V. M. Stuart [illegible]

San Francisco.

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THE

RELATIVE PROPORTIONS

OF THE

STEAM-ENGINE:

BEING

A RATIONAL AND PRACTICAL DISCUSSION OF THE DIMENSIONS OF EVERY DETAIL OF THE STEAM-ENGINE.

BY

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WITH NUMEROUS DIAGRAMS.

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PREFACE.

It is a source of regret to the author of these Lectures that none of the distinguished writers upon Mechanics or the Steam-engine have undertaken to give, in a simple and practical form, rules and formulæ for the determination of the relative proportions of the component parts of the steam-engine.

The authors of the few works as yet published in the English language either entirely ignore the proportions of the steam-engine or content themselves with scanty and general rules—Rankine excepted, who in the attempt to be brief is sometimes obscure, leaving many gaps in the immense field which he has attempted to cover. This deficiency in the literature of the steam-engine is remarkable, because the problem which the mechanical engineer is most frequently called upon to solve is the determination of the dimensions of its various parts.

From time to time hand-books of the steam-engine have been published giving practical (?) rules, the result of observation of successful construction; and with these rules the practising engineer, who has little time for original investigation, has had to content himself. It is of course reasonable to limit the correctness of these rules to cases in which all the conditions are the same, as in the case or cases from which these rules have been derived, thus placing a serious obstruction in the way of improvement or alteration of design, and rendering the rules worse than useless—even dangerous in many cases.

"The usual resource of the merely practical man is precedent, but the true way of benefiting by the experience of others is not by blindly following their practice, but by avoiding their errors, as well as extending and improving what time and experience have proved successful. If one were asked, What is the difference between an engineer and a mere craftsman? he would well reply that the one merely executes mechanically the designs of others, or copies something which has been done before, without introducing any new application of scientific principles, while the other moulds matter into new forms suited for the special object to be attained, and lets his experience be guided and aided by theoretic knowledge, so as to arrange and proportion the various parts to the exact duty they are intended to fulfil.

"'For this is art's true indication,
When skill is minister to thought,
When types that are the mind's creation
The hand to perfect form hath wrought.'"

STONEY'S *Theory of Strains.*

Zeuner, in his elegant *Treatise on Valve Gears,* translated by M. Müller, has laid the foundation for the treatment of slide-valve motions for all time, and in his *Mechanische Wärmetheorie* has carried the application of the mechanical theory of heat to the steam-engine as far as the present state of the science of Thermo-dynamics will permit.

Poncelet, in his *Mécanique appliquée aux Machines*, has most thoroughly treated some members of the steam-engine, neglecting others of as great practical importance.

Hirn, in his *Théorie mécanique de la Chaleur*, gives us, besides a very able treatise on the science of Thermo-dynamics, a valuable series of experiments upon the steam-engine itself, confirming Joule's results.

A translation of *Der Constructeur*, by F. Reuleaux, would, if made, add much to our knowledge of the proper proportions of the steam-engine, as well as of other machines.

A rational and practical method of determining the proper relative proportions of the steam-engine seems as yet to be a desideratum in the English literature of the steam-engine; and these Lectures have been written with that feeling, purposely omitting the consideration of such topics as have already in many cases been over-written, and considering only those which have not received the attention which their importance demands.

In the choice of a factor of safety—a matter wherein opinions widely differ—the author, guided by considerations set forth in Weyrauch's *Structures of Iron and Steel*, translated by DuBois, has fixed upon *10* as being the most correct value. If any of our readers should prefer a different factor, the formulæ deduced will be correct if the actual steam-pressure per square inch is divided by *10* and multiplied by the preferred factor of safety, and the result used in the place of the actual steam-pressure.

In reducing all the required dimensions of parts of the steam-engine to functions of the boiler-pressure or mean steam-pressure in the cylinder per square inch, of the diameter of the steam-cylinder, length of stroke, number of strokes per minute, and horse-power, he trusts that he has put the formulæ in the simplest possible form for immediate use.

It is indeed in this transformation of the formulæ for the strength of materials that the usefulness of the book lies; for the practitioner, once satisfied of their correctness, has but to insert quantities fixed at the commencement of his design, and derive from the formulæ the required dimensions, being relieved of many formulæ and details connected with the applications of statics to the strength and elasticity of materials.

The constant references to the fourth section of Weisbach's *Mechanics of Engineering* are necessary, as it is no part of the author's plan to discuss the strength and elasticity of materials any further than it is necessary to do so in their application to the steam-engine. Those unacquainted with this branch of mechanical engineering will nowhere find it treated with greater simplicity and thoroughness. Other references have been made for the purpose of directing the reader to such sources as have been drawn upon in the consideration of topics discussed in this work.

"We who write at this late day are all too much indebted to our predecessors, whether we know it or not, to complain of those who borrow from us;" and each of us is only able to make his relay, taking up his work where others have left it.

The lack of accurate experimental data has, in many cases, forced the writer to make, perhaps, bold assumptions which may not prove entirely correct; however, as in all cases the method of reasoning is given, the reader, where he is in possession of more accurate data, can modify by substitution.

The accidental loss of all of the original manuscript and drawings of these Lectures, and the necessity of rapidly re-writing them for use in daily instruction, have caused the work to be more abbreviated than was originally intended.

Deeply sensible of the many unavoidable deficiencies of this little work, even in the limited field covered, its author still hopes that it will aid in the diffusion and advancement of real knowledge, upon whose progress the prosperity of our civilization depends.

W. D. M

UNIVERSITY OF PENNSYLVANIA,
Philadelphia, 1878.

PREFACE TO THE SECOND EDITION.

THE kind reception of the First Edition of this work has afforded the author an unexpected pleasure, for which he is very grateful.

The pleasure is the greater because it has come from an unlooked-for direction. Those actively engaged in designing engines rarely turn to books for instruction, and are quite frequently heard to complain that they find in them a good deal that they do not care to know, and nothing that they can utilize. Among the added lectures the author wishes particularly to call the attention of readers to *The Cheapest Point of Cut-Off* and to *The Errors of the Zeuner Diagram.* The former will prove of interest to all, whether designers or users. The latter is not written with a desire to detract from the honor due to Prof. Zeuner, but rather to perfect the great discovery of the law of valve-motions due to his genius.

W. D. M.

UNIVERSITY OF PENNSYLVANIA, 1883.

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THE

RELATIVE PROPORTIONS

OF THE

STEAM-ENGINE.

CHAPTER I.

(1.) **Introductory.**—In the present work, unless specially stated, the single-cylinder, double-acting steam-engine only will be the subject of discussion.

In making use of the following rules and formulæ, if a non-condensing engine is under consideration, the pressures per square inch above the atmosphere, or as registered by an ordinary steam-gauge, must be used. If a condensing engine be considered, fifteen pounds per square inch must be added to the pressures above the atmosphere.

While it is impossible to take up every form of steam-engine which has been invented, the formulæ are sufficiently general to admit of adaptation to any form of engine which the engineer may wish to devise.

(2.) **The Steam-Cylinder.**—In many cases occurring in practical Mechanics other considerations than economy of steam determine either the stroke or the diameter of the steam-cylinder. When, however, these dimensions are not fixed by other considerations, that of economy of steam should have the precedence, as being a constant source of gain; and it being demonstrable by the differential calculus that the surface of any cylinder closed at the ends and enclosing a given volume is a minimum when the diameter of that cylinder is equal to its length,* it follows that any given volume of steam has a minimum of surface of condensation, and consequently loses less by condensation than it would in any other cylinder of equal volume and different relative dimensions; and therefore that the best relative proportions of the stroke and diameter of the steam-cylinder are attained when they are equal.

The importance of the action of the walls of the steam-cylinder in condensing steam, and the inability of a steam-jacket to do more than keep the cylinder warm, without actually communicating any appreciable amount of heat to the enclosed steam, are becoming more clearly recognized among engineers, and are forcing them to adopt higher piston-speeds,† to take means of reducing the surface of con-

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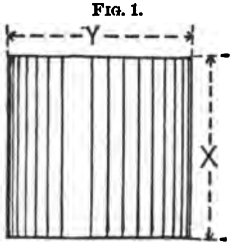
* *Demonstration.*—Let the surface of the cylinder $= S = \pi yx + \frac{\pi y^2}{2}$. (1)

" " volume " " $= V = \frac{\pi y^2 x}{4}$. (2)

From eq. (2) we have $\pi x = \frac{4V}{y^2}$ or $\pi xy = 4Vy^{-1}$.

FIG. 1.



Substituting in eq. (1), we have

$$S = 4Vy^{-1} + \frac{\pi y^2}{2}.$$

Differentiating, we have

$$\frac{dS}{dy} = -4Vy^{-2} + \pi y = 0,$$

giving $$y = \sqrt[3]{\frac{4V}{\pi}}.$$

Substituting this value of y in eq. (2), we have also $x = \sqrt[3]{\frac{4V}{\pi}}$, showing, since the second diff. coef. is positive, a minimum.

† For a full discussion of the consumption of steam, see *A Treatise on Steam*, chap. iv., Graham; "Experiments with a Steam-Engine," translated from *Bullétin de la Société Industriel de Mulhouse*, in *Journal of the Association of Eng'g Societies*, May, 1883, Hirn-Smith; also Chapters XV. and XVI.

densation in the cylinder, and to superheat the steam to a limited extent before its introduction into the engine.

If the cylinder and heads be of a uniform thickness, the quantity of metal required to form a cylinder of the desired volume is nearly a minimum: we say "nearly," because sufficient length must be added to the cylinder to provide for the thickness of the piston-head and the required clearance at the ends.

The use of shorter cylinders than has hitherto been customary has the advantage of reducing the piston-speed, and consequently the wear upon the piston-packing, for any given number of revolutions, although the wear upon the interior of the steam-cylinder is constant for any constant number of strokes per minute.

(3.) **Indicated Horse-Power.**—In the present work the indicated horse-power only will be referred to or made use of.

Let (HP) = the indicated horse-power.
" P = the mean pressure of steam on the piston-head in pounds per square inch.
" L = the length of stroke in feet.
" A = the area of the piston-head in square inches.
" N = the number of strokes (= twice the number of revolutions of the crank) per minute.

We have the well-known formula,

$$(HP) = \frac{PLAN}{33000}. \tag{1}$$

If in formula (1) we make the length of stroke in inches equal to the diameter of the steam-cylinder, it becomes, letting d = the diameter of cylinder in inches,

$$(HP) = \frac{Pd\left(\frac{\pi d^2}{4}\right)N}{12 \times 33000};$$

and reducing, we have for the common diameter and stroke of a steam-cylinder of any assumed horse-power,

$$d = 79.59\sqrt[3]{\frac{(HP)}{PN}}. \qquad (2)$$

This formula gives, for any assumed horse-power, mean pressure, and number of strokes per minute, the common diameter and stroke of the steam-cylinder in *inches*.

The reduction in size, and the consequent economy in using steam, resulting from assuming the pressure per square inch, P, and the number of strokes per minute, N, as large as circumstances will permit, in designing an engine of any desired horse-power (HP), will at once be perceived upon inspection of formula (2). The advantages resulting from high pressures, early cut-off, and rapid piston-speed will be more thoroughly discussed in Art. 35 of this work

Example.—In a given cylinder,

Let $L = 4$ ft. $= 48$ inches.
" $d = 32$ inches.
" $P = 40$ lbs. per square inch.
" $N = 40$ per minute $= 20$ revolutions of the crank.

Using formula (1), we have

$$(HP) = \frac{PLAN}{33000} = \frac{40 \times 4 \times 804.25 \times 40}{33000} = 156, \text{ approx.}$$

If now we assume the horse-power $(HP) = 156$, and

Let, as before, $N = 40$ per minute,
$P = 40$ lbs. per square inch,

we have, using formula (2),

$$d = 79.59\sqrt[3]{\frac{(HP)}{PN}} = 79.59\sqrt[3]{\frac{156}{40 \times 40}} = 36.73 \text{ inches,}$$

for the required common stroke and diameter of a cylinder of equal power with the first.

(4.) **Thickness of the Steam-Cylinder.**—Steam-cylinders are usually made of cast iron; and in order that the engine may be durable, this casting should be made of as hard iron as will admit of working in the shop. Steel lining-cylinders for ordinary cast-iron cylinders have sometimes been used, and have well repaid in durability their greater cost.

Large steam-cylinders should always be bored in either a horizontal or vertical position, similar to that in which they are to be placed when in use.

Weisbach, in Art. 443, Vol. ii., of the *Mechanics of Engineering*, gives the following formula for the thickness of steam-cylinders:

Let t = the thickness in inches of cast-iron cylinder-walls.
" P_b = the boiler-pressure in pounds per square inch.
" d = the diameter of the cylinder in inches.

Then

$$t = 0.00033 P_b d + 0.8 \text{ inch}, \qquad (3)$$

which makes 0.8 inch the least possible thickness of a steam-cylinder.

Van Buren, in *Strength of Iron Parts of Steam-Machinery*, page 58, establishes the following formula by means of a discussion of a 72-inch English steam-cylinder which had been found to work well:

$$t = 0.0001 P_b d + \sqrt{.0223d + \frac{d^2 P_b^3}{1000000000}}. \qquad (4)$$

Reuleaux, in *Der Constructeur*, page 561, gives the following empirical formula for the completed thickness of steam-cylinders:

$$t = 0.8 \text{ inch} + \frac{d}{100}. \qquad (5)$$

2*

Inspection of these differing formulæ, all founded upon successful practice, would lead to the conclusion that it is best first to calculate the thickness necessary to withstand the pressure of the steam, and then to make an addendum sufficient to provide for boring* and re-boring, and also to give the cylinder perfect rigidity in position and form.

Good cast iron has an average tensile strength of 18,000 pounds per square inch cross-section, and with a factor of safety of 10 gives 1800 pounds per square inch as a safe strain. That this factor is not too large will be conceded when we consider that with some forms of valve-motion the admission of steam to the cylinder partakes of the nature of a veritable explosion.

From Weisbach's *Mechanics of Engineering*, Vol. i., Sec. vi., Art. 363, we have, if we take safe strain = 1800 pounds,

$$t = \frac{P_b d}{3600} = 0.00028 P_b d. \qquad (6)$$

Example.—For a locomotive cylinder,

Let $P_b = 150$ pounds per square inch.
" $d = 20$ inches.

From formula (3) we have $t = 1.8$ inches.
" " (4) " " $t = 1.34$ "
" " (5) " " $t = 1.00$ "
" " (6) " " $t = .83$ "

Any of the thicknesses given would probably serve successfully, and about 1¼ inches is the best practice. It is not

* The best means of securing an approximately true cylinder is to finish to size with a shallow broad cut, giving a rapid feed to the lathe or boring-mill tool. If the cylinder is bored with a fine feed and deep cutting-tool, the gradual heating and subsequent cooling are apt to make the interior tapering in form, as well as to require the running of the shop out of hours in order to avoid stopping, and thereby causing a jog in the cylinder.

advisable to make a steam-cylinder of less than 0.75 inch thickness under any circumstances.

In deciding upon the thickness to be given to any cylinder, the method of fastening it, as well as the distorting forces that are likely to occur, should be carefully considered.

(5.) **Thickness of the Cylinder-Heads.**—It is demonstrated in Weisbach's *Mechanics of Engineering*, Vol. i., Sec. vi., Art. 363, that if the cylinder-heads were made of a hemispherical shape, they would need to be of only half the thickness of the cylinder-walls; and in designing, the attempt is sometimes made to attain greater strength by giving to the cylinder-heads the form of a segment of a sphere.

In considering rectangular plane surfaces subjected to fluid pressure in Weisbach's *Mechanics of Engineering*, Vol. ii., Sec. ii., Art. 412, the following formula is deduced for square plane surfaces, which will of course be, with greater safety, true for a circular inscribed surface:

Let t_1 = the thickness in inches of the cylinder-heads.
" P_b = the boiler-pressure in pounds per square inch.
" d = the diameter of the steam-cylinder in inches.

$$t_1 = 0.003d\sqrt{P_b}. \qquad (7)$$

In large cylinders the heads are stiffened by casting radial ribs upon them.

Example.—

Let $d = 20$ inches.
" $P_b = 150$ pounds per square inch.

We have, from formula (7),

$$t_1 = 0.003d\sqrt{P_b} = 0.003 \times 20\sqrt{150} = 0.73 \text{ inches.}$$

Comparing this with the result derived from formula (6), we observe it to be less; but we also find, comparing formulæ (7) and (6), that

$$\frac{t_1}{t} = \frac{.003d\sqrt{P_b}}{.00028dP_b} = \frac{10}{\sqrt{P_b}}, \text{ approximately.} \qquad (8)$$

From formula we see that $t_1 = t$ when $P_b = 100$ pounds; that t_1 is greater than t when $P_b < 100$ pounds; and that t_1 is less than t when $P_b > 100$ pounds.

A good practical rule for engines in which the pressure does not exceed 100 pounds per square inch is to make the thickness of the cylinder-heads one and one-fourth that of the steam-cylinder walls.

(6.) **Cylinder-Head Bolts.**—Having assumed a convenient width of flange upon the steam-cylinder, the diameter of the bolt should be assumed at one-half that width, and thoroughfare bolts used preferentially to stud bolts, as a stud bolt is likely to rust and stick in place, and be broken off in the attempt to remove it.

The bolts fastening the cylinder-head to the cylinder should not be placed too far apart, as that would have a tendency to cause leaks.

Taking 5000 pounds per square inch of the nominal area of a bolt as the safe strain,* in order to cover fully the strain upon the bolt due to screwing its nut firmly home, as well as the strain due to the steam-pressure, and dividing the total pressure of the steam upon the cylinder-head by it, we will obtain the area of all the bolts required; and divid-

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* The Baldwin Locomotive Works use eleven ⅞-inch stud bolts to secure the head of an 18-inch steam-cylinder. If we assume the greatest steam-pressure to be 150 pounds per square inch, we have for the stress per square inch of nominal area of the bolts about 5800 pounds, and we are therefore well within limits which have been found thoroughly practical by a tentative process.

ing this latter area by the area of one bolt of the assumed diameter, we have the number of bolts required.

Let P_b = the boiler-pressure in pounds per square inch.
" d = the diameter of the steam-cylinder in inches.
" c = the area of a single bolt of the assumed diameter in square inches.
" b = the number of bolts required.

We have

$$b = \frac{0.7854 d^2 P_b}{5000 c} = 0.0001571 \frac{d^2 P_b}{c}. \qquad (9)$$

Example.—In a given steam-cylinder,

Let $d = 32$ inches.
" $P_b = 81$ pounds per square inch.
" $c = 0.442$ square inches (diameter of bolt $\frac{3}{4}$ inch).

We have, from formula (9),

$$b = 0.0001571 \frac{1024 \times 81}{0.442} = 30,$$

showing about 30 three-quarter inch bolts to be required. With so wide a margin as is given by the assumption of 5000 pounds per square inch as a safe strain, considerable variation may be made from this number. Mr. Robert Briggs, in a paper in the *Journal* of the Franklin Institute for February, 1865, page 118, says: "Ordinary wrought iron, such as is generally used in bolts, can be stated to be reliable for a maximum load under 20,000 pounds per square inch, and the absolute (ultimate?) tensile strength of any bolt may be safely estimated on that basis."

CHAPTER II.

(7.) **Standard Screw-Threads for Bolts.**—The standard American pitch and dimensions of head and nut of bolts as now used in all the mechanical workshops of the United States was first proposed by Mr. Wm. Sellers. (See Report of Proceedings, *Journal* of the Franklin Institute for May, 1864; Report of Committee, *Journal* of the Franklin Institute, January, 1865. For a full critique and comparison with other systems, see *Journal* of the Franklin Institute, February, 1865; On a Uniform System of Screw-Threads, Robert Briggs.)

The advantages of uniformity of dimensions in an element of a machine so frequently occurring do not need discussion.

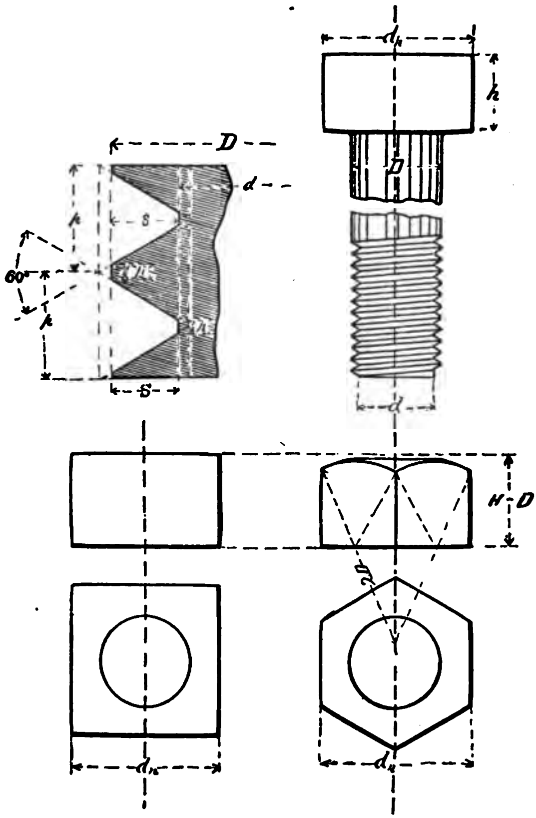
The "Committee on a Uniform System of Screw-Threads" reported as follows:

"*Resolved*, That screw-threads shall be formed with straight sides at an angle to each other of 60°, having a flat surface at the top and bottom equal to one-eighth of the pitch. The pitches shall be as follows, viz.:

Diameter of bolt..	¼	5/16	⅜	7/16	½	9/16	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞
No. threads pr. in.	20	18	16	14	13	12	11	10	9	8	7	7	6	6	5½	5	5
Diameter of bolt..	2	2¼	2½	2¾	3	3¼	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾	6
No. threads pr. in.	4½	4½	4	4	3½	3½	3¼	3	3	2⅞	2¾	2⅝	2½	2½	2⅜	2⅜	2¼

"The distance between the parallel sides of a bolt head and nut, for a rough bolt, shall be equal to one and a half diameters of the bolt plus one-eighth of an inch. The thickness of the heads for a rough bolt shall be equal to one-half the distance between its parallel sides. The thickness of the nut shall be equal to the diameter of the bolt. The thickness of the head for a finished bolt shall be equal to the thickness of the nut. The distance between the parallel sides of a bolt head and nut and the thickness of the nut shall be one-sixteenth of an inch less for finished work than for rough."

FIG. 2.



The required dimensions of bolts and nuts can be best expressed in a general way by means of formulæ and Fig. 2.

Let D = the nominal diameter of any bolt in inches.
Then p = the pitch $= 0.24\sqrt{D+0.625} - 0.175$.

n = the number of threads per inch $= \frac{1}{p}$.

d = the effective diameter of the bolt—*i. e.*, at root of thread $= D - 1.3p$.

S = the depth of thread $= 0.65p$.

H = the depth of nut $= D$.

d_n = the short diameter of hexagonal or square nut $= \frac{3}{2}D + 0''.125$.

h = the depth of the head of bolt $= \frac{3}{4}D + \frac{1}{16}$ inch.

d_h = the short diameter of the head of bolt $= \frac{3}{2}D + 0.125$.

The value of the pitch p, in terms of D, was derived from a graphical comparison of the then existing threads as used in the most prominent workshops in the United States. The depth of the thread S is deduced as follows:

Since the angles of a *complete* V thread are each $= 60°$, its sides and the pitch would form an equilateral triangle, and we would have for its depth $p \sin 60° = 0.866p$; but in the actual thread $\frac{1}{8}$ is taken off at the top and bottom, leaving only $\frac{3}{4}$ of the depth of a complete V thread.

Therefore, $$S = \frac{3}{4}0.866p = 0.65p.$$

Were it possible or always convenient to have uniformly close work, such an accident as the stripping of the thread from a bolt with the dimensions stated would be impossible.

The proportions established are the result and an average of the practical requirements of machine-shop practice, and are therefore to be preferred to proportions which might be established from a theoretical consideration of the strength of materials. The appended table of dimensions (pp. 26, 27) is furnished by Messrs. Wm. Sellers & Co.

(8.) **The Steam-Chest.**—In deciding upon the dimensions of the steam-chest, it must be borne in mind that it ought to be as small as the dimensions and travel of the valve will permit, in order to avoid loss by condensation of steam.

The chest is subject to many modifications of form, but usually consists of the ends and two sides of a cast-iron box, resting upon a rim surrounding the valve-face, upon which a flat cover is placed, and the whole firmly secured to the steam-cylinder by means of stud-bolts passing through the cover outside of the sides of the box, in order to avoid rusting of the bolts as well as to diminish the contents of the box.

The number of bolts required can be determined, as shown in Art. (6), from a consideration of the steam-pressure upon the steam-chest cover.

It is customary to make the sides and cover of the steam-chest of the same material and thickness as the cylinder-walls, sometimes strengthening the cover by casting ribs upon it.

To deduce the theoretical thickness, we have, from Weisbach's *Mechanics of Engineering*, Vol. ii., Sec. ii., Art. 412, the following formula:

Let l = the longest inside measurement of chest in inches.
" b = the breadth of chest in inches.
" P_b = the boiler-pressure in pounds per square inch.
" T = the safe tensile strain upon cast iron per square inch = 1800 pounds.
" t_2 = the required thickness of the steam-chest cover in inches.

TABLE I.

PROPORTIONS FOR

SCREW THREADS, NUTS AND BOLT-HEADS.

Nominal diameter of bolt. D	Threads per inch. n	Diame'r at root of thread. d	Width of flat. $=\frac{1}{8}p$	Safe strain on bolts.	Nuts: Hexagonal. Short diame'r. Rough. d_n	Nuts: Hexagonal. Short diame'r. Finished.	Nuts: Hexagonal. Long diameter. Rough.	Nuts: Square. Long diameter. Rough.	Nuts: Thickness. Rough. H	Nuts: Thickness. Finished.	Bolt-heads: Hexagonal. Short diame'r. Rough. d_n	Bolt-heads: Hexagonal. Short diame'r. Finished.	Bolt-heads: Hexagonal. Long diameter. Rough.	Bolt-heads: Square. Long diameter. Rough.	Bolt-heads: Thickness. Rough. h	Bolt-heads: Thickness. Finished.
$\frac{1}{4}$	20	.185	.0062	276 lbs.	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{37}{64}$	$\frac{7}{10}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{37}{64}$	$\frac{7}{10}$	$\frac{1}{4}$	$\frac{3}{16}$
$\frac{5}{16}$	18	.240	.0074	450 "	$\frac{19}{32}$	$\frac{17}{32}$	$\frac{11}{16}$	$\frac{10}{12}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{19}{32}$	$\frac{17}{32}$	$\frac{11}{16}$	$\frac{10}{12}$	$\frac{19}{64}$	$\frac{1}{4}$
$\frac{3}{8}$	16	.294	.0078	700 "	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{51}{64}$	$\frac{63}{64}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{51}{64}$	$\frac{63}{64}$	$\frac{11}{32}$	$\frac{5}{16}$
$\frac{7}{16}$	14	.344	.0089	1,000 "	$\frac{25}{32}$	$\frac{23}{32}$	$\frac{9}{10}$	$1\frac{7}{64}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{25}{32}$	$\frac{23}{32}$	$\frac{9}{10}$	$1\frac{7}{64}$	$\frac{25}{64}$	$\frac{3}{8}$
$\frac{1}{2}$	13	.400	.0096	1,250 "	$\frac{7}{8}$	$\frac{13}{16}$	1	$1\frac{15}{64}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{7}{8}$	$\frac{13}{16}$	1	$1\frac{15}{64}$	$\frac{7}{16}$	$\frac{7}{16}$
$\frac{9}{16}$	12	.454	.0104	1,550 "	$\frac{31}{32}$	$\frac{29}{32}$	$1\frac{1}{8}$	$1\frac{23}{64}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{31}{32}$	$\frac{29}{32}$	$1\frac{1}{8}$	$1\frac{23}{64}$	$\frac{31}{64}$	$\frac{1}{2}$
$\frac{5}{8}$	11	.507	.0113	2,000 "	$1\frac{1}{16}$	1	$1\frac{7}{32}$	$1\frac{1}{2}$	$\frac{5}{8}$	$\frac{9}{16}$	$1\frac{1}{16}$	1	$1\frac{7}{32}$	$1\frac{1}{2}$	$\frac{17}{32}$	$\frac{9}{16}$
$\frac{3}{4}$	10	.620	.0125	3,000 "	$1\frac{1}{4}$	$1\frac{3}{16}$	$1\frac{7}{16}$	$1\frac{49}{64}$	$\frac{3}{4}$	$\frac{11}{16}$	$1\frac{1}{4}$	$1\frac{3}{16}$	$1\frac{7}{16}$	$1\frac{49}{64}$	$\frac{5}{8}$	$\frac{11}{16}$
$\frac{7}{8}$	9	.731	.0138	4,000 "	$1\frac{7}{16}$	$1\frac{3}{8}$	$1\frac{21}{32}$	$2\frac{1}{32}$	$\frac{7}{8}$	$\frac{13}{16}$	$1\frac{7}{16}$	$1\frac{3}{8}$	$1\frac{21}{32}$	$2\frac{1}{32}$	$\frac{23}{32}$	$\frac{13}{16}$
1	8	.837	.0156	5,500 "	$1\frac{5}{8}$	$1\frac{9}{16}$	$1\frac{7}{8}$	$2\frac{19}{64}$	1	$\frac{15}{16}$	$1\frac{5}{8}$	$1\frac{9}{16}$	$1\frac{7}{8}$	$2\frac{19}{64}$	$\frac{13}{16}$	$\frac{15}{16}$
$1\frac{1}{8}$	7	.940	.0178	7,000 "	$1\frac{13}{16}$	$1\frac{3}{4}$	$2\frac{3}{32}$	$2\frac{9}{16}$	$1\frac{1}{8}$	$1\frac{1}{16}$	$1\frac{13}{16}$	$1\frac{3}{4}$	$2\frac{3}{32}$	$2\frac{9}{16}$	$\frac{29}{32}$	$1\frac{1}{16}$
$1\frac{1}{4}$	7	1.065	.0178	8,000 "	2	$1\frac{15}{16}$	$2\frac{5}{16}$	$2\frac{53}{64}$	$1\frac{1}{4}$	$1\frac{3}{16}$	2	$1\frac{15}{16}$	$2\frac{5}{16}$	$2\frac{53}{64}$	1	$1\frac{3}{16}$
$1\frac{3}{8}$	6	1.160	.0208	10,000 "	$2\frac{3}{16}$	$2\frac{1}{8}$	$2\frac{17}{32}$	$3\frac{3}{32}$	$1\frac{3}{8}$	$1\frac{5}{16}$	$2\frac{3}{16}$	$2\frac{1}{8}$	$2\frac{17}{32}$	$3\frac{3}{32}$	$1\frac{3}{32}$	$1\frac{5}{16}$
$1\frac{1}{2}$	6	1.284	.0208	13,000 "	$2\frac{3}{8}$	$2\frac{5}{16}$	$2\frac{3}{4}$	$3\frac{23}{64}$	$1\frac{1}{2}$	$1\frac{7}{16}$	$2\frac{3}{8}$	$2\frac{5}{16}$	$2\frac{3}{4}$	$3\frac{23}{64}$	$1\frac{3}{16}$	$1\frac{7}{16}$
$1\frac{5}{8}$	$5\frac{1}{2}$	1.389	.0227	15,000 "	$2\frac{9}{16}$	$2\frac{1}{2}$	$2\frac{31}{32}$	$3\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{9}{16}$	$2\frac{9}{16}$	$2\frac{1}{2}$	$2\frac{31}{32}$	$3\frac{5}{8}$	$1\frac{9}{32}$	$1\frac{9}{16}$
$1\frac{3}{4}$	5	1.491	.0250	17,000 "	$2\frac{3}{4}$	$2\frac{11}{16}$	$3\frac{3}{16}$	$3\frac{57}{64}$	$1\frac{3}{4}$	$1\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{11}{16}$	$3\frac{3}{16}$	$3\frac{57}{64}$	$1\frac{3}{8}$	$1\frac{11}{16}$
$1\frac{7}{8}$	5	1.616	.0250	20,000 "	$2\frac{15}{16}$	$2\frac{7}{8}$	$3\frac{13}{32}$	$4\frac{5}{32}$	$1\frac{7}{8}$	$1\frac{13}{16}$	$2\frac{15}{16}$	$2\frac{7}{8}$	$3\frac{13}{32}$	$4\frac{5}{32}$	$1\frac{15}{32}$	$1\frac{13}{16}$

SCREW THREADS, NUTS AND BOLT-HEADS.

					NUTS						BOLT-HEADS					
Nominal diameter of bolt. D	Threads per inch. n	Diame'r at root of thread. d	Width of flat. $= \frac{1}{8}p$	Safe strain on bolts.	Hexagonal. Short diame'r. Rough. d_n	Hexagonal. Short diame'r. Finished.	Hexagonal. Long diameter. Rough.	Square. Long diameter. Rough.	Thickness. Rough. H	Thickness. Finished.	Hexagonal. Short diame'r. Rough. d_n	Hexagonal. Short diame'r. Finished.	Hexagonal. Long diameter. Rough.	Square. Long diameter. Rough.	Thickness. Rough. h	Thickness. Finished.
2	4 1/2	1.712	.0277	23,000 lbs.	3 1/8	3 1/16	3 5/8	4 27/64	2	1 15/16	3 1/8	3 1/16	3 5/8	4 27/64	1 9/16	1 15/16
2 1/4	4 1/2	1.962	.0277	30,000 "	3 1/2	3 7/16	4 1/16	4 61/64	2 1/4	2 3/16	3 1/2	3 7/16	4 1/16	4 61/64	1 3/4	2 3/16
2 1/2	4	2.176	.0312	37,000 "	3 7/8	3 13/16	4 1/2	5 31/64	2 1/2	2 7/16	3 7/8	3 13/16	4 1/2	5 31/64	1 15/16	2 7/16
2 3/4	4	2.426	.0312	46,000 "	4 1/4	4 3/16	4 29/32	6	2 3/4	2 11/16	4 1/4	4 3/16	4 29/32	6	2 1/8	2 11/16
3	3 1/2	2.629	.0357	54,000 "	4 5/8	4 9/16	5 3/8	6 17/32	3	2 15/16	4 5/8	4 9/16	5 3/8	6 17/32	2 5/16	2 15/16
3 1/4	3 1/2	2.879	.0357	60,000 "	5	4 15/16	5 13/16	7 1/16	3 1/4	3 3/16	5	4 15/16	5 13/16	7 1/16	2 1/2	3 3/16
3 1/2	3 1/4	3.100	.0384	75,000 "	5 3/8	5 5/16	6 7/64	7 39/64	3 1/2	3 7/16	5 3/8	5 5/16	6 7/64	7 39/64	2 11/16	3 7/16
3 3/4	3	3.317	.0413	87,000 "	5 3/4	5 11/16	6 21/32	8 1/8	3 3/4	3 11/16	5 3/4	5 11/16	6 21/32	8 1/8	2 7/8	3 11/16
4	3	3.567	.0413	100,000 "	6 1/8	6 1/16	7 3/32	8 41/64	4	3 15/16	6 1/8	6 1/16	7 3/32	8 41/64	3 1/16	3 15/16
4 1/4	2 7/8	3.798	.0435	113,000 "	6 1/2	6 7/16	7 9/16	9 3/16	4 1/4	4 3/16	6 1/2	6 7/16	7 9/16	9 3/16	3 1/4	4 3/16
4 1/2	2 3/4	4.028	.0454	127,000 "	6 7/8	6 13/16	7 31/32	9 3/4	4 1/2	4 7/16	6 7/8	6 13/16	7 31/32	9 3/4	3 7/16	4 7/16
4 3/4	2 5/8	4.256	.0476	142,000 "	7 1/4	7 3/16	8 13/32	10 1/4	4 3/4	4 11/16	7 1/4	7 3/16	8 13/32	10 1/4	3 5/8	4 11/16
5	2 1/2	4.480	.0500	158,000 "	7 5/8	7 9/16	8 27/32	10 49/64	5	4 15/16	7 5/8	7 9/16	8 27/32	10 49/64	3 13/16	4 15/16
5 1/4	2 1/2	4.730	.0500	175,000 "	8	7 15/16	9 9/32	11 21/64	5 1/4	5 3/16	8	7 15/16	9 9/32	11 21/64	4	5 3/16
5 1/2	2 3/8	4.953	.0526	190,000 "	8 3/8	8 5/16	9 23/32	11 7/8	5 1/2	5 7/16	8 3/8	8 5/16	9 23/32	11 7/8	4 3/16	5 7/16
5 3/4	2 3/8	5.203	.0526	214,000 "	8 3/4	8 11/16	10 5/32	12 3/8	5 3/4	5 11/16	8 3/4	8 11/16	10 5/32	12 3/8	4 3/8	5 11/16
6	2 1/4	5.423	.0555	230,000 "	9 1/8	9 1/16	10 19/32	12 15/16	6	5 15/16	9 1/8	9 1/16	10 19/32	12 15/16	4 9/16	5 15/16

$$\text{Then } t_2 = l\sqrt{\frac{l^2b^2}{l^4+b^4}\,\frac{P_b}{2T}} = \frac{l^2b}{60}\sqrt{\frac{P_b}{l^4+b^4}}. \qquad (10)$$

Example.—In a given steam-chest,

Let $l = 40''$.
" $b = 20''$.
" $P_b = 81$ pounds.

Then, substituting in formula (10), we have

$$t_2 = \frac{40}{60}\sqrt{\frac{1600\times 400}{2560000+160000}\,81} = 3 \text{ inches}.$$

(9.) **The Steam-Ports.***—D. K. Clark, in *Railway Machinery*, page 108, states that with a piston-speed of 600 feet per minute a port area equal to one-tenth of the piston area is found sufficient to admit steam to the cylinder with sufficient facility for all practical purposes, and with but a slight reduction of pressure. The size of the port may be increased or diminished proportionally to the piston-speed from these data with good success, bearing in mind that all tortuousness of direction that can be should be avoided in ports.

Example.—Let the piston-speed = 500 feet per minute;

* For a thorough treatise on link- and valve-motions, as well as independent cut-offs, see Zeuner's *Treatise on Valve-Gears.* Auchincloss (*Link- and Valve-Motions*) treats the same subject in a simpler manner, avoiding the use of formulæ as much as possible. King (*Steam, Steam-Engine Propellers, etc.*) gives good descriptions of some purely American forms of valves. Burgh (*Link-Motions*) gives a large number of examples of English link-motions which have been constructed. The author refrains from entering upon the wide field of valve-motions, as one which has been thoroughly covered by the splendid work of Zeuner mentioned above.

then the port area should be $= \frac{5}{6} \times \frac{1}{10}$ piston area $= \frac{1}{12}$ piston area.

(10.) **The Piston-Head.**—It is very doubtful if any formula can be derived which will give a correct value for the thickness of the piston-head under all circumstances. If the steam-cylinder be laid horizontally, other things being equal, the piston-head should be broader than if the cylinder is vertical, and an extra breadth of piston should be given in all cases of rough usage or very rapid piston-speed.

The writer offers as a guide only the following formula:

Let L = the length of stroke in inches.
" d = the diameter of cylinder in inches.
" t_3 = the thickness of the piston-head in inches.

$$\text{Then} \qquad t_3 = \sqrt[4]{Ld}, \qquad (11)$$

$$\text{or if } L = d, \; t_3 = \sqrt{d}. \qquad (12)$$

Example.—Let $L = 24$ inches.
" $d = 20$ inches.

$$\text{Then, } t_3 = \sqrt[4]{24 \times 20} = 4\tfrac{1}{2} \text{ inches, approx.}$$

(11.) **The Piston-Rod.**—The piston-rods of the best engines are made of steel, although in many instances hammered wrought iron is still used. The rod, being fastened at one end to the piston-head and at the other keyed to the cross-head, sustains an alternate thrust and pull while passing accurately through the stuffing-box and gland at one end of the cylinder.

It is obvious that the rod must, in addition to being strong enough to sustain these stresses, be possessed of sufficient rigidity not to bend in the slightest degree while in action.

As the piston-head is aided in holding its position across the cylinder by means of the rod, and the cross-head is always liable to have a slight amount of lateral play, due to imperfections of workmanship or wearing of the guides, we must regard it as a solid column fixed at one end and loaded at the other, which is free to move sideways.

(12.) **Wrought-Iron Piston-Rod.**—For the purpose of determining the proper diameter of the piston-rod, either of the two following formulæ may be used, both taken from Sec. iv. of Weisbach's *Mechanics of Engineering:*

Let S = the stress upon the whole piston-head due to the pressure of the steam.
" d_1 = the diameter of the rod in inches.
" F = the cross-section of the piston-rod in square inches.
" K = the ultimate crushing strength per square inch of wrought iron = 31000 pounds (Weisbach).
" W = the measure of the moment of flexure of a round rod $= \frac{\pi d_1^4}{64}$.
" E = the modulus of elasticity of wrought iron = 28000000 pounds per square inch.
" l = the length of the rod in inches = L = the stroke.

We have, for columns breaking by direct crushing,

$$S = FK, \tag{13}$$

and for long columns tending to break by bending—*i. e.*, buckling—

$$S = \left(\frac{\pi}{2l}\right)^2 WE. \tag{14}$$

Formula (13) must be used when the resulting diameter of the rod is greater than one-twelfth of its length. If the use of formula (13) results in a diameter less than one-twelfth the length of the rod, formula (14) must be used.*

If we assume 5 as the least allowable factor of safety for materials subjected to a constant strain in one direction only, then will 10 be the proper factor for members subject to alternate and equal stresses in opposite directions—*i. e.*, tension and compression. (See Weyrauch, *Structures of Iron and Steel*, translated by Du Bois, chapters iv. and xiii.†)

As before,

Let P_b = the steam-pressure in boiler in lbs. per square inch.
" d = the diameter of the steam-cylinder in inches.

The greatest stress which the piston-rod has to sustain is that due to the initial pressure of the steam in the cylinder, which can be taken as equal to the boiler-pressure.

Introducing a factor of safety of 10 and substituting in formula (13), we have,

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* To determine the ratio of the length to the diameter of a wrought-iron rod at which its tendency to crush is equal to its tendency to break by buckling, we place formula (13) = formula (14). Let d = the diameter of rod in inches, $\frac{\pi d^2}{4}K = \left(\frac{\pi}{2l}\right)^2 \frac{\pi d^4}{64} E$. Therefore, $\frac{l}{d} = \frac{\pi}{8}\sqrt{\frac{E}{K}}$, and substituting the values of E and K, we have $\frac{l}{d} = 12$.

† The ever-varying chemical constituents of the various brands of wrought iron; the fact now fully ascertained that, however carefully made and worked, wrought iron may be heterogeneous in its composition; that the reduction of the area from pile to bar has a very great influence upon its strength (the greater the reduction, the greater its strength); that a high tensile and compressive strength is often obtained by loss, to a great extent, of ductility and welding power—all being considerations of a purely practical nature, as well as the theoretical one mentioned above,—point to a higher factor of safety than 6 or 8, which is generally recommended.

$$10P_b\frac{\pi d^2}{4}=\frac{\pi d_1^2}{4}31000.$$

Therefore, $$d_1=0.0179d\sqrt{P_b}. \qquad (15)$$

Or supposing the pressure P_b uniform throughout the stroke, we have from formula (1)

$$P_b\frac{\pi d^2}{4}=\frac{33000(HP)}{LN}\times 12,$$

if we take the stroke in inches. Therefore, from formula (15)

$$\frac{(HP)}{LN}\frac{33000\times 12\times 4}{3100\times \pi}=d_1^2=162.6\frac{(HP)}{LN}.$$

Therefore, $$d_1=12.753\sqrt{\frac{(HP)}{LN}}. \qquad (16)$$

Note that the stroke is taken *in inches.*

Substituting in formula (14) the values as stated above, we have, using a factor of safety of 10,

$$\frac{10P_b\pi d^2}{4}=\frac{\pi^2}{4L^2}\frac{\pi d_1^4}{64}E.$$

Therefore, $$d_1=\sqrt[4]{\frac{640}{\pi^3\times 28000000}}\sqrt[4]{d^2L^2P_b}=$$

$$0.03901\sqrt[4]{d^2L^2P_b} \text{ inches.} \qquad (17)$$

If we take $L=d$, formula (17) becomes

$$d_1=0.03901d\sqrt[4]{P_b} \text{ inches.} \qquad (18)$$

If we suppose the pressure of the steam to be uniform, formula (17) becomes, in terms of the horse-power (HP),

$$d_1=1.039\sqrt[4]{\frac{(HP)L}{N}} \text{ inches.} \qquad (19)$$

From the consideration of the preceding formulæ we deduce the following rule for wrought-iron piston-rods:

Deduce the diameter of the piston-rod by means of formula (15) or (16), as may be most convenient. Should this diameter be less than one-twelfth of the stroke, then use the most convenient of formulæ (17), (18), and (19).

Example.—What should be the diameter of a piston-rod for a steam-cylinder? Data as follows:

$$L = 48 \text{ inches.}$$
$$d = 32 \text{ inches.}$$
$$P = 40 \text{ pounds per square inch.}$$
$$N = 40 \text{ per minute.}$$
$$(HP) = 156, \text{ approx.}$$

Using formula (15), we have

$$d_1 = 0.0179 \times 32 \sqrt{40} = 3.6 \text{ inches.}$$

Using formula (16), we have

$$d_1 = 12.753 \sqrt{\frac{156}{48 \times 40}} = 3.63 \text{ inches.}$$

As the diameter, 3.6 inches, is less than one-twelfth of the stroke, 48 inches, we must use formula (17) or (19).

Substituting in formula (17), we have

$$d_1 = 0.03901 \sqrt[4]{40 \times 32^2 \times 48^2} = 3.84 \text{ inches.}$$

Substituting in formula (19), we have

$$d_1 = 1.039 \sqrt[4]{\frac{156 \times 48}{40}} = 3.84 \text{ inches.}$$

NOTE.—It must be borne in mind that the piston-rod will almost always be proportioned from the initial or boiler-pressure of the steam, P_b, and not from the mean pressure, P, as in all the better forms of engine the mean pressure is much less than the initial pressure.

(13.) **Steel Piston-Rod.**—When steel is the material used for piston-rods, its greater modulus of ultimate strength and differing coefficient of elasticity will alter the numerical values in the formulæ given above.

Hodgkinson, in his experiments on long columns of wrought iron and steel, found that a steel column of the same dimensions as a wrought-iron column would bear one and one-half times as great a load.

A consideration of formula (14) shows that these experiments would require that E, the modulus of elasticity for steel (which is 28,000,000 pounds for wrought iron), should become 42,000,000 pounds.

This value agrees very nearly with that given by Reuleaux in *Der Constructeur*, page 4, for cast steel.

Kupffer, Styffe and Fairbairn place the value of E at between 30 and 31 million pounds per square inch for steel.

As, however, we shall use the value $E = 42,000,000$ lbs. under exactly the same conditions as the experiments from which it is derived, it is probably the most correct value for our purposes.

The crushing strength of steel is so much greater than its tensile strength, where the material is not permitted to deflect, that it need not be taken into consideration in formula (13).

As a mean of eleven experiments made by Kirkaldy on steel taken at random from merchants' stores, in all cases the extension being upward of 10 per cent., we have for the tensile breaking strain (equal to K) about 90,000 pounds per square inch.

The elongation of 10 per cent. or upward would indicate steel of sufficient toughness for machinery purposes.

In the absence of reliable experiments determining the ultimate crushing strength of *soft* steel, we are safe in assuming that it will bear 100,000 pounds per square inch *without deformation,* which assumption makes the crushing strength of steel equal to that of cast iron, and which it probably much exceeds.

Substituting in formula (13) and using the same notation as in Art. (12), we have

$$\frac{10P_b\pi d^2}{4} = \frac{\pi d_1^2}{4}90000.$$

$$\text{Therefore,} \quad d_1 = 0.0105d\sqrt{P_b}. \qquad (20)$$

Or if we suppose the pressure uniform throughout the whole stroke, we have, in terms of the horse-power,

$$d_1 = 7.481\sqrt{\frac{(HP)}{LN}}. \qquad (21)$$

Substituting in formula (14), we have

$$\frac{10P_b\pi d^2}{4} = \frac{\pi^2}{4L^2} \quad \frac{\pi d_1^4}{64} 42000000;$$

and reducing, we have

$$d_1 = 0.03525\sqrt[4]{d^2L^2P_b}; \qquad (22)$$

or if in (22) we suppose $L = d$,

$$d_1 = 0.3525d\sqrt[4]{P_b}. \qquad (23)$$

If we suppose the initial pressure to be uniform throughout the whole stroke, we have

$$d_1 = 0.9394\sqrt[4]{\frac{(HP)L}{N}}. \qquad (24)$$

Formulæ (20) and (21) should be used when the resulting diameter of the piston-rod is greater than $\frac{1}{8}$ of the length of the stroke; if it is less, then formula (22), (23) or (24) must be used.*

Example.—What should be the diameter of a steel piston-rod for steam-cylinder? Data as before.

$L = 48$ inches.
$d = 32$ inches.
$P = 40$ pounds per square inch.
$N = 40$ per minute.
$(HP) = 156$ approx.

Using formula (20), we have $d_1 = 0.0105 \times 32\sqrt{40} = 2.12$ inches. Using formula (21), we have $d_1 = 7.48\sqrt{\frac{156}{48 \times 40}} = 2.13$ inches.

We observe that the diameter of rod resulting from formulæ (20) and (21) is less than $\frac{1}{8}$ of the stroke, and we must therefore use formula (22) or (24).

Substituting in formula (22), we have

$$d_1 = 0.03525\sqrt[4]{32^2 \times 48^2 \times 40} = 3.47 \text{ inches.}$$

Substituting in formula (24), we have

$$d_1 = 0.9394\sqrt[4]{\frac{156 \times 48}{40}} = 3.47 \text{ inches.}$$

* Referring to note to Art. (12), we find the following general formula: $\frac{l}{d} = \frac{\pi}{8}\sqrt{\frac{E}{K}}$. Substituting in this the *assumed* values of E and K for steel, $\frac{E = 42000000}{K = 100000}$, we have $\frac{l}{d} = 8$.

CHAPTER III.

14. Comparison and Discussion of Wrought-Iron and Steel Piston-Rods.—If, considering the formulæ for rupture by crushing or tearing only for wrought iron and steel, we divide (21) by (16), we have

$$\frac{\text{For steel } d_1}{\text{For wrought iron } d_1} = \frac{7.48\sqrt{\frac{(HP)}{LN}}}{12.75\sqrt{\frac{(HP)}{LN}}} = 0.58. \quad (25)$$

That is, the steel rod would have but 0.58 the diameter of the wrought-iron rod, and 0.34 the area.

If we treat the formulæ (24) and (19) for rupture by buckling in a similar manner, we have

$$\frac{\text{For steel } d_1}{\text{For wrought iron } d_1} = \frac{0.9394\sqrt[4]{\frac{(HP)L}{N}}}{1.039\sqrt[4]{\frac{(HP)L}{N}}} = 0.90. \quad (26)$$

That is, the steel rod has 0.9 the diameter and 0.81 the area of a wrought-iron rod working under the same conditions.

In either case the use of steel is productive of economy in weight, varying from 34 to 81 per cent.

Letting V = the volume of the piston-rod in cubic inches,
" y = the weight of a cubic inch of wrought iron or steel = 0.27 lb., approx.,

we have, for purposes of comparison,

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$$V=\frac{\pi d_1^2}{4}L. \qquad (27)^*$$

Letting C represent the numerical constant, we have, by substituting for d_1^2 its value, derived equation (16) or (21) for crushing,

$$V=\frac{\pi}{4}C^2\frac{(HP)}{N}, \qquad (28)$$

and we see from (28) that the volume, and consequently the weight, of a piston-rod are inversely as the number of strokes per minute for any assigned horse-power.

Substituting formulæ (19) and (24) for rupture by buckling in (27), we have

$$V=\frac{\pi}{4}C^2\sqrt{\frac{(HP)L^3}{N}}. \qquad (29)$$

If in this we assume $L=d$, we have from equation (2), letting C_1 represent the constant in that equation,

$$V=\frac{\pi}{4}C^2C_1^{\frac{3}{2}}\frac{(HP)}{N}\quad\frac{1}{\sqrt{P}}, \qquad (30)$$

giving an expression similar to (28), but affected by the steam-pressure, and showing an economy in weight to be derivable from high pressure as well as piston-speed for this particular case—that is, when the stroke and diameter of the steam-cylinder are equal, and the diameter of the rod does not exceed for wrought iron $\frac{1}{12}$ and for steel $\frac{1}{8}$ of its length.

If we multiply the results of formula (28), (29) or (30) by $y=0.27$, and by the factor representing the ratio of the

* This statement is not accurately true, as the piston-rod must be somewhat longer than the stroke. Multiplying (27) by a factor of from $\frac{3}{2}$ to 2 would give an approximate result.

total length of the piston-rod to the stroke, we obtain its weight in pounds for *a uniform* pressure.

Referring to formula (15) or (20) for crushing or tearing, and to formula (17) or (22) for rupture by buckling, we see that the diameter of the piston-rod increases with the square root of the pressure, or its area increases directly as the pressure of the steam in the first case—*i. e.*, for crushing or tearing—and that the diameter of the piston-rod increases with the fourth root of the steam-pressure, or its area with the square root of the pressure, in the second case.

All of these formulæ alike show that a rapid increase in the boiler-pressure does not cause a correspondingly rapid increase in the diameter of the rod, and explain the success of empirical rules giving a constant ratio between the diameters of piston-rods and steam-cylinders regardless of the steam-pressure.

For the purpose of showing how little variation in the diameters of piston-rods results from great changes in the steam-pressure, the following short table of second and fourth roots of the usual steam-pressures is given:

TABLE II.

Number.	Square root.	Fourth root.
25	5.00	2.236
50	7.07	2.659
75	8.66	2.949
100	10.00	3.163
125	11.18	3.344

Thus we see that for a pressure increased 5 times formulæ (15) and (20) increase the diameter of the rod but a little over 2 times, and formulæ (17) and (22) increase the diameter of the rod but a little less than one-half.

Generally it will be found that the formula for crushing (15) will give the greatest diameter for a wrought-iron piston-rod, and that formula (22) will give the greatest diameter of a steel piston-rod.

(15.) Keys and Gibs.—Reserving keys for shafts for discussion in Article (40), we will consider that form of key used in making the connection between the piston-rod, piston-head and cross-head, and also for the connecting-rod.

If we wish to avoid weakening the piston-rod at the point where the key passes through it—a precaution not found to be practically necessary in most cases—we must increase the diameter of the rod at that point.

It is customary to thicken the rod where it enters the piston-head, but for convenience not to do so where the piston-rod enters the cross-head. If it is desired to thicken the rod at both ends, it is best to make it of a uniformly enlarged diameter from end to end.

Let d' = the diameter of the enlarged end of rod in inches.
" d_1 = the diameter of the piston-rod, as derived in Art. (12), (13) or (14), in inches.

The average dimensions of keys are assumed as follows:

h = the breadth of the key $= d'$.

t = the thickness of the key $= \frac{d'}{4}$.

Therefore, $2ht = \frac{d'^2}{2}$, which nearly equals the cross-section

of the enlarged end $= \frac{\pi d'^2}{4} - \frac{d'^2}{4} = 0.5354d'^2$.

Bearing in mind that there are two shearing-sections for every through-key, and assuming the shearing strength of wrought iron equal to its tensile or compressive strength, as is customary in practice.

We can place, if the enlarged end of the rod weakened by the key-way and the key are to be of equal strength,

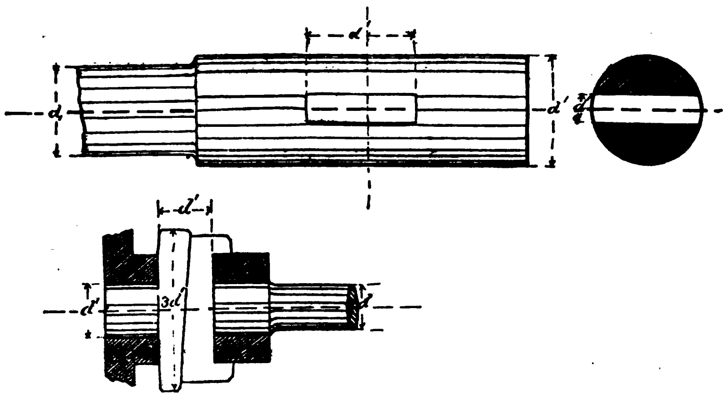
$$\frac{\pi d'^2}{4} - \frac{d'^2}{4} = \frac{\pi d_1^2}{4}.$$

Reducing, $2.1416d'^2 = 3.1416d_1^2$.

Therefore, $d' = 1.211d_1$. (31)

These various dimensions and measurements are shown in Fig. 3.

FIG. 3.



If one or two gibs are used in connection with the wedge-form of key, their mean area must be used in the calculations as for a single key. Various methods are used to prevent keys from falling out, as split pins, set screws, and bolts attached to one end of the key and held by a nut on the gib.

(16.) **Wrought-Iron Keys.**—The method given in the preceding paragraph, although the usual one in practice, is liable to cause some error in results.

Let F = the mean area of a key in square inches.
" K = the shearing strength (ultimate) of wrought iron per square inch = 50000 pounds.
" P_b = the boiler-pressure in pounds per square inch.
" d = the diameter of the steam-cylinder in inches.
" 10 = the factor of safety.

Recollecting that every through-key has two shearing surfaces, we have

$$2 \times 5000 \times F = \frac{\pi}{4} d^2 P_b.$$

$$\text{Therefore, } F = 0.000078 d^2 P_b. \qquad (32)$$

Or, if we assume the steam-pressure to be uniform throughout the stroke, and take the length of stroke *in inches*, we have

$$F = 39.6 \frac{(HP)}{LN}, \qquad (33)$$

and placing $2F = \frac{\pi d'^2}{4} - \frac{d'^2}{4}$,

we have $d' = 1.9\sqrt{F}$. (34)

Example.—Let P_b = the boiler-pressure = the pressure throughout the stroke = 40 pounds per square inch.
" d = 32 inches.
" L = 48 inches.
" N = 40 per minute.
" (HP) = 156, approx.

From formula (32) we have

$$F = 0.000078 \times 1024 \times 40 = 3.19 \text{ square inches.}$$

From formula (33) we have

$$F = 39.6\frac{156}{48 \times 40} = 3.21 \text{ square inches.}$$

Therefore, from formula (34), we have

$$d' = 1.9\sqrt{3.2} = 3.40 \text{ inches.}$$

Comparing $d' = 3.40$ inches with d_1 in the example at the end of Art. 12, we see that we are able to key the rod without thickening it at either end, if we suppose the safe shearing and tensile strength equal, and = 5000 pounds per square inch.

(17.) **Steel Keys.**—Adopting the same notation as in the preceding paragraph, we have only to change the value of K. The shearing strength of steel is equal to three-fourths of its tensile strength; therefore, $K = \frac{3}{4}\,90000 = 67500$ pounds per square inch area.

We have $$2 \times 6750 \times F = \frac{\pi}{4}d^2P_b.$$

$$\text{Therefore, } F = 0.000058d^2P_b. \qquad (35)$$

Or if we suppose the steam-pressure to be uniform throughout the length of the stroke, and take the stroke in inches,

$$F = 29.33\frac{(HP)}{LN}. \qquad (36)$$

Comparing formulæ (33) and (36), we have

$$\frac{\text{For steel, } F}{\text{For wrought iron, } F} = \frac{29.33}{39.6} = 0.74,$$

and therefore that the mean area of a steel key is 74 per cent. of the mean area of a wrought-iron key.

Bearing in mind that the shearing strength of steel is but three-fourths of its tensile strength, we can place Art. (15),

$$2\tfrac{3}{4}F = \frac{\pi d'^2}{4} - \frac{d'^2}{4} = \frac{2.1416}{4}d'^2.$$

$$\text{Therefore, } d' = 1.68\sqrt{F}. \qquad (37)$$

Example.—Let $P_b = 40$ pounds per square inch = uniform pressure.
" $d = 32$ inches.
" $L = 48$ inches.
" $N = 40$ per minute.
" $(HP) = 156$, approx.

We have, from formula (35),

$$F = 0.000058 \times 1024 \times 40 = 2.37 \text{ square inches.}$$

We have also, from formula (36),

$$F = 29.33\frac{156}{48 \times 40} = 2.37 \text{ square inches.}$$

From formula (37) we have

$$d' = 1.68\sqrt{2.37} = 1.73 \text{ inches.}$$

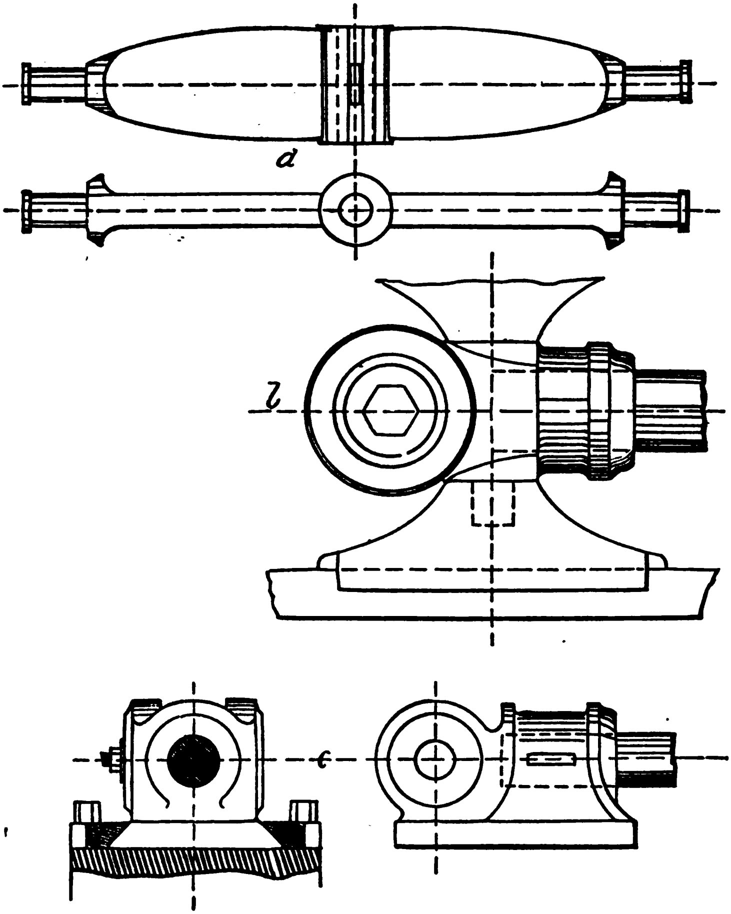
Comparing this latter result with the value of d_1 in the example Art. 13 = 3.47, we find that no enlargement of the ends of the rod is needed.

(18.) **The Cross-Head.**—The cross-head—or motion-block, as it is sometimes called—to one end of which the piston-rod is keyed, and extending laterally from which are the slides or slide, which by pressure upon the guides preserve the rectilinear motion of the end of the piston-rod after leaving the cylinder, and to the other end of which the connecting-rod is attached by means of a pin, usually of the same diameter as the crank-pin, though not necessarily so, has many different forms.

The proportions of the cross-head or motion-block are with a few exceptions rather a matter of experience and good taste than of calculation. Reuleaux, in *Der Con-*

structeur, page 546 *et seq.*, gives many very good examples of forms of cross-head. A few of the more common forms

FIG. 4.



are shown in Fig. 4, *a*, *b* and *c*, although the determination of dimensions, rather than the suggestion of ingenious arrangements, is the purpose of this work. Arthur Rigg, in *A Practical Treatise on the Steam-Engine*, suggests (Plate 41) one or two good forms of cross-head.

The material used in the construction of cross-heads is, according to circumstances, cast or wrought iron or steel.

Example *a* is a form of cross-head used for direct-acting pumping and blowing engines, and its dimensions can be calculated from the formula for a beam fixed at one end and loaded at the other, given in Table VI., the load being one-half the stress due to the steam-pressure upon the piston-head.

Example *b* is a common form for engines having but two guides.

Example *c*, sometimes called the slipper form, having but one guide, is well adapted to engines running in one direction only.

Many other forms of cross-head exist, and in fact almost every new construction demands special adaptation of the form of cross-head.

(19.) **Area of the Slides.**—When a horizontal engine "throws over" while the piston-head is moving toward the main shaft, all of the pressure and wear comes upon the lower slide. If the motion of the engine be reversed, all of the strain and wear comes upon the upper slide.

Since lubricants spread and flow over the lower guide more easily than the upper, and since it is easier to resist a strain in compression than tension by means of fastenings to the bed-plate, it is customary and proper to cause engines having motion in one direction only to throw over rather than under. The slipper-guide (see Fig. 4, *c*) can be used in this case with great propriety.

The greatest pressure upon a slide occurs when it is near the middle of its stroke (assumed at the middle for convenience), and can be deduced with little trouble from the triangle of forces.

Let S = the total steam-pressure on the piston-head in pounds $= \frac{\pi}{4} d^2 P_b$.

" S_1 = the pressure upon the guide in pounds.

" l = the length of the connecting-rod in inches.

" r = the length of the crank $= \frac{L}{2}$ in inches.

" $n = \frac{l}{r}$ = the rates of the length of the connecting-rod to the crank.

" P_b = the boiler-pressure in pounds per square inch.

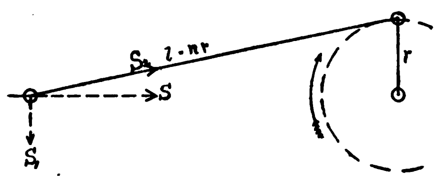
" d = the diameter of the steam-cylinder in inches.

" L = the length of stroke in inches.

" (HP) = the indicated horse power.

Referring to Fig. 5,

FIG. 5.



we have $S : S_1 :: \sqrt{n^2r^2 - r^2} : r$.

Therefore, $$S_1 = \frac{S}{\sqrt{n^2 - 1}}. \qquad (38)$$

The value of n commonly varies between 4 and 8. For $n = 4$, formula (38) becomes

$$S_1 = \frac{1}{\sqrt{16 - 1}} S = 0.2582 S.$$

For $n = 8$, it becomes

$$S_1 = \frac{1}{\sqrt{64-1}} S = 0.126S.$$

If in this we substitute the value $S = \frac{\pi}{4} d^2 P_b$, we have

$$S_1 = \frac{0.7854 d^2 P_b}{\sqrt{n^2 - 1}}. \qquad (39)$$

Or, supposing the pressure per square inch, P_b, to be uniform throughout the stroke, we have

$$S_1 = \frac{396000}{\sqrt{n^2-1}} \frac{(HP)}{LN}. \qquad (40)$$

We have, then, from either equation (39) or (40) the maximum pressure upon the slides.

One hundred and twenty-five pounds per square inch is as high a pressure per square inch as should be used, and the most modern English locomotive practice takes forty pounds per square inch as the proper pressure, the practical result being a great diminution in the wear upon both guides and slides.

Let A = the area of a slide in square inches.
" b = the pressure per square inch allowed.

Then will we have for the area of a slide,

$$A = \frac{S_1}{b}. \qquad (41)$$

Example.—Let $P_b = P = 40$ pounds per square inch.
" $d = 32$ inches.
" $N = 40$ per minute.
" $L = 48$ inches.
" $(HP) = 156$ horse-power.
" $b = 125$ pounds per square inch.
" $n = 5$.

Substituting in formula (40), we have

$$S_1 = \frac{396000}{\sqrt{24}} \quad \frac{156}{48 \times 40} = 6568 \text{ pounds.}$$

Substituting in formula (41), we have

$$A = \frac{6568}{125} = 52.5 \text{ square inches.}$$

CHAPTER IV.

(20.) Stress on and Dimensions of the Guides.*— The first requisite of a guide is that it shall be perfectly rigid under all circumstances. In many cases the guides are so attached to the bed-plate or frame-work of the engine as to require no calculation of their rigidity.

Cast iron is used for guides where they are firmly fastened throughout their length to a bed-plate or framing.

Under other circumstances wrought iron or steel is to be preferred as having greater moduli of elasticity, and consequent rigidity.

When it is considered necessary to calculate the dimensions of a guide, the following method will give a result which is safe:

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* Parallel motions, which take the place of guides in some engines, are discussed in Willis's *Principles of Mechanism*, pp. 350–363. *How to Draw a Straight Line*, by A. B. Kempe, is a particularly interesting little book, giving all the later discoveries in parallel motions which have followed the invention of Peaucellier's perfect parallel motion.

Let S_1 = the stress upon the guide [see formulæ (39), (40) Art. (19)] in lbs.
" l' = the length of guide in inches.
" W = the measure of the moment of flexure of the guide.
" E = the modulus of elasticity, { For wro't iron = 28000000 lbs. For steel = 30000000 "
" a = the deflection in inches.

We have (Wëisbach's *Mechanics of Engineering*, Sec. iv., Art. 217), for a beam supported at both ends and loaded in the middle,

$$a = \frac{1}{48} \frac{S_1 l'^3}{WE}. \tag{42}$$

Since perfect rigidity is unattainable, let us concede a deflection, $a = \frac{1}{100}$ of an inch, and formula (42) becomes for wrought iron,

$$W = \frac{S_1 l'^3}{13440000}. \tag{43}$$

Assuming a rectangular cross-section for the guide,

Let b = the breadth in inches.
" h = the depth in inches.
Then $W = \frac{bh^3}{12}$,

and formula (43) becomes

$$h^3 = \frac{12 S_1 l'^3}{13440000 b}. \tag{44}$$

Substituting the value of S_1, derived from equation (39), and extracting the cube root, (44) becomes

$$h = 0.00889 l' \sqrt[3]{\frac{d^2 P_b}{b\sqrt{n^2 - 1}}}, \tag{45}$$

and substituting the value of S_1 from equation (40),

$$h = 0.7071 l^2 \sqrt[3]{\frac{(HP)}{bLN\sqrt{n^2 - 1}}}. \qquad (46)$$

Example.—Let $l = 60$ inches.
" $d = 32$ inches.
" $L = 48$ inches.
" $P_b = P = 40$ lbs. per square inch.
" $b = 4$ inches.
" $(HP) = 156$ indicated horse-powers.
" $N = 40$ per minute.
" $n = 5$.

Using formula (45), we have

$$h = 0.00889 \times 60 \sqrt[3]{\frac{32^2 \times 40}{4\sqrt{25 - 1}}} = 6.82 \text{ inches.}$$

Using formula (46), we have

$$h = 0.7071 \times 60 \sqrt[3]{\frac{156}{4 \times 48 \times 40\sqrt{25 - 1}}} = 6.82 \text{ inches.}$$

(21.) **Distance between Guides.**—It is important to know at what angle of the crank the connecting-rod requires the greatest distance between the guides, if the plane of vibration of the connecting-rod intersects them.

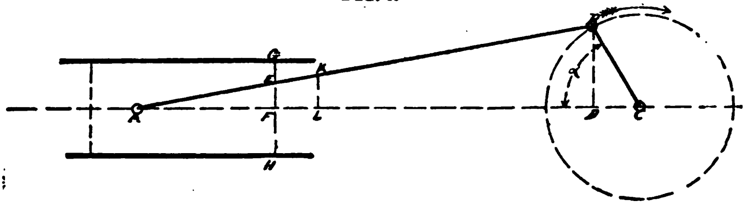
We can then determine the least possible distance between the guides, or that position of the connecting-rod which, clearing all parts of the engine, will make it impossible for it to touch with the crank at a different angle.

Solution.—(Approximate.)

Let r = radius of crank = CB.
" $l = nr$ = length of connecting-rod = AB.

We have $EF : BD :: [2r - r(1 - \cos a)] : \sqrt{l^2 - r^2 \sin^2 a}$.

FIG. 6.



Let $EF = x$, we have $BD = r \sin a$,

then $$x = \frac{r^2 \sin a\,(1+\cos a)}{r\sqrt{n^2 - \sin^2 a}},$$

and neglecting for the present $\sin^2 a$ in the denominator,

we have $$x = \frac{r}{n} \sin a\,(1+\cos a).$$

But $\sin a = 2 \sin \frac{a}{2} \cos \frac{a}{2}$ and $(1+\cos a) = 2 \cos^2 \frac{a}{2}$, which gives $x = \frac{4r}{n} \sin a \cos^3 \frac{a}{2}$. Differentiating, we have

$$\frac{dx}{da} = \frac{4r}{n}\left[-3 \sin^2 \frac{a}{2} \cos^2 \frac{a}{2} + \cos^4 \frac{a}{2}\right],$$

and placing this equal to 0 and dividing by $\cos^2 \frac{a}{2}$,

$$3 \sin^2 \frac{a}{2} = \cos^2 \frac{a}{2},$$

$$\text{or} \quad \tan^2 \frac{a}{2} = \tfrac{1}{3},$$

$$\tan \frac{a}{2} = \sqrt{\tfrac{1}{3}} = .578,$$

$$\frac{a}{2} = 30^\circ \text{ and } a = 60^\circ \text{ approximately.}$$

Showing that when the crank forms an angle of 60° with the centre line of the cylinder, we have the maximum distance from that centre line required to make the centre line of the connecting-rod clear the guides,

$$x = r\frac{1.5 \times 0.866}{\sqrt{n^2 - .75}} = \frac{1.3r}{\sqrt{n^2 - .75}} \quad \text{or} \quad \frac{1.3r}{n}.$$

To get the whole distance between the guides we multiply by 2, giving $GH = 2.6\frac{r}{n}$.

To this value must be added the thickness of the connecting-rod. For any point, as K, on the connecting-rod, we have, knowing the angle $a = 60°$, the proportion,

$$AL : AD :: KL : BD,$$

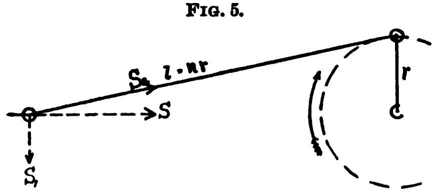
$$KL = \frac{AL \times BD}{AD},$$

to which we must add the half thickness of the connecting-rod and multiply by 2 to get the whole distance between the guides.

It must be borne in mind, when arranging the guides, that room must be made for the introduction of the key into the stub end of the connecting-rod, if that end cannot be slid outside of the guides at one end of the stroke.

(22.) **The Connecting-Rod.**—The connecting-rod of a steam-engine is usually made from 4 to 8 times the length of the crank—that is, 2 to 4 times the length of the stroke of the steam-cylinder.

FIG. 5.



Referring to Fig. 5, we see that when the crank is at right angles to the centre line of the piston-rod, the strain upon the connecting-rod is a maximum.

Let S = the total steam-pressure upon the piston-head in pounds.
" S_2 = the stress upon the connecting-rod in pounds.
" l = the length of the connecting-rod in inches.
" r = the radius of the crank.
" $n = \frac{l}{r}$ = the ratio of the length of the connecting-rod to the crank.

We have $S : S_2 :: \sqrt{n^2r^2 - r^2} : nr.$

Therefore, $$S_2 = S\frac{n}{\sqrt{n^2-1}}. \qquad (47)$$

If in (47) we let $n = 4$, we have

$$S_2 = S\frac{4}{\sqrt{16-1}} = 1.0328S.$$

If we let $n = 8$, we have

$$S_2 = S\frac{8}{\sqrt{64-1}} = 1.0008S.$$

These numerical results show the rapidity with which the value of the maximum stress on the connecting-rod approaches the constant stress upon the piston-rod as the ratio of the connecting-rod to the crank is increased, and further by what a small percentage = .03 the stress upon the connecting-rod is greater in the extreme case for the value of $n = 4$.

A relatively short connecting-rod is productive of economy of material, and the increased pressure upon the sides can be provided for by increased area. (See Art. 19.)

The connecting-rod, being free to turn about its pins at either end, must be regarded as a solid column not fixed at either end, but neither end free to move sideways.

To determine the point at which a column of this character has an equal tendency to rupture by crushing or buckling, we place the formulæ for crushing and buckling equal to each other (Weisbach's *Mechanics of Engineering*, sec. iv., art. 266), and letting d_2 = the diameter of the connecting-rod,

$$S_2 = FK = \frac{\pi^2}{l^2} WE,$$

or, substituting the values of F and W,

$$\frac{\pi d_2^{\,2}}{4} K = \frac{\pi^2}{l^2} \quad \frac{\pi d_2^{\,4}}{64} E,$$

which gives $\dfrac{l}{d_2} = .7854\sqrt{\dfrac{E}{K}}$.

$$\text{For wrought iron,} \quad \frac{l}{d^2} = .7854\sqrt{\frac{28000000}{31000}} = 23\tfrac{1}{2}, \quad (48)$$

$$\text{For steel,} \quad \frac{l}{d_2} = .7854\sqrt{\frac{42000000}{100000}} = 16, \quad (49)$$

and we see that a *wrought-iron column* will rupture by crushing when its diameter is greater than $\frac{1}{24}$ of its length, and will rupture by buckling when its diameter is less than $\frac{1}{24}$ of its length, and that a *steel column* will rupture by crushing when its diameter is greater than $\frac{1}{16}$ of its length, and will rupture by buckling when its diameter is less than $\frac{1}{16}$ of its length.

(23.) **Wrought-Iron Connecting-Rod.**—Let F = the cross-section of the rod in square inches.

Let K = the ultimate crushing strength of wrought iron per square inch.

Referring to formula (47), we have for crushing,

$$S_2 = \frac{n}{\sqrt{n^2-1}} S = FK. \qquad (50)$$

Let P_b = the boiler-pressure in pounds per square inch.
" d_2 = the diameter of the connecting-rod in inches.
" d = the diameter of the steam-cylinder in inches.
" the factor of safety be 10, as before.

Then formula (50) becomes

$$10P_b\frac{\pi d^2}{4} = \sqrt{\frac{n^2-1}{n^2}} \quad \frac{\pi d_2^2}{4} 31000.$$

$$\text{Therefore,} \quad d_2 = 0.0179d\sqrt{P_b}\sqrt[4]{\frac{n^2}{n^2-1}}. \qquad (51)$$

In this formula for $n = 4$ we have

$$\sqrt[4]{\frac{n^2}{n^2-1}} \qquad \sqrt[4]{\frac{16}{15}} = 1.016,$$

and as the value of n increases this quantity becomes more and more nearly equal to unity, and can therefore be neglected in all cases in which $n = 4$ or a greater number.

Formula (51) then becomes

$$d_2 = 0.0179d\sqrt{P_b}. \qquad (52)$$

Let (HP) = the indicated horse-power.
" L = the length of the stroke in inches.
" N = the number of strokes per minute.

Formula (52) becomes, in terms of the horse-power,

$$d_2 = 12.753\sqrt{\frac{(HP)}{LN}}. \qquad (53)$$

The formula for rupture by buckling for long solid columns, not fixed at either end, is Weisbach's *Mechanics of Engineering*, sec. iv., art. 266.

Letting W = the measure of the moment of flexure,
" E = the modulus of elasticity = 28000000 pounds per square inch,
" l = the length of the rod in inches $= nr = n\frac{L}{2}$,

$$S_2 = \left(\frac{\pi}{l}\right)^2 WE. \qquad (54)$$

Substituting in this the values of $S_2 = \frac{n}{\sqrt{n^2-1}}S$, $W = \frac{\pi d_2^4}{64}$ and E, we have, with a factor of safety, 10,

$$\left(\frac{n}{\sqrt{n^2-1}}\right)\left(10P_b\frac{\pi d^2}{4}\right) = \frac{\pi^2}{l^2}\;\frac{\pi d_2^4}{64}\,28000000.$$

Reducing and neglecting the term $\sqrt[8]{\frac{n^2}{n^2-1}}$, we have

$$d_2 = 0.02758\sqrt[4]{d^2l^2P_b}\text{ inches.} \qquad (55)$$

If in this we substitute the value of $l = n\frac{L}{2}$, we have

$$d_2 = 0.0195\sqrt[4]{n^2d^2L^2P_b}\text{ inches.} \qquad (56)$$

If $L = d$, (56) becomes

$$d_2 = 0.0195d\sqrt[4]{n^2P_b}\text{ inches.} \qquad (57)$$

If we suppose the pressure of the steam to be uniform throughout the stroke, formula (56) becomes, in terms of the horse-power,

$$d_2 = 0.5196\sqrt[4]{\frac{n^2(HP)L}{N}}\text{ inches.} \qquad (58)$$

The following rule may be given for the determination of the diameters of round wrought-iron connecting-rods:

Deduce the diameter of the connecting-rod by the use of formula (51) *or* (53), *as may be most convenient. Should this diameter be less than* $\frac{1}{24}$ *of the length of the rod, then use formula* (55), (56), (57) *or* (58), *as may be most convenient.*

Example.—Let $L = 48$ inches.
" $d = 32$ inches.
" $n = 5$.
" $P_b = P = 40$ pounds per square inch.
" $N = 40$ per minute.
" $(HP) = 156$ approximately.

Substituting in formula (52), we have

$$d_2 = 0.0179 \times 32\sqrt{40} = 3.62 \text{ inches},$$

or substituting in formula (53), we have

$$d_2 = 12.753\sqrt{\frac{156}{48 \times 40}} = 3.63 \text{ inches}.$$

Since the length of the connecting-rod $= \frac{48}{2} \times 5 = 120$ inches, we see that its diameter is less than $\frac{1}{24}$ of its length, and that we must use formula (56) or (58).

Substituting in (56), we have

$$d_2 = 0.0195\sqrt[4]{25 \times 1024 \times 2304 \times 40} = 4.30 \text{ inches}.$$

Substituting in formula (58), we have

$$d_2 = 0.5196\sqrt[4]{\frac{25 \times 156 \times 48}{40}} = 4.31 \text{ inches}.$$

CHAPTER V.

(24.) Steel Connecting-Rod.—The same course of reasoning as that followed in Art. (13) gives us, for the proper diameter of a steel piston-rod to resist safely a strain in tension,

$$d_2 = 0.0105 d\sqrt{P_b} \text{ inches,} \qquad (59)$$

which is the same as formula (20), or

$$d_2 = 7.481\sqrt{\frac{(HP)}{LN}} \text{ inches,} \qquad (60)$$

which is the same as formula (21) for a constant pressure of the steam throughout the length of the stroke.

Considering next the strength of a connecting-rod to resist rupture by buckling, and letting $E = 42000000$ pounds per square inch for steel in formula (54), substituting in a similar manner to that in Art. (23) and reducing, we have

$$\left(\frac{n}{\sqrt{n^2-1}}\right)\left(10P_b\frac{\pi d^2}{4}\right) = \frac{\pi^2}{l^2}\frac{\pi d_2^4}{64}42000000.$$

Therefore, neglecting the term $\sqrt[8]{\frac{n^2}{n^2-1}}$,

$$d_2 = 0.02492\sqrt[4]{d^2l^2P_b}. \qquad (61)$$

Substituting $l = nr = n\frac{L}{2}$, we have

$$d_2 = 0.0176\sqrt[4]{n^2L^2d^2P_b}. \qquad (62)$$

If in (62) we make

$n = 4$, or $l = 2L$, we have $d_2 = 0.0352\sqrt[4]{d^2L^2P_b}$; (63)

$n = 5$, or $l = 2\frac{1}{2}L$, " $d_2 = 0.0394\sqrt[4]{d^2L^2P_b}$; (64)

$n = 6$, or $l = 3L$, " $d_2 = 0.0432\sqrt[4]{d^2L^2P_b}$; (65)

$n = 7$, or $l = 3\frac{1}{2}L$, " $d_2 = 0.0466\sqrt[4]{d^2L^2P_b}$; (66)

$n = 8$, or $l = 4L$, " $d_2 = 0.0496\sqrt[4]{d^2L^2P_b}$. (67)

It must be remembered that the term $\sqrt[8]{\frac{n^2}{\sqrt{n^2-1}}}$ is neglected in these formulæ, and that it should be taken into consideration when n is less than 4.

If in the above formulæ we let $L = d$, the radical reduces to the form $d\sqrt[4]{n^2 P_b}$.

If we consider the steam-pressure uniform throughout the stroke, and substitute for $P_b L d^2$ in (62) its value in terms of the horse-power, we have

$$d^2 = 0.469\sqrt[4]{\frac{n^2(HP)L}{N}}. \qquad (68)$$

From their experiments upon steel, Kupffer and Styffe have arrived at the conclusion that the percentage of carbon has no effect upon the modulus of elasticity of steel, and give as its value $E = 30000000$ per square inch.

In the present discussion we have assumed the value of $E = 42000000$, as given by Reuleaux, and the ultimate strength to resist crushing $K = 100000$ pounds per square inch—a value far less than would be considered safe from the experiments of W. Fairbairn on the mechanical properties of steel. (*Report of the British Association for 1867*, or Stoney's *Theory of Strains*, vol. ii., page 480.)

It is more than probable, from a consideration of these facts, that the ratio of l to d_2 should be much less than 16, and require the use in all cases of formulæ (61) to (68), inclusive, for the determination of the diameter of steel connecting-rods.

Comparing formula (22) for rupture by buckling of steel piston-rods with formulæ (63) to (67), inclusive, for rupture of steel connecting-rod, we find that when

(69) $n=4$, the diameter of the connecting-rod $=1.00$ times the diameter of the piston-rod;

(70) $n=5$, the diameter of the connecting-rod $=1.12$ times the diameter of the piston-rod;

(71) $n=6$, the diameter of the connecting-rod $=1.28$ times the diameter of the piston-rod;

(72) $n=7$, the diameter of the connecting-rod $=1.30$ times the diameter of the piston-rod;

(73) $n=8$, the diameter of the connecting-rod $=1.44$ times the diameter of the piston-rod.

If the length of the connecting-rod be taken = twice the stroke, the diameters of piston- and connecting-rod are equal.

The following rule may be given for the determination of the diameters of round steel connecting-rods:

Deduce the diameter of the connecting-rod from formula (59) or (60), as may be most convenient. If the resulting diameter be less than one-sixteenth of the length of the rod, then use some one of the formulæ (61) to (73), inclusive.

Example.—Let $L=48$ inches.
" $d=32$ inches.
" $n=5$ inches.
" $P_b=P=40$ pounds per square inch.
" $N=40$ per minute.
" $(HP)=156$.

Substituting these values in formula (59), we have

$$d_2=0.0105\times 32\sqrt{40}=2.12 \text{ inches},$$

or, substituting in formula (60), we have

$$d_2=7.481\sqrt{\frac{156}{48\times 40}}=2.13 \text{ inches}.$$

$\frac{1}{16}$ of $24\times 5=120=7\frac{1}{2}$ inches, and we see that we must use one formula, (61) to (73).

Substituting in formula (64), we have for $n=5$

$$d_2 = 0.0394 \sqrt[4]{1024 \times 2304 \times 40} = 3.88 \text{ inches};$$

or, substituting in formula (68), we have

$$d_2 = 0.469 \sqrt[4]{\frac{25 \times 156 \times 48}{40}} = 3.89 \text{ inches.}$$

Referring to example for Art. 13, we find the diameter of a steel piston-rod to be $d_1 = 3.47$ inches, and substituting in formula (70), we have

$$d_2 = 1.12 d_1 = 3.47 \times 1.12 = 3.89 \text{ inches.}$$

(25.) General Remarks concerning Connecting-Rods.—The discussion of Art. (14) will apply with little modification to connecting-rods.*

If we take formula (27), as before, for the volume of a connecting-rod, we have

$$V_1 = \left(\frac{\pi d_2^2}{4}\right)\left(\frac{nL}{2}\right). \qquad (74)$$

Substituting in this the value of d_2 from equation (68), we have, letting C = constant,

$$V_1 = C^2 \frac{\pi}{4} \sqrt{\frac{n^2 (HP) L}{N}} \; \frac{nL}{2}. \qquad (75)$$

$$\text{Reducing} \quad V_1 = C_1 n^2 \sqrt{\frac{(HP) L^3}{N}}. \qquad (76)$$

Equation (76) shows that the volume of the connecting-rod is affected by the varying conditions in a similar manner to a piston-rod, excepting that the square of the ratio n enters in and affects the volume of the rod.

* It is customary to make round connecting-rods with a taper of about one-eighth of an inch per foot from the centre to the necks, which should be of the calculated diameter. Experiment does not show an increased strength from a tapering form.

Thus, if we first take $n=4$, and then $n=8$, we see that the volume in the first case is but $\frac{1}{4}$ of the volume in the latter case.

The increase of the area of the slides to provide for shorter connecting-rods is quite slow (see Art. 19). It is customary to make connecting and parallel rods of a rectangular cross-section in locomotive practice. When this is done it will be safe to make the smaller of the rectangular dimensions equal to the diameter of a round rod suitable to withstand the strains to which it will be subjected. The resistance of a long rectangular column to rupture varies as the cube of its smallest dimension of cross-section. (See Weisbach's *Mechanics of Engineering*, sec. iv., art. 266.)

(26.) **Connecting-Rod Straps.**—By means of gibs and keys the straps draw the brasses solidly against the stub-end of the connecting-rod.

If necessary the uniform cross-section of the straps can be preserved by thickening them at the point where they are slotted to receive the gib and key. (See Art. 15.)

Fig. 7 *a* shows the usual form of strap for locomotives, and Fig. 7 *b* a solid stub-end in which the keys are used only to set the brasses without moving the strap. Fig. 7 *c* represents the ordinary form of strap in which the brasses and strap are held by means of the gib and key.

(27.) **Wrought-Iron Strap.**—

Let F_1 = the area of one leg of the strap in square inches.
" the safe strain per square inch = 5000 pounds.
" d = the diameter of the steam-cylinder in inches.
" P_b = the pressure per square inch of the steam.

We have $2 \times 5000 F_1 = 0.7854 P_b d^2$;

therefore $F_1 = 0.000078 P_b d^2$ square inches. (77)

or if we assume the pressure to be uniform, and

Let L = the length of the stroke in inches,
" N = the number of strokes per minute,
" (HP) = the horse-power (indicated),

$$F_1 = 39.6 \frac{(HP)}{LN} \text{ square inches.} \qquad (78)$$

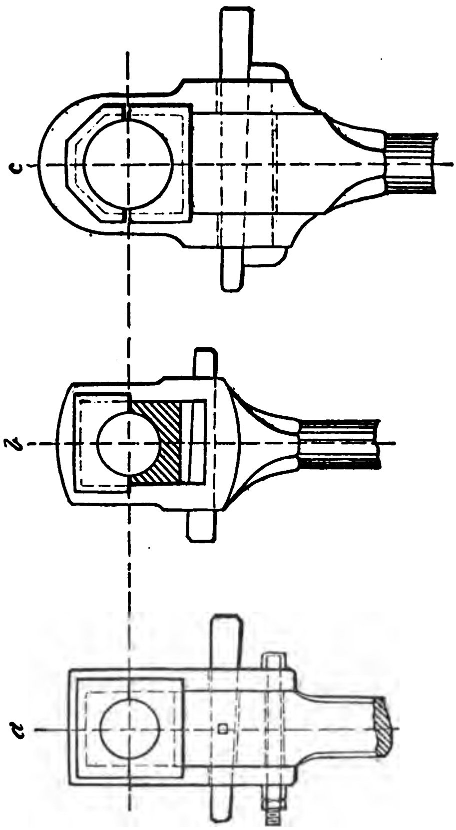


Fig. 7.

Example.—Let $d = 32$ inches.
" $L = 48$ inches.
" $N = 40$ per minute.
" $P_b = P = 40$ pounds per square inch.
" $(HP) = 156$ indicated horse-powers.

Substituting in formula (77), we have

$$F_1 = 0.000078 \times 40 \times 1024 = 3.19 \text{ square inches,}$$

or substituting in formula (78), we have

$$F_1 = 39.6 \frac{156}{48 \times 40} = 3.21 \text{ square inches.}$$

(28.) **Steel Strap.**—Let 9000 pounds per square inch equal the safe working-strain in tension.

Let F_1 = the area of one leg of the strap in sq. inches.
" d = the diameter of the steam-cylinder in inches.
" P_b = the steam-pressure per square inch.
" L = the length of stroke in inches.
" N = the number of strokes per minute.
" (HP) = the indicated horse-power.

We have, as in the preceding article,

$$2 \times 9000 F_1 = 0.7854 d^2 P_b,$$

therefore
$$F_1 = 0.0000437 d^2 P_b \text{ square inches,} \qquad (79)$$

or assuming the stream-pressure to be uniform throughout the stroke, and substituting for $d^2 P_b$ its value in terms of the horse-power,

$$F_1 = 22.034 \frac{(HP)}{LN} \text{ square inches.} \qquad (80)$$

Dividing formula (80) by formula (78), we find that the cross-section of a steel strap is but $\frac{5}{9}$ of the cross-section of a wrought-iron strap of equal strength.

Example.—Let the data be the same as in the example appended to the preceding article.

Substituting in formula (79), we have

$$F_1 = 0.0000437 \times 1024 \times 40 = 1.77 \text{ square inches.}$$

Substituting in formula (80), we have

$$F_1 = 22.034 \frac{156}{48 \times 40} = 1.78 \text{ square inches.}$$

CHAPTER VI.

(29.) **The Crank-Pin and Boxes.**—The crank-pin has ever been one of the most troublesome parts of the steam-engine to the mechanical engineer. The mere determination of its proportions, so that it will not break under the strain put upon it by the pressure of the steam upon the piston-head, does not suffice, and often results in trouble from heating when the engine is at work. It therefore becomes a first consideration to so proportion crank-pins as to prevent heating, their strength being a matter of secondary importance, to be afterward investigated if it is deemed necessary to do it.

Before taking up the mathematical part of our consideration, it will be of practical value to quote, from the writings of General Morin, the following remarks:

"But it is proper to observe that from the form itself of the rubbing body (cylindrical) the pressure is exerted upon a less extent of surface according to the smallness of the diameter of the journal, and that unguents are more easily expelled with small than with large journals. This circumstance has a great influence upon the intensity of friction, and upon the value of its ratio to the pressure.

"The motion of rotation tends of itself to expel certain unguents and to bring the surfaces to a simply unctuous state. The old mode of greasing, still used in many cases, consisted simply in turning on the oil or spreading the lard or tallow upon the surface of the rubbing, and in renewing the operation several times in a day.

"We may thus, with care, prevent the rapid wear of journals and their boxes; but with an imperfect renewal of the unguent, the friction may attain .07, .08, or even .10, of the pressure.

"If, on the other hand, we use contrivances which renew the unguent, without cessation, in sufficient quantities, the rubbing surfaces are maintained in a perfect and constant state of lubrication, and the friction falls as low as .05 or .03 of the pressure, and probably still lower.

"The polished surfaces operated in these favorable conditions became more and more perfect, and it is not surprising that the friction should fall far below the limits above indicated." (Bennett's *Morin*, pp. 307, 308.)

If the unguents are expelled by extreme pressure, so that the surfaces are simply unctuous, the friction increases rapidly, and the surfaces begin to heat and wear immediately.

These statements apply with equal force to cast iron and cast iron; cast iron and wrought iron; cast iron and brass or Babbitt's metal; or with steel or wrought iron in the place of cast iron.

The supposed superiority of brass or Babbitt's metal lined boxes over iron boxes in positions very liable to heating lies in their greater softness and conductivity for heat. Brass will conduct heat away from two to four times as rapidly as iron. However, the film of unguent interposed may render the conductivity of brass of less avail than is generally supposed, and the advantage lies only in the fact that, being a softer metal, in case of heating, the surface of the softer metal receives the principal damage. Phosphor bronze, which is a patented alloy, being nothing more than brass or gun-metal in which the formation of oxide has been prevented by the introduction of phosphorus, is coming into general use for positions in which the wear is very great.

It is, perhaps, good practice to use brass or soft metal wherever the pressure exceeds 125 pounds per square inch of projected area. At lower pressures a good lubricating oil may be relied upon to form a film and run without breaking at ordinary speeds. (The continuity of the film of lubricant is affected by so many different conditions that it is impossible to fix any exact limit of pressure.)

With soft metal or brass bearings good results can be obtained at pressures of 1000 pounds or more per square inch of projected area. (See *Hand-Book of the Steam-Engine*, Bourne, page 183, where 1400 pounds per square inch is given as the greatest pressure per square inch of projected area allowable on crank-pins. Arthur Rigg, *A Practical Treatise on the Steam-Engine*, page 147.) If, however, the film of unguent does break at these higher pressures, heating begins almost instantly; and if the surfaces in contact are both of hard metal, as iron and iron, injury to both at once results, while, if the boxes are brass or some of the softer metals, the continuity of the surface film may be restored by increased lubrication or by stopping and cooling as soon as heating is observed.

Several expedients are used to keep bearings which have a tendency to heat cool until they have worn smooth.

The introduction of rotten stone or sulphur with oil is perhaps the best. Quicksilver or lead-filings, introduced with oil, coat the rubbing surfaces and diminish the heating where the rubbing surfaces are very much scored. (See *The Working Engineer's Practical Guide*, pages 48 and 49, Joseph Hopkinson.)

A great increase of the velocity of the rubbing surfaces renders bearings more liable to heat than a great increase in pressure, although the total amount of work done by friction is the same in both cases, and is probably accounted for by the more rapid expulsion of the lubricant.

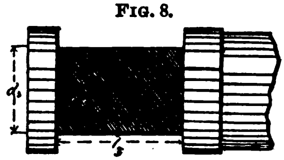
As the cause of the heating of bearings, when they are of tolerably good workmanship, is the transformation of the work of friction into heat, we see that it is necessary to reduce the friction as much as possible by the perfect smoothness of surfaces in contact, the interposition of lubricants, and the reduction of the speed and pressure upon the rubbing surfaces.

In all machines there is a limit below which we cannot reduce the speed and pressure of the rubbing surfaces, and we must, therefore, so proportion journal-bearings as to cause no more work due to friction—*i. e.*, heat—to be produced than can be conveyed away by the unguents, the atmosphere and the conductivity of the metals without raising the temperature of the bearing appreciably.

From the statistics of the working of the crank-pins of four screw propellers in the United States Navy (Van Buren, *Strength of Iron Parts of Steam Machinery*, page 24) we take the following statement and table:

"The crank-pins of these vessels worked cool, giving but little trouble, which is the exception rather than the rule for screw-engines."

FIG. 8.



The projected area of crank-pin journal, given in column 7, is that rectangular area formed by a central section of the crank-pin journal in the direction of its length. Shown cross-hatched in Fig. 8.

Columns 1, 2, 4, 5 and 6 are given. Columns 3, 7, 8, 9, 10 and 11 are calculated from them.

In calculating column 9 from columns 3 and 8 we have assumed the coefficient of friction at .05, which is the highest value given by General Morin for constant lubrication, and probably greater in the present cases.*

Column 11 is derived from columns 3 and 7. Column 10 is derived from columns 7 and 9.

TABLE III.

NAME OF VESSEL.	1. Diameter of steam-cylinder.	2. Pressure per square inch.	3. Total force of steam pressure on piston-head.	4. Number of strokes per minute.	5. Length of crank-pin.	6. Diameter of crank-pin.	7. Projected area of crank-pin.	8. Velocity of rubbing surfaces in feet per min.	9. Total work of friction in foot-pounds per min.	10. Work of friction per sq. inch of projected area in foot-pounds per min.	11. Pressure per sq. inch of projected area in lbs.
	in.	lbs.	lbs.		in.	in.	sq. in.				
Swatara........	36	40	40716	160	12	8.5	102.	178.	362372	3552.6	399.5
Saco..............	30	40	28274	180	9	7.5	67.5	176.7	249801	3700.7	419.
Wampanoag...	100	40	314160	62	27	16.	432.	129.8	2038898	4719.7	727.2
Wabash.........	72	28	114002	100	16	15.	240.	196.3	1118910	4662.1	475.
Averages........										4159.	505.

From column 10 of the table we find the average amount of work per square inch of projected area of crank-pin journal, which, in the cases cited, has been borne without heating, to be 4159 foot-pounds per minute; and in making use of this quantity in our subsequent calculations, we are on the safe side if the coefficient of friction (assumed at .05) has not been taken too small.

(30.) **The Length of Crank-Pins.**—Let d = the diameter of the piston-head in inches, P = the mean pressure in the steam-cylinder in pounds per square inch.

* It is probable that the coefficient of friction, for crank-pins of marine propellor-engines under ordinary conditions, is 9 or 10 times greater than the assumed 0.05.

Then $7854d^2P$ = the mean pressure on the piston-head in pounds.

Let f = the coefficient of friction.

" l_3 = the length of the crank-pin journal in inches.

" d_3 = the diameter of the crank-pin journal in inches.

The mean *force* of friction at the rubbing surfaces of any crank-pin journal per square inch of projected area is

$$= .7854 f \frac{Pd^2}{l_3 d_3}.$$

Let N = the number of strokes per minute (equal twice the number of revolutions).

" w = the work of friction per minute.

The space passed over by the force due to friction in one minute

$$= \frac{\pi}{2} N d_3 = 1.5708\ N d_3 \text{ inches,}$$

and we have for the work of friction—*i. e.*, heat—per minute,

$$w = 1.5708 \times .7854 f \frac{PNd^2}{l_3} \text{ inch-pounds.}$$

From this formula the diameter of the crank-pin journal (d_3) has vanished. Why it has vanished will be understood when we observe that the force per square inch of projected area due to friction is inversely as the diameter of the journal; while the space passed over by this force is directly as its diameter.

Replacing w in the last formula by the mean value derived from column 10 of the table, equal 4159 foot-pounds equal 49908 inch-pounds, we have

$$49908 = 1.2337 f \frac{PNd^2}{l_3}.$$

Therefore, $$l_3 = .0000247 f PNd^2 = 12.454 f \frac{(HP)}{L}. \quad (81)$$

Considering formula (81), we see that the length of the crank-pin increases and decreases with the coefficient of friction, the mean steam-pressure per square inch the number of strokes per minute, and with the square of the diameter of the steam-cylinder.

A consideration of the component formulæ of (81) shows that as the crank-pin journal decreases in size the pressure per square inch becomes greater; but if this reduction in size is obtained by a diminution of the diameter (d_3) of the crank-pin journal, the work per square inch of projected area is not increased, for the velocity of the rubbing surfaces by this means is decreased in the same ratio as the pressure is increased.

Within reasonable limits as to pressure and speed of rubbing surfaces, the general law may be enunciated:

The longer any bearing which has a given number of revolutions and a given pressure to sustain is made, the cooler it will work, and its diameter has no effect upon its heating.

Example.—Let $d = 30''$, $N = 180$, and $P = 40$ pounds per square inch. We have, by substitution, in formula (81),

$$l_3 = .0000247 \times f \times 40 \times 180 \times 900 = 160.f.$$

If in this we take $f = .03$ to $.05$ for perfect lubrication,

we have $l_3 = 4.8''$ to $8''$.

If we take $f = .08$ to $.10$ for imperfect lubrication,

we have $l_3 = 12.8''$ to $16''$.

The results show the great advantages arising from constant oiling of bearings and smoothness of surfaces.

NOTE.—If 0.05 be taken as the coefficient of the force of friction, we obtain the average length of the crank-pins quoted in Table III., Art. (29). About one-quarter of the length required for propellor crank-pins will serve for the pins of side-wheel engines with good results, and one-tenth for locomotive or stationary engines.

(31.) Locomotive Crank-Pins, Length and Diameter.—If for locomotive crank-pin journals we assume N, the number of strokes per minute = 600.

P the pressure per square inch in pounds = 150, the formula (81) being changed to

$$l_3 = .00000247 f P N d^2,$$

by removing the decimal point one place to the left, will give the length of journal commonly assumed in successful practice, if we assume the coefficient of friction at .06.

The above formula then becomes

$$l_3 = .013 d^2. \qquad (82)$$

This formula would prove the amount of heat per square inch of projected area conveyed away from the crank-pins of locomotives to be ten times greater than in the case of marine engines, did not the variations of speed and frequent stoppages of a locomotive prevent comparison.

Example.—Let $d = 18$ inches. We have

$$l_3 = .013 \times 324 = 4.21 \text{ inches}.$$

The diameters of locomotive crank-pins are usually taken equal to their length.

CHAPTER VII.

(32.) Diameter of a Wrought-Iron Crank-Pin for a Single Crank.*—It is necessary, first, to determine its length by formula (81), and with this length to determine the proper diameter.

Let a = the deflection of pin under stress in inches.
" S = the stress on pin in pounds.
" E = the modulus of elasticity of wrought iron = 28000000 pounds.
" W = the measure of the moment of flexure of the pin.
" l_2 = the length of journal in inches.

The deflection of a beam fixed at one end and loaded at the other (Weisbach's *Mechanics of Engineering*, sec. iv., art. 217) is

$$a_1 = \frac{Sl^3}{3WE}.$$

If, again, the beam be supposed to be uniformly loaded and fixed at one end, its deflection will be (Weisbach's *Mechanics of Engineering*, sec. iv., art. 223)

$$a_2 = \frac{1}{8}\frac{Sl^3}{WE};$$

and if for the load at the end we concede a deflection of $\frac{1}{100}$ of an inch, we have for the same load under the two cases above mentioned

$$a_1 = .01 \text{ inch},$$
$$a_2 = .0038 \text{ inch}.$$

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* In Arts. 54 and 55 will be found a discussion of the stresses on crank-pins for double and triple cranks.

Then, taking the most unfavorable case—*i. e.*, the load at the end—we have,

letting $$S = \frac{\pi d^2}{4} P_b,$$

letting $P_b =$ the greatest pressure of steam in cylinder equal the boiler-pressure,

letting $$W = \frac{\pi d_3^4}{64},$$

$$\frac{1}{100} = \frac{1}{3} \frac{\frac{\pi d^2}{4} P_b l_3^3}{\frac{\pi d_3^4}{64} 28000000} = \frac{16 P_b l_3^3 d^2}{84000000 d_3^4},$$

$$d_3^4 = \frac{1600}{84000000} P_b l_3^3 d^2, \qquad (83)$$

$$d_3 = .066 \sqrt[4]{P_b l_3^3 d^2} = 1.758 \sqrt[4]{\frac{(HP)}{LN} l_3^3}, \qquad (84)$$

for a constant steam-pressure.

Example.—Let $P_b = 60$ pounds per square inch.
" $l_3 = 8$ inches.
" $d = 30$ inches.

Substituting in formula (84), we have

$$d_3 = .066 \sqrt[4]{60 \times 512 \times 900} = 4.79 \text{ inches.}$$

There is no need of an investigation of the strength of a crank-pin, as the condition of rigidity gives a great excess of strength.

(33.) **Steel Crank-Pins.**—The length of a steel crank-pin is just the same as that of a wrought-iron pin, and the modulus of elasticity of steel is so nearly equal to that of wrought iron as to make formula (84) serviceable for both steel and wrought iron alike.

The advantages of steel crank-pins over wrought iron are their greater strength, and the possibility of obtaining a much smoother surface because of the homogeneous structure of steel. Their disadvantage is their liability to sudden fracture when not working truly for any reason, as inaccuracy of workmanship or wrenching as in a marine-engine.

(34.) Diameters of Crank-Pins from a Consideration of the Pressure upon them.—Referring to the table, we find the average pressure per square inch of projected area to be about 500 pounds.

If now we divide the whole pressure upon the crank-pin by 500, we obtain the projected area required when this limit is not to be exceeded.

The equation $d_3 l_3 = \frac{\pi d^2 P}{4 \times 500}$ gives $d_3 = .00157 \frac{d^2 P}{l_3}$. (85)

Example.—Let $P = 40$ pounds, $l_3 = 8''$, $d = 30''$,

$$d_3 = .00157 \frac{900 \times 40}{8} = 7.06 \text{ inches.}$$

This latter method is perhaps the most practical, and has the advantage of limiting the pressure. It will almost always give larger results than the preceding method.

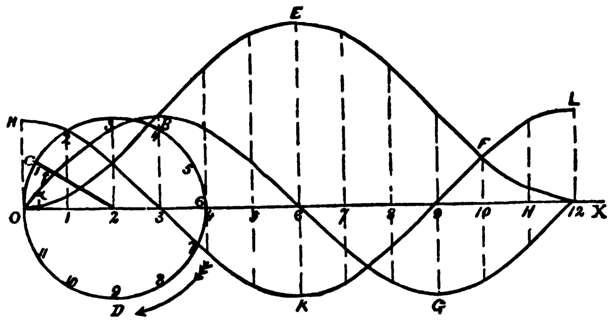
If we assume the pressure to be uniform throughout, the stroke (85) becomes, in terms of the horse-power,

$$d_3 = 791.6 \frac{(HP)}{l_3 LN}. \qquad (86)$$

(35.) Of the Action of the Weight and Velocity of the Reciprocating Parts.—By the reciprocating parts we mean the piston-head, piston-rod, cross-head or motion-block, and the connecting-rod; also, in a vertical engine, the working-beam if one is used.

We are obliged to neglect the action of friction from the impossibility of determining it, and we will also at first neglect the influence of gravity and the angular position of the connecting-rod—*i. e.*, suppose it to be of infinite length.

FIG. 9 (A).



(A)

In order to clearly comprehend the motion of the reciprocating parts in a horizontal direction for a horizontal engine, Fig. 9 (A), lay down a horizontal line, O X, and divide it into 12 equal spaces, O, 1, 2, 3, etc., to 12; at these points draw ordinates at right angles to O X.

With any centre upon the line O X, as 2, and any radius as 2 C, describe the circle O C B 4 D O, and beginning at O divide the circumference of this circle likewise into 12 equal parts.

Let 2 C represent the position of the centre line of the crank, let 2 be the centre of the crank-shaft, and let C be the centre of the crank-pin.

Let the angle $a = C\,2\,O$ be the variable angle formed by the centre line of the crank with the horizontal O X.

Let r = the radius of the crank 2C.
" S = the space passed over by the piston-head (necessarily in a horizontal direction O X) in the time t.
" V = the angular velocity of revolution of the crank (assumed constant).
" T = the time of one revolution of the crank.

We have
$$V = \frac{2\pi}{T},$$
$$S = r(1 - \cos a). \tag{87}$$

Differentiating (87), we have
$$dS = r \sin a \, d a. \tag{88}$$

Letting v represent the velocity of the piston-head in a horizontal direction, and dividing (88) by dt, we have
$$\frac{dS}{dt} = r \sin a \frac{da}{dt}, \tag{89}$$
and since
$$\frac{dS}{dt} = v \text{ and } \frac{da}{dt} = V = \frac{2\pi}{T},$$
$$v = \frac{2\pi r}{T} \sin a. \tag{90}$$

Differentiating equation (90), we have
$$dv = \frac{2\pi r}{T} \cos a \, d a, \tag{91}$$
and dividing by dt, as before, we have for the acceleration
$$\frac{dv}{dt} = \frac{2\pi r}{T} \cos a \frac{da}{dt} = \left(\frac{2\pi}{T}\right)^2 r \cos a. \tag{92}$$

If we assume the angular velocity $= V = \frac{2\pi}{T} = 1$, we can graphically compare the curves of these equations, Fig. 9 (A).

The ordinates to the curve O B E F 12 show the distance of the piston-head from its starting-point during one revolution, which can be calculated also from equation (87). The ordinates to the curve O B 6 G 12 show the velocities of the piston-head during one revolution, which can also be calculated from equation (90). The ordinates to the curve H 3 K 9 L show the accelerations of the velocity of the piston-head during one revolution, which can also be calculated from equation (92). All of the reciprocating parts are supposed to move in conjunction with the piston-head.

If now we wish to determine the acceleration of the piston-head for every position, Fig. 9 (B), on the line O X, we lay off from O toward X the ordinates to the curve of distances for six points, and at these points erect ordinates taken from the same positions and equal to the ordinates to the curve of accelerations. The extremities of these ordinates can be joined by a straight line, A B. For, if we substitute in equation (87) the value of cos a derived from equation (92) we have, letting $y = \frac{dv}{dt} =$ the acceleration,

$$S = r\left(1 - \frac{y}{V^2 r}\right).$$

Therefore
$$S = r - \frac{y}{V^2}, \tag{93}$$

which is the equation of a straight line cutting O X at a distance r from the origin.

Referring to equation (92), we observe that the acceleration varies with the cosine of a, and therefore is a maximum for cos $a = 1$. This gives

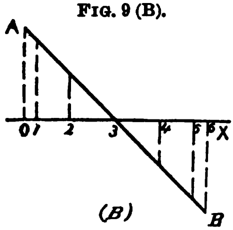
$$y = V^2 r, \tag{94}$$

which is the expression for the acceleration due to centrifugal force. If, therefore, we wish to balance a horizontal engine *at its dead points*, we must use a counter-weight so placed that its statical moment is equal to the statical moment of the reciprocating parts supposed to be concentrated at the centre of the crank-pin. The engine *cannot* be balanced for any other than its dead points; and when the crank is at right angles to the centre line of the cylinder, nearly the full centrifugal force of the counter-weight is felt. In engines driven at widely different speeds—as, for instance, a locomotive—the use for counter-weights seems to be the only practical method, and therefore the reciprocating parts should be made as light as is consistent with sufficient strength to resist the stresses coming upon them.

In the case of engines running at a constant speed, the weight and velocity of the reciprocating parts affect the stress upon the crank-pin in a manner which can be determined, and are therefore worthy of consideration.

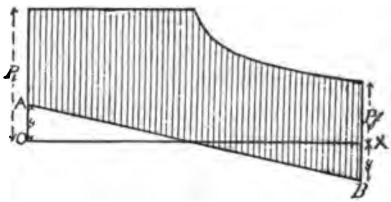
Referring to Fig. 9 (B), we see that the piston resists, leaving each end of the steam-cylinder with a force equal to its centrifugal force (equation 94), and further, that the intensity of this force diminishes uniformly from the end to the centre of stroke 3, where it is zero. It will, therefore, be at once recognized that an amount of work represented by the area of the triangle A 3 O is subtracted from the work impressed upon the piston by the steam during the first half stroke, and that an equal amount, represented by the area of the triangle 3 X B, is added to the work impressed upon the piston during the last half of the stroke.

FIG. 9 (B).



In an ordinary indicator diagram, Fig. 10, we have the means of measuring the force acting upon the piston-head

FIG. 10.



at every point of the stroke. (For a thorough discussion and explanation of the steam-engine indicator refer to *The Richards Steam-engine Indicator*, Porter, or *The Engine-Room, and who should be in it.*)

Let the line O X represent the atmospheric line of an indicator diagram taken from a non-condensing engine cutting off at one-half stroke.

Let P_i = the initial pressure of the steam upon the piston-head in pounds per square inch.

" P_f = the final pressure of the steam upon the piston-head in pounds per square inch.

" y = the resistance due to the inertia of the reciprocating parts in pounds per square inch.

If now we impose the condition that the initial and final pressures upon the crank-pin be equal, we must have

$$P_i - y = P_f + y. \tag{95}$$

Therefore

$$y = \frac{P_i - P_f}{2}. \tag{96}$$

Let A = the area of the piston-head in square inches.

" G = the weight of the reciprocating parts.

We have, equating equations (94) and (96),

$$A\left(\frac{P_b - P_f}{2}\right) = \frac{G}{g} V^2 r. \qquad (97)$$

Transposing and substituting the values,

$A = 0.7854d^2$ square inches,
d = diameter of steam-cylinder in inches,
$g = 32.2$ feet per second,
N = the number of strokes per minute,
$V = \frac{\pi}{60} N$ feet per second,
r = the radius of the crank in feet,

we have

$$G = \frac{d^2 \frac{\pi}{4}\left(\frac{P_b - P_f}{2}\right) g}{V^2 r} = 4612.3 \frac{(P_b - P_f)d^2}{N^2 r}. \qquad (98)$$

Considering formula (98), we see that if the initial and final steam-pressure P_b and P_f become nearly or quite equal, the weight of the reciprocating parts should be nothing. Of course, the only method in such a case is to make the reciprocating parts as light as possible. We further observe that the weight of the reciprocating parts increases as the difference between P_b and P_f increases, and is inversely as the square of the number of strokes.

A glance at the cross-hatched portion of Fig. 10 shows that the pressure upon the crank-pin is by no means uniform, being greatest at the point of cut-off.

We further know that the angular position of the connecting-rod causes the straight line A B to become a curve. The effect of the connecting-rod is discussed at considerable length in *The Richards Steam-engine Indicator.* The curve departs more widely from the straight line as n, the ratio of the connecting-rod to the crank, becomes less. We see then that, for early cutting off of the steam, either the reciprocating parts should be made heavy or the number of revolutions increased or decreased until the assumed weight G of the reciprocating parts is obtained.

Transposing equation (98), we have

$$N = 67.914d\sqrt{\frac{(P_b - P_f)}{Gr}}, \qquad (99)$$

which enables us to determine approximately the number of strokes per minute for any assumed pressure and weight of reciprocating parts, which will give a nearly uniform stress on the crank-pin.

P_f may be determined from P_b with sufficient approximation for practical purposes from Mariotte's or Boyle's law for gases that the pressures are inversely as the volumes.

The initial and final pressures can be taken from an indicator diagram, and should be determined after the erection of the engine.

Example.—To determine the proper weight of the reciprocating parts of a horizontal engine, data as follows (non-condensing cut-off $= \frac{1}{5}$ stroke):

$P_b = 125$ pounds per square inch.
$P_f = 25$ pounds per square inch.
$d = 12$ inches.
$N = 200$ per minute.
$r = 6'' = \frac{1}{2}$ foot.

Substituting in formula (98), we have the weight =

$$G = \frac{4612.3 \times 100 \times 144}{40000 \times \frac{1}{2}} = 3321 \text{ pounds.}$$

Noting that this weight is very large, and assuming 800 pounds as the approximate weight of the reciprocating parts of a steam-engine of one foot stroke and one foot diameter of cylinder, we have, substituting in formula (99) the proper number of strokes,

$$N = 67.914 \times 12 \sqrt{\frac{100}{800 \times \frac{1}{2}}} = 407 \text{ strokes per minute.}$$

In an unbalanced vertical engine the weight of the reciprocating parts lessens the resistance with which the piston leaves the upper end of the cylinder and adds to the resistance with which it leaves the lower end of the cylinder, thus shifting the line A B, Fig. 10, parallel to itself above or below, but not introducing any greater irregularity of pressure upon the crank-pin during one stroke.

CHAPTER VIII.

(36.) **The Single Crank.**—The crank is made of either cast or wrought iron or steel; the first-named metal is but little used, and should be avoided in any but the very roughest machinery, because it is very liable to hidden defects, is much weaker and has a lesser modulus of elasticity, thus requiring to be heavier than wrought iron or steel. Further, it will not admit of the crank-pin being "shrunk in" to the eye without great danger of cracking.

This latter fact is illustrated very practically by an occurrence described by William Pole in his *Lectures on Iron as a Material of Construction*, p. 123. The italics are ours:

"You have probably heard of the process of drawing lead tube by forcing it in a semi-fluid (or sometimes in a nearly solid) state through a small annular hole. The lead is contained in a cylinder and pressed upon by a piston, and the force required is enormous, amounting to 50 or 60 tons per square inch.

"The practical difficulty of getting any cylinder to withstand the pressure was almost insurmountable. Cast iron cylinders 12 inches thick were quite useless; *they began to open in the inside, the fracture gradually extending to the outside, and increased thickness gave no increase of strength.*

"Cylinder after cylinder thus failed, and the makers (Messrs. Eaton & Amos) at length constructed a cylinder of wrought iron 8 inches thick; after using this cylinder the first time, the internal diameter was so much increased by the pressure that the piston no longer fitted with a sufficient closeness. A new piston was made to suit the enlarged cylinder; and a further enlargement occurring again and again with renewed use, the constant requirement of new pistons became almost as formidable an obstacle as the failure of the cast-iron cylinder.

"The wrought-iron cylinder was on the point of being abandoned, when Mr. Amos, having carefully gauged the cylinder both inside and out, found to his surprise that *although the internal diameter had increased considerably the exterior retained precisely its original dimensions.* He consequently persevered in the construction of new pistons, and found ultimately that the cylinders enlarged no more, and so the last piston continued in use for many years.

"Here, therefore, the permanent set operated first in the internal portions of the metal; as they expanded it was then gradually extended to the surrounding layers, and so at last sufficient material was brought into play with perfect elasticity, not only to withstand the strain, but to return back to the normal state every time after its application, and thus, by the spontaneous and unexpected operation of what was then an unknown principle, an obstacle apparently insurmountable, and which threatened at one time to render much valuable machinery useless, was entirely overcome."

It is this very property of wrought iron or soft steel which renders it useful for cranks; the metal when heated possesses increased ductility or viscidity, and adapts itself to the strain put upon it, while shrinking in a very perfect manner.

The following expansions in length for one degree Fahrenheit are given in Table XIV., p. 15, of Box's *Practical Treatise on Heat:*

Cast iron	.000006167
Steel	.000006441
Wrought iron	.000006689

Neglecting cast iron as being unsuitable for cranks, let us take up the case of a wrought-iron crank.

The value of E for wrought iron—*i. e.*, the hypothetical weight which would stretch a bar one square inch in area to twice its original length if it were perfectly elastic—is 28000000 pounds.

If we take the rupturing strain in tension for wrought iron at ordinary temperatures at 50000 pounds per square inch area,

Letting $x =$ the extension at the point of rupture, we have $1 : x :: 28000000 : 50000$, and

$$x = \frac{5}{2800} = .0017857 \text{ of its length.}$$

If further we divide this amount by the lengthening of a bar for one degree Fahr., we have

$$\frac{.0017857}{.000006689} = 267 \text{ degrees Fahr.,}$$

and we see that if it were not for the viscosity of wrought iron, a bar heated 267° and fastened so as not to be able to contract would rupture in cooling, and that further a difference of 26.7 degrees Fahr. would in the contraction resulting strain a bar 5000 pounds per square inch area.

Example.—Let us assume the diameter of that part of the crank-pin which is to be inserted into the eye of the crank at 5 inches, and further that the eye of the crank is bored to a diameter of 4.98 inches—an accuracy which it is quite practicable to obtain in any good machine-shop.

We then have for the required number of degrees through which the eye of the crank must be raised

$$\frac{.02}{.000006689 \times 5} = 630 \text{ degrees Fahr.}$$

More accurately, 4.98 should have been used in the place of 5 inches.

Knowing that at a temperature of 977 degrees Fahr. iron is just visibly red, we may use the following formula to determine the difference in the diameters of the eye of the crank and of the inserted part of the crank-pin:

Let d_c = diameter of inserted part of pin,
" d_e = required diameter of eye;

we have, with sufficient approximation,

$$\frac{d_c - d_e}{.000006689 d_e} = 900,$$

and
$$d_e = \frac{d_c}{1.0060201} = .99402 d_c. \qquad (101)$$

Should this formula give a greater difference than is necessary in practice, reduce it according to judgment by using less heat, unless a particularly strong grip of the eye upon the pin is desired. A high heat is apt to warp a forging, and no more should be used than is necessary. This formula cannot be regarded as accurate, and should be used as a guide only; it applies with sufficient approximation to machinery steel.

If the entering part of the crank-pin be made very slightly taper and larger than the eye of the crank, and the pin be forced in by strong hydraulic pressure, we avoid the risk of warping occasioned by heat, and secure almost as good a result so far as strength of grip is concerned.

Crank-pins are sometimes fastened by a key "cutter" at the back, and sometimes held by a nut.

Special care must be taken to have a sufficient thickness of metal around the eye to hold the pin firmly. In practice the thickness of the ring around the eye is usually taken equal to one-half the diameter of the eye. The depth of the eye is usually from 1 to 2 times its diameter, thus giving a cross-section of ring around the eye of from $1\frac{27}{100}$

to $2\frac{54}{100}$ greater area than that of the inserted part of the crank-pin at right angles to its axis.

In some cases the pin, crank and crank-shaft are forged in one solid piece out of wrought iron or made of cast steel in one solid mass; this method is more expensive, but more satisfactory.

The web of the crank is that part connecting its hub and eye, and is made of many differing shapes, and always with its cross-section increasing from the eye to the hub. Economy of material, as will presently be shown, dictates that this cross-section be increased by an increase of the breadth of the face of the crank—*i. e.*, in a direction at right angles to the centre line of the crank-pin, rather than in the direction of the crank-pin.

If now we assume that the crank has a constant thickness in the direction of the centre line of the crank-pin, the longitudinal profile of the web of the crank should be a parabola (Weisbach's *Mechanics of Engineering*, sec. iv., art. 251–253), with its vertex at the centre of the pin, and, providing we neglect the torsional strain on the web due to the moment of the crank-pin, will be of uniform strength.

The torsion of the web due to the force acting at the crank-pin is much slighter, in fact, than might at first appear, as the pin is fastened in the connecting-rod as well as the crank; and if the pin is a little loose, as it never should be, its springing tends, by throwing the centre of pressure nearer the crank, to reduce the moment. We can, therefore, neglect the torsion of the web, and consider it as a beam required to be of uniform strength, fixed at one end and loaded at the other.

To trace the parabolic outline of the crank would be tedious and result in an ugly shape, but the property of the tangent of a parabola permits us, with an increase of less than six per cent. of material, to locate correctly the straight sides of the web.

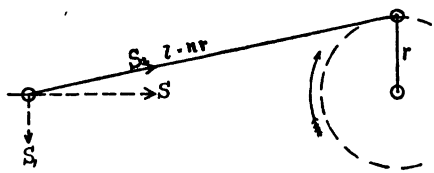
The interior diameter of the hub of the crank depends upon the diameter of the main or crank-shaft; the depth of the hub is usually made equal to the diameter of the shaft, or somewhat greater, and the thickness of metal around the shaft equal to $\frac{1}{2}$ of its diameter.

These proportions are varied very frequently, and are to a great extent a matter of judgment with the designer.

The crank is frequently shrunk on the shaft, just as described in the case of the crank-pin, and in such a case similar precautions must be taken.

(37.) **Wrought-Iron Single Crank.**—Referring to Fig. 5, we see that in addition to a constant stress S in a

FIG. 5.



horizontal direction the crank is subjected to an alternate maximum stress in tension and compression, $S_1 = \frac{S}{\sqrt{n^2 - 1}}$, which, for the values of $n = 4$ to 8, gives $S_1 = 0.26$ to $0.13\ S$. See formula (38), Art. 19.

Let S = the stress in pounds upon the crank at extremity, tending to cause flexure.

" S_1 = the stress in pounds upon crank, tending to cause extension or compression.

" F = the cross-section of the crank in inches.

" b = the assumed thickness of the crank in inches at right angles to its face.

Let v = the variable width of the face of the crank in inches.
" x = the variable length of the crank from the centre of the eye.
" W = the measure of the moment of flexure of the cross-section of the crank at any point.
" T = the safe strain per square inch upon the crank = 5000 pounds.
" e = the half width of the face of the crank = $\frac{v}{2}$.
" r = the radius of the crank in inches = $\frac{L}{2}$.

We have, if we suppose, as is usually assumed, the connecting-rod to be of infinite length (*i. e.*, $n = \infty$), $S_1 = 0$, but S_1 is too large to be neglected in practice.

Placing $$T = \frac{S_1}{F} + \frac{Sxe}{W}. \qquad (102)$$

See Weisbach's *Mechanics of Engineering*, sec. iv., art. 272.

Therefore, $$F = \frac{S_1}{T} + \frac{Sx}{T}\,\frac{Fe}{W}. \qquad (103)$$

For a rectangular cross-section $\frac{Fe}{W} = \frac{6}{v}$.

Therefore, $$F = \frac{S}{T}\left(\frac{1}{\sqrt{n^2-1}} + \frac{6x}{v}\right). \qquad (104)$$

Substituting for F its value bv, we have

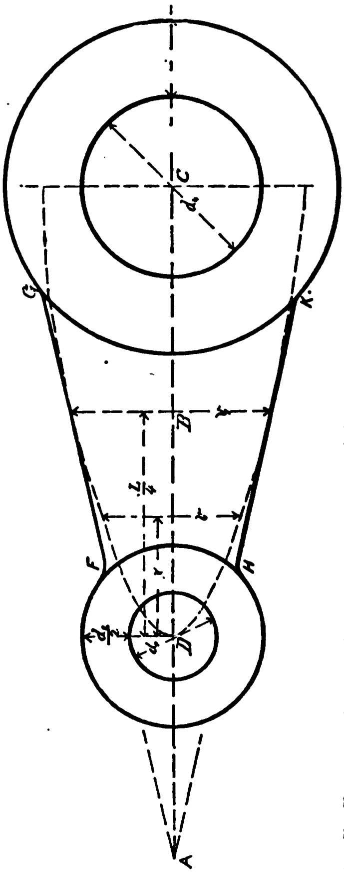
$$bv = \frac{S}{T}\left(\frac{1}{\sqrt{n^2-1}} + \frac{6x}{v}\right).$$

Therefore, $$v = \frac{S}{2bT\sqrt{n^2-1}} + \sqrt{\frac{S^2}{4b^2T^2(n^2-1)} + \frac{6xS}{bT}}. \qquad (105)$$

We have then the means of computing the value of V for a series of assigned values of b and x.

We see further that the first two terms of the second member of equation (105) will give small values which do not greatly affect the value of v. If in (105) we assume $x=\frac{L}{4}-\frac{r}{2}$, we have the width of the face of the crank at a point midway between the centres of the eye and hub, and equation (105) becomes

FIG. 11.



$$v_1=\frac{S}{2bT\sqrt{n^2-1}}+\sqrt{\frac{S^2}{4b^2T^2(n^2-1)}+\frac{3SL}{2bT}}. \quad (106)$$

If we suppose $n=\infty$, equation (106) becomes

$$v_1=\sqrt{\frac{3SL}{2bT}}. \quad (107)$$

In making use of equations (105) and (106) it is most convenient to calculate the value

$$S=0.7854P_bd^2$$

separately. If in (107) we substitute the values of S and T, we have

$$v_1=0.0153d\sqrt{\frac{P_bL}{b}}, \quad (108)$$

and for a uniform pressure P we have, in terms of the horse-power,

$$v_1 = 10.864\sqrt{\frac{(HP)}{bN}}. \qquad (109)$$

If now we assume the value b = the thickness of the web as uniform throughout its length, and recollecting that the tangent to a parabola intersects the axis of abscissa at a distance from the origin (vertex) equal to the abscissa of the point of tangency, we have a ready means of graphically constructing the web of the crank, Fig. 11.

From equation (106) or (108) (the first is the more exact) calculate the value of v_1. Draw the centre line A B C through the centres of the eye and hub. Bisect D C in B and lay off D A = D B; at the point B erect the double ordinate v_1, and through the extremities of this ordinate and through the point A draw the lines A F G and A H K to intersections with eye and hub.

CHAPTER IX.

(38.) **Steel Single Crank.**—Formulæ (105) and (106) apply in the same manner, with the exception that $T = 9000$ pounds per square inch for steel. Formula (107) becomes for this value of T, and the substitution of the value of S,

$$v_1 = 0.0114d\sqrt{\frac{P_bL}{b}}, \qquad (110)$$

and for an uniform pressure P we have, in terms of the horse-power,

$$v_1 = 8.095\sqrt{\frac{(HP)}{bN}}. \qquad (111)$$

Examples.—Let $L = 48$ inches.
" $d = 32$ inches.
" $P_b = P = 40$ pounds per square inch.
" $n = 5$.
" $(HP) = 156$ approximately.
" $N = 40$ per minute.
" $b = 7$ inches.

We have $S = 0.7854 \times 1024 \times 40 = 32170$ pounds.

For a *wrought-iron crank*, $T = 5000$ pounds per square inch.

Substituting in formula (106), we have

$$v_1 = \frac{32170}{2 \times 7 \times 5000 \times 4.899}$$

$$+\sqrt{\frac{(32170)^2}{4 \times 49 \times 25000000 \times 24} + \frac{3 \times 32170 \times 48}{2 \times 7 \times 5000}}$$

$$= 0.094 + \sqrt{0.0088 + 66.18} = 0.094 + 8.25 = 8.344 \text{ inches.}$$

If we substitute in formula (109), we have

$$v_1 = 10.864\sqrt{\frac{156}{7 \times 40}} = 8.11 \text{ inches.}$$

We see that the difference between the results of formulæ (106) and (109) is trifling, and this difference will be less for all values of $n > 5$; it will be greater for all values of $n < 5$.

For a *steel crank* we should take $b = 6''$ and $T = 9000$ pounds per square inch.

Substituting in formula (111), we have

$$v_1 = 8.095\sqrt{\frac{156}{6 \times 40}} = 7 \text{ inches.}$$

The use of these values of v_1 in the graphical construction of the web of the crank, Fig. 11, Art. 37, is obvious.

The use of formula (105) for the computation of a series of values of v for assigned values of x and b is tedious, but easy to understand.

(39.) **Cast-Iron Cranks.**—Although subject to a greater liability to accidental fracture than wrought iron or steel, cast-iron cranks are frequently used in the cheaper forms of engines.

FIG. 12.

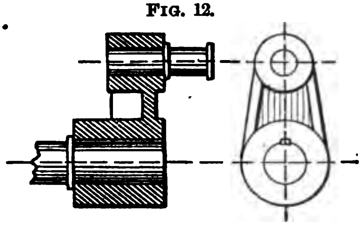
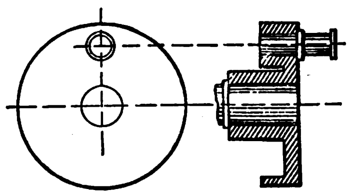


Fig. 12 gives a rather neat-looking unbalanced crank, and Fig. 13 is an example of the disc crank, which allows of being balanced with great ease by means of a counter-weight placed on the opposite side from the pin.

FIG. 13.



The proportions of cast-iron cranks depend to a great extent upon the character of the iron used. Formulæ can hardly be applied to so uncertain a material as cast iron.

(40.) **Keys for Shafts.**—In every case a key is used to prevent the crank from slipping on the shaft. The best position for a key is in line with and between the centres of the eye and hub of the crank. If more than one key is used, their combined shearing strength should be equal to that of a single key. Great care should be taken that the sides of the key fit the keyway perfectly, and that these sides be perfectly parallel, *not taper*. The very slight taper given to the key should be at the top.

Considering the case of a single key,

Let b_1 = its breadth in inches.
" l = its length in inches.
" t = its thickness in inches.
" K = the safe shearing strain per square inch area.
" d = the diameter of the steam-cylinder in inches.
" L = the length of stroke in inches.
" P_b = the steam-pressure per square inch.
" d_4 = the diameter of the shaft in inches.

Since the moment of the safe shearing strain of the key should be equal to the maximum moment upon the crank, we have

$$\frac{b_1 l K d_4}{2} = \frac{0.7854}{2} d^2 P_b L.$$

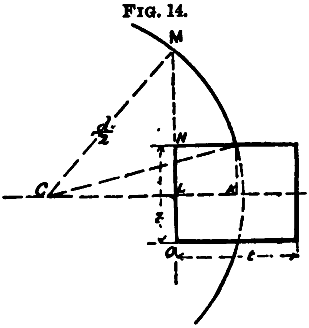
Therefore, $$b_1 = 0.7854 \frac{d^2 P_b L}{l d_4 K}. \qquad (112)$$

Or, if we suppose the steam-pressure to be uniform, we have, in terms of the horse-power (HP), and number of strokes per minute N,

$$b_1 = 396000 \frac{(HP)}{N l d_4 K}. \qquad (113)$$

To determine the proper thickness of the key t we may adopt the following method:

FIG. 14.



Let the key be sunk one-half of its thickness in the shaft, Fig. 14, and let the thickness of the key be such that the shearing surface of the shaft on a line with the inside surface of the key N M equals the shearing surface of the key O N. If t be not determined graphically, it can be determined approximately in an analytical way.

We have $$t = 2KL = 2(CK - CL). \qquad (114)$$

$$CK = \sqrt{\left(\frac{d_4}{2}\right)^2 - \left(\frac{b_1}{2}\right)^2} = \frac{d_4}{2}\left[1 - \left(\frac{b_1}{d_4}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{d_4}{2}\left[1 - \tfrac{1}{2}\left(\frac{b_1}{d_4}\right)^2 - \tfrac{1}{8}\left(\frac{b_1}{d_4}\right)^4 \text{ etc.}\right]$$

$$CL = \sqrt{\left(\frac{d_4}{2}\right)^2 - \left(\frac{3b_1}{2}\right)^2} = \frac{d_4}{2}\left[1 - \left(\frac{3b_1}{d_4}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{d^4}{2}\left[1 - \tfrac{9}{2}\left(\frac{b_1}{d^4}\right)^2 - \frac{81}{8}\left(\frac{b_1}{d_4}\right)^4 \text{ etc.}\right].$$

If now we substitute these values of CK and CL in formula (114), we have

$$t = d_4\left[4\left(\frac{b_1}{d_4}\right)^2 + 10\left(\frac{b_1}{d_4}\right)^4 \text{ etc.}\right],$$

and reducing we have, with sufficient approximation,

$$t = \frac{4b_1^2}{d_4} + \frac{10b_1^4}{d_4^3} + \text{etc.} \qquad (115)$$

Example.—Let $d_4 = 16$ inches.
" $b_1 = 3$ inches.

Substituting in formula (115), we have

$$t = \frac{4 \times (3)^2}{16} + \frac{10 \times (3)^4}{(16)^3} = 2.25 + \frac{810}{4096} = 2.45 \text{ inches.}$$

Where more than one key is used the sum of their widths should equal the width of a single key fitted to withstand the given stress.

The thickness of the keys will be much less, as, for instance, two keys $1\frac{1}{2}$ inches broad and $\frac{9}{16}$ of an inch thick will take the place of the single key given in the example with equal safety.

It is a point worthy of particular notice that, neglecting the last term of equation (115), we see that the thickness of keys for shafts varies directly as the square of their breadth, and therefore that the use of two or more keys is attended with great economy of material in the keys, and less reduction of the cross-section of the shaft, while the safety is equally as great.

In establishing the proper thickness of metal for the hub of the crank the reduction due to the keyways must be considered.

(41). **Wrought-Iron Keys for Shafts.**—If in formula (112) we place $K=5000$ pounds per square inch, we have

$$b_1 = 0.000157\frac{d^2P_bL}{ld_4}. \tag{116}$$

Letting $K=5000$ pounds per square inch, we have, from formula (113),

$$b_1 = 79.2\frac{(HP)}{Nld_4}. \tag{117}$$

Example.—Let $d=32$ inches.
" $P_b=P=40$ pounds per square inch.
" $L=48$ inches.
" $l=20$ inches.
" $d_4=16$ inches.
" $N=40$ per minute.
" $(HP)=156$ approximately.

Substituting in formula (116), we have

$$b_1 = 0.000157\frac{1024\times40\times48}{16\times20} = 0.96 \text{ inches.}$$

Substituting in formula (117), we have

$$b_1 = 79.2\frac{156}{40\times20\times16} = 0.96 \text{ inches.}$$

We have also for the thickness of the key, from formula (115),

$$t = \frac{4 \times 0.92}{16} = 0.23 \text{ inches.}$$

(42.) **Steel Keys for Shafts.**—Since the shearing strength of machinery steel is found to be $\frac{3}{4}$ of its tensile strength, we have for a safe shearing stress $\frac{3}{4}$ of 9000 pounds = 6750 pounds per square inch. Substituting this value for K in formulæ (113) and (114), we have

$$b_1 = 0.000116 \frac{d^2 P_1 L}{l d_4}, \qquad (118)$$

or

$$b_1 = 58.667 \frac{(HP)}{N l d_4}. \qquad (119)$$

Comparing formulæ (117) and (119), we find

$$\frac{b_1 \text{ for steel}}{b_1 \text{ for wrought iron}} = \frac{58.667}{79.2} = 0.74.$$

Example.—Data as before in Art. (41). We can calculate as there shown, or abbreviate by taking 74 per cent. of the breadth of a wrought-iron key for the breadth of a steel key of equal strength,

$$b_1 = 0.74 \times 0.96 = 0.71 \text{ inches.}$$

The thickness would be the same as before for a wrought-iron shaft, and for a steel shaft, formula (115), we have

$$t = \frac{4 \times 0.50}{16} = 0.125 \text{ inch.}$$

This latter value is too small for convenience, and the key should be made of $\frac{1}{4}$ inch thickness, or as would be advisable greater in both cases.

(43.) **The Crank or Main Shaft.**—The first requisite of the crank-shaft is rigidity. Any flexure will be accompanied by injury and final destruction to the bearings of the crank-pin and of the shaft itself.

The stresses to which the crank-shaft is subjected are as follows:

(1.) At the beginning of each stroke it receives the thrust due to the pressure of steam upon the piston-head, which in some cases is a veritable blow, as will be noticed when the journals are not properly fitted, and also with some valve-motions without compression.

(2) At the middle of the stroke the crank-shaft is subjected to a maximum torsional stress (whose moment is measured by the length of crank multiplied by the steam-pressure on the whole of the piston), in addition to the thrust due to the whole steam-pressure on the piston.

(3.) When a fly-wheel propellor or paddle-wheels are attached to the shaft it is deflected by their weight, as well as strained by the torsional stress mentioned above.

These three cases will be discussed separately.

(44.) **Shaft Subjected to Flexure only.**—Round shafts only are used for steam-engine crank-shafts, and will therefore be the only form of shaft considered.

Shafts are subject to stress in flexure only, either when at rest or in some cases at the dead points—that is, when no fly-wheel is used. The parts subject to flexure are the overhanging ends of the shaft to which the crank propellor or paddle-wheel is attached, or between those bearings which support the fly-wheel propellor or paddle-wheel.

Let T = the safe stress per square inch upon the exterior fibres of the shaft.

" W = the measure of the moment of flexure for a round-shaft $= \dfrac{\pi d_4^4}{64}$.

" d_{4f} = the required diameter of the shaft to resist flexure, in inches.

Let l = the length of that part of the shaft under consideration in inches.

" G = the load upon the shaft in pounds.

We have (Weisbach's *Mechanics of Engineering*, sec. iv., art. 235), for a shaft fixed at one end and loaded at the other,

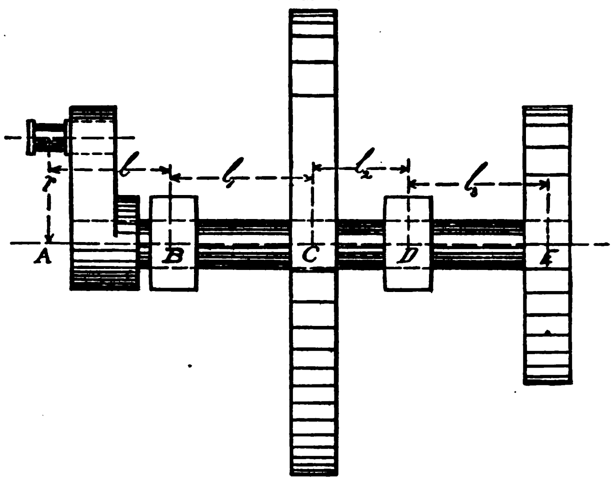
$$G = \frac{\pi}{32} \; \frac{d_4^{\,3} T}{l}.$$

Transposing,

$$d_4^{\,3} = \frac{32}{\pi} \; \frac{Gl}{T}. \qquad (120)$$

$$d_4 = 2.168 \sqrt[3]{\frac{Gl}{T}}. \qquad (121)$$

FIG. 15.



(45.) **Wrought-Iron Shaft Flexure only.**—In the case of a wrought-iron shaft $T = 5000$ pounds per square inch, and formula (120) becomes

$$d_{lf}^3 = .002037\,Gl. \qquad (122)$$

Formula (121) becomes

$$d_{lf} = 0.1268\sqrt[3]{Gl.} \qquad (123)$$

In the case of an overhanging weight, as at E, Fig. 15, these two formulæ can be applied directly by substituting E for G and l_3 for l.

In the case of a load C between two supports B and D, we can determine the reactions at B and D as follows:

Let l_1 = the distance BC in inches.
" l_2 = the distance CD in inches.
" C = the weight in pounds.
" B = the reaction at support B in pounds.
" D = the reaction at support D in pounds.

We have $$Cl_1 = D(l_1 + l_2).$$

Therefore, $$D = \frac{Cl_1}{(l_1 + l_2)}. \qquad (124)$$

We have then to substitute the derived value of D for G and l_2 for l, in equation (122) or (123), in order to determine the diameter for flexure of the part CD of the shaft. Similarly, we can treat the part BC of the shaft.

The overhanging part AB of the shaft should be estimated from the centre of the crank-pin, and for G the maximum pressure of the steam upon the piston-head $= 0.7854P_b d^2$ should be substituted.

Making these substitutions in formula (122), we have

$$d_{lf}^3 = 0.0016P_b d^2 l. \qquad (125)$$

If we suppose the pressure P_b uniform, we have

$$d_{lf}^3 = 806.7\frac{(HP)l}{LN}. \qquad (126)$$

CHAPTER X.

(46.) Steel Shaft Flexure only.—For a steel shaft $T = 9000$ pounds per square inch, and formula (120) becomes

$$d_{tf}^3 = 0.00113\ Gl. \qquad (127)$$

Formula (121) becomes

$$d_{tf} = 0.1042\sqrt[3]{Gl.} \qquad (128)$$

The method of applying these two formulæ is the same as in Art. (45), equation (124), etc.

For the overhanging part of the shaft AB, Fig. 15, to which a single crank is attached, we have by substitution in formula (127)

$$d_{tf}^3 = 0.000889\ P_b d^2 l. \qquad (129)$$

If in this we suppose the steam-pressure P_b to be uniform, we have

$$d_{tf}^3 = 448.18\ \frac{(HP)l}{LN} \qquad (130)$$

(47.) Shaft Subjected to Torsion only.—Shafts are subjected to a stress in torsion from the point where the power is received to the point at which it is given off, and not beyond that point.

The torsional stress in crank-shafts is zero at the dead points, and a maximum when the crank stands at right angles to the centre line of the steam-cylinder for single cranks. (For double and treble cranks see Arts. 54 and 55.)

We need only to consider the maximum torsional stress upon the shaft, leaving out of consideration the deflecting stress occurring at the point of maximum torsional stress, and due to the angular position of the connecting-rod. (See Art. 37.)

Let r = the length of the crank in inches $= \frac{L}{2}$.

" d_u = the required diameter of the shaft to resist torsion, in inches.

" T = the safe stress per square inch upon the exterior fibres of a round shaft.

" $S = 0.7854 P_b d^2$ = the stress at the extremity of the crank—*i. e.*, upon the crank-pin.

We have (Weisbach's *Mechanics of Engineering*, sec. iv., art. 264)

$$Sr = 0.1963 d_u^3 T \text{ and } d_u^3 = \frac{16Sr}{\pi T}.$$

Therefore, substituting for S and r the values given above,

$$d_u^3 = 2.\frac{P_b d^2 L}{T}, \qquad (131)$$

and

$$d_u = 1.26 \sqrt[3]{\frac{P_b d^2 L}{T}}. \qquad (132)$$

If in formula (131) we consider the pressure of the steam to be uniform, we have, in terms of the horse-power,

$$d_u^3 = 1008660.\frac{(HP)}{NT}. \qquad (133)$$

(48.) **Wrought-Iron Shaft Torsion only.**—For wrought iron we have, with a factor of safety of 10, $T = 5000$ pounds per square inch, and formula (131) becomes

$$d_u^3 = 0.0004 P_b d^2 L, \qquad (134)$$

and formula (132) becomes

$$d_u = 0.07368 \sqrt[3]{P_b d^2 L}. \qquad (135)$$

Formula (133) becomes

$$d_u^3 = 201.73 \frac{(HP)}{N}. \qquad (136)$$

(49.) **Steel Shaft Torsion only.**—Since the stress upon a steel shaft at the outer layer of fibres is of the nature of a shearing stress, we can take, with a factor of safety of 10, $T = 6750$ pounds per square inch.

Formula (131) becomes

$$d_{4}^{3} = 0.0002964 P_{b} d^{2} L. \qquad (137)$$

Formula (132) becomes

$$d_{4} = 0.06667 \sqrt[3]{P_{b} d^{2} L}. \qquad (138)$$

Formula (133) becomes

$$d_{4}^{3} = 149.43 \frac{(HP)}{N}. \qquad (139$$

(50.) Shaft Submitted to Combined Torsion and Flexure.—The most frequent case for crank-shafts is where they are submitted at the same time to stresses of flexure and torsion.

Let T = the safe stress per square inch.
" S = the whole stress of the pressure of the steam upon the piston-head, in pounds.
" G = the load causing flexure, in pounds.
" l = the distance in inches of the point of application of the load to the point of support.
" r = the length of the crank in inches $= \frac{L}{2}$.
" d_{4} = the required diameter of the shaft, in inches, to withstand the combined stresses of flexure and torsion.

We have (Weisbach's *Mechanics of Engineering*, sec. iv., art. 277) the following approximate formulæ:

$$d_{4} = \frac{2\sqrt[3]{\frac{2Sr}{\pi T}}}{\left[1 - \frac{32Gl}{\pi T d_{4}^{3}}\right]^{\frac{1}{3}}} \qquad (140)$$

or
$$d_4 = \frac{2\sqrt[3]{\frac{4Gl}{\pi T}}}{\left[1 - \left(\frac{16Sr}{\pi T d_4^3}\right)^2\right]^{\frac{1}{3}}} \qquad (141)$$

Considering formulæ (140) and (141), we observe that $2\sqrt[3]{\frac{2Sr}{\pi T}} = d_{4t}$. Refer to Art. (47); and that $\frac{32\,Gl}{\pi T} = d_{4f}^3$. Refer to Art. (44).

Substituting these values, we have

$$d_4 = \frac{d_{4t}}{\left[1 - \frac{d_{4f}^3}{d_4^3}\right]^{\frac{1}{3}}} \qquad (142)$$

or
$$d_4 = \frac{d_{4f}}{\left[1 - \left(\frac{d_{4t}^3}{d_4^3}\right)^2\right]^{\frac{1}{3}}} \qquad (143)$$

As in both formulæ (142) and (143) d_4 is greater than either d_{4f} or d_{4t}, we must, in order to obtain the first approximation, substitute for d_4 in the second member the greater resulting diameter from a consideration of the stresses in flexure and torsion singly.

A single approximation will generally be sufficient for practical purposes.

Therefore we have the following simple rules for calculating the diameter of a shaft submitted simultaneously to torsion and flexure:

FIRST. *Calculate the diameters for torsion and flexure singly.*

SECOND. *If the diameter due to torsion be the greater, divide it by the sixth root of the expression, Unity minus the quotient of the cube of the diameter due to flexure divided by the cube of the diameter due to torsion, and the result will be the required diameter.*

THIRD. *If the diameter due to flexure be the greater, divide it by the cube root of the expression, Unity minus the quotient of the sixth power of the diameter due to torsion divided by the sixth power of the diameter due to flexure, and the result will be the required diameter.*

Claudel, *Formules a l'usage de l'Ingenieur*, p. 288, gives the following rule: "Calculate the diameter of the shaft to resist each strain separately. Take the greatest of the two values. If the largest diameter is given by the effort of torsion, augment it by $\frac{1}{8}$ to $\frac{1}{10}$." This rule gives too small values.

Formulæ (142) and (143) can be expanded into a series giving an approximate value. Thus:

$$\left[1 - \left(\frac{d_{4f}}{d_4}\right)^3\right]^{-\frac{1}{6}} = \left[1 + \frac{1}{6}\frac{d_{4f}^3}{d_4^3} + \frac{7}{72}\left(\frac{d_{4f}}{d_4}\right)^6 + \text{etc.}\right].$$

Therefore (142) becomes when torsion gives the greater diameter

$$d_4 = d_{4t}\left[1 + \frac{1}{6}\frac{d_{4f}^3}{d_{4t}^3} + \frac{7}{72}\left(\frac{d_{4f}}{d_{4t}}\right)^6 + \text{etc.}\right], \qquad (144)$$

and in the same manner (143) becomes when flexure gives the greater diameter

$$d^4 = d_{4f}\left[1 + \frac{1}{3}\frac{d_{4t}^6}{d_{4f}^6} + \frac{2}{9}\left(\frac{d_{4t}}{d_{4f}}\right)^{12} + \text{etc.}\right]. \qquad (145)$$

(51.) **Flexure and Twisting of Shafts.**—The shaft is deflected by the load placed upon it, and also twisted through a greater or less angle by the action of the crank.

Let d_4 = the diameter of the shaft in inches.
" G = the load in pounds at the extremity of the part considered.
" l = the length of that part of the shaft under consideration in inches.
" E = the modulus of elasticity = 28000000 pounds per square inch for wrought iron.
" a = the deflection in inches.

We have (Weisbach's *Mechanics of Engineering*, sec. iv., art. 217), for a beam fixed at one end and loaded at the other,

$$a = \frac{64}{3\pi E} \frac{Gl^3}{d_4^4}, \qquad (146)$$

giving the deflection from a straight line.

If the weight of the shaft be taken into consideration, we must add or subtract it, as the case may require.

For the angle through which a wrought-iron shaft will be twisted (Weisbach's *Mechanics of Engineering*, sec. iv., art. 263) we have, letting a° = the number of degrees and the notation be as before,

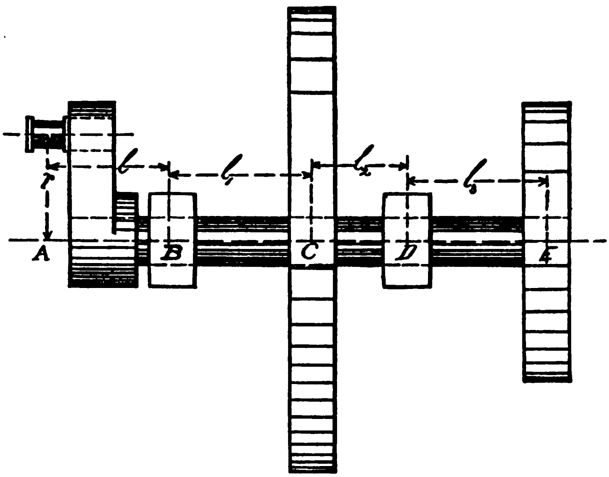
$$a^\circ = 0.0000254 \frac{d^2 P_b L l}{d_4^4}, \qquad (147)$$

or supposing the pressure P_b constant,

$$a^\circ = 12.807 \frac{(HP) l}{N d_4^4}. \qquad (148)$$

Example.—Fig. 15, to determine the proper diameter of a wrought-iron shaft having a fly-wheel C, weighing 70000 pounds, supported between the plumber-blocks B and D.

FIG. 15.



Let	$l = 36$ inches.
"	$l_1 = 48$ inches.
"	$l_2 = 36$ inches.
"	$P_b = P = 40$ pounds per square inch.
"	$d = 32$ inches.
"	$L = 48$ inches.
"	$N = 40$ per minute.
"	$(HP) = 156$ approximately.

The Overhanging Part AB.—From formula (126) we have for deflection

$$d_4^3 = 806.7\frac{156 \times 36}{48 \times 40} = 2359.6.$$

From formula (136) we have

$$d_{4t}^3 = 201.\frac{73}{100}\;\frac{156}{40} = 786.75.$$

Observing that the diameter due to flexure is the greater, and extracting the cube root, we have

$$d_{4f} = 13.31 \text{ inches},$$

and using formula (143), or, more conveniently, (145), we have

$$d_4 = 13.31\left[1 + \tfrac{1}{3} \times \left(\frac{786.75}{2359.6}\right)^2\right] = 13.8 \text{ inches},$$

giving the required diameter for the part AB of the shaft.

The Part BC of the Shaft.—From Art. (45) we have the reaction at

$$B = \frac{Cl_2}{l_1 + l_2} = \frac{70000 \times 3}{7} = 30000 \text{ pounds}.$$

We then have, formula (122),

$$d_{4f}^3 = 0.002037 \times 30000 \times 48 = 2933.28,$$

$$d_{4f} = 14.31 \text{ inches}.$$

Using formula (145), we have

$$d_4 = 14.31\left[1 + \tfrac{1}{3}\left(\frac{786.75}{2933.28}\right)^2\right] = 14.6 \text{ inches},$$

giving the required diameter of the part BC of the shaft.

The Part CD of the Shaft.—By substitution in formulæ (124) and (123) we find the required diameter of the part CD to be $= 14.31$ for flexure alone, if we suppose the power to be taken off the fly-wheel and neglect the stress from a belt or gearing.

The Part DE.—If we suppose the weight at $E = 40000$

pounds and $l_3 = 36$ inches, we have, by substitution in formula (123), $d_4 = 14.31$ inches for flexure alone, or, by substitution in formula (145), if we suppose the shaft submitted to torsion also,

$$d_4 = 14.6 \text{ inches.}$$

For torsion only $d_{4t} = \sqrt[3]{786.75} = 9.23$ inches.

(52.) **Comparison of Wrought-Iron and Steel Crank-Shafts.**—Comparing formulæ (128) and (123), we see

$$\frac{\text{For steel } d_{4f}}{\text{For wrought iron } d_{4f}} = \frac{0.1042}{0.1268} = 0.82.$$

Therefore, a steel shaft, to withstand the same stress in flexure, requires to be but 0.82 of the diameter and 0.67 of the weight of a wrought-iron shaft.

Comparing formulæ (138) and (135), we have

$$\frac{\text{For steel } d_{4t}}{\text{For wrought iron } d_{4t}} = \frac{0.06667}{0.07368} = 0.904.$$

Therefore, a steel shaft withstanding the same stress in torsion requires to be 0.90 the diameter and 0.81 the weight of a wrought-iron shaft.

Because of the near equality of the moduli of elasticity of wrought iron and steel, the deflection and torsional angle of wrought iron and steel under stress will be practically the same in all cases.

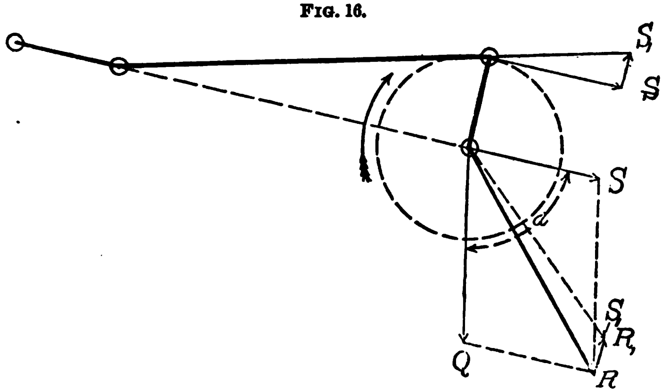
(53.) **Journal-Bearings of the Crank-Shaft.**—In the various hand-books of mechanical engineering, giving empirical rules for the length of the journals of shafts, we are advised to make the journal-bearing from $1\frac{1}{4}$ to 2 times the diameter of the shaft. In Art. (30), formula (81), we have the means of determining the least allowable length of journal of shaft under the most favorable circumstances—that is, when the shaft is submitted to torsion only, and does not bear the additional weight of a fly-wheel, screw-propellor or paddle-wheels.

If to the stress due the steam-pressure on piston be added the weight of a fly-wheel, etc.,

Let Q = reaction at bearing due to weight.
" S = stress due steam-pressure on piston.

Referring to Fig. 16, we see that the force S always acts in the direction of the centre line of the cylinder, and that the force Q acts downward, and further that the small force S, for ordinary lengths of connecting-rods, causes the re-

FIG. 16.



sultant R of the forces Q and S to vibrate between the positions and values R and R_1 when the crank moves in the direction of the arrow—*i. e.*, throws over and tends to lift the shaft from its bearings. If the crank throws under, the small force S_1 tends to press the shaft down upon its bearings.

We have for the value of the resultant force, neglecting S_1,

$$R = \sqrt{S^2 + Q^2 + 2QS \cos a.} \qquad (149)$$

For the angle $a = 90$ degrees—that is, for a horizontal engine—we have

$$R = \sqrt{Q^2 + S^2}.$$

For $a = 0$ degrees—that is, for a vertical engine—we have

$$\left.\begin{array}{l} R = Q+S, \text{ or} \\ \text{For } a = 180 \text{ degrees, we have} \\ R = Q-S. \end{array}\right\}. \qquad (151)$$

Let R = pressure on journal from above formulæ.
" f = coefficient of friction.
" d_4 = diameter of journal of shaft.
" l_4 = length of journal of shaft.
" W = work (or heating) allowed per square inch of projected area per minute = 49908 inch-pounds.
" N = number of strokes per minute.

Then, by a similar course of reasoning to that in Art. 30, we have

$$l_4 = \frac{1.5708}{49908} fRN = .0000325 fRN. \qquad (152)$$

Example.—Let us take a horizontal cylinder.

Let $S = 60000$ pounds.
" $Q = 30000$ pounds. We have

$$R = \sqrt{Q^2+S^2} = \sqrt{4500000000} = \text{approx.} 6700 \text{ lbs.}$$

Let $N = 40$ per minute and $f = .08$.

Then, by formula (152), we have

$$l_4 = .0000325 \times .08 \times 67000 \times 40 = 6.9 \text{ inches,}$$

which is the minimum length of shaft-journal allowable.

The importance of the influence of the number of turns upon the length of the bearing has not hitherto been noticed, and the use of empirical rules has resulted in bearings much too long for slow-speeded engines and too short for high-speeded engines.

To cover the defects of workmanship, neglect of oiling, and the introduction of dust, it is probably best to take $f = .16$, or possibly even greater if we make use of formula 152.

Five hundred pounds per square inch of projected area may be allowed for steel or wrought-iron shafts in brass bearings with good results, if a less pressure is not attainable without inconvenience.

Babbit or soft-metal linings, which are moulded by pouring in the metal around the shaft and allowing it to fit in cooling, are used in some forms of engine with great economy and good results.

For great pressures the shaft is sometimes cased in gun-metal and the casing run in lignum-vitæ bearings, which are lubricated with water. This expedient is commonly used for propellor-shafts, and an aperture communicating with the hold of the vessel causes a constant stream of water to flow through the bearing.

Hollow or "lantern" brasses, through the interior of which a constant stream of water is kept flowing in order to convey away the heat, are also used for great pressures with good success.

It is best, where great pressures are used, to have some means of feeling of the shaft as it turns, and any spot which feels rough ("ticklish") should at once be lubricated by means of a long-nosed oil-can, with a wick in the end of the nose placed in contact with it.

By means of these expedients pressures of 1000 pounds per square inch of projected area have been successfully used.

With very slow speed even greater pressures are sometimes used.

Arthur Rigg gives in *A Practical Treatise on the Steam-engine* many forms of plumber-blocks and bearings from English models.

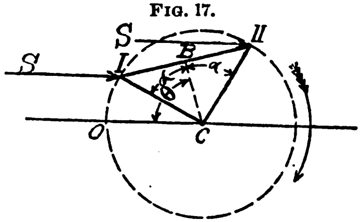
Warren's *Elements of Machine Construction and Drawing* gives some good forms of the French and American types. Inspection of Fig. 16 shows at what points the bearings are liable to the greatest wear, being the points at which the resultant R intersects the circumference of the bearings.

CHAPTER XI.

(54.) **Double Cranks.**—For the purpose of obtaining greater regularity of revolution of the crank-shaft, as well as to enable the engine to start in any position, two cranks at right angles are frequently used.

We shall neglect the obliquity of the connecting-rod, and also consider the pressure upon the crank-pin to be uniform throughout the stroke, as has been shown possible to render it approximately in Art. (35).

FIG. 17.



Let α = the half angle between the two cranks I and II.

" θ = the variable angle B C O formed by the line C B, which bisects the angle I C II.

" S = the force acting upon each crank-pin.

" r = the radius of the crank.

Then, Fig. 17, we have for the combined moments of the crank = y, when both cranks are on one side of the line O C,

$$y = Sr[\sin(\theta - \alpha) + \sin(\theta + \alpha)] \quad (153)$$

$$= 2Sr \sin\theta \cos\alpha. \quad (154)$$

Differentiating, and equating with 0, we have

$$\frac{dy}{dt} = 2Sr \cos\theta \cos\alpha = 0,$$

giving a maximum for $\theta = 90°$ or $270°$. When the cranks C I and C II are on opposite sides of the line O C, we have

$$y = Sr\,[\sin(\theta - a) + \sin(\theta + a - 180)] \qquad (155)$$
$$= 2Sr \cos\theta \sin a. \qquad (156)$$

Differentiating, and equating with 0, we have

$$\frac{dy}{d\theta} = -2Sr \sin\theta \sin a = 0,$$

giving a maximum for $\theta = 0°$ or $180°$.

Further, it will at once be seen, if we consider a as a variable as well as θ, that the value of expressions (154) and (156), which express all values of the combined moments, will be a maximum, and therefore the possible minimum value of the expressions differ least from the maximum values if we make $\sin\theta = \cos a$ and $\cos\theta = \sin a$—a condition which can only be fulfilled by letting $\theta = a = 45°$, giving for the angle between the cranks 90°, and for the position of least moment $\theta = 45°$—that is, when one of the cranks is at its dead point.

We have then for a maximum value of equations (154) and (156)

$$y = 2Sr \times .7071 = 1.414Sr, \qquad (157)$$

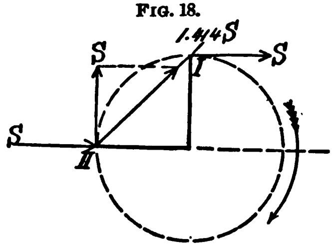
and for a minimum value

$$y = 2Sr \times .5 = Sr. \qquad (158)$$

If, as in the case of a marine-engine, the power of the first crank is communicated through a second bent crank, we see that that crank and the shaft leading from it at no time sustain twice the torsional stress exerted by one crank, but at a maximum 1.414 times as much; and in determining the dimensions of the after-crank and of the shaft, we should regard the stress as that upon the first multiplied by 1.414.

Multiplying the numerical coefficient of equation (132) by $\sqrt[3]{1.414}$, we have, with the same notation for the diameter of the shaft

FIG. 18.



$$d_4 = 1.414\sqrt[3]{\frac{P_b d^2 L}{T}}. \quad (159)$$

In the case of double engines it is customary to make the after-crank-pin of the same size as the shaft, for the reason that it is subjected to many unforeseen stresses.

The length of the after-crank-pin should be the same as that of the forward pin. See Art. (30).

The stress upon the after-pin due to its cylinder may be regarded as constant, and in the direction of the centre line of its cylinder.

The stress upon the after-pin from the forward crank-pin is a maximum in a tangential direction to its circle of revolution, and equal to S when the forward pin makes an angle of 90° with its cylinder centre line. Fig. 18.

If we make the supposition, as may happen to be the case, that both these stresses act simultaneously at the forward end of the after-pin without support from the forward crank,* we have for the maximum stress upon the pin $\sqrt{2}S = 1.414S$, and for the diameter of the after-pin, from formula (84), Art. (32),

$$d_3 = 0.072\sqrt[4]{P_b l_3^3 d^2}; \quad (160)$$

or, if we regard the steam-pressure as constant,

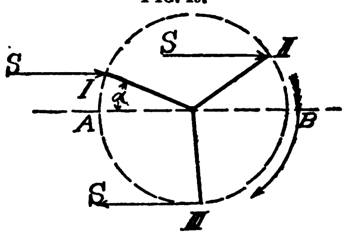
$$d_3 = 1.917\sqrt[4]{\frac{(HP)}{LN} l_3^3}. \quad (161)$$

* This supposition does not give greater results than are often found practically necessary by an expensive tentative process.

It should, however, be remembered that, although the most unfavorable suppositions possible for the known stresses have been made in the present case, unforeseen stresses are liable to occur, which can be guarded against only by making the after-pin the same size as the shaft if possible.

(55.) **Triple Cranks.**—Triple cranks 120 degrees apart are sometimes used to attain still greater regularity of motion in the engine. (Fig. 19.)

FIG. 19.



Let α = the angle made by one of the cranks with the line AB.

" S = the pressure on the piston-head in lbs.

" r = the radius of the crank in inches.

We have for the torsional moment of the three cranks, $= y$,

$$y = Sr[\sin\alpha + \sin(\alpha + \tfrac{2}{3}\pi) + \sin(\alpha + \tfrac{4}{3}\pi)], \qquad (162)$$

in which the sines are taken as positive because the cylinders are double-acting.

Further, we see that the sum of the sines is not increased when we increase each of them by $\frac{\pi}{3} = 60$ degrees, and it is then only necessary to consider the equation for the angle α between the limits 0 and $\frac{\pi}{3}$.

Reducing equation (162), we have

$$y = Sr[\sin\alpha + \sqrt{3}\cos\alpha]. \qquad (163)$$

Neglecting Sr, differentiating, and placing the first differential coefficent = 0, to find the maximum and minimum values, we have

$$\frac{dy}{da} = \cos a - \sqrt{3}\sin a = 0,$$

$$\frac{d^2y}{da^2} = -\sin a - \sqrt{3}\cos a,$$

giving for a maximum value

$$\cos a = \sqrt{3}\sin a.$$

Therefore, $\tan a = \frac{1}{\sqrt{3}}$; therefore, $a = 30° = \frac{\pi}{6}$.

The minimum values of equation (163) occur when $a = 0$ and $= \frac{\pi}{3} = 60°$.

We see that that part of the shaft attached to the third crank is subjected to a maximum torsional stress twice as great as that due to one cylinder. Its diameter can be most readily calculated by doubling the actual steam-pressure in equation (132).

In the same manner, the proper proportions of the crank can be calculated by Art. (37).

In the case of three cranks, in order to find the maximum stress to which the second crank and shaft may be submitted, we must find the maximum of the expression

$$y = Sr\left[\sin a + \sin\left(a + \frac{2}{3}\pi\right)\right] \qquad (164)$$

Reducing, and neglecting Sr,

$$y = \frac{3}{2}\sin a + \frac{\sqrt{3}}{2}\cos a.$$

Differentiating, and placing $= 0$, we have

$$\frac{dy}{da} = \frac{3}{2}\cos a - \frac{\sqrt{3}}{2}\sin a = 0.$$

Therefore $\tan \alpha = \sqrt{3}$, therefore $\alpha = 60° = \frac{\pi}{3}$,

gives the greatest maximum to which they are subjected, and equation (164) becomes

$$y = Sr[0.866 + 0.866] = 1.732\ Sr,$$

which is also the minimum value of the torsional stress for 3 cranks together.

We can calculate the proper proportions of the crank, shaft and crank-pin by multiplying the steam-pressure by 1.732 in equation (132).

(56.) **The Fly-Wheel.**—Before taking up the subject of the fly-wheel mathematically, it will perhaps be best to give a general idea of its function.

It is impossible to control the speed of any engine for any considerable length of time by means of a fly-wheel, or to render the motion of any engine *exactly* uniform for any period of time.

It is, however, possible, by properly proportioning the weight, diameter and speed of rim of a fly-wheel to the work given out by the steam-cylinder, to confine the variation of the speed of the engine from any assigned mean speed during the time of one stroke, within any assigned limits.

A fly-wheel serves to store up work, or to give it out when required, just as a mill-pond fed by a stream of variable discharge serves to store up water for the mill-wheel; the larger the pond, the less the effect upon it of any sudden increase or diminution of the water flowing into it, and so with the fly-wheel: the larger it is, and the more rapid its motion, the more steadily it will run, so that it would hardly be possible, where a uniform motion in one direction only is desired, to make a fly-wheel too large, were it not for the circumstances that the loss of work due to increased friction and its greater cost limit us in that direction.

In considering the weight and speed of a fly-wheel, that only of its rim will be considered, and it is also proper to state here, to avoid leading our readers astray, that, unless specially noted, the formulæ to be subsequently established do not take cognizance of the variation in work given out by the steam-cylinder produced by the angular position of the connecting-rod; that the suppositions are also made that the mean pressure of the steam is uniform throughout the stroke; and that the work given out by the fly-wheel is given out uniformly.

The variation in the work given out by the steam-cylinder, produced by the angularity of the crank, is specially considered.

When any body of a weight W is in motion with a velocity v, it has stored up in it work $= w = \dfrac{Wv^2}{2g}$, and all of this work must be given out before it can come to rest. If this weight is not entirely brought to rest, but its speed reduced to v_1, it will, while being retarded, give out work

$$w = W\left(\frac{v^2 - v_1^2}{2g}\right). \qquad (165)$$

Or if the body be moving with a velocity v_1, and by the action of a force its speed be increased to a velocity v, it will store up work

$$w = W\left(\frac{v^2 - v_1^2}{2g}\right). \qquad (165)$$

Now, $(v^2 - v_1^2) = (v + v_1)(v - v_1)$, and if we take the mean uniform speed of the body $= u = \dfrac{v + v_1}{2}$, when v and v_1 are supposed not to differ greatly, and denote by m the fractional part of the mean velocity u, by which v and v_1 are allowed to differ, we have $mu = v - v_1$, and we have

$$(v^2 - v_1^2) = (v + v_1)(v - v_1) = 2mu^2,$$

and formula (165) takes the following form:

$$w = m\frac{Wu^2}{g}, \qquad (166)$$

in which $g = 32.2$, and which gives the amount of work gained or lost by the body when its speed is increased or decreased by $(v - v_1) = mu$.

For the present it will suffice to say that m is taken from $\frac{1}{5}$ to $\frac{1}{100}$, according to the degree of regularity required, and that u is the *mean speed per second in feet.*

We will next take up the amounts of work lost and gained by the fly-wheel while receiving work from the steam-cylinder in periodically varied quantities, and parting with it, on the other hand, in uniform quantities to the machinery driven.

If the work is not given out in uniform quantities, as is sometimes the case, special provision should be made for it by increasing the weight of the fly-wheel, so as to meet and overcome this source of irregularity, or, better, by the use of a fly-wheel at the point at which the variations occur, calculated to meet and overcome the irregularities at that point.

With an early cut-off of the steam, the irregularity due to the variable pressure on the piston is superadded to that due to the angular position of the crank and the connecting-rod, and will be specially noted hereafter in Table IV. for such cases as may occur in which the weight of the piston and appurtenances, is not or cannot be proportioned to the speed of the reciprocating parts and the steam-pressure. See Art. (35).

To establish a clear understanding between the reader and ourselves, we here define ***work to be force multiplied by the space passed over by that force.***

(57.) **Fly-Wheel, Single Crank.**—Let P = the steam-pressure per square inch upon the piston-head.

Let d = the diameter of the steam-cylinder in inches.
" S = the total pressure upon the piston-head.

The force pressing upon the piston-head is, as before stated, for a single cylinder,

$$S = \frac{\pi d^2}{4} P. \tag{167}$$

(Assuming the engine to have attained its average speed, and neglecting the small variation produced by the connecting-rod,)

Letting r = radius of crank,
" a = angle formed by centre line of crank with the centre line of cylinder,
" s = space passed over by the piston,

the distance passed over by the piston is

$$s = r(1 - \cos a). \tag{168}$$

Therefore, we have for the work derived from the piston = w_1

$$w_1 = Sr(1 - \cos a). \tag{169}$$

To find the increments of work corresponding to each increment of arc, we differentiate (169), giving

$$dw_1 = Sr \sin a da. \tag{170}$$

The total amounts of work gained and lost by the fly-wheel during one stroke equal SL. [L = the length of the stroke.]

As, however, the work is gained in varied increments, as shown by (170), and on the other hand is lost in assumed uniform quantities to the machinery driven, there are points at which the increments of the gained and the lost work are equal.

The lost work during the passage of the crank through the angle a is

$$w_2 = S\frac{2}{\pi}ra. \qquad (171)$$

Since $\frac{4Sr}{2\pi}$ = the lost work for the unit of arc, and differentiating (170), we have

$$dw_2 = \frac{2}{\pi}Srda. \qquad (172)$$

Placing equations (170) and (172) equal to each other, we have

$$Sr \sin a = \frac{2}{\pi}Sr.$$

Therefore,

$$\sin a = \frac{2}{\pi} = 0.636618, \qquad (173)$$

which gives the following values,

$$\left.\begin{aligned} \text{Angle } a &= 39^\circ\ 32'\ 25'' \\ \text{or } &= 140^\circ\ 27'\ 35'' \\ \text{or } &= 219^\circ\ 32'\ 25'' \\ \text{or } &= 320^\circ\ 27'\ 35'' \end{aligned}\right\}. \qquad (174)$$

We see from equation (170) that the increment of gained work is also equal to 0 for $a = 0^\circ$ or 180°, and a maximum for $a = 90^\circ$ or 270°, while the increment of lost work, equation (172), is a constant.

If now we substract the value of equation (169) for

$a = 39° 32' 25''$ from its value for $a = 140° 27' 35''$, we have

$$w_1 = Sr[(1 + \cos a) - (1 - \cos a)];$$

therefore, $w_1 = 2Sr \cos a = 1.54232Sr,$ (175)

and (175) is the total amount of work received from the steam-cylinder by the fly-wheel from the point where the increment of gained work is equal to the increment of lost work until they are again equal. In order to find the surplus of work absorbed by the fly-wheel, we must subtract the total amount of lost work (lost uniformly) during the same interval.

If now we subtract the value of equation (171) for $a = 39° 32' 25''$ from its value for $a = 140° 27' 35''$, we have

$$w_2 = 2S\frac{r}{\pi}\left(\frac{100°.91945}{180°}\right)\pi = 1.12133Sr. \quad (176)$$

Substracting (176) from (175), we have for the work stored by the fly-wheel between the two values of a, above stated, $= w_3$,

$$w_3 = (w_1 - w_2) = 0.42099Sr,$$

and substituting for r its value $\frac{L}{2}$, we have, for a single cylinder,

$$w_3 = .21049SL, \quad (177)$$

or about .21 of the work done during the whole stroke.

If we substitute for S its value given in equation (167), and take L *in feet*, we have, in terms of the pressure diameter of steam-cylinder, letting c represent the numerical coefficient derived from calculation or Table IV.,

$$w_3 = .7854Pd^2cL. \quad (178)$$

To prove that the work lost by the fly-wheel, in passing from crank-angle 140° 27′ 35″ to 219° 32′ 25″, is equal to that gained in passing from crank-angle 39° 32′ 25″ to 140° 27′ 35″.

Referring to equation (175), we see that the total work gained between the limits $a = 140° \, 27' \, 35''$ and $a = 219° \, 32' \, 25''$, if we properly alter the signs, is

$$w_1 = 2Sr(1 - \cos a), \qquad (179)$$

and, referring to equation (171), proceed as in equation (176), we have for the total lost work of the fly-wheel

$$w_2 = 2S\frac{r}{\pi}\left(\frac{79°.0805}{180°}\right)\pi = 2Sr(0.4393). \qquad (180)$$

$$\cos a = 0.77116, \quad \therefore (1 - \cos a) = .22884.$$

Subtracting equation (179) from (180), we have

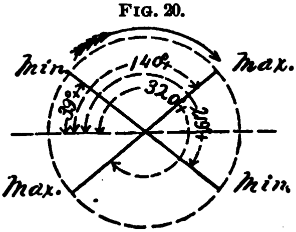
$(w_2 - w_1) = 2Sr(0.4393 - .2288) = .421\,Sr = w_3$ the work lost,

and we see that the amount of work gained by the fly-wheel in an arc of 100.92 degrees is equal to that lost in an arc of 79.08 degrees.

Now, for the sake of a clear understanding of this topic, let us follow the crank through one revolution. (See Fig. 20.)

From crank-angle 39° 32′ 25″ to 140° 27′ 35″ the angular velocity of the crank and fly-wheel increases, attaining a maximum at 140°+, the fly-wheel storing up work. From 140°+ to 219°+ the angular velocity of the crank and fly-wheel decreases, reaching a minimum at 219°+, the fly-wheel giving out work equal in amount to that stored up in the first arc mentioned; from 219°+ to 320°+ the fly-wheel again stores work, and from 320°+ to 39°+ gives the same amount up.

FIG. 20.



(58.) **Fly-Wheel, Double Crank, Angle 90°.**—The most advantageous angle between the cranks being 90°, as shown in Art (54), the total work of the two cranks will be, if we

Let a = the angle between the centre line of the cylinder and the first crank,
" S = the pressure in pounds upon each piston-head,
" r = the radius of the crank,
" w_1 = the work given out by the two cylinders,

$$\text{since } \cos(90+a) = -\sin a,$$

$$w_1 = Sr[(1-\cos a)+(1+\sin a-1)]. \qquad (181)$$

Reducing and differentiating, we have the increment of gained work for each increment of arc

$$dw_1 = Sr(\sin a+\cos a)da. \qquad (182)$$

Multiplying equation (172) by 2, we have for the increment of the lost work for each increment of arc

$$dw_2 = \frac{4}{\pi}Srda. \qquad (183)$$

Equating equations (182) and (183), we have

$$\sin a+\cos a = \frac{4}{\pi}.$$

Squaring both members,

$$\sin^2 a+\cos^2 a+2\sin a\cos a = \left(\frac{4}{\pi}\right).$$

$$\text{Therefore, } \sin 2a = \left(\frac{4}{\pi}\right)^2 - 1 = 0.62114. \qquad (184)$$

Giving $2a = 38°\ 24'$
and $a = 19°\ 12'$.

Remembering that after passing through 90° of arc the same variations are repeated, we have for the various values of a, when the increments of lost and gained work are equal,

19° 12′	
	51° 36′
70° 48′	
	38° 24′
109° 12′	
	51° 36′
160° 48′	
199° 12′	
250° 48′	
289° 12′	
340° 48′, etc.	

From (182) we further see that the increment of gained work is a maximum for $a = 45°$ and a minimum for $a = 0°$ or 90°.

To determine the excess of work lost or gained, we have, equation (181) reduced,

$$w_1 = Sr[1 - \cos a + \sin a].$$

For $a = 70° 48'$ this equation.................... $= Sr \times 1.615509$
For $a = 19° 12'$ this equation.................... $= Sr \times 0.384491$

The total amount of work gained........... $= Sr \times 1.231018$

The total amount of work lost is $4Sr\frac{51.°6}{180} = Sr \times 1.146667$

Giving for the gained or lost work............ $Sr \times 0.084351$

If for r we substitute its value $\frac{L}{2}$, we have

$$w_2 = 0.0422SL. \qquad (185)$$

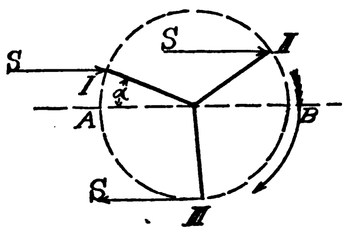
As in the preceding article, we see that the angles determined correspond to points of minimum and maximum angular velocity of the fly-wheel, and it is easy to prove that the work gained by the fly-wheel in passing from say 19° 12′ through 51° 36′ is equal to the work lost in passing from 70° 48′ through an arc of 38° 24′.

CHAPTER XII.

(59.) **Fly-Wheel, Triple Crank, Angle 120°.**—Letting the notation be the same as in the preceding article, we have, for the work given out by the three cranks,

$$w_1 = Sr[(1 - \cos a) + (1 - \cos (a + \tfrac{2}{3}\pi) - \tfrac{3}{2}) + 1 - \cos (a + \tfrac{4}{3}\pi) - \tfrac{1}{2})], \qquad (186)$$

FIG. 19.



which, after reduction, becomes

$$W = Sr[1 - \cos a + \sqrt{3} \sin^{\cdot} a]. \qquad (187)$$

Differentiating, and neglecting the constants, we have

$$\frac{dw}{da} = \sin a + \sqrt{3} \cos a, \qquad (188)$$

which can be placed equal to $\frac{6}{\pi}$ = the work lost in each element of arc, to find the points at which the increments of the gained and the lost work are equal.

$$\sin a + \sqrt{3} \cos a = \sin a \cos 60° + \cos a \sin 60°,$$

$$\text{since} \quad \frac{\sin 60°}{\cos 60°} = \sqrt{3} \text{ and } \cos 60° = \tfrac{1}{2}.$$

We then have

$$\sin(a+60^\circ)=\frac{3}{\pi}.$$

Therefore, $(a+60)=\ 72^\circ\ 44'$ and $a=12^\circ\ 44'$,
or $(a+60)=107^\circ\ 16'$ and $a=47^\circ\ 16'$.

Since the same phases are repeated after the cranks have passed through an angle of 60 degrees, we need consider this equation only between 0 and 60 degrees.

Expression (188) becomes a minimum for $a=0$ or $=60^\circ$, and is a maximum for $a=30^\circ$.

To determine the amount of work lost or gained by the fly-wheel, we substitute in equation (187) the values $a=47^\circ\ 16'$ and $a=12^\circ\ 44'$.

For $a=47^\circ\ 16'$ equation (187) becomes..........	$Sr\times 1.5936$
For $a=12^\circ\ 44'$ equation (187) becomes..........	$Sr\times 0.4063$
The total amount of work gained is...............	$Sr\times 1.1873$
The total amount of work lost is for an arc of $34^\circ\ 32'=34.^\circ 533$..........................	$Sr\times 1.1511$
Giving for the gained work absorbed by the fly-wheel in an arc of $34.^\circ 533$.............	$Sr\times 0.0362$

To find the work lost by the fly-wheel as a check, we have

For the work lost in an arc of $25^\circ\ 28'$............	$Sr\times 0.8488$
For the work gained in an arc of $25^\circ\ 28'$.........	$Sr\times 0.8126$
Giving for the lost work given out by the fly-wheel in an arc of $25^\circ\ 28'$..................	$Sr\times 0.0362$

If for r we substitute its value $\frac{L}{2}$, we have

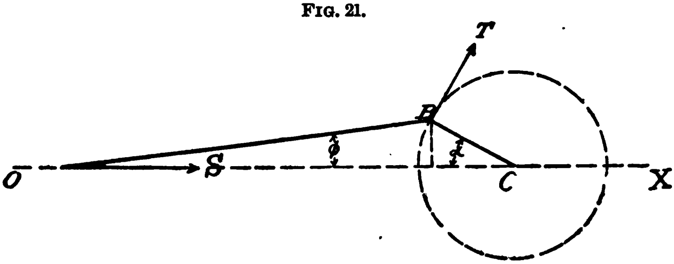
$$w_s=0.0181SL. \qquad (189)$$

We can, in a similar manner to that already explained, determine the six points of maximum and the six points of minimum velocity of the fly-wheel.

(60.) Of the Influence of the Point of Cut-off and the Length of the Connecting-Rod upon the Fly-Wheel.—

Let S = the total pressure of the steam upon the piston-head.
" r = radius of crank.
" $l = nr$ = the length of the connecting-rod.
" α = the angle between the crank and the centre line O X.
" φ = the angle between the connecting-rod and the centre line O X. (See Fig. 21.)

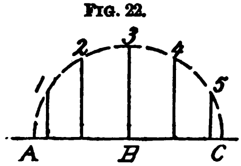
FIG. 21.



Referring to equation (170), Art. (37), we see that the increments of work given out with a varying velocity are proportional to sin α, the force S being assumed *constant;* as the crank-pin may be assumed to move in a tangential direction with a *constant* velocity, the condition of the equality of the increments of work at the points O and B* requires that the tangential or torsional force T shall vary as the sin α. (This can be proved graphically also.)

CASE I.—Let S be constant and the length of the connecting-rod be assumed infinite.

FIG. 22.



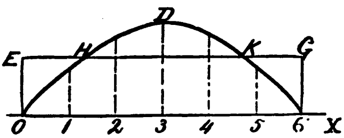
With a radius A B, Fig. 22, representing the constant force S acting on the piston to a convenient scale, describe the semicircle A 3 C. Divide this semi-perimeter into any number

* Theorem of virtual velocities.

of equal spaces, as 1, 2, 3, 4, 5, C. From these points drop verticals to the line A C; these verticals will represent the tangential forces T at these points.

If we lay off a horizontal line, as O X, Fig. 23, representing the semi-perimeter of the crank-circle to any convenient scale, and divide it into six equal parts, and upon these divisions erect verticals of equal length to the verticals in Fig. 22, we obtain the curve of sines O D 6, and the area of this figure, O D 6 O, represents the work done in one stroke. The work lost by the engine, being assumed lost uniformly, can be represented by the rectangle O E G X O, in which the vertical O E = G 6 = the work done in one stroke divided by the distance O 6, and the sum of the areas of E H O + K G 6 = area D K H.

FIG. 23.



CASE II.—Let S be variable (steam cut-off) and the length of the connecting-rod assumed infinite.

The different pressures caused by cutting off steam can be calculated by Mariotte's law, or more correctly taken from an indicator diagram, as shown in Fig. 24, by laying off versines, say for each 30 degrees of arc on a diameter equal to the length of the indicator diagram.

FIG. 24.

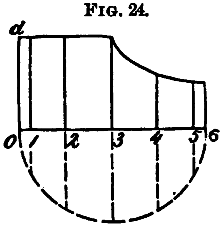
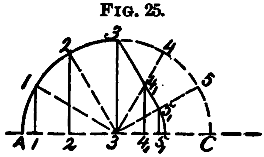
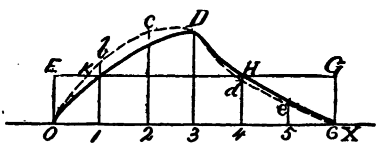


FIG. 25.



With the greatest force as a radius = O a, Fig. 24, describe the semi-circle A 3 C with 3 as a centre, Fig. 25; divide this into six equal arcs and draw the radii 1, 2, 3, 4, 5 to 3; upon these radii lay off the forces as measured from the corresponding ordinate of the indicator diagram, Fig. 24. Connecting these extremities we have the curve A 1, 2, 3, 4_1, 5_1, 6_1, and the verticals 11, 22, 33, 4_1, 4, 5_1, 5; from the intersections of this curve with the radii to the line A C give the tangential forces acting upon the crank.

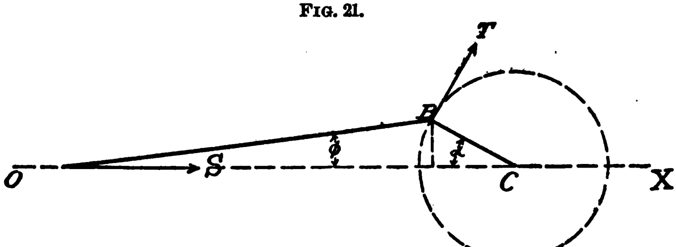
Fig. 26.



If we divide the line O X, Fig. 26, representing the length of half the crank-circle to any convenient scale, into six equal parts, and at the points of division erect verticals equal in length to the verticals in Fig. 25, we obtain the irregular solid curve O D X. The area of this figure, O D X O, equals the work done in one stroke.

Case III.—Let the force S be variable and the length of the connecting-rod be taken into consideration.

Fig. 21.



Referring to Fig. 21, we see that the effect of the connecting-rod of finite length is to cause the piston to move farther than the crank-pin in a horizontal direction in the first and fourth half strokes, and to move a less distance than the crank in a horizontal direction in the second and third half strokes. This can be shown in the following manner:

We have, for the work done in the cylinder with a *variable* velocity and on the crank-pin with a *constant* velocity,

$$w = Sr[(1 - \cos \alpha) + n(1 - \cos \varphi)]; \qquad (190)$$

$$\sin^2 \varphi = \frac{\sin^2 \alpha}{n^2}, \text{ since } r \sin \alpha = nr \sin \varphi;$$

$$\cos \varphi = \sqrt{1 - \sin^2 \varphi} = \sqrt{1 - \frac{\sin^2 \alpha}{n^2}} = 1 - \tfrac{1}{2}\frac{\sin^2 \alpha}{n^2} - \tfrac{1}{8}\frac{\sin^4 \alpha}{n^4};$$

$$w = Sr\left[1 - \cos \alpha + \frac{\sin^2 \alpha}{2n}\right].$$

Neglecting quantities containing greater than the second power of $\frac{\sin \alpha}{n}$, and differentiating, we have

$$\frac{dw}{d\alpha} = Sr\left[\sin \alpha + \frac{1}{2n} 2 \sin \alpha \cos \alpha\right],$$

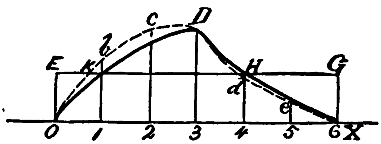
and the tangential force which is proportional to the first differential coefficient of the work, since the velocity of the crank-pin is constant, is proportional to

$$\sin \alpha + \frac{\sin 2\alpha}{2n}. \qquad (191)$$

Attention must be paid to the signs of the circular functions.

Laying down a straight line O X, Fig. 26, dividing it, and erecting ordinates as before, we lay off the length of these ordinates, which in the first quadrant are greater and in the second quadrant less by $\frac{\sin 2a}{2n}$ than for the solid curve O D X.

FIG. 26.



In the third quadrant the ordinates will be less and in the fourth quadrant greater by $\frac{\sin 2a}{2n}$ than in Case II., giving the broken line curve O b c d e 6.

The work lost, being lost uniformly, can be represented by the rectangle O E G X O, and the height of this rectangle = O E = G X is equal to the work done in one stroke, represented by the figure O b D d X O, divided by half the length of the crank-circle = O X.

The areas of these figures, as also the equal amounts of work lost and gained by the fly-wheel, can be calculated by means of Simpson's rule, or more conveniently by the use of the polar planimeter.

Table IV. gives the work lost and gained by the fly-wheel in terms of the fractional part of the whole work done by *one* cylinder in one stroke, or, what is the same thing, the value of the coefficient c in equation (178) for the common values of $n = \frac{l}{r}$, and the usual points of cut-off when the influence of the weight and velocity of the reciprocating parts is disregarded.

If we wish to use this table when the indicated horse-power of an engine is given, divide the horse-power of *one* cylinder by the number of strokes per minute, and multiply it by 33000 foot-pounds, since the work done in one stroke

$$SL = 33000\frac{(HP)}{N},$$

and multiply the result by the coefficient, given in the table.

TABLE IV.

[From *Des Ingenieurs Taschenbuch*, page 379.]

$n = \frac{l}{r}$	Engine without expansion. Single crank.	Two cranks.	Three cranks.
4	0.2717	0.1672	0.0693
5	0.2577	0.1422	0.0594
6	0.2489	0.1256	0.0504
7	0.2489	0.1136	0.0453
8	0.2384	0.1046	0.0414
Infinite	0.2105	0.0422	0.0181

Engine with expansion. Steam cut off at—	$\frac{2}{3}L$	$\frac{1}{2}L$	$\frac{1}{3}L$	$\frac{1}{4}L$	$\frac{1}{5}L$	$\frac{1}{6}L$
Single crank	0.3741	0.4076	0.4372	0.4523	0.4625	0.4702
Two cranks	0.2044	0.2250	0.2412	0.2496	0.2552	0.2594
Single crank	0.3252	0.3594	0.3916	0.4088	0.4216	0.4300
Two cranks	0.0652	0.0720	0.0786	0.0820	0.0846	0.0862

It is the best plan to make use of the coefficient of single cranks for double cranks, and of the coefficient for double cranks for treble cranks, since it is sometimes necessary to disconnect one cylinder for repairs, which we should be able to do without creating such irregularities as will render the engine unserviceable.

If we take the pressures upon the crank-pin as altered by the weight and velocity of the reciprocating parts, as shown in Fig. 10, Art. 35, and treat them as explained in this article, we can obtain an exact representation of the work lost and gained by the fly-wheel. For ordinary practice, with the weight and velocity of the reciprocating parts properly adjusted, we can use the coefficients for engines without expansion in Table IV.

CHAPTER XIII.

(61.) The Weight of the Rim of Fly-Wheels.—Referring to equation (166), we see that the amount of work which any body can store up without having its velocity increased, or give out without having its velocity decreased more than the fraction m of the mean velocity u, is

$$w = \frac{m}{g} Wu^2.$$

If now we place this amount of work, equal to the amount given by equation (178), as the excess or deficiency of work to be stored or given out by the fly-wheel, we have

$$\frac{m}{g} W_f u^2 = .7854\, cPLd^2;$$

therefore, $$W_f = 25.29 \frac{c}{m} \cdot \frac{PLd^2}{u^2}, \qquad (192)$$

in which L is the length of stroke *in feet*, and u the mean velocity of the rim of the wheel in *feet per second.*

If for u we wish to substitute the mean diameter of the fly-wheel rim D and the number of strokes per minute N,

$$u = \frac{\pi DN}{2 \times 60} = .02618\, DN.$$

Squaring and substituting in formula (192), we have

$$W_f = 36899. \frac{c}{m} \cdot \frac{PLd^2}{D^2N^2}. \qquad (193)$$

The mean diameter of a fly-wheel D is usually taken at from 3 to 10 times the length of the stroke. Inspection of formula (193) shows that, other things being equal, it will diminish the weight and cost of a fly-wheel to increase the number of strokes the diameter of the wheel, or to increase the fractional coefficient m.

(62.) Value of the Coefficient of Steadiness m. —Rankine recommends taking $m = \frac{1}{32}$ for ordinary machinery, and $= \frac{1}{50}$ to $\frac{1}{60}$ for machinery requiring unusual steadiness.

Watt's rule, given by Farey, and regarded by him as giving sufficient regularity for the most delicate purposes, is, "Make $\frac{1}{2}$ the *vis viva*—*i. e.*, the work—stored in the fly-wheel equal to the work done by the engine in $3\frac{3}{4}$ strokes," and gives $m = \frac{1}{36}$.

Bourne's rule for all engines is, "Make the work stored in the fly-wheel equal to that developed by the steam-cylinder in six strokes," which would give m little greater than $\frac{1}{60}$, a quantity much too small for ordinary purposes, but nearer right than he is usually. Nystrom suggests making m, in practice, to vary between $\frac{1}{10}$ and $\frac{1}{100}$.

Morin gives, for engines requiring great regularity, the value of m at from $\frac{1}{37}$ to $\frac{1}{45}$, which conforms with the best practice, for engines of great steadiness of motion.

$m = \frac{1}{20}$ is a good value for engines in which small temporary fluctuations of speed are of little consequence.

The following values of m are taken from *Des Ingenieurs Taschenbuch*, Hütte, page 378:

For machines which will permit a very uneven motion, as for hammer-work, $m = \frac{1}{5}$.

For machines which permit some irregularity, as pumps, shearing-machines, etc., $m = \frac{1}{20}$ to $\frac{1}{30}$.

For machines which require an approximation to a uniform speed, as flour-mills, $m = \frac{1}{25}$ to $\frac{1}{35}$.

For machines which demand a tolerably uniform speed, as weaving-machines, paper-machines, etc., $m = \frac{1}{30}$ to $\frac{1}{40}$.

For machines which demand a very uniform speed, as cotton-spinning machinery, $m = \frac{1}{40}$ to $\frac{1}{60}$.

For spinning machinery for very high yarn numbers, $m = \frac{1}{100}$.

Example.—Let $n = \frac{l}{r} = 5$; then from column first of the table we have $c = .2577 = \frac{1}{4}$ approx.

Let the uniform pressure be $P = 61.5$ pounds per square inch, $m = \frac{1}{20}$, $L = 1$ foot, $d = 12$ inches, $D = 5$ feet, $N = 100$ strokes per minute.

Substituting in formula (193), we have

$$W_f = \frac{36899 \times 20 \times 61.5 \times 144}{4 \times 25 \times 10000} = 6535 \text{ pounds.}$$

Upon reflection, we see that this would be comparatively a very great weight, and, if possible, it will be best to increase the diameter of the fly-wheel, so as to lessen its weight. Letting $D = 10$ feet, we have

$$W_f = 1634 \text{ pounds,}$$

with an equal coefficient m of steadiness.

(63.) Area of the Cross-Section of the Rim of a Fly-Wheel.—*Given the weight and mean diameter of the rim of a fly-wheel to determine the area of its cross-section.*

Let D be the mean diameter in feet of a fly-wheel rim.
" W_f be the weight in pounds.
" F be the cross-section of a fly-wheel rim in square inches.

We have for the mean perimeter in inches

$$12\pi D,$$

and for the volume of the rim in cubic inches

$$12\pi DF.$$

One cubic inch of iron weighs about .26 of a pound, and we therefore have

$$W_f = .26 \times 12\pi FD.$$

Therefore, $$F = .10176 \frac{W_f}{D}. \qquad (194)$$

Examples.—Let $W_f = 6535$ pounds.
" $D = 5$ feet.

We have $$F = .10176 \frac{6535}{5} = 133.05 \text{ square inches.}$$

Again, Let $W_f = 1634$ pounds.
" $D = 10$ feet.

We have $$F = .10176 \frac{1634}{10} = 16.63 \text{ square inches.}$$

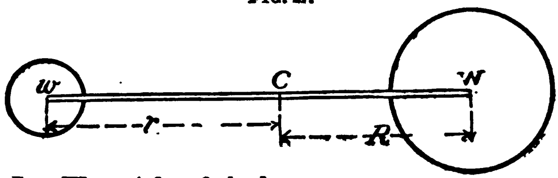
(64.) **Balancing the Fly-Wheel.**—When the fly-wheel is erected, great care should be taken that its centre of gravity coincides with the centre of the shaft upon which it is placed.

It is not necessary that it be *perfectly* circular, so long as its centre of gravity coincides with its axis, as will presently be shown.

With regard to a plane passing through its centre of gravity at right angles to the axis, the wheel must be perfectly symmetrical. If it is not, the centrifugal force will give rise to a couple tending to place it in that position, and of course straining the shaft and the fly-wheel unnecessarily. In both cases these disturbing forces will increase with the square of the velocity.

If any two masses, as W and w, are in statical equilibrium with regard to an axis C; if caused to revolve, they will have no tendency to leave their common axis.

FIG. 27.



Let W = weight of the larger mass.
" w = weight of the smaller mass.
" R = distance of the larger mass W from C.
" r = distance of the smaller mass w from C.

Since these bodies are assumed in statical equilibrium, we have

$$WR = wr.$$

The centrifugal force of the smaller body $= \frac{wv^2}{gr}$.

The centrifugal force of the larger body $= \frac{WV^2}{gR}$.

Letting N represent the number of turns of the connected bodies about C per second, we have

$$V = 2\pi RN, \qquad v = 2\pi rN,$$

and $$\frac{wv^2}{gr} = \frac{4w\pi^2r^2N^2}{gr}, \quad \text{and} \quad \frac{WV^2}{gR} = \frac{4W\pi^2R^2N^2}{gR}.$$

And dividing, we have

$$\frac{\frac{wv^2}{gr}}{\frac{WV^2}{gR}} = \frac{wr}{WR}.$$

But $wr = WR$. Hence, the centrifugal forces of the two masses are equal and opposite, which was to be shown.

The fly-wheel is sometimes purposely erected out of balance, in order to force the crank over some particular point, but this method of proceeding results in strains upon the shaft and its bearings, which tend to injure them by causing wear and vibration, and should not be used when avoidable. Refer to Art. (35).

(65.) Speed of the Rim of a Fly-Wheel.—It has been stated in article (61) that increasing the speed of the rim of a fly-wheel, or, what is the same thing, increasing its diameter and number of revolutions, one or both, is productive of economy in its weight.

There is, however, a limit beyond which the speed of rim cannot be driven without bursting the rim.

If we suppose the rim to be solid, and neglect the support that it receives from its arms, we first have the case in which centrifugal force acts in a similar manner to the outward bursting-pressure of water. (Weisbach's *Mechanics of Engineering*, sec. vi., art. 363.)

Letting T = the tensile strength per foot of area,

" $\frac{D}{2}$ = mean radius of rim in feet,

" u = velocity of rim in feet per second,

" G = weight per cubic foot,

" p = radial force of each cubic foot due to centrifugal force,

we have for the centrifugal force of each cubic foot $= p = \frac{2Gu^2}{gD}$, and also, as shown in Weisbach's *Mechanics of Engineering*, the resisting force is $p = \frac{2T}{D}$. Equating these values, we have

$$T = \frac{Gu^2}{g}.$$

Therefore,

$$u = \sqrt{\frac{g}{G}T}. \qquad (195)$$

Examples.—For a cast-iron rim, assuming $T=2592000$ pounds per square foot, and its weight $G=450$ pounds per cubic foot, we have for its bursting speed

$$u=\sqrt{\frac{32.2\times 2592000}{450}}=430.7 \text{ feet per second.}$$

If we use a factor of safety of 10, we have

$$u=\sqrt{\frac{32.2\times 259200}{450}}=136.2 \text{ feet per second}$$

for a safe speed.

The speed of rim of fly-wheels is in some cases pushed to about 80 feet per second, but is probably not often exceeded.

It is of interest to note that if 5000 pounds per square inch be regarded as a safe strain for a railway tire when subjected to shocks occurring while in motion, the greatest speed of a locomotive with safety may be deduced.

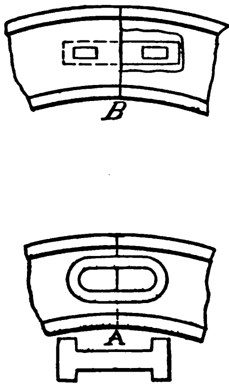
We have $u=\sqrt{\frac{32.2\times 720000}{490}}=217.\frac{1}{2}$ feet per second,

or about 2.47 miles per minute.

CHAPTER XIV.

(66.) Centrifugal Stress on the Arms of a Fly-Wheel.—In the case just discussed the fly-wheel has been supposed to rely entirely upon the strength of its solid rim, regardless of any support it might have from the arms; when, however, large wheels are constructed, the difficulties attendant upon handling them, as well as the expense of making large castings, make it necessary to cast the rim in sections, which are put together in place, the arm sometimes being cast with its section of rim, which is the best method when practicable, and sometimes in a separate piece. These segments on being put in place are caused to abut firmly upon each other, and held in position by means of a prisoner and keys, as shown at B, or by means of links, as shown at A, Fig. 28. A double-headed bolt sometimes takes the place of the link. Both link and bolt when used are heated and then allowed to contract in place, drawing the ends of the segments solidly together.

FIG. 28.



For various practical mechanical reasons, the strength of the bolts, prisoners or links cannot be relied upon, and they should only be considered as fastenings to preserve the form of the fly-wheel.

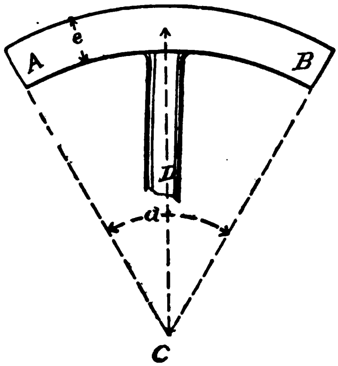
The arm for each segment should be so proportioned as to support with perfect safety its weight when at its lowest position, and also the stress due to its centrifugal and tangential force, a slight taper being given to the arm, increasing from the rim to the hub, to allow for the weight and centrifugal force of the arm itself.

The centrifugal force of any mass is exactly what would result if its whole mass is supposed concentrated at its centre of gravity and the motion of this point alone considered. We have (Weisbach's *Mechanics of Engineering*, sec. iii., art. 115), for the distance from the centre of the wheel to the centre of gravity of the segment, $= x$,

$$x = \frac{\sin\frac{a}{2}}{a}\left[1 - \frac{1}{3}\left(\frac{e}{D}\right)^2\right] D,$$

in which e is the thickness of rim, a = the angle A C B, and D is its mean diameter. Fig. 29.

FIG. 29.



Let W_f = the weight of the rim in pounds.

" M = the number of sections = number of arms into which the rim is divided.

" C = the centrifugal force of each segment.

" u = the velocity of the rim in feet per second.

" v = the velocity of the centre of gravity of each segment.

We have $$C = \frac{W_f}{M} \; \frac{v^2}{gx};$$

and since $$u : v :: D : 2x,$$

we have $$v^2 = \frac{4x^2u^2}{D^2}.$$

Substituting the value of x given above, we have

$$C = \frac{W_f}{M}\left(\frac{4xu^2}{gD^2}\right) = 4\frac{W_f}{M} \; \frac{u^2}{gD}\left\{1+\tfrac{1}{3}\left(\frac{e}{D}\right)^2\right\}\frac{\sin\frac{a}{2}}{a}.$$

In this latter formula, if e is very small in comparison to D, we can neglect the small term $\frac{1}{3}\left(\frac{e}{D}\right)^2$, and the equation becomes

$$C = 4\frac{W_f}{M}\ \frac{u^2 \sin\frac{a}{2}}{gDa}.$$

If to this expression for the intensity of the centrifugal force of each segment we add its weight for its lowest position, we have, calling Y the strain on any arm in the direction of the radius of the wheel,

$$Y = \frac{W_f}{M}\left\{1 + 4\frac{u^2 \sin\frac{a}{2}}{gDa}\right\}; \qquad (196)$$

and since $u^2 = .0006853924\,D^2N^2$, we have

$$Y = \frac{W_f}{M}\left\{1 + \frac{.00274156\,DN^2 \sin\frac{a}{2}}{ga}\right\}, \qquad (197)$$

in which $g = 32.2$ feet per second.

If we divide the intensity of the force Y by the safe strain in tension per square inch of the material of the arm, we obtain the required cross-section to resist rupture from the centrifugal force and weight of each segment.

Example.—Let $W_f = 1634$ pounds. Let $M = 6$—*i. e.*, the rim be divided into six parts, each having its arm. Let $D = 10$ feet and $N = 100$ strokes per minute.

We also have $\frac{a}{2} = 30°$, and $a = \frac{2\pi}{6} = 1.0472.$

$$Y = 272.37\left\{1 + \frac{.00274156 \times 10 \times 100^2 \times .5}{32.2 \times 1.0472}\right\} = 5.096 \times 272.37 =$$

1388 pounds approximately.

(67.) Tangential Stress on the Arms of a Fly-Wheel for a Single Crank.—In addition to the stress on the arm due to its weight and centrifugal force, each arm sustains a tangential strain at its extremity due to the inertia of the rim, which, in case of sudden stoppages, is sometimes of very great intensity.

For the case of any ordinary fly-wheel, *whose only office is to equalize the work* given out in each element of time or arc,

Letting R = the mean radius of the rim in feet,
" X = the sum of the tangential forces at the extremities of the arms in pounds,

we have, for the work given out or absorbed for each element da of arc,

$$XRda.$$

We also have, equation (33), for the work received from the steam-cylinder for each element da of arc,

$$Sr \sin a \, da,$$

and, equation (35), for the work given out uniformly for each element da of arc, $Sr\frac{2}{\pi}da$.

We see that the rim is called upon at certain points or during certain arcs to assist or resist the work given out by the steam-cylinder.

If we place

$$XRda = Sr \sin a \, da - Sr\frac{2}{\pi}da, \qquad (198)$$

or

$$X = \frac{Sr}{R}\left(\sin a - \frac{2}{\pi}\right),$$

we obtain the measure of the tangential force X.

$$\text{For } a = \left\{\begin{array}{r} 39^\circ\ 32'\ 25'' \\ 140^\circ\ 27'\ 35'' \\ 219^\circ\ 32'\ 25'' \\ 320^\circ\ 27'\ 35'' \end{array}\right\} X = O.$$

For $a = 90° = 270°$ we have

$$X = +.36338\frac{Sr}{R}.$$

For $a = 0° = 180°$ we have

$$X = -.636618\frac{Sr}{R}. \qquad (199)$$

This last value of X represents the maximum value of X, being its value for the two dead points; and if we divide this by the number of arms, we obtain the force at the extremity of each arm which tends to bend or to break it at its junction with the hub of the wheel, or the rim. We have (Weisbach's *Mechanics of Engineering*, sec. iv., art. 272) the following equation for a beam fixed at one end and loaded at the other:

$$T = \frac{Y}{F} + \left(\frac{X}{M}\right)\frac{le}{W},$$

in which M = the number of arms; F = cross-section of an arm in square inches; W = its measure of the moment of flexure; l = the length of arm in inches; T = the proof (or safe) stress per square inch; Y = the radial stress on each arm; X = the tangential stress on each arm; and e = the *half diameter of the arm in the plane of the fly-wheel;* or inversely,

$$F = \frac{1}{T}\left\{ Y + \frac{Fe}{W}\left(\frac{X}{M}\right)l \right\}. \qquad (200)$$

In this formula, for round and elliptical arms

$$\frac{Fe}{W} = \frac{4}{e}.$$

For rectangular arms,

$$\frac{Fe}{W} = \frac{3}{e}.$$

(Weisbach's *Mechanics of Engineering*, Art. 236, Sec. iv.) In formula (200) the value of e remains to be determined approximately. This can be done by substituting in either of the two following formulæ, and taking the greater value,

$$e = \frac{WTM}{Xl}. \qquad (201)$$

(Weisbach's *Mechanics of Engineering*, Art. 235, Sec. iv.)

$$F = \frac{Y}{T}. \qquad (202)$$

Example.—Let us assume the shape of the arm of the fly-wheel already discussed to be elliptical.

$$Y = 282 \text{ pounds}, \quad \frac{X}{M} = 141 \text{ pounds}.$$

We have, equation (202),

$$\pi eb = F = \frac{Y}{T}.$$

Therefore, $$e = \frac{Y}{\pi b T}.$$

in which b = the smaller ½-diameter of the arm assumed, and, equation (201), since $W = \frac{\pi e^2 b}{4}$ for an ellipse,

$$e = 2\sqrt{\frac{Xl}{\pi b M T}}.$$

(Weisbach's *Mechanics of Engineering*, Art. 231, Sec. iv.)

Let $b = 1''$, $T = 1800$, and $l = 60$ inches.

$$e = \frac{282 \times 7}{22 \times 1800} = .05 \text{ inch},$$

or $$e = \sqrt{\frac{141 \times 60 \times 7}{22 \times 1800}} = 2\sqrt{\frac{592.2}{396}} = 2.44 \text{ inches}.$$

The second value of $e = 2.44$ inches must be substituted in formula (200), and we have

$$F = \frac{1}{T}\left\{Y + \frac{4}{e}\left(\frac{X}{M}\right)l\right\} = \frac{1}{1800}\left\{282 + \frac{4 \times 141 \times 60}{2.44}\right\}$$

$$= 7.862 \text{ square inches.}$$

We have assumed $b = 1$ inch; and since $F = \pi eb$, we have

$$e = \frac{7}{22} \times 7.86 = 2.5 \text{ inches.}$$

We thus see that each arm of a fly-wheel of the dimensions indicated should be of an elliptical form, whose major and minor axes are respectively 5 and 2 inches.*

It is customary to give the arms a slight taper from the hub to the rim.

(68.) Work Stored in the Arms of the Fly-Wheel.—If we wish to take into account the weight of the arms in estimating the work stored in the fly-wheel, we have, letting u = velocity of rim, W_a = the total weight of the arms, and w = work stored,

$$w = 0.325 \frac{W_a}{2g} u^2, \text{ approximately,}$$

which can be added to the work stored in the rim.

For a more general and less practical analytical discussion of fly-wheels, reference may be made to the works of Morin, Dulos, Poncelet and Resal.

Dr. R. Proel, in his *Versuch einer Graphischen Dynamik*, gives very clear and elegant graphical methods of represent-

* If the power of the engine is conveyed by means of a band or geared fly-wheel, we must calculate the tangential stress upon the arms by means of the theorem of moments, regarding the crank as the short lever at whose extremity the whole steam-pressure acts.

ing the work lost and gained by a fly-wheel under various conditions.

It perhaps appears superfluous to some of our readers to enter into detail to so great an extent as has here been done, but the danger and loss resulting from the accidental breakage of a fly-wheel demand the most painstaking care in establishing its dimensions.

(69.) **The Working-Beam.**—The working-beam is becoming less used as the speed of the steam-engine is increased; it is preferably constructed of wrought iron or steel, or, if made of cast iron, is in many instances bound around with wrought iron. Its form, if solid, should be parabolic, with the vertex at the point where the connecting-rod joins it, and the load at that point is the total pressure of the steam upon the piston-head. (Weisbach's *Mechanics of Engineering*, sec. iv., arts. 251–52–53.)

The working-beam is supposed to be fixed at its central bearing, and thus becomes a beam fixed at one end and loaded at the other. See Table VI.

Where web-bracing is used in working-beams, the graphical method will afford the simplest solution. (See *Graphical Statics*, Du Bois.)

(70.) **General Considerations.**—The recent improvements in parallel motions will probably lead to their more general use in the place of guides and slides. A most interesting and instructive little work, *How to Draw a Straight Line*, by A. B. Kempe, suggests to the mechanician many forms which can be adapted to the steam-engine with little trouble.

In the foundation and framework of engines every precaution must be taken to obtain RIGIDITY and immovability.

Too much stress cannot be laid upon this point; an in-secure foundation inevitably injures, and perhaps ruins, the engine upon it.

The centrifugal governor has not been considered, because it forms one of the principal topics in almost every work on the steam-engine.

The practical defects of the centrifugal governor are insurmountable when a perfectly regular speed is desired of the engine. They are as follows:

(1.) The engine must go fast in order to go slow, or the reverse, since the balls cannot move without a change of speed in the engine and themselves.

(2.) The opening of the steam-valve is dependent upon the angle which the arms attached to the balls form with the central spindle around which they revolve.

Thus, an engine having its full amount of work, and governed by an ordinary ball-governor, will be kept at a uniform speed by the governor so long as the average resistance to be overcome by the engine remains constant; but whenever any of the work is taken off, the speed of the engine will be increased to a higher rate, corresponding to the diminished work, and at this faster speed the engine will then run uniformly under the mastery of the governor so long as the work continues without further alteration. This arises from the fact that the degree of opening of the steam-valve is directly controlled by the angle to which the governor-balls are raised by their velocity of revolution, the steam-valve being moved only by a change of speed, and consequently by a change of the angle of suspension of the governor-balls; whence it follows that a larger supply of steam for overcoming any increase of work can be obtained only in conjunction with a smaller angle of the suspension-rods of the governor-balls, and consequently with a slower speed, and that a larger angle of the ball-rods, and consequently a higher speed, must be attained in order to reduce the supply of steam for meeting any reduction of work to be done by the engine.

(3.) The governor must be sensitive—*i. e.*, quick to act. This result is usually attained in the centrifugal governor by giving to the balls a speed much greater than that of the engine, so that a slight variation of speed in the engine is multiplied in the governor many times.

A high speed, however, is attended with the disadvantage of rapid wear, and, in the case of an ordinary governor, wear such as to admit of any lost motion is attended with much trouble to the engineer and sudden variations of speed in the engine.

(4.) The governor must have power, which means an even and sure motion of the valve notwithstanding the almost unavoidable defects of workmanship, such as the sticking of the valve or the binding of the valve-stem through careless packing of the stuffing-box. In the ordinary governor this power is sought to be obtained either by a high speed, the defects of which have already been pointed out, or by means of very heavy balls, which results in a very cumbersome and large machine, besides adding largely to the expense.

Thus we see that not only is the speed of the steam-engine entirely different with different loads, but also that with a constant load the speed varies between limits which are determined by the sensitiveness of the governor and is at no time regular.

The necessity of a very sensitive governor is done away with by the use of a properly proportioned fly-wheel.

The use of the governor to determine the point of cut-off, as shown in the Corliss engines, if the fly-wheel be of the proper weight and size, is attended with great regularity of speed and economy of steam. Siemens' chronometric governor, in which first the inertia of a pendulum and afterward hydraulic resistance were used as a *point d'appui* to move the valve from, produces a very regular speed of engine (*Proceedings of the Institution of Mechanical Engineers*, January, 1866), but is too costly for general use.

Marks' isochronous governor (patented), in which the motion of the valve precedes any change of speed in the governor-balls, or, as since altered, in the hydraulic cup, subserves the same purpose, and is much cheaper than the former. (*Journal of the Franklin Institute*, May, 1876.)

A vast number of forms of governor of varying merit have been invented, this portion of the steam-engine appearing to be the most attractive to, and the most considered by, mechanics and engineers.

The only test of beauty and elegance of design in an engine is fitness and perfect proportion to the stresses placed upon the various parts.

The severest simplicity of design should be adhered to. Every pound of metal, where it does not subserve some useful purpose, every attempt at mere ornament, is a defect, and should be avoided.

The well-educated engineer should combine the qualities of the practical man and the physicist; and the more he blends these together, making each mould and soften what the other would seem to dictate if allowed to act alone, the more will his works be successful and attain the exact object for which they are designed.

The steam-boiler and its construction will be found to be very thoroughly treated in Professor Trowbridge's *Heat and Heat Engines*, in Wilson's *Steam-boilers*, and in *The Steam-Engine*, by Professor Rankine.

(71.) **Note on the Taper of Connecting-Rods.**—The following remark, page 62, needs qualification:

"It is customary to make round connecting-rods with a taper of about one-eighth of an inch per foot, from the centre to the necks, which should be of the calculated diameter. Experiment does not show an increased strength from a tapering form."

Passed Assistant Engineer C. H. Manning,* in a pleasant correspondence with the writer, differed from him, deeming it a better method to taper connecting-rods from the crank-pin end to the cross-head end, as "experience had shown that connecting-rods usually failed at the crank-pin."

Led by this discussion to make a more thorough investigation into the stress upon connecting-rods due to their own inertia than he had before deemed necessary, the writer submits the results, hoping they may be of interest to engineers engaged in the designing of mechanism.

The connecting-rod, if of *wrought iron*, must be considered either, 1st, as a short column, tending to rupture by crushing, or 2d, as a long column, tending to rupture by buckling. The tendency to fail in tension can be neglected, as being much less than in the two cases mentioned.

In the first case it is obvious that tapering will not add to its strength if we neglect the stress due to its inertia and weight.

In the second case, theoretically, if we disregard the stress in flexure due to the inertia and weight of the connecting-rod, the increase in its diameter will be a maximum at its centre. (See Weisbach's *Mechanics of Engineering*, Sec. iv., Art. 267.)

The connecting-rod, if of *steel*, may be considered, 1st, as a tension-rod, tending to fail in tension, or 2d, as a long column, tending to rupture by buckling. The ability of steel to withstand a much greater stress in compression than in tension avoids the necessity of considering it as a short

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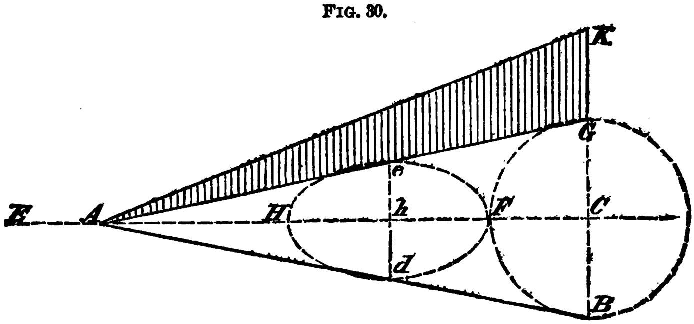
column, failing by crushing. In both cases the inertia and weight of the rod are disregarded.

These are the general conditions which have controlled the mechanical engineer in the consideration of the proportions of wrought-iron or steel connecting-rods. In addition, it has been customary, in order to meet an unknown stress in flexure due to the inertia of the rod, to increase the diameter of the rod at the middle, or latterly to give an increased diameter at the crank-pin, and taper from this to the smallest dimension of the rod at the necks or neck, this increase being purely empirical.

FIG. 30.



Another correspondent says: "Where iron and steel are used, the figures adopted as constants must vary greatly, especially with steel. The value of E (modulus of elasticity) varies *enormously;*" with which opinion the writer agrees perfectly, only regretting that the very basis of all correct calculations is thus taken from us, and we are forced to be contented with results which have a reasonable probability of correctness if we assume average values for the safe stresses per square inch and the moduli of elasticity of the material with which we are dealing.

If a straight line, A B, Fig. 30, have its extremities A and B respectively caused to move reciprocally upon the straight line E A H and in the perimeter of the circle B F G, any point upon the line A B as d will trace an approximate ellipse as d H e F.

If, now, we let the length of the line $AB = l$; the radius CB of the circle $= r$; the variable distance of the point d from the point $A = Ad = x$; we have the semi-major axis of this ellipse $hF = hH = r$, and the semi-minor axis $hd = he = \frac{x}{l} r$, and the radius of curvature of the osculatory circle at the vertex d of the semi-minor axis,

$$R = \frac{l}{x} r. \tag{203}$$

The maximum resistance due to its inertia of any small mass m at the end B of the rod A B, to motion in the direction B C, is equal to its centrifugal force,

$$c = \frac{mv^2}{r}, \tag{204}$$

in which $v =$ the linear velocity of the point B in feet per second.

(For a demonstration of this refer to a paper by F. A. P. Barnard, *Transactions of the American Institute.*)

Observing now that in the position shown in the figure every point on the line A B as d is moving with a velocity v in an arc of an osculatory circle whose radius is $R = \frac{l}{x} r$, equation (203), we have the means of determining the resistance due to the inertia of each element of mass m.

For the point d it would be

$$\frac{mv^2}{R} = \frac{mxv^2}{lr} \tag{205}$$

for the point A, since $x = 0$, $R = \infty$, and the resistance of a mass m would be $=0$. We thus see, equation (205), that the resistance of each element of mass due its inertia to motion in a direction at right angles to E A C is directly proportional to its distance $=x$ from the point A.

If, now, for the line A B we substitute a rod of *uniform* cross-section $=F$, we have

$$m = \frac{F\gamma dx}{g}, \qquad (206)$$

in which γ = weight per cubic unit, and $g = 32.2$ feet per second, F being taken in square units.

Substituting this value of m in equation (205), and integrating between the limits $x=0$ and $=l$, we have, denoting the whole resistance of the rod $A\,B$ by P,

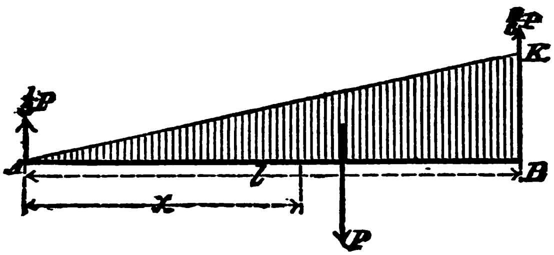
$$P = \int_0^l \frac{F\gamma v^2}{glr}\, x dx = \frac{F\gamma v^2}{2gr}\, l,$$

or since the weight of the rod $= G = F\gamma l$,

$$P = \frac{Gv^2}{2gr}. \qquad (207)$$

And since this load upon the rod due to the resistance of its own inertia increases uniformly from the end A to the end B, the rod can, with sufficient approximation, be supposed

Fig. 31.



in the condition of a horizontal beam loaded with a triangularly-shaped load, as shown, Fig. 31, in the cross-hatched portion A K B.

For a vertical engine we can neglect the weight of the connecting-rod, considering only the stress due to its inertia.

For the moment of flexure of any cross-section at a distance x from the extremity A we have, letting c = the load per unit of length at the point K,

$$\tfrac{1}{3}Px - \tfrac{1}{6}\frac{c}{l}x^3 = \tfrac{1}{3}P\left(x - \frac{x^3}{l^2}\right), \qquad (208)$$

which becomes a maximum for $x = l\sqrt{\tfrac{1}{3}} = 0.578l$, at which point, therefore, the maximum cross-section of the connecting-rod should be placed.

In horizontal engines it is necessary to take into consideration the weight of the rod, as well as its inertia; the weight of the rod may be regarded as a uniformly distributed load always acting in one direction.

For the cross-section, at a distance x from the extremity A, the moment of flexure is, letting G = whole weight of rod $= \gamma l$,

$$(\tfrac{1}{3}P + \tfrac{1}{2}G)x - \left(\gamma\frac{x^2}{2} + \tfrac{1}{6}\frac{c}{l}x^3\right)$$

$$= (\tfrac{1}{3}P + \tfrac{1}{2}G)x - \frac{G}{2l}x^2 - \frac{P}{3l^2}x^3, \qquad (209)$$

which is a maximum for

$$x = -\frac{Gl}{2P} \pm l\sqrt{\frac{1}{3} + \frac{G}{2P} + \frac{G^2}{4P^2}}. \qquad (210)$$

But we have, from equation (207),

$\frac{G}{2P} = \frac{gr}{v^2}$, in which g = acceleration of gravity, v = the linear velocity of the crank-pin, B, in feet per second, and r = the radius of the crank in feet. Substituting this value in equation (210), we have

$$x = l\left\{ -\frac{gr}{v^2} \pm \sqrt{\frac{1}{3} + \frac{gr}{v^2} + \left(\frac{gr}{v^2}\right)^2}, \right. \qquad (211)$$

which shows that as the velocity of the crank-pin increases the value of x approaches more nearly to $l\sqrt{\frac{1}{3}} = 0.578l$, but can never quite equal it.

Referring to equation (209), and taking P = zero, we find the maximum value of $x = \frac{l}{2} = 0.5l$.

We thus see that the greatest moment of flexure of any connecting-rod lies between the limits $0.5l$ and $0.578l$, measured from the point A.

It is of interest further to note that the crank-pin takes one-half the stress due to the weight of the rod, and two-thirds of the stress due to the inertia of the rod.

The end, A, takes one-third of the stress due to the inertia of the rod, alternately increasing and decreasing the stress upon the guides to this amount, and one-half the stress due to its weight.

In engines having a large number of revolutions per minute P becomes worthy of notice. In slow-moving engines it is very small, and may be neglected.

From these considerations the writer is of the opinion that the failure of connecting-rods at the neck nearest the crank-pin, *if they are properly proportioned*, is probably due to the crank-pins being out of truth, or to the seizing of heated boxes on the crank-pin, rather than to the weight or inertia of the rods themselves.

The assumption of a connecting-rod of uniform cross-section is manifestly incorrect, and only serves to prove generally a principle which should be recognized.

In the case of a practical application, graphical methods taking cognizance of the usual changes in cross-section must be used for a series of approximations. (Vide Du Bois's *Graphical Statics*, or Reuleaux, *Der Constructeur*.)

CHAPTER XV.

(72) **The Limitations of the Steam-Engine.**—Perhaps there is no more unsafe proceeding in science than to attempt to predict the limitation of the development of any of its results. Yet the theory of the steam-engine has so far been developed as to seem to permit us to mark out the boundaries of its progress with tolerable accuracy.

At least, we trust that a full criticism of its present deficiencies, and a deduction of its limitations from known laws, will interest such of our readers as are engaged in the improvement of this machine.

In this lecture we will consider the steam-engine alone, as it is in no wise responsible for the economy of the boiler which supplies the steam, or for the losses incurred in conveying the steam from the boiler to the engine.

The boiler should be considered apart from the engine; and it is well to incidentally remark here that the performances of the best types of boiler leave but little room for improvement so far as evaporation is concerned.

It is rather in the utilization of the steam after it reaches the engine that we must look for progress, and in this point, we take it, lies the value of a thorough investigation of the machine itself.

The well-known formula for the horse-power of a steam-engine* contains three factors, which can be varied at will

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* $\frac{PLAN}{33000} = (HP)$; in which P = mean effective pressure per square

—the mean effective pressure, the volume of the cylinder, and the number of strokes per minute.

Concentration of power in a small space is the greatest attribute of the steam-engine; it is its power to concentrate the strength of thousands of horses in the space of an ordinary room that renders the steam-engine so useful and indispensable to us.

For this reason we cannot go on increasing the volume of the steam-cylinder indefinitely without a proportionate increase in power; and it is only necessary to remark to mathematicians, with regard to the proportions of the cylinder, that that cylinder whose stroke equals its diameter contains the maximum of volume with the minimum of condensation-surface.*

The importance of reducing condensation-surface is becoming more and more appreciated among engineers, and shortening the stroke will prove less of a bugbear when the laws of inertia, as applied to the reciprocating parts of a steam-engine, are better understood.

Shortening the stroke has the further advantages of shortening the space passed over by the piston-head during a given number of strokes, and consequently of reducing the wear on the piston-head, as well as of rendering the engine more compact for a given power.

The following formula gives for any assumed horse-power, mean effective pressure, and number of strokes per minute,

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inch; L = length of stroke in feet; A = area of piston in square inches; N = number of strokes per minute; (HP) = indicated horse-powers; $\frac{AL}{144} = V$ = volume of cylinder in cubic feet.

* Should the internal condensation of cylinder-surface prove to be directly proportional to the time of exposure, as well as to the area exposed, the stroke should be twice the diameter; but the writer doubts the practical accuracy of this statement in high-speed engines.

the required common diameter and stroke of cylinder, *in inches:*

$$d = 79.59\sqrt[3]{\frac{(HP)}{NP}}. \qquad (2)$$

Concentration being attained, the next important consideration that arises is *economy of steam;* and this is the battle-ground upon which the struggle for supremacy among our engine-builders is to-day being practically fought out, while the physicists are quite as hotly engaged in a dispute as to the relative theoretical merits of steam-, hot air-, and electric-engines, into which we will not enter.

Practically, we think that, as far as the steam-engine is concerned, the impossibility of supplying to the steam in the cylinder any appreciable amount of heat by means of steam or hot-air jackets is acknowledged by all, and the jacket is only expected to keep the cylinder warm and prevent it from abstracting heat from the steam inside the cylinder.

The jacket proving imperfect, we must find other means for preventing condensation by cooling; and they are these: Diminution of condensation-surface, as already stated above, and shortening of the duration of the time for condensation, which means an increase in the number of strokes of the piston. The effect upon the structure of the engine of increased rotative speed is not injurious to the engine, as so positively stated by many engineers; but, if attempted, rapid speed requires a far greater knowledge of the dynamics of the steam-engine than has as yet been applied to it by many constructors, and will almost inevitably shake to pieces a faultily designed structure.

The length of the crank-pin and shaft-bearings must also increase directly with the number of revolutions made, while their diameter, within ordinary limits as to pressure and speed, has no appreciable effect upon the heating of the bearing; but when we consider the practical fact that the eccentrics, possibly because of greater speed of rubbing surfaces, are always sources of large frictional losses, it would seem best to make the diameters of all bearing journals just as small as the pressures allowable upon the surfaces of contact will permit. Much yet remains to be determined with regard to the laws controlling bearings.

It has been shown conclusively that the coefficient of friction is much affected and reduced by the state of the rubbing surfaces, by the method of lubrication, and by the quality of the lubricant.

Perfect, almost mirror-like, surfaces reduce the coefficient of friction far below the 3 to 5 per cent. formerly stated by Morin as a minimum, provided a continuous lubricating apparatus is used with good sperm oil; and it will be found that a costly lubricant, such as sperm oil, is cheapest where care is exercised not to waste in its application.*

The important influence of the number of turns upon the length of bearings has not been clearly enough understood to prevent accidents from heating of the crank-pins and bearings, which is one of the most annoying mishaps which

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* Let l = the length of the crank-pin, in inches.
" L = " stroke, in feet.
" (HP) = the indicated horse-power.

Then, for marine propellor engines a practical formula, giving safe lengths, would be

$$l = 0.622 \frac{(HP)}{L}.$$

One-fourth of this length will serve safely for side-wheel or stationary engines.

For locomotive engines practice seems to prove that $l = .013d^2$ is a safe value, in which d = diameter of the steam-cylinder, in inches.

For crank-shaft bearings, letting R = the whole maximum weight on bearings, and N = the number of strokes per minute, we have $l = .000005\ RN$.

can befall an engine, and at times requires the utmost vigilance of the engineer for its prevention.

The great difficulty attendant upon making bearing and shaft press uniformly throughout the length of the journal has almost forced the use of Babbitt or soft metal for high-speed engines; and where proper care is taken in proportioning the results have justified its use.

Crank-pins shrunk in by heating the crank are apt to be out of truth, no matter how carefully the work has been done, as heat is apt to warp the crank.

Pins should be forced in cold, or the crank and pin be forged in one piece, as in the case of most small double cranks.

High rotative speed, while increasing the power of the steam-engine, also renders it more compact and diminishes the weight of the fly-wheel necessary to obtain regularity of speed.

Rotative speed has, however, its limitations, independently of the trouble sometimes caused by the heating of bearings.

For instance, it can be shown that if the safe stress in tension upon cast iron equals 1800 pounds per square inch, a cast-iron ring can be revolved safely at a maximum lineal speed of about 8000 feet per minute, which at once places a limitation upon the rotative speed of the fly-wheel, and demands reduction of its diameter as the rotative speed is increased.

Let N = the number of revolutions per minute.
" D = the diameter of cast-iron ring, in feet.

We have

$$\pi DN = 8000,$$

or

$$DN = 2546.$$

This formula would limit the diameter of the fly-wheel, if used, of a pair of engines to make 600 revolutions per minute, lately proposed by the distinguished engineer, Mr. Charles T. Porter, to about 4 feet.

Of course, as the tensile strength of the material of which the fly-wheel is made is increased, the lineal speed at which it can be driven is increased.

If we take 5000 pounds per square inch as the safe tensile strength of the steel tire of a locomotive, we find its lineal speed to be limited to about 2½ miles per minute, or 150 miles per hour, and it only remains for our engineers to provide boiler-capacity and road-bed adapted to such speeds in order that they may be realized by a properly constructed locomotive.

This speed would shorten the time between New York and Philadelphia (90 miles) to 36 minutes.

High rotative speed necessitates special precautions as to the weight of the reciprocating parts.

If we impose the condition that the initial and final stresses upon the crank-pin be equal, we must make the centrifugal force of the reciprocating parts (piston, piston-rod, cross-head, and connecting-rod), supposed to be concentrated at the crank-pin, equal to half the difference of the initial and final steam-pressures upon the whole piston-head.

High rotative speed has also demanded a change in the form of the connecting-rod, which was formerly largest at its mid-length, but has of late years been made largest at the crank-pin end; a consideration of the stress upon it due to its own inertia would place the point of greatest stress at 0.578 of its length from the cross-head end; but the accidental stresses due to the twisting of the crank-pin in its boxes recommends as a measure of safety that the rod be made largest at the crank-pin neck, and tapered down from that to the cross-head.

In a properly proportioned engine of any assumed horse-power the *only* method of reducing the weight of the moving parts is to increase the number of strokes per minute; lengthening the stroke will not do it.

The points of interest with regard to the volume of the steam-cylinder and the rotative speed of the engine having been touched upon, there remains to be considered the mean effective pressure of the steam.

In the *Journal of the Franklin Institute* for June, 1880, the writer has shown, under the assumption, approximately correct, that the steam-expansion curve is an equilateral hyperbola, that the point of cut-off giving greatest economy of steam, but not of money, for steam-engines, is determined by dividing the absolute back pressure by the absolute initial pressure of the steam.

Indicators do not give such curves, even if in perfect order, unless the steam is a little wet, but the approximation is sufficiently close for all practical purposes.

A more accurate way would be to say that the most economical point of cut-off is attained when the final steam-pressure equals the back pressure; but our first statement is more convenient in form, and will serve our purpose with all the practical accuracy necessary.

Let P = the mean effective pressure of steam in pounds per square inch.

" P_b = the initial absolute pressure of steam in pounds per square inch.

" B = the back absolute pressure on piston in pounds per square inch.

" e = the fractional part of the stroke at which the steam is cut off.

" V = the volume of the steam-cylinder in cubic feet.

Then we have

$$P = eP_b\left[1 + \text{nat. log}\,\frac{1}{e}\right] - B \qquad (212)$$

Let (HP) = the indicated horse-power.

Let S = the specific volume of the steam for a pressure, P_b.

" W = the weight of water used per horse-power.

We have

$$(HP) = \frac{PLAN}{33000}, \qquad (1)$$

in which L = the length of stroke in feet.

N = the number of strokes per minute.

A = the area of the piston-head in square inches.

Therefore

$$(HP) = \frac{\left\{ eP_b \left[1 + \text{nat. log } \frac{1}{e} \right] - B \right\} VN}{33000} \times 144, \qquad (213)$$

since $144\ V = LA$.

The quantity of water used during one minute is

$$62\tfrac{1}{2} \frac{eVN}{S}, \qquad (214)$$

$62\frac{1}{2}$ pounds being taken as the weight of one cubic foot of water, and S representing the specific volume of the steam for the pressure P_b.

Therefore, the weight of water used per horse-power is

$$W = \frac{62\tfrac{1}{2} \dfrac{eVN}{S}}{\dfrac{144}{33000} \left\{ eP_b \left[1 + \text{nat. log } \frac{1}{e} \right] - B \right\} VN}$$

$$= \frac{14323e}{S \left\{ eP_b \left[1 + \text{nat. log } \frac{1}{e} \right] - B \right\}}. \qquad (215)$$

If in this last formula we substitute $\frac{B}{P_b}$ for e, we obtain the theoretical minimum quantity of water to be evaporated per horse-power, and

$$W = \frac{14323}{P_bS.\ \text{nat. log}\ \frac{P_b}{B}} = \frac{6220.3}{P_bS\ \log \frac{P_b}{B}}. \qquad (216)$$

For one-horse-power per hour we would have, multiplying by 60,

$$60\,W = \frac{373218}{P_bS\ \log \frac{P_b}{B}}. \qquad (217)$$

As, for instance, for a non-condensing engine having a gauge-pressure of 45 pounds, and cutting off at $\frac{1}{4}$ the stroke, we find the minimum limit of evaporation of water required to be about 24 pounds of water per horse-power per hour.

Mr. Charles E. Emery has experimentally realized as low as 39 pounds of water per horse-power per hour with the given pressure in small non-condensing engines.

If in the last formula we regard the product of the pressure and the specific volume, P_bS, as constant, which it is with a considerable degree of approximation, we observe that the economy of steam varies directly as the log $\frac{P_b}{B}$, showing that both Watt and Oliver Evans were partially right in their attempts to increase the economy of the steam-engine. Watt was right in perfecting more and more the vacuum obtained, and Evans in increasing the steam-pressure at the boiler. Neither of them were wholly astray, but were soon met and their progress stopped by the slow increase of the economy beyond certain limits, and by the practical difficulties arising from surface-condensation in the pursuit of a partially apprehended law.

It is evident that we are limited as to the steam-pressure, and that a great increase of that, or a great decrease of back pressure by reason of more perfect vacuums, will render the point of cut-off so early, the mean effective pressure so low, and the condensation so great that the necessary increase of volume in the steam-cylinder will prevent that compactness and concentration of power so desirable in the steam-engine; and, although it has already been shown that high rotative speed aids in rendering the engine compact, we are also limited in that direction.

Regularity of speed is an imperative condition in all engines used to drive high-speeded machinery; under such circumstances regularity becomes the first requisite, and to it all other considerations must be subordinated.

Mr. G. H. Corliss, with his automatic cut-off engine, was the first to satisfactorily solve this problem.

With a constant steam-pressure and a constant load an engine will run, under the control of a throttling governor, with an approximation to regularity determined by the sensitiveness of the governor. With a throttling governor every change of the steam-pressure, every change in the load, causes a change in the speed of the engine, no matter how well proportioned it may be.

The laws regulating the centrifugal governor are such as to prevent it from ever becoming a very perfect mechanism, for the engine *and governor* must go fast in order to go slow, and *vice versâ*. With an automatic cut-off governor the speed will be held with greater regularity under all steam-pressures and loads within the engine's capacity, the approximation to perfection attained being determined by the proportions of the fly-wheel and the sensitiveness of the governor.

Governors in which the inertia of the balls is utilized to check any attempted change of speed, without an appreciable change of speed in the governor itself, will ultimately supersede the centrifugal governor, which is full of radical defects.

Valve-motions actuated by a weight or spring released automatically by the *centrifugal* governor are comparatively so slow in their motions as to prevent high rotative speeds and short strokes. This form of valve-motion has been elaborated with great skill and ingenuity on such engines as the Corliss, Harris-Corliss, Reynolds-Corliss, Wheelock, and Brown, which, so long as long stroke and slow rotation is adhered to, leave but little to be desired on the score of regularity of *average* speed or of perfection of workmanship.

To a different class belong the Porter-Allen and the Buckeye engines: in these engines, which are the exponents of high rotative speeds and short strokes, the cut-off of the engine is actuated by means of a link or an eccentric, whose position is regulated by means of a *centrifugal* governor, and, whatever the speed, the valves must keep pace with the engine in its motions.

A large number of valve-motions are in use on the various forms of locomotive, marine, and stationary engines throughout the civilized world, and it is perhaps worth our while to practically note the important points of some of the older forms.

The plain hollowed slide-valve is the most commonly used on the rougher forms of engines, because of its simplicity; as, however, the earliest point at which it can cut off steam without choking the exhaust is about $\frac{5}{8}$ of the stroke, it is not economical with the high pressures and large engines of the present day or with the vacuums ordinarily attained in condensing engines.

When a reversing gear is used the Stephenson link-motion is commonly used, where simplicity of mechanism rather than economy of steam is a point to be gained.

With open rods it has a variable lead, decreasing from the dead point of the link; with crossed rods it has a variable lead, increasing from the dead point of the link.

For crossed rods the position of the piston is nearly constant at the opening of the steam-ports; this is a decided advantage. Crossed rods permit a slightly greater length of eccentric rods.

Fastening the link to the ends of the eccentric rods at a point behind the centre line of the link, as is usually done in American practice, introduces irregularities into the point of cut-off, but does not affect the lead.

When the ends of the eccentric rods are fastened to the link at points on the centre line of the link, this error does not occur, but larger eccentrics are required, which are sometimes inconvenient.

The mode of suspension of the link is far more important than is usually supposed, as imperfect suspension not only introduces serious errors in the valve-motion, but also affects the durability of the construction by producing excessive slip in the block.

Where a constant lead is desired, using different angles of advance for the two eccentrics will produce a constant lead in one direction of motion, but ruins the action of the valve-motion in the opposite direction.

The Gooch link-motion is difficult to fit into an engine, because of the distance required between the centre of the crank-shaft and the end of the radius-rod; it is very easy to handle, and should therefore be used where frequent reversing is necessary, as in the cases of yard locomotives and hoisting-engines, even though requiring considerable trouble to fit in.

If a constant lead be a great advantage, this link-motion has this advantage. The suspension-link for the link proper should be made as long as possible, and the suspension-link for the radius-rod should be attached as closely as possible to the link-block. The tumbling-shaft should be placed on the opposite side of the link from the crank-shaft.

Allan's link-motion, having a straight link, can be more easily fitted up than motions having a curved link; it has all the disadvantages of the Gooch link-motion, but, unlike it, has not a constant lead, and has a greater number of parts. Particular attention must be paid to the relative lengths of the tumbling-shaft arms for simultaneously raising and lowering the link and radius-rod. The Allan link-motion does not handle as easily as the Gooch.

Heusinger von Waldegg's link-motion is the easiest of all to handle; it can be so arranged as to have very little slip in the block, but cannot be easily attached to all forms of cross-head. It is the simplest reversing gear yet mentioned, and has a constant lead.

Pius Fink's link-motion is the simplest reversing gear in existence which at the same time can be adjusted as a cut-off; it has a constant lead, but a limited range of action, and in this point is inferior to all the others, as well as being irregular in its variation from its calculated positions. However, if carefully designed, it will do good work, and is very easy to handle and very durable.

The fact that it is impossible to avoid choking the exhaust, and the consequent compression of the steam, thereby reducing the power of the steam-engine whenever the attempt is made to cut off early in the stroke with a single valve, has led engineers to devise double valve-motions which will effect an early cut-off of the steam, independently of the motion of the hollow of the slide-valve relatively to the exhaust.

To this type belong the valve-motions of the Porter-Allen*

* The Porter-Allen valve-motion is essentially Pius Fink's.

and Buckeye engines, both of which show a wide range of action for motion in one direction, and great simplicity of structure.

The Gonzenbach link-motion, with its double steam-chest and excessively complicated outside gear, is now almost obsolete; it will not cut off sharply and well, except for very early points, and that, too, only for motion in one direction; and when properly arranged for motion in one direction gives a double admission of steam for motion in the opposite direction; it is difficult to handle, and its complication renders it very liable to break down.

The Polonceau link-motion, while it has not the double steam-chest or as much complication in its outside valve-gear, cannot cut off steam later than $\frac{1}{8}$ of the stroke, and is therefore not very well adapted to general locomotive use, where economy is sacrificed to concentration of power. It is not very difficult to handle, but, like the Gooch, presents some difficulty in adapting to the narrow limits offered by a locomotive.

Meyer's link-motion seems to have met with the greatest favor from builders of marine and stationary engines, as it presents a wide range for motion in one direction, and a greater range when used as a reversing motion than any of the others. The screws which move the expansion-blocks are liable to rust or wear, and cause trouble; the expansion-valve should have separate eccentrics to work best; it is intricate in its construction, and liable to break down. It is also difficult to handle, and absorbs much power unless the distribution-valve is balanced.

An ingenious modification of the Meyer valve-motion is the Ryder cut-off, in which a cylindrical expansion-valve, whose edges approach each other after the manner of an isosceles triangle wrapped around a cylinder, is moved around its axis by means of the governor, so as to have the same effect as that of increasing or decreasing the distance between the expansion-blocks of the Meyer cut-off.

In order that we may make ourselves clearly understood, we recapitulate:

A *really good* steam-engine should possess the following qualities: *Concentration of power, economy of steam, regularity of speed, simplicity of design, and durability of construction.*

We have endeavored to mark the limitations of the first three attributes of a good engine; the last two must be worked out in the shop.

We are perhaps too bold in our prediction, but it seems to us that the engine of the future will have a cylinder whose stroke is equal to its diameter; its speed will be limited by the tensile strength of its revolving parts; its bearings will be of great length; its cut-off will be determined, approximately at least,* by the ratio of the back pressure to the initial pressure; its valve-motion will be rigidly in connection with the crank-shaft; its reciprocating parts will be so proportioned as to put a nearly constant pressure on the crank-pin; its fly-wheel, if it has one, will be proportioned so as not to admit of more than a predetermined variation of speed during one stroke; its governor will no longer be centrifugal, but a revolving pendulum, almost isochronous, and holding the engine, with all necessary approximation, within fixed limitations, to a constant speed.

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* For reasons for varying cut-off see Lecture XVI., on The Cheapest Point of Cut-Off.

CHAPTER XVI.

(73) **The Cheapest Point of Cut-Off.**—In the *Journal of the Franklin Institute* for June, 1880, the writer published a brief paper, determining, in an approximate way, from a purely dynamical point of view, the most economical point of cut-off for steam engines.* This inquiry confined itself *entirely to the ratio of the indicated horse-power to the steam used*, and provoked an amount of criticism in German, English, and American serials which was as surprising to him as the misapprehension of the limitations of the paper, together with the way in which the intent of the writer was gratuitously assumed.

As a question of finance the subject becomes more complicated, for the engine-owner asks not, How can I save the most steam? but, All the circumstances being taken into consideration, how can I get the useful work which I require most cheaply?

So far as the delivery of useful work by the engine alone is concerned, a method has been given by Professor Rankine, and elaborated by himself as well as others; but, perhaps because of insufficiently profound study of the question from a financial point of view, or because the subject was hardly deemed worthy of his thought, the question was never exhausted by him or his followers.

The ship-owner says, How can I obtain the power to drive the propellor most cheaply?

The mill-owner says, How can I obtain the power to drive the mill-stones most cheaply? and the shop-owner says, How can I obtain the power to drive my machinery most cheaply? and, as they use the engine for the purpose of making money, wish to have it designed for that pur-

* Appended to this Lecture.

pose, and care nothing at all for purely scientific considerations.

It is to these men that the writer will endeavor to make reply, giving to a most perplexing question, involving many considerations, at least an approximately correct reply, and indicating a method which, by elaboration and a detailed consideration of the thermodynamic questions raised, will, we trust, enable an engineer to reach an economy of useful power as yet not *knowingly* obtained by other means than an inspection of the Profit and Loss account at the end of the year.

In "The Limitations of the Steam-Engine" the writer has stated as the five points of an engine: (1) concentration of power, (2) economy of steam, (3) regularity of speed, (4) simplicity of design, and (5) durability of construction.

When we do not restrict ourselves to economy of Nature's forces, economy of steam becomes economy of money.

The following assumptions are made and particulars must be understood in the discussion which follows.

The expansion-curve of steam is assumed to be, with sufficient practical accuracy, an equilateral hyperbola.

The steam made by the boiler is to the steam shown by the indicator diagram as 4 to 3. This, certainly, is not correct under all circumstances, but is an approximation derived from the experiments under favorable conditions upon the Reynolds-Corliss, the Harris-Corliss, and the Wheelock engines at the Millers' Exhibition, Cincinnati, June, 1880 (Report of J. T. Hill, pages 77 and 79), and is *nearly the same for both condensing and non-condensing engines.* Those in possession of more accurate experimental data can substitute other ratios in each case.

A percentage of the true stroke must be added at each end of the sketch which is made to allow for the clearance, which must be determined. The cost of all charges upon the engine and machinery is taken in steam for the sake of convenience; and this proceeding is perfectly proper, since money and steam are convertible.

For the present all reference to the saving of fuel resulting from the diminished number of heat-units required to increase the pressure of steam is *premeditatedly* omitted, because we are practically limited by expense to low pressures in ordinary cases.

The constant charges which come upon engine-boilers and machinery are as follows:

(1) Wages of attendants upon engine, boilers, and machinery.

(2) Interest upon cost of engine, boilers, and machinery.

(3) Depreciation of engine, boilers, and machinery.

(4) Repairs to engine, boilers, and machinery.

(5) Cost of lubrication of engine, boilers, and machinery.

(6) Taxes and insurance upon engine, boilers, and machinery.

(7) Interest upon cost of shelter and room for engine, boilers, and machinery.

Many other charges may exist which the writer has not mentioned, and some of the charges mentioned are not applicable in cases where methods of charging cost may differ from that of works and factories engaged in the production of some staple article or of a ship where the engine serves only for propulsion. In every case the distribution of cost must form an individual problem.

The only variable charge when the engine is already established is the cost of the steam—*i. e.*, fuel and water. When engaged in the design of an engine and plant most of the constant charges may be regarded as variables, but not according to any uniform law, and must be considered as separate problems which must be solved from known precedents which vary in different localities.

Let P = the mean effective steam-pressure in pounds per square inch.

" C = the constant charges in dollars and cents for any assumed time.

" P_b = the absolute initial pressure in the cylinder in pounds per square inch.

" B = the absolute back pressure in the cylinder in pounds per square inch while the exhaust-port is open.

" e = the fraction of the volume at which steam is cut off.

" V = the volume of the steam-cylinder.

" c = the factor of the volume of steam proportional to the ratio of the constant charges to the cost of steam for any assumed time.

" b = the fraction of the volume at which compression begins, being measured from the opposite end from which e is measured.

" k = the fraction of the volume allowed for clearance.

We can then write:

$$\frac{\text{Useful work}}{\text{Cost of work in steam}} = \frac{PV}{\left[\frac{4}{3}e+c-b\frac{B}{P_b}\right]V} = \frac{P}{\frac{4}{3}e+c-b\frac{B}{P_b}}. \quad (218)$$

$$\text{But } P = eP_b\left(1+\text{nat. log }\frac{1}{e}\right) - B\left[1-b\left(1-\text{nat. log }\frac{b}{k}\right)\right]. \quad (219)$$

This value of P could be much more accurately determined by the careful use of an indicator.

Substituting in equation (218), we have

$$\frac{\text{Useful work}}{\text{Cost of work}}=\frac{eP_b\left(1+\text{nat. log }\frac{1}{e}\right)}{\frac{4}{3}e+\frac{C}{D}\left\{eP_b\left(1+\text{nat. log }\frac{1}{e}\right)-B\left[1-b\left(1-\text{nat. log }\frac{b}{k}\right)\right]\right\}-b\frac{B}{P_b}}$$

$$-\frac{B\left[1-b\left(1-\text{nat. log }\frac{b}{k}\right)\right]}{\frac{4}{3}e+\frac{C}{D}\left\{eP_b\left(1+\text{nat. log }\frac{1}{e}\right)-B\left[1-b\left(1-\text{nat. log }\frac{b}{k}\right)\right]\right\}-b\frac{B}{P_b}}. \quad (220)$$

Differentiating with respect to e and seeking a maximum, we have

$$e=\frac{B\left[1-b\left(1-\text{nat. log }\frac{b}{k}\right)\right]}{P_b}-\frac{3}{4}\frac{Bb}{P_b}\text{ nat. log }\frac{1}{e}. \quad (221)$$

The natural logarithm can be obtained by multiplying the common logarithm by 2.3026, and this transcendental equation must be solved by a series of approximations, beginning with an assumption that

$$\text{nat. log }\frac{1}{e}=\text{nat. log }\frac{P_b}{B\left[1-b\left(1-\text{nat. log }\frac{b}{k}\right)\right]}, \quad (222)$$

and substituting the nearer value of $\frac{1}{e}$ again in the second member of equation (221), and so on until two successive values of e nearly agree.

As logarithms do not vary rapidly, the approximations required to obtain all the accuracy justified by the data or realizable in practice will be few.

Perhaps how to deduce the value of c is not clear.

Determine the constant charges, in dollars and cents, upon engine and machinery for one day. Regardless of the power, determine approximately, from the first term only of the second member of equation (221), the most economical point of cut-off for steam alone.

With the following formula determine the weight of water required per horse-power per hour:

W = weight of water per H.-P. per hour.

S = specific volume of steam for pressure P_b.

$$W=\frac{4}{3}\frac{859380e}{S\left\{eP_b\left[1+\text{nat. log}\frac{1}{e}\right]-B\left[1-b\left(1-\text{nat. log}\frac{b}{k}\right)\right]\right\}}.\quad(223)$$

(See "The Limitations of the Steam-Engine.")

With this determine the cost of fuel and water for the required horse-power per day, remembering that $\frac{4}{3}$ is an assumed quantity which, at the best, is only approximately correct.

The value of c given in formula (220) is deduced as follows:

$$c:e::C:\frac{De}{eP_b\left[1+\text{nat. log}\frac{1}{e}\right]-B\left[1-b\left(1-\text{nat. log}\frac{b}{k}\right)\right]}.^{*}$$

Therefore,

$$c=\frac{C}{D}\left\{eP_b\left[1+\text{nat. log}\frac{1}{e}\right]-B\left[1-b\left(1-\text{nat. log}\frac{b}{k}\right)\right]\right\}.\quad(224)$$

An inspection of equation (221) reveals many interesting facts.

We observe that an increase of the initial pressure or a diminution of the back pressure renders the cut-off of steam earlier. Also that a diminution of the back pressure has much more influence than an increase of the initial pressure.

If we have no compression [$b=0$], equation (221) becomes

$$e=\frac{B}{P_b}.$$

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* See (223) $D=\dfrac{859380}{S}$.

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If with these conditions we substitute in equation (223), we have

$$W=\frac{4}{3}\,\frac{859380}{P_bS\ \text{nat. log}\ \dfrac{P_b}{B}},$$

and for any assumed initial pressure we can say,

$$W=\text{constant}\times\frac{1}{\text{nat. log}\ \dfrac{P_b}{B}}.$$

The following tabulation will approximately show the relative economy of successively increased expansions with the cheapest point of cut-off per horse-power per hour. At what particular point this gain is met and annulled by the losses due to internal condensation cannot be said with our present experimental knowledge. The demand for concentration of power, which is the greatest attribute of the steam-engine, will not permit us to logically accept these results, as they would give very slight initial pressures for condensing engines or require impossible points of cut-off with present engines. A consideration of equation (223) will point out to us several methods of escape. The specific volume of steam steadily decreases as its pressure increases; therefore for a fixed point of cut-off and back pressure the economy of steam effected is greater as the pressure is greater. A reduction of the back pressure effects a *rapid* increase in

$e=\frac{B}{P_b}$	$\frac{1}{\text{nat. log}\ \frac{P_b}{B}}$	Saving.	Per cent saved. Successive vols.	Per cent saved. Of the first volume.
$\frac{1}{2}$	3.222			
$\frac{1}{3}$	2.096	1.226	37	37
$\frac{1}{4}$	1.661	0.435	21	49
$\frac{1}{5}$	1.431	0.230	14	56
$\frac{1}{6}$	1.286	0.145	10	60
$\frac{1}{7}$	1.182	0.104	8	63
$\frac{1}{8}$	1.107	0.075	6	66
$\frac{1}{9}$	1.048	0.059	5	69
$\frac{1}{10}$	1.000	0.048	5	70
$\frac{1}{11}$	0.961	0.039	4	70
$\frac{1}{12}$	0.927	0.034	3	71
$\frac{1}{13}$	0.898	0.029	3	72
$\frac{1}{14}$	0.873	0.025	2½	73

saving, since it is multiplied by (S) the specific volume, usually a large quantity.

Very few condensing engines have good vacuums, but for the purpose of giving approximate results we will neglect the back pressure. Equation (223) then becomes

$$W = \frac{\text{constant}}{SP_b\left[1 + \text{nat. log}\,\frac{1}{e}\right]}.$$

If the point of cut-off (e) is fixed, we can still effect some saving by increasing the steam pressure, since the product SP_b is a slowly increasing quantity as the initial pressure is increased.

If, again, we fix upon some particular value of P_b in the preceding equation, it becomes

$$W = \text{constant}\,\frac{1}{1 + \text{nat. log}\,\frac{1}{e}}.$$

Tabulating the results of insertion of decreasing values of e, we have—

e	$\frac{1}{1 + \text{nat. log}\,\frac{1}{e}}$	Saving.	Per cent saved.	
			Successive vols.	Of the whole volume.
1	1			
$\frac{1}{2}$	0.591	0.409	41	41
$\frac{1}{3}$	0.476	0.115	19	52
$\frac{1}{4}$	0.419	0.057	12	58
$\frac{1}{5}$	0.383	0.036	9	62
$\frac{1}{6}$	0.358	0.025	7	64
$\frac{1}{7}$	0.339	0.019	5	66
$\frac{1}{8}$	0.324	0.015	4	68
$\frac{1}{9}$	0.312	0.012	$3\frac{1}{2}$	69
$\frac{1}{10}$	0.303	0.009	3	70

These rudely approximate figures — which, however, are probably quite as accurate as any that can be realized in practice—show the imperative need that exists for experimental demonstration of the causes of condensation in the cylinder, and the application of a remedy for it before further intelligent progress can be made in the economical use of steam. A comparison of the two tables shows that in the absence of condensation, expansion could probably be carried further with profit in the former than in the latter case. The expansion necessary under existing conditions to effect the greatest saving of steam per horse-power has already been exceeded in many cases. Besides expansion, the condensation certainly is a function of the temperatures and of the surfaces exposed, and of the conductivity of the surfaces, as well as of their time of exposure, and perhaps of the relative time of exposure of the interior surfaces of the steam-cylinder. It should be remembered that an increase of horse-power is the only way to decrease the constant charge per horse-power.

We have obtained but 13 per cent. of the power in coal, and even now we are at the limits of commercial economy in the use of the steam-engine.

With our present types of boilers and engines it does not pay to use coal more economically. It is cheaper to waste the 87 per cent. of the power of the coal than to go on increasing the cost of the engine in the endeavor to save coal.

Of course if we wish only to conserve Nature's forces, and disregard the money it costs to produce a diminished consumption of coal, we can, and probably will, succeed in obtaining larger results.

But money derives its value from the labor of humanity, and for this reason should be saved in preference to Nature's forces.

There are some avenues of escape open to us from this dilemma, and I will mention them.

Taking the best type of engines of to-day as a starting-point, we must depart in the following directions:

We do not particularly need to increase the efficiency of the boiler as an evaporator, but we must increase its ability to withstand pressure without increasing its cost.

We must decrease the condensation *inside* of the steam-cylinder by using a non-conducting surface or by superheating to a small extent.

We must decrease the friction of the engine and of the machinery of transmission to the point where the useful work is delivered.

We must produce better vacuums in the condenser, and diminish its cost.

We must diminish the cost of the engine.

We must diminish the cost of the attendance on engines, boilers, and machinery, and of lubrication.

We must increase the durability of engines, boilers, and machinery.

Coal is too cheap even now to admit of increased economy of it at the cost of increased outlay for plant and attendance. We would be saving coal without saving money, or rather spending more money in the difference in constant charges, such as interest, deterioration, and attendance are, than we would save in the difference in the coal-bill.

The reason why automatic cut-off engines have produced such favorable results is this: The condition in every case is that a point of cut-off shall be determined, and then the engine designed for some certain power with that cut-off. The demands of the shop or mill will not permit such a condition of uniformity to be fulfilled; and here the automatic cut-off steps in, and, with the added advantage of its carefully regulated speed, uses less steam, although not so economically, per horse-power per hour, whenever the demand for power is lessened; and so at the end of the month a real saving is apparent in the coal-bill, although the engine may not have cut off at the point of least cost for a good part of the time.

The engine-owner must have an engine which is adequate to the greatest demand for power which may be made upon it, but he should not choose an engine which would cut-off too early under its average load.

Economy of Steam Alone without Compression. [From *J. F. I.*, June, 1880.]

Let P = the mean pressure in pounds per square inch.

" P_b = the initial pressure in pounds per square inch (absolute).

" B = the back pressure in pounds per square inch (absolute).

" e = the fractional part of the stroke at which the steam is cut off by the valve-motion.

" V = the volume of the steam-cylinder.

" E = the economy.

Now, it will further be acknowledged that the economy increases directly as the work done during one stroke and inversely as the steam used during one stroke. Therefore we have

$$E = \frac{PV}{eV} = \frac{P}{e}. \qquad (225)$$

We have further the well-known formula for the mean effective pressure of steam used expansively, supposed to expand according to Mariotte's law:

$$P = eP_b \left[1 + l.\frac{1}{e}\right] - B. \qquad (226)$$

While this does not accurately represent the law of expansion of steam, it does it with close approximation for all practical purposes. Substituting the value of P from equation (226) in equation (225), we have

$$E = P_b \left[1 + l.\frac{1}{e}\right] - \frac{B}{e}. \qquad (227)$$

Differentiating with respect to e, and seeking the maximum, we find it to be

$$e = \frac{B}{P_b}. \qquad (228)$$

CHAPTER XVII.

(74) **The Errors of the Zeuner Diagram as Applied to the Stephenson Link-Motion.*** — I. INTRODUCTION. The mathematical elegance of Professor Gustav Zeuner's *Treatise on Valve-Gears* is due to the fact that he has shown that the equation representing the distance of a slide-valve, controlled by an eccentric or by means of a link, is in all cases with greater or less approximation the polar equation of a circle. Deservedly, his work has met with a most gratifying acceptance from all intelligent engineers, as not only being the most correct, but also the only method which, without the aid of models or templates, enables the practitioner to devise and study any desired form of valve-gear.

As shown in the drawing (Fig. 32), upon the top of a standard behind the section of the cylinder a pulley of equal size with the crank-shaft was connected with the crank-shaft by means of a steel saw-band running upon thin gutta-percha strips glued to the shaft and pulley surfaces. This steel band was kept very taut by means of a stretching pulley about the middle of its length.

Upon the end of the pulley-shaft and just back of the valve a drawing-board was so attached as to permit a pencil, attached to the valve and kept pressed against the paper

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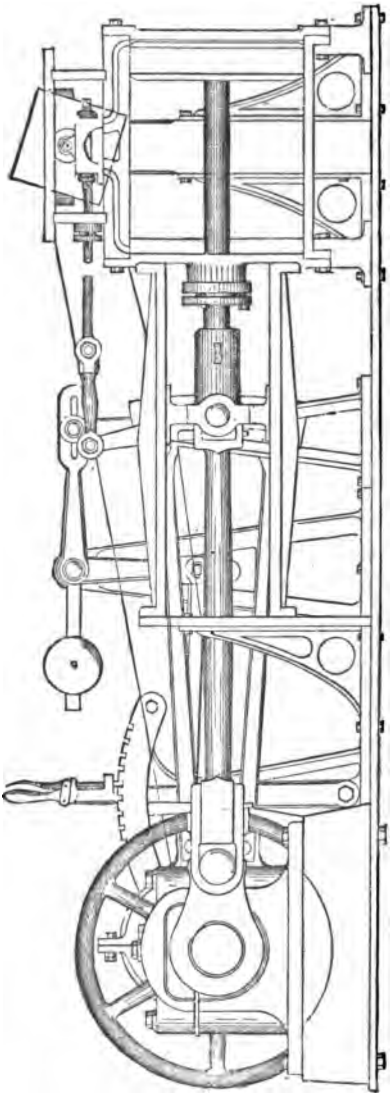
* The drawings for this chapter were made by Mr. G. H. Lewis, a graduate of the department of Dynamical Engineering, and used by him as a part of a thesis.

The mathematical treatment is my own. Mr. Lewis's drawings have been somewhat added to, in order to give graphical methods of determining the errors of the diagram. I am indebted to Mr. Lewis for many ingenious and thoughtful suggestions and much accurate and painstaking work in tracing the diagrams.

FIG. 82.



stretched upon the drawing-board, to trace the curve, showing the distance of the valve from its central position.

Had the drawing-board been attached directly to the crank-shaft, and a rod having a pencil in the end been attached to the link-block or any point on the valve or valve-stem, and carried back to the centre of the board, it would have been more serviceable for scientific purposes, as eliminating some of the possible sources of error due to the imperfections of the model.

This model was constructed of iron, brass and mahogany, and every possible precaution was taken to obtain rigidity and avoid shrinkage; it was constructed full size from the dimensions stated by Zeuner in his *Treatise on Valve-Gears*, page 78.

Eccentricity $= r = 2.36$ inches.

Angular advance $= \delta = 30°$.

Length of the eccentric-rods $= l = 55.1$ inches.

Half length of the link $= c = 5.9$ inches.

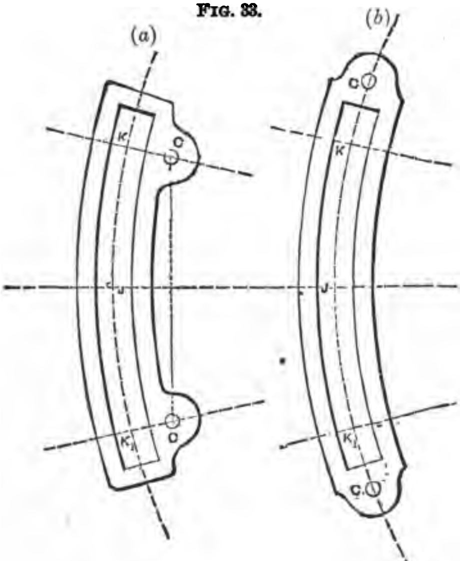
Outside lap $= e = 0.94$ inches.

Inside lap $= i = 0.27$ "

Open eccentric-rods and equal angles of advance were taken. The link was so attached to the eccentric-rods as to permit the link-block to be placed immediately in front of the ends of the eccentric-rods; in other words, so that the variable distance u of the link-block from the centre of the link could at its maximum be made equal to the half length of the link c.

This form of link is shown in Fig. 33 *a*.

FIG. 33.



The diagrams taken upon this model clearly showed that some greater sources of error existed than the so-called "Missing Quantity" of Zeuner.

Acceptance of authority is a great preventive of advancement of knowledge, and it will be our task to show clearly what points have been overlooked by Professor Zeuner, with, we hope, the result of making even more clearly understood this construction, so simple in its mechanism and so intricate in its action.

II. THE SIMPLE SLIDE-VALVE. CONSIDERATION OF THE MISSING QUANTITY IN THE SIMPLE SLIDE-VALVE. SETTING VALVE FOR EQUAL LEADS EQUIVALENT TO ALTERING THE LAPS OF THE VALVE. For the sake of simplicity let us first consider the simple slide-valve.

On page 11 of his *Treatise on Valve-Gears*, Zeuner gives for the distance of a simple slide-valve from its centre of motion ξ,

$$\xi = r \sin (w + \delta) + \frac{r^2}{2l} \sin (2\delta + w) \sin w. \qquad (229)$$

The first term of the second member of this equation is the polar equation of a circle, with the origin in its circumference and its diameter forming an angle equal to δ; with the axis of ordinates O Y (see Fig. 34) w is the angle which the crank forms with the axis of abscissas O X. All of this can readily be understood from the explanations given in the book.

It is with the second term of the second member ("the missing quantity") that we shall have particularly to deal, for Zeuner has considered it as inappreciable in most cases; which is not practically true, for many cases occur in which of necessity the eccentric-rods are comparatively short.

Dr. Zeuner fixes the central position of the slide-valve by taking the mean of the two positions of the valve when the *crank is on its dead points. He does this on the assumption that the valve will be set for equal leads; which is always the proper method.*

This central position differs from the true central position by a quantity $= \frac{r^2 \cos^2 \delta}{2l}$ (230), for the true central point of the valve travel is a mean between the extreme positions of the valve, and farther away from the crank-shaft, a distance equal to the above-stated quantity; therefore at one extreme the valve's distance from Zeuner's centre $= r + \frac{r^2 \cos^2 \delta}{2l}$, and at the other extreme $= r - \frac{r^2 \cos^2 \delta}{2l}$.

If now we can convert the missing quantity into a function of the theoretical valve distance from its centre for equal leads (Zeuner's centre), we can much more conveniently lay down the irregular curve of the valve-circle for the case of a short eccentric-rod.

According to the diagram, $\xi = r \sin (w + \delta)$ (231). Page 43, *Z. T. V. G.*,* the missing quantity is given as

$$z = \frac{r^2}{2l} [\cos^2 \delta - \cos^2 (w + \delta)], \qquad (232)$$

or

$$z = \frac{r^2}{2l} (\cos^2 \delta - 1) + \frac{r^2 \sin^2 (w + \delta)}{2l}, \qquad (233)$$

or

$$2lz = r^2 (\cos^2 \delta - 1) + r^2 \sin^2 (w + \delta). \qquad (234)$$

Letting $C = r^2 (\cos^2 \delta - 1)$ and substituting ξ for its value, we have

$$\xi^2 = 2lz - C, \qquad (235)$$

the equation of a parabola whose ordinates are the theoretical travels of the valve from its centre of motion, and whose abscissas are the missing quantities for the same.

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* Abbreviation of Zeuner's *Treatise on Valve-Gears.*

The radius of curvature of this parabola at its vertex $= \frac{1}{2}$ the latus rectum or parameter, and is equal to l, the length of the eccentric-rod, and we can substitute an arc of a circle with the radius l for this parabola without appreciable error.

For the travel $\xi = o$,

$$z = \frac{C}{2l} = \mp \frac{r^2}{2l} \sin^2 \delta. \tag{236}$$

For $\xi = r$,

$$z = \pm \frac{r^2}{2l} \cos^2 \delta. \tag{237}$$

For $z = o$,

$$\sin^2 \delta = \sin^2 (w + \delta). \tag{238}$$

Therefore $$w = o.$$

That is to say, the "missing quantity" disappears on the dead points, since the valve is *actually set for different leads.*

To lay down the actual curves of valve travel, the "missing quantity" being taken into account.

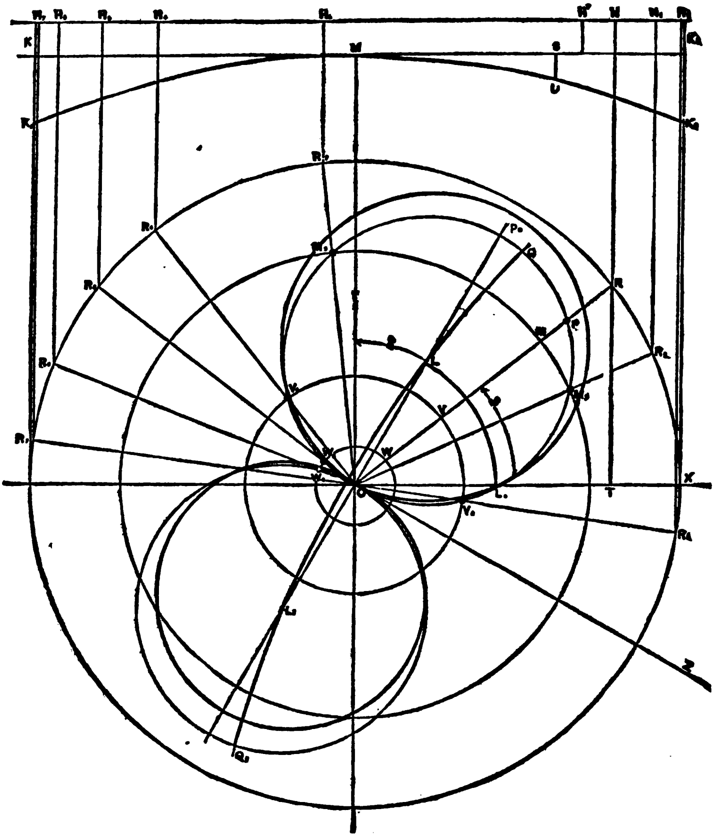
Fig. 34.—With a radius $O L_0$ and the centre O describe an arc L_0L to intersection L with the diameter of the valve circle $O P_0$. At the point O, and at right angles with $O P_0$, draw the indefinite line O Z.

With a radius of compass $= l$, and with the centre on the line O Z, describe through L and L_1 the arc $Q L L_1 Q_1$. The ordinates to this arc from the line $O P_0$ measure in quantity and direction the values of the "missing quantity," which must be added to or subtracted from the theoretical radius vector in order to obtain the true curve of the motion of the valve.

Fig. 34, for the purpose of showing an extreme case, has been laid down to scale as follows:

Eccentricity $= r = 2$ inches.
Angular advance $= \delta = 30°$.

FIG. 34.



Outside lap $= e = 0.82$ inches.
Width of port $= a = 0.75$ inches.
Length of eccentric-rod $= l = 8$ inches.

The construction for the missing quantity, for the sake of clearness, has all been added in heavy lines.

The effect of the "missing quantity," when considerable enough to be noticed, is, when the piston-head moves toward the crank-shaft, the cylinder being at the right hand,

(1) To delay slightly the pre-admission of steam.

(2) To increase the over-travel.

(3) To hasten the cut-off of the steam (very slightly).

(4) To hasten the compression of the steam.

(5) To hasten the release of the steam.

When the piston-head moves away from the crank-shaft,

(1) To hasten the pre-admission.

(2) To diminish the over-travel.

(3) To delay the cut-off (very slightly).

(4) To delay the compression.

(5) To delay the release.

A glance at the diagram at once reveals the fact that equalizing the lead very nearly equalizes the cut-off.

It is only when the valve is set for equal extreme travels from the centre that different laps are required. No attention has been paid to the variation in position of the piston due to the obliquity of the connecting-rod.

III. THE PISTON'S POSITION. The effect of the obliquity of the connecting-rod is to keep the piston nearer to the crank-shaft when it is moving away from it, and to draw it closer to the crank-shaft when it is moving toward it, than it would be if the connecting-rod was constantly parallel to the centre line of the cylinder.

At the dead points, the connecting-rod being in the centre line of the cylinder, this action ceases.

Letting w = angle of the crank,
" R = radius " "
" L = the length of the connecting-rod,

we would have, if the connecting-rod were constantly parallel to the centre line of the cylinder, for the space passed over by the piston-head = S,

$$S = (1 - \cos w)\,R; \qquad (239)$$

and when we take the obliquity of the connecting-rod into consideration,

$$S_1 = R(1 - \cos w - L)\left(1 - \frac{\sqrt{L^2 - R^2 \sin^2 w}}{L}\right). \quad (240)$$

Then for the difference d between the two positions we have

$$d = S - S_1 = L\left(1 - \frac{\sqrt{L^2 - R^2 \sin^2 w}}{L}\right), \quad (241)$$

or, expanding,

$$d = \frac{R^2}{2L} \sin^2 w, \text{ approximately.} \quad (242)$$

Fig. 34.—The positions H_1 to H_7 can be corrected by laying down in the opposite direction from the cylinder, from the points as already found, the values of d.

It will be observed that the equation for d is the equation of a parabola whose semi-latus rectum is equal to L. Further, for $w = o$ or $180°$ $d = o$. If for this parabola we substitute the osculatory circle of a radius L to its vertex, we are practically close enough.

If now with a radius of compass $= L$, with one point in M and the other on the line Y O bisecting the cylinder, we describe the arc K_2K_3, we have, with sufficient approximation, the desired parabola.

Taking off for the position O R of the crank the distance $RT = R \sin w$, and laying it off from M to S, we have the correction S U of the position of the piston H, which, if we consider the cylinder at the right-hand side, should be laid off to the left of H, giving the true position of the piston-head at H′.

Thus we can lay down graphically the actual positions of the piston and the true distances of the slide-valve from its centre of motion, *when set for equal leads* for every position of the crank and for any proportions of the mechanism.

For the sake of emphasis we again repeat: *Different laps are not necessary when the valve is set for equal leads, when the piston position is disregarded.*

Altering the laps will alter the leads. If the piston position is regarded and the alteration in the leads is disregarded for the sake of a very accurate cut-off, the lap should be shortened on the side toward the crank and lengthened on the side away from the crank. These amounts can be determined from the diagram.

It is only in the case of a very short connecting-rod that such a procedure is necessary; short eccentric-rods do not require it.

IV. THE STEPHENSON LINK-MOTION. ERROR DUE TO AN IMPERFECT MODE OF ATTACHING THE LINK TO THE ECCENTRIC-RODS. On pages 56–98 of *Z. T. V. G.* the Stephenson Link-Motion is very fully treated for both open and crossed rods, and for both forms of link shown in Fig. 33 *a* and *b*, no distinction being made between them.

In Fig. 33 *a* it will be observed that the rods are attached on the concave side at the points C and C_1, introducing an error which we will next endeavor to determine.

Fig. 35 is the Zeuner diagram carefully laid down for the dimensions already given of the model, on which was used a form of link shown in Fig. 33 *a*.

The method of making the slide-valve describe its own diagram has already been explained. It is only necessary to add that as the drawing-board turns synchronously with the crank, the valve-circles (curves) will both be on the same side of the origin instead of on opposite sides, as drawn for the sake of clearness in Fig. 34.

The object of making the link of the form Fig. 33 *a* is twofold: first, to reduce the eccentricity; second, to enable us to place the valve wholly under the control of one eccentric-rod.

FIG. 35.

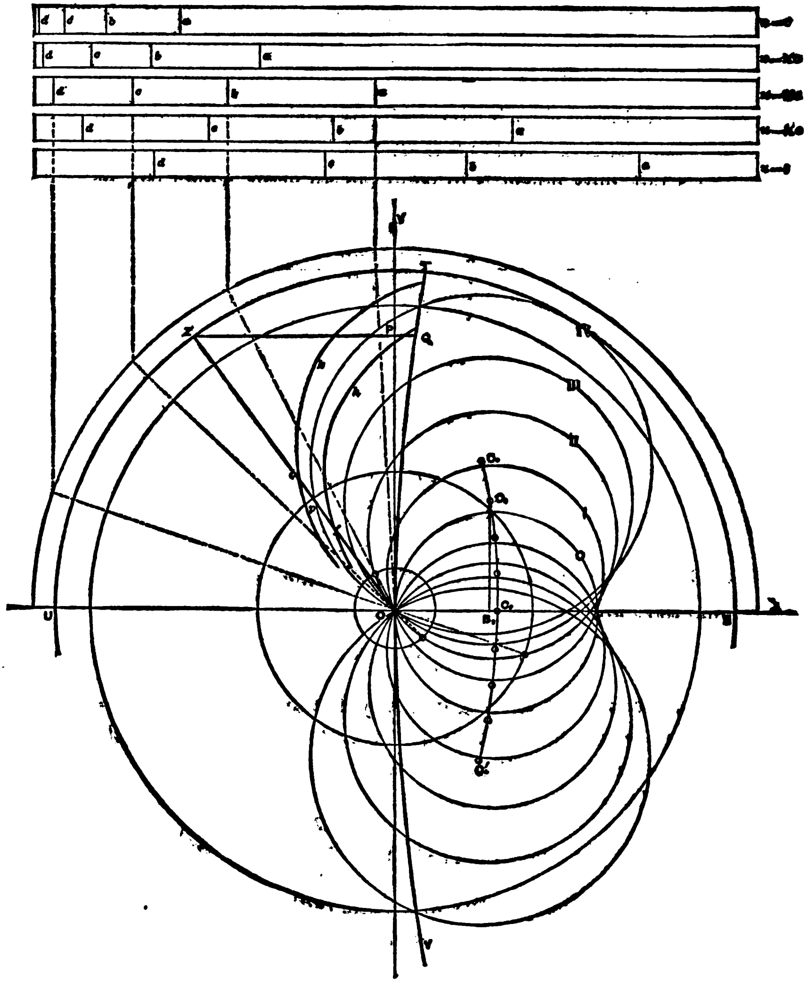
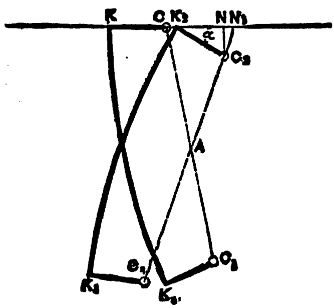


Fig. 36 is a centre-line sketch of Fig. 33 *a*, similarly lettered. It will be observed that as the suspended link sweeps to and fro with a scythe-like motion, the line K C forms an angle with the horizontal line K N_1, which is approximately equal to the angle a, which the chord of the link forms with the vertical.

The value of sin a is given with very close approximation on page 61, equation (11), *Z. T. V. G.*

As our only object is to point out an error which can be

FIG. 36.



avoided, we will make use of the principal term of this quantity, and take

$$\sin a = \frac{r}{c} \cos \delta \sin w. \qquad (243)$$

Let us denote the missing quantity due to this error by $z_1 = NN_1$, its effect being to keep the link closer to the crank-shaft except where it equals zero.

Let $KC = q$:

$$NN_1 = z_1 = q(1 - \cos a) = q\left(1 - \sqrt{1 - \frac{r^2}{c_2} \cos^2 \delta \sin^2 w}\right), \quad (244)$$

or, expanding the quantity under the radical, and neglecting terms containing greater than the second power of the circular functions, we have

$$z_1 = \frac{qr^2}{2c^2} \cos^2 \delta \sin^2 w. \qquad (245)$$

For $w = 90°$ this quantity is a maximum, and for $w = 0°$ it is equal to zero. That is, it does not appear in the lead when the valve is set for equal leads, but it does attain its maximum near the point of usual cut-off, and is particularly pernicious there and at the point of exhaust closure. The reason that it has remained unperceived hitherto is probably because it does not appear in the lead.

Transposing, we have

$$r^2 \sin^2 w = \frac{2c^2}{q \cos^2 \delta} z_1. \qquad (246)$$

The equation of a parabola whose ordinates are $r \sin w$ and whose abscissas are z_1, its semi-latus rectum is $\frac{c^2}{q \cos^2 \delta}$, which is also the radius of curvature of the osculatory circle to its vertex.

A moment's reflection will convince the reader that the error due to Zeuner's missing quantity is inappreciable (where of any consequence) in the present case. See Fig. 34 and explanation.

To determine the error z_1, through O (Fig. 35), with a centre on O X produced, describe an arc of a circle T O V with a radius $= \frac{c^2}{q \cos^2 \delta}$. With O as a centre and the radius r describe an arc S T U. Draw any position of crank as O Z to intersection Z with the arc S T U. Parallel to O X draw through Z the line Z Q. The distance $PQ =$ the error, which can be laid off both inside and outside the theoretical valve-circle, as at *pf pb*. In the model $q = 3$ inches, $\delta = 30°$, and $c = 5.9$ inches.

Therefore

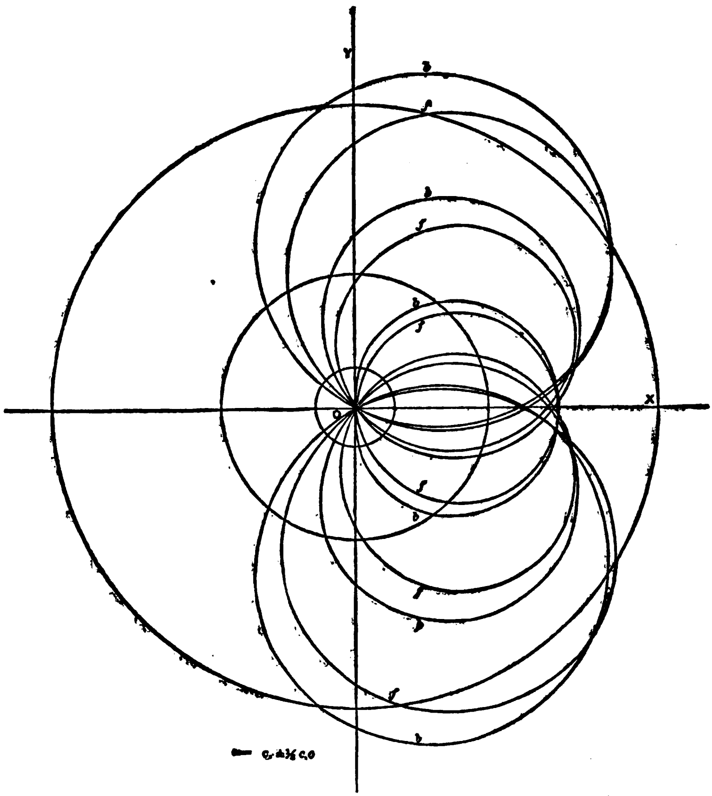
$$\frac{c^2}{q \cos^2 \delta} = 15.47 \text{ inches},$$

which is the radius of the arc V O Q T.

Laying down after the manner described the arcs *bk fh*, we have the corrected circles for the valve-motion at the IV grades. These arcs are laid down for the neighborhood of the point of cut-off only.

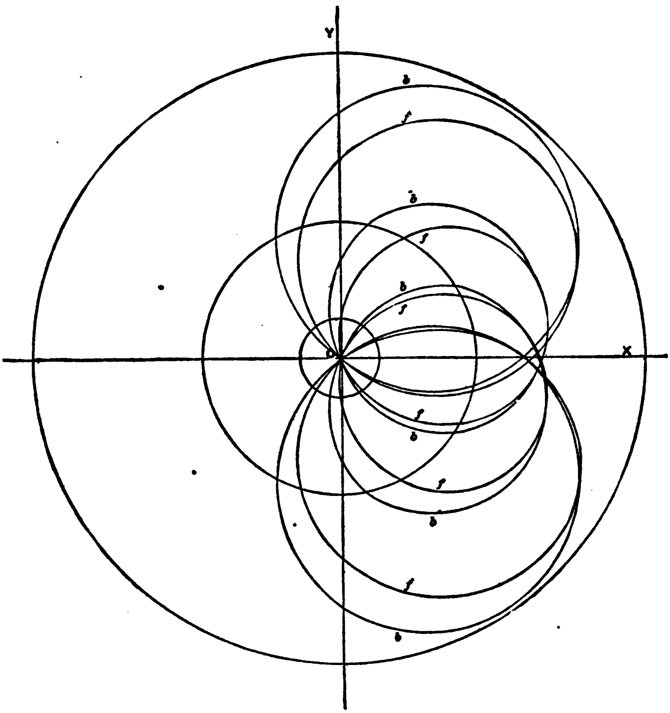
This most pernicious error can be avoided by the use of

FIG. 37.



the link (Fig. 33 *b*), although a larger eccentric is required, and, therefore, it is sometimes difficult to fit into confined spaces. Certainly it is of great importance to avoid so faulty a construction if it be possible.

FIG. 38.



Figs. 37 and 38 are diagrams automatically traced by the working model.

To avoid the errors due to suspension, the link-block was clamped in the link for each grade, and the link, therefore, swung upon the rocker-shaft arm.

To avoid the errors due to the "lost motion," the valve-circles were traced twice by reversing the direction of the motion, and the mean between the two circles traced with pen and ink by hand. The difference was very slight, if any at all.

It will be seen that these actual valve-curves verify with great accuracy the corrected valve-circles (Fig. 35) for the fourth grade.

Similar corrections can be made for each grade of link if desired.

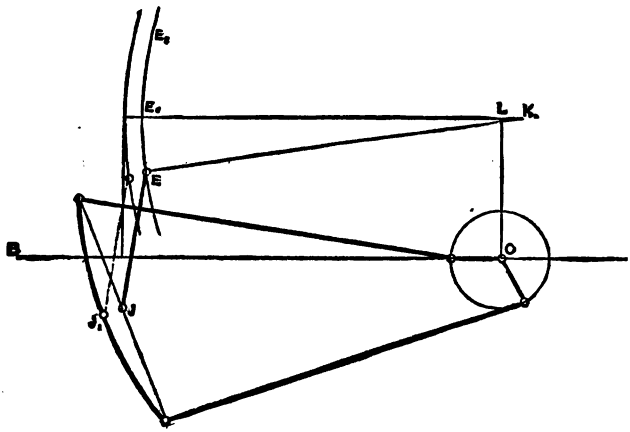
The letters *f* and *b* refer to the direction of motion of the piston-head, forward (*f*) meaning toward the crank-shaft, and backward (*b*) meaning away from the crank-shaft, a rocker-shaft intervened reversing the direction of motion of the valve.

When the form of link shown (Fig. 33 *b*) is used, the increased eccentricity required will increase the "missing quantity" given by Prof. Zeuner, and it must therefore be guarded against, particularly in extreme cases.

Cases may occur when it will prove advantageous to attach the eccentric-rods to the link at points nearer its centre than the extreme limits of the travel of the link-block, but special pains should be taken to place the centre of the pin-joint on the central arc of the link; this method of attachment, however, will result in increasing the slip of the link-block.

V. Slip of the Link-Block. Zeuner gives two cases of the suspension of the link, by means of a hanger attached at the centre of the chord of the link and at the bottom of the link; in the first case the upper end of the hanger should theoretically move in an arc of a circle which has for a radius the length of the eccentric-rod, and whose centre is above the centre line a distance equal to the length of the hanger. The lower end of the hanger should be attached at the centre of the link and on the central arc of the link, thus placing the origin of the arc of suspension at a horizontal distance equal to the length of the eccentric-rod from the centre of the crank-shaft. Fig. 39 will render this clear. J_1, and not J, should be the point of attachment of the hanger, and L, not K_1,

FIG. 39.



should be the centre of the arc in which the upper end of the hanger E should move.

We can thus avoid increasing the slip by the quantity $\overline{J J_1} \tan a$ in one direction, and decreasing it by the same amount in the other direction.

Fig. 40 shows the second method of suspension of the link by a hanger attached to the bottom.

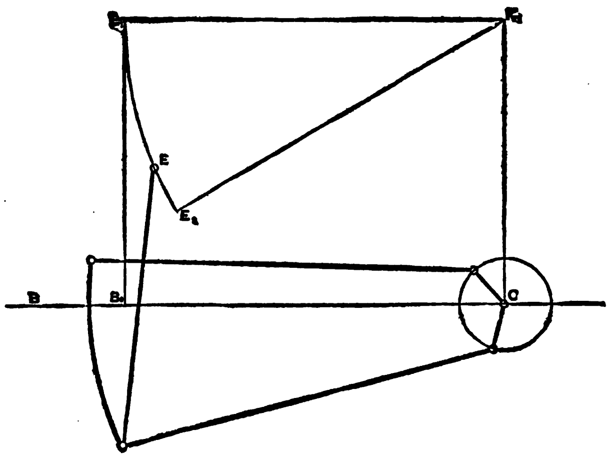
Both of these methods are fully explained by Zeuner, and the reader is referred to his work for further details.

It will be perceived that the hanger is but a rude approximation to a parallel motion, used only because of its simplicity and lightness when the link-block is forced to move to and fro in a straight line.

When the link-block is attached to the end of a rocker-shaft arm, as is commonly the case for American locomotives, if the hanger is made the same length as the rocker-shaft arm, there will be no slip when the link-block coincides with the point of suspension; the slip for other positions of the link-block will be due to the angular position of the link.

Assuming, when the block is forced to move in a straight line, that some method has been adopted to force the point of suspension of the link to move in an at least very close

FIG. 40.



approximation to a straight line, and, further, that when the link-block moves in an arc of a circle of a given radius the hanger is of the same length as this radius, we can consider the slip as due only to the angular position of the link.

Of course these conditions cannot always be fulfilled, but it is best to know what ought to be done, even if we cannot exactly do it.

Fig. 41 shows the two positions of the links $K K_1$ and K_2K_3, for which the slip is zero; and letting s equal the amount of slip for all other positions, if we suppose the hanger attached to the middle point of the link, and u the distance of the link-block from that point, we will have

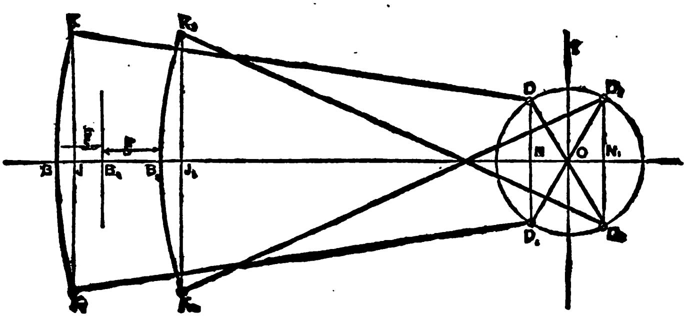
$$s = (\sec a - 1)\, u, \tag{247}$$

or, since the angle a is always very small,

$$s = u\,(1 - \cos a) = u\,(1 - \sqrt{1 - \sin^2 a})\,; \tag{248}$$

and substituting for $\sin a$ its value, and expanding and neg-

FIG. 41.



lecting all terms containing higher powers than the square of the circular functions, we have

$$s = \frac{ur^2}{2c^2} \cos^2 \delta \sin^2 w. \tag{249}$$

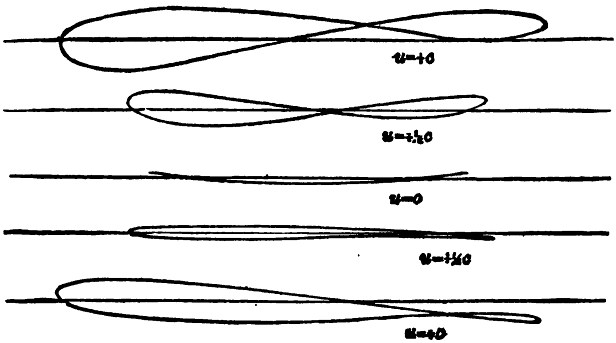
We thus see that the effect of the slip does not appear in the lead, but, being a maximum for $w = 90°$ or $270°$, will affect the points of cut-off and exhaust closure.

Increasing the angular advance diminishes the slip, as also does increasing the length of the link. The tendency of the slip is to increase the travel of the valve by an amount V:

$$V = \frac{ur^2}{2c^2} \cos^2 \delta \sin^2 w. \tag{250}$$

This amount is very small for a well-proportioned valve-gear, but it increases directly as the distance *u* from the point of attachment of the hanger to the link.

FIG. 42.



When the link is suspended from the bottom the value *u* must be replaced by $(c + u)$.

We thus see that for general usage at all points suspending the link at the middle is the best, while if one particular point is intended to be constantly used, and the other points only exceptionally, it is best to attach the hanger to the link at that point.

If the tumbling-shaft arm cannot be made equal to the length of the eccentric-rods (and for obvious reasons it rarely can be so proportioned), the centre of the tumbling-shaft must be so placed as to make an arc, struck with its arm as a radius, intersect the theoretical arc at the point or points of greatest usage.

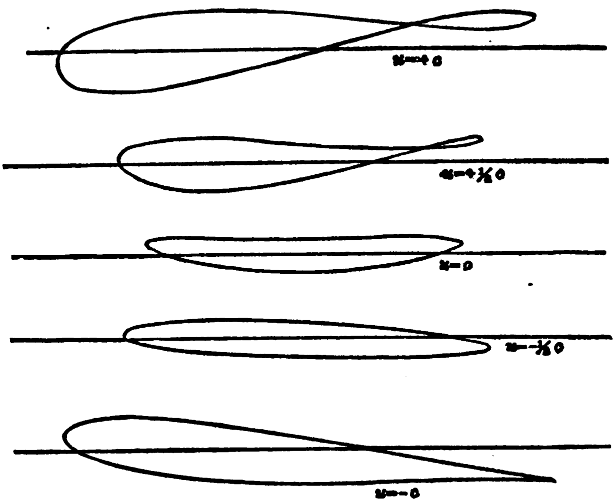
From what has been said about the position of the arc of suspension, it will readily be perceived that its length on either side of the horizontal line E_0K_1 (Figs. 39 and 40) is determined by the point of attachment of the hanger to the link.

The link-motion on which experiments were made with a view to testing the correctness of these results had the following dimensions:

Length of eccentric-rods $= l = 18$ inches.
Radius of eccentricity $= r = 1\frac{1}{4}$ "
Length of link $= 2c = 6$ "
Angular advance $= \delta = 30°$.
Open rods.

These results verify the above theory only in a qualitative

Fig. 43.



way, as the upper end of the hanger was not always kept on the true arc of suspension.

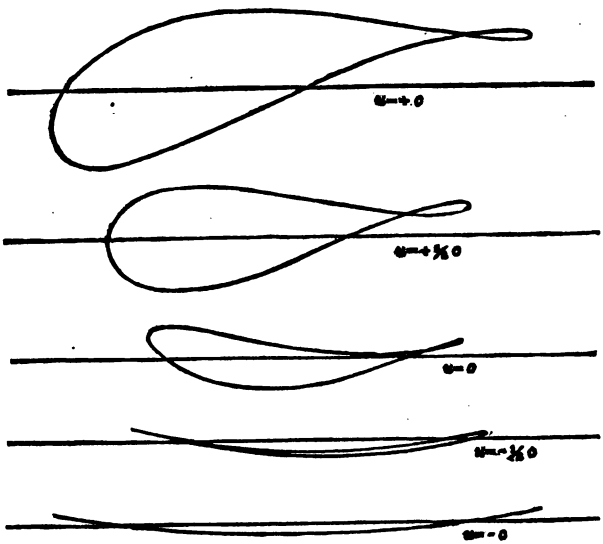
The link being suspended after the manner described, a pencil was fixed in the link-block, and the block successively fixed at different grades, the pencil being allowed to trace on a paper back of it the curves of slip.

Case I. *The link suspended at the centre of its arc.*—Fig.

42 shows that the slip of the block increases both ways from its centre, as had been predicted. The arc for $u = o$ is the standard with which the other curves must be compared.

Case II. *The link being suspended at the centre of its chord.*—Fig. 43 is inserted merely for the purpose of showing an imperfect mode of suspension. All of the slip-curves are bad, and at no point is there any cessation of the slip.

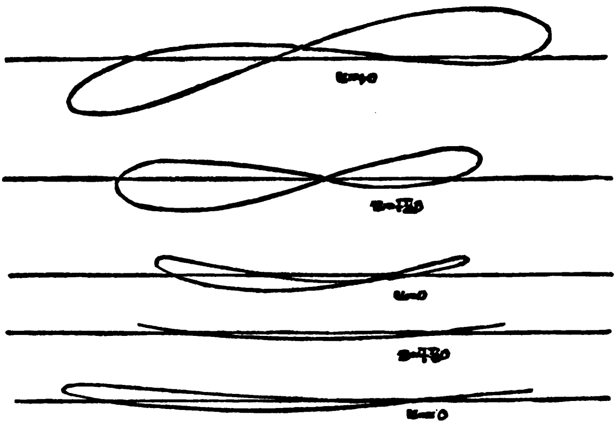
Fig. 44.



Case III. *The link suspended at the bottom.*—Fig. 44 reveals the fact that the slip becomes very great for the upper end of the link—so great as to seriously affect the distribution of the steam. The lower half of the link only can be relied upon for accurate work.

Case IV. *The link suspended halfway between the bottom and centre.*—Fig. 45 shows a better average result than any of the others, and is undoubtedly the best mode of hanging the link when the grade $u = -\frac{1}{2}c$ is to be generally used. Viewed from a practical point, slip is of great importance, being the cause of the wear upon links, which soon unfits

FIG. 45.



them for accurate work. Great pains are taken to reduce this wear by case-hardening the links or using steel in the place of wrought iron.

A proper mode of suspension is the most important point to be attained when the durability of the link-motion is under consideration.

TABLES.

THE following four tables, condensed from the fourth section of Weisbach's *Mechanics of Engineering*, which has formed the basis of this work, are inserted as a means of ready reference for ordinary problems in the strength and elasticity of materials. The same notation as that used by Weisbach is retained, in order to avoid confusion in referring to his work.

TABLE V.

(75) Elasticity and Strength of Extension and Compression. (Arts. 201–214.)

(Art. 204.) To find increase or decrease in length under a strain of extension or compression.

Where the weight of the body under strain is not considered—

λ = the amount of the extension in inches.
P = the weight acting in pounds.
l = the length of the body acted upon in inches.
F = the area of the cross-section in square inches.
E = the modulus of elasticity in pounds per square inch.

$$\lambda = \frac{Pl}{FE}.$$

Let G = the weight of the body under strain.

(Art. 207.) Where the weight of the body under the strain is also taken into account,
$$\lambda = \frac{(P \pm \frac{1}{2}G)l}{FE}.$$

(Art. 205.) To find the proof-strength of a body to be submitted to strain.

T = the proof-strength for extension per square inch in pounds.
T_1 = the proof-strength for compression per square inch in pounds.
K = the ultimate strength for extension per square inch in pounds.
K_1 = the ultimate strength for compression per square inch in pounds.

For a pull, $P = FT$. For a thrust, $P_1 = FT_1$.
To find the ultimate strength of a body to be submitted to strain.
To tear the body asunder, $P = FK$. To crush it, $P_1 = FK_1$.

(76) TABLE VI.—Elasticity and Strength of Flexure or Bending. (Arts. 214 to 257.)

Notation.—P = the load in pounds. W = the measure of the moment of flexure.
T = the proof-strength in pounds per square inch. e = the half depth of the beam in inches.
l = the length of the beam in inches. E = the modulus of elasticity in pounds per square inch.
a = the deflection in inches.

Bearing.	Proof load.	Deflection.		Values of W and $\frac{W}{e}$.
Fixed at one end and loaded at the other.........	$P = \frac{WT}{le}$. See Art. 235.	$a = \frac{Pl^3}{3WE}$ See Art. 217.	b = breadth of the beam in inches. h = height of beam in inches.	For a solid girder with rectangular cross-section: $W = \frac{bh^3}{12}$. $\frac{W}{e} = \frac{bh^2}{6}$,
Fixed at one end and uniformly loaded..............	$2P$. See Art. 240.	$\frac{3}{8}a$. See Art. 223.		
Supported at both ends and loaded in the middle	$4P$. See Art. 240.	$\frac{1}{16}a$. See Art. 217.	b_1 = inside breadth of beam in inches. h_1 = inside height of the beam in inches. b and h as before.	For a hollow or a single-webbed girder with a rectangular cross-section: $W = \frac{bh^3 - b_1h_1^3}{12}$. $\frac{W}{e} = \frac{bh^3 - b_1h_1^3}{6h}$.
Supported at both ends and uniformly loaded...	$8P$. See Art. 240.	$\frac{5}{128}a$. See Art. 223.		
Fixed at both ends and loaded in the middle....	$8P$. See Art. 246.	$\frac{1}{64}a$.	r = radius in inches. d = diameter in inches.	For a solid girder with a circular cross-section: $W = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$, $\frac{W}{e} = \frac{\pi r^3}{4} = \frac{\pi d^3}{32}$,
Fixed at both ends and uniformly loaded.........	$12P$. See Art. 246.	$\frac{1}{128}a$.		
Fixed at one end and supported at the other; loaded in the middle..	$5\frac{1}{3}P$. See Art. 247.	$\frac{45}{100}$ of $\frac{1}{16}a$.	r_1 = exterior radius in inches. r_2 = interior radius in inches.	For a hollow girder with a circular cross-section: $W = \frac{\pi}{4}(r_1^4 - r_2^4)$. $\frac{W}{e} = \frac{\pi}{4r_1}(r_1^4 - r_2^4)$.
Fixed at one end and supported at the other, and uniformly loaded......................................	$8P$. See Art. 247.	$\frac{1}{2}$ of $\frac{1}{16}a$.		

For the determinations of the values of the measure of the Moment of Flexure, W and $\frac{W}{e}$, for any form of cross-section of beam, refer to Arts. 224 to 232, inclusive, and Art. 236. *Recollect always to take the weight of the beam into account.*

TABLE VII.

(77) The Elasticity and Strength of Torsion. (Arts. 262–265.)

Notation.—P = the load in pounds. a = the lever-arm of the load in inches. T = the modulus of proof-strength for shearing in pounds per square inch. W = the measure of moment of torsion. e = the greatest distance in inches of any element of the cross-section from the neutral axis.

l = the length in inches submitted to stress.
d = the diameter of round shafts in inches.
b = the length of one side of a square shaft in inches.
a° = the angle through which the body is twisted in degrees.

Form of body.	Proof-strength.	Angle of torsion.
For any form of cross-section..................	$P=\frac{TW}{ae}$. See Art. 264.	
For a solid round shaft...	$P=0.1963\frac{d^3T}{a}$. See Art. 264.	Cast iron (Art. 263). $a^\circ=0.0002053\frac{Pal}{d^4}$. Wro't iron (Art. 263). $a^\circ=0.0000648\frac{Pal}{d^4}$.
For a solid square shaft..	$P=0.2357\frac{b^3T}{a}$. See Art. 264.	Cast iron (Art. 263). $a^\circ=0.0001211\frac{Pal}{b^4}$. Wro't iron (Art. 263). $a^\circ=0.0000382\frac{Pal}{b^4}$.

TABLE VIII.

(78) The Proof-Strength of Long Columns. (Arts. 265 to 270.)

l = the length of the column in inches.
d = the diameter of the column in inches.

When the length of columns is so increased as to cause rupture by first bending and then breaking across (buckling).

The following formulæ will apply approximately:

Method of adjustment.	Force necessary to rupture by buckling.	Remarks.
Column fast at the lower end, load applied at the upper end, which is free to move sideways......................	$P=\left(\frac{\pi}{2l}\right)^2 WE.$ See Art. 265.	W and E are the same as in the case of flexure.
Column not fixed at either end, but neither end free to move sideways...........	$4P$. See Art. 266.	
Column fixed at both ends, and not free to move sideways.........................	$16P$. See Art. 266.	According to Hodgkinson's experiments, we have only $12P$.

(Art. 266.) For a solid cylindrical pillar not fixed at either end, but neither end free to move sideways, whose diameter is d and whose length is l, we must take the formulæ for buckling, in preference to that for compression.

In the case of cast iron when $\frac{l}{d} = 9$ or a larger number.

In the case of wrought iron when $\frac{l}{d} = 22$ or a larger number.

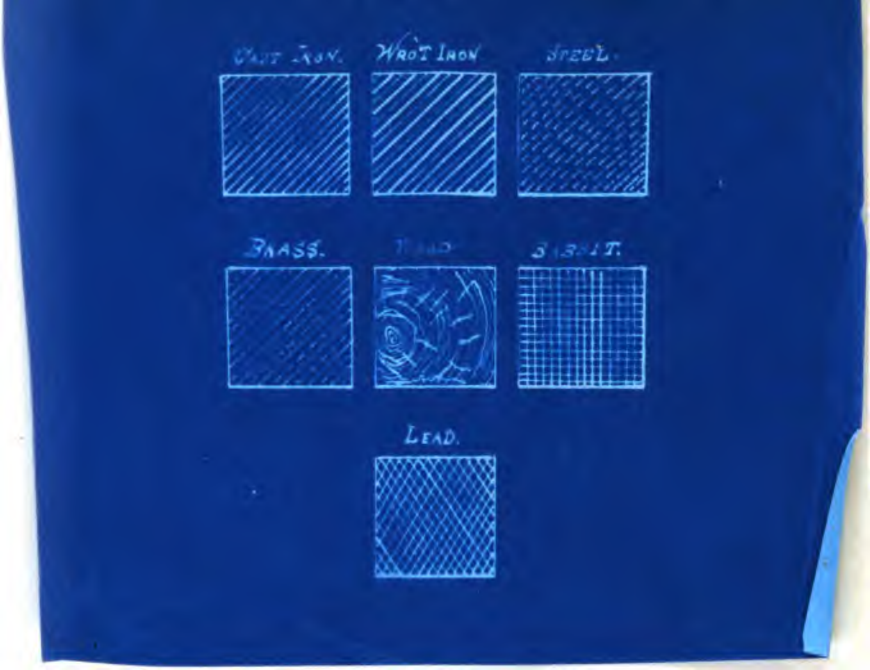
in ... the diameter of the
link in eighths and point off one place.
... 1" chain ... 6.4 ...

[illegible]

[illegible]

[illegible] x

[illegible]


CAST IRON.
WROT IRON
STEEL.
BRASS.
LEAD.