \alpha_1 = EA \cot \alpha_2 + S \cot \alpha_2 \quad \text{where } AB = S.$$

or $EA (\cot \alpha_1 - \cot \alpha_2) = S \cot \alpha_2$

$$EA = \frac{S \cot \alpha_2}{\cot \alpha_1 - \cot \alpha_2}$$

$$EA = \frac{S \tan \alpha_1}{\tan \alpha_2 - \tan \alpha_1} \qquad ...(13.32)$$

$\because$ $H = AE \cot \alpha_1$

$\therefore$ $$H = \frac{S}{\tan \alpha_2 - \tan \alpha_1} \qquad ...(13.33)$$

or $$H = S \cos \alpha_1 \cos \alpha_2 \, cosec \, (\alpha_2 - \alpha_1) \qquad ...(13.34)$$

Eq. (13.34) is a useful form of the Eqn. (13.33) for logarithmetical calculations.

Elevation formula :

R.L. of D = R.L. of trunning axis + $EA - AD$

$$= \text{R.L. of trunning axis } + \frac{S \tan \alpha_1}{\tan \alpha_2 - \tan \alpha_1}$$

$$- \text{ height of lower target above } D \qquad ...(13.35)$$

Case 2. Distance and Elevation Formulae (Fig. 13.21)

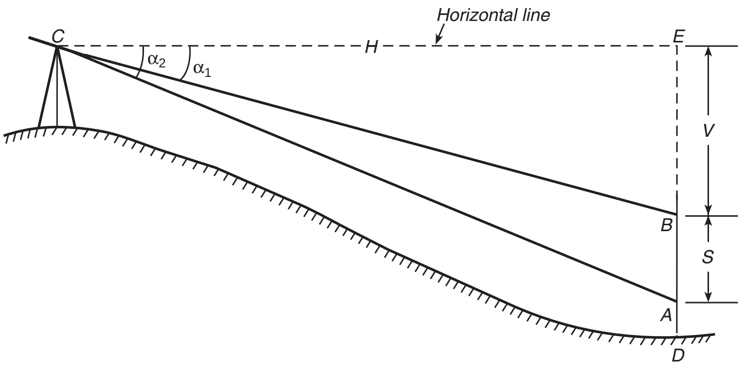


Fig. 13.21. Both vertical angles are depressions.

Use of the same notations as in case 1.

Horizontal distance formula :

From Δs ACE and BCE, we get

$$H = AE \cot \alpha_2 = BE \cot \alpha_1$$

$$(AB + BE) \cot \alpha_2 = BE \cot \alpha_1$$

$$AB \cot \alpha_2 + BE \cot \alpha_2 = BE \cot \alpha_1$$

$$AB \cot \alpha_2 = BE (\cot \alpha_1 - \cot \alpha_2$$

$$BE = \frac{S \cot \alpha_2}{\cot \alpha_1 - \cot \alpha_2} = \frac{S \tan \alpha_2}{\tan \alpha_2 - \tan \alpha_1}$$

But $$H = BE \cot \alpha_1$$

$\therefore$ $$H = \frac{S}{\tan \alpha_2 - \tan \alpha_1} \quad ...(13.36)$$

$$= S \cos \alpha_1 \cos \alpha_2 \operatorname{cosec} (\alpha_2 - \alpha_1) \quad ...(13.37)$$

Elevation formula :

R.L. of D = R.L. of trunnion axis --EB--BD

$$= \text{R.L. trunning axis} - \frac{S \tan \alpha_2}{\tan \alpha_2 - \tan \alpha_1}$$

–height of the upper target above D. ...(13.38)

Case 3. Distance and Elevation Formulae (Fig. 13.22)

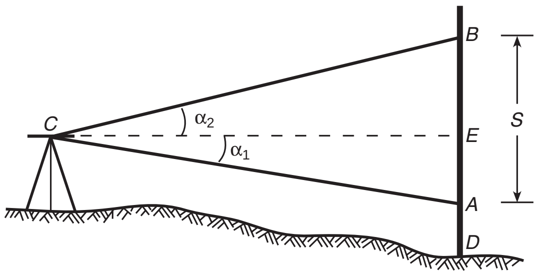


Fig. 13.22. One angle is elevation and other depression.

Use the same notations in case 1.

Horizontal distance formula :

From Δs ACE and BCE, we get

$$H = AE \cot \alpha_1 = BE \cot \alpha_2$$

or $$AE \cot \alpha_1 = (S - AE) \cot \alpha_2$$

or $$AE \cot \alpha_1 = S \cot \alpha_2 - AE \cot \alpha_2$$

or $$AE (\cot \alpha_1 + \cot \alpha_{2)} = S \cot \alpha_2$$

or $$AE = \frac{S \cot \alpha_2}{\cot \alpha_1 + \cot \alpha_2}$$

$$= \frac{S \tan \alpha_1}{\tan \alpha_1 + \tan \alpha_2}$$

But $H = AE \cot \alpha_1$

$$\therefore \quad H = \frac{S}{\tan \alpha_1 + \tan \alpha_2} \qquad \text{...(13.39)}$$

$$= S \cos \alpha_1 \cos \alpha_2 \operatorname{cosec} (\alpha_1 + \alpha_2) \qquad \text{...(13.40)}$$

Elevation Formula :

R.L. of D = R.L. of the trunnion axis $- AE - AD$

$$= \text{R.L. of the trunnion axis} - \frac{S \tan \alpha_1}{\tan \alpha_1 + \tan \alpha_2}$$

$-$ height of *lower* target ...(13.41)

13.13. DISADVANTAGES OF THE TANGENTIAL METHOD

The tangential method of tacheometry has the following disadvantages :

1. It lacks speed.
2. It involves more computations for reducing distances and elevations.
3. Two vertical angles are observed for computing a distance.
4. During the interval of observing vertical angles, the instrument might get disturbed unnoticed.

Due to above disadvantages, tangential method is considered inferior to the stadia method and is generally not adopted.

Note. The most common method of tacheometry is a fixed hair stadia method, using a staff held vertically.

Example 13.19. *A surveyor wishes to obtain the height of his instrument by observing a staff held upon a Bench Mark which is at a lower level than the instrument. He takes two observations on the staff, the readings being 3.260 and 1.090 and the corresponding angles of depression being 9°* 36′ and *10°* 48′*. Calculate the elevation of the instrument if that of the bench mark is 85.658.*

Solution. (Fig. 13.23)

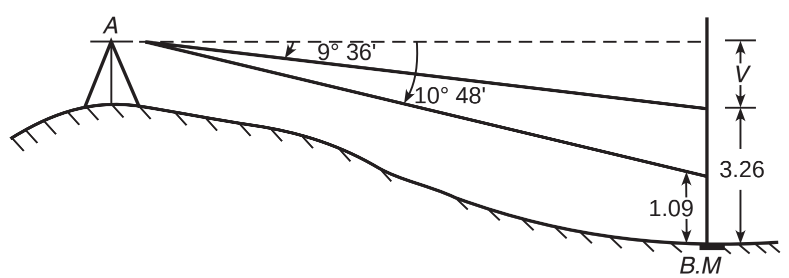


Fig. 13.23.

From Eq. (13.36) we know

$$H = \frac{S}{\tan \alpha_2 - \tan \alpha_1}$$

Here $S = 3.26 - 1.09 = 2.17$ m $\qquad \alpha_2 = 10° \ 48' \ ; \alpha_1 = 9° \ 36'$

$$\therefore \qquad H = \frac{2.17}{\tan 10° \ 48' - \tan 9' \ 36'} = \frac{2.17}{0.190764 - 0.169137}$$

$$= \frac{2.17}{0.021623} = 100.36 \text{ m}$$

Again, $\qquad V = H \tan \alpha_1 = 100.36 \times \tan 9° \ 36'$

$$= 100.36 \times 0.1679137 = 16.97 \text{ m}$$

R.L. of $\qquad A$ = R.L of B.M. + Reading of upper mark + V

$$= 85.685 + 3.26 + 16.97 = 105.888 \text{ m}$$

The reduced level of the instrument axis = 105.888 m **Ans.**

Example 13.20. *Two observations are taken upon a vertical staff by means of a theodolite, the reduced level of the trunnion axis of which is 154.3 m. In the case of the first observation the line of sight is directed to give staff reading of 1.0 m and the angle of elevation as* 4° 58′. *In the second observation the staff reading is 3.5 m, and the angle of elevation is* 5° 44′. *Compute the reduced level of the staff station and its horizontal distance from the instrument.*

Solution. (Fig. 13.24).

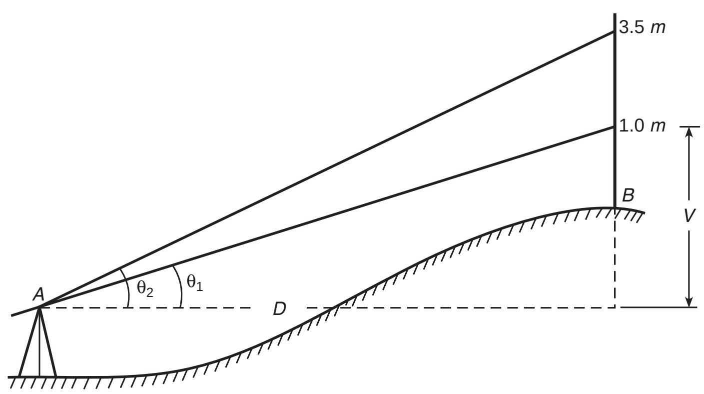


Fig. 13.24.

From Eq. (13.33), we know

$$D = \frac{S}{\tan \alpha_2 - \tan \alpha_1}$$

Here $\qquad S = 3.5 - 1.0 = 2.5$ m ; $\alpha_1 = 4° \ 58'$; $\alpha_2 = 5° \ 44'$

$$D = \frac{2.5}{\tan 5° 44' - \tan 4° 58'} = \frac{2.5}{0.100401 - 0.0869025}$$

$$= \frac{2.5}{0.0134985} = 185.20 \text{ m} \quad \textbf{Ans.}$$

Again $V = D \tan \alpha_1 = 185.20 \times \tan 4° 58'$

$= 185.20 \times 0.0869025 = 16.39$ m

R.L. of B = R.L. of trunnion axis + v − 1.0.

$= 154.30 + 16.09 - 1.0 = 169.39$ m **Ans.**

Example 13.21. *The vertical angles to vanes fixed at 0.5 m and 3.5 m above the foot of the staff held vertically at a point were* +2° 30′ and 4° 12′ *respectively. Find the horizontal distance and the reduced level of the point if the level of the instrument axis is 125.35 metres above data.*

Solution.

Let D = horizontal distance

$$S = 3.5 - 0.5 = 3.0 = \frac{S}{\tan \alpha_2 - \tan \alpha_1}$$

$$\therefore \quad D = \frac{3.0}{\tan 4° 12' - \tan 20° 30'}$$

$= 100.76$ m **Ans.**

R.L. of the point = R.L of inst. axis + D tan 2° 30′ − 0.5

$= 125.35 + 100.76 \times \tan 2° 30' - 0.5$

$= 29.28$ m. **Ans.**

13.14. THE BEAMAN'S STADIA ARC

The Beaman's stadia arc is a mechanical attachment fitted to the vertical circle of theodolites and telescopic alidades. Its use facilitates the determination of horizontal distances and the differences of elevation without the use of tacheometric tables or reduction diagrams. The arc contains two scales H and V, having their central points marked 0 and 50 respectively. The arc moves with the movement of telescope in a vertical plane and the readings are observed with a common index.

Principle of the Beaman's stadia arc. We know that the vertical component between the staff position and instrument station may be obtained by using tacheometric formula $V = AS \times \frac{\sin 2\theta}{2}$. In this equation, A is a multiplying constant whose value is generally kept 100. By assuming suitable value of θ, the value of $\frac{1}{2} \sin 2\theta$ may be made equal

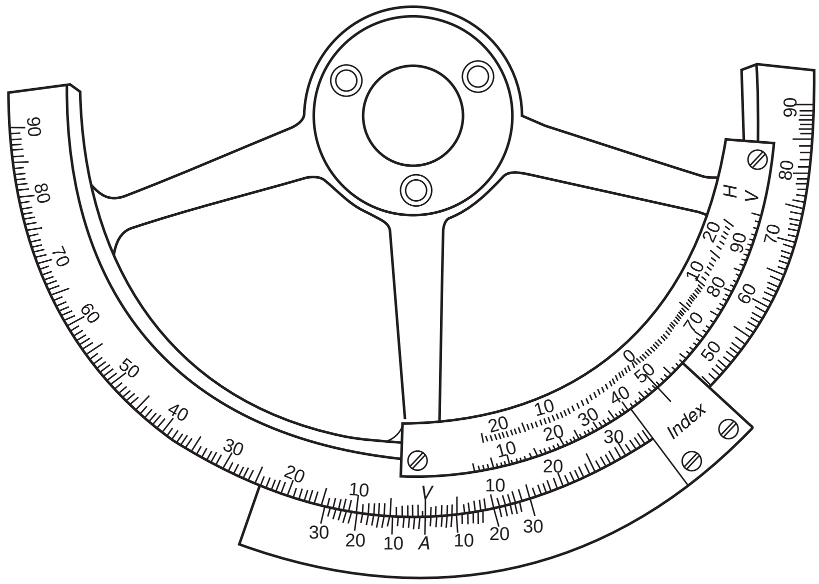


Fig. 13.25.

to 0.01, 0.02, 0.03, etc. Solving trigonometrical equation ½ sin 2 θ = k, and by assuming the values of *k* as 0.01, 0.02, 0.03, etc. the different values of θ may be obtained as shown in Table 13.1.

Table 13.1

$\frac{1}{2}$ sin 2 θ	θ *to nearest second*			$\frac{1}{2}$ *sin 2* θ	θ *to nearest second*		
0.01	0°	31′	23″	0.11	6°	21′	30″
0.02	1	8	46	0.12	6	56	30
0.03	1	43	12	0.13	7	04	45
0.04	2	17	39	0.14	8	07	45
0.05	2	52	11	0.15	9	43	45
0.06	3	26	46	0.16	9	43	45
0.07	3	01	26	0.17	10	56	00
0.08	4	36	12	0.18	10	33	00
0.09	5	11	6	0.19	11	10	00
0.10	5	46	7	0.20	11	47	30

The graduations of *V*-scale correspond to the value of $\frac{1}{2}$ sin 2θ such that each division is a multiple of 0.01. When the telescope is elevated till the index mark is against a reading 55 (say), then the angle of elevation (θ) is equal to $\frac{1}{2}\sin^{-1} 0.05 = 2° \, 52' \, 11''$.

Let staff intercept = 5m

Multiplying constant = 100

Additive constant = 0.

$\therefore$ Vertical component $(V) = \text{A.S.}\ \frac{1}{2}\ \sin 2\theta = 100 \times S \times 0.05$

$V = 5 \times S$

$= S \times (\text{Reading on } V\text{–scale} \pm 50)$

Similarly the vertical components for other vertical angles may be obtained by multiplying the staff intercept by the V-scale reading.

On the horizontal scale (H), the divisions of such values are to represent a reading which when multiplied by the staff intercept gives the necessary horizontal correction. *To obtain the corrected horizontal distance, the horizontal corrction is subtracted from the distance reading*.

Note. The following points may be noted.

(*i*) By sighting the staff, the index may not be agains a division of V-scale. The exact coincidence of the index mark with the nearest division may be made by using the tangent screw of the vertical circle. By tilting the line of sight slightly for this operation, there will be no significant change in the value of S.

(*ii*) When the index is against 50, line of sight is horizontal. A reading less than 50 indicates the angle of depression and more than 50, an angle of elevation.

Example 13.22. *Calculate the horizontal distance and vertical component between two stations from the following observations made on a Beaman's stadia arc fitted on a telescopic alidade.*

Central hair reading = 2.565 m

Staff intercept (S) = 1.585 m

Reading on V-scale = 57

Reading on H-scale = 5

Elevation of the instrument axis= 500.00 m.

Solution

Assuming $A = 100$; $B = 0$

Vertical component $(V) = S \times (V \text{ scale reading} - 50)$

$= 1.585 \times (57 - 50)$

$= 1.585 \times 7 = 11.095\text{m}.$

As the V-scale reading is more than 50, the vertival angle is an elevation.

Elevation of the staff station $= 500.00 + 11.095 - 2.565$

= 508.530m **Ans.**

Horizontal correction = $S \times$ (H scale reading)

= $1.585 \times 5 = 7.795$m

$\therefore$ Horizontal distance = $1.585 \times 100 - 7.925$

= $158.5 - 7.925$

= 150.575m. **Ans.**

Example 13.23. *A plane-table alidade with a tacheomtric telescope is provided with Beaman's stadia arc which has engraved on it two scales V and H. The scale V gives values of sin* θ *cos* θ *and the scale H gives 100* $(l - \cos^2 \theta)$. *The zero or horizontal reading of the former is 50.*

If the intercept is 1.750 m, the axial reading is 1.275 m and the instrument fitted with an anallatic lens, has a constant 100, find the vertical distances of the staff stations above or below the instrument axis when the V scale readings are 68 and 32 respctively. What are the corresponding reaidngs on the H-scale, and what are the horizontal distances to the staff ?

Solution.

Here, staff intercept = 1.750 m

V-scale reading = (68 – 50) = 18

and = 50 – 32 = 18

$\therefore$ Vertical component (V) = $18 \times 1.750 = 31.5$ m

Difference in elevation of second station

= 31.500 – 1.275 = 30.225 m

and difference in elevation of second station

= – 31.5 – 1.275 = – 32.775 m

Again, vertival component $V = 100 \times S \times \frac{\sin 2\,\theta}{2}$

or $31.5 = 100 \times 1.750 \times \frac{\sin 2\,\theta}{2}$

$$\sin 2\theta = \frac{2 \times 31.5}{175} = 0.3600$$

$\therefore$ Horizontal distance $D = 100 \times S \times \cos^2\theta$

$= 100 \times 1.75 \times (0.983096)^2$

= 169.13m

Let n be the H-scale reading.

Horizontal correction $= n \times 1.75$

or $169.13 = 100 \times 1.75 - n \times 1.75$

$1.75n = 175 - 169.13 = 5.87$

$\therefore$ Horizontal scale reading $n = \dfrac{5.87}{1.75}$

$= 3.35$ **Ans.**

Example 13.24. *When the telescopic alidade was aligned on a staff held vertically at A the telescope was elevated until the vertical scale of the Beaman's arc reads 25 and the horizontal scale read 6.7. The stadia staff readings were 3.645, 3.140 and 2.635 m. Calculate the difference in height and the horizontal distance between the point A and the plane table staton.*

Solution.

Here $S = 3.645 - 2.635 = 1.010$ m

Let multiplying constant $= 100$

V-scale reading $= 25$ (Given) : H-scale reading $= 6.7$ (Given)

$\therefore$ V-component $= 25 \times 1.01 = 25.25$ m

$\therefore$ The difference in height $= 25.25 - 3.14$

$= 22.11$ m **Ans.**

Horizontal correction $= 6.7 \times 1.01 = 6.767$m

Horizontal distance $= 100 \times 1.010 - 6.767$

$=$ **94.233 m** **Ans.**

13.15 FERGUSON'S PERCENTAGE UNIT SYSTEM

Mr. J.C.Ferguson has devised a system for the divisions of the vertical circle of theodolites. The unequal divisions are reckoned in terms of their natural tangents, expressed as percentage. With this system, computation of horizontal and vertical distances may be made, using tangential tacheometry without referring to Log Tables. (Fig. 13.26).

1. Method of percentage divisions. The vertical circle is inscribed in a square and is divided into eight equal parts. Each length of eight tangents is sub-divided into 100 equal parts. These points are joined to the centre of the circle. Thus each octant (1/8 th portion of the circle) is divided into 100 unequal parts. Each part subtends an angle

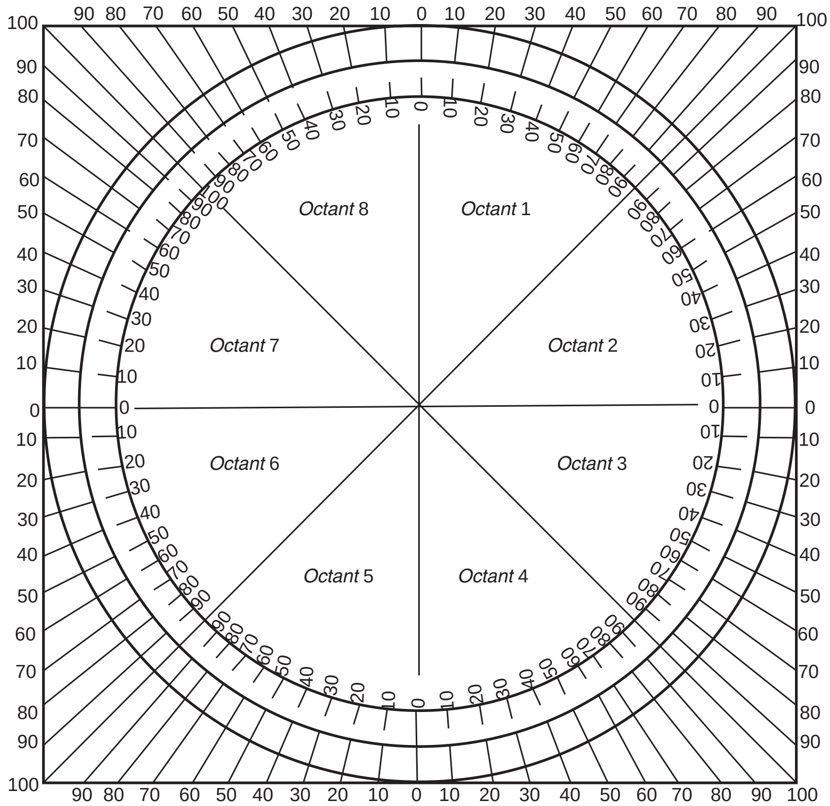


Fig. 13.26.

whose natural tangent is 1/100. The points of division on the circle are numbered from 0 on the horizontal and vertical lines and 100 on the corners of the square.

Magnitude of any angle. (Fig. 13.27)

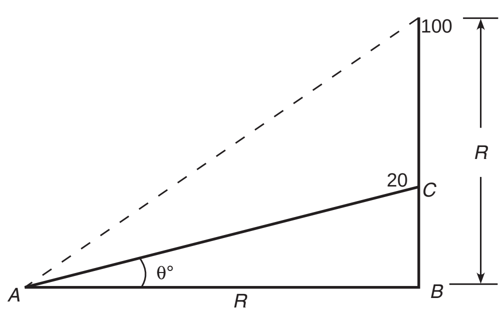


Fig. 13.27. Magnitude of any angle.

Suppose the angle subtended by 20th division and zero division at the centre is $\theta°$. From the right angled triangle *ABC*

$$\tan \theta = \frac{BC}{AB} = \frac{R \times 20}{100} \times \frac{1}{R} = 0.2$$

Again, 20th division represents $\frac{1}{100} \times 20 = 0.2$

Similarly, it can be shown that each point of the division represents a percentage.

Note : A vernier cannot be used to read the sub-divisions of unequal percentage divisions. To overcome this difficulty a spiral drum micrometer is usd to read up to second place of decimal of a unit.

2. Modification of the tangential formula. With percentage units, the system of tangential tacheometry may be applied by making observations either on a levelling staff or on two targets fixed at known distance. A levelling staff is more convenient for reduction. A reading on the lower portion of the staff is taken by setting the vertical circle to a whole percentage division. The telescope is then elevated through an exact number say n units and the upper staff reading is noted.

The tangential tacheometry formula, reduces to

$$H = \frac{100\,S}{\alpha_2\% - \alpha_1\%} = \frac{100\,S}{n}$$

$$= H \tan \alpha_1 = H \tan \alpha_2 \quad ...(13.42)$$

But $\tan \alpha_1 = \alpha_1\%$; $\tan \alpha_2 = \alpha_2\%$

$\therefore$ Vertical component $H\,\alpha_1\% = H\alpha_2\%$ ...(13.43)

Alternative method (Fig. 13.28)

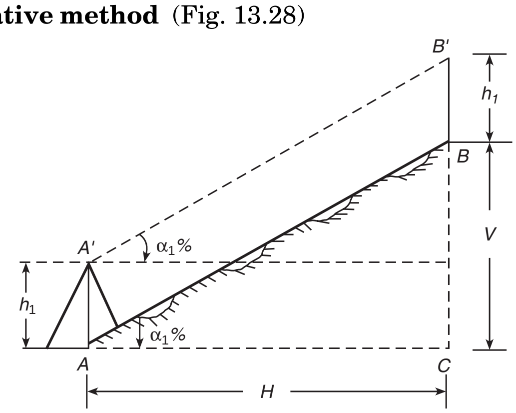


Fig. 13.28. Reading a percentage theodolite.

Let h_1 be the height of the trunnion axis above ground A and $\alpha_1\%$ be the reading on the vertical circle to the mark on the leveling staff h_1 m above ground level at B. From the Fig. 13.18 it is seen that $AA'\,B'B$ is a parallelogram and $A'B'$ is paralell to AB, the gradient between the instrument station and the staff station.

Difference of elevation $(V) = H.\alpha_1\%$. ...(13.44)

Example 13.25. *Staff readings observed with a percentage theodolite corresponding to angles of elevation of 4% and 5% are 1.525 and 2.925 respectively. If the vertical angle on sighting the staff reading equal to the height of the trunnion axis above ground, was 4.5%, calculate:*

(*ii*) *The elevation of staff station if that of the instrument station was 493.700.*

Solution. (Fig. 13.29).

Fig. 13.29.

$$\tan \alpha_1 = 4\% \; ; \; \tan \alpha_2 = 5\% = 0.05$$

$$S = 2.925 - 1.525 = 1.400 \text{ m}$$

The horizontal distance

$$H = \frac{S}{\tan \alpha_2 - \tan \alpha_1} = \frac{1.4}{0.05 - 0.04} = 140 \text{ m} \quad \textbf{Ans.}$$

Fig. 13.29.

Vertical component $V = H \tan \alpha_1 = 140 \times 0.04 = 5.6$ m

Let the angle of elevation on sighting the staff reading equal to the height of the trunnion axis above ground be α.

i.e. $\tan \alpha = 4.5\% = 0.045$

If S' is the corresponding staff intercept between α_1 and α readings,

$$H = \frac{S'}{\tan \alpha - \tan \alpha_1} = \frac{S'}{0.045 - 0.04}$$

$$S = H\,(0.045 - 0.04)$$

$$= 140 \times 0.005$$

$= 0.7$ m

Let h be the height of trunnion axis above ground, then

$$h = 1.525 + 0.70 = 2.225 \text{ m}$$

$\therefore$ R.L. of staff station

$= $ R.L. of Inst. station $+ h + V -$ lower staff reading

$= 493.7 + 2.225 + 5.600 - 1.525$

$= 500$ m **Ans.**

Example 13.26. *An observation with a percentage theodolite gave staff readings of 1.052 and 2.620 for angles of depression of 5% and 4% respectively. On sighting the graduation corresponding to the height of the instrument axis above the ground, the vertical angle was 4.25%. Calculate the horizontal distance and the elevation of the staff station if the instrument station has an elevation of 505.35 m.*

Solution. (Fig. 13.30.)

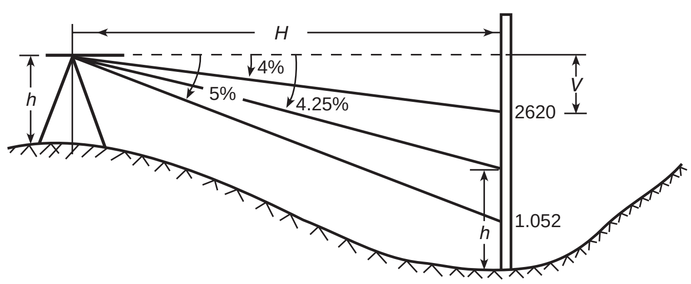


Fig. 13.30.

$$\tan \alpha_1 = 4\% = 0.04 \; ; \tan \alpha_2 = 5\% = 0.05$$

$$S = 2.620 - 1.052 = 1.568 \text{ m}$$

$$H = \frac{S}{\tan \alpha_2 - \tan \alpha_1} = \frac{1.568}{0.05 - 0.04} = 156.8 \text{ m} \qquad \textbf{Ans.}$$

$$V = H \tan\alpha_1 = 156.8 \times 0.004 = 6.272 \text{ m}$$

Let the angle of depression to the graduation corresponding to the height of the instrument axis above the ground be α.

$$\therefore \qquad \tan \alpha = 0.0425$$

Let S' be the corresponding staff intercept between α_1 and α.

$$H = \frac{S'}{\tan \alpha_2 - \tan \alpha_1} = \frac{S'}{0.0425 - 0.0400} = \frac{S'}{0.0025}$$

$$\therefore \qquad S' = H \times 0.0025 = 156.8 \times 0.0025 = 0.392$$

$\therefore$ Height of trunnion aixs (h) above ground

$$= 2.620 - 0.392 = 2.228 \text{ m}$$

$\therefore$ R.L. of the staff station.

$$= \text{R.L. of trunnion axis } - (V + \text{ upper target reading})$$

$$= 505.350 + 2.228 - (6.272 + 2.620)$$

$$= 498.686 \text{ m} \quad \textbf{Ans.}$$

13.16. REDUCTION OF READINGS

Even though a theodolite might be fitted with an anallatic lens, the reduction of distances by tacheometric formulae involves laborious computation. To avoid these computations and to achieve speed in reduction of readings, "Tacheometric Tables and Reduction Diagrams" are generally used.

13.17. TACHEOMETRIC TABELS

Different forms of tacheometric tables are available in market. Tacheometric tables which are in common use have been given in Appendix-I. These tables have been prepared for horizontal distances and vertical and components for one metre intercept on the staff held vertically, assuming the instrument constants as 100 and zero.

We know that the tacheometric formulate for

$$\text{Horizontal distance } (H) = \frac{f}{i} S \,.\, \cos^2 \alpha \quad \text{...(13.45)}$$

$$\text{and vertical component } (V) = \frac{f}{i} S . \frac{\sin 2\alpha}{2} \quad \text{...(13.46)}$$

where α is a vertical angle.

Substituting the value of S as one metre in Eq. (13.45) and (13.46), we get

$$H = \frac{f}{i} \cos^2 \alpha = 100 \cos^2 \alpha$$

$$V = \frac{f}{i} \frac{\sin 2\alpha}{2} = 100 \frac{\sin 2\alpha}{2}$$

By substituting different values of α varying from 0° to 15° , a table of horizontal distances and vertical components may be prepared.

Use of a tacheometric table. The use of a tacheometric table can be understood by the following example :

Suppose, the vertical angle

$$= 4° \, 30'$$

Staff intercept $= 1.40$ m.

Refer to tacheometric table and note down the horizontal distance (H) and vertical component (V) against 4° 30′

i.e. $H = 99.38m$; $V = 7.82$ m.

As the tacheometric tables are prepared by using staff intercept as 1 metre, correct distances will be obtained by multiplying these values by actual staff intercept, *i.e.,* 1.40 m.

The horizontal distance (H) $= 1.40 \times 99.38 = 139.132$ m

The vertical component (V) $= 1.40 \times 7.82 = 10.948$ m

13.18. REDUCTION DIAGRAMS

A graphically constructed scale which is used to measure corrections for horizontal distances and vertical components, is known a *reduction diagram.* Various forms of reduction diagrams are available but simple forms of the reduction diagrams are discussed here :

Correction to horizontal distance. We know that for any inclined sight, if the staff intercept is S and the vertical angle is α, the horizontal distance $H = A.S.\cos^2\alpha$, assuming that the instrument is fitted with an anallatic lens, Again, if the line of sight is horizontal, the horizontal distance (H) = $A.S.$

i.e. $$H = A.S. \quad ...(13.47)$$

$$H' = A.S.\cos^2\alpha \quad ...(13.48)$$

Subtracting Eq. (13.48) from Eq. (13.47), we get

Horizontal correction $= H - H'$

$$= A.S. - A.S.\cos^2\alpha$$

$$= A.S.(1 - \cos^2\alpha)$$

$$= A.S.\sin^2\alpha \quad ...(13.49)$$

In equation (13.49), the product of the multiplying constant A and staff intercept S, is known as *distance reading.*

Preparation of a reduction diagram for horizontal corrections (Fig. 13.31.)

Procedure : Following steps are involved.

1. Draw a horizontal staright line AB say 20 cm to represent 200 m and divid it into 8 equal parts, each part representing 25 m.
2. Erect a perpendicular BC at B.
3. Work out different values of $AS\sin^2\theta$, *i.e.* $200\sin^2\theta$ for varying values of θ from 0° to 10° at an interval fo 30′
4. Mark off these values along the perpendicular BC on any convennient scale.

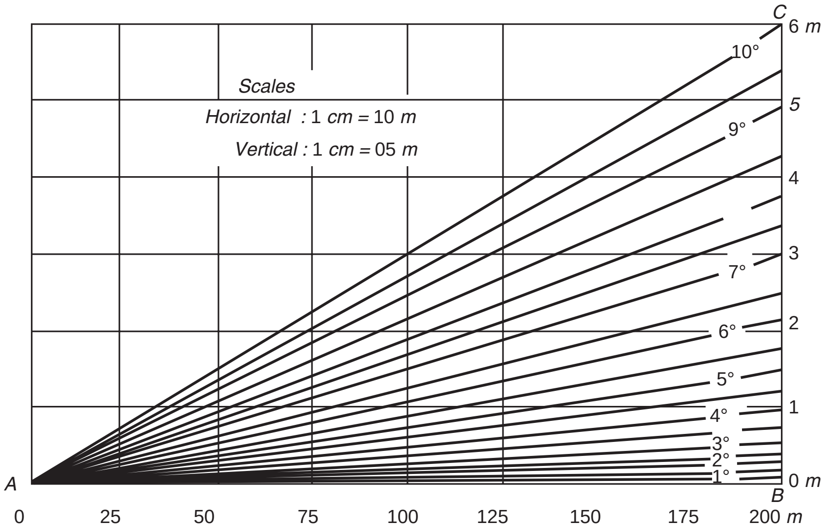


Fig. 13.31. Reduction diagram for horizontal corrections.

5. Join each point on the perpendicular *BC* to the origin *A* to get radial lines.
6. Draw lines parallel to *BC* passing through the 25 m horizontal distance points.
7. Similarly, draw lines parallel to *AB* through full metre correction points on BC.
8. A graphical scale bounded by *ABCD* is the required reduction diagram.

Application of a reduction diagram for horizontal corrections.

Let multiplying constant = 100

Staff intercept = 1.25 m

Vertical angle = 5°

$\therefore$ Distance reading = $100 \times 1.25 = 125$ m

From the reduction diagram, the horizontal correction for 125 m = 0.95m.

$\therefore$ Corrected horizontal distance = $125 - 0.95 = 124.05$ m.

Prepration of a reduction diagram for Vertical Components

Reduction diagram for vertical components for different vertical angles is constructed as under :

1. Draw a horizontal line *AB* say 20 cm to represent 200 m and divide it into 8 equal parts, each part representing 25 m. (Fig. 13.32)

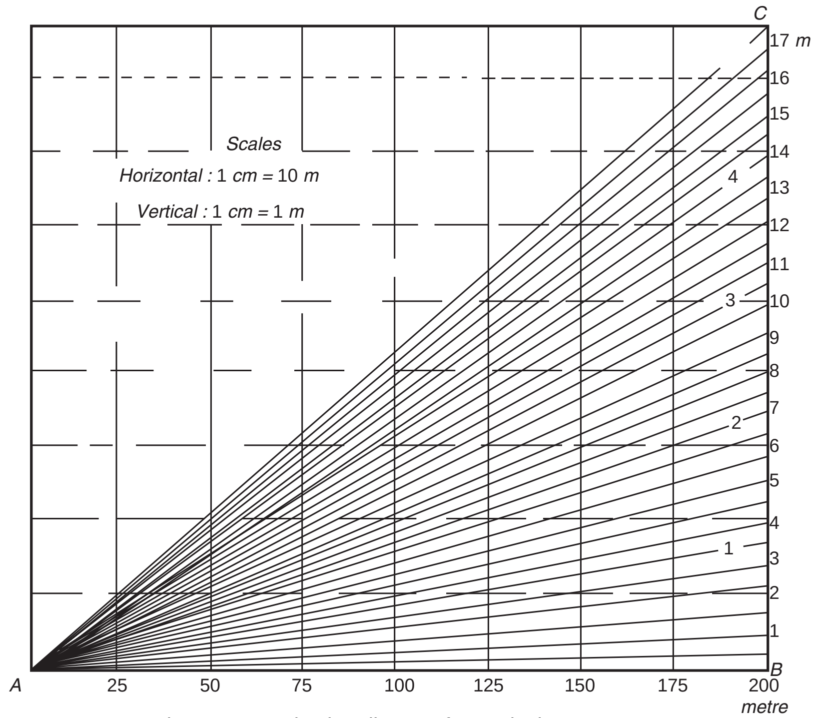


Fig. 13.32. Reduction diagram for vertical components.

2. Erect a perpendicular *BC* at *B*.
3. Work out different values of $A.S. \dfrac{\sin 2\theta}{2}$, *i.e.* $\dfrac{200 \sin 2\theta}{2}$ for varying values of θ from 0° to 10°. Upto 5°, the values of vertical component are calculated at 10 minute interval and beyond θ = 5°, these are calculated at an interval of 1°.
4. Mark off the calculated values of vertical components along the perpendicular *BC* on any convenient scale.
5. Join different points marked off on the perpendicular *BC* with the origin *A*.
6. Draw lines parallel to *AB* and *BC*.
7. A graphical scale bounded by *ABCD* is the required reduction diagram.

Application of a reduction diagram for vertical components

Let the multiplying constant = 100

Staff intercept = 1.25 m ; Vertical angle = 5°

∴ Distance reading = 1.25 × 100 = 1.25 m

From the reduction diagram, the vertical component for 125 m = 10.85m

Note. The following points may be noted.

(*i*) From reduction diagram for horizontal distances, horizontal corrections are obtained for different vertical angles and these corrections are substracted from the distance readings.

(*ii*) From reduction diagram for vertical components, the vertical components are *directly obtained* and no correction is necessary.

(*iii*) It may be appreicated that once the reducation diagrams are prepared, the observations can be reduced rapidly with the help of these diagrams.

13.19. TACHEOMETRY AS APPLIED TO SUBTENSE MEASUREMENT

In tangential method, two readings on a vertically held staff are taken by two pointings of a theodolite. Similarly much time is spent in movable method of tacheometry. To attain speed in field operations, the subtense method of tacheometry is extensively adopted to compute the horizontal distances.

In this method, a base of known length is kept in a horizontal plane and the horizontal angle subteneded at any point on its perpendicular bisector is measured by a theodolite. The distance between the theodolite station and the centre of the base is inversely proporational to the subtended angle. The length of the base is kept constant and the subtended angle is measured by the method of repitition to get a most probable value of the angle.

1. The Subtense Bar. (Fig. 13.33). The subtense bar is an instrument used for measuring horizontal distances in the areas where direct chaining becomes difficult due to undulations or other obstructions.

It consists of a metal tube of length varying from 3 m to 4 m. Two discs 20 centrimetres in diametre, painted either black or red on one side and white on the other, each with a 7.5 cm white or black centre

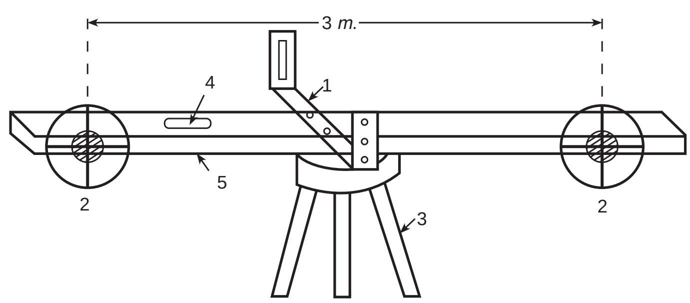


Fig. 13.33. A subtense bar.

are placed 3 metres apart. Red or black faces of discs are kept towards the theodolite.

At the centre of the bar, an alidade perependicular to the axis of the bar is attached. The bar is mounted on a speical tripod provided with a cup and ball socket which enables the surveyor to level the bar with a small bubble without shifting the legs. When properly aligned with the alidade and levelled with a bubble tube, the bar is clamped in position by clamping the ball and socket.

2. Computation of Subtense bar Distances. The horizontal distance *D* between the instrument station *O* and the subtense bar station *C*, may be calculated as under :

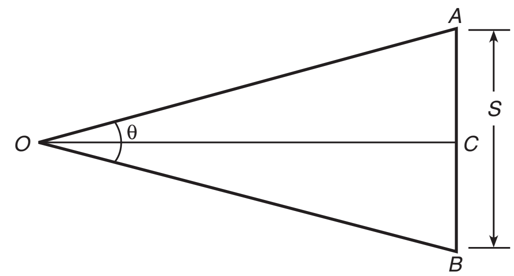


Fig. 13.34. Computation of distances.

Let θ be the horizontal angle subtended by the discs *A* and *B*, fixed at *S* distance apart.

From Δs *OBC* or *OAC*, we get

$$D \tan \frac{\theta}{2} = \frac{S}{2}$$

or $$D = \frac{1}{2} S \ . \ \cot \frac{\theta}{2} \qquad ...(13.50)$$

$$= \frac{1}{2} S . \ \frac{2}{\theta} \quad \text{when } \theta \text{ is in radians}$$

$$= \frac{1}{2} S \times \frac{206265}{\theta} \times 2 \text{ when } \theta \text{ is in seconds}$$

or $$D = \frac{S \times 206265}{\theta} \qquad ...(13.51)$$

"To compute the horizontal distance, multiply the constant 206265 by the lenght in metres between the centres of the discs and divide the product by the subtended angle in seconds.

3. Effect of Angular Error on Horizontal Distances

In the subtense bar distance formular $D = \frac{S \times 206265}{\theta}$, *D* is inversely proportional to θ.

In other words, if θ decreases, D increases and vice versa.

Let the negative error in θ be $\delta\theta$ and the positive error in D be δD.

From Eqn. (13.50), we get

$$\frac{1}{2} S = D \tan \frac{\theta}{2} = D \frac{\theta}{2}$$

or $$S = D \cdot \theta \qquad ...(13.52)$$

Substituting the values of D and θ in Eqn. (13.52.)

$$S = (D + \delta D)\ (\theta - \delta\theta) \qquad ...(13.53)$$

Equating Eqns. (13.52) and (13.53), we get

$$(D + \delta D)\ (\theta - \delta\theta) = D \cdot \theta$$

or $$\frac{D + \delta D}{D} = \frac{\theta}{(\theta - \delta\theta)} \qquad ...(13.54)$$

By cross-multiplication and on simplification, we get

$$\delta D = \frac{D \cdot \delta\theta}{(\theta - \delta\theta)}$$

Similarly if δD is negative, it can be proved that the resulting error $\delta\theta$ which is positive is given by

$$\delta D = \frac{D\delta\theta}{(\theta + \delta\theta)} \qquad ...(13.55)$$

However, if $\delta\theta$ is too small as compared to θ, we have

$$\delta D = \frac{D\delta\theta}{\theta} \qquad ...(13.56)$$

4. Effect of Sag of the Bar on Distances.

The horizontal distance between the centres of the discs must be accurately measured with a properly calibrated steel tape. An error of 3 mm in 3-metre long subtense bar, generally gives an error of 1 metre in about 800 metres.

5. Measurement of Horizontal Angle Between the Discs.

The angle subtended by the discs, at the station of observation is measured by the method of repetition as follows :

1. Clamping the upper plate, intersect the centre of the left hand disc by moving the lower plate.
2. Clamping the upper plate, intersect the centre of the left hand disc with the upper tangent screw. Read both the verniers of the horizontal circle and find out the mean reading.
3. Swing the telescope with the *the upper tangent screw only* and intersect the right hand disc. Read one vernier and calculate

the first measure of the subtended angle to provide a check against a gross error.

4. Keeping both plates clamped, bring the telescope back with the lower tangent screw and intersect the left hand disc again.
5. Move the telescope with the upper tangent screw and intersect the right hand disc. Now the vernier reads twice the measure of the subtended angle.
6. Steps 4 and 5 are then repeated till a fixed number of measures of the angle, *i.e.* 5, 10, or 20 are taken.
7. The final intersection should be on the right hand disc. Both the verniers are read and the mean of the readings calculated.
8. Subtract the mean value of the first readings from the mean value of the final reading and divide the difference by the number of measures made, to give the final value of the subtended angle.
9. To guard against the wrong countings of the number of repetitions, check the final value of the subtended angle against the first measure.

Note. The following points may be noted.

(*i*) The number of repetitions of the measurement should vary with the magnitude of the subtended angle. A convenient rule is to repeat observations on the 3-metre bar until the accumulated angle amounts to about a degree.

(*ii*) It is not necessary to measure the whole length of a traverse leg by subtense bar alone. If a part of the leg may be measured by chaining, the other part may be measured with a subtense bar. The sum of two parts is equal to the length of the traverse leg.

(*iii*) The advantages of the system of repetition over a number of separate measures of the subtended angle are that the error of reading the graduated circle, enters inthe repetition system twice into each group of measures. On the other hand, the error occurs twice in each measure when separate measures are taken.

(*iv*) The computed distance is always a horizontal distance and should not be corrected for the slope of the ground.

(*v*) Every bisection of either disc of the bar must be made with equal care.

(*vi*) In every bisection it is important that the final adjustment is made by turning the tangent screw against the spring to avoid back-lash error.

(*viii*) Distances over 800 metres should not be measured with a substense bar as the subtended angle becomes less than 13′.

Example 13.27. *The horizontal angle subtended at a theodolite by a subtense bar with targets 3 m apart is 18′. Compute the horizontal distance between the instrument and the bar.*

Deduce the error of horizontal distance if the bar were 1° from being normal to the line joining the instrument and bar station.

Solution. (Fig. 13.35)

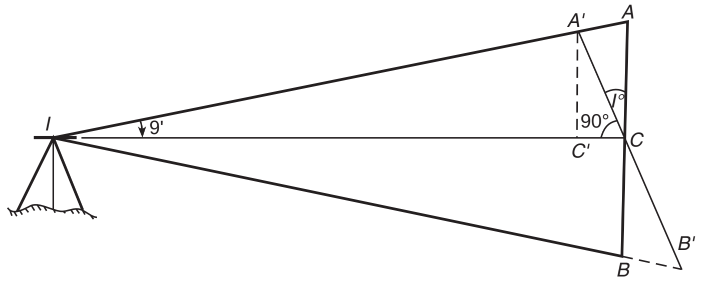


Fig. 13.35.

From Δ *IAC*, we get

$$IC = H = \frac{AC}{\tan AIC}$$

$$= \frac{1.5}{\tan 9'} = 1.5 \times 381.96805$$

∴ Horizontal distance

$$H = 572.95 \text{ m} \quad \textbf{Ans.}$$

Now, the bar is 1° from normal *i.e.* anlge *ACA*=1°

$$H' = \frac{A'\,C'}{\tan 9'} = \frac{AC \times \cos 1°}{\tan 9'} = \frac{1.5 \times 0.999848}{0.00261802}$$

$$= 572.86 \text{ m.}$$

∴ Error $= H - H' = 572.95 - 572.86$

$$= 0.09 \text{ m.} \quad \textbf{Ans.}$$

Alternaitve Method. From Eqn. (13.51), we get

$$D = \frac{S \times 206265}{\theta} \qquad ...(i)$$

Here $S = 3\text{m}$; $\theta = 18 \times 60 = 1080$ sec.

Substituting the values in eqn. (*i*), we get

$$D = \frac{3 \times 206265}{1080}$$

$$= 572.96 \text{ m.}$$

When the bar is not perpendicualr to the line joining the substense bar station and instrument station,

$$D' = \frac{S' \cos \alpha \times 206265}{1080} = 572.87\text{m}.$$

$\therefore$ Error = 572.96 – 572.87 = 0.09 m. **Ans.**

13.20. TACHEOMETRIC PLANE TABLING

The method of plane tabling in which a telescopic alidade fitted with stadia haris and a levelling staff are used, is known as *tacheometric plane tabling.* In this method distance between the plane tbale station and other ground points are computed by tacheometric distance formulae and graphical construction of the angles between these points at the plane table station is made by drawing rays along the fiducial edge of the telescopic alidade. (Fig. 13.36.)

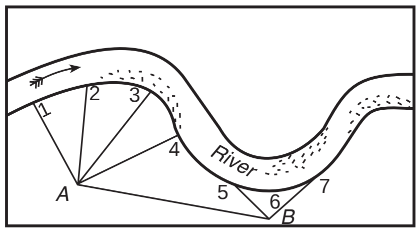


Fig. 13.36. Tacheometric plane tabling.

Procedure. Following steps are followed :

(*i*) Set up the plane table at the starting station *A* and centre its location *a* over the ground station point carefully.

(*ii*) Orient the table in true magnetic or arbitrary meridan as convenient.

(*iii*) Pivoting the telescopic alidade about *a*, sight a levelling staff held vertical at the detail points and draw rays twoards them.

(*iv*) Keeping the bubble of the level tube central, observe stadia readings on the staff held at the detail points.

(*v*) Compute the horizontal distances and the vertical components for each point by using tacheometric formulae or by refering to standard Tacheometric Tables.

(*vi*) Plot the horizontal distances after reducing to scale along the rays drawn to the respective details points from the plane table station.

(*vii*) Compute the reduced levels of each point and enter them against their locations.

(*viii*) After completing the detail survey, contours may now be interpolated between spot levels.

(*ix*) Similarly, the location of other details and required contours are surveyed in the vicinity of the plane table station.

(*x*) The plane table is then shifted to next convenient station *B* and survey is completed. A number of stations may be occupied to cover the entire area.

13.21. THREE POINT PROBLEM IN TACHEOMETRIC PLANE TABLING

The location of the station occupied by the plane table may be obtained by making tacheometric observation as under :

Let *A, B* and *C* be three ground control points whose locations on a plane table section are at *a, b* and *c* respectively. (Fig. 13.37). Set the table over station *D*. Observe stadia readings on the staff held vertically at *A, B* and *C* in turn and compute the horizontal ditances, *AD, BD* and *CD* by using tacheometric formulae. With *a* and *b* as centres and radii equal to *AD* and *BD* (reduced to scale of survey) draw arcs to intersect at *d*. With *c* as centre and radius equal to *CD* (reduced to scale of survey) draw another arc to intersect the arcs drawn from *a* and *b*. If the three arcs intersect at a point, the observations are correct and if not, re-observe the stadia readings and check their computations.

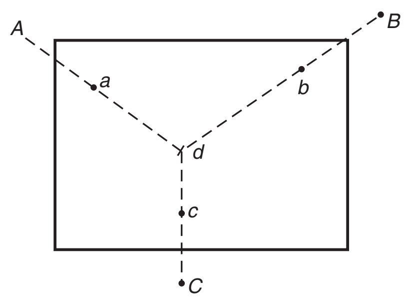


Fig. 13.37. Three points problem in Tacheometric plane tabling.

Place the fiducial edge of the alidade along the line joining the farthest point (say *b*) with *d* and rotate the table until the staff held at *B* is bisected by the vertical hair of the telescope. Clamp the table and draw resectors from *A* and *C*, which will intersect at *d*, if the work is accurate.

13.22. FIELD WORK FOR TACHEOMETRIC SURVEYING

Tacheometric surveys which are generally conducted for contouring and detail survey in undulating ground, are specially suitable for surveying narrow belts generally required for the alignments of highways, railways, etc. Tacheometric surveys involve two steps *i.e.*,

(*i*) Running a traverse enclosing the area.

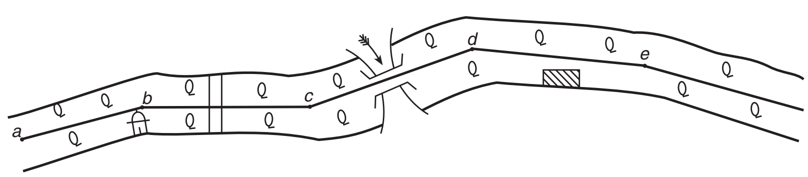


Fig. 13.38. Tacheometric survey of a narrow belt.

(*ii*) Locating details and contours from the traverse stations.

When the area to be surveyed lies in a narrow belt, the traverse stations are selected along the centre line of the belt such as *a, b, c, d, e,* and *f* (Fig. 13.39).

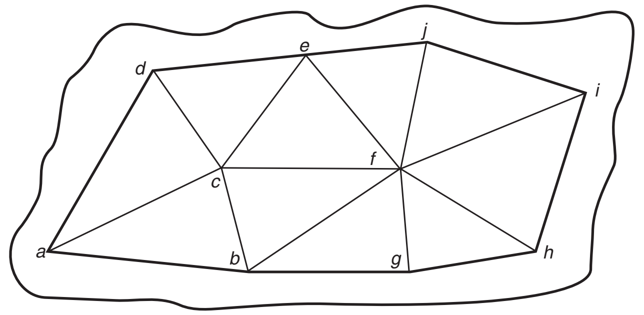


Fig. 13.39. Triangles and polygons for tacheometric survying.

While measuring the traverse angles, chain pins (arrows) held vertically on the station marks should be sighted.

Procedure. The following steps are followed :

1. Set up the instrument over the first station, centre and level it accurately.
2. Measure the vertical distance from the top of the station peg to the centre of trunnion axis of the tacheometer.
3. Orient the instrument correctly with reference to a fixed station whose distance and bearing from the starting station, are pre-determined.
4. Sight the staff held vertical on the nearest available bench mark to determine the reduced level of the starting station. If the bench mark is considerably higher or lower than the turnnion axis of the tacheometer and the staff may not be read with horizontal line of sight, then tacheometric distance formulae may be used compute the distance and difference in elevation between B.M. and the first station.
5. Locate the detail points around the station as explained here :

(*i*) Observe the horizontal angle between the reference station and the point.

(*ii*) Observe the vertical angle to the central horizontal wire of the diaphragm.

(*iii*) Observe the staff readings of the stadia hairs.

The observations made to locate a detail point completely, are known as *side shots.* For speedy observations, it is preferred to observe the staff stations on radia lines through the instrument station with an angular interval of 10′ to 30′. depending upon the nature of the terrain.

6. Take a foresight on the second traverse station and observe the following :
   (*i*) the traverse angle
   (*ii*) the vertical angle
   (*iii*) the staff readings of stadia hairs.
7. Transfer the instrument to the second station. Centre and level it carefully.
8. Measure the height of the trunnion axis of the tacheometer above the station peg.
9. Sight the staff on the first station and observe the following :
   (*i*) the traverse angle
   (*ii*) the vertical angle
   (*iii*) the staff readings of the stadia hairs.
10. Computer the traverse leg between first station and second station and their difference in elevation from observations made from both stations. If these observations agree within the limits of accuracy, the average of the two values may be accepted as the value of the distance and elevation of the station. If not, repeat the observations at both the stations.
11. Locate the detail points around second station, within the range of the instrument.
12. Take a foresight on the third station and make the necessary observations.
13. Repeate the procedure at succeeding stations till the entire work is completed.

A specimen field book for tacheometric surveys, is shown on page 721.

Note. The following points may be noted :

(*i*) Selection of a tacheometer station should be made so that it commands a clear view of the area within the range of observation around it and use of large vertical angle is avoided.

(*ii*) While measuring the traverse angles, chain pins held vertically on the station marks should invariably be sighted.

13.23. ADVANTAGES AND DISADVANTAGES OF TACHEOMETRIC PLANE TABLING

Advantages. The following are the advantages :

1. Distances computed by tacheometric observations need no corrections to get horizontal distances.
2. Required number of points to survey details, may be provided from single station.
3. Accuracy of the survey may be checked by observing check points without loss of time.

4. Details survey and contouring proceed simultaneously.
5. Location of points may be obtained without resorting to intersections from two or more stations.
6. Accuracy of detail survey and contouring is better than that obtainable with a clinometre.

Disadvantages. The following are the disadvantages.

1. An extra assistant and a levelling staff are required.
2. The telescopic alidade is more cumbersome than an ordinary alidade to use in the field.

13.24. DIRECT READING TACHEOMETER

To overcome the difficulty of using complicted tacheometric formulae, various types of direct reading tacheometres have been invented.

Dr. H.H. Jeffcott invented a direct reading tacheometer whose diaphragm is provided with three horizontal pointers by means of which staff readings are taken. The central pointer remains fixed and the other two moved by a system of cams and levers as the telescope is elevated or depressed so as to introduce th factors $\cos^2 \theta$ and $\sin \theta \cos \theta$ automatically. The right hand movable pointer, is called the *distance pointer.* Its position remeains below the fixed pointer. The staff intercept between the distance pointer and the fixed pointer, multiplied by 100, gives the horizontal distance between the instrument station and staff position. The left hand movable pointer is called the *height pointer*. It occupies its position according to the vertical angles of the line of collimation. For angles of elevation, the height pointer occupies its position below the fixed pointer. For angles of deprssion, it occupies its position above the fixed pointer. For horizontal line of sight, the left hand movable pointer faces the fixed pointer. The staff intercept between the height pointer and the fixed pointer, multiplied by 10, gives the vertical component between the instrument station and staff position. Direct reading tacheometers are fitted with anallatic lens. If the reading of the fixed pointer is more than that of height pointer, the component is *positive* and *vice versa*.

While making observations with this tacheometer, the fixed pointer is usually set at a full metre or decimetre mark and other movable pointers are read afterward. This facilitates the subtraction of the pointer readings mentally.

Illustration. Let the readings of the upper, central and lower pointers be 2.860, 2.000 and 0.800 respectively.

Horizontal distance = 100 (2.860 – 2.000) = 86 m

Vertical component = 10 (2.000 – 0.800) = 12 m

Reduced level of the staff station may be obtained as under :

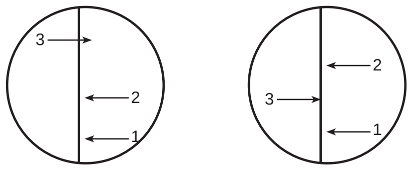


Fig. 13.40. As seen in diaphragm.

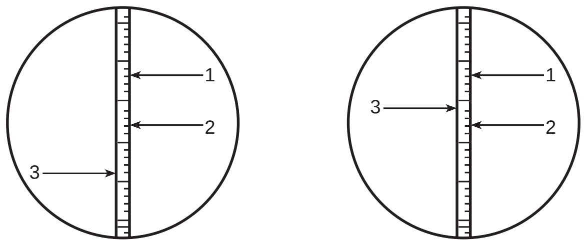


Fig. 13.41. Telescopic View of Staff

For angle of elevations :

R.L. of staff station =

R.L. of Inst. station + H.I. + V – fixed pointer reading.

For angle of depressions :

R.L. staff station =

R.L. of Inst. station + $H.I.$ – V – fixed pointer reading.

A specimen tacheometry field book for using a direct reading tacheometer is shown on page 714.

Example 13.28. *A direct reading tacheometer was set up at station A and the readings observed on a staff held vertically on a B.M. and staion B are tabulated*:

		Stadia			
Stn. Inst.	*Staff Stn.*	*Right pointer*	*Fixed pointer*	*Left pointer*	*Remarks*
A	*B.M.*	*3.850*	*2.500*	*1.845*	*505.555*
A	*B*	*2.985*	*2.100*	*2.755*	

Calculate the horizontal distance from A to B and also the R.L. of station B.

Solution. Staff held at B.M.

Vertical component V = 10 (fixed ponter reading – left pointer reading)

$$= 10\ (2.500 - 1.845) = +6.55$$

∴ R.L. of instrument axis = R.L. of B.M. + fixed pointer reading vertical component.

Specimen field book for direct reading tacheometers

Tacheometer No......

Inst. Stn. & Ht of axis	Staff Stn.	Horizontal Circle reading	Angle	Stadia readings					Horizontal distance	Vertical component	Remarks
		° ′ ″	° ′ ″	Right pointer (l)	Fixed pointer (m)	Left pointer (n)	1 − m	m − n	100 (1 − m)	10 (m − n)	
A	Trav. stn No. 1	0 00 00	...	1.555	1.070	1.255	0.485	0.0185	48.5	− 1.85	
	A_1	4 50 20	4 50 20	1.170	0.605	0.870	0.565	0.265	56.5	− 2.65	
	A_2	15 37 00	10 46 40	2.580	1.350	1.615	1.230	0.265	123.0	− 2.65	
	A_3	26 36 00	10 59 00	3.650	2.675	2.260	0.975	+ 0.415	97.5	+ 4.15	
	A_4	37 32 40	20 56 40	2.380	1.560	1.215	0.820	+0.435	82.0	+ 4.35	
	A_5	48 35 00	11 10 20	2.850	2.430	2.215	0.420	+0.615	42.0	+ 6.15	
	...	...	...	...	...	...	...	...	...	...	
	...	...	...	...	...	...	...	...	...	...	
	...	...	...	...	...	...	...	...	...	...	
	...	...	...	...	...	...	...	...	...	...	
	Trav. Stn. No. 1	360 00 00									
	Stn. B	195 35 00	195 35 00	2.140	1.420	1.125	0.720	+ 0.405	72.0	+ 4.05	
	Stn. B	31 10 20	195 35 20								

Date of observation

Observer's signatures

$= 505.555 + 2.500 - 6.550$

$= 501.505$ m.

Staff held at station B

Horizontal distance = 100 (right pointer reading - fixed pointer reading)

$= 100\ (3.850 - 2.500) \quad = 135.0$ m **Ans.**

Vertical component = 10 (fixed pointer reading - left pointer reading)

$= 10\ (2.100 - 2.755) \quad = (-)\ 6.55$ m.

$\therefore$ R.L. of B = R.L. of Inst. axis - V - fixed pointer reading.

$= 501.505 - 6.550 - 2.100 = 492.855$ m. **Ans.**

13.25. ERRORS IN STADIA SURVEYING

The main causes of errors in stadia surveying are as discussed below:

I. Instrumental Errors. The instrumental errors include the following :

1. Imperfect adjustment.
2. Irregular divisions of stadia rod.
3. Incorrect value of the multiplying constant.

Out of the above three sources of error, the third source is of very serious nature. It is always advisable to determine the accurate value of the multiplying constant before making field observations.

II. Errors due to manipulation and sighting. These errors include the following :

1. Inaccurate centering and levelling of the instrument.
2. Non-verticality of the stadia rod. The magnitude of the error due to non-verticality of the stadia rod varies with the magnitude of observed vertical angles. This may be eliminated by using a plumb line or a small circular spirit level.
3. Inacurate estimation of the stadia intercept. This error may be minimised by taking the folowing percautions :

(*i*) Focuss the eye piece and objective properly, so as to remove parallax.

(*ii*) Read all the three hairs. The central hair reading should equal the mean of the other two hairs.

(*iii*) Ensure that, while making observations, the axial hair is not mistaken to be the staida hair.

III. Errors due to natural causes. These errors include the errors caused due to the following, natural agencies:

1. High winds.

2. Unequal refraction. This error is cumulative. The density of the air up to one metre above ground is much more than that of the air above it. The ray of light passing through this air stratum gets bent and hance the observed intercept is less than what it should be. This error can be minimised by taking observations in the morning and evening. Observations at noon time should be avoided.

3. Unequal expansion. This error is caused due to unequal expansion of the instrument when exposed to hot sun. To minimise this error, the instrument should be protected with an umbrella.

13.26. PRECISION OF STADIA SURVEYING

The permissible errors in stadia surveying are, as detailed below :

1. Single horizontal distance should be correct to 1 in 500.
2. Single vertical distance should be correct to 0.1 m.
3. Average error in distance should be from 1 in 600 to 1 in 850.
4. Error of closure in elevation should be within 0.08 $\sqrt{M}$ to 0.25 $\sqrt{M}$ where p is the perimeter of the traverse in metres.
   where p is the perimeter of the traverse in metres.

Example 13.29. *Two observations are taken upon a vertical staff by means of a theodolite of which R.L. of trunnion axis is 154.30. In the first case the line of sight is directed to give staff reading of 1.00 m and angle of elevation 4° 58′. In the second case the staff reading is 3.66 m and angle of elevation 5° 44′. Compute the horizontal distance of staff station from the instrument and R.L. of staff station.*

Solution.

Given $\alpha = 5° \, 44'$; $\beta = 4° \, 58'$

Since both angles α and β are angles of elevation, we use

$$D = \frac{S}{\tan \alpha - \tan \beta} \text{ and } h = \frac{S \tan \beta}{\tan \alpha - \tan \beta}$$

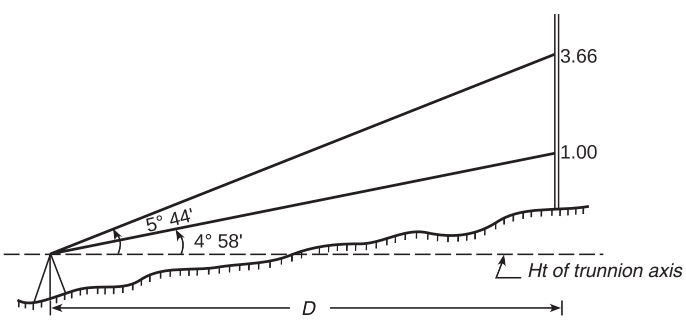


Fig. 13.42.

Here $S = 3.66 - 1.00 = 2.66$ m

$$\therefore \quad D = \frac{2.66}{\tan 5°44' - \tan 4°58'}$$

$$= 197.037 \text{ m}$$

$$h = D \tan \beta = 197.037 \times \tan 4° 58' = 17.12 \text{ m}$$

$\therefore$ R.L. of staff station = 154.30 + 17.12 – 1.00

= 170.42 m **Ans.**

Example 13.30. *A tacheometer was placed over station A, RL 1639.40 m. The trunnion axis of the telescope being 1.7 m above the ground surface. Determine the R.L. of the staff station B when the staff was kept normal to the line of sight and the angle of depression was – 6° 12′. The three stadia readings are 1.06, 1.57, 2.07. Given* $\frac{f}{i} = 100$; $(f + d) = 1.0$

Solution.

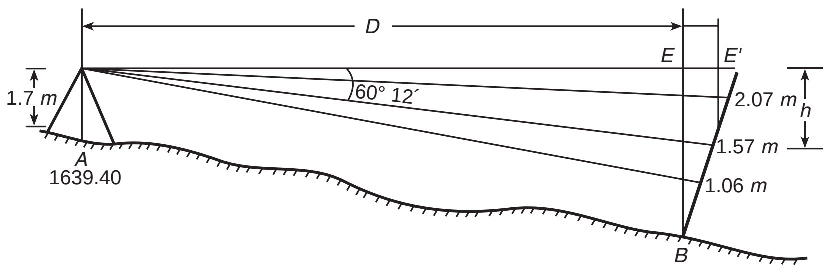


Fig. 13.43.

R.L. of instrument axis = 1639.40 + 1.70 = 1641.1 m

$$S = 2.07 - 1.06 = 1.01 \text{ m}$$

$$h = \left[S\frac{f}{i} + (f + d) \right] \times \sin \theta$$

$$= [1.01 \times 100 + 1] \sin 6° 12'$$

$$= 11.016 \text{ m}$$

R.L. of B = 1641.1 – 11.016 – 1.57 cos 6° 12′

= 1628.524 m **Ans.**

Example 13.31. *The elevation of point P is to be determined by making observations from two adjacent stations of a tacheometric survey. The staff was held vertically upon a point and the instrument is fitted with an anallactic lens of constant 100. Compute the mean elevation of* *the point P from the following data:*

Inst. stn	*Ht of axis*	*Staff point*	*Vertical angle*	*Staff readings*	*Elevation of stn*
A	*1.8 m*	*P*	$+3° 12'$	*1.33, 2.15, 2.97*	*250.56*
B	*1.6 m*	*P*	$-5° 36'$	*1.84, 2.52, 3.20*	*318.90*

Solution.

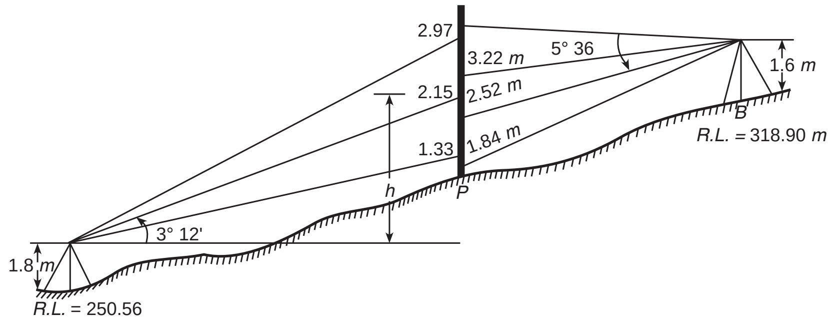


Fig. 13.44.

Observation from station A

Vertical component, $h = S.\frac{f}{i}.\frac{\sin 2\theta}{2}$ $S = 2.97 - 1.33 = 1.64$ m

$$h = 1.64 \times 100 \times \frac{\sin 6° 24'}{2} = 9.14 \text{ m}$$

R.L. of $P = 250.56 + 1.8 + 9.14 - 2.15 = 259.35$ m ...(*i*)

Observation from station B

Vertical component, $h = S.\frac{f}{l}.\frac{\sin 2\theta}{2}$ $S = 3.20 - 1.84 = 1.36$ m

$$h = 1.36 \times 100 \times \frac{\sin 11° 12'}{2} = 13.21 \text{ m}$$

R.L. of $P = 318.90 + 1.6 - 13.21 - 2.52$

$= 304.77$ m ...(*ii*)

Mean elevation of $P = \frac{259.35 + 304.77}{2} = 282.06$ m **Ans.**

Example 13.32. *The readings on a vertical staff held upon a BM. whose R.L. = 100.00 were 2.51, 1.59, 0.67, while the angle of elevation was* $+1°$. *The readings at station P were 1.60, 1.06, 0.52, while the vertical angle was* $-1°$. *Compute the horizontal distance from its instrument station X to P and its R.L. if the tacheometric constants are 100 and 0.3.*

Solution.

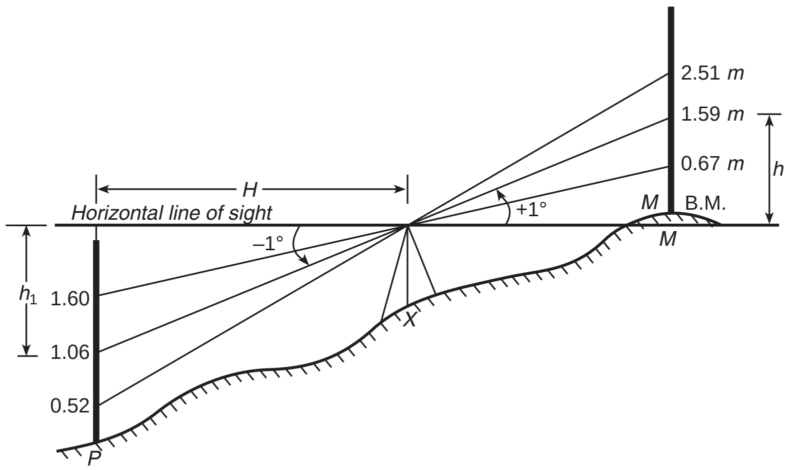


Fig. 13.45.

Observation on staff on B.M.

R.L. of horizontal line of sight OM

$$= R.L \text{ of } BM + \text{middle hair reading} - h$$

where $h = S\ \dfrac{f}{l} \cdot \dfrac{\sin 2\theta}{2} + (f + d) \sin\theta$

$$S = 2.51 - 0.67 = 1.84 \text{ m}$$

$\therefore$ $$h = 1.84 \times 100 \times \frac{\sin 2^\circ}{2} + 0.3 \sin 1^\circ$$

$$= 3.216 \text{ m}$$

$\therefore$ R.L. of horizontal line of sight = 100 + 1.59 − 3.216

$$= 98.374 \text{ m}$$

Observations on staff at *P*

$$H = S.\frac{f}{i}\cos^2\theta + (f + d)\cos\theta$$

$$= 1.08 \times 100 \times \cos^2 1^\circ + 0.3 \cos 1^\circ$$

$$S = 1.06 - 0.52 = 1.08 \text{ m}$$

$$= 108.267 \text{ m}$$

$$h_1 = H \tan\theta = 108.267 \times 0.017 = 1.840 \text{ m}$$

$\therefore$ R.L. of P = height of horizontal line of sight $- h_1$ − middle hair

= 98.374 − 1.840 − 1.060 = 95.474 m. **Ans.**

Example 13.34. *The constants* $\left(\frac{f}{i}\right)$ *and (f + d) for a certain tacheometer were 100 and 0.3 m respectively. Readings of three diaphram wires on a staff held at a distant object were found 3.44, 2.40, 1.60 m. The telescope being horizontal.*

Find the horizontal distance of the staff from the instrument axis and the R.L. of the staff point if the R.L. of instrument axis is 80.00 metres. (Fig. 13.46.)

Solution. (Fig. 13.46)

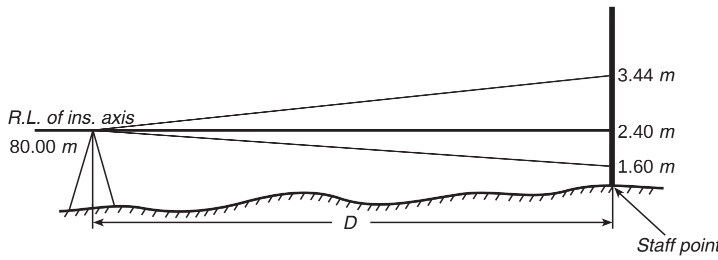


Fig. 13.46.

Let D = horizontal distance.

$S = 3.44 - 1.60 = 1.84$ m

$$D = S.\frac{f}{i} + f + d$$

$$= 1.84 \times 100 + 0.3 = 1.84 + 0.3 = 184.3 \text{ m}$$

R.L. of the staff point = R.L. of instrument axis – central hair reading

= 80.00 – 2.40 = 77.60 m **Ans.**

Example 13.35. *In order to determine the constants of a tacheometer distances 201 and 400 m were accurately measured and readings on stadia rod on upper and lower wires were as follows:*

at 201 m	*2.00*	*4.00*
at 400 m	*0.50*	*4.50*

Determine the values of the constants and find the distance when the readings of stadia wires were 1.5, 4.5 m. The line of sight being horizontal in all cases.

Solution.

Here $S_1 = 4.00 - 2.00 = 2$ m

$S_2 = 4.50 - 0.50 = 4$ m

We know $$D = S.\left(\frac{f}{i}\right) + (f + d) \quad ...(i)$$

Substituting the values in eqn (*i*) we get

$$201 \text{ m} = 2.0 \times \left(\frac{f}{i}\right) + f + d$$

$$400 \text{ m} = 4.0 \times \left(\frac{f}{i}\right) + f + d$$

Let $\frac{f}{i} = a; f + d = b$

or $201 = 2a + b$ ...(*ii*)

$400 = 4a + b$ ...(*iii*)

Solving eqns. (*iii*) and (*ii*), we get,

$\therefore$ $a = 99.5$

$b = 2.00$

Now $S = 4.5 - 1.5 = 3.00$

$\therefore$ The horizontal distance

$$D = S\left(\frac{f}{i}\right) = 3 \times 99.5 + 2 = 300.5 \text{ m}$$ **Ans.**

EXERCISE 13

1. Fill in the blanks from suitable word (s)given in the brackets.

(*i*) Tacheometry is the branch of surveying in which measurement of distances is made by .... (chaining, pacing, computation)

(*ii*) The accuracy of tacheometric measurements is ...... as compared to direct chaining in undulating grounds. (same, less, more)

(*iii*) A tacheometer is an instrument like a ......fitted with stadihairs. (level, theodlite, clinometer)

(*iv*) Tacheometry is best suited to survey a ...... ground. (level, undulating, congested)

(*v*) Tacheometric measurments require...... correction. (slope, tension, temperature, no)

(*vi*) A stadia diaphragm consists of......vertical and ......horizontal hairs. (four, three, two, one)

(*vii*) Two hairs placed equidistant from the central hair on either side..........to the horizontal hair, are called stadia hairs. (perpendicular, parallel)

(*viii*) The stadia hairs are ......in the plane containing horizontal and vertical cross hairs. (not kept, kept)

(*ix*) A theodolite may be used as a tacheometer if it is provided with a diaphragm containing......horizontal hairs. (one, two, three) (*x*) An anallatic lens is provided in ......focussing telescopes to simplify the computation of horizontal distance between instrument station and the staff. (external, internal)

(*xi*) The introduction of an anallatic lens between the object glass and eye piece, simplifies the computation of ......distances. (horizontal, vertical or both)

(*xii*) Multiplying constants of the theodolites / tacheometers, are kept......by the manufacturers. (50, 100, 200)

(*xiii*) In fixed hair method of tacheometry, the observation on the staff is made when it is held...... (vertical, inclined, or both)

(*xiv*) In tangential method of tacheometry......pointings of the telescope are made on a staff to calculate the horizontal distance between the instrument station and the staff. (one, two, three)

(*xv*) The mulitplying constant of a theodolite is......

$$\left[\frac{f}{i}, \frac{f^2}{i}, (f+d), \left(\frac{f}{i}+d\right)\right]$$

(*xvi*) For a horizontal line of sight, horizontal distance between instrument and staff positions is given by

$$\left[\frac{f}{i}S-(f+d); \frac{f}{i}S+(f+d); \frac{f}{i}S+(f+d)^2; \frac{f^2}{i}S+d\right]$$

(*xvii*) In case of an inclined line of sigth with the staff held vertical, the horizontal distance formula by tacheometry is D =......

$$\left[\frac{f}{i}S\sin\theta+(f+d)\sin\theta; \frac{f}{i}\cos\theta+(f+d)\cos\theta; \frac{f}{i}S\cos^2\theta+(f+d)\cos\theta\right]$$

(*xviii*) In case of an elevated line of sight when a staff is held normal to the line of sight, the horizontal distance formula is D = ...

$$\left[\frac{f}{i}S\cos^2\theta+(f+d)\cos\theta+h\cos\theta; \frac{f}{i}S\sin\theta+(f+d)\sin\theta+h\sin\theta;\right.$$

$$\left.\frac{f}{i}S\cos\theta+(f+d)\cos\theta+h\sin\theta\right]$$

(*xix*) If the angles of elevation and depression observed by a theodolite on a vertically held staff are kept the same, the horizontal distance computed by observations will be...... (different, same)

(*xx*) The additive constant of a theodolite is determined by adding the distance along the telescope from the centre of the theodolite to the......and its focal length. (eyepiece, objective)

(*xxi*) In a tacheometer......constant is more important than the......constant. (multiplying, additive)

(*xxii*) Anallatic lens is fitted nearer to the ...... (objective glass, eye piece)

(*xxiii*) If the vertical circle of a theodolite is fitted with a Beaman's stadia arc, the horizontal and vertical distances are obtained by the use of......(tacheometric table, reduction diagram, neither of the two)

(*xxiv*) In movable hair method, the distance between......is moved.

(stadia hairs, staff intercept)

(*xxv*) The multiplying constant of a subtense theodolite varies between...... (0 to 50, 50 to 100, 100 to 500, 600 to 1000)

2. Explain how would you obtain in the fild the constants of a tacheometer.

3. Derive an expression for the horizontal distance of a vertical staff from a tacheometer if the line of sight is horizontal.

4. (*a*) What do you understand by a tacheometer ? Discuss the errors in 'stadia surveying'. What is the utility of an anallatic lens in a tacheometer?

(*b*) Obtain the expression for horizontal distance and elevation when the instrument is fitted with a stadia diaphragm only and the line of sight is horizontal.

5. Derive an expression for the horizontal distance D of a vertical staff from a tacheometer if the line of sight is inclined. How do you determine the constants of a tacheometer ?

6. Prove that in case of a telescope fitted with an anallatic lens, the additive constant becomes zero and expression for the distance reduces to D = *ms*, where m is a multiplying constant and s is the staff intercept.

7. In case of a subtense bar, justify the following statement.

"To compute the horizontal distance, multiply the constant 206265 by the length in metres between the centres of the discs and divide the product by the subtended angle is seconds".

8. Describe in detail with a neat sketch, the construction and working of a subtense bar.

9. (*a*) Explain how would you carry out tacheymetric plane tabling in an undulating ground.

(*b*) What are the advantages and disadvantages of tacheometric plane tabling as compared to simple plane tabling.

10. Describe in detail, how tachcometric surveys are conducted in the field.

11. Write short notes on :

(*i*) Subtense bar

(*ii*) Tangential tacheometry

(*iii*) Tacheometric plane tabling

(*iv*) Tachcometric tables

(*v*) Anallatic lens

(*vi*) Direct reading tacheometer

12. (*i*) Explain different systems of tacheometry and discuss their relative merits, (*ii*) Derive an equation for the horizontal and vertical distances by the tangential method when both the angles are angles of depression.

13. Two distances of 50 and 75 metres were accurately measured on a fairly level ground. The intercept on the staff held vertical were accurately measured on a fairly level ground. The intercept on the staff held vertical were 0.495 and 0.745 metres respectively. Calculate the tacheometric constants of the instrument.

14. A tacheometer reads 1.284 and 1.780 metres corresponding to the stadia hairs on a vertical staff 50 metres away. If the focal length of the object glass is 25 cm and the distance from the object glass to the trunnion axis of the tacheometer is 15 cm. Calculate the stadia interval.

15. A vertical staff was held at a distance of 90 m from an external focussing theodolite and the readings taken with the horizontal line of sight were 1.558, 2.055 metres. If the focal length of the object glass was 20 cm and its distance from vertical axis was 10 cm. Catculate :

(*i*) stadia interval.

(*ii*) multiplying constant of the theodolite.

(*iii*) additive constant of the theodolite.

16. A staff held vertically at a distance of 50 m and 100 m from the centre of a theodolite fitted with stadia hairs, the staff intercepts with the telescope horizontal were 0.495 and 0.990 respectively. The instrument was then set up near a station *A* of R.L. 1950.85 m and the reading on staff held on the B.M. was 1.585 m. The hair readings on a staff held vertically at station *B* were 1.005, 1.855 and 2.705 m while line of sight is horizontal. Calculate the horizontal distance of *AB* and the reduced level of station *B*.

17. A tacheometer is set up at an intermediate point on a traverse leg *AB* and the following observations are made on a vertically held staff :

Staff station	*Vertical angle*	*Staff readings*
A	+ 5° 42′	1,756, 2.506, 3.256
B	3° 36′	0.855, 1.255, 1.655

The instrument is fitted with an anallatic lens and the multiplying constant is 100. Compute the length *AB* and the reduced level of *B* if R.L. of *A* = 500.0 m.

18. The following observations were made on a vertfcally heId staff with a tacheometer set up on an intermediate point on a straight line *CD* :

Staff station	*Vertical angle*	*Staff intercept(m)*	*Axial hair reading(m)*
C	+ 8° 36′	2.880	2.505
D	– 8° 36′	1.655	2.850

The instrument was fitted with an anallatic lens and had a constant of 100. Compute the length *CD* and the R.L. of *D*, given that *C* has a reduced level of 527.63 metres.

19. *A, B, C* are three points on the centre line of a proposed road. A tacheometer fitted with an anallatic lens and having its multiplying constant as 100, was set over the point *A* and the observations were made :

Staff stn.	*Vertical angle*	*Staff intercept*	*Axial hair reading(m)*
B	0° 00′ 00″	0.255	1.857
C	+ 3° 15′ 00″	1.755	1.585

Calculate the sloping distance BC and the pereentage of the gradient between points *B* and *C*.

20. A theodolite fitted with stadia wires and having an additive constant of 30 cm is used for contouring. At the first station the height of the instrument was 1.535 m, and the following readings were taken :

1	0.525	1.155	1.785	+ 5° 18′	*Theodolite was set up over B.M. 75.855 m above M.S.L.*
2	0.755	2.575	3.395	+ 6° 24′	
3	0.150	1.955	2.760	+ 7° 36′	
4	0.458	1.685	2.912	+ 8° 42′	

Calculate the reduced level of the stations above M.S.L. Assume that the staff was held vertical and the multiplying constant of the theodolite was 100.

21. A fixed hair tacheometer fitted with an anallatic lens and having its constant 100, was set up at station *C* and the following observations were made :

Stn. sighted	*Bearing*	*Readings of stadia hairs*	*Readings of axial hairs*	*Vertical angles*
A	320° 40′	0.915 2.585	1.750	+ 10° 36′
B	50° 40′	0.765 3.655	2.210	+ 8° 54′

Calculate the gradient from the point *A* to the point *B*.

22. Determine the gradient from a point *M* to a point *N* from the following observations made with a fixed hair tacheometer fitted with an anallatic lens, the constant of instrument being 100 :

	Azimuth	*Reading of stadia hair*	*Reading of axial hair*	*Vertical Angle*
To *M*	55° *W*	1.584 2.950	2.267	+ 5° 36′
To *N*	35° *E*	0.852 2.384	1.618	– 5° 36′

23. During course of tacheometric traverse from A to D, the following observations were made with the theodolite fitted with an anallatic lens.

Line	*Bearing*	*Vertical angle*	*Staff reading*
AB	35° 45′	+ 5° 30′	1.055, 1.955, 2.885
BC	110° 36′	+ 6° 42′	1.250, 2.115, 2.980
CD	210° 48′	– 3° 12′	1.325, 2,190, 3.055

If the multiplying constant of the instrument was 100, calculate the bearing and distance from *A* to *D*. Assume that the staff was held vertical in each case.

24. The stadia intercept read by a fixed hair theodolite on a vertically held staff is 1.255 m, the angle of elevation being 5° 24′. The constants of the instrument were 100 and 0.30. What would be the total number of turns registered on a movable hair theodolite at the same station for a 1.450 m intercept on a staff held on the same point, the vertical angle in this case being 7° 42′ and the constants 1000 and 0.4.

25. Two observations were taken upon a vertically held staff by means of a theodolite whose trunnion axis is 1.856 m above a Bench Mark whose reduced level is 857.750 m above M.S.L. For first observation, the line of sight is directed to give a staff reading 1.000 m and the angle of elevation is 5° 36′. In the second observation the staff reading is 3.5 m and the angle of elevation is 6° 18′. Compute the reduced level of the staff station and its horizontal distance from the instrument station.

26. It was required to ascertain the difference in elevations of two points *A* and *B* on opposite banks of a river. Following observations were made with a theodolite fitted with an anallatic lens, having multiplying constant 100.

Inst. Stn.	*Staff station*	*Readings of stadia hairs*	*Readings of axial hair*	*Vertical angle*
P	*A*	1.856 1.844	1.850	0° 0′
	B	2.886 1.528	2.207	3° 54′

where *P* is on the line *BA* produced.

Calculate the difference in elevation of the points *A* and *B*.

27. An observation with a percentage theodolite gave staff readings of 1.052 and 2.502 metres for angles of elevation of 5 and 6 percent respectively. On sighting the staff graduation corresponding to the height of instrument axis above the instrument station, the vertical angle was 5.25 per cent.

Compute the horizontal distance and the elevation of the staff station if the instrument station has an elevation of 942.552 metres.

28. A tacheometer is set up at in intermediate point on a traverse course *PQ* and the following observations are made on a vertically held staff :

Staff Station	*Verical Angle*	*Staff Intercept*	*Axial hair Reading*
P	+ 9° 30′	2.250	2.105
Q	+ 6° 00′	2.055	1.875

The instrument is fitted with an anallatic lens and the multiplying constant is 100. Compute the length *PQ* and the reduced level of *Q* if R.L. of *P* = 350.50 m.

29. Following observations were made with a telescopic alidade which was fitted with Beaman's stadia arc and its multiplying constant was 100:

Staff intercept	1.585 m
Central hair reading	2.545 m
Reading on V-scale	59 m
Reading on H-scale	5 m

If the elevation of the telescope was 500.525 m above datum, calculate the horizontal distance and the elevation of the point sighted.

30. A subtense bar having three targets fixed at 3 metres and 2 metres apart was used to measure the horizontal distance between station A and B on the opposite banks of a river and following observations were made :

The repeated angle	Distance between
0° 14′ 30″	3 metres
0° 09′ 40″	2 metres

Calculate the horizontal distance between A and B.

If the angle of depression to station B is 2° 36′, calculate the reduced level of B. Assume the R.L. of A to be 1000.0 metres and also that the height of the trunnion axis above the ground level is equal to height of the aildade sighted above the ground at B.

ANSWERS

1. (*i*) computations (*ii*) same (*iii*) theodolite

(*iv*) undulating (*v*) no (*vi*) one ; three

(*vii*) parallel (*viii*) kept (*ix*) three

(*x*) external (*xi*) both (*xiii*) 100

(*xiii*) vertical (*xiv*) two (*xv*) f/i (*xvi*) $\frac{f}{i} s + (f + d)$

(*xvii*) $f/i\ S \cos^2 \theta + (f + d) \cos$

(*xviii*) $f/i\ \ S\ \cos \theta + (f + d) \cos \theta - h \sin \theta$

(*xix*) same (*xx*) objective (*xxi*) multiyling

(*xxii*) eye piece (*xxiii*) neither (*xxiv*) stadia hairs

(*xxv*) 600 to 1.000.

13. 100 ; 0. **14**. 2.5 mm

15. 2 mm ; 100; 0.3 **16.** 171.72 m, 1950.580 m

17. AB = 228.20 m, R.L. of B = 491.440 m

18. CD = 443.358 m' R.L. of D = 460.233 m

19. BC = 150.06 m, gradient 1 in 14.64

20. 87.824 m, 104.059 m 109.651 m, 112.397 m

21. Gradient 1 in 24. **22.** Gradient 1 in 7.393

23. AD = 185.85 m, Bearing of AD = 109° 57′ 10″

24. 11.464 turns **25.** 202.429 m, 878.454 m

26. 8.858 m **27.** 1.450 m, 950.165 m

28. PQ = 422.12 m, R.L. = 335.467 m

29. 150.575 m, 512.245 m

30. AB = 711.259, R.L. of B = 967.702 m.

14

Trigonometrical Levelling

14.1. INTRODUCTION

The branch of levelling in which difference of elevations of two points is determined from the observed vertical angles and measured horizontal distance, is called *trigonometrical levelling*. The vertical angles are generally observed by a theodolite and horizontal distances are either measured directly or computed trigonometrically.

Depending upon the horizontal distance between stations, trigonometrical levelling, may be classified into two categories :

(*i*) Observations of heights and distances as plane surveys.

(*ii*) Observations of heights and distances as geodetic surveys.

1. Heights and distances as plane surveys. When the distances between the stations is not large, the distance between the stations measured on the surface of the earth or computed trigonometrically may be assumed as a horizontal distance and the amount of correction due to curvature of the earth surface, is ignored. The following three cases are involved :

Case 1. Base of the object is accessible.

Case 2. Base of the object inaccessible and instrument stations and the elevated object are in the same vertical plane.

Case 3. Base of the object inaccessible and instrument stations and the elevated object are not in the same vertical plane.

14.2. BASE OF THE OBJECT ACCESSIBLE

Assumptions. The horizontal distance between the instrument stations C and the base of object A can be measured accurately.

Let D = horizontal distance between C and A.

S = reading on the levelling staff held vertically on a bench mark with the line of sight horizontal.

h = height of the instrument at C.

α = angle elevation of B.

The horizontal line of sight meets the vertical through B at F.

From $\Delta\, EBF$, we get

$$FB = EF \tan \alpha = D \tan \alpha$$

$\therefore$ R.L. of B = R.L. of instrument axis + FB

or R.L. of B = R.L. of B.M. + $S + D \tan \alpha$ ...(14.1)

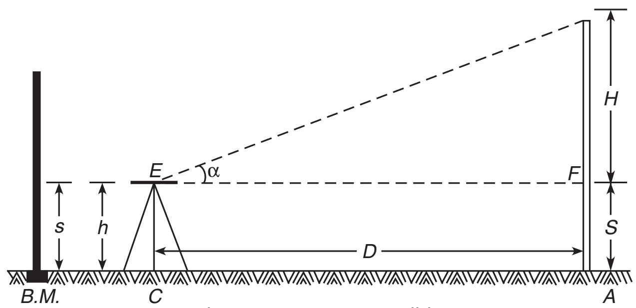


Fig. 14.1. Base Accessible.

14.3. BASE OF AN INCLINED OBJECT ACCESSIBLE. (FIG. 14.2)

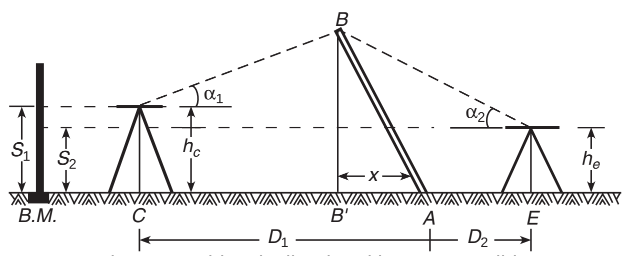


Fig. 14.2. Object inclined and base accessible.

Assumptions. Following assumptions are made :

1. The horizontal distance between the foot of the object and the instrument stations can measured.
2. Foot of the object and instrument stations are in one line.

Let, AB be an inclined tower.

x is the distance between the foot of the tower and the projected point B' of its top B on ground surface. C and E are instrument stations such that C, A and E are in one straight line. D_1 and D_2 are the horizontal distances of the foot of tower from the stations C and E respectively.

Let S_1 and S_2 be the readings of the staff held on the same benchmark from two instrument stations.

α_1 and α_2 be the angles of elevation from stations C and E respectively.

From equation (14.1), we get

R.L. of B = R.L. of B.M. + $S_1 + (D_1 - x) \tan \alpha_1$ ...(*i*)

Also, R.L. of B = R.L. of B.M. + $S_2 + (D_1 + x) \tan \alpha_2$ ...(*ii*)

Equating equations (*i*) and (*ii*), we get

$S_1 + (D_1 - x) \tan \alpha_1 = S_2 + (D_2 + x) \tan \alpha_2$

or $(S_1 - S_2) + D_1 \tan \alpha_1 - x \tan \alpha_1 = D_2 \tan \alpha_2 + x \tan \alpha_2$

or $(S_1 - S_2) + D_1 \tan \alpha_1 - D_2 \tan \alpha_2 = x(\tan \alpha_1 + \tan \alpha_2)$

or $$x = \frac{(S_1 - S_2) + D_1 \tan \alpha_1 - D_2 \tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \quad ...(14.2)$$

Quantities on the right side of the equation (14.5) being known, the value of x can be calculated.

$\therefore$ R.L. of B = R.L. of B.M. + $S_1 + (D_1 - x) \tan \alpha_1$

or = R.L. of B.M. + $S_2 (D_1 + x) \tan \alpha_2$ ...(14.3)

14.4. R.L. OF THE ELEVATED POINTS WITH INACCESSIBLE BASES

Depending upon the terrain, three cases may arise :

1. When the instrument axes at both stations A and B are at the same level.
2. When the instrument axes at stations A and B are at different levels but the difference in their level is small.
3. When the instrument axes at stations A and B are at different levels and the difference in their level is more.

If the horizontal distance from the instrument station to the bottom of the object can not be measured due to obstructions, two instrument stations in a plane containing the elevated point may be used with a known distance between them.

1. Instrument Axes at the same Level (Fig. 14.3)

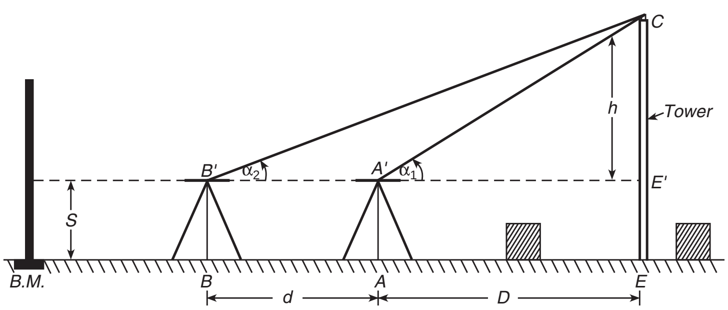


Fig. 14.3. Instrument axes as the same level.

Assumptions. Instrument stations and the elevated point are in the same plane

Let α_1 = angle of elevation of C at A

α_2 = angle of elevation of C at B

S = reading on the staff held on a Bench Mark from both stations.

d = horizontal distance measured between the instrument stations A and B.

D = horizontal distance between instrument station A and bottom of tower E.

$h = CE'$, height of C above A' or B'

From $\Delta A'CE'$ $\quad h = D \tan \alpha_1$ ...(*i*)

From $\Delta B'CE'$ $\quad h = (D + d) \tan \alpha_2$ ...(*ii*)

Equating equations (*i*) and (*ii*), we get

$$D \tan \alpha_1 = (D + d) \tan \alpha_2$$

or
$$D = \frac{d \tan \alpha_2}{(\tan \alpha_1 - \tan \alpha_2)}$$

But,
$$h = D \tan \alpha_1$$

$$= \frac{d \tan \alpha_2 . \tan \alpha_1}{\tan \alpha_1 - \tan \alpha_2} \quad ...(14.4)$$

$$= \frac{d \tan \alpha_1 . \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \quad ...(14.5)$$

or R.L. of C = R.L. of B.M. + S + h

Procedure : Following steps are followed:

1. Set up the theodolite at *A,* level it carefully and observe the angle of elevation α_1.
2. With the vertical vernier set to zero reading and bringing the altitude bubble at the centre of its run, take a reading on a staff held vertically on a bench mark. Let the reading be *S*.
3. Transit the telescope so that the line of sight is reversed.
4. Mark a point *B* in the line of sight at a convenient distance *d*. Measure the distance *d* accurately with a steel tape.
5. Shift the theodolite to the point *B*. Set up and centre it to have same instrument height as at *A*. Carefully level the instrument and observe the angle of elevation α_2.
6. With the vertical vernier-set to zero reading and bringing the altitude bubble at the centre of its run, take a reading on the staff held vertically on the bench mark. Its reading should be *S* as before.

Note. Equation (14.4) is generally preferred to when a clinometer is used instead of a theodolite.

2. Instrument Axes at Different Levels

Assumptions. Difference in levels being small.

Depending upon the terrain, the following two cases may arise :

Case I. Instrument axis at B is higher

Case II. Instrument axis at A is higher.

Let S_1 and S_2 be the staff readings from A and B on the bench mark. C and C' are the projections of C' on the horizontal lines through trunnion axis A' and B' respectively.

d is the distance between two instrument positions.

Case I. Instrument axis at B is higher. (Fig. 14.4)

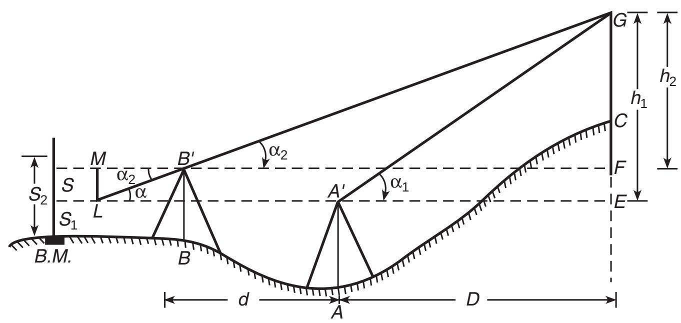


Fig. 14.4. Instrument axis at B is higher.

Assume $EG = h_1$ and $FG = h_2$

From $\Delta A'EG$, $h_1 = D \tan \alpha_1$ ...(*i*)

From $\Delta B'FG$ $h_2 = (D + d) \tan \alpha_2$ ...(*ii*)

Subtracting Eq. (*ii*) from Eqn. (*i*), we get

$$h_1 - h_2 = D \tan \alpha_1 - (D + d) \tan \alpha_2$$

But $h_1 - h_2 = EF = S_2 - S_1 = S$ say

$\therefore$ $S = D \tan \alpha_1 - (D + d) \tan \alpha_2$

or $D (\tan \alpha_1 - \tan \alpha_2) = S + d \tan \alpha_2$

$$D = \frac{S + d \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

or

$$D = \frac{(d + S. \cot \alpha_2) \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

Again $h_1 = D \tan \alpha_1$

$\therefore$

$$h_1 = \frac{(d + S \cot \alpha_2) \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

$$= \frac{(d + S \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)}$$

$\therefore$ R.L. of G = R.L. of B.M. + $S_1 + h_1$

$$= \text{R.L. of B.M.} + S_1 + \frac{(d + S \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \quad ...(14.6)$$

Alternative Method. (Fig. 14.4)

Construction : Produce GB' to cut the horizontal line through A' at L. Drop LM perpendicular to horizontal line through B'.

Proof :

Angle $GB'F$ = angle $LB'M = \alpha_2$

At point L, the level of instrument axis B is the same as at A.

The distance $A'L = d + B'M = d + S \cot \alpha_2$

Now, the problem may be considered same as in the case of instrument being at the same level. Substituting $(d + S \cos \alpha_2)$ for d in Eqn. (14.4), we get

$$h_1 = \frac{(d + S \cot \alpha_2) \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2}$$

$$= \frac{(d + S \cot \alpha_2). \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)}$$

$$\therefore \quad \text{R.L. of } G = \text{R.L. of B.M.} + S_1 + \frac{(d + S \cot \alpha_2). \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)}$$

This is the same as Eqn. (14.6).

Case II. Instrument axis at A is higher (Fig. 14.5)

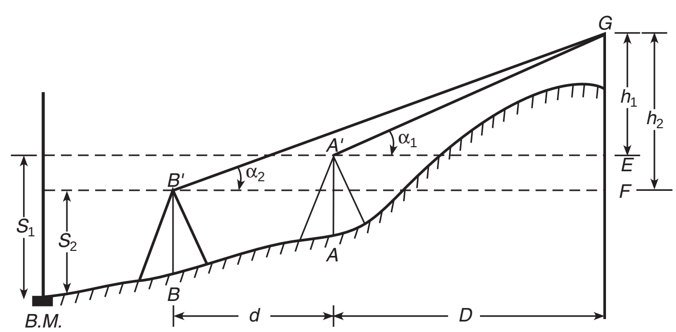


Fig 14.5. Instrument axis at *A* is higher.

From $\Delta GA'E$, $h_1 = D \tan \alpha_1$ ...(*i*)

From $\Delta GB'F$, $h_2 = (D + d) \tan \alpha_2$ ...(*ii*)

Subtracting Eqn. (*i*) from Eqn. (*ii*), we get

$$h_2 - h_1 = (D + d) \tan \alpha_2 - D \tan \alpha_1$$

But $$h_2 - h_1 = EF = S_1 - S_2 = S \text{ say}$$

$$S = (D + d) \tan \alpha_2 - D \tan \alpha_1$$

or $$D = \frac{d \tan \alpha_2 - S}{\tan \alpha_1 - \tan \alpha_2}$$

$$= \frac{\left(d \tan \alpha_2 - S.\, \dfrac{\tan \alpha_2}{\tan \alpha_2}\right)}{\tan \alpha_1 - \tan \alpha_2}$$

or $$D = \frac{(d - S \cot \alpha_2) \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \qquad ...(14.7)$$

But $$h_1 = D \tan \alpha_1$$

∴ $$h_1 = \frac{(d - S \cot \alpha_2) \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \qquad ...(14.8)$$

$$= \frac{(d - S \cot \alpha_2).\, \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \qquad ...(14.9)$$

or $$\text{R.L. of } G = \text{R.L. of B.M} + S_1 + h_1 \qquad ...(14.10)$$

The general expressions for the distance D and difference in height h_1 for both the cases may be written as

$$D = \frac{(d \pm S \cot \alpha_2) \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \qquad ...(14.11)$$

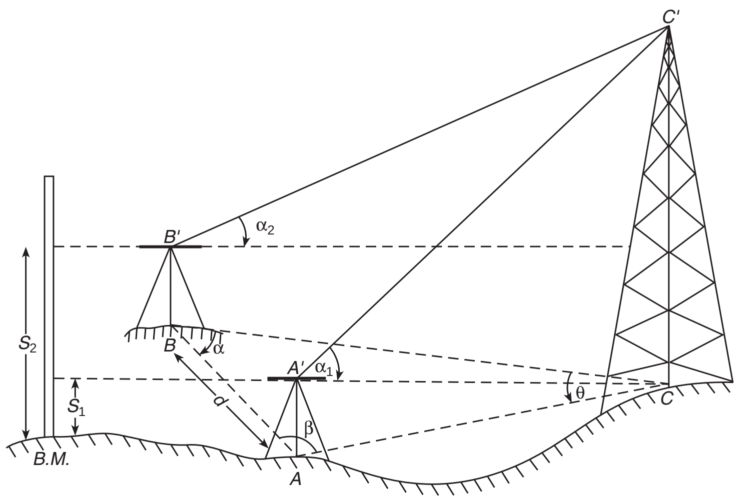


Fig. 14.6. Instrument stations and the object not in the same vertical plane.

$$h = \frac{(d \pm S \cot \alpha_2).\ \sin \alpha_1.\ \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \qquad ...(14.12)$$

Use a + sign if station A nearer to the object, is lower and a negative sign if it is higher as compared to other station B.

14.5. INSTRUMENT AXES AT DIFFERENT LEVELS

Assumptions. Instrument stations and the elevated object are not in the same plane.

Procedure : Following steps are followed :

1. Set up the theodolite at station A, centre it carefully and measure horizontal angle BAC. Let it be β.
2. Sight C', the top of the object and observe the angle of elevation α_1, ensuring that the altitude bubble is central of its run.
3. Setting the vertical vernier to zero take a reading on a staff held vertically on a B.M. Let the reading be S_1.
4. Shift the theodolite to station B and centre it over the mark and observe the the horizontal angle CBA. Let it be α.
5. Sight C', the top of the object and observe the angle of elevation α_2, ensuring that the altitude bubble is central of its run.
6. Setting up the vertical vernier to zero, take a reading on a staff held vertically on the same bench mark. Let the reading be S_2.
7. Measure the horizontal distance d between stations A and B.

Calculations :

In horizontal triangle ABC, we know

$$\angle ACB = 180° - (\alpha + \beta) = \theta$$

Applying sine rule to the triangle ABC, we get

$$\frac{AC}{\sin ABC} = \frac{BC}{\sin BAC} = \frac{AB}{\sin ACB}$$

$$\therefore \qquad AC = \frac{AB.\sin ABC}{\sin ACB} = \frac{d \sin \alpha}{\sin \theta}$$

$$\text{and} \qquad BC = \frac{AB.\ \sin BAC}{\sin ACB} = \frac{d \sin \beta}{\sin \theta}$$

Knowing horizontal distances AC and BC, the vertical components h_1 and h_2 may be calculated as under :

$$h_1 = AC \tan \alpha_1\ ;\ h_2 = BC \tan \alpha_2$$

$\therefore$ R.L. of the elevated object C' = R.L. f of B.M. $+ S_1 + h_1$

$$= \text{R.L. of B.M.} + S_1 + \frac{d \sin \alpha \tan \alpha_1}{\sin \theta} \qquad ...(14.13)$$

or R.L. of the elevated object C' = R.L. of B.M. $+ S_2 + h_2$

$$= \text{R.L. of B.M.} + S_2 + \frac{d \sin \beta . \tan \alpha_2}{\sin \theta} \qquad ...(14.14)$$

Note. The following points may be noted:

(*i*) The accuracy of the work depends upon the accuracy with which horizontal and vertical angles are measured and also on the accuracy of the horizontal distance beteen the instrument stations *A* and *B*.

(*ii*) It is assumed that the staff held vertically on B.M. can be read clearly from both stations with line of sight horizontal.

Example 14.1. *An instrument was set up at A and the angle of elevation of the top of an electric pole BC was 24° 36′. The horizontal distance between A and B, the foot of the pole was 600 m. Determine the reduced level of the top of the pole, if the staff reading held on a B.M. (R.L. 100.00 m) was 2.532 m, with the telescope in horizontal plane.*

Solution. (Fig. 14.7)

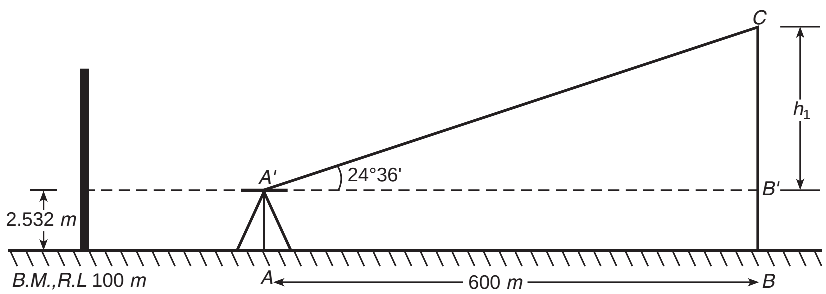


Fig. 14.7

From $\Delta A'B'C$, we get

$$h_1 = A'B' \tan 24° 36'$$
$$= 600 \tan 24° 36' = 600 \times 0.457836$$
$$= 274.702 \text{ m.}$$

$\therefore$ Reduced level of C = R.L. of B.M. $+ S_1 + h_1$

$$= 100.00 + 2.532 + 274.702$$
$$= 377.234 \text{ m.} \quad \textbf{Ans.}$$

Example 14.2. *To determine the reduced level of the top of a television tower surrounded by transmission buildings and other administrative blocks, a theodolite was set up at A in an adjoining park and the angle of elevation α_1, was measured and found to be 33′ 17. The reading on a staff held on a B.M. with telescope in horizontal plane, was 1.585 m. The theodolite was transited and another point B 100 m away* *was fixed on the ground surface. The angle of elevation of the top was measured at B and found to be 26° 24′. The reading on the staff held on the B.M. was again 1.585 m. If the reduced level of B.M. is 294.819 m, calculate the R.L. of the top of the television tower.*

Solution. (Fig. 14.8)

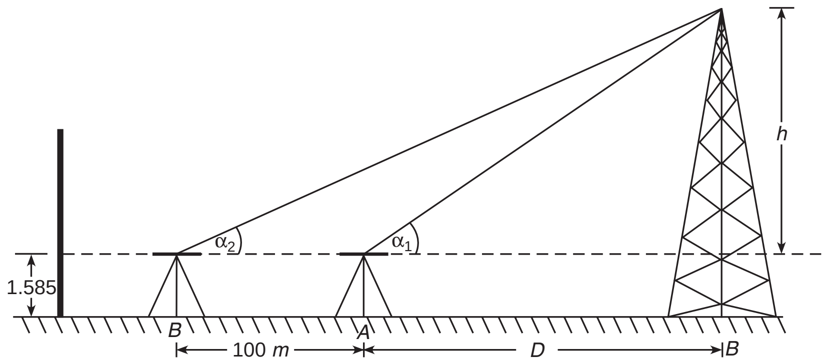


Fig. 14.8.

We know that if the axes of the theodolite at both stations are at the same level, then from Eqn. (14.4)

$$h = \frac{d.\tan \alpha_1.\tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \qquad ...(i)$$

where letters carry their usual meanings.

Here $d = 100$ m ; $\alpha_1 = 33° 17'$; $\alpha_2 = 26° 24'$

Substituting the values in Eq. (i), we get

$$= \frac{100 \times \tan 33° 17' \times \tan 26° 24'}{\tan 33° 17' - \tan 26° 24'}$$

$$h = \frac{100 \times 0.656461 \times 0.496404}{0.656461 - 0.495404}$$

= 203.596 m.

∴ R.L. of the top of the transmission tower

= R.L. of B.M. + S_1 + h = 294.819 + 1.585 + 203.596

= 500.0 m. **Ans.**

Example 14.3. *To determine the elevations of the top of a chimney the following observations were made :*

Station	*Reading on B.M.*	*Angle of elevation*
P	*2.870*	*28° 42′*
Q	*3.750*	*18° 06′*

The top of chimney and the stations P and Q are in the same vertical plane, PQ is 100 m. If the R.L. of the B.M. is 100.000, determine the elevation of the top of the chimney.

Solution. (Fig. 14.9)

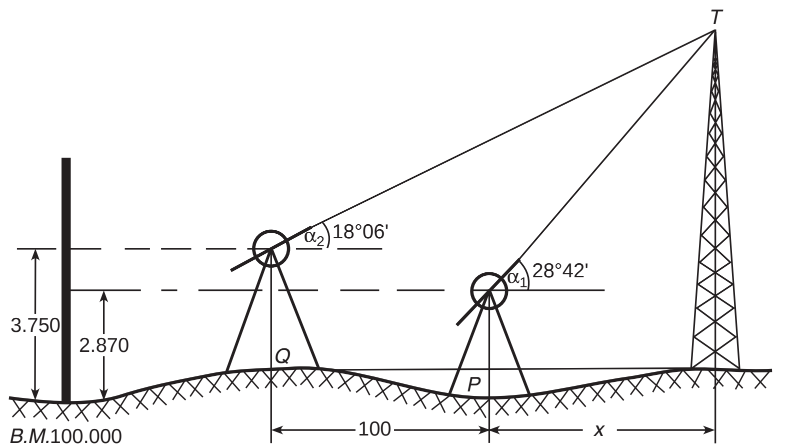


Fig. 14.9

From eqn (14.6) we get

R.L. of the chimney top T,

$$= \text{R.L. of B.M.} + S_1 + \frac{(d + S \cot \alpha_2) \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \quad ...(i)$$

Given: $S_1 = 2.870$ m

$S_2 = 3.75$ m

$S = 3.750 - 2.870 = 0.88$ m

$d = 100$ m

$\alpha_1 = 28° \ 42'$

$\alpha_2 = 18° \ 06'$.

Substituting the values in Eq. (*i*), we get

R.L. of the chimney of top

$$= 100.000 + 2.870 + \frac{(100 + 0.88 \cot 18° \ 06') \sin 28° \ 42' \times \sin 18^{o} \ 06'}{\sin(28° \ 42' - 18° \ 06')}$$

$$= 100.000 + 2.870 + \frac{(100 + 0.88 \times \ 3.0595039) \ 0.4802235 \times \ 0.3106764}{0.1839513}$$

$= 100.000 + 2.870 + 83.29$

$= 186.16$ m. **Ans.**

Example 14.4. *Find the reduced level of a church spire C from the following observations taken from two stations A and B, 50 m apart.*

Angle BAC = 60° : Angle ABC = 50°

Angle of elevation from A to the top spire = 30°

Angle of elevation from B to the top spire = 29°

Staff reading from A, on bench mark of reduced level 20.00 m = 2.500 m.

Staff reading from B to the same bench mark = 0.500 m.

Solution. (Fig. 14.6). In ΔABC, angle BCA

$$= 180° - (\angle ABC + \angle BAC)$$

$$= 180° - (50° + 60°) = 70°$$

Applying sine formula to the triangle ABC, we get

$$\frac{AC}{\sin ABC} = \frac{BC}{\sin BAC} = \frac{AB}{\sin ACB}$$

or
$$\frac{AC}{\sin 50°} = \frac{BC}{\sin 60°} = \frac{50}{\sin 70°}$$

$$AC = \frac{50 \sin 50°}{\sin 70°} = \frac{50 \times 0.766044}{0.939693} = 40.76 \text{ m}$$

$$BC = \frac{50 \sin 60°}{\sin 70°} = \frac{50 \times 0.866025}{0.939693} = 46.08 \text{ m.}$$

$\therefore$
$$h_1 = AC \tan \alpha_1 = 40.76 \times \tan 30° = 40.76 \times 0.57735$$

$$= 23.533 \text{ m}$$

and
$$h_2 = BC \tan \alpha_2 = 46.08 \times \tan 29° = 46.08 \times 0.554309$$

$$= 25.543 \text{ m}$$

R.L. of the church spire = R.L. of B.M. + $S_1 + h_1$

= R.L. of B.M. + $S_2 + h_2$

= 20.000 + 2.500 + 23.533 = 46.033 m.

Again, R.L. of the church spire

= R.L. of B.M. + $S_2 + h_2$

= 20.000 + 0.500 + 25.543 = 46.043 m.

$\therefore$ Mean reduced level of the church spire

$$= \frac{46.033 + 46.043}{2} = 46.038 \text{ m.}$$ **Ans.**

Example 14.5. *A theodolite was set up in between two towers X and Y. The distance of the theodolite station from X is 60 m and from Y, 120 m. Observations were taken from the theodolite to the top of towers X and Y and were recorded as 33° 26′ 20″ and 30° 50′ 40″ respectively, telescope focussed upward for both the cases.*

The R.L. of the trunnion axis of the theodolite was 139.675 m above mean sea level. Calculate the R.L. of the top of the tower X and that of Y.

Solution. (Fig. 14.10)

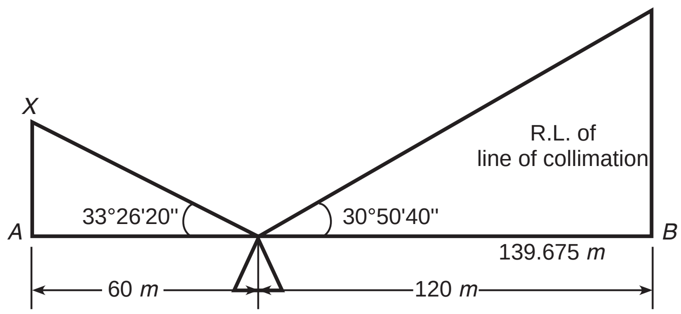


Fig. 14.10.

Let AB be the horizontal line through the trunnion axis of the theodolite, T.

From ΔXAT, $AX = AT \tan 33° 26' 20''$

$= 60 \times \tan 33° 26' 20''$

$= 60 \times 0.6603527$

$= 39.621$ m

From ΔYBT, $YB = BT \tan 30° 50' 40''$

$= 120 \tan 30° 50' 40''$

$= 120 \times 0.5971714$

$= 71.660$ m

R.L. of the top of tower

X = R.L. of trunnion axis + AT

$= 139.675 + 39.621$

$= 179.296$ m. **Ans.**

R.L. of the top of tower

Y = R.L. of trunnion axis + BY

$= 139.675 + 71.660$

$= 211.335$ m. **Ans.**

Example 14.6. *The top T of a tower is observed from two stations A and B, 50 m apart such that angles TAB and TBA are 60° and 50° respectively. The angles of elevation of T from A and B are 30° 15′ and 29° respectively. The readings on a staff held on a B.M. of R.L. 30.000 were 2.270 and 0.5000 from A and B respectively. Find the R.L. of the top of the tower, T.*

Solution. (Fig. 14.11)

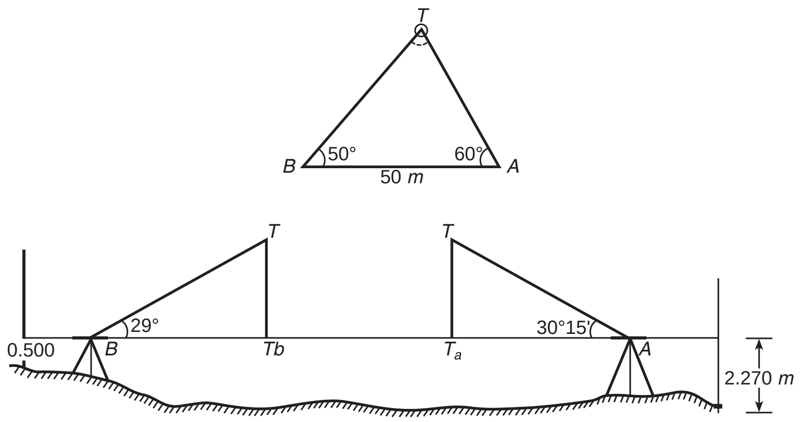


Fig. 14.11.

From Δ ABT, we get

$$\frac{BT}{\sin 60°} = \frac{AT}{\sin 50°} = \frac{AB}{\sin [180° - (50° + 60°)]} = \frac{50}{\sin 70°}$$

$$\therefore \quad BT = \frac{50 \sin 60°}{\sin 70°}\ ; \quad AT = \frac{50 \sin 50°}{\sin 70°}$$

$$= \frac{50 \times 0.866025}{0.939693}\ ; \quad = \frac{50 \times 0.766044}{0.939693}$$

$$= 46.08 \text{ m}\ ; \quad = 40.76 \text{ m.}$$

Observations from station A

Vertical component $TT_a = AT \tan 30° 15'$

$= 40.76 \times 0.583183 = 23.77$ m.

∴ R.L. of the tower,

R.L. of B.M. + Staff reading + TT_a

$= 30.000 + 2.270 + 23.770$

$= 56.04$ m. **Ans.**

Observations from station B

Vertical component $TT_b = BT \tan 29°$

$= 46.08 \times 0.554309$

$= 25.543$ m.

R.L. of the tower, T = R.L. of B.M. + Staff reading $+ TT_b$

$= 30.000 + 0.5000 + 95.543$

$= 56.03$ m.

$\therefore$ R.L. of the top of tower, $T = \dfrac{56.04 + 56.04}{2}$

$= 56.04$ m. **Ans.**

Example 14.7. *An instrument was set up at P and the angle of elevation to a vane 4 m above the foot of staff held at Q was 9° 30′. Horizontal distance between P and Q was known to be 2000 m.*

Determine the RL of staff at station Q, given that the RL of the instrument axis was 2650.38 m. Take the correction for curvature and refraction into account.

Solution. (Fig. 14.12) Given: Horizontal distance D = 2000 m; $\alpha = 9° 30'$

Height of the vane above instrument axis

$= D \tan \alpha = 2000 \tan 9° 30'$
$= 334.68$ m

We know that the combined correction for curvature and refraction

$= 0.0673\, D^2$ m

where D is in kilometres.

$= 0.0673 \times \left(\dfrac{2000}{1000}\right)^2$

$= + 0.27$ m positive for the angle of elevation

$\therefore$ the height of vane above inst. axis

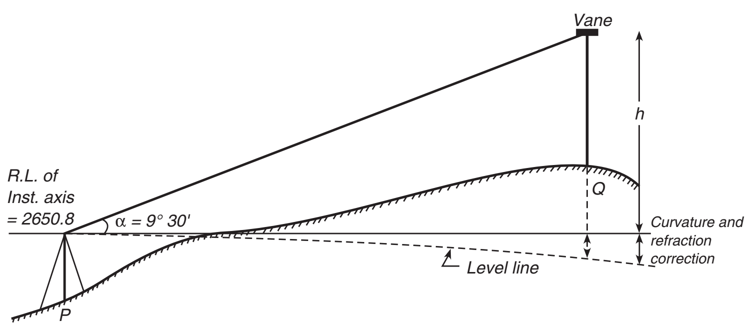


Fig. 14.12.

$= 334.68 + 0.27 = 334.95$ m

R.L of the vane = R.L. of inst. axis + height of vane above inst. axis

$= 2650.38 + 334.95 = 2985.33$ m.

R.L. of staffstation $Q = 2985.33 - 4.00 = 2981.33$ m. **Ans**

Example 14.8. *In order to ascertain the elevation of top Q of a signal on a hill, observations were made from two instrument stations P and R at a horizontal distance of 100 m apart, the stations P and R being in line with Q. The angles of elevation of Q at P and R were 28°42′ and 18°06′ respectively. The staff readings upon the BM of elevation 287.28 m were respectively and 3.750 m when the instrument was at P and R, the telescope being horizontal*

Determine the elevation of the foot of the signal if the height of the signal above its base is 3 m.

Solution. (Fig. 14.13) Given: Instrument positions at P, R and the signal are in vertical plane.

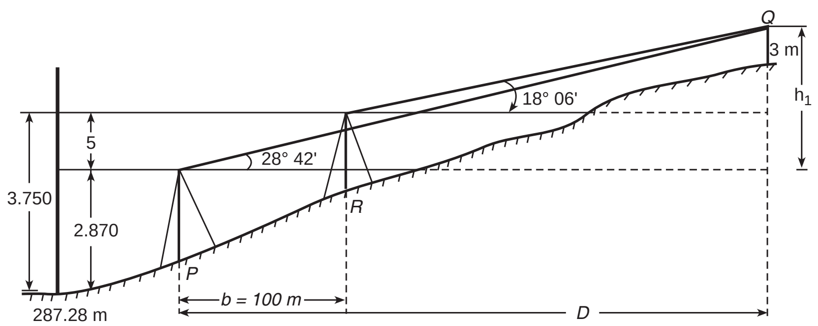


Fig. 14.13.

$S =$ vertical distance between instrument axis.

We know, if $b =$ distance between instrument stations.

$\alpha_1 =$ angle of elevation from lower station of observation

$\alpha_2 =$ angle of elevation from higher station of observation

then
$$D = \frac{(b + S \cot \alpha_2) \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \quad ...(i)$$

and
$$h_1 = D \tan \alpha_1$$

$$\therefore \quad S = 3.50 - 2.870 = 0.88 \text{ m}$$

By substituting the values in eqn (i) we get

$$D = \frac{(100 + 0.88 \cot 18° \, 06') \tan 18° \, 6'}{\tan 28° \, 42' - \tan 18° \, 06'} = 152.1 \text{ m}$$

$$h_1 = D \tan \alpha_1 = 152.1 \times 28°42' = 83.264 \text{ m}$$

R.L. of foot of signal = R.L. of instrument axis at $P + h_1$ – ht of signal

$$= 287.28 + 2.870 + 83.264 - 3.00$$

$$= 290.15 + 83.264 - 3.00 = 370.414 \text{ m} \quad \textbf{Ans.}$$

Example 14.9. *The top Q of a chimney was sighted from two stations P and R at very different levels, the station P and R being in line with the top of the chimney. The angle of elevation from P to the top of the chimney was 38°21′ and that from R of top of chimney was 21°18′. The angle of elevation from R to a vane 2 m above the foot of the staff hold at P was 15°11′. The heights of the instrument at P and R were 1.87 m and 1.64 m respectively. The horizontal distance between P and R was 127 m. R.L. of R = 112.78 m.*

Find the R.L. of top of chimney and horizontal distance from P to the chimney.

Solution. Fig. (14.14)

Let b = distance between stations of observation

We know, if r = staff reading at P

i = height of instrument axis above P

S = staff intercept = $b \tan \alpha - r + i$

then
$$D = \frac{b \tan \alpha_2 - S}{\tan \alpha_1 - \tan \alpha_2}$$
$$h = D \tan \alpha_1$$
$$S = b \tan \alpha - r + i$$

Given:

$\alpha_1 = 38° \ 21'$; $\alpha_2 = 21° \ 18'$

$r = 2$ m $\qquad \alpha = 15° \ 11'$

$i = 1.87$ m $\qquad i = 1.64$m

$\beta = 127$m $\qquad$ R.L. of $R = 112.78$ m

Substituting the values we get

$$\therefore \quad S = 127 \tan (15° \ 11') - 2.00 + 1.87$$
$$= 34.47 - 2.00 + 1.87 = 34.34 \text{ m}$$
$$D = \frac{b \tan \alpha_2 - S}{\tan \alpha_1 - \tan \alpha_2}$$
$$= \frac{127 \tan (21° \ 18') - 34.34}{\tan (38° \ 21') - \tan (21° \ 18')} = 37.8 \text{ m}$$

and
$$h_1 = D \tan \alpha_1 = 37.8 \tan (38°21')$$

= 29.92 m

R.L. of Q = R.L. of instrument axis at $P + h_1$

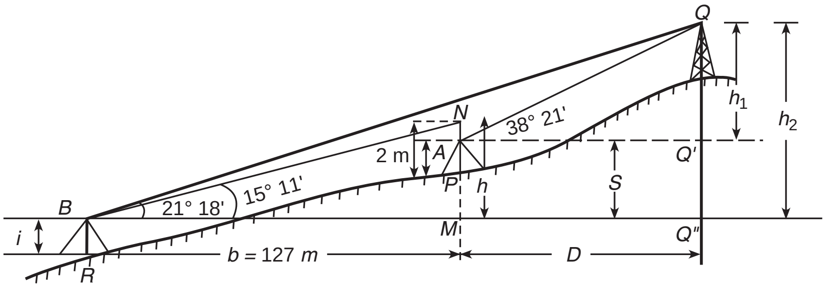


Fig. 14.14.

Computation of R.L. of instrument axis at h_1

= *R.L.* of $R + i + S$

= 112.78 + 1.64 + 34.34 = 148.76

R.L. of top of chimney Q = 148.76 + h_1

= 148.76 + 29.92 m

= 178.68 m **Ans.**

Example 14.10. *To determine the elevation of top Q of a hill flag staff of 2 m height was erected and observations were made from stations P and R, 60 m apart. The horizontal angle measured at P between R and the top of the flag staff was 60°30′ and that measured at R between the top of the flag staff and P was 68°18′. The angle of elevaion to the top of the flag staff from P = 10°12′ and from R = 10°48′. Staff readings on BM when instrument was at P = 1.965 m and that with instrument at R = 2.055 m. Calculate the elevation of the top of the hill if BM was 435.065 m?*

Solution. (Fig. 14.15)

Given: Base b = 60 m

$\angle RPM = \theta_1$ = 60°30′ where m is the foot of the flag staff in horizontal plane.

$\angle PRM = \theta_2$ = 68°18′

Angle of elevation α_1 from P to Q = 10° 12′

Angle of elevation α_2 from R to Q = 10° 48′

Staff reading on BM from P = 1.965 m

Staff reading on BM from R = 2.005 m

Height of flag staff on hill = 2.0 m

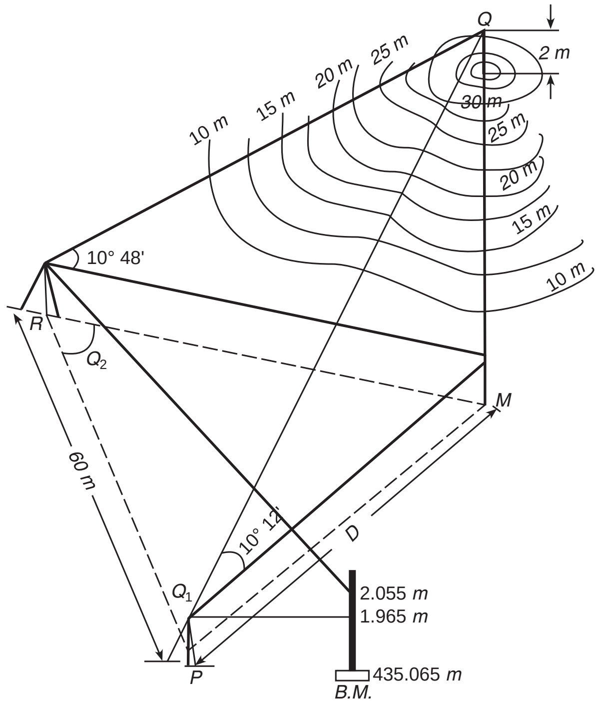


Fig. 14.15.

Let horizontal distance $PM = D$.

Applying the sine rule to ΔPRM we get,

$$D = \frac{b \sin \theta_2}{\sin (\theta_1 + \theta_2)} = \frac{60 \times \sin 68° \ 18'}{\sin (68° \ 36' + 60° \ 18')}$$

$$h_1 = D \tan \alpha_1$$

$$= \frac{60 \times \sin 68°18'}{\sin (128° \ 54')} \times \tan 10° \ 12'$$

$$= \frac{60 \times 0.92913}{0.77824} \times 0.17993 = 12.89 \text{ m.}$$

R.L. of Q = R.L. of instrument axis at P + h_1

$$= 435.065 + 1.965 + 12.89 = 449.92 \text{ m}$$

$\therefore$ Elevation of the hill top = 449.92 – 2.00 = 447.92 m **Ans.**

Similarity, the elevaion of hill top may be computed with the help of observations from R and is formed equal to 449.92 m **Ans.**

EXERCISE 14

1. A theodolite was set up at A and the angle of elevation to the top of a tree was 8° 36′. The horizontal distance between the vertical axis of the theodolite and the projected position of the top of the tree is 200 m. Determine the reduced level of the tree if the reduced level of the instrument axis was 1525.350 m.

2. A theodolite was set up at A and the angle of elevation to a target 5 m above the top of a tower was 15° 12′. The horizontal distance between the instrument and the target was known to be 250 m. Determine the reduced level of the top of the tower if the reduced level of the trunnion axis of the theodolite ws 2560.600 m.

3. An instrument was set up at a point 200 m away from a transmission tower. The angle of elevation to the top of the tower was 30° 42′, whereas the angle of depression to the bottom was 2° 30′. Calculate the total height of the transmission tower.

4. To ascertain the reduced levels and the horizontal distance between two inaccessible hill tops A and B, the following observations were made by a theodolite from the ends of a base CD 150 m long.

Angle ACD = 90° ; Angle ADC = 40° ; Angle BCD = 50°

Angle BDC = 78° ;

Angle of elevation to A from C = 22°

Angle of elevation to B from C = 19°

R.L. of station C = 248.0 m

Height of the instrument at C = 1.4 m

Calculate the reduced levels of A and B and also the horizontal distance AB.

5. In order to ascertain the elevation of a hill top C, observations were made to a target, from two instrument stations A and B at a horizontal distance 100 metres apart, the stations A and B are in line with C. The angle of elevation from A and B to the target were 18° 6′ and 28° 42′ respectively. The staff readings upon the bench mark of elevation 345.580 m were respectively 2.550 m and 1.670 m when the instrument was set upon at A and B, keeping the telescope horizontal. Determine the reduced level of the top of the hill if the height of the target was 5 metres.

6. The top (T) of a radio transmission tower was sighted from two small hill tops A and B which were in line with T. The angle of elevation from A to the top of tower was 38° 21′ and that from B to the top of tower was 21° 18′. The angle of elevation from B to a target 3 m above the ground level at A was 14° 56′. The heights of the theodolite at A and B were 1.80 m and 1.50 m respectively. The horizontal distance between A and B was 150 m and the reduced level of B was 121.50 m. Calculate the R.L. of the top of the tower and the horizontal distance from A to the tower.

7. The top of a hill subtends an angle 9° 30′ at a point A. The same point on the top of the hill subtends an angle of 12° 30′ at point B, which is in direct line joining point A and top of the hill. Distance AB was measured and found to be 1600 m. Determine the elevation of the top of the hill and its horizontal distance from point A, given elevation of point A is 430.650 m.

ANSWERS

1. 1555.597 m

2. 2623.524 m

3. 110.02 m

4. 300.253 m, 313.511 m, 120.84 m

5. 424.748 m

6. 200.606 ; 49.05 m

7. 1092.11 m.

15

Simple Curves

15.1. INTRODUCTION

It is neither practicable nor feasible to have straight railways or highways in a country. Their alignments require some changes in direction due to the nature of the terrain, cultural features or other unavoidable reasons. Such changes in direction cannot be at sharp corners but have to be gradual which necessitates the introduction of curves in between the straights. To explain this, in detail, the following example may be considered.

Suppose A and B are two towns which are to be connected by a new rail/ road project. During the reconnaissance survey, it is noticed that there is marshy land in between them. (Fig. 15.1)

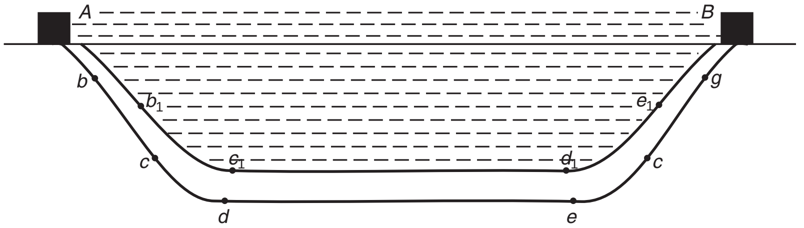


Fig. 15.1. Two alignments of a proposed road.

There may be a number of alignments that may be followed to avoid the marshy land. Here only two alternative alignments are considered just to give an idea why curves are introduced.

First Alternative Alignment. It may be followed as shown by the path *A, b, c, d, e, f, g, B* where *Ab, cd, ef* and *gB* are circular arcs of equal or different radii and *bc, de* and *fg* are straight portions.

Second Alternative Alignment. It may be followed as shown by A, b_1, c_1, d_1, e_1, B. In this alignment, the curve Ab_1 deflects to right, $b_1\,c_1$ deflects to left. After straight $c_1\,d_1$ the curve. $d_1\,e_1$ deflects to the left and finally the curve $e_1\,B$ deflects to right. In second alignment the straights between the curves $A\,b_1$, b_1c_1 and d_1e_1, e_1B have been eliminated.

First alternative alignment is most commonly employed where high speed traffic is expected. The use of a reverse curve is generally avoided for the following reasons :

1. Sudden change of direction is uncomfortable to the passengers.
2. There is no opportunity to raise the outer edge of the road or railway track as required for compensating the centrifugal force.
3. The change of superelevation from one curve to another is sudden near the point of reverse curvature.

Sometimes replacing a simple circular curve by a reverse curve or two simple curves of opposite curvature with a straight in between them, may be adopted to change the direction of the forward straight through 180°. The best example illustrating this on Indian Railways is the old and new alignments at Ghaziabad.

The main line from Delhi (A) to Moradabad (C) connecting Ghaziabad (B) is shown by ABC. The old alignment of the rail track from Meerut was as shown by the dotted line.

When a train followed the old track from Meerut, the face of the engine at Ghaziabad used to be towards Moradabad as can be seen from Fig. 15.2. The face of the engine had to be changed by a *turn table* and the engine was then attached on the other end of the train. In this process, considerable time used to be involved. To avoid this delay, a reverse curve with a straight cd was introduced and a new alignment was followed. With this alignment the face of the engine, when it reaches Ghaziabad, remains towards Delhi.

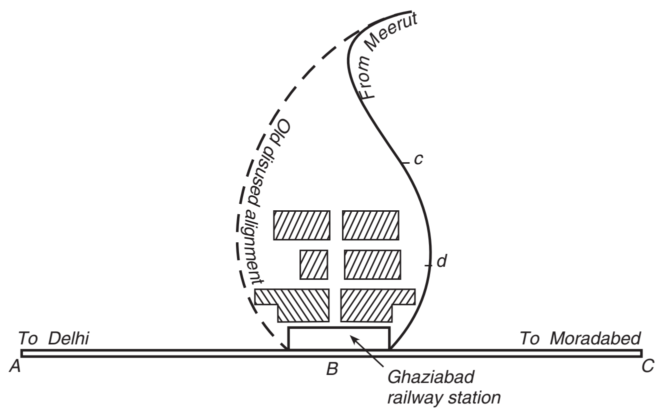


Fig. 15.2. Two alignments at Ghaziabad.

15.2. CURVES

A regular curved path followed by a railway or highway alignment is called a *curve*. A curve may be either circular, parabolic or spiral and is always tangential to the two straight directions at its ends. The curves may be further classified as :

(i) Simple (ii) Compound (iii) Reverse (iv) Transition.

1. Simple curve. The curve which is a single arc of a circle, is known as *simple curve* or a *simple circular curve*. It is tangential to both the straights AT_1 and CT_2. (Fig. 15.3).

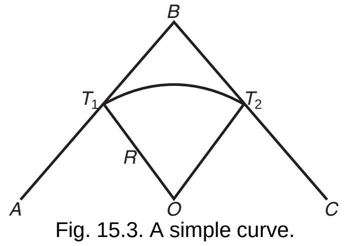


Fig. 15.3. A simple curve.

2. Compound curve (Fig. 15.4). A curve which consists of two or more arcs of different circles with different radii having their centres on the same side of the common tangent in succession, each bending in the same direction, is known as a *compound curve*.

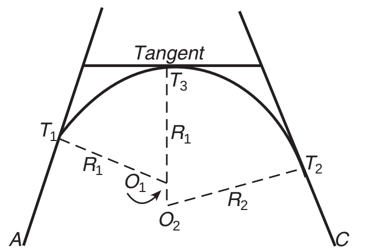


Fig. 15.4. A compound curve.

3. Reverse curve (Fig. 15.5). A curve which consists of two opposite circular arcs of same or different radii, is known as a *reverse curve*. In such curves the centres of the arcs are on the opposite sides of the curve. The two arcs turn in opposite directions with a common tangent at the junction of the two arcs.

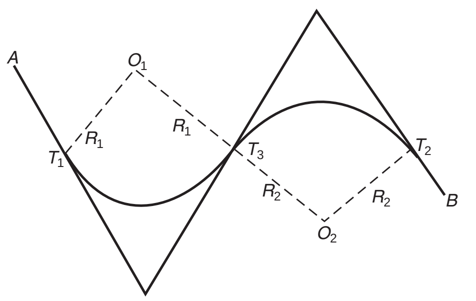


Fig. 15.5. A reverse curve.

4. Transition curve. A curve of varying radius introduced between a straight and a circular curve, is called a *transition curve*.

15.3. ELEMENTS OF A CURVE

1. **Back tangent**. The tangent T_1I at T_1, the point of commencement of the curve, is called '*back tangent*'.
2. **Forward tangent.** The tangent $I\,T_2$ at T_2, the end point of the curve, is called '*forward tangent'*.
3. **Point of intersection.** The point I where back tangent when produced forward and the forward tangent when produced backward meet, is called the *point of intersection.*
4. **Angle of Intersection.** The angle between the back tangent IT_1 and the forward tangent $I\,T_2$, is called the *angle of intersection* of the curve.
5. **Angle of Deflection.** The angle through which forward tangent deflects, is called *angle of deflection* of the curve. It may be either to the right or to the left of the back tangent.
6. **Point of commencement.** The point T_1 where the curve originates from the back tangent, is called the *point of commencement* of the curve. It is also sometimes known as *point of the curve*.
7. **Point of tangency.** The point T_2 where the curve joins the forward tangent, is called *point of tangency*.
8. **Deflection angle to any point on the curve.** The angle between the back tangent and the chord joining the point of commencement to that point on the curve, is called *deflection angle of the point*. In Fig 15.6 the deflection angle to the point A is $I\,T_1A$ which is generally denoted by δ.
9. **Tangent distances.** The distance between the point of intersection and point of commencement of the curve, or the distance between the point of intersection and point of tangency, are called the *tangent distances*.
10. **Length of the curve.** The total length of the curve from the point of commencement to the point of tangency, is called *length of the curve*.

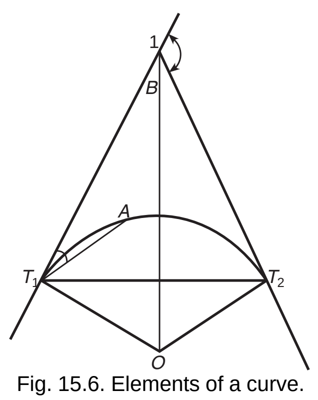


Fig. 15.6. Elements of a curve.

11. **Long chord.** The chord joining the point of the commencement and point of tangency, is called *long chord*.
12. **Mid-ordinate.** The ordinate joining the mid point of the curve and long chord, is called *Mid-ordinate*.
13. **Normal chord.** A chord between two successive regular pegs on the curve, is called a *normal chord*.
14. **Sub-chord.** When a chord is shorter than the normal chord, it is called a *sub-chord'*. These sub-chords generally occur at the beginning and at the end of the curve.

15.4. GEOMETRICS OF A CIRCLE

To understand the geometrics of curves, the important properties of a circle are discussed with reference to Fig. 15.7.

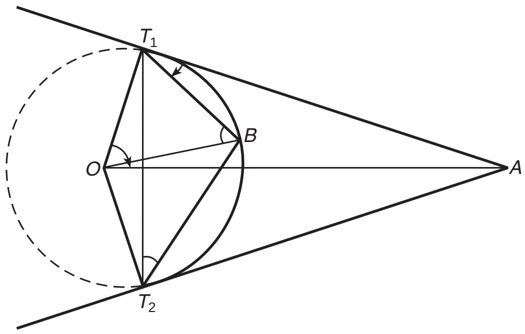


Fig. 15.7. Geometric of circle.

1. Length of tangents $AT_1 = AT_2 = R \tan \theta$; where θ is the angle $T_1\,O\,A$ and R is the radius of the circle.
2. Long chord $T_1T_2 = 2\,R \sin \theta$.
3. The angle subtended by any chord at the centre of the circle is twice the angle between the chord and tangent.

i.e. $\angle AT_1B = \frac{1}{2} \angle T_1OB$

Proof :

Let $\angle AT_1B = \delta$,

then $\angle BT_1O = 90°o - \delta = \angle T_1BO$

$\therefore$ Angle T_1OB subtended by the chord T_1B at the centre O

$$= 180° - 2\,(90° - \delta) = 2\delta$$

or $\angle AT_1B = \frac{1}{2}\, T_1OB.$

4. The angle subtended by a chord at any point on the circumference is equal to the angle between the chord and the tangent, *i.e.*

$$\angle AT_1B = \angle T_1T_2B$$

Proof : $\angle BT_2T_1 = \frac{1}{2} \angle BOT_1 = \angle BT_1A$ from property No. 3.

15.5. DEGREE OF CURVE

Degree of curve may be defined either with respect to a fixed length of an arc of the curve or with respect to a fixed length of a normal chord of the curve.

Fixed length of an arc. The degree of curve may be defined as the central angle of the curve that is subtended by an arc of 30 metres (or 100 ft). This definition is generally adopted for railway curves.

Fixed length of a chord. The degree of a curve may be defined as fixed central angle of the curve that is subtended by a chord of 30 metres (or 100 ft) length. This definition is generally adopted for the road curves.

Derivation of the formula. Let D^o be the angle subtended by an arc of 30 m length of a circle whose radius is R.

The total circumference of the circle $= 2\pi R$

$\therefore$ 30 m arc of the circumference makes an angle

$$D^\circ = \frac{360^\circ}{2\pi R} \times 30$$

$$= \frac{10800}{2\pi R} \text{ degrees}$$

or

$$\frac{1718.9}{R} = D^\circ \quad \text{(By definition)} \qquad ...(15.1)$$

15.6. RELATIONSHIP BETWEEN THE RADIUS AND DEGREE OF A CURVE

1. **Based on fixed length of an arc**

We know that $\frac{1718.9}{R} = D^\circ$

or

$$R = \frac{1718.9}{D^\circ} \qquad ...(15.2)$$

For a 1° curve, the radius

$$R = 1718.9 \text{ m} \qquad ...(15.2a)$$

2. **Based on the fixed length of chord**

Let PQ be a chord of length 30 m. D° is the angle subtended by the chord at the centre O. (Fig. 15.8),

From ΔOPM, we get

$$\sin\frac{1}{2}D = \frac{PM}{OP} = \frac{15}{R}$$

or
$$R = \frac{15}{\sin\frac{1}{2}D°o} = \frac{15}{\frac{1}{2}D°}$$

$$= \frac{15}{\frac{D}{2} \times \frac{\pi}{180°}}$$

$$= \frac{15 \times 360}{D \times \pi}$$

$$= \frac{5400}{D°\pi}$$

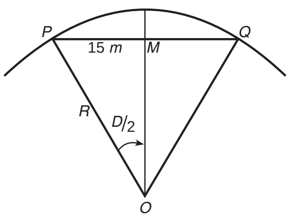


Fig. 15.8. Degree of a curve.

or
$$R = \frac{1718.9}{D°} \quad \text{...(15.3)}$$

15.7. CALCULATION OF VARIOUS ELEMENTS OF A SIMPLE CURVE

Given Data :

1. Chainage of the point of intersection.
2. Either the angle of intersection, I or angle of deflection Δ.
3. The radius of the curve.

To find :

1. The length of back tangent = tangent of forward tangent

$$= R \tan \frac{\Delta}{2} \quad \text{...(15.4)}$$

2. The length of curve $= \frac{2\pi R \Delta°}{360°}$

$$= \frac{\pi R \Delta°}{180°} \quad \text{...(15.5)}$$

3. Chainage at the point of commencement T_1 = Chainage at point of intersection--length of the back tangent ...(15.6)

4. Chainage at the point of tangency T_2 = Chainage at the point of commencement + length of the curve. ...(15.7)

15.8. SETTING OUT A SIMPLE CIRCULAR CURVE

A simple circular curve may be set out on the ground be any one of the following methods.

1. Offsets from the tangents
2. Offsets from the long chord.
3. Successive bisection of the chords.

4. Offsets from chords produced.
5. Deflection angles from the point of commencement and normal chords.
6. Deflection angles from the point of commencement and point of tangency, using two theodolites.

Methods 1 to 4 are known as linear methods whereas the other two methods, are known as angular methods.

1. Offsets from the Tangents

Limitation of the Method. This method can be used conveniently if the deflection angle and radius of the curve are comparatively small.

The offsets from the tangents may be either, perpendicular or radial.

1. **Perpendicular Offsets.** (Fig. 15.9).

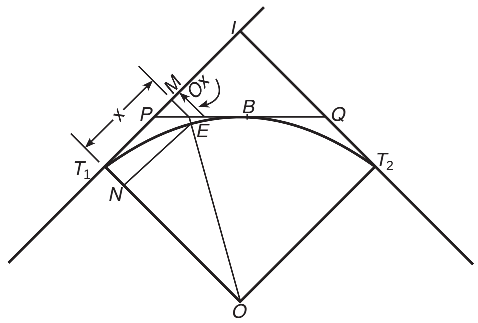


Fig. 15.9. Perpendicular Offsets.

Let any point M on the back tangent of a curve of radius R be at a distance of x from T_1. Length of the perpendicular offset ME from tangent T_1, I to the curve be Ox.

Construction. Drop EN perpendicular to OT_1.

Now $$OE^2 = NE^2 + NO^2$$

$$R^2 = x^2 + (R - Ox)^2$$

$$(R - Ox)^2 = R^2 - x^2$$

$$R - Ox = \sqrt{R^2 - x^2}$$

$$Ox = R - \sqrt{R^2 - x^2} \qquad \text{(Exact)} \qquad ...(15.8)$$

$$Ox = R - R\left[1 - \left(\frac{x}{R}\right)^2\right]^{\frac{1}{2}}$$

$$= R - R\left(1 - \frac{x^2}{2R^2} - \frac{x^4}{8R^4}\right) \text{(Expanding binomially)}$$

$$= \frac{Rx^2}{2R^2} - \frac{Rx^4}{8R^4}$$

or $$Ox = \frac{x^2}{2R} \text{ (Approx.)} \qquad ...(15.8a)$$

ignoring higher powers of x.

Note : The following points may be noted.

(i) One-half of the curve may be conveniently set out from the back tangent $T_1 I$. The other half of the curve is to be set out from the forward tangent $T_2 I$.

(ii) If the curve is long, the offsets will also be long. In such cases it is advisable to set the middle third of the curve by calculating offsets from a tangent at the mid point B of the curve.

Field Operations. Before setting out a curve of radius say 250 m, a table of offsets corresponding to a number of points on the tangents may be made as shown in Table 15.1.

Table 15.1

S. N.	*X (metres)*	*Ox (metres)*
1	10	0.20
2	20	0.80
3	30	1.80
4	40	3.20
5	50	5.00

Procedure. From the point of commencement T_1, measure distances x_1, x_2, x_3, etc., along the tangent $T_1 I$. Erect perpendiculars equal in lengths of the offsets corresponding to distances x_1, x_2, x_3, etc., with the help of an optical square.

As the offsets of the points, equidistant from point of commencement T_1 and point of tangency T_2 are equal, the table 15.1 may also be used for offsets from the forward tangent.

2. Radial offsets. (Fig. 15.10).

Let M be any point on the tangent at a distance x from the point of commencement T_1. Ox be the radial offset from M to the curve. R be the radius of the curve with O as its centre.

Now, $$(R + Ox)^2 = R^2 + x^2$$

$$R + Ox = \sqrt{R^2 + x^2}$$

or $$Ox = \sqrt{R^2 + x^2} - R \quad \text{...... (Exact)} \qquad ...(15.9)$$

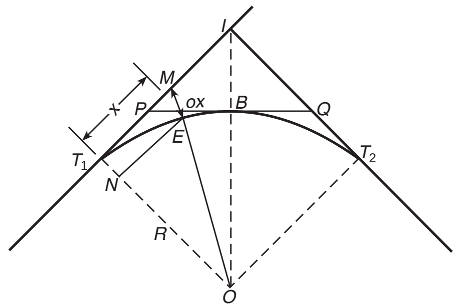


Fig. 15.10. Radial offsets.

$$= R\left[1 + \left(\frac{x}{R}\right)^2\right]^{\frac{1}{2}} - R$$

$$= R\left[1 + \frac{x^2}{2R^2} + \frac{x^4}{8R^4} + \ldots\right] - R$$

or $$Ox = \frac{x^2}{2R} \quad \text{(Approximate).} \qquad \ldots(15.10)$$

ignoring higher powers of x.

Note. The following points may be noted.

(*i*) The lengths of the offsets, whether perpendicular or radial, are the same when approximate formulae are accepted.

(*ii*) Half the curve may be set out from the back tangent and the other half from the forward tangent.

(*iii*) As the distance x increases, the offset length also increases.

(*iv*) For the mid-point of the curve, the length of the offset is largest.

(*v*) When the length of the curve is large, the middle third of the curve may be set out from the tangent at the mid-point of the curve.

Field Operations : Following steps are followed.

1. Fix ranging rods at T_1, I, T_2 and O.
2. Measure a distance x along $T_1 I$ and fix a point M.
3. From M measure a distance equal to the calculated offset length along the line joining the point M and the centre of the curve O.
4. Similarly, locate other points on the first half of the curve.
5. The other half of the curve is similarly set out from the forward tangent $T_2 I$.
6. To avoid large offsets, the setting out of the middle third of the curve, may be done from the tangent PQ, at the mid-point B of the curve.

2. Offsets from the Long Chord

Let T_1 and T_2 be the points of commencement and tangency of the curve. Radius of the curve is R having O as centre. (Fig. 15.11)

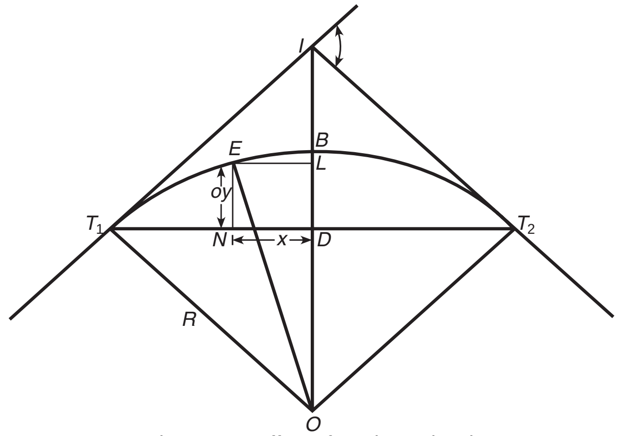


Fig. 15.11. Offsets from long chord.

Construction. Join $T_1 T_2$. Divide $T_1 T_2$ at D. Join ID which intersects the curve at B. The maximum length of the offset from the long chord $T_1 T_2$ is BD, whose value may be calculated as under.

From $\Delta\, T_1 DO$,

$$T_1 O^2 = T_1 D^2 + OD^2$$

or
$$R^2 = (L/2)^2 + (OB - BD)^2$$

where L is the length of the long chord.

$$R^2 = (L/2)^2 + (R - Ox)^2$$

or
$$(R - Ox)^2 = R^2 - (L/2)^2$$

$$R - Ox = \sqrt{(R^2 - (L/2)^2}$$

or
$$Ox = R - \sqrt{R^2 - (L/2)^2} \quad \text{......(Exact)} \qquad \text{...(15.11)}$$

To find the ordinate Oy at any point N at a distance x from mid point D, drop EL perpendicular to BD.

Now,
$$Oy = EN = LD = LO - OD$$

or
$$Oy = LO - (R - Ox) \qquad \text{...(15.12)}$$

From $\Delta\, ELO$.

$$LO = \sqrt{OE^2 - EL^2} = \sqrt{R^2 - x^2}$$

Substituting the value of LO in Eqn. (15.12).

$$Oy = \sqrt{R^2 - x^2} - (R - Ox) \qquad \text{...(15.13)}$$

where Ox is the ordinate at the mid-point of the long chord.

Field Operations. To set out a circular curve with offsets from the long chord, the following steps are followed.

1. Erect ranging rods at T_1, D and T_2.
2. Divide the long chord $T_1 T_2$ in equal parts of suitable length.
3. Calculate the lengths of the offsets corresponding to distances from the mid-point of the chord.
4. Erect perpendiculars with the help of an optical square and measure the calculated offset lengths.

3. Successive Bisection of Chords

Let $T_1 T_2$ be the long chord of a curve whose angle of deflection is Δ. (Fig. 15.12).

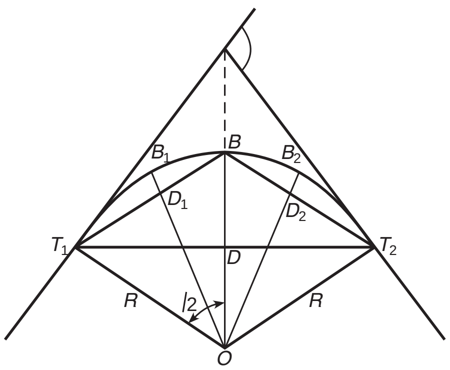


Fig. 15.12. Successive bisection of chords.

Construction : Divide $T_1 T_2$ at D. Join OD and produce it to intersect the curve at B.

Now $\quad BD = OB - OD$

$= R - R \cos \Delta/2$

or $\quad BD = R\,(1 - \cos \Delta/2)$. ...(15.14.)

To obtain the position of the point B, erect a perpendicular offset equal to $R(1 - \cos \Delta/2)$ at D. Now, consider $T_1 B$ and $T_2 B$ independent portions of the curve having $T_1 B$ and $T_2 B$ as long chords. Divide $T_1 B$ and $T_2 B$ at D_1 and D_2 respectively.

It can be proved that offsets $B_1 D_1$ and $B_2 D_2$ are each equal to $R(1 - \cos \Delta/4)$ where angles T_1OD_1 and T_2OD_2 are each equal to $\Delta/4$.

To locate B_1 and B_2, erect perpendicular offsets equal to $R(1 - \cos \Delta/4)$ at D_1 and D_2.

By further successive bisection of the chords T_1B_1, B_1B, BB_2 and B_2T_2 we may obtain the locations of other points on the curve.

Field Operations : To set out a curve by successive bisection of chords, the following steps may be followed :

1. Locate the positions of T_1 and T_2.
2. Measure $T_1\,T_2$ and find its mid-point D.
3. Set out the perpendicular offset DB with an optical square equal to $R(1 - \cos \Delta/2)$.
4. Measure chords $T_1\,B$ and $T_2\,B$ and find their mid-points D_1 and D_2 respectively.
5. Set out the perpendicular offsets $D_1\,B_1$ and $D_2\,B_2$, each equal to $R(1 - \cos \Delta/4)$ with an optical square.
6. The process may be continued till sufficient number of points on the curve are fixed.

Note. The following points may be noted.

(*i*) Accuracy of the work depends upon the number of bisections of chords.

(*ii*) The length of $T_1\,B = T_2B = \sqrt{T_1\,D^2 + BD^2}$.

4. Offsets from Chords Produced

This method is commonly employed when a theodolite is not available and it is necessary to set out a curve only with a chain or a tape. The curve is divided into a number of chords normally 20 or 30 m in length. As continuous chainage is required along the curve, two sub-chords generally occur, one at the beginning and the other at the end of the curve. The length of the first subchord is equal to the difference of the chainage of the full number of chains *just after* the commencement and the chainage of the point of commencement. Similarly, the length of the last sub-chord is equal to the chainage of full number of chains *just before* the point of tangency and the chainage of point of tangency. Offsets from chords produced may be computed with the help of the formula derived under :

Derivation of the formulae.

Let AI be the back tangent, $T_1a = C_1$ be the first sub-chord and the angle IT_1a between the tangent $T_1\,I$ and the sub-chord T_1a be δ. (Fig. 15.13).

Let R be the radius of the curve having its centre at O.

$$OT_1 = Oa = \text{radius } R$$

$$\angle\, OaT_1 = \angle\, OT_1a = 90° \;\angle\, \delta$$

$$\therefore \qquad \angle\, T_1Oa = 180° - 2\,\angle\, OT_1a$$

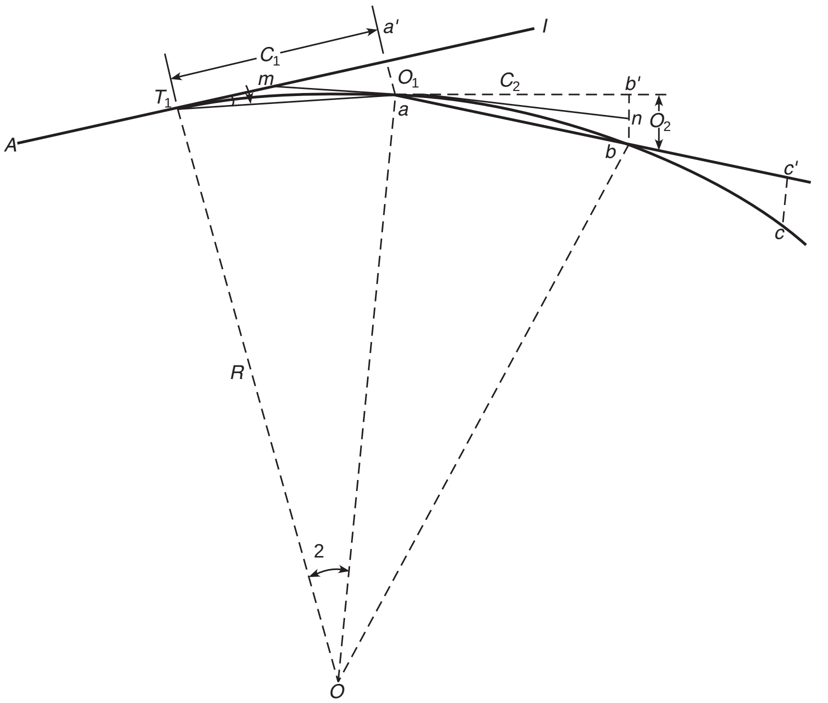


Fig. 15.13. Offsets from chords produced.

$$= 180° - 2(90° - \delta) = 2\,\delta.$$

The chord T_1a being very nearly equal to arc T_1a

$$T_1a = R\ .\ 2\,\delta$$

or $$\delta = \frac{T_1a}{2R} \quad ...(15.15)$$

Similarly, the chord $aa'$is very nearly equal to the arc aa',

or $$aa' = T_1a\ .\ \delta \quad ...(15.16)$$

Substituting the value of δ from Eq. (15.15) in Eq. (15.16), we get

$$aa' = \frac{(T_1a)\,(T_1a)}{2R} = \frac{(T_{1a})^2}{2R}$$

or offset, $$O_1 = \frac{C_1^2}{2R} \quad ...(15.17)$$

where O_1 is the first offset length for the first sub-chord of length C_1. With a as centre, draw an arc of radius $ab = C_2$ to cut the line T_1a produced at b'. Draw a tangent mn at a, cutting $T_1\,I$ at m and bb' at n.

Now $mT_1 = ma$, both being tangents.

$\therefore \qquad \angle mT_1a = \angle maT_1 = \angle b'an.$

$\Delta s\ a'\ T_{1a}$ and $b'an$ are both nearly isoscles triangles and hence similar.

$$\therefore \qquad \frac{b'n}{ab'} = \frac{aa'}{T_1a'}$$

or
$$b'n = \frac{ab'\,.\,aa'}{T_{1a'}} = \frac{C_2\,O_1}{C_1}$$

Substituting the value of O_1 from Eq. (15.17), we get

$$b'n = \frac{C_2\,C_1^2}{C_1.\,2R} = \frac{C_2\,C_1}{2R}$$

But, $\quad bn = \dfrac{C_2^2}{2R}$ as obtained in Eq. (15.17).

$\therefore$ The second offset, $\quad O_2 = bn + nb'\ O_2 = \dfrac{C_2^2}{2R} + \dfrac{C_2\,C_1}{2R}$

or
$$O_2 = \frac{C_2\,(C_2 + C_1)}{2R} \qquad ...(15.18)$$

Similarly, we may get the value of third offset O_3 *i.e.*

$$O_3 = \frac{C_3\,(C_3 + C_2)}{2R} \qquad ...(15.19)$$

All the chords, excepting the sub-chords are generally equal, *i.e.* $C_2 = C_3 = C_4 = ... = C_{n-1}$

$$\therefore \qquad O_2 \text{ to}_{(n-1)} = \frac{C^2}{R}$$

where C is the length of normal chords.

The offset for the last sub-chord C_n.

$$O_n = \frac{C_n\,(C_n + C_{n-1})}{2R} \qquad ...(15.20)$$

Field operations : The field work is carried out in the following steps :

1. Locate the point of the commencement T_1 and the point of tangency T_2.
2. Calculate the chainage of T_1 by subtracting the back tangent length from the chainage of the point of intersection I.
3. Calculate the length of first sub-chord.
4. Calculate the length of the curve and find the length of last sub-chord.

5. Calculate the lengths of the offsets for the sub-chords and normal chords.
6. Mark the point a' along the tangent $T_1 I$ such that $T_1 a'$ is equal to the first sub-chord.
7. With the zero end of the chain (or tape) at T_1 and radius equal to T_1a, draw an arc $a' a$. Cut off $a'a = O_1$. The first point a on the curve is thus fixed.
8. Pull the chain forward in the direction of $T_1 a$ produced. Measure ab' equal to the second chord C_2 (*i.e.* a normal chord).
9. Hold the zero end of the chain at a, swing an arc of a radius equal to the length of the chord C_2. Cut off $b'b$ equal to second offset O_2. Point b is thus fixed on the curve.
10. Continue the process until the point of tangency T_2 is located which should agree with the position already fixed by measuring IT_2 equal to the forward tangent length.

Note. The following points may be noted :

(*i*) If the error in the location of the point of tangency is more than 2 m, the whole work is revised.

(*ii*) If the error is less than 2 m, all the curve pegs are moved *sideways by an amount proportional to the square of their distances* from the point of commencement T_1 so that the error is distributed among all the points of the curve.

5. Rankine's Method of Tangential Deflection Angles

The angle between the back tangent and the chord joining the point of commencement of the curve and the next point on the curve, is generally known as *deflection angle of the chord.*

In Fig. 15.14 angles IT_1a, IT_1b, IT_1c, etc. are deflection angles of the

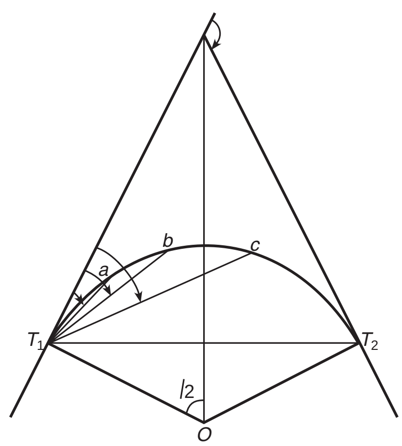


Fig. 15.14. Rankine's tangential angles.

chords T_1a, T_1b, T_1c, etc. These deflection angles are not to be confused with the angle of deflection of the curve. The latter being the total angle through which the forward tangent of the curve deflects with respect to the back tangent.

Derivation of the formula

Assumption. When the radius of the curve is large, the small arc of the circle approximates to its chord, *i.e.* the arc T_1a is very nearly equal to chord T_1a. (Fig. 15.15)

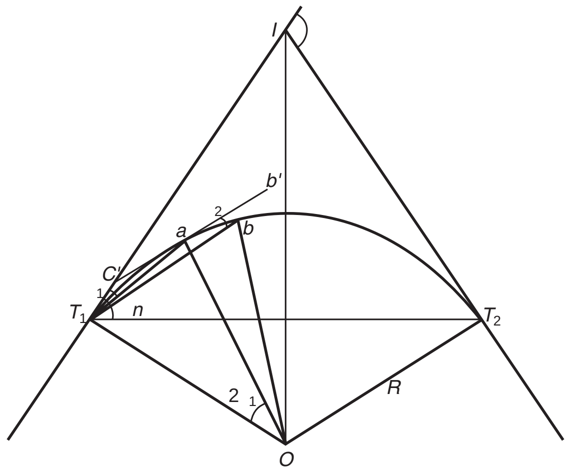


Fig. 15.15. Rankine's method of deflection angles.

Let the deflection angle for the chord T_1a be δ_1

$$\angle OT_1a = 90° - \delta_1 = \angle OaT_1$$

∴ Angle subtended by the arc T_1a at the centre O

$$= 180° - 2\,(90° - \delta_1) = 2\delta_1,$$

Draw a tangent $c'b'$ at a and let angle $b'ab$ be δ_2.

Now, angle aT_1b = angle $b'ab = \delta_2$.

∴ Total deflection angle IT_1b for chord T_1b

$$= \text{Angle } IT_1a + \text{Angle } aT_1b$$

or $\angle IT_1b = \delta_1 + \delta_2$.

Similarly, values of deflection angles for remaining chords may be calculated.

In general practice, the first and last sub-chords are not equal. These are designated as C_1 and C_2 whereas normal chords are designated by C.

The length of the arc $C_1 = \dfrac{2\pi R \times 2\delta_1}{360^\circ}$

or $$\delta_1 = \frac{360^\circ}{4\pi R}.C_1 \text{ degrees} = \frac{90^\circ \times C_1}{\pi R} \text{ degrees}$$

$$= \frac{90 \times C_1}{\pi \times R} \times 60 \text{ minutes}$$

or $$\delta_1 = 1718.9\,\frac{C_1}{R} \text{ minutes} \qquad ...(15.21)$$

And, for normal chords, $\delta = 1718.9\,\dfrac{C}{R}$ minutes.

For subchord of length C_2, $\delta_2 = 1718.9\,\dfrac{C_2}{R}$ minutes.

If $\delta_1, \delta_2, \delta_3, \delta_4, ...\delta_n$ are the angles for the first subchord, normal chords and last sub chord with their tangents, then the total deflection angles for various chords with the back tangent are :

$$\Delta_1 = \delta_1$$

$$\Delta_2 = \delta_1 + \delta_2$$

$$\Delta_3 = \delta_1 + \delta_2 + \delta_3$$

$$\cdots \quad \cdots \quad \cdots$$

$$\cdots \quad \cdots \quad \cdots$$

$$\cdots \quad \cdots \quad \cdots$$

$$\Delta_n = \delta_1 + \delta_2 + \delta_3 + ... + \delta_n.$$

For a check on calculations, angle IT_1T_2 must be equal to $\Delta_n = \dfrac{1}{2}\Delta$.

Field operations : Rankine's method of tangential (or deflection) angles involves the use of a theodolite and a chain. This method is most commonly adopted for setting out the railway and metro curves. The following steps are involved :

1. Set up the theodolite at the point of commencement of the curve, the location of which is already fixed by chaining the length of back tangent from the point of intersection.
2. Let the chainage of the point of commencement finally occurs at M chains and n links.
3. Calculate the length of the first sub-chord to have a continuous chainage *i.e.* length of the first sub-chord = chainage of the next full chain just after the point of commencement -- chainage of the point of commencement.

4. Calculate deflection angle for the first sub-chord, normal chords and the last sub-chord. Tabulate the values of deflection angles.
5. Centre the theodolite over the point of commencement and level it carefully.
6. Sight the point of intersection I and make the vernier of the vernier theodolite to read zero degree, zero minute and zero second.
7. Unclamp the upper plate of the theodolite and set the vernier to read the deflection angle Δ_1 for the first sub-chord C_1. Now, the line of sight of the telescope is along the first sub-chord.
8. With T_1 as centre and C_1 as radius, swing the chain and fix a chain pin in the line of sight.
9. Set the vernier to read the value of the deflection angle Δ_2 for the first normal chord, so that the telescope points in the direction of the other end of the first normal chord.
10. Keeping one end of the chain at the point a previously fixed on the curve, swing the chain until other end of the chain falls in the line of sight of the telescope. Pull the chain straight and fix a chain pin to locate the next point b on the curve.
11. This procedure may be continued till the point of tangency is located.

In case, there is no error in calculations, observations and chaining the distances, the last point should normally coincide with the point of tangency.

6. Two Theodolite Method.

Let T_1 and T_2 be the point of commencement and point of tangency respectively, I is the point of intersection and O is the centre of the circular curve. (Fig. 15.16).

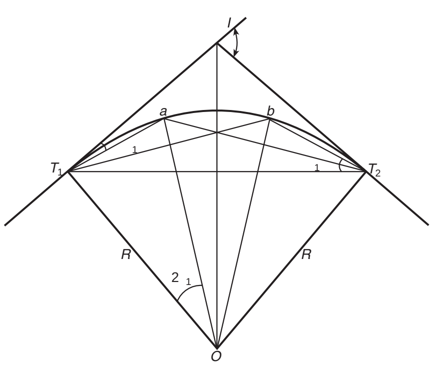


Fig. 15.16. Two theodolite method.

Let $T_1 a$ be the first sub-chord of length C_1 and δ_1 be its deflection angle.

From the properties of a circle, we know

$$\angle IT_1a = \frac{1}{2} \angle T_1Oa$$

But $$\angle T_1T_2a = \frac{1}{2} \angle T_1Oa$$

$\therefore$ $$\angle IT_1a = \angle T_1T_2a = \delta_1.$$

Similarly, it can be shown that

$$\angle IT_1b = \angle T_1T_2\, b \text{ and so on.}$$

Field operations. For setting out a curve with two theodolites, the following steps are involved :

1. Prepare a table of deflection angles for the first sub-chord, normal chords and the last sub-chord.
2. Set up one theodolite at T_1 and other theodolite at T_2.
3. Theodolite at T_1 should be directed towards the point I. Theodolite at point T_2 should be directed towards T_1.
4. The verniers of both the theodolites should read zero.
5. Set the first deflection angle δ_1 on both theodolites so that their telescopes are in the direction of T_1a and T_2a respectively.
6. Ask the attendent to move in the line of sight of one theodolite with a ranging rod. The observer of the other theodolite keeps on viewing through the telescope till the ranging rod is intersected by the vertical wire of his theodolite. The position of the ranging rod is the required location on the curve.
7. Set the second value of the deflection angle on both the theodolites and repeat the steps (6) to get the location b on the curve.
8. Continue the process for obtaining the locations of other points in a similar manner.

Note : The following points may be noted :

(*i*) When the curve is large and terrain is undulating, an experienced attendent can find the ranging rod position without spending much time.

(*ii*) As chaining is completely disposed off, this method may be usefully employed in undulating ground.

15.9. DIFFICULTIES IN RANGING A SIMPLE CURVE

While setting out simple curves, the following difficulties may be encountered :

1. Point of intersection is inaccessible.

2. Point of commencement is inaccessible.
3. Point of tangency is inaccessible.
4. Both the points of commencement and tangency are inaccessible.
5. The complete curve cannot be set out from the point of commencement.
6. An obstacle intervenes the curve.

1. Point of intersection Inaccessible.

When the curve is located near lakes, large rivers or wooded areas, point of intersection may become inaccessible. (Fig. 15.17).

Let T_1I and T_2I be two straights intersecting at I, the point of intersection of two straights. T_1 and T_2 are the points of commencement and tangency of curve. Let us assume that point of intersection I is falling in the middle of a perennial river.

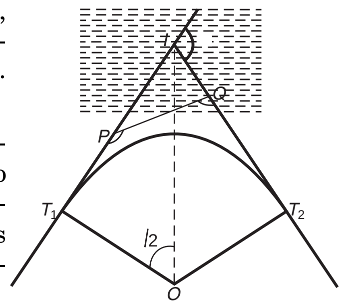


Fig. 15.17. Point of intersection inaccessible.

Procedure : The following steps are followed:

1. Select two intervisible points P and Q on the tangents T_1I and T_2I respectively. Measure the distance PQ.
2. Measure angle T_1PQ and PQT_2 by setting up a theodolite at P and Q respectively. Let the angles be α and β respectively.
3. Calculate the angles of intersection and deflection as under :

$$\angle IPQ = 180° - \angle T_1PQ = 180° - \alpha$$

$$\angle IQP = 180° - \angle PQT_2 = 180° - \beta.$$

$\therefore$ Angle of intersection

$$= 180° - [(180° - \alpha) + (180° - \beta)]$$

$$= \alpha + \beta - 180°$$

Angle of deflection $\Delta = 180° - (\alpha + \beta - 180°)$

$$= 360° - (\alpha + \beta)$$

4. By applying the sine formula to the triangle PQI, calculate PI and QI.

i.e.
$$\frac{PI}{\sin PQI} = \frac{QI}{\sin IPQ} = \frac{PQ}{\sin PIQ}$$

or
$$IP = \frac{PQ \sin PQI}{\sin PIQ}$$

and $$QI = \frac{PQ \sin IPQ}{\sin PIQ}$$

5. Calculate the tangent lengths

$$= R \tan \frac{\Delta}{2}$$

6. Calculate distances $T_1 P$ and $T_2 Q$

i.e. $$T_1 P = \text{Tangent length} - PI$$

and $$T_2 Q = \text{Tangent length} - QI.$$

7. Locate the point of commencement by measuring a distance equal to PT_1 from P along the back tangent.
8. Locate the point of tangency by measuring a distance equal to QT_2 from Q along the forward tangent.

Note. In case, two intervisible points P and Q are not available, a small traverse may be run between P and Q and the length and bearing of PQ may be calculated from the independent co-ordinates of P and Q. Knowing the bearings of $T_1 I$, PQ and $Q T_2$, the angles $T_1 PQ$ and PQT_2 may be computed. For detailed procedure refer to 'Chapter 11 Practical Problems' of Author's Text Book of Advanced Surveying.

2. Point of Commencement is Inaccessible

Let R be radius of the curve, Δ angle of deflection of curve and T_1 be the point of commencement which falls in a water body. (Fig. 15.18).

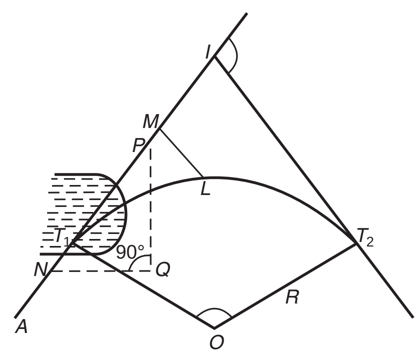


Fig. 15.18. Point of commencement inaccessible.

Procedure : The inaccessibility of the point of commencement may solved as under :

1. Measure a distance IP clear of the obstacle.

$$PT_1 = T_1I - PI$$

2. Select a point Q clear of the obstacle such that angle PQN is a right angle.
3. Measure distances PQ and QN and calculate the distance

$$PN = \sqrt{PQ^2 + QN^2}$$

4. Chainage of N = chainage of $I - (IP + NP)$.
5. Chainage of T_1 = the chainage of $N + NP - PT_1$.
6. Chainage of the T_2 = the chainage of $T_1 + \dfrac{\pi R\Delta}{180°}$
7. Set out the curve from T_2 in the reverse direction.
8. Measure a perpendicular offset LM from the last located point say L, which should be approximately equal to $\dfrac{T_1 M^2}{2R}$ to provide a check on the accuracy of work.

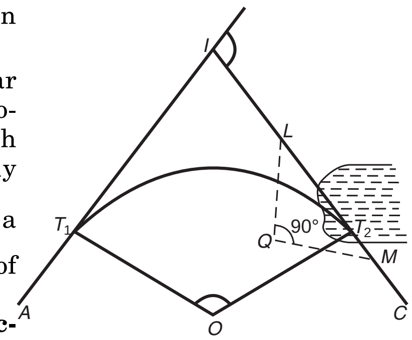


Fig. 15.19. Point of tangency inaccessible.

3. Point of Tangency is Inaccessible

Let the point of the tangency T_2 falls in a lake. (Fig. 15.19).

Procedure : The following steps are followed:

1. Calculate the chainage of the point of commencement T_1.
2. Calculate the chainage of the point of tangency *i.e.,* the chainage of the point of commencement + length of the curve.
3. Select a point L on the forward straight CI, clear of the obstacle.
4. Select another point M on the forward straight on the other side of the obstacle.
5. Calculate the length of LM across the obstacle by any method.
6. Chainage of M = chainage of $T_2 + T_2 M$

$$= \text{chainage of } T_2 + (LM - LT_2).$$
$$= \text{chainage of } T_2 + LM - (IT_2 - IL)$$
$$= \text{chainage of } T_2 + LM + IL - IT_2.$$

From the point M, locate the first full chain on the forward straight and proceed further.

4. Both the Points of Commencement and Tangency are Inaccessible

It is assumed that the point of intersection is either available or its inaccessibility is solved as already explained with (Fig. 15.20).

Procedure : The following steps are followed;

1. Calculate the chainage of T_1 as explained earlier.
2. Select any point P clear of the obstacle towards the point of

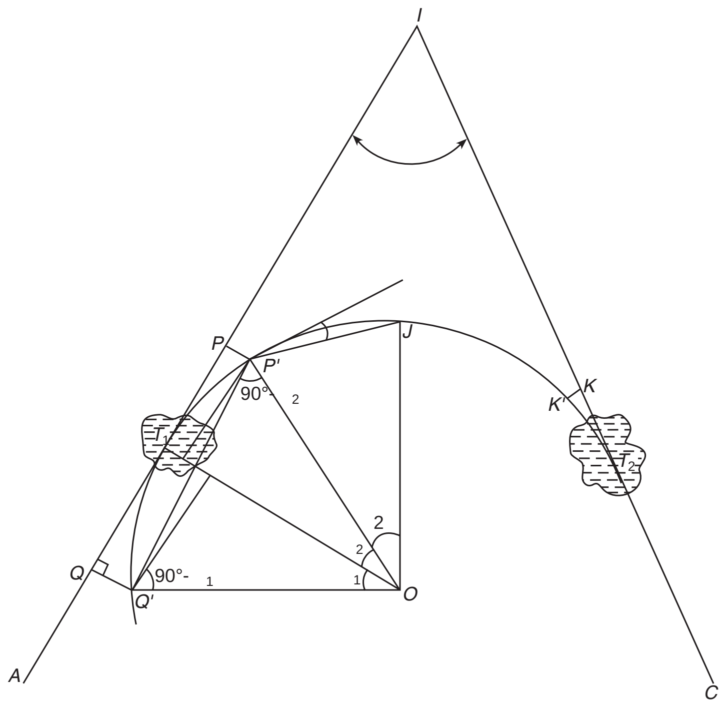


Fig. 15.20. Points of commencement and tangency inaccessible.

intersection and another point Q along the straight line on the other side of the obstacle.

3. Calculate the distances $T_1 P$ and $T_1 Q$.
4. Calculate the perpendicular offsets PP' and QQ'.

Now $$PP' = R - \sqrt{R^2 - T_1 P^2} = R - \sqrt{(R + T_1 P)(R - T_1 P)}$$

and $$QQ' = R - \sqrt{R^2 - T_1 Q^2} = R - \sqrt{(R + T_1 Q)(R + T_1 Q)}$$

5. Calculate the angles α_1 and α_2 subtended by the arc $Q' T_1$ and $T_1 P'$ at the centre of the curve. *i.e.*

$$\alpha_1 = \sin^{-1} \frac{T_1 Q}{R}$$

and $$\alpha_2 = \sin^{-1} \frac{T_1 P}{R}$$

6. Calculate the length of the circular arc $T_1 P'$ *i.e.*

$$\text{arc } T_1 P' = \frac{\pi R \alpha_2}{180°}$$

7. Chainage of P' = Chainage of T_1 + Length of the arc $T_1 P'$.
8. Calculate the length of the first sub-chord with respect to P'.
9. Calculate the Rankine's angles of deflection for the sub-chord

and normal chords from the formula.

10. Locate the line of sight tangential to the curve at P' as under :

Set up the theodolite and centre it accurately over the point P'. Sight Q' and turn the instrument through an angle equal to.

$$180° + \frac{1}{2}(\alpha_1 + \alpha_2) + \delta.$$

Proof :

In $\Delta OQ'P'$, $\angle OP'Q' = \angle OQ'P'$

$$= \frac{180° - (\alpha_1 + \alpha_2)}{2} = 90° - \frac{1}{2}(\alpha_1 + \alpha_2)$$

From the triangle $P'OJ$,

$$\angle JP'O = 90° - \delta$$

where δ is the deflection angle for the chord PJ.

$$\therefore \quad \angle JP'Q' = \angle OP'Q' + \angle JP'O$$

$$= 90° - \frac{1}{2}(\alpha_1 + \alpha_2) + 90° - \delta$$

$$= 180° - \frac{1}{2}(\alpha + \alpha_2) - \delta.$$

$\therefore$ Angle to be set on theodolite

$$= 360° - [180° - \frac{1}{2}(\alpha_1 + \alpha_2) - \delta]$$

$$= 180° + \frac{1}{2}(\alpha_1 + \alpha_2) + \delta \qquad ...(15.22)$$

11. Locate the pegs on the curve till last peg K, clear of the obstacle at the point of tangency is located.
12. Measure the perpendicular offset $K'K$ and compare its value with its computed value, *i.e.*

$$KK' = R - \sqrt{R^2 - T_2K^2} = R - \sqrt{(R + T_2K)(R - T_2K)}$$

13. If the error exceeds the permissible limit, reset the entire curve.

Alternative Method. In case, point of intersection is accessible, the curve may be set out as under :

1. Calculate the chainage of the point of commencement T_1 as explained earlier.
2. Calculate the chainage of the point of tangency T_2 by adding the length of the curve to the chainage of T_1, (Fig. 15.21).
3. Set up the theodolite at the point of intersection I and bisect the angle T_1IT_2.

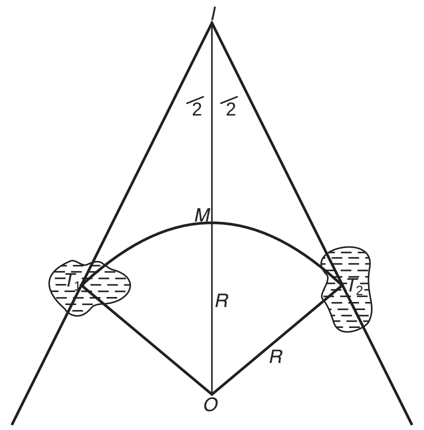


Fig. 15.21. Points of commencement and tangency inaccessible.

4. Locate the apex point M on the curve, by measuring a distance along the bisector, $IM = \sqrt{R^2 + T_1 I^2} - R$.
5. Chainage of M = The chainage of T_1 + half the length of the curve.
6. Set up the theodolite at M and orient it by sighting to I, by the methods discussed earlier.
7. Set out the curve in both directions from M by any one of the methods discussed earlier.

5. Complete curve can not be set out from the point of commencement.

When the length of a curve is large, it may not be possible to peg all station marks. On such curves, the instrument may be required to be set up at intermediate points. The following two cases may arise.

1. Point of commencement is visible from an intermediate point.
2. Point of commencement is not visible from any intermediate point.

Case 1. The point of commencement visible from an intermediate point. There are two methods of setting out the curve to overcome this difficulty.

First Method. (Fig. 15.22). Let C be the last point located from the point of commencement T_1 and Δc be its angle of deflection. For setting out the remaining portion of the curve, proceed as under :

Procedure : The following steps are followed.

1. Set up the theodolite at C.
2. Set the vernier to read 0° on face right and sight T_1 accurately.
3. Transit the telescope so that line of sight is directed along T_1C produced and the face of the theodolite is left.
4. Loosen the upper plate and set the vernier to read the deflec-

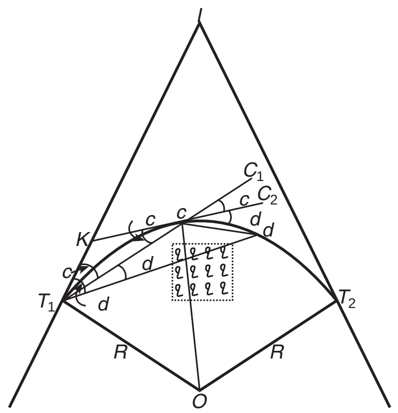


Fig. 15.22. Point of Commencement visible from an intermediate point.

tion angle Δd of the next point d. Locate the position of d.

Proof.

Draw a tangent at C to intersect the back straight at K.

KT_1 and KC being tangents to the circular curve.

$$\angle KT_1 C = \angle KCT_1 = \Delta c = \angle C_1 CC_2.$$

Add the angle δd on both the sides

$$\angle KT_1C + \Delta d = \angle C_1 CC_2 + \Delta d$$

or $$\angle KT_1 d = \angle C_1 Cd$$

Second Method. (Fig. 15.22). In the first method, it is assumed that the instrument is in perfect adjustment. If it is not, the curve can be set out as explained under.

Procedure : The following steps are followed.

1. After locating the last point C, fix a point C_1 in the direction $T_1 C$ produced roughly 200 metres away.
2. Set up the theodolite at C. Clamping both the plates, set the vernier to read zero and sight the point C_1 accurately. The instrument is thus correctly oriented.
3. Loosen the upper plate and set vernier to read deflection and Δd. Locate the peg d.

2. Point of commencement not visible from any intermediate point

Let C be the last point located from point T_1. From point C, the last point located is D. The rest of the curve is to be set out from D. If D and T_1 are not intervisible due to an obstacle, proceed as under (Fig. 15.23).

Procedure : The following steps are followed.

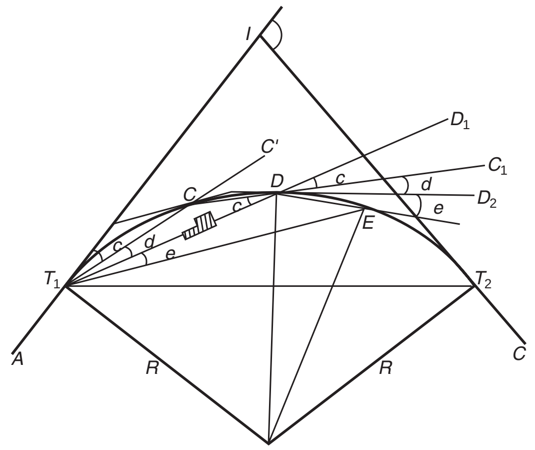


Fig. 15.23. Point of commencement not visible from any intermediate point.

1. Set up the theodolite at D.
2. Set the vernier to read Δc, the deflection angle for peg C and sight C.
3. Transit the telescope to have the line of sight along DC_1.
4. Loosen the upper plate and set the vernier to read the deflection angle of the next point E.

Proof.

Angle $IT_1E = \angle IT_1C + \angle CT_1D + \angle DT_1E$

$$= \Delta c + \Delta d + \Delta e$$

or $\angle IT_1E = \angle D_1DC_1 + \angle C_1DD_2 + \angle D_2DE$

$$= \angle D_1DE.$$

5. The angle D_1DE is correctly now set out by the theodolite.
6. Establish the remaining points from D.

6. An obstacle intervenes on the curve

When an obstacle intervenes the curve, it cannot be chained across. In such cases it is necessary to omit the location of the curve across it until the obstacle is removed. (Fig. 15.24).

Let C be the last point located clear of the obstacle.

Points beyond the obstacle may be located as under :

Procedure : The following steps are followed.

1. Set out the deflection angle of E and ensure that a clear line of sight is obtained.
2. Let T_1E be the line of sight clear of obstacle whose angle of deflection is ΔE.

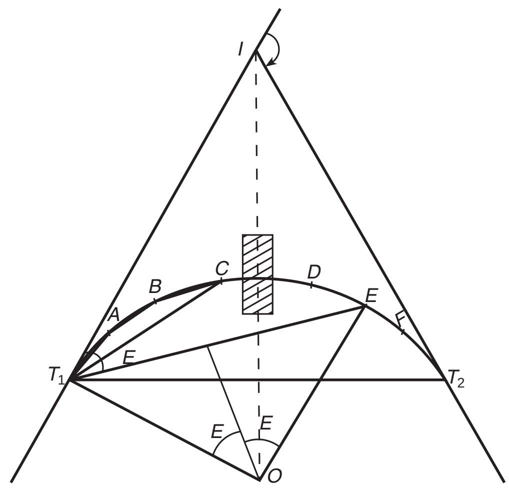


Fig. 15.24. An obstacle on the curve.

3. Calculate the length of the long chord $T_1 E$, *i.e.*
   Long chord $T_1E = 2\,R \sin \Delta E$.
4. Measure the distance T_1E in the line of sight and fix the point E.
5. Proceed till the point of tangency is located.

15.10. TYPICAL FIELD PROBLEMS IN SETTING OUT SIMPLE CURVES

During reconnaissance, it may be decided that the curve should be aligned so as to fulfill some required conditions.

1. Calculation of the radius of a curve tangential to three straight lines.

Let *AB, BC* and *CD* be the given straight lines, α and β be the angles of deflection and distance $BC = d$. (Fig. 15.25).

Let T_1, E and T_2 be the three tangent points.

From arc T_1E, $\quad BE = BT_1 = R \tan \dfrac{\alpha}{2}$ ...(15.23)

From arc ET_2, $\quad CE = CT_2 = R \tan \dfrac{\beta}{2}$ ...(15.24)

Adding Eqns. (15.23) and (15.24), we get

$$d = BE + CE$$

$$= R\left(\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}\right)$$

or
$$R = \frac{d}{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}} \qquad ...(15.25)$$

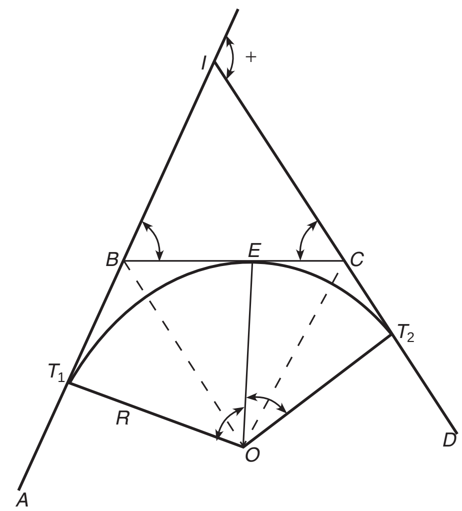


Fig. 15.25. A curve tangential to these straight.

2. Calculation of the radius of a curve passing through a fixed point given:

1. Deflection angle, Δ.
2. Directions of two straights.
3. Fixed point *P* through which the curve must pass.

Required : Radius of the curve and lengths of tangents.

Procedure : The following steps are followed.

1. Measure the angle T_1IP. Let it be α. (Fig. 15.26).

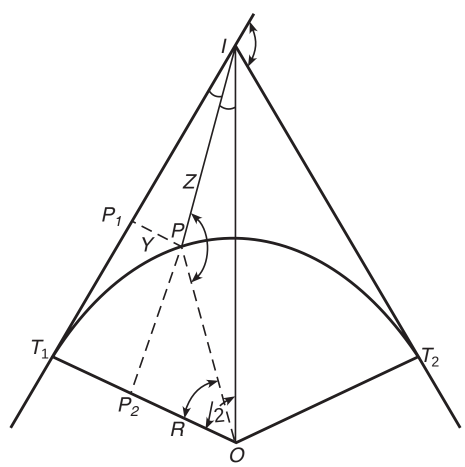


Fig. 15.26. A curve passing through a given point.

2. Measure *IP*. Let it be *z*.
3. Assume the angle $T_1OP = \theta$.

Construction. Drop PP_1 perpendicular to $T_1 I$ and a perpendicular PP_2 to T_1O.

4. From ΔPOI, we get

$$\angle PIO = \frac{1}{2}(180° - \Delta) - \alpha$$

$$= 90° - \left(\alpha + \frac{\Delta}{2}\right)$$

and $$\angle POI = \frac{\Delta}{2} - \theta$$

$\therefore$ $$\angle OPI = 180°o - \left[\left\{90° - \left(\alpha + \frac{\Delta}{2}\right)\right\} + \left\{\frac{\Delta}{2} - \theta\right\}\right]$$

$$= 90° + (\alpha + \theta)$$

Applying sine rule to ΔPOI, we get

$$\frac{\sin OPI}{IO} = \frac{\sin PIO}{PO}$$

$$\frac{\sin OPI}{\sin PIO} = \frac{IO}{PO}$$

$$\frac{\sin\,[90° + (\alpha + \theta)]}{\sin\left\{90° - \left(\alpha + \frac{\Delta}{2}\right)\right\}} = \frac{R \sec \frac{\Delta}{2}}{R}$$

$$\frac{\cos\,(\alpha + \theta)}{\cos\left(\alpha + \frac{\Delta}{2}\right)} = \sec \frac{\Delta}{2}$$

or $$\cos\,(\alpha + \theta) = \frac{\cos\,(\alpha + \frac{\Delta}{2})}{\cos \frac{\Delta}{2}} \qquad ...(15.26)$$

5. Calculate the value of θ from Eq. (15.26).
6. Calculate the radius of the curve as under :

$$R = R \cos\theta + T_1 P_2 = R \cos\theta + z \sin\alpha$$

or $$R = \frac{z \sin\alpha}{1 - \cos\theta} \qquad ...(15.27)$$

7. Calculate the tangent length

$$IT_1 = R \tan \frac{\Delta}{2} \qquad ...(15.28)$$

Example 15.1. *Two straights intersect at chainage 2056.44 m and the angle of intersection is 120° . If the radius of the simple curve to be introduced is 600 m, find the following :*

(*i*) *Tangent distances*

(*ii*) *Chainage of the point of commencement*

(*iii*) *Chainage of the point of tangency*

(*iv*) *Length of the long chord.*

Solution. (Fig. 15.27).

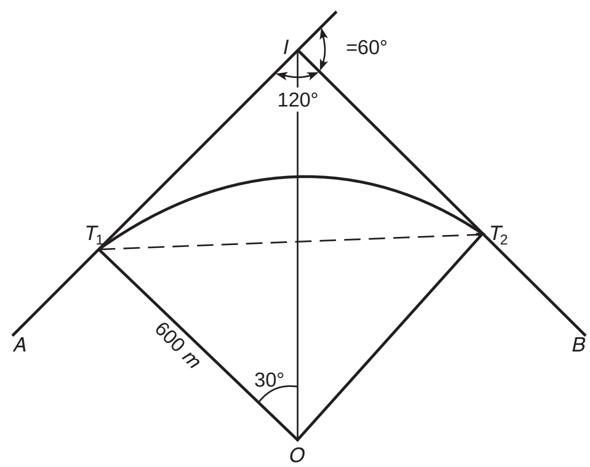


Fig. 15.27.

The deflection angle $\Delta = 180° - 120°o = 60°$; $R = 600$ m (Given)

(*a*) Tangent distances

$$= R \tan \frac{\Delta}{2}$$

$$= 600 \times \tan 30° = 600 \times 0.57735$$

= **346.41 m. Ans.**

(*b*) The length of the curve

$$= \frac{\pi R \Delta}{180°}$$

$$= \frac{\pi \times 600 \times 60°}{180°} = 628.32 \text{ m.}$$

(*c*) Chainage at the point of commencement, T_1

= Chainage at the point of intersection *I*

– tangent length

= 2056.44 – 346.41

$= \mathbf{1710.03\ m.}$ **Ans.**

(d) Chainage at the point of tangency

$= $ Chainage at point of commencement

$+$ length of the curve

$= 1710.03 + 628.32 = 2338.35$ m. **Ans.**

(e) Length of the long chord

$$= 2R \sin \frac{\Delta°}{2} = 2\,R \sin 30°$$

$$= 2 \times 600 \times \frac{1}{2} = \mathbf{600\ m.}$$ **Ans.**

Example 15.2. *Two roads meet at an angle of 127° 30′. Calculate the necessary data for setting out a curve of 15 chains radius to connect two straight portions of the road if it is intended to set out the curve by chain and offsets only. Explain carefully how you would set out the curve in the field. Assume the length of chain as 20 metres.*

Solution. (Fig. 15.28).

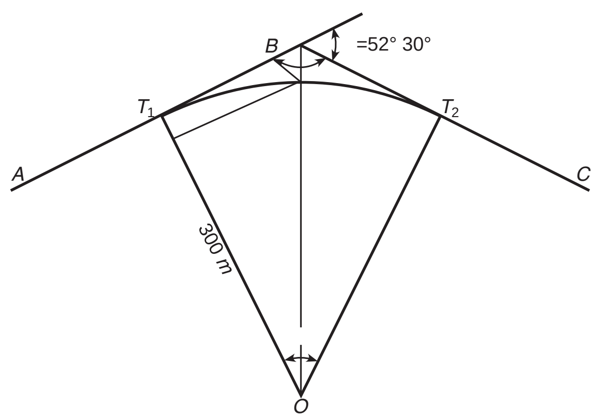


Fig. 15.28.

The length of the radius $= 15 \times 20 = 300$ m

Angle of deflection $\Delta = 180° - 127°o\ 30' = 52°\ 30'$

The length of tangent $T_1 B = R \tan \dfrac{\Delta°o}{2}$

$= 300 \tan 26°\ 15' = 300 \times 0.493145 = 147.94$ m.

Calculation of offsets

(a) **Radial offsets :** (Exact)

From equation (15.9), we get

$$O_x = \sqrt{R^2 + x^2 - R}$$

$$O_{20} = \sqrt{300^2 + 20^2} - 300 = 300.67 - 300 = 0.67 \text{ m}$$

$$O_{40} = \sqrt{300^2 + 40^2} - 300 = 302.66 - 300 = 2.66 \text{ m}$$

$$O_{60} = \sqrt{300^2 + 60^2} - 300 = 305.94 - 300 = 5.94 \text{ m}$$

$$O_{80} = \sqrt{300^2 + 80^2} - 300 = 310.48 - 300 = 10.48 \text{ m}$$

$$O_{100} = \sqrt{300^2 + 100^2} - 300 = 316.23 - 300 = 16.23 \text{ m}$$

$$O_{120} = \sqrt{300^2 + 120^2} - 300 = 323.11 - 300 = 23.11 \text{ m}$$

$$O_{140} = \sqrt{300^2 + 140^2} - 300 = 331.06 - 300 = 31.06 \text{ m}$$

$$O_{147.94} = \sqrt{300^2 + 147.94^4} - 300 = 334.49 - 300 = 34.49 \text{ m}$$

Other half of the curve may be set out from the second tangent.

(*b*) **Perpendicular offsets :** (Exact)

From equation (15.7) we get

$$O_x = R - \sqrt{R^2 - x^2}$$

$$O_{20} = 300 - \sqrt{300^2 - 20^2} = 300 - 299.33 = 0.67 \text{ m}$$

$$O_{40} = 300 - \sqrt{300^2 - 40^2} = 300 - 297.32 = 2.68 \text{ m}$$

$$O_{60} = 300 - \sqrt{300^2 - 80^2} = 300 - 289.14 = 6.06 \text{ m}$$

$$O_{80} = 300 - \sqrt{300^2 - 80^2} = 300 - 289.14 = 10.86 \text{ m}$$

$$O_{100} = 300 - \sqrt{300^2 - 100^2} = 300 - 282.84 = 17.16 \text{ m}$$

$$O_{120} = 300 - \sqrt{300^2 - 140^2} = 300 - 274.95 = 25.05 \text{ m}$$

$$O_{140} = 300 - \sqrt{300^2 - 140^2} = 300 - 265.33 = 34.67 \text{ m}$$

$$O_{147.94} = 300 - \sqrt{300^2 - 147.94^2} = 300 - 260.99 = 39.01\text{m}$$

The distance x of the point from T_1 for locating the apex point

$$x = R \sin \frac{\Delta°}{2} = 300 \times \sin 26° \, 15'$$

$$x = 300 \times 0.442289 = 132.69 \text{ m}$$

$$O_{132.69} = 300 - \sqrt{300^2 - 132.69^2}$$

$$= 300 - 269.06 = \mathbf{30.94 \text{ m}}$$

The other half of the curve, may be set out from the second tangent.

Example 15.3. *Tabulate the necessary data for setting out circular curve with the following data :*

Angle of intersection = *144°*

Chainage of point of intersection = *1390 m*

Radius of the curve = *300 m*

The curve is to be set out by offsets from chords produced with pegs at every 20 m of through chainage.

Solution.

Angle of deflection $= 180° - 144° = 36°$

The length of the tangents $= R \tan \dfrac{\Delta}{2}$

$= 300 \times \tan 18°$

$= 300 \times 0.324920 = 97.48$ m

The length of the curve $= \dfrac{\pi R \Delta}{180}$

$= \dfrac{\pi \times 300 \times 36°}{180°} = 188.50$ m

Chainage at the point of commencement $= 1390.00 - 97.48$

$= 1292.52$ m

The chainage at the point of tangency $= 1292.52 + 188.50$

$= 1481.02$ m

The length of first sub-chord

$= 1300 - 1292.52 = 7.48$ m

The length of the last sub-chord

$= 1481.02 - 1480.00 = 1.02$ m

No. of normal chords $= \dfrac{188.50 - (7.48 + 1.02)}{20} = 9$

$\therefore$ Total number of chords $= 2 + 9 = 11$

Offset for the first sub-chord $= \dfrac{C_1^2}{2R}$

$= \dfrac{(7.48)^2}{2 \times 300} = 0.09$ m

Offset for the second chord

$$= \frac{C_2 (C_2 + C_1}{2R} = \frac{20 (20 + 7.48)}{2 \times 300} = 0.92 \text{ m}$$

Offset for the 3rd to 10th chords

$$= \frac{20 (20 + 20)}{2 \times 300} = 1.33 \text{ m}$$

Offset for the last sub-chords

$$= \frac{1.02 (1.02 + 20)}{2 \times 300} = 0.040 \text{ m}$$

Necessary data is tabulated hereunder :

Chainage	*Length of the chord (m)*	*Length of the offset (m)*
1300	7.48	0.09
1320	20.00	0.92
1340	20.00	1.33
1360	20.00	1.33
1380	20.00	1.33
1400	20.00	1.33
1420	20.00	1.33
1440	20.00	1.33
1460	20.00	1.33
1480	20.00	1.33
1481.02	1.02	0.04
Length of curve = 188.50		

Example 15.4. *Two tangents intersect at chainage 1190 m, the deflection angle being 36° . Calculate all the data necessary for setting out a curve with a radius of 300 m by deflection angle method. The peg interval is 30 m.*

Solution. (Fig. 15.29).

The tangent length $= R \tan \frac{\Delta° o}{2} = 300 \times \tan 18°$

$= 300 \times 0.324920 = 97.48$ m.

The length of the curve $= \frac{\pi R \, \Delta°}{180°}$

$$= \frac{3.1416 \times 300 \times 36°}{180°}$$

$= 188.50$ m.

The chainage at the point of intersection = 1190.00 m (Given)

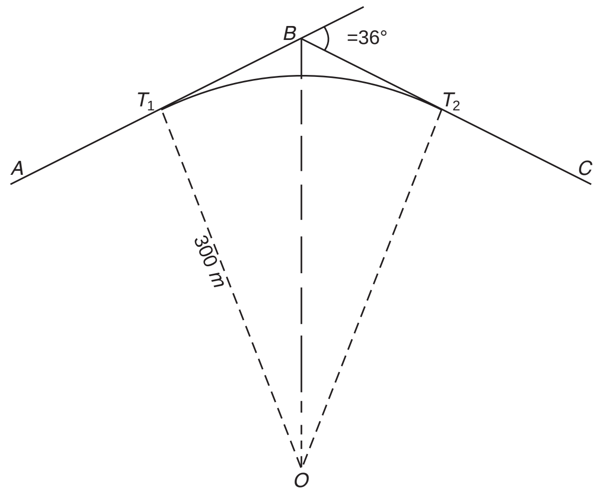


Fig. 15.29.

Subtract the tangent length	$= -97.48$
$\therefore$ Chainage of the point of tangency T_1	$= 1092.52$ m
Add the length of the curve	$= +188.50$
$\therefore$ Chainage at the point of tangency T_2	$= 1281.02$ m
The length of the first sub-chord	$= 1110 - 1092.52$
	$= 17.48$ m
The length of the last sub-chord	$= 1281.02 - 1260.00$
	$= 21.02$ m
No. of normal chords	$= \dfrac{188.50 - (17.48 + 21.02)}{30}$
	$= \dfrac{150}{30} = 5.$

Calculation of deflection angles :

The deflection angle $\delta_1 = \dfrac{1718.9 \times C_1}{R}$

$$= \frac{1718.9 \times 17.48}{300} = 100'.155$$

$$= 1° \; 40' \; 09''$$

The deflection angles, δ_2 to δ_6

$$= \frac{1718.9 \times 30}{300} = 171.89$$

$= 2° 51' 53''$

The deflection angle $\delta_7 = \dfrac{1718.9 \times 21.02}{300} = 120.44$

$= 2° 00' 26''$

The total deflection angle $\Delta_1 = 1°o\ 40' 09''$

$\Delta_2 = 1° 40' 09'' + 2° 51' 53''$

$= 4° 32' 02''$

$\Delta_3 = 4° 32' 02'' + 2° 51' 53''$

$= 7° 23' 55''$

$\Delta_4 = 7^0\ 23' 55' + 2°o\ 51' 53''$

$= 10° 15' 48''$

$\Delta_5 = 10° 15' 48'' + 2° 51' 53''$

$= 13° 07' 41''$

$\Delta_6 = 13° 07' 41'' + 2° 51' 53''$

$= 15° 59' 34''$

$\Delta_7 = 15° 59' 34'' + 2° 00'' 26'$

$= 18° 00' 00''$

Check : The total deflection angle for the point of tangency T_2

$= \dfrac{1}{2}$ deflection angle of the curve

$= \dfrac{1}{2} \times 36°$

$= 18° 0' 0''$ O.K.

Example 15.5. *Two roads BA and CA intersect at a point A which falls in the bed of a river. These are to be connected by a simple circular curve of radius 200 m. To do this, a line MN connecting these tangents at points M and N respectively is measured to be 170 m. The angles ∠ BMN = 105°o* and *∠CNM = 135°o . The chainage of point M is 1815 m. Determine the chainages of tangent joints and the length of the curve.*

Solution. (Fig. 15.30)

In ΔAMN we get

$\angle AMN = 180° - 105°o = 75°$

$\angle ANM = 180° - 135°o = 45°$

$\therefore \quad \angle MAN = 180° - (75° + 45°) = 60°$

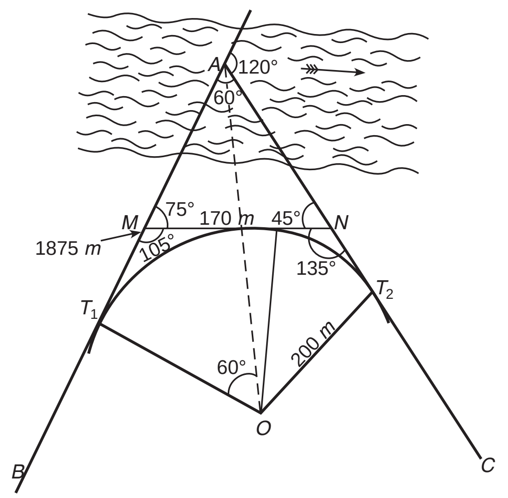


Fig. 15.30

and angle of deflection $= 180° - 60° = 120°$.

Let T_1 and T_2 be the points of tangency.

$$\therefore \quad AT_1 = R \tan \frac{\Delta}{2} = 200 \times \tan \frac{120°}{2} = 346.41 \text{ m}$$

Applying sine rule to ΔMAN we get

$$\frac{AM}{\sin 45°} = \frac{170}{\sin 60°}$$

$$\therefore \quad AM = \frac{170 \sin 45°}{\sin 60°} = 138.80 \text{ m}$$

Chainage of A = chainage of $M + MA$

$= 1815 + 138.80 = 1953.8$ m

Chainage of T_1 = Chainage of A – back tangent AT_1

$= 1953.80 - 346.41 = 1607.39$ m.

Length of the curve $= \dfrac{\pi R\Delta}{180} = \dfrac{\pi \times 200 \times 120^0}{180^0} = 418.88$ m

Chainage of T_2 = chainage of T_1 + length of the curve

$= 1607.39 + 418.88 = 2026.27$ m.

Chainage of $T_1 = 1607.39$ m

Chainage of $T_2 = 2026.27$ m **Ans.**

Length of curve $= 418.88$ m

Example 15.6. *Two straights AB and BC are to be connected by a right hand circular curve. The bearings of AB and BC are 70°* and *140° respectively. The curve is to pass through a point P such that BP is 120 metres and the angle ABP is 40°. Determine the radius of the curve (30 metre chain is used). If the curve is to be set out on the ground, determine the chainage of the tangent points and total deflection angles for the first two pegs on the curve at through chainages of 30 m ; the chainage of the intersection point being 3,000 metres.*

Solution. (Fig. 15.31).

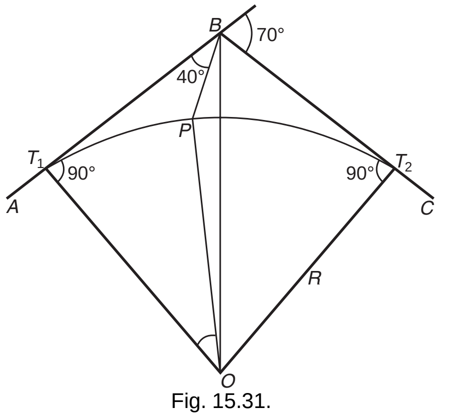


Fig. 15.31.

From equation (15.26) we know, that

$$\cos(\alpha + \theta) = \frac{\cos\left(\alpha + \frac{\Delta°}{2}\right)}{\cos\frac{\Delta}{2}} \qquad ...(i)$$

Here $\alpha = 40°$

and Δ = Bearing of BC − Bearing of AB

$= 140° - 70° = 70°o$

Substituting the values, in Eqn. (i), we get

$$\therefore \qquad \cos(\alpha + \theta) = \frac{\cos(40° + 35°)}{\cos 35°} = \frac{\cos 75°}{\cos 35°}$$

$$= \frac{0.258819}{0.819152} = 0.31595967$$

$$\alpha + \theta = 71°\ 34'\ 52''$$

or $\theta = 71°\ 34'\ 52'' - 40° = 31°\ 34'\ 52''$

From Eqn. (15.27), we get

$$R = \frac{Z \sin \alpha}{1 - \cos \theta} \qquad ...(ii)$$

Here $Z = 120$ m.

Substituting the values in Eqn. (*ii*), we get

$$R = \frac{120 \sin 40°}{1 - \cos 31° 34' 52''} = \frac{120 \times 0.642788}{1 - 0.851900}$$

$$= 520.83 \text{ m.}$$

Tangent length $= R \tan \frac{\Delta}{2}$

$$= 520.83 \times \tan 35°$$

$$= 520.83 \times 0.700207$$

$$= 364.69 \text{ m.}$$

The length of the curve

$$= \frac{\pi R \Delta°}{180°}$$

$$= \frac{\pi \times 520.83 \times 70°}{180°}$$

$$= 636.31 \text{ m.}$$

Chainage of the point of intersection

$= 3000$ m (given)

$\therefore$ Chainage of the first point of tangency T_1

$$= 3000 - 364.69 = 2635.31 \text{ m}$$

Chainage of the second point of tangency T_2

$$= 2635.31 + 636.31$$

$$= 3271.62 \text{ m}$$

The length of the first sub-chord

$$= 2640.00 - 2635.31 = 4.69 \text{ m}$$

The angle of the deflection for the first peg

$$\delta_1 = \frac{1718.9 \times 4.69}{520.83} = 15'.478$$

$$= 0° 15' 29''$$

The angle of deflection for second peg

$$\delta_2 = \frac{1718.9 \times 30}{520.83} = 99'.009 = 1° 39' 00''$$

Total deflection angle of first peg

$= 0° 15' 29''.$ **Ans.**

Total deflection angle of second peg

$= 0° 15' 29'' + 1° 39' 00''$

$= 1° 54' 29''.$ **Ans.**

Example 15.7. *Two tangents to the railway curve meet at an angle of 140° . Owing to the position of a building, a curve is to be choosen which will pass through a point P, whose co-ordinates are x = 10, y = 20 m measured along the bisector. Find the radius of the circular curve which shall be tangential to two straights and will pass through P. What will be the radius if P lies on the bisector.*

Solution. (Fig. 15.32).

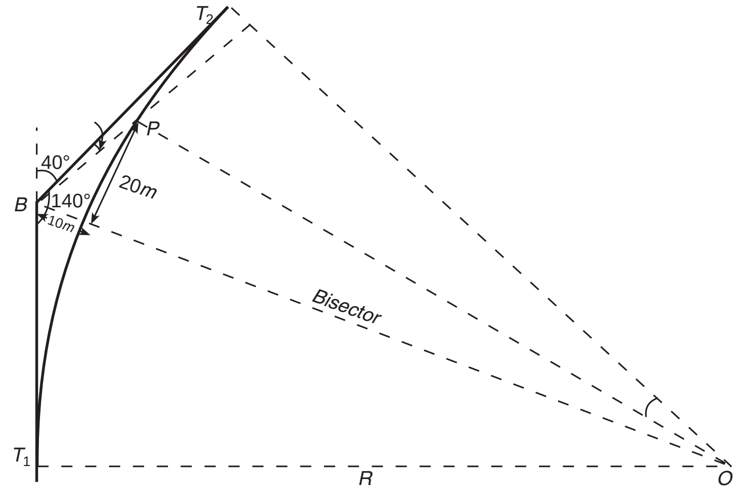


Fig. 15.32.

(*i*) Deflection angle $\Delta = 180 - 140° = 40°$

Distance $BP = \sqrt{10^2 + 20^2} = 22.36$ m

$$\angle PBO = \tan^{-1} \frac{20}{10} = \tan^{-1} 2 = 63° 26' 06''$$

$$\alpha = \angle PBT_2 = 70° - \angle PBO$$

or $$\alpha = 70° - 63° 26' 06'' = 6° 33' 54''$$

Let θ be the angle POT_2,

Substituting the values in eqn. (15.26)

$$\cos(\alpha + \theta) = \frac{\cos(\alpha + (\Delta/2))}{\cos \frac{\Delta}{2}}$$

$$= \frac{\cos (6° 33' 54'' + 20°)}{\cos 20°} = \frac{\cos 26° 33' 54''}{\cos 20°}$$

$$= \frac{0.894428}{0.939693} = 0.95183$$

$$\therefore \qquad \alpha + \theta = 17° 51' 22''$$

$$\theta = 17° 51' 22'' - 6° 33' 54''$$

$$= 11° 17' 28''$$

From Eqn. (15.27), we get

$$R = \frac{Z \sin \alpha}{1 - \cos \theta}$$

where $Z = 22.36$ m

$$\therefore \qquad R = \frac{22.36 \times \sin 6° 33' 54''}{1 - \cos 11° 17' 28''} = \frac{22.36 \times 0.11433}{1 - 0.980645}$$

or $$R = \frac{22.36 \times 0.11433}{0.019355} = 132.08 \text{ m.}$$ **Ans.**

(*ii*) When the point P lies on the bisector then,

$$(R + 10) \cos 20° = R$$

$$\therefore \quad R \cos 20° + 10 \cos 20° = R$$

or $$R (1 - \cos 20°) = 10 \cos 20°$$

or $$R = \frac{10 \cos 20°}{1 - \cos 20°} = \frac{10 \times 0.939693}{1 - 0.939693}$$

or $$R = \frac{9.39693}{0.060307} = 155.82 \text{ m.}$$ **Ans.**

Example 15.8. *Two straights AB and BC, having bearings respectively of 100°* and *160°o 12′ are to be connected by a right hand circular curve. The curve is to pass through a point D such that BD is 47.66 metres long and its bearing is 245° 24′. Determine the radius of the curve.*

If the chainage of the point of intersection is 3447.50 metres, determine the tangential angles to be set on a theodolite reading to 20″ for setting out the first two pegs of the curve at through chainages of 20 metres.

Solution. (Fig. 15.33)

Bearing of $BC = 160° 12'$

Bearing of BB_1 = Bearing of $AB = 100°$

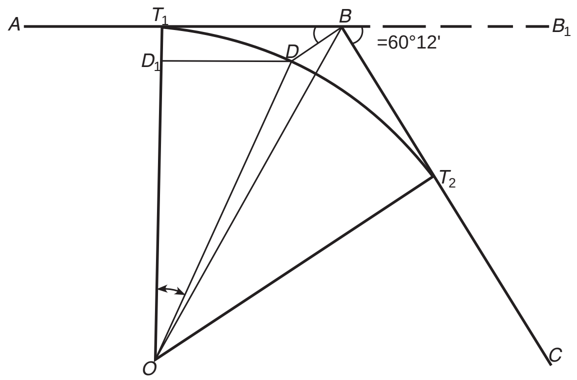


Fig. 15.33

$\therefore$ The angle of deflection $\Delta = 160^\circ\ 12' - 100^\circ = 60^\circ\ 12'$

Bearing of $BA = 100^\circ + 180^\circ = 280^\circ$

Bearing of $BD = 245^\circ\ 24'$

$\therefore \qquad \alpha = 280^\circ - 245^\circ\ 24' = 34^\circ\ 36'$

Let the angle $T_1OD = \theta$

From Eqn. (15.26), we get

$$\cos(\alpha + \theta) = \frac{(\cos \alpha + \Delta/2)}{\cos \Delta/2} \qquad ...(i)$$

Substituting the values in eqn. (*i*)

$$\cos(\alpha + \theta) = \frac{\cos(34^\circ o\ 36' + 30^\circ\ 06')}{\cos 30^\circ\ 06'}$$

$$= \frac{\cos 64^\circ\ 42'}{\cos 30^\circ\ 06'} = \frac{0.427358}{0.865151} = 0.49396926$$

$$\alpha + \theta = 60^\circ\ 23'\ 54''$$

$$\theta = 60^\circ\ 23'\ 54'' - 34^\circ\ 36' = 25^\circ\ 47'\ 54''$$

From Eqn. (15.27), we get

$$R = \frac{Z \sin \alpha}{1 - \cos \theta} \qquad ...(ii)$$

Substituting the values in eqn. (*ii*) we get

$$= \frac{47.66 \sin 34^\circ\ 36'}{1 - \cos 25^\circ\ 47'\ 54''}$$

or
$$R = \frac{47.66 \times 0.567844}{1 - 0.900332} = 271.54 \text{ m}$$

Length of the tangent $= R \tan \Delta/2$

$= 271.54 \tan 30° 06'$

$= 271.54 \times 0579680$

$= 157.41$ m

The chainage of the point of intersection

(Given) $= 3447.50$ m

Deduct the tangent length $= 157.41$

$\therefore$ Chainage of the point of commencement

$= 3290.09$ m

The length of first sub-chord,

$= 3300.00 - 3290.09 = 9.91$ m

The angle of deflection,

$$\delta_1 = \frac{1718.9 \times 9.91}{271.54} = 62' \ 7322$$

$= 1° \ 02' \ 44''$

The angle of deflection,

$$\delta_2 = \frac{1718.9 \times 20}{271.54} = 126' \ 604$$

$= 2° \ 06' \ 36''$

$\therefore$ Tangential angles to be set on the theodolite

$\Delta_1 = 1° \ 02' \ 44'' = 1° \ 02' \ 40''$

$\Delta_2 = 1° \ 02' \ 44'' + 2° \ 06' \ 36'' = 3° \ 09' \ 20''$

Radius of the curve $= 271.54$ m

Tangential angle $\Delta_1 = 1° \ 02' \ 40''$ **Ans.**

Tangential angle $\Delta_2 = 3° \ 09' \ 20''$

Example 15.9. *The centre line of a road is to be tangential to each of the following lines* :

Line	*W.C.B.*	*Length*
AB	*0°*	...
BC	*90°*	*450.24 m*
CD	*143° 12″*	...

Calculate the radius of the curve and the tangent lengths.

Solution. (Fig. 15.34).

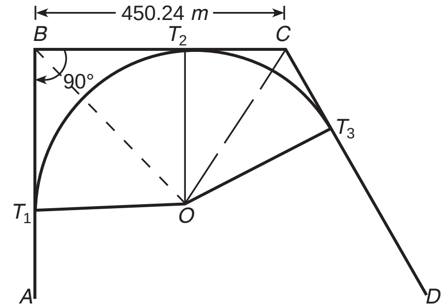


Fig. 15.34

Let the bisectors of the angles ABC and BCD intersect at O, which is the required centre of the curve.

From O drop perpendiculars OT_1, $O\,T_2$, and OT_3 to AB, BC and CD respectively. T_1, T_2 and T_3 are therefore tangent points,

Angle ABC = Bearing of BA – Bearing of BC = 180° – 90° = 90°

Angle BCD = Bearing of CB - Bearing of CD = 270° – 143° 12′ = 126° 48′

In $\Delta\, OBC$, $\angle\, OBC = 45°$

$$\angle\, OCB = \frac{126°\ 48'}{2} = 63°\ 24'$$

$\therefore$ $\angle\, BOC = 180°o - (45° + 63°\ 24') = 71°o\ 36'$

Applying sine rule to ΔOBC, we get

$$OB = \frac{BC}{\sin BOC} \times \sin OCB$$

$$= \frac{450.24 \times \sin 63°\ 24'}{\sin 71°\ 36'}$$

or $$OB = \frac{450.24 \times 0.894154}{0.948876} = 424.27 \text{ m}$$

Similarly, $$OC = \frac{BC}{\sin\ BOC} \times \sin OBC$$

$$= \frac{450.24 \times \sin 45°}{\sin 71°\ 36'}$$

or $$OC = \frac{450.24 \times 0.707107}{0.948876} = 335.52 \text{ m}$$

Radius of the curve $= OB \cos 45°$

$= 424.27 \times 0.707107 = 300.0$ m. **Ans.**

Tangent length $BT_1 = BT_2 = OB \cos 45°$

$= 424.27 \times 0.707107$

$= 300.0$ m.

Tangent length $CT_2 = CT_3 = OC \cos 63° 24'$

$= 335.52 \times 0.447758$

$= 150.23$ m

Radius of the curve $= 300.01$ m

Tangent $T_1B = 300.0$ m **Ans.**

Tangent $T_3C = 150.2$ m

Example 15.10. *Two straights AX and XB having bearings of 146° 36′* and *86° 06′ respectively intersect at X and are connected by a circular curve of 250 metres radius. The co-ordinates of A and B are as under :*

	Co-ordinates	
Point	*North (m)*	*East (m)*
A	*312.6*	*160.4*
B	*200.2*	*586.8*

Calculate the chainage of first and second points, if chainage of A is 9564.00 metres.

Solution. (Fig. 15.35)

Difference in Northings of A and B,

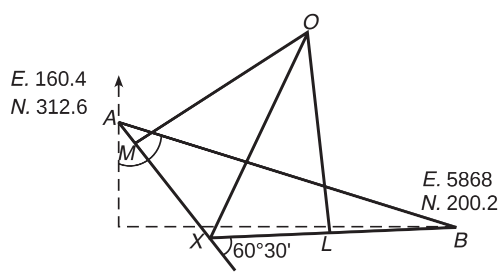


Fig. 15.35.

$= 312.6 - 200.2 = 112.4$ m

Difference in Eastings of A and B,

$= 586.8 - 160.4 = 426.4$ m

$\therefore \quad \tan \theta = \frac{426.4}{112.4} = 3.7935943$

or $\theta = 75° 13' 57''$

$\therefore$ Bearing of $AB = 180° - 75° 13' 57'' = 104° 46' 03''$

And $AB = \sqrt{(112.4)^2 + (426.4)^2}$

$= 440.97$ m.

In ΔAXB, $\angle BAX$ = Bearing of AX – Bearing of AB

$= 146° 36' 00'' - 104° 46' 03''$

$= 41° 49' 57''$

$\angle ABX$ = Bearing of BA – Bearing of BX

$= (104° 46' 03'' + 180°) - (86° 06' 00'' + 180°)$

$= 284° 46' 03'' - 266° 06' 00''$

$= 18° 40' 03''$

Interior Angle AXB = Bearing of XA – Bearing of XB

$= (146° 36' + 180° 0') - 86° 06'$

$= 326° 36' - 86° 06'$

$= 240° 30'$

or Interior Angle $AXB = 360° - 240° 30'$

$= 119° 30'$

Angle of deflection $\Delta = 180° - 119° 30' = 60° 30'$

Applying sine rule to ΔAXB, we get

$$\frac{AX}{\sin ABX} = \frac{BX}{\sin BAX} = \frac{AB}{\sin AXB}$$

$$AX = \frac{AB \sin ABX}{\sin AXB}$$

$$= \frac{440.97 \times \sin 18° 40' 03''}{\sin 119° 30'}$$

$$= \frac{440.97 \times 0.320076}{0.870356}$$

$= 162.17$ m

and $$BX = \frac{AB \sin BAX}{\sin AXB} = \frac{440.97 \times \sin 41° 49' 57''}{\sin 119° 30'}$$

$$= \frac{440.96 \times 0.666955}{0.870356}$$

or $BX = 337.92$ m.

Length of the curve $$= \frac{\pi R \Delta}{180°o} = \frac{\pi \times 250 \times 60.5}{180°}$$

$= 263.98$ m

Length of the tangent $= R \tan \frac{\Delta}{2} = 250 \times \tan 30° \, 15'$

$= 250 \times 0.583183$

$= 145.80$ m.

Chainage of X = Chainage of $A + AX$

$= 9564.00 + 162.17$

$= 9726.17$ m

Chainage of 1st point of tangency M

= Chainage of X ––taugent length

$= 9726.17 - 145.80$

$= 9580.37$. **Ans.**

Chainage of second point of tangency (L)

= Chainage of M + length of the curve

$= 9580.37 + 263.98$

$= 9844.35$ m. **Ans.**

Example 15.11. *Two straights meet at an apex angle 126° 48′ and are to be joined by a circular curve of 300 m radius. Calculate the data necessary to set out the curve using a 30 m chord.*

Tabulate the data properly for field use.

Solution.

Note. In the question, the chainage of the point of intersection is not given. After calculating the length of tangent, the point of commencement is located and deflection angles are calculated for 30 m chords. Only a sub-chord at the end of the curve is used.

The angle of intersection $= 126° \, 48'$ (Given)

$\therefore$ The angle of deflection, $\Delta = 180° - 126° \, 48' = 53° \, 12'$

The tangent length $= R \tan \frac{\Delta}{2}$

$= 300 \times \tan 26° \, 36'$

$= 300 \times 0.500763 = 150.23$ m

The length of the curve $= \frac{\pi R \Delta}{180°}$

$$= \frac{\pi \times 300 \times 53.2}{180}$$

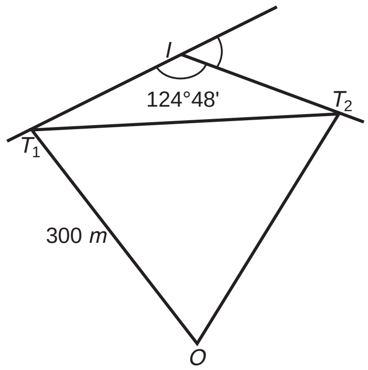


Fig. 15.35(*a*)

$= 278.55$ m

No. of peg intervals $= \dfrac{278.55}{30} = 10$

Length of sub-chord at the end $= 278.55 - 30 \times 9$

$= 8.55$ m

Calculation of deflection angles

$$\delta_1 \text{ to } \delta_9 = 1718.9 \times \frac{30}{300} = 171.89 \text{ minutes}$$

$$= 2° \ 51' \ 53'' .4$$

$$\Delta_{10} = 1718.9 \times \frac{8.55}{300} = 48' .989 = 0° \ 48' \ 59''.4$$

$$\Delta_1 = 2° \ 51' \ 53''.4$$

$$\Delta_2 = \Delta_1 + \delta_2 = 5^o \ 43' \ 46''.8$$

$$\Delta_3 = \Delta_2 + \delta_3 = 8° \ 35' \ 40''.2$$

$$\Delta_4 = \Delta_3 + \delta_4 = 11^o \ 27' \ 33''.6$$

$$\Delta_5 = \Delta_4 + \delta_5 = 14^o \ 19' \ 27''.0$$

$$\Delta_6 = \Delta_5 + \delta_6 = 17^o \ 11' \ 20''.4$$

$$\Delta_7 = \Delta_6 + \delta_7 = 20^o \ 03' \ 13''.8$$

$$\Delta_8 = \Delta_7 + \delta_8 = 22^o \ 55' \ 07''.2$$

$$\Delta_9 = \Delta_8 + \delta_9 = 25^o \ 47' \ 00''.6$$

$$\Delta_{10} = \Delta_9 + \delta_{10} = 26^o \ 36' \ 00''.0$$

Check : Total deflection angle $= \dfrac{1}{2} \Delta = 26° \ 36' \ 00''$

$\Delta_1 = 2° 51' 53''.4$ $\Delta_6 = 17° 11' 20''.4$

$\Delta_2 = 5° 43' 46''.8$ $\Delta_7 = 20° 03' 13''.8$

$\Delta_3 = 8° 35' 40''.2$ $\Delta_8 = 22° 55' 07''.2$ **Ans.**

$\Delta_4 = 11° 27' 33''.6$ $\Delta_9 = 25° 47' 00''.6$

$\Delta_5 = 14° 19' 27''.0$ $\Delta_{10} = 26° 36' 00''.0$

Example 15.12. *If the approximate perpendicular offset for the mid-point of a circular curve deflecting through 76° 38′ is 96.1 m. Calculate the radius of the curve.*

Solution. (Fig. 15.36).

Angle of deflection $\Delta = 76° 38'$

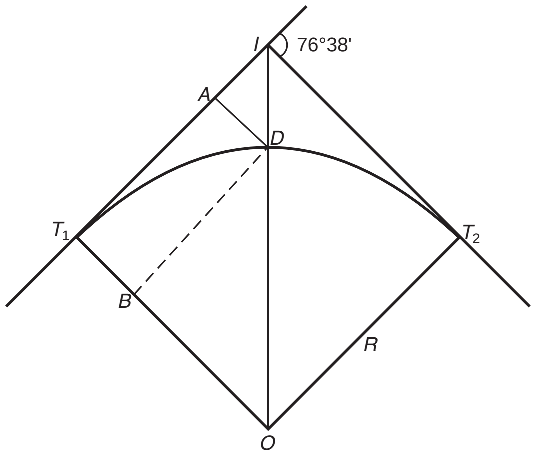


Fig. 15.36

Angle $T_1OI = \frac{1}{2}\ \Delta = 38°o\ 19'$

Let $T_1A = x$; $AD = O_x$; radius $= R$

$$\therefore \quad O_x = \frac{x^2}{2R}$$

$$96.1 = \frac{x^2}{2R}$$

or $$x^2 = 2R \times 96.1 \qquad ...(i)$$

From $\Delta\, BOD$, $T_1A = BD = R \sin 38° 19'$

or $$x = R \sin 38° 19' \qquad ...(ii)$$

Dividing Eq. (*i*) by Eq. (*ii*), we get

$$x = \frac{2 \times 96.1}{\sin 38^\circ\ 19'} = \frac{192.2}{0.620007} = 310 \text{ m}$$

Substituting the value of x in Eq. (i), we get

$$R = \frac{310^2}{192.2} = 500 \text{ m.} \quad \textbf{Ans.}$$

Example 15.13. *Calculate the radius of a 877 m simple circular curve deflecting right if the bearings of its back tangent and long chord are 60° 30′* end *100° 45′ respectively.*

Solution. (Fig. 15.37)

Given : Bearing of $T_1T_2 = 110° 45'$

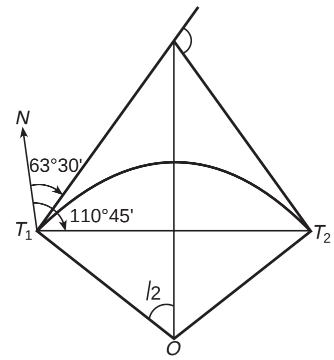


Fig. 15.37.

Bearing of $T_1B = 60° 30'$

Length of the curve $= 877$ m

Angle BT_1T_2 $=$ Bearing of $T_1 T_2$ – Bearing of T_1B

$= 110° 45' - 60° 30'$

$= 50° 15'$

Angle of deflection $\Delta = 2 \times (50°o\ \ 15')$

$= 100° 30'$

The length of curve $= \dfrac{\pi R \Delta°}{180}$

$$\therefore \quad \frac{\pi R \times 100.5}{180°} = 877 \text{ m} \quad \text{(Also given)}$$

or

$$R = \frac{877 \times 180}{\pi \times 100.5} = 500 \text{ m.} \quad \textbf{Ans.}$$

Example 15.14. *The apex distance of a circular curve is one tenth of the distance between the point of intersection B and the centre of the* *curve O. The co-ordinates of the point of intersection and the centre of the curve are as under:*

Point	*Easting (m)*	*Northing (m)*
B	*1550*	*1900*
O	*1000*	*1000*

Calculate the length of the curve, the chainages of the points of tangencies and also deflection angles for setting out the first half of the curve with a 1″ theodolite. Assume the chainage of the point of intersection as 1559.75 m, with 30 m peg interval.

Solution. (Fig. 15.38)

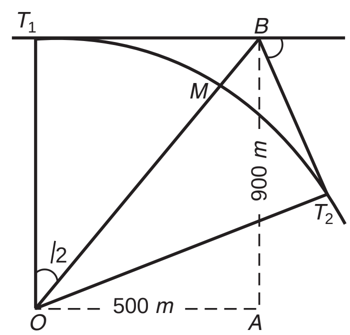


Fig. 15.38.

Let T_1 and T_2 be the points of tangencies and M, the mid-point of the curve.

Difference of Eastings of B and O = 1550 − 1000 = 550 m

Difference of Northings of B and O = 1900 − 1000 = 900 m

$\therefore \quad OB = \sqrt{550^2 + 900^2} = 1054.75$ m

Let $BM = X\,;\, OB = Y$

and $OM = R$

From ΔBT_1O,

$$(R + X)\cos\frac{\Delta}{2} = R$$

$$R\left(1 - \cos\frac{\Delta}{2}\right) = X\cos\frac{\Delta}{2} \qquad ...(i)$$

And
$$R\sec\frac{\Delta}{2} = Y \qquad ...(ii)$$

Dividing Eq. (*i*) by Eq. (*ii*) we get

$$\frac{1 - \cos \dfrac{\Delta}{2}}{\sec \dfrac{\Delta}{2}} = \frac{X \cos \Delta/2}{Y}$$

or $$1 - \cos \frac{\Delta}{2} = \frac{X}{Y} = \frac{0.1\,Y}{Y} = 0.1$$

or $$\cos \frac{\Delta}{2} = 1 - 0.1 = 0.9$$

or $$\frac{\Delta}{2} = 25° \, 50' \, 31''$$

or $$\Delta = 51° \, 41' \, 02''$$

Substituting the values of Δ and Y in Eq. (*ii*), we get

$$R = 1054.75 \cos 25° \, 50' \, 31''$$

or $$R = 949.275 \text{ m}$$

$\because$ Length of the curve $= \dfrac{\pi R \Delta}{180°}$

$$= \frac{\pi \times 949.275 \times 51.683888}{180°}$$

= 856.30 m. **Ans.**

Length of tangent $= R \tan \Delta/2$

$= 949.275 \times \tan 25° \, 50' \, 31''$

= 459.75 m

Chainage of the point of intersection

= 1559.75 m (given)

Chainage of the point of commencement T_1

= 1559.75 – 459.75 = 1100 m. **Ans.**

Chainage of the mid-point of curve M

= 1100 + 428.15 = 1528.15 m

Chainage of the point of tangency T_2

= 1528.15 + 428.15 = 1956.30 m. **Ans.**

Length of the first sub-chord

= 1110 – 1100 = 10 m

Length of the sub-chord before M

= 1528.15 – 1500 = 28.15 m

No. of normal chords

$$= \frac{428.15 - (10 + 28.15)}{30} = 13$$

$\therefore$ Total number of chords = 2 + 13 = 15

$$\delta_1 = \frac{1718.9 \times 10}{949.275} = 0^o\ 18'\ 06''.45'$$

$$\delta_2 \text{ to } \delta_{14} = \frac{1718.9 \times 18'}{949.275} = 0°\ 54'\ 19''.\ 35$$

$$\delta_{15} = \frac{1718.0 \times 28.15}{949.275} = 0°\ 50'\ 58''.\ 36$$

Total deflection angle for the mid-point *M*

$$= \delta_1 + 13\ \delta + \delta_{15}$$

$$= 0°\ 18'\ 06''.45 + 13° (0°\ 54'.19''\ 35 + 0°\ 50'\ 58''.36$$

$$= 0°\ 18'\ 06''.45 + 11°\ 46'\ 11''.55 + 0°\ 50'\ 58''.36$$

$$= 12°\ 55'\ 16''.36$$

$\frac{1}{4}$th of the deflection angle for the curve

$$= \frac{1}{4} \times 51°\ 41'\ 02''$$

$$= 12°\ 55'\ 15''.5. \qquad \text{O.K.}$$

Deflection angles		*Theodolite Readings*	
$\Delta_1 = \delta_2$	0° 18′ 06″.5	0° 18′ 06″.5	0° 18′ 07″
$\Delta_2 = \Delta_1 + \delta_2$	0° 18′ 06″.5 + 0° 54′ 19″.4	1° 12′ 25″.9	1° 12′ 26″
$\Delta_3 = \Delta_2 + \delta_3$	1° 12′ 25″.9 + 0° 54′ 19″.4	2° 06′ 45″.3	2° 06′ 45″
$\Delta_4 = \Delta_3 + \delta_4$	2° 06′45″.3 + 0° 54′ 19″.4	3° 01′ 04″7	3° 01′ 05″
$\Delta_5 = \Delta_4 + \delta_5$	3° 01′ 04″.7 + 0° 54′ 19″.4	3° 55′24″1	3° 55′ 24″
$\Delta_6 = \Delta_5 + \delta_6$	3° 55′ 24″.1 + 0° 54′ 19″.4	4° 49′ 43″.5	4° 49′ 44″
$\Delta_7 = \Delta_6 + \delta_7$	4° 49′ 43″.5 + 0° 54′ 19″.4	5° 44′ 02″.9	5° 14′ 03″
$\Delta_8 = \Delta_7 + \delta_8$	5° 44′ 02″.9 + 0° 54′ 19″.4	6° 38′ 22″.3	6° 38′ 22″
$\Delta_9 = \Delta_8 + \delta_9$	6° 38′ 22″.3 + 0° 54′ 19″.4	7° 32′ 41″.7	7° 32′ 42″
$\Delta_{10} = \Delta_9 + \delta_{10}$	7° 32′ 41″.7 + 0° 54′ 19″.4	8° 27′ 01″.1	8° 27′ 01″
$\Delta_{11} = \Delta_{10} + \delta_{11}$	8°*o* 27′ 01″.1 + 0° 54′ 19″.4	9° 21′ 20″ .5	9° 21′ 21″
$\Delta_{12} = \Delta_{11} + \delta_{12}$	9°*o* 21′ 20″.5 + 0° 54′ 19″.4	10° 15′ 39″ .9	10° 15′ 40″
$\Delta_{13} = \Delta_{12} + \delta_{13}$	10°*o* 15′ 39″.9 + 0° 54′ 19″.4	11° 09′ 59″.3	11° 09′ 59″
$\Delta_{14} = \Delta_{13} + \delta_{14}$	11°*o* 09′ 59″.3 + 0° 54′ 19″.4	12° 04′ 18″.7	12° 04′ 19″
$\Delta_{15} = \Delta_{14} + \delta_{15}$	12°*o* 04′ 18″.7 + 0° 50′ 58″.4	12° 55′ 17″.1	12° 55′ 17″

Example 15.15. *For preparing a track for national games, following theodolite traverse was conducted :*

Side	*Length (m)*	*Bearing*
AB	*592.65*	*20°*
BC	*501.47*	*85°*
CD	*455.88*	*140°*
DE	*410.29*	*190°*
EF	*501.57*	*245°*
FA	*638.24*	*310°*

Calculate the minimum radius of the circular curves to be introduced at the traverse stations so that the length of the track is exactly 3000 kilometres.

Solution. (Fig. 15.39)

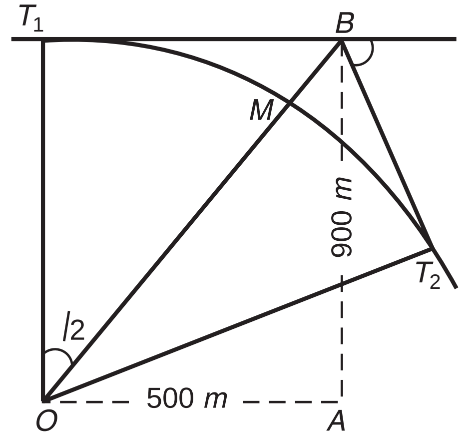


Fig. 11,39.

Deflection angle Δ_1 = Bearing of BC = Bearing of AB

$= 85° - 20° = 65°$

Δ_2 = Bearing of CD – Bearing of BC

$= 140° - 85° = 55°$

Δ_3 = Bearing of DE – Bearing of CD

$= 190° - 140° = 50°$

Δ_4 = Bearing of EF – Bearing of DE

$= 245° - 190° = 55°$

Δ_5 = Bearing of FA – Bearing of EF

$= 310° - 245° = 65°$

Δ_6 = Bearing of AB – Bearing of FA

$= 20° - 310° = 70°$

Let R be the radius of the circular curves.

and $l_1, l_2, \ldots l_6$ be the lengths of the curves.

$$AB + BC + CD + DE + FF + FA + l_1 + l_2 + l_3 + l_4 + l_5 + l_6$$

$$-\left(2R \tan\frac{\Delta_1}{2} + 2R\tan\frac{\Delta_2}{2} + 2R\tan\frac{\Delta_3}{2} + 2R\tan\frac{\Delta_4}{2} + 2R\tan\frac{\Delta_5}{2} + 2R\tan\frac{\Delta_6}{2}\right) = 3000$$

or
$$592.65 + 501.47 + 455.88 + 410.29 + 501.57 + 638.24 + \frac{\pi R\,\Delta_1}{180} + \frac{\pi R\,\Delta_2}{180} + \frac{\pi R\Delta_3}{180} + \frac{\pi R\,\Delta_4}{180} + \frac{\pi R\,\Delta_5}{180} + \frac{\pi R\,\Delta_6}{180}$$
$$- 2R\left(\tan\frac{\Delta_1}{2} + \tan\frac{\Delta_2}{2} + \tan\frac{\Delta_3}{2} + \tan\frac{\Delta_4}{2} + \tan\frac{\Delta_5}{2} + \tan\frac{\Delta_6}{2}\right) = 3000$$

or
$$3100 + \frac{\pi R}{180}(65° + 55° + 55° + (65° + 70°o)$$
$$- 2R\left(\tan\frac{65°}{2} + \tan\frac{55°}{2} + \tan\frac{50°}{2} + \tan\frac{55°}{2} + \tan\frac{65°}{2} + \tan\frac{70°}{2}\right) = 3000$$

or
$$3100 + \frac{\pi R}{180} \times 360° - 2R\left(\tan\frac{65°}{2} + \tan\frac{55°}{2} + \tan\frac{50°}{2} + \tan\frac{55°}{2} + \tan\frac{65°}{2} + \tan\frac{70°}{2}\right) = 3000$$

or
$$2\pi R - 2R(0.6370700 + 0.5205670 + 0.4663080 + 0.630700 + 0.5205670 + 0.7002070) = 3000 - 3100$$

or
$$2R(\pi - 3.481789) = -100$$

or
$$R = \frac{-50}{\pi - 3.481789}$$

or
$R = 146.97$ m. **Ans.**

Example 15.16. *To connect two straights AB and CB, by two right handed simple circular curves separated by a straight MN, 120 m long, following data was recorded ;*

Bearing of AB = 35° 30′

Bearing of MN = *85° 30′*

Bearing of CB = *295° 30′*

Chainage of B = *35 chains + 35 links of 30 m chain*

Total length of both curves including straight is 600 m.

Radius of the first curve is half that of second.

Calculate :

(*i*) *the radii of curves*

(*ii*) *the chainages of the points of tangencies.*

Solution. (Fig. 15.40).

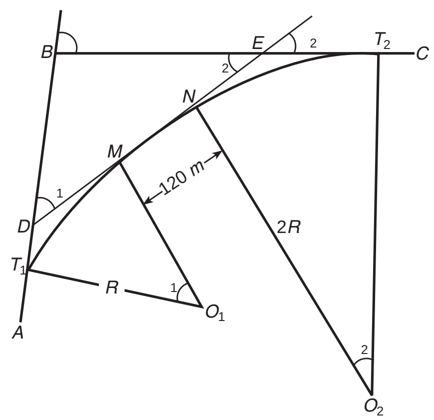


Fig. 15.40.

Construction : Extend *MN* on either side to intersect *AB* and *BC* at *D* and *E* respectively.

Let the radius and deflection angle of the first curve be R and Δ_1, radius and deflection angle of the second curve be $2R$ and Δ_2.

$$\Delta_1 = \text{Bearing of } MN - \text{Bearing of } AB$$

$$= 85° \, 30' - 35° \, 30' = 50° \, .$$

$$\Delta_2 = \text{Bearing of } CB - \text{Bearing of } NM$$

$$= 295° \, 30' - (85° \, 30' + 180°) = 30°$$

Deflection angle of the main curve

$$= \Delta_1 + \Delta_2 = 50° + 30° = 80°$$

Length of the first curve $= \dfrac{\pi R \, \Delta_1}{180°} = \dfrac{\pi R 50°}{180°}$

Length of the second curve $= \dfrac{\pi . \, 2 \, R \, \Delta_2}{180°}$

$$= \frac{2\pi R\ 30°}{180°}$$

$\therefore$ Total length of the curve

$$= \frac{\pi R\ 50°}{180°} + \frac{2\pi R\ 30°}{180°} + 120 = 600 \text{ (Given)}$$

or $$R\left[\frac{50\pi}{180} + \frac{60\ \pi}{180}\right] = 600 - 120 = 480$$

$$R = \frac{480 \times 180}{110\ \pi} = 250 \text{ m}$$

Radius of the first curve = 250 m

Radius of the second curve = 500 m

Length of the tangent, $T_1D = R \tan \frac{\Delta_1}{2}$

$= 250 \tan 25° = 116.58$ m

Length of the tangent $T_2 E = 2R \tan \frac{\Delta_2}{2}$

$= 500 \tan 15° = 133.97$ m

$\therefore$ $DE = 116.58 + 120.0 + 133.97 = 370.55$ m

By solving $\Delta\, BDE$,

$$DB = \frac{DE \sin 30°}{\sin 100°}$$

$$= \frac{370.55 \sin 30°}{\sin 100°} = \frac{370.55 \times 0.5}{0.984808}$$

$= 188.13$ m

$\therefore$ $T_1B = T_1D + DB$

$= 116.58 + 188.13 = 304.71$ m

Chainage of B = 35 chains + 35 links

$= 35 \times 30 + 35 \times 0.2$

$= 1057$ m.

Chainage of T_1 = chainage of $B - T_1B$

$= 1057 - 304.71$

$= 752.29$ m. **Ans.**

Chainage of M = chainage of T_1 + length of 1st curve

$$= 752.29 + \frac{\pi \times 250 \times 50}{180°o}$$

$$= 752.29 + 218.17 = 970.46 \text{ m.} \quad \textbf{Ans.}$$

Chainage of N = chainage of M + 120

$$= 970.46 + 120 = 1090.46 \text{ m.} \quad \textbf{Ans.}$$

Chainage of T_2

$$= \text{chainage of } N + \text{length of second curve}$$

$$= 1090.46 + \frac{\pi \times 500 \times 30°}{180°}$$

$$= 1090.46 + 261.80$$

or Chainage of T_2 $= 1352.26$ m. **Ans.**

Example 15.17. *The parallel sides of a trapezoidal plot of land are 500 m and 800 m, the other sides measure 1500 m each. Calculate the perimeter of the largest sized pond to be dug out, having its two ends circular.*

Solution. (Fig. 15.41).

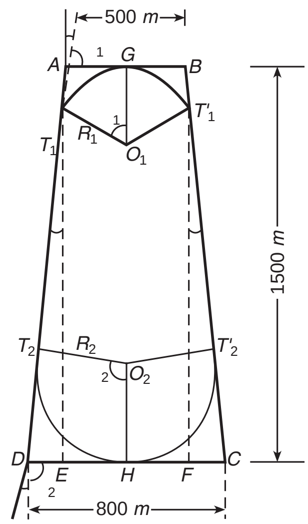


Fig. 15.41.

Construction : Drop AE and BF perpendiculars to DC.

Let $\angle EAD = FBC = \theta$

$$\sin \theta = \frac{150}{1500} = 0.1$$

$$\theta = 5° \, 44' \, 21''$$

Let O_1 and O_2 be the centres of the circular ends which are tangential to sides AB and CD respectively.

Let the radii be R_1 and R_2.

Apparently $\angle T_1O_1G$

= angle of deflection of the straight T_1A and AB.

$$= 90° - 5° 44' 21''$$

$$= 84° 15' 39''$$

Similarly, $\angle T_2O_2H =$ Angle of deflection of straight T_2D and DH

$$= 90° + 5° 44' 21'' = 95° 44' 21''$$

Tangent $$AT_1 = R_1 \tan \frac{84° 15' 39''}{2} = 250 \text{ m}$$

G being mid-point of AB

or $$R_1 = \frac{250}{\tan 42° 07' 50''} = 276.38 \text{ m}$$

$$T_2D = \frac{R_2 \tan 95° 44' 21''}{2} = 400 \text{ m}$$

or $$R_2 = \frac{400}{\tan 47°o\ 52' 10''} = 361.81 \text{ m}$$

$$T_1T_2 = T_1'T_2' = 1500 - (276.38 + 361.81)$$

$$= 861.81 \text{ m}$$

Length of circular end $T_1GT_1' = \dfrac{2\pi R_1 \Delta_1}{180°}$

$$= \frac{2\pi \times 276.38 \times 84.260833}{180}$$

$$= 812.90 \text{ m}$$

Length of circular end T_2HT_2'

$$= \frac{2\pi R_2 \Delta_2}{180°} = \frac{2\pi \times 361.81 \times 95.739166}{180°}$$

$$= 1209.14 \text{ m}$$

$\therefore$ Total perimeter of the pond

$$= 2 \times 861.81 + 812.90 + 1209.14$$

$$= 3745.66 \text{ m.}$$ **Ans.**

Example 15.18. *In order to layout a pond in a public park, two perpendiculars AD and BC of 40 m and 80 m length respectively were* *erected at either end of a line AB of length 240 m. If the pond is to have straight sides laying along AB and DC, the ends being formed of circular arcs to which AB, DC and end perpendiculars are tangential. Calculate:*

(*i*) *The radii of the sub circular arcs.*

(*ii*) *The perimeter of the pond.*

Solution. (Fig. 15.42).

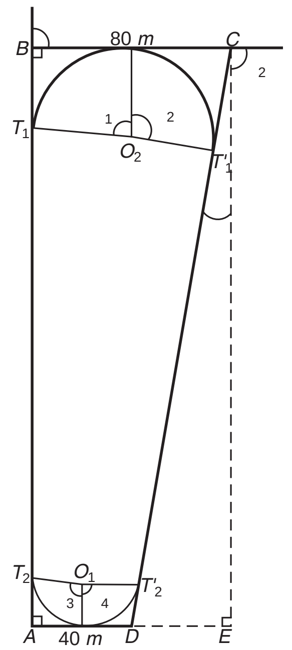


Fig. 15.42

Drop $\perp$ *CE* on *AD* produced,

In ΔDCE

$$\tan \theta = \frac{DE}{CE} = \frac{40}{240} = 0.166667$$

or $$\theta = 9° \; 27' \; 44''$$

Deflection angle of the straights

AB and *BC*, $\Delta_1 = 90°$

Deflection angle of the straights

DC and *CB*, $\Delta_2 = 90° + 9° \; 27' \; 44''$

$$= 99° \; 27' \; 44''$$

Let R_2 be the radius of the circle of centre O_2

$$R_2 \tan \frac{90°}{2} + R_2 \tan \frac{99° 27' 44''}{2}$$

$$= 80 \text{ m}$$

or R_2 (tan 45° + tan 49° 43′ 52″) = 80

$$R_2 = \frac{80}{1 + 1.18046} = 36.69 \text{ m.} \quad \textbf{Ans.}$$

Deflection angle of straights BA and AE, $\Delta_3 = 90°$

Deflection angle of straights CD DA, Δ_4

$$= 90° - 9° 27' 44'' = 80° 32' 16''$$

Let R_1 be the radius of the circle of centre O_1

then $$R_1 \tan \frac{90°}{2} + R_1 \tan \frac{80° 32' 16''}{2} = 40$$

or R_1 (tan 45° + tan 40° 16′ 08″) = 40

or $$R_1 = \frac{40}{1 + 0.847128} = 21.66 \text{ m.} \quad \textbf{Ans.}$$

$$T_1 T_2 = 240 - (R_1 + R_2)$$

$$= 240 - (36.69 + 21.66)$$

$$= 181.65 \text{ m}$$

$$T_1' T'_2 = CD - (CT_1' + DT_2')$$

$$= \frac{CE}{\cos \theta} - \left(R_2 \tan \frac{\Delta_2}{2} + R_1 \tan \frac{\Delta_4}{2}\right)$$

$$= \frac{240}{\cos 9° 27' 44''} - (36.69 \tan 49° 43' 52'' + 21.66 \tan 40° 16' 08'')$$

$$= 243.31 - (43.31 + 18.35)$$

$$= 181.65 \text{ m}$$

Length of the circular arc $T_1 T_1'$

$$= \frac{\pi R_2 (\Delta_1 + \Delta_3)}{180°}$$

$$= \frac{\pi \times 36.69 \times (90° + 99°o\ 27' 44'')}{180°}$$

$$= \frac{\pi \times 36.69 \times 189.46222}{180°o}$$

$$= 121.32 \text{ m}$$

Length of arc $T_2\,T_2'$

$$= \frac{\pi R_1\,(\Delta_3 + \Delta_4)}{180°}$$

$$= \frac{\pi \times 21.66 \times (90° + 80'\,32'\,16'')}{180°}$$

$$= \frac{\pi \times 21.66 \times 170°\,32'\,16''}{180°}$$

$$= \frac{\pi \times 21.66 \times 170.53777}{180°o}$$

$$= 64.47 \text{ m}$$

Perimeter of the pond

$$= T_2\,T_1 + \text{arc } T_1\,T_1' + T_1'\,T_2' + \text{arc } T_2'\,T_2$$

$$= 181.65 + 121.32 + 181.65 + 64.47$$

$$= 549.09 \text{ m.}$$ **Ans.**

Example 15.19. *Two straights AB and BC which intersect at B are connected by a circular curve passing through a point D. Independent co-ordinates of the points A, B, C and D are as under :*

Point	*Easting*	*Northing*
A	*515*	*215*
B	*1725*	*1005*
C	*2325*	*621*
D	*1425*	*528*

Calculate the radius of the curve. Also, calculate the length and bearing of the mid-point of the curve from D.

Solution. (Fig. 15.43).

Difference in eastings of *A* and *B*

$$= 1725 - 515 = 1210 \text{ m}$$

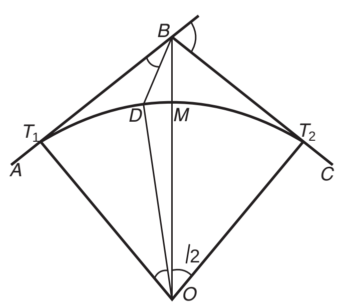


Fig. 15.43

Difference in northings of A and B

$$= 1005 - 215 = 790 \text{ m}$$

Let θ_1 be the reduced bearing of AB

$$\tan \theta_1 = \frac{1210}{790} = 1.5316455$$

$$\theta_1 = 56° \; 51' \; 35''$$

$\therefore$ Bearing of $BA = 180° + 56° \; 51' \; 35''$

$$= 236° \; 51' \; 35''$$

Difference in eastings of B and D

$$= 1725 - 1425 = 300$$

Difference in northings of B and D

$$= 1005 - 528 = 477 \text{ m}$$

Let θ_2 be the reduced bearing of DB

$$\tan \theta_2 = \frac{300}{477} = 0.6289308$$

$$\theta_2 = 32° \; 10' \; 02''$$

Bearing of BD $= 180° + 32° \; 10' \; 02''$

$$= 212° \; 10' \; 02''$$

Angle $\alpha =$ Bearing of BA – Bearing of BD

$$= 236° \; 51' \; 35'' - 212^o \; 10' \; 02''$$

$$= 24° \; 41' \; 33''$$

$$BD = \sqrt{300^2 + 477^2}$$

$$= 563.50 \text{ m}$$

Difference in eastings of B and C

$$= 2325 - 1725 = 600 \text{ m}$$

Difference in northings of B and C

$$= 1005 - 621 = 384 \text{ m}$$

Let the reduced bearing of BC be θ_3

$$\tan \theta_3 = \frac{600}{384} = 1.562500$$

$$\theta_3 = 57° \; 22' \; 51''$$

Bearing of BC $= 180° - 57° \; 22'.51''$

$= 122° 37' 09''$

Angle of Deflection $\Delta =$ Bearing of BC-- Bearing of AB

$= 122° 37' 09'' - 56^o 51' 35''$

$= 65° 45' 34''$

Let $\angle T_1OD = \theta$, then Eqn. (15.26)

$$\cos(\alpha + \theta) = \frac{\cos\left(24°o\ 41'\ 33'' + \dfrac{65°\ 45'\ 34''}{2}\right)}{\cos \dfrac{65°\ 45'\ 34''}{2}}$$

$$= \frac{\cos 57°\ 34'\ 20''}{\cos 32°\ 52'\ 47''}$$

$$= \frac{0.536236}{0.8398120} = 0.6385191$$

or $\alpha + \theta = 50° 19' 07''$

$\theta = 50° 19' 07'' - 24° 41' 33''$

$= 25° 37' 34''$

From Eqn. (15.27)

$$R = \frac{Z \sin \alpha}{1 - \cos \theta}$$

or

$$R = \frac{563.5 \sin 24°\ 41'\ 33''}{1 - \cos 25°\ 37'\ 34''}$$

$$= \frac{563.5 \times 0.417748}{1 - 0.901636} = 2393.16 \text{ m.}$$ **Ans.**

Appex distance $BM = R\left(\sec \dfrac{\Delta}{2} - 1\right)$

$$= 2393.16\left(\sec \frac{65°\ 45'\ 34''}{2} - 1\right)$$

$= 2393.16\ (1.1907426 - 1)$

$= 456.48$ m. **Ans.**

Bearing of BM = Bearing of $BC + \dfrac{1}{2}(180° - 65° 45' 34'')$

$= 122° 37' 09'' + 57^o 07' 13''$

$= 179° 44' 22''$

$\angle DBM =$ Bearing of BD – Bearing of BM

$= 212° \ 10' \ 02'' - 179° \ 44' \ 22''$

$= 32° \ 25' \ 40''$

Applying cosine formula to ΔBDM, we get

$$DM = \sqrt{BD^2 + BM^2 - 2BD \cdot BM \cos DBM}$$

$$= \sqrt{(563.5)^2 + (456.48)^2 - 2 \times 563.5 \times 456.48 \times \cos 32° \ 25' \ 40''}$$

$$= \sqrt{525906.24 - 434233.28}$$

$$= 302.78 \text{ m.}$$

Applying sine rule to ΔBDM we get

$$\sin BDM = \frac{BM \sin DBM}{DM}$$

$$= \frac{456.48 \sin 32° \ 25' \ 40''}{302.78}$$

$$= 0.80844507$$

or angle $BDM = 53° \ 56' \ 40''$

$\therefore$ Bearing of DM = Bearing of $DB + \angle BDM$

$= 212° \ 10' \ 02'' - 180° + 53° \ 56' \ 40''$

$= 86° \ 06' \ 42''$

Distance of $DM = 302.78$ m

Bearing of $DM = 86° \ 06' \ 42''$ **Ans.**

Example 15.20. *A portion of an existing highway consists of three circular curves and two straight portions in between them as under :*

Arc / straight	*Deflection*	*Radius*	*Length*
AB	*70° right*	*257.07 m*	-
BC	-	-	*185 m*
CD	*70° left*	*121.39 m*	-
DE	-	-	*75 m*
EF	*55° right*	*172.89*	-

It is now decided to replace these by a circular arc by shifting the points of tangencies A and F by equal amount.

Calculate the radius of the curve. Assuming the chainage of A as 20 km, calculate the revised chainage of F.

Solution. (Fig. 15.44).

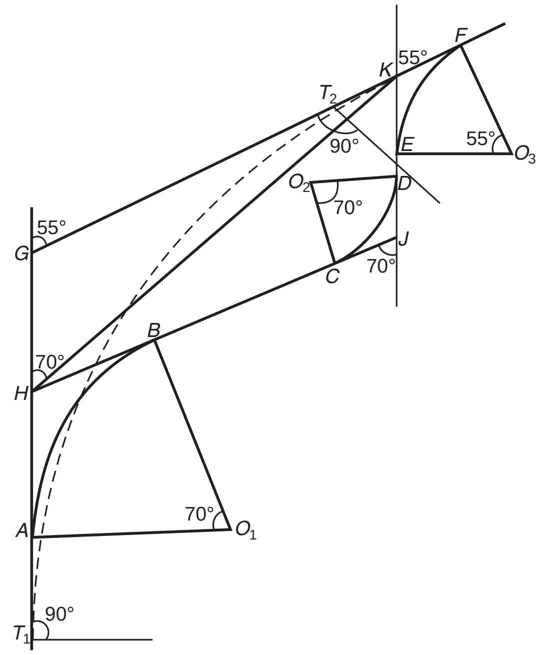


Fig. 15.44

$$AH = HB = R_1 \tan \frac{\Delta_1}{2}$$

$$= 257.07 \tan 35° = 180 \text{ m}$$

$$CJ = JD = R_2 \tan \frac{\Delta_2}{2}$$

$$= 121.39 \tan 35° = 85 \text{ m}$$

$$EK = KF = R_3 \tan \frac{\Delta_3}{2}$$

$$= 172.89 \tan 27° 30' = 90 \text{ m}$$

In ΔHJK,

$$HJ = HB + BC + CJ = 180 + 185 + 85 = 450 \text{ m}$$

$$JK = JD + DE + EK = 85 + 75 + 90 = 250 \text{ m}$$

$$\angle HJK = 180° - 70° - 110°$$

Applying consine formula

$$HK = \sqrt{HJ^2 + JK^2 - 2HJ \,.\, JK \cos 110°}$$

$$= \sqrt{450^2 + 250^2 - 2 \times 450 \times 250 \times \cos 110°}$$

$$= \sqrt{265000.0 + 76954.5} = 584.77 \text{ m}$$

Applying sine formula,

$$\sin JHK = \frac{JK \sin HJK}{HK}$$

$$= \frac{250 \sin 110°}{584.77} = 0.40173615$$

or $$\angle JHK = 23° \, 41' \, 12''$$

and $$\angle HKJ = 180° - (110° + 23° \, 41' \, 12'')$$

$$= 46° \, 18' \, 48''$$

In ΔGHK, $\angle GHK = \angle HKJ = 46° \, 18' \, 48''$

$$\angle GKH = \angle GKJ - \angle HKJ$$

$$= 55° - 46° \, 18' \, 48'' = 8° \, 41' \, 12''$$

Applying sine rule to ΔGHK

$$GH = \frac{HK \sin 8° \, 41' \, 18''}{\sin 125°}$$

$$= \frac{584.77 \sin 8° \, 41' \, 18''}{\sin 125°} = 107.84 \text{ m}$$

$$GK = \frac{HK \sin 46° \, 18' \, 48''}{\sin 125°}$$

$$= \frac{584.77 \sin 46° \, 18' \, 48''}{\sin 125°} = 516.22 \text{ m}$$

Tangent $GF = GK + KF = 516.22 + 90.0 = 606.22$ m

Tangent $GA = GH + HA = 107.84 + 180.0 = 287.84$ m

Mean tangent length $= \dfrac{606.22 + 287.84}{2} = 447.03$ m

Let R be the radius of the required curve

$$R \tan \frac{55°}{2} = 447.03$$

or $$R = \frac{447.03}{\tan 27° \, 30'}$$

$$= 858.74 \text{ m.}$$ **Ans.**

Length of arc $AB = \dfrac{\pi R_1 \Delta_1}{180°} = \dfrac{\pi \times 257.07 \times 70°}{180°} = 314.07$ m

Length of arc $CD = \dfrac{\pi R_2 \Delta_2}{180°}$

$$= \frac{\pi \times 121.39 \times 70°}{180°} = 148.31 \text{ m}$$

Length of arc $EF = \frac{\pi R_3 \Delta_3}{180°}$

$$= \frac{\pi \times 172.89 \times 55}{180°} = 165.96 \text{ m}$$

Length of arc $T_1 T_2 = \frac{\pi R \Delta}{180°}$

$$= \frac{\pi \times 858.74 \times 55}{180°} = 824.33 \text{ m}$$

Existing Chainage of F = Chainage of A + arc AB + BC + arc CD + DE + arc EF

= 20,000 + 314.07 + 185 + 148.31 + 75 + 165.96

= 20,888.34 m

Revised chainage of F

= Chainage of $A - A\,T_1$ + arc $T_1\,T_2 + T_2\,F$

= Chainage of A + arc $T_1\,T_2$ $(AT_1 = TF_2)$

= 20,000 + 824.33 m

= 20,824.33 m.

∴ Change in the chainage of F

= 20,888.34 – 20824.33

= 64.01 m. **Ans.**

Example 15.21. *Two straights AB and BC of a railway curve intersect at B and deflects through 75°. The length of the curve is 523.60 m. To shift the curve atleast by 200 m to avoid an obstacle, the following operation is made. A peg P is fixed on the back straight AB such that* T_1P *is 225.07 m. Another peg Q is fixed on the forward straight BC such that* T_2Q *is 175.07 m from* T_2*. Two circular curves of radius 200 m each are inserted at the end of the line PQ and straights. Ascertain whether desired shift is available.*

Assuming that a 15 km stone is just located at the point of commencement T_1 *of the existing curve, calculate the chainages of salient points of the re-aligned curve.*

Solution. (Fig. 15.45).

Let R be the radius of existing circular curve

The length of main circular curve

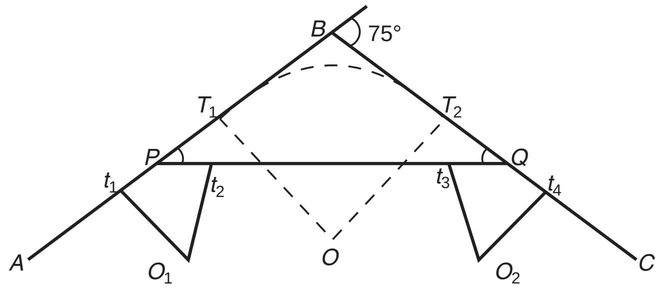


Fig. 15.45.

$$\frac{\pi R\Delta}{180°o} = 523.60$$

or $$R = \frac{523.60 \times 180°}{\pi \times 75°} = 400.00 \text{ m}$$

Length of tangent $$T_1 B = R \tan \frac{1}{2}\Delta = T_2 B$$
$$= 400 \tan 37° 30'$$
$$= 306.93 \text{ m.}$$

In ΔPBQ, we get

$$PB = BT_1 + PT_1 = 306.93 + 225.07 = 532.00 \text{ m}$$
$$BQ = BT_2 + QT_2 = 306.93 + 175.07 = 482.00 \text{ m}$$

$\angle BPQ = \alpha$; $BQP = \beta$; $\angle PBQ = 105°$

$$PQ^2 = PB^2 + QB^2 - 2PB \,.\, QB \,.\, \cos 105°$$
$$= 532^2 + 482^2 - 2 \times 532 \times 482 \times \cos 105°$$
$$= 515348.00 + 132734.80$$

or $$PQ = 805.04 \text{ m}$$

Applying sine rule to ΔPBQ, we get

$$\frac{\sin\alpha}{BQ} = \frac{\sin\beta}{BP} = \frac{\sin 105°}{PQ}$$

or $$\sin\alpha = \frac{\sin 105°}{805.04} \times 482 = 0.57832695$$

or $$\alpha = 35° 20' 00''$$

and $$\sin\ \beta = \frac{\sin 105° \ \times 532}{805.04} = 0.63831937$$

or $$\beta = 39° 40' 00''$$

Length of tangent $t_1p = pt_2$

$$= R \tan \frac{\Delta_1}{2} = 200 \times \frac{\tan 35° \, 20'}{2}$$

$$= 200 \tan 17° \, 40' = 63.70 \text{ m}$$

Length of tangent $t_3Q = t_4Q$

$$= 200 \tan \frac{39° \, 40'}{2} = 200 \tan 19° \, 50'$$

$$= 72.14 \text{ m}$$

Length of curve t_1t_2 $= \dfrac{\pi \times 200 \times 35.3333}{180°} = 123.34$ m

Length of curve t_3t_4 $= \dfrac{\pi \times 200 \times 39.66667}{180°} = 138.46$ m

Chainage of T_1 = 15,000.00 m

Chainage of t_1 = 15000 – (225.07 + 63.70)

= 14711.23

Chainage of t_2 = 14711.23 + 123.34 = 14,834.57 m

Chainage of t_3 = 14,834.57 + [805.04 – (63.70 + 72.14)

= 14834.57 + (805.04 – 135.84)

= 14,834.57 + 669.20

= 15,503.77 m

Chainage of t_4 = 15,503.77 + 138.46

= 15,642.23 m.

Check :

The apex point of the existing curve

$$= \left(\sec \frac{\Delta}{2} - 1\right) = 400 \left(\sec \frac{75°}{2} - 1\right)$$

= 104.19 m

The nearest point of revised alignment

$= BP \sin \alpha$

= 532 sin 35° 20′

= 307.67 m

∴ Shift in alignment

= 307.67 – 104.19

= 203.48 m greater than 200 m, safe.

Example 15.22. *The bearings of two straights AB and CD which intersect at E are 65° and 110° respectively. Calculate the radius of two similar circular curves to be introduced between AEC and BED so that the perimeter of the number 8 so formed, is 1000 m.*

Solution. (Fig. 15.46).

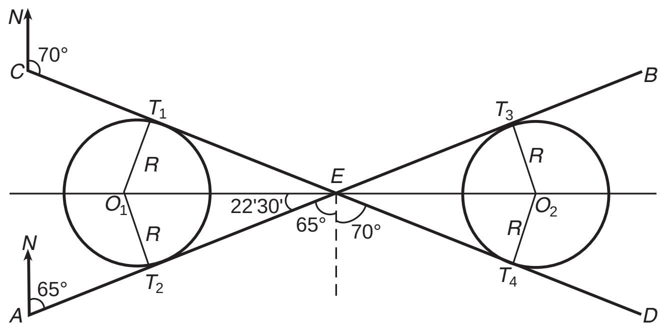


Fig. 15.46

Angle AED = Angle $CEB = 65° + 70° = 135°$

$\therefore$ Angle $T_1 ET_2 = 180° - 135° = 45°$

or Angle $O_1ET_2 = \dfrac{45°}{2} = 22° \ 30'$

Angle $T_1O_1T_2 = T_3O_2T_4 = 180° - 45° = 135°$

Exterior angle $T_1O_1T_2$ = Exterior angle $T_3O_2T_4$

$= 360° - 135° = 225°o$

From the Fig. 15.46 the perimeter of number 8

$= 2(ET_2 + ET_3) + (\text{arc } T_1 T_2 + \text{arc } T_3 T_4) = 1000 \text{ m}$

$$= 4 \times \frac{R}{\tan 22° \ 30'} + 2 \frac{\pi R \Delta}{180°} = 1000$$

or $$= R\left(\frac{2}{\tan 22° \ 30'} + \frac{\pi \times 225°}{180}\right) = 500$$

or $$R = \frac{500}{\dfrac{2}{\tan 22°o \ 30'} + \dfrac{\pi \times 225}{180°}} = \frac{500}{4.828422 + 3.9269907}$$

$R = 57.11$ m. **Ans.**

Example 15.23. *The following theodolite traverse is carried out :*

Side	*Length (m)*	*Deflection angle*
$T_1 B$	*80*	0°
BE	*80*	70° *right*
EC	*80*	0°

CF	*80*	120° *left*
FD	?	0°
DG	?	140° *right*

Calculate the radii of three circular curves to be inserted between T_1 and *E, E and F, and F and G. Also, calculate the radius of simple circular curve to be inserted between straights* T_1D *and DG produced, originating from* T_1 *and passing through F.*

Solution. (Fig. 15.47)

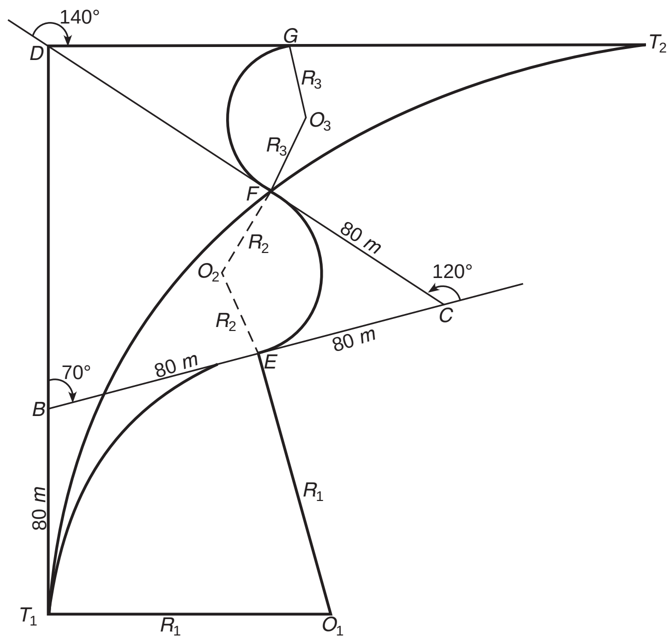


Fig. 15.47.

Let R_1 be the radius for arc T_1E

$$\therefore \quad R_1 \tan \frac{70°}{2} = 80$$

or
$$R_1 = \frac{80}{\tan 35°} = 114.25 \text{ m}$$

Let R_2 be the radius for arc *EF*

$$\therefore \quad R_2 \tan \frac{120°}{2} = 80$$

or
$$R_2 = \frac{80}{\tan 60°} = 46.19 \text{ m}$$

Let R_3 be radius of arc *FG*

Applying sine rule to ΔBCD we get

$$\frac{DC}{\sin 70°} = \frac{BC}{\sin [180° - (70° + 60°)]} = \frac{160°}{\sin 50°}$$

or $$DC = \frac{160 \sin 70°}{\sin 50°} = 196.27 \text{ m}$$

$\therefore$ $$DF = 196.27 - 80 = 116.27 \text{ m}$$

or $$R_3 \tan \frac{140°}{2} = 116.27$$

or $$R_3 = \frac{116.27}{\tan 70°o} = 42.32 \text{ m}$$

From Eqn. (15.26), we know

$$\text{Cos } (\alpha + \theta) = \frac{\cos\left(\alpha + \frac{\Delta}{2}\right)}{\cos \frac{\Delta}{2}} = \frac{\cos (50° + 45°)}{\cos 45°}$$

$$\cos (\alpha + \theta) = \frac{\cos 95°}{\cos 45°} = -1.2325673$$

$$\alpha + \theta = 97° \, 04' \, 48''$$

$$\theta = 97° \, 04' \, 48'' - 50° = 47° \, 04' \, 48''$$

From Eqn. (15.27) we get

$$R = \frac{Z \sin \alpha}{1 - \cos \theta} = \frac{116.27 \sin 50°}{1 - \cos 47° \, 04' \, 48''}$$

or $R = 279.19$ m. **Ans.**

Example 15.24. *The following theodolite traverse has been carried out for designing the layout of a reverse curve.*

Side	*Bearing*	*Length*
AB	*N 20° W*	*?*
BC	*N 20° E*	*900 m*
CD	*S 70° W*	*?*

If the distance of point of reverse curvature E on side BC, is 500 m from B and chainage of point B is 1350 m,

Calculate :

(*i*) *Radii of arcs of reverse curve*

(*ii*) *Chainages of point of tangency, point of reverse curve and the point of intersection of AB (produced) and CD.*

Solution. (Fig. 15.48)

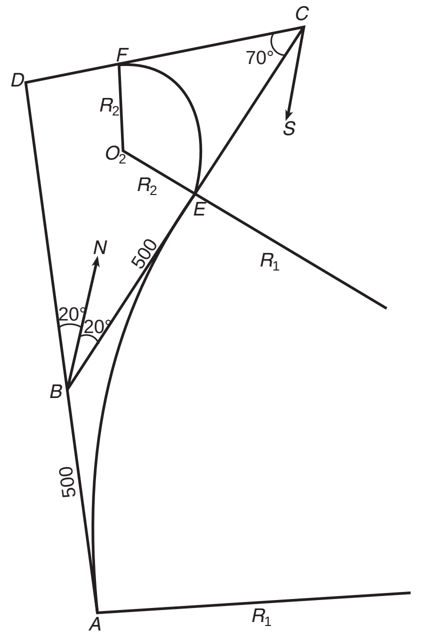


Fig. 15.48.

From ΔBCD, angle $BCD = 70° - 20° = 50°$

$$\text{angle } CDB = 180° - (50°o + 40°) = 90°$$

$$BE = BA = 500 \text{ m} \qquad \text{(Given)}$$

Let R_1 be the radius of the curve AE.

$$R_1 \tan \frac{\Delta}{2} = 500 \qquad \text{where } \Delta = 40°$$

or $$R_1 = \frac{500}{\tan 20°} = 1373.74 \text{ m}$$

Let R_2 be the radius of arc EF

$$R_2 \tan \frac{\Delta}{2} = 400 \qquad \text{where } \Delta = 180° - 50° = 130°$$

or $$R_2 = \frac{400}{\tan 65°} = 186.52 \text{ m}$$

Length of curve

$$AE = \frac{\pi R_1 \Delta}{180°} = \frac{\pi \times 1373.74 \times 40°}{180°} = 959.05 \text{ m}$$

$$\text{Length of curve } EF = \frac{\pi R_2 \, \Delta}{180^\circ} = \frac{\pi \times 186.52 \times 130^\circ}{180^\circ} = 423.20 \text{ m}$$

Chainage of $B = 1350$ m

Chainage of $A = 1350 - 500 = 850$ m. **Ans.**

Chainage of $E = 850 +$ length of curve AE

$= 850 + 959.05 = 1809.05$ m. **Ans.**

Chainage of $F = 1809.05 +$ length of curve EF

$= 1809.05 + 423.20 = 2232.25$ m. **Ans.**

From right anlged triangle BDC we get

$CD = BC \cos 50^\circ$

$= 900 \cos 50^\circ = 578.51$ m

$FD = 578.51 - 400 = 178.51$ m

Chainage of $D = 2232.25 + 178.51 = 2410.76$ m. **Ans.**

Example 15.25. *Two straight roads BA and CA intersect at A. They are intersected by a line EF, making angles 140° and 150° with BA and CA respectively. The curve TP is OF 10 chain radius and curve T_1P is of 15 chain radius.*

Calculate the lengths AT, AT_1, EF and AF.

Solution. (Fig. 15.49)

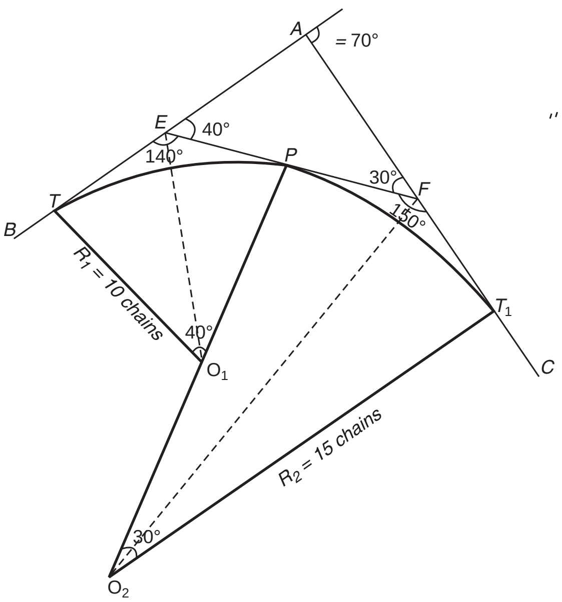


Fig. 15.49.

$$\angle AEF = 180° - 140° = 40°$$

$$\angle AFE = 180° - 150° = 30°$$

$\therefore$ Angle of deflection

$$\Delta = 40° + 30° = 70°$$

Now, $$EP = ET = R_1 \tan \frac{40°}{2} = 3.64 \text{ chains}$$

$$FP = FT_1 = R_2 \tan \frac{30^2}{2} = 4.02 \text{ chains}$$

$EF = EP + PF = 3.64 + 4.02 = 7.66$ chains **Ans.**

$AT = AE + ET = AE + 3.64$ chains **Ans.**

But, $$AE = EF \times \frac{\sin 30°}{\sin 110°} = 4.08 \text{ chains } \textbf{Ans.}$$

and $$AF = EF\frac{\sin 40°}{\sin 110°} = 5.24 \text{ chains } \textbf{Ans.}$$

$AT_1 = AF + FT_1 = 5.24 + 4.02 = 9.26$ chains **Ans.**

EXERCISES 15

1. Fill in the blanks :

(*i*) Curves are introduced between straights for the change in.......

(*ii*) A curve consisting of two or more arcs of different circles with different radii and bending in the same direction, is known as.....curve.

(*iii*) the angle of deflection and angle of intersection of a simple are .....angles.

(*iv*) The angle subtended by the long chord of a simple curve at its centre is equal to angle of......

(*v*) The length of the tangent of a curve whose radius is R and the angle of deflection is Δ = ...

(*vi*) The length of long chord of a curve having angle of deflection Δ and radius R = : ....

(*vii*) The distance between the mid-points of the curve and the long chord is known as ......

(*viii*) The angle subtended by an arc of chord of 30 metres of a curve at its centre, is known as......of the curve.

(*ix*) If the degree of a curve is 3°, the radius of the curve is ......

(*x*) If the radius of a curve is 343.78 m, the degree of the curve is......

(*xi*) The approximate formula for perpendicular and radial offsets from the tangents is......

(*xii*) General formula for the offsets from the chords produced is given by the equation.....

(*xiii*) If C is the length of a chord and R is the radius of the curve, the Rankine's angle of deflection of the point, is......

(*xiv*) Location of points on the curve by two theodolite method involves no......measurements.

(*xv*) A simple circular curve is designated by......or......of the curve.

2. Pick up the correct word(s) from the brackets to fill up the blanks.

(*i*) The angle between the tangents is known as angle of......
(intersection, deflection)

(*ii*) The centres of two arcs of a compound curve lie on the......side of the common tangent. (same, opposite)

(*iii*) If the angle of intersection of a curve is $\alpha°$, the angle of deflection is equal to...... $(\alpha° + 180°, 180^o - \alpha°, 90° - \alpha°o)$

(*iv*) If R is the radius of a curve and $\Delta°$ is its angle of deflection, its tangent length, is...... $\left(R \sin \frac{\Delta}{2}, R \cot, \frac{\Delta}{2}, R \tan \frac{\Delta}{2}\right)$

(*v*) Perpendicular offsets from the tangent of a simple curve are......
$\left(\frac{x}{2R}, \frac{2x}{R}, \frac{x^2}{2R}\right)$

(*vi*) Offsets from the chords produced of a simple curve, are calculated from the for mula......
$\left(\frac{C_n (C_n + C_{n-1})}{2R}, \frac{C_n (C_{n-1} - C_n)}{2R}, \frac{C_{n-1} (C_n + C_{n-1})}{3R}\right)$

(*vii*) Rankines's formula for deflection angles is......
$\left(\delta = \frac{1718.9C}{R}, \frac{17.189C}{R^2}, \frac{171.89}{R^3}\right)$

(*viii*) Setting out a simple curve by two theodolite method eliminates ...... measurements. (linear angular; both)

(*ix*) A simple circular curve is designated by ......
(degree of curve, radius of curve, both)

(*x*) If the radius of a circular curve is 500 m, the degree of the curve is....... (38.378, 3.4378, 0.34378)

3. Draw a neat sketch of a circular curve and show the following elements there on.

(*a*) back tangent (*b*) forward tangent
(*c*) point of commencement (*d*) point of tangency
(*e*) point of intersection (*f*) angle of deflection
(*g*) angle of intersection (*h*) central angle
(*i*) radius of the curve (*j*) long chord.

4. What are the elements of a simple circular curve ? Give their relationships.

5. What is meant by degree of a curve ? Derive its relationship with the radius of the curve.

6. Describe a method of setting out simple circular curves by perpendicular offsets from tangents.

7. Describe the method of setting out simple circular curves by offsets from chords produced.

8. Derive the formula for calculating offsets from chords produced for setting out a simple circular curve.

9. Describe the method of setting out a simple circular curve by method of deflection angles, using a chain and a theodolite.

10. Establish the following relationship

$$\delta = \frac{1718.9C}{R} \text{ minutes}$$

where δ = deflection angle of the chord, C = length of the chord and

R = radius of the curve.

11. Describe briefly the method of setting out a circular curve using two theodolites.

12. Describe the method of setting out a simple curve using the knowledge of tacheometry.

13. While setting out a simple circular curve, it is found that the point of intersection of the tangents falls in deep water of a river. Describe the method to locate the points of commencement and tangency.

14. If the tangents to a circular curve having 500 m radius intersect at an angle of 120° and the chainage of the point of intersection, is 1520.5 m, calculate.

(*i*) Tangent distance (*ii*) Degree of the curve

(*iii*) Length of the long chord (*iv*) Length of the curve

(*v*) Chainage of the points of commencement and tangency.

15. Calculate the perpendicular offsets at 20 m intervals along the tangents to set out first five pegs of a simple circular curve of 250 m radius.

16. Two straights of a road intersect at a chainage 2565.0 m having their angle of intersection equal to 115°. Calculate the chainage of the point of commencement, the point of tangency and the mid-point of the curve if the degree of the curve is 5°.

17. Calculate the ordinates from a 150 m long chord at 10 m interval to set out a simple circular curve of 8°.

18. Tabulate the necessary data to set out a right handed simple circular curve of 600 m radius to connect two straights intersecting at a chainage 3605 m by Rankines method of deflection angles. The angle of deflection of the curve is 25° and the peg interval 30 m.

19. Two straights intersect at chainage 3500.5 m with an angle of intersection of 156°. These two straights are to be connected by a simple circular curve of 200 m radius. Calculate the data necessary by the method of offsets from the chords produced with a peg interval of 20 m. Explain, also the procedure to set out the curve.

20. Two straights *AB* and *BC* intersect at an inaccessible point *B*. To connect them by a simple circular curve of 500 m radius, two points *M* and *N* are selected on *AB* and *BC* respectively and the following data obtained:

$\angle AMN = 153° \, 36'$; $\angle CNM = \angle \, 165° \, 42'$; $MN = 225$ m.

Calculate : (*a*) the chainage of the tangent points, (*b*) the necessary data for setting out the curve by method of deflection angles, given that the chain used is 30 m and the chainage of the point M is 1820.70 m.

21. Two straights *AB* and *BC* intersect at a chainage 1500 m. These are intersected by a third straight $LM = 200$ m long, such that $\angle ALM = 155° \, 18'$ and $\angle LMC = 148° \, 54'$. Calculate the radius of the simple circular curve which will be tangent to the three straights *AB*, *BC* and *LM*. Also, calculate the chainage of the point of tangency and the deflection angle to the mid-point of the curve.

22. Two straights *AB* and *BC* of a circular curve intersect at angle of 150°. Find the radius of the curve so that the curve may pass through a point *P* making the angle *ABP* equal to 35° and *BP* = 60.0 m.

23. In setting out a circular railway curve it was found necessary that the curve should pass through a point at a distance of 50 m from the point of intersection and equidistant from the tangents. If the chainage of the point of intersection is 1000 m and the angle of deflection is 25°, calculate the radius of the curve and the chainages of the points of commencement and tangency.

24. Two roads meets at an angle of 127° 30′. Calculate the necessary data for setting out a curve of 15 chains (30 m) radius to connect two straight portions of the road :

(*a*) if it is intended to set out the curve by chain and offsets only ; and

(*b*) if a theodolite is available.

25. Tabulate the data necessary for setting out the first five pegs of a circular curve with the following data :

Angle of intersection of the straights	= 145°
Chainage of the point of intersection	= 1580 m
Radius of the curve	= 400 m

The curve is to be set out by the method of deflection angles, with pegs at every 30 metres of through chainage with a theodolite having a least count of 20″.

26. Two straights *AB* and *BC* are to be connected by a right hand circular curve. The whole circle bearings of *AAB* and *AC* are 65° and 130° respectively. The curve is to pass through a point *P* such that *BP* is 150 metres and the bearing of the line *BP* is 217° 30′. Determine the radius of the curve in metres and the chainages of the point of commencement and tangency if that of point of intersection is 1500 m.

27. The whole circle bearings of two straights which intersect at a chainage of 10,000 m are 40° and 140° respectively. If the length of the long chord is limited to 700 m, calculate the chainage of the beginning and end of the curve and also the least distance of the curve from the point of intersection.

28. Two straights AV and BV meet at V on the other side of a river. On the near side of the river, a point E was selected on the straight AV and a point F on the straight BV, and the distance from E to F was found to be 102 m.

The angle AEF was found to be 165° 36′ and the angle BFE 168° 44′. If the radius of a circular curve joining the straights is 600 m, calculate the distance along the straights from E and F to the tangent points.

ANSWERS

1. (*i*) direction (*ii*) compound (*iii*) supplementary

(*iv*) deflection (*v*) $R \tan \frac{\Delta}{2}$ (*vi*) $2R \sin \frac{\Delta}{2}$

(*vii*) Apex distance (*viii*) degree (*ix*) 572.97 m

(*x*) 5° (*xi*) $\frac{\alpha^2}{2R}$ (*xii*) $\frac{C_n (C_n + C_{n-1})}{2R}$

(*xiii*) $\frac{1718.9C}{R}$ (*xiv*) Linear (*xv*) degree, radius

2. (*i*) intersection (*ii*) same (*iii*) 180° – α

(*iv*) $R \tan \frac{\Delta}{2}$ (*v*) $\frac{x^2}{2R}$ (*vi*) $\frac{C_n (C_n + C_{n-1})}{2R}$

(*vii*) $\delta = \frac{1718.9\, C}{R}$ (*viii*) Linear (*ix*) both

(*x*) 3.4378.

14. (*i*) 288.67 m, (*ii*) 3.4378 m, (*iii*) 500.00 m,

(*iv*) 523.60 m, (*v*) 123 1.83 m, 1755.43 m

15. 0.8 m ; 3.2 m ; 7.2 m ; 12.8 m ; 20.0 m

16. 2346 m, 2,736 m; 2541 m

17. O_1 = 3.44, O_2 = 6.35, O_3 = 8.74, O_1 = 10.64,

O_5 = 12.05, O_6 = 12.99, O_7 = 13.45

18. Chainage of points of tangencies 3471.98 m, 3733.78 m

19. Chainage of points of tangencies 3457.99 m; 3541.77 m

20. (*a*) 1720.47 ; 2075.64 m

(*b*) δ_1 = 1° 07′ 08″, δ_2 to δ_{21} = 1°*o* 43′ 08″, 0° 09′ 05″

21. 402.24 m ; 1287.02 m, 1678.76 m

22. 1286.53 m

23. 2059.35 m., 543.45 m, 1442.01 m

24. Tangent length = 221.92 m, tangent of curve = 412.33 m

25. $\Delta_1 = 1° 09' 20''$, $\Delta_2 = 3° 18' 20''$, $\Delta_3 = 5° 27' 00''$,
$\Delta_4 = 7° 36' 00''$, $\Delta_5 = 9° 45' 00''$

26. 677.14 m ; 1068.61 m ; 1836.80 m

27. 9455.50 m, 10252.93 m, 253.90 m

28. 90.56 m, 78.12 m.

16

Compound and Reverse Curves

16.1. INTRODUCTION

A compound curve consists of two or more arcs of circles of different radii that deviate in the same direction and join at a common tangent point. These curves may be two-centered, three-centered or so, according to the number of simple arcs these are composed of. For understanding the component parts of a compound curve, a two centered compound curve is discussed in this chapter.

16.2. TWO CENTERED COMPOUND CURVE

Two straights AI and BI (when produced) intersect at I, the point of intersection. A two centered compound curve T_1CT_2 is inserted between them. Two circular arcs T_1C and CT_2 have O_1 and O_2 as their centres. T_1 and T_2 are point of commencement and point of tangency of the curve respectively. (Fig. 16.1).

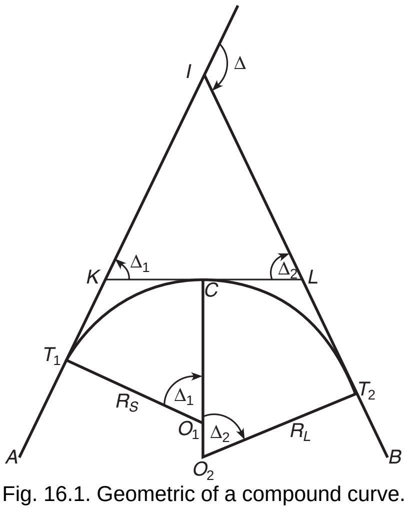


Fig. 16.1. Geometric of a compound curve.

Draw a tangent KL at C the junction point of the circular arcs which meet AI and BI at K and L respectively.

The essential components of a compound curve are :

Δ = total deflection angle

Δ_1 = deflectionangle IKL

Δ_2 = deflection angle ILK

Rs = radius of arc (small) T_1C

R_L = radius of arc (large) T_2C.

T_S = total tangent length (small), T_1I

T_L = total tangent length (large), T_2I.

Angle $T_1 O_1 C = 180° - \text{angle } T_1 KC = \Delta_1$

Angle $CO_2 T_3 = 180° - \text{angle } CLT_2 = \Delta_2$

$$T_1 K = KC = t_1 \text{ and } CL = LT_2 = t_2$$

$$KL = t_1 + t_2$$

Applying sine rule to $\Delta\, IKL$, we get

$$\frac{KI}{\sin \Delta_2} = \frac{IL}{\sin \Delta_1} = \frac{KL}{(\sin 180° - \Delta)}$$

$$\therefore \quad KI = KL.\frac{\sin \Delta_2}{\sin \Delta} = (t_1 + t_2)\frac{\sin \Delta_2}{\sin \Delta}$$

or

$$IL = KL.\frac{\sin \Delta_1}{\sin \Delta} = (t_1 + t_2)\frac{\sin \Delta_1}{\sin \Delta}$$

Hence, total tangent length $T_S = T_1K + KI$

$$= t_1 + (t_1 + t_2)\frac{\sin \Delta_2}{\sin \Delta} \quad ...(16.1)$$

Total tangent length $T_L = T_2L + LI$

$$= t_2 + (t_1 + t_2)\frac{\sin \Delta_1}{\sin \Delta} \quad ...(16.2)$$

16.3. RELATIONSHIP BETWEEN DIFFERENT PARTS OF A COMPOUND CURVE

There are seven essential parts of a two centered compound curve *i.e.* Δ, Δ_1, Δ_2, T_S, T_L, R_S and R_L. If any four of these quantities including at least *one angle* are known, the remaining quantities may be calculated. (Fig. 16.2).

The angle of deflection Δ between the straights may either be calculated from their bearings or may be measured in the field. Tangent lengths R_S and R_L are estimated from the layout plan. The fourth quantity out of Δ_1, Δ_2, T_S and T_L is determined from the plan according to the requirement. Calculation of the rest of three quantities is made as under :

In all, five cases may arise and the formulae for each case are detailed below :

Case I. (Fig. 16.2)

Given : Δ, R_S, R_L, Δ_1 or (Δ_2)

Required : T_S, T_L, Δ_2 or (Δ_1)

Note. This is the most common case.

$$\Delta_2 = \Delta - \Delta_1 \qquad ...(16.3)$$

or $$\Delta_1 = \Delta - \Delta_2$$

Substituting the values in Eqns. (16.1) and (16.2), we get

$$T_S = R_S \tan\frac{\Delta_1}{2} + \left(R_s \tan\frac{\Delta_1}{2} + R_L \tan\frac{\Delta_2}{2}\right)\frac{\sin\Delta_2}{\sin\Delta} \qquad ...(16.4)$$

$$T_L = R_L \tan\frac{\Delta_2}{2} + \left(R_s \tan\frac{\Delta_1}{2} + R_L \tan\frac{\Delta_2}{2}\right)\frac{\sin\Delta_1}{\sin\Delta} \qquad ...(16.5)$$

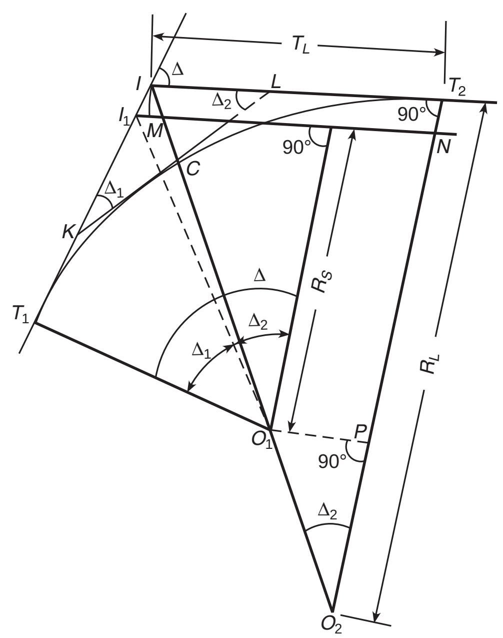


Fig. 16.2.

Case II. (Fig. 16.2)

Given : Δ, R_S, R_L and T_S

Required : Δ_1, Δ_2, T_L

From the formula

$$\cos \Delta_2 = \frac{R_L - R_S - \left(T_S - R_S \tan \frac{\Delta}{2}\right) \sin \Delta}{R_L - R_S.} \quad \text{...(16.6)}$$

Δ_2 can be calculated.

and $\quad \Delta_1 = \Delta - \Delta_2$

$$T_L = (R_L - R_{S)} \sin \Delta_2 + R_S \tan \frac{\Delta}{2} - \left(T_S - R_S \tan \frac{\Delta}{2}\right) \cos \Delta \quad \text{...(16.7)}$$

Proof :

Let curve T_1C when produced cut O_1U parallel to O_2T_2 at U.

Construction. Draw I_1N parallel to IT_2 through U.

Drop IM perpendicular to I_1N and $O_1 P$ perpendicular to O_2N.

CL and LT_2 are the tangents to the curve of radius R_L, having angle of deflection Δ_2.

The central angle $CO_2T_2 = \Delta_2$.

and $\quad$ angle $II_1M = \Delta$

From $\Delta\, O_1O_2\, P$ we get

$$\cos \Delta_2 = \frac{O_2P}{O_1O_2} \quad \text{...}(i)$$

But $\quad O_2P = O_2T_2 - PT_2$

$$= O_2T_2 - (PN + NT_2) = R_L - R_S - IM$$

$$= R_L - R_S - I_1I \sin \Delta$$

or $\quad O_2\, P = R_L - R_S - \left(T_S - R_S \tan \frac{\Delta}{2}\right) \sin \Delta$

and $\quad O_1O_2 = CO_2 - CO_1 = R_L - R_S$

Substituting the values in Eqn. (i), we get

$$\cos \Delta_1 = \frac{R_L - R_S - \left(R_L \tan \frac{\Delta}{2}\right) \sin \Delta}{R_L - R_S} \text{ Proved.}$$

Again, $T_L = IT_2 = MN$

Again, $T_L = IT_2 = MN$

$$= NU + UI_1 - I_1M$$

$$= O_1P + UI_1 - I_1M$$

or $$T_L = (R_L - R_S) \sin \Delta_2 + R_S \tan \frac{\Delta}{2} - \left(T_S - R_S \tan \frac{\Delta}{2}\right) \cos \Delta \text{ (Proved)}$$

Case III. (Fig. 16.3)

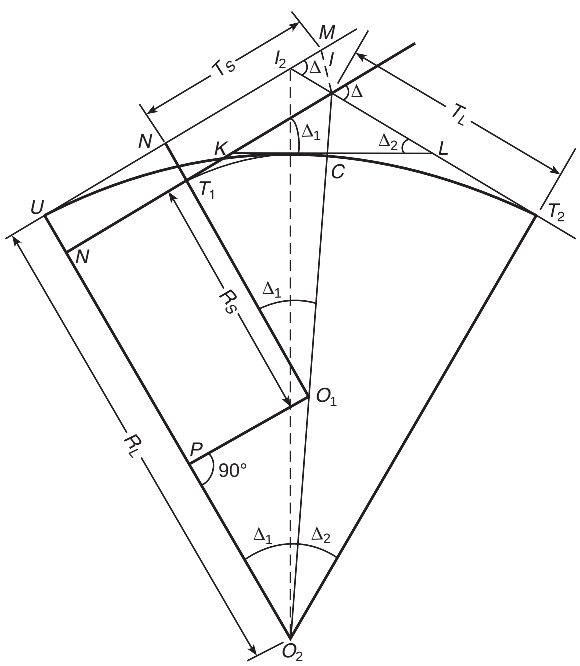


Fig. 16.3.

Given : Δ, R_S, R_L, and T_L

Required : Δ_1, Δ_2 and T_S

From the formula,

$$\text{Cos } \Delta_1 = \frac{R_L - R_S - \left(R_L \tan \frac{\Delta}{2} - T_L\right) \sin \Delta}{R_L - R_S} \quad \text{...(16.8)}$$

Δ_1 can be calculated

$\therefore \quad \Delta_2 = \Delta - \Delta_1$

$$T_S = R_L \tan \frac{\Delta}{2} + \left(R_L \tan \frac{\Delta}{2} - T_L\right) \cos \Delta - (R_L - R_S) \sin \Delta_1 \text{...(16.9)}$$

Proof :

Construction. Draw the curve T_2C to U

Drop a perpendicular IM on UI_2 (produced) and O_1P perpendicular to UO_2.

I_2U and I_2T_2 are tangents to curve UCT_2, having angle of deflection $MI_2I = \Delta$

KT_1 and KC are tangents to curve T_1C, having angle of deflection Δ_1

$\therefore$ Central angle $T_1O_1C = \Delta_1$

But, O_1T_1 is parallel to O_2U

$\therefore$ Angle $UO_2O_1 = \Delta_1$

$$\cos \Delta_1 = \frac{O_2P}{O_1O_2} \quad ...(i)$$

But
$$O_2P = O_2U - UP$$
$$= O_2U - (NP + UN)$$
$$= O_2U - (NP + MI)$$
$$= O_2U - NP - II_2 \sin \Delta$$

or
$$O_2P = R_L - R_S - \left(R_L \tan \frac{\Delta}{2} - T_L\right) \sin \Delta$$

and
$$O_1O_2 = CO_2 - CO_1$$
$$= R_L - R_S$$

Substituting the values of O_2P and O_1O_2 in Eqn. (i), we get

$$\cos \Delta_1 = \frac{R_L - R_S - \left(R_L \tan \frac{\Delta}{2} - T_L\right) \sin \Delta}{R_L - R_S} \quad \text{Proved.}$$

Again,
$$T_S = IT_1 = MN$$
$$= MU - NU$$
$$= I_2U + I_2M - UN$$

or $T_S = R_L \tan \frac{\Delta}{2} + \left(R \tan \frac{\Delta}{2} - T_L\right) \cos \Delta - (R_L - R_S) \sin \Delta_1$ **Proved.**

Case IV. (Fig. 16.4).

Given : Δ, T_S, T_L and R_S

Required : Δ_1, Δ_2, R_L.

From the formula

$$\tan\frac{\Delta_2}{2} = \frac{\left(T_S - R_S \tan\frac{\Delta}{2}\right)\sin\Delta}{T_L - R_S\tan\frac{\Delta}{2} + \left(T_S - R_S\tan\frac{\Delta}{2}\right)\cos\Delta} \qquad ...(16.10)$$

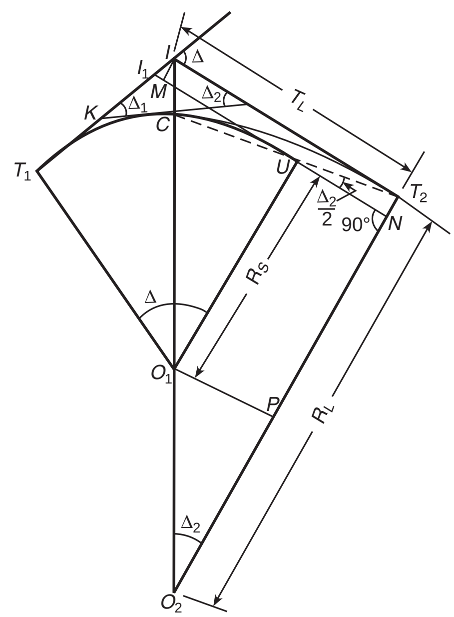


Fig. 16.4.

Δ_2 can be calculated.

$\Delta_1 = \Delta - \Delta_2$

$$R_L = R_S + \frac{T_L - R_S\tan\frac{\Delta}{2} + \left(T_S - R_S\tan\frac{\Delta}{2}\right)\cos\Delta}{\sin\Delta_2} \qquad ...(16.11)$$

Proof. Join CT_2 passing through U.

In Δ O_2CT_2, Angle T_2CO_2 = Angle CT_2O_2

$$= \frac{180° - \Delta_2}{2} = 90° - \frac{\Delta_2}{2}$$

∴ Angle $T_2UN = \frac{\Delta_2}{2}$

Now, $\tan\frac{\Delta_2}{2} = \frac{T_2 N}{UN}$ ...(*i*)

But, $T_2N = IM$

$= II_1 \sin \Delta$

$= (T_1I - T_1I_1) \sin \Delta$

$$= \left(T_S - R_S \tan \frac{\Delta}{2}\right) \sin \Delta$$

and $UN = MN + I_1M - I_1U$

$$= T_L + II_1 \cos \Delta - R_S \tan \frac{\Delta}{2}$$

$$= T_L + \left(T_S - R_S \tan \frac{\Delta}{2}\right) \cos \Delta - R_S \tan \frac{\Delta}{2}$$

or
$$UN = T_L - R_S \tan \frac{\Delta}{2} + \left(T_S - R_S \tan \frac{\Delta}{2}\right) \cos \Delta \qquad ...(ii)$$

Substituting the values in Eqn. (*i*), we get

$$\tan \frac{\Delta_2}{2} = \frac{\left(T_S - R_S \tan \frac{\Delta}{2}\right) \sin \Delta}{T_L - R_S \tan \frac{\Delta}{2} + \left(T_S - R_S \tan \frac{\Delta}{2}\right) \cos \Delta} \quad \text{Proved.}$$

Again, $R_L = R_S + O_1O_2$

$$= R_S + \frac{UN}{\sin \Delta_2} \text{ from Eqn. } (ii)$$

$$= R_S + \frac{T_L - R_S \tan \frac{\Delta}{2} + \left(T_S - R_S \tan \frac{\Delta}{2}\right) \cos \Delta}{\sin \Delta_2} \quad \text{Proved.}$$

Case V. (Fig. 16.5)

Given : Δ, T_S, T_L and R_L

Required : Δ_1, Δ_2, and R_S

From the formula

$$\tan \frac{\Delta_1}{2} = \frac{\left(R_L \tan \frac{\Delta}{2} - T_L\right) \sin \Delta}{R_L \tan \frac{\Delta}{2} + \left(R_L \tan \frac{\Delta}{2} - T_L\right) \cos \Delta - T_S} \qquad ...(16.12)$$

Δ_1 can be calculated

Now $\Delta_2 = \Delta - \Delta_1$

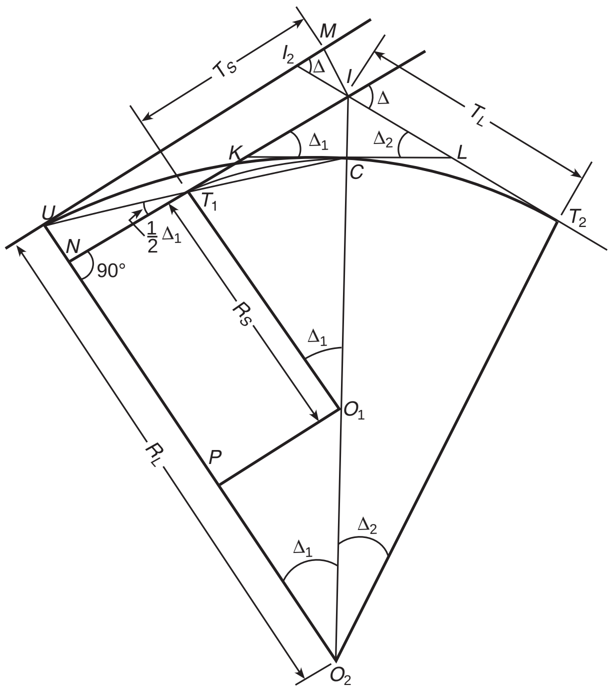


Fig. 16.5.

and $$R_S = R_L - \frac{R_L \tan\frac{\Delta}{2} - T_S + \left(R_L \tan\frac{\Delta}{2} - T_L\right)\cos\Delta}{\sin\Delta_1} \qquad ...(16.13)$$

Proof : Join UC which will pass through T_1

In $\Delta\ UCO_2$, = Angle CUO_2 = Angle UCO_2

$$= \frac{180° - \Delta_1}{2} = 90° - \frac{\Delta}{2}$$

$\therefore$ Angle $UT_1N = \frac{\Delta_1}{2}$

Now $$\tan\frac{\Delta_1}{2} = \frac{UN}{NT_1} \qquad ...(i)$$

$$UN = IM = II_2 \sin\Delta$$

$$= \left(R_L \tan\frac{\Delta}{2} - T_L\right)\sin\Delta$$

and $$NT_1 = NI - T_1I$$

$$= UI_2 + I_2M - T_1I$$

$$= R_L \tan\frac{\Delta}{2} + \left(R_L \tan\frac{\Delta}{2} - T_L\right)\cos\Delta - T_S$$

Substituting these values in Eqn. (*i*), we get

$$\tan\frac{\Delta_1}{2} = \frac{\left(R_L \tan\frac{\Delta}{2} - T_L\right)\sin\Delta}{R_L \tan\frac{\Delta}{2} + \left(R_L \tan\frac{\Delta}{2} - T_L\right)\cos\Delta - T_S} \qquad \text{Proved.}$$

Again, $R_S = R_L - O_1O_2$

$$= R_L - \frac{O_1P}{\sin\Delta_1}$$

$$= R_L - \frac{R_L \tan\frac{\Delta}{2} - T_S + \left(R_L \tan\frac{\Delta}{2} - T_L\right)\cos\Delta}{\sin\Delta_1} \qquad \text{Proved.}$$

16.4. SETTING OUT A COMPOUND CURVE

As a compound curve consists of two or more simple curves, its setting out involves setting out of two or more simple curves of different radii in continuation. Compound curves may be set out by any one of the methods explained earlier. To achieve better accuracy it is recommended that compound curves may be set out by the method of deflection angles, using a theodolite.

Office Work : Proceed as under.

1. Calculate all the seven component parts of the curve *i.e.* four known and three unknown components. (Fig. 16.1).
2. Locate the point of intersection I, the point of commencement T_1 and point of tangency T_2.
3. Calculate the chainage of the point of commencement *i.e.* chainage of T_1 = chainage of I – tangent length T_1I.
4. Calculate the chainage of the point of compound curvature *i.e.*, chainage of C = chainage of T_1C + length of the arc T_1C which is equal to $\frac{\pi R_S \Delta_1}{180°}$.
5. Calculate the chainage of the point of tangency *i.e.* chainage of T_2 = chainage of C + length of the arc T_2C which is equal to $\frac{\pi R_L \Delta_2}{180°}$.
6. Calculate the deflection angles for both the arcs from their tangents *i.e.* $\delta = \frac{1718.9\ C}{R}$ where C is the chord length and R is the radius of the arc.

Field Word : Proceed as under.

1. Set up the theodolite at the point of commencement T_1 and set out the arc T_1C as already explained in simple curves.
2. Shift the theodolite and set it up at point C.
3. Set the vernier A to read $\left(360^o - \frac{\Delta_1}{2}\right)$ or $\frac{\Delta_1}{2}$ according as the curve is right handed or left handed.
4. Take a back sight on the point T_1 and transit the telescope so that the line of sight is along line T_1C produced.
5. Swing the telescope in clockwise direction through $\frac{\Delta_1}{2}$ so that the telescope directs along KCL and the vernier A reads zero.
6. Set the vernier to read the first deflection angle of the second arc CT_2 for its first sub-chord.
7. Continue the process till point of tangency T_2 is located.

16.5. CHECKS ON FIELD WORK

Following two checks ascertain the accuracy of setting out of the work.

1. The point of tangency located from the point of common curvature should coincide with the location already fixed by measuring the distance equal to the total tangent length from the point of intersection.

2. Angle $$T_1CT_2 = 180° - \left[\frac{\Delta_1}{2} + \frac{\Delta_2}{2}\right] = 180° - \frac{\Delta_1 + \Delta_2}{2}$$

$$= 180° - \frac{\Delta}{2}$$

Example 16.1. *Two straights AI and IC whose respective bearings are 60°* and *132° interest at I. Two points P and Q are located on AI and IC respectively such that the bearing of PQ is 112° and the distance between them is 141.63 m. Determine the length of short tangent of a right handed compound curve having radii 200 m and 250 m respectively, if the point of compound curvature falls on PQ.*

If the chainage of 1 is 1000 m, also calculate the chainage of the point of tangency.

Solution. (Fig. 16.6)

The angle of deflection Δ = Bearing of IC – Bearing of AI

$$= 132° - 60° = 72°$$

The angle of deflection of the first arc

$$\Delta_1 = \text{Bearing of } PQ - \text{Bearing of } AI$$

$$= 112 - 60° = 52° .$$

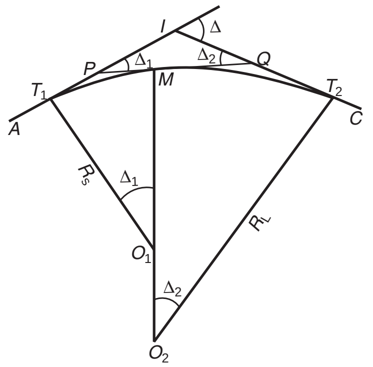


Fig. 16.6.

$\therefore \quad \Delta_2 = \Delta - \Delta_1 = 72° - 52° = 20°$

Also $\quad R_s = 200$ m ; $R_L = 250$ m. (Given)

The problem refers to Case I.

Substituting the values in Eqn. (16.4), we get

$$T_S = R_S \tan \frac{\Delta_1}{2} + \left(R_S \tan \frac{\Delta_1}{2} + R_L \tan \frac{\Delta_2}{2} \right) \frac{\sin \Delta_2}{\sin \Delta}$$

$$= 200 \tan 26° + (200 \tan 26^o + 250 \tan 10°) \frac{\sin 20°o}{\sin 72°}$$

$$= 200 \times 0.487733 + (200 \times 0.487733 + 250 \times 0.176327) \times \frac{0.34202}{0.951056}$$

$$= 97.547 + (97.547 + 44.082)\ 0.3596213$$

$$= 97.547 + 50.933$$

$= 148.48$ m. **Ans.**

Length of the first arc $= \frac{\pi R_S \Delta_1}{180°} = \frac{\pi \times 200 \times 52°}{180°}$

$= 181.51$ m

Length of the second arc $= \frac{\pi R_L \Delta_2}{180°} = \frac{\pi \times 250 \times 20°}{180°}$

$= 87.27$ m

Chainage of point of commencement

= Chainage of point of intersection – T_S

$= 1000 - 148.48$

$= 851.52$ m

Chainage of point of double curvature

= Chainage of point of commencement

+ length of the first arc

= 851.52 + 181.51 = 1033.03 m

Chainage of the point of tangecy

= Chainage of point of double curvature

+ length of the second arc

= 1033.03 + 87.27 = 1120.30 m. **Ans.**

Example 16.2. *A compound curve is to consist of an arc of 19 chains radius followed by another of 38 chains radius and is to connect two straights with a deflection angle of 60°o. At the intersection point the chainage along the first tangent is 100 chains and the starting point of the curve is selected at chainage 80 chains. Calculate the chainages at the point of junction of two curves and at the end of the curve.*

Solution. (Fig. 16.6)

Given : T_S, R_S, R_L and Δ

Required : The chainages of the point of double curvature and the second point of tangency.

This problem refers to Case II

From Eqn. (16.6), we get

$$\cos \Delta_2 = \frac{R_L - R_S - \left(T_S - R_S \tan \dfrac{\Delta}{2}\right) \sin \Delta}{R_L - R_S} \qquad \text{...}(i)$$

Here R_L = 38 chains, R_S = 19 chains

T_S = 100 – 80 = 20 chains, Δ = 60°

Substituting the values in Eqn. (*i*), we get

$$\cos \Delta_2 = \frac{38 - 19 - (20 - 19 \tan 30°) \sin 60°}{38 - 19}$$

$$= \frac{38 - 19 - (20 - 19 \times 0.57735) \times 0.866025}{19}$$

$$= \frac{38 - 12 - 7.82051}{19} = \frac{11.1795}{19} = 0.5883943$$

$\therefore$ $\Delta_2 = 53° \ 57' \ 24''$

and $\Delta_1 = 60° - 53^o \ 57' \ 24'' = 6° \ 02' \ 36''$

Length of arc $T_1M = \frac{\pi R \Delta_1}{180°}$

$$= \frac{\pi \times 19 \times 6.0433}{180°o} = 2.004 \text{ chains}$$

Length of arc $T_2M = \frac{\pi \times 38 \times 53.95667}{180°} = 35.785$ chains

Chainage of the first point of tangency = 80.000 chains

Add the length of the curve = 2.004 chains

Chainage of point of the double curvature = 82.004 chains **Ans.**

Add the length of the second arc = 35.785 chains

∴ Chainage of the end of the curve = 117.789 chains. **Ans.**

Example 16.3. *A compound curve consists of two simple curves whose radii are 300 m and 700 m. It is inserted between two straights intersecting at chainage 5265 m and their angle of deflection being 70° . The curve is to start at chainage 4955 m. Calculate the chainages of the point of junction of the curves and the end of the curve.* (*A.M.I.E., 1992 Winter*)

Solution. (Fig. 16.7).

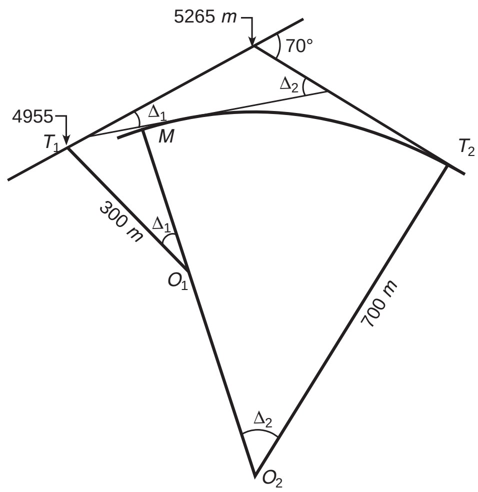


Fig. 16.7.

This problem refers to Case II

Given : Deflection angle

$$\Delta = 70°$$

$$R_S = 300 \text{ m}$$

$$R_L = 700 \text{ m}$$

$$T_S = 5265 - 4955 = 310 \text{ m}$$

We know from eqn. (16.6)

$$\cos \Delta_2 = \frac{R_L - R_S\left(T_S - R_S \tan \frac{\Delta}{2}\right)\sin \Delta}{R_L - R_S} \quad ...(i)$$

Substituting the values in Eq. (i), we get

$$\cos \Delta_2 = \frac{700 - 300 - (310 - 300 \tan 35^o) \sin 70^\circ}{700 - 300}$$

$$= \frac{400 - (310 - 210.06) \times 0.9396926}{400}$$

$$\cos \Delta_2 = \frac{306.09}{400} = 0.765225$$

$$\Delta_2 = 40^\circ .72987$$

$$\Delta_1 = 70^\circ - \Delta_2 = 29^o .927013$$

Length of the first arc TM

$$= \frac{\pi R \Delta_1}{180^\circ} = \frac{\pi \times 300 \times 29.927013}{180^\circ}$$

$$= 156.70 \text{ m}$$

Length of the second arc MT_2

$$- \frac{\pi R \Delta_2}{180^\circ} = \frac{\pi \times 700 \times 40^\circ .07987}{180^\circ}$$

$$= 489.58 \text{ m.}$$

Chainage at M, the junction point

= Chainage at T_1 + length of first arc

= 4955 + 156.70 = 5111.70 m. **Ans.**

Chainage of T_2, the end at the curve

= 5111.70 + 489.58 = 5601.28 m. **Ans.**

Example 16.4. *A compound curve is to consist of an arc of 250 m radius followed by another of 300 m radius and is to connect two straights with a deflection angle of 80° . At the intersection point, the chainage along the short tangent is 1500 m, and the length of the long tangent is 243.05 m. Calculate the chainage of the point of tangency.*

Solution.

Given : R_s = 250 m, R_L = 300 m, Δ = 80° , R_L = 243.05 m.

This problem reters to Case III

Let Δ_1 and Δ_2 be the deflection angles of two arcs.

From Eqn. (16.8), we get

$$\cos \Delta_1 = \frac{300 - 250 - (300 \tan 40^\circ o\ - 243.05) \sin 80^\circ}{300 - 250}$$

$$= \frac{50 - (251.73 - 243.05) \times 0.984808}{50}$$

$$= \frac{50 - 8.5481334}{50} = 0.8290373$$

$\because \quad \Delta_1 = 34^\circ$

$\Delta_2 = 80^\circ - 34^\circ = 46^\circ$

From Eqn. (16.9), we get

$$T_S = 300 \tan 40^\circ + (300 \tan 40^\circ o\ - 243.05) \cos 80^\circ - (300 - 250) \sin 34^\circ$$

$$= 251.73 + (251.73 - 243.05)\ 0.173648 - 50 \times 0.719340$$

$= 251.73 + 1.51 - 27.96 = 225.28$ m. **Ans.**

Length of the first arc

$$= \frac{\pi \times 250 \times 34^\circ}{180^\circ} = 148.35 \text{ m}$$

Length of the second arc

$$= \frac{\pi \times 300 \times 46}{180^\circ} = 240.86 \text{ m}$$

Chainage of the point of intersection = 1500 m (Given)

Chainage of the point of commencement = 1500 – T_S

= 1500 – 225.28

= 1274.72 m

Chainage of point of tangency = 1274.72 + 148.35 + 240.86

= 1663.93 m. **Ans.**

Example 16.5. *A compound curve is to connect two straights having a deflection angle of 80° . As determined from the plan, the lengths of the two tangents are 225.28 m and 243.05 m respectively. Calculate the lengths of the two arcs if the radius of the first curve is to be 250 m.*

Solution.

Given : T_S = 225.28 m, T_L = 243.05 m, Δ = 80° , R_S = 250 m.

This problem refers to Case IV.

Let Δ_1 and Δ_2 be the deflection angles of two arcs.

From Eqn. (16.10), we get

$$\tan \frac{\Delta_2}{1} = \frac{(T_S - R_S \tan \Delta/2) \sin \Delta}{T_L - R_S \tan \Delta/2 + (T_S - R_S \tan \Delta/2) \cos \Delta}$$

$$= \frac{(225.28 - 250 \tan 40°) \sin 80°o}{243.05 - 250 \tan 40° + (225.28 - 250 \tan 40°) \cos 80°}$$

$$= \frac{(225.28 - 209.775) \sin 80°}{243.05 - 209.775 + (225.28 - 209.755) \cos 80°}$$

$$= \frac{15.269448}{243.05 - 209.775 + 2.6924122}$$

$$= \frac{15.269448}{35.967410} = 0.42453565$$

$$\frac{1}{2} \Delta_2 = 23° \; 00' \; 11''$$

$$\Delta_2 = 46° \; 00' \; 22''$$

$$\therefore \quad \Delta_1 = 80° - 46° \; 0' \; 22'' = 33° \; 59' \; 38''$$

From Eqn. (16.11), we get

$$R_L = R_S + \frac{T_L - R_S \tan \Delta/2 + (T_S - R_S \tan \Delta/2) \cos \Delta}{\sin \Delta_2}$$

$$= 250 + \frac{243.05 - 250 \tan 40° + (225.28 - 250 \tan 40°) \cos 80°}{\sin 46° \; 00' \; 22''}$$

$$= 250 + \frac{243.05 - 209.775 + (225.28 - 209.775) \cos 80°o}{\sin 46° \; 00' \; 22''}$$

$$= 250 + 50 = 300 \text{ m.}$$

Length of first curve $= \dfrac{\pi R_S \Delta_1}{180°}$

$$= \frac{\pi \times 250 \times 33°.993888}{180^o} = 148.33 \text{ m.}$$ **Ans.**

Length of second curve $= \dfrac{\pi R_L \Delta_2}{180°}$

$$= \frac{\pi \times 300 \times 46°.00611}{180^o} = 204.89 \text{ m.}$$ **Ans.**

Example 16.6. *A compound curve is to connect two straights having a deflection angle of 80° . As determined from the plan, the lengths of two tangents arc 225.28 m and 143.05 m respectively. Calculate the length of short radius if the long radius is 300 m.*

Solution.

Given : $T_S = 225.28$ m, $T_L = 243.05$ m, $\Delta = 80°$, $R_L = 300$ m.

This problem refers to case V.

From Eqn. (16.12), we get

$$\tan\frac{1}{2}\Delta_1 = \frac{\left(R_L \tan\frac{1}{2}\Delta - T_L\right)\sin\Delta}{R_L \tan\frac{1}{2}\Delta + \left(R_L \tan\frac{1}{2}\Delta - T_L\right)\cos\Delta - T_S}$$

$$= \frac{(300 \tan 40° \;- 243.05) \sin 80^o}{(300 \tan 40° + (300 \tan 40^o - 243.05) \cos 80° \;- 225.28}$$

$$= \frac{8.5481334}{251.7300 + 1.5072646 - 225.28}$$

$$= \frac{8.5481334}{27.957260} = 0.3075719$$

$$\frac{1}{2}\Delta_1 = 17° \; 00' \; 05''$$

$$\Delta_1 = 34° \; 00' \; 10''$$

From Eqn. (16.13), we get

$$R_S = R_L - \frac{R_L \tan\Delta - T_S + \left(R_L \tan\frac{1}{2}\Delta - T_L\right)\cos\Delta}{\sin\Delta_1}$$

$$= 300 - \frac{300 \tan 40° - 225.28 + (300 \tan 40° - 243.05) \cos 80°}{\sin 34° \; 00' \; 10''}$$

$$= 300 - \frac{251.73 - 225.28 + 1.5072646}{0.559193}$$

$$= 300 - 50$$

$= 250$ m. **Ans.**

Example 16.7. *A 200 m length of a straight connects two circular curves deflecting to the right. The radius of the first curve was 250 m and that of the second curve was 200 m. The central angle for the second curve was 15° 58′. The combined curve is to be replaced by a single circular curve between the same tangent points.*

Find the radius of curve.

Solution. (Fig. 16.8).

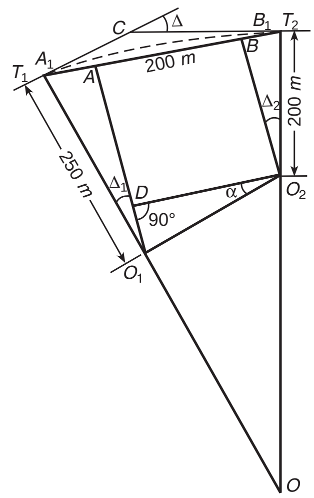


Fig. 16.8.

Let $T_1 A$ and BT_2 be the two circular curves separated by a straight AB = 200 m.

Let O be the centre of single circular curve replacing the two curves and the intervening straight.

Let R be the radius of circular curve

Join $O_1 O_2$. Drop $O_2 D \perp AO_1$

Then $\quad O_1 O_2 = \sqrt{O_1 D^2 + O_2 D^2}$

Here $\quad O_1 D = AO_1 - AD = 250 - 200 = 50$ m

$O_2 D = AB = 200$ m.

$\therefore \quad O_1 O_2 = \sqrt{50^2 + 200^2} = 206.16$ m

$$\alpha = \tan^{-1} \frac{O_1 D}{O_2 D} = \tan^{-1} \frac{50}{200} = 14° \ 02' \ 10''$$

In $\Delta \ O_1 O_2 O$, $\angle \ \Delta O_1 O_2 O = 180°o - (\alpha + 90° + \Delta_2)$

$= 180° - (14° \ 02' \ 10'' + 90° + 15° \ 58' = 59° \ 59' \ 50''$

$O_1 O = R - 250$

and $\quad O_2 O = R - 200$

Applying consine rule to $\Delta \ O_1 O_2 O$, we get

$$O_1 O^2 = (O_1 O_2)^2 + (O \ O_2)^2 - 2 \ O_1 O_2 \times OO_2 \times \cos 59°o \ 59' \ 50''$$

or $(R - 250)^2 = (206.16)^2 + (R - 200)^2 - 2 \times (206.16) \ (R - 200) \times \cos 59° \ 59' \ 50''$

$$R^2 - 500\,R + 62500 = 42501.95 - 400\,R + 40000 - 412.32 \times (R - 200) \times 0.500042$$

$$-\,500\,R + 62500 = 42501.95 - 400\,R + 40000 - 206.18\,R + 41235.46$$

$$-\,500\,R + 400\,R + 206.18\,R = 42501.95 + 40000 + 41235.46 - 62500$$

$$106.18\,R = 61237.41$$

$$R = 576.73 \text{ m.}$$ **Ans.**

Example 16.8. *The bearings of two straights are 50°* and *110°. Three circular arcs of following specifications were introduced between them.*

Arc	*Deflection*	*Length*	*Radius*
$T_1\,T_2$	*right*	*174.533*	*200 m*
$T_2\,T_3$	*left*	*261.800*	*250 m*
$T_3\,T_4$	*right*	*366.520*	*300 m*

Later on a compound curve was introduced between T_1 and T_4 *such that arc tangential to* T_4 *is having a radius of 500 m. Calculate the radius of the other arc.*

Solution. (Fig. 16.9).

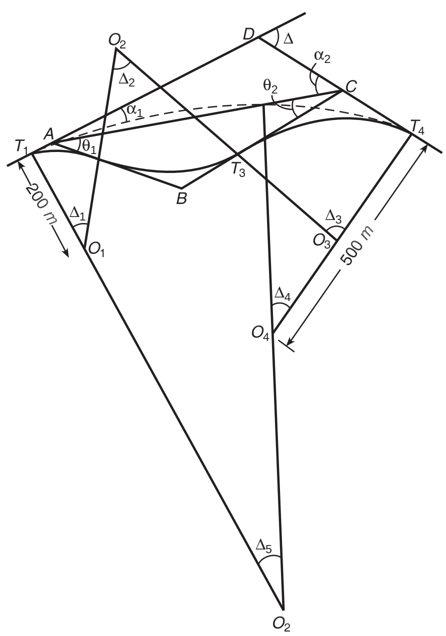


Fig. 16.9.

Let the common tangents to the arcs $T_1 T_2$ and $T_2 T_3$ and that to the arcs $T_2 T_3$ and $T_3 T_4$ meet the straights T_1D and T_4D A and C respectively and these intersect at B.

From the arc T_1T_2 we get

$$\frac{\pi \times 200\ \Delta_1}{180°} = 174.533$$

or $$\Delta_1 = \frac{174.533 \times 180°}{\pi \times 200} = 50°$$

$\therefore$ $$T_1 A = AT_2 = R \tan \frac{\Delta_1}{2}$$

$$= 200 \tan 25^{\circ} = 93.262 \text{ m}$$

From the arc T_1T_3 we get

or $$\frac{\pi R_2\ \Delta_2}{180°} = 261.800$$

or $$\Delta_2 = \frac{261.800 \times 180°}{\pi \times 250} = 60°$$

$\therefore$ $$T_2B = BT_3 = R_2 \tan \frac{\Delta_2}{2}$$

$$= 250 \tan 30 = 144.338 \text{ m}$$

From the arc T_3T_4 we get

or $$\frac{\pi \times 300 \times \Delta_3}{180° o} = 366.520$$

or $$\Delta_3 = \frac{366.520 \times 180°}{\pi \times 300} = 70°$$

$$T_3\ C = CT_4 = R_3 \tan \frac{1}{2}\ \Delta_3$$

$$= 300 \tan 35° = 210.062 \text{ m}$$

$$AB = AT_2 + T_2\ B = 93.262 + 144.338 = 237.6 \text{ m}$$

$$BC = BT_3 + T_3\ C = 144.338 + 210.062 = 354.4 \text{ m}$$

Deflection angle of the main curve

$$= \text{Bearing of } DT_4 - \text{Bearing of } T_1\ B$$

$$= 110° - 50° = 60°$$

In triangle ABC,

$$\angle ABC = 180° - 60° = 120°$$

Applying cosine formula to ΔABC, we get

$$AC = \sqrt{(237.6)^2 + (354.4)^2 - 2 \times 237.6 \times 354.4 \cos 120°}$$
$$= \sqrt{182053.12 + 84205.44}$$
$$= 516.0 \text{ m}$$

Let $\angle CAB = \theta_1$ and $\angle ACB = \theta_2$

Applying sine rule to ΔABC, we get

$$\sin \theta_1 = \frac{BC \sin 120°}{AC}$$
$$= \frac{354.4 \sin 120°}{516}$$
$$= 0.59480476$$

or $\theta_1 = 36° \, 29' \, 55''$

$$\theta_2 = 180° - (120° + 36° \, 29' \, 55'')$$
$$= 23° \, 30' \, 05''$$

In ΔDAC, let $\angle DAC = \alpha_1$ and $\angle DCA = \alpha_2$

$$\alpha_1 = 50° - \theta_1 = 50° - 36° \, 29' \, 55'' = 13° \, 30' \, 05''$$
$$\alpha_2 = 70° - \theta_2$$
$$= 70° - 30° \, 05'' = 46° \, 29' \, 55''$$

Applying sine rule to $\Delta\, ACD$, we get

$$\frac{DC}{\sin \alpha_1} = \frac{AD}{\sin \alpha_2} = \frac{AC}{\sin 120°}$$

or $$DC = \frac{516 \sin 13° \, 30' \, 05''}{\sin 120°}$$
$$= \frac{516 \times 0.233469}{0.866025} = 139.11 \text{ m}$$

and $$AD = \frac{516 \sin 46° \, 29' \, 55''}{\sin 120°}$$
$$= \frac{516 \;\; 0.725358}{0.866025} = 432.19 \text{ m}$$

$\therefore$ $$DT_1 = DA + AT_1$$
$$= 432.19 + 93.262 = 525.452 \text{ m}$$

and $$DT_4 = DC + CT_4$$
$$= 139.11 + 210.062 = 349.172 \text{ m}$$

Let the radius of the other arc of the compound curve be R_L with central angle Δ_5.

From Eqn. (16.10), we get,

$$\tan\frac{\Delta_5}{2} = \frac{(DT_4 - R_S \tan \Delta / 2 \sin \Delta)}{DT_1 - R_S \tan\dfrac{\Delta}{2} + \left(DT_4 - R_S \tan\dfrac{\Delta}{2}\right)\cos \Delta°}$$

$$= \frac{(349.172 - 500 \tan 30°)\sin 60°}{525.452 - 500\tan 30° + (349.172 - 500 \tan 30°)\cos 60°}$$

$$= \frac{52.391914}{525.452 - 288.675 + 30.4285}$$

$$= \frac{52.391914}{267.0255}$$

$$= 0.19620565$$

$$\frac{\Delta_5}{2} = 11° \, 06.02.5''$$

$$\Delta_5 = 22° \, 12' \, 05''$$

Substituting the values in Eqn. (16.11).

$$R_L = R_S + \frac{DT_1 - R_S \tan\dfrac{\Delta}{2} + \left(DT_2 - R_S \tan\dfrac{\Delta}{2}\right)\cos \Delta}{\sin \Delta_5}$$

$$= 550 + \frac{525.452 - 500 \tan 30° + (389.172 - 500 \tan 30°)\cos 60°}{\sin 22° \, 12' \, 05''}$$

$$= 500 + \frac{525.452 - 288.675 + (349.172 - 288.675) \times 0.5}{0.377863}$$

$$= 500 + 706.67$$

$= 1206.67$ m. **Ans.**

Example 16.9. *The angle of deflection of two straights is 80° . Design a compound curve of the following specifications.*

The central angle of first arc = *36°*

The central angle of second arc = *44°*

The length of the short tangent = *562.04* m

The radius of second arc is twice the radius of first arc.

Solution. (Fig. 16.10).

Let *AB* and *BC* be the straights deflecting through 80° .

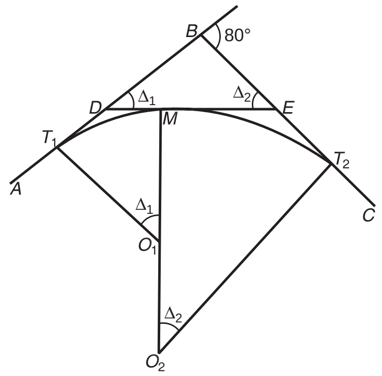


Fig. 16.10

Let Rs be the radius of the short arc.

From Eqn. (16.4) we know

$$T_S = R_S \tan\frac{\Delta 1}{2} + \frac{\left(R_S \tan \Delta\frac{1}{2} + R_L \tan\frac{\Delta^2}{2}\right)\sin \Delta 2}{\sin \Delta} \quad ...(i)$$

Substituting the values in Eqn. (i)

$$562.04 = R_S \tan 18° + \frac{(R_S \tan 18° + R_L \tan 22°)\sin 44°}{\sin 80°}$$

$$562.04 = R_S \{\tan 18° + \frac{(R_S \tan 18° + R_S \tan 22°)\sin 44°}{\sin 80°}$$

$$562.04 = R_S \left[\tan 18° + \frac{(\tan 18° + 2\tan 22°)\sin 44^o}{\sin 80°}\right]$$

$$562.04 \sin 80° = R_S[\tan 18° \sin 80° + (\tan 18° + 2\tan 22^o)\sin 44°]$$

or $$R_S = \frac{562.04 \sin 80°}{\tan 80° \sin 80° + (\tan 18^o + 2\tan 22°)\sin 44°}$$

$$= \frac{562.04 \times 0.984808}{0.32492 \times 0.984808 + (0.32492 + 2 \times 0.0404026) \times 0.694658}$$

$$= \frac{553.5015}{0.3199838 + 0.78702806}$$

$= 500$ m. **Ans.**

and $R_L = 2 \times 500 = 1000$ m. **Ans.**

$$T_L = R_L \tan 22° + \frac{(R_L \tan 22° + R_S \tan 18°)\sin 36°}{\sin 80°}$$

$$= R_L \tan 22° + \frac{1000 \times \tan 22° + 500 \times \tan 18° \sin 36°}{\sin 80°}$$

$$= 404.026 + \frac{404.026 + 95.491}{0.984808}$$

$$= 404.026 + 507.273 = 911.25 \text{ m.} \quad \textbf{Ans.}$$

16.6. MISSING DATA METHOD

It may sometimes be easier to determine the unknown component parts of a compound curve by using the principle of missing data. As discussed in chapter 12 Theodolite Traversing we know that in a closed traverse, algebraic sum of the latitudes and departure equals to zero separately.

Depending upon the given data, six cases may arise, which have been discussed hereunder.

Case I. (Fig. 16.11).

Given : Δ_1, Δ_2, T_S and R_S.

To calculate : T_L and R_L

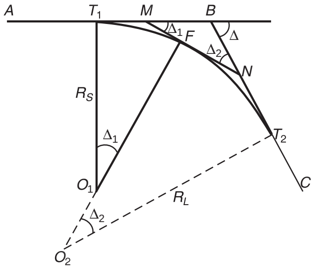


Fig. 16.11.

Procedure. Proceed as under.

1. Calculate $\Delta = \Delta_1 + \Delta_2$
2. Assume $O_1 T_1$ parallel to the meridian and point of intersection B as the origin of co-ordinates.
3. Calculate the co-ordinates of F from the traverse $BT_1 O_1 F$.
4. Calculate the length $BM = T_S - R_S \tan \dfrac{\Delta_1}{2}$
5. Solve the triangle BMN to obtain the length of BN.
6. Calculate the co-ordinates of N from the known distance and bearing.
7. Calculate the length of FN from their independent co-ordinates ($FN = NT_2$).
8. Calculate $T_L = BT_2 = BN + NT_2$

9. Calculate $R_L = FN \div \tan\dfrac{\Delta_2}{2}$.

Case II (Fig. 16.12).

Given : Δ_1, Δ_2, R_L, R_S

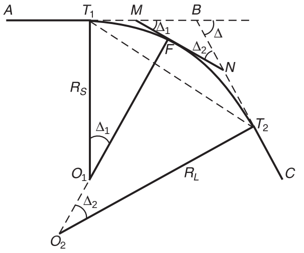


Fig. 16.12.

To calculate : T_S and T_L.

Procedure : Proceed as under.

1. Calculate $\Delta = \Delta 1 + \Delta 2$.
2. Assume O_1T_1 parallel to the meridian and T_1 as the origin of co-ordinates.
3. Calculate the co-ordinates of T_2 from the traverse $T_1O_1O_2T_2$.
4. Calculate the length and bearing of T_1T_2 from known co-ordinates T_1 and T_2.
5. Solve the triangle T_1T_2B to obtain the lengths of BT_1 and BT_2.
6. From known distance and bearing of T_1B, calculate the co-ordinates of B.

Case III. (Fig. 16.13).

Given : Δ_1, $\Delta_2 T_S$, T_L.

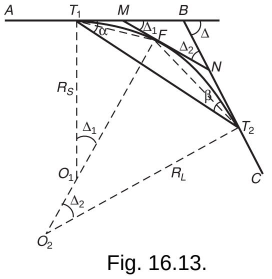


Fig. 16.13.

To calculate : R_S and R_L.

Procedure : Proceed as under

1. Calculate $\Delta = \Delta_1 + \Delta_2$
2. Assume O_1T_1 as meridian and T_1 as the origin of co-ordinates.
3. Calculate the co-ordinates of T_2 from the traverse T_1BT_2.
4. Calculate the distance and bearing of T_1T_2 from known co-ordinates of $T_1\ T_2$.
5. Calculate angles BT_1T_2 and T_1T_2B say α and β respectively.
6. In $\Delta\ FT_1T_2$, $\angle FT_1\ T_2 = \alpha - \frac{1}{2}\Delta_1$ and $\angle\ T_1T_2F = \beta - \frac{1}{2}\Delta_2$.
7. Calculate the lengths of T_1F and T_2F by applying sine rule to $\Delta T_1\ T_2F$.
8. Calculate $R_S = T_1\ F \div 2 \sin \frac{1}{2}\Delta_1$
9. Calculate $R_L = TF_2 \div 2 \sin \frac{1}{2}\Delta_2$

Case IV. (Fig. 16.14).

Given : Δ_1, Δ_2, T_S and R_L.

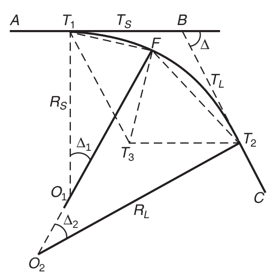


Fig. 16.14.

To calculate : T_L and R_S.

In this case missing lengths of T_L and R_S are not adjacent.

Construction : To make the missing lengths adjacent, draw T_1T_3 equal and parallel to BT_2 so that $T_3\ T_2 = T_S$.

Procedure : Proceed as under.

1. Calculate $\Delta = \Delta_1 + \Delta_2$.
2. Assume O_1T_1 parallel to the meridian and T_3 as the origin of co-ordinates.
3. Calculate co-ordinates of F from the traverse $T_3\ T_2\ O_2\ F$.
4. From the known co-ordinates, calculate the length and bearing of $T_3\ F$.

5. In $\Delta\ T_1\ T_3\ F$, all the angles and side $T_3\ F$ are known *i.e.*
   Angle $T_1\ T_3\ F =$ Bearing of $T_3\ F -$ Bearing of $T_3\ T_1\ (= T_2\ B)$.
   Angle $FT_1\ T_3 =$ Angle $BT_1\ T_3 -$ Angle $BT_1\ F = \Delta - \frac{1}{2}\Delta_1$.
6. Calculate the side T_1F and T_1T_3 by applying sine rule.
7. $T_L = T_1\ T_3$ and $R_S = T_1\ F \div\ 2 \sin \frac{1}{2}\Delta_1$

Case V. (Fig. 16.15).

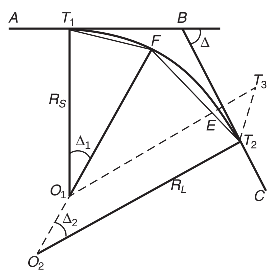


Fig. 16.15.

Given : $\Delta, T_s\ T_L, R_S$.

Calculate : R_L and Δ_1, Δ_2.

Construction : Draw O_1T_3 parallel to O_2T_2 and T_2T_3 parallel to O_2O_1.

Procedure. Proceed as under.

1. Assume O_1T_1 as meridian and O_1, the origin of co-ordinates.
2. Calculate the co-ordinates of T_2 from the traverse $O_1T_1BT_2$.
3. Point E on the arc FE must fall at the intersection of FT_2 and $O_1\ T_3$.
4. Calculate the coordinates of E, knowing $O_1\ E = R_S$ and angle $T_1\ O_1\ E = \Delta$
5. Calculate the length and bearing of ET_2
6. Calculate the angle $F\ T_2B =$ Bearing of $T_2B -$ Bearing of T_2F.
7. Calculate angle $\Delta_2 =$ Twice angle $FT_2\ B$.
8. Calculate angle $\Delta_1 = \Delta - \Delta_2$.
9. Calculate the bearing of $O_1\ F =$ Bearing of $O_1\ E -$ Angle Δ_2.
10. Calculate the co-ordinates of F.

11. Calculate the length and bearing of FT_2.
12. Calculate $R_L = FT_2 \div 2 \sin \frac{1}{2} \Delta_2$.

Case VI. (Fig. 16.16).

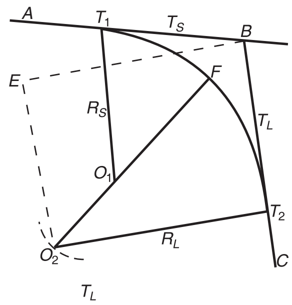


Fig. 16.16.

Given : Δ, T_S, R_S and R_L,

To Calculate : T_L

Construction : Draw BE parallel and equal to $O_2 T_2$.

Procedure : Proceed as under.

1. Assume O_1T_1 as the meridian and O_1 as the origin of co-ordinates.
2. Calculate the co-ordinates of E from the traverse $O_1 T_1 BE$.
3. Calculate the co-ordinates of O_2 where $O_1 O_2 = R_L - R_S$.
4. Calculate the bearing of $O_1 O_2$.
5. Calculate the angle $\Delta_2 =$ Bearing of $O_1 B$ − Bearing of $O_2 O_1$.
6. $\Delta_1 = \Delta - \Delta_2$.
7. Calculate the length of O_1E from known coordinates of O_1 and E.
8. Calculate $O_1E = T_2 B = T_L$.

16.7. THREE CENTERED COMPOUND CURVE

The component parts of a three centered compound curve, are shown in Fig. 16.17. Computation of tangent lengths can be done by combining the two arcs from either end.

In a three-centred curve, the first curve is introduced from the back straight whose radius is R_1 and the centre is at O_1. From the end T_2 of the first circular curve, the second circular curve having radius as R_2 and the centre at O_2 is introduced. The points T_2, O_1 and O_2 lie in a straight line. From the end of the second curve T_3, the third circular curve is introduced to end at the point of tangency T_4.

The layout of the three-centered compound curve may also the considered from right to the left where, the radii go on decreasing.

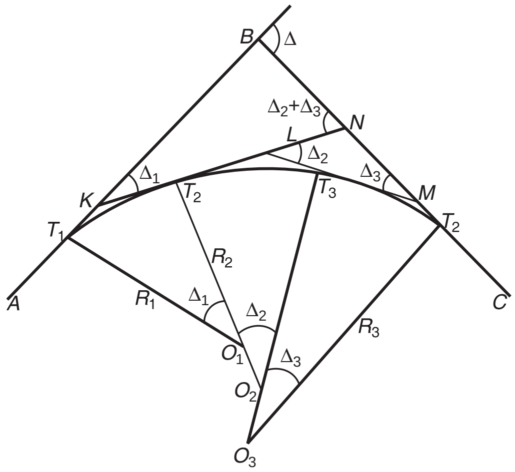


Fig. 16.17. A three centered compound curve.

16.8. REVERSE CURVES

A reverse curve consists of two circular arcs of same or different radii having their centres on the opposite sides of the common tangent at the point of reverse curvature.

16.9. NECESSITY OF PROVIDING REVERSE CURVES

Reverse curves are generally provided when the straights are either parallel or the angle between them is very small. Serpentine road or railway curves in mountainous regions, are generally reverse curves. In such case, a loop of a curve in a valley is immediately followed by another loop round the shoulder of a ridge of opposite curvature. Reverse curves are also frequently required in cities where roads turn in different directions in succession or where roads approach flyovers.

16.10. DISADVANTAGES OF A REVERSE CURVE

When high speed vehicles ply on highways and main railway lines, use of reverse curves should be avoided for the following reasons :

1. Sudden change of superelevation is required from one side to the other.
2. At the point of reverse curvature, no superelevation is provided.
3. Steering is dangerous in the case of highways. Unless driver is cautious, there are chances of overturning the vehicle.
4. Sudden change of directions causes great discomfort to the passengers.

It is therefore recommended to avoid the reverse curves by inserting a small length of straight between the circular arcs.

16.11. ELEMENTS OF A REVERSE CURVE (FIG. 16.18)

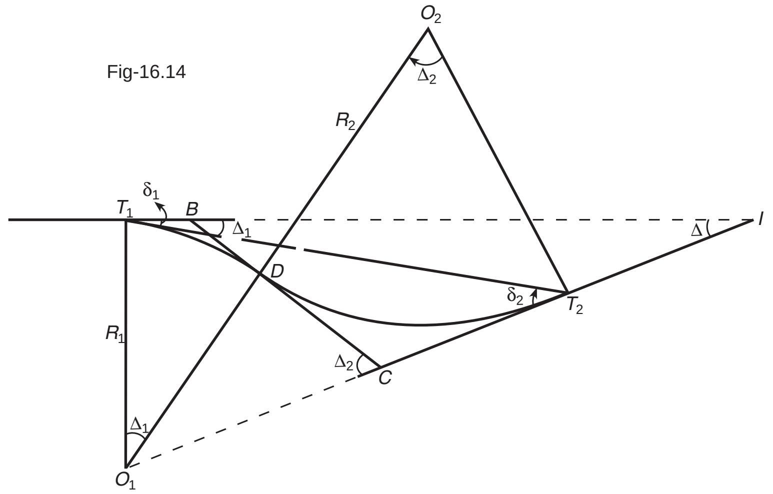


Fig. 16.18. Elements of a reverse curve.

The various components of a reverse curve are :

1. Radii R_1 and R_2 of two circular arcs.
2. Angle of total deflection (Δ) of the straights.
3. Angles of deflection (Δ_2, Δ_2) of the common tangent.
4. Angle (δ_1, δ_2) between the straights and the line joining the points of commencement and tangency.

Assumptions. It is not possible to determine the elements of a reverse curve unless following conditions are specified.

1. R_1 and R_2 are equal.
2. Δ_1 and Δ_2 are equal.
3. The length of the line joining the tangent points is known.

16.12. RELATIONSHIP BETWEEN ELEMENTS OF A REVERSE CURVE

Depending upon the values of angles of deflection (Δ_1, Δ_2) of the common tangent, three cases may arise :

Case I. When the two straights are non-parallel and Δ_1 Δ_2.

Point of intersection. It may be obtained by producing the forward straight backward to intersect the back-straight.

Given Data. (*i*) The length of the line joining the tangent points T_1 and T_2. (Fig. 16.19).

(*ii*) The angles δ_1 and δ_2 between the line T_1 T_2 and back straight T_1B and forward straight CT_2 respectively.

Required. To find the common radius R.

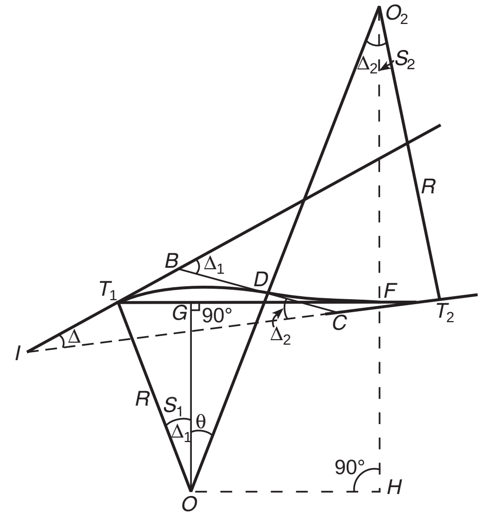


Fig. 16.19. Non-parallel straights $\Delta_1 > \Delta_2$.

Specified Conditions. $R_1 = R_2 = R$

Let T_1 and T_2 be the two tangent points.

O_1 and O_2 be the centres of the circular arcs.

Distance $T_1\, T_2$ be equal to L.

Construction. Draw O_1G and O_2F perpendiculars to T_1T_2 and Q_1H perpendicular to O_2F (produced).

Let the angle $O_2\, O_1\, H$ be θ.

Now $$T_1\, T_2 = L = T_1\, G + GF + FT_2 \qquad \text{...(16.14)}$$

$$O_1G = R \cos \delta_1 = FH$$

$$O_2F = R \cos \delta_2$$

$$O_1O_2 = 2R$$

$$\sin \theta = \frac{O_2\, H}{O_1\, O_2} = \frac{O_3\, F + FH}{O_1\, O_2} = \frac{R \cos \delta_1 + R \cos \delta_2}{2R}$$

$$\theta = \sin^{-1} \frac{\cos \delta_1 + \cos \delta_2}{2} \qquad \text{...(16.15)}$$

Again, $T_1G = R \sin \delta_1$

$$GF = O_1\, H = 2R \cos \theta$$

$$FT_2 = R \sin \delta_2$$

Substituting the values in Eqn. (16.14)

$$L = R \sin \delta_1 + 2R \cos \theta + R \sin \delta_2$$

or $$R = \frac{L}{\sin \delta_1 + 2 \cos \theta + \sin \delta_2} \quad ...(16.16)$$

The central angle for the first arc

$$\Delta_1 = \delta_1 + (90° - \theta)$$

The central angle for the second arc

$$\Delta_2 = \delta_2 + (90° \ \theta)$$

Now, the lengths of the curve may be computed.

Case II. When the straights are non-parallel and $\Delta_1 < \Delta_2$.

Point of intersection. It may be obtained by producing the back straight. (Fig. 16.20)

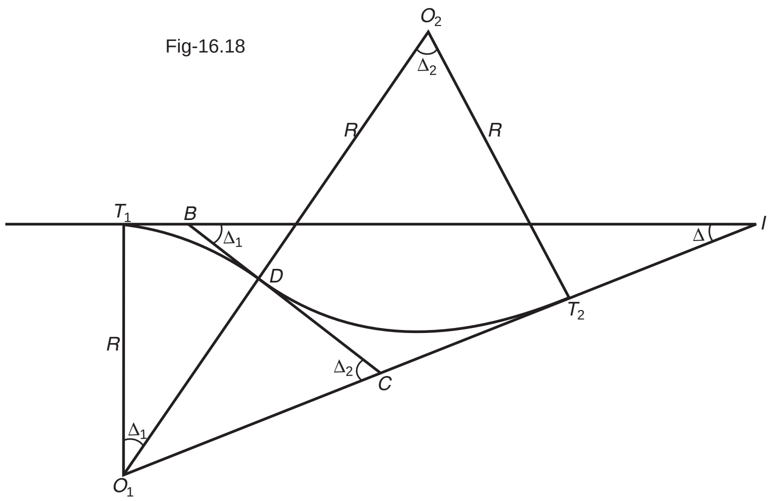


Fig. 16.20. Non-parallel straights $\Delta_1 < \Delta_2$.

Given Data : The central angle (Δ_1, Δ_2) and the length (L) of the common tangent BC

Required : To find the length of the common radius R and the chainages of T_1, D and C if that of I is given.

Specified Condition : $R_1 = R_2 = R$.

Let BC be the common tangent of length L.

$\therefore$ $T_1 B$ and BD are tangent to the first arc $T_1 D$.

T_2C and DC are tangent to the second arc $T_2 D$.

$\therefore$ $T_1 B = BD$; $CD = CT_2$.

But $$BD = R \tan \frac{\Delta_1}{2} \text{ and } CD = R \tan \frac{\Delta_2}{2}$$

$$\therefore \quad BC = L = R \tan \frac{\Delta_1}{2} + R \tan \frac{\Delta_2}{2}$$

or $$R = \frac{L}{\tan \frac{1}{2}\Delta_1 + \tan \frac{1}{2}\Delta_2} \quad ...(16.17)$$

Knowing R, Δ_1 and Δ_2, the lengths of the two arcs, can be calculated.

We also know that, $\Delta_2 = \Delta_1 + \Delta$

$$\therefore \quad \Delta = \Delta_2 - \Delta_1$$

Applying sine rule to the triangle BCI, we get

$$\frac{BI}{\sin \angle BCI} = \frac{BC}{\sin \angle BIC}$$

or $$BI = BC \frac{\sin \Delta_2}{\sin \Delta} = L. \frac{\sin \Delta_2}{\sin \Delta}$$

$$\therefore \quad T_1 I = BT_1 + BI = R \tan \frac{\Delta_1}{2} + L \frac{\sin \Delta_2}{\sin \Delta}$$

Chainage of T_1 = Chainage of $I - TI_1$

Chainage of D = Chainage of T_1 + Length of first arc $T_1 D$.

Chainage of T_2 = Chainage of D + Length of second arc $T_2 D$

where $$\text{arc } T_1 D = \frac{\pi R \Delta_1}{180°} \text{ and } T_2 D = \frac{\pi R \Delta_2}{180°}$$

Field Operations :

The first arc $T_1 D$ is set out from T_1 and the second arc $T_2 D$ from D by any one of the methods explained in chapter 15 Simple Curves.

Case III. When the straights are Parallel. (Fig. 16.21).

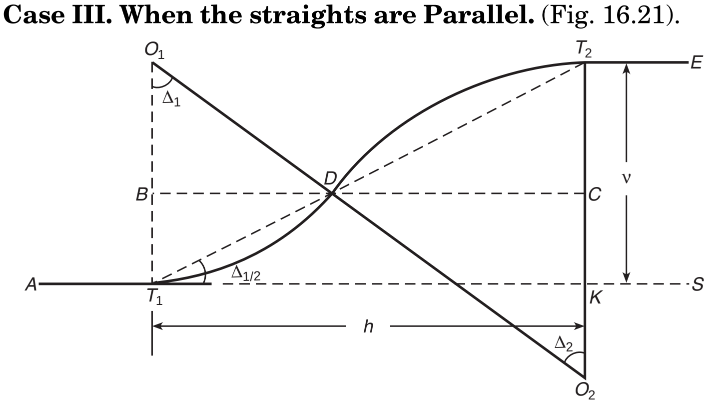


Fig. 16.21. Parallel straights.

Given Data : Two radii (R_1, R_2) and central angles (Δ_1, Δ_2)

Required : (*i*) The distance (L) between the tangent points T_1 and T_2.

(*ii*) Perpendicular distance v between two given straights.

(*iii*) Distance h between perpendiculars at T_1 and T_2.

Specified Condition : $\Delta_1 = \Delta_2$

Let AS and $T_2 E$ be two parallel straights,

R_1 = Smaller radius,

R_2 = Larger radius

Δ_1 = Central angle for the arc of radius R_1

Δ_2 = Central angle for the arc of radius R_2

L = Distance between T_1 and T_2

v = Perpendicular distance between two straights.

h = Distance between perpendiculars at T_1 and T_2

D = Point of reverse curvature.

Through D draw a line BC parallel to two straights

$\therefore$ $O_1 T_1$ is parallel to $O_2 T_2$

But $\Delta_1 = \Delta_2$

Now $v = T_1 B + CT_2$

$$\therefore \quad T_1 B = O_1T_1 - O_1 B$$

$$= R_1 - R_1 \cos \Delta_1 = R_1 (1 - \cos \Delta_1)$$

$$= R_1 \text{ versine } \Delta_1$$

Similarly, $CT_2 = R_2$ versine Δ_2

$$v = R_1 \text{ versine } \Delta_1 + R_2 \text{ vernier } \Delta_2$$

or
$$V = (R_1 + R_2) \text{ versine } \Delta_1 \qquad ...(16.18)$$

Again $T_1 T_2 = T_1 D + DT_2$

But $T_1 D = 2 R_1 \sin \dfrac{\Delta_1}{2}$ and $DT_2 = 2 R_1 \sin \dfrac{\Delta_2}{2}$

$$\therefore \quad L = 2 (R_1 + R_2) \sin \frac{\Delta_1}{2} + 2 R_1 \sin \frac{\Delta_2}{2}$$

or
$$L = 2 (R_1 + R_2) \sin \frac{\Delta_1}{2} \qquad ...(16.19)$$

From $\Delta\, T_1 T_2 K$, we get

$$\sin \frac{\Delta_1}{2} = \frac{T_2 K}{T_1 T_2} = \frac{v}{L}$$

Substituting the value of $\sin \frac{1}{2} \Delta$, in Eqn. (16.19), we get

$$L = \frac{v.\ 2\,(R_1 + R_2)}{L} \qquad ...(16.20)$$

$$L^2 = 2v\,(R_1 + R_2)$$

or $$L = \sqrt{2v\,(R_1 + R_2)} \qquad ...(16.21)$$

Again $$h = BD + DC$$

$$= R_1 \sin \Delta_1 + R_2 \sin \Delta_2$$

or $$h = (R_1 + R_2) \sin \Delta_1 \qquad ...(16.22)$$

Special Case : If $R_1 = R_2 = R$

From Eqn. (16.19), $$L = 4\,R \sin \frac{\Delta_1}{2} \qquad ...(16.23)$$

From Eqn. (16.21), $$L = \sqrt{4Rv} \qquad ...(16.24)$$

From Eqn. (16.18), $$v = 2R\,(1 - \cos \Delta_1) \qquad ...(16.25)$$

From Eqn. (16.22) $$h = 2R \sin \Delta_1 \qquad ...(16.26)$$

Example 16.10. *Two straights AB and CD both when the produced intersect at V. Angle CBV = 30°* and *angle BCV = 120° . It is proposed to introduce a reverse curve consisting of two circular arcs AT and TD. T lying on BC. Length BC is 791.71 m, and radius of arc AT is 800 m, chainage of B is 1000 m.*

Calculate :

(*i*) *Radius of arc TD,* (*ii*) *Length of arc AT,*

(*iii*) *Length of arc TD,* (*iv*) *Chainage of point D.*

Solution. (Fig. 16.22)

(*i*) Let the radius of arc *TD* be *R*

From $\Delta\, O_1 BT_1$, $BT = O_1 T \tan 15° = 800 \tan 15° = 214.36$ m

$$CT = 791.71 - 214.36 = 577.35 \text{ m}$$

From $\Delta\, O_2\, TC$, $TC = R \tan 30°$

or $$R = \frac{TC}{\tan 30°} = \frac{577.35}{0.57735} = 1000 \text{ m.} \quad \textbf{Ans.}$$

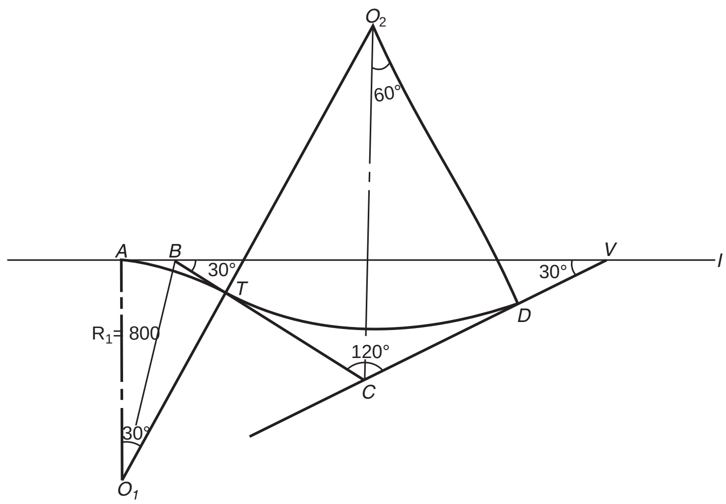


Fig. 16.22.

(*ii*) Length of $AT = \dfrac{\pi R \Delta_1}{180°}$ where $\Delta_1 = 30°$; $R = 800$

$$= \frac{\pi \times 800 \times 30°}{180°} - 418.88 \text{ m.} \quad \textbf{Ans.}$$

(*iii*) Length of arc $TD = \dfrac{\pi R_2 \Delta_2°}{180°}$ where $\Delta_2 = 60°$ $R_2 = 1000$ m

$$= \frac{\pi \times 1000 \times 60°}{180°} = 1047.20 \text{ m.} \quad \textbf{Ans.}$$

(*iv*) Chainage of point

B = 1000 (m. (given)

∴ Chainage of point A = Chainage of point $B - AB$

= 1000 − 214.36 = 785.64 m

Chainage of point T = Chainage of point A + arc AT

= 785.64 + 418.88 = 1204.52 m

Chainage of point D = Chainage of point T + arc TD

= 1204.52 + 1047.20 = 2251.72 m. **Ans.**

Example 16.11. *Two parallel lines which are 469 m apart are to be joined by a reverse curve ABC which deflects to the right by an angle of 30° from the first straight. If the radius of the first arc is 1400 m and the chainage of A is 2500 m, calculate the radius of the second arc and the chainages of the points B and C.*

Solution. (Fig. 16.23).

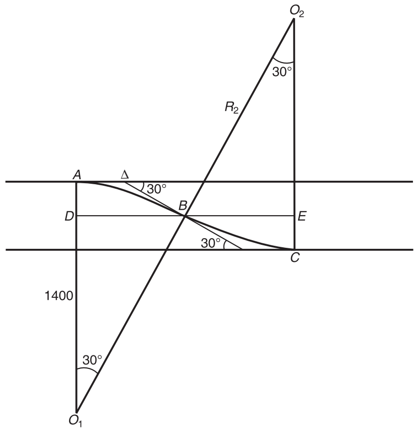


Fig. 16.23.

Let A and C be the points of tangencies and B, the point of reverse curvature.

The distance between the lines is 469 m.

We know from Eqn. (16.18), distance between parallel straights.

$$v = (R_1 + R_2) \text{ versin } \Delta_1 \quad \text{...}(i)$$

Substituting the values in Eqn. (i), we get

$$469 = (1400 + R_2)\ (1 - \cos 30°\ ) = (1400 + R_2)\ (1 - 0.866026)$$

$$\therefore \quad R_2 = \frac{469}{0.133974} - 1400 = 2100.68 \text{ m.}$$ **Ans.**

The chainage of B = The chainage of A + Length of arc AB

$$= 2500 + \frac{\pi \times 1400 \times 30°}{180°}$$

$$= 2500 + 773.04 = 3233.04 \text{ m.}$$ **Ans.**

The chainage of C = The chainage of B + Length of arc BC

$$= 3233.04 + \frac{\pi \times 2100.68 \times 30°}{180°}$$

$$= 3233.04 + 1099.91 = 4332.95 \text{ m.}$$ **Ans.**

Example 16.12. *A reverse curve AB is to be set out between two parallel railway tangents 32 m apart. If the two arcs of the curve are to have same radius and the distance between the tangent points A and B* *is 160 m, calculate the radius. The curve is to be set out from AB at 10 m intervals along that line. Calculate the lengths of the offsets.*

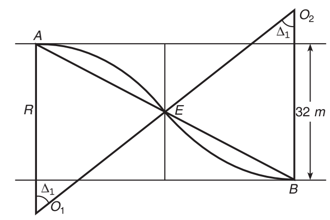


Fig. 16.24.

Solution. (Fig. 16.24).

We know from Eqn. (16.24) that the length (L) between tangent points A and B.

$$L = \sqrt{4Rv} \quad ...(i)$$

Substituting the values, in Eqn. (i), we get

$$160 = \sqrt{4 \times R \times 32}$$

$$R = \frac{160 \times 160}{4 \times 32} = 200 \text{ m.} \qquad \textbf{Ans.}$$

The maximum ordinate at the mid-points of AE and BE

$$O_0 = R - \sqrt{R^2 - \left(\frac{80}{2}\right)^2}$$

$$= 200 - \sqrt{200^2 - 40^2} = 4.04 \text{ m.}$$

The various offsets may be calculated from Eqn. (15.13).

$$O_m = \sqrt{R^2 - x^2} - (R - O_0)$$

x being measured from the mid-points of AE and EB.

$$O_{10} = \sqrt{200^2 - 10^2 - (200 - 4.04)}$$

$$= 199.75 - 195.96 = 3.79 \text{ m}$$

$$O_{20} = \sqrt{200^2 - 20^2} - (200 - 4.04)$$

$$= 199.00 - 195.96 = 3.04 \text{ m}$$

$$O_{30} = \sqrt{200^2 - 30^2} - (200 - 4.04)$$

$$= 197.74 - 195.96 = 1.78 \text{ m}$$

$$O_{40} = \sqrt{200^2 - 40^2} - (200 - 4.04)$$

$$= 195.96 - 195.96 = 0.000 \text{ m}$$

The ordinates of the other half of the arc AE are the same as above.
For each branch, offsets are :
0.00, 1.78, 3.04, 3.79, 4.04, 3.79, 3.04, 1.78, 0.00 m. **Ans.**

Example 16.13. *The lengths and bearings of a traverse carried out for the alignment of a highway are :*

Line	*Distance*	*Bearing*
AB	*238.36 m*	*60°*
BC	*Not known*	*160°*
CD	*643.35 m*	*30°*

A reverse curve AED is introduced having P.R.C. (point of reverse curvature) E on the line BC. If the radius of the first arc AE is 200 m, calculate the radius of the second arc ED.

Solution. (Fig. 16.25).

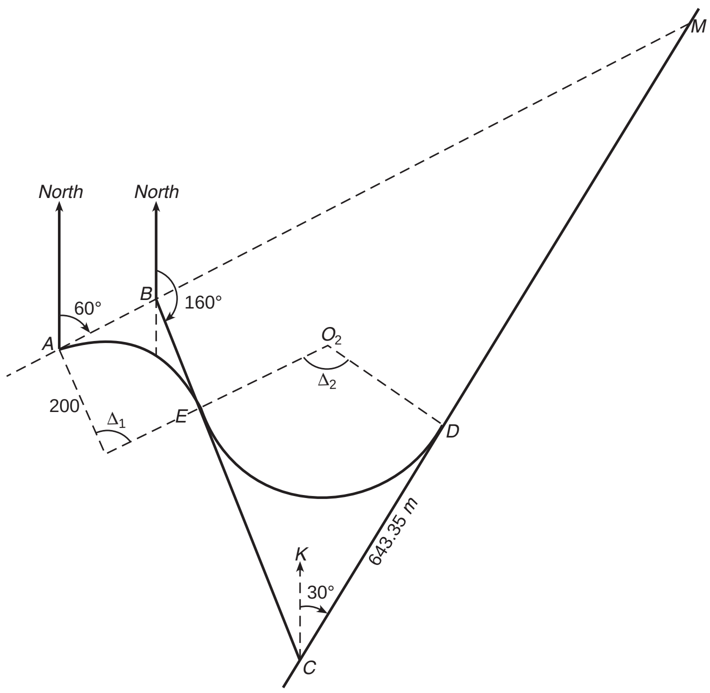


Fig. 16.25.

Let the two straights meet at M

$$\angle MBC = \text{Bearing of } BC - \text{Bearing of } AB.$$

$$= 160° - 60° = 100°$$

$\therefore$ Central angle $\Delta_1 = 100°$

and $$AB = BE = R_1 \tan 50°$$

$$= 200 \times 1.19175 = 238.35 \text{ m}.$$

Again, exterior angle ECD = Bearing of CB--Bearing of CD

$$= 340° - 30° = 310°o$$

$\therefore$ Interior $\angle ECD = 360° - 310° = 50°$

$\therefore$ Central angle $\Delta_2 = 180° - 50° = 130°$

$EC = CD = R \tan 65° = R \times 2.14451$

Substituting the value of $EC = CD = 643.35$, we get

or $$R = \frac{643.35}{2.14451} = 300 \text{ m.} \quad \textbf{Ans.}$$

Example 16.14. *A reverse curve AB is to be set out between two parallel railway tangents 30 metres apart. If the two arcs of the curve are to have the same radius and the distance between the tangent points A and B is to be 240 metres. Calculate the radius. The curve is to be set out by means of offsets from long chord AB at 20 metre intervals along that line. Calculate the lengths of the offsets.*

Solution. (Fig. 16.26).

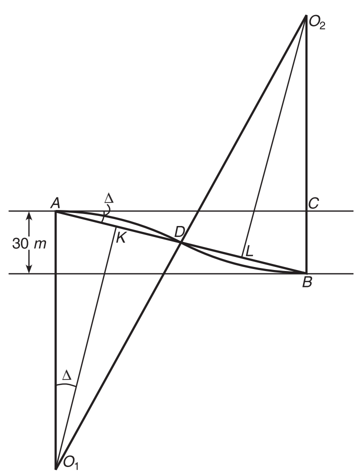


Fig. 16.26.

From ΔABC, we get

Sine $$CAB = \sin \Delta = \frac{BC}{AB} = \frac{30}{240} = \frac{1}{8} = 0.125$$

$\therefore$ $$\Delta = 7° \ 10' \ 51''$$

No. of offset intervals

$$= \frac{120}{20} = 6$$

Apex distance $\quad O_0 = R - R \cos \Delta = R\,(1 - \cos 7° \, 10' \, 51'')$

$$= 480 - 480 \times 0.992157$$

$$= 480 - 476.24 = 3.76 \text{ m.}$$

Offsets from long chord may be calculated from the formula

$$O_{10} = \sqrt{R^3 - x^2} - (R - O_o)$$

$$O_{20} = \sqrt{480^2 - 20^2} - (480 - 3.76) = 3.34 \text{ m}$$

$$O_{40} = \sqrt{483° - 40^2} - (480 - 3.76) = 2.09 \text{ m}$$

$$O_{60} = \sqrt{480 - 60^2} - (480 - 37.6) = 0.00\text{m}$$

Offsets from the chord AB are :

O_{20} = 2.09 m left $\quad O_{140}$ = 2.09 m right

O_{40} = 3.34 m left $\quad O_{160}$ = 3.34 m right

O_{60} = 3.76 m left $\quad O_{180}$ = 3.76 m right

O_{80} = 3.34 m left $\quad O_{200}$ = 3.34 m right

O_{100} = 2.09 m left $\quad O_{220}$ = 2.09 m right

O_{120} = 0.00 m left $\quad O_{240}$ = 0.00 m right

Example 16.15. *Two parallel railway tracks were to be connected by a reverse curve, each section having the same radius. The distance between their centre lines was 20 m. The distance between tangent points measured parallel to the track was 80 m. Determine the radius of the curve.*

If the radii were to be different, calculate :

(*i*) *The radius of the second if that of the first was 90 m.*

(*ii*) *The lengths of both branches of the curve.*

Solution. (Fig. 16.26)

Let A and B be the points of tangencies.

O_1 and O_2 be the centres of the arcs.

From $\Delta\, ABC$

$$AB = \sqrt{BC^2 + AC^2}$$

or

$$L = \sqrt{20^2 + 80^2} \qquad \text{...}(i)$$

But from Eqn. (16.24), we get

$$L = \sqrt{4R_v} \qquad ...(ii)$$

Equating Eqn. (*i*) and (*ii*), we get

$$\sqrt{4R_v} = \sqrt{20^2 + 80^2}$$

or

$$R = \frac{20^2 + 80^2}{4 \times 20}$$

$$= 85 \text{ m. } \textbf{Ans.}$$

From ΔACB

$$\tan \Delta = \frac{BC}{AC}$$

$$= \frac{20}{80} = 0.25$$

$$\Delta = 14° \, 02' \, 10'' \text{ and } 2\,\Delta$$

$$= 28° \, 04' \, 20''$$

Substituting the values in Eqn. (16.21), we get

$$(80^2 + 20^2) = 2 \times 20 \, (90 + R_{2)}$$

or

$$R_2 = \frac{80^2 + 20^2}{2 \times 20} - 90$$

$$= 170 - 90$$

$$= 80 \text{ m. } \textbf{Ans.}$$

Length of the first branch *AD*

$$= \frac{\pi \, R_2 \, \Delta}{180°}$$

$$= \frac{\pi \times 90 \times 28.072221}{180°}$$

$$= 44.10 \text{ m. } \textbf{Ans.}$$

Length of the second branch *DB*

$$= \frac{\pi R_2 \, \Delta}{180°}$$

$$= \frac{\pi \times 80 \times 28.072221}{180°}$$

$$= 39.20 \text{ m. } \textbf{Ans.}$$

EXERCISE 16

1. Pick up the correct word(s) from the brackets to fill in the blanks.

(*i*) A compound curve may consist of two or more arcs which deviate in......direction (same, opposite)

(*ii*) A compound curve tangential to three straights and consisting of arcs of different radii, is known as......centred compound curve (three, two, one)

(*iii*) A reverse curve is generally provided if straights are either parallel or the angle between them is very...... (small, large)

(*iv*) The use of a reverse curve on high ways is avoided because sudden change of super elevation is required at the point of...... (commencement, tangency, reverse curvature)

(*v*) In a reverse curve, superelevation at the point of reverse curvature is...... (nil, least, greatest)

(*vi*) If Δ_1 and Δ_2 are the angles of deflection of the arcs of a reverse curve, L is the distance between tangent points, the radius of the arcs will be

$$\left(\frac{L}{\sin \Delta_1 + \sin \Delta_2}, \frac{L}{\tan \frac{\Delta_1}{2} + \tan \frac{\Delta_2}{2}}, \frac{L}{\cos \frac{\Delta_1}{2} + \cos \frac{\Delta_2}{2}}\right)$$

(*vii*) If R_1 and R_2 are the radii of two arcs of a reverse curve introduced between two parallel straights separated by v, the distance between tangent points is

$$\sqrt{2v\,(R_1 + R_2)},\ \sqrt{v\,(R_1 + R_2)},\ v^2\,(R_1 + R_2),\ 2v\,(R_1 + R_2)$$

(*viii*) If R_1 and R_2 are the radii of two arcs of a reverse curve introduced between two parallel straights, and Δ is the central angle of each arc, then the distance between points of tangencies measured along straight is

$$\left[(R_1 + R_2) \sin \Delta, \frac{(R_1 + R_2)}{\sin\ \Delta}, \frac{R_1 + R_2}{\cos \Delta}, (R_1 + R_2) \tan \Delta\right]$$

(*ix*) If the distance between two parallel straights is 25 m, and the radii of each arc is 100 m, the distance between points of tangencies is (100 m, 50 m, 125 m, 75 m)

2. A compound curve consists of two simple curves whose radii are 300 m and 700 m. It is inserted between two straights which intersect at 5265 m and their angle of deflection is 70°. The curve is to start at chainage 4955 m. Calculate the chainage of the point of junction of the curves and at the end of the curve.

3. A railway siding is deflected through an angle of 80° . The two straights intersect at a chainage of 555.0 m. To avoid an existing building, a compound curve of two arcs having radii equal to 250 m and 350 m is proposed. The length of the first tangent distance is kept as 230 m. Calculate the length of the whole curve and the length of the second tangent length.

4. To set out a three centered compound curve between the straights AI and BI which intersect at an angle of intersection 45° and at a chainage of 1000 m, the following data was obtained.

Bearing of T_1K = 45° , Bearing of KL = 110°

Bearing of LM = 150° , Bearing of MT_2 = 180°

Distance T_1I = 600 m, Distance T_1K = 100 m

Distance KL = 250 m, Distance LM = 200 m

Calculate the length of the curve and the chainage of end of the curve.

5. The angle of intersection between two straights is very acute. If the length of the common tangent to the reverse curve to be introduced between straights is 100 m, and the deflection angle of the common tangents from straights are 30° and 40° , calculate the length of the common radius and the length of the whole curve.

6. The angle of intersection of the straights T_1I and I_2I is 25° . A reverse curve T_1OT is to be set out such that the length of the common tangent BC = 450° m and the central angles of the two circular arcs are 35° and 60° respectively. If the radius of the curve originating from T_1 is 675 m, calculate the distance between their centres and also the length of the whole curve.

7. Two straights AB and BC intersect at B deflecting through 22° right. A reverse curve is introduced with its point of commencement B. The ends of the common tangents are selected at 225 m and 60 m from B. If the radius of the first arc is 400 m, calculate the radius of the second arc.

Later on, it was decided to replace the reverse curve by a simple circular curve of same length. Calculate its radius.

ANSWERS

1. (*i*) same

(*ii*) two

(*iii*) small

(*iv*) reverse curvature

(*v*) nil

(*vi*) $\dfrac{L}{\tan \frac{1}{2}\,\Delta_1 + \tan \frac{1}{2}\,\Delta_2}$

(*vii*) $\sqrt{2v\,(R_1 + R_2)}\,(R_1 + R_2)\sin \Delta/$

(*ix*) 100 m.

2. 5111.70 m, 5601.20 m

3. 413.28 m, 266.50 m

4. 563.49 m, 963.49 m

5. 158.25 m, 193.34 m

6. 1085.80 m, 842.52 m

7. 2597.72 m, 400.57 m

17

Transition Curves

17.1. INTRODUCTION

A non-circular curve introduced between a straight and a circular curve, is known as a *transition Curve*. The curvature of such a curve varies from zero at its beginning to a definite value at its junction with the circular curve.

17.2. NECESSITY OF A TRANSITION CURVE

As soon as a moving vehicle starts negotiating a curve, it is acted upon by the centrifugal force which tends to overturn the moving vehicle. Sudden change of curvature from zero to a definite value at the point of commencement of the curve, causes lurching of the moving vehicle and also causes great discomfort to the passengers. The effect of the centrifugal force may be neutralised if the outer edge of the track is raised. The raising of the outer edge of the track or the outer rail of the railways, is called *superelevation* or *cant*. The amount of the superelevation depends on the speed of the moving vehicles and the radius of the curve. In case of railways, the maximum superelevation should not exceed 15 cm. Thus, it is evident that outer rail should be raised by 15 cm at the point of commencement, which is highly objectionable. The effect of the centrifugal force may be reduced and gradual increase of the super-elevation may be made by introducing a transition curve between the straight and the curve. Introduction of transition curves has following advantages :

1. It enables to introduce super-elevation in proportion to the rate of change of curvature.
2. It avoids the danger of derailment at the point of commencement if full amount of super-elevation is suddenly applied at the point.
3. It avoids over turning and side slipping of the moving vehicles.
4. It eliminates discomforts caused to the passengers while negotiating a curve.

17.3. TYPES OF TRANSITION CURVES

There are mainly three types of transition curves, namely :

(*i*) Cubical spiral
(*ii*) Cubic parabola
(*iii*) The Lemniscate curve.

Cubical spiral and cubic parabolic transition curves are best suited to railway curves and Lemniscates for highway curves.

1. Cubical Spiral. The standard equation of a cubical spiral (Fig. 17.1) is given by

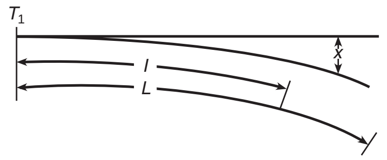


Fig. 17.1. Cubic parabola.

$$x = \frac{l^3}{6\,RL} \qquad ...(17.1)$$

where L = Total length of the transition curve

R = radius of the circular curve.

l = distance measured along the curve

x = perpendicular offset from the tangent.

2. **Cubic parabola.** The standard equation of a cubic parabola (Fig. 17.2) is given by :

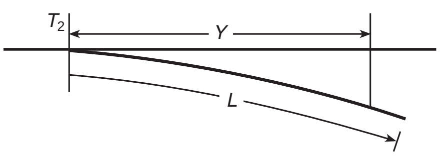


Fig. 17.2. Cubic parabola.

$$x = \frac{y^3}{6\,RL} \qquad ...(17.2)$$

where x = perpendicular offset from the tangent

y = distance measured along the tangent

R = radius of the circular curve.

L = length of the transition curve

3. **The Lemniscate curve.** The standard equation of a lemniscate curve (Fig. 17.3) is given by :

$$r = \frac{\rho}{3 \sin 2\alpha} \qquad ...(17.3)$$

where r = radius of the curvature

ρ = polar ray of any point

α = polar deflection angle, i.e.,

angle between the polar ray and the straight.

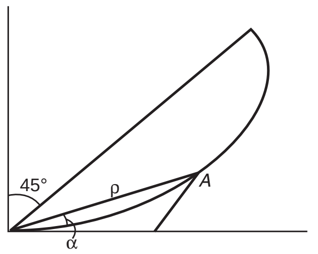


Fig. 17.3. Lemniscate Curve.

17.4. SUPER-ELEVATION

When a vehicle moves from a straight to a curve, it is acted upon by centrifugal force in addition to its own weight. The centrifugal force acts through the centre of gravity of the moving body horizontally away from the centre of the curve and tends to push the vehicle off the track. In order to balance this force, outer rail on railway or outer edge of highways, is raised above the inner one. The difference in the top levels of outer and inner rails is known as *super elevation*, which depends upon the radius of the curve and the speed of the vehicles. As all the vehicles do not run with the same speed, the average of the fastest and slowest speeds of the vehicles is used for the purpose of calculation of the required amount of super elevation.

17.5. DERIVATION OF THE FORMULA FOR SUPER-ELEVATION

Let O be the centre of the curve

R be the radius of the curve

v be the speed of the vehicle

t be the time required to travel an arc PP'

θ be the angle subtended by the arc PP' at the centre O.

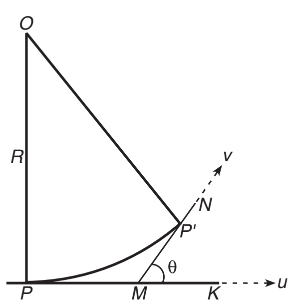


Fig. 17.4.

As the vehicle moves along the curve from P to P', the direction of the speed after time t becomes along MN, where $\angle POP'$ is small and equals to θ. (Fig. 17.4).

Resolving the speed v parallel and perpendicular to PO.

Component along PO is $v \sin \theta$.

Component along PK is $v \cos \theta$.

$\because$ The radial acceleration along PO

$$= \frac{\text{changed velocity}}{\text{time}} = \frac{v \sin \theta}{t}$$

$$= \frac{v.\,\theta}{t} \qquad (\sin \theta = \theta \text{ if } \theta \text{ is small})$$

$$= \frac{v}{t} \cdot \frac{\text{arc } PP'}{R} \qquad \text{...(17.4)}$$

Again, $$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} = v \text{ m/sec} \qquad \text{...(17.5)}$$

Substituting the value of $\frac{\text{arc } PP'}{t} = v$ in Eqn. (17.4), we get

Radial acceleration $= \frac{v\,.\,v}{R} = \frac{v^2}{R}$

But force $=$ mass $\times$ acceleration

$\therefore$ Radial force $= m \cdot \frac{v^2}{R}$

where m is the mass of the vehicle

$$= \frac{W}{g} \times \frac{v^2}{R}$$

where W is the weight of the vehicle.

According to Newton's third law of motion, an equal and opposite force is created.

$\therefore$ Centrifugal force P acting along the radius but away from the centre

i.e. $$P = \frac{W}{g} \cdot \frac{v^2}{R} \qquad \text{...(17.6)}$$

or Centrifugal ratio $= \frac{\text{centrifugal force}}{\text{weight of the vehicle}}$

$$= \frac{\frac{W}{g} \cdot \frac{v^2}{R}}{W} = \frac{v^2}{gR} \qquad ...(17.7)$$

Let O be the centre of gravity of the vehicle. (Fig. 17.5) a and c be tops of the rails

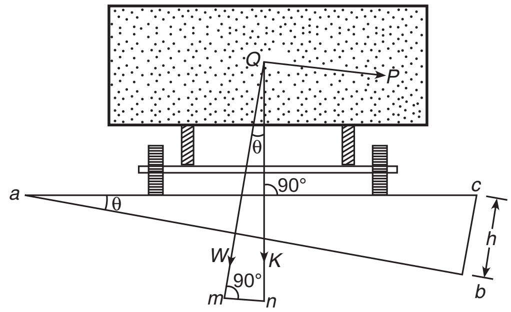


Fig. 17.5. Superelevation.

h, be the superelevation

θ angle *cab*

P, be centrifugal force

W, be the weight of the vehicle

Δ*s abc* and *Omn* are similar. Centrifugal force *P,* weight *W* and their resultant *K* may be represented by the side *mn, Om* and *On* of the triangle *Omn*.

Again $\tan\theta = \frac{bc}{ab} = \frac{mn}{Om} = \frac{P}{W}$

Substituting the value of θ from Eqn. (17.6), we get

$$\tan\theta = \frac{\frac{Wv^2}{gR}}{W} = \frac{v^2}{gR}$$

But $h = ab\tan\theta = \frac{ab.v^2}{gR}$

$$\therefore \text{ Superelevation } (h) = \frac{G.v^2}{g.R} \qquad ...(17.8)$$

where G is the gauge of the railway track.

17.6. LENGTH OF A TRANSITION CURVE

The transition curve is introduced between a straight and a circular curve in order to introduce superelevation gradually from zero at the point of commencement of the transition curve and the full amount at the junction of transition curve and the circular curve. The length of the transition curve may be calculated from one of the following considerations.

1. Definite rate of superelevation. Keeping in view the type of moving vehicles, the length of the transition curve may be calculated by assuming a definite rate of superelevation say 1 in 300 to 1 in 500.

If h is the amount of superelevation in centimetres

1 in n is the rate of superelevation over the transition curve

L is the length of the transition curve in metres.

then $$L = \frac{n \times h}{100} \text{ metres} \qquad ...(17.9)$$

Example 17.1. *Calculate the length of a transition curve to be introduced between a straight and a curve such that 15 cm superelevation may be introduced over the circular curve. Assume the rate of superelevation as 1 in 500.*

Solution.

From Eqn. (17.9), we know

$$L = \frac{n \times h}{100} \qquad ...(i)$$

Here $n = 500, \quad h = 15$ cm

Substituting the values in Eqn. (i), we get

$$L = \frac{500 \times 15}{100} = 75 \text{ m.}$$ **Ans.**

2. Arbitrary rate of superelevation in centimetres per second. The length of the transition curve may be calculated by assuming a suitable time rate say "x" cm per second.

Let v be the average speed of vehicles in m / sec

h be the amount of superelevation in centimetres

x be the time rate, where x varies from 2.5 to 5.0 cm per second

L be the length of the transition curve in metres.

Time taken by the vehicle to travel a distance $L = \frac{L}{v}$ sec

Superelevation = time rate × time taken by the vehicle *i.e.*

$$h = x \times \frac{L}{v}$$

or $$L = \frac{h.v}{x} \qquad ...(17.10)$$

Example 17.2. *Calculate the length of a transition curve to be inserted between a straight and a circular curve such that a superelevation of 15 cm over a circular curve may be attained. Assume the rate of attaining superelevation as 2.5 cm per second and average speed of the vehicles as 60 km / hour.*

Solution. From Eqn. (17.10), we know

$$L = \frac{h.v}{x} \qquad \text{...}(i)$$

Here $h = 15$ cm, $x = 2.5$ cm/sec.

$$v = \frac{60 \times 1000}{60 \times 60} = \frac{100}{6} \text{ m/sec}$$

Substituting the values in Eqn. (*i*), we get

$$L = \frac{15 \times 100}{2.5 \times 6} = 100 \text{ m.} \quad \textbf{Ans.}$$

3. Definite rate of change of radial acceleration, say 30 cm/sec^2.

Let v be the average speed of the vehicles in metres/sec

R be the radius of the circular curve in metres

$\frac{v^2}{R}$ be radial acceleration.

L be the length of the transition curve in metres

'c' is the rate of development of radial acceleration.

Time taken to travel length $L = L/v$ secs.

Time required to attain the maximum radial acceleration

$$= \frac{v^2}{R.c} \text{ secs}$$

Comparing the values of time taken, we get

$$\frac{L}{v} = \frac{v^2}{R.c}$$

or

$$L = \frac{v^3}{R.c} \qquad \text{...(17.11)}$$

Example 17.3. *The maximum allowable speed on a curve is 80 km / hour and the rate of change of radial acceleration is 30 cm / sec^2. Calculate the length of the transition curve if the radius of the circular curve is 200 metres.*

Solution. From Eqn. (17.11), we get

$$L = \frac{v^3}{R.c} \qquad ...(i)$$

Here $v = \frac{80 \times 1000}{60 \times 60} = 22.22 \text{ m/sec}$

$R = 200$ metres, $c = 0.3$ m/sec^2

Substituting the values in Eqn. (*i*), we get

$\therefore$ $L = \frac{(22.22)^3}{200 \times 0.3} = 182.9$ m **Ans.**

4. The length of the transition curve, may arbitrarily be assumed say 100 m, 120 m etc.

17.7. REQUIREMENTS OF AN IDEAL TRANSITION CURVE

An ideal transition curve should meet the following requirements :

1. An ideal transition curve should be tangential to the straight as well as to the circular curve.
2. The curvature of an ideal transition curve should be zero at its origin on the straight.
3. The radius of an ideal transition curve at the junction of the circular curve should be same as that of the circular curve.
4. The length of an ideal transition curve should be such that the required superelevation is attained at its junction with the circular curve.
5. The rate of increase of curvature along an ideal transition curve should be same as that of superelevation.

Note. Following points may be noted.

(*i*) First three requirements are precise and definite. The rate of change of curvature and length of transition curve depend on the rate at which the superelevation is introduced.

(*ii*) It has been universally accepted that superelevation is introduced at a uniform rate and curvature of the transition curve at any point is kept proportional to its distance from the beginning of the transition curve.

17.8. EQUATION OF AN IDEAL TRANSITION CURVE

As the superelevation is required to be introduced uniformly, the centrifugal force must be increased at a constant rate and hence it must vary with time. On the other hand, the speed of the vehicle is also kept constant. The distance travelled along the transition curve must also, therefore vary with time.

Centrifugal force is proportional to the length of the transition curve.

or $\frac{Wv^2}{gr} \propto I$

But W, v and g are constants for particular vehicle.

Hence $l \propto \frac{1}{r}$

or $$Ir = a \text{ constant} = LR \qquad ...(17.12)$$

where L = total length of the transition curve

R = radius of the transition curve at its junction with the circular curve.

As the superelevation (h) increases at a constant rate, it must be proportional to l.

i.e. $$h \propto I \propto \frac{Gv^2}{gr}$$

But G, v and g are constants.

$\therefore$ $$I \propto \frac{I}{r}$$

or $$Ir = \text{a constant} = LR \qquad ...(17.13)$$

Equations (17.12) and (17.13) suggest that the fundamental requirement of a transition curve is that its radius of curvature (r) at any point, may be inversely proportional to the distance (l) from the beginning of the transition curve to the point.

Curves fulfilling the above requirements, are known as "*Clothoids*" or "*True spirals*".

17.9. INTRISIC EQUATION OF AN IDEAL TRANSITION CURVE. (FIG. 17.6)

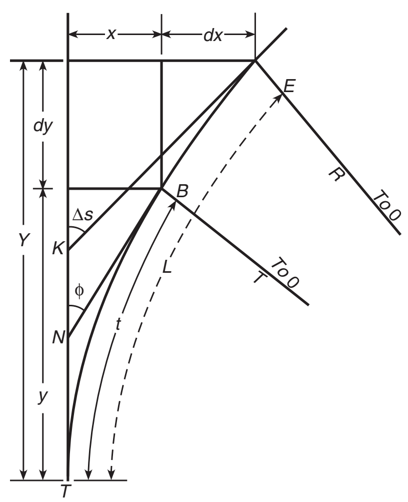


Fig. 17.6. An intristic equation of an ideal curve.

Let O be the centre of the main circular curve

R be the radius of the main circular curve

E be the junction of the transition curve and circular curve

B be any point on the transition curve, distant l from the point of commencement T.

Construction. Draw tangents to the curve at E and B to meet the back tangent at K and N respectively.

From Eqn. (17.13) we get

$$\frac{1}{r} = \frac{I}{LR}$$

But $$\frac{1}{r} = \text{curvature of the curve } = \frac{d\phi}{dl}$$

$\therefore$ $$\frac{d\phi}{dl} = \frac{l}{LR}$$

or $$d\phi = \frac{l.dl}{LR} \qquad ...(17.14)$$

Integrating Eqn. (17.14), we get

$$\phi = \frac{l^2}{2LR} + C$$

where C is a constant of integration

When $l = 0, \quad \phi = 0 \qquad \therefore \; C = 0$

or $$\phi = \frac{l^2}{2LR} \qquad ...(17.15)$$

Eqn. (17.15) is the required intrinsic equation of an ideal transition curve.

Equation (17.15) may be expressed as

$$I = K\phi^{1/2} \qquad ...(17.16)$$

where $K = \sqrt{2LR}$

When $l = L, \quad \phi = \Delta s$

Substituting the values in Eqn. (17.15), we get

$$\Delta_S = \frac{L^2}{2LR} = \frac{L}{2R} \qquad ...(17.17)$$

The angle Δ_S is generally known as *spiral angle*.

17.10. CARTESIAN CO-ORDINATES OF THE POINTS ON A TRANSITION CURVE

To set out the transition curve by perpendicular offsets from the back tangent, the cartesian co-ordinates (or rectangular co-ordinates) may be computed as under. (Fig. 17.7).

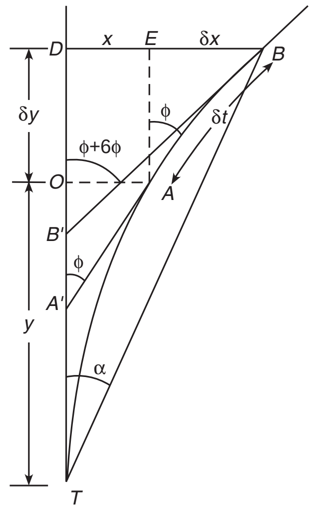


Fig. 17.7.

Assumptions. The back tangent is assumed as *Y*-axis, a line perpendicular to it as the *X*-axis, and point of commencement *T* as the origin of coordinates.

Let *A* and *B* be two points on the curve separated by a distance δl.

ϕ be the angle between the tangent *AA′* drawn at *A* and the back tangent.

$\phi + \delta\phi$ be the angle between the tangent *BB′* drawn at *B* and the back tangent *TD*.

(x, y) be the cartesian coordinates of *A*.

$(x + \delta x, y + \delta y)$ be the cartesian co-ordinates of *B*.

Construction : Draw *BD* and *AO* perpendiculars to the back tangent *TD* to meet at *D* and *O* respectively.

Draw *AE* perpendicular to *BD* to meet at *E*.

From the right angled triangle *AEB*, we get

$AE = \delta y$, $EB = \delta x$, $AB = \delta l$, angle $EAB = \phi$.

Now $$\frac{EB}{AB} = \frac{\delta x}{\delta l} = \sin\phi$$

$$\delta x = \delta l \sin\phi$$

or $$\delta x = \delta l \left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \ldots \right) \qquad \ldots(17.18)$$

Differentiating the Eqn. (17.16), we get

$$\delta l = \frac{K}{2\phi^{1/2}} \, . \, \delta \phi \qquad \ldots(17.19)$$

Substituting the value of δl in Eqn. (17.18), we get

$$\delta x = \frac{K}{2} \left(\delta^{1/2} - \frac{\delta^{5/2}}{3.2.1} + \frac{\phi^{9/2}}{5.4.3.2.1} - \ldots \right) d\phi$$

or $$\delta x = \frac{K}{2} \left(\phi^{1/2} - \frac{\phi^{5/2}}{6} + \frac{\delta^{9/2}}{120} - \ldots \right) d\phi$$

Integrating the above equation, we get

$$x = K \left(\frac{\phi^{3/2}}{3} - \frac{\phi^{7/2}}{42} + \frac{\phi^{11/2}}{1320} - \ldots \right)$$

$$= K\phi^{3/2} \, \frac{1}{3} \left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440} - \ldots \right)$$

Substituting the value of $l = K\phi^{1/2}$, and $\phi = \dfrac{l^2}{K^2}$, we get

$$x = \frac{l^3}{3K^2} \left(1 - \frac{l^4}{14\,K^4} + \frac{l^8}{440\,K^8} - \ldots \right)$$

Substituting the value $K = \sqrt{2RL}$ we get

or $$x = \frac{l^3}{6RL} \left(1 - \frac{l^4}{56R^2\,L^2} + \frac{l^3}{7040\,R^4\,L^4} - \ldots \right) \qquad \ldots(17.20)$$

From the right angled triangle AEB

$$\frac{\delta\, y}{\delta\, l} = \cos \delta$$

or $$\delta\, y = \delta\, l \cos \phi = \delta l \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4\,!} - \ldots \right)$$

Substituting the value of δl from Eqn. (17.19)

$$\delta y = \frac{K}{2} \left(\phi - \frac{1}{2} \, \frac{\phi^{3/2}}{2!} + \frac{\phi^{7/2}}{4!} - \ldots \right) \delta\, \phi$$

Integrating the above equation

$$y = K \left(\phi^{1/2} - \frac{\phi^{5/2}}{2\,! \times 5} + \frac{\phi^{9/2}}{4\,! \times 9} - \ldots \right)$$

$$= K\phi^{1/2}\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \dots\right)$$

Substituting the value of K, we get

$$y = l\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \dots\right) \quad \text{...(17.21)}$$

Substituting the value of ϕ from Eq. (17.16) in Eq. (17.21)

$$y = l\left(1 - \frac{l^1}{10\,K^4} + \frac{l^3}{216\,K^3} - \dots\right)$$

Substituting the value of $K = \sqrt{2RL}$, we get

$$y = l\left(1 - \frac{l^4}{10 \times 4R^2\,L^2} + \frac{l^8}{216 \times 16R^4\,L^4} - \dots\right)$$

or

$$y = l\left(1 - \frac{l^4}{40\,R^2\,L^2} + \frac{l^8}{3456\,R^4\,L^4} - \dots\right) \quad \text{...(17.22)}$$

From equations (17.20) and (17.22) the cartesian co-ordinates of various points on the transition curve may be calculated.

17.11. MODIFICATION OF THE IDEAL TRANSITION CURVE

As the computation of cartesian co-ordinates from Eqn. (17.20) and (17.22) is not simple, modifications are made to the ideal transition curve to suit our requirements.

1. The cubic spiral. If we assume $\sin \phi = \phi$ and ignore other terms of its value, the equation (17.20) reduces to

$$x = \frac{l^3}{6RL} \quad \text{...(17.23)}$$

which is the equation of the cubic spiral.

2. The cubic parabola. If we neglect all the terms except the first one of Eqn. (17.21), we get

$$y = l \quad \text{...(17.24)}$$

Similarly from Eqn. (17.20), we get

$$x = \frac{l^3}{6RL} \quad \text{...(17.25)}$$

Substituting the value of $l = y$ in Eqn. (17.25), we get

$$x = \frac{y^3}{6RL} \quad \text{...(17.26)}$$

which is the equation of the cubic parabola, generally known as *Froude's transition equation.*

17.12. DEFLECTION ANGLES FOR TRANSITION CURVES

In case transition curve is set out with a theodolite, deflection angles for various points on the curve from the point of commencement, may be calculated as under. (Fig. 17.7).

Let α be the polar deflection angle for any point.

$$\tan\alpha = \frac{x}{y} = \frac{K\left(\frac{\phi^{3/2}}{3} - \frac{\phi^{7/2}}{42} + \frac{\phi^{11/2}}{1320} - \ldots\right)}{K\left(\phi^{1/2} - \frac{\phi^{5/2}}{10} + \frac{\phi^{9/2}}{216} - \ldots\right)}$$

$$= \frac{\phi}{3} + \frac{\phi^3}{105} + \ldots \qquad \ldots(17.27)$$

This is resembling very closely with the expression

$$\tan\frac{\phi}{3} = \frac{\phi}{3} + \frac{\phi^3}{81} + \ldots$$

i.e. $\tan\alpha \approx \tan\phi/3$

or $\alpha = \frac{\phi}{3}$ in radians ...(17.28)

$$= \frac{1}{3}\cdot\frac{l^2}{2RL} = \frac{l^2}{6RL} \text{ radians} \qquad \ldots(17.29)$$

$$= \frac{l^2}{6RL}\cdot\frac{180°}{\pi}$$

$$= \frac{30\,l^2}{\pi RL} \text{ degrees} \qquad \ldots(17.30)$$

$$= \frac{1800\,l^2}{\pi RL} \text{ minutes} \qquad \ldots(17.31)$$

or $\alpha = \frac{573\,l^2}{RL}$ minutes ...(17.31*a*)

Note. The following points may be noted.

(*i*) In above equations l is the distance of the point from the point of commencement measured along the transition curve.

(*ii*) R is the radius of the main circular curve.

(*iii*) L is the length of the transition curve.

17.13. CHARACTERISTICS OF A TRANSITION CURVE

The transition curves are introduced at both the ends of a circular curve by shifting the main curve inwards. To design a transition curve and to calculate the necessary data for its setting out, one should understand various components of the transition curve, which are discussed here under. (Fig. 17.8).

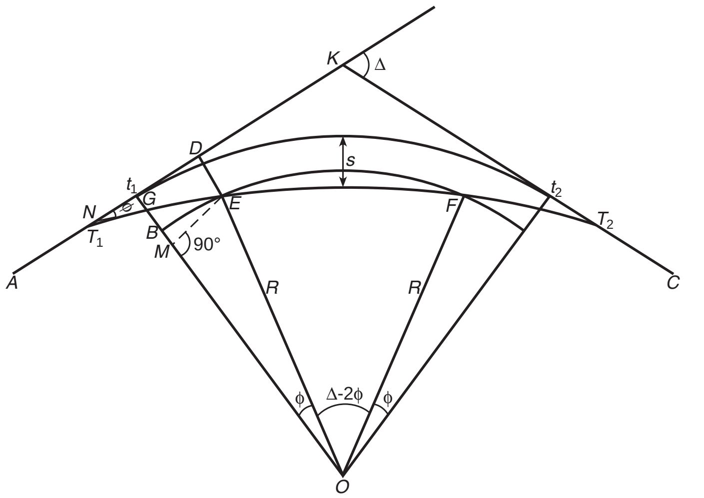


Fig. 17.8.

Let *AK* and *KC* to two straights.

Δ, the angle of deflection

t_1 ,t_2, the points of tangencies of the original curve

T_1 , T_2 the points of tangencies of transition curve

E and *F*, the junction points of the transition curves with the circular curve.

R, the radius of the circular curve.

S, the shift of the circular curve

EN, the tangent at *E* meeting back tangent *AK* at *N*

ø the spiral angle

O, the centre of the main circular curve

Construction. Drop *EM* perpendicular to *O* t_1 and *ED* perpendicular to *AK*.

1. **The spiral angle.** The angle between the back tangent and tangent at the junction of the transition curve with the circular curve, is called *spiral angle.*

From Δ*EMO*, we know

$$\angle MOE = 90° - \angle MEO$$
$$= \angle MEN = END = \phi$$

2. **Shift.** The distance through which main circular curve is shifted inward to accommodate the transition curves, is known as *shift.* Its value is $\dfrac{L^2}{24R}$.

Proof : Assume T_1, the point of commencement as origin of the coordinates.

Back tangent as Y-axis and a perpendicular to back tangent as X-axix.

Let $DE = X$ and $T_1, D = Y$

Construction. Prolong the shifted circular curve beyond E upto B.

Now, Angle DNE = angle NEM = angle $EOM = \phi$.

The length of arc $EB = R.\phi$

or $$= R.\ \frac{L}{2R}$$

As EG is approximately equal to EB.

$$EG = \frac{L}{2}$$

i.e. the shift $t_1\, B$ bisects the transition curve at G.

Shift
$$S = t_1\, B$$
$$= t_1\, M - BM = DE - (OB - OM)$$
$$= X - (R - R \cos \phi) = X - R\,(1 - \cos \phi)$$
$$= X - R.2 \sin^2 \frac{\phi}{2}$$
$$= X - 2\,R\left(\frac{\phi}{2}\right)^2 \qquad (\phi/2 \text{ being small})$$

Substituting the values of $X = L^3/6LR$ and $\phi = L/2R$, we get

$$S = \frac{L^3}{6RL} - 2\,R \times \left(\frac{L}{4R}\right)^2$$
$$= \frac{L^2}{6R} - \frac{L^2}{8R}$$

or $$S = \frac{L^2}{24\,R} \qquad ...(17.32)$$

i.e. the shift of the main curve is directly proportional to the square of the length of the transition curve and inversely proportional to the radius of the circular curve.

3. **The tangent length of the combined curve**

Total tangent length $= T_1 K = T_1 t_1 + t_1 K$

But, $t_1 K = (R + S) \tan \frac{\Delta}{2}$ and $T_1 t_1 = \frac{L}{2}$

$\therefore$ Total tangent length $= (R + S) \tan \frac{\Delta}{2} + \frac{L}{2}$ ...(17.33)

4. **Length of the combined curve**

The central angle for the circular curve $= \Delta - 2\phi$

The length of the circular curve $= \dfrac{\pi R (\Delta - 2\phi)}{180^\circ}$

$\therefore$ Total length of the combined curve

$= L$ + length of the circular curve $+ L$

$$= \frac{\pi R (\Delta - 2\phi)}{180^\circ} + 2L \qquad ...(17.34)$$

5. **Chainages of main points of the curve.**

1. The chainage at the point of commencement (T_1) of the combined curve

= the chainage at the point of intersection (K)–total tangent length.

2. The chainage at the first junction point (E)

= Chainage at the point of commencement (T_1) + length of the transition curve (L).

3. The chainage at the second junction point (F).

= chainage of the first junction point (E)

+ length of the circular curve.

4. The chainage at the point of tangency (T_2)

= chainage of the second junction point (F)

+ length of the transition curve (L).

Check : The chainage at the point of tangency (T_2) must be equal to the chainage at the point of commencement (T_1) + length of the combined curve *i.e.* $\left[\dfrac{\pi R (\Delta - 2\phi)}{180^\circ} + 2L\right]$.

Example 17.4. *Two straights AB and BC intersect at the chainage 2635.22 m, the deflection angle being 48° 24′. It is proposed to insert a circular curve of 300 m raidus with two transition curves, 80 m long at each end.*

Calculate the shift of the main curve, the spiral angle of the transition curve and the chainage at the point of commencement.

Solution. (Fig. 17.8).

Given data : L = 80 m, length of the transition curve

R = 300 m, radius of the curve

Δ = 48° 24′, deflection angle of the straights.

2635.22 m = chainage at the point of intersection

Calculations :

(*i*) Shift $S = \dfrac{L^2}{24R} = \dfrac{80^2}{24 \times 300}$

$= 0.888$ m. **Ans.**

(*ii*) Spiral angle $\phi = \dfrac{L}{2R} = \dfrac{80}{2 \times 300}$ radians

$= \dfrac{2}{15} \times \dfrac{180°}{\pi}$

$= 7.6394$ degrees $= 7° \, 38' \, 22''$. **Ans.**

(*iii*) Total tangent length of the combined curve

$= (R + S) \tan \dfrac{\Delta}{2} + \dfrac{L}{2}$

$= (300 + 0.888) \tan \dfrac{48° \, 24'}{2} + \dfrac{80}{2}$

$= 300.888 \tan 24° \, 12' + 40$

$= 175.22$ m.

(*iv*) Chainage at the point of intersection = 2635.22 m

Substract the length of total tangent = 175.22 m

∴ Chainage at the point of commencement

= 2460.00 m. **Ans.**

Example 17.5. *The angle of intersection between two straights is 140° . The spiral angle for each transition curve is 5° . If the radius of the main curve is 400 m, calculate the length of the transition curve and the length of the circular curve.*

Solution. (Fig. (17.8).

Given data : $\phi = 5°$, $R = 400$ m,

Calculations :

Angle of deflection,

$$\Delta = 180° - 140° = 40°$$

(*i*) Spiral angle $\phi = \frac{L}{2R}$

or $$L = 2\,R\,\phi = 2 \times 400 \times 5 \times \frac{\pi}{180°}$$

$$= 69.81 \text{ say } 70 \text{ m.} \quad \textbf{Ans.}$$

(*ii*) The length of the circular curve

$$= \frac{\pi\,R\,(\Delta - 2\phi)}{180°}$$

$$= \frac{\pi \times 400 \times (40 - 2 \times 5)}{180°}$$

$$= \frac{\pi \times 400 \times 30°}{180°} = 209.44 \text{ m.} \quad \textbf{Ans.}$$

Example 17.6. *Calculate the tangential offsets for setting out a transition curve of 80 m length with 20 m peg interval. Assume the radius of the circular curve as 300 m.*

Solution.

Cubic Parabola : The coordinates may be calculate from the formula.

$$x = \frac{y^3}{6RL}.$$

For $y = 0$ $\quad x = \frac{0}{6RL} = 0$ m

$y = 20$ $\quad x = \frac{20^3}{6 \times 300 \times 80} = 0.055$ m

$y = 40$ $\quad x = \frac{40^3}{6 \times 300 \times 80} = 0.444$ m

$y = 60$ $\quad x = \frac{60^3}{6 \times 300 \times 80} = 1.500$ m

$y = 80$ $\quad x = \frac{80^3}{6 \times 300 \times 80} = 3.555$ m

Check :

Shift for the circular curve,

$$S = \frac{L^2}{24\,R} = \frac{80^2}{24 \times 300} = \frac{8}{9} \text{ m}$$

Perpendicular offset for the junction point = 4 × shift

$$= 4 \times \frac{8}{9} = 3.555 \text{ m.} \qquad \text{O.K.}$$

Cubical spiral : The coordinates may be calculated from the formula

$$x = \frac{l^3}{6\,RL}$$

for $l = 0 \quad x = \dfrac{0^3}{6 \times 80 \times 300} = 0.0$ m

$l = 20 \quad x = \dfrac{20^3}{6 \times 80 \times 300} = 0.055$ m

$l = 40 \quad x = \dfrac{40^3}{6 \times 80 \times 330} = 0.444$ m

$l = 60 \quad x = \dfrac{60^3}{6 \times 80 \times 300} = 1.500$ m

$l = 80 \quad x = \dfrac{80^3}{6 \times 80 \times 300} = 3.555$ m.

Example 17.7. *It is proposed to insert a transition curve of 90 m length between a straight and a circular curve of 300 m radius. Calculate the deflection angles for setting out the transition curve if the peg interval on the transition curve is 15 m and chainage at the point of commencement is 1810.4 m.*

Solution.

Let *A, B, C, D, E* and *F* be the points on the transition curve whereas *T* and *L* be the point of commencement and junction point of the transition curve with the circular curve.

Chainage at $T = 1810.4$ m

Chainage at $A = 1815.0$ m

Chainage at $B = 1815.0 + 15.0 = 1830.0$ m

Chainage at $C = 1830.0 + 15.0 = 1845.0$ m

Chainage at $D = 1845.0 + 15.0 = 1860.0$ m

Chainage at $E = 1860.0 + 15.0 = 1875.0$ m

Chainage at $F = 1875.0 + 15.0 = 1890.0$ m

Chainage at $L = 1890.0 + 10.4 = 1900.4$ m.

$\therefore$ The distances of the various points *A, B, C,* .... etc. from the point of commencement are :

$$TA = 1815.0 - 1810.4 = 4.6 \text{ m}$$
$$TB = 1830.0 - 1810.4 = 19.6 \text{ m}$$
$$TC = 1845.0 - 1810.4 = 34.6 \text{ m}$$
$$TD = 1860.0 - 1810.4 = 49.6 \text{ m}$$
$$TE = 1875.0 - 1810.4 = 65.6 \text{ m}$$
$$TF = 1890.0 - 1810.4 = 79.6 \text{ m}$$
$$TL = 1900.4 - 1810.4 = 90.0 \text{ m}$$

Deflection angles may be calculated from the formula

$$\alpha = \frac{573 l^2}{RL} \text{ minutes}$$

Deflection angle for A $\quad \alpha_1 = \dfrac{573 \times (4.6)^2}{300 \times 90} = 0° \ 0' \ 27''$

Deflection angle for B $\quad \alpha_2 = \dfrac{573 \times (19.6)^2}{300 \times 90} = 0° \ 08' \ 09''$

Deflection angle for C $\quad \alpha_3 = \dfrac{573 \times (34.6)^2}{300 \times 90} = 0° \ 25' \ 24''$

Deflection angle for D $\quad \alpha_4 = \dfrac{573 \times (49.6)^2}{300 \times 90} = 0° \ 52' \ 13''$

Deflection angle for E $\quad \alpha_5 = \dfrac{573 \times (64.6)^2}{300 \times 90} = 1° \ 28' \ 34''$

Deflection angle for F $\quad \alpha_6 = \dfrac{573 \times (79.6)^2}{300 \times 90} = 2° \ 14' \ 28''$

Deflection angle for F $\quad \alpha_7 = \dfrac{573 \times (90)^2}{300 \times 90} = 2° \ 51' \ 54''$

Check : Total deflection angle $= \dfrac{1}{3} \times$ spiral angle

Spiral angle $= \dfrac{L}{2R} = \dfrac{90}{2 \times 300} = \dfrac{3}{20} \times \dfrac{180}{\pi} = 8° \ 34' \ 40''$

$$\therefore \qquad \alpha_7 = \frac{8° \ 35' \ 40''}{3} = 2° \ 51' \ 53''. \qquad \text{O.K.}$$

Example 17.8. *Two straights of a proposed road intersect at a chainage of (78 + 34) chians in 20 m units with a deflection angle 40° 30° (right). It is proposed to lay out a circular curve of 10 chains radius with transition curve 1.5 chains long at each end.*

Calculate the chainages of both tangent points and both junction points and, also tangential offset for setting out the first junction point.

Solution.

Given data : R = 10 chains = 200 m.

L = 1.5 chains = 30 m.

$\Delta = 40° \ 30'$

Chainage at the point of commencement

= (78 + 34) chains

= 78 × 20 + 34 × 0.2 = 1566.80 m

(*i*) Shift $(S) = \dfrac{L^2}{24R} = \dfrac{30^2}{24 \times 200} = 0.1875$ m.

(*ii*) Total tangent length $= (R + S) \tan \dfrac{\Delta}{2} + \dfrac{L}{2}$

$$= (200 + 0.1875) \tan \frac{40° \ 30'}{2} + \frac{30}{2}$$

$$= 200.1875 \tan 20° \ 15' + 15 = 88.85 \text{ m.}$$

(*iii*) Spiral angle $(\phi) = \dfrac{L}{2R}$ radians

$$= \frac{30}{2 \times 200} \times \frac{180}{\pi} = 4.29718$$

or $\phi = 4° \ 17' \ 50''$

(*iv*) The length of the circular curve

$$= \frac{\pi R \ (\Delta - 2\phi)}{180°}$$

$$= \frac{\pi \times 200 \ (40° \ 30' - 8° \ 35' \ 40'')}{180°}$$

$$= \frac{\pi \times 200 \times 31° \ 54' \ 20''}{180°} = 111.37 \text{ m}$$

(*iv*) Tangential offset for first junction point $= \dfrac{y^3}{6LR}$

$$= \frac{30^3}{6 \times 200 \times 30} = 0.75 \text{ m.} \quad \textbf{Ans.}$$

Check : Tangential offset for the junction point

$$= 4 \times \text{shift} = (4 \times 0.1875) = 0.75 \text{ m.}$$

Chainage at the point of intersection	= 1566.80 m.	
Substract total tangent length	− 88.85 m.	
Chainage at the point of commencement	= 1477.95 m.	**Ans.**
Add the length of the transition curve	+ 30.00 m	
Chainage at the first junction point	= 1507.95 m.	**Ans.**
Add the length of the circular curve	+ 111.37 m	
Chainage at the second junction point	= 1619.32 m.	**Ans.**
Add the length of the transition curve	+ 30.00	
Chainage at the point of tangency	= 1649.32 m.	**Ans.**

Example 17.9. *A national highway curve of 625 m radius is to be set out connect two straights. The maximum speed of the moving vehicles on this curve is restricted to 90 km/hour. Transition curves are to be introduced at each end of the curve.*

Calculate :

(*i*) *A suitable length of the transition curve*

(*ii*) *The necessary shift of the circular curve*

(*iii*) *The chainage at the beginning and at the end of the curve*

(*iv*) *Tangential offset of the first junction point.*

(*v*) *Total deflection angle of the first junction point.*

Assuming : a peg interval of 20 m on circular curve and 10 m on the transition curve.

Angle of intersection $= 130° 24'$

Rate of change of acceleration $= 0.25$ *m/sec*^{3}

Chainage at the point of intersection = *1092.5 m.*

Solution.

(*i*) On the basis of radial acceleration, the length of the transition curve is given by Eqn. (17.11) is

$$L = \frac{v^3}{RC}$$

Here $R = 625$ m ; $C = 0.25$ m/sec^3 ; $v = \frac{90 \times 1000}{60 \times 60} = 25$ m /sec.

$\therefore \quad L = \dfrac{25^3}{625 \times 0.25} = 100$ m. **Ans.**

(*ii*) Shift of the circular curve,

$$S = \frac{L^2}{24R}$$

or $= \dfrac{100^2}{24 \times 625} = 0.667$ m. **Ans.**

Here $\Delta = 180° - 130° \, 24' = 49° \, 36'$

Total tangent length $= (R + S) \tan \dfrac{\Delta}{2} + \dfrac{L}{2}$

$\therefore$ Total tangent length

$$= (625 + 0.67) \tan \frac{49° \, 36'}{2} + \frac{100}{2}$$

$$= 625.67 \tan 24° \, 48' + 50 = 339.10 \text{ m}$$

$\therefore$ Chainage at the point commencement of the curve

= 1092.50 – total tangent length

= 1092.50 – 339.10

= 753.40 m. **Ans.**

(*iv*) Length of the combined curve

Spiral angle $\phi = \dfrac{L}{2R}$ radians

$$= \frac{L}{2R} \times \frac{180°}{\pi} \text{ degrees}$$

$$= \frac{100 \times 180}{2 \times 625 \times \pi} = 4° \, 35'$$

$\therefore$ Central angle of the circular curve

$= 49° \, 36' - 9° \, 10'$

$= 40° \, 26'$

Total length of the combined curve

$$= \frac{\pi \times 625 \, (40.433)}{180} + 200$$

$$= 441.06 + 200 = 641.06 \text{ m.}$$

$\therefore$ The chainage at the end point

$$= 753.40 + 641.06 = 1394.46 \text{ m.} \quad \textbf{Ans.}$$

(v) Tangentical offset for first junction point

$$x = y^3/6RL$$

or
$$x = \frac{100^3}{6 \times 625 \times 100} = 2.667 \text{ m.} \quad \textbf{Ans.}$$

Check : Tangential offset = 4 × shift = 4 × 0.0667 = 2.667 m.

(vi) Total deflection angle of the first junction point

$$\alpha = \frac{573 l^2}{RL} \text{ minutes} = \frac{573 \times 100^2}{625 \times 100} \text{ minutes.}$$

$$= 91'.68 \text{ minutes} = 1° \, 31' \, 41''. \quad \textbf{Ans.}$$

Deflection angle $= \dfrac{\text{spiral angle}}{3} = \dfrac{4° \, 35'}{3} = 1° \, 31' \, 40''$. O.K.

Example 17.10. *A road bend which deflects 80° is to be designed for a maximum speed of 120 kmph, maximum centrifugal ratio of 1/4, and a maximum rate of change of acceleration of 30 cm / sec^2, the curve consisting of a circular arc combined with two cubic spirals. Calculate (i) the radius of the circular arc (ii) the requisite length of transition and (iii) total length of the compositive curve.*

Solution.

Let R be the radius of the circular arc.

The speed of the vehicles

$$= \frac{120 \times 1000}{60 \times 60} = 33.33 \text{ m/sec.}$$

(i) The centrifugal ratio

$$\frac{v^2}{gR} = \frac{1}{4} \qquad \text{(given)}$$

$$\therefore \quad \frac{v^2}{gR} = \frac{1}{4}$$

or
$$R = \frac{4 \times (33.33)^2}{9.81} = 4.52.96 \text{ say } \textbf{453 m.} \quad \textbf{Ans.}$$

(ii) Here rate of change of acceleration = 0.3 m/ sec^2.

Length of transition $= \dfrac{v^3}{C.R} = \dfrac{(33.33)^2}{0.3 \times 453} = 272.44977.$

(iii) Spiral angle $\phi = \dfrac{L}{2R} \times \dfrac{180}{\pi}$ degrees

$$= \frac{272.5 \times 180}{2 \times 453 \times \pi} \text{ degree} = 17.233$$

Length of circular arc

$$= \frac{\pi R (\Delta - 2\phi)}{180} = \frac{\pi \times 453 (80 - 1)(17.233)}{180°} = 496.26 \text{ m.}$$

∴ Total length of composite curve

$$= 496.26 + 2 \times 272.5 = 1041.26 \text{ m.} \quad \textbf{Ans.}$$

Example 17.11. *A transition curve is required for a railway circular curve 400 m radius, the gauge being 1.676 m and maximum superelevation restricted to 15 cm. The transition curve is to be designed for a velocity such that no lateral pressure is imposed on the rails when the rate of gain of radial acceleration is 30 cm / sec^3. Calculate the required length of the transition curve and the design speed.*

Solution. From Eqn. (17.8), we know

$$h = \frac{Gv^2}{gR} \quad ...(i)$$

Here $H = 15$ cm, $G = 1.676$ m

$R = 400$ m, $2 = 9.81$ m/sec^2

Substituting the values in Eqn. (i), we get

$$0.15 = \frac{1.676 \times V^2}{9.81 \times 400}$$

or

$$v^2 = \frac{0.15 \times 9.81 \times 400}{1.676}$$

$$v = 18.74 \text{ m/sec.}$$

∴ Length of transition curve $= \dfrac{V^3}{CR}$

$$\therefore \quad = \frac{(18.74)^3}{0.3 \times 400} = 54.84. \quad \textbf{Ans.}$$

Designed speed $= \dfrac{18.74 \times 60 \times 60}{1000}$

$$= 67.46 \text{ km/hour.} \quad \textbf{Ans.}$$

Example 17.12. *Two straights AB and BC intersect at the chainage 1400.00 metres, the deflection angle being 40° . It is proposed to insert a right handed circular curve 400 metres radius with a cubic parabola of 90 metre length at each end. The circular curve is to be set out with pegs* *at 20 metre intervals and the transition curve with pegs at 10 metre intervals of through chainage.*

Find the chainages :

1. *At the beginning and at the end of the combined curve.*
2. *At the junctions of the transition curves with the circular arc.*
3. *Tangential angles for the first two points on the first transition curve.*
4. *Tangential angles for the first two points on the circular curve.*

Solution. (Fig. 17.8).

Given data : $L = 90$ m, $R = 400$ m, $\Delta = 40°$

1400.00 m = chainage of point of intersection.

(*i*) Shift $S = \dfrac{L^2}{24\,R}$

$$= \frac{90^2}{24 \times 400} = \frac{81}{66} = 0.844 \text{ m}$$

(*ii*) Spiral angle $\phi = \dfrac{L}{2R}$

$$= \frac{90}{2 \times 400} = \frac{9}{80} \text{ radians}$$

$$= \frac{9}{80} \times \frac{180}{\pi} = 6.4457 \text{ degrees}$$

$$= 6° \, 26' \, 45''$$

(*iii*) Tangent length of the combined curve

$$= (R + S) \tan \frac{\Delta}{2} + \frac{L}{2}$$

$$= (400 + 0.844) \tan 20° + 45$$

$$= 400.844 \times 0.36397 + 45 = 190.90 \text{ m}$$

(*iv*) Length of the circular curve

$$= \frac{\pi\, R\, (\Delta - 2\,\phi)}{180°}$$

$$= \frac{\pi \times 400\, (40° - 12°\, 53'\, 30'')}{180°}$$

$$= \frac{\pi \times 400 \times 27°\, 06'\, 30''}{180°}$$

$$= \frac{\pi \times 400 \times 27.3}{180} = 189.25 \text{ m.}$$

(*v*) Chainage at point of intersection	= 1400.00 m
Deduct length of the tangent	– 190.90
Chainage at first point of tangency	= 1209.10 m
Add length of transition curve	+ 90.00 m
Chainage at first junction point	= 1299.10 m
Add length of circular curve	+ 189.25
Chainage at second junction point	= 1488.35 m
Add length of transition curve	+ 90.00 m
Chainage at second point of tangency	= 1578.35 m
Length of the sub-chord	= 1210.0 – 1209.1 = 0.9 m

(*vi*) Tangential angles for the first two points on the transition curve

$$\delta_1 = \frac{572\, l^3}{RL} = \frac{573 \times (0.9)^2}{400 \times 90} = 0° \ 0' \ 08''$$

$$\delta_2 = \frac{573\, l^2}{RL} = \frac{573 \times (10.9)^2}{400 \times 90} = 0° \ 01' \ 53''$$

(*vii*) Tangential angles for the point on circular curve.

Length of the sub-chord = 1300.00 – 1299.10 = 0.90 m

Length of the second chord = 20 m

Substituting the values of C and R in the Rankine's formula

$$\delta = \frac{1718.9\, C}{R} \text{ minutes, we get}$$

$$\therefore \quad \delta_1 = \frac{1718.9 \times 0.90}{400} = 0° \ 3' \ 52''$$

and $$\delta_1 = \frac{1718.9 \times 20}{400} = 85'.95 = 1° \ 25' \ 57''$$

i.e., $$\Delta_1 = \delta_1 = 0° \ 03' \ 52''$$

$$\Delta_2 = \delta_1 + \delta_2 = 0° \ 03' \ 52'' + 1° \ 25' \ 57''$$

$$= 1° \ 29' \ 49''.$$ **Ans.**

Example 17.13. *It is required to join two straights having a total deflection angle 18° right by a circular curve of 500 m radius, having cubic spiral transition curves at each end. The design velocity is 72 km per hour and the rate of change of radial acceleration along the transition* *curve is not to exceed 25 cm / sec^3. Chainage at the point of intersection is 840.0 m. Assume peg interval along transition carve 10 m, and along circular curve 20 m, calculate the necessary data required for setting out the curve.*

Solution. (Fig. 17.7).

Given data : Design velocity (v) = 72 km/hour = 20 cm/sec

Rate of change of radial acceleration

a = 0.25 m /sec^3

Radius of the circular curve = 500 m.

(i) From Eqn. (17.11), length of transition curve

$$L = V^3/R.a$$

or

$$L = \frac{20 \times 20 \times 20}{500 \times 0.25} = 64 \text{ m}$$

(ii) Shift of the curve $= \dfrac{L^2}{24R} = \dfrac{64^2}{24 \times 500} = 0.3413$ m

(iii) Total tangent length $= (R + S) \tan \Delta/2 + \dfrac{L}{2}$

$$= (500.0 + 0.3413) \tan 9^\circ + \frac{64}{2}$$

$$= 500.3413 \times 0.158385 + 32$$

$$= 79.25 + 32 = 111.25 \text{ m}$$

Chainage at the point of commencement

$$= 840.0 - 111.25 = 728.75 \text{ m}$$

Chainage at first junction point

$$= 728.75 + 64.0 = 792.75 \text{ m.}$$

(iv) Cubic spiral transition curve with 10 m chord lengths. To locate a point on the transition distance *l metres* from A, from Eqn. (17.31a), we get

$$\alpha = \frac{573}{RL} \times l^2 \text{ min.}$$

$$\alpha = \frac{753}{500 \times 64} \times l^2 \text{ min.}$$

$$\alpha = 0.01790625\, l^2 \quad \text{min.}$$

The various deflection angles are calculated in a tabular form below:

Chord (m)	*l (m)*	*Chainage (m)*	*Deflection Angle*	*Angle set on the Instrument*
0	0	748.75		0″ 0.0
1.25	1.25	730.00	$0.01790625 = (125)^2 = 0.028'$	0′ 02″
10.00	11.25	740.00	$0.01790265\ (11.25)^2 = 2.267'$	2′ 16″
10.00	21.25	750.00	$0.01790265\ (21.25)^2 = 8.086'$	8′ 05″
10.00	31.25	760.00	$0.01792265\ (31.25)^2 = 17.487'$	17′29″
10.00	41.25	770.00	$0.01790265\ (41.25)^2 = 30.469'$	30′ 28″
10.00	51.25	780.00	$0.01790265\ (51.25)^2 = 47.032'$	47′ 02″
10.00	61.25	790.00	$0.01790261\ (61.25)^2 = 67.176'$	1° 07′ 11″
2.75	64.00	792.75	$0.01790265\ (64.00)^2 = 73.34'$	1° 13′ 21″

(*v*) Circular arc $T_1\ T_2$

Spiral angle $\quad \phi_1 = 3\delta_1 = 3 \times 73.344 = 219.987' = 3°\ 667$

or $\quad \alpha_1 = \dfrac{L}{2R} \times \dfrac{180°}{\pi} = \dfrac{64 \times 180}{500 \times 2 \times \pi} = 3.667.$ O.K.

Angle subtended by the circular arc

$$= 18° - 2\phi_1 = 18° - 7.334 = 10°\ 666$$

Length of circular arc $= \dfrac{\pi R}{180}\ (\Delta - 2\phi_1)$

$$= \frac{\pi \times 500}{180} \times 10.666 = 93.08 \text{ m}$$

∴ Chainage at the end of circular curve

$$= 792.75 + 93.08 = 885.83 \text{ m}$$

Chainage at the point of tangency $= 885.83 + 64 = 949.83$ m

Chord length for circular arc is kept less than $\dfrac{R}{20}$

$$= \frac{500}{20} = 25 \text{ m}$$

The given chord length being less than 25 m, hence O.K.

Chord (m)	*Chainage (m)*	*Deflection Angle* $= \frac{1718.9C}{R}$	*Total Deflection Angle at* T_1	*Angle set on 20″ theodolite*
0	792.75	0	0	0
7.25	800.00	$1718.9 \times \frac{7.25}{500} = 24.924$	24.924 = 24′55″	0°25′00″
20.00	820.00	$1718.9 \times \frac{20}{500} = 68.756$	93.680 = 1°33′39″	1°33′40″

20.00	840.00	$1718.9 \times \frac{20}{500} = 68.756$	162.436 = 2°42′26″	2°42′20″
20.00	860.00	$1718.9 \times \frac{20}{500} = 68.756$	231.192 = 3°51′12″	3°51′20″
20.00	880.00	$1718.9 \times \frac{20}{500} = 68.756$	299.948 = 4°59′57″	5°0′00″
20.00	885.83	$1718.9 \times \frac{20}{500} = 20.042$	319.990 = 5°20′0″	5°20′00″

(*vi*) The second Transition curve is to set out from point of tangency. For through chainage, pegs may be placed as shown in the table below:

Chainage (m)	*Distance from B (m)*	*Chord (m)*	*Deflection Angle* $= 0.01790625\, l^2$	*Angle set on 20″ theodolite*	*Offset* $x = \frac{l^3}{6LR}$
885.83	64.00	4.17	$0.01790625 \times 64^2 = 73.344$ = 1° 13′ 21″	358° 46′ 40″	1.365
890.00	59.83	10.00	$0.01790625 \times (59.83)^2$ = 64.098 = 1° 04′06″	358° 56′00″	1.111
900.00	49.83	10.00	$0.01790625 \times (49.83)^2$ = 44.462 = 0°44′28″	359° 15′40″	0.644
919.00	39.83	10.00	$0.01790265 \times (39.83)^2$ = 28.407 = 0° 28′24″	359° 31′40″	0.329
920.00	29.83	10.00	$0.01790625 \times (29.83)^2$ = 15.933 = 0° 15′56″	359° 44′00″	0.138
930.00	19.83	10.00	$0.01790625 \times (19.83)^2$ = 7.041 = 0° 07′02″	359° 53′00″	0.041
940.00	9.83	9.83	$0.01790625 \times (9.83)^2$ = 1.730 = 0° 01′44″	359° 58′20″	0.005
949.83	0.00	0	0 = 0° 00′00″	360° 00′00″	0.000

17.14. METHOD OF SETTING OUT A COMBINED CURVE

The setting out of a combined curve is carried out in two stages :

(*i*) setting out of transition curves.

(*ii*) setting out of circular curve.

1. Setting out of Transition curves by tangential offsets

The first transition curve is set out from T_1, the point of commencement and the second transition curve from T_2, the point of tangency whose positions are fixed on the ground by measuring the total tangent lengths from the point of intersection.

Steps : The following steps are followed.

1. Either assume a convenient length of the transition curve or calculate its length by any one of the methods discussed in art. 17.6.

2. Calculate the amount of the shift *i.e.* $S = \dfrac{L^2}{24R}$

3. Calculate the total tangent length *i.e.* $(R + S) \tan \dfrac{\Delta}{2} + \dfrac{L}{2}$

4. Locate the points of commencement and tangency by measuring the total tangent lengths from the point of intersection along their straights.

5. Calculate the tangential offsets for the transition curve by the following formulae :

(*i*) $x = \dfrac{y^3}{6RL}$ for a cubic parabolic transition curves

(*ii*) $x = \dfrac{l^3}{6RL}$ for a cubic spiral transition curves

Values for the calculated offsets may be tabulated as under :

Sl. No.	*Distancey along tangent*	*Perpendicular offset to the tangent*	*Remarks*
0	0	0,	
1	10 m	$10^3/6RL$	
2	20 m	$20^3/6RL$	
3	30 m	$30^3/6RL$	
4	40 m	$40^3/6RL$	
5	50 m	$50^3/6RL$	

(A) **Field work for setting out a cubic parabola.**

Procedure : Following steps are involved. (Fig. 17.9).

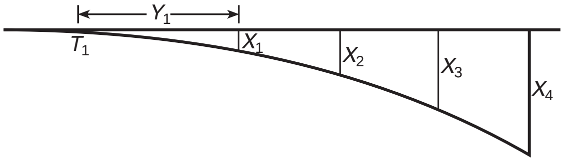


Fig. 17.9. Setting out a cubic parabola

1. Measure a distance equal to first peg interval (y_1) from the point of commencement T_1 along the tangent. Set out a perpendicular offset (x_1) equal to its calculated value.

2. Measure the distance equal to first two peg interval from the point of commencement (T_1) and set out the perpendicular offset (x_2) equal to its calculated value.

3. Proceed in a similar way till last offset is set out. This locates the junction point of the transition curve with the circular curve.

4. Join the ends of the offsets so located to fix the points on the transition curve on the ground.

(B) **Field work for setting out cubic spirals**

Procedure : Following steps are involved.

1. Measure the distance equal to first peg interval (l_1) along the transition curve and fix a peg in such a position that perpendicular offset to the tangent equals the calculated value (x_1). (Fig. 17.10).

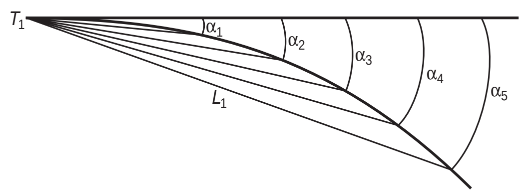


Fig. 17.10. Setting out a cubic spiral.

2. Measure the next distance equal to second peg interval (l_2) along the transition curve from the previously fixed point and swing the leading end of the chain till the perpendicular offset to the tangent equals the calculate value (x_2) *i.e.* $\frac{(l_1 + l_2^3)}{6RL}$.
3. Proceed in a similar way till the last offset is set out, which locates the junction point of the transition curve with the circular curve.
4. Join the points thus located to define the required transition curve on the ground.

2. Setting out of Transition curves by deflection angles

Knowing the peg interval, total deflection angles for various pegs, may be calculated from the formula $\alpha = \frac{573\, l^2}{RL}$ minutes.

The data may be tabulated as under :

Sl. No.	*Leg distance along curve*	*Deflection angle in minutes*	*Remarks*
1	l_1	$\alpha_1 = \frac{573\, l^2}{RL}$	
2	l_2	$\alpha_2 = \frac{573\, l^2}{RL}$,	
3	l_3	$\alpha_3 = \frac{573\, l^2}{RL}$,	

A. Field work for setting out the transition curves by deflection angles (Fig. 17.11).

Procedure : Following steps are involved.

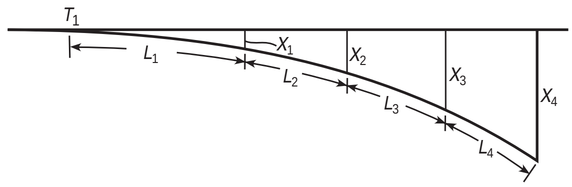


Fig. 17.11. Setting out transition curve with deflection angles.

1. Locate the points of commencement and tangency.
2. Centre the theodolite over the point of commencement T_1 and sight the point of intersection accurately, keeping verniers to read zero.
3. Unclamp the upper plate and set the first value of deflection angle for the peg interval (l_1).
4. Measure the distance (l_1) in the line of sight and locate the first point on the transition curve.
5. Next set the second value of the deflection angle (α_2) and measure the distance (l_2) in the line of sight from the point of commencement (T_1).
6. Proceed in a similar manner to locate the remaining points of the transition curve.
7. For setting out the second transition curve from the point of tangency, the deflection angles are, subtracted from 360° before these values are set on the theodolite.

Note. It may be noted that for each setting, the distance is measured from the point of commencement.

17.15. METHOD OF SETTING OUT THE CENTRAL CIRCULAR CURVE

After fixing the junction points, the circular curve is set out between the transition curves by any one of the methods, discussed in chapter 15 simple curves. The accuracy of the field work is checked at the junction of the circular curve with the beginning of the second transition curve, whose position is already fixed from the point of tangency.

Procedure : The field work may be completed in the following steps:

1. Shift the theodolite at the first junction point and centre it over the point accurately.
2. Set the vernier to read $\left(360° - \frac{2}{3}/\phi\right)$ for a right hand curve and ($2/3\phi$) for a left hand curve, where ϕ is the spiral angle.
3. Unclamp the lower plate to sight the point of commencement (T_1). Accurate bisection is made with lower tangent screw.
4. Loosen the upper plate and swing the theodolite clockwise through an angle $2/3\phi$ so that the line of sight lies along the common tangent at the junction point. The vernier now reads either 360° or 0° .

5. Transit the telescope. Now the telescope points towards the forward direction of the common tangent at the junction point *i.e.* along the tangent to the circular curve.
6. Set the main circular by setting-out deflection angles *i.e.*

$$\delta = 1718.9 \frac{C}{R}$$

or offsets from the chords produced *i.e.*

$$O_n = \frac{C_n\,(C_n + C_{n-1})}{2R}$$

7. Revise the entire work. If the second junction point does not coincide the position already fixed.

17.16. SPIRALLING COMPOUND CURVES

If the radii of the arcs of a compound curve differ considerably, the super elevation at the junction point changes abruptly. To avoid this, transition curves are sometimes introduced between these arcs. This is particularly required for railway curves. (Fig. 17.12).

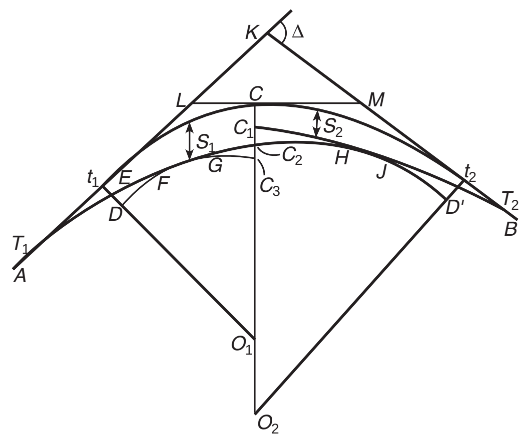


Fig. 17.12. Spiralling compound curves.

Procedure : Following steps are followed.

1. For the given designed speed v, calculate the super elevations e_1 and e_2 for each arc from the formula, $e = \frac{Gv^2}{gR}$.
2. Calculate the length L_1 and L_2 of transition curves from the relation $L = ne$ where n is the rate of application of the super-elevation.
3. Calcultate the amount of shifts S_1 and S_2 for each arc from the formula $S = \frac{L^3}{24R}$

The distance $C_1 C_3$ between the tangents of the shifted curves $= S_1 - S_2$

4. Calculate the length of the transition curve at the common tangent point C_2 from the relation $L = n\,(e_1 - e_2)$. The length of the transition curve may also be assumed arbitrarily.
5. Determine the chainages of various points of the compound curve introduced with transition curve as follows :

From the given chainage of point of intersection, determine the chainage of T_1 by subtracting the total tangent length.

(*i*) Chainage of T_1 = chainage of $K - \left[(R_1 + S_1) + \tan\Delta + \dfrac{L_1}{2}\right]$

(*ii*) Chainage of F = Chainage of $T_1 + L_1$

(*iii*) Chainage of G = Chainage of F + length of 1st arc $-\dfrac{L'}{2}$

(*iv*) Chainage of C_2 = Chainage of $G + \dfrac{L'}{2}$

(*v*) Chainage of H = chainage of $C_2 + \dfrac{L'}{2}$

(*vi*) Chainage of J = chainage of H + length of second arc $-\dfrac{L'}{2}$

(*vii*) Chainage T_2 = chainage of $J + L'/2$.

6. Calculate the deflection angles for the points on the transition curve from the formula $\alpha = \dfrac{573\, l^3}{RL}$ minutes
7. Calculate the offsets from the tangents from the formulae $x = \dfrac{y^3}{6RL_1}$ or $x = \dfrac{4S_2}{L_1^3}\,y^3$, where S_1 is the shift of the first curve.
8. Calculate the deflection angles for the circular arcs from the formula

$$\delta = 1718.9\,\frac{C}{R}\ \text{minutes}$$

9. Calculate the offsets for the transition curves at the common tangent by the relation $x = \dfrac{4(S_1 - S_2)}{L_3}\,y^3$.
10. Locate the junction points of the transition curve with the curve by setting out $L/2$ from C_2 in each direction.

The method is illustrated in example 17.14.

Note : The transition curve at the common point of tangency is bisected at C_2, being equidistant from C_1 and C_3.

Example 17.14. *Two straights AK and BK intersect at K. Two points L and M on straights AK and BK are selected so that angles ALM and LMB are 140°* and *150° respectively. It is proposed to introduce a compound curve tangential to AB, LM and KB, the radii of two branches of the curve are 1000 and 1200 m respectively. The maximum speed is 108 km per hour, and the chainage of K = 5525.0 m. Compute the necessary data for the location of the curve, if the transition curves are to be inserted between the two arcs and at the junctions with the straights AK and BK. The distance between the centres of rails = 1.676 m.*

Solution. (Fig. 17.12).

(*i*) 108 km per hour $= \dfrac{108 \times 1000}{30 \times 60} = 30$ m per second

Superelevation for 1st arc $e_1 = \dfrac{Gv^2}{gR} = \dfrac{1.676 \times 30^2}{9.81 \times 1000}$

$= 0.1538$ m

Superelevation for second arc $e_2 = \dfrac{Gv^2}{gR} = \dfrac{1.676 \times 30^2}{9.81 \times 1200}$

$= 0.1281$ m

(*ii*) **Length of the transition curves :**

Assuming that the superelevation is applied at a uniform rate of 1 in 400, we have

Length of the first transition curve at the junction of the straight *AK*.

$$L_2 = ne_1 = 400 \times 0.1538$$

$$= 61.2 \text{ say } 60 \text{ m}$$

Length of the second transition curve at the junction of the straight *BK*.

$$= L_2 = ne_2 = 400 \times 0.1281$$

$$= 51.2 \text{ say } 50 \text{ m}$$

Length of the third transition curve at the junction *C* of the two curves

$$L_3 = (e_1 - e_2) = 400\ (0.1538 - 0.1281)$$

$$= 10.28 \text{ say } 10 \text{ m}$$

(*iii*) **Shifts**

For the first curve, $S_1 = \dfrac{60^2}{24 \times 1000} = 0.15$ m

For the second curve, $S_2 = \dfrac{50^2}{24 \times 1200} = 0.09$ m

(*iv*) **Tangent lengths :**

Deflection angle of first curve $\Delta_1 = 180° - 140° = 40°$

Deflection angle of second curve, $\Delta_2 = 180° - 150° = 30°$

$$LC = (R_1 + S_1) \tan \frac{\Delta_1}{2} = (1100 + 0.15) \tan 20°$$
$$= 364.02 \text{ m}$$

$$CM = (R_2 + S_2) \tan \frac{\Delta_2}{2} = (1200 + 0.09) \tan 30°$$
$$= 692.87 \text{ m}$$

$$\therefore \quad LM = LC + CM = 364.02 + 692.87$$
$$= 1056.89 \text{ m}$$

Applying sine rule to ΔKLM, we get

$$KL = \frac{1056.89 \sin 30°}{\sin 110°} = 562.36 \text{ m}$$

and $$KM = \frac{1056.89 \sin 40°}{\sin 110°} = 722.96 \text{ m}$$

$$\therefore \quad Kt_1 = KL + Lt_1$$
$$= 562.36 + 364.02 = 926.38 \text{ m}$$
$$Kt_2 = KM + Mt_2$$
$$= 722.96 + 692.87 = 1415.83 \text{ m}$$

Chainage of $K = 5525.0$ (given)

Chainage of $t_1 = 5525.0 - 926.38 = 4598.62$ m

Chainage of $T_1 = 4598.62 - 30.0 = 4568.62$ m

Chainage of F, the end of the first transition curve

$$= 4568.62 + 60.0 = 4628.62 \text{ m}$$

Spiral angle $$\phi_1 = \frac{L}{2R_1}$$
$$= \frac{60}{2 \times 1000} \times \frac{180°}{\pi} = 1° \, 43' \, 08''$$

Length of the first arc $= \dfrac{\pi R \, (\Delta - \phi)}{180°} - \dfrac{L_2}{2}$

$$= \frac{\pi \times 1000 \times (40 - 1° .7189)}{180°} - 5$$

$$= \frac{\pi \times 1000 \times 38.2811}{180°} - 5 = 663.13 \text{ m}$$

Spiral angle $\quad \phi_2 = \frac{L_2}{2R_1}$

$$= \frac{50}{2 \times 1200} \times \frac{180°}{\pi} = 1.1937$$

Length of second arc

$$= \frac{\pi \times 1200\ (30 - 1.1937)}{180°} - 5 = 598.32 \text{ m}$$

Chainage of G = 4628.62 + 663.13 = 5291.75 m

Chainage of H = 5291.75 + 10 = 5301.75 m

Chainage of J = 5301.75 + 598.32 = 5900.07 m

Chainage of T_2 = 5900.07 + 50 = 5950.07 m

(v) The offsets for setting out the first transition curve L_1 at 10 m interval from the formula $x = \frac{4S_1y^3}{L_1^3}$

$$x_1 = \frac{4 \times (0.15) \times 10^3}{60^3} = 0.00278 \text{ m} = 2.78 \text{ mm}$$

$$x_2 = \frac{4 \times (0.15) \times 20^3}{60^3} = 0.022 \text{ m} = 2.2 \text{ mm}$$

$$x_3 = \frac{4 \times (0.15) \times 30^3}{60^3} = 0.075 \text{ m} = 7.5 \text{ mm}$$

$$x_4 = \frac{4 \times (0.15) \times 40^3}{60^3} = 0.178 \text{ m} = 0.178 \text{ mm}$$

$$x_5 = \frac{4 \times (0.15) \times 50^3}{60^3} = 0.347 \text{ m} = 0.347 \text{ mm}$$

$$x_6 = \frac{4 \times (0.15) \times 60^3}{60^3} = 0.600 \text{ m} = 0.600 \text{ mm}$$

Similarly, the offsets for locating the second transition curve (L_2) may be calculated.

17.17. BERNOULLI'S LEMNISCATE CURVE

Bernoulli's Lemniscate Curve is generally used as transition curve for modern highways. Because of its symmetry, it is well adopted when angle of deflection of the straights is large. It is particularly suitable if a curve transitional throughout, is required to be introduced between straights, having no intermediate circular curve. The lemniscate resembles the clothoid very closely up to a polar angle of 5°. (Fig. 17.13)

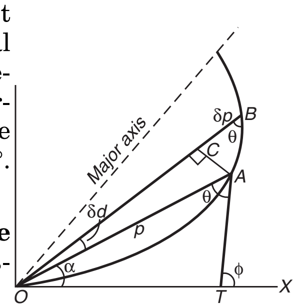


Fig. 17.13.

Advantages of Bernoulli's Lemniscate : The following are the advantages of lemniscate.

1. Rate of change of curvature is less than that of the clothoid.
2. Rate of increase of curvature diminishes near the end of the transition curve with circular curve.
3. Shape of lemniscate resembles the path actually traced by an automobile when turning freely.

There is practically no difference between a clothoid, lemniscate and cubic parabola upto 12° . For large angles, the cubic parabola leaves the other two curves and its radius reaches a minimum when ø is approximately 24° 06′. Thereafter it starts increasing again. The radius of curvature of a clothoid is minimum at 60° and that of lemniscate is at 135° .

The polar equation of the lemniscate curve, is

$$p = K\sqrt{\sin 2\alpha} \qquad ...(17.35)$$

Relationship between Deviation angle and polar angle

Let A and B be two points on the lemniscate curve very close to each other. p and $p + \delta p$ be their distances from O.

α be the angle between OA and OX. AT is tangent at A.

Draw AC perpendicular to OB

$$AC = p.\, \delta\alpha.$$

Let angle $OAT \approx$ angle $OBT = \theta$

From right angled ΔACB

$$\tan\theta = \frac{AC}{BC}$$

or

$$\tan\theta = \frac{p.\,\delta\,\alpha}{\delta P} \qquad ...(17.36)$$

Differentiating Eqn. (17.35)

$$\frac{\delta\, p}{\delta\,\alpha} = \frac{K\cos 2\alpha}{\sqrt{\sin 2\alpha}}$$

Substituting the values in Eqn. (17.36), we get

$$\tan\ \theta = \frac{K.\ \sqrt{\sin 2\alpha}.\ \sqrt{\sin 2\alpha}}{K \cos 2\alpha}$$

$$\tan \theta = \tan 2\alpha$$

or $$\theta = 2\alpha$$

From ΔOAT, $\alpha + \theta = \phi$

or $$\alpha + 2\alpha = \phi$$

Hence $$\phi = 2\alpha \qquad ...(17.37)$$

i.e., the deviation angle (ϕ) *is equal to three times the polar deflection angle* (α). It is a rigorously true relationship for the lemniscate.

The radius of curvature r at any point of the curve with polar co-ordinates is given by

$$r = \frac{\left[p^2 + \left(\frac{\delta p}{\delta \alpha}\right)^2\right]^{\frac{3}{2}}}{\left[p^2 + 2\left(\frac{\delta p}{\delta \alpha}\right)^2 - p\,\frac{\delta^2 p}{\delta \alpha^2}\right]} \qquad ...(17.38)$$

From Eqn. (17.35), we get

$$p^2 = K^2 \sin 2\ \alpha$$

Differentiating the above Eqn. in succession,

$$\frac{\delta p}{\delta \alpha} = K \cos 2\alpha.\ (\sin 2\alpha)^{-1/2}$$

and $$\frac{\delta^2 p}{\delta \alpha^2} = -K \cos^2 2\alpha\ (\sin 2\alpha)^{-3/2} - 2K\ (\sin 2\alpha)^{1/2}$$

Substituting the values in Eqn. (17.38), we get

$$r = \frac{\left[K^2 \sin 2\alpha + K^2 (\cos 2\alpha)^2 (\sin 2\alpha)^{-1}\right]^{3/2}}{K^2 \sin 2\alpha + \frac{2K^2 \cos^2 2\alpha}{\sin 2\alpha} + K\sqrt{\sin 2\alpha}\left[K \cos^2 2\alpha\ (\sin 2\alpha)^{-3/2} + 2K \sin 2\alpha)^{1/2}\right]}$$

$$= \frac{K^3\left[\sin 2\alpha + (\cos 2\alpha)^2\ (\sin 2\alpha)^{-1}\right]^{3/2}}{K^2\left[\sin 2\alpha + \frac{2 \cos^2 2\alpha}{\sin 2\alpha} + \cos^2 2\alpha\ (\sin 2\alpha)^{-1} + 2\ (\sin 2\alpha)\right]}$$

$$= \frac{K\left(\sin 2\alpha + \dfrac{\cos^2 2\alpha}{\sin 2\alpha}\right)^{3/2}}{\left(3 \sin 2\alpha + \dfrac{3 \cos^2 2\alpha}{\sin 2\,\alpha}\right)}$$

or $$r = \frac{K\left(\sin 2\alpha + \dfrac{\cos^2 2\alpha}{\sin 2\alpha}\right)^{3/2}}{3\left(\sin 2\alpha + \dfrac{\cos^2 2\alpha}{\sin 2\alpha}\right)}$$

$$= \frac{K\sqrt{\sin 2\alpha + \dfrac{\cos^2 2\alpha}{\sin 2\alpha}}}{3} = \frac{K\sqrt{\dfrac{\sin^2 2\alpha + \cos^2 2\alpha}{\sin 2\alpha}}}{3}$$

or $$r = \frac{K}{3\sqrt{\sin 2\alpha}} \qquad ...(17.39)$$

Substituting the value of K from Eqn. (17.35) in Eqn. (17.39), we get

$$r = \frac{p}{3 \sin 2\alpha} \qquad ...(17.40)$$

From Eqn. (17.39), we get

$$K = 3r\sqrt{\sin 2\alpha} \qquad ...(17.41)$$

Substituting the value of $\sqrt{\sin 2\alpha}$ from Eqn. (17.35) in Eqn. (17.41), we get

$$K = 3r \times \frac{p}{K}$$

or $$K = \sqrt{3rp} \qquad ...(17.42)$$

Let l be the length of the curve for a deviation angle ϕ

$$r = \frac{dl}{d\phi} = \frac{K}{3\sqrt{\sin 2\alpha}} \qquad ...(17.43)$$

Integrating Eqn. (17.43), we get

$$l = \frac{K}{\sqrt{2}}\left(2 \tan^{1/2} \alpha - \frac{1}{5} \tan^{5/2} \alpha + \frac{1}{12} \tan^{9/2} \alpha - \frac{5}{104} \tan^{13/2} \alpha + \ldots\right)$$

This series does not convege very rapidly unless α is relatively small.

Prof. F.G. Royal Dawson made a thorough study of lemniscate curves and has suggested the following empirical formula for the lengths of lemniscate curve.

i.e. $$l = \frac{2K\alpha}{\sqrt{\sin\alpha}} \cdot \cos K\alpha$$

$$= 6\, r\alpha \sqrt{\cos\alpha} \cdot \cos K\alpha$$

where K is a coefficient whose approximate value ranges from 0.195 to 0.160.

For small angles,

$$l = 6\, r\, \alpha \qquad \text{...(17.44)}$$

where α is in radians

and $$l = \frac{r\,\alpha}{9.55} \qquad \text{...(17.45)}$$

where α is in degrees.

At the end of the curve, $r = R$; $l = L$; $\phi = \phi_1 = 3\alpha$ (max).

Minor Axis of Lemniscate : To locate the position of the minor axis of the curve, following steps are involved. (Fig. 17.14).

1. Draw the polar ray OB_1 making an angle of 15° with the tangent OX.
2. Draw the polar ray OA making an angle of 45° with the tangent OX.
3. Drop $B_1 B_2$ perpendicular to OA meeting it at C.

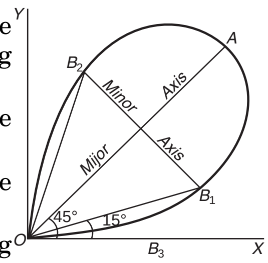


Fig. 17.14.

4. Draw $B_1 B_3$ a tangent to the curve such that angle $B_1 B_3 X$ is 45°. Alternatively draw $B_1 B_3$ parallel to OA.
5. Apparently $\Delta OB_1 B_2$ is an equilateral triangle.

$$B_1 B_2 = OB_1 = K\sqrt{\sin 30°} = \frac{K}{\sqrt{2}}$$

But $$OA = K\sqrt{\sin 90°} = K \qquad \text{...(17.46)}$$

$\therefore$ $$B_1 B_2 = \frac{OA}{\sqrt{2}}$$

or $$B_1 B_2 = \frac{1}{\sqrt{2}} = \frac{1}{1.4142}$$

From Eqn. (17.39), radius of curvature at A

$$= \frac{K}{3} = \frac{OA}{3}$$

It may be seen that radius of curvature decreases gradually from infinity at the origin O to a minimum $\frac{1}{3} OA$ at A for which polar ray makes angle of 45°.

Length of the Lemniscate Curve

The length of any curve (r, θ)

$$dl = \int_0^{\theta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ .\ d\theta \qquad ...(17.47)$$

The equation of lemniscate curve is

$$r = K\sqrt{\sin 2\theta} \qquad ...(17.48)$$

The curve $OB_1 A$ can be traced between 0° to $\pi/4$

Differentiating Eqn. (17.48), we get

$$\frac{dr}{d\theta} = K\frac{\cos 2\theta}{\sqrt{\sin 2\theta}}$$

Substituting the values in Eqn. (17.47), we get

$$dl = \int_0^{\pi 4} \sqrt{K^2 \sin 2\theta + K^2 \frac{(\cos 2\theta)^2}{\sin 2\theta}\, d\theta}$$

$$= \int_0^{\pi 4} \sqrt{\frac{K^2}{\sin 2\theta}}\ d\theta$$

or

$$dl = \int_0^{\pi 4} K\sqrt{\text{cosec } 2\,\theta\ .\ d\theta} \qquad ...(17.49)$$

But $\sqrt{(\text{cosec } 2\theta)} = \frac{1}{\sqrt{2\theta}}\left[1 + \frac{2}{3}\theta^2 + \frac{14}{45}\theta^4 + \frac{124}{945}\theta^6 + \frac{254}{4725}\theta^8 + \ldots\right]^{\frac{1}{2}}$

Expanding the terms in the brackets binomially, we get

$$\sqrt{(\text{cosec } 2\theta)} = \frac{1}{\sqrt{2\theta}}\left[1 + \frac{1}{3}\theta^2 + \frac{1}{10}\theta^4 + \frac{61}{1890}\theta^6 + \frac{1261}{113400}\theta^8 + \ldots\right]$$

Substituting the value of $\sqrt{\text{cosec } 2\theta}$ in Eqn. (17.49) we get

$$dl = K\int_0^{\pi/4} \frac{1}{\sqrt{2\theta}}\left[1 + \frac{1}{3}\theta^2 + \frac{1}{10}\theta^4 + \frac{61}{1890}\theta^6 + \frac{1261}{113400}\theta^8 + \ldots\right] d\theta$$

Integrating this equation we get

$$l = K\left[\sqrt{2}\ \theta\left(1 + \frac{1}{15}\theta^2 + \frac{1}{90}\theta^4 + \frac{61}{24570}\theta^6\right) + \frac{1261}{19{,}27{,}800}\theta^5 + \ldots\right]_0^{\pi/4}$$

$$= K\left[\sqrt{\pi/2}\left(1 + \frac{1}{15}\right)\left(\frac{\pi}{4}\right)^2 + \frac{1}{90}\left(\frac{\pi}{4}\right)^4 + \frac{61}{24570}\theta^6 + \frac{1261}{1{,}927{,}800}\theta^8 + \ldots\right]$$

$= K[1.253313608\ (1 + 0.04112328 + 0.0042278 + 0.00058272 + \ldots]$

or $\quad l = 1.31115\ K = 1.31115\ OA$ from Eqn. (17.46)

While inserting lemniscate curves between the straights, two cases may arise.

Case I. Lemniscate be transitional throughout

Let AB and CB be two straights which intersect at B, having a deflection angle Δ. (Fig. 17.15)

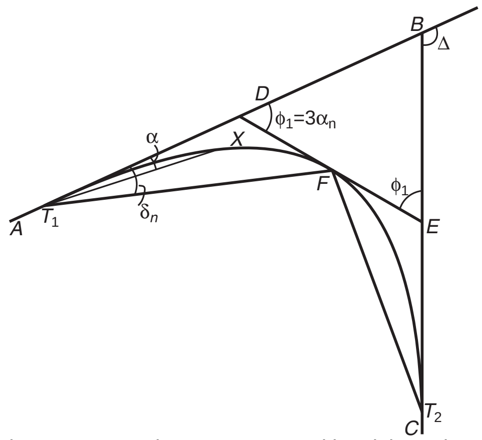


Fig. 17.15. Lemniscate curve transitional throughout.

DFE be tangent to both branches of the curve.

a_n be polar deflection angle BT_1F

©$_1$ be angle FEB = angle BDF

As the curve T_1FT_2 is placed symmetrical, BF becomes normal to the curve and bisects the angle DBE

Angle $\quad DBE = 180° - \Delta$

Angle $\quad DBF = \dfrac{1}{2}(180° - \Delta) = 90° - \dfrac{\Delta}{2}$

Angle $\quad BDF = 90° - \left(90° - \dfrac{\Delta}{2}\right) = \dfrac{\Delta}{2} = \angle BEF = \phi_1$

But $\quad \phi_1 = 3\alpha_n$

$$\therefore \quad \alpha^n = \frac{\phi_1}{3} = \frac{\Delta}{6} \qquad \ldots(17.50)$$

i.e., to have a curve transitional throughout, the maximum polar deflection angle must be equal to 1/6th of the deflection angle of the curve.

The three angles of $\Delta\ T_1BF$ are as under :

Angle $T_1BF = 90° - \frac{\Delta}{2}$

Angle $BT_1F = \frac{\Delta}{6}$

Angle $T_1FB = 180° - \left(90° - \frac{\Delta}{2} + \frac{\Delta}{6}\right) = 90° + \frac{\Delta}{3}$

Setting out of the curve. Apex distance and deflection angle being known.

Steps : Following steps are followed.

1. Applying the sine rule to $\Delta\, T_1BF$, the lengths of sides T_1B and T_1F may be calculated.
2. Measuring tangent length BT_1, the point of commencement T_1 can be located.
3. As $T_1B = BT_2$, the point of tangency T_2, can also be located.
4. Having located the tangent points T_1 and T_2 the arc T_1F may be set out from T_1. The arc T_2F may be set out from T_2.
5. Prepare a table of polar rays for successive values of polar deflection angles from the equation
$$p = K\sqrt{\sin 2\alpha}$$
6. The length of the polar ray T_1F can be calculated from the equation
$$p = 3r \sin 2\alpha_n \qquad \text{from Eqn. (17.39)}$$
if r, radius at F and deflection angle are given,
7. Knowing the polar ray T_1F, the lengths of the tangent T_1B, and apex distance BF can be calculated by applying sine rule.

Case II. Lemniscate to be transitional at the ends of a circular curve.

In case, the value of α_n is less than 1/6 Δ, a circular curve is inserted between two lemniscates. (Fig. 17.16).

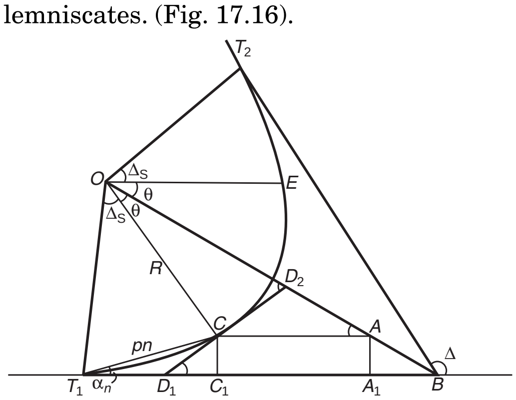


Fig. 17.16. Transition curves at the ends of a circular curve.

Apparently angle $T_2OT_1 = 180° - T_2BT_1 = \Delta$

Angle subtended by circular curve CE at 0 is 2θ

$$\therefore \quad 2\theta = \Delta - 2\,\Delta\, s$$

$$\therefore \quad = \Delta - \alpha_n$$

Draw D_1CD_2 tangent at the junction (C) of the transition curve T_1C and circular curve CE.

Draw CA parallel to T_1B to intersect OB at A.

Drop CC_1 and AA_1 perpendiculars to T_1B.

The tangent $T_1B = T_1C_1 + C_1A_1 + A_1B$ ...(17.51)

From $\Delta\, T_1CC_1$,

$$T_1C = CT_1 \cos \alpha_n \quad = p_n \cos \alpha_n$$

$$CC_1 = CT_1 \sin \alpha_n \quad = P_n \sin \alpha_n = AA_1$$

In the triangle OCA,

$$\angle COA = \theta$$

$$\angle CAO = \text{Angle } A_1BA = \frac{180° - \Delta}{2} = 90° - \frac{\Delta}{2}$$

$OC = R$, radius of circular arc.

In the triangle D_1D_2B

$$\angle D_1D_2O = \angle D_2D_1B + \angle D_1BD_2$$

$$= \Delta s + 90° - \frac{\Delta}{2} \qquad ...(i)$$

From $\Delta\, OCD_2\ \angle CD_2O = 90° - \theta$ ...(ii)

Equating Eqn. (i) and Eqn. (ii) we get

$$\Delta s + 90° - \frac{\Delta}{2} = 90° - \theta$$

or

$$\theta = \frac{\Delta}{2} - \Delta s$$

Now applying the sine rule to ΔCOA,

$$CA = \frac{OC \sin COA}{\sin CAO}$$

$$\theta = \frac{R \sin \theta}{\sin\left(90° - \frac{\Delta}{2}\right)}$$

$$= \frac{R \sin\left(\frac{\Delta}{2} - \Delta s\right)}{\cos \frac{\Delta}{2}}$$

or
$$CA = \frac{R\left(\sin \frac{\Delta}{2} \cos \Delta s - \cos \frac{\Delta}{2} \sin \Delta s\right)}{\cos \frac{\Delta}{2}}$$

or
$$CA = C_1 A_1 = R\left(\cos \Delta s \tan \frac{\Delta}{2} - \sin \Delta s\right)$$

Again
$$A_1B = AA_1 \cot\left(90° - \frac{\Delta}{2}\right)$$
$$= p_n \sin \alpha_n \tan \frac{\Delta}{2}$$

Substituting the values in Eqn. (17.51), we get

$$T_1B = p_n \cos \alpha_n + R (\cos \Delta s \tan \Delta/2 - \sin \Delta s) + p_n \sin \alpha_n \tan \Delta/2 \quad ...(17.52)$$

Example 17.15. *Two straights intersect at an angle of 72°. If the apex distance is 20 m, calculate necessary data for setting out a curve transitional throughout.*

Solution. (Fig. 17.15)

$$\Delta = 180° - 72° = 108°$$

In $\Delta\, T_2BF$, $BF = 20$ m

$$\angle FBT_2 = 90° - \frac{\Delta}{2} = 90° - 54° = 36°$$

$$\angle BT_2F = \frac{\Delta}{6} = \frac{180°}{6} = 18° = \alpha_u$$

$$\angle T_2FE = 2\alpha_n = 36°$$

$$\angle T_2FB = 90° + 36° = 126°$$

Applying sin rule to ΔT_2FB we get

$$T_2 B = \frac{BF \,.\, \sin T_2FB}{\sin FT_2B}$$

$$= \frac{20 \sin 126°}{\sin 18°} = 52.361 \text{ m}$$

$$T_2F = \frac{BF \,.\, \sin FBT_2}{\sin FT_2B}$$

$$= \frac{20 \sin 36°}{\sin 18°} = 38.0422 \text{ m}$$

From Eqn. (17.35), we get

or $$K = \frac{P}{\sqrt{\sin 2\alpha}}$$

$$= \frac{38.0422}{\sqrt{\sin 36°}} = 49.620 \text{ m}$$

The polar equation of the curve, is

$$P = 49.720 \sqrt{\sin 2\alpha}$$

Now calculate the values of p for different values of α, and tabulate the results as below :

α	*p in metres*	α	α
2°	13.13	12°	31.71
4°	18.55	14°	34.07
6°	22.67	16°	36.19
8°	26.10	18°	38.12
10°	29.08		

Example 17.16. *In example 17.15 if the minimum radius of curvature of the curve is 50 m, calculate the necessary data for setting out the curve.*

Solution. (Fig. 17.15).

$$\alpha_n = \frac{\Delta}{6} = \frac{180°}{6} = 18°$$

From Eqn. (17.39), we get

$$K = 3r \sqrt{\sin 2\alpha_n}$$

$$= 3 \times 50 \sqrt{\sin 36°} = 115.00$$

From Eqn. (17.35).

$$p_n = 115 \sqrt{\sin 36°} = 88.17 = T_2F$$

Tangent length T_2B $$= \frac{T_2 F \sin 126°}{\sin 36°}$$

$$= \frac{88.17 \sin 126°}{\sin 36°} = 121.36 \text{ m}$$

The values of p for different values of α may now be calculated from the equation.

$$p = 115.00 \sqrt{\sin 2\alpha}$$

EXERCISE 17

1. Fill in the blanks with suitable word(s) to complete the following sentense.

(*i*) A transition curve is introduced to change......gradually.

(*ii*) The raising of outer rail is known as........

(*iii*) Main types of the transition curves are

(*a*).....................................(*b*)...........................(*c*)..........................

(*iv*) The ratio of the centrifugal force and the weight of the moving vehicle, is known as...............

(*v*) Spiral angle of transition curve is..................where L is the length of transition curve and R the radius of circular curve.

(*vi*) Shift of a curve is equal to............where L is length of transition curve and R radius of the circular curve.

(*vii*) The tangential perpendicular offset for a cubic parabolic transition curve is...............where L and R have their usual meanings.

2. Pick up the correct words from the brackets.

(*i*) Transition curves are provided to change........... (direction/grade)

(*ii*) The amount of superelevation to be provided along the curves depends upon.......... (speed of vehicle, radius of the curve, both)

(*iii*) Lemniscate transitional curves are generally provided along............. (railways, highways)

(*iv*) Shift of the curve........... $\left(\frac{L^2}{24R} \cdot \frac{L^2}{34R}, \frac{R}{24R}\right)$

(*v*) Spiral angle of transition curve, is............., $\left(\frac{L^2}{2R}, \frac{L}{2R}, \frac{L}{R}\right)$

(*vi*) The standard equation of a cubic parabola, is.............

$$\left(x = \frac{y^3}{6LR}, x = \frac{y^3}{24\,RL} \;\; x = \frac{y^3}{6L}\right)$$

(*vii*) Deflection angles for setting out a transition curve, are calculated by the formula................

$$\left(\alpha = \frac{1718.9\, l^2}{RL}, \alpha = \frac{573\, l^2}{RL}, \alpha = \frac{573\, l}{RL}\right)$$

(*viii*) Total deflection angle of a transition curve is equal to...........of the spiral angle. (1/4, 13, 12)

3. Define a transition curve. Explain its importance on railways and highways.

4. What is meant by a transition curve ? Give reasons for introducing a transition curve between a straight and the circular curve on a road or railway.

5. Derive an expression for the shift of a curve.

6. What are the various methods of determining the length of a transition curve. Explain each in brief.

7. (*a*) What is meant by a transition curve and what are their forms ?

(*b*) What is meant by 'shift' of a curve. Derive an expression for the same.

8. Explain how you would set out a transition curve (*a*) by tangent offsets (*b*) by deflection angles.

9. (*a*) With diagrams explain a transition curve.

(*b*) Derive expressions for the length and shift for a transition curve required for a first class road.

10. Explain different forms of transition curves. Derive an expression for an ideal transition curve.

11. Describe in detail the method of setting out the combined curve by the method transition angles.

12. Two straights intersect at a chainage 425.8 m having angle of deflection of 40°. The two straights are to be connected by a circular curve of radius 220 m and two transition curves, the length of each being 50 metres. Calculate the chainage at the beginning and end of the combined curve.

13. Two straights of road intersect at chainage 18 km and 350 m with a right deflection of 40° . It is proposed to introduce circular curve of 300 m radius with a cubic spiral of 90 m length at each end. Calculate the necessary data to set out the curve by the method of deflection angles.

14. The whole circle bearings of two straights *AB* and *BC* of a railway line are 65° and 90° respectively. The chainage at the point of intersection *B* is 55,800 m. The two straights are to be connected by a circular curve of radius 300 m and two transition curve the length of each being 45 m. Calculate the chainage at the beginning and end of the main curve.

15. A road curve of 190 m radius is to be set out to be connect two tangents. The maximum speed allowed on the curve will be 50 km/hour. Transition curves are to be introduced at each end of the curve. Find a suitable length for the transition curve and calculate :

(*a*) The necessary shift of the circular curve.

(*b*) The chainage at the beginning and at the end of the curve.

(*c*) The value of the deflection angle of the first, junction point.

Assume the angle of intersection as 64° 24′ and the rate of change of acceleration as 30 cm/sec^3, the chainage at the intersection point being 1200.00 m.

Also calculate the maximum amount of superelevation if the width of the road is 10 m.

16. Two straights AB and BC intersect at an inaccessible point B. Two points M and N are selected on AB and BC respectively, and the following data is obtained :

$\angle AMN = 155° \ 36'$; $\angle CNM = 163° \ 42'$; $MN = 220$ m.

A circular curve of 500 m radius and two transition curve 60 m each at each end are to be introduced. Calculate the chainages at the beginning and end of the curve if the chainage at the point M is 1825.5 m.

ANSWERS

1. (*i*) Direction (*ii*) Superelevation

(*iii*) Cubic spiral, cubic parabola, Lemniscate

(*iv*) Centrifugal ratio (*v*) $\frac{L}{2R}$

(*vi*) $\frac{L^2}{24\,R}$ (*vii*) $\frac{Y^3}{6\,RL}$

2. (*i*) Direction (*ii*) Both

(*iii*) Highways (*iv*) $\frac{L^2}{24\,R}$

(*v*) $\frac{L}{2R}$ (*vi*) $x = \frac{Y^3}{6RL}$

(*vii*) $\frac{573\,l^2}{RL}$ (*viii*) $\frac{1}{3}$

12. 320.55 m, 524.14 m

13. 18,195.4 m, 18494.79 m

14. 55710.93 m ; 55886.83 m

15. 47 m (*a*) 0.484 m, (*b*) 874.02 m, 1304.36 m, 2° 21′44″ , 1035 m

16. 1704.63 m ; 2119.80 m.

18

Vertical Curves

18.1. INTRODUCTION

While crossing low ridges or valleys, the railways and highways face a change of gradients. In case of a ridge, a rising gradient (*i.e.* up 2 gradient) is followed up to the highest point, the summit and then a down-gradient is followed on the other side of the ridge. In case of a valley, a down gradient is followed by an up-gradient. Sometimes two up-gradients or two down-gradients may also follow in quick succession. Movement of vehicles along such tracks causes great discomfort to the passengers unless the tracks are smoothened in such a way that there is a gradual change from the up gradient to down-gradient and *vice versa*. Sufficient amount of clearance is also provided for safe driving on the summit. To provide safety, comfort and clearance along such tracks either circular or parabolic arcs are introduced. A parabolic arc is preferred to a circular one due to simplicity of calculating offsets for setting out vertical curves. A parabolic curve also provides the best riding qualities as the rate of change in grade is uniform throughout along a parabola having its axis vertical *i.e.,* the rate of change of slope of a parabola is constant.

Proof. The general equation of a parabola having vertical axis, is

$$y = ax^2 + bx \qquad ...(18.1)$$

The slope of the curve

$$\frac{dy}{dx} = 2ax + b \qquad ...(18.2)$$

The rate of change of grade or slope

$$\frac{d_2 y}{dx^2} = 2a \qquad ...(18.3)$$

As the R.H.S. of Eqn. (18.3) does not contain any variable, it is proved that the rate of change of grade along a vertical parabola is always uniform.

18.2. THE GRADE OF VERTICAL CURVE

The gradient or grade may be defined as a proportional rise or fall between two points along a straight line. It is expressed either as a percentage or as a ratio.

1. **As a percentage (%).** Vertical rise or fall per 100 horizontals *e.g.* 1%, 2%, 5%, etc.
2. **As a ratio.** One vertical rise or fall in *n* horizontals *e.g.* 1 in 200, 1 in 500 etc.

The grades are further classified into following two categories :

(*a*) Up-grades or positive grades

(*b*) Down grades or negative grades

A grade is classified as an *upgrade* if elevations along it increase whereas it is classified as *downgrade* if the elevations decrease. It is important to note that these classifications depend upon the direction of the movement of the vehicles. An up-grade becomes a down grade if the direction of motion of the vehicle is reversed.

18.3. RATE OF CHANGE OF GRADE

From equations (18.1) and (18.2), it is noticed that the gradient (or slope) along a parabolic curve changes from point to point but the rate of change of grade remains constant at every point.

It has been recommended that the rate of change of gradient on the *railways* should be 0.06% per 20 metre stations at *summits* and 0.03% per 20 metre stations in valleys. Twice of these values are recommended for branch railway lines.

Explanation. Suppose a 0.5% grade is to be smoothened @0.05% per 20 metre stations. The gradients at different stations on the curve will be as under:

Station	*Distance from the beginning*	*Gradient*
0	0	0.50%
1	20 m	0.45 %
2	40 m	0.40%
3	60 m	0.35%
4	80 m	0.30%
5	100 m	0.25%
6	120 m	0.20%
7	140 m	0.15%
8	160 m	0.10%
9	180 m	0.05%
10	200 m	0.00%

18.4. TYPES OF VERTICAL CURVES

Depending upon the different combinations of different grades, the following six types of vertical curves, are generally met while executing a rail or highway project.

1. An up-grade (+ g_1%) followed by a downgrade ($-g_2$%) (Fig. 18.1).

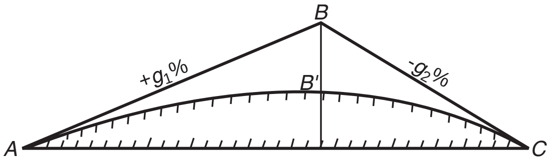


Fig. 18.1. An upgrade followed by a downgrade.

2. A downgrade ($-g_1$%) followed by an up-grade ($+g_2$) (Fig. 18.2).

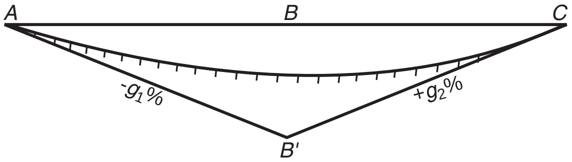


Fig. 18.2. A downgrade followed by an up-grade.

3. An up-grade ($+g_1$%) followed by another up-grade ($+g_2$%), $g_2 > g_1$. (Fig. 18.3).

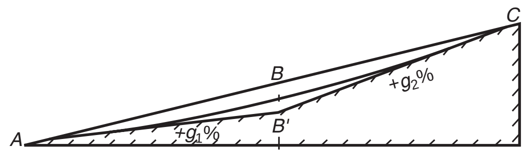


Fig. 18.3. An up-grade followed by another up-grade.

4. An up-grade ($+g_1$%) followed by another up-grade ($+g_2$), $g_1 > g_2$. (Fig. 18.4).

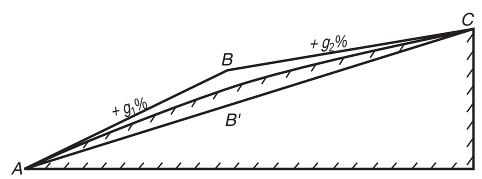


Fig. 18.4. An up-grade followed by another up-grade.

5. A downgrade ($-g_1$%) followed by another downgrade (g_2%), $g_2 > g_1$. (Fig. 18.5).

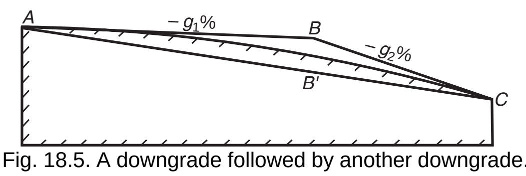


Fig. 18.5. A downgrade followed by another downgrade.

6. A down grade $(-g_1\%)$ followed by an other downgrade $(-g_2\%)$, $g_1 > g_2$. (Fig. 18.6).

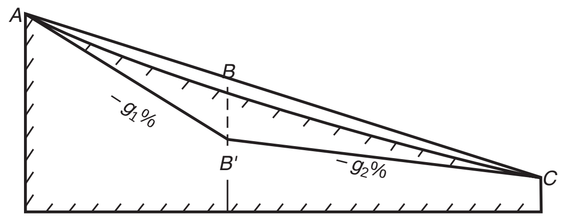


Fig. 18.6. A downgrade followed by another downgrade.

18.5. LENGTH OF VERTICAL CURVES

The length of a vertical curve may be defined as the length from the point of commencement of the curve *i.e.,* the point of rising to the point of tangency of the curve *i.e.,* the point where curve meets horizontal straight again.

Let l_1 be the length of the curve from the point of commencement to summit ; $g_1\%$ be the upgrade ; r be the rate of change of grade,

Then $$l_1 = \frac{\text{percentage of up–grade}}{\text{rate of change of grade}} = \frac{g_1}{r}$$

Similarly, the length of the curve from the summit to the point of tangency.

$$l_2 = \frac{\text{percentage of downgrade}}{\text{rate of change of grade}} = \frac{g_2}{r}$$

∴ Total length of the vertical curve

$$L = \frac{g_1}{r} + \frac{g_2}{r} \qquad \text{...(18.4)}$$

As upgrades are treated positive and downgrades negative, equation (18.4) may be written as

$$L = \frac{g_1 - (-g_2)}{r} \qquad \text{...(18.5)}$$

Hence, *the length of the vertical curve may be obtained by dividing the algebraic difference of the grades by the rate of change of grade with due regard to the sign of each grade.*

Note. The following points may be noted :

(*i*) In general practice, nearest full number of chains is adopted and half of the length is set out to the either side of the summit or valley in the case of railways.

(*ii*) In case of highways the calculated length of the vertical summit curve is also checked for sight distances as described in article 18.9.1. later.

Example 18.1. *Calculate the length of a vertical curve if an upgrade g_1 = 1.4% is followed by a downgrade g_2 = 0.6% and the rate of change of grade, is recommended as 0.1% per 20 m chain.*

Solution.

From Eqn. (18.5),

$$L = \frac{g_1 - g_2}{r} \quad ...(i)$$

Here $g_1 = +\ 1.4\%, g_2 = -\ 0.6\%$

r = per 20 m

Substituting the values in Eqn. (i), we get

$$= \frac{1.4 - (-\ 0.6)}{0.1}$$

$$= 20 \text{ chains} = 20 \times 20 = 400 \text{ m.}$$ **Ans.**

Example 18.2. *Calculate the length of a vertical curve if an upgrade g_1 = itali2% is followed by an upgrade g_2 = 1.5%. Assume the recommended rate of change of grade as 0.1 per 30 m chain.*

Solution.

From Eqn. (18.5), we get

$$L = \frac{g_1 - g_2}{r} \quad ...(i)$$

Substituting the values in Eqn. (i)

$$L = \frac{2.0 - 1.5}{0.1} = 5 \text{ Chains}$$

$$= 5 \times 30 = 150 \text{ m.}$$ **Ans.**

18.6. GEOMETRICS OF A VERTICAL CURVES

Assumptions. As vertical curves are generally flat, the distances along the curve are measured horizontally and the offsets from the tangents to the curve are measured vertically without introducing any appreciable error. The total length of the vertical without introducing any appreciable error. The total length of the vertical curve is therefore its horizontal projection. (Fig. 18.7).

Let OX' and OY be the axes of the cartesian co-ordinates, passing through the point of commencement of the vertical curve to be introduced between two grades g_1 and g_2.

OA is a tangent at O, having $+g_1\%$ slope

AB is a tangent at B, having $-g_2\%$ slope

A is the summit where two grades meet

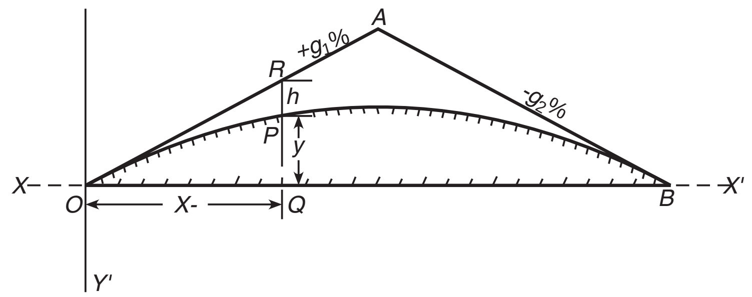


Fig. 18.7. Geometrics of a vertical curve.

P is any point on the curve whose coordinates are : (x, y).

Construction. Drop a perpendicular *RQ* through *P* to axis *OX*′.

The general equation of a vertical parabola having its axis parallel to *Y*-axis is

$$y = ax^2 + bx \qquad \text{...(18.6)}$$

Differentiating Eqn. (18.6), we get

$$\frac{dy}{dx} = 2ax + b \qquad \text{...(18.7)}$$

At $\quad x = 0, \dfrac{dy}{dx} = +g_1$

Substituting these values in Eqn. (18.7), we get

$$g_1 = 2a\ (0) + b$$

or $$g_1 = b \qquad \text{...(18.8)}$$

Substituting the value of *b* in Eqn. (18.6), we get

$$y = ax^2 + g_1 x \qquad \text{...(18.9)}$$

Let $RP = h$, the vertical distance from the tangent *OA* to the curve at *P*.

$$h = RQ - PQ$$

But $\quad RQ = x \times g_1$, the rise in *x* distance and $PQ = y$

$\therefore$ $$h = xg_1 - y \qquad \text{...(18.10)}$$

Substituting $\quad xg_1 - y = ax^2$ (Eqn. 18.9), we get

$$h = -ax^2 = Cx^2 \text{ where } C \text{ is a constant}$$

or $$h = KN^2 \qquad \text{...(18.11)}$$

where *K* is a constant and *N* is the distance measured from *O*, the beginning of the curve.

i.e. the difference in elevations of the points between a vertical curve and a tangent varies as the square of their horizontal distances from the point of tangency. This difference in elevations, is known as tangent correction.

The value of the constant 'K' (Fig. 18.8).

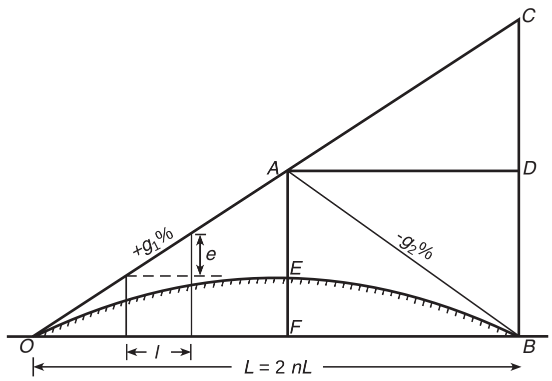


Fig. 18.8. Calculation of constant K.

Let $2n$ be total number of equal chords, each of length l

$g_1\%$ and $g_2\%$ be the upgrade and downgrade respectively.

e_1 and e_2 be corresponding rise or fall per chord length.

Construction. Produce the tangent *OA* till it meets the vertical through *B* at *C*. Drop *AD* perpendicular to the vertical *BC*.

If $$OA = AC$$

$$\Delta\, OAF = \Delta ACD$$

$\therefore$ $$AF = n_1\, e = CD$$

Similarly $$AF = -\, n_{e2} = BD$$

$\therefore$ $$BC = BD + DC = n_{e1} - n_{e2}$$

$$BC = n(e_1 - e_2)$$

But, from equation (18.11), we get

$$BC = KN^2 = K(2n)^2$$

or $$K(2n)^2 = n(e_1 - e_2)$$

or $$K = \frac{(e_1 - e_2)}{4n} \qquad \text{...(18.12)}$$

Knowing the values of e_1, e_2 and n, the value of the constant K can be calculated.

Note. In the equation (18.12), proper signs of e_1 and e_2 should be used for obtaining the value of the constant K.

Example 18.3. *Calculate the value of the constant K when an upgrade 2.5% is followed by a downgrade 1.5%. Assume the rate of change of grade is 0.05% per 20 m chain.*

Solution.

$g_1 = +2.5\%, ; g_2 = -1.5\% ; r = 0.05\%$

$\therefore$ Length of the vertical curve

$$L = \frac{g_1 - g_2}{r} = \frac{2.5 + 1.5}{0.05} = 80 \text{ chains}$$

$\therefore$
$$n = \frac{80}{2} = 40$$

$$e_1 = \frac{2.5}{100} \times 20 = 0.5$$

$$e_2 = -\frac{1.5}{100} \times 20 = -0.3$$

or
$$K = \frac{e_1 - e_2}{4n} = \frac{0.5 + 0.3}{4 \times 40} = \frac{1}{200}. \qquad \textbf{Ans.}$$

18.7. SETTING OUT A VERTICAL CURVE BY TANGENT CORRECTIONS

Principle. Knowing the value of the constant k, the required tangential corrections for various points on the curve, may be calculated from the equation $h = k\,n^2$.

Assumption. The reduced level and chainage at the point of grade separation *i.e.* the summit or valley, are generally known.

Let n be the number of chords on either side of the summit or valley.

l be the length of each chord.

Chainage of point of commencement is known.

e_1 , e_2 be the rise or fall per chord length.

The reduced levels and chainages of various points on the curve, may be computed as under :

Procedure : Proceed as under.

1. Chainage at the point of commencement
   B = chainage of $A - nl$.
2. Chainage at the point of tangency
   C = chainage of $A + nl$.
3. R.L. of point of commencement B = R.L. of $A - ne_1$

4. R.L. of point of tangency C = R.L. of $A - ne_2$
5. In a parabola, $AF = EF$ where AE is the axis of the parabola.
6. Calculation of tangent corrections.

$$h = kN^2$$

Substituting the value of N as 1, 2, 3......n, we get

$$h_1 = 1.k$$

$$h_2 = 4.k$$

$$h_3 = 9.k$$

....

....

$$h_n = (2n)^2 .k.$$

Example 18.4. *Calculate the reduced levels of the various station pegs on a parabolic vertical curve which is to be set to connect two uniform grades 0.6% and – 0.7%. The chainage and required level of the point of intersection are 2525 m and 335.65 m respectively. Assume the rate of change of grade to be 0.05 per 20 m chain.*

Solution. (Fig. 18.9).

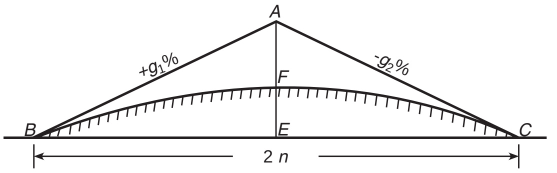


Fig. 18.9

Here $g_1 = 0.6\%$; $g_1 = -0.7\%$: $r = 0.05$

The total length of the curve

$$L = \frac{g_1 - g_2}{r} = \frac{0.6 - (0.7)}{0.05} = 26 \text{ chains}$$

The length of the curve on either side

$$= 13 \text{ chains} = 13 \times 20 = 260 \text{ m.}$$

Chainage at point of intersection A = 2525 (given)

Chainage at point of commencement

$$B = 2525 - 260 = 2265 \text{ m}$$

Chainage at point of tangency

$$C = 2525 + 260 = 2785 \text{ m}$$

R.L. of point of intersection A = 335.65 m (given)

R.L. of point of commencement

$$B = 335.64 - \frac{0.6}{100} \times 260$$

$$= 335.65 - 1.56 = 334.09 \text{ m}$$

R.L. of point of tangency

$$C = 335.65 - \frac{0.7}{100} \times 260$$

$$= 335.65 - 1.82 = 333.83 \text{ m}$$

R.L. of the point

$$E = \frac{\text{R.L. of } B + \text{R.L. of } C}{2}$$

$$= \frac{334.09 + 333.83}{2} = 333.96 \text{ m}$$

R.L. of the point

$$F = \frac{\text{R.L. of } E + \text{R.L. of } A}{2}$$

$$= \frac{1}{2}\,(333.96 + 335.65) = 334.805 \text{ m}$$

Difference in R.Ls. of point of intersection and the apex of the curve

$$= 335.650 - 334.805 = 0.845 \text{ m}$$

Check : $$e_1 = \frac{g_1}{100} \times 20 = \frac{0.6}{100} \times 20 = 0.12$$

and $$e_2 = \frac{g_2}{100} \times 20 = \frac{0.7}{100} \times 20 = -\,0.14$$

$\therefore$ $$k = \frac{e_1 - e_2}{4n} = \frac{0.12 + 0.14}{4 \times 13} = \frac{0.26}{52} = 0.005$$

or $$AF = kn^2 = 0.005 \times 13^2 = 0.845 \text{ m.} \qquad \text{O.K.}$$

Tangential elevation of the first peg

$$= \text{R.L. of point of commencement} + e_1$$

$$= 334.09 + 0.12 = 334.21 \text{ m}$$

Tangent correction of the first peg

$$= kn^2 = 0.005 \times 1 = 0.005 \text{ m}$$

$\therefore$ R.L. of the first peg

$$= 334.210 - 0.005 = 334.205 \text{ m}$$

Tangential elevation of the second peg

$$= 334.09 + 2 \times 0.12 = 334.33 \text{ m}$$

Tangent correction of second peg

$$= kn^2 = 0.005 \times 2^2 = 0.020$$

$\therefore$ R.L. of the second peg

$$= 334.33 - 0.02 = 334.31 \text{ m}$$

R.Ls. of the remaining pegs, may be calculated in a similar manner as tabulated here.

Sl. No.	*Chainage (m)*	*Tangent Elevation (m)*	*Tangent Correction (m)*	*R.L. (m)*
0	2265	334.090	0.000	334.090
1	2285	334.210	0.005	334.205
2	2305	334.330	0.020	334.310
3	2325	334.450	0.080	334.405
4	2345	334.570	0.080	334.490
5	2565	334.690	0.125	334.565
6	2385	334.810	0.180	334.630
7	2405	334.930	0.245	334.685
8	2425	335.050	0.320	334.730
9	2445	335.170	0.405	334.765
10	2465	335.290	0.500	334.790
11	2485	335.410	0.605	334.805
12	2505	335.530	0.720	334.810 Apex
13	2525	335.650	0.845	334.805
14	2545	335.770	0.980	334.790
15	2565	335.890	1.125	334.765
16	2585	336.010	1.280	334.730
17	2605	336.130	1.445	334.685
18	2625	336.250	1.620	334.630
19	2645	336.370	1.805	334.565
20	2665	336.490	2.000	334.490
21	2685	336.610	2.205	335.405
22	2705	336.730	2.420	334.310
23	2725	336.850	2.645	334.205
24	2745	336.970	2.880	334.090
25	2765	337.090	3.125	333.965
26	2785	337.210	3.380	333.830

Example 18.5. *A horizontal grade meets a –2.5% grade at 3035 metre chainage and 218.905 m elevation. A vertical curve of 16 metre length with 4 metre peg intervals is to be introduced. Calculate the necessary elevations on the curve.*

Solution. Fig. (18.10).

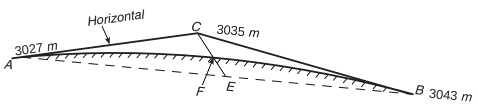


Fig. 18.10

Let C be the point of intersection of two grades having 218.905 m elevation. The length of the curve is 16 metres and peg interval is 4 metres.

$\therefore$ Number of chords $n = \dfrac{16}{4} = 4$

Chainage at the beginning of the curve

$= 3035 - 8 = 3027$ m

R.L. of B $= \text{R.L. of } C - 2.5\% \text{ of } 8$

$= 218.405 - \dfrac{2.5}{100} \times 8$

$= 218.905 - 0.200 = 218.705$ m

R.L. of A $= \text{R.L of } C = 218.905$ m

(AC being horizontal)

R.L. of E $= \dfrac{1}{2}(218.705 + 218.905) = 218.805$ m

R.L. of F $= \dfrac{1}{2}(\text{R.L. of } C + \text{R.L. of } E)$

$= \dfrac{1}{2}(218.905 + 218.855) = 218.855$ m

$\therefore$ $CF = \text{R.L of } C - \text{R.L. of } F$

$= 218.905 - 218.855 = 0.05$ m.

Now $e_1 = O \,;\, e_2 = \dfrac{-2.5}{100} \times 4 = -0.1$ m

and $2n = 4$ or $n = 2$

$$k = \frac{e_1 - e_2}{4n} = \frac{O - (-0.1)}{4 \times 2} = \frac{0.1}{8} = 0.0125 \text{ m}$$

The tangential correction $h = kn^2$

$\therefore$ $h_1 = 0.0125 \times (1)^2 = 0.0125$ m

$$h_2 = 0.0125 \times (2)^2 = 0.0500 \text{ m}$$

$$h_3 = 0.0125 \times (3)^2 = 0.1125 \text{ m}$$

$$h_4 = 0.0125 \times (4)^2 = 0.2000 \text{ m}$$

R.Ls. of the various points of the curve, are tabulated under.

S. No.	*Chainage (m)*	*Grade Elevation (m)*	*Tangential Correction (m)*	*Curve Level (m)*
0	3027 m	218.905	0.000	219.905
1	3031 m	218.905	0.013	218.892
2	3035 m	218.905	0.050	218.855
3	3039 m	218.905	0.113	218.792
4	3043 m	218.905	0.2000	218.705

Example 18.6. *A – 2.5% grade meets another –2.0% grade at chainage 3145 m. The straight length of road along –2.5% grade is 110 metres and the elevation at the chainage 3035 metres is 218.905 metres. A vertical curve of 60 metre length with 10 metre peg interval is to be introduced. Calculate the necessary elevations on the curve. Assume through chainage from right to left.*

Solution. (Fig. 18.11).

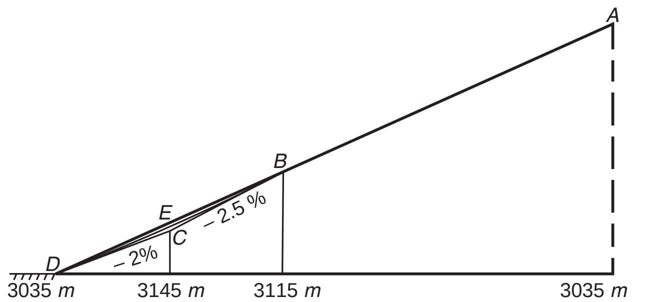


Fig. 18.11

Let A be the point at chainage 3035 m

B and D be the tangent points and C the point of intersection

Elevation of C $= 218.905 - \dfrac{2.5}{100} \times 110$

$= 218.905 - 2.750 = 216.155$ m

Elevation of D $= 216.155 - \dfrac{2}{100} \times 30 = 215.555$ m

Elevation of B $= 216.155 + \dfrac{2.5}{100} \times 30 = 216.905$ m

Elevation of E $= \dfrac{215.555 + 216.905}{2} = 216.230$ m

Elevation of F $= \dfrac{216.230 + 216.155}{2} = 216.193$ m

$$CF = 216.193 - 216.155 = 0.038 \text{ m}$$

i.e. number of peg intervals = 60 over 10 = 6

$$2n = 6 \quad \text{or} \quad n = 3$$

$$e_1 = \frac{2}{100} \times 10 = +0.20 \qquad \text{(From left to right)}$$

Again $$e_2 = \frac{2.5}{100} \times 10 = +0.25$$

$\therefore$ $$k = \frac{e_1 - e_2}{4n} = \frac{0.20 - 0.25}{4 \times 3} = \frac{-0.05}{12} = -0.0042$$

The value of k can also be calculated as under

$$CF = Kn^2$$

or $$-0.038 = k.n^2$$

or $$K = \frac{0.038}{3^2} = -0.0042. \quad \text{O.K.}$$

Tangent correction $h = k.n^2$

$$h_1 = 0.0042 \times (1)^2 = 0.004$$

$$h_2 = 0.0042 \times (2)^2 = 0.017$$

$$h_3 = 0.0042 \times (3)^2 = 0.038$$

$$h_4 = 0.0042 \times (4)^2 = 0.067$$

$$h_5 = 0.0042 \times (5)^2 = 0.105$$

$$h_6 = 0.0042 \times (6)^2 = 0.151$$

R.Ls. of various points on the curve are tabulated here under :

S. No.	*Chainage (m)*	*Grade elevation (m)*	*Tangent correction +(m)*	*Curve level*
0	3175	215.555	0	215.555 m
1	3165	215.755	0.004	215.759 m
2	3155	215.955	0.017	215.972 m
3	3145	216.155	0.038	216.193 m
4	3135	216.355	0.067	216.422 m
5	3125	216.555	0.105	216.660 m
6	3115	216.755	0.151	216.906 m

Ans.

Example 18.7. *A –2.0% grade of a railway meets a +3.0% grade at a chainage 11520 metres and 2568.655 m elevation. A valley curve it to be designed such that the rate of chainage of grade not to exceed 0.05% per 20 m chain. Calculate the curve levels at 100 metre peg intervals.*

Solution. (Fig. 18.12).

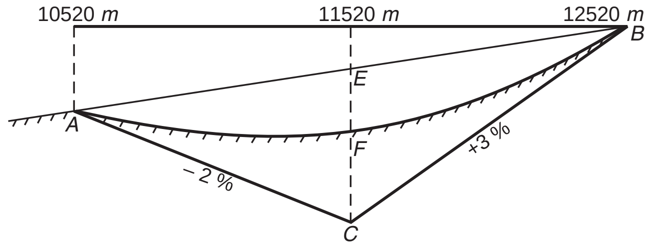


Fig. 18.12

The length of the curve

$$= \frac{\text{total change in grade}}{\text{rate of change of grade}} = -\frac{-2.0-3.0}{-0.05} = \frac{5.0}{0.05}$$

$$= 100 \text{ chains} = 100 \times 20 = 2000 \text{ m}$$

Number of peg intervals = (2000/100) = 20

$$2n = 20 \quad \text{or} \quad n = 10$$

$$e_1 = -\frac{2.0}{100} \times 100 = -2.0$$

$$e_2 = \frac{3.0}{100} \times 100 = +3.0$$

$$k = \frac{e_1 - e_2}{4n} = \frac{-2-3}{4 \times 10} = -\frac{5}{40} = -0.125$$

R.L. of A = R.L. of C + 2% of 1000

= 2568.655 + 20.000 = 2588.655 m

R.L. of B = R.L. of C + 3% or 1000

= 2568.655 + 30.000 = 2598.655 m

R.L. of E $= \frac{1}{2}$ [2588.655 + 2598.655] = 2593.655 m

R.L. of F $= \frac{1}{2}$ [2593.655 + 2568.655] = 2581.155 m

$$CF = 2568.655 - 2581.155 = -12.5 \text{ m}$$

Check :

$$k = \frac{-12.5}{10^2} = \frac{-12.5}{100} = -0.135 - 0.125$$

Tangent corrections may be calculated from the Eqn. $h = k.n^2$

$h_1 = -\,0.125 \times 1 = -\,0.125$ $\quad h_2 = -\,0.125 \times (2)^2 = -\,0.500$

$h_3 = -\,0.125 \times (3)^2 = -\,1.125$ $\quad h_4 = -\,0.125 \times (4)^2 = -\,2.000$

$h_5 = -\,0.125 \times (5)^2 = -\,3.125$ $\quad h_6 = -\,0.125 \times (6)^2 = -\,4.500$

$h_7 = -\,0.125 \times (7)^2 = -\,6.125$ $\quad h_8 = -\,0.125 \times (8)^2 = -\,8.000$

$h_9 = -\,0.125 \times (9)^2 = -\,10.125$ $\quad h_{10} = -\,0.125 \times (10)^2 = -\,12.500$

$h_{11} = -\,0.125 \times (11)^2 = -\,15.125$ $\quad h_{12} = -\,0.125 \times (12)^2 = -\,18.000$

$h_{13} = -\,0.125 \times (13)^2 = -\,21.125$ $\quad h_{14} = -\,0.125 \times (14)^2 = -\,24.500$

$h_{15} \quad -\,0.125 \times (15)^2 = -\,28.125$ $\quad h_{16} = -\,0.125 \times (16)^2 = -\,32.000$

$h_{17} = -\,0.125 \times (17)^2 = -\,36.125$ $\quad h_{18} = -\,0.125 \times (18)^2 = -\,40.500$

$h_{19} = -\,0.125 \times (19)^2 = -\,45.125$ $\quad h_{20} = -\,0.125 \times (20)^2 = -\,50.000$

Necessary data is tabulated here under:

Sl. No.	*Chainage (m)*	*Grade elevation (m)*	*Tangent correction +(m)*	*Curve levels (m)*
0	10520	2588.655	0.000	2588.655
1	10620	2586.655	0.125	2586.780
2	10720	2584.655	0.500	2585.155
3	10820	2582.655	1.125	2583.780
4	10920	2580.655	2.000	2582.655
5	11020	2578.655	3.125	2581.780
6	11120	2576.655	4.500	2581.155
7	11220	2574.655	6.125	2580.780
8	11320	2572.655	8.000	2580.655
9	11420	2570.655	10.125	2580.780
10	11520	2568.655	12.500	2581.155 valley
11	11620	2566.655	15.125	2581.780
12	11720	2564.655	18.000	2582.655
13	11820	2562.655	21.125	2583.780
14	11920	2560.655	24.500	2585.155
15	12020	2558.655	28.125	2586.780
16	12120	2556.655	32.000	2588.655
17	12220	2554.655	36.125	2590.780
18	12320	2552.655	40.500	2593.155
19	12420	2550.655	45.125	2595.780
20	12520	2548.655	50.000	2598.655

Example 18.8. *A gradient of +1% meets a gradient of +4% at the intersection point C, the chainage and reduced level of which are 500 m and 261.30 m respectively. A 100 m long vertical curve is to be inserted between the straights. Calculate the corrected grade elevations i.e. levels on the curve at 25 m intervals.*

Solution. (Fig. 18.13).

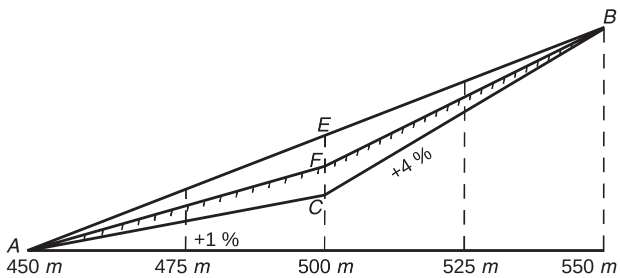


Fig. 18.13

R.L. of C $= 261.30$ m (given)

R.L. of A $= 261.30 - \frac{1}{100} \times 50 = 260.80$ m

R.L. of B $= 261.30 + \frac{4}{100} \times 50 = 263.30$ m

R.L. of E $= \frac{260.80 + 263.30}{2} = 262.05$ m

R.L. of F $= \frac{262.05 + 261.30}{2} = 261.675$ m

$\therefore$ $CF = 261.300 - 261.675 = -0.375$ m

Hence $-0.375 = K. (2)^2$

or $k = -\frac{0.375}{4} = -0.09375$ m

Again $e_1 = \frac{1}{100} \times 25 = 0.25$; $e_2 = \frac{4}{100} \times 25 = 1$

or $k = \frac{e_1 - e_2}{4n} = \frac{0.25 - 1.0}{4 \times 2}$

$= \frac{-0.75}{8} = -0.09375.$ O.K.

Tangential correction $h = k(n)^2$

$$h_1 = -0.9375 \times (1)^2 = -0.094 \text{ m}$$

$$h_2 = -0.9365 \times (2)^2 = -0.375 \text{ m}$$

$$h_3 = -\,0.09375 \times (3)^2 = \,-\,0.844 \text{ m}$$

$$h_4 = -\,0.09375 \times (4)^2 = \,-\,1.500 \text{ m}$$

Grade elevations :

Grade at chainage 450 m = 260.80 + 0 = 260.80 m

Grade at chainage 475 m = 266.80 + 0.25 = 261.05 m

Grade at chainage 500 m = 261.05 + 0.25 = 261.30 m

Grade at chainage 525 m = 261.30 + 0.25 = 261.55 m

Grade at chainage 550 m = 261.55 + 0.25 = 261.80 m

Necessary data is tabulated here under

S. No.	*Chainage (m)*	*Grade Elevation (m)*	*Tangent Correction +*	*Curve level (m)*
0	450	260.800	0.000	260.80
1	475	261.050	0.094	261.144
2	500	261.300	0.375	261.675
3	525	261.550	0.844	262.394
4	550	261.800	1.500	263.300

18.8. SETTING OUT A VERTICAL CURVE BY CHORD GRADIENTS

Chord gradient. *The difference in elevation between the ends of a chord joining two adjacent stations is known as choral gradient*

In this method, the successive difference in elevations between the points on the curve are calculated and R.L. of each point is obtained by adding the *chord gradient* to the R.L. of the preceding point.

Let AC and BC be two grades meeting at C, where A and B are the points of tangency. (Fig. 18.14).

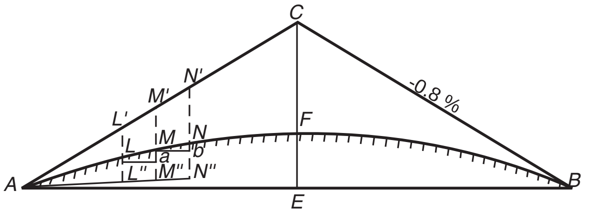


Fig. 18.14. Elevation by chord gradients.

Consider any two adjacent points L and M on the vertical curve AFB.

Construction. Through A draw a horizontal line $AL'' M''$.

Through L and M drop perpendiculars $L' L''$ and $M' M''$. Through L and M draw horizontal lines La and Mb respectively.

Let e_1 and e_2 be the respective rise and fall of the tangents per chord length.

Assuming L to be the first peg at an interval of l, then

$$e_1 = L'\,L''$$

$\therefore$ Tangent correction

$$L'\,L = K.N^2 = 1.k$$

$\therefore$ R.L. of L $= e_1 - k$

Chord gradient for the first chord

$$AL = L'\,L'' - L'\,L = e_1 - 1.k$$

where $k = \dfrac{e_1 - e_2}{4.n}$

Similarly, $M'\,M'' = 2e_1$

Tangent correction $M'\,M = (2)^2\,k = 4k$

$\therefore$ R.L. of $M = 2e_1 - 4k$

or chord gradient for the second chord

$$LM = \text{R.L. of } M - \text{R.L. of } L$$

$$= (2e_1 - 4k) - (e_1 - k) = (e_1 - 3k)$$

Similarly, $N'\,N'' = 3e_1$

Tangent correction $N'\,N = (3)^2\,k = 9k$

R.L. of $N = 3e_1 - 9k$

Chord gradient for the chord MN = R.L. or N – R.L. of M

$$= (3e_1 - 9k) - (2e_1 - 4k) = e_1 - 5k.$$

Hence, nth chord gradient $= e_1 - (2n - 1)k$ ...(18.13)

Knowing the chord gradients for different pegs, their R.L.s may be easily calculated as under :

The R.L. of 1st peg = R.L. of tangent point
+ First chord gradient

The R.L. of second peg = R.L. of 1st peg
+ Second chord gradient

The R.L. of third peg = R.L. of second peg
+ third chord gradient.

and so on.

Example 18.9. *A –1.0% grade meets a +2.0% grade at a station C whose chainage is 1550 m and elevation is 555.555 m. A vertical curve of length 100 m is required to be introduced with pegs at 10 m intervals. Calculate the elevations of the points on the curve by*

(*a*) *Tangent corrections.* (*b*) *Chord gradients.*

Solution. (Fig. 18.15).

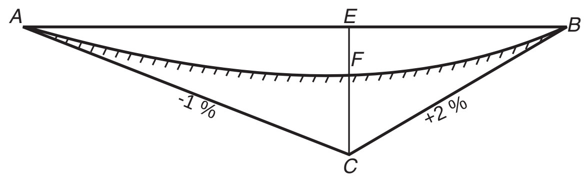


Fig. 18.15

(*a*) **Reduced Levels by Tangent Correction Method**

Total number of stations at 10 m interval

$$= \frac{100}{10} = 10$$

Chainage at A = 1550 – 50 = 1500 m

Chainage at B = 1550 + 50 = 1600 m

Number of stations to each side of apex = 5

Again $$e_1 = \frac{g_1}{100} \times 10 = \frac{-1}{100} \times 10 = -0.10 \text{ m}$$

$$e_2 = \frac{g_2}{100} \times 10 = \frac{2}{100} \times 10 = +0.20 \text{ m}$$

R.L. of A = R.L. of C + 1% of 50 m

= 555.555 + 0.500 = 556.055 m

R.L. of B = 555.555 + 2% of 50 m

= 555.555 + 1.000 = 556.555 m

R.L. of $$E = \frac{1}{2}[556.055 + 556.555] = 556.305 \text{ m}$$

R.L. of $$F = \frac{1}{2}[556.305 + 555.555] = 555.930 \text{ m}$$

$$CF = 555.555 - 555.930 = -0.375 \text{ m}$$

Again $$K = \frac{e_1 - e_2}{4n} = \frac{-0.10 - 0.20}{4 \times 5}$$

$$= \frac{-03}{20} = -0.015$$

or $$K = \frac{CF}{n^2} = \frac{-0.375}{(5)^2} = -0.015. \quad \text{O.K.}$$

Tangent corrections :

$h_1 = -0.015 \times (1)^2 = -0.015$ m $h_6 = -0.015 \times (6)^2 = -0.540$ m

$h_2 = -0.015 \times (2)^2 = -0.060$ m $h_7 = -0.060 \times (7)^2 = -0.735$ m

$h_3 = -0.015 \times (3)^2 = -0.135$ m $h_8 = -0.135 \times (8)^2 = -0.960$ m

$h_4 = -0.015 \times (4)^2 = -0.240$ m $h_9 = -0.240 \times (9)^2 = -1.215$ m

$h_1 = -0.015 \times (5)^2 = -0.375$ m $h_{10} = -0.375 \times 10.^2 = -1.500$ m

Necessary data is tabulated here under :

Sl. No.	*Chainage (m)*	*Tangent grade (m)*	*Tangent correction + (m)*	*Curve levels (m)*
0	1500	556.055	0.000	556.055
1	1510	555.955	0.015	555.970
2	1520	555.855	0.060	555.915
3	1530	555.755	0.135	555.890
4	1540	555.655	0.240	555.895
5	1550	555.555	0.375	555.930
6	1560	555.455	0.540	555.995
7	1570	555.355	0.735	556.090
8	1580	555.255	0.960	556.215
9	1590	555.155	1.215	556.370
10	2600	555.055	1.500	556.555

(*b*) **Reduced Levels by Chord Gradient Method**

From Eqn. (18.13), we know

Chord gradient $= e_1 - (2n - 1) K$

Here $e_1 = -0.10$; $K = -0.015$

1. For the first point, chord gradient

$= -0.10 - (2 - 1) \times (-)\ 0.015$

$= -0.10 + 0.015 = -0.085$

R.L. of first point = R.L. of A + chord gradient

$= 556.055 - 0.085 = 555.970$ m

2. For the second point, chord gradient

$= -0.10 - (4 - 1) \times (-)\ 0.15$

$= -0.1 + 045 = -0.055$

$\therefore$ R.L. of the second point = R.L. of first point + chord gradient

$$= 555.970 - 0.055 = 555.915 \text{ m}$$

3. For the third point, chord gradient

$$= -0.10 - (6-1)(-)\ 0.015$$
$$= -0.1 + 0.075 = -0.025 \text{ m}$$

$\therefore$ R.L. of the third point = R.L. of second point + chord gradient

$$= 555.915 - 0.025 = 555.890 \text{ m}$$

4. For the fourth point, chord gradient

$$= -0.1 - (8-1) \times (-)\ 0.15$$
$$= -0.1 + 0.105 - +0.005 \text{ m}$$

$\therefore$ R.L. of the fourth point = R.L. of 3rd point + chord gradient

$$= 555.890 + 0.005 = 555.895 \text{ m}$$

5. For the fifth point, chord gradient

$$= 0.100 - (10-1) \times (-)\ 0.015$$
$$= -0.100 + 0.135 - +0.035 \text{ m}$$

$\therefore$ R.L. of the fifth point = R.L. of 4th point + chord gradient

$$= 555.895 + 0.035 = 55.930 \text{ m}$$

6. For the sixth point, chord gradient

$$= -0.100 - (12-1) \times (-)\ 0.015$$
$$= -0.100 + 0.165 - +0.065 \text{ m}$$

$\therefore$ R.L. of the sixth point = R.L. of 5th point + chord gradient

$$= 555.930 + 0.065 = 555.995 \text{ m}$$

7. For the seventh point, chord gradient

$$= -0.100 - (14-1) \times (-)\ 0.015$$
$$= -0.100 + 0.195 - +0.095 \text{ m}$$

$\therefore$ R.L. of seventh point = R.L. of 6th point + chord gradient

$$= 556.995 + 0.095 - 556.090 \text{ m}$$

8. For the eighth point, chord gradient

$$= -0.100 = (16-1) \times (-)\ 0.015$$
$$= -0.100 + 0.225 - +0.125 \text{ m}$$

∴ R.L. of the eighth point = R.L. of 7th point + chord gradient

$$= 556.090 + 0.125 - 556.215 \text{ m}$$

9. For the ninth point, chord gradient

$$= -0.100 - (18 - 1) \times (-)\ 0.015$$

$$= -0.100 + 0.255 = 0.155 \text{ m}$$

∴ R.L. of the ninth point = R.L. of 8th point + chord gradient

$$= 556.215 + 0.155 = 556.370 \text{ m}$$

10. For the tenth point, chord gradient

$$= -0.100 - (20 - 1) \times (-)\ 0.015$$

$$= -0.100 + 0.285 = +0.185$$

∴ R.L. of the tenth point = R.L. of 9th point + chord gradient

$$= 556.370 + 0.185 = 556.555 \text{ m.}$$ **Ans.**

Example 18.10. *A vertical curve is to be set out by pegs driven at 30 metre interval to connect two uniform gradients of +0.5% and –0.5%. Calculate the R.Ls. of the first six station pegs. The chainage and R.L. at the point of intersection are 600 m and 430.750 m respectively. Take the rate of change of grade as 0.1% per 30 metres.*

Solution. Fig. (18.16).

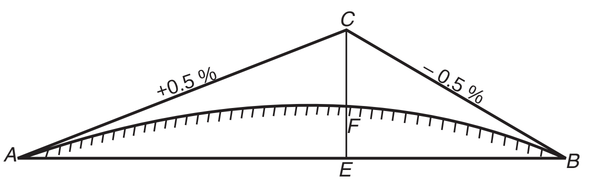


Fig. 18.16.

$g_1 = +0.5\%$; $g_2 = -0.5\%$; $r = 0.1\%$ per 30 m

∴ Length of the curve AB

$$= \frac{0.5 - (0.5)}{0.1} = 10 \text{ chains.} = 10 \times 30 = 300 \text{ m}$$

Peg intervals on either side of the apex $= \dfrac{150}{30} = 5$

Again $$e_1 = \frac{0.5}{100} \times 30 = +0.15$$

$$e_2 = \frac{-(0.5)}{100} \times 30 = -0.15$$

∴ $$K = \frac{e_1 - e_2}{4n} = \frac{0.15 + 0.15}{4 \times 5} = \frac{0.3}{20} = 0.015$$

R.L. of $A = \text{R.L. of } C \, \frac{0.5}{100} \times 150 = 430.75 - 0.75 = 430.00 \text{ m}$

R.L. of $B = \text{R.L. of } C - \frac{0.5}{100} \times 150 = 430.75 - 0.75 = 430.0 \text{ m}$

R.L. of $E = \frac{430.0 + 430.0}{2} = 430.0$

R.L. of $F = \frac{430.75 + 430.00}{2} = 430.375 \text{ m}$

$\therefore \quad CF = 430.750 - 430.375 = 0.375 \text{ m}$

Check : $K = \frac{CF}{n^2} = \frac{0.375}{(n)^2} = \frac{0.375}{25} = 0.015.$ O.K.

Tangent corrections :

$h_1 = 0.015 \times (1)^2 = 0.015 \text{ m}$ $\quad h_4 = 0.015 \times (4)^2 = 0.240 \text{ m}$

$h_2 = 0.015 \times (2)^2 = 0.060 \text{ m}$ $\quad h_5 = 0.015 \times (5)^2 = 0.375 \text{ m}$

$h_3 = 0.015 \times (3)^2 = 0.135 \text{ m}$ $\quad h_6 = 0.015 \times (6)^2 = 0.540 \text{ m}$

Sl. No.	*Chainage (m)*	*Tangent grade (m)*	*Tangent correction –(m)*	*R.L. (m)*
0	450	430.000	0.000	430.000
1	480	430.150	0.015	430.135
2	510	430.300	0.060	430.240
3	540	430.450	0.135	430.315
4	570	430.600	0.240	430.360
5	600	430.750	0.375	430.375
6	630	430.900	0.540	430.360

Example 18.11. *A 3% rising gradient meets a 2% down gradient at a chainage of 2600 m, the R.L. of the point of intersection being 1300.00 m. A vertical parabola is to be set out connect two grades with pegs at 20 metre intervals. The rate of change of grade allowed is 0.5% per 20 metre chain.*

Tabulate the chainages and R.L.'s of the station pegs for setting out the vertical curve.

Solution. (Fig. 18.17).

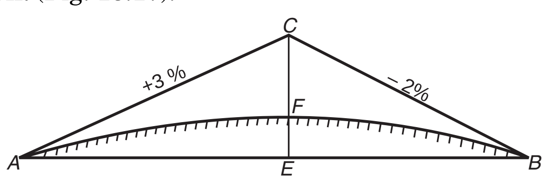


Fig. 18.17.

Here, $g_1 = +3\%$; $g_2 = -2\%$ $\quad r = 0.5\%$ per 20 m.

$\therefore$ Length of the curve $AB = \dfrac{3-(-2)}{0.5} = 10$ chains

$$= 10 \times 20 = 200 \text{ m}$$

Peg intervals on either side of the apex

$$= \frac{100}{20} = 5$$

Again $\quad e_1 = \dfrac{3}{100} \times 20 = 0.6$ m

$$e_2 = \frac{2}{100} \times 20 = -0.4 \text{ m}$$

$\therefore \quad K = \dfrac{e_1 - e_2}{4n} = \dfrac{0.6 + 0.4}{4 \times 5}$

$$= \frac{1}{20} = 0.05$$

R.L. of $\quad A = \text{R.L. } C - \dfrac{3}{100} \times 100$

$$= 1300.00 - 3.00 = 1297.00 \text{ m}$$

R.L. of $\quad B = \text{R.L. } C - \dfrac{2}{100} \times 100$

$$= 1300.00 - 2.0 = 1298.00 \text{ m}$$

R.L. of $\quad E = \dfrac{1297 + 1298}{2} = 1297.5$ m

R.L. of $\quad F = \dfrac{1300 + 1297.5}{2} = 1298.75$ m

$$CF = 1300.00 - 1298.75 = 1.25 \text{ m}$$

Check : $\quad K = \dfrac{CF}{n^2} = \dfrac{1.25}{n^2} = \dfrac{1.25}{25} = 0.05.$ O.K.

Tangent corrections :

$h_1 = 0.05 \times (1)^2 = 0.05$ m $\quad h_6 = 0.05 \times (6)^2 = 1.80$ m

$h_2 = 0.05 \times (2)^2 = 0.20$ m $\quad h_7 = 0.05 \times (7)^2 = 2.45$ m

$h_3 = 0.05 \times (3)^2 = 0.45$ m $\quad h_8 = 0.05 \times (8)^2 = 3.20$ m

$h_4 = 0.05 \times (4)^2 = 0.80$ m $\quad h_9 = 0.05 \times (9)^2 = 4.05$ m

$h_5 = 0.05 \times (5)^2 = 1.25$ m $\quad h_{10} = 0.05 \times (10)^2 = 5.00$ m

Results are tabulated here under :

Sl. No.	*Chainage (m)*	*Tangent grade (m)*	*Tangent correction (--)*	*Curve levels (m)*
0	2500	1297.00	0.00	1297.00
1	2520	1297.60	0.05	1297.55
2	2540	1298.20	0.20	1298.00
3	2560	1298.80	0.45	1298.35
4	2580	1299.40	0.80	1298.60
5	2600	1300.00	1.25	1298.75
6	2620	1300.60	1.80	1298.80
7	2640	1301.20	2.45	1298.75
8	2660	1301.80	3.20	1298.60
9	2680	1302.40	4.05	1298.35
10	2700	1303.00	5.00	1298.00

Example 18.12. *A parabolic vertical curve is to connect two uniform grades of 1.0% and –0.8%. The chainage at the point of intersection is 9864 metres and its RL is 988.765 metres. The rate of change of grade is 0.06% per chain of 20 metres. Calculate the R.L.'s and chainages at beginning, vertex and end of the curve.*

Solution. (Fig. 18.18).

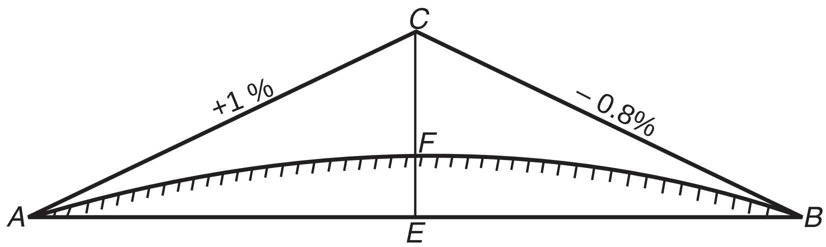


Fig. 18.18.

Here, $g_1 = 1.0\%$ $g = -0.8\%$, $r = 0.06$ per 20 m

Length of the curve $= \dfrac{g_1 - g_2}{r} = \dfrac{1.0 + 0.8}{0.06} = 30$ chains

$= 30 \times 20 = 600$ m

Length of the curve on either side = 300 m

R.L. of A = R.L. of $C - \dfrac{1}{100} \times 300$

$= 988.765 - 3.000 = 985.765$ m

R.L. of B = R.L. of $C - \dfrac{0.8}{100} \times 300$

$= 988.765 - 2.400 = 986.365$ m

R.L. of $E = \dfrac{955.765 + 986.365}{2} = 986.065$ m

R.L. of $F = \dfrac{986.065 + 988.765}{2} = 987.415$ m

Chainage at C = 9864 m Given

Chainage at A = chainage *at* $C - \dfrac{1}{2}L$

= 9864 − 300 = 9564 m

Chainage at B = Chainage of $C + \dfrac{1}{2}L$

= 9864 + 300 = 10,164 m.

	Point	*Chainage (m)*	*R.L. (m)*
Result :	A	9564 m	985.765
	C	9864 m	988.765
	B	10164 m	986.365

Example 18.13. *Calculate the reduced levels of the various station pegs on a vertical curve connecting two uniform grades of 0.5% and --0.7%. The chainage and the reduced level of the point of intersection are 500 m and 350.75 m respectively. Take the rate of change of grade as 0.1% per 30 m.*

Solution. (Fig. 18.19).

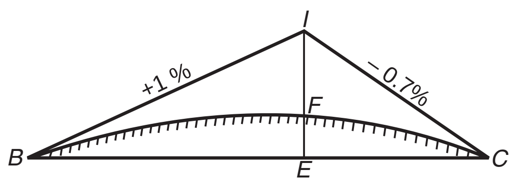


Fig. 18.19.

$g_1 = 0.5\%$; $g_2 = -0.7\%$; $r = 0.1\%$ per 30 m

The length of the curve $\dfrac{e_1 - g_2}{r} = \dfrac{0.5 + 0.7}{0.1} = 12$ chains

Length of the curve on either side = 6 chains = 180 m

Chainage at the point of intersection I = 500.00 m

Chainage at B = 500 − 180 = 320 m

Chainage at C = 500 + 180 = 680 m

Reduced level of the point of intersection = 350.75 m

$$e_1 = \frac{g_1}{100} \times 30 = \frac{0.5}{100} \times 30 = 0.15$$

$$e_2 = \frac{g_2}{100} \times 30 = \frac{-0.7}{100} \times 30 = -0.21$$

R.L. of the point of tangency

$= \text{R.L. of } I - ne_1$

$= 350.75 - 6 \times 0.15 = 349.85$ m

R.L. of the second point of tangency

$= \text{R.L of } I - ne_2$

$= 350.75 - 6 \times 0.21 = 349.49$ m

R.L. of $E = \frac{1}{2}$ (R.L. of B + R.L. of C)

$= \frac{1}{2}(349.85 + 349.49) = 349.67$ m

R.L. of $F = \frac{1}{2}$ (R.L. of I + R.L. of E)

$= \frac{1}{2}(350.75 + 349.67) = 350.21$ m

Difference in R.L.s. of I and the apex F of the curve

$= 350.75 - 350.21 = 0.54$ m

Check : $IF = kn^2$

$$k = \frac{e_1 - e_2}{4n} = \frac{0.15 + 0.21}{4 \times 6}$$

$$= \frac{0.36}{24} = 0.015$$

or $IF = 0.015 \times 36 = 0.54$ m checked.

Tangential correction for first peg $= kn^2$

$= 0.015 \times (1)^2 = 0.015$ m

Tangent elevation for first peg = R.L. of $B + e_1$

$= 349.85 + 0.15 = 350.00$ m

∴ R.L. of the first station peg

$= 350.00 - 0.015 = 349.985$ m

The R.L.'s of the station pegs are calculated as tabulated on next page .

S. No.	Chainace (m)	Grade Elevation (m)	Tangential correction (–) (m)	R.L. (m)
0	320 m	349.85	0.00	349.85
1	350 m	350.00	0.02	349.98
2	380 m	350.15	0.06	350.09
3	410 m	350.30	0.14	350.16
4	440 m	350.45	0.24	350.21
5	470 m	350.60	0.38	350.22
6	500 m	350.75	0.54	350.21
7	530 m	350.90	0.74	350.16
8	560 m	351.05	0.96	350.09
9	590 m	351.20	1.22	349.98
10	620 m	351.35	1.50	349.85
11	650 m	351.50	1.82	349.68
12	680 m	351.65	2.16	349.49

18.9. LENGTH OF A VERTICAL CURVE WITH REGARD TO SIGHT DISTANCE

Definitions : The following definitions may be understood.

(*i*) **Sight distance.** It is the length of the roadway ahead of the driver. Sight distance depends upon the speed of the moving vehicles.

(*ii*) **The stopping sight distance.** It is the minimum length of the roadway which is required for stopping a moving vehicle. It is the total distance travelled during the following three time intervals before a vehicle stops.

(*a*) The time for the driver to perceive the obstruction.

(*b*) The time for the driver to react.

(*c*) The time for stopping the vehicle after brakes are applied.

Necessity of sight distances

While negotiating a summit curve, the driver finds difficult to see approaching vehicles or other obstructions on the other side of the summit, due to curvature. A minimum sight distance for the safe driving of vehicles, is therefore, kept in mind while designing a summit curve. According to A.A.S.H.O. the recommended sight distances for the various design speeds are as under :

Design speed	*Stopping sight distance*
48 km / hour	61 m
64 km / hour	84 m
80 km / hour	107 m
96 km / hour	145 m

Formula for the sight distance when entirely on the curve. (Fig. 18.20).

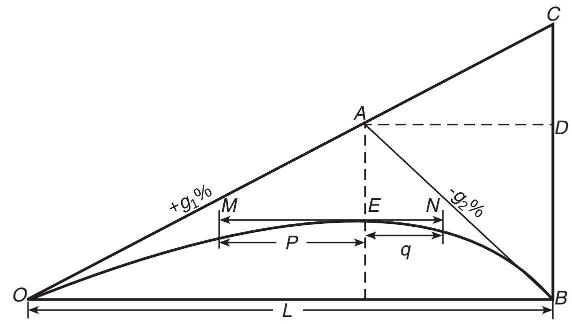


Fig. 18.20. Sight distance formula.

Let g_1 = up grade

g_2 = down grade

h_1 = height of the driver's eye M above road level

h_2 = height of the obstruction N above road level

S = minimum sight distance

L = length of the curve.

MN is the line of sight of the driver, which touches the curve tangentially at E.

From Eqn. (18.11) tangential correction,

$h = kx^2$ where x is the distance of M from O

When , $x = L$

$$h = kL^2 = BC.$$

We also know that

$$BC = \frac{g_1 - g_2}{100} \cdot \frac{L}{2}$$

or
$$\frac{g_1 - g_2}{100} \cdot \frac{L}{2} = kL^2$$

or
$$k = \frac{g_1 - g_2}{200\,L} \qquad ...(18.14)$$

Again $h_1 = kp^2$ and $h_2 = ka^2$

$\therefore$ Sight distance, $S = p + q = \sqrt{\frac{h_1}{k}} + \sqrt{\frac{h_2}{k}}$

$$= \frac{1}{\sqrt{k}}\,(\sqrt{h_1} + \sqrt{h_2})$$

Substituting the value of k from Eqn. (18.14), we get

$$S = \frac{14.14\sqrt{L}}{\sqrt{g_1 - g_2}}(\sqrt{h_1} + \sqrt{h_2}) \quad ...(18.15)$$

Squaring both sides of equation (18.15), we get

$$S^2 = \frac{14.14^2 .L}{g_1 - g_2}(\sqrt{h_1} + \sqrt{h_2})^2$$

$$L = \frac{S^2 (g_1 - g_2)}{200(\sqrt{h_1} + \sqrt{h_2})^2} \quad ...(18.16)$$

Taking $h_1 = h_2 = h$ we get

$$L = \frac{S^2 (g_1 - g_2)}{800h} \quad ...(18.17)$$

Note : The units of L, S and h should be kept same.

Example 18.14. *An upgrade (+1.5%) meets a downgrade (--1%). If the height of the driver's eye is 1.37 m, the height of obstruction is 0.10 m and the minimum sight distance required is 200 m, calculate the length of the curve.*

Solution.

From Eqn. (18.16), we get

$$L = \frac{S^2 (g_1 - g_2)}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{200^2 (1.5 + 1.0)}{200(\sqrt{1.37} + \sqrt{-0.10})^2}$$

or $$L = \frac{200 \times 2.5}{2.21} = 226.2 \text{ m.} \quad \textbf{Ans.}$$

Example 18.15. *A 2% (1 in 50 elevation) gradient meets a –0.25% (1 in 400 fall) gradient at a chainage of 3560 ft and at the reduced level of 251.15 ft. If the sight distance be 1000 ft determine the length of the vertical curve and the reduced levels of the tangent points and the highest point on the curve. Assume that the eye level of the driver is 3 ft 9 in. above the road surface.*

Solution. (Fig. 18.21).

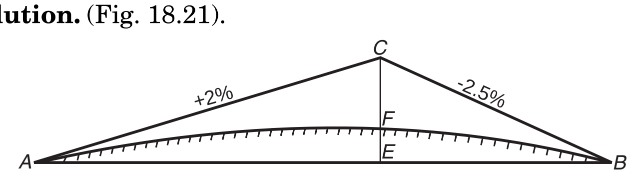


Fig. 18.21.

(The problem has been solved in metric units after converting the given data to merit units).

Chainage at the apex = 1085 m

R.L. of the apex = 76.55m

Sight distance S = 304.8m

Height of the driver's eye = 1.143 m

Height of the obstruction = 0.1 m

Minimum length
$$L = \frac{S^2 (g_1 - g_2)}{200 (\sqrt{h_1} + \sqrt{h_2})^2}$$

$$= \frac{304.8 \times 304.8 \, (2 + 0.25)}{200 (\sqrt{1.143} + \sqrt{0.1})^2}$$

$$= \frac{64.52 \times 2.25}{1.069 + 0.316}$$

or
$$L = \frac{1045.17}{1.9182} = 544.87 \text{ say } 550 \text{ m}$$

$\therefore$ The length of the curve on either side = 275 m. **Ans.**

R.L. of A $= \text{R.L. of } C - \dfrac{2}{100} \times 275$

$= 76.55 - 5.50 = 71.05$ m.

R.L. of B $= \text{R.L. } C - \dfrac{(0.25)}{100} \times 275$

$= 76.55 - 0.698 = 75.85$ m

R.L. of the point $E = \dfrac{71.05 + 75.86}{2} = 73.46$ m

R.L. of the apex $= \dfrac{76.55 + 73.46}{2} = 75.00$ m

Length of the curve = 550 m

R.L. of the first tangent point = 71.05 m

R.L. of the second tangent point = 75.86 m

R.L. of the highest point of curve = 75.00 m **Ans.**

Example 18.16. *A down grade of 1% is followed by an upgrade of 1.4%. The reduced level of the point of intersection is 150.00 m and its chainage 450 m. Calculate the reduced levels on the points on the curve and the staff readings required if the pegs are to be driven with their tops at the formation of the curve. Assume the rate of change of grade to be 0.1 per 10 m and the height of collimation 153.000 m .*

Solution. (Fig. 18.15)

The length of curve $= \dfrac{g_1 - g_2}{r} = \dfrac{-1.0 - 1.4}{0.1}$

$= 24$ chains of 10 m = 240 m

Number of peg interval $= \dfrac{240}{10} = 24$

$$2n = 24 \text{ or } n = 12$$

$$e_1 = \frac{-1.0}{100} \times 10 = -0.10$$

$$e_2 = \frac{-1.4}{100} \times 10 = 0.14$$

$$K = \frac{e_1 - e_2}{4n} = \frac{-10 - 0.14}{4 \times 12}$$

$$K = -0.005$$

Chainage of A = Chainage of C – half length of the curve

= 450 – 120 = 330 m

Chainage of B = Chainage of C + half length of the curve

= 450 + 120 = 570 m

R.L. of A = R.L. of C + 1% of 120

= 150.00 + 1.20 = 151.20 m

R.L. of B = R.L. of C + 1.4% of 120

= 150.00 + 1.68 = 151.68 m

R.L. of E $= \dfrac{1}{2}(151.20 + 151.68) = 151.44$ m

R.L. of F $= \dfrac{1}{2}(151.44 + 150.00) = 150.72$ m

$$CF = 150.00 - 150.72 \text{ m} = -0.72$$

or $$K = \frac{-0.72}{12^2} = -0.005. \quad \text{O. K.}$$

Tangent correction $h = Kn^2$

$h_1 = -0.005 \times 1 = -0.005$ $\quad$ $_{13} = -0.005 \times 169 = -0.845$

$h_2 = -0.005 \times 4 = -0.020$ $\quad$ $h_{14} = -0.005 \times 196 = -0.980$

$h_3 = -0.005 \times 9 = -0.045$ $\quad$ $h_{15} = -0.005 \times 225 = -1.125$

$h_4 = -0.005 \times 16 = -0.080$ $\quad$ $h_{16} = -0.005 \times 256 = -1.280$

$h_5 = -0.005 \times 25 = -0.125$ $\quad$ $h_{17} = -0.005 \times 289 = -1.445$

$h_6 = -0.005 \times 36 = -0.180$ $\quad$ $h_{18} = -0.005 \times 324 = -1.620$

$h_7 = -0.005 \times 49 = -0.245$ $h_{19} = -0.025 \times 361 = -1.805$

$h_8 = -0.005 \times 64 = -0.320$ $h_{20} = -0.005 \times 400 = -2.000$

$h_9 = -0.005 \times 81 = -0.405$ $h_{21} = -0.005 \times 441 = -2.205$

$h_{10} = -0.005 \times 100 = -0.500$ $h_{22} = -0.005 \times 484 = -2.420$

$h_{11} = -0.005 \times 121 = -0.605$ $h_{23} = -0.005 \times 529 = -2.645$

$h_{12} = -0.005 \times 144 = -0.720$ $h_{24} = -0.005 \times 576 = -2.880$

Calculation of staff readings may be done as shown in the following table.

Stn.	*Chainage*	*Grade Elev.*	*Tangent Correction*	*Curve Elev.*	*Ht. of collimation*	*Staff readings*	*Remarks*
A	330	151.200	0	151.200	153.000	1.800	Beginning of curve
	340	151.100	– 0.005	151.105		1.895	
	350	151.000	– 0.020	151.020		1.980	
	360	150.900	– 0.045	151.945		1.055	
	370	150.800	– 0.080	150.880		2.120	
	380	150.700	– 0.125	150.825		2.175	
	390	150.600	– 0.180	150.780		2.220	
	400	150.500	– 0.245	150.745		2.255	
	410	150.400	– 0.320	150.720		2.280	
	420	150.300	– 0.405	150.705		2.295	
	430	150.200	– 0.500	150.700		2.300	
	440	150.100	– 0.605	150.705		2.295	
	450	150.000	– 0.720	150.720		2.280	Vertex
	460	149.900	– 0.845	150.745		2.255	
	470	149.800	– 0.980	150.780		2.220	
	480	149.700	– 1.125	150.825		2.175	
	490	149.600	– 1.280	150.880		2.120	
	500	149.500	– 1.445	150.945		2.055	
	510	149.400	– 1.620	151.020		1.980	
	520	149.300	– 1.805	151.105		1.895	
	530	149.200	– 2.000	151.200		1.800	

	540	149.100	– 2.205	151.305		1.695	
	550	149.000	– 2.420	151.420		1.580	
	560	148.900	– 2.645	151.545		1.455	
B	570	148.800	– 2.880	151.680		1.320	End of curve

18.10. DESIGN OF PAVEMENT CROWNS

The parabolic curve is generally used in the design of crowned pavements. After deciding the total rise or crown at the centre of the pavement and also its width, the elevations of various intermediate points along a cross section of the roadway, may be calculated by the following formula. (Fig. 18.22).

$$O_x = \frac{dx^2}{(W/2)^2} \times O_c \qquad ...(18.18)$$

where O_x is the required vertical offset

dx is the distance of the point of offset from centre

W is the total width of the pavement

O_c is the crown, the height of the centre of the pavement above gutter elevation.

Example 18.17. *A 10.8 m wide pavement of a highway is provided a 10 cm crown. Calculate the reduced levels for the road surface at 2 m and 4 m on either side of the centre line if the centre line reduced level is 153.545 m.*

Solution. (Fig. 18.22).

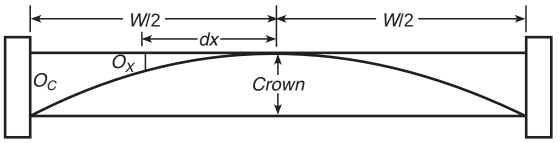


Fig. 18.22.

The offsets at

2 m distance $= \frac{2^2}{(5.4)^2} \times 10 = 1.37$ cm

4 m distance $= \frac{4^2}{(5.4)^2} \times 10 = 5.49$ cm

The reduced level at centre of the pavement

= 153.545 m

R.L. at 2 m either side = 153.545 -- 0.137 = 153.408 m. **Ans.**

R.L. at 4 m either side = 153.545 -- 0.549 = 152.996 m. **Ans.**

Example 18.18. *The reduced level of the top of a pavement on the centre line at a chainage 450 m is 285.840 m. The grade is –1.5%, the pavement is 10 m face to face of the kerbs, and the crown provided is 12 cm.*

Calculate the grade elevation of a point at chainage 510 m and 3 m distant, at right angles from the centre line of the pavement.

Solution.

The distance between sections = 510 – 450 = 60 m

The reduced level at chainage 510 m

$$= \text{R.L. of chainage 450 m} - \frac{1.5}{100} \times 60$$

$$= 285.840 - 0.900 = 284.940 \text{ m}$$

The offset at 3 m from the centre

$$= \frac{3^3}{(5)^2} \times 12 = 4.32 \text{ cm} = 0.043 \text{ m}$$

∴ R.L at 3 m from the centre line at chainage 510 m

$$= 284.940 - 0.043 = 284.897 \text{ m.}$$ **Ans.**

EXERCISE 18

1. Prove that a parabolic curve provides a rate of chainage of grade at a uniform rate.

2. Derive an expression for the tangent corrections on a vertical curve.

3. Derive an expression for calculating the minimum length of a vertical curve for providing a safe sight distance.

4. Calculate the length of the vertical curves connecting two uniform grades from the following data :

(*i*)	– 0.5% and + 1%	r = 0.5% per 30 m
(*ii*)	+ 2% and – 1.5%	r = 0.1% per 20 m
(*iii*)	+ 1.5% and – 1.5%	r = 0.15% per 20 m

5. Vertical curve is to be set out by pegs driven at 30 m interval to connect two uniform gradients of 2.5% and -1.5%. Calculate the length of the curve and the R.L.'s of the first five station pegs. The chainage and R.L. of the point of intersection are 1250 m and 825.85 m respectively. Assume the rate of change of grade as 0.1% per 30 metres.

6. A 2% rising gradient meets a 3% down gradient at a chainge of 2500 metres, the R.L. of the point of intersection being 875.00 metres. A vertical parabola is to be set out to connect the two grades with pegs at 20 metre intervals. The rate of chainage of grade allowed is 0.5% per 20 metre chain. Tabulate the chainages and R.L. of the station pegs for setting out the vertical curve.

7. A down grade of 1.5% is followed by an upgrade of 2.5%. The reduced level of the point of intersection is 90.00 m and its chainages 450 m. A vertical parabolic curve 180 m long is to be introduced to connect the two grades. The pegs are to be fixed at 20 m intervals. Calculate the elevations of the points on the curve.

8. A downgrade of 2.5% is followed by an up grade of 1.5%. The reduced level of the point of intersection is 100.00, and its chainage 560 m. A vertical parabolic curve 150 m long is to be introduced to connect the two grades. The pegs are to be fixed at 15 m intervals. Calculate the elevations of the points on the curve by (*a*) tangent corrections, and (*b*) chord gradients.

Also calculate the staff readings if the pegs are to be driven with their tops at the formation of the curve.

9. Design a vertical curve connecting two gradients +2% and –1.5% at a point whose chainage is 850 m and R.L. is 705.0 m. The curve is to be such that two points 300 m apart and 1.25 m above the curve are intervisible.

10. Design a vertical sag curve 600 m long connecting a falling gradient of 2% with a rising gradient of 1.25%. The R.L. of the intersection point of the gradients is 203.350 m above datum and its chainage is 2750 m.

11. A parabolic vertical curve of length 300 m is formed at a summit between grades of 0.7% up and 0.8% down. The length of the curve is to be increased to 900 m retaining as much as possible for the original curve and adjusting the gradients on both sides to be equal. Determine this gradient.

12. A vertical parabolic curve 500 m long is introduced between two gradients of 1 in 100 and 1 in 150. If the reduced level of the tangent point of steeper gradient is 830.120 m, calculate the reduced levels of points at 100 m intervals along the curve including that of second tangent point. Find, also the chainage and reduced level of the highest point of the curve.

ANSWERS

4. (*i*) 900 m (*ii*) 700 m (*iii*) 400 m

5. 1080 m; R.L. of first point, 812.45 m, R.L. of end point 817.75 m and R.L. of highest point 820.45 m

6.

Chainage	R.L.	Chainage	R.L.	Chainage	R.L.
3607.	91.35 m	450 m	90.908 m	540	92.25 m

11. 1 precent.

12. 83.120, 83.96. 84.4b, 84.62, 84.45. 83.953 m, 300 m, 84.62 m.

19

Setting out Works

19.1. INTRODUCTION

Setting out of works may be defined as marking the out-lines of excavation on the ground for the guidance of the contractor and the labour. On completion of the estimates from the approved plan, excavation for the foundations is required to be made on the ground. To minimise the cost of digging foundation trenches, it is very necessary to define the out-lines of excavation stakes accurately. It may be appreciated that setting out the exact position of each corner of the structures is useless because these would be disturbed during excavation of the foundations.

Setting out of different works may be made by different methods. Setting out of few important structures, is only discussed in this chapter.

19.2. SETTING OUT THE BUILDINGS

Let *ABCDEFGHA* defines the external lines of the excavation whereas $A_1\,B_1\,C_1\,D_1\,E_1\,, F_1\,G_1\,H_1\,A_1$ represents the internal lines of the building. For setting out this building on the ground, it is required to locate accurately all the exterior corners *i.e., A, B, C, D,* etc. and also all the interior corners A_1, B_1, C_1, D_1, etc. on the ground so that necessary excavation for the foundation may be made in between these lines up to the required depth. There are two methods of setting out of buildings.

1. Setting out by circumscribing rectangles.
2. Setting out by rectangles formed by the centre lines.

First Method. In this method reference lines *ab, bc, cd, de, ef, fg, gh,* and *ha* are set out at a known distance (say 2.5 m) and parallel to the lines *AB, BC, CD, DE,* etc. The actual procedure for setting out is explained here under.

Illustration. Set out the building shown in (Fig. 19.1), assuming the following dimensions.

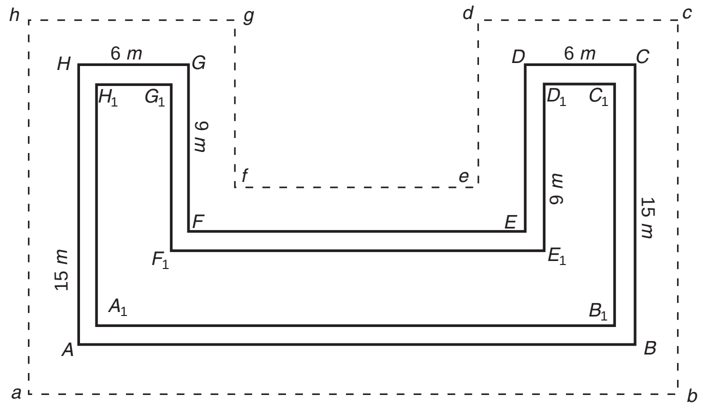


Fig. 19.1. Setting out of buildings.

Side $AB = 30$ m

Side $CD = GH = 6$ m

Side $BC = HA = 15$ m

Side $DE = FG = 9$ m

Width of foundation trench = 1 m

Distance of reference rectangles out side the limits of excavation = 2.5 m

Sides of the reference rectangles may be calculated as under :

Side $ab = 2.5 + 30.0 + 2.5 = 35$ m

Side $bc = 2.5 + 15.0 + 2.5 = 20$ m

Side $cd = 2.5 + 6.0 + 2.5 = 11$ m

Side $de = 2.5 + 9.0 - 2.5 = 9$ m

Side $ef = 2.5 + 30.0 + 2.5 - 2\,(2.5 + 6.0 + 2.5) = 13$ m

Side $fg = 2.5 + 9.0 - 2.5 = 9$ m

Side $gh = 2.5 + 6.0 - 2.5 = 11$ m

Side $ha = 2.5 + 15.0 + 2.5 = 20$ m

Procedure : Following steps may be followed :

1. Fix two stakes a and b accurately at the required distance *i.e.*, 35 m.
2. Stretch a cord and secure its ends to the wire nails driven in the centres of stakes.
3. At a set out a line perpendicular to the line ab. For small buildings, setting out of the perpendiculars may be made with a tape by 3-4-5 method. For large buildings, perpendiculars must invariably by set out with a transit.

4. On the perpendicular line, fix a stake at a distance equal to the distance between *a* and *h i.e.* 20 m.
5. Measure the diagonal *bh* with a tape accurately and compare it with its calculated length *i.e.*

$$ah = \sqrt{35^2 + 20^2} = 40.31 \text{ m.}$$

The error if any, should be corrected.

6. Similarly fix the stake *c* on a line perpendicular to *ab* at *b*, at a distance equal to *bc i.e.* 20 m.
7. Measure the diagonal *ac* which should equate 40.31 m as in step 5.
8. Measure the distance between stakes *c* and *h* which should be equal to 35 m if work is accurate.
9. Measure a distance equal to 11 m from stakes *c* and *h* along *ch* and fix stakes *d* and *g* respectively.
10. Set out perpendicular lines at *d* and *g* to the cord *ch*.
11. Measure the distances *de* and *gf* equal to 9 m along perpendicular lines and fix stakes *e* and *f*.
12. Measure the distance between stakes *e* and *f* which should be equal to 13 m.
13. Measure the diagonals *af* and *be* which should be equal to $\sqrt{11^3 + 11^2} = 15.56$ m.
14. After checking the sides of the reference rectangles, stretch a cord round the periphery of the circumscribing rectangles.
15. With reference to the corners of the reference rectangles, fix stakes at the outer and inner corners of the building.
16. Stretch a cord round the outer corners and also another cord round the inner corners of the buildings.
17. Spread lime among the outside of the cords.
18. Excavate the foundation in between the lines.

Second Method. (Fig. 19.2).

In this method, the rectangle *ABCD* formed by the centre lines of the outside walls of the building is set out accurately with a tape. The corners of the building are then located by measuring their distances with reference to this rectangle. As the excavation proceeds, stakes, *A, B, C* and *D* will get disturbed and hence they will no longer be available for reference. To overcome this difficulty reference stakes 1 to 8 on the prolongation of the sides of this rectangle are established outside the limits of excavation say 2 to 3 m from the building line. These reference stakes are protected by surrounding them with drain pipes. In uneven ground, the required reference points, may be transferred on the ground by using a plumb bob.

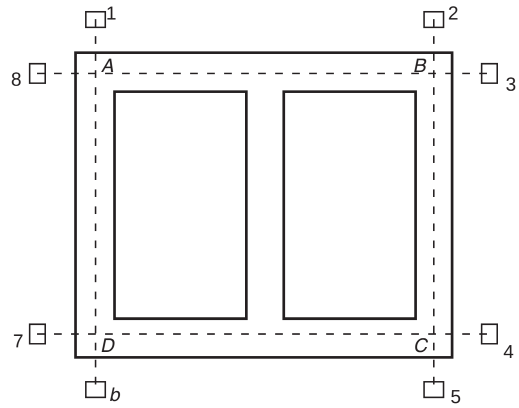


Fig. 19.2. Setting out a building.

19.3. SETTING OUT OF CULVERTS (FIG. 19.3)

Setting out a culvert involves locating the corners of the abutments and wing walls with reference to the centre lines of the road or railway, and the drainage crossed over. These centre lines are used as axis of co-ordinates and their point of intersection as the origin of the co-ordinates. The site engineer responsible for the execution of the project, is provided with a tracing of the plan of the culvert foundation. On this plan, co-ordinates of the corners of the abutments and wing walls, are given in a tabular form.

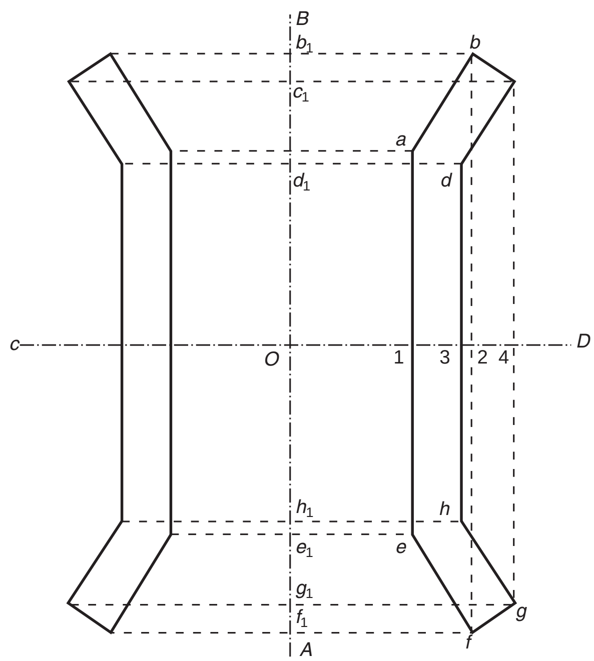


Fig. 19.3. Setting out culverts.

Let *AB* represents the centre line of the road.

CD represents the centre of the drainage.

O represents the centre of the culvert.

Then, co-ordinates of the corners of the wing walls, may be tabulated as under :

Corner	*Easting*	*Northing*
a	O1	1 *a*
b	O2	2 *b*
c	O4	4 *c*
d	O3	3 *d*
e	O1	1 *e*
f	O2	2 *f*
g	O4	4 *g*
h	O3	3 *h*

Procedure : Following steps are involved.

1. Fix a peg at *O*.
2. Set up a theodolite over *O*.
3. Bisect the point *B* with a transit on the central alignment and fix a number of points along *OB*.
4. Transit the telescope and fix a number of point along *OA*.
5. Fix chain pins or arrows or perforated pegs at these points located on *AOB*.
6. Stretch a chord through the eyes of the arrows or through the holes of the pegs, which defines the line *AB*.
7. Set out a line *CD* at right angles to *AB* and fix a number of points necessary to define the line *CD*.
8. Stretch a cord through the eyes of the arrows fixed at the points on the line *CD*.
9. Set off the distances, *O1, O2, O3, O4,* etc. along *CD*. On side of *O* and 1*a*, 2*b*, 3*c*, 4*d*, etc. along *AB* on either side of *O*. Fix arrows at these points.
10. Take two tapes and put their rings together.
11. Direct the chairman to pull the tapes together tight while one of the tapes is held by the engineer and other by his assistance at arrows 1 on *CD* and a_1 on *AB* with their respective readings $O1$, Oa_1. This fixes the position of the corner *a* which is then marked with a peg.
12. In a similar manner, other corners of the wing walls and abutments, may be marked by their co-ordinates and pegs are driven.
13. Stretch a cord round the periphery of each abutment and two wing walls as *a, b, c, d, h, g, f, e* and mark the out line of the foundations by cutting a narrow trench along the line.

14. Determine the reduced levels of the pegs for the purpose of determining depths of excavations and estimate the correct quantity of earth work.

Note. The following points may be noted.

(*i*) If the wing wall is curved, then the points on the curve may be set out by perpendicular offsets to the chord. The offsets and the distances along the chord may be scaled off from the plan.

(*ii*) Similar procedure may be followed for setting out the bridge foundations.

(*iii*) In case of skew bridges, the procedure is exactly similar except that the line *AOB* and *COD* are first set out at the desired angle and the procedure repeated.

19.4. SETTING OUT OF A BRIDGE

While setting out the bridges, following two problems are encountered.

1. To determine the accurate distance between a point on one bank of a river and a point on the other bank, both the points being predetermined on the centre line of the road or railway.
2. To locate the central points for piers of the bridge.

1. Determination of the distance between end points

The length of the centre line of a short span bridge may be measured directly with a standardised tape duly compared with a standard tape. Necessary corrections are made to the measured length to obtain absolute distance. The procedure is similar to measuring the length of a base line for tertiary triangulation. For detailed procedure, refer to Chapter 2 'Triangulation' of Author's text book Advanced Surveying, 2nd Eqn. 1986.

In case of long bridges; the lengths are usually determined by triangulation.

Procedure. (Fig. 19.4).

Following steps are involved :

1. Let A and B be the end points on the centre line on either bank of the river.
2. Set AD perpendicular to the centre line AB with a transit.
3. Measure AD carefully.
4. Measure angle ADB accurately by the method of repetition, making observations on both faces. Let its value be θ_1.

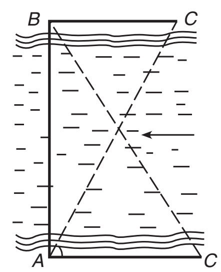


Fig. 19.4.

5. Calculate $AB = AD \tan \theta_1$.
6. Similarly measure a base BC along the other bank.

7. Measure the angle *BCA* by the method of repetition on both faces. Let its value be θ_2.
8. Calculate $AB = BC \tan \theta_2$.
9. The mean of the two values of *AB* if well within limits, may be accepted as the true value of *AB*.

2. Locating the pier positions

There are two methods, as discussed here.

First Method. (Fig. 19.5).

The following steps are involved.

1. Lay one base line on each bank exactly at right angles to the centre line *AB*, extending on either side of the centre line.
2. Measure the accurate length of centre line *AB* by triangular as explained in first method.

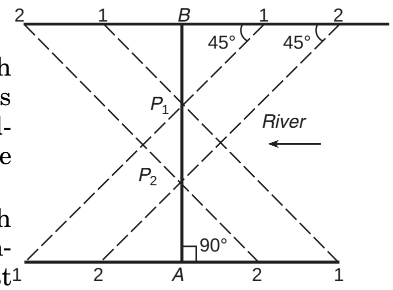


Fig. 19.5.

3. Locate the positions of the piers on the centre line as under :

(*i*) Calculate the distances BP_1, P_1P_2 , P_2A between the abutments and piers.

(*ii*) Measure distances *B*1, *B*2 equal to BP_1 and BP_2 respectively on the base line perpendicular at *B* on either side.

(*iii*) Similarly, measure distances *A*2, *A*1 equal to AP_2 and AP_1 on the base line perpendicular at *A* on either side. The intersecting lines 1—1 and 2––2 make angles of 45° with the base lines on opposited banks and also with the centre line *AB*.

(*iv*) The position of pier 1 may be located by simultaneously sighting at the intersection of the two intersecting lines 1—1.

(*v*) Similarly, the position of pier 2 be may be located by simultaneously sighting at the intersection of the two intersecting lines 2-2.

Second Method.

Following steps are involved :

1. Set out two base lines approximately perpendicular to the centre line *AB*. (Fig. 19.6).
2. Measure the base line *AC* and *BD* accurately. The lengths of each base line should preferably be equal to the centre-line *AB* but in no case less than three fourth of this.
3. Measure the angles of the triangles *ABC* and *ABD* by method of repetition and the triangular error should not exceed 5″ and for important work 2″.

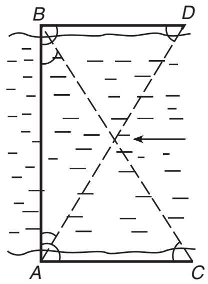


Fig. 19.6.

4. Compute the length of AB by solving $\Delta\, ABC$ and also by solving $\Delta\, ABD$.
5. If the two values do not differ much, take their mean is the most probable length of the centre line AB.

For locating the position of piers, the following steps are involved.

1. After obtaining the length of centre line AB, calculate the interdistances Ap_1, $p_1\, p_2$ and $p_2\, B$ where p_1 and p_2 are the centre positions of the piers.
2. Apply sine formula to $\Delta_S\ ACp_1$, ACp_2, BDp_2, p_1 and BDp_2 to get the values of angles ACp_1, ACp_2, BDp_1 and BDp_2.
3. Set up transits at A and C. Instrument at C is directed to A and an angle ACp_1 is set so that telescope sights towards p_1.
4. By simultaneous observations, locate the position of p_1.

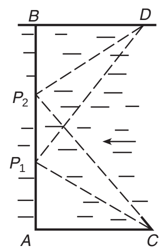


Fig. 19.7.

5. Set up a theodolite at D and set up angle BDp_1. If the location of p_1 is correct, the line of sight Dp_1 shall pass though p_1.
6. Locate the pier p_2 in a similar way.
7. Establish reference points on the banks in line of Cp_1, Cp_2 and Dp_2, Dp_1 for reference during construction.

19.5. SETTING OUT SEWER GRADES

For efficient working of sewers, the inverts are laid with proper grades. The reduced level of the bottom of inner surface of the sewer pipe, is called *invert*. The procedure for the laying the sewers is as explained under. (Fig. 19.8).

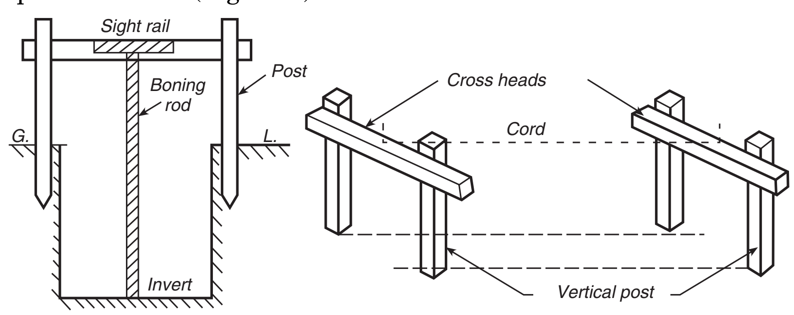


Fig. 19.8. Setting out sewer grades.

1. Fix stakes on the ground at the centre line of the proposed sewer, at 20 m intervals.

2. Set out a parallel line to the proposed centre line of the sewer on one side at a distance apart so that it may not get disturbed during excavation.
3. Fix stakes flushed with the ground surface, on the parallel line at 20 m intervals. On hard surfaces, nails, spikes, etc. may be driven flushed with surface of ground. In case of road pavements, chisel marks may be suitably marked.
4. Excavate the sewer trench of desired width and depth.
5. Erect cross heads at 30 metre apart and at each change of gradient and direction.

A cross head consists of two vertical posts 1 m to 1.5 m high, firmly embedded into the ground, one on each side of the trench and a horizontal sight rail nailed to the vertical posts across the trench.

A sight rail (or batter board) which is a horizontal timber beam (15 cm side and 5 cm thick), is set across the trench and properly nailed to the vertical posts of cross heads. The sight rails are white washed.

6. Set the top edge of each sight rail truly horizontal with the help of a carpenter's level.
7. Set the top of each sight rail accurately by a level at a fixed whole number of metres *i.e.* 2, 3, 4 above the invert of the sewer. Prepare a boning rod of the same length.

A *boning rod* is a T-shaped wooden frame. Its top piece which is generally 10 cm × 3 cm in section and 40 cm in lengths, is nailed to the vertical leg. The length of the boning rod is kept same for each sewer section. The top of the boning rod is pained black.

8. Drive in a nail on the top edge at the centre of the sight rail to define the centre line of the sewer.
9. Establish the gradient of the line joining top edge of two consecutive sight rails as that of the invert of the sewer.
10. Stretch a fine cord between the nails.
11. Excavate the exact depth at intermediate sections by sighting the boning rod.

Note. The setting up of the sight rails from the lower-end of the sewer.

Example 19.1. *For setting out a trunk sewer between sections A and B, following data is available :*

Gradient of the sewer : 1 in 200

Depth of the invert at lower end A is 2.650 m below the peg A.

Distance between Sections A and B = 75 m.

Staff reading on peg A = 1.855 m.

Staff reading on peg B = 2.350 m.

Height of collimation of the level set up near by = 250.000 m

The length of the boning rod = 4.0 m.

Make necessary calculations for fixing the sight rails at A and B.

Solution. (Fig. 19.9).

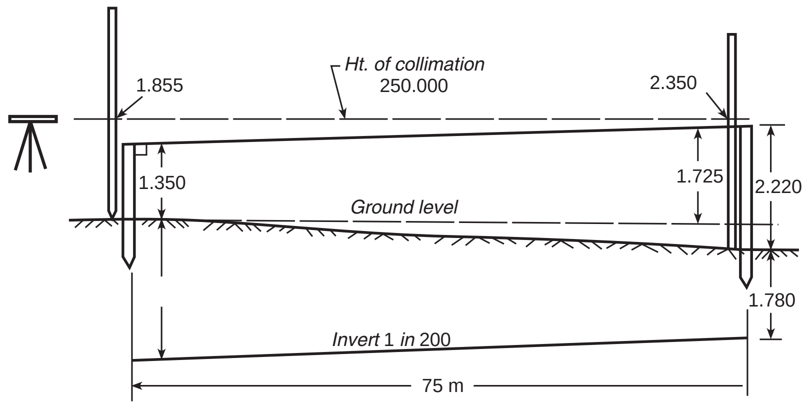


Fig. 19.9.

Let A' and B' be the locations of the inverts at section A and B respectively.

Fall of the invert $= \dfrac{75}{200} = 0.375$ m.

The top edge of the sight rail at A above invert level at A'

= 4 m. (being length of boning rod)

The top edge of the sight rail at A above peg A.

= 4.000 − 2.650 = 1.350 m.

The edge of sight rail a B above peg A

= top edge of sight rail at A above peg A + fall of invert.

= 1.350 + 0.375 = 1.725 m.

Difference in level of pegs A and B

= Staff reading on peg B–staff reading on peg A.

= 2.350 − 1.855 = 0.495 m, A being higher.

Top edge of sight rail above peg B.

= 1.725 + 0.495 = 2.220 m.

Depth of invert below peg B

= 4.000 − 2.220 = 1.780 m.

Height of collimation = 250.000 m (given)

Staff reading on peg A = 1.855

∴ R.L. of the peg A = 248.145 m.

Staff reading on peg B = 2.350 m

∴ R.L. of the peg B = 247.650 m

R.L. of the top of sight rail at A

= R.L. of peg A +ht. of sight rail abovepeg A

= 248.145 + 1.350 = 249.495 m.

R.L. of the top of sight rail at B

= R.L. of peg B + ht. of sight rail above peg B.

= 247.650 + 2.220 = 249.870 m.

Calculation of staff reading for fixing the tops of sight rails.

at B = 250.000 − 249.495 = 0.505 m

at B = 250.000 − 249.870 = 0.130 m.

For fixing the sight rails to the vertical posts, proceed as under :

1. Move the staff up and down along the vertical post at A until a reading of 0.505 m is obtained.
2. Mark a line on the vertical post at A at the foot of the levelling staff.
3. Similarly, mark a line on the other vertical post at the same level.
4. Nail the sight rail to the vertical posts, ensuring that its top edge is in exact coincidence with the lines marked on the vertical posts.
5. Fix another sight rail at B exactly in the similar way.

Example 19.2. *A contractor is asked to set out a trunk sewer grades and is supplied with the trace of working drawing shown in Fig. 19.10. Calculate the heights of the sight rails above surface pegs A, B and C. Also, suggest a suitable length of the boning rod.*

Solution.

(*i*) R.L. of the sight rail at D

= R.L. of surface peg at D+ ht. of sight rail at D.

= 168.75 + 1.75 = 170.50 m.

Invert at D = 166.50 m. (given)

∴ Height of boning rod = 170.50 − 166.50 = 4.0 m. **Ans.**

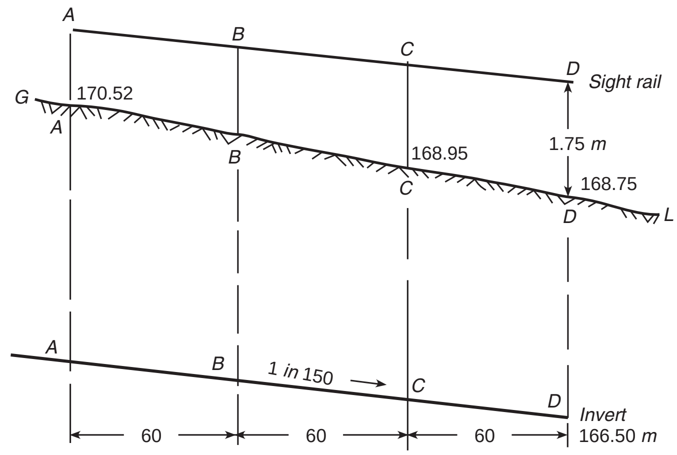


Fig. 19.10.

(*ii*) Gradient from D to A = 1 in 150

Invert level at $C = 166.50 + \dfrac{60}{150} = 166.90$ m.

$B = 166.50 + \dfrac{120}{150} = 167.30$ m.

$A = 166.50 + \dfrac{180}{150} = 167.70$ m.

(*iii*) R.L. of sight rail at

C = Invert level at C + length of boning rod

$= 166.90 + 4.00 = 170.90$ m.

R.L. of sight rail at $B = 167.30 + 4.00 = 171.30$ m

R.L. of sight rail at $A = 167.70 + 4.00 = 171.70$ m.

(*iv*) Height of sight rail above surface peg at C

= R.L. of sigh rail – R.L. of surface peg C

$= 170.90 - 168.95 = 1.95$ m.

Height of sight rail above peg B

$= 171.30 - 169.45 = 1.85$ m.

Height of sight rail above peg A

$= 171.70 - 170.52$

$= 1.18$ m. **Ans.**

19.6 SETTING OUT GRADE STAKES.

A stout or a post-sharpened at one end enabling it to drive into the ground to mark the alignment of a highway or a railway project, is called a **stake**. The stakes which are driven into the ground, keeping the level of their tops at the required grade, are known as grade stakes. A grade is defined as the rate of rise/fall of a slope. The stakes are driven into the ground such that their top level represent the required gradient. When it is not found possible to drive in the stakes so that their tops are at the desired elevation, these are suitabley driven to a convenient height above the ground level surface such taht the required depth of the cut or to fill is a whole number of metres (or decimetres) below or above the top of the stakes. The required depth is usually indicated on the side of the stake. Cuts are indicated by a plus (+) sign and fills by a negative (–) sign. The grade stakes fixed along the proposed alignment of the project indicate either a rising grade, or a falling grade or a horizontal line.

The reduced levels of the grade stakes fixed at desired locations along the proposed alignment are obtained with a dumpy level and levelling staff, as briefly, described under:

(*i*) Take a back sight (BS) on a benchmark already established nearby.

(*ii*) Obtain the height of instrument (height of collination) by adding the back staff reading to the reduced level of the bench mark (B.M.). *i.e.* if the back staff reading is 1.35 and the reduced level of the bench mark is 98.65m, then, the height of instrument (or collimation)

$$= 98.65 + 1.35 = 100.00 \text{ m}$$

Now, the grade stakes are driven into the ground one by one sequentially so that their tops levels represent the formation level of the route alignment. The difference between the height of instrument and the reduced level of the top of each stake, is computed before hand and calibrated.

The stake is driven into the ground till the calculated staff reading is obtained on the levelling staff held on the top of the stake.

Illustrative example.

Let us consider the following readings for setting up the stake grade:

Case 1. The height of the instrument = 100 m.

The height of the formation level = 97.25 m

The required staff reading

$$= 100.00 - 97.25 \text{ m} = 2.75 \text{ m}$$

Case 2. The height of instrument = 100.00 m

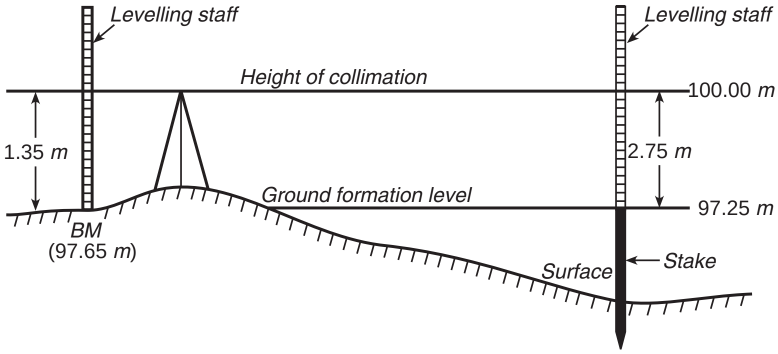


Fig. 19.11.

The formation level = 95.50 m

The required staff reading = 100.00 – 95.50 = 4.5 m

This reading is not possible as the length of levelling staff is 4.0 m.

Add 3.00 m to the formation level i.e. 95.50 + 3.00 = 98.50

The adjusted staff reading = 100.00 – 98.50 = 1.50 m

After fixing the grade stake, to the height of 98.50 mm, we have to indicate + 3 m on the side of the stake.

Case 3. The height of the instrument = 100.00 m

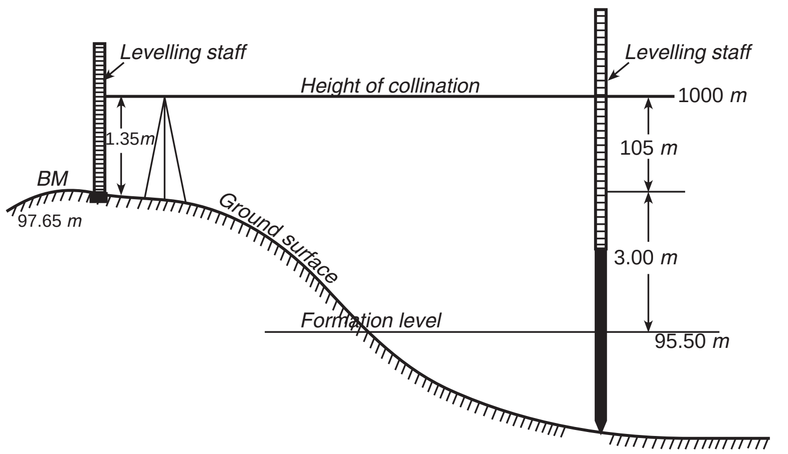


Fig. 19.12.

The formation level = 102.50 m

The required staff reading = 100 – 102.50 m = – 2.5 m

The adjusted staff reading = 3.00 – 2.50 = 0.50 m

In the case we have to indicate – 3.00 m outside the stake.

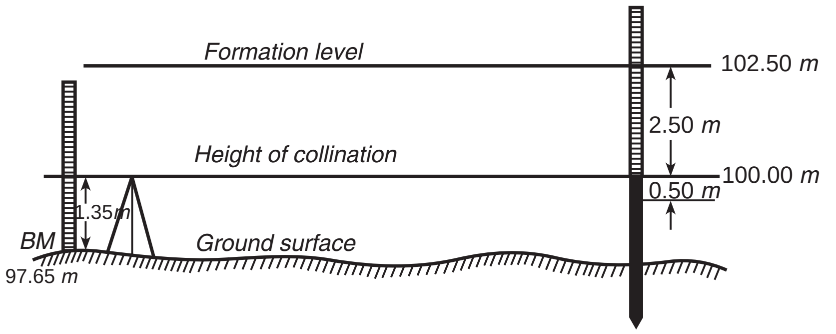


Fig. 19.13.

EXERCISE 19

1. What are the methods of setting out buildings ? Describe each of them in details.

2. Describe the method of setting out of culverts.

3. How will you measure the length of centre line of a long bridge.

4. How will you locate the piers on the centre line of a long bridge.

5. Describe the procedure of setting out a two storied building with 200 mm thick load bearing walls, all round. The foundation width is 750 mm. The building is absolutely rectangular and outside dimensions above plinth level, are 12000 mm × 18000 mm.

Draw the foundation of the outer walls showing all the dimensions (not to scale).

In the above plan, show the positions of different pegs required for set out of the above building.

6. To determine the angle accurately between two adjacent portions AB and AC of a compound wall of a large factory, a theodolite was set up at D 10 m from A such that points A, B, C are in sight. Following readings were taken.

Angle $BDA = 43° 26' 30''$, angle $CDA = 40° 52' 30''$.

Distance AB = 162.51 m, Length AC = 115.14 m.

Assuming that walls AB and AC are dead straight and corners A, B and C are clearly visible. Calculate the angle BAC.

ANSWERS

6. 90°

20

Map Projections

20.1. INTRODUCTION

A map is a graphical representation of the locations of individual features on the surface of the earth. Since the surface of the earth is spherical and the surface of the map is a plane, it is difficult to represent a given area without some distortion. If the area is small, the earth's surface may be regarded as plane and a map may be constructed without any appreciable distortion. The maps by plane surveying method are constructed assuming the earth's surface as plane and by using either rectangular co-ordinates or by horizontal angles and distances. As the size of the area increases, the various types of projections are employed to minimise the effect of map distortion. On such a map, spherical co-ordinates of control points are used. Since the spherical co-ordinates of a point are its latitude and longitude, it is necessary to show meridians and parallels on the finished map. The maps of states and Indian continent as well as those of other countries are constructed in this manner.

An ideal map free from distortion must fulfill the following conditions :

(*i*) All distances and areas must have correct relative magnitude.

(*ii*) All azimuths and angles must be correctly shown.

(*iii*) All the great circles must appear as straight lines.

(*iv*) Geographical latitudes and longitudes of all points must be correctly shown.

20.2. SPHERICAL CO-ORDINATES

To understand various types of projections and their construction, it is essential to study the spherical co-ordinates, *i.e.,* latitude and longitude.

1. **Latitude.** Rotation of earth gives an idea about its imaginary axis of rotation. The upper end of this axis is called *North pole* and the

lower end is called *South pole*. An imaginary circle around and the globe, equidistant from either pole, is called *equator*. The imaginary lines drawn round the earth parallel to the equator, are called *latitudes.* The angular distance of a point on the earth's surface north or south of the equator, and measured from the centre of the earth, is called *latitude* of the point. The imaginary line which joins the points of same latitude on the earth's surface, is called a *parallel* of the latitude. In fact, the parallels are not lines but circles which are imagined to be drawn parallel to the equator in east west direction. The latitudes are also imaginary circles with the two poles as centres. The radii of the latitudes decrease as the distance of the point increases from the equator. At north and south poles, the radius becomes zero.

The parallels of latitude are marked off in ninety degrees from the equator to each of the poles. The equator which is the *prime latitude,* represents 0° latitude. The latitude of the north pole is 90° N and that of the south pole is 90° S latitude. Each degree is subdivided into 60 minutes and each minute into 60 seconds. Degrees of latitude are approximately the same linear distance *i.e.*, 111 kilometres (69 miles). However, at the poles, the parallels are slightly longer.

2. **Longitude.** The angular distance measured along the equator, between the standard or prime meridian and the meridian through a given point, is called the *longitude.* The meridian through the Royal Astronomical observatory at Greenwich near London, has been internationally adopted as the *prime meridian* Longitudes are measured in degrees west or east of Greenwich from 0° to 180° . The Greenwich meridian represents 0° and the meridian 180°W coincides with the meridian 180° E. The complete angular distance round the equator is

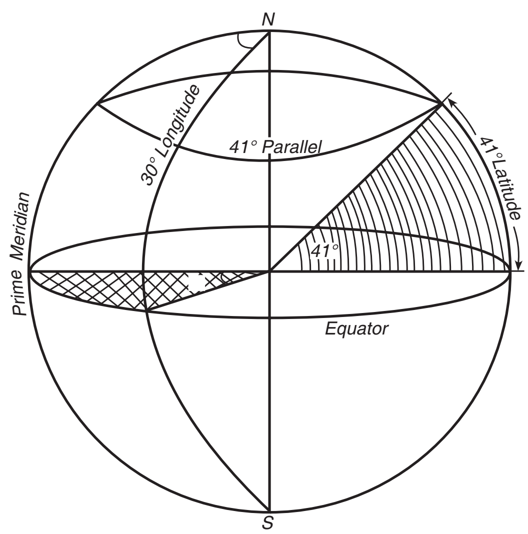


Fig. 20.1. Latitude and longitude.

360°. Hence, 360° meridians may be drawn from each pole and they will lie 1° of longitude apart. The meridians converge to north and south poles. The longest distance of 1° longitude is at the equator where it approximates to the distance of one degree of latitude. Longitude is represented either in time, or in arc. The longitude may also be expressed as the difference of local times of the place and the Greenwich.

3. **Projection.** The system of representing the earth's parallels and meridians with respect to which control points in spherical terms, are plotted on the map, is called a *map projection.*

It is impossible, however, to construct a map of a considerable portion of the earth's surface on a flat surface without some distortion of shapes, relative areas and directions. A projection involves the construction on the plane surface of a graticule formed by two intersecting systems of lines, corresponding to the parallels of the latitude and the meridians of longitude on the earth surface. Some projections aim at showing directions correctly, the other areas or shapes and for general purposes the most useful projections are those which take all these factors into consideration.

20.3. CLASSIFICATION OF PROJECTIONS

Depending upon the overlapping properties, the various types of projections may be classified as under :

(*i*) **Equal area projection.** The map projection which maintains a constant ratio of areas, is called *equal area projection.* On such a map one square centimeter taken at any location on the map represents a certain constant portion of the earth's surface. In other words, the ratio between any area on the map and the corresponding area on the globe is constant. It does not mean necessarily that the shape of the area contained in the square centimetre is as it really exists on the earth's surface. This is due to the fact that the scale in a north-south direction is different than that in on east-west direction. Equal area projection is useful for political maps.

(*ii*) **Conformal or orthomorphic projection.** The type of map projection in which the shape of any very small area on the map is the same as the shape of the corresponding small area on the earth, is called *conformal* or *orthomorphic projection.* On such a map an angle between any pair or short intersecting lines appears in correct and small areas appear in correct shape. This means that at any point, the scale along the parallel and the meridian is the same. Due to variance of scale from point to point, the shapes of larger areas are incorrect.

(*iii*) **Azimuthal or zenithal projection.** The map projection in which a portion of the globe is projected upon a plane tangent to it, is called *azimuthal* or *zenithal projection.* The tangent plane on which the projection is made is not necessarily tangent at the pole. As the name azimuthal indicates, all points have their true compass directions from the centre of the map. For an example, if New Delhi were the central point, the map should give true directions to all the other cities. Directions on the map will however not be accurate at points other than the central point of the map.

20.4. COMMON TYPES OF MAP PROJECTIONS

There are many types of map projections employed world over. The important ones are discussed here under :

1. Polyconic projection. As the surface of a cone can be developed into a plane without distortion, a number of conical projections have been devised using sometimes a cone tangent along a parallel of latitude and some time a cone cutting the earth sphere along two parallels. In case of polyconic projection, a series of conical surfaces are used and the points on the surface of each are considered as projected to a series of frustums of cone which are fitted together. The conical surfaces are developed on either side of the central meridian. As the radii of the succeeding cones go on decreasing, the resulting strips do not exactly fit together when these are laid flat. The spaces between the strips increase in width as the distance from the central meridian, increases as in Fig. 20.2.

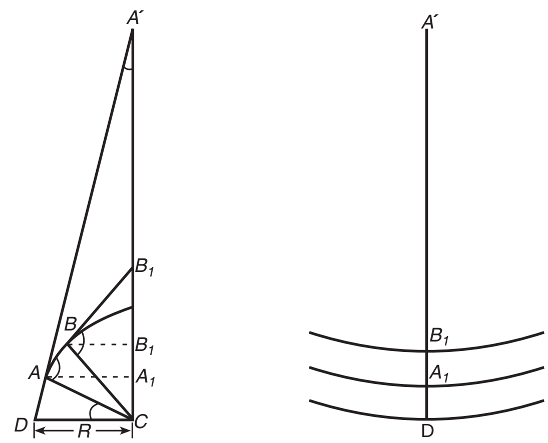


Fig. 20.2. Polyconic projection.

Let C be the earth's centre [Fig. 20.2(*i*)]

A and B be two points on earth's surface

R be the radius of the earth

λ be the latitude of the point A.

Construction : Proceed as under:

Draw AA' and BB' tangents at A and B respectively.

Draw AA_1 and BB_1 parallel to the equator DC.

Apparently $A'AC$ and $B'BC$ are right angled triangles at A and B respectively.

From $\Delta A'AC$, we get

$$AA' = R \cot \lambda \qquad ...(20.1)$$

Now, distances DA_1, A_1B_1, etc. [Fig. 20.2 (*ii*)] along the central meridian DA' are true scale representation of corresponding distances DA, AB, etc. in the meridian section.

In practice, the parallels drawn on the map are calculated with a small difference in latitude, each one is drawn with its own radius.

Characteristics of the Polyconic projection. The following characteristics of polyconic projection may be understood carefully.

(*i*) The meridians are curved, concave toward the central meridian.
(*ii*) If the parallels are drawn close enough together, each meridian may consist of straight lines from parallel to parallel.
(*iii*) On the central meridian, there is little error in the map.
(*iv*) The error in map scale increases in proportion to the square of the difference in longitude along any parallel.
(*v*) The variance in scale error with difference in latitude is not in direct proportion.
(*vi*) The map is true to scale along the central meridian and also along every parallel. The scale changes along other meridians.
(*vii*) The parallels and meridians intersect at right angles near the central meridian.
(*viii*) Polyconic projection is sufficiently accurate for maps of considerable areas.

2. **Lambert conformal conic projection.** This is one of the most important projection based on the projection of earth's surface on to a cone. In this case a cone is imagined to cut the surface of the earth along two parallels of latitudes, called *standard parallels*. There is a decrease in the scale between the standard parallels and an increase in the scale outside the standard parallels. The developed cone cut along the central meridian and laid flat, shows the meridians as straight lines which appear to meet at a point outside the map and parallels appear as concentric arcs. On this projection east-west strip of the earth's surface for a short distance on either side of the standard parallel, is shown correct to scale. The distortion of the map increases north or south of the standard parallel. If the distance between the standard parallels is decreased, the effective belt is narrowed and the distortion of the scale becomes less. The upper and lower limits projecting away from standard parallels is normally kept equal to one-fourth the distance between the standard parallels.

Characteristics of Lambert projection. From the conditions stated above, the characteristics of this projection are as under :

(*i*) The longitude scale is exact along the standard parallels.

(*ii*) The projection is practically conformal, because the conical surface is nearly coincident with the earth's surface. On a conformal projection, the latitude and longitude scales are so nearly exact, that angles between lines on the map are nearly the same as the angles between the corresponding lines on the earth's surface.

(*iii*) The scale of the zone between standard parallels is too small. Similarly, for the zones north and south of standard parallels, the scale is too large.

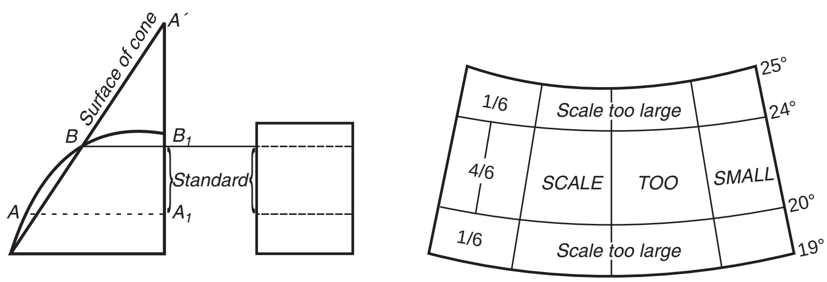


Fig. 20.3. Lambert conformal conic projection.

(*iv*) The projection may be extended indefinitely in an east-west direction, without affecting the accuracy of the map. The extension of the projection in a north-south direction, affects the scales in a rapidly increasing ratio.

3. **Mercator projection.** This map projection was invented by Richard Marcator in 1569. In this projection, the points on the globe surface are projected on a cylinder touching the globe along the equator. The curved surface of the cylinder may be developed into a plane by cutting along any one of the equi-spaced meridians. All the meridians appear as straight parallel lines and the parallels of latitude have the same length as the equator. There is an east-west stretching on this projection everywhere except at the equator and this stretching increases with distance from the equator. On the globe, the parallels of latitudes at 60° is one-half the length of the equator, so at that latitude stretching is two-fold. To compensate for this, a two-fold stretching of the meridians is also made. The ratio of the areas of 1°} square at 60° latitude and at equator is four. As the stretching both east-west and north-south becomes so great that Mercator's projection are seldom taken beyond 80° latitude, because at this latitude stretching becomes about 6 times in each direction.

The change of scale along the parallel, varying with the latitude, may be easily computed by means of the formula

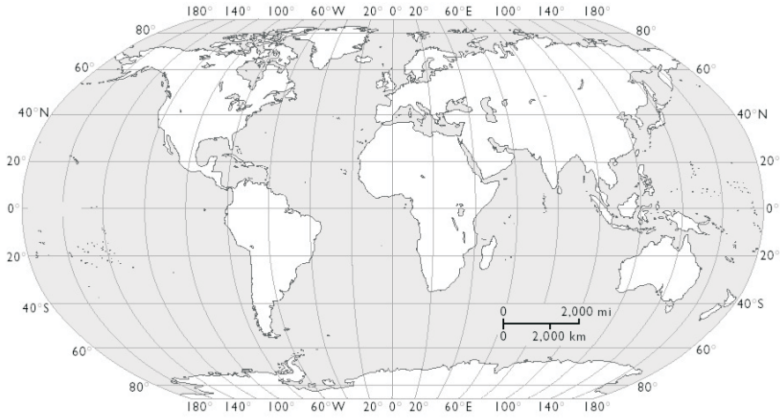


Fig. 20.4. Mercator projection.

$$S' = S \cos \lambda$$

where S is the scale at the equator and S' is the scale at any latitude λ. Though the scale varies from point to point on this projection, at any given point the scale is the same in all directions. The map is therefore, conformal.

Due to excessive distortion of distances, areas, shapes of land surfaces and oceans on this projection, it is not suitable for general use. However, it is commonly used when the entire surface of the globe needs be shown. On this map projection, a line making a constant angle with all meridians, called a *Rhumb line* or *loxodrome* appears as straight line. That's why it is of great value for preparation of nautical charts for navigational purposes.

4. International Projection. In 1895, International Geographical Congress recommended the Polyconic projection for the preparation of international maps* on scale 1 : 1000,000. Later in 1909, International Maps Committee recommended the use of the polyconic projection with

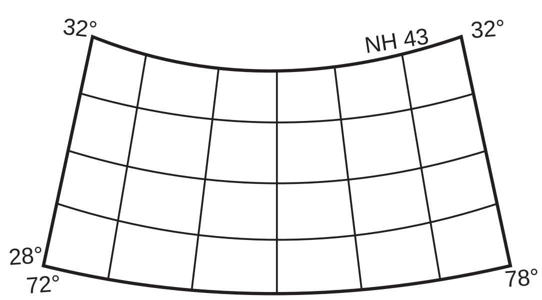


Fig. 20.5. Sheet No. NH 43.

*International map Carte International DU Monde (international Map of the World) Series.

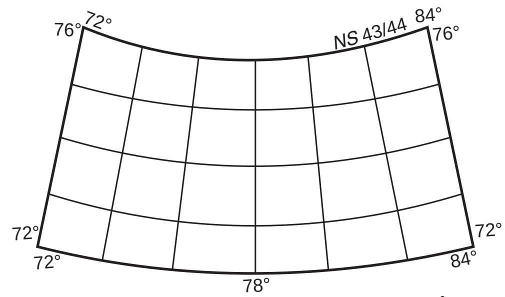


Fig. 20.6. Sheet No. NS 43/44. (lying beyond 60° latitude)

few modifications. The modified projection suggested by G.T. Macaw is called, International Projection.

On this projection, the entire globe has been divided into 2222 sheets. Sheets in the belt of 0° latitude to 60° latitude in northern and southern hemispheres are bounded by 4° latitude and 6° longitude and between 60°. Latitude to 88° Latitude, the sheets are bounded by 4° latitude and 12° longitude. The north and south polar regions are circular sheets of 4° latitudes.

20.5. INTERNATIONAL SERIES SHEET NUMBERING

The meridian of Greenwich near London is universally accepted as prime meridian. 180° *E* or *W* meridian diametrically opposite the prime meridian is called *International Date line.* The entire equator is divided into 6° equal parts. These parts starting from international date line, *i.e.,* 180° E (or, W) longitude westernly are numbered from 1 to 60. Similarly the belts of the area covering 4° latitudes from the equator on either side upto 88° latitudes are serially numbered by English alphabets *A* to *V* and Polar areas are lettered *Z*. The individual sheet is designated as under :

NB 36 ; NH 43 ; SB 25 ; SL 35 ; etc.

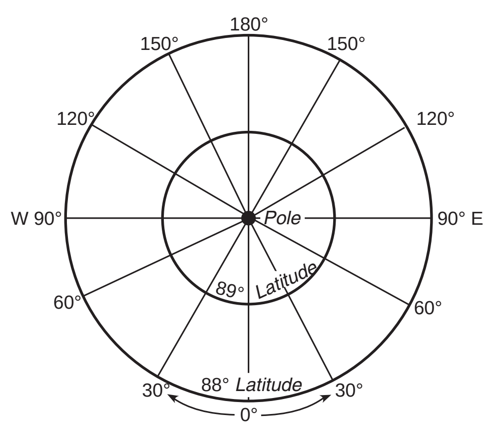


Fig. 20.7. Polar sheet Z.

In the international sheet numbering, the letter *N* is used for the sheets falling in northern hemisphere and *S* is used for those in southern hemisphere. It may be noted that the sheet number, is composed of zone letter and the sector number corresponding to its location preceded by the letter *N* if the sheet is in the northern hemisphere or *S* in the southern hemisphere. A combination of two or more sheets in the same zone is indicated as NP 31/32.

Each sheet is further divided into 24, 1° sheets and numbered from *A* to *X* starting from north-west corner row-wise as shown in Fig. 20.8.

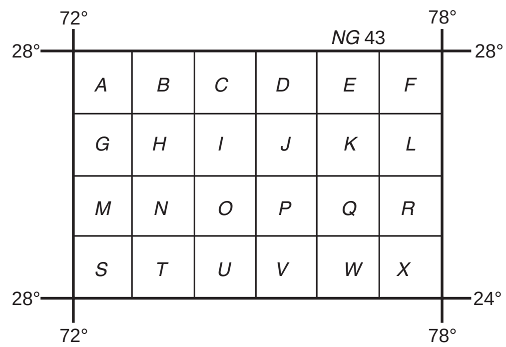


Fig. 20.8. International sheet numbering system.

The degree sheet covering an area between latitudes 26° to 27° and longitude 75° to 76°, shown hatchured in Fig. 20.8, is numbered as NG43J.

Illustration. Find out the international sheet number for the project site whose location is defined by the spherical co-ordinates as under:

Latitude	Longitude
25° 30′ 45″ North	80° 35′ 30″ East

Solution. The project site is falling in the region bounded by 24° to 28° latitude and 78° to 84° longitude.

(*i*) The project site falls in the northern hemisphere. Hence, the first letter is N.

(*ii*) The English alphabet is obtained by dividing the upper boundary latitude by 4°.

i.e., $\frac{28}{4}$ = 7th letter or G.

(*iii*) The region number is obtained by dividing the longitude of eastern boundary by 6.

i.e., $\frac{84°}{6} = 14$

(*iv*) Add 30 for the east of Greenwich

i.e., $30 + 14 = 44$

∴ The international sheet number in which project site is falling, is NG44.

20.6. MODIFICATIONS TO POLYCONIC PROJECTION

The following modifications have been made to the polyconic projection for the International projection.

(*i*) The scale is kept exact at meridians 2° on either side of the central meridian and *not* on the central meridian.

(*ii*) All the meridians are straight lines. These are obtained by joining the corresponding points on the bounding latitudes. In case of the polyconic projection, all the meridians other than the central meridian, are curved.

(*iii*) Values of the bounding latitudes of each zone are obtained from the published mathematical tables. The various parallels are arcs of different circles.

(*iv*) The remaining parallels between the bounding latitudes are obtained by dividing all the meridians and joining the corresponding points. These are also arcs of circles.

Table 20.1. True length between 4° latitudes at mid-meridian in millimetres on scale: 1 : 1000,000.

Latitude	*Exact length*	*Latitude*	*Exact length*
0° 4°	442.00	32° 36°	443.50
4° 8°	442.04	36° 40°	443.81
8° 12°	442.14	40° 44°	444.14
12° 16°	442.28	44° 48°	444.47
16° 20°	442.45	48° 52°	434.81
20° 24°	442.67	52° 56°	445.13
24° 28°	442.91	56° 60°	445.44
28° 32°	443.18		

Construction. For the construction of international projection on scale 1: 1,000,000, the following steps are followed :

(*i*) From the mathematical table 20.1, obtain the correct length between the given latitudes say 24° to 28° (442.91 mm).

(*ii*) Draw a line along *N-S* direction on the drawing sheet and mark *A* and *B* 442.91 mm apart.

(*iii*) Erect perpendiculars at *A* and *B* and extend on either side.

(*iv*) From the mathematical table 20.2, obtain the values of cartesian co-ordinates (*x, y* for the cutting points with reference to latitude on the central meridians, *i.e.* x_1, y_1; x_2, y_2; x_3, y_3.

(*v*) Plot these co-ordinates with reference to the central meridian.

(*vi*) Join the various points to get curved parallels of 24° and 28°.

(*vii*) Join the corresponding points to get the straight meridian lines.

(*viii*) Divide each meridian into equal parts and join the various points to get the arcs of the intermediate parallels.

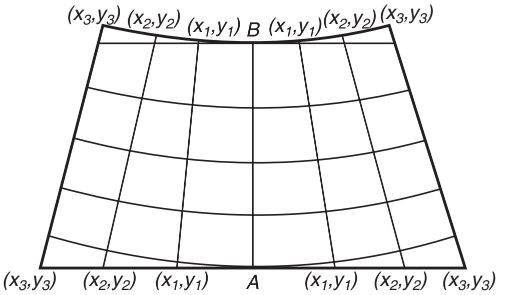


Fig. 20.9. International sheet projection.

20.7. PROPERTIES OF THE INTERNATIONAL PROJECTION

The international projection possesses the following properties :

(*i*) Scale on the central meridian is too small whereas on the bounding meridians, it is too large. The meridians, at 2° from the central meridian are of exact scale.

(*ii*) This is most simple for the construction because the lengths are obtained from available published mathematical tables world over.

(*iii*) The individual sheets may be mosaiced easily and a composite map of larger area may be obtained. A minimum of 9 sheets may be mosaiced without any appreciable error.

(*iv*) The error in shapes and areas is too less and for all practical purposes, it may be termed as a projection of true area.

Table 20.2. Co-ordinates of cutting points in millimetres for the Latitudes and Longitudes.

Latitudes	*Co-ordinates*	*Longitudes from the central meridian*		
		1°	*2°*	*3°*
0	*x*	111.32	222.74	333.96
	y	0.00	0.00	0.00
4°	*x*	111.05	222.10	333.16
	y	0.07	0.27	0.61
8°	*x*	110.24	222.49	330.74
	y	0.13	0.54	1.21
12°	*x*	108.90	217.81	326.71
	y	0.20	0.79	1.78
16°	*x*	107.04	214.07	321.10
	y	0.26	1.03	2.32
20°	*x*	104.65	209.29	313.93
	y	0.31	1.25	2.81
24°	*x*	101.75	203.50	305.36
	y	0.36	1.45	3.25

28°	x	98.36	195.72	295.06
	y	0.40	1.61	3.63
32°	x	94.50	188.98	283.45
	y	0.41	1.95	3.93
36°	x	90.16	180.32	270.46
	y	0.46	1.85	3.16
40°	x	85.40	170.78	256.14
	y	0.48	1.92	4.81
44°	x	80.21	160.40	240.51
	y	0.49	1.95	4.48
48°	x	74.63	149.24	223.83
	y	0.48	1.94	4.36
52°	x	68.68	137.34	205.98
	y	0.47	1.89	4.25
56°	x	62.39	124.77	187.13
	y	0.45	1.81	4.06
60°	x	55.80	111.59	167.35
	y	0.42	1.69	3.80

EXERCISE 20

1. Define a map projection. Enumerate the ideal conditions of an ideal map.

2. Define the following terms :

(*i*) Latitude (*ii*) Longitude

(*iii*) Equator (*iv*) Prime meridian

(*v*) International date line (*vi*) Parallel of latitude

(*vii*) Rhumb line or Loxodrom. (*vii*) Rhumb line

3. Give classification of map projections. Describe each in detail.

4. Give salient features of a polyconic projection. Also, discuss its main characteristics.

5. How Mercator projection is made. Discuss its suitability for the navigational charts.

6. How is International sheet numbering made? Give 24 sheet divisions of International sheet number NG 45.

7. Compare the polyconic projection with the International projection on scale 1000,000.

21

Triangulation

21.1 INTRODUCTION

To prevent accumulation of errors, it is necessary to provide a number of control points all over the area, which will form a frame work on which entire survey is to be based. Provision of such control points can be made either by one or a combination of both the following methods.

1. Theodolite Traverse
2. Triangulation.

Triangulation is more accurate than theodolite traverse, as there is less accumulation of error the that in theodolite traversing.

21.2 TRIANGULATION

It is process of measuring the angles of a chain or a network or triangles formed by stations marked on the surface of the earth.

21.3 PRINCIPLE OF TRIANGULATION

If all the three angles and the length of one side of a triangle are known, then, by simple trigonometry, the lengths of the remaining sides of the triangles can be calculated. Again, if the co-ordinates of nay vertex of the triangle and azimuth of any side are also known, then the co-ordinates of the remaining vertices may be computed. The side of the first triangle, whose length is predetermined is called the *base line* and vertices of the individual triangles are known as triangulation stations. To minimise accumulation of errors in lengths, subsidiary bases at suitable intervals are provided and to control error in azimuth of stations, astronomical observation are made at intermediate stations. The triangulation stations at which astronomical observations for azimuth are made, are called Laplace stations.

The method of surveying by triangulation was first introduced by the Dutchmen Snell in 1615.

21.4 PURPOSE OF TRIANGULATION SURVEYS

Triangulation surveys are carried out for the following purposes:

(*i*) Establishment of accurate control points for plane and geodetic surveys of large areas, by ground methods.
(*ii*) Establishment of accurate control points for photogrammetric surveys of large areas.
(*iii*) Accurate location of engineering works *i.e.,*
(*a*) Fixing the centre line, terminal points and shaft for long tunnels.
(*b*) Fixing centre line and abutments of long bridges over large rivers.
(*c*) Transferring the control points across wide sea channels, large water bodies, etc.
(*d*) Finding the direction of the movement of clouds.

21.5 CLASSIFICATION OF TRIANGULATIONS

On the basis of quality, accuracy and purpose, triangulations are classified as :

1. Primary Triangulation or First order Triangulation.
2. Secondary Triangulation or Second order Triangulation.
3. Tertiary Triangulation or Third order Triangulation.

1. **Primary Triangulation.** It is the highest grade of triangulation system which is employed either for the determination of the shape and size of the earth's surface or for providing precise planimetric control points on which subsidiary triangulations are connected. The stations of first order triangulation are generally selected 16 to 150 km apart. Every possible precaution is taken in making linear, angular and astronomical observation, and also in their computation.

2. **Secondary Triangulation.** It is triangulation system which is employed to connect two primary series and thus to provide control points closer together than those of primary triangulation. If any triangulation series which is carried out as primary does not attain the standard of accuracy of that class, due to unfavorable conditions, may also be classified as triangulation of second order.

3. **Tertiary Triangulation.** It is the triangulation system which is employed to provide control points between stations of primary and second order series. In the department of Survey of India, tertiary triangulations, known as topo triangulation, form the immediate control for topographical surveys on various scales.

21.6 LAYOUT OF TRIANGULATION

The arrangement of the triangles of a series is known as the layout of triangulation. A series of triangulation may consists of either of the following orders :

(*a*) Simple triangles in chain (Fig. 21.1).
(*b*) Braced quadrilaterals (Fig. 21.2).

(*c*) Centered triangles and polygons (Fig. 21.3).

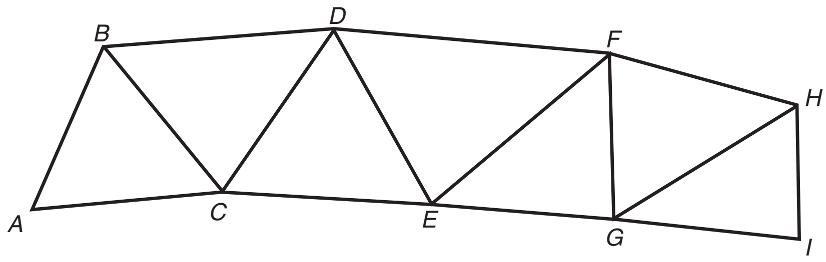


Fig. 21.1. Simpli triangles.

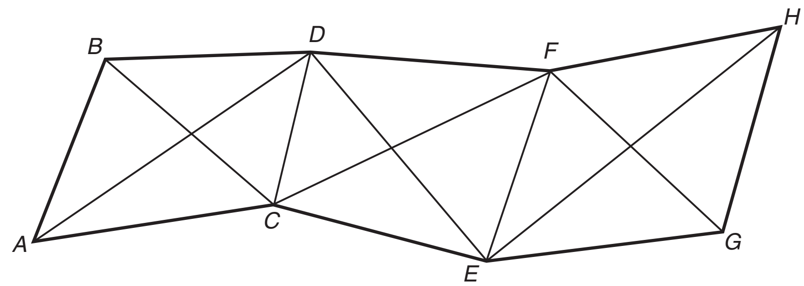


Fig. 21.2. Braced quadrilaterals.

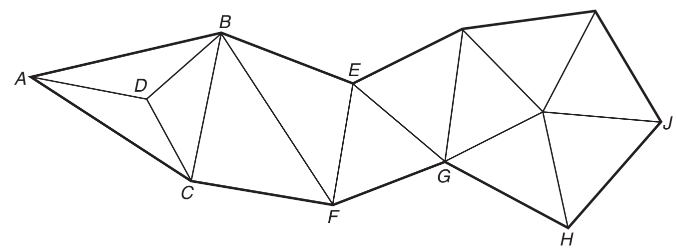


Fig. 21.3. Centered triangle and polygon.

1. **Simple Triangles in Chain.** (Fig. 21.4) this layout of triangulation is generally used when control points are provided in a narrow strip of terrain such as a valley between ridges. This system is rapid and economical due to its simplicity of sighting only four other stations, and does not involve observations of long diagonals. On the other hand, simple triangles of a triangulation system do not provide any check on the accuracy of observations as there is only one route through which distances can be computed. To avoid excessive accumulated errors, check base lines and astronomical observations for azimuth at frequent intervals are therefore very necessary, in this layout.

2. **Braced Quadrilaterals.** (Fig. 21.5) A triangulation system which consists of figures containing four corner stations and observed diagonals is known as a layout of braced quadrilaterals.

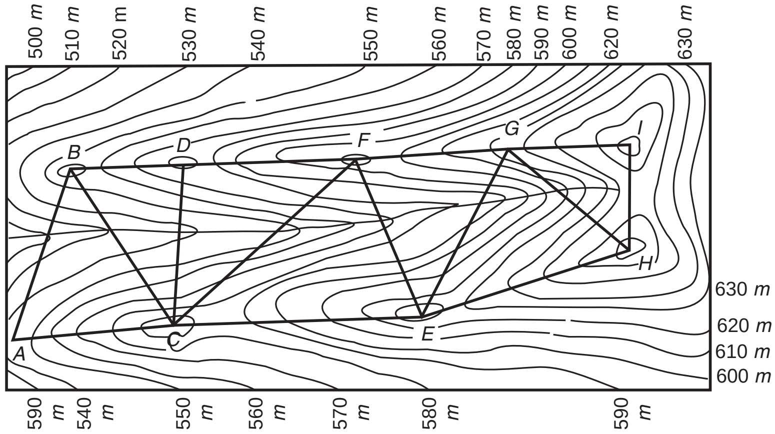


Fig. 21.4. Simple triangles in chain.

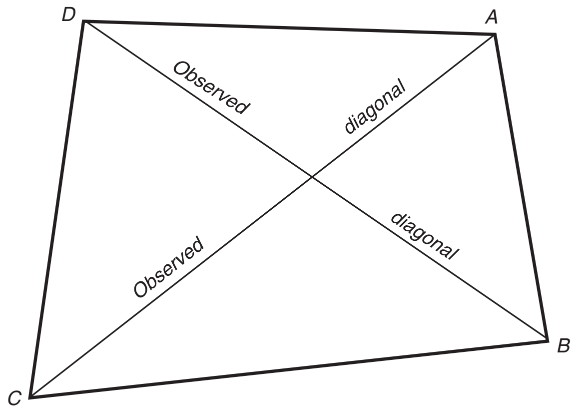


Fig. 21.5. A braced quadrilateral.

This system is treated to be the best arrangement of triangles as it provides a means of computing the lengths of sides using different combination of sides and angles.

3. **Centered Triangles and Polygons.** (Fig. 21.3) A triangulation system which consists of figures containing centered polygons and centered triangles is known as centered triangles and polygons. This layout of triangulation is generally used when vast area in all dimensions is required to be covered. The centered figures generally are quadrilaterals, pentagons or hexagons with central stations. Though this system provides proper check on the accuracy of the work, the progress of the work is generally low due to the fact that more settings of the instrument are required.

21.7 IDEAL FIGURES FOR TRIANGULATION

The under-mentioned factors need be kept in mind while deciding and selecting a particular figure in any triangulation system.

1. Simple triangles should be preferably equilateral.
2. Braced quadrilaterals should be preferably square.
3. Centered polygons should be regular.
4. No angle of the figure, opposite a known side should be small, which ever end of the series is used for computation.
5. Angles of simple triangles should not be less than 45° and in case of quadrilaterals no angle should be less than 30°. In case of centered polygons, no angle should be less than 40°.
6. The sides of the figure should be of comparable length.

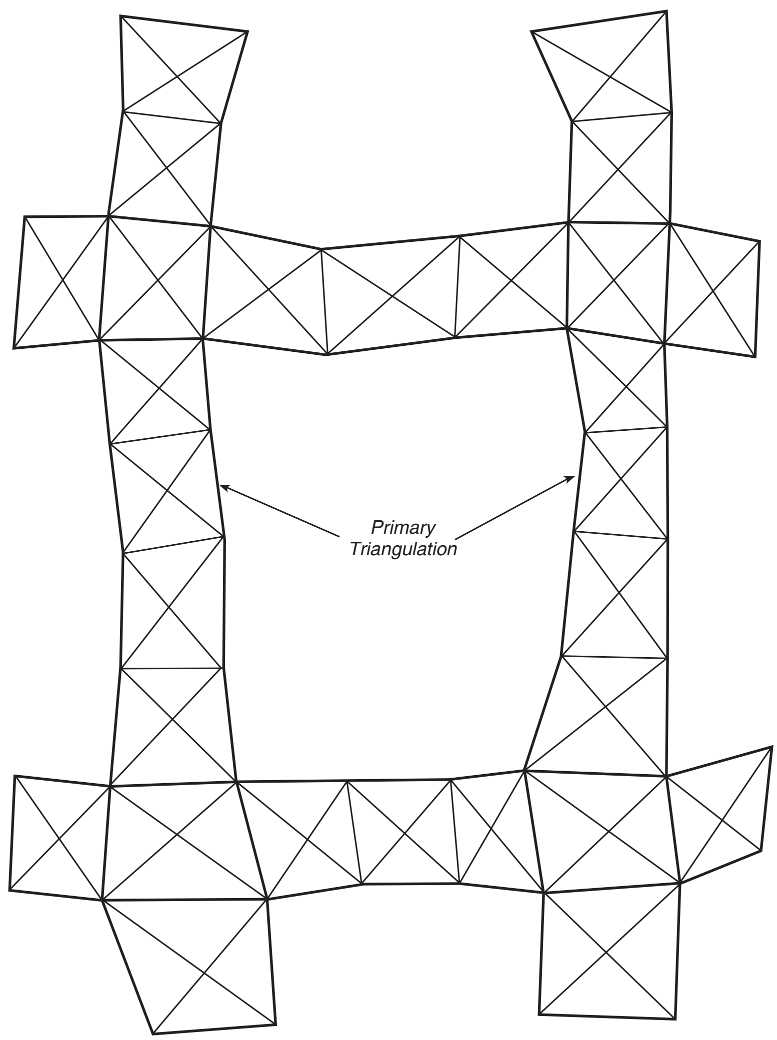


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Fig. 21.6. Grid system of triangulation.

Note. It may be noted that if a very small angle of a triangle does not opposite the known side, it does not affect the accuracy of triangulation, *i.e.,* neither the sides not the azimuths.

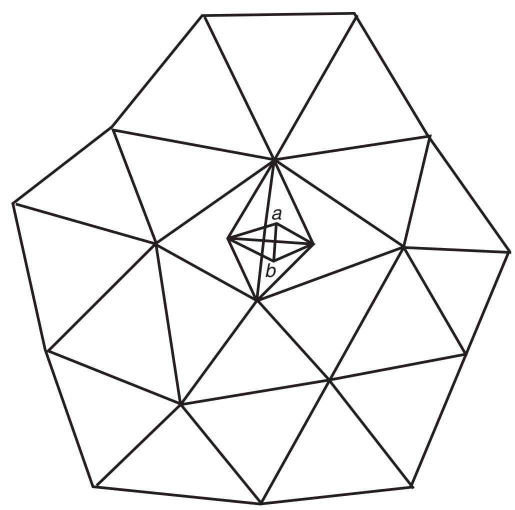
21.8 LAYOUT OF PRIMARY TRIANGULATION FOR LARGE COUNTRIES

In large countries, the frame work of triangulation may be provided by the following two methods :

(*a*) Grid iron system, (*b*) Centered system.

1. **Grid Iron System** (Fig. 21.6). In this system, the primary triangulation is laid in series of chains of triangles, which usually run roughly along meridians (north-south) and along perpendiculars to the meridians (east-west), throughout the country, the distance between two such chains vary from 150 to 250 kms. The area between the parallel and perpendicular series of primary triangulation is filled by the secondary and tertiary triangulation systems. Grid iron system has been adopted in India and other foreign countries like, Austria, Spain, France, etc.

2. **Centered System** (Fig. 21.7). In this system, the whole area of the survey is covered by a net work of primary triangulation extending outwards in all directions from the initial base line, which is generally laid at the centre of the country. This system is generally used for the survey of an area of moderate extent and has been adopted in United Kingdom.



ab is the initial base line

Fig. 21.7. Centered system of triangulation.

21.9 ROUTINE OF TRIANGULATION SURVEY

The entire routine of triangulation survey may be broadly divided into two stages.

(*a*) Field work of triangulation (*b*) Computation of triangulation.

21.10 FIELD WORK OF TRIANGULATION

To carry out fields work of triangulation, following steps are involved:

(*i*) Reconnaissance.
(*ii*) Erection of signals.
(*iii*) Measurement of the base lines.
(*iv*) Measurement of horizontal angles.

1. **Reconnaissance.** Preliminary field inspection of the entire area to be covered by triangulation is known as 'reconnaissance'. During reconnaissance, the surveyor goes over the area and decides the best plan of working, keeping in view the main principle of surveying, *i.e.*, working, from the whole to the part. The reconnaissance survey, thus requires great experience, judgement and skill. The accuracy and economy of triangulation depends upon the points reconnaissance survey. In includes the following operations :

1. Proper examination of the terrain.
2. Selection of suitable positions for base lines.
3. Selection of suitable positions of triangulation stations.
4. Determination of intervisibility of triangulation stations.
5. Selection of conspicuous well defined natural points to be used as intersected points, the points observed from two or three stations.

Reconnaissance may be effectively carried out in the office if accurate topographical maps of the area are available. General study of the area may also be made with the help of properly mosaiced vertical aerial photographs. If neither the maps, nor the photographs of the area are available, necessary reconnaissance may carried out on a plane table on one fourth the scale of the proposed survey. In skilled hands, the plane table reconnaissance may become a fairly accurate map of the area which very much facilitates to decide the well conditioned triangles. Accurate bisection of the intersected points indicates their proper identification by the surveyor. On completion of the reconnaissance most suitable triangulation scheme is selected.

Selection of Triangulation Stations. Triangulation stations are selected, keeping in view the following considerations.

1. Intervisibility of triangulation stations. For this purpose, stations are placed on the highest point of elevated places such as hill tops, house tops, etc.
2. Easy access to the stations with instruments.
3. Various triangulation stations should form well conditioned triangles.

4. Stations should be useful for providing intersected points and also for details survey.
5. For plane surveys, excessively distant stations should be avoided.
6. Stations should be on commanding situations so that these may be used for further extension of the triangulation system.
7. Grazing rays (line of sights) should be avoided and no line of sight should pass over the industrial areas to avoid irregular atmospheric refraction.

1. **Marking of Triangulation Stations.** During reconnaissance the exact positions of various triangulation stations are permanently marked on the ground so that the theodolite and signal may be centred accurately over them. Following requirements should be met while making a station.

(*i*) It should be marked on perfectly stable foundation. Station mark on rock *in-situ* is generally preferred to if the size of the rock is so large that the theodolite may rest on the rock itself while should be observations.

(*ii*) The mark should be distinctive and indestructable. Generally, a hole 10 cm to 15 cm deep is drilled into a rock and a copper or iron bolt is fixed with cement. If no rock is available, a large stone should be embedded about 1 metre under ground with a circle and dot cut on it and a second stone with a circle and dot flushed with the top surface of the platform is placed vertically above the first. An old pipe 2.5 cm diameter driven vertically into ground up to a depth of one metre also serves a good station mark.

(*iii*) The exact location of the station mark should be marked with arrows in the surrounding rocks.

(*iv*) The station mark with a vertical pole placed centrally should be covered with a conical heap of stones placed symmetrically. This arrangement of marking station, is known as placing a cairn.

(*v*) Two to three reference marks at some distance on fairly permanent features should be established to locate the station mark if the top mark is disturbed or removed.

(*vi*) Surrounding the station mark a platform 3 m × 3 m × 0.5 m should be built up of earth.

2. **Erection of Signals.** To define exact position of triangulation station during observations from other stations, signals are used. Various types of signals are centered vertically over the station marks and observations are made to these signals. It is very necessary to ensure that signals whenever used are truly vertical centered over the station marks. The accuracy of triangulation is entirely dependent on the degree of accuracy of centering the signals. Greatest care of center-

ing the transit over the station mark will be useless, unless some degree of care of centering the signals is impressed upon.

Classification of Signals. The signals may be classified as under:

(*a*) luminous signals, (*b*) opaque signals.

1. Luminous Signals. Luminous signals are further divided into two categories *i.e.*, sun signals and night signals.

(*a*) **Sun signals.** Those signals reflect the rays' of the sun towards the station of observations, are known as heliotropes. Apparently, such signals can easily be used in clear weather.

(*b*) **Heliotropes.** It consists of a circular plane mirror with a small hole a centre which reflect the sun's rays and a sight vane with an aperture carrying a cross hairs. The circular mirror can be rotated horizontally through 360° and vertically through few degrees. The heliotrope is centred over the station mark and the line of sight is directed towards the station of observation. The attendent looks through the hole and adjusts the sight vane till the flashes given from station of observation fall at the centre of cross. Once this is done, heliotrope is not disturbed. Now the heliotropes attendent rotates the frame carrying the mirror so that the black shadow of the small central hole of the plane mirror falls exactly at the cross of sight vane. The reflected beam of rays will be seen at the station of observation. Due to motion of the sun this small shadow also moves and hence the attendant remains on constant alert to ensure that the shadow always remains at the cross.

The following points may be noted :

(*i*) Heliotropes do not give better results as compared to opaque signals.

(*ii*) Heliotropes are found useful when the signal station is in plains and the station of the observation is on elevated ground.

(*c*) **Night signals.** These signals are used where astronomical observations are made to stars. It consists of a patromax with glowing light on the station mark.

2. Opaque Signals. Opaque, or non-luminous signals used during day are of various forms and most commonly used ones are the following:

(*i*) *Pole Signals* (Fig. 21.8) It consists of a round pole painted black and white in alternate and is supported vertically over the station mark on a tripod.

(*ii*) *Target Signals* (Fig. 21.9) It consists of a pole carrying two square or rectangular targets placed at right angles to each other. The targets are generally made of cloth stretched on wooden frames.

Pole signals and target signals are useful for sights less than 6 km.

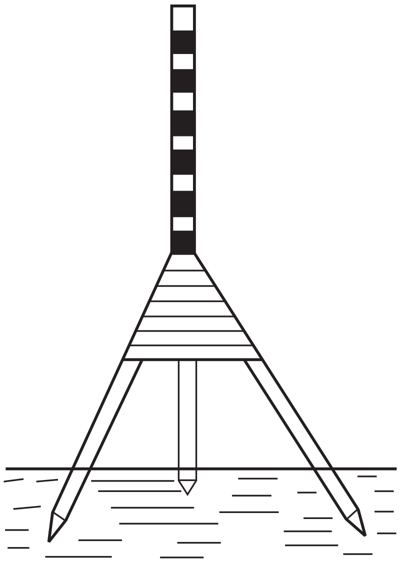


Fig. 21.8. A pole signal.

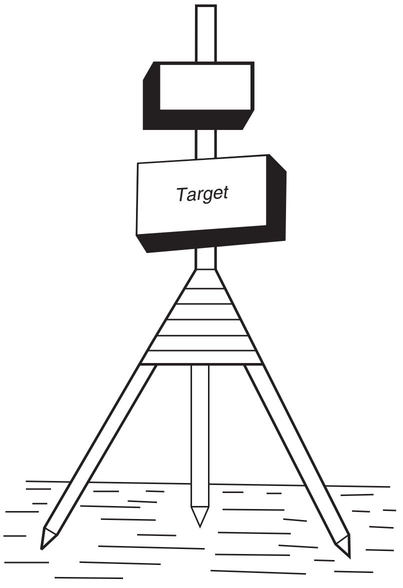


Fig. 21.9. A target signal.

(*iii*) *Pole and Brush Signal* (Fig. 21.10). It consists of a straight pole about 2.5 metres long with a bunch of long grass tied symmetrically around the top, making a cross. The signal is erected vertically over the station mark by heaping a pile of stones, upto 1.7 metres round the pole. A rough coat of white wash is given to make it more conspicuous to be seen against black ground.

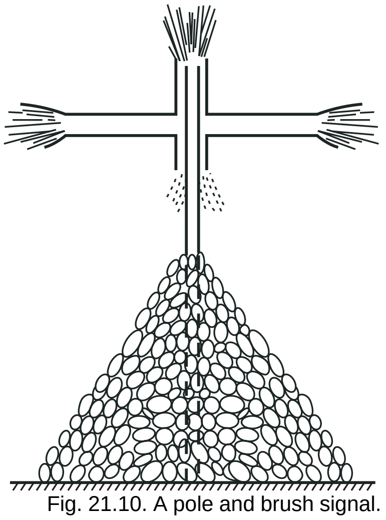


Fig. 21.10. A pole and brush signal.

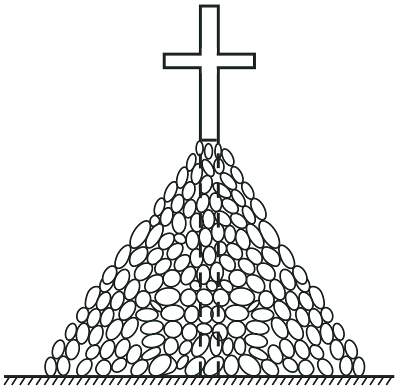


Fig. 21.11. A stone cairn.

The pole and brush signal is a very useful opaque signal and must be erected over every station of observation during reconnaissance.

(*iv*) *Stone Cairn* (Fig. 21.11). It consists of stones built up to a height of 3 metres in a conical shape. This white washed opaque signal is very useful if the background is dark and the sighted station is at far distance.

(*v*) *Beacons* (Fig. 21.12). It consists of red and white cloth tied around the three straight poles. This can be easily centered over the station marks with a plumb bob.

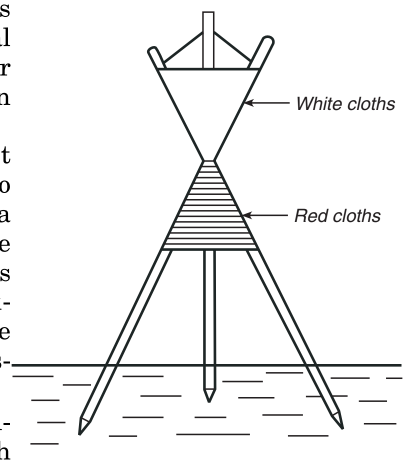


Fig. 21.12. A beacon.

Beacons are useful when simultaneous observations are made at both stations.

Requirements of an Ideal Signal. A good signal should fulfill the following requirements :

(*i*) It should be conspicuous *i.e.*, it should be clearly visible from a distance against any background.
(*ii*) It should provide easy and accurate bisection by telescope.
(*iii*) It should be capable of being accurately centred over the station mark.
(*iv*) It should exhibit very little phase error of bisection of the signal.

3. **Measurement of Base Lines.** The accuracy of any order of triangulation is dependent upon the measurement of the base line. Hence, in triangulation a base line is of prime importance. The length of a base line depends upon the grade of the triangulation. In India, for the net work of first order triangulation, the bases were whose lengths vary from 1.7 miles (2.83 km) to 7.8 miles (13 kms). For topographical triangulation, the length of the base line used is generally 88 yards (80.467 m).

Selection of a Site for a Base Line. Following factors must be considered for the selection of site for a base line for any triangulation system.

(*i*) The ground at the site should be fairly level, or uniformly sloping or gently undulating.
(*ii*) It should be free from obstructions throughout of its length.
(*iii*) The ground should be firm and smooth.

(*iv*) The extremities of the base line should be intervisible at ground level.

(*v*) The site should be such that well triangles can be obtained while connecting its end stations to the stations of the main triangulation.

Generally following types of base measuring equipments are used.

1. **Standardised Tapes.** For measuring short bases in plain areas standardised tapes are generally used. After having measured the length, the correct length of the base is calculated by applying the required corrections. For details of correcting, refer to Chapter 2 of Author's text book of surveying and levelling.

2. **Hunter's Short Base** (Fig. 21.13). It is a short base which was devised by Dr. Hunter who was a director in the Survey of India. It consists of four chains 22 yards (20.117 m) linked together and supported at every chain, by two legged stands. The extreme ends are fixed to two targets which are fixed to three legged stands. A 1 kg weight is suspended at the end of an arm so that the chains assume straightness. The correct length of the individual chain is supplied by the observatory. The lengths of the joints at intermediate supports are measured directly on the ground with the help of a graduated scale. To obtain correct length between the centres of the targets, usual corrections for sag, temperature, M.S.L. etc. are applied.

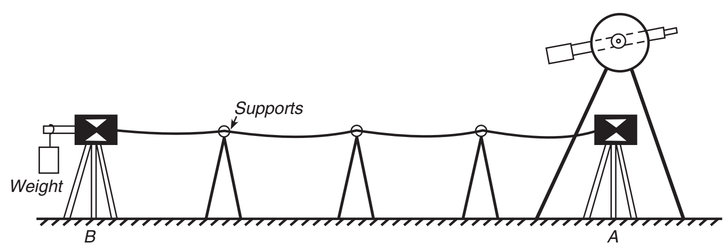


Fig. 12.13. A Hunter's short base.

Setting up of the Hunter's Short Base. The base has two ends, *A* and *B*. To identify them easily, *A* is marked in red and *B* is marked in green colours. The stand of *A* end is centered on the ground mark and the target is fitted with a clip. The target *A* is made truly vertical so that the notch on its top side is centered on the ground mark. One end of the base is hooked with the plate *A* a and it is spread carefully till its other end is reached. In between, at every joint of the chains two legged supports are fixed to carry the base. *B* end is fixed to the *B* target which is fixed to a support at *B* stand and the 1 kg weight is attached at the end of the lever. While fixing the end supports *A* and *B*, it should be ensured that their leg should face each other under the base. Approximate alignment of the base is then done by eye judgement.

For aligning the base theodolite, following steps are involved.

(*i*) Set up the theodolite accurately over the notch of the plate *A*.
(*ii*) Centre and level the transit accurately.
(*iii*) Bisect the target *B* or the end of the tape with the vertical hair.
(*iv*) The intermediate supports are brought in line of the vertical hair.
(*v*) Ensure that, all the intermediate supports and the target *B* are in one line.

Measuring the slope angles. In case the base is spread along undulating ground, slope correction is applied. The slope angles of individual supports are observed as under:

(*a*) Fix a target to a long iron rod such that it is as high above the tape at *A* as the trunnion axis of the theodolite.
(*b*) Hold the rod vertically at each support and read the vertical angles.

Computation of the length of the base line. The measured length is applied all the corrections and accurate length of the base is obtained.

3. Tacheometric Base Measurement. In undulating ground, small length of the base line can be precisely measured by tacheometrical observations. For details refer to Chapter 13 of this textbook.

4. Electronic Apparatus. For very precise triangulation work, electronic apparatus such as tellurometers, geodimeters, etc. are used now-a-days. A base line upto 35 km can be measured with an accuracy of 1 in 1000,000. For details refer to Authors, textbook of Advance Survey.

5. Measurements of Horizontal Angles. The instruments for triangulation surveys require great degree of refinement. To achieve this small glass arc or double reading theodolites are generally used. With wild universal theodolites horizontal and vertical circles can be read upto 1 second of arc. Recently, other types of glass arc theodolites have also been manufactured such as kern, ziess, etc. For details, refer to chapter 23 of this text book.

The salient features of modern theodolites are as follows :

(*i*) These are small in dimensions and light in weight.
(*ii*) The graduations are engraved on glass circles and are much finer.
(*iii*) The mean of the two readings on the opposite sides of the circle can be read directly through an eye piece attached close to the telescope. This saves the observations time.
(*iv*) There is no necessity to adjust the micrometers.
(*v*) These are water-proof and dust proof.

(*vi*) These are provided with electrical arrangement for illumination during nights if necessary.

Method of Observation of Horizontal Angles. The horizontal angles of a triangulation system can be observed by the following methods:

1.The Repetition method.

2. The Reiteration method or direct method.

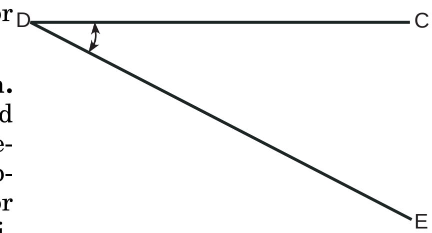


Fig. 21.14.

1. **Method of repetition.** (Fig. 21.14) This method is used when angle is too small and specially for measuring the angle subtended by base at first station for its extension. If any angle of a triangulation series is also less than 20°, the method of repetition be used.

Repetition Method Using a Vernier Theodolite. (Fig. 21.14) Following steps are involved :

(*i*) Centre the theodolite over the station of observation say *D* and level it accurately.

(*ii*) With the help of upper clamp and upper tangent screw, set the vernier *A* to read 0° on face left.

(*iii*) Loosen the lower plate and swing the telescope to bisect the station *C*. Clamp the lower plate and disect *C* accurately using lower tangent screw. Read both the verniers *A* and *B*.

(*iv*) Unclamp the upper and swing the telescope in clockwise direction till station *E* is sighted. Clamp the upper clamp and bisect accurately with the upper tanget screw. Note down the readings of both the verniers to get the approximate value of the angle *CDE*.

(*v*) Loosen the lower clamp and swing the telescope clockwise to sight *C*. Accurate bisection to be done using lower tangent screw. With this setting the vernier readings do not change.

(*vi*) Unclamp the upper plate and swing the telescope clockwise to sight the section *E*. Bisect *E* accurately by me upper tangent screw. The range is now repeated twice.

(*vii*) Repeat the process till the angle is repeated five to seven times, depending upon its value.

(*viii*) Calculate the average angle with face left by dividing the difference of final reading on the station *E* and the first reading on the section *C*, by the number of repetitions.

(*ix*) Change the face and make the same number of repetitions of the angle *CDE* on the face right and calculate the average angle.

(*x*) The mean value of the average angles so obtained in steps (*viii*) and (*ix*) is the required value of the angle *CDE*.

Repetition Method Using a Wild Universal Theodolite (Fig. 21.14)

Following steps are involved :

(*i*) Centre the theodolite over the ground mark of station *D*.
(*ii*) Clamp the horizontal circle and intersect the station *C* with the tangent screw and record the horizontal reading. Ensure that horizontal clamp is not released till the whole observations involving total number of repetitions are completed.
(*iii*) Swing the telescope using the tangent screw only and intersect the station *E*. Record the reading and calculate the approximate measure of the angle.
(*iv*) Keeping the telescope still intersecting station *E*, change the circle reading by roughly 180°/N where *N* is the total number of expected repetitions, using base plate screw.
(*v*) Again intersect the station *E* and record the reading leaving a space for recording the reading of station *C*.
(*vi*) Using the upper plate tangent screw intersect the station *C* again. Record the reading and obtain the second measure.
(*vii*) Keeping the telescope intersecting the station *C*, swing the telescope in the same direction as before through another angle of 180°/*N* and proceed as above. The above steps are repeated till the required number of measure of the angle *CDE* have been obtained. The mean of the measures is the require value of the angle *CDE*.

2. **The Reiteration Method.** (Fig. 21.15) In the reiteration method, the signals of the stations are bisected successive and a value is obtained for each direction at each of the several round of observation.

Following steps are involved :

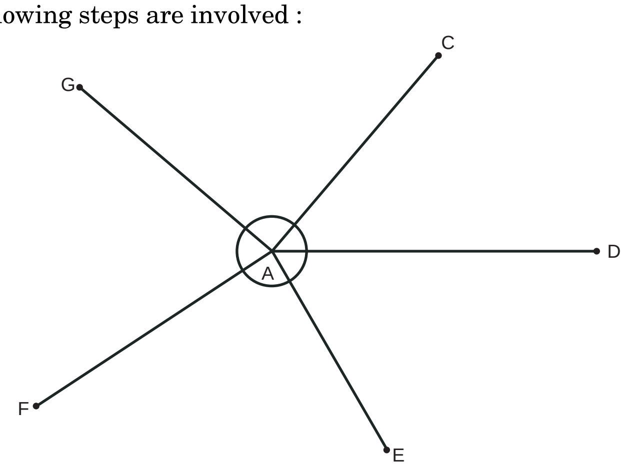


Fig. 21.15 Reiteration method.

(*i*) Set up the instrument with face left over the station of observation say *O* and level it accurately.
(*ii*) Make the vernier *A* to read a few minutes over 0°.
(*iii*) Unclamp the lower plate and turn in azimuth until a well defined station *G* known as initial or reference station, is brought in the field of view.
(*iv*) Clamp the lower plate and bisect the signal *G* accurately using the lower tangent screw.
(*v*) Loosen the upper plate and swing the telescope in anticlockwise direction until *G* again enters the field, without overshooting it.
(*vi*) Clamp the upper plate gently and intersect the signal *G* using the upper tangent screw.
(*vii*) Read both the verniers *A* and *B* and record their readings.
(*viii*) Unclamp the upper plate gently and swing the telescope clockwise towards *C* till it is brought in the field of view.
(*ix*) Clamp the upper plate gently and intersect the signal *C* using the upper tangent screw.
(*x*) Repeat the steps (*viii*) and (*ix*) for stations *D*, *E*, *F* and finally for *G*.
(*xi*) Unclamp the upper plate, change the face of telescope and intersect the signal *G* accurately with the upper tangent screw. Read both the verniers *A* and *B*.
(*xii*) Swing the telescope in anticlockwise direction towards stations *F*, *E* etc and finally signal *G*.

Note: The following points may be noted :
(*i*) The final intersection should be made against the spring of the slow-motion screw.
(*ii*) The final intersection of the signals must e made from the left to the right on face left and from the right to the left on face right.
(*iii*) The instrument must be rotated by holding the supports and not the telescope.
(*iv*) The comparison of two readings of stations *G* will test the stability of the instrument during observations.
(*v*) The best zero station is one that remains visible clearly all days.
(*vi*) The horizontal angles must be observed in early mornings or late evenings.

21.11 NUMBER OF ZEROS

To eliminate the error due to inaccurate graduations of the horizontal circle, the measures of the horizontal angles to the triangulation stations should be taken on three different zeros.

Zero to be used are :

F.L.	0°	F.L	60°	F.L	120°
F.R.	180°	F.R.	240°	F.R.	300°

21.12 TYPES OF TRIANGULATION STATIONS

The triangulation stations may be categorised as under :

(*i*) Main stations

(*ii*) Subsidiary stations

(*iii*) Satellite stations

(*iv*) Pivot stations.

1. **Main Stations.** The triangulation stations which are used to carry forward the network of triangulation are known as *main stations*. Observations on two zeros only.

2. **Subsidiary Stations.** Triangulation stations which are used only to provide additional rays to intersected points, are known as *Subsidiary stations*. Observation of the horizontal angles for *subsidiary stations* are made on two zeros only.

3**. Satellite Stations.** The stations which are selected close to the main triangulation stations to avoid intervening obstructions, are knowns as *satellite* or *eccentric* or *false stations*. Observations at ***satellite stations*** are made with the same precision as for the main stations.

4. Pivot Stations. The stations at which no observations are made but, the angles at those stations are used for the continuity of a triangulation series, are known as *pivot stations.* As there is no check on the observations of the triangles involving a pivot station, it is recommended to avoid them in the normal triangulation net.

In Fig. (21.16) *A, B, C, D, E, G, H, I, J* are main stations.

K, L are the subsidiary stations.

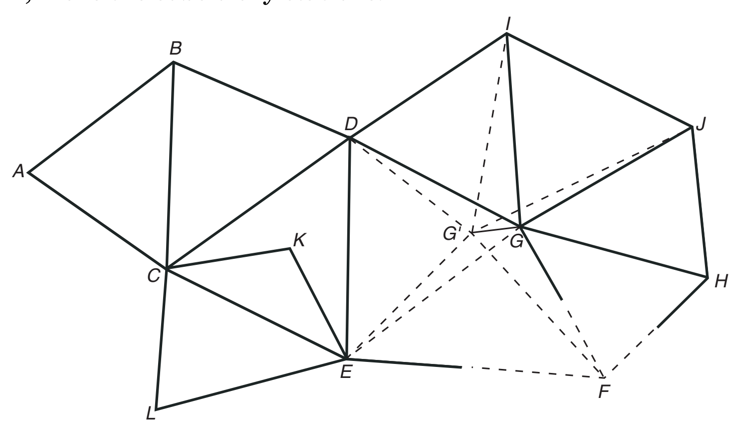


Fig. 12.16. Triangulation stations.

G is the satellite stations.

F is the pivot stations.

21.13 TRIANGULATION COMPUTATIONS

It is includes the following :

(*i*) Checking the means of the observed angles.

(*ii*) Checking the triangulation errors.

(*iii*) Checking the total round at each station.

(*iv*) Computation of the length of the base line.

(*v*) Computation of the side of the main triangle.

(*vi*) Computation of the latitudes and departures of each side of the triangulation net.

1. **Means of the Observed Angles.** The means of the readings of two verniers for each measures and the means of the angles obtained from various rounds are checked before entering the angles in the abstracts at each station.

2. **Checking the Triangular Errors.** The sum of three angles of each triangle must be equal to 180°. If there is any difference, it should be distributed to three angles in the ratio of their magnitudes.

3. **Checking the total Round at Station.** The sum of the individual angles obtained from various rounds, must be equal to 360°. If the discrepancy at any station is large, fresh round of angles must be obtained at particular station.

4. **Computation of the Length of the Base Line.** The absolute length of the base line is obtained by applying different corrections. Refer to chapter 2, of this textbook of 'Surveying and Levelling'.

5. **Computation of the Sides of a Plane Triangle.** As one side and three angles of each triangle are known, the length of the remaining two sides of the triangle may be obtained by applying sine formula, *i.e.,*

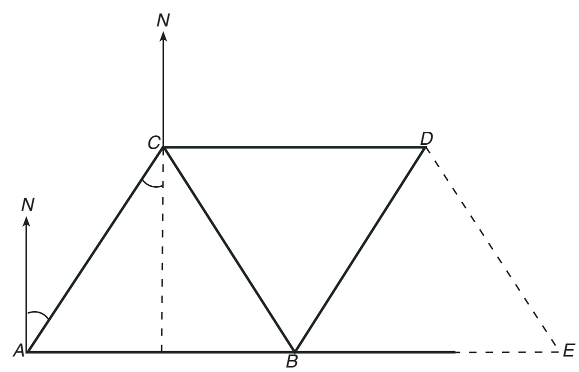


Fig. 21.17

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} .$$

6. **Computation of Latitudes and Departures.** (Fig. 21.17). Knowing all the three sides, all the three angles of each triangles, and the bearing of one side, the bearing of other sides can be computed as under :

Let ABC be the triangle in which bearing of the side CA is known.

Hence, Bearing of side AC = Bearing of $CA - 180°$

Bearing of side AB = Bearing of $AC + \angle CAB$

Bearing of side BC = Bearing of $AB + 180° + \angle ABC$

If the sum less than 180°, add 180°, if more than 180° subtract 180°.

Knowing the co-ordinates of A, the co-ordinate of B and C can be computed as under :

Latitude of $C = AC$ consine of the reduced bearing of AC

Departure of $C = AC$ sine of the reduced bearing of AC

Example 21.1. *Following observations were made from two triangulation stations C and D, 200 m apart to two inaccessible points A and B.*

$$\angle ADC = 29° \, 37' \, 25'', \angle CDB = 16° \, 17' \, 38''$$

$$\angle DCA = 136° \, 54' \, 33'', \angle DCA = 157° \, 06' \, 34''$$

Calculate the distance of AB.

Solution. (Fig. 21.18)

In ΔACD, $\angle ADC = 29° \, 37' \, 25''$

$\angle DCA = 136° \, 54' \, 33''$

$\therefore \angle DAC = 180° - [29° \, 37' \, 25'' + 136° \, 54' \, 33'']$

$= 13° \, 28' \, 02''$

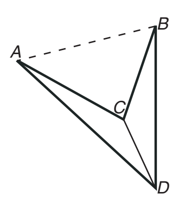


Fig. 12.18.

In ΔBCD,

$\angle CDB = 16° \, 17' \, 38''$

$\angle BCD = 157° \, 06' \, 34''$

$\therefore \quad \angle CBD = 180° - [16° \, 17' \, 38'' + 157° \, 06' \, 34'']$

$= 6° \, 35' \, 48''$

In $\Delta \, ACB$,

$\angle ACB = 360° - [136° \, 54' \, 33'' + 157° \, 06' \, 34'']$

$$= 65° \, 58' \, 53''$$

Now, $$AC = \frac{200 \sin 29° \, 37' \, 25''}{\sin 13° \, 28' \, 02''}$$

$$= 424.494 \text{ m}$$

and $$BC = \frac{200 \sin 16° \, 17' \, 38''}{\sin 6° \, 35' \, 48''}$$

From $\Delta\, ACB$,

$$AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cdot \cos \theta$$

$$= (424.494)^2 + (488.451)^2 - 2 \times 424.494 \times 488.451 \cos 45° \, 58' \, 53''$$

$\therefore$ $AB = 499.99\text{m}$ **Ans.**

Example 21.2. *Following observation were made from two inter-visible stations C and D to stations A (E 15000, N 8000) and B (E 15600, 8800)*

∠ACD = 50° 26′ 10″, ∠CDA = 30° 00′ 00″, ∠ADB = 15° 00′ 00″

If the bearing of CA is 63° 26′ 00″ *and point D is roughly southeast of C, calculate the distance CD and the co-ordinates of the point D.*

Solution. (21.19)

Difference in Easting of A and B

$$= 15600 - 15000 = 600 \text{ m}$$

Difference in Northings of A and B

$$= 8800 - 8000 = 800 \text{ m}$$

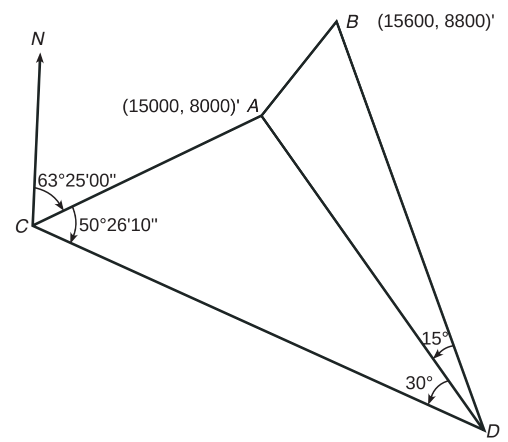


Fig. 21.19.

$\therefore \quad AB = \sqrt{600^2 + 800^2} = 1000$ m

Bearing of $AB = \tan^{-1} \dfrac{\Delta E}{\Delta N} = \dfrac{600}{800} = .7500$

$= 36° \ 52' \ 12''$

Bearing of CD = Bearing of CA + Angle ACD

$= 63° \ 26' \ 00'' + 50° \ 26' \ 10'$

Bearing of $DC = 113° \ 52' \ 10'' + 180° = 293° \ 52' \ 10''$

Bearing of DB = Bearing of DC + Angle CDB

$= 293° \ 53' \ 10'' + 45°$

Bearing of $DB = 338° \ 52' \ 10''$

$\therefore$ Bearing of $BD = 338°.52' \ 10'' - 180° = 158° \ 52'10''$

Bearing of $BA = 216° \ 52' \ 12''$

$\therefore$ Angle ABD = Bearing of BA − Bearing of BD

$= 58° \ 00' \ 02''$

In ΔACD, $\angle CAD = 180° - (50° \ 26' \ 10'' + 30° \ 00' \ 00'')$

$= 99° \ 33' \ 50''$

In ΔABD, $\angle BAD = 180° - (58° \ 00' \ 02'' + 15° \ 0' \ 00'')$

$= 106° \ 59' \ 58''$

Applying sin rule to ΔBAD

$$AD = \frac{AB \sin ABD}{\sin ADB} = \frac{1000 \times \sin 58° \ 00' \ 02''}{\sin 15°}$$

or

$$AD = \frac{1000 \times 0.848053}{0.258819} = 3276.63 \text{ m}$$

Again, from ΔACD

$$CD = \frac{AD \sin 99° \ 33' \ 50''}{\sin 50° \ 26' \ 10''} = \frac{3276.63 \times 0.986.1010}{0.770915}$$

$= 4191.24$ m. **Ans.**

Reduced Bearing (θ) of

$AD = 180° - [36° \ 52' \ 12'' + 106° \ 59' \ 58'']$

$= 36° \ 07' \ 50''$

Latitude of $AD = AD \cos \theta$

$= 3276.63 \cos 36° \ 07' \ 50''$

$= 2646.46$ m

Departure of $AD = AD \sin \theta$

$= 3276.63 \sin 36° \, 07' \, 50'' = 1931.99$ m

Easting of D = Easting of A + Departure of AD

= 15000 + 1931.99 = 169310.99 m

Northing of D = Northing of A + Latitude of AD

= 8000 – 2646.46

= 5353.54 **Ans.**

21.14 E.D.M. INSTRUMENTS

E.D.M. instruments for surveying may be divided into two classes :

(*i*) Those in which a beam of light is the carrier and which is reflected back from a mirror located at the other end. Such instruments are less expansive because one active instrument and a battery are only needed at one end and the instrument at the other end is simply a reflecting mirror centered over the ground station mark. Such instruments are used for shorter distances only. When the distance between the stations is comparatively large, a telescope is used for aiming the beam at the mirror and laser is used as carrier, because this has a very small angle of spread.

(*ii*) Those which transmit short radio waves (microwaves) as carrier. In this type of instruments, an active instruments and operator are needed at each end of the line. The signal is retransmitted on another carrier wave back to the originating instrument. The wavelengths of the carrier of such instruments varies from 8 to 10 mm. These instruments do not require precise directional setting because of spread angle of carrier waves is between 5° to 10°. Instruments using microwaves are generally used for the long distance required in primary triangulation and precise traversing.

Geodimeter. This instrument was developed by Dr. Bergstrand of Sweden in 1950. It consists of an electromagnetic optical instrumentation unit at one end of the line and an optical instrumentation unit at the other end of the line. The electromagnetic optical unit is known as the Geodimeter which transmits a highly collimated, electrically modulated light beam to the optical unit centred over the other end of the line. The optical unit consists of a reflecting element which returns the light beam to the Geodimeter. The reflecting elements consist of retro-directive prisms which possess the property of returning the incident light beam to its source, for short distance, ordinary flat mirrors are used to reflect the light beam, after careful orientation. A comparison is made between the light beam returned to the Geodimeter and the one transmitted by the Geodimeter. From the comparison, for two or three separately used modulating frequencies, the distance between the two instrument stations can be determined.

1. **The light Beam.** The light beam used in the Geodimeter is obtained from an in candescent light source. The beam is made to pass through an electric shutter consisting of a pair of crossed polaroids and a kerr cell. By creating an electric field across the kerr cell, the electronic shutter can be controlled to transmit beam in any desired fashion. The modulated light beam may be considered as a number of chain lengths, placed end to end, moving through the distance with the velocity of light. The rate of emission of these chanin lengths is equal to the frequency of the kerr cell modulation. The length of these chain lengths is equal to the velocity of the light. The rate of emission of these chain lengths is equal to the velocity of the light divided by the frequency of the modulated light beam, *i.e.,* the modulation wavelength λ. The beam of chain lengths is returned to the Geodimeter by the reflector located at the other end of the line. (Fig. 21.20).

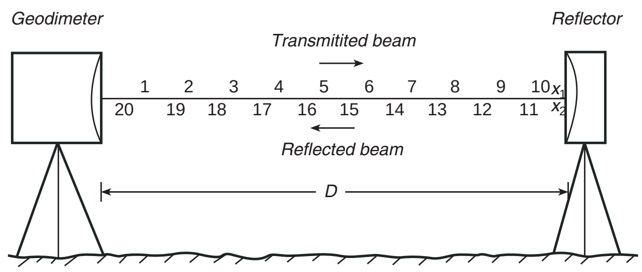


Fig. 21.20. A Geodimeter.

Let us assume that successive chain lengths are 1, 2, 3 , 4, ..., 20 when the Geodimeter has just emitted a full chain length. In this case, the transmitted beam contains 10 full chain lengths and distance x_1 whereas the reflected beam contains 9 full chain lengths and a distance x_2 near reflector and d near the Geodimeter. It may be noted that $x_1 + x_2 \pm \lambda_1$ is a full chain length and hence, if D is the distance between the ends of the line, the number of full chain lengths is 20 over a path length of $2D - d$ where d is the remaining fraction of a chain length. In a generalised case, if n is the number of full chain lengths and d is the fraction of a chain length near the Geodimeter, then $D = \pm \frac{1}{2}$ $(n\lambda + d$, where λ is the chain length in both transmitted and reflected beams.

The Geodimeter compares the transmitted and reflected beams and thus determine only d. It may be clearly understood that there is no provision in the Geodimeter to determine the full chain lengths between the end points of the line. The number of full chain lengths has to be determined independently of Geodimeter measurements. The number of the full chain lengths may be determined if the distance $2D$ is known in advance to within $\pm \frac{1}{2}\lambda$ or D is known i advance to with in $\pm \frac{1}{4}\lambda$. If the two ends of the line can be located on a topographic map (say on scale 1 : 250, 000), the distance can be ascertained correct to 62.5 m easily, assuming 0.25 mm as the plotable error. Nowadays, modulating frequency being used is 10 megacycles to which the chain length approximately corresponds to 30 metres in length. Apparently, the distance between the ends of the line should be known before hand within ± 7.5 metres.

2. **The full wavelengths.** A practical method of ascertaining the number of full chain lengths is to provide two or three, separately used, modulating frequencies. The length of the primary frequency will be denoted by λ and that of secondary by λ'. Corresponding fractional lengths will be denoted by d and d'. If one adjusts the frequencies such that $100\ \lambda = 101\ \lambda'$, then it is only necessary to know the distance D within $\pm \frac{1}{4}\ (100\ \lambda) = 25\ \lambda$ in advance. Hence, we may say that for 10 mega cycles primary modulating frequency, the distance between the stations being measured by Geodimeter need be known within 750 metres. By reducing the frequency of the modulated beams, by 1/10th, the distances between stations need be known within 75 metres. And further reducing the frequency by 1/100th the distance needed only correct to 7.5 metres. If the wavelength is reduced by 1/1000, the reading of distance will be 0.75 metre. This method is used in E.D.M. systems.

The Tellurometer. The main principle of working of this instrument is similar to that of the Geodimeter with the exception that in it high frequency radio waves (microwaves) are used instead of light waves. With this modification, tellurometers can be used during day and nights and even in hazy weather, whereas the Geodimeters are used in nights only. The tellurometer is considered to be first portable electronic distance measurement instrument with a light weight power supply of 12 or 24 volts. The disadvantage of the tellurometer is however, that two similar instruments are required, each will a skilled operator at either end of the line to e measured. These are called as master or control set and remote or slave set. By switching, master and remote sets can be reversed easily.

The master instrument emits radio waves at a frequency of 3000 Mcs (3×10^9 c.p.s.), which are superimposed by waves of frequency of 10 Mcs. The lower frequency waves are used for the measurements, because it is difficult to make accurate measurements with short wavelengths (10 cm) of higher frequency. The higher frequency waves which can be propagated in straight line paths over long distance, are termed as the carrier wave. Those high frequency waves are, therefore, said to be modulated by the low frequency pattern waves. The combined wave *i.e.*, high frequency wave and low frequency wave, is reflected by the remote station and is recovered again by the master instrument which ultimately separates them again as carrier wave and low frequency pattern wave. The pattern wave is only used for the measurements of the distances. The phase delay or the change in phase of the low frequency wave between its emission and reception gives a measure of the fractional part of a wavelength in excess of the unknown number of wavelengths between the master and remote stations. To determine unknown distances upto 15 km, four low frequencies are generally employed.

1. **Ambiguity resolution in distance.** (Fig. 21.21). Let A be the radiated signal, whose reflected signal is shown as a dotted line. To understand it clearly, the signal is shown as a continuation of a full line, which is the reflected wave of the wave emitted by the remote station to a second master station located at equal distance, but in the opposite direction. Now the phase delay is measured at master station and the corresponding proportion of the wavelength computed. To over come the ambiguity of whole numbre of wavelengths, a second low frequency wave (pattern B, is transmitted, reflected and recovered again at different frequency but using this some carrier waves.

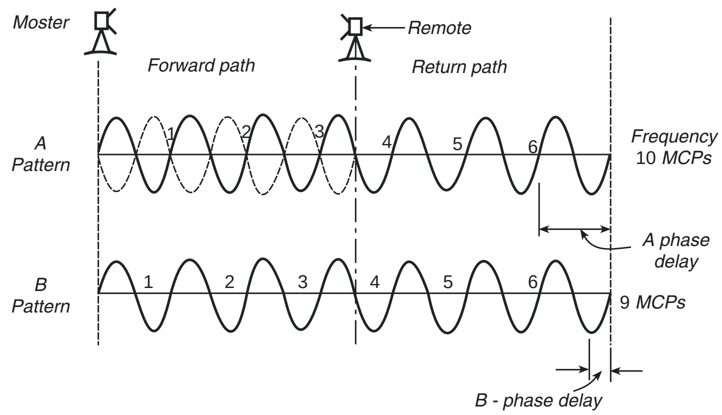


Fig. 21.21. Ambiguity resolution in distances.

Let the frequency of the second signal B be 9 Mcs, then 9 wavelengths of pattern B (approximate 33 m) = 10 wavelengths of pattern A (Approximate 30 m, i.*e.,* 10 Mcs).

If the phase delay of second signal is B, then $A - B$ gives a partial solution of the ambiguity of resolution in distances, as explained under.

The value of $A - B$ will differ and will be zero at every tenth wavelength of pattern A and ninth wavelength of pattern B, *i.e.,* 10×30 or 9×33 = approximately 300 metres for the both way journey or roughly 150 m for the distance between the master and remote stations. It means that a phase delay of $A - B$ will correspond to a distance between 0 and 150 metres.

Again let the phase delay of a third signal of frequency 9.9 Mcs be C, then with the same reasoning the value of $A - C$ will correspond to a distance between 0 and 1500 metres. Similarly, a fourth signal of frequency 9.99 Mcs with phase delay D will enable to solve the ambiguity of distances upto 1500 m. In actual practice, the four phase delays are measured separately and B,C and D are subtracted from A in turn. Phase delays are expressed in time units and observed on oscilloscope scale which is divided into 100 equal parts. The four final readings of A, $A - B$, $A - C$ and $A - D$ are recorded in millimicro seconds (10^{-9}seconds) and then finally converted to distances using a value of velocity of propagation of the radio waves which is generally 299,792.5 km per second.

Distance obtained by tellurometers are corrected for temperatures, pressure and atmospheric relative humidity.

2. **Field operation with a tellurometer.** A master and a Remote station are set up, one at each end of the line to be measured. The observations, are made by the operator of master station, who is also linked by radio telephone to the operator of the remote station. They first align their instruments roughly and by means of indicated signals, strength, accurate alignment is made. The aerials of the instruments are parabolic mirrors with transmitting and receiving dipoles placed at their foci. A modulated radio wave is transmitted by the master, which is reflected by the slave and is finally received by master, where phase delay in observed. The accuracy of the measurements of distance is similar to that of Geodimeter.

Notes :

(*i*) The reflections from features in the beam path do not cause significant errors.

(*ii*) The reflection from water surfaces, lack of vegetation sandy country cause appreciable errors.

(*iii*) No error is caused if the indirect ray path does not differ greatly in length from the direct path.

(*iv*) The worst reflecting surface usually cause little error of the line of sight between station is less than 70 metres above these surfaces.

(*v*) If the beams gaze the ground at the intermediate points, it produce fuzzy displays on the cathode ray tube.

(*vi*) The rays should be kept to a moderate distance above the ground.

(*vii*) The components (aerials or lenses) which transmit and receive the signals remain separate from the main instrument, only connected by flexible wires. These components can be raised on poles to get the beam clear of obstructions.

(*viii*) By attaching these comments above a theodolite, angles and distance can be measured at one set up.

21.15 COMPARISONS OF THE GEODIMETER AND TELLUROMETER

Geodimeter	Tellurometer
1. It consists of one instrument only set up at one end and at other end, mirror is used as reflector.	1. It consists of two identical instruments, one at each end of the line.
2. It is a heavier instrument.	2. It is a lighter instrument.
3. The observations are limited to nights only.	3. Observations may be made in day as well as in nights.
4. Light radio waves of 10^{17} c.p.s. are used.	4. High frequency radio waves (micro waves) are used.

21.16 DM 502 (ELECTRO-OPTICAL DISTANCE METER) (FIG. 21.22)

In this instrument manufactured by Kern & Co. Ltd., Switzerland, the intensity of the infrared radiation supplied by the light emitting semiconductor diode is modulated with two frequencies. The instrument

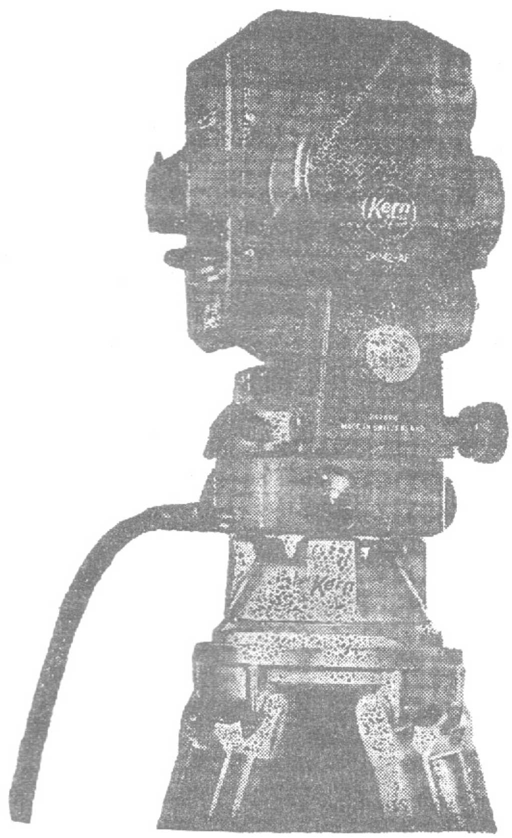


Fig. 21.22. DM 502 (Electro-optical Distance Meter)
(Photograph by the Courtesy of Kern & Co. Ltd., Switzerland)

measures the phase difference between the transmitted and reflected measuring signal from which the slope distance is calculate and displace on the LCD (Liquid-crystal Display) in digital form. Due to its symmetrical design, the DM 502 is balanced on the telescope of the opto-electrical or electronic theodolite without cumbersome counter weights.

With this instrument distance measurement is fully automatic and only following operations are necessary :

(*i*) Switch on the instrument, and

(*ii*) Press the start button for measurement release.

The gain of the received signal is adjustment automatically. The signals received from the distance meter get decoded and the data in digital form right at the reflection site is displayed. If the instrument is operated in conjunction with the E_1, electronic theodolite, it also transmits directly the horizontal distance and elevation difference.

The remote receiver. It is designed with a liquid crystal display and rotary switch for selecting the desired data. If features and bayonet lock for direct reflector mounting. If the receiver is located within the transmission beam of the distance meter, an acoustic signal is generated. Under favourable atmospheric condition, distances of more than 1200 metres can be measured with one reflector. Three reflectors are required for distances upto 2000 metres and for greater distances, upto five reflectors are used.

The reflector is set up on a versatile tripod which has two adjustable struts with lever ratchet clamps. It allows the reflector to be set up at any height from 20 cm to 2 m. For greater heights upto 3 m, extension tubes are used.

Measuring procedure. The three following operations are required for measuring distance, vertical angles and horizontal directions with only one pointing.

1. Point the telescope cross hairs on the target of the reflector.
2. Rotate the function switch to 'Measure'.
3. Press the starting button marked 'Measure'.

In about 8 seconds, the slope distance is displayed in millimeters by six liquid-crystal digits. The vertical angle and horizontal direction are read in the circle reading eye piece of the theodolite.

The determination of horizontal distance, difference in elevations may be made directly in the field by using a pocket calculator.

EXERCISE 21

1. Write a short note on E.D.M. electronic distance measurement.
2. Explain the main principle of electronic measuring devices.
3. Describe the working of a geodimeter.
4. Describe the working of a tellurometer.
5. How does a tellurometer differ a geodimeter.
6. Explain the method of trilateration.

22

Photogrammetric Surveying

22.1 INTRODUCTION

Photogrammetric surveying or photogrammetry is the branch of surveying in which maps are prepared from photographs taken from ground or air stations. With an advancement of the photogrammetric techniques, photographs are also being used for the interpretation of geology, classification of soils and crops.

In 1859 Frenchman Colonel A. Laussedat took photographs from a phototheodolite and exhibited a plan of Paris prepared by photographic surveys in 1857. Almost simultaneously and independently a German Scientist A. Meydonbour used tow photographs for the measurements of architectural details of a building. S. Finster Walder published his book, "The Fundamental Geometry of Photogrammetry" which explained the basic principles of photogrammetry in the perspective sense, in 1889. Aerial photogrammetry made a headway with the development of aeroplanes in twentieth century. The first aerial photographs taken from an aeroplane was made on 24th April 1909 by Willer Wright over Italy.

In 1915, Oscar Messter built the first aerial camera in Germany and J.W. Bogloyad A. Brock manufactured the first aerial cameras in U.S.A.

22.2 AERIAL PHOTOGRAPHS

These photographs are taken from camera stations in the air with the axis of the camera vertical or nearly vertical.

According to the direction of the camera axis at the time of exposure, aerial photographs are further divided into the following mainclasses :

(*i*) Vertical photographs.

(*ii*) Oblique photographs.

(*i*) **Vertical photographs.** These photographs are taken from the air with the axis of the camera vertical or nearly vertical. A truly vertical photograph closely resembles a map. These are utilised for the compilation of topographical and engineering surveys on various scales.

(*ii*) **Oblique photographs.** These photographs are taken from air with the axis of the camera intentionally tilted from the vertical. An oblique photograph covers larger area of the ground but clarity of details diminishes towards the far end of the photograph. Depending upon the angle of obliquity, oblique photographs may be further into two categories.

(*a*) **Low oblique photographs.** An oblique photographs which does not show the horizon, is known as low oblique photograph. Such photographs are generally used to compile reconnassiance maps of inaccessible areas.

(*b*) **High oblique photograph.** An oblique photograph which is sufficiently tilted to show the horizon, is known as high oblique photograph. Such photographs were previously used for the extension of planimetric and height control in areas having scantly ground control. With the introduction of radial line assumptions theory, oblique photographs are no longer used.

22.3 PRINCIPLE OF PHOTOGRAMMETRY AND ITS LIMITATIONS

The principle of photogrammetric survey in its simplest form is very similar to that of the plane survey. The only difference is that the most of the work which in plane table survey is executed in the field, is done in office. The principal point of each photograph is used as a fixed station in plane table surveys and rays are drawn to get points of intersections very similar to those used in plane table.

Photogrammetry is particularly suitable for topographical or engineering surveys and also for those projects demanding higher accuracy. The photogrammetry is rather unsuitable for dense forest and flat-sands due to the difficulty of identifying points upon the pair of photographs. It is also unsuitable for flat terrian where contour plans are required because interpretation of contours becomes difficult in the absence of spirit levelled heights. Considering these factors, it is evident that photogrammetry may be most suitably employed for mountaineous and hilly terrain with little vegetation.

22.4 AERIAL PHOTOGRAPHY

Mapping of large areas from aerial photographs is faster and cheaper than any other method yet developed. With the aerial photographs, more complete and accurate topographic maps, can be prepared on various scales ranging from 1 : 500 to 1000,000 Contours can be accurately surveyed upto 50 cm vertical interval.

Aerial photogrammetry, the science of measurement by means of aerial photographs, consists of four stage.

1. Air flights
2. Photography

3. Ground control
4. Photographic compilation.

(*i*) **Air flights** Aerial surveying is generally carried out for large areas, involve elaborate photography and heavy expenditure. Such surveys are therefore made by the government organisation or large private companies. In India, photographs are taken by M/S. Air Survey Coy of India and are supplied to the Survey of India for compilation.

The ground area is photographed from the aeroplane as it flies back and forth along parallel flight lines. The photographs taken in one flight line constitute a 'strip'.

Planning for flights. It consists of laying down the flight lines and start and finish points for each run on the flight map. Depending upon the scale of the photographs, flying height of the air craft, is decided, and knowing the speed of the aircraft, the exposure interval may be computed. Planning of flights, including :

(*i*) Marking of each run on the flight map.
(*ii*) Computing the flying height of the aircraft.
(*iii*) Determining the number of linear distance of flying.
(*iv*) Determining the exposure intervals and number of exposures in each run.
(*v*) Making a choice of proper aircraft with due regard to its ceiling heights, type of camera and scale of photographs.

Design of photographic coverage. Each photograph covers the ground are depending upon the height of the aircraft. Proper size of an aerial photograph is decided on the following factors :

(*i*) Scale of photography
(*ii*) Type of camera lens
(*iii*) Percentage of overlap
(*iv*) Topography of the terrain
(*v*) Scale of the final map
(*vi*) Vertical contour interval of finished map
(*vii* Accuracy achievable by the available instruments.

The photographs of each strip are taken with a certain amount of overlap which is known as forward overlap.

The overlap between any two adjacent strips is known as 'lateral overlap'.

For stereoscopic plotting of aerial photographs, at least 50% forward overlap is required. But to avoid risk of short overlap, normally 60% forward overlap is provided. Normally a lateral overlap of 20% is sufficient but in mountaineous terrain at least 30% is preferred, to avoid risk of short overlap due to large relief variations.

Photography. To have aerial photographs, a suitable camera fitted in the aircraft is used. Air survey cameras are fully automatic and carry roll films several metres long.

Aerial survey cameras. The consist essentially of the following parts :

(*i*) Lens assembly which includes lens, diaphragm and shutter
(*ii*) Camera cone
(*iii*) Focal plane
(*iv*) Camera body
(*v*) Drive mechanism
(*vi*) Magazine.

(*i*) **Lens assembly.** The main part of this assembly is the camera lens. A camera lens may be either wide angle, normal angle of narrow angle.

A wide angle lens has a field of the order of 95° and exposes photographs which show distortion and loss of illumination at the picture edges. This type of lens gives a maximum economy of photography 'Avigon' is the wide angle lens manufactured by the Wild Co.

A normal angle lens has a field of the order of 60° and is generally used for large scale work. Such lenses are free from distortion. The Ross 30 cm and Wild Avitor are the examples of the type of lenses which provide pictures of 23 cm × 23 cm and 18 cm × 18 cm respectively, the focal length of the latter being 21 cm.

A narrow angle lens has a field of the order of 40° or less. Its pictures are most accurate plans of the ground and hence, are very useful for urban surveys.

The Ross 25 m is a good example of this type.

The cameras and lens available in India are :

Camera	*Focal length of lens*	*Formal size*
Wild R.C. 5a	11.5 cm and 21 cm	18 cm × 18 cm
Wild R.C. 8	11.5 cm	18 cm × 18 cm
RMK 15/23	15 cm	23 cm × 23 cm
Eagel IX	15 cm and 30 cm	18 cm × 18 cm

(*ii*) **Focal plane.** As the distance of the flying aircraft is always considerably large, the focal planes of aerial cameras are at fixed locations. All aerial cameras are therefore a fixed type.

(*iii*) **Camera cone.** It supports the entire lens assembly including the filter. Collimation marks which define the focal plane of the camera are provided on its top. This cone is made up of material of least coefficient of the thermal expansion to ensure relative positions of the collimation marks and the lens system of the camera to remain the same various operational temperatures.

(*iv*) **Camera body.** The camera body forms the integral part of the camera cone, which preserves the interior orientation, *i.e.,* relative position of the lens, the lens axis, the focal plane and the collimation marks.

(*v*) **The drive mechanism.** It is used for winding the film and is either operated manually or automatically.

(*vi*) **The magazine.** It holds the exposed and unexposed films. The film is flattened at the focal plane by creating vacuum or pressing it against the register glass which carries collimating marks which appear on the photographs. On each photographs, following data is recorded.

(*i*) The time of exposure
(*ii*) Date of photography
(*iii*) A spirit level indicating the amount of tilt
(*iv*) The number of the strip
(*v*) The digital number of the photograph
(*vi*) The flying height of the camera
(*vii*) The fiducial or collimating marks.

For various flying height, the following aircrafts are generally used in India.

Type of Aircraft	*Ceiling Height*
Canberra	15,500 m
Dakota	7,300 m
Cessna	8,000 m
Dominie	5,200 m

22.5 TECHNICAL TERMS USED IN AERIAL SURVEYING (Fig. 22.1)

1. **Vertical Photograph.** An aerial photograph taken with a camera having its optical axis truly vertical, downward along the direction of gravity, is known as vertical photograph.

2. **Tilted Photograph.** An aerial photograph taken with a camera having its optical axis tilted usually less than 3° from the vertical is known as Tilted Photograph.

3. **Exposure (or Air) Station (0).** The exact position of the front nodal point of the lens in the air at the instant of exposure, is known as exposure station or air station.

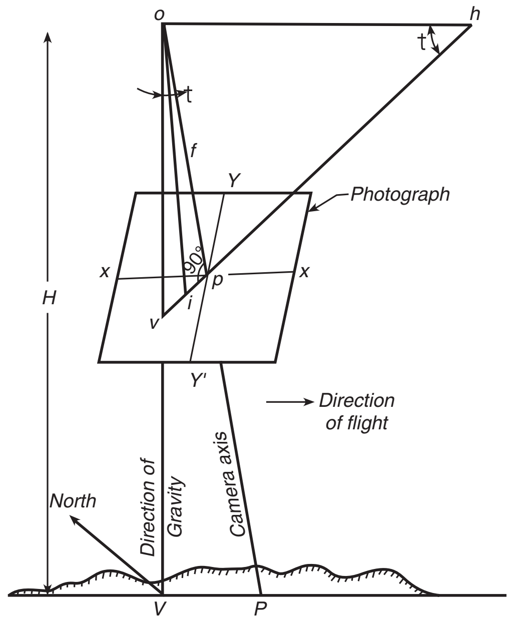


Fig. 22.1. Technical terms used in Photogrammetry.

4. **Flying height (H)** The elevation of the air station above mean sea level, is known as flying height of the aircraft.

5. **Line of Flight.** A line which represents the track of aircraft on an existing map is known as the line of flight.

6. **Focal Length (OP).** The distance from the front nodal point of the lens to the plane of the photograph, or the distance of the image plane from the rear nodal point, is known as the focal length.

7. **Perspective Projection.** In this projection, straight lines radiating from a common (or selected) point and passing through points on the sphere are projected to the plane of projection. A photograph represents a perspective plane.

8. **Perspective Centre.** The real or imaginary point of the origin of bundles of perspective rays is known as perspective centre. In an aerial camera, there are two points—one perspective centre which relates to points in the photograph and the other which relates to the objects photographed.

9. **Principal Point (P).** The point where a perpendicular dropped from the front nodal points strikes the photograph, is known as 'principal point' of the photograph.

or

The point where a perpendicular dropped from the rear nodal point strikes the image plane, is known as principal point of the negative.

The point where the plate perpendicular strikes the ground is known as 'Ground Principal Point' (*P*).

10. **Nadir Point.** (or Plump Point). The where a plumb line dropped from the front nodal point strikes the photograph is known as Nadir point or 'Plumb Point'.

The Plumb point *(v)* of the photograph defines the location of the ground point vertically beneath the air station.

11. **Ground Nadir Point or Ground Plumb Point** *(V)*. It is the point on the ground point vertically beneath the station of exposure.

12. **Principal Plane.** The plane which contains the plumb line through the rear node and the plate perpendicular is known as principal plane, *i.e.*, plane *v.o.p.*

13. **Tilt.** The angle *vop* between the plumb line *ov* and the optical axis of the lens *op* is known as **tilt**.

or

The deviation of a plate from the horizontal plane at the time of exposure is know as tilt.

14. **Iso Centre** *(i)*. The point in which the bisector of the angle of tilt meets the photograph is known as *iso* centre. It lies on the principal line at a distance of $f \tan \frac{t}{2}$ from the principal point. It is the point in the plane of the tilted photograph at which all angles in the photograph are equal to their respective traces on the ground.

On a truly vertical photograph, the *iso* centre the plumb point and the principal point coincide.

15. **Swing** (α). The horizontal angle measured clockwise in the plane of the photograph from the positive *Y*-axis to the plumb point is known as swing.

16. **Horizon Point** (*h*). The point of intersection of the principal line *(vip)* and the horizontal line *(oh)* through the perspective centre *O*, is known as horizon point.

22.6. RELATION BETWEEN THE PRINCIPAL POINT, PLUMB POINT AND ISO-CENTRE OF A TILTED PHOTOGRAPH. (Fig. 22.1)

Following relations are important.

(*i*) The distance of the plumb point from the principal point it equal to $f \tan t$.

Proof

In right angled triangle *vpo*, we get

$$\angle vop = t, \text{ the angle of tilt}$$

$\angle vpo = 90°$

and side $op = f$, the focal length

$\therefore \quad vp = op \,.\, \tan t$

or $\quad = f \tan t \qquad \ldots(22.1)$

(*ii*) The distance of the *iso* centre from the principal point is its equal to $f \tan \frac{t}{2}$.

Proof. In right angled triangle *ipo*, we get

$\angle ipo = 90°$

$\angle iop = \frac{t}{2}$

side $op = f$

$\therefore \quad ip = op \tan \frac{t}{2}$

$= f \tan \frac{t}{2} \qquad \ldots(22.2)$

Again, when t is small, $\tan \frac{t}{2} = \frac{t}{2}$

i.e., plumb point distance from the principal point is twice the distance of the iso centre.

(*iii*) The distance of the horizon point along the principal line from the principal point is equal to f cor t.

Proof. In right angled Δ *oph*,

$\angle oph = 90°, \angle pho = t$

$\therefore \quad ph = op \cot t$

$= f \cot t \qquad \ldots(22.3)$

22.7 SCALE OF A VERTICAL PHOTOGRAPH, (FIG. 22.2)

Let, N be the perspective centre

H be the flying height of the camera

B be the top of a hill of height h above M.S.L.

V be the ground plumb point

b be the image of the top B on the photograph

v be the plumb point of the photograph

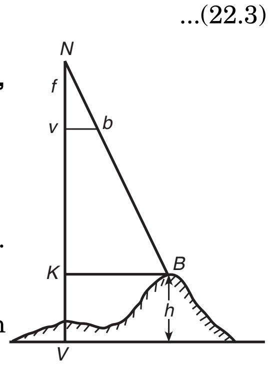


Fig. 22.2 Scale of a photograph.

f be the focal length of the camera

Construction :

Draw BK perpendicular to NV meeting at k.

Now Δ $sNvb$ and NKB are similar

$$\therefore \text{ Scale of the photograph} = \frac{f}{H-h} \qquad ...(22.4)$$

If the terrain is perfectly plane at the M.S.L. the scale of the photograph is equal to $\frac{f}{H}$. It is generally known as Datum Scale.

If average height of the terrain is substituted in eqn. (22.4) the scale of the photograph is known as Average Scale.

Note. The scale of a photograph varies according to the elevation and hence the scale of photograph is not constant.

Example 22.1. *Vertical photographs were taken from a height of 3500 metres above the terrain with a camera of 15 cm focal length. Calculate the scale of the photograph.*

Solution.

Here $H - h = 3500$ m

$f = 15$ cm

Substituting the values in eqn. (22.4), we get

Scale of the photography

$$= \frac{0.15}{3500} = \frac{1}{23333.3} \quad \textbf{Ans.}$$

22.8 DISPLACEMENT OF PHOTO IMAGE DUE TO HEIGHT (FIG. 22.3)

Let, N be the camera station

AB be the hill

f be the focal length of the camera

h be the height of the hill above M.S.L.

H be the flying height of the camera

D be the horizontal distance of A from plumb point V.

Then, the perspective rays from A and B will appear on the photographic plane at a and b respectively. The displacement *(ab)* of the image of point B with respect to the image of the point A is known as *height displacement.*

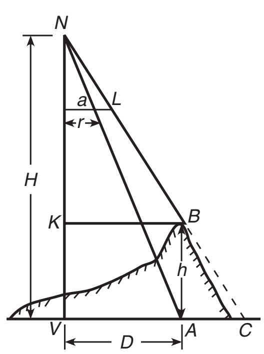


Fig. 22.3. Height Displacement.

Construction. Through B, draw BK parallel to AV to meet NV at K. joint NB and product it to meet the horizontal line VA at C.

From similar Δ *sNab* and NAC, we get

$$\frac{ab}{AC} = \frac{Na}{NA} = \frac{Nv}{NV} = \frac{f}{H}$$

$$ab = \frac{AC \cdot f}{H} \qquad \text{....(22.5)}$$

Again, from similar Δ*sNKB* and BAC we get

$$\frac{AC}{KB} = \frac{AB}{NK} = \frac{h}{H - h}$$

or $$AC = \frac{KB \cdot h}{H - h} = \frac{D \cdot h}{H - h} \qquad \text{...(22.6)}$$

Substituting the value of AC in eqn. (22.5)

$$ab = \frac{D \cdot h}{H - h} \cdot \frac{f}{H} = \frac{Dhf}{H\,(H - b)} \qquad \text{...(22.7)}$$

Again, from similar Δ *sNKB* and Nvb, we get

$$\frac{KB}{vb} = \frac{(H - h)}{f} = \frac{D}{r}$$

where r is radial distance of the top image from the plumb point.

$\therefore$ $$D = \frac{r\,(H - h)}{f} \qquad \text{...(22.8)}$$

Substituting the value of D in Eqn. (22.7)

$$ab = \frac{\dfrac{r\,(H - h)}{f} \times h \times f}{H\,(H - b)}$$

$$= \frac{rh}{H}$$

or $$ab = \frac{rh}{H} \qquad \text{...(22.9)}$$

Hence, the height displacement of a point is proportional to its height above M.S.L. and the distance of its top image from the plumb point.

Note: The following points may be noted:

(*i*) The height displacement increases as the horizontal distance from the principal point increases.

(*ii*) The height displacement decreases as the flying height increases.

(*iii*) The height displacement is positive, (*i.e.,* radially outward) for points above the datum.

(*iv*) The height displacement is negative, (*i.e.,* radially inward) for the points below the datum.

(*v*) The height displacement is zero for the point vertically below the air station.

22.9 DETERMINATION OF THE HEIGHT OF TOWERS, PILLARS ETC.

Let $I : S$ be the scale of photograph

f be the focal length of the camera

x be the height displacement of the tower

h be the height of the tower above assumed datum

H be the height of the camera, above assumed datum

r be the radial distance of the top of the tower.

We know from Eqn. (22.9) that

or $$ab = \frac{rh}{H} \qquad ...(22.10)$$

or $$x = \frac{rh}{H}$$

or $$h = \frac{xH}{r} \qquad ...(22.11)$$

Knowing the value of x, H and r, the value of h can be computed.

Example 22.2. *The distance of an image of a triangulation station 250 m above mean sea level form the principal point is 3.20 cm. Calculate the height displacement if the flying height of the camera is 2000 m.*

Solution. From Eqn. (22.11) we get

$$x = \frac{rh}{H} \qquad ...(i)$$

Substituting the values of r, h and H in Eqn. *(i)*

$$x = \frac{3.20 \times 250}{2000} = 0.40\text{cm} = 4 \text{ mm. } \textbf{Ans.}$$

22.10 FLIGHT PLANNING

While designing the flights, following considerations are kept in mind :

(*i*) Layout of the area (*ii*) Photo scale

(*iii*) Flying height (*iv*) Over laps.

(*i*) **Layout of the area.** The layout of the area to be photographed indicating north, south, east and west limits are marked on an existing 1,250,000 maps of the area. Aerial photographs are flown in strips to cover the specified area. The flight direction of main strips which are called filling-in strips is, always kept, along the length of the area. When economy in ground control is desired, photographic control strips called 'tie strips' are flown at right angles to the filling-in-strips. The spacing of the tie trips depends on the accuracy of survey desired.

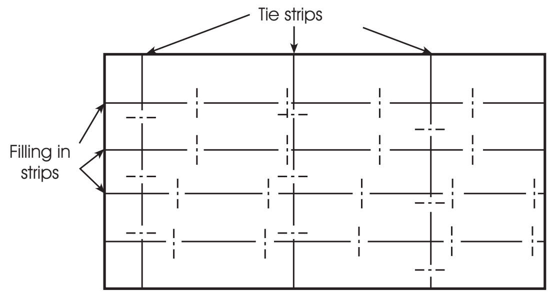


Fig. 22.4. Plan showing layout of filling in and tie strips.

(*ii*) **Consideration for photo scale.** The photo scale is directly linked with the accuracy of the map which is the final product. After deciding the map accuracy by the map indentor, the scale of the aerial photography is suggested by the mapping agency. For topographical map of small and medium scales, the height accuracy attainable and the image quality, are the governing factors for the choice of the scale, whereas for large scale maps planimetric accuracy is more important. The largest scale of map that can be produced photogrammetrically in and economic way is 1 : 500. It is therefore essential to ensure that both planimetric and height accuracy requirements are met with the decided photo scales.

The standard error in spot height measurements that can be achieved by graphical adjustment of aerially triangulated strips can be anything up to 1% of the flying height. Knowing the required spot height

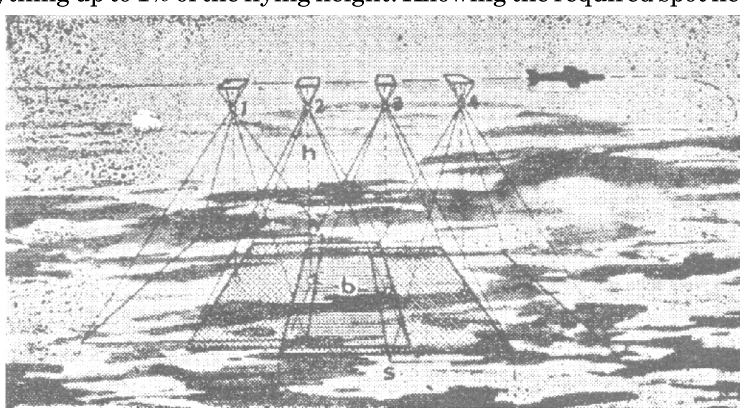


Fig. 22.5. A flight line.
(Photograph by the Courtesy of Carl Ziess, West Germany)

accuracy or the contour interval, it is possible to decide the flying height and photo scale.

(*iii*) **Flying height.** If planimetric accuracy is important, the flying height depends on the photo scale chosen to meet the planimetric accuracy. If, on the other, the height accuracy is of prime importance, the flying height may be fixed first depending on the required contour interval. The flying height is often related to the contour interval of the final map. The process is rated by its co-factor which is the number by which the contour interval is multiplied to obtain the flying height. Thus, flying height = (contour interval) X cofactor.

(*iv*) **Over laps.** The normal longitudinal overlap is generally kept 60% or more along the flight line and at least 20% between the adjacent strips.

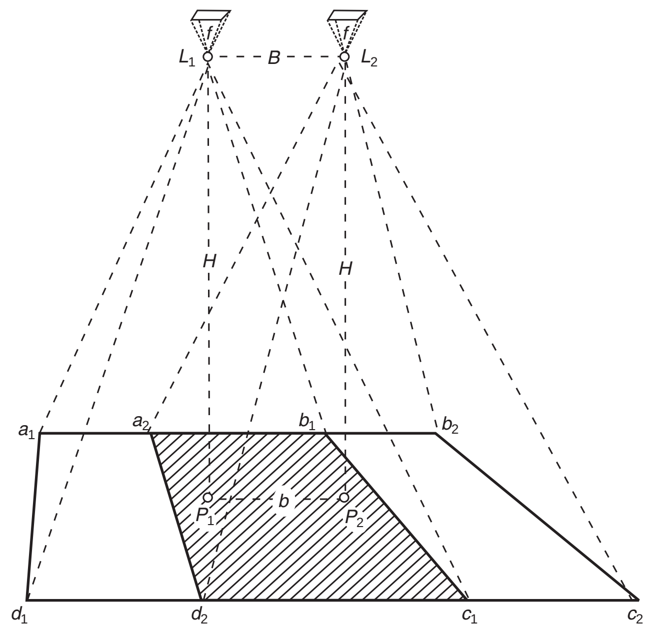


Fig. 22.6. Overlaps.

In mountainous terrain overlaps run short due to height distortion and it is always preferred to increase the overlaps to avoid rephotography of the area.

22.11. STEREOSCOPY AND CONTOURING

1. **The human eye.** The human eye essentially resembles a photographic camera. In eye the lens system comprises, of the cornea, the interior chamber and crystalline lens. The retina on which an image is formed, corresponds to the negative plane of the camera. Any object to be clearly distinguished by the retina should subtend at the eye an

angle larger than one minute of arc. This characteristic of the human eye enables the viewer to determine precisely stereo-photogrammetric distance differences which in photogrammetry correspond to elevation variations. Farthest stereoscopic vision is only possible upto 600 metres because the parallactic angle is too small beyond that distance and nearest stereoscopic vision with normal naked eye is also not possible closer than 25 cm without the help of pair of lenses.

2. **Natural Stereoscopic Vision.** The main advantage of aerial photogrammetry is that it presents a three dimensioned model of the terrain by stereoscopic vision. Stereoscopic vision is an essential prerequisite for the application of photogrammetric methods of survey. In photogrammetry with binocular vision, the surveyor obtains a three dimensioned model fromed by two overlapping photographs of the same object taken from two different positions.

The two images of an object are perceived by eye and are co-ordinated in mind. The two image halves are not exactly identical and the two eyes are separated laterally and occupy different view points. This lateral shift of the two images is known as 'binocular parallax'. Natural stereoscopic vision is based on the fact that horizontal parallaxes occur between the image halves because of the depth differences in the viewed object.

In photogrammetry instead of viewing in actual space two aerial photographs of the same area taken from two different view points (Air Stations) presenting two dissimilar views of the same area, are viewed. Under suitable conditions a stereoscopic three dimensioned model similar to that of natural ground may be obtained from two aerial photographs.

3. **Stereoscopic Vision.** Stereoscopic vision may be either orthoscopic view or pseudoscopic view.

(*i*) **Orthoscopic view.** If the overlaps of the stereo pair are kept inwards and observed under a stereoscope an orthoscopic view is obtained in which elevations and depths appear their natural order.

(*ii*) **Pseudoscopic view.** If the overlaps of the stereoscopic view are kept outwards, pseudoscopic view is obtained in which the natural order is reversed.

4. **The essential condition for stereoscopic vision.** The following conditions are essential for stereoscopic vision.

(*i*) Separate images must be presented practically simultaneously to each eye but must exhibit parallax, *i.e.*, views must be taken from two different points.

(*ii*) The direction of vision to corresponding image points must intersect in space.

(*iii*) If images are to be viewed with naked eye, corresponding distant points of the two images should not have any greater relative separation than the interocular distance. (*i.e.,* the distance between the eye).

5. **Requirement of a stereoscopic pair.** In order to produce a spatial model, the two overlapping photographs must fulfill the following conditions :

(*i*) The two photographs must have adequate common overlap, *i.e.,* a minimum of 60%.
(*ii*) The camera axes at the time of exposures should be approximately co-plancer.
(*iii*) The convergence of the optical axes of the cameras must not be too large and no greater convergence should be experienced than natural vision.
(*iv*) The scale of the adjoining photographs should be approximately the same, even though difference in scale of photographs upto 15% can be tolerated. When the scale difference exceeds considerably, the continuous observation and measurement become difficult.

22.12 STEREOSCOPES

Instruments used for viewing stereo pairs are called stereoscopes.

There are two types of stereoscopes generally used for viewing the aerial photographs.

(*i*) Lens Stereoscopes.
(*ii*) Mirror Stereoscopes.

1. **Lens stereoscopes.** (Fig. 22.7) A lens stereoscopes consists of two single convex lenses with focal length of 250 mm and which are mounted on a frame. The distance between the two lenses is generally kept 65 mm, which is equal to the normal eye base. In adjustable

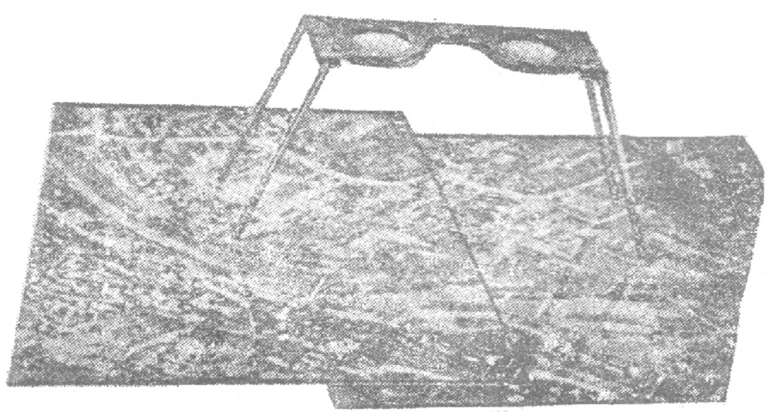


Fig. 22.7. A lens stereoscope.

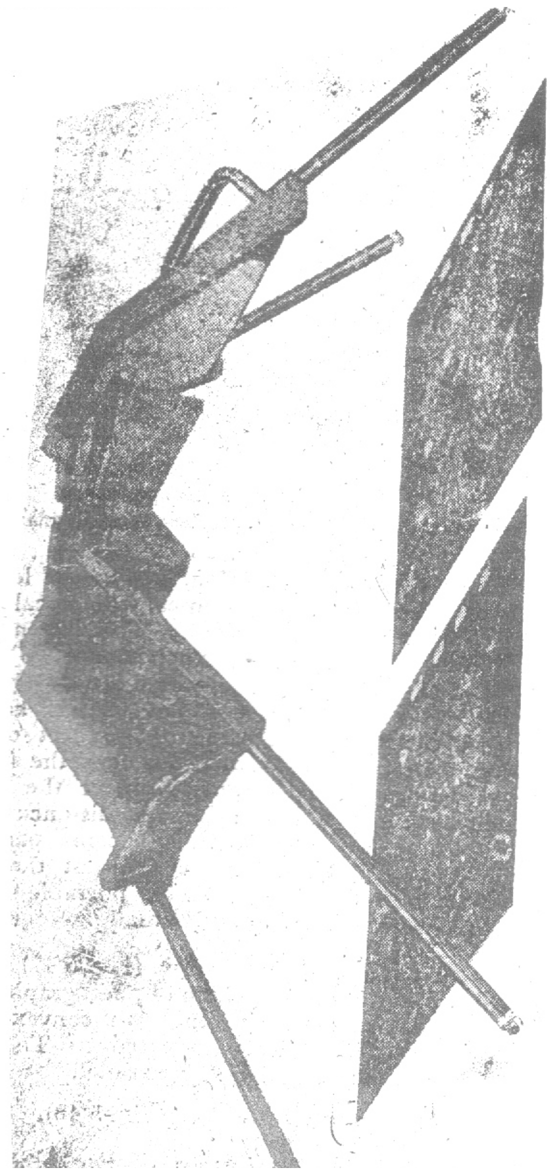


Fig. 22.8. A mirror stereoscope.
(Photograph by the courtesy of Carl Ziess, West Germany)

Fig. 22.8. A mirror stereoscope.
(Photograph by the courtesy of Carl Ziess, West Germany)

stereoscopes eye base can be adjusted from 58 mm to 72 mm. The pocket stereoscopes usually have planoconvex lenses. The distance between the nodal point of the lens and the plane of the photographs depends upon the focal length of the lens. The photographs are so placed under the stereoscope that corresponding details lie at about the same distance from each other. The rays from the image detail become parallel after passing through the lens, and the eyes can adjust themselves for sharp vision. This is a cheap and easily portable in pockets. The lens stereoscope is best suited only for small photographs.

2. **The Mirror Stereoscope.** (Fig. 22.8) Two pairs of plane mirrors inclined 45° to the plane of photographs with their reflecting surface facing each other and two convex lenses are the main components of a mirror stereoscope. The refractive power of the lens makes the out going rays parallel.

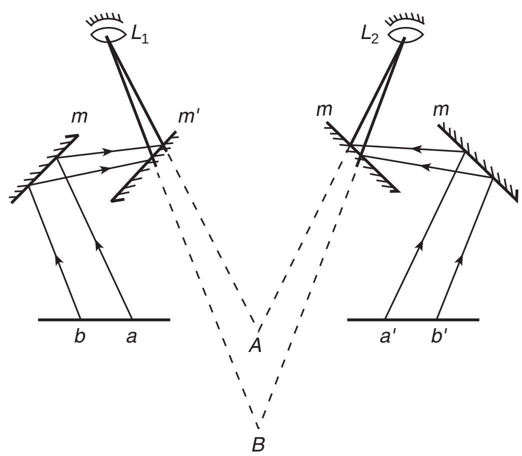


Fig. 22.9. Principle of mirror stereoscope.

Let *a',b'* and *a' b'* be the images of two objects on two successive photographs. Rays from these points on two photographs get reflected twice at plane mirrors *m* and *m'* and finally enter lenses L_1 and L_3. The fused images will appear at *A* and *B*. Mirror stereoscopes are generally provided with binoculars with 4 to 6 times magnification. These are usually used for continuous viewing of large photographs.

22.13. APPLICATION OF AERIAL PHOTO INTERPRETATION

The art of photo interpretation is being used for many disciplines of civil engineering. Important ones are briefly described below.

1. **Application to Geology.** The geological data and other surface features of the earth are shown on the aerial photographs. Now-a-days, geological study on an area is made with the help of aerial photographs and the art of examining geological feature is known as photogeology. The analysis of surface forms is made by the application of the principles of geomorphology. Detail study of aerial photographs is made by the stereoscopic examination of stereo pairs of photographs.

In the three dimension model when viewed under stereoscope, it is possible to identify geologic features, *i.e.*, folds, faults, dikes, veins etc. It is also possible to study the effect of weathering agents such s wind, water and glaciers.

For a geologist, aerial photographs are best maps, on which he plots directly geologic data. With the help of aerial photographs. geological exploration and field checking are considerably reduced. As various minerals usually occur in their specific morphological complexes, photographs which do not show the associative morphology can be eliminated by a systematic examination under stereoscope. 1 : 20,000 scale vertical photography is usually used for geological interpretation.

2. **Application to soil survey.** Aerial photographs are of great help for soil survey and mapping. With the help of images it is possible to classify the soils on the surface of the earth. These aerial photographs are also best base maps on which soil boundaries can be delineated under stereoscopic viewing. The aerial photographs provide an over all view of a large area, uninterrupted by vegetative cover, topographical features and other cultural details. General classification of soils of the earth's surface may be done with aerial photographs and for detailed study of the soils, field checking and laboratory test are essentially required.

A systematic analysis of pedological elements on the aerial-photographs is made to study the soils. After marking the boundaries of various soil complexes, tentative classification of the mapping units are made. The result of photo interpretation are incorporated on a photo-analytical map which is finally checked in the field and necessary corrections made. The photo-analysis thus reduces the field work by 75%.

The correctness of the results of photo interpretation for soil survey largely depends upon the quality and interpretability of aerial photographs and also on the scale of the photography, the nature of the terrain and atmospheric conditions at the time of photography. For soil survey and mapping, the photography is generally made on 1 :20,000 scale and the contact print 1 : 10,000 scale are used in the field.

EXERCISE 22

1. (*a*) Describe in detail how ground photogrammetry is conducted in field and in office. For what type of country photogrammetry is suited?

(*b*) Explain the following terms in connection with the aerial survey.

(*i*) Scale (*ii*) Overlap

(*iii*) Flight lines.

2. How does an aerial photograph differ from a map ?

3 Explain the following terms :

(*i*) Mean principal base (*ii*) Iso-Centre

(*iii*) Height distortion.

4. Write short notes on the following :

(*a*) Stereoscopy (*b*) Oblique photography

(*c*) Photo mosaic (*d*) Non-vertical photograph

(*e*) Scale of vertical photograph.

5. Define the following terms :

(*i*) Air base (*ii*) Principal point

(*iii*) Plumb point (*iv*) Iso-centre

(*v*) Tilt (*vi*) Vertical photograph.

6. Explain, with the help of neat sketch, the component parts of a phototheodolite. Also, describe the steps to plot the survey.

7. Define 'tilt distortion'."In a tilted photograph, tilt distortion is radial from the Iso-centre". Explain this.

8. (*a*) Write the important stages of the radial line methods of plotting.

(*b*) Describe the slotted template method of combination.

9. Write in brief what you know about photo interpretation.

10. Name the automatic photogrammetric plotting machines used now-a-days.

23

Modern Surveying Instruments

23.1 AUTOMATIC CONSTRUCTIONS LEVEL GKO-A

The Kern GKO-A is an automatic construction level. It is ruggedly built and is capable to withstand shock and vibration without affecting its operation. The toughness of the instrument is due to compact-die cast housing of high strength with corrosion-proof light metal.

The objective of the level is protected by extension of housing whereas the eyepiece is strongly built. The telescope is exceptionally achromatic and produces a sharp-high contrast image. The compensator which is provided to eliminate the levelling of the insturment's line of sight, before reading each staff, is supported by a strong precision ball bearing on a steel axis. Shock and vibration do not disturb the GKO-A compensator. The horizontal slow motion screw may be operated by either hand.

Observation with GKO-A. The GKO-A level is very simple to use. Every observer quickly becomes familiar with the instrument. As the instrument is not provided with foot screws, but a jointed-head, it is very convenient to make observations. The following steps are followed:

(*i*) Place the instrument on the tripod head and secure it with the fastening screw.

(*ii*) Shift the instrument over the spherical surface of the tripod head until the bull's-eye mounted within the housing, is centered.

(*iii*) Tighten the fasting screw.

The level GKO-A is shown in Fig. 23.1.

Laying out angles with GKO-A. For measurement and layout of angles, the GKO-A is provided with a 360° or 400 g horizontal circle and a reading magnifier. The instruments which are not provided horizontal circles, are equipped with an optical square to enable to get horizontal sights at right angles to the line of collimation on either side.

Fig. 23.1. Kern GKO-A level
(Courtsy of Kern and Co. Ltd., Switzerland)

Specifications of GKO-A level. The salient-specifications of the GKO-A Kern level are the following :

1.	Mean error in 1 km (double run)	...	+ 5 mm
2.	Telescope magnification	...	21 ?
3.	Objective aperture	...	30 mm
4.	Shortest focusing distance	...	0.75 m
5.	Diameter of field at 1000 ft. 305 (m)	...	30 ft. (9 m)

6.	Stadia multiplication constant	...	100
7.	Stadia addition constant	...	0
8.	Sensivity of bull's level	...	20′ per 2 mm
9.	Compensator's working range	...	± 30′
10.	Compensator centering precision	...	± 3″
11.	Weight of instrument	...	1.9 kg
12.	Weight of carrying case	...	0.8 kg

23.2 AUTOMATIC ENGINEER'S LEVEL GK1-A

The Kern GK1-A is an automatic engineer's level. It is exceptionally elegant, handy and accurate. The magnetic compensator suspension of GK1-A its a a special feature. The pendulum compensator is suspended in the field of force of a permanent magnet. The conical ends of the pendulum axis are centered between the equally conical shaped poles of the magnet. As three is no friction involved. It provides an exceptionally high balancing accuracy.

The folded ray path of the telescope has enabled the manufacturer to reduce the dimensions of the instrument. The housing consists of two parts and is carefully sealed for effective protection against moisture and dust. As it meets all the needs of engineering and construction survey, all engineering prefer to use this instrument in preference to spirit level type.

The main characteristics of GK1-A. The main characteristics of GK-1 are sd under :

1. Its reliable accuracy and problem-free handling.
2. There is no need for the time-consuming centering of a sensitive telescope bubble as a compensator is provided to take care automatically of the levelling of the line of sight to a constant accuracy of ± 1″. The oscillation of the pendulum is affectively braked by a pneumatic damping system.
3. The pendulum which is designed symmetrically, ensures that its centre of gravity does not vary with changes in temperature.
4. Its permanent adjustment remains unaffected.
5. Its objective is protected by the extensive projection of the objective hood against a fall and also allows unimpeded sighting even in sunshine and rain.
6. Its high-powered telescope is exceptionally achromatic and produce a sharp high contrast image.

Observations with GK1-A. The following procedure is adopted to make observations with Kern GK1-A :

(*i*) Place the instrument on the tripod head and secure it with the fastening screw.

(*ii*) Shift the instrument over the spherical surface of the tripod head until bull's-eye level is centered.

(*iii*) Tighten the fastening screw.

(*iv*) Use the horizontal slow-motion screw for convenient and exact-pointing of the telescope on the levelling staff.

Specifications of GK1-A Level. The salient specifications of the GK-A Kern level are detailed belows :

1. Mean error in 1 km (double run) ... ? 2.5 mm

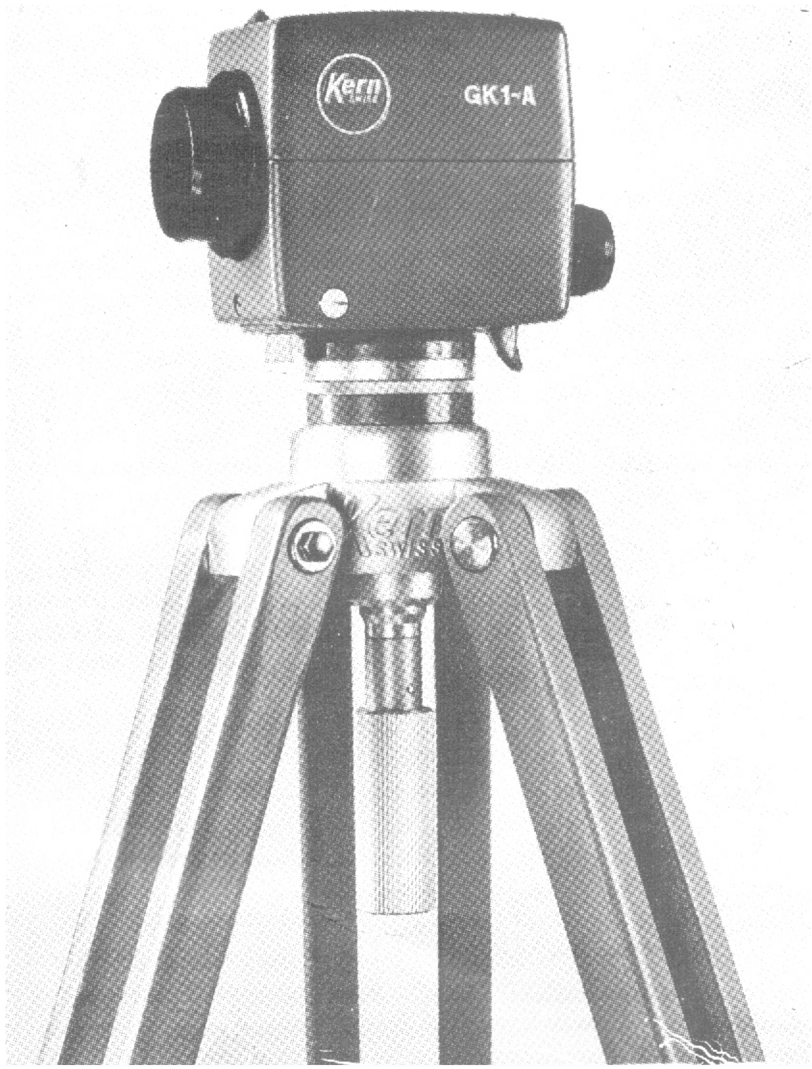


Fig. 23.2. Kern GK1-A level.
(Courtsy of Kern and Co. Ltd., Switzerland)

2.	Telescope magnification	...	25 ×
3.	Objective aperture	...	45 mm
4.	Shortest focusing distance	...	2.3 m
5.	Diameter of field of view at 1000 ft.	...	25 ft.
6.	Stadia multiplication constant	...	100
7.	Stadia additive constant	...	0
8.	Sensitivity of bull's-eye level	...	12′ to 15′ per 2 mm
9.	Compensator working range	...	± 10′
10.	Compensator centering accuracy	...	± . 15″ to 1.5″
11.	Diameter of horizontal circle	...	60 mm
12.	Circle reading with magnifier estimation to	...	0.1°

23.3 KERN GK2-A PRECISE AUTOMATIC LEVEL

The Kern GK2-A is most precise and reliable level. It is primarily designed by using a pendulum compensator with its very high centering accuracy, the superior telescope optical micrometer which makes use of the accuracy of the compensator to its fullest. Its pendulum compensator is suspended in the field of a permanent magnet and not by sensitive metal strips or wires. Due to frictionless suspension, the compensator achieves extraordinary, centering accuracy ± 0.3″. As the pendulum is of symmetrical design, the centre of gravity is not affected by the change in temperature and thus its adjustment remains more or less constant. The pendulum motion is quickly retarted by a pneumatic damping device. The compensator includes magnetic system, pendulum and damper as a component easily replacable by the service station whenever a need arises.

Telescope. The Kern GK2-A is fitted with a powerful telescope of 32.5 X magnification, and capable of producing an upright, clear image with excellent contrast. All the components of the optical system are properly coated for anti-reflection. The focussing system of the telescope is designed with three lenses.

GK2-A as Universal Level. As this instrument does not use any micrometer and provides sufficient precision for all height measurement in engineering, it is worth calling GK2-A as 'universal level'. By using a micrometer and an invar level rod, the GK2-A can be used as a precision level.

Applications of GK2-A : Without micrometer, the GK2-A can be used as engineer's level for the following purpose :

(*i*) For the establishment of bench marks.

(*ii*) For transferring bench-marks elevations at construction sites.

(*iii*) Determination of elevations for engineering projects, highway construction and hydrographic projects.

(*iv*) For industrial survey projects with micrometer, the precision level can be used for :

(*a*) Determination of precise reference elevations (BMs) of highways, railways, bridges, tunnels, and power plant.

(*b*) Settlement and deformation surveys at construction sites and installation of heavy machinery.

(*c*) Determination of highly precise elevations, vertical control and alignment points in industrial projects.

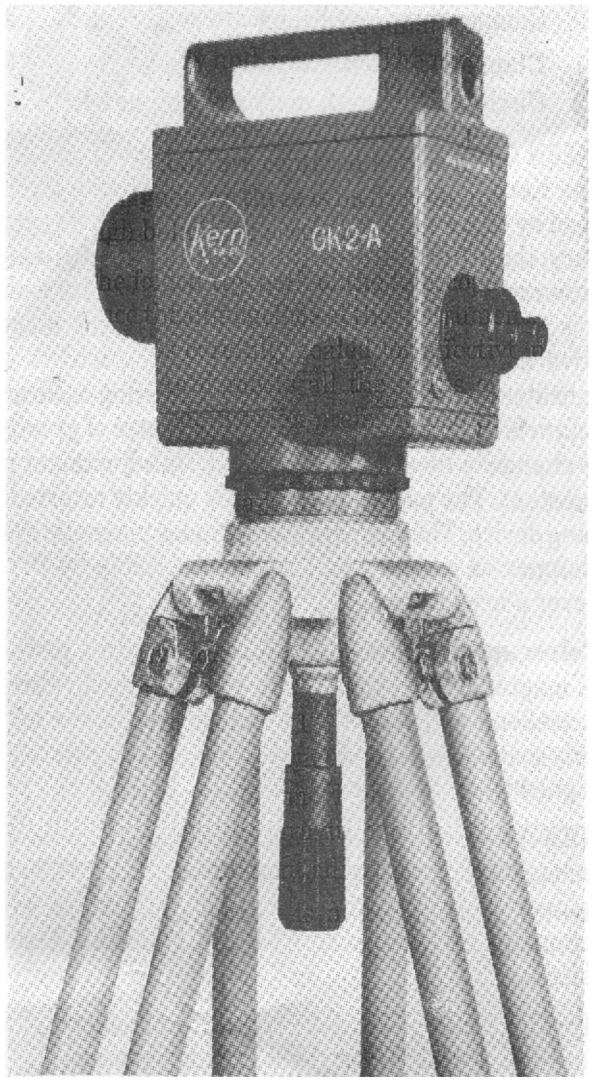


Fig. 23.3. Kern GK2-A precise automatic level.
(By courtesy Kern Company Ltd.)

Simple and economical operation and observation. It is very convenient to operate and use the GK2-A level.

For ease of operation of the instrument the focussing knob 1. and micrometer knob 2. are provided on the right side of the instrument.

The continuous slow motion (3) may be operated from either side.

The bull's-eye level (4), telescope eyepiece (5) circle reading eyepiece (6) micrometer reading eyepiece (7) can be viewed conveniently from one and the same position.

Focusing the telescope. The image of the levelling rod is rapidly and approximately brought into focus with the fast motion. The reversal of the motion automatically engages the slow motion for precise focussing of the object.

Self levelling of line of sight. The GK2-A does not require time to level it as in other conventional types of level. Its compensator automatically levels the line of sight with a constant precision of 0.3″.

For making an observation, the surveyor needs only level the instrument coarsely, point the levelling rod, focus the telescope and finally take a reading.

23.4 PRECISION LEVEL WILD N_3

This instruments is well suited to precision and reliability. The classical use of the instrument is for geodesy particularly for the measurement of national levelling networks. It is also suitable for the highest accuracy requirements in engineering and for the control of structures. With the advanced technology, a precision level of the highest class is found necessary in industries, laboratories and for research for making special measurements. The N_3 is most suitable in such situations.

Keeping in view the inherent qualities of a well designed precision that an automatic level can hardly match a spirit level for producing a basic stability, the N_3 level is designed as a spirit level instrument. Of course, the Wild NA2 with parallel plate micrometer, attains the levelling accuracies approaching that of the Wild N_3, but there are circumstances and fields of application where the spirit level instrument has an edge over others. The N_3 is capable of attaining reliable results even in windy conditions because of its tubular level and the stability of the instrument. This instrument is also relatively unaffected by vibrations and strong permanent magnetic fields in laboratories and research establishments.

Suitability of Wild N_3. The Wild N_3 level is specially suited for the following :

1. **First order geodetic levelling.** It is generally used for first order geodetic levelling for national networks for establishing the geodetic bench-marks with 0.2 mm standard deviation for 1 km double

run. It also facilities to carry level lines over wide rivers for determining crystal movements and height measurements for scientific studies.

2. **Engineering and deformation surveys.** It is a most suitable level for providing height control for engineering projects, deformation, subsidence measurements and checking bridge and structure, etc.

3. **Optical tooling, industry and special tasks.** It is also most suitable for the following tasks:

(*i*) Measuring the heights of machine components.

(*ii*) Setting base plates and rollers horizontal and at the required height.

(*iii*) Investigation the deformation of bearings and drive shafts.

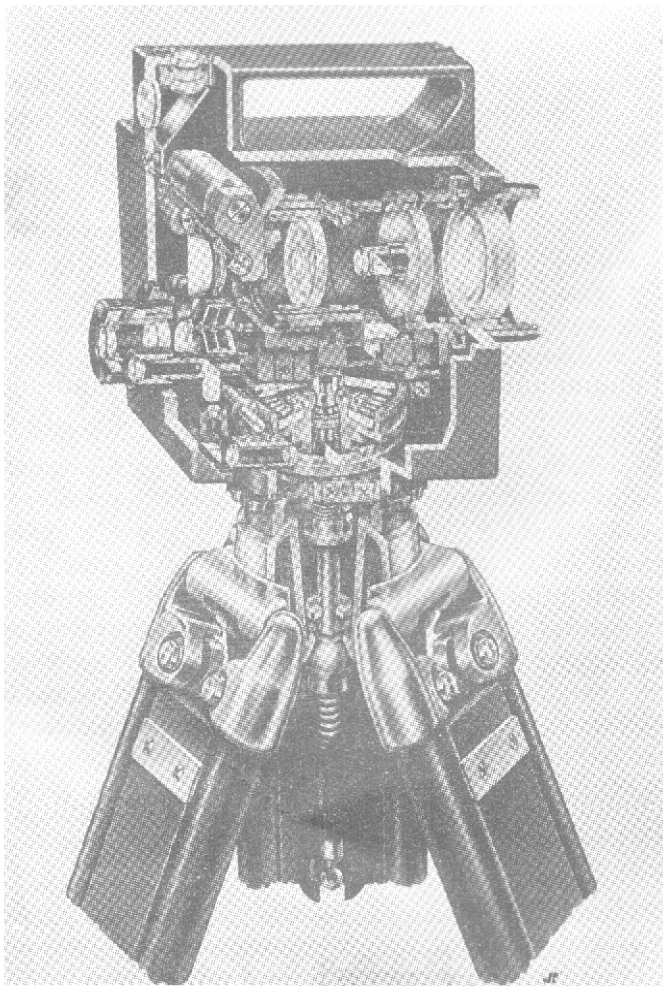


Fig. 23.4. Sectional view of internal structure of GK2-A.

(*iv*) Measuring the flatness of beds, block and plates.

(*v*) Aligning and positioning of components.

(*vi*) Checking the straightness of axes and rails.

(*vii*) Establishing and calibrating optical and mechanical control system.

Special features of Wild N_3 level. The special features of the instrument are as under :

(*i*) A large, light-gathering objective of the telescope provides a bright, high contrast image, even in poor light. Magnification power of the telescope is well suited for the highest precision work.

(*ii*) In contrast to anallactic telescope, both the magnification and the field of view of the Wild N_3 telescope vary with focussing distance.

(*iii*) A target 30 cm in front of the cover glass can be brought into perfect focus, *i.e.,* it has extremely short minimum-focussing distance. A short minimum focus is essential for measurements in industry and laboratories.

(*iv*) The bore of the telescope tube, the centring of the optics and the run of the focussing lens are all to a high degree of exactness. These factors and the inherent stability of the instrument, make the N_3 as an alignment telescope.

(*v*) For quick setting of the bubble ends in coincidence, an arrow us provided to indicate how to turn the tilting screw.

(*vi*) The graduated tilting screw provides the geodetic engineer a means to carry 1st-order level lines over wide rivers and gorges. One interval of the tilting screw drum corresponds to 1 : 100,000. One drum has 50 intervals and the range of the tilting drum is 8 revolutions.

Observations with the Wild N_3 level. The following procedure is adopted for making observations with the Wild N_3 level :

(*i*) Setup the level at a comfortable height for making observations.

(*ii*) View the circular bubble via the penta-prism and center it by turning the rapid action foot screws.

(*iii*) Put the tilting screw to the approximately required position.

(*ii*) See the ends of the spilt bubble.

(*v*) Set the line of sight horizontal by turning the tilting screw slightly.

(*vi*) Take the reading of the levelling staff.

Levelling staves for the N_3 level. The invar staves which are used for making observations with the N_3, have the following specifications.

(*i*) The invar strip which is graduated to 1 cm graduations, has negligible coefficient of expansion (1 micron per 1 m per 1°C).

(*ii*) The invar strip lies in a groove in the wooden staff.

(*iii*) The invar strip is attached in such a way that the graduated strip is not affected with any expansion contraction.

(*iv*) The invar staff are capable of holding by hand or by the telescope supporting struts according to the desired accuracy.

Specifications of precision level Wild N_3. The salient specifications of Wild N_3 are as under :

(*i*)	Standard deviation for 1 km double run levelling	... ± 0.2 mm
(*ii*)	Telescope with panfocal optics clear objective aperture	... 52 mm
(*iii*)	Shortest focussing distance : Standing axis to target	... 45 cn
	Cover glass to target	... 28 cm
(*iv*)	Setting accuracy of split bubble circular bubble sensivity per 2 mm	... 2′
(*v*)	Parallel plate micrometer with glass scale	... Range Interval 10 mm 0.1 mm
(*vi*)	Environmental range	... – 30° to + 60°C

The N_3 Wild level is shown in Fig. 23.5.

23.5 WILD T1 MICROMETER THEODOLITE

The wild T1 is a micrometer theodolite which is a successor of the Wild T1A. It is completely redesigned with a new reading system and more powerful telescope. The Wild T1 provides accuracy, comfort, and 100% free from error digital readings. With its wide range of accessories, it is an ideal micrometer theodolite for all types of survey work.

Optical sight and coarse-focussing. The T1 has an optical sight for easy and quick aiming the target. By pointing its bright white cross, the target comes in the telescope's field of view.

Focussing of its telescope is very easy. By simply turning the sleeve, the target instantly comes into view. By giving a fine motion a perfect, parallel-force setting is obtained. Arrows on the sleeve are provided to indicate infinity. The shortest distance of this theodolite is 1.7. m. As the telescope is capable of transting at both ends, observations can be taken in both faces.

Bright-high-contrast errect image. This instrument has a large objective that gathers maximum light. As the optics have anti-reflection coatings, the powerful telescope gives a bright, high contrast errect image and enables perfect target bisection even in poor conditions. The stadia hairs have also been provided for distance measurements.

A standard eyepiece of 30 X magnification is provided for most applications. The eyepieces of other magnification are also available, which can be interchanged in few seconds easily if required.

Digital Reading. The T1 reading system is digital and it is almost impossible to make mistakes by the observer. Both the circles (horizontal and vertical) are seen simultaneously in the reading microscope. To differentiate between horizontal and vertical, the horizontal circle appears yellow and the vertical circle appears white. As the large mirror throws an enormous amount of light into the instrument, the illumination of the reading microscope. To differentiate between horizontal and vertical, the horizontal circle appears yellow and the vertical circle into the instrument, the illumination of the reading microscope is excellent.

Fig. 23.5. Precision level wild N_3.

Graduations of circles and micrometer. Both the circles are graduated to 1° intervals whereas each micrometer interval is 6″ (or 0.2^g) and numbered. While reading, the circle graduation line is first set between the double index and the reading is read directly in figures. The estimation can be made easily to half an interval *i.e.*, 3″ (0.1^g).

With the help of the automatic index, the vertical angle measurement is not only quick but also accurate. Automatic index is achieved with the help of a liquid compensator which attains excellent damping properly. The setting accuracy is ± 1″.

Instrument control system. The clamps and drives of the wild T1 are so arranged that the instrument can be handled with one or both hands. For example while tracking a star for astronomical observation one hand is used for moving the instrument in azimuth while other hand to used for raising or lowering the telescope in vertical plane. The drives have long ranges of about 6°. As the centre of the range is marked, the observer is able to set the screw fore equal run on either side. Overturning is prevented by clamps.

With the help of the upper and lower plate clamps and drives, it is very easy to set zero or initial reading. These controls are also very useful to make observation by the method of repetition or to carry the bearings from station to station while carrying out theodolite traversing. Control knobs of different shapes are provided to prevent to confusion while making observation.

The optical plummet is provided, which focusses from 50 mm to infinity. At the base of the instrument, a centring flange with three holding studs ar provided for forced-centering in a Wild tribarch.

The instrument's centring flange fits exactly into the tribarch dish. Unintensional release of the instrument is prevented with the help of a locking knob. Levelling up of the instrument is quick and positive.

The T1 and D135 Distomat. The T1 and D135 Distomat can be easily combined to form an electronic reduction tacheometer. The combination provides a complete. EDM survey system for control traverses, trigonometrical heights, cadastral surveys, contouring profiles setting out works, etc.

The T1 + D135 combination is highly versatile. Both horizontal and vertical angles are measured in seconds. On tapping the vertical angle, the horizontal distance and the difference in height are displayed. By entering the bearing of the line, it is possible to get the co-ordinate differences, ΔX and ΔY.

Technical specifications of the T1

(*i*) Telescope ... erect image

(*ii*) Magnification with standard eyepiece ... 30 X

(*iii*)	Clear objective aperture	... 42 mm
(*iv*)	Field of view at 1000 m	... 27 mm
(*v*)	Shortest focussing distance	... 1.7 m
(*vi*)	Multiplication factor	... 100
(*vii*)	Additive constant	... 0
(*viii*)	Bubble sensitivity per 2 mm run	
	Circular level	... 8′
	Plate level	... 30″
(*ix*)	Automatic vertical index	
	Setting accuracy	... ± 1″
	Working range	... ± 2″
(*x*)	Glass circles graduation diameter of horizontal and vertical circles	... 79 mm
	Graduation interval of horizontal and vertical circles	... 1°
(*xi*)	Direct reading on micrometer	... 6″
(*x*)	By estimation	... 3″

(*By Courtsy of Wild Heerbrug Ltd. Switzerland*)

The Wild T-1 theodolite is shown in Fig. 23.6.

Uses of the Wild T1. This theodolite can be used for all routine survey work in civil engineering, construction industry and land survey as mentioned below :

(*i*) Traversing and minor triangulation
(*ii*) Trigonometrical levelling
(*iii*) Civil engineering setting out works
(*iv*) Mining and tunneling
(*v*) Alignment measurements in industry
(*vi*) Cadastral surveys and probe divisions
(*vii*) Tacheometric surveys
(*viii*) Field astronomy
(*ix*) Compass traversing

23.6 WILD T2 UNIVERSAL THEODOLITE

It is the most popular one-second reading theodolite used throughout the world. Steel, the basis of the T2 gives strength and stability. As it expands and contracts almost like optical glass, the precision throughout the widest temperature range is maintained. Special types of oils and greases keep the instrument in perfect working orders in all climates.

Fig. 23.6. The wild T-1 theodolite

Telescope. A powerful, erect image telescope possessing the superb light-collecting properties, is fitted in this instrument. The loses of the telescope are almost totally colour-correct through reduction of the secondary spectrum and are provided special anti-reflection coatings. These lenses produce a bright, high contrast image fro correct pointing even in poor light.

Eyepiece. Its standard eyepiece gives 30 X magnification. Arrows on the focussing sleeves indicate the direction to infinity.

Transiting the telescope. The telescope transits at either end with objective and eyepiece accessories for observations in both faces. Two optical sights, one above and one below are provided to facilitate rapid target location in either telescope position.

For steep sighting, eyepiece prisms and diagonal eyepieces are also provided as accessories.

Horizontal and vertical reading circles. A selector knob is provided to change the required reading circle *i.e.,* horizontal or vertical as seen in the reading microscope. The horizontal circle appears yellow and the vertical circle appears. The viewing of one circle at a time is advantageous because the circle graduation lines are seen at the centre of the microscope field of view. The centre of the microscope is the optimum position for an eye to judge a perfect coincidence setting.

When either circle is read, the value read is the mean reading of two diametrically opposite points to ensure the elimination of the influence of any residual circle eccentricity. The paths of the light rays for illumination and image formation of both circle positions are provided symmetrical to ensure to see graduation lines in the reading microscope of same strength and quality, an essential for exact coincidence setting.

The digital reading which is quick and error free, is provided in the theodolite. After setting the circle graduations in coincidence by turning the micrometer knob, the observer reads directly in figures with only the single seconds being from the scale.

For setting the horizontal circle for different zeroes, a circle drive knob is used. The vertical circle readings in face left are zenith angles. The two reading points for the vertical circle are brought horizontal automatically by a pendulum compensator.

Applications. The T2 is most suitable for the following :

(*i*) Triangulation up to 2nd order

(*ii*) Precise traversing.

Distance measurement with Wild T2. The T2 reticle has stadia hairs on the vertical and horizontal cross hairs. The staff intercept cut by the stadia hairs gives the distance to about 10 cm if sights are upto about 100 m. For obtaining horizontal distance from inclined sights tacheometric tables are used.

The T-2 may be easily combined with the D135 distomat to form a distance/angle measurement of high precision. On entering the vertical angle, the horizontal distance and difference in height are displayed. By tapping the horizontal angle, coordinate differences are displayed.

The T2 + D135 is an electronic reduction tacheometer for almost every type of survey work such as precise traversing, trigometrical heights, cadastral surveys, layouts, detail surveys, etc.

Technical Data of Wild T2

1.	Telescope	... erect image
2.	Magnification with standard eyepiece	... 30 X

3. Clear objective aperture ... 40 cm
4. Field of view at 1000 m ... 29 m
5. Shortest focussing distance ... 2.2 m
6. Multiplication factor ... 100
7. Additive constant ... 0
8. Bubble sensivity per 1 mm run circular level, standard ... 20″
9. Graduation diameter
   Horizontal circle ... 90 mm
   Vertical circle ... 70 mm
10. Graduation interval of horizontal and vertical circle ... 20′
11. Smallest interval of optical micrometer ... 1″

(By courtesy of Wild Heerburg Switzerland)

Application of Wild T2 theodolite. The wild T2 is a most suitable theodolite whenever 1″ accuracy is needed as mentioned under :

1. Triangulation and traversing
2. Precise construction
3. Tunnelling and mining
4. Cadestral surveys
5. Deformation studies
6. Optical tooling and laboratories measurements
7. Field astronomy

The Wild T2 theodolite is shown in Fig. 23.7.

23.7 THE KERN DKM2A ONE-SECOND THEODOLITE

The Kern DKM2A one-second theodolite has been designed to attain high measuring accuracy, simple operation and maximum stability under any environmental conditions.

Telescope. The DKM2A theodolite is fitted with a high resolution telescope that produce a sharp image with maximum contrast.

Reading horizontal and vertical circles. The digital read out eliminates all possibilities of reading errors. Its digital read out is explicit. The operating knobs are in logical sequence and as such assure operating convenience. Both tangent screws are equipped with coarse and fine drive enabling precise pointing. An automatic vertical compensator is used to increase read out efficiency and precision of the vertical angle.

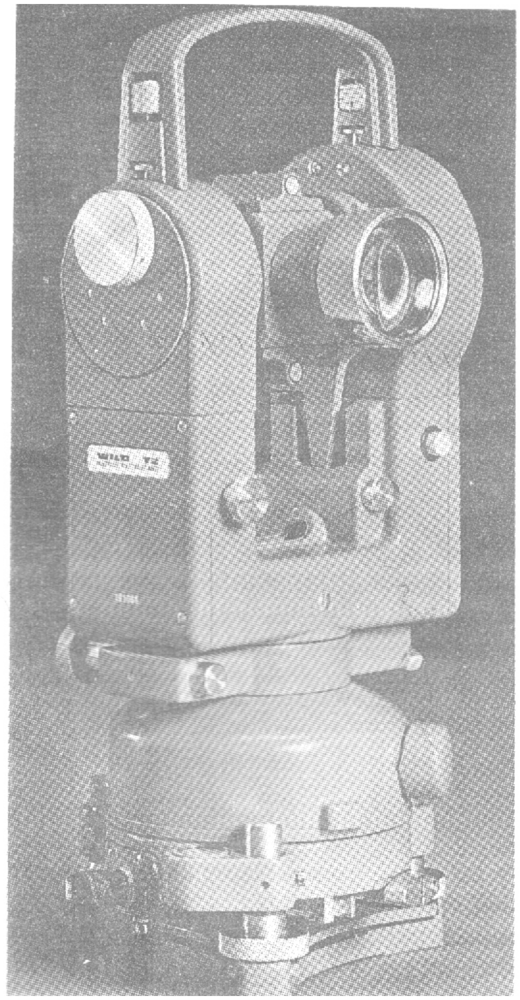


Fig. 23.7. The wild T_2 theodolite

The read out for both horizontal and vertical circles appears simultaneously in the circle reading eyepiece fitted with green filter. The one-second theodolite is designed with double-circle reading principle. The summetry "setting" is used estimate equal distance between two narrow vertical lines. The horizontal circle is designated with H and that of vertical circle by V.

Upto and including the 10 seconds graduation, all values are read directly in digits. The horizontal circle is quickly rotated through 360? with the coarse drive and set easily and precisely to any desired value with the fine drive. Both drives are protected by a hinged cover against accidental operation.

The DKM2A theodolite is equipped an optical plummet which is located in the alidade and can be self-checked and adjusted by alidade rotation. The focussing range of the optical plummet is from 0.7 m to infinity.

Technical data

1.	Telescope magnification	... 32 X
2.	Objective aperture	... 45 mm
3.	Shortest focussing distance	... 1.5 mm
4.	Field of view at 1000 mm	... 27 mm
5.	Multiplication constant	... 100
6.	Additive constant	... 0
7.	Diameter of horizontal circle	... 80 mm
8.	Diameter of vertical circle	... 74 mm
9.	Circle reading direct	... 1″
10.	Circle reading by estimation	... 0.1″
11.	Sensitivity of plate level per 2 mm	... 20^2
12.	Focussing range of optical plummet	... 0.7 m to ∞

(*By Courtsy of Wild Heerburg Switzerland*)

The Kern DKM2A level is shown in Fig. 23.8.

23.8. TOTAL STATIONS

The era of purely mechanical and optical instruments is over and that has been replaced by an era having digital instruments in all spheres of life including in surveying instruments. Electronic theodolites and total stations are replaced by conventional theodolites. Total stations have dramatically enhanced work efficiency by reducing operation the and eliminating observer fatigue when sighting the survey signals or targets.

Z_1 –8060 **Auto pointing total stations.** While making observations with this total stations, roughly pointing at the prism is made and the measurement key is pressed. Looking through the telescope for accurate sighting if the target is not required. Auto pointing function works with a single APO1 prism from 2 m to 800 m with an accuracy of 2.5 mm upto 100 m. The handling of total stations is so easy that beginner and an experienced surveyor are able to make measurements with the same accuracy and speed. Moreover, in cloudy and hazy climatic atmosphere, observations with a total station are made with the same speed as clear sunny days. The guide line leads the prism along the telescope sighting for setting out. Highly practical and functional softwares are available for enhancing the survey field work. Internal memory of total stations has sufficient storage-roughly 20,000 points of data.

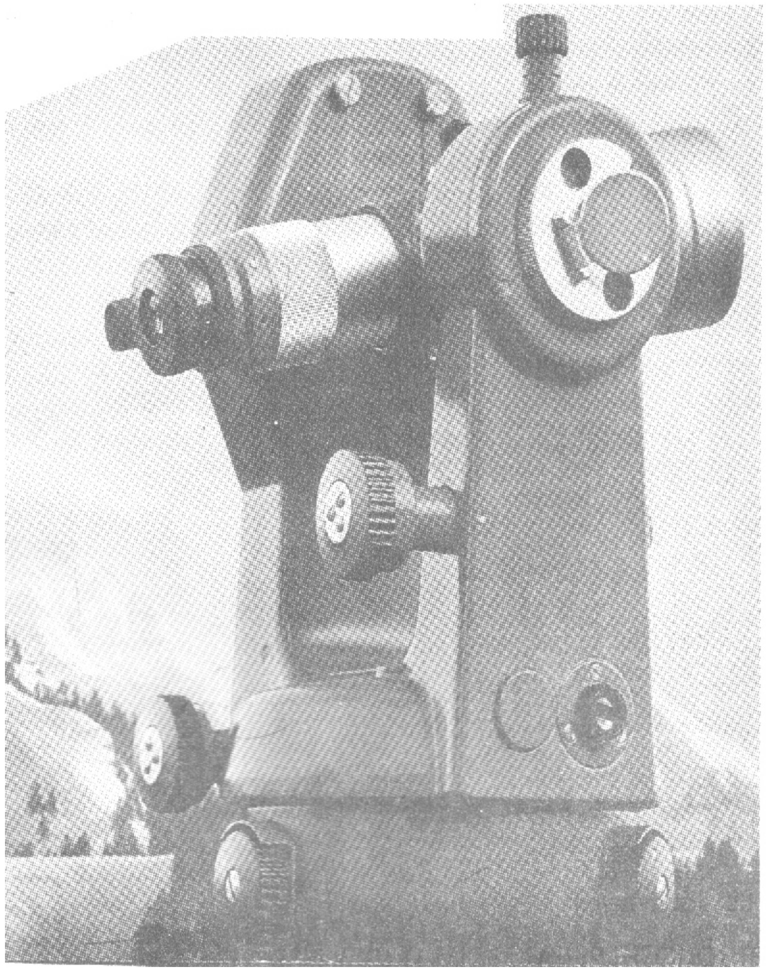


Fig. 23.8. The Kern DKM2A one second theodolite.

By courtesy, Zeal International, 1, Netaji Subhash Marg, Daryaganj, New Delhi-110002 (India)

For quick centering the total station over the ground station, a laser plummet is provided. Focus and laser plummet intensity can be adjusted as pr the requirements. The laser spot falling on the ground surface is clear enough to be seen under the sunlight. Separate provision to turn off the laser power is provided separately from the system power.

Total station observations are made in the same way as by other conventional theodolite. In total stations observations computation of data is done by the instrument itself. The net result is displaced on the key pad.

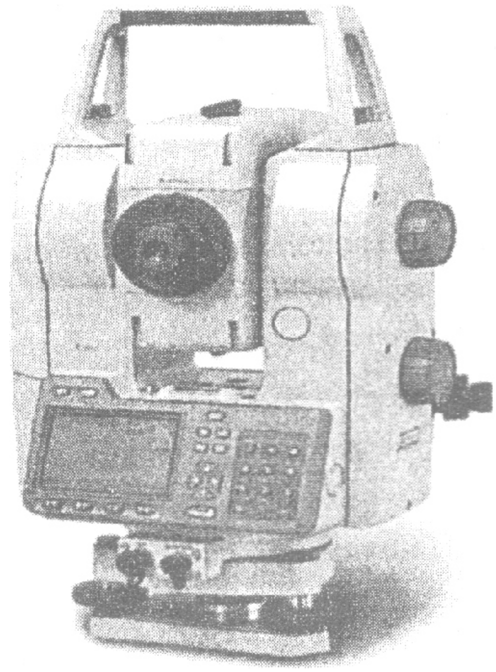


Fig. 23.9. Z1 ?8060 auto printing total station.

Method of resection of ground station by making observations to two control points is explained to familiarise the net result explained to familiarise the observers.

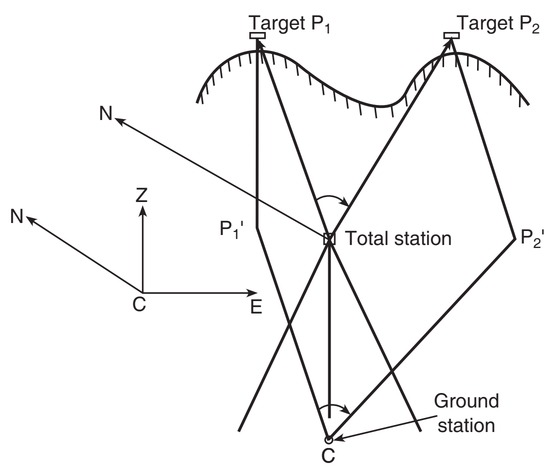


Fig. 23.10. Setting up the total station for obtaining the co-ordinates of the station of observations C.

Choose two control stations P_1 P_2. Set up the total station over the desired ground station. Observe the horizontal angle P_1CP_2. Press the key pad. Obtain the co-ordinates of station of observation (Fig. 23.10)

Z_1 ?8061 **Non-Prism Total Stations.** In this type of total stations, no prism is provided. This total station measures the distance with or without reflectors. This type of total station is an ideal instrument for making measurements to inaccessible location or in high traffic regions, structural measurements of buildings, surveying management for strip mining, etc. It accurately measures distance to 85 metres without a reflector. Reflection sheet targets extend the measuring distance to about 500 metres (Fig. 13.11).

Fig. 23.11. Z_1 8061. Non prism total station.

This instrument provides precise measurement even to objects that are at inclination to the line of right. Highly practical and functional softwares are also available to enhance the survey work. Nickel-metal hydride battery having a service of 5.5 hours is provided.

The operation of the total station is similar to that of Z_1 8060 total station.

By Courtsey Zeal International, 1, Netaji Subhash Marg, Daryaganj, New Delhi-110002 (India)

EXERCISE 23

1. Write a brief description of a modern automatic level.

2. Describe the main characteristics of modern levels

3. Write the special features of Wild N_3 level and describe the process of observations with it.

4. Write a brief note on Wild T2 Universal Theodolite.

5. What is a total station? Why is it preferred in surveying these days, Explain briefly.

24

Remote Sensing System

24.1. INTRODUCTION

Remote sensing is a terminology which refers to any method adopted for gathering information about an object without actually coming in contact with it. In a broader sense, the term 'remote sensing' is used more commonly to denote identification of earth features by detecting their characteristics with the help of electromagnetic radiations either reflected or emitted by the earth surface features. The advent of satellites for weather forecasting, for communications, for studying the earth and for studying the space, is one of the most exciting developments of the modern times with an extensive application of remote sensing for resource management. The United Nations has defined the remote sensing on December 3, 1986 as under:

Remote sensing means seeing the earth's surface by making use of the properties of electromagnetic waves emitted, reflected or diffracted by the sensed objects for the purpose of improving the natural resource management, land use and protection of the environment.

The remote sensing techniques by aeroplanes and satellites in combination with the ground surveys, has revolutionised topographical surveys. The technique of remote sensing has made available to man visible to nacked eye macroscopic, affording a comprehensive mental synoptic visualized view for time, in several bands to electromagnetic spectrum.

Space instruments and remote sensing tools are used to look at the earth with new eyes. Both the types of tools have helped the man to an extended vision of the earth's surface. Remote sensing in simple language means the sensation received from a distance, without any direct contact. It is a technology for sampling the electromagnetic radiations to acquire and interpret geospatial data to extract and develop information about features, objects and classes on the surface of the earth, and oceans and also in atmosphere. Detection and measurement of electromagnetic energy emanating from distant objects made of various materials to identify and categorize these objects by either class or type, is the prime job of the remote sensing.

24.2. FUNDAMENTAL PRINCIPLE OF REMOTE SENSING

The fundamental principle of remote sensing methods, is to measure the varying energy levels of a single entity, the *photon,* a quantum of electromagnetic energy proportional to the frequency of its radiation. The photon is the fundamental unit in electromagnetic (abbreviated as EM) force field.

The phenomenon of interference and polarization require *EM* radiation to behave like a wave while for some interaction such as in the photo electric effect, the radiation behaves like particles. In other words it may be said that *EM* radiation has duel nature-wave and particle. The particulate nature of the *EM* radiation is generally explained in terms of the Quantum Theory. According to this theory, the *EM* radiation propagates in space as discrete packets or quanta of energy propagating with the same speed (e_v) and direction defined by the wave theory. The energy of the photons is related by their frequencies *i.e.,*.

$$e_b = hv$$

where h is the Plank's constant (6.63×10^{-34} Ws^2).

Hence, the radiometric quantities differ at different wave lengths to enable to diagnose the material. Remote sensing of the earth's surface traditionally uses reflected energy in the visible spectrum (0.4 μ to 0.7 μ) and emitted energy in the thermal infrared energy in infrared and microwave regions together with radiation. Both the reflected energy and emitted energy can be analysed numerically and suitably used to generate images. The gathering to a range of wave lengths is termed as *multispectral remote sensing*. The images obtained from varying intensity signals show variations in grey tones in black and white images. In colour images, they differ in terms of *hue, saturation* and intensity.

24.3. BRIEF HISTORY OF INDIAN REMOTE SENSING (IRS)

India has a glorious tradition in space science since Vedic period. *Brahamanda* Sarah, is a treatise written by Vyasya in space studies. During *Vedic period*, Indians made observations to the sun, the moon, the planets and stars. The Vedic period calender predicted the transit of planets and eclipses accurately. Brahamanda, the famous atronomer of India was born in 476 AD.

Raja Swai Jai Singh (1686-1743) built observatories in five cities *i.e.,* Jaipur, Delhi, Mathura, Ujjain and Varanasi for making astronomical observations. A two-stage sounding rocket was launched in 1963 from a small village Thumba, on the west coast close to the equator in the state of Kerla.

In 1970's, India demonstrated the space applications such as Satellite Instructional Television Experiment (SITE) and Satellite Telecommunication Experiment with the help of the satellites belonging to other countries. Simultaneously, the satellite Aryabhata Bhaskara, APPLE and Rohini Series were built and also experimental Satellite Launch Vehicles (SLV-3, ASLV) were developed. In 1983, INSAT was successfully developed. The birth of Indian Remote sensing took place in 1988. Polar Satellite Launch Vehicle (PSLV) capable to launch 1250 kg pay load into 820 km polar orbit, was also developed.

Even though, it has been an endevour for man to map the configuration of the earth from tree tops, mountains and even from high rising balloons, the earth was viewed in totality only from satellites. In the last three decades, space borne remote sensing capabilities have grown to such an extent that space-based observations have become the prime source of information on earth's resources and its environment.

Remote sensing enables synoptic observations of large areas of the earth surface on repetitive basis. With a very high speed the satellite provides a global coverage with the same sensor or a set of sensors. In view of several beneficial applications, the Indian space efforts put considerable emphasis on realising an operational remote sensing programme.

In the area of remote sensing, India has the largest number of remote sensing satellites in operation. The most important ones are : IRS-IC and IRS-ID, the best civilian remote sensing satellites. IRS-P4 (OCEANSAT-1) launched in May 1999 is used to monitor ocean resources and for understanding the atmospheric conditions over the oceans. Satellites for cartographic applications are also launched..

Uses of IRS Satellite data. The data obtained from IRS satellite is used for the following purpose:

1. To estimate the acreage and yield of important crops.
2. To survey the dense forest coverage.
3. To forecast the drought conditions.
4. To map the flood area and to demarcate flood-risk zone.
5. To map the areas for land use and land cover for agro-climatic planning.
6. To map the waste land and to classify them.
7. To develop irrigation command area.
8. To survey the snow cover areas and the snow-melt run-off estimation of the Himalayan rivers.

Data from IRS is also usefully employed in other spheres including the sustainable development at micro level.

24.4. IMAGERIES VERSUS AERIAL PHOTOGRAPHS

1. **Aerial photographs.** The photographs taken from a camera station occupied by an aircraft in the air with axis of camera vertically

down, are called *aerial photographs.* Aerial photographs are described in chapter 22, photogrammetic survey in the textbook.

2. **Satellite imageries.** The photographs of the earth taken from space by satellite are called *imageries.* Because of very high flying heights of the satellite, imageries are on very small scale. Individual features of the earth are not easily discerned. For reading or viewing the satellite imageries, one has to study various types of tints represented by different features on the earth.

Formation of satellite images. The satellite images are collected by the satellite sensors on board a satellite and the same is relayed to earth as a series of electronic signals which on processing by a computer, produce an image of the earth's surface.

24.5. SIGNATURES

The technique of remote sensing is used to recognize an object or a feature from its surrounding on the surface of the earth. To achieve this, we should be familiar with the characteristics which distinguish them from their surroundings. The characteristic features which help to identify or recognise an object or a feature, is called *signature.* If an object or a feature is identified through the difference in the reflectance/emittance characteristics, with respect to wave length, then it is called *spectral signature.*

Characteristics of ground features. There are many other characteristics which are useful for identification of objects. The important ones are mentioned below:

1. Differences in scattering cross section with respect to polarisation in the microwave region.
2. Measurement of the temperature using thermal *IR* region. The thermal inertia provides signatures to identify certain objects.
3. Temporal variation like the growth profile difference of the plants also acts as signatures to differentiate the crops in agriculture remote sensing.

Parameters of signatures are however not completely determinisitic. They are statistical in nature. They have some mean value and some dispersion around.

For defining a sensor system, its time and frequency of observation and also to interpret it, one should understand the physical basis of signatures. We shall only deal with the signatures of the earth surface targets.

24.6. CLASSIFICATION OF THE FEATURES ON THE EARTH SURFACE

The features on the earth surface are broadly classified in the following categories :

1. Vegetation

2. Soil, rock, minerals
3. Water bodies
4. Snow
5. Man-made features.

Keeping in view, the interaction mechanism in the various regions of electromagnetic spectrum, the above five main features of the earth surface are dealt in three broad regions:

(*i*) Reflective optical infrared region (0.4 μm – 3 μm)

(*ii*) Thermal infrared region (8.0 μm – 14 μm)

(*iii*) Microwave region (1 cm – 30 cm)

1. Signatures in the Reflective O/R region. As the source of energy in this region is the sun, we first discuss the physical processes that give rise to signatures for various features on the earth surface.

(*i*) **Vegetation.** Vegetations as agricultural crops, or flora such as forest cover, bushes, shrubs, etc., are of great importance for the existence of human and animal life. Vegetation also plays an important role in regulating the carbondioxide (CO_2) through the process of photosynthesis and also in balancing solar radiation in the atmosphere. Solar radiation adversely affects the weather and overall climate of the region. Keeping in view these important factors, monitoring of vegetation has become most important application of the remote sensing technology.

From the space, through the eye of a remote sensor, vegetation is seen as the integrated effect of leaves, stems, branches, flowers, buds and appendages of plants and trees along with the back ground soil. The scattering and reflection due to vegetative cover largely change the direction and spectral composition of the incident radiation in a complex manner. The leaves of a vegetative canopy contribute to the major effect to the reflected energy as leaves of vegetation, cover the entire body of the tree or plant.

Due to large range of variations in the shape, size and surface including internal characteristics of leaves, there occurs large differences in spectral characteristics of the plants.

Plants absorb very efficiently the required energy throughout the ultravoilet and visible regions of the spectrum for the process of photosynthesis. Plants have very little absorption in the near *IR* region of spectrum. They absorb good amount of energy at wavelengths greater than about 2.5 μ m. It may be noted that solar radiation obtained is not much at spectrum having longer wavelengths. However, radiation obtained from the surroundings gets reflected by plants as they are good radiators and absorbers. Plant cells themselves help by radiation to control temperature.

2. Spectral reflectance mechanism. In spectral reflectance mechanism of a typical leaf of a tree is briefly described below :

(*a*) **Visible spectral region** (0.4 μm – 0.7 μm)**.** Absorption by leaf pigments dominates the reflectance characteristics in the region (0.4 μm – 0.7 μm) of the spectrum as explained under :

The blue (–0.45 μm) region and red (–0.67μm) in the visible spectrum absorb the incident radiation corresponding to the absorption bands of chlorophyll. Due to absorption by other pigments (other than chlorophyll) in the reflectance spectra, a green leaf shows a characteristics peak at about 0.55 μm. Leaves with low chlorophyll content have entirely different reflectance characteristics.

(*b*) **Near-infrared region (0.7 μm – 1.3 μ m).** The internal structure of various leaves show marked differences in the reflectance in the (0.7μm – 1.3 μ m) region. In this region about 40 to 50% reflectance takes place with less than 5% of the incident energy absorption. The reflection/transmission in this region mainly takes place in the mesophyll structure of the leaf. The internal structure of leaves of different types of plants and trees differs considerably. Accordingly, their reflectances are also different in the near *IR* region than in visible region.

3. **Short wave infrared region (1.3 μm – 2.7 μm).** There are three strong water absorption bands at (1.4, 1.9 and 2.7) μm in this region. Water content of the leaf greatly influences the refectance and transmission on characteristics of solar energy. Absorption of solar energy is proportional to the equivalent water thickness which depends on the moisture content of the leaf and its thickness. As the moisture content of a leaf decreases, the reflection activity increases in this region.

Reflectance from the Vegetative Cover. The reflectance characteristics of a leaf or a vegetation canopy as seen by a remote sensor is complex. This complexity is due to the interaction of solar radiation with different parts of the plant, soil back ground and shadow. The vegetation incident radiation duly affected by these variables of the tree canopy when measured by the sensor, is called a *canopy reflectance.* The effect of multiple leaves of a canopy is maximum in the near-*IR* region, the effect is practically negligible in the visible region and marginal in the short wave *IR* region. It may be summarised that reflectance depends primarily on the following factors :

(*i*) Pigment of the leaf
(*ii*) Internal structure of leave cell
(*iii*) Equivalent water content of the leaf.

Effect of the Background on Vegetative Reflectance Spectra. In vegetative cover reflectance sensed by remote sensing sensor, is a mixture of reflectance from the vegetation soil, underneath the plants and their shadow. In low vegetation cover, the background reflectance affects the canopy reflectance. While applying the remote sensing technique to discriminate various type of vegetations, the underlying ground effects need be taken into account especially in agriculture. In crops planted in rows, the reflection of solar radiation is mainly due to the bare soil. As the crop grows with time, it covers the bare soil and the net reflection of the solar radiation is mainly from vegetative cover. Moreover, the relative contribution to the received radiation from plants, soil and plant shadows in the background are dependent on the direction of the sunrays and also due to the view point of the sensor. The most important parameters to be considered for determining the reflectance of a vegetation canopy are as described below :

(*i*) Transmittance of leaves.

(*ii*) Characteristics of components of vegetative canopy.

(*iii*) Number and arrangements of leaves.

(*iv*) Characteristics of the background.

(*v*) Solar zenith angle.

(*vi*) Azimuth angle.

(*vii*) Look angle.

The special reflectance data of vegetation are related to the plant parameters. To correlate with the plant variables, a normalised difference vegetation index is used.

$$NDVI = \frac{IR - RED}{IR + RED}$$

where IR = reflectance in infrared region
RED = reflectance in red region

The value of NDVI may vary from – 1 to +1, depending upon the relative value of RED and IR reflectance.

24.7. ELECTROMAGNETIC RADIATION

Electromagnetic waves are produced due to the motion of an electric charge. For understanding the electromagnetic theory, we have to study the relationship between electricity and magentism. It is a well known fact that the light (the rays forming the visible spectrum) which exites the sensation of vision, consists of a transverse wave motion in vacuum. The violet rays of the spectrum possesses the shortest wave length and highest frequency whereas the rays of red colour have have longest wavelength and lowest frequency. Oscillation of the charged particles sets up changing electric fields which induce changing magnetic fields in its surrounding medium. The changing magnetic fields further set up more changing electric fields and thus a chain reaction continues endlessly. The electromagnetic wave is self propagating. The net result is that the wave energy consisting of both the magnetic and electric fields travels, across the space. When the magnetic and electric waves propagate in a homogeneous and isotropic media, the directions of the two fields, are at right angles to each other and both the fields are at right angles to the direction of propagation Fig. 24.1.

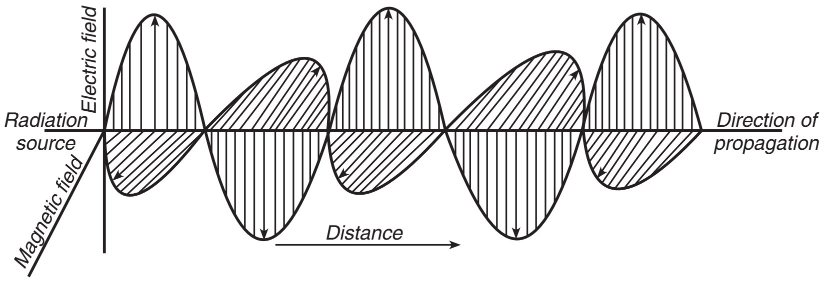


Fig. 24.1. Relationship between electric and magnetic fields and also the direction of propagation.

The electromagnet waves are characterised by frequency, wavelength, intensity, direction of travel and the plane of polarisation.

The distance between successive crests of the wave is called wavelength. It is usually represented by λ and is measured in metres and part thereof. The height of the crest above the mean position of the wave is called a **amplitude**. The energy transported by the wave is proportional to the square of the amplitude. The electromagnetic spectrum extends the frequency [wavelength from gamma rays

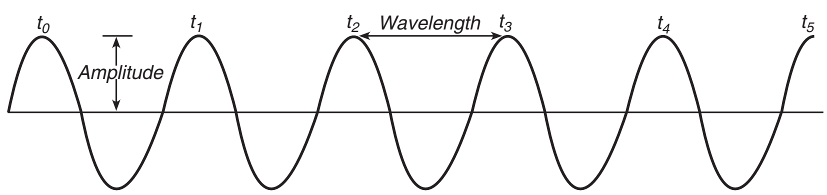


Fig. 24.2. Propagation of electromagnet waves.

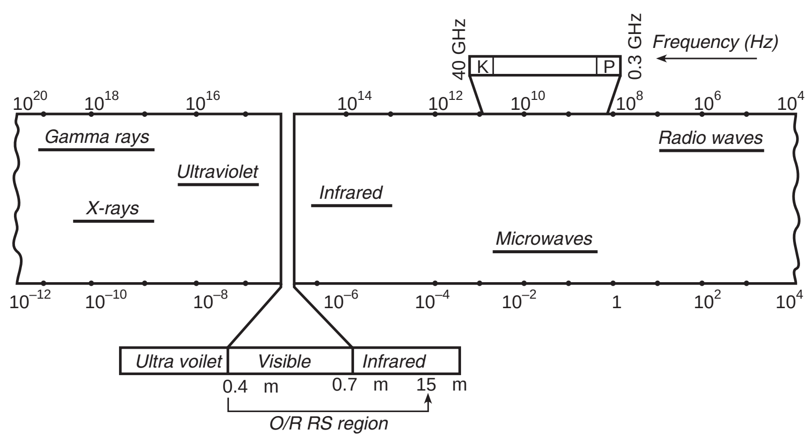


Fig. 24.3. Electromagnetic Spectrum

(wavelength < 10^{-10}m) to radio waves (wavelength 1 m)]. The visible spectrum, called **light waves** occupies only a small portion of the electromagnetic spectrum with wavelengths between 0.40 μm to 0.70 μm.

Optical Infrared Region (OIR). The salient features of optical infrared region are described below:

Visible region ..... 0.4 μ*m* to 0.7 μ*m*

Near-infrared (N/R) region ..... 0.7 μ*m* to 1.5 μ*m*

Short wave infrared region ..... 1.5 μ*m* to 3 μ*m*

Mid-wave infrared region ..... 3 μ*m* to 8 μ*m*

Long wave infrared region ..... 8 μ*m* to 15 μ*m*

Far Infrared (F/R) region ..... Beyond 15 μ*m*

Microwaves. The salient features of microwaves are described below :

P band	0.3 to 1 GHz (30 cm to 100 cm)
L band	1 to 2 GHz (15 cm to 30 cm)
S band	2 to 4 GHz (7.5 cm to 15 cm)
C band	4 to 8 GHz (3.8 cm to 2.5 cm)
X band	8 to 12.5 GHz (2.4 cm to 3.8 cm)
K_u band	12.5 to 18 GHz (1.7 cm to 2.4 cm)
K band	18 to 26.5 GHz (1.1 cm to 1.7 cm)
K_a band	26.5 to 40 GHz (0.75 cm to 1.1 cm)

where 1 GHz = 10^9 Hz

Velocity of Electromagnetic Radiation. By using the well known Maxwell's equations, a relation between the velocity of the electromagnetic wave and the properties of the medium, may be established as under:

Let ε be the electric permittivity of the medium
μ be the magnetic permeability of the medium
C_m be the velocity of electromagnetic radiation then,

$$C_m = \frac{1}{\sqrt{\varepsilon\mu}} \qquad ...(i)$$

In vacuum, $\varepsilon = \varepsilon_0 \approx 8.85 \times 10^{-12}$ Farad/m

$\mu = \mu_0 \approx 4\pi \times 10^{-7}$ Henry/m

Substituting these values for vacuum in eqn. (*i*), we get

$$C_{\text{vacuum}} = \frac{1}{\sqrt{\varepsilon_0\,\mu_0}} = \frac{1}{\sqrt{8.85 \times 10^{-12} \times 4\pi \times 10^{-7}}}$$

$$= \frac{1}{\sqrt{\frac{8.85 \times 4\pi \times 10^{-6}}{10^3}}}$$

$$= \frac{1}{\sqrt{\frac{8.85 \times 4\pi \times 10^{-8}}{10}}} = \frac{1}{0.3335} \times 10^8$$

$= 2.9985 - 3 \times 10^8$ /s, equal to velocity of light.

Again $\varepsilon = \varepsilon_r\, \varepsilon_0$ and $\mu = \mu_r\, \mu_0$ where ε_r is the relative permittivity (*i.e.*, dielectric constant) and μ_r is the relative permeability.

Substituting these value in eqn. (*i*), we get

$$C_m = \frac{1}{\sqrt{\varepsilon_r\, \varepsilon_0\, \mu_r\, \mu_0}} = \frac{1}{\frac{\sqrt{\varepsilon_r\, \mu_i}\; C\, \sqrt{\varepsilon_0\, \mu_0}}{n}} \qquad ...(ii)$$

where n is the refractive index

The velocity of electromagnetic radiation is reduced by a factor $\frac{1}{\sqrt{\varepsilon_r\, \mu_r}}$ as compared to that in vacuum.

Relationship between wavelength, frequency and velocity of Electromagnetic wave.

Let λ = wavelength
v = frequency
c = velocity

then $c = v\lambda$ = frequency $\times$ wavelength

Wave number $K = 2\pi/\lambda$

Period of wave $T = \frac{1}{v}$

Angular frequency $w = 2\pi\, v$.

Polarisation. In the application of remote sensing especially in the microwave region, the polarisation of the electromagnetic radiation plays an important role. As already discussed the electromagnetic wave is made up of electric and magnetic fields both being mutually orthogonal and transverse in the direction of propagation. For the study of the phenomenon of polarisation, use of electric vector is made use of, Polarisation defines the orientation of the fields.

If the electromagnetic wave is considered to move in the z direction, its electric vector lies in $X - Y$ plane with any orientation. As the radiation of the electromagnetic waves which is composed of many waves, have their electric vectors randomly oriented with respect to each other, such a radiation is called **unpolarized** or **randomly polarized.** When the electric field oscillates with the direction of the electric vector in a definite direction, a linearly polarized wave is generally referred to as a polarized plane. When the electric vector lies in the plane of incidence, it is called **vertical polarization**. When the electric vector is at right angles to the plane of incidence, it is called **horizontal polarization**. The plane containing the incident ray and normal to the reflecting surface at the point of incidence, is called the **plane of incidence**.

Coherent radiation. The waves which have a constant phase relationship both in time and space, are called **coherent** with each other.

The phase of a wave is expressed as a fraction of a period with reference to the previous passage through zero points from the negative to the positive direction, such as *M* and *N* (Fig. 24.4).

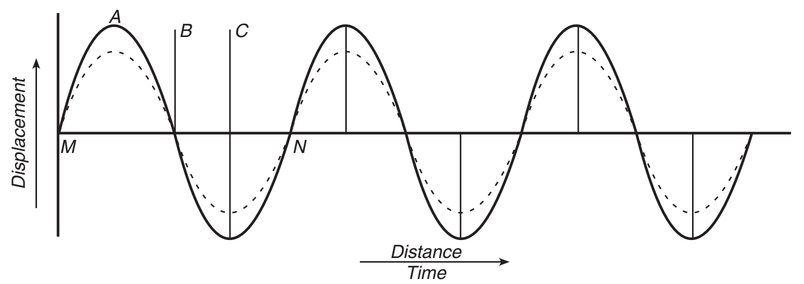


Fig. 24.4. Wave motion showing amplitude variations.

It is usually specified by angular measures with one period taken as 360° (or 2π radians). The amplitude of crest *A* is 90° (or 2/π) whereas that of trough is 270° (or 3π/2). The crest *A* and trough *C* of the wave motion are said to be 180° out of the phase. The surface defined by the locus of the points that have the same phase in an isotropic medium*, is called a **wave front**. The wave front travels perpendicular to the wave motion.

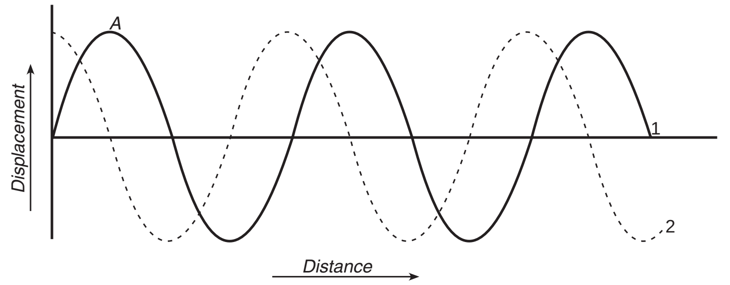


Fig. 24.5. A phase difference.

*Isotropic medium. The medium in which speed of the wave is same along any direction.

Coherence and Phase Difference. Consider two waves 1 and 2 as shown in Fig. 24.5.

At A two waves 1 and 2 have a phase difference of $\frac{\pi}{2}$ which remains the same throughout their travel. Two electromagnetic waves are said to be coherent if their phase difference is constant both in time and space. For a complete coherent radiation, the beam of wave should have a single frequency and radiation monochromatic and the direction of motion of wave parallel.

The coherences may be of the following types :

1. **Temporal Coherence.** This type of coherence takes place only if the phase difference is maintained even after travelling a distance. Monochromatic waves generally experience the temporal coherence.

2. **Spatial Coherence.** This type of coherence takes place only in pure plane waves (parallel beams).

Interference of Waves. Superposition of coherent waves is called **interference.**

Consider two waves of equal frequency and amplitude travelling with the same speed along x-axis and having a phase difference ϕ between them. The equations of the two waves are :

$$y_1 = y_m \sin (k_x - \omega t - \phi) \quad ...(i)$$

and

$$y_2 = y_m \sin (k_x - \omega t) \quad ...(ii)$$

Eqn. (i) may be written in two different forms as under :

$$y_1 = y_m \sin \left[k\left(x - \frac{\phi}{k} \right) - \omega t \right] \quad ...(iii)$$

$$y_1 = y_m \sin [k_x - \omega(t + \phi/\omega)] \quad ...(iv)$$

From eqns. (iii) and (ii) it is observed that two waves at any time t, are displaced from one another along x-axis by a constant $\frac{\phi}{k}$.

Similarly, from eqns. (vi) and (ii) it is observed that a point x, two waves give rise to two simple harmonic motions and have a constant time difference $\frac{\phi}{\omega}$.

For superposition, we add eqns. (i) and (ii).

$$y = y_1 + y_2 = y_m \, [sin \, (k_x - \omega t - \phi) + \sin (k_x - \omega t)]$$

$$= y_m \left[2 \sin \left(k_x - \omega t - \frac{\phi}{2} \right) \cos \frac{\phi}{2} \right]$$

$$\Rightarrow \quad y(2\, y_m \cos \phi/2) \sin (k_x - \omega t - \phi/2) \quad ...(v)$$

i.e., the resultant wave has the same frequency but with an amplitude

$$2\, y_m \cos \frac{\phi}{2}$$

Note. The following points need special attention :

1. If ϕ is too small compared to 180°, the resultant amplitude approximates to $2\, y_m$.

2. If ϕ is zero, the two waves have same phase every where. It means that the crest of one wave corresponds to the crest of the other. Similarly, their troughts also correspond. The resultant coherence of two waves is called **constructive interference** and the resultant amplitude is just the double of the initial amplitude of the either wave.

3. If ϕ is near, 180°, the resultant amplitude approximates to zero. When ϕ is exactly 180°, the crest of one wave corresponds exactly to the trough to the other. Such a coherence of two waves is called **destructive interference** and the resultant amplitude is zero.

From the above we may summerise that when the phase difference is zero (or an integral multiple of 2π), a constructive interference takes place. When the phase difference is π (or odd multiple of π), a destructive interference takes place.

In actual practice, no monochromatic electromagnetic radiation is obtained as the central frequency (υ) or wave length (λ) will have a band width $\Delta\upsilon$ (or $\Delta\lambda$). The coherence length may be defined as the length over which there is strong correlations between the amplitudes such that

$$l_{\text{coh}} = \frac{c}{\Delta\upsilon} = \frac{\lambda}{\Delta\lambda}$$

The coherence time Δt_{coh} may be defined as $\Delta t_{\text{coh}} = \frac{1}{\Delta\upsilon}$.

Propagation of Electromagnetic waves from one medium to another. When an electromagnetic wave falls on to a boundary of two

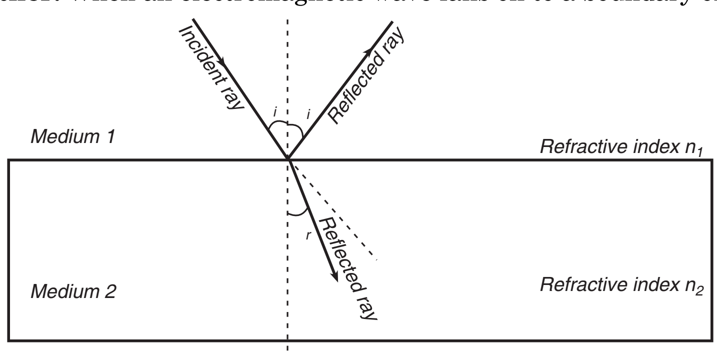


Fig. 24.6. Reflection and refraction of EM radiation.

homogenous mediums with different refractive indices, a part of the wave is reflected back to the first medium and the rest gets transmitted to the second medium.

The angle of incidence (θ_i) is defined as the angle between the incident ray and the normal to the boundary surface of separation at the point of incidence.

The angle of refraction (θ_r) is defined as the angle between the refracted ray and normal to the boundary surface of separation at the point of the incidence. When a radiation takes place from a rare medium to denser medium, the angle of refraction is lesser than that of angle of incidence. On the other hand if the radiation takes place from the denser medium to rarer medium, the angle of refraction is greater than the angle of incidence. The angle of incidence is equal to the angle of reflection. The angle of refraction in (θ_r) is given by the Snell's law.

$$\sin \theta_r = \frac{n_1}{n_2} \sin \theta_1$$

where n_1 and n_2 are the equation, if $\frac{n_1}{n_2} \sin \theta_1 = 1$, the refracted ray, emerges tangentially to the boundary of separation. This is possible only when the incident ray passes through a denser medium to rarer medium in which the refracted ray deviates away from the normal at the point of incidence. (Fig. 24.7).

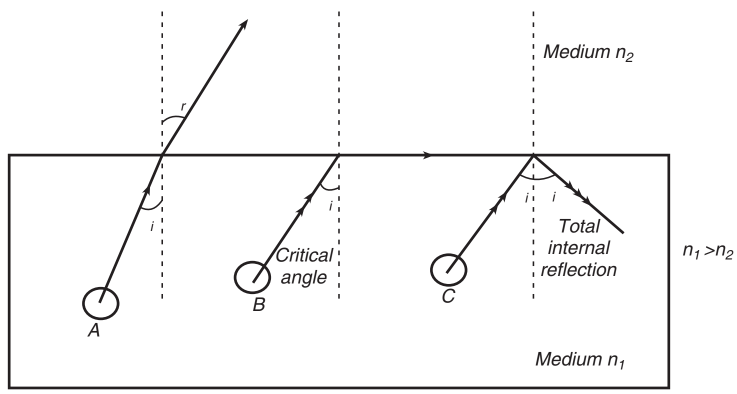


Fig. 24.7. Critical angle and total internal reflection.

The angle of incidence is called a **critical angle** when its angle of refraction becomes 90°. When the angle of incidence exceeds 90°, its ray of incidence gets reflected in the medium. This phenomenon is called **total internal reflection**.

Dispersion. The refractive index (*RI*) of a medium depends on the wave length of the radiation. The variation of the refractive index with wavelength is called **dispersion**. The wavelengths of different colours of the spectrum are different. While passing through a prism, the rays of different colour bends through different angles while entering and leaving the prism. This is the reason why colours of white light get splitted into seven bends. Through a prism the yellow light gets most deviated whereas the red light is deviated least.

Reflection and transmission of electromagnetic radiation between two ideal dielectric media. To study the phenomenon of reflection and transmission, we should carefully define these terms.

1. **Reflectance.** The ratio of the reflected flux to the incident flux is known as reflectance (ρ). Its value lies between 0 and 1.

2. **Transmittance.** The reflectance and transmittance of flux depend on the wavelength, angular distribution and the polarisation of the incident ray.

Let ρ_v = vertical polarization

ρ_h = horizontal polarization

θ_1 = angle of incident

θ_r = angle of reflection.

then
$$\rho_v = \frac{\tan^2 (\theta_1 - \theta_r)}{\tan^2 (\theta_1 + \theta_r)} \quad ...(i)$$

$$\rho_h = \frac{\sin^2 (\theta_1 - \theta_r)}{\sin^2 (\theta_1 + \theta_r)} \quad ...(ii)$$

Applying the Snell's law and using several trigonometric transformations, equns. (*i*) and (*ii*) may be written as under :

$$\rho_v = \left[\frac{n_2 \cos \theta_1 - n_1 \cos \theta_r}{n_2 \cos \theta_1 + n_1 \cos \theta_r}\right]^2$$

$$\rho_h = \left[\frac{n_1 \cos \theta_1 - n_1 \cos \theta_r}{n_1 \cos \theta_1 + n_2 \cos \theta_n}\right]^2$$

Now, the reflactance angle $\rho = \frac{\rho_v + \rho_h}{2}$

For the wave incident to the boundary between the two media,

$$\rho = \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2}$$

For a normal incidence, the transmittance

$$t = \frac{4n_1^2}{(n_1 + n_2)^2}$$

But, $\theta_i + \theta_r = \frac{\pi}{2}$

$$\therefore \qquad \rho_v = \frac{\tan^2(\theta_1 - \theta_r)}{\tan^2 90°} = \frac{\tan^2(\theta_1 - \theta_2)}{\propto} = 0$$

or $\rho_v = 0.$

24.8. ATMOSPHERIC WINDOWS

The sun is a most important source of electromagnetic radiation used in passive optical remote sensing. The sun's radiation covers ultravoilet, visible, infrared and radio frequency regions. Maximum radiation from the sun occurs around 0.55 μm which lies in visible region of the optical spectrum. Solar radiation before reaching the surface of the earth, gets modified by the atmospheric effects. All bodies at temperatures above absolute zero degree emit electromagnetic radiation at different wave lengths as per the following Plank's Law :

$$M_\lambda = \frac{2\pi\, hc^2}{\lambda^5\,[e_x\,(ch/\lambda kT) -]} \qquad ...(i)$$

where M_λ = spectral radiant existence ($W\,m^{-2}\,\mu m^{-1}$)

h = Plank's constant ($6.6256 \times 10^{-34}\,Ws^2$)

c = Velocity of light (2.9979×10^8 m s^{-1})

K = Boltzman's constant ($1.3805 \times 10^{-23}\,Ws\,K^{-1}$)

T = Absolute temperature in Kelvin

λ= Wavelength in microns in Kelvin

By integrating the Planks eqn. (*i*) from $\lambda = 0$ to $\lambda = \alpha$

Total emission $M_{\text{total}} = \sigma\, T^4\, W\,\text{m}^{-2}$ ...(*ii*)

According to Wien's Displacement Law,

$\lambda_{\max}.\, T$ = constant

By expressing $\lambda_{\max}$ in micrometre, T in Kelvin, the value of constant reduces to 2897

$$\therefore \qquad \lambda_{\max} = \frac{2897}{T}$$

Assuming the source on earth at 300 K temperature.

$$\lambda_{\max} = \frac{2897}{300} = 9.66\ \mu\text{m}$$

i.e., if the earth is treated as a black body at 300 K, it emits electromagnetic radiation with a peak value at about 9.5 μm.

As the solar radiation passes through the atmosphere, before reaching the earth's surface, the radiation gets scattered by atmospheric gases and particulates. The maximum absorption occurs at wavelength shorter than 0.3 μm due to the effect of ozone. There are certain spectral regions of the electromagnetic radiation which pass through the atmosphere without much attenuation. Such regions of the spectral regions, are called *atmospheric windows*.

Remote sensing of the earth's surface is mainly confined to the wavelength (0.4μm to 1.3 μm ; 1.5 μm to 1.8 μm, 2.2 μm to 2.6 μm, 3.0 μm to 3.6 μm, 4.2 μm to 5.6 μm, 7.0 μm to 15.0 μm and 1cm to 300cm.)

24.9. PLATFORMS

For taking measurements from a satellite of the ground features, remote sensors are supported in space by a suitable arrangement. Such an arrangement which could be a simple tripod or a complex space craft is called a *platform.* For the same remote sensor, the spatial resolution of the ground features decreases with the increase of the height of observation but the area of observation increases with the increase of the height of observation. The design of the remote sensor changes with the type of the platform. Initially, balloons and rockets were used to carry the platforms of the remote sensors. In the present days due to technological development, aircrafts and satellites are being used to provide suitable platforms for the remote sensor. The balloon at which was used as a platform to carry the remote sensor, was made of polethylene at zero pressure and whose volume was 684 cm^3. This balloon floated at a constant height of about 30 km and took contnuous photogrpahs for about 4 hours.

In sixties, aircrafts were traditionally used for aerial photography for mapping the ground surface by air survey and photogrammetric methods. Air flights are suitably used as the necessary ground data is easily collected over a specified area with a short time notice. Depending upon the photographic scale requirements, the same aerial photographs may be used for remote sensing. The major draw-back of aerial photographs is, its very high cost. Repetitive photography of the same area, has to be taken, ensuring a sufficient fore-aft and lateral overlaps.

Space-born remote sensing satellites have proved an ideal choice for repetitive global coverage as they can move along different paths or orbits around the earth. The path which is followed by a satellite in the space, is called its *orbit*. The basic principle involved in the motion of a satellite and the main characteristics of different types of orbits of satellites, are described in the following paragraphs.

24.10. PRINCIPLE OF SATELLITE MOTION

Satellites are the moving objects around gravitational force of a central mass much as our earth. Satellite's motion obeys the Kepler's laws. Kepler, based on the collected data, advocated that the movements of planets (celestial bodies) are well organised and follow certain laws which are now known as Kepler's laws.

Kepler's first law. It states, "*The path followed by each planet is an ellipse with the sun at one of the foci.*" In case of earth satellites, the earth is at one of the foci of the elliptical orbit of the satellite.

The nearest point of the earth on the satellite orbit, is called perigee whereas the farthest point is called apogee. (Fig. 24.8).

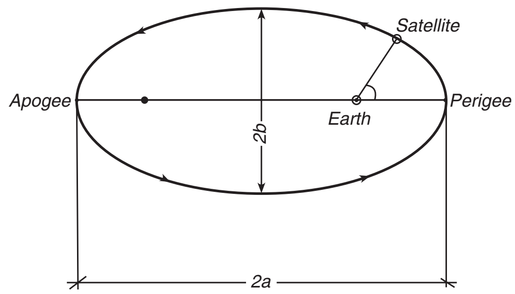


Fig. 24.8. Satellite orbit.

The straight line joining the apogee and perigee, is called the *line apsides.*

The size of the satellite orbit is decided by the distance between the apogee and perigee which is twice the semi-major axis of the ellipse. When the two foci coincide, the elliptical orbit becomes the special case of a circular orbit. The deviation of the elliptical orbit from the circular orbit is given by a term eccentricity (e). Its value is given by the relation.

$b^2 = a^2 (1 - e^2)$ where value of e is less than 1.

Kepler's second law. It states, "*The line joining the planet to the sun sweeps out equal areas in equal times.*"

The sweeping of the area by the planet (or satellite) depends upon its distance from the sun. As the planet (or satellite) at the perigee is at the least dsitance form the sun (or earth) it has to move fast so that the area swept in equal time (t) is equal to the area swept when it is at the apogee, the farthest point. The same analogy is true in this case of satellite and the earth.

As the time of travel (t) of the satellite is the same, the area swept at perigee is equal to that at apogee, the velocity of the satellite is more at perigee as compared to that at apogee. (Fig. 24.9).

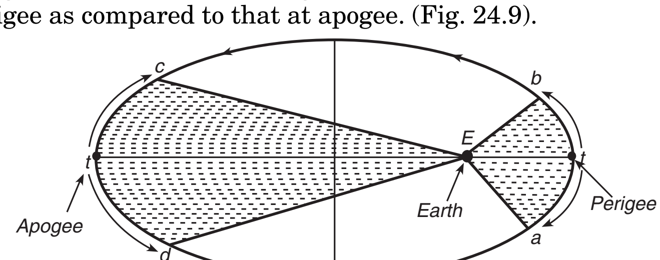


Fig. 24.9. Diagram depicting the Keplar's second law.

Kepler's third law. It states. "*The square of the period of the planet is proportional to the cube of the semi-major axis.* The period of a satellite or planet is the time taken by the satellite or planet for going around the earth/sun once.

According to Kepler's third law,

$$T^2 \propto a^3$$

where T is the period of the satellite

a is the semi major axis of the elliptical orbit of the satellite

It must be clearly understood that Kepler's laws describe how planets move in space whereas Newton's laws of motion describe why the planets (or satellites) move in space.

The condition necessary for a body (celestial body or satellite) to orbit around the earth is that earth's gravitational force is equal to the centrifugal force of the orbiting body.

We know that earth's gravitational force $= \dfrac{G\,M_e\,m}{r^2}$ ...(*i*)

and centrifugal force of the orbiting body $= \dfrac{mv^2}{r}$ ...(*ii*)

where G = the universal gravitational constant

M_e = the mass of the earth

m = the mass of the satellite

r = distance of the body from the earth.

$= r_e + h$, where r_e is the radius of earth and h is the height of body from the earth's surface.

Comparing Eqns. (*i*) and (*ii*), we get

$$GM_e \frac{m}{r^2} = \frac{mv^2}{r} \Rightarrow GM_e = rv^2 \qquad ...(1)$$

24.11. TYPES OF SATELLITE VELOCITIES

According to the position of the satellite with respect to the earth, its velocities are defined as under :

1. *First astronautical velocity.* The velocity required for a satellite moving around the earth close to the surface of earth *i.e.,* in a circular orbit, it is called the first astronautical velocity of the satellite. It is roughly 7.9 km/s.

2. *Secondary astronautical velocity.* The velocity of satellite to be attained to move along its orbit (ellptical) at which it escapes the earth's gravity and leaves its orbit, is called the secondary astronautical velocity or *escape velocity.* It is roughly 11.2 km/s.

From eqn. (1), the orbital period (T) *i.e.,* the time required to complete one revolution for a circular orbit,

$$T = \frac{2\pi}{\sqrt{GM_e}} (r_e + h)^{3/2} - 10^{-2} (r_e + h)^{3/2} \qquad ...(2)$$

when both r_e and h are in kilometres and T is in seconds.

Velocity of a Satellite in Circular Orbit

Let the radius of the earth = r_e

The height of satellite above earth surface = h

$\therefore$ The circular orbit of the satellite = $2\pi(r_e + h)^{3/2}$ ...*(i)*

Circular orbital time $T = \frac{2\pi}{\sqrt{GM_e}} (r_e + h)^{3/2}$ ...*(ii)*

Velocity of the satellite $V_s = \frac{\text{Circumference}}{\text{Orbital time}}$

$$= \frac{2\pi(r_e + h)}{\frac{2\pi}{\sqrt{GM_e}} (r_e + h)^{3/2}} = \frac{2\pi(r_e + h)\sqrt{GM_e}}{2\pi(r_e + h)^{3/2}}$$

or $$V_s = \left[\frac{GM_e}{r_e + h}\right]^{1/2}$$

During the same duration of time, the sub-satellite point moves through a smaller distance as compared to the satellite distance movement because of the difference in height, *i.e.,* $(r_e + h)$ and r_e. Applying the theorem of proportionality, we get

i.e., Velocity of sub-satellite point $V_g = V_s \left[\frac{r_e}{r_e + h}\right]$.

Example 24.1. *Calculate the velocity of a satellite moving at a height of 1000 km. Also, find the sub-satellite velocity. Assume radius of earth = 6380 km.*

Solution. We know that

$$V_s = \left[\frac{GM_e}{r_e + h}\right]^{1/2} \qquad ...(i)$$

Substituting the values, $GM_e = 398601$, $r_e = 6380$ km, $h = 1000$ km, we get

$$V_s = \left[\frac{398601}{6380 + 1000}\right]^{1/2}$$

or $$V_s = \left[\frac{398601}{7380}\right]^{1/2} = 7.35 \text{ km/s.} \quad \textbf{Ans.}$$

Velocity of sub-satellite

$$V_g = V_s \left[\frac{r_e}{r_e + h}\right] = 7.35 \left[\frac{6380}{6380 + 1000}\right]$$

or $$V_g = \frac{7.35 \times 6380}{7380} = 6.354 \text{ km/}s. \quad \textbf{Ans.}$$

24.12. INERTIAL COORDINATE SYSTEM

For locating a satellite orbit also to locate the position of the satellite on its orbit, the Keplerian elements are generally used by inertial coordinates system. This coordinate system is as explained under :

1. The axis of this coordinate system is aligned with the earth's axis of rotation.

2. The x-axis is aligned along the line joining the centre of the earth and the sun at the moment of Vernal Eauinox, the moment sun crosses the equatorial plane from the southern to northern hemisphere.

3. The y-axis is chosen as right angles to the x-axis so that a right handed coordinate system is obtained.

Technical terms relevant to the location the satellite orbit

The following terms need be understood carefully.

1. **Celestial sphere.** The sphere centered on the inertial coordinate system, is called the *celestial sphere*.

2. **Inclination of the orbit** (θ)**.** The angle between the satellite orbital plane and the earth's equatorial plane is defined as the *inclination of the orbit.* The orbital inclination of the satellite may vary from 0° to 180°.

If the satellite's orbital plane and the earth's equatorial plane coincide, the satellite rotates in the same direction as the earth. If the satellite rotates in the opposite direction to the earth's rotation, its inclination would be 180°.

Prograde and retrograde orbits. If the satellite's inclination is less than 90°, its orbit is called prograde orbit. When the satellite's inclination is between 90° and 180°, its orbit is called retrograde orbit.

3. **Right Ascension of Ascending Node (RAAN)** (Ω)**.** The point where the satellite crosses the earth's equator from the southern hemisphere to the northern hemisphere, is called the *ascending node.* The point where the satellite crosses the earth's equator from northern to southern hemisphere, is called the *descending node.*

The line joinin the two nodes, *i.e.,* ascending node and descending node, is called the *line of nodes.*

The right ascension of ascending node (RAAN) is the angle between the x-axis (Vernal Equinox) and it is measured in the anti-clockwise direction from the x-axis. The value of the RAAN is a number which may lie between 0° and 360°.

4. **Argument of perigee (ARGP) (*w*).** The angle between the line of nodes and the major axis of the satellite orbit, (the line of apsides) measured along the orbital plane from the ascending node to the perigee, is called the *argument of perigee.* It also lies between 0° and 360°. When the perigee and ascending node coincide, the argument of perigee is 0°.

With the help of these definitions, the inclination, right ascension of ascending node and argument of perigee (w), the satellite's orbit may be defined in inertial spares. The eccentricity (e) and semi-major axis (a) decide the shape and size of the satellite's orbit.

Eccentricity (*e*). It defines the shape of the orbit.

Semi-major axis. It defines the size of the orbit.

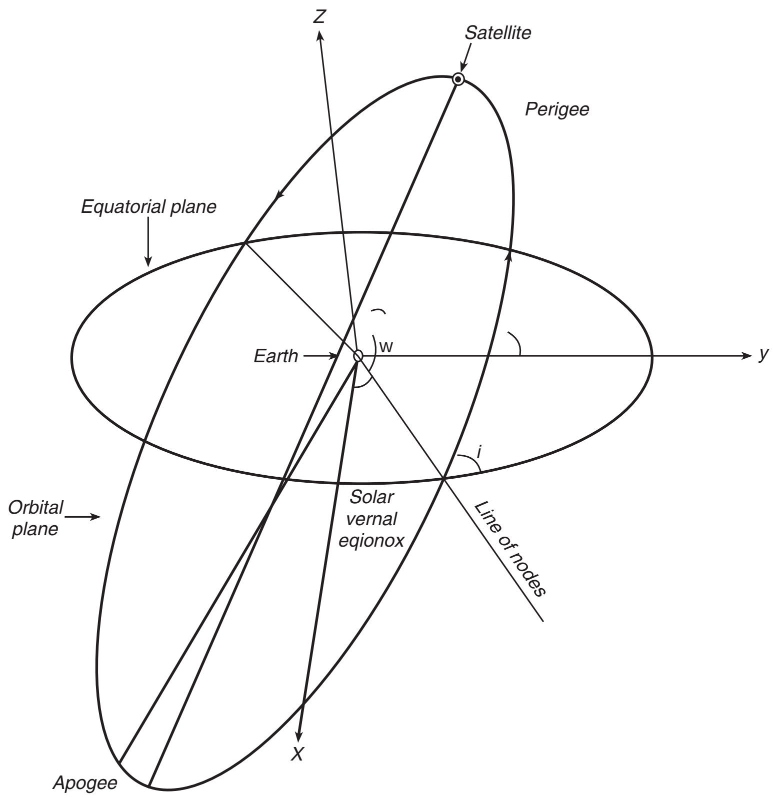


Fig. 24.10. Interial coordinate system.

True Anomaly α. It is the angle between the line joining the satellite and the centre of the earth and the major axis measured counter-clockwise from the perigee along the orbital plane.

The inclination θ, Ω and *w* define to orbit of satellite in inertial space.

24.13. CLASSIFICATION OF SATELLITE ORBITS.

The plane containing the orbit of the satellite and passing through the centre of the earth, is called the *orbital plane.* Depending upon the inclination of the orbital plane with the earth's equatorial plane, the orbits of the satellites are of the following types :

1. **Equatorial orbit.** When the orbital inclination is either 0° or 180°, the orbital plane lies in the equatorial plane. Such an orbit is called an *equatorial orbit.*

2. **Polar orbit.** When the orbital inclination is 90°, the orbital plane passes through the poles, *i.e.* north pole, centre of the earth and the south pole all lie in the orbital plane. Such an orbit is called a *polar orbit*.

3. **Inclined orbit.** The satellite orbit whose angle of inclination is between 0° and 180°, is called an inclined orbit.

4. **Near polar orbit.** The satellite orbit having the angle of inclination very close to 90°, is called a near *polar orbit*.

Different types of satellite orbits are shown in Fig. 24.11.

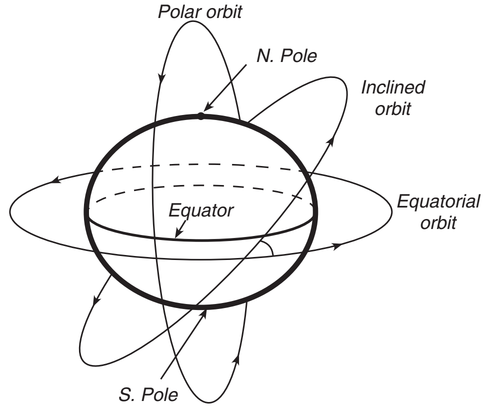


Fig. 24.11. Different types of satellite orbits.

Crossing of equator by polar orbit. The polar orbit crosses the equator twice while moving from the north pole to south pole and again while moving from the south pole to the north pole.

(*a*) *Descending node crossing.* When the satellite moves from north to south, the orbit crossing the equator, is called a *descending node crossing*.

*Siderial day. It refers to the time taken by the earth to rotate 360° and is equal to 23 hours, 56 minutes and 4.019 seconds. For details please refere to the Author's text book of Advanced Surveying.

(*b*) *Ascending node crossing.* When the satellite moves from south to north, the orbit crossing the equator, is called an *ascending node crossing.*

(*c*) *Nadir trace.* The path traced on the surface of the earth by the sub-satellite point, an intercept on the earth by the line joining the centre of the earth to the satellite, is called the ground track on *nadir trace*.

Geosynchronous and Geostationary Orbits of Satellite. These types of satellites need special mention of their characteristics.

1. **Geosynchronous orbit.** As the height of the satellite increases, the time taken by the satellite for one revolution (period) also increases as it has to cover more distance. At about 35,786 km height from the ground surface, the period of rotation is equal to one siderial*. The orbit along which the orbital period is synchronized with the rotational period of the earth, is called synchronous orbit.

2. **Geostationary orbit.** When a satellite is in a circular orbit at geosynchronous orbit, it appears stationary with respect to the earth over the equator. Such an orbit is called *geostatioary orbit.*

Uses of geostationary Orbit. Geostationary orbits are used for the following :

1. For communication purposes
2. For weather studies and forecasting
3. For study of dynamic phenomenon, *i.e.*, cyclone movement, cloud movement, earthquakes, floods, etc. The angle between the sun-earth

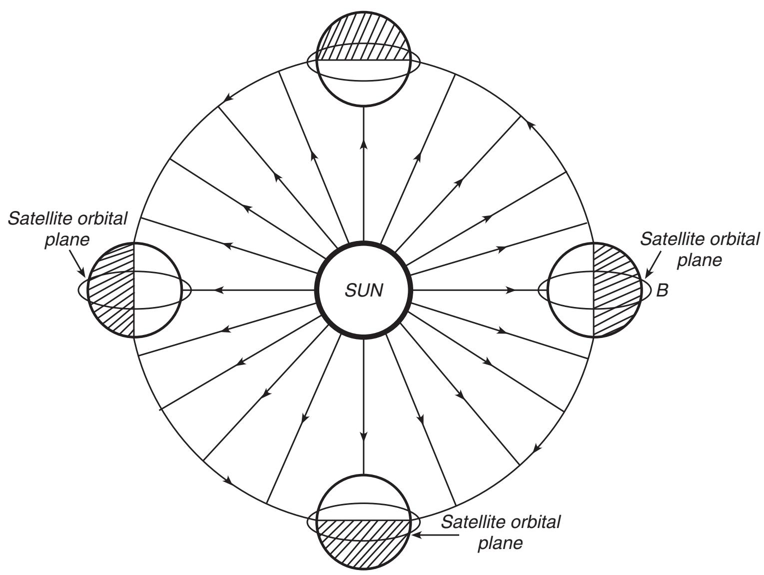


Fig. 24.12 Sun-synchronous orbit.

line and orbital plane, goes on changing with the movement of the earth around the sun.

In Fig. 24.12 the sun-earth line is at right angles to the orbital plane at A whereas at B, it is parallel to the orbital plane. Solar illumination varies as satellite revisits that area. To overcome this difficulty, the orbital plane may be rotated exactly to compensate the movement of the earth around the sun at a rate of $\frac{36°}{365.25} = 0°.9856$ per day. It means that the orbital plane need be rotated (to regress) around the earth at a rate of 0°.9856/day. The oblate spheroidal shape of the earth (the average polar radius is about 6356 km and equatorial radius is 6378 km) makes the orbital plane to rotate about the axis of the earth, except in case of polar orbit. The precession rate of the orbit is a function of the satellite altitude and the orbital inclination. By suitably adjusting these two parameters *i.e.* altitude of the satellite and its orbital inclination, the precession rate can be made to compensate the mean rate of the earth about the sun. In this way the satellite is able to see the same point on the earth surface. Such an orbit is called a sunsynchronous orbit. It is always near the poles.

24.14. VISIBILITY OF EARTH FROM ORBIT

The maximum visibility of the earth is from the horizon to horizon. If $2\phi_v$ is the angel of view from the orbit, then

$$\sin \phi_v = \frac{AD}{CD} = \frac{r_e}{r_e + h}$$

$$\Rightarrow \qquad \phi_v = \sin^{-1}\left[\frac{r_e}{r_e + h}\right]$$

where r_e = radius of the earth

h = height of the satellite above the earth's surface.

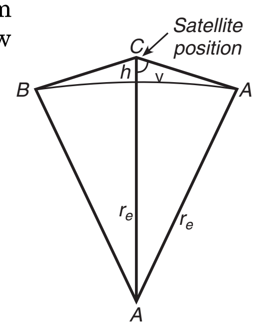


Fig. 24.13. Visibility.

From equation (i) we observe that if the height of satellite increases, the view angle decreases but the maximum distance on earth, *i.e.,* from horizon to horizon increases. Because of the proximity of points to the horizon due to earth curvature, the resolution substantially degrades near the the horizon.

For a gesynchronous orbit, the maximum earth coverage is about 42% of the earth's sphere coverage at latitude of about ± 81°.

Example 24.2. *Find the height of a satellite if its angle of view is 80°. Assume the radius of the earth as 6380 km.*

Solution. Let S be the position of the satellite, O is the centre of earth whose radius is 6380 km.

Angle of view of hte satellite = 80° (given)

SO bisects the angle *ASB*

$$\therefore \qquad \angle OSB = \frac{80°}{2} = 40°$$

Let h be the height of satellite above *M.S.L.*

From Δ *OSB*, we have

$$\sin OSB = \sin 40° = \frac{OB}{OB + h}$$

$$= 0.642788$$

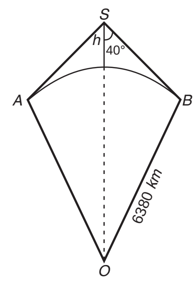


Fig. 24.14.

$$\Rightarrow \qquad 6380 + h = \frac{6380}{0.642788} = 9925.0 \text{ km}$$

$$\Rightarrow \qquad h = 9925 - 6380 = 3545 \text{ km.}$$

The height of the satellite, = 3545 km.

Example 24.3. *If remote sensing satellite weights 63 kN on the surface of the earth, find the gravitational force on it due to earth at a height equal to half the radius of the earth.*

Solution. Let g be the acceleration of the earth on its surface,

g' be the acceleration of the earth on the satellite at a height of half radius of the earth.

We know, $g' = g\left(1 + \frac{\frac{1}{2}r}{r}\right)^{-2}$ where r is the radius of the earth.

$$\Rightarrow \qquad g' = g\left(1 + \frac{1}{2}\right)^{-2}$$

$$= \frac{g}{2.25} \qquad ...(i)$$

The weight of the satellite on the earth's surface

= mass of earth × acceleration due to gravity

= mg = 63 kN

Putting the value of g in eqn. (i), we get

$$g' = \frac{63 \times 10^3}{m} \times \frac{1}{2.25}$$

Gravitational force on the satellite = mg'

$$= m \times \frac{63 \times 10^3}{m} \times \frac{1}{2.25}$$

$28 \times 10^3 = 28$ kN. **Ans.**

24.15. REMOTE SENSORS

The instrument that measures the properties of electromagnetic radiations leaving a surface/medium due to scattering or emission, is called a *remote sensor*.

Remote sensors are of two types :

1. Passive sensors, 2. Active sensors

1. **Passive sensors.** The remote sensors, which sense natural radiations either emitted or reflected from the earth, are called *passive sensors.*

2. **Active sensors.** The remote sensors which produce their own electromagnectic radiation of specific wavelength or a band of wavelength, as a part of the sensor system, are called, active *remote sensors*.

As the technology involved in developing sensors throughout the electromagnetic spectrum, is not the same, the remote sensors, are also classified as under :

(*i*) The remote sensors which operate in the optical infrared region.

(*ii*) The remote sensors which operate in the optical microwave region.

Both the optical infrared and microwave sensors may be either imaging or non-imaging sensors. The imaging remote sensors give a two-dimensional spatial distribution of the emitted or reflected intensity of radiation (such as a photographic camera). The non-imaging sensors measure the intensity of radiation, within the field of view. Vertical temperature profiling radiometer (VTPR) is a type of non-imaging sensor.

Sensor on Board Satellite Scaning the Ground Surface. Sensors mounted on aircraft or satellite platforms measure the amounts of

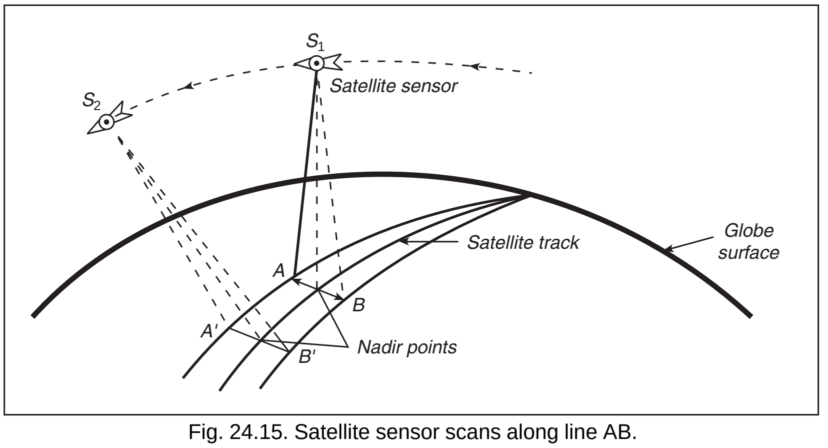


Fig. 24.15. Satellite sensor scans along line AB.

energy reflected from or emitted by earth's surface. The sensor measures the ground surface along the one dimensional profile on the ground below the platform on either side of the ground track of the platfrom. The sensor scans the ground below the satellite platform. Movement of the platform in forward direction, enables the sensor to build an image of the earth's surface (Fig. 24.15).

The satellite sensor scans the ground surface along line *AB*. Each scan line of a remotely sensed image is a digital or numerical record of radiance measurements made at regular intervals along the line. Consecutive scan lines form an image. Depending upon the direction of principal axis of the sensor, the sensors are called Nadir looking or side looking sensor.

A Nadir looking sensor images the ground area on either side of the satellite platform. Whereas a side-looking sensor images the earth's surface lying on one side of the satellite track.

Selection of sensor's parameters. As the information collected from the remote sensor is primarily meant to identify and map the various earth features, the parametres of the remote sensor are judged according to its mapping accuracy requirements. The most suitable remote sensor is the one that is able to detect small difference in the remmitance/reflectance of the features on the earth's surface in a number of spectral bands. Remote sensor's parameters are considered under the following domains :

1. Spatial resolution
2. Spectral resolution
3. Radio metric resolution
4. Temporal resolution

We shall now discuss these four domains briefly for the purpose of mapping the earth surface.

1. **Spatial resolution.** Spatial resolution may be defined as a measure of the remote sensor's ability to image closedly by spaced objects on the earth surface so that they can be distinguished a separate object. A remote sensor having a 1 m spatial resolution reproduces finer details of the site features imaged by the remote sensor compared to a sensor with a 25 m resolution. Resolving the objects on the surface of the earth by an imaging system, say a sensor is due to the property of diffraction, a phenomenon of electromagnetic radiation (EMR).

The property of bending of waves around an obstacle, is called *diffraction*. As electromagnetic radiation is a wave and it obeys the rule of diffraction. It also bends around the obstacle. Diffraction is an important physical property inherent in all wave-phenomenon.

Diffraction of a wave by an aperture depends on the ratio λ/d, where λ is the wavelength of the wave and d is the diameter of the aperture. From the ratio λ/d, it is evident that the diffraction of a wave is directly proportional to its wavelength (a constant for the wave) and inversely proportional to the diameter of its aperture.

Diffraction is more prominent if the value of λ/d is large.

Pattern of the different. Pattern of the diffraction for a circular slit consists of a bright central maximum with secondary maxima and minima on either side. The intensity of each succeeding secondary maximum goes on decreasing as the distance from the centre of the circular slit increases (Fig. 24.16).

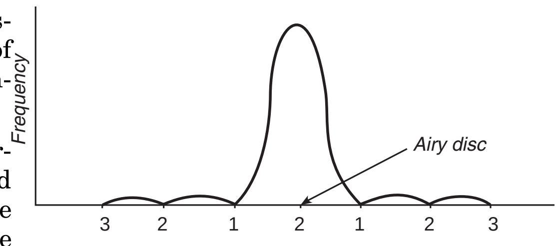


Fig. 24.16. Pattern of the diffraction.

The bright is surrounded by bright and dark rings which are called *airy pattern*. The central bright disc is called **airy disc**. If Fig. 24.16 the distribution of energy in the airy pattern due to a circular aperture is illustrated. The main peak is followed by a number of minima and maxima. The diameter of the minima and maxima may be the following relationship :

$$d = 2.44\ (\lambda/D)\ f$$

where D is the diameter of the aperture and f is the focal length and λ is wavelength.

Two objects separated by 1.22 $\lambda/D\ f$ is just resolved, even though this is the theoretical resolving limit of two objects. The resolution may be defined in angular resolution. This is 1.22 λ/D radians. It may however be noted that the term resolving power and resolution are not the same. The resolving power refers to the components of the imaging system such as lens, film, etc. whereas resolution applies to the image produced by the imaging system. The ratio of the radiation between the adjacent areas, maximum (L_{max}) to minimum (L_{min}) is known as *contrast ratio*.

2. **Spectral resolution.** In multi-spectral remote sensing, the variation in reflected/emitted special radiation, is used to distinguish various features. As it is difficult to get continuous spectral information, the reflected/emitted spectrum are sampled by making measurements at a few selected wavelengths. The wavelength region of observation is called *spectral band*. The spectral bands are defined in terms of a central wavelength (λc) and a band width (Δλ).

Spectral band selection is made on the following factors:

(*a*) Location of the central wavelength

(*b*) The band width

(*c*) The total number of bands.

Band width. It is defined by a lower λ_1 and an upper λ_2 cut off wavelength. The spectral resolution $\Delta\lambda$ which is given by $(\lambda_2 - \lambda_1)$ describes the wavelength interval in which observation is made. Smaller the $\Delta\lambda$, higher is the spectral resolution.

Location of spectral bands. The most important criterion for the location of spectral bands is that they should be in the atmosphere window and away from the absorbtion bands of atmospheric constituents.

As far as possible, selected bands should be un-correlated since correlated bands provide redundant informations.

3. **Radiometric resolution.** The radiometric resolution is a measure of the capability of the sensor to differentiate the smallest change in the spectral reflectance/emittance between various targets.

Radiometric quality of the image depends primarily on radiometric resolution calibratio-accuracy and modulation transfer function. Resolution in general, is the minimum difference between two discrete values that can be distinguished by a measuring device. Accuracy is a measure of how close the measurement is to the true value.

4. **Temporal resolution.** Satellite remote sensing enables repeatedly observations of a scence at regular intervals. Temporal resolution is also called *repetivity*. Repetivity depends on orbit characteristics and swath. Larger swath provides higher temporal resolution.

Higher temporal resolution enables monitoring occurence of rapid changes such as forest fire, floods, etc. Moreover, after each repition, every image is taken with the same instrument view angle for any location, which is necessary for bi-directional reflectance distribution function.

24.16 OPTICAL INFRARED SENSORS

The sensors whose response covers a wave length region from about 0.4 μm to 20 μm, are called *optical infrared sensors*. In this type of sensors, both the radiation reception and analysis are carried out by the instruments built on optical technology *i.e.,* a combination of lenses, mirrors, prisms, etc.

Classification of the optical infrared sensors. Optical infrared sensors are classified as under :

1. **Photographic sensor systems.** In this type of sensors, the images of the ground features are formed directly on to a sensilized film.

2. **Electro-optical sensors.** In this type of sensors, the optical image of the ground features, is first converted into an electrical signal and processed to transmit the data.

Quality of the image formed by optical systems. An ideal imaging system must be able to map every point in the object space to a well defined print in the image plane on a reduced size and keeping inter distances between the points in the image plane as those in the objects space. Moreover, it should reproduce the relative intensity distribution in the objects space.

24.17. GEOMETRY OF A SIMPLE LENS

A lens is formed by two curved surfaces. The imaginary straight line that coincides the axis of the symmetry of the spherical curved surfaces, is called the *optical axis* of the lens. The imaginary line which passes through the centres of curvature of the lens surfaces, is called the *principle axis*. In case of a compound lens composed of a number of simple lenses, the optical axis is the line formed by the coinciding principle axes of the series of the optical elements. The rays close and parallel to the optical axis converge to a point on the principal axis, called the *focus point*. A plane at right angles to the principal axis passing through the focal point (*i.e.,* focus) is called a *focal plane*.

The point on the optical axis of the refractive optical element through which rays pass without any deviation, is called the *optical centre*. The point of intersection of the optical axis and principal axis is called the *principal point*. The distance between the *principal point* and the focal point is known as focal length.

The focal length may be measured accurately by measuring the displacement of the image in the focal plane due to small till (θ) to the parallel ray. If x is the shift in the image in the focal plane, the focal length f is given by $f = \frac{x}{\tan\theta}$.

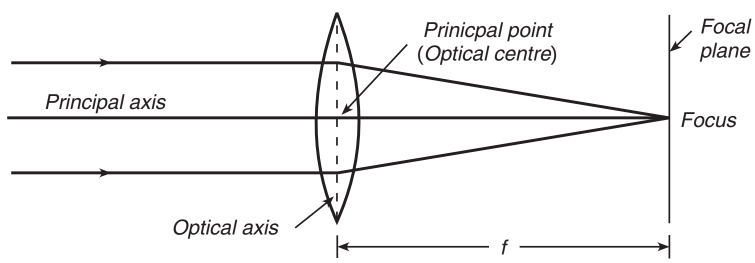


Fig. 24.17. Geometry of a lens.

Every optical system is provided with an aperture stop for limiting the amount of light reaching the imaging area. The image of the aperture stop as seen from a point on the principal axis from the objects space, is called *exit pupil*. The ratio of the effective focal length to the entrance pupil diameter is called f-number. It is usually written as $\frac{f}{10}$, meaning that the entrance diameter is 1/10th of the focal length. An optical system with smaller f-number is called a faster system and with large f-number, is called a slow system.

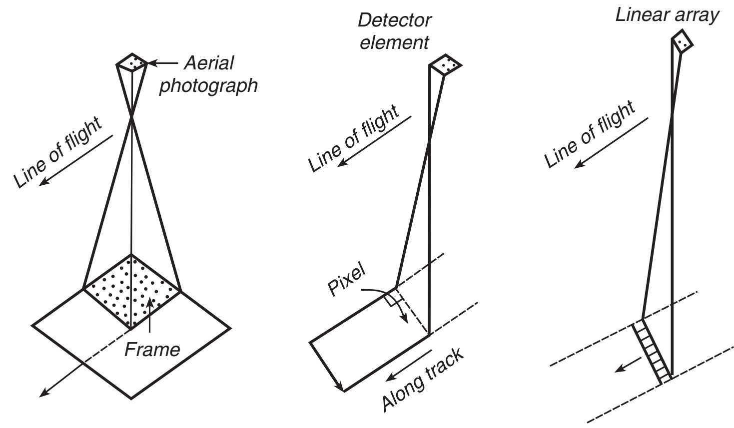
24.18. IMAGING MODES

The imaging process of an area may be carried in the following ways:

1. **Frame by frame system.** In this system of imaging an area, a snapshot is taken at one instant covering a portion of certain area on the earth surface, depending upon the sensor's characteristics and the height of the sensor's platform. A typical example of frame by frame mode is the conventional photographic camera. Successive frames image a strip of terrain along and also on either side of the line of light, having a certain overlap in successive frames.

2. **Pixel by pixel mode.** In this system of imaging, the sensor collects the radiation from pixel at a time. To achieve this, the scan mirror directs the sensor to the next pixel in the cross track direction and by the scan mirror motion, one cross track line of width equal to one pixel, is imaged. The motion of the platform produces the successive lines. This method is generally called the *whisk broom scanning.*

3. **Line by line mode.** In this system of imaging, the sensor collects radiation from one line in the cross track direction at one instant. Successive lines are created by the motion of the platform. This mode of imaging is generally known as *push broom scanning*. All the three modes are showing Fig. 24.18.



(*a*) *Frame imaging* (*b*) *Frame imaging* (*c*) *Line by line framing*

Fig. 24.18. Modes of imaging

24.19. PHOTOGRAPHIC FILMS

A photographic film consists of a photosensitive photographic emulsion coated on to a base for support. The emulsion consists of silverhalide crystals of different size, embeded in a gelatin matrix. When light is allowed to fall on the emulsion, a photo-chemical reaction takes place and a latent image is formed. From the area of the film where light has not fallen, the silverhalide gets dissolved during the developing process and the area remains transparent. A negative image is formed. Positive images are produced on paper and a transparent positive is obtained.

Types of films used for aerial/space photography. Three types of films are used *i.e.,*

1. Black and white film
2. True colour film
3. Colour infrared film

1. **Black and white (B and W) films.** Films which are only sensitive to visible light are called **panchromatic films**. For obtaining a black and white film, a minus-blue filter is used to eliminate the short wavelength blue light. Short wavelength blue light causes haze. Black and white infrared (B and WIR) films are sensitive to spectral region covering both visible and near IR region (0.4 μ/m to about 0.9 μ/m). In black and white infrared films also, a minus-blue filter is used to eliminate the blue light. In both the type of films, gray shades depict the intensity of light.

Difference between B & W and B & WIR photographs. Due to different portions of spectral region covered by them, the photographs obtained from the films are also different. As vegetation of the earth surface has a higher reflectance in near IR region, it is depicted in brighter tone in B and WIR photographs as compound to B and W photographs. The boundary between water bodies and adjoining land is also much sharper in B and WIR photographs as compared to conventional B and WIR photographs.

The Human eye. The human eye has three separate light receptors (sensors) in its retina for responding to blue, green and red lights. The colour of an object is perceived by the optical nerve, depending on the combination of sensors excited and also on their relative excitement. Blue, green and red colours are called **additives primaries.** With the help of these additive primaries, various types of colours are produced such as yellow, cyan and magenta. These colours are called **subtractive primaries**. These subtrative primaries are produced by subtracting one of the primary colour from the visible white light. For example, cyan colour is obtained by removing red colour from white light.

24.20 CONSTRUCTION OF A COLOUR FILM

A colour film (negative) consists of three layers of dye. The top layer is sensitive to blue light, second layer is sensitive to green light and the third layer is sensitive to red. As second and third layers are also sensitive to blue, a blue absorbing filter is introduced between the first

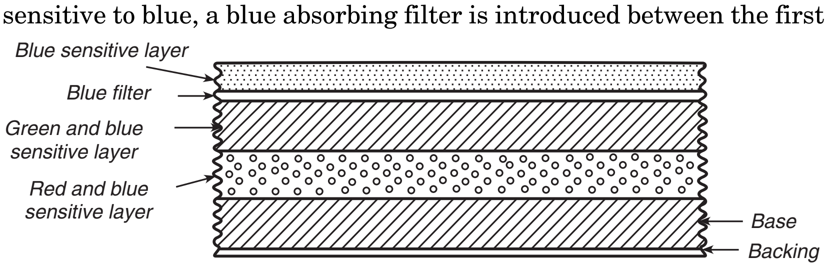


Fig. 24.19. Cross section of a colour film.

and second emulsion layers. The cross section of a colour film is shown in Fig. 24.19.

Development of a Colour Film. After developing the exposed film, the red sensitive bottom layer produces a complementary cyan image of the red portion of the ground surface.

Blue and green sensitive layers produce magenta and yellow images respectively.

White light falling on the film exposes all the three layers and thus has no transmitting light.

Black object produces a transparent image in the absence of exposing none of the rays.

After fully developing an exposed colour film, a negative colour film is obtained.

Use of Colour IR film in Remote Sensing. For mapping the ground surface by the remote sensing techniques, coloured infrared film (Colour IR-CIR), is most suitable as a colour IR film is designed to record green and red. The emulsion of such films is so sensitized that after developing the colour IR prints, give blue images for the objects which primarily reflect in the green wavelength. Similarly, objects which primarily reflect the red wavelength appear green while a red image is produced by objects which primarily reflect in the near-IR region. By using a yellow filter over the lens, it is possible to cut off blue light.

Remember. All the three layers are sensitive to blue. The net resultant of the colours is not the true reproduction of colour as seen by human eye, it is called a *false colour film*.

24.21 RELIEF DISPLACEMENT

The light rays from the objects on the earth surface pass through a single point, known as perspective centre of the lens, before falling on a sensitized film. The geometry of viewing vertical photographs has two distinctive features *i.e.,*

(*i*) Tall objects block the view of the nearby objects away from the principal point.

(*ii*) The land surface features lying above or below the horizontal datum get displaced from their true planimetric positions. Such a displacement of the objects, is called the **relief displacement**.

The effects of relief displacement are as under :

(*i*) The ground features of elevation above datum, experience the displacement outward *i.e.,* away from the principal point.

(*ii*) The ground features of elevation below datum, experience the displacements inward *i.e.,* towards the principal point.

The images of the ground objects on a vertical photographs differ in size, shape and location compared to their locations on a map, which is a vertical projection of the features *i.e.,* top view.

Fig. 24.20 explains the geometrical construction of the height displacement.

From Fig. 24.20 it is observed that the relief displacement (d) of the tower of height (h) is directly proportional to height of the object.

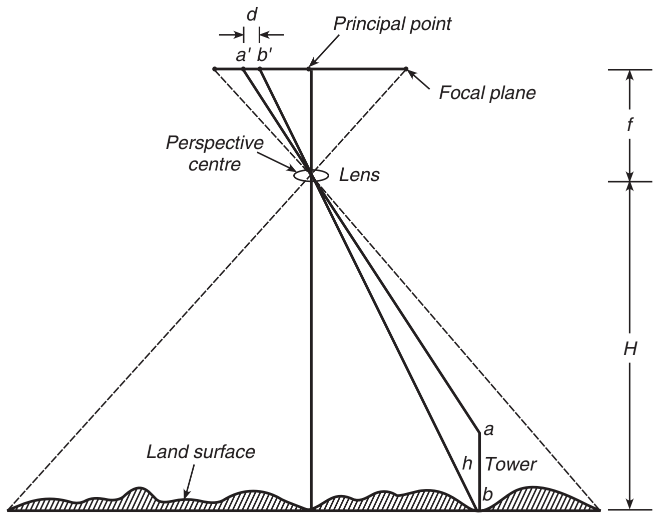


Fig. 24.20. Relief displacement of a tall tower of height *h*

$d \propto h$, where h is the height of the tower ab

$\propto r$, where r is the radial distance of the tower from the principal point of the vertical photograph

$\propto \frac{1}{H}$ where H is the flying height of the camera above datum

Hence,

$$d = \frac{h_r}{H} \qquad ...(i)$$

where constant of proportionality is 1

From eqn. (i), it is apparent that for the same ground features on a photo imagery taken from a satellite, the relief displacement will be much smaller than that on a vertical aerial photograph taken from an aerial camera. The main reason of such a discrepancy is that the satellite flies at elevation much higher than the aerial camera.

The relief displacement of a point on a photograph may be used to evaluate its height or difference in elevation between objects from a single imagery.

Scale of aerial photograph. The scale of an aerial photograph is a function of a number of factors as detailed below :

1. Focal length of the camera
2. Flying height of the camera
3. Ground elevation above sea level.

As aerial photographs are distorted due to ground topography, tilt of the camera axis, relief displacement, etc., the photographs can not be used for mapping unless corrected by a process called or the **rectification**. This process transfers the central projection of the photograph into orthogonal view of the ground features.

Limitations of a remote sensor camera. The remote sensor camera has the following limitations:

(*i*) Only limits spectral response upto about 0.9 μm.

(*ii*) Inability to photograph the short wave IR and thermal IR regions.

(*iii*) Reproductibility of quality of the imagery degrades the quality of the pictures.

(*iv*) Dimensional stability applied to size changes, is caused due to change in humidity, temperature and processing. Aging is also difficult to control.

24.22 MICROWAVES SENSORS

The wavelength of microwaves may extend from a few micro metres to metres. Sensors operating in the microwave region, are classified as active as well as passive sensors. As sun radiation does not play any direct part, the microwave sensors can operate during day as well as at night. Microwaves are more transparent in atmosphere compared to optical rays. This is the reason why microwave sensors provide an all weather monitoring capability.

Uses of microwave spectrum. The microwave spectrum is used for the following :

(*i*) Remote sensing technique

(*ii*) Communication satellites

(*iii*) Terrestrial radars

The microwave emission from the earth's surface is very weak. The passive sensing of the earth is vulnerable to interference from the active sensor emission. The transmission frequency and out-of-band emissions are mostly responsible for the interference. The allocation of the microwave frequency spectrum is governed by the world Administrative Radio Conference (WARC).

Even though both the light and microwaves are electromagnetic radiation, the technique involved in both for realizing the receiving and transmitting system, are quite different. In case of microwaves, an antena is used for receiving and transmitting systems. The basic function of the antena is to collect the electromagnetic radiation and concentrate it on to a detector or to produce a collimated beam from a source as a focus and also acts as a transducer to convert an electrical signal to EM radiation and vice-versa. To understand the functioning of microwaves, we need to know the basic information of its antena.

24.23 WORKING OF AN ANTENA

The working of an antena for transmitting electromagnetic waves, was developed by Guglielmo Marconi in 1897. In a transmitting antena, the signal from an electric circuit causes electrons in the antena to oscillate. The moving electric charges generate electromagnetic radiation which propagates in space. If the conductive structure of the antena carries a time varying electrical current, it radiates electromagnetic field. Similarly, if a conductor intercepting an EM field, carries an electrical current, it radiates electromagnetic field. To meet today's requirement, different types of antenas are designed.

Technical Terms used in antenas. For understanding the proper working of an antena, the following technical terms must be clearly studied :

1. **Radiation pattern.** Radiation pattern describes the relative strength of the radiated field with the angle at a fixed distance. It is equally applicable for both the transmission and receiving data. Radiation patterns are three dimensional. In actual practice, they are usually measured in two orthogonal planes. The radiation patterns are plotted either by polar or Cartesian coordinate systems. The patterns are normalised to the maximum value at 0 dB. In antena, for remote sensing, the pattern has a main lobe for concentrating the most of the energy. Side lobes may also be provided but these are undesirable.

2. **Antena gain.** The ability of the antena to focus the radiation in a particular direction is known a its **antena gain**. The antena gain refers to the direction of maximum radiation. It is not possible to change the total power of the antena. It is only possible to redistribute the energy in a particular direction. It means that the power can be both reduced or increased in any direction as required. Antena gain is expressed relative to the performance of an ideal antena which radiate the energy in all directions equally and expressed in ducible (dB). If the antena gain is referred to an isotropic direction, *i.e.*, equally in all directions, the antena gain G with losses is given by correlation

$$G = \frac{4\,\tau\,\eta\,A}{\lambda^2}$$

where A is the aperture area of the antena

λ is the wavelength

η is the efficiency *i.e.* total power radiated divided by net power fed $\frac{\pi D^2}{4}$.

Beam width. Two points on either side of the peak power of the main lobe in the radiation pattern, are referred to as *half power point*. The angular distance between the half power points travelling through the peak, is known as **beam width**.

The beam width (BW) is a function of the wavelength (λ) and the antena aperture (*D*), *i.e.,*

$$BW \propto \frac{\lambda}{D} \qquad ...(i)$$

i.e., for larger antena dimension, a narrower beam width is obtained. Larger antena, therefore, has higher gain but narrow width.

24.24. TYPES OF ANTENAS

Antenas are of two types :

1. **Paraboloid antena.** It is the most commonly used antena for microwave radiometers. Its reflector (dish) is a section of a surface formed by rotating a parabola about its axis. The reflector is illuminated by a horn suitably located at the focus of the reflector. It gives a good directionability.

2. **Horn antena.** It consists of a wave guide section whose cross section area increases towards an over end known as the **aperture.** The maximum response is along the axis of the horn. In a horn antena of rectangular cross section, the beam width both in azimuth and elevation can be adjusted in a desired direction. Horn antenas are generally used as **reflectors**.

24.25 DATA RECEPTION

The reception of data from a satellites is as under :

Data received from satellite sensors is in the form of electrical signals (digital format) which are amenable for further computer processing. Electrical signals produced by the sensors are recorded on board in the satellite and then transmitted to the ground station, by using a suitable communication system.

Signal formatting. To understand the principles of signal formatting, let us consider an example of a signal band Charged Couple Device (CCD) camera. A Charged Coupled Device (CCD) essentially consists of a linear or two dimensional (area) array of detectors.

Working principle of CCDs. Light energy incident on the photosite of the CCD gets converted into electrons. These electrons generated by light photons get accumulated in the associated potential wells and remain confined there. The potential wells are produced by applying well defined voltages on to a set of electrodes which are connected to a number of different groups to a logic drive clock. During the exposure period, the sensor accumulated a special charge distribution throughout the detector array corresponding to the intensity distribution of the incident light photons. After exposure period, in a linear CCD image sensor, the charge from the detector elements are first transferred to the parallel inputs of a number of analogue shift register. (Fig. 24.21).

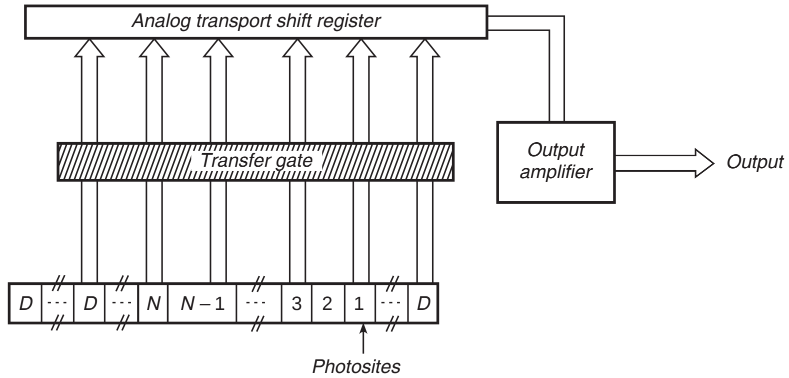


Fig. 24.21. Line diagram of a CCD.

The detector elements are now ready to accumulate a new line of data while previous one present in the analogue shift resisters gets transferred serially by the transport clock to the sensor's Read-out stage. Each clock sequence introduces successive charge packet into a reverse-biased diode circuit and converts it into an analog voltage.

With the help of appropriate clocks, information from the analog shift finally contains the data from all the pixels connected to the shift register. The outputs of the odd and even shift registers are combined with the output device to produce a single video output per device.

There are many array architectures used for reading out the data. The most useful architecture for imaging from the satellite is a frame transfer architecture.

Working of along-track or push broom scanner system (Fig. 24.22)

A strip of the terrain is focussed by a system of lenses on the linear detector mounted on to a moving platform. Thus, we get successive scan lines due to the motion of the platform. This mode of scanning is sometimes known as *push broom scanning*.

Suppose there are '*n*' elements detectors to generate the image data for *n* pixel. Each pixel gives an output voltage proportional to the energy falling on the respective detector element. The output voltage is digitised to provide a word of say '*m*' bits. Binary data of *n* words is made, each

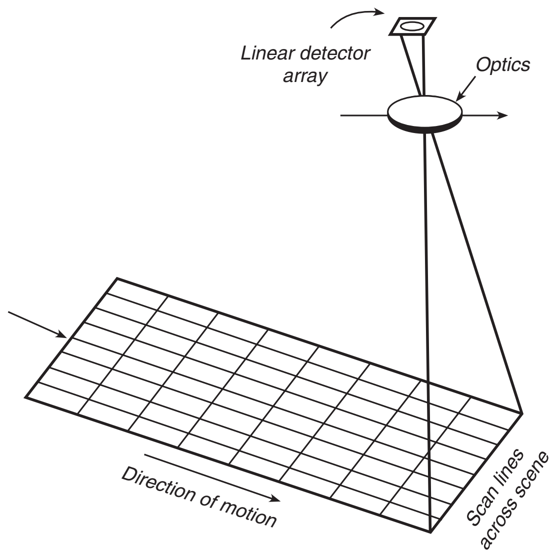


Fig. 24.22. Push broom camera.

having words of '*m*' bits. Auxiallary information related to the space craft (temperature, orbit and altitude data) is also made available. On line of data is now ready to be transmitted. A unique word (Sync word) is entered preceding each line data, taking care of that syn words are not confused with the main data. One frame of telemetry data represents one cross-track image and successive image lines produce successive telemetry frame.

In push broom systems, scanners record multiband image data along a swath, beneath an aircraft. As the aircraft advances in the forward direction, the scanner scans the earth with respect to the designed swath to build a two-dimensional image by recording successive scanlines that are at right angles to the direction of the aircraft. In this system, a linear array of detectors is used and not a rotating mirror. This is the main difference between along-track and across track scanners.

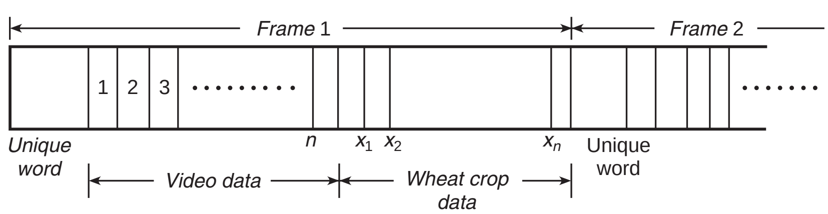


Fig. 24.23. A single band CCD telemetry format.

We have described a single band CCD telemetry format. There may, however, be additional bands which are formatted suitably.

The output from all satellite sensors is in the form of electrical signals. The electrical outputs from the sensors may be directly related with the reflectance/emittance from the surface of the earth as in optomechanical sensors. The electrical outputs is also a complex function similar to SAR, which requires a processing method to generate the image. Moreover, the loaded data may contain various errors/distortions which might have been introduced by the sensors platforms, atmosphere, etc.

Duly corrected data from various sensors are presented in a proper format with the desired radiometric and geometric accuracy, easy to read by various application of scientists/users to process it by computers. The data presented to the users, is called *data products.*

Working of Across-track or whisk broom scanner systems. The built-up two-dimensional images of the terrain for a swath beneath the platform, may also be done by across-track system, also known as *whisk broom scanner system*. In this type of scanning system, the terrain along scan lines is scanned at right angles to the direction of the aircraft platform. (Fig. 21.24)

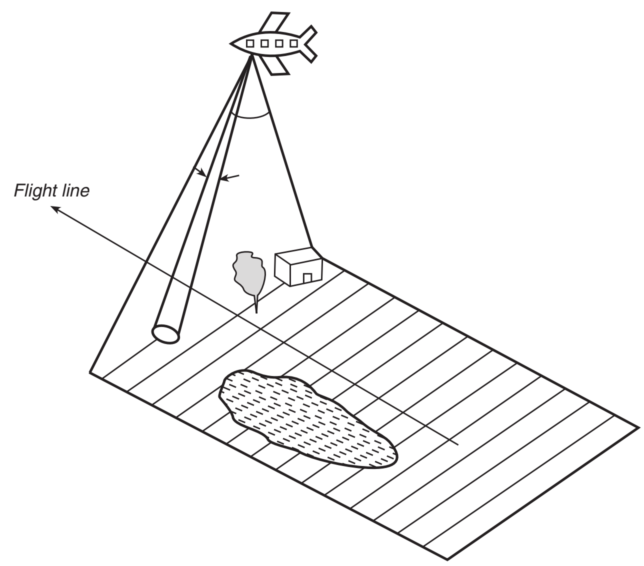


Fig. 24.24. A cross-track or whisk broom scanner system.

In this type of scanning system, scanner repeatedly measures the energy from one side of the aircraft of the other within an arc below in aircraft typically of 90° and 120°. Successive adjacent lines are covered as the aircraft moves forward. A series of narrow strips of observations comprising a two dimensional image of rows and columns are obtained.

Data product generation. The remote sensing data provided to the users should represent the geometric and radiometric properties of the surface of the ground. The data received at the ground station possess a number of errors produced by the sensor itself coupled with those produced by the platform, intervening atmosphere, and data transmission/reception system. Data products need be corrected for geometric/radiometric errors. Systematic errors are eliminated by suitable operations on the data received from the satellite.

Errors in data products. The important errors of data product are as made :

1. Radiometric Errors. Radiometric errors are introduced in the remote sensing data, due to sensor characteristics, the intervening atmosphere and noise introduced during signal generation, transmission/reception. Due to common radiometric error in a photographic camera, even for uniform land surfaces, the intensity at the edge of the field, decreases as compared to the on-axis intensity, Due to radiometric erros, images are produced in which equal input and intensity produces different gray lines, known as *shading*. Radiometric errors may be corrected by using a light transfer function evaluated by elaborate caliberation on the ground.

2. Geometric Errors. Due to various geometric errors, the imaging system does not produce an exact representation of the earth surface. The geometric errors are mainly due to the sensor and platform. In case of optomechanical scanner, the major cause of distortion is due to the geometry of scanning. In such cases, the data samples are taken at regular intervals of time. It means that each data value refers to equal angular interval increments.

In case of satellite sensor, the ground distance swept is proportional to $\sec^2 \theta$ where θ is the angle of scan measured from the Nadir.

24.26 GLOBAL POSITIONING SYSTEM (GPS)

The global positioning system (GPS) consists of a network of 24 satellites in roughly 12 hour-orbits, each carrying atomic clocks on board. The orbital radius of satellites is roughly twice the earth diameter. The orbits are nearly circular, with eccentricity of less than 1%. Orbital inclination its the earth's equator is roughly 55 degrees. The satellites have orbital speeds of about 3.9 km/s in a frame centered on the earth. Normally, satellites occupy one of six equally spaced orbital planes. Four satellites occupy each plane spread at roughly 90 degree intervals around the earth in that plane. The precise orbital periods of the satellites are close to 11 hours and 58 minutes so that ground tracks of the satellites repeat day after day.

The satellite atomic clocks are correct to about 1 nano second (ns). As the speed of light is about one foot (0.3048 m) per nano second, the system is capable of desired accuracy in locating any detail object on earth. The satellite clocks are fully synchronized with the ground atomic clocks. Knowing the instant a signal is sent from a satellite and the time delay for that signal in reaching a ground receiver, the accurate distance between the satellite and the ground receiver can be determined.

By using four satellites to triangulate with these determined distances, the position of the unknown location can be determined with good precision. It may be noted that GPS operates by sending atomic clock signals from the orbital altitudes to the ground receiver. The total distance is covered in 0.08 seconds whereas it is very long when it is measured in nano second *i.e.,* 0.08 second is read as 80,000,000 ns by an atomic clock.

Location of points on a plane surface is determined by using the Cartesian system of coordinates. This system of coordinates was first introduced by the great philosopher Desecrates. In this system *OX* and *OY* are two orthogonal fixed straight lines in the plane of the paper. The line *OX* is called the x-axis, the line *OY* is called the *y*-axis whilst two axes together are called the axes of coordinates. The point of intersection of the axes is called, the origin (Fig. 24.25).

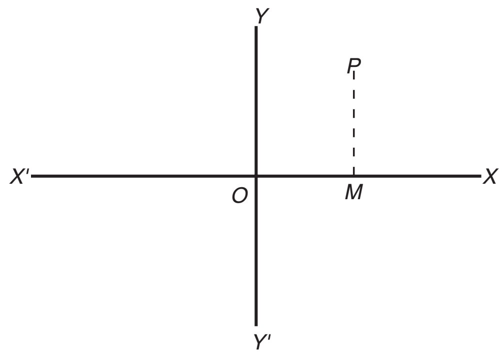


Fig. 24.25. Cartesian coordinate system.

Consider a point *P* on the plane *XY*. The distance *OM* along the *x*-axis is called the abscissa and the distance *MP* measured parallel to *y*-axis is called the ordinate of *P*. Both the abscissa and ordinate of the point *P* are called its coordinates.

Spherical Coordinate System. The location of a point on the spherical surface of the earth is determined by using the spherical coordinates, *i.e.*, latitude and longitude.

(*i*) **Latitude.** The angular distance of the point north or south of the equator measured in degrees along the longitude of the point, is called the **latitude**. The values of latitude vary from 0° to 90° North and South of the equator, Latitude in the northern hemisphere are treated positive.

(*i*) **Longitude.** The angular distance of the point east or west of the Prime Meridian (*i.e., Greenwhich meridian*) measured in degrees, is called the **longitude**. The values of longitude vary from 0° 180° E or W of the Prime meridian.

Spherical coordinates are determined by making geodetic observations to the Celestial bodies and are commonly used for triangulation.

24.27 LOCATION OF A POINT BY GLOBAL POSITIONING SYSTEM

Global Positioning System (GPS) is a satellite navigation system based on the principle of trilateration. The trilateration is a method used to find the location of a point if its distances from at least three other stations of known coordinates, are predetermined.

Location of a point with the help of three stations, in two dimensional space, may be obtained as explained under :

Let A, B and C be any three stations whose coordinates are predetermined. Let the position of the station to be located be marked as P.

The distance of stations A, B and C from point P be x_1, x_2 and x_3 respectively. Proceed as explained below stepwise :

(*i*) Plot the location of the given points (stations) to a suitable scale.

(*ii*) Taking A as the centre and x_1 as radius, draw a circle.

(*iii*) Taking B as the centre, and x_2 as radius draw another circle to intersect the first circle at two points say a and b.

(*iv*) Taking C as the centre and x_3 as radius, draw a circle which passes through either a or b.

The point through which three circles pass, is the required position of the station P.

Satellite based navigation method is an extension of this principle of trilateration. (Fig. 24.26)

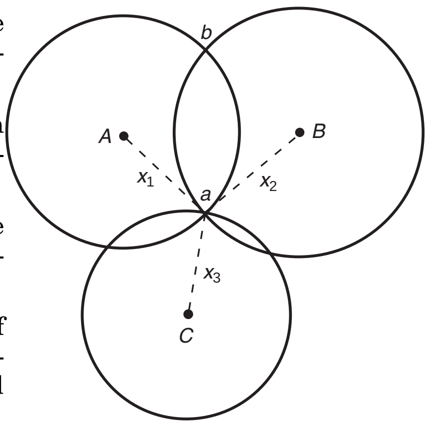


Fig. 24.26. Trilateration method of interpolation.

For locating the position of the observear on the sphere, the following steps are involved :

(*i*) Recognise the locations of three satellites in the space forming a well conditioned spherical triangle.

(*ii*) Name them as S_1, S_2 and S_3.

(*iii*) Draw a sphere with S_1 as centre and its distance from the ground position P, as radius. Similarly, draw two spheres with S_2 and S_3 as centres and their respective distances from P as radii.

(*iv*) The position on the earth's surface where three spheres intersect is the required position *P*, the observer's location.

Fundamental Components of GPS. The fundamental components of Geographical positioning System (GPS) are described below:

1. Space. This segment of GPS consists of the orbiting satellites making up the constellations composed of 24 satellites and the design of the orbit, and the spacing of the satellites orbital plane.

Every user is able to access at least six or more satellites orbiting in the space.

2. Control. This segment consists of ground station of observation from where user oversees the building, launching, orbital positioning and monitoring of the system.

3. The User. (GPS receiver). The space segment consists of 24 satellites which revolve about the earth at an altitude of about 20,000 km having a period of about 12 hours with a 55° inclination and placed in six differential orbital planes, having four satellites in each plane. Every satellite transmits two *L* band signals (L_1 with 1575.42 MHz and L_2 with 1227.60 MHz). These signals are duly modulated by Pseudo random binary codes (*PN*). GPS receiver reads the code of each satellite even though all the satellites transmit in the same frequency with the help of *PN* code, it is possible to determine the time taken by the signal to reach the receiver. By knowing the accurate time taken by the signal, the distance between the signal and receiver in each case, can be determined accurately.

24.28. ADVANTAGES OF GPS

The basic advantages of GPS are as explained under :

1. **Identification of spherical coordinates.** GPS helps to identify or define geographical coordinates on the satellite image and also to reduce distortions and positional accuracy of the images. By identifying three or four well defined points, the locations of the satellite image on the ground can be obtained by the method of resection. The GPS receivers collect accurate geographical coordinates of these locations. The remaining images are located both on the satellite image and on the ground. The remaining image points are filled in between the locations of the control points by normal method of air surveying to obtain real-world coordinates.

2. **Truthing of satellite images.** In case on a satellite image, there appears a region of unusual or unrecognised reflectivity or back scatter, the coordinates of such region may be reloaded in a GPS receiver.

3. **Cost effective tool.** GPS is a cost effective tool for updating GIS or computer aided design (CAD) systems. The users of GPS equipment

simply move from one location to the other along the surface of the earth using the geographical coordinates.

GPS has proved to be an excellent tool for data collection by the users in clear sky environments.

EXERCISE 24

1. Define remote sensing. Also, narrate its fundamental principle.

2. Describe the various uses of IRS satellite data.

3. Describe the various characteristics of ground features which help for their identification.

4. Write a short note on electromagnetic radiation.

5. Explain the mechanism of propagation of electromagnetic waves from one medium to another.

6. Write short notes on the following:

(*a*) Signatures

(*b*) Polarisation

(*c*) Atmospheric windows

(*d*) Platforms

7. State the kepler's laws of satellite motion.

8. Differentiate between apogee and perigee of the satellite path.

9. Explain various types of satellite orbits with neat sketches.

10. Establish a relation between the angle of view, radius of the earth and height of the satellite above the earth's surface.

11. Describe the various types of remote sensors used in satellites.

12. Write a detailed note on imaging modes of an area.

13. Explain the working of along-track (push broom) scanner system.

14. Explain the working of across track (whisk broom) scanner system.

15. Explain the working of global positioning system (GPS).

16. What are the advantages of global positioning system (GPS)?

25

Geographical Information System (GIS)

25.1 INTRODUCTION

The underlying principle of Geographical Information System was in existence centuries ago in the form of geographical informations compiled from the land surveys prepared by conventional methods such as chain surveying and planetabling. With these two conventional methods, the details (or objects) on the surface of the earth were usually depicted on a suitable scale. For preparation of multi-coloured maps, the details of each colour were drawn on separate transparent sheets. According to the details they represented, these transparent sheets were called as **outline original**, **contour original**, **green original**, **blue origin**, etc. With the help of topographical maps (generally on scale 1:50,000), the geographical maps showing different themes were complied on reduced scale. Thus, the geographical maps were used to depict the required information for the map users.

The Survey of India published the following thematic maps:

1. **Physical map.** It depicts the physical configuration of earth's surface of Indian territory. It mainly includes the mountains, hills, plateaus, network of rivers.

2. **Polical map.** It depicts all the states and their boundaries. It also includes the district and tahsil boundaries and important rivers.

3. **Railway map.** It depicts the network of Indian railways of all types of guage and important rivers, cities.

4. **Road map.** It depicts the network of roads, highways and national highways, important cities and rivers.

5. **Soil map (or Geologic map).** It depicts various types of soils and general configuration of the earth surface.

With passage of time, these originals on transparent sheets were replaced by scribed originals with details of each colour separately.

Geographers and other geographical information users found great difficulty in deriving correct information in the form of spatial data (ground features occupying some space). It used to involve voluminous data and a lot of time. Moreover, suitable techniques were not available then to process these data. With the invent of photogrammetry and remote sensing techniques, GIS has been developed as a useful tool for the geographical analysis for the following purposes :

1. **To search a suitable site.** For the search for optimal sites such as new schools, bus terminuses, airports, a large number of topo sheets on scale 1 : 50,000 were used for the analysis. In those days, the pressure of population was less. In the present days, population explosion has made small towns as cities and cities have been upgraded to metropolitans. The increase in population in big cities, has pressurised the local governments to tackle a number of formidable problems. One of these problems is disposal of huge mass of solid waste generated by city population. It is collected by municipal corporation and disposed off at far fluing area. GIS comes forward to help the local administration to select suitable sites for solid waste disposal at an environmentally safe location. For proper identification of suitable sites for the solid waste disposal, a thorough analytical and comprehensive studies of the physical characteristics of the site, are of prime importance.

Conventional of methods for site investigation and their identification lack both comprehensiveness and objectiveness of the project. The study of a large number of toposheets containing various types of data is not feasible with a conventional approach and generally results in inappppropriate conclusions. Moreover, this kind of approach for tackling the problem may prove simply more time consuming. It also does not help to analyse a large number of criteria required to be filled for the final selection of the site.

GIS techniques offer an alternative approach to facilitate quick and easy remodelling for the slight changes in siting criteria if required. GIS helps to produce suitable maps for the analysis and presentation. The GIS uses the flow chart as shown in Fig. 25.1 for the identification of a suitable site such as for the solid waste disposal.

In the first stage, a number of spatial data sets such as the land use, type of soil, geological structure and transport facilities for each site are generated by a thorough study of details collected by the conventional and remote sensing sources. The selected spatial data sets are then digitized.

In the second stage, with the help of secondary data layers, *i.e.,* the slope of the general terrain, their nearness to water bodies and major roads leading to the sites, are generated by processing the spatial data. The data sets pertaining to the sites so selected should represent the specified site criteria. For the generation of the slope layer, the contour map of the site is studied to ensure proper slope necessary for the site and also to avoid steep slope, if any. A detailed map showing areas in close proximity to the water bodies is studied so that the sites may be excluded from the scope of the site idenfication.

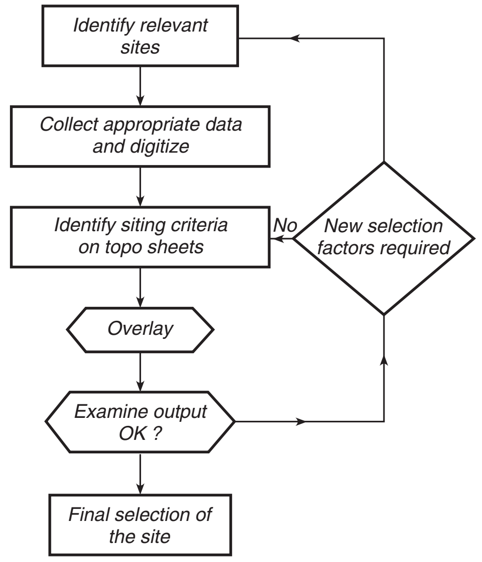


Fig. 25.1. A flow chart.

GIS software may be used to superimpose the data layers with other data layers representing the suitable site criteria. The final map thus obtained represent the suitable site criteria. The final map thus obtained represents a map showing the locations of a number of sites suitable for solid waste disposal catering to the specified site criteria. The main advantage of using GIS for performing the selection of sites is that the siting criteria can be altered/amended and new criteria be loaded in the analysis without much difficulty and with least wastage of time.

2. Soil conservation planning. India is a large country, extending from Gujarat in the West to Nagaland in East; from Jammu & Kashmir in the North to Tamil Nadu in the South. Such a large country is expected to face the problem of land degradation and soil erosion especially in mountainous regions. Moreover, improper land use practices such as deforestation and unregulated farming techniques affect the soil fertility. The government of India as well as the state governments have taken a number of water-shed management projects to tackle the land degradation problems. Hilly areas are more prone to degradation because of steep slopes and unstable soil profiles. Land slides so often occur in hills particularly in rainy season. Unregulated land use practices also affect the ecological and environmental imbalance and cause land slides.

GIS has proved a very useful tool for planning and administration of land degradation. In GI systems data from aerial photographs, topographical maps, satellite imageries and socio-economic surveys of the area are mapped and overlaid to locate the most problematic areas. With the help of these maps, the GIS planners easily pin-point the areas of soil erosion, suitable sites for check dams and land use practice in the areas to prevent further land degradation.

For correct mapping of the area, the project planners are required to know the characteristics of the land surfaces, type of soil, slope of the ground, present land use practices, geologic formations, etc. GIS helps to bring together data from various sources as stated above, to solve the environmental problems. It may however be noted that the accuracy of GIS mapping mainly depends upon the accuracy of the source data, scale of the source data and the quality of the source data.

3. **Search of a new rented of house.** Search of a new rented house in a big city like Delhi or Kolkata by a tenant largely depends on one's income, life style, culture, etc. It becomes cumbersome and labourious task, if the family members of the home seeker work in different offices situated at different places in the city. While searching a suitable home, the requirements of the working couple, their schools going children and elderly parents need be kept in mind. The chosen home should preferably be nearer to children' schools and also nearer to a good hospital for the sake of old parents. Nearness to a major road and a railway/Metro station is also an essential factor for the selection of the newly rented house.

A suitable home satisfying the essential criteria may be selected by using the GIS decision-support with location information. While making a final decision, the occupant has to decide the preference of his choices, *i.e.,* proximity to schools, railway stations, major roads, shopping centres, hospitals, etc.

The various factors of the necessities need be allocated weights and scores reflecting their levels of importance. At the very initial stage of selection, water-logged areas, under-developed areas and cluster of huts are rejected out-rightly. With the weight allocation process, the data selected need be combined in GIS, using a multi-criteria modelling techniques. In modelling techniques, the information layer with the highest weight greatly influences the final selection. The resulting map consisting of a number of layers is finally studied to help in tracking a new home.

25.2. DEVELOPMENT OF MAPPING TECHNIQUES

Collection of data about special features on the earth's surface has been an endevour of all organised societies from the inception of civilization. Spatial data collected by surveys, geographers, cartogrophers and navigators was transformed into pictorial form by map makers. During the eighteenth century the government of India felt the necessity of systematic mapping of the earth's surface of Indian territory. The department of Survey of India, a National Government Institution was established in the year 1767. The survey of India was entrusted the responsibility of preparing the topographical maps for the entire country.

Initially, the conventional method of planetabling was used to survey the physical and cultural details on the earth's surface in horizontal plane. The method of chain survey was used to survey details of comparatively small areas in flat terrain. Chain survey maps were largely used for the revenue and cadastral surveys. With the passage of time, it was felt necessary to show elevations of the important features on the map. Consequently, the method of levelling was adopted. The spot levels at convenient points were recorded on the map prepared by the plane table method. Visualising a three dimensional-effect of the earth's surface with spot levels was difficult. The term contour was perhaps coined at this stage. The contour line may be defined as the imaginary line joining the points of the same level above an assumed datum. For the national maps, the mean sea level at Mumbai is adopted. With the help of a plane table and level, it was found possible to prepare a contour map with other cultural details (Fig. 25.2).

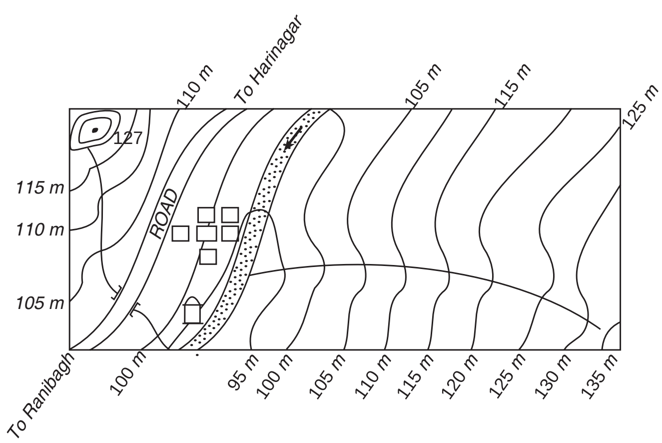


Fig. 25.2 A contour map

25.3. DEVELOPMENT OF TOPOSHEETS

All countries worldover decided to divide the earth's surface into belts bounded by latitudes and longitudes. The area of the earth surface bounded by 1° latitude and 1° longitude was termed a degree sheet. All the countries including India surveyed their areas and published as degree sheets. Prior to 1957, degree sheets were published on scale 1/4″ = 1 mile (1:253440). Accordingly, these sheets were known as quarter inch sheets. After 1957, the metric system was adopted in India and the degree sheets were surveyed and published on scale 1:250,000. Each degree sheet is further divided into 16 sheets on scale 1:50,000.

53J

$53\frac{J}{1}$	$53\frac{J}{5}$	$53\frac{J}{9}$	$53\frac{J}{13}$
$53\frac{J}{2}$	$53\frac{J}{6}$	$53\frac{J}{10}$	$53\frac{J}{14}$
$53\frac{J}{3}$	$53\frac{J}{7}$	$53\frac{J}{11}$	$53\frac{J}{15}$
$53\frac{J}{4}$	$53\frac{J}{8}$	$53\frac{J}{12}$	$53\frac{J}{16}$

Fig. 25.3. A degree topo sheet with 1:50,000 sheets

With an advancement of technology, the methods of surveying were also upgraded. The conventional method of planetabling was replaced by Air survey method. For carrying out air survey of an area, the entire area need be covered by vertical aerial photographs, having more than 50% overlap along the line of flight and at least 30% overlap between adjacent flights. Maps were compiled by graptical method. The maps so prepared by using the aerial photographs, were verified on the ground to pick up the names of natural as well as man-made features. This is known as *ground verification survey*.

The 'air survey' method was replaced by photogrammetric method in which either the negatives or the positives of the aerial photographs were mounted on sophisticated photogrammetric machines to view a three-dimensioned model of the terrain in air-conditioned rooms. With great advancement in the surveying techniques and the cartographic methods, Laurni and Thompson (1992) coined the term "**Geomatics**" to cover the different methods of surveying and to help in formulating and understanding spatial information systems. The earliest traditional method of storing, analysing and presenting data were only maps. A map may be said as the starting point of the analysis and to be used in presentation of results of operational projects. A map user always appreciates the map which provides high quality information even though it is complied by any method, *i.e.,* cartography, photogrammetry or geographical information systems (GIS). Keeping in view these facts in mind, it may be inferred that maps and their production by using any modern technologies, is an important and essential starting point and necessary tools to study the characteristics of spatial phenomenons. A topographical map is an essentially representation of the earth's features drawn to scale and thus it is a reduction of the real configuration of the earth's surface. The scale of the map, however, limits the type and manner of the data being depicted.

25.4. TYPES OF FEATURES ON EARTH'S SURFACE

The surface of the earth is covered by a number features of different characteristics. The features which occupy some area, are called spatial data. Spatial data are classified into two categories.

(*a*) **Natural features.** Hills, rivers, water bodies, forest, etc., are classified as natural features.

(*b*) **Man-made features.** Townships, highways, railways, airports, canals, metro tracks, etc., are classified as man-made features. Spatial data on the earth attracted the attention of navigators, geographers, cartographers and land surveyors from the very beginning of civilization. Initially, maps were prepared depicting spatial data at far off places such as hills, rivers, etc., to provide an aid to navigation and military movements. With passage of time, the National government started preparation of tropographical maps. A topographical map is a map on a sufficiently large scale to enable the individual features shown on it, be identified on the ground by their shape and position. On the other hand, a geographical map is on such a small scale that the individual features shown on it, are suitably generalized and the map gives a picture of the country as whole and not a strict representation of its individual features.

The Survey of India developed strategies relating to the areas such as natural resources management and development, tourism department, area of military importance from defence point of views. With the technological development, many more areas have been identified to study these innovative technologies for preparation of the general purpose maps With the invention of the remote sensing techniques, GIS and GPS methods have been developed to study the spatial data and to locate individual locations respectively.

25.5. SPATIAL DATA

Spatial data are characterized by the information about its position, connections with other features and details of non-spatial characteristics. The method of referencing the spatial data is also very important. In appropriate referencing method proves to be a failure in future.

Geographical information systems (GIS) are infact computer representations of some aspect of the earth surface. The simplified view of the earth surface adopted by GIS is generally known as a *model*. A model is a synthesized data that enables us to grasp mentally the configuration of the earth surface. Infact, a special model emphasizes, on reasoning, about the earth surface by means of translation in space. This phenomenon helps to solve the geographical problems by GIS.

Before understanding how special models are constructed by using a GIS, one needs to distinguish between two terms data and information, although these two terms are so often used interchangeably. Data is nothing but the raw numbers listed either in a table or a column, and having no particular meaning. The users always need to know what the data listed in the table, refer to and which unit of measurement is used to record the data. Once these attributes are added to the data, the data become information. A per Hanold (1972), the information is data with meaning and context added.

25.6. DATA SOURCES

The data sources may be categorized as primary source and secondary source. Number of air crafts flying in the sky may be termed as primary whereas secondary source indudes the same number of aircrafts with their types.

All primary and secondary data have three modes or dimensions: temporal, thematic and spatial. For each data one of the three modes must be clearly identified. The temporal dimension of data provides a record of the data and time when the data was collected. The thematic dimension of the data describes the character of the earth surface it refers to. In GIS, the thematic data is generally referred to as non-spatial or *attribute data*. The spatial dimension of data, conveys to the user the information about the location of the ground features being observed. All spatial data used in GIS are given a mathematical spatial reference to locate their positions. The most commonly used mathematical spatial reference, is the Cartesian Coordinate System (x, y).

GIS uses spatial dimension for converting data into information which becomes very useful to understand the geographic phenomena.

25.7. INFLUENCE OF MAPS ON THE CHARACTER OF SPATIAL DATA

From the ancient times, maps have been used for storing, analysing and presenting spatial data. For GIS, a map is a fundamental unit as a source data, a structure of storing data and a device for analysis and display. For exploring the characteristics of spatial data, one need to know the fundamentals of maps and the basis of their production. Maps are published on different scales. Topographic and thematic maps are more complex. Thematic maps depict data relating to a particular theme such as forest area, a populated area, an irrigated area, etc. Topographic maps on the other hand are general purpose maps and depict a device set of data on different themes. Infact a topographic map is a repilica of the earth surface it represents. In other words, a topographic map is a composite map of different kinds of maps.

The cartographic process includes the following:

(*a*) The purpose of the map

(*b*) The scale of the map

(*c*) The selection of earth features to be shown on the map

(*d*) The method of representation of the earth features.

(*e*) The generalisation of the features for representation in two-dimensions.

(*f*) A proper selection of the map projection.

(*g*) To provide a suitable spatial referencing system for locating relative positions of each other detail.

(*h*) To provide explanatory notes to facilitate the use of the map.

25.8. TOPOLOGY

A cartographer produces a map on the principles of topology which is based on the geometric relationships of objects (features). Topology may be defined as the study of geometrical properties and spatial relations unaffected by the continuous change of shape or size of figures. The topological characteristics of an object, are independent of scale of measurement. Topology consists of three elements : *adjacency, containment* and *connectivity.* The adjacency describes the adjoining data. The common boundaries of a municipal corporation and the contonment at a place is the example of adjacency. The cantonment may be treated as the extension of the adjacency theme with the difference that it describes the area features wholly contained within another area feature. An oasis in the desert, an island in a lake and the area of a cantonment within the boundary of the municipal corporation are few examples of the containment. The connectivity is a geometric property which is used to describe the linkages between line features. A network of streams and river is a good example of the connectivity. Similarly, a network of canal system starting from the main canal to distributaries and finally its minors, is also a good example to illustrate the connectivity. An understanding of the geometric relationships between spatial entities is of prime importance for analysis and integration in GIS.

It may be remembered that a point is the simplest entity which can be represented by vector by topology.

25.9. SCALES OF THE SPECIAL DATA

The main purpose of all types of maps and other sources of spatial data is to convert data into information. A single purpose map may not be as accurate as the topographic maps prepared by the Survey of India. Utility companies generally do not use the topographic maps as they are not much interested in accurate maps.

Spatial data is generally depicted on a smaller scale than their existence on the grounds surface. Scale of a map may be defined as the ratio of the distance on the map to the corresponding distance on the ground. Scales of the maps may be expressed in one of the following ways:

(*i*) A ratio scale

(*ii*) A verbal scale

(*iii*) A graphical scale

(*i*) **A ratio scale.** The ratio scale of a map expresses the distance between two points on the map and that of between their corresponding points on the ground as a ratio. The examples of ratio scale are the topographical sheets such as 1:25,000; 1:50,000 ; 1:250,000.

(*ii*) **A verbal scale.** A verbal scale expresses, the scale of the map in words. For example, 1 cm represents 100 m or 1 cm = 100 m.

(*iii*) **Graphical scale.** The scale which is drawn on the map during its preparation to illustrate the distance visually, is called a graphical scale. Graphical scales are drawn usually on hard copies to eliminate the likehood of misinterpretation of the scale in the event of variations in paper size.

1. Small scale maps. The maps on scale 1:250,000; 1:000,000, etc., are called small scale maps.

2. Large scale maps. The maps on scale 1:1000; 1:5000; 1:10,000, 1:25,000 and 1:50,000 are called large scale maps. It may be mentioned here that small scale maps cover large areas as compared to large scale maps, provided the actual dimensions of maps in both the cases remain the same.

In the case of aerial photographs and satellite imageries the scale is not immediately known and it needs be calculated by the users. Moreover, the scale on these sources i.e. aerial photographs and satellite imageries, differs from point to point on them specially so if there is large difference in elevation from one part to the other of the photograph.

25.10 COMPONENTS OF A GIS

In the simplest form, a GIS may be considered as a software whose components being the tools which help to enter, manipulate, analyse and output data. But in broader sense, the various components of the GIS are as explained under:

(*a*) The computer system (hardware and operating system)

(*b*) The software

(*c*) The spatial data

(*d*) The data management

(*e*) The analysis procedures

(*f*) The people operate the GIS

1. The computer systems and software

GIS may be run on all types of computers ranging from portable personal computers (PCs) to multi-user super computers and may be programmed in a wide variety of software languages. Some systems use dedicated and expensive work stations with monitors and digitizing tables built-in whereas some systems may be easily run on a personal computer (PCs). Burrough (1986) classified the various elements that are essential for effective GIS operations, as under:

- A powerful processor to run the software
- A sufficiently large storage memory for data
- A good quality, high-resolution colour monitor
- Input and output devices such as digitizers, scanners, keyboard, printers and plotters.

2. Spatial data. Spatial data or geographical data on the earth surface, are characterized by informations about their position, connections and other features and also details of non-spatial characteristics. For example, the spatial data for one metrological station in New Delhi may include the following:

(*i*) Latitude and longitude of the station to provide a geographical reference. The geographical reference is used to deduce the relationships with nearby features of interest. If the spherical coordinates of a rain gauge station, are known, the relative positions of other rain gauge stations can be ascertained easily.

(*ii*) Connection details such as roads, metro train station, foot paths and other facility factors to provide an access to the train gauge station.

(*iii*) Non-spatial (or attributes) data such as rainfall, temperature, elevation, wind speed and its direction.

The spatial referencing. Spatial referencing of spatial data is very important. It must be considered at the outset of the GIS project. Selection of an inappropriate referencing system sometimes proves to restrict its future use. It is therefore recommended to adopt a flexible and lasting referencing system as the expectation of a GIS may last many years.

The most prevailing traditional method to represent the geographic space occupied by a spatial data, is as a series of the themic layers, *i.e.,* layers showing different themes. Let us consider traditional cartographic maps that might be available for a limited small area. There may be a map showing forest cover, a map showing desert sand dunes, a topographical map showing cultural and other environmental features on the earth surface.

The other method to represent the ground reality in a computer may be thought of a space populated by discrete objects. Let us consider that the Department of Horticulture of a Municipal Corporation of a city needs a map to manage the development of the parks. Each park may be regarded as a discrete object, having empty space between the objects. This method is popularly known as the *object-oriented method.*

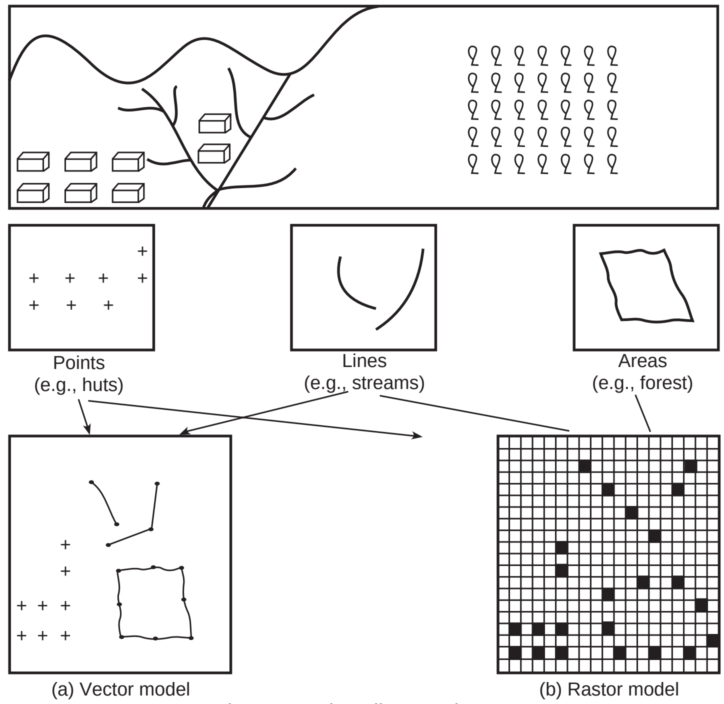


Fig. 25.4 Points, lines and areas

Before storing the spatial data represented by either layers or objects must be simplified. To achieve this, all geographic features are converted into three basic entity types i.e. points, lines and areas (Fig. 25.4).

Points are used to represent indivisual entities such as houses, temples, mosques, restaurants, pylones, fire stations, etc. Lines are used to represent entities whose length is more as compared to their widths, such as, roads, railways, metro railways, streams, etc. Area features are used to represent geographical zones. Such as protected reserve forests, mines, land fills, parks, gardens, etc. Sometimes, points and lines together are used to detect the surface in three-dimensions. For example, a sloping ground may be represented by points as shown in Fig. 25.5 for each points, its elevation is written on its left. The arrows indicate the direction of the ground slope.

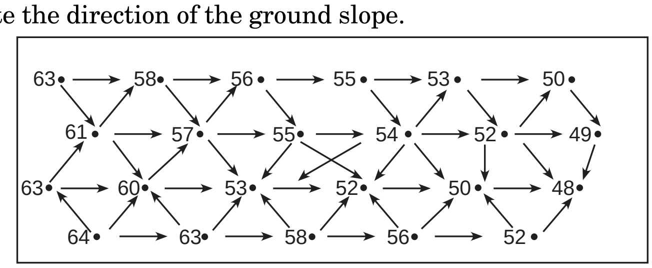


Fig. 25.5. Points with reduced levels

Similarly, the slope of the ground surface may be visualized mentally by a series of lines (or contour lines) having constant vertical interval as shown in Fig. 25.6.

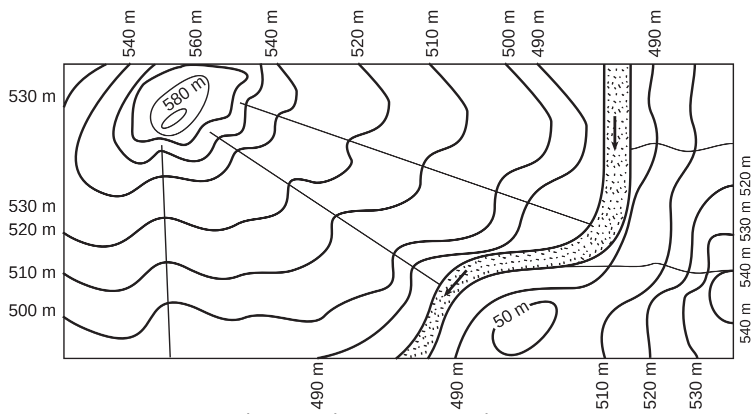


Fig. 25.6 Lines represent slope.

Representations of the real-world phenomenon, are normally held in GIS either as a raster, sometimes called tesseral (a small square or block) or a vector.

25.11 DATA INPUT AND UPDATING

Data input and its updating are the most important parts of a GIS project. Both the parts are expansive and time consuming. Eighty per cent of the total duration of a large scale GIS project, is consumed in data input and its management.

Data base management system (DBMS) facilitates the storage, organization and retrieval of data. A DBMS is a set of computer programs for organizing the information, the most important being a database. An ideal GIS DBMS is most efficient to provide a support for multiple users and multiple databases. It allows efficiently updating to minimise the redundant information, data independence security and integrity. The most important data base models used by GIS are as under:

(*i*) Relational data bases (*ii*) Flat files.

1. Data bases. The data collected by remote sensing satellites from automatic environmental monitoring equipment, by institutions during automated business transactions and by individuals engaged in research and survey work are managed in *data bases.* Data, if transformed into information become valuable and can be sold for public utility. In the present age, information resource is boundless, linking data bases to GIS. Additional spatial capability enhances the value of data to provide useful information to the user.

Government agencies, multinational companies and large organizations have adopted GIS for various purposes. The existing data bases, *i.e.,* managing customer data bases, inventories of parts or financial information are relational as a relational data base approach has been successfully used by all sectors of the business world. As the relational data base is most commonly used in the present days, an establishment of a relational data base must be thoroughly understood before using it in GIS.

As said earlier, data are rare facts and the information is derived from data by giving its attributes. In a data base, data can be ordered, re-ordered, summarized and combined, to obtain with the help of a database, the following operations,

• To sort a range of temperature values into ascending/descending order.

• To calculate the maximum and minimum values of given numbers and also the average.

• To convert degree Celsius to degree Fahrenheit.

Decision-makers of an organisation, using GIS need the information, not data. It is the data base that offers one method of providing the necessary information.

The most important capability of GIS is to transform spatial data from one entity type (*i.e.,* points, lines and areas) to another type and also helps to perform spatial analysis. The process of changing the representation of a single identity or a whole set of data,is termed as *transformation.* In GIS, transformation is used to change the projection of a map layer or correction of systematic errors from digilizing. Transformation also helps to convert data held as rasters to vectors or vice versa.

25.12 GIS ANALYSIS

In GIS, the user converts data into information and utilizes the information to decision-making. To achieve this goal, the user makes use of the data analysis. Measurement techniques, attribute queries, proximity analysis, overlay operations and analysis of models of surfaces and net works are some of the GIS packages. Practical applications of data analysis is described in the following paragraphs. The task of data analysis is to measure distances and answer queries in GIS. Proximity, neighbourhood and reclassification functions are marked before integrating data using overlay functions.

To understand spatial data analysis in GIS, the following terms are generally referred:

• **Entity.** An individual point, line or area is known as an entity.

• **Attribute.** The data that describe an entity is known as an attribute.

• **Feature.** The object in the real word to be encoded in a GIS data base is referred to as a feature.

• **Data layer.** A data set for the area of interest in a GIS, is called data layer.

• **Image.** A data layer in a raster GIS, is known as the image.

• **Cell.** An individual pixel in a raster image, is called a **cell**.

• **Algorithm.** The computer implementation of a sequence of actions designed to solve a problem, is called an algorithm.

Various spatial and non-spatial analysis for making models and their implementation in GIS, are briefly described here under:

1. **Measurements.** The main aim of GIS is to calculate lengths, perimeters and areas. Measuring the distance between two entiteis from a digital map is a straight forward task. However, the user obtains different measurements from different types of GIS (raster or vector) by using the different methods. It must be clearly understood that the measurements from a GIS are approximate..

The raster GIS measurements are of five types, *i.e.,* The Pythagorean distance, the Manhattan distance, the proximity distance, the perimeter and the area.

1. **The Pythagorean distance.** The shortest distance (also called euclidean distance) is calculated by drawing a straight line between end points of a line and constructing a right triangle so that the pythagorean geometry can be used.

2. Manhattan method. It is the distance along raster cell sides from one point to the other. In this case the number of sides of the cells traversed by the user equals the distance between end points. It is always greater than the Pythogorean distance.

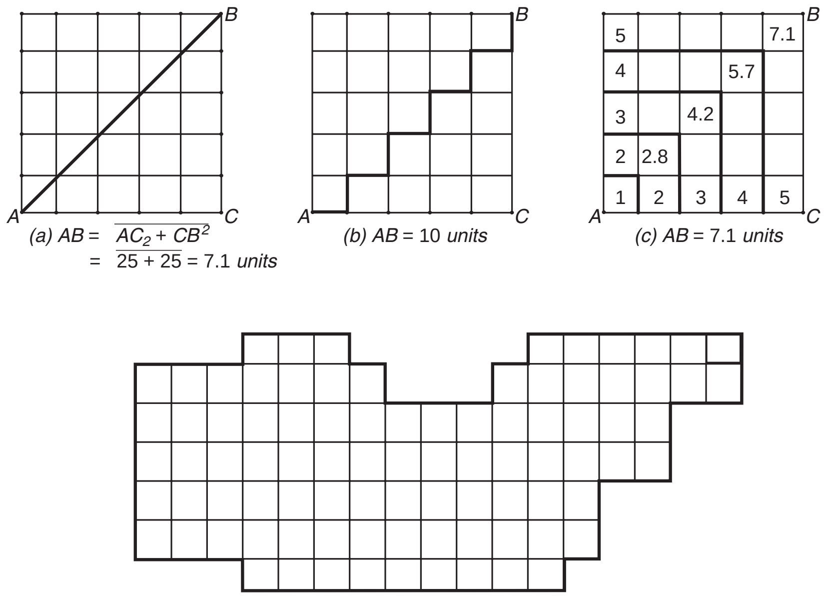
3. Proximity method. In this method, concentric equidistant zones are established around the start point. The resulting image equals the shortest straight line distance from every point on the map. Thus, the distance between points can be ascertained.

The perimeter measurement in a raster GIS is equal to the number of cell sides (or pixel sides) that makeup the boundary of the feature and multiplied by the known resolution of the raster grid.

The area measurement of a spatial features is the number of cells it occupies. When these cells multiplied by the known area of an individual grid cell, the area is obtained.

Vector GIS distance. In a vector GIS the distances are measured using Pythagorean's theorem to obtain straight (or eludian) distance.

For calculation of perimeter and area, simple geometry is used. In vector GIS, the length the perimeter and the area of a feature can be stored as attributes in a data base and is permanently saved.



(a) $AB = \sqrt{AC_2 + CB^2}$
$= \sqrt{25 + 25} = 7.1$ units

(b) $AB = 10$ units

(c) $AB = 7.1$ units

(d) Perimeter = 52 units
(e) Area = 88 units^2

Fig. 25.7.

Queries. Query may be defined as the question that enables users to quickly search through a set of information regarding spatial or non-spatial data. Performing queries on a GIS database to retrieve data is an essential part of most GIS projects. Queries are useful at all stages of GIS analysis for checking the quality of figure. Spatial and aspatial types of queries are most performed with GIS. Aspatial queries are the questions about the attributes of features. Spatial queries are the questions about the location of the spatial data. Asking for the number of luxury hotels in a city like New Delhi is an aspatial query, whereas asking for the locations of luxury hotels in New Delhi is a spatial query. Spatial queries may be made more complex by a combination with questions about their distances from Palam Airport, areas and perimetres.

25.13 THE PARTS OF GIS ANALYSIS

The procedure of the GIS procedure may be divided into the following three parts:

(*i*) The analysis procedures which are used for storage and retrieval. For example, presentation capabilities may allow to display the catchment area of a river system (*ii*) The constrained queries which allow the users to look at the patterns of stream flow in their data using queries, the sites of land slides could be selected for viewing for further analysis.

(*iii*) The modelling procedures or functions that help to predict what data might be at different times and places in future. Predictions might be made about which land slides would be vulnerable to erosion with passage of time.

Data in GIS are normally held in a series of layers. For example, a 1:50,000 topographic map could be digitized to create a series of layers. One layer could be for highways and roads, one for the built-up area, one for parks and picnic spots, one for ground configuration (*i.e.* contours) and one for water bodies. A data layer normally depicts data of only one entity type *i.e.,* points. lines, or areas data. Data analysis can be done either on one layer at a time or a number of layers in combination lying one over the other. Most GIS output is in the form of maps.

25.14 DATA STRUCTURES

A variety of data structures of the earth's surface are used in GIS. The data structures can be classified either as raster data or vector data.

Raster data In the raster world, individual cells (tesserae) are used as the building blocks for creating images of points, lines, areas, network and surface entities. The relief of the area is modelled by giving every cell in the raster image an altitude value. The altitude values are grouped and shaded to provide a three dimension effect as obtained by contours. The size of the raster grid cell is very important as it affects the appearance of the entity. With the decrease in the cell size, more details of the entity can be defined and vice versa.

In a raster data structure, a spatial entity is encoded so that it can be represented in the computer. In a most simplified and straight forward method, the cell in each line of the image are mirrored by an equivalent row of numbers in the file structure. The first line of the file tells the computers that the image of the spatial data contains 10 rows and 10 columns and that the maximum cell value is 3. (Fig. 25.8)

In a raster data set, a value is recorded and stored for each cell in the image. A complex image made up of a mosaic of different features, requires the same amount of storage space as a similar raster map showing the location of a single water body.

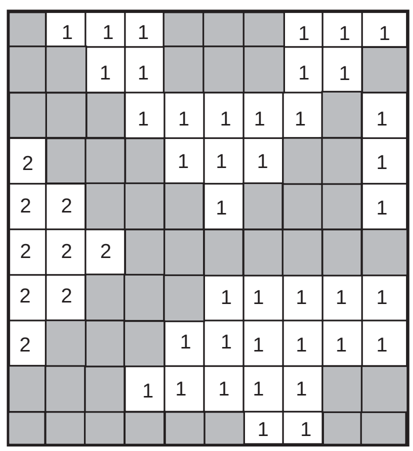


Fig. 25.8(*a*).

Vector data structure. The different types of entities of special features of the earth surface may be represented and stored in the computer by using the coordinate geometry (x, y). The simplest vector

3	1	1	1	3	3	3	1	1	1
3	3	1	1	3	3	3	1	1	3
3	3	3	1	1	1	1	1	3	1
2	3	3	3	1	1	1	3	3	1
2	2	3	3	3	1	3	3	3	1
2	2	2	3	3	3	3	3	3	3
2	2	3	3	3	1	1	1	1	1
2	3	3	3	1	1	1	1	1	1
3	3	3	1	1	1	1	1	3	3
3	3	3	3	3	3	1	1	3	3

(b)

10, 10, 3,

3,	1,	1,	1,	3,	3,	3,	1,	1,	1,
3,	3,	1,	1,	3,	3,	3,	1,	1,	3,
3,	3,	3,	1,	1,	1,	1,	1,	3,	1,
2,	3,	3,	3,	1,	1,	1,	3,	3,	1,
2,	2,	3,	3,	3,	1,	3,	3,	3,	1,
2,	2,	2,	3,	3,	3,	3,	3,	3	3,
2,	2,	3,	3,	3,	1,	1,	1,	1,	1,
2,	3,	3,	3,	1,	1,	1,	1,	1,	1,
3,	3,	3,	1,	1,	1,	1,	1,	3,	3,
3,	3,	3,	3,	3,	3,	1,	1,	3,	3,

(c)

Fig. 25.8 Encoaling of spatial features by raster method.

data structure that can be used to reproduce a geographical image in the computers is a file containing pairs of coordinates (x, y) corresponding to the location of individual point features (or points used to construct lines or areas). With this coordinate system, the periphery of a park may be represented by coordinate pairs that define the corner points of the polygon. (Fig. 25.9.)

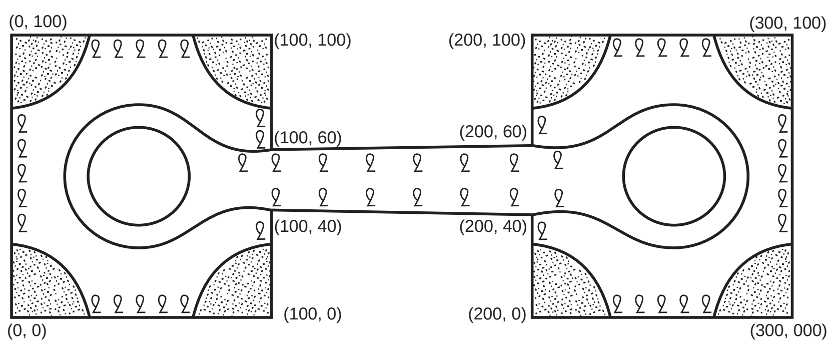


Fig. 25.9. Shows a vector data structure for NDMC, park.
The outer boundary the park is defined by coordinates pairs.

25.15 PROJECTIONS

To locate simple spatial entities (points, lines and areas) by GIS analysis, these entities should be projected on plane surface so that two dimensions may be used. The method of laying earth's surface flat is a map projection. Projection may be defined as the representation on a plane surface of any point of the surface of the earth or a celestial sphere. With the help of a map projection, it is possible to transfer the spherical earth onto a two-dimensional surface. Map projection introduces errors into spatial data, the character of the error depends on the type of projection. Some projections maintain the linear distances of the spatial entities whereas some projections maintain the shapes of the spatial entities with the cost of accuracy of dimensions.

The most suitable projection developed by Ferdin and Hassler, the first Superintendent of the Coast and Geodetic Survey of the USA is polyconic projection in which parallels are represented by a system of non-concentric circular area with centres lying on the straight line representing the central meridian. The distortion in shape is minimal along this line and increases towards east and west of the meridian. The scale remains true along the central meridian and also along each parallel. The Survey of India topographical sheets on scale 1:25,000, 1:50,000 and 1:250,000 are published on polyconic projection.

Spatial referencing. The method by which spatial entities are located on earth's surface or on the map, is known as spatial referencing. The following methods of spatial referencing are mostly adopted by GIS users.

- Geographic coordinate system
- Rectangular coordinate system
- Non-coordinate system

1. Geographic coordinate systems. This system uses the latitude and longitude of the place i.e. spatial entity.

Latitude of a point on earth surface may be defined as the angular distance north or south of the equator, as measured from the centre of the earth. Parallels of latitude are lines drawn around the earth parallel to the earth equator. Parallels of latitude are marked off in ninety divisions or degrees from the equator to each of the poles. The equator represents 0° latitude whereas north and south poles are 90° N and 90° S respectively. Each degree is subdivided into 60 minutes and each minute into 60 seconds. Each degree of latitude is 69 miles (111.045 kms) every where on the earth surface except near the poles where they are slightly longer due to flattening of the earth.

Longitude of a point on the earth surface may be defined as the angular distance measured along the equator between the meridian and Prime meridian of Greenwich. The longitudes vary form 0° to 180° east or west of Greenwich. Each degree is sub-divided into 60 minutes and each minute is sub-divided into 60 seconds. The length of a degree of longitude is longest at the equator where its value is approximately equal to that of a degree of latitude.

Lines of latitude orthogonally intersect the lines of longitude and run parallel to one another. The circumference of the circle of the latitude goes on decreasing as the latitude goes on shifting towards the poles. The circle with the greatest circumference of the latitude is known as the equator. At the two poles, the lines of latitude reduce to a single point, the **pole.**

The location of a point on the earth's surface may be located by referencing its latitude and longitude in degrees, minutes and seconds. Let the spherical coordinates of a point P be 28° 35′ 17″ N, 77° 15′ 23″ E. The first set of numbers 28° 35′ 17″ N represents latitude where N indicates that the point is north of the equator. Similarly 77° 15′ 23″ E indicates that the given point is to the east of the Prime meridian passing through Greenwich. (Fig. 25.10.)

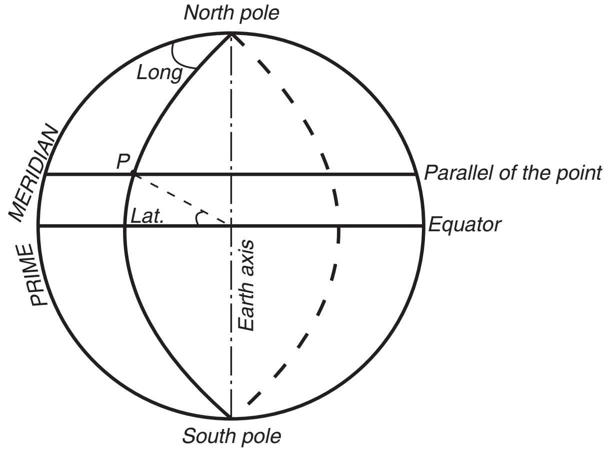


Fig. 25.10. Latitude and Longitude of points *P*.

Using the latitude and longitude of points, all features on the surface of the earth can be located relative to one another and distance between them calculated. Please refer to Author's textbook on Advanced Surveying. The short distance between two points on the earth surface is known as the great circle distance. The great circle is the circle that passes through given two points and the centre of the earth.

The concept of latitude and longitude coordinate referening system is based on the assumption that the earth is a perfect sphere. In fact, the earth is an oblate spheroid resembling an orange with flatter poles and outward bulges in the equatorial region. Moreover, the earth's surface is not smooth. It is covered by vast stretch of Indo-Gangetic plane, coastal plane, the desert of Rajasthan and high mountainous terrain of Himalyas, the Deccan trap, the Western Ghat and Southern Nilgiri hills. On small scale maps such variations hardly matter but on large scale maps of a small portion of the earth surface, it is essential to make corrections for these factors. Good Child and Yan'g (1989) has dealt with these irregularities on the earth's surface by the Quanternary Triangular mesh referencing systems.

2. Rectangular coordinate system. The most of the available GIS spatial data exist in two-dimensional form. For locating the spatial data with two dimension, a rectangular coordinates system is required. When the small surface of a sphere is projected onto a flat surface, the image on the outer periphery becomes distorted. When large areas of the globe are projected onto a flat surface, the grid of lines gets stretched or torn at places. This is the reason way rectangular coordinate systems are adopted to mapping of specific geographical regions. The universal transverse mercator (UTM) plane grid system in which the earth is divided into 60 vertical zones that are 6 degrees of longitude wide, ignoring both the poles. This system has been adopted by many countries for remote sensing, topographic mapping and natural resource management.

In mercator's projection, all the parallels of latitude have the same length as the equator. The east-west stretching increases with distance from the equator. On the globe the parallel of latitude 60° is one half of the length of the equator. To compensate this, a two-fold north-south stretching is done. As the stretching both east-west and north-south becomes so much greater in the vicinity of the pole, maps on this projection are generally limited to 80°. At this latitude the stretching is about six-fold in each direction. The main advantage of Mercator's projections are the following:

(*i*) Due to equal stretching, compass bearings, are truly represented.

(*ii*) Very useful for marine charts.

(*iii*) Very useful for climate logical map.

3. **Non-coordinate systems.** In this system, spatial references are made by descriptive code rather than the coordinates. Postal code widely used worldover is a good example. Some postal codes may be fully numeric and some may be alpha-numeric. Postal codes are very useful to increase the efficiency of mail sorting and delivery. This system is not useful for GIS users.

25.16 DATA BASE MANAGEMENT

A centrally controlled and integrated collection of logically organised data, usually stored as single or multiple files by an organisation, is called its *data base.* The maintenance of centrally controlled data in the organisation is very important for efficient management. It is also essential that various pieces of data within the database must have logical connections among themselves for proper integration of the data base management system. (DBMS) may be defined as the system which allows for the definition, creation, updating, readings, maintenance and protection of the data base. The process of management takes information as input to decision-making.

GIS is primarily dependent on being able to retrieve data from their databases, processes these data and comes up with useful information. The special dependence upon the information which, in turn, depends on data, GIS users have always paid special attention to their databases.

A DBMS is a set of computer programs for organisationing information from the database. An ideal GIS DBMS enables to support for multiple-users and multiple databases allow efficient updating, minimize redundant information and allow data independence. The main difference between management information system (MIS) and computer aided design (CAD) and the GIS is that it is able to transform one type entity spatial data such as points, lines and areas to another and also to perform spatial analysis. Data in GIS are normally held in a series of layers. For instance, a 1:25,000 topographic map can be suitably digitized to create a series of the following layers:

- A layer for net work of road
- A layer for built-up area
- A layer for parking and picnic sites
- A layer for contours
- A layer to show water bodies.

The vector model is found more suitable for mapping discrete geographic entities such as roads, railways, rivers and administrative bound aries.

25.17. THE DATA MANAGEMENT AND ANALYSIS

The main functions of GIS may be divided into the following sub heads: Data input, storage management, transformation, analysis and output.

(*a*) **Data input.** The process of converting data from the existing form (raw data) to one that can be used by the GIS, is known as data input. It is the procedure of encoding data into a computer utilization form and writing the data to the GIS data base. The data input also includes the ground verification field work to authenticate the correctness of data and transformation procedures to use data from different sources. GIS handles two types of data: geographical data and non-spatial data. The geographical data specify the spatial characteristics of the natural features on the earth's surface. Non-spatial attribute data specify the types of features being described. They tell the computers what a particular sets of entities they represent. Data input and updating are the most important parts of GIS as both are most expensive and time consuming. In GIS projects, about 80% of its duration is required for proper data input and updating.

The data management functions are also necessary in any GIS to facilitate the storage, organistaion and retrieval of data of making use of the database management system (DBMS). It may be noted that databases contain data not information. Information may be obtained by processing the data from the database.

(*b*) **Data output.** Data output may be obtained as per the requirement of the users. Data may be output in digital form for transfer to another software package. Mostly, GIS output is in the form of maps which may be easily displayed on screen for immediate communication to users.

25.18 ADVANTAGES AND DISADVANTAGES OF RASTER OR VECTOR DATA

The following are the advantages and disadvantages of raster or vector data:

1. Raster data sets record a value for all points in the covered area which may require more storage space than representing data in a vector format

2. Raster data allows easy implementation of overlay operations which prove to be more difficult with vector data.

3. Vector data can be displayed as vector graphics used on two dimensional maps whereas the raster data appear as an image that may have a blocky appearance for object boundaries.

4. Vector data can be easier to register, scale, and to reproject which simplifies covering vector layers from different sources.

5. Vector data is more compatible with relational data base environments where they can be a part of relational table as a normal column and processed by using a multitude of operators.

6. Vector file sizes are usually smaller than raster data. Raster files can be 10 to 100 times larger than vector data.

7. Vector data is simpler to update and maintain whereas a raster image will have a completely reproduction.

8. Vector data allows much more anlaysis capability specially for networks such as roads in cities, power lines, railways, etc.

25.19 METHODS OF ENTERING DATA INTO A GIS

The most common method is scanning, generally used to enter data into a GIS in a digital format. Scanning is used to convey a map data in raster data which may further be processed to produce vector data. Survey data may be directly entered into GIS from digital data collection system on survey instruments, using a coordinate geometry. The data from global positioning system (GPS) can also be entered directly into a GIS. The digital data is mostly obtained from photo interpretation of aerial photographs.

Satellite remote sensing also provides spatial data. In this method, the satellite uses different sensors packages to passively measure the reflectance from parts of electromagnetic spectrum that were sent out from an active sensor such as a radar. The raster data collected by Remote sensing can be further processed to identify objects and class such as the earth surface.

After entering data in GIS, it is further edited to remove errors. The vector data is made topologically correct before it can be used or processed. For example in a network of canal system, lines must connect with nodes at the intersection. Under shoots/over shoots errors must be removed.

Raster to Vector translation. The raster data may be translated to vector data structure by generating lines around the cells with same classification while determining the coordinates and relationship-such as adjacency.

25.20 USES OF GEOGRAPHICAL INFORMATION (GIS)

Data management systems, Geographical Information System, Global Positioning System (GPS). Remote sensing and satellite communication system, etc indivisually or in combination of two or more than two, now a days, tackle the mounting problems for the maintenance of infrastructure services. It is a well known fact for the maintenance of the present day infrastructure service, fairly accurate and up-to-date information regarding the physcial structure and related land parameters are necessary. Technical advancement in the field of remote sensing and computer, based automated GIS, have been proved very useful tools to provide infrastructure solutions.

The use of Information Technology applications in infrastructure planning, has made urban areas more prosperous and efficient. Information technology helps to plan precision planning, interlinking various infrastructure servies, quick decision making etc. and at the same time provide savings in both time and cost.

The use of GIS and remote sensing in the field of Civil Engg. for various projects are described below:

1. **Construction of Power transmission projects.** The present day pace of development of tranmission lines can not be accelerated by using conventional methods of land surveying. Recently, 'Power Grid'

an organisation of Government of India has adopted the GIS in route surveying to improve efficiencies of major projects in respect of the project concept, detailing and execution. GIS has proved very useful for tranmission projects through geographical tough regions such as forests, deserts, hilly terrain etc. In a transmission project, generation projects (Power Stations) have fixed locations whereas the tranmission project runs cross-country through thousands of Kilometers. The importance of carrying out surveying for designing an optimal tranmission line route convering multiple locations through in topographical variations, needs no mention.

Transmission towers to be erected at suitable intervals are also significant parts of a transmission line projects.

The transmission towers in fairly plane areas are erected at about 800 metre intervals for double circuit lines. Erection of towers at specified locations are mini projects of the main project. For successful erection of towers, it is absolutely necessary to have detailed survey to identify all the constraints while planning the erection of the towers before hand. The main constraints for erection of towers are mainly due to in-accessibility, construction feasibility, conservation of environment, right of way and technical restrictions.

Due to inaccessible terrain in high mountaneous regions, (such as Himalayan region), extensive ground surveys are not possible by normal ground methods. However, detailed and accurate surveys are absolutely necessary to judiciously allocate the resources, to determine the costs of the project in advance and to avoid future surprises at the actual construction stage. In the absence of information technology and geographical information system, the base material *i.e.*, topographical sheets prepared by Survey of India were used for the reconnaisance of the transmission lines. The Survey of India topo sheets generally remain out-dated. Moreover, these topo sheets do not show new constructions that might have come up subsequently. As toposheets are not digitised they are not suitable to update them.

As compared to toposheets, aerial surveys for large extent of areas are preferred to get a more acccurate picture of the project terrain. In aerial survey, human judgement plays a vital role. The National Remote Sensing Agency (NRSA) provides up-dated topographical maps that too in digitised form. The digitised maps provided by NSRA are not found for optimisation of transmission, on routes, especially in forest covers or snow cover in higher reaches.

Keeping in view, these constraints, Power Grid Corporation of India Ltd., now uses high resolution satellite imageries obtained from NRSA combined with updated Survey of India topo sheets. With the help of satellite imageries and topo sheets, digital terrain modelling with contours is made available. Digital terrain model helps to provide appropriate locations for the towers. Final alignment profile is studied for the GPS positioning at the site. By processing the digitised information and with the help of appropriate software, the tower positions are decided.

2. GIS in Road Planning. GIS technology can also be usefully employed for a variety of purposes in road planning.

This is especially suitable for the following purposes:

(*i*) Rural road network planning and managements,

(*ii*) Traffic assignment and traffic information system

(*iii*) Pavement information and management,

(*iv*) Pollution profile mapping and transport planning aspects,

(*v*) Identification for ropeways in hilly regions,

(*vi*) Disaster mitigation and management plan.

1. Rural road network planning and management. The Indian rural roads comprise of other district roads and village roads. The Pradhan Mantri Gram Sadak Yojna (PMGSY) is the biggest rural road project. Under this project, all villages with population exceeding 500 in plain and 250 in hilly areas will be provided connectivity by all weather roads. GIS technology in rural roads is primarily used for geographic anaylsis as well as in planning and measurement of rural road accecibility.

Methodology adopted for rural network planning through GIS. This methodology involves the following types:

(*i*) A detailed data base is prepared for rural roads at the district level. The data base comprises information such as population density, sex ratio, connected and unconnected villages, available growth centres in the district such as education facilities, etc.

District rural road planning map of the district is made based on the available data base on scale 1:50,000, on the bases of GIS available guide lines.

The GIS base map is used to indentify the core network on population links for implementation. Based on the Habitation Index (HINDEX), priority of providing connectivity of unconnected villages is decided. The Core network which consists of through routes and link routes provides an access to all habitations/villages for qualifying population with all weather road connectivity. Thorough routes collect traffic from several link roads passing through a number of villages and leading to market centres, either directly or thorough a national highway, state highway or a major district road. Link routes may be defined as the roads connecting a single habitation or a group of habitations to thorough routes or a major link roads leading to the market centre directly.

If the network and routing modules are used in the GIS software, the preparation of the core network is very much simplified. After identification of the core network, the shortest distance from the unconnected village to the nearest centre along the existing network may be calculated. The best links further selected on the basis of the access benefit and cost ratio, are selected for implementation.

(*ii*) **Traffic Assignment and Traffic Information System**. The knowledge of GIS is also used to study the traffic composition on a road or a flyover, or a grade separtor. The procedure involves identifying suitable locations for conducting traffic surveys, preparation involves identifying suitable locations for conducting traffic surveys, preparation of a traffic data base or a traffic flow diagram. Based on the data and flow diagram, we can compute the traffic composition on a particular road portion. The study of such traffic composition helps in effective monitoring of the traffic.

2. Pollution Profile Mapping and Transport Planning. GIS technology also helps to study the pollution profile of a city and also enables transport planning. As metros have become highly polluted due to high degree of private vehicle usage, study of pollution profile of metros has become the necessity of the day.

The pollution study is carried out at a road network in terms of link volumes categorized by vehicle types. The operating speed of each vehicle type is the related to the observed pollution level in terms of different pollutants. The pollution study is usually made on an hourly basis by dividing the day in 24 hours and then into three or four shifts. Such population levels are used for predicting the level of pollution with different speed.

The GIS is thus used to evaluate the traffic planning and management options at a road network both in terms of traffic carried by the road and interms of environmental parameters.

3. Identification of locations for ropeways, waterways and tunnelling. Development models for the plains are unsuitable for hilly regions. That is why special attention is needed for hilly regions such as Jammu and Kashmir, Himachal Pradesh and Uttrakhand and also for North Eastern states. Because of difficult terrain and due to ecological reasons, it is not feasible to connect each habitation with all-weather roads. With the advanced technology, GIS provides alternative transport system such as ropeways, water ways or through tunnels. To install ropeways, waterways and tunnels in hilly regions, the planners must possess the knowledge of topographic nature *i.e.*, geography and geological condition, population and its characteristics, natural resources, land use pattern, rainfall and climatic conditions of the region.

GIS helps to select the area for providing rope-way and also to examine the existing road network system including foot paths and mule-tracks, the present road transportation and also topography of the region. Based on the thorough study of the region, preliminary design and a rough alignment for an alternative transport system need be identified. For final selection of the alignment, an economic comparison of various alternative transportation system is done. Thus, GIS technology provides an authentic data base for the road network.

4. Urban infrastructure and utility planning. GIS has been found very useful in planning urban infrastructure. GIS is being used for the following perposes in urban cities:

(*i*) Laying pipelines for gas distribution.

(*ii*) Identifying locations for equipment installation and repairs in electricity distribution.

(*iii*) Setting up new power/gas transmission projects.

(*iv*) Developing water supply networks

(*v*) Providing drainage/sanitation projects

(*vi*) Laying roads and directing traffic.

(*vii*) Managing telecom networks

(*viii*) Urban planning.

In a gas or electricity project, GIS provides a support to energy procurement by developing geographic descriptions for load estimation, daily load forecasting, weather integration, etc. In transmission planning and projects GIS provides inputs for right-of-way modelling.

In initial stages of developing GIS, efforts were directed towards particular engineering projects. Now, the trend is shifting towards integrating these models with other business applications. The new generation GIS models of today provide more impacting visualisation with data on many levels.

The typical uses of Geographic Information System in urban infrastructure planning may be summarised as under:

1. Development and planning
2. Land and estate
3. Land requistions
4. Water supply
5. Drainage and sanitation
6. Traffic and roads
7. Electricity
8. Telecommunication.

EXERCISE 25

Multiple Choice Type Questions

1. GIS helps in the geographical analysis of the following:
(*a*) to search suitable sites (*b*) soil conservation planning
(*c*) to find a new home (*d*) all of these

2. GIS helps to search optimal site for
(*a*) a new school (*b*) a new bus terminus
(*c*) a new aerodrome (*d*) a new sewer treatment plant
(*e*) all of these.

3. The following steps are involved for the identification of a suitable solid waste disposal site:

1. Generation of spatial data sets for each site i.e. land use, soil, geology and transport facilities from conventional and remote sensing resources.
2. Generation of slope layer from a contour map or a digital elevation model
3. Generation of secondary data layer *i.e.* slope of the terrain, water bodies in the vicinity and major approach roads

The correct sequence of the steps is:
(*a*) 1, 3, 2 (*b*) 1, 2, 3
(*c*) 2, 1, 3 (*d*) 3, 2, 1

4. Which of the following is the criterion that causes land degradation:
(*a*) the type of soil and its characteristics
(*b*) the terrain characteristics i.e. steepness and drainage pattern
(*c*) the type of present land use
(*d*) the geology and geomorphology of the area
(*e*) all of these

5. Various criteria that cause land degradation are identified from:
(*a*) aerial photographs (*b*) satellite images
(*c*) topographical sheets (*d*) attribute data
(*e*) all of these

6. GIS has the following component:
(*a*) computer hardware (*b*) software *i.e.* computer programs
(*c*) appropriate techniques for task implementation
(*d*) all of these

7. Pick up the correct statement for the following:
(*a*) GIS is used to add a value to spatial data
(*b*) GIS creates useful information for decision making

(*c*) GIS acts as a spatial decision support system

(*d*) all of these

8. Geographic information technologies includes:

(*a*) global position system (GPS)

(*b*) remote sensing

(*c*) geographic information systems

(*d*) all of these

9. Computer cartography uses a computer for:

(*a*) generation of maps (*b*) storage of maps

(*c*) editing of maps (*d*) all of these

10. Which of the following elements is essential for effective GIS:

(*a*) A processor with sufficient power to run the software

(*b*) Sufficient memory for the storage of large volume of data

(*c*) A good quality high resolution colour graphic screen

(*d*) Data input and output devices

(*e*) all of these

11. Spatial data is characterized by the following information:

(*a*) position of the data

(*b*) connections of the data with other features

(*c*) details of non-spatial characteristics.

(*d*) all of these

12. In GIS

(*a*) points are used to represent the locations of the spatial features

(*b*) lines are used to represent linear features such as canals, roads, rivers, etc.

(*c*) area features are used to represent geographical zones

(*d*) points, lines and areas are used to represent surfaces

(*e*) all of these

13. Pick up the correct statement from the following:

(*a*) The raster method is useful for representation of real world where remotely sensed images are used.

(*b*) The vector model method is used for discrete geographic entities.

(*c*) Both (*a*) and (*b*) are correct. (*d*) Neither (*a*) nor (*b*) is correct

14. Data input in GIS:

(*a*) converts data from its existing form to suitable form for GIS.

(*b*) is the procedure of encoding data into a computer readable form and writing the data to the GIS database.

(*c*) includes verification procedures to check the correctness of the data transfered from different sources

(*d*) all of the above

15. GIS handles:

(*a*) the geographical data that describe the spatial characteristics of the real-world features.

(*b*) non-spatial attribute data that describe the features they represent

(*c*) Both (*a*) and (*b*) are correct

(*d*) Neither (*a*) nor (*b*) is correct

16. An ideal GIS DBMS (Data base management system)

(*a*) provides supports for multiple users and multiple databases

(*b*) allows efficient updating

(*c*) minimises repeated information

(*d*) allows data independence, security and integrity

(*e*) all of these

17. Which one of the following techniques is able to transform spatial data to another and to perform spatial analysis:

(*a*) MIS (Management information system)

(*b*) GIS (Geographical information system)

(*c*) CAD (Computer aided design)

(*d*) GPS (Global positioning system)

18. In GIS, transformation of spatial data involves:

(*a*) changing the projection of the map layer

(*b*) correction of systematic errors resulted during digitizing

(*c*) conversion of data held as rasters to vectors or vice versa

(*d*) all of these

19. The output of GIS is in the form of maps that may be

(*a*) displayed on the computer screen

(*b*) photographed

(*c*) stored digitally

(*d*) plotted to produce permanent hard copies

(*e*) all of these

20. Pick up the correct statement from the following:

(*a*) The simplified view of the real world adopted by GIS is a model

(*b*) A model is 'a synthesis of data' used as a means of getting to grip with systems (*c*) Models contain the users ideas about how or why the elements of real world interact in a particular way.

(*d*) All of these

21. The model or dimensions of the primary data is:

(*a*) temporal (*b*) thematic

(*c*) spatial (*d*) all of these

22. The mode or dimension of the secondary data is:

(*a*) temporal (*b*) thematic

(*c*) spatial (*d*) all of these

23. The three modes of data of a cloud burst in Leh are:

(*a*) temporal ... 6, August 2010 (*b*) thematic ... cloud burst

(*c*) spatial ... Leh valley (*d*) all of these

24. Match list I with list II and select a suitable answer using the codes given below the lists

	List I	*List II*
	(mode)	*(Characteristic)*
A.	Temporal	1. location of the feature
B.	Thematic	2. data of recording
C.	Spatial	3. real world feature

Codes:

	A	B	C
(*a*)	1	2	3
(*b*)	2	3	1
(*c*)	2	1	3
(*d*)	3	2	1

25. Which of the following statements is correct?

(*a*) Temporal data describes data organised and analysed according to time

(*b*) Thematic data organised and analysed by time

(*c*) Spatial data organised and analysed by location

(*d*) all of these

26. For turning data into information, GIS places great emphasis on the use of:

(*a*) spatial dimension (*b*) thematic dimension

(*c*) temporal dimension (*d*) all of these

27. The map is of fundamental importance in GIS as a

(*a*) source of data

(*b*) structure for storing data

(*c*) device for analysis and display
(*d*) all of these

28. Pick up the correct statement from the following
(*a*) Thematic maps show data relating to a particular theme
(*b*) Topographic maps contain a diverse set of data on different themes
(*c*) Both (*a*) and (*b*) are correct
(*d*) Neither (*a*) nor (*b*) is correct

29. Scale of the map:
(*a*) may be defined as the ratio of a distance on the map to the corresponding distance on the ground
(*b*) gives an indication of how much smaller than reality, a map is
(*c*) is the order of level of generalization at which phenomena are perceived
(*d*) can be expressed in one of the ways as a ratio scale, a verbal scale or a graphical scale
(*e*) all of these

30. 1 cm represents 50 m, is;
(*a*) a verbal scale (*b*) a ratio scale
(*c*) a geographical scale (*d*) none of these

31. A point on GIS maps is represented by:
(*a*) one dimensions (x) (*b*) two dimensions (x, y)
(*c*) three dimensions (x, y, z) (*d*) none of these

32. A line on GIS maps:
(*a*) is an ordered set of points
(*b*) is a string of (x, y) coordinates joined together in an order
(*c*) may be isolated
(*d*) all of these

33. Areas on GIS maps:
(*a*) are represented by a closed set of lines
(*b*) are used to define features such as fields, buildings
(*c*) are often referred to as polygons
(*d*) all of these

34. Pick up the correct statement from the following:
(*a*) As the scale increases, the cartographer has a greater scope for including more details
(*b*) The relationship between scale and detail is refereed to as scale-related generalization

(*c*) For the sake of clarity, the cartographer has to be selective about drawing map features

(*d*) All of these

35. Pick up the correct statements

(*a*) Map projections transfer the spherical earth onto a two dimensional surface

(*b*) Map projection introduces errors into spatial data.

(*c*) A cylindrical projection is very suitable for making maps of an area which has only small extent in longitude

(*d*) all of these

36. The polyconic projection is used by the Survey of India for the following maps:

(*a*) 1:25,000 topo sheets (*b*) 1:50,000 topo sheets

(*c*) 1:250,000 topo sheets (*d*) all of these

37. In polyconic projection:

(*a*) the distortion in shape is minimal along standard layers

(*b*) the distortion in shape increases towards east and west of the meridian

(*c*) the scale of the map remains true along the central meridian and along each parallel

(*d*) all of these

38. Which one of the following spatial referencing represents

(*a*) geographical coordinate system

(*b*) rectangular coordinate system

(*c*) non-coordinate system

(*d*) polar coordinate system

39. In a polyconic projection:

(*a*) Lines of longitude are widest apart at the equator and closed together at the poles

(*b*) The relative distance between lines of longitude where they intersect lines of latitude is always equal

(*c*) Lines of latitude lie at right angles to lines of longitude and run parallel to one another

(*d*) The circle with the greatest circumferences is known as the equator

(*e*) At the two poles, the lines of Latitude are provided by a single point, the pole

(*f*) All of these.

Answers

1. (*d*)	2. (*e*)	3. (*a*)	4. (*e*)	5 (*e*)
6. (*d*)	7. (*d*)	8. (*d*)	9. (*e*)	10 (*e*)
11. (*d*)	12. (*e*)	13. (*c*)	14. (*d*)	15 (*c*)
16. (*e*)	17. (*b*)	18. (*d*)	19. (*e*)	20 (*d*)
21. (*d*)	22. (*d*)	23. (*d*)	24. (*b*)	25 (*d*)
26. (*a*)	27. (*d*)	28. (*c*)	29. (*e*)	30 (*a*)
31. (*c*)	32. (*d*)	33. (*d*)	34. (*d*)	35 (*d*)
36. (*d*)	37. (*d*)	38. (*a*)	39. (*d*)	

Objective Type Questions

I. *Fill in the blanks / complete the sentences:*

1. The longest chain line passing through the centre of the area is known as ....
2. The angle between two plane mirrors of an optical square is ....
3. For accurate measurement of distances .... is used.
4. .... meridian of a place changes its position.
5. .... is the horizontal angle between true north and magnetic north at the place.
6. Four operations needed at every plane table station are: (*a*) .... , (*b*) .... , (*c*) .... and (*d*) ....
7. A levelling instrument essentially consists of the following parts (*a*) .... , (*b*) .... , (*c*) .... and (*d*) ....
8. The combined effect of curvature and atmospheric refraction is .... where d is in km.
9. The point at which both back sight and fore sight readings are taken is known as ....
10. The contour interval of a map depends on ....
11. Ring contours of higher values inside represent ....
12. Face left and face right observations eliminate .... errors.
13. The angle between the prolongation of preceding line and forward line of a traverse, is called ....
14. The measurement of several angles at a common station is made by .... method.
15. Multiplying constant of most of theodolites/tachometers is kept .... by the manufactures.
16. A stadia diaphragm consists of .... vertical and .... horizontal hairs.
17. The distance between the midpoint of the curve and the long chord, is known as ....

18. General formula for the offset from the chord produced is given by the equation ....

19. Planimeter is used for measuring ....

20. A pantograph is an instrument used for ....

21. If the length of a chain line along a slope of $\theta°$ is l, the required slope correction is ....

22. Accidental or compensating errors of length l are proportional to ....

23. Determining the difference in elevation between two points on the surface of the earth is known as ....

24. In reciprocal levelling, the error which is not completely eliminated is due to ....

25. An imaginary line joining the points of equal elevation on the surface of the earth, represents ....

26. The operation of revolving a plane table about its vertical axis so that all lines on the sheet become parallel to corresponding lines on the ground is known as ....

27. The horizontal angle between true meridian and magnetic meridian, is known as ....

28. The zero of the graduated circle of a prismatic compass is located at ....

29. Whole circle bearing of a line is preferred to the quadrantal bearing merely because ....

30. The angle of intersection of a curve is the angle between ....

31. The chord of a curve less than the peg interval, is known as ....

32. The area of any irregular figure on a plotted map is measured with ....

33. The profile levelling is usually done for determining ....

34. Box sextant is used for the measurement of ....

35. The Indian tangent clinometer is used in combination with ....

36. The smaller horizontal angle between the true meridian and a survey line, is known as ....

37. Removal of parallax may be achieved by focusing ....

38. A traverse deflection angle is ....

39. In levelling operation if the first reading is less than second, it represents a ....

40. Contours of different elevations may cross each other only in case of ....

41. Long chord of a chord is the chord joining points .... to ....

42. The longest of the chain lines used in making a survey is generally regarded as the ....

43. If the object is round, an offset should be taken to the ....

44. The angle between two .... mirrors of an optical square is 45°.

45. A triangle may be defined as well conditioned when the triangle is ....

46. The combined effect of curvature and refraction is .... d^2 when d is in Km.

(*a*) 0.0685 (*b*) 0.0755

(*c*) 0.0673 (*d*) 0.0692

47. .... levelling avoids the error due to refraction and curvature.

48. The 'changing of zero' eliminates the error due to ....

49. Slip occurs due to .... arrangement.

50. The additive constant of the anallatic lens tachometer is ....

51. For tacheometry survey the instruments required are as (*a*) ...... and (*b*) ....

52. Reduced bearing is defined when the whole circle bearing of a line when it exceeds ....

53. The direction indicated by a freely suspended and properly balanced magnetic needle, is called as ....

54. The vertical distance between any two consecutive contours is called as ....

55. Plane tabling is a .... method of surveying.

56. Plane table essentially consists of (*a*) .... and (*b*) ....

57. The mid span sag of a suspended tape is equal to ....

58. Planimeter is used for measuring ....

59. RL of point = RL of .... + Backsight.

60. Contour lines close together near the top of the hill, represent very .... ground.

II. *Pick up the correct answer from the following:*

1. For ranging a line, the number of ranging rods required is:

(*a*) at least two (*b*) at least three

(*c*) at least four (*d*) at least five

2. Compensating error is proportional to:

(*a*) L (*b*) $\sqrt{L}$

(*c*) $2L$ (*d*) L^2

3. The working principle of an optical square is based on:

(*a*) reflection (*b*) refraction

(*c*) double reflection. (*d*) double refraction

4. If a wooded area obstructs a chain line, then it is crossed by the:

(*a*) profile line (*b*) random line

(*c*) projection line (*d*) None of these

5. At the equator, the dip of the magnetic needle is:

(*a*) 180° (*b*) 0°

(*c*) 90°

6. In a prismatic compass, the zero is marked on the :

(*a*) north end (*b*) south end

(*c*) west end (*d*) east end

7. The principle of plane table is based on :

(*a*) traversing (*b*) triangulation

(*c*) parallelism (*d*) radiation

8. The north line of the map is marked on the :

(*a*) right-hand bottom corner (*b*) left-hand top corner

(*c*) right-hand top corner (*d*) left-hand south corner

9. When contour lines touch one another in a particular zone, it indicates a :

(*a*) level surface (*b*) vertical cliff

(*c*) horizontal surface (*d*) Steep slope

10. When the higher value contour is inside the loop, it indicates a :

(*a*) high ground (*b*) level ground

(*c*) depression (*d*) None of these

11. The alignment of a highway is generally taken along :

(*a*) ridge line (*b*) valley line

(*c*) across the contour line (*d*) along the contour

12. The included angles of a traverse are measured :

(*a*) clockwise (*b*) anti-clockwise

(*c*) either way (*d*) None of these

13. Fine adjustment in a theodolite is done by the :

(*a*) tangent screw (*b*) clamping screw

(*c*) focusing screw (*d*) None of these

14. Balancing of a theodolite traverse is done according to :

(*a*) transit rule (*b*) prismodial rule

(*c*) trapezoidal rule (*d*) Simpson's rule

15. An ideal transition curve is also known as :

(*a*) clothoid curve (*b*) cubical curve

(*c*) parabolic curve (*d*) elliptical curve

16. The radius of a one-degree curve is :

(*a*) 1760 m (*b*) 1.719 m

(*c*) 176.0 m (*d*) 1719 m

17. The intrinsic equation of an ideal transition curve, is given by the expression :

(*a*) $\phi \dfrac{L^2}{2R}$ radian (*b*) $\phi \dfrac{L}{2R}$ radian

(*c*) $\phi \dfrac{L^2}{2R^2}$ radian (*d*) $\phi \dfrac{L^3}{2R}$ radian

18. The multiplying constant of a tacheometer, is denoted by :

(*a*) $\dfrac{f}{i}$ (*b*) i/f

(*c*) $i \times f$ (*d*) $f \times i^2$

19. If a 20 m chain gets displaced from the correct alignment by a perpendicular distance d m, then the error is given by :

(*a*) $\dfrac{d^2}{40}$ (*b*) $\dfrac{d^2}{60}$

(c) $\frac{d^2}{80}$ (d) $\frac{d^2}{100}$

20. The accuracy with which the instrument station can be established is known as the :

(a) strength of fix (b) strength of accuracy

(c) strength of solution (d) strength of station

21. A chain may get elongated due to :

(a) change in temperature (b) difference in pull

(c) openings of rings (d) kinks in links

22. A Gunter's chain is

(a) 100 ft long (b) 22 yards

(c) 30 m long (d) 33 ft long

23. To erect a perpendicular to a chain line from a point on it :

(a) 3-4-5 method is used (b) a ranging rod is used

(c) a trough compass is used (d) a clinometer is used

24. Isogonic lines are the lines having same :

(a) elevation (b) bearing

(c) dip (d) declination

25. Strength of fix is poor when :

(a) station is on the great circle

(b) station is within the great triangle

(c) station is outside the great circle

(d) station is within great circle but outside great triangle.

26. The dip of the magnetic needle is :

(a) 90° at equator (b) 0° at equator

(c) 180° at equator (d) unpredictable

27. Advantage of plane tabling is :

(a) it is intended for accurate work

(b) field book is not necessary

(c) suitable for preparing large-scale map

(d) it does not require a skilled worker

28. A bench Mark is :

(a) a reference point
(b) a point of known elevation
(c) the very first station in levelling
(d) last station where survey closes

29. In a levelling work Σ Rise = zero, then the ground is :

(a) continuously falling (b) continuously rising
(c) undulating (d) perfectly level

30. A 15 cm theodolite means

(a) length of telecscope is 15 cm
(b) height of standards is 15 cm
(c) diameter of lower plate is 15 cm
(d) radius of upper plate is 15 cm

31. Anallatic lens is a :

(a) convex lens (b) concave lens
(c) concavo-convex lens (d) plain lens

32. When line of sight is inclined at 0° to the horizontal and staff is held vertical, the horizontal distance :

(a) $\frac{f}{i} S + (f + d)$ (b) $\frac{f}{i} S \cos \theta + (f + d)$
(c) $\frac{f}{i} S \cos^2 \theta + (f + d)$ (d) None of these

33. If the radius of a circular arc is 100 m, deflection angle is 90°, the length of backward tangent is :

(a) zero (b) 70.7 m
(c) ∝ (d) 100 m

34. Line of collimation of the theodolite is the line:

(a) joining the optical centre of the objective and eyepiece
(b) about which telescope can be rotated
(c) joining intersection of cross hairs to the optical centre of the objective
(d) which is horizontal

35. A clinometer is used to measure :

(a) distance approximately (b) the angle of a slope
(c) reduced level of a plane (d) bearing of a line.

36. When a tape is used for the measurement of a distance along a slope, the correction for slope is given by:

(*a*) $\sum \frac{h}{2l}$ (*b*) $\sum \frac{h^2}{l^2}$

(*c*) $\sum \frac{h^2}{2l}$ (*d*) $\sum \frac{h^2}{2l^2}$

37. A planimeter is used for measuring :

(*a*) inclination of a slope (*b*) altitude of a place

(*c*) area of a map (*d*) speed of an automobile

38. The odd instrument out of the following is :

(*a*) Theodolite (*b*) Prismatic compass

(*c*) Box sextant (*d*) Dumpy level

39. When temperature rises, the length of bubble of the bubble tube :

(*a*) remains unaltered (*b*) decreases

(*c*) increases (*d*) is uncertain

40. A boning rod which is used to set out of sewer pipe is :

(*a*) *L* shaped (*b*) *A* shaped

(*c*) *T* shaped (*d*) *I* shaped

41. Chain survey is recommended :

(*a*) in a city area

(*b*) in a forest area without local attraction

(*c*) in a fairly open area

(*d*) in a small area

42. In an optical square, the mirrors are fixed at an angle of :

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 75°

43. In chain surveying the area is divided into :

(*a*) rectangles (*b*) squares

(*c*) triangles (*d*) any type of polygon

44. In the prismatic compass, zero graduation starts from :

(*a*) North (*b*) South

(*c*) East (*d*) West

45. In a prismatic compass survey the local attraction affects the measurement of included angles.

(*a*) true (*b*) false

(*c*) not always true (*d*) may or may not

46. The sum of the interior angles of a closed transverse is equal to:

(*a*) $(2n - 4) \times 90$ (*b*) $(2n - 4) \times 90$

(*c*) $(n - 4)\ 90$ when n is the number of sides

47. In a Plane Table Survey, locations of objects are plotted by :

(*a*) traversing method (*b*) radiation method

(*c*) resection method (*d*) All of these

48. Accuracy with which a plane table is located on the map in a revision survey is known as :

(*a*) strength of station (*b*) strength of fix

(*c*) strength of accuracy (*d*) accuracy

49. A plane table map can be plotted on a different scales if there is:

(*a*) field book (*b*) level book

(*c*) angle book (*d*) cannot be plotted

50. Mean Sea Level (MSL) adopted by the Survey of India, is located at :

(*a*) Kolkata (*b*) Mumbai

(*c*) Karachi (*d*) Delhi

51. Water surface at rest is a :

(*a*) level surface (*b*) horizontal surface

(*c*) tangential surface

52. Relative movement between cross hair and the staff reading, is due to

(*a*) error in the line of collimation

(*b*) error in the trunnion axis

(*c*) parallax error

53. A contour surface is a :

(*a*) level surface (*b*) horizontal surface

(*c*) plane surface (*d*) inclined surface.

54. When contour lines at different elevations meet at a point or a line it indicates

(*a*) overhanging cliff (*b*) vertical cliff
(*c*) river valley

55. The contour gradient is a line of :
(*a*) constant altitude (*b*) constant slope
(*c*) minimum slope (*d*) maximum slope

56. Closer contour lines indicate :
(*a*) steeper slope (*b*) flatter slope
(*c*) maximum slope (*d*) No slope

57. By method of repetition, observational errors eliminated are :
(*a*) line of collimation (*b*) trunnion axis
(*c*) graduational error (*d*) All of these

58. The intrinsic equation of an ideal transition curve is given by the expression

(*a*) $\phi = \frac{l}{2R}$ (*b*) $\phi = \frac{l^2}{2RL}$

(*c*) $\phi = \frac{l^3}{2RL^2}$

where ϕ = tangential angle at any point P on the transition curvel

l = distance of P

R = minimum radius

L = total length of the transition curve

59. The offset distance of a point of the transition curve, is given by the expression :

(*a*) $\frac{l^3}{6RL}$ (*b*) $\frac{l^2}{6R}$

(*c*) $\frac{l^2}{24R}$

60. The nature of a vertical curve is :
(*a*) circular (*b*) parabolic
(*c*) elliptical (*d*) clothoid

61. A tie line is run :
(*a*) to check the accuracy of fieldwork
(*b*) to locate details which are far away from the chain line
(*c*) between main survey stations

(*d*) parallel to the survey line

62. For a well-conditioned triangle, no angle should be less than :

(*a*) 45° (*b*) 30°

(*c*) 15° (*d*) 60°

63. The allowable length of an offset depends upon :

(*a*) scale of plotting

(*b*) degree of accuracy requirement

(*c*) both (*a*) and (*b*)

(*d*) neither (*a*) nor (*b*)

64. A negative declination shows that the magnetic meridian is on :

(*a*) West of the true meridian (*b*) East of the true meridian

(*c*) South of the true meridian (*d*) None of the above

65. The horizontal angle between the true meridian and the magnetic meridian is called :

(*a*) dip (*b*) azimuth

(*c*) declination (*d*) none of the above

66. The vertical angle between the longitudinal axis of a freely suspended magnetic needle and the horizontal line, is called :

(*a*) declination (*b*) dip

(*c*) azimuth (*d*) none of the above

67. Two-point and three-point problems are the methods of :

(*a*) traversing (*b*) resection

(*c*) resection and orientation (*d*) orientation

68. The instrument which is used in plane table surveying for determining horizontal distances without actual measurement, is called:

(*a*) plane alidade (*b*) telescopic alidade

(*c*) tacheometer (*d*) clinometer

69. An imaginary line lying throughout the ground surface and having a constant inclination to the horizontal is :

(*a*) contour line (*b*) ridge line

(*c*) contour gradient (*d*) none of the above

70. In reciprocal levelling, the error which is not fully eliminated is due to:

(*a*) earth's curvature (*b*) inclined line of sight

(*c*) refraction (*d*) both (*a*) and (*c*)

71. The final setting of the theodolite plates when taking a sight is achieved by using :

(*a*) upper clamp screw (*b*) upper tangent screw

(*c*) lower clamp screw (*d*) lower tangent screw

72. The horizontal angle between two survey lines is generally measured:

(*a*) Clockwise from the forward station.

(*b*) Clockwise from the back station.

(*c*) Counter clockwise from the forward station.

(*d*) Counter clockwise from the back station.

73. The prismoidal formula for the volume is :

(*a*) $V = \frac{D}{6}(A_1 + 2A_m + A_2)$ (*b*) $V = \frac{D}{3}(A_1 + 4A_m + A_2)$

(*c*) $V = \frac{D}{6}(A_1 + 4A_m + A_2)$ (*d*) $V = \frac{D}{3}(A_1 + 2A_m + A_2)$

where D is the distance between end sections 1 and 2.

74. The latitude of a line is obtained by multiplying its length by :

(*a*) tan θ (*b*) sin θ

(*c*) cos θ (*d*) cot θ

where θ is the reduced bearing of the line.

75. Ceylon Ghat Tracer is used to measure :

(*a*) distance approximately (*b*) reduced level of a place

(*c*) bearing of a line (*d*) the angle of a slope

76. The prismoidal correction is always :

(*a*) negative (*b*) positive

(*c*) uncertain (*d*) none

77. In a theodolite traverse for land surveying, the length of the traverse lines are generally measured with :

(*a*) chain (*b*) metallic tape

(*c*) steel tape (or band) (*d*) invar tape

78. The parallex can be removed by :

(*a*) focusing the objective

(*b*) focusing the eyepiece

(*c*) focusing both the objective and eyepiece

(*d*) none of the above.

79. The most accurate method of plotting a theodolite traverse is by:

(*a*) consecutive coordinates (*b*) independent coordinates

(*c*) included angles (*d*) tangent method.

80. The difference between the arc length and chord length for a distance of 18 km on earth is :

(*a*) 5 cm (*b*) 10 cm

(*c*) 15 cm (*d*) 1 cm

81. The curvature of the earth is usually taken into account when the extent of the area is more than :

(*a*) 50 km^2 (*b*) 100 km^2

(*c*) 200 km^2 (*d*) 250 km^2

82. Chain survey is recommended when the area is :

(*a*) crowded (*b*) undulating

(*c*) simple (*d*) level

83. At the equator, the dip of the needle is :

(*a*) 180° (*b*) 0°

(*c*) 90° (*d*) 45°

84. The angular error of closure should not exceed :

(*a*) $15 \sqrt{N}_{minutes}$ (*b*) $30 \sqrt{N}_{minutes}$

(*c*) $\sqrt{N}_{minutes}$ (*d*) $45 \sqrt{N}_{minutes}$

85. In the Quadrantal Bearing (QB) system, a line is said to be free from local attraction, if the forward bearing and backward bearing are :

(*a*) numerically equal with same quadrant

(*b*) numerically equal with opposite quadrant

(*c*) difference is 90° algebraically

(*d*) the WCB's are same

86. In the WCB (Whole Circle Bearing) system, a line is said to be free from local attraction if the differences between FB and BB is :

(*a*) 270° (*b*) 90°

(*c*) 0° (*d*) 180°

87. The U-fork and plumb bob are required for :

(*a*) radiation (*b*) levelling

(*c*) orientation (*d*) centering

88. The working edge of the alidable is known as the :

(*a*) fiducial edge (*b*) bevelled edge

(*c*) parallel edge (*d*) cutting edge

89. The operation of levelling across any river or water body is termed as :

(*a*) profile levelling (*b*) raciprocal levelling

(*c*) compound levelling (*d*) fly levelling

90. The contour interval is inversely proportioned to the :

(*a*) contour value (*b*) extent of the area

(*c*) scale of the map (*d*) steepness of the area

91. The volume computed by the prismoidal method is considered to be:

(*a*) exact (*b*) approximate

(*c*) average (*d*) weighted mean

92. If f be focal length of the objective and f_1 that of the eyepiece, then the magnifying power is given by :

(*a*) $\frac{f_1}{f}$ (*b*) $\frac{f}{f_1}$

(*c*) $f \times f_1$ (*d*) $\frac{f_1}{f^2}$

93 . The length of a transition curve is given by the relation:

(*a*) $\frac{V^3}{CR}$ (*b*) $\frac{V^2}{CR}$

(*c*) $\frac{V}{CR}$ (*d*) $\frac{V^3}{C^2R}$

Answers of Objective Type Questions

1. *Fill in the blanks/complete the sentences.*

1. Base line 2. 45° 3. invar tape
4. Magnetic 5. declination
6. (*a*) setting, (*b*) levelling, (*c*) centring and (*d*) orientation
7. (*a*) Tripod, (*b*) levelling head (*c*) telescope and (*d*) levelling staves
8. 0.673 d^2 9. Change point
10. topography 11. Broken tops
12. axis of plate levels and line of collimation.
13. Deflection angle 14. Reiteration 15. 100
16. One, three 17. Mid-ordinate
18. $\dfrac{C_n\,(C_n - C_{n-1})}{2R}$ 19. area 20. Reducing/enlarging
21. $l\,(1 - \cos\theta)$ 22. $\sqrt{l}$ 23. Levelling
24. Refraction 25. contour 26. Orientation
28. South 29. North merdian
30. tangents 31. sub chord 32. planimeter
33. difference in heights 34. horizontal angles
35. plane table 36. azimuth 37. telescope & eyepiece
38. the angle between the prolongation of preceding line and forward line
39. fall 40. overhanging cliff
41. Commencement, tangency 43. centre
44. plane 45. equilateral 46. 0.007
47. Reciprocal
48. in equal graduations of the horizontal plate
49. fine motion 50. 100
51. tacheometer, levelling staff
52. 90° 53. magnetic meridian
54. Contour interval 55. graphical
56. Planetable and alidade 57. $\dfrac{L}{24}\left(\dfrac{W}{p}\right)$
59. BM 60. steep

Answers of Multiple Choice Type Questions

1. (*c*)	2. (*b*)	3. (*c*)	4. (*b*)	5. (*b*)
6. (*b*)	7. (*c*)	8. (*a*)	9. (*b*)	10. (*a*)
11. (*a*)	12. (*b*)	13. (*a*)	14. (*a*)	15. (*a*)
16. (*d*)	17. (*c*)	18. (*a*)	19. (*a*)	20. (*a*)
21. (*c*)	22. (*b*)	23. (*a*)	24. (*d*)	25. (*a*)
26. (*b*)	27. (*b*)	28. (*b*)	29. (*d*)	30. (*c*)
31. (*b*)	32. (*a*)	33. (*d*)	34. (*c*)	35. (*b*)
36. (*c*)	37. (*c*)	38. (*d*)	39. (*c*)	40. (*c*)
41. (*c*)	42. (*b*)	43. (*c*)	44. (*b*)	45. (*b*)
46. (*a*)	47. (*b*)	48. (*b*)	49. (*d*)	50. (*b*)
51. (*a*)	52. (*c*)	53. (*c*)	54. (*b*)	55. (*b*)
56. (*a*)	57. (*c*)	58. (*a*)	59. (*a*)	60. (*b*)
61. (*b*)	62. (*b*)	63. (*c*)	64. (*a*)	65. (*c*)
66. (*b*)	67. (*c*)	68. (*d*)	69. (*c*)	70. (*c*)
71. (*b*)	72. (*b*)	73. (*c*)	74. (*c*)	75. (*d*)
76. (*a*)	77. (*c*)	78. (*c*)	79. (*b*)	80. (*b*)
81. (*d*)	82. (*c*)	83. (*b*)	84. (*d*)	85. (*b*)
86. (*d*)	87. (*d*)	88. (*a*)	89. (*b*)	90. (*a*)
91. (*a*)	92. (*b*)	93. (*a*)		

Classified Questions

1. LINEAR MEASUREMENTS

1. Explain the method of reciprocal ranging. What errors does this method eliminate ?

2. Explain how you overcome an obstacle in the form of a

(*i*) Lake, (*ii*) River

3. (*a*) Describe a steel band. State the suitability of steel band for measurements.

(*b*) How will you test the chain ? What is the tolerance for a 30 metre chain?

(*c*) What is a Hypotenusal allowance ? Derive the expression for the same.

4. (*a*) While carrying out ranging it is observed that the two selected stations are not intervisible. How will you range the intermediate points between these two points ?

(*b*) A line measured on a rising gradient of 1 in 12 was found to be 450.5 m. It was afterwards found that the 30 metre chain used for the measurement was 5 cm too short. Find the correct horizontal length of the line.

5. (*a*) Describe various types of tapes in detail. Which type of the tape you have used for your project ? Why ?

6. (*a*) What are the circumstances under which the following instruments are used for linear measurements :

(*i*) Odometer (*ii*) Metric chain

(*iii*) Steel band (*iv*) Invar tape

(*b*) What do you understand by "ranging" ? Explain a method used for ranging when the ends of a line are not intervisible.

(*c*) The length of a line measured by means of a 20 m chain was found to be 610.2 m is known to be 612.0 m. What was the actual length of the chain ?

7. (*a*) It is required to find the height of a building with the help of two ranging rods and a tape only. Explain how you will carry out the survey work.

(*b*) How will you test the chain ?

(*c*) Write a short note on a steel band

8. Write short notes on the following :
(*i*) Errors in chaining
(*ii*) Reciprocal ranging
(*iii*) Chain angle

2. CHAIN SURVEYING

9. (*a*) What are the different obstacles in chaining ? How will you overcome the same ?

(*b*) A man standing on the bank of a river sees a tree on the other side of a river in a direction N 30° W. He walks a distance of 100 metres towards west and finds that the direction of the tree is N 30°E. The river flows east west. Find out the width of the river.

10. What do you understand by limiting length of the offset ?

Derive the expression for a limiting length of offset for combined errors in length and direction of offsets.

11. (*a*) Stations A and B are located on two slopes having a valley in between. Indicate how will you Range the line AB.

(*b*) If two points are situated on a steep slope, how will you measure the distance ?

12. A line CAB crosses a river. A and B are on near and distant banks of the river respectively. Perpendiculars AD and CE are 30.50 m and 50.50 m respectively, such that B,D and E are in a straight line. If the chainage of C and A are 505.50 and 550.50, calculate the chainge of B.

13. Describe the construction and working of an optical square.

14. Find the limiting length of the offset if the combined error in laying out an offset is not to exceed 0.25 mm. The accuracy of measuring the offset was 1 in 200 and the scale is 1 cm = 40 cm. What is the permissible angular error in this case.

15. A chain line ABC crosses a river, B and C being on the near and distant banks respectively. A perpendicular BD 52.85 m long is set out at B on the left of the chain line. The respective bearings of C and A taken at D are 67° 30′ and 157° 30′. Find the chainage of C given that AB = 25.52 m and chainage of B = 312.50 m.

16. Find the maximum length of an offset so that the displacement on paper does not exceed 0.025 cm. The offset was laid out 3° from its true perpendicular position. The scale of plotting was R.F. = 1/2000.

3. COMPASS SURVEY

17. (*a*) Define the following terms :

(*i*) Azimuth (*ii*) Agonic lines (*iii*) Isogonic lines (*iv*) Magnetic declination, (*v*) Isoclinic lines

(*b*) The magnetic bearing of the sun at noon is 170°. Draw a sketch and find the variation.

(*c*) The bearing of a side AB of a square ABCD is 30° 30′. Compute the bearings of the remaining sides and diagonals.

18. (*a*) The following readings were observed in a running traverse ABCDA

Line	*Bearing*
AB	N 42°–30′ E
BC	S 57°10′ E
CD	S 02°–10′ W
AD	S 36°–30′ E

Find the included angles A, B, C, D.

If point A is affected by Local attraction, which of the calculated angles will be wrong ?

(*b*) Define the following terms :-

Agonic lines, Angle of Dip, Declination Azimuth.

19. (*a*) How will you test the prismatic compass before using it ? Explain in detail its adjustments.

(*b*) Sketch the surveyor's compass and indicate its various parts.

20. (*a*) State the surveying principles you have used in chain and compass survey.

(*b*) In a closed traverse with a compass it is observed that not a single line gives exact difference of 180° between F.B. and B.B.

Describe how will you proceed to solve the problem for local attraction at each station.

21. Explain the construction of a prismatic compass with a neat sketch.

22. (*a*) What is local attraction ? How is it detected and eliminated?

(*b*) The following bearings were observed on a traverse :

Line	*F.B.*	*B.B.*
AB	80° 45′	260° 0′
BC	130° 30′	311° 35′
CD	240° 15′	60° 15′
DA	290° 30′	110° 10′

Find the corrected bearings of the lines and included angles.

Also, find the true bearings if the declination at the place is 1° 30′ W.

23. (*a*) Explain the terms :

(*i*) True meridian

(*ii*) Magnetic meridian

(*iii*) Arbitrary meridian

(*b*) What is local attraction ? How is it detected and corrected ?

(*c*) If the respective bearings of two lines PQ and QR are 154° 22′ and 53° 12′, find the angle PQR.

24. The following bearings were taken during traversing with a compass at a place where local attraction was suspected.

Line	*Fore Bearing*	*Back Bearing*
AB	S 45° 30′ E	N 45° 30′ W
BC	S 60° 0′ E	N 60° 40′ W
CD	S 5° 30′ E	N 3° 20′ W
DA	N 83° 30′ W	S 85° 0′ E

Locate the local attraction and determine the corrected bearings if the declination at the place is 1‘30’ W. Determine the true bearings.

25. Write short notes on the following :

(*i*) Permanent adjustments of a compass
(*ii*) Surveyor’s compass.

(*iii*) Plane Table Surveying

26. Describe the resection in plane table.

27. (*a*) Describe advantages of plane table survey.

(*b*) How will you carry out orientation in plane table survey ?

(*c*) Write a short note on the importance of centring in the plane table survey.

(*d*) Describe two, point problem in plane table survey.

28. What do you understand by the orientation of a plane table ? Describe the two methods of orientation stating their relative merits and demerits. What is the effect of bad orientation on traverse ?

29. What is 'three point problem' in plane table survey ? How is it solved by Bessel's method ?

30. Describe the Lehmann's Rules in plane table survey.

5. LEVELLING

31. (*a*) Explain the procedure of carrying out fly levels with neat sketches.

(*b*) Fly levels were taken from station X (R L – 115.0) to station Y for establishing a temporary B.M. at Y. Readings are as follows : 3.35, 1.07, 3.06, 3.34, 3.47, 3.86 , 1.22 and 1.31. The first reading is on station X and last reading on station Y. Find R.L. of station Y. Use collimation method and apply the usual arithmetic checks ?

(*c*) Show how the profile levelling and cross sectioning are plotted?

32. (*a*) Derive with the help of a sketch the formula for curvature correction.

(*b*) Ten pegs are driven at 100m intervals with their tops at a particular gradient. Find their gradient from the following observations :

Fly levels are first taken from station A to C 1.65 (B.M. 100.00), 0.06, 2.53, 1.31, 1.85. This was followed by six intermediate sights and a foresight with the staff held on top of the pegs.

0.82, 1.09, 1.39, 1.65, 1.92, 2.19, 2.48.

The instrument was shifted and the levels taken are a back sight followed by readings on the remaining three pegs.

0.58, 0.84, 1.12, 1.41.

33. The line of sight through two observation stations P and Q just touches the horizon at sea level. If the height of stations P and Q above horizon are 4500 m and 6000 m respectively, find the distance PQ.

34. (*a*) The following observations are taken to determine the sensitivity of a bubble tube. Find the radius of curvature of bubble tube and sensitivity. Staff readings at a distance of 35 m with bubble central and two divisions out of centre = 1.015, 1.115 respectively, one division of bubble = 2 mm.

(*b*) The following observations are made in a levelling work to establish grade points.

Back sights	3.215	1.03	1.295	1.855
Fore sights	1.225	3.29	2.085	
R.L. of B.M.	393.705			

Six pegs are to be set from the last position of the instrument at 25 m intervals at 1 in 100 falling gradient, R.L. of first peg is 384.50 Find grade rod readings for all the pegs.

35. (*a*) Calculate the missing readings shown by a cross (X) in the following observations :

Station	*B.S.*	*I.S*	*F.S.*	*Rise*	*Fall*	*R.L*
A	0.915					X
B	1.400		X	0.335		X
C	X		2.00		0.600	X
D	1.370		0.975		0.300	X
E		0.550		X		30.00
F	0.915		X	0.300		X
G	X		0.645	X		X
H	0.380		2.60		0.640	X
I			1.28		X	X

Apply the arithmetical checks.

(*b*) Explain the need and method of determining the sensitivity of a bubble tube.

36. The following readings were taken with a dumpy level 1.25, 2.00, 2.75, 4.00, 1.50, 2.50, 0.75, 0.50, 4.00, 3.20, 1.75, 0.75, T.B.M. The level was shifted after 4th and 8th reading.

(*a*) Reduce the levels by rise and fall method.

(*b*) Calculate the level of T.B.M. if the line of collimation was tilted upwards at an angle of 6 min and each back sight length was 100 m and the foresight length 25.0 m.

(*c*) Calculate the level of T.B.M. if the staff was not held up right but leaning backwards at 5° to the vertical in all cases.

37. Describe fully temporary adjustments of a dumpy level. Why these adjustments are necessary ?

38. The centre line of a proposed work was marked out by pegs at every 100 metres on a gradually falling ground. From the last position of the instrument station 6 pegs are to be driven. The first peg is to have a R.L. of 48.55. Work out the staff readings for setting out the pegs at 25 metres interval at a falling gradient of 1 in 50.

The following are the readings upto the last position. First reading on B.M. of R.L. 49.90 :

B.S.	1.60	2.41	1.15	0.71
F.S.	0.80	1.50	2.20	-

39. The following staff readings were observed with a dumpy level in taking profile levelling :

3.975, 2.457, 3.943, 2.963, 3.156, 3.749, 2.653, 2.945, 1.975 and 1.776 (m)

The level was shifted after 5th and 8th readings. The first reading was taken on a B.M. of R.L. 150.250. Calculate the R.L's of remaining points and apply usual checks.

40. The following staff readings were observed successively with a level, the instrument having been moved forward after the second, fourth and eighth readings :

0.875, 1.235, 2.310, 1.385, 2.930, 3.125, 4.125, 0.120, 1.875, 2.030, 3.765.

The reading on the bench mark is 132. 135. Enter the readings in level book and reduce the levels. Find the difference between first and last point. Apply the checks.

41. (*a*) List the permanent adjustments of a dumpy level. Explain the adjustment for any one of these.

(*b*) A dumpy level was set up midway between two staff stations A and B, 75 m apart. The staff readings on A and B are respectively 1.800 and 1.300. The level is now shifted a point 40 m away from A and on the line AB produced. Staff readings on A and B are 1.72 and 1.32 respectively.

Find the correct staff readings for the second position of the instrument and the true difference of level between A and B.

42. (*a*) Name the different types of levels and mention where each type is used.

(*b*) What are temporary adjustments of a level ? How are these accomplished ?

(*c*) A solid dumpy level was set up midway between two staff stations P and Q which are 100 m apart. The staff readings on P and Q are respectively 2.100 and 1.600 m. The level is now shifted to a point 25 m away from P on the line PQ produced and the staff readings on PQ are 1.900 and 1.500 m. Find the correct staff readings for the second position of the instrument and the true difference of level between P and Q.

43. (*a*) Explain with neat sketches how you overcome obstacles during levelling.

(*b*) The following observations are made in a profile levelling survey. The instrument is shifted after the 3rd, 6th and 10th readings. Find the reduced level of all the points by the line of collimation method and also Rise and Fall method.

1.915	2.625	3.115	2.755	1.625	1.915
2.050	2.755	3.225	3.165	3.225	2.520

44. (*a*) An observer 6 m above the horizon just sees the top of a light house which is 100 m above the horizon. Find the distance of the observer from the light house.

(*b*) The following observations are made to determine the sensitivity of a bubble tube :

Reading on staff at a distance of 50 m with bubble central = 1.115 m.

Staff reading with bubble three divisions off centre - 1.145 m. Find radius of curvature of bubble tube and sensitivity if one division of bubble = 2 mm.

45. Two pegs P and Q on either bank of a river are 800 m apart. The following observations were made :

Level near	*Staff Readings*	
P	1.245	1.84
Q	0.630	1.19

Find the true difference of level and the collomation error.

46. (*a*) List the temporary adjustments of a dumpy level and explain how these are accomplished.

(*b*) From a bench mark of R.L. 200.00, fly levels were taken and the following staff readings are obtained.

Back sights	2.900	1.315	0.910	1.895
Fore sights	1.325	1.800	2.165	2.270

From the last position of the instrument, six pegs at 20 m intervals are to be set out on an uniformly rising gradient of 1 in 80, the first peg is to have a R.L. of 199.50. Work out the staff readings for setting the tops of pegs to the given gradient. Apply usual check.

47. (*a*) Explain with a neat sketch, the errors you can eliminate by using reciprocal levelling.

(*b*) Two points A and B are located on either side of a river. The following observations are made to find the difference of level between these points.

Instrument Station	*Staff Station*
	A B
A	2.405 3.055
B	1.215 1.875

Find difference of level and state whether line of collimation is inclined upwards or downwards.

48. The following observations are made to determine the sensitivity of a bubble tube on a staff held 100 m away. Staff reading with bubble central 0.785, Staff reading with bubble 2 divisions out of centre = 0.805. If length of one division is 2 mm. Find angular value of one division of bubble tube and its radius of curvature.

49. A passenger travelling in a ship sees a light house which is just visible. The top of the house is 50 m above sea level. If the height of the passenger's eye is 4.5 m above sea level, find the distance of the passenger from the light house.

50. (*a*) Distinguish between :

(*i*) Datum and Bench Mark

(*ii*) Level line and horizontal line.

(*b*) The following readings are successively taken from a level, the instrument having been shifted after 4th and 7th readings.

0.350	0.683	0.581	1.758	1.895
2.830	1.780	0.345	0.682	1.230

Enter the readings in the from of a level field book and calculate the reduced levels of all the points. Assume R.L. of the first point as 85.00 m. Apply usual checks.

51. (*a*) Explain clearly how the procedure of reciprocal levelling eliminates the errors due to curvature, refraction and in-adjustment of line of sight ?

(*b*) The following consecutive readings were taken with a dumpy level and 4 m staff on continuously sloping ground on a straight line at a common interval of 20 m 0.680, 1.325, 2.320, 2.625, 3.825, 1.055, 2.850, 3.540, 1.530, 2.565, 3.830.

The reduced level of the first point was 85.560. Calculate the reduced levels of all the points and also the gradient of the line joining the first and last points.

52. (*a*) What is the sensitivity of the bubble tube ? Explain how it is determined in the field ?

(*b*) Find the radius of curvature of a bubble tube whose length of one division is 2 mm and the angular value of its one division :

(*i*) 20″

(*ii*) 10″

(*c*) A tower house 60 metres in height was just visible to a passenger standing on the deck of a ship. Find the distance between the ship and the tower.

53. (*a*) What are Indirect Methods of Levelling?

(*b*) Describe the various difficulties in levelling work. Explain in detail how you will overcome any one of them.

54. (*a*) State the precautions you will take while carrying out of levelling work.

(*b*) Write a short note on precisian of levelling.

(*c*) Give reasons :-

(*i*) The three screw head is commonly adopted in levelling work.

(*ii*) The correction for curvature of earth's surface is always substractive.

55. Following readings were taken with a dumpy level and 4.0 m staff on a continuously sloping ground.

1.68, 2.47, 3.55, 0.680, 1.20, 2.05, 3.800, 1.20, 1.60, 1.85, 3.60, 1.8, 2.5, 3.50.

Rule out a complete page of a level book and-

(*i*) Fill it correctly

(*ii*) Find out Rise and fall for each point

(*iii*) Find R.L. of each point

56. Write short notes on the following :

(*i*) Temporary adjustments of level.

(*ii*) Rise and fall and the line of collomation methods.

(*iii*) Optics in a levelling instrument.

(*iv*) Profile levelling.

(*v*) Correction for curvature and refraction.

(*vi*) Internal focussing telescope.

(*vii*) Bench marks.

(*viii*) Difficulties in levelling work.

(*ix*) Self reading staff.

(*x*) Reciprocal levelling.

6. CONTOURING

57. (*a*) Show the following ground features by contours-

(*i*) Steep slopes (*ii*) Saddle or by pass (*iii*) Overhanging cliff

(*iv*) Vertical cliff.

(*b*) List uses of contour maps.

58. (*a*) Show by means of neat sketches the following contour features :

(*i*) Ridge and valley line (*ii*) A depression (*iii*) A saddle (*iv*) An overhanging cliff.

(*b*) Explain the different methods of contouring.

59. (*a*) List the methods of plotting Contouring Surveys and explain any one method in detail.

(*b*) Explain the engineering and other uses of contour maps.

60. (*a*) Describe in detail the field and office works for block contouring an area of 200 × 200 m. Contours are required at 0.5 m intervals. Each block size can be taken as 4 m × 4 m. Show a typical page of the field book showing the observations.

(*b*) Show by means of neat contour sketches atleast four different ground features.

61. (*a*) Draw representative contours for-

(*i*) Hill top (*ii*) Overhanging cliff (*iii*) Plateau (*iv*) Ridge (*v*) Valley.

62. Show with neat sketches the characteristic features of contours for the following :

(*i*) overhanging cliff (*ii*) ridge (*iii*) pond.

63. Explain any four uses of contours. Also, draw contours of four different situations of ground slopes.

64. (*a*) Show by contour lines the following features :

(*i*) Plane area (*ii*) Saddle (*iii*) Plateau (*iv*) Valley.

(*b*) Find out the contour interval for a scale of 1 cm = 80 metres.

65. Write short notes on the following :

(*i*) Methods of contouring.

(*ii*) Uses of contours.

66. What are the characteristics of contours ? Give the uses of contour maps in Civil Engineering Projects.

7. AREAS

67. (*a*) State the precautions to be taken while using a planimeter.

(*b*) A figure is drawn with horizontal scale 1 cm = 10 m and vertical scale 1 cm = 0.50 m. It is traversed by a planimeter with the anchor point out side the figure and the following readings are obtained :-

Initial reading 8.745; Final reading 2.232

The zero of the dial passed the fixed index mark once in the clockwise direction. If M = 60 cm^2, calculate the area of the figure.

(*c*) What is meant by "Zero circle"? Describe the method of determining zero circle area.

68. It is required to carry out survey for finding out the area of a irregular figure in the open field. Describe fully how you will carry out the survey and find the area, of the open field.

69. (*a*) It is required to find the area of an irregular field. Describe in detail how you will carry out survey work for the same.

(*b*) Using Simpson's rule find the area enclosed between the chain line, the irregular boundary and first and last ordinates. The distances along chain line in metres and the lengths of offsets in metres are as below :

Distance :	0	15	30	45	60	90	120 m
Offset :	2.7	4.1	6.2	7.1	9.0	8.1	7.8 m

70. (*a*) Define the zero circle of a planimeter. How is its area determined?

(*b*) The tracing arm of a planimeter is so set that the roller reads 0.625 for an area of 62.5 sq cm. The perimeter of area is traversed clockwise first with anchor point outside and then anchor point inside. The corresponding differences in reading are 2.085 and 0.678. What is the planimeter constant ? What is the area of zero circle ?

71. (*a*) What is 'zero circle' of a planimeter ? How do you determine the area of zero circle ?

(*b*) How do you find out area with the help of planimeter, if the anchor point is kept

(*i*) inside the area (*ii*) outside the area.

How do you find instrumental constant of planimeter, if it is not, given ?

72. The areas enclosed by the contours in a lake are as under :

Contour (m)	270	275	280	285	290
Area (m^2)	2050	8400	16300	24600	31500

Calculate the total volume of water in the lake

73. The following perpendicular offsets were taken at 10.0m intervals from a survey line :

3.82 4.37 6.82 5.26 7.59 8.9 9.52 8.42 6.43

Calculate the area in sq. metres enclosed between the survey line, the irregular boundary line and the first and last offsets by the application of (*i*) Simpson's rule, (*ii*) Trapezoidal rule.

74. A Figure drawn with a scale of 1 cm = 5 m. It is traversed by a planimeter and the following readings were obtained :-

Initial Reading	6.745
Final Reading	1.325

The zero of the dial passed the fixed index once in clockwise direction. If multiplying constant is 100 cm^2, Find the area of the figure.

8. THEODOLITE

75. Write an exhaustive note on permanent adjustments of a Theodolite ?

76. Explain how you will measure the horizontal angles by a theodolite by the methods of repetition and reiteration.

77. State the errors that are eliminated by 'Method of Repetition' in Theodolite Traverse. How will you measure the horizontal angle by this method ?

78. Write short notes on the following :

(*i*) Method of repetition.

(*ii*) Transit theodolite

(*iii*) Measurement of horizontal angles with methods of repetition and reiteration

(*iv*) Permanent adjustments of a theodolite.

9. THEODOLITE TRAVERSING

79. (*a*) A closed traverse ABCDE is running in anti-clockwise direction.

Following observations were recorded :-

Line	*Length*	*Bearing*
AB	217.5 m	120° 15′
BC	318.0 m	62° 30′
CD	375.0 m	322° 24′
DE	283.5 m	235° 18′
EA	....	.....

Calculate length and bearing of line EA ?

(*b*) Explain in brief Bowditch 'Method of balancing of theodolite traverse.'

80. (*a*) Explain the methods of balancing a theodolite traverse.

(*b*) Following lengths and bearings were recorded in running a theodolite traverse A B C D . There are obstacles which prevent direct measurement of the bearing and length of line A D :

Line	Length in m	Bearing
AB	485	341° 48′
BC	1725	16° 24′
CD	1050	142° 06′

Calculate the length and bearing A D.

81. A closed traverse as indicated below is carried out. Balance the traverse and calculate independent co-ordinates in a form Gale's Traverse Table.

Line	*Distance in Meters*	*Angle*	*Remarks*

AB	1618	*A* 55° 29′	Bearing of *AB* is 37° 18′
BC	1050	*B* 104° 47′	
CD	1725	*C* 54° 17′	
DA	485	*D* 145° 23′	

82. Draw a format of a Gale's traverse table and explain how you fill up.

83. (*a*) Define traversing. What are the different methods of traversing ? Mention the situations where each of them is most suitable.

(*b*) The following data is a part of theodolite survey.

Line	*Length*	*Bearing*
AB	200 m	24° 54′
BC	400 m	298° 06′
CD	300 m	255° 42′

If P is a mid-point on AB and Q is a point on CD at two third from D, find the length and bearing of PQ.

84. Explain deflection angle in the open traverse. What are the checks for the open traverse ?

85. The notes taken in the field of a part of a traverse are recorded as under:

Line	*Length in m*	*Bearing*
AB	405	N 12° 24 E
BC	376	N 15° 36′ W
CD	530	N 20° 12′ W

Determine the length and bearing of the closing line DA.

86. The sketch shows unadjusted field data of a closed traverse. Compute the bearing of line AB using the corrected angular data.

A = 107° 33′ 45″

B = 64° 21′ 15″

C = 206° 35′ 15″

D = 64° 53′ 30″

E = 96° 38′ 45″

87. (*i*) What are the field checks in (*a*) a closed traverse and (*b*) an open traverse ?

(*ii*) It is impossible to observe the length and bearing of a line AB directly, and the following are the observations made from two stations C and D.

Line	*Length in m*	*Bearing*
CA	129.0	S 68° 24′ W
CD	294.0	N 20° 37′ E
DB	108.0	N 60° 18′ W

Compute the length and bearing of AB and also the angles CAB and DBA.

9. Tacheometry

88. (*i*) To detemine R.L. of station Q and distance PQ by tacheometry, observations are to be taken from station P which is higher than station Q. Stadia hairs of the tacheometer are missing . Explain a suitable method and derive the formulae.

(*ii*) The following observations were made with a telescopic alidade fitted with Beaman's stadia arc. Staff intercept = 1.26 m, Central hair reading = 2.545 m Reading on V - scale = 45, Reading on H - scale = 6.

If the multiplying constant is 100 and the elevation of instrument axis is 200.62 m above datum, determine the horizontal distance and elevation of the point sighted.

89. (*a*) What is the purpose of providing an anallactic lens in a tacheometer?

(*b*) The following notes to a traverse run by a tacheometer fitted with an anallactic lens. The multiplying constant of the instrument was 100 and staff was held vertical.

Line	*Bearing Vertical*	*Staff Readings*
AB	30° 24′ + 5° 6′	1.054- 1.992- 2.930
BC	300° 48′ + 3° 48′	1.250- 1.973- 2. 696
CD	226° 12′ – 2° 36′	1.324- 1.687- 2.050

Find the length and bearing of DA. If the R.L. of instrument axis at C is 225.32 m, find the R.L. of D.

90. The stadia intercept read by means of a tacheometer on a vertically held staff was 1.266 m and the angle of elevation was 7^o 42′. The constants of the instrument were 100 and 0.3. Find the total number of turns registered on a movable hair instrument at the same station, if the intercept on the staff held on the same point was 1.650 m, the angle of elevation being 7° 36′. The constants of the movable hair instrument were 1000 and 0.45.

91. (*a*) What is an anallatic lens ? Show that by introducing an anallatic lens in an external-focusing telescope of a tacheometer the distance D of a vertically held staff from the instrument axis can be expressed by D = K.S where K is a constant and S is the staff intercept.

(*b*) A tacheometer has a diaphragm with three cross hairs spaced at a distance of 0.125 cm from each other. The focal length of the object glass is 25 cm and the distance between the object glass and trunion axis is 7.5 cm. A staff is held vertically at a point A, the R.L. of the ground at A being 520.00 m. A tacheometer is set up at a point B and the readings taken on the staff at a point A are 1.600, 1.450, 1300. The angle of elevation recorded on vertical circle is 9°

Determine :

(*i*) horizontal distance AB

(*ii*) R.L. of the ground at B, if the height of trunnion axis of the instrument is 1.5 m

92. (*a*) Derive expressions for horizontal and vertical distances of a vertical staff from tacheometer.

(*b*) The following observations were made on a vertically held staff with a tacheometer set up on an intermediate point on a straight line CD.

Staff station	*Vertical angle*	*Staff intercept*	*Axial hair reading (m)*
C	+ 8° 36′	2.880	2.505
D	–8° 36′	1.655	2.850

The instrument was fitted with an anallactic lens and had a constant of 100. Compute the length CD and R.L. of D, given that the R.L. of C = 527.63 m.

93. (*a*) Explain different systems of tacheometry and discuss their relative merits.

(*b*) Determine the gradient from a point A to a point B from the following observations made with a tacheometer fitted with an anallactic lens. The constant of the instrument was 100 and the staff was held vertically.

Instrument Station	*Staff station*	*Bearing*	*Vertical Angle*	*Staff reading (m)*
P	A	340°	+ 14° 36	1.9, 2.6, 3.3
	B	70°	+ 948°	1.6, 2.2, 2.8

94. (*a*) What do you understand by "tacheometry" ? What are the instruments used in it ? List out the advantages of this method compared to chain survey.

(*b*) What is an anallatic lens? Where it is used ? What are the merits and demerits of using such lens ?

(*c*) The following data is extracted from the notes relating to a line levelled tacheometrically with an anallatic tacheometer whose multiplier of was 100:

Instrument station	*Ht. of axis*	*Staff at B.M.*	*Vertical angle*	*Hair readings*
A	1.53	23.60	–6° 30′	0.80 1.67 2.53
A	1.53	B	+ 6° 30′	0.93 1.73 2.53
C	1.50	B	+ 12° 30′	0.43 1.37 2.30

Determine the elevations A, B and C.

95. Two sets of tacheometer readings were taken from an instrument station A, the reduced level of which was 100.06 m to a staff station B.

(*a*) Instrument P-multiplying constant 100, additive constant 0.06, staff held vertically.

(*b*) Instrument Q-multiplying constant 90, additive constant 0.06 m, staff held normal to the line of sight.

Inst.	*At*	*To*	*Ht. of Inst.*	*Vertical angle*	*Stadia Readings*
P	A	B	1.5 m	26°	0.755 1.005 1.255
Q	A	B	1.45 m	26°	?

What should be the stadia readings with instrument ?

96. (*a*) The following observations were made in a traverse run to find the distance between A and C

Line	*Distance*	*Deflection*	*Bearing*
AB	160 m	...	N 30 E
BC	250 m	70° E	...

Find the distance AC and bearing of AC.

10. TRIGONOMETRICAL LEVELLING

97. A Theodolite was set-up at a distance of 200 m from a tower. The angle of elevation to the top of the tower was 8°18. While the angle of depression to its foot was 2° 24′. The staff reading on the B.M. of R.L. = 250.00 m with the telescope horizontal was 1.385 m. Find the height of the tower and the R.L. of its top.

98. Find the elevation of top of a chimney from the following data :

Instrument station	*Reading on B.M.*	*Angle of elevation*	*Remarks*
A	0.860 m	18° 36′	1. R.L. of B.M. = 511.460 m
B	1.220 m	10° 12′	2. Distance AB = 50 m

Stations A,B and the top of the chimney are in the same vertical plane.

99. To determine the elevation of the top of a flag post, the following observations are made :

Insr. Stn.	*Reading on B.M.*	*Angle of elevation*
A	2.065	11° 15′
B	1.815	17° 10′

Distance AB = 40 m A and B are in the same vertical plane. R.L. of B.M. = 100 find elevation of tower and its distance from A.

100. (*a*) Define the Trigonometrical levelling and differentiate it from ordinary levelling.

(*b*) Top of chimney has been sighted from two instrument stations A and B. 100 m part and in the same line with the chimney. The angles of elevation to the top of the chimney were 45° and 20° from A and B respectively. The heights of instrument axis above ground level at both the stations were 1.5 m R.L. of station A was 200 m and the reading on the staff held on station 'A' with telescope horizontal at station 'B' was 2.5 m. Also, when the telescope was horizontal at station B, the bottom of chimney was exactly bisected. Calculate the distance between station A and chimney and the height of chimney.

11. CURVES

101. (*a*) Enlist various linear and instrumental methods of setting out a simple circular curve.

(*b*) Two tangents meet at chainage 1000 m, the deflection angle being 36°. A circular curve of radius 300 m is to be introduced in between the two tangents, calculate the following :

(*i*) Tangent length

(*ii*) Length of circular curve

(*iii*) Chainages of tangent points

(*iv*) Deflection angles for setting out the first three pegs and the last peg on the curve by Rankine's method.

Pegs are to be fixed at 20 m interval.

102. Two straights intersect at the chainage 1800.00 m with a right deflection angle of 36° 24′. It is proposed to insert a circular curve of 450 m radius with cubic parabolic transitions at each end, between these straights. The maximum speed on the curve is 65 Kmph and the rate of radial acceleration is 30 cm / sec^3. Find suitable length for the transition curves and calculate the chainage at the beginning and end of the combined curve.

If the combined curve is to be set out by the method of deflection angles with a peg interval of 30 m, just illustrate how you will calculate the deflection angles and set out the curve ?

103. A compound curve is to consist of an arc of 900 m radius followed by one of 1200 m radius and is to connect two straights intersecting at an angle of 84° 32′. At the intersection point, the chainage if continued along the first tangent, would be 2329.20 m and the starting point of the curve is selected at chainage 1354.20 m. Calculate the chainages at the point of junction of two branches and at the end of the curve.

104. (*a*) What is super elevation ? Given the speed of the vehicle and the radius of the curve, derive an expression for the amount of super elevation to be provided.

(*b*) Enlist the various methods of setting out transition curves and explain any one of them in detail.

(*c*) Define the shift in case of a transition curve.

105. Enlist various methods of setting out simple circular curves and explain in detail any one of them.

106. (*a*) In a certain curve the maximum allowable speed is 120 km per hour and a rate of change of radial acceleration is 25 cms/sec^3. Calculate the length of the transition curve if radius of circular curve is 600 meters.

(*b*) Two straights of a road intersect at an angle of 120°. They are to be connected by a circular curve of 400 m radius. The chainage at the point of intersection is 100 chains and 25 metres. The length of a chain is 30 metres.

Calculate :-

(*i*) Lengths of tangents

(*ii*) Lengths of long chord

(*iii*) Offsets from long chord at 30 and 60 meters from the centre of long chord.

107. (*a*) Explain the method of setting out a simple curve with one theodolite. Derive the formulae for calculating angles for setting out the curve.

(*b*) Sketch in detail a reverse curve. Explain its various components.

Give formulae for connecting different parameters.

108. Two straights intersect at a chainage of 656.2 m having an angle of deflection of 40°. They are to be connected by a circular curve of 220 m radius and two transition curves, the length of each being 50 m. Calculate the chainages at the beginning and end of the curve.

Explain, without actual calculations, how you would lay out the combined curve by deflection angles.

109. (*a*) Expalin the necessity of a transition curve and methods of finding out length of transition curve.

(*b*) Explain any one of the methods for setting out a transition curve.

110. (*a*) What is a transition curve? Explain its necessity.

(*b*) Explain different forms of transition curves. Derive an expression for an ideal transition curve.

(*c*) Two straights intersect at chainage (612 + 15) with a deflection angle of 40°. It is proposed to insert a circular curve of 300 m radius with a cubic spiral of 90 m length at each end, Calculate-

(*i*) shift (*ii*) total tangent length (*iii*) spiral angle (*iv*) length of circular arc

(*v*) chainages- upto tangent point - T1 (second tangent point)

(Note-one chain is 30 m).

111. (*a*) Derive an expression for setting a simple curve by the method of offsets from the long chord.

(*b*) Two roads meet at an angle of 130°. Calculate the necessary data for setting out a curve of 15 chains radius to connect the two straight portions of the road, if it is intended to set out the curve by chain and offsets from the long chord. The chain used is a 20 m chain.

112. (*i*) A simple circular curve is to be set out by deflection angle method. Starting from the point of commencement, two pegs A and B are fixed on the curve and instrument is shifted to B. How will you set the remaining curve ?

(*ii*) Calculate the offsets from long chord of 180 m length at 45 m interval for setting out a simple circular curve of 8 degree (based on 30 m chord length).

(*iii*) What are different forms of a transition curve ?

(*iv*) A vertical curve is to be set out to connect two grades + 0.8 % and –0.6 % at a 0.5 % rate of change of grade per 20 m. chain. R.L. of second station on a curve is 135.20 m. Calculate the R.L. of third station.

113. In a proposed road alignment, a rising gradient of 1 in 50 is followed by a falling gradient of 1 in 40. Chainage of point of intersection of two grades is 625 m and its R.L. is 60.20 m. Design a vertical curve joining the two grades such that two points 185 m apart and 1.25 m above the curve are intervisible.

114.(*a*) What is a transition curve ? Why and where such a curve is used?

(*b*) Reverse curves should be avoided on highways and main railway lines. Why ?

(*c*) Two straights BA and AC are intersected by a line EF. The angles BEF and EFC are 140° and 145° respectively. The radius of first arc is 600 m and that of second arc 400 m. Find the Chainages of the tangent points and the point of compound curvature given that the chainage of the intersection point A is 3415 m.

115. The internal angle ABC between two tangents is 120° and these are to be connected by a circular curve. Owing to the presence of buildings it was found necessary that the curve should pass through a point D the length of the perpendicular DE on to the tangent AB being 24 m the distance BE being 500 m. Find the radius and tangent distance of a suitable curve.

116. Two straights intersect at chainage 18018 m, the deflection angle being 45° and the straights are to be connected by a circular curve of 300 m radius with cubic parabolas at ends. The curve is to be designed for a speed of 72km/h with the rate of change of radial acceleration of 30 cm/sec^3. Determine the required length of the cubic parabolas and find out the details to set out the 1st cubic parabola and the circular curve by the method of deflection angles. The circular curve is to be set out with pegs at 30 m intervals and the cubic parabolas with pegs at 15 m interval of through chainge.

117. (*a*) What is a transition curve ? What are the objects of introducing a transition curve.

(*b*) The following data refer to a composite curve. Deflection angle = 60° 30′, maximum speed 90 km per hour, centrifugal ratio = 1/4 ; max. rate of change of acceleration = 0.3 m per sec^3, chainage of intersection point = 2570 m.

Find :-

(*a*) the radius of the circular curve

(*b*) the length of the transition curve and (*c*) the chainages of the beginning and end of the transition curves and of the junctions of the transition curves with the circular arc.

118. (*a*) Describe the method of setting out simple circular curve by offsets from long chord.

(*b*) A road bend which defects 75°, is to be designed for a maximum speed of 90 km/hr. A maximum centrifugal ratio of 1/4 and a maximum rate of change of radial acceleration of 40 cm/sec^3, the curve consists of a circular arc with two transition curves at the ends.

Calculate :

(*a*) radius of the circular arc

(*b*) the required length of transition and the

(*c*) total length of the composite curve

119. (*i*) Explain how will you set out a circular curve of given radius between two roads meeting at right angle to each other.

Appendix-A

1. ACCURACY OF LINEAR MEASURMENTS

The final error in liner measurements is assumed to be composed of two portions:

(*i*) **Cumulative errors.** The errors which cause the observed lengths to be either in excess or fall short of the true distances, are called *cumulative errors.* Cumulative errors are proportional to the length of the base (L). Assuming the partial error which is cumulative as d links per chain, the total cumulative error in a length of n chains.

$$= d.n \text{ links} \quad ...(1)$$

(*ii*) **Compensating errors.** The errors which obey no definite rule and cause the observed lengths to be in excess and to fall short of the true distances, are called *compensating errors.* The compensating errors are proportional to square root of the length of the base (L).

Assuming the partial error which is compensating as f links per chain the total compensating error in a length of n chains.

$$= f.\sqrt{n} \text{ links} \quad ...(2)$$

The resultant error due to both the cumulative and compensating partial errors

$$x = \sqrt{d^2n^2 + f^2n} \quad ...(3)$$

The error obtained from eqn (3) is an average value.

2. PERMISSIBLE ERROR

The greatest allowable deviation of the measured length from the length determined trignometrically, should conform to the following :

(*i*) 1 in 1000 for $\frac{1}{2500}$ or larger plans

(*ii*) 1 in 500 for $\frac{1}{2500}$ or smaller maps.

The accuracy of the measurements obtained with long steel tapes, is expected up to 1 in 2000.

By applying a constant pull and rough corrections for temperature, to a standardised steel tape, the permissible accuracy is 1 in 5000.

3. ERRORS IN AREAS

The errors in the linear measurments also affects the accuracy of the calculated areas.

(1) Rectangle

Let a = the long side of the rectangle

b = the short side of the rectangle

S = the area of the rectangle

$\therefore$ $Sp = ab$ ...(4)

LetδS_1 = the small error produced in S due to small error δa in a

δS_2 = the small error produced in S due to small error in δb in b

By differentiating (4) we get

$$\delta S_1 = b.\,\delta a \text{ or } \frac{\delta S_1}{S} = \frac{\delta a}{a}$$

Similarly, $$\frac{\delta S_2}{S} = \frac{\delta b}{b}$$

If δa and δb are very small actual errors, the resultant fractional error due to both errors

$= \pm \frac{\delta a}{a} \pm \frac{\delta b}{b}$ with due regard to the signs of δa and δb respectively.

But $\pm\,\delta a$ and $\pm\,\delta b$ are the probable errors in the sides a and b.

$\therefore$ Propable errors in S ...(5)

$$= \frac{\delta S}{S} = \pm \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2}$$

or $$\delta S = \pm \sqrt{a^2b^2\left(\frac{\delta a}{a}\right)^2 + a^2b^2\left(\frac{\delta b}{b}\right)^2}$$

$$= \pm \sqrt{(b\delta a)^2 + (a\delta b)^2}$$

(a) If δa and δb are proportional to a and b respectively, $\frac{\delta a}{a}$ and $\frac{\delta b}{b}$ are constants, *i.e.,* the fractional error $\frac{\delta S}{S}$ remains unaltered by the variation in the ration of a to b.

(*b*) If δa and δb are proportional to $\sqrt{a}$ and $\sqrt{b}$ respectively, then $\frac{\delta a}{\sqrt{a}} = \frac{\delta b}{\sqrt{b}} = K$ say.

or $$\delta a = K\sqrt{a} \text{ and } \delta b = K\sqrt{b}$$

Substituting these values in Eqn. (5), we get

$$\frac{\delta S}{S} = \pm\sqrt{\left(\frac{K\sqrt{a}}{a}\right)^2 + \left(\frac{K\sqrt{b}}{b}\right)^2} = \pm\sqrt{K^2\left(\frac{1}{a}+\frac{1}{b}\right)}$$

or $$\frac{\delta S}{S} = \pm K\sqrt{\frac{a+b}{ab}}$$

...(6)

The right hand side value of eqn. (6) is minimum if $a = b$ *i.e. the rectangle is a square.*

(2) Triangle

(*i*) Let $b =$ the base of the triangle

$h =$ the altitude of the triangle

$S =$ the area of the triangle

$$\therefore \quad S = \frac{1}{2}bh$$

and $$\frac{\delta S}{S} = \pm\sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta h}{h}\right)^2} \qquad \text{...(7)}$$

Fig. 1.

(*ii*) The area of ΔABC

$$S = \frac{1}{2}bc\sin A \qquad \text{...(8)}$$

Let $\delta S_1 =$ the error produce in S by the small error δb in b

$\delta S_2 =$ the error produce in S by the small error δc in c

$\delta S_3 =$ the error produce in S by the small error δA in A

By differentiating eqn. (8) we get

$$\delta S_1 = \frac{1}{2}\delta bc\sin A$$

or $$\frac{\delta S_1}{S} = \frac{\delta b}{b} \qquad \text{...(9)}$$

Similarly, $$\frac{\delta S_2}{S} = \frac{\delta C}{C} \qquad \text{...(10)}$$

Also $\delta S_3 = \frac{1}{2} bc \cos A \,.\, \delta A$

$$\therefore \quad \frac{\delta S_3}{S} = \cot A \,.\, \delta A \qquad \text{...(11)}$$

The value of $\frac{\delta S_3}{S}$ is a *minimum* when the angle A is 90°. Treating $\pm\, \delta b$, $\pm\, \delta c$, $\pm\, \delta A$ as the probable errors in *b, c,* and A respectively, the probable error in S is:

$$\frac{\delta S}{S} = \pm \sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta c}{c}\right)^2 + (\cot A \,.\, \delta A)^2}$$

If δb and δc are proportional to $\sqrt{b}$ and $\sqrt{c}$, to fractional error $(\delta S)/S$ will be minimum when $b = c$ and $A = 90°$ *i.e., the triangle is a right angled isosceles triangles.*

If δb and δc are proportional to b and$c$ respectively, the value of $(\delta S)/S$ is not affected by the ratio of b to c, but it is minimum when A is 90°.

Example 1. *If the probable error in the linear dimesions of a rectangle is* $\pm \frac{1}{1500}$, *calculate the probable error in the area.*

Solution. The probable error in the area

$$\frac{\delta S}{S} = \pm \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \qquad \text{...}(i)$$

Assuming $\frac{\delta a}{a} = \frac{\delta b}{b} = \frac{1}{1500}$, and substituting in eqn. (*i*), we get

$$\frac{\delta S}{S} = \pm \sqrt{\left(\frac{1}{1500}\right)^2 + \left(\frac{1}{1500}\right)^2} = \frac{\sqrt{2}}{1500}$$

$= \pm$ 1 in 1060.7. **Ans.**

Example 2. *If the probable error in the linear measurements in the sides is* ± 24, *calculate the probable error in the area of* ΔABC. *Given :* $AB = 350$ *m,* $AC = 400$ *and* A = 83° 24′

Solution. The probable error in the area of ΔABC. (Fig. 1)

$$\delta S = \pm S \sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta c}{c}\right)^2 + (\cot A \,.\, \delta A)^2}$$

The area $S = \frac{1}{2} \times 400 \times 350 \sin 83°24' = 69536.1 \text{ m}^3$

$$\therefore \quad \frac{\delta S}{S} = \pm 69536.1 \sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{1000}\right)^2} + (0.115704 \times 0.00698)^2$$

$$= \pm 69536.1 \times .00162857 = \pm 113.2 \text{ m}^3 \textbf{ Ans.}$$

Example. *A plot of land* 60 *m* × 20 *m is measured with stell tape. If the standard error of length width measurement is taken as* ±1 *cm, the standard error of the area of the plot will be:*

(*a*) ± 0.1414 m^2 (*b*) ± 0.566 m^2 (*c*) ± 0.632 m^2 (*d*) ± 0.8484 m^2.

(*U.P.S.C Civil Services Exam*. 1993)

Solution. Probable error in rectangular plot

$$\delta S = \pm \sqrt{(b\ \delta a)^2 + (a\ \delta b)^2}$$

where *a, b* are the sides of the rectangle

δa, δb are the small actual errors.

Substituting the values in eqn. (1), we get

$$\delta S = \pm \sqrt{(60 \times 0.01)^2 + (20 \times 0.01)^2}$$

$$= \pm \sqrt{0.36 + 0.04} = 0.632 \text{ m}^2$$

∴ Correct answer is (c).

Example. *If a 30 m length can be taped with a precision of* ±0.01 *m, then the standard error in measuring 1.08 km with the same precision will be*

(*a*) ± 0.54 *m* (*b*) ± 0.45 *m* (*c*) ± 0.36 *m* (*d*) ± 0.06 *m*.

(*U.P.S.C Civil Services Exam*. 1993)

Solution. The distance measured = 1.08 km = 1080 m

$\therefore$ Number of chains = $\frac{1080}{30} = 36$

The compensating error is ∂ to $\sqrt{length}$

$\therefore$ error $= \pm 0.01 \sqrt{36} = \pm 0.06$ m. ∴ The correct answer is (*d*).

Accuracy of Angular Measurments

An ordinary 6 inch (15 cm) vernier theodolite is graduated to read 20". The maximum error due to reading with such a theodolite, would be about 10". The graduation reading more than 10" is recorded as 20".

The probable error for a single vernior is ±5″. Assuming an error of 2 or 3 seconds for imperfect graduations, the probable error is taken as ±8″.

For two verniers, the probable error is $\pm 8''/\sqrt{2}$.

For face right and face left observations, the probable error of the four readings $= \pm 4''$.

Errors due to Centering of the Theodolite

The error due to cetering varies inversely with the length of the side. The error in the bisectionof the signal also varies with the nature of the signal sighted and the length of the side.

Let the probable error due to reading of two verniers $= 8''/\sqrt{2}$.

Let the probable error due to inaccurated bisection $= \pm 6''$, then the probable error $= \pm \sqrt{(5.7)^2 + 6^2} = \pm 8''$.

With a single face right and face left set of readings the probable error

$$= \pm \frac{11.5}{\sqrt{2}} = \pm 8''$$

The probable error is not the *maximum value*. It may be more or less.

Note. The following points may be noted.

(*i*) For n reiterations, the probable errors is about $\pm 8'' + \sqrt{n}$.

(*ii*) For n repetitions on one face, the probable error due to inaccurate reading is about $\pm(8\sqrt{2})/\sqrt{2n}$ seconds.

(*iii*) For n repetition on one face, the probable error due to inaccurate bisections is about $\pm \dfrac{6\sqrt{2n}}{n}$ seconds.

(*iv*) The probable error in the value of the angle with n repetitions due to inaccurate reading and bisection is about $\pm \sqrt{\left(\dfrac{8}{n}\right)^2 + \left(\dfrac{6\sqrt{2n}}{n}\right)^2}$ seconds.

(*v*) If the triangular error is $\pm 26''$, the average angular error is $\pm \dfrac{26''}{\sqrt{3}} = \pm 15''$.

USEFUL MATHEMATICAL FORMULAE

$\pi = 3.141,592,65$

Arc of circle substending $\theta° = \frac{\pi R\theta}{180°}$

Chord of a circle subtending $\theta° = 2R \sin\frac{\theta}{2}$

Perimeter of circle $= 2\pi R$

$\text{Sin } (A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\text{Cos } (A \pm B) = \cos A \cos B \pm \sin A$ and B

$\text{Tan } (A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$

$\text{Sin } 2A = 2 \sin A \cos A$

$\text{Cos } 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\text{Sin } A + \sin B = 2 \sin\frac{A+B}{2} \cos\frac{A-B}{2}$

$\text{Sin } A - \sin B = 2 \cos\frac{A+B}{2} \sin\frac{A-B}{2}$

$\text{Cos } A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

$\text{Cos } B + \cos A = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

$1 + \cos A = 2\cos^2 \frac{A}{2}$

$1 - \cos A = 2\sin^2\frac{A}{2}$

$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c}$$

$$\text{Cos } A = \frac{b^2 + c^2 + a^2}{2bc}$$

$$c^2 = a^2 + b^2 - 2\,ab\,\cos C$$

$$\text{Tan } \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}$$

Area of triangle $= \sqrt{s(s-a)\,(s-b)\,(s-c)}$ where $s = \dfrac{a+b+c}{2}$

$$\sin \theta = \theta - \frac{\theta^3}{L^3} + \frac{\theta^5}{L^5} - \frac{\theta^7}{L^7} + \ldots..$$

$$\cos \theta = 1 - \frac{\theta^2}{L2} + \frac{\theta^4}{L4} - \frac{\theta^6}{L6} + \ldots$$

$$\tan \theta = \theta + \frac{\theta^3}{1.3} + \frac{2\theta^5}{3.5} + \frac{17\,\theta^7}{5.7.9} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{L2} + \frac{x^3}{L3} + \ldots$$

$$\log_e (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{2} - \frac{x^4}{4} + \ldots$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)\,(n-2)}{1.2.3}x^3 + \ldots$$

(Binomial Theorem)

Equation of circle $x^2 + y^2 = a^2$

Equation of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Equation of parabola $y^2 = 4ax$

$$\text{Radius of curvature } = \frac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}$$

Mensuration Formulae

Area of a rectangle = length × breadth

Area of a triangle $\dfrac{1}{2}$ × base × height

Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$ where a is the side

Area of regular hexagon $= \frac{3\sqrt{3}}{2} a^2$ where a is the side

Area of trapezium $= \frac{1}{2}$ (sum of parallel sides) $\times$ perpendicular distance between them.

Area of circle $= \pi r^2$ where r is the radius.

Area of circle $= \frac{\pi d^2}{4}$ where d is the diameter

Curved surface of cylinder $= 2\pi rh$ where h is the height

Curved surface of sphere $= 4\pi r^2$ where r is the radius

Lateral surface of pyramid $= \frac{1}{2}$ (perimeter) $\times$ slant height

Curved surface of cone $= \pi rl$ where l is the slant height

Volume of cylinder $= \pi r^2 h$

Volume of sphere $= \frac{4}{3} \pi r^3$

Volume of pyramid $= \frac{1}{2}$ (area of base) $\times$ height

Volume of cone $= \frac{1}{3} \pi \text{ (radius)}^2 \times \text{height} = \frac{1}{3} \pi r^2 h$

Differential Calculus Formulae

$\frac{d}{dx}(x^n) = nx^{n-1}$ $\qquad$ $\frac{d}{dx}(\log x) = \frac{1}{x}$

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\qquad$ $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cos x$

$\frac{d}{dx}(a^n) = a^n \log_e a$ $\qquad$ $\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}(\cot x^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\text{cosec}^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

Integral Calculus Formulae

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \qquad \int \frac{1}{x} \, dx = \log x$$

$$\int e^x \, dx = e^x \qquad \int a^x \, dx = \frac{a^x}{\log_e a}$$

$$\int \sin x \, dx = -\cos x \qquad \int \cos x \, dx = \sin x$$

$$\int \tan x \, dx = \log \sec x \qquad \int \cot x \, dx = \log \sin x$$

$$\int \sec x \, dx = \log(\sec x + \tan x) = \log \tan\left(\frac{x}{x} + \frac{\pi}{4}\right)$$

$$\int \text{cosec}\, x \, dx = \log(\text{cosec}\, x - \cot x) = \log \tan\frac{x}{2}$$

$$\int sec\ x \, dx = \tan x$$

$$\int \text{cosec}^2 x \, dx = -\cot x$$

$$\int sec^2\, x \tan x \, dx = sec\ x$$

$$\int \text{cosec}\, x \cot x \, dx = -\text{cosec}\, x$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$-\int \frac{dx}{\sqrt{a^2-x^2}} = \cos^{-1}\frac{x}{a}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$-\int \frac{dx}{a^2+x^2} = \frac{1}{a}\cot^{-1}\frac{x}{a}$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}$$

$$-\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\text{cosec}^{-1}\frac{x}{a}$$

Greek Letter Chart

Form	*Greek*	
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
S	ξ	Xi
Ο	ο	Omicron
Π	π	Pi
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	φ	Phi
X	χ	Chi
Ψ	ψ	Psi
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