PERGAMON MATERIALS ADVISORY COMMITTEE

CALCULATIONS
IN
FURNACE TECHNOLOGY

BY

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To my wife Marion

Preface

THIS is intended as a course book for students taking examinations under the broad subject heading "Furnace Technology". It should be particularly useful to students of Metallurgy taking Furnace Technology at both part II and part IV of the Institution of Metallurgists examinations. It does, however, cover the requirements of a large number of professional, technical and university courses. For this reason, many of the worked examples have been given in detail. It has been the author's experience that such a treatment is very necessary and not an over-simplification.

It is expected that students will be conversant with the appropriate subject-matter of the book, and will have studied, or will be studying, the more theoretical and practical aspects of the subject-matter.

Acknowledgements

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The author also acknowledges the assistance of the Institute of Fuel, the Institution of Metallurgists, the Institution of Heating and Ventilating Engineers, the City and Guilds of London Institute, the Swansea Education Committee, and the University of Wales, in granting permission for the use of past examination questions. The author accepts full responsibility for any errors in reproduction of questions and for the solutions given.

Since the change to SI units is imminent a number of the examples have been changed to what they would probably have been in this system of units.

Finally, I would like to thank J. N. Harris and D. H. Davies for helpful suggestions, and D. W. Hopkins, without whose considerable help and encouragement this book might never have been completed.

Introduction

FURNACE technology involves the detailed study of solid, liquid, gaseous, and nuclear fuels and electric heating as well as the design of furnaces and other heat utilization devices. In order to compare the economics of different sources of heat the efficiency of utilization processes must be determined. Furnaces and boilers using the same or different fuels may be compared one with another, or with the theoretical thermodynamic heat requirement for the operation. In order to arrive at efficiencies of utilization, various calculations have to be made. Thus, the available heat in fuels (calorific values), the rate of combustion, and the product of these two, calorific intensity, need to be determined in order to decide whether sufficient heat of the required level of intensity is theoretically available in the fuel to carry out the specified duty.

In evaluating efficiencies, it is necessary to measure the quantity of fuel used, of air entering, and of flue gases leaving the plant, and the heat lost to the surroundings. In addition, the effect of insulation upon furnace structures must be calculated before it is actually installed. For example, in the open-hearth steel furnace, while heat conservation by roof insulation appears desirable, any attempt to do so would raise the temperature of the "cold end" to such a value that the refractory would fail by deformation under the existing compression stresses. In all instances of furnace insulation there is an optimum beyond which the additional cost of lagging exceeds the value of the heat saved.

When the design of furnaces is under consideration, it is necessary to calculate the size of flues, combustion space, and chimneys, and the rating of fans required to supply sufficient air for combustion. It is also necessary to calculate the pressure losses that occur throughout the system. Temperatures and temperature differences must also be known as the basis for choice of materials and for installation of heat recovery devices.

CHAPTER 1

Introductory

1.1. INTRODUCTION

The fossil fuels—coal, oil, and gas—are at present the most important sources of energy, although atomic energy is likely to assume increasing importance. Atomic energy has to be converted into a usable form, generally electricity, and this, at present, involves normal modes of heat transference.

The instrinsic value of any fuel as a source of heat is related to the heat which would be produced by combustion under isothermal and ideal gas conditions.

Coal and oil are defined by the geologist as sedimentary rocks but they differ from all other sedimentary rocks by being organic (chemical) and it is the release of the heat of combustion of the organic material that provides us with a source of energy. In most coalfields there is a geological continuity from peat through brown coals to the anthracites. This change in rank can be measured by a number of parameters, perhaps the best function being the increase in carbon content of the vitrain maceral, vitrain being one of the four banded constituents that can be observed by macroscopic examination of coals. Just as rocks contain minerals, so coals contain macerals.

No chemical formula can be applied to coal or oil; the former is a complex organic molecule whose structure has not yet been elucidated, while oil is a complex mixture of many different organic molecules. However, both can be assigned elemental formulae which indicate the percentages of carbon, hydrogen, oxygen, nitrogen, sulphur, and other elements, and this information is generally sufficient to assess the value of the product as a fuel. In both cases it is necessary to consider the effect of any mineral matter which is present, before any calculations on the basis of elemental composition can be made, since in many cases the mineral matter can either interfere with the mechanical processes of combustion or have deleterious effects on the structure of the combustion chamber.

1.2. PRESENTATION OF ANALYSIS

Solid fuels contain inorganic matter and are usually burned containing moisture, and the combustion engineer requires to know the composition and characteristics of the material being burned—the "as-fired" analysis. Sampling and analysis of fuels at this stage is generally inconvenient and expensive and it is necessary to devise a method of presentation which will enable allowances to be made for variations resulting from changes in sources of supply and from consequences of storage in the open.

Analysis may be carried out on "air-dried" fuel, i.e. fuel in approximate equilibrium with the atmosphere at the prevailing temperature and humidity, or on the "dry basis", i.e. after heating to 105°C for at least one hour *in vacuo*, or in an atmosphere of nitrogen. Solid fuel for industrial purposes is purchased against specified values for heating capacity, moisture and ash content, chemical composition in respect of certain elements, and physical condition relative to size. The mineral matter in coal as mined is not identical with the ash content as determined by combustion under standard conditions. But there is a comparatively simple relationship: the King–Maries–Crossley formula (KMC) where mineral matter (MM) is given by:

$$\text{MM} = 1{\cdot}13\ \text{ash} + 0{\cdot}5\ \text{pyritic S} + 0{\cdot}8\,\text{CO}_2 - 2{\cdot}8\ \text{S in ash} + 2{\cdot}8\ \text{SO}_4 + 0{\cdot}5\ \text{Cl}.$$

Alternatively, the British Coal Utilization Research Association (BCURA) formula, which requires less data, may be used:

$$\text{MM} = 1{\cdot}10\ \text{ash} + 0{\cdot}53\ \text{total S} + 0{\cdot}74\,\text{CO}_2 - 0{\cdot}32.$$

It is most important that the ash determination is carried out according to BS 1016, because ash is not an intrinsic property of the fuel, it is an assay, and unless the conditions of test are strictly adhered to, then a different value may result.

The results of analysis may be expressed on the above-mentioned basis, but it is frequently found to be more effective for comparison of fuels to present them on either the dry, ash-free (d.a.f.) or dry, mineral-matter free (d.m.m.f.) basis. Care must be taken when using the various forms of analysis for combustion data. For example, if the calorific value (CV) of a sample of coal containing ash and moisture is determined, the d.a.f. or d.m.m.f. value will be higher than the found value (there will be more carbonaceous matter).

$$\text{Hence, CV (d.a.f)} = \frac{\text{CV as determined} \times 100}{100 - \text{moisture} - \text{ash}}.$$

If the fuel supplier quotes the analysis of a fuel on a d.a.f. or d.m.m.f. basis, then the properties of the fuel as charged to the furnace have to be evaluated before the quantities of air, sizes of furnaces, etc., can be calculated. If the CV (d.a.f.) of a fuel is given, then the CV as fired = CV (d.a.f.)$\times$(100 – moisture – ash), i.e. less than the d.a.f. value due to the presence of moisture and ash. If P is a parameter (other than volatile matter) then:

$$\text{d.a.f analysis} = \frac{P \times 100}{100 - \text{moisture} - \text{ash}}.$$

The volatile matter (VM) has to be further corrected for the evolution of inorganic volatiles (e.g. carbon dioxide from carbonates). A suitable correction that may be applied is:

$$\text{Correction to VM} = 0{\cdot}13\ \text{ash} + 0{\cdot}2\ \text{S} + 0{\cdot}7\ \text{CO}_2 - 0{\cdot}13.$$

With liquid fuels there is no complication as regards ash and mineral matter. However, analysis is normally reported on a weight basis. Where the analysis is given on a volume basis it is important that the temperature of volume measurement is quoted.

With gaseous fuels, analysis is generally reported on a volume basis. Where it is required on a mass basis then the following procedure is adopted. Consider the following gas with volumetric analysis as indicated: CO, 8; CO_2, 2; H_2, 60; CH_4, 30%. To convert to weight analysis the volumetric percentage is multiplied by the molecular weight, and the percentage analysis recalculated on the new total, e.g.:

		Weight (%)
$CO = 8 \times 28 = 224$	÷ 912	24·5
$CO_2 = 2 \times 44 = 88$		9·6
$H_2 = 60 \times 2 = 120$		13·3
$CH_4 = 30 \times 16 = 480$		52·6
912		100·0

1.3. DEFINITIONS AND UNITS

Before carrying out calculations it is necessary to state the system of units to be employed and to define certain properties and standards.

Temperature

This can be stated for some purposes on the basis of the International Practical Temperature Scale of 1948 (IPTS) and the revision of that scale (1960). This is based upon a number of fixed and reproducible points, and defines the instruments and equations to be used over its range.

A practical scale such as this suffers from the scientific disadvantage that it does not depend on any fundamental understanding such as is involved in the thermodynamic definition of temperature. Kelvin defined a thermodynamic temperature scale which was independent of the thermometer used. This scale follows the relation $Q_1/Q_2 = T_1/T_2$, where Q_1 is the heat absorbed by an ideal engine at the higher temperature T_1, and

Q_2 the heat rejected at the lower temperature T_2, when operating a Carnot cycle.

In 1954 the Tenth Conference of Weights and Measures adopted a resolution which defined the "triple point" of water to be 273·16 degrees Kelvin (K) and eliminated the steam point from the thermodynamic scale. In 1948 the IPTS had been defined with its zero at 0·0100°C below the triple point of water, so that the practical scale and the thermodynamic scales are in agreement by definition at this one temperature.

In 1948 Celsius rather than centigrade was chosen to describe the practical scale (°C). Common units in combustion science are °C (Celsius), °F (Fahrenheit), K (Kelvin), and R (Rankine).

$$0°\text{C} = 32°\text{F}; \qquad 100°\text{C} = 212°\text{F};$$

$$t°\text{F} = 5/9(t-32)°\text{C};$$

$$t°\text{C} = (32+1{\cdot}8t)°\text{F};$$

$$1\ \text{K} = 1{\cdot}8\ \text{R};$$

$$t\,\text{K} = t°\text{C}+273{\cdot}16 \quad \text{and} \quad t\ \text{R} = t°\text{F}+459{\cdot}67.$$

Normal temperature and pressure (NTP) is taken as 0°C (32°F) and 760 mmHg pressure. For technical purposes in Great Britain, gases are measured saturated with water vapour at 60°F, under a total pressure of 30 in.Hg and this is often referred to as standard temperature and pressure (STP).

It is proposed that the future standard reference conditions should be 15°C and 1013 mbar and dry (STP). This will coincide with the introduction of natural gas into Great Britain.

In the future it is possible that NTP may be revised to 0°C and 1 bar (1000 mbar); 1 bar = 750 mmHg.

Quantity of heat

Heat can be measured by the amount of energy required to raise the temperature of a specified weight of water through a specified temperature range.

The pound-Fahrenheit scale measures heat in terms of the British thermal unit (Btu). One Btu is the quantity of heat required to raise the temperature of 1 lb of water from 60° to 61°F. The mean Btu is 1/180th of the heat required to raise the temperature of 1 lb of water from 32° to 212°F without conversion to vapour.

In the metric system, the basic unit is the gram-calorie which is the amount of heat required to raise the temperature of 1 g of water from 15° to 16°C. The mean calorie is 1/100th of the amount of heat required to raise the temperature of 1 g of water from 0° to 100°C without conversion to vapour.

Another unit, the pound-calorie or Centigrade heat unit (Chu) is often used. It is the amount of heat required to raise the temperature of 1 lb of water through 1°C.

In SI units the basic unit of heat is the absolute Joule (J). The Joule is the work done when the point of application of a force of one Newton (N) is displaced through a distance of one metre (m) in the direction of the force:

$$\mathrm{J} = \mathrm{N\,m}.$$

The Newton (N) is a unit of force which can be defined as the product of mass (kg) and an acceleration ($\mathrm{m/s^2}$):

$$\mathrm{N} = \mathrm{kg\,m/s^2}.$$

$$\text{Thus } \mathrm{J} = \mathrm{kg\,m^2/s^2}.$$

$$1 \text{ cal} = 4{\cdot}1868 \text{ J}; \qquad 1 \text{ Btu} = 1055{\cdot}06 \text{ J}.$$

$$1 \text{ Btu} = 252 \text{ cal} = 0{\cdot}556 \text{ lb-cal}.$$

$$1 \text{ Kcal} = 1000 \text{ cal}.$$

$$1 \text{ therm} = 100{,}000 \text{ Btu}.$$

$$1 \text{ thermie} = 4{\cdot}1868 \text{ MJ}.$$

Specific heat

The specific heat of a substance is the amount of heat expressed in thermal units required to raise unit mass or volume through one degree of temperature.

The specific heats of all substances vary with temperature. Since all substances vary in volume or pressure with changes in temperature, it is necessary to distinguish between the specific heats at constant volume and constant pressure, denoted by C_v and C_p respectively.

Liquids and solids undergo only small volume changes over a temperature change of one degree and C_v and C_p can be taken as equal for most purposes. For gases, the value at constant pressure is higher than the value at constant volume, because of the work done against the atmosphere during expansion while being heated.

The instantaneous specific heat of a substance is the amount of heat that must be added to unit mass (volume) of such a substance at some definite temperature to increase its temperature one degree under specified conditions of volume or pressure. The relationship between instantaneous specific heat and temperature is of the form:

$$C_p = A+BT+DT^2+ET^3+ \ldots .$$

It is convenient to express specific heats in terms of the mean value between two temperature ranges (e.g. 0° and t°C). The mean specific heat of a substance, over a given temperature range, is the value by which such a range must be multiplied to give the quantity of heat necessary to raise unit weight (volume) through the range under the conditions of pressure or volume that exist. In combustion calculations the mean specific heats should always be used.

With the exception of water vapour, the variation with pressure in the specific heats of gases encountered in most combustion work is small, and can be neglected. The range of pressures encountered in combustion problems is normally quite low, so that gases may be assumed at constant pressure. Hence calculations based upon the mean specific heats at constant pressure will give results well within the accuracy of most combustion computations.

Sensible heat

This is the term given to the heat content of fuels and their combustion products over and above a specified datum level. Sensible heat is heat that can be sensed, that is, if heat is applied to a system then it is immediately "sensed" by a rise in temperature, or if heat is removed it is "sensed" by a fall in temperature. A quantitative expression for sensible heat is: Sensible heat = mass (volume) × MC_p × temperature difference. MC_p is the mean specific heat at constant pressure. For gaseous mixtures it is found by multiplying the percentage of each constituent by its mean specific heat and dividing the sum by 100.

With gases it is more convenient to measure volumes than masses so that MC_p on a volume basis is used instead of a mass basis. It is evident that although the mass of a gas is always the same, irrespective of temperature, the volume will vary considerably with temperature. Thus in the formula for sensible heat, it is clear that the temperature at which the volume is measured will be important, as also is MC_p. Hence, by convention, the MC_p on a volume basis is calculated on the basis of a normal cubic foot (metre), that is a cubic foot (metre) of gas measured at NTP. Hence the volume to be used in the calculations is the volume of gases measured at NTP.

Latent heat

When a material undergoes a change of state at constant pressure heat is either absorbed or evolved, and during this change the temperature of the system remains constant. It cannot be "sensed"; e.g. when 1 lb of water is converted into steam at 1 atm pressure and at 212°F then 970·1 Btu are absorbed; the temperature of the system remains constant during the change.

SI units

The United Kingdom is changing to the metric system at a time when a degree of standardization of the system has itself just been achieved. The SI sets out the basic and derived units which have been agreed internationally. In the main the units in Tables 1–5 have already been adopted in metric-using countries.

TABLE 1. BASIC SI UNITS

Quantity	Unit	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
electric current	ampere	A
luminous intensity	candela	cd

TABLE 2. DERIVED UNITS WITH SPECIAL NAMES

Quantity	Units	Symbol
force	newton	$N = kg\ m/s^2$
work energy	joule	$J = N\ m$
power	watt	$W = J/s$
electrical potential	volt	$V = W/A$
luminous flux	lumen	$lm = cd\ sr$
illumination	lux	$lx = lm/m^2$

TABLE 3. DERIVED SI UNITS WITH COMPLEX NAMES

Quantity	Units	Symbol
area	square metre	m^2
volume	cubic metre	m^3
frequency	hertz	$Hz = s^{-1}$
density	kilograms per cubic metre	kg/m^3
velocity	metre per second	m/s
pressure, stress	newton per square metre	N/m^2
thermal conductivity	watt per metre kelvin	$W/m\ K$
kinematic viscosity	square metre per second	m^2/s

TABLE 4

Multiplication factor		Prefix	Symbol
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
100	10^2	hecto	h
10	10^1	deca	da
0·1	10^{-1}	deci	d
0·01	10^{-2}	centi	c
0·001	10^{-3}	milli	m
0·000 001	10^{-6}	micro	μ

TABLE 5. CONVERSION FACTORS

Length	1 in. 1 Å (angstrom)	25·4 mm 10^{-10} m
Mass	1 lb 1 ton	0·453 592 37 kg 1016·06 kg
Force	1 pdl 1 dyn	0·138 26 N 10^{-5} N
Energy (work, heat)	1 ft lbf 1 ft pdl 1 erg 1 cal (int.) 1 Btu 1 kWh 1 therm	1·355 8 J 0·042 14 J 10^{-7} J 4·186 8 J 1·055 06 kJ 3·6 MJ 105·51 MJ
Calorific value	1 Btu/ft^3 1 Btu/lb	37·259 kJ/m^3 2·326 kJ/kg
Velocity	1 ft/s	0·304 8 m/s
Pressure	760 mmHg 1 in. water 1 mmHg (1 torr) 1 lbf/in^2	1013 mbar 249·09 N/m^2 133·32 N/m^2 6·894 8 kN/m^2
Heat transfer coefficient	1 Btu/h ft^2 °F	5·678 3 W/m^2 K

Specific heat	1 Btu/lb °F	4·186 8 kJ/kg K
	1 Btu/ft^3 °F	67·066 kJ/m^3 K
Thermal conductivity	1 Btu/h ft °F	1·730 7 W/m K
Power	1 hp (British)	745·7 W
	1 ft lbf/s	1·355 8 W
	1 Btu/h	0·293 07 W
	1 erg/s	10^{-7} W
Density	1 lb/ft^3	16·019 kg/m^3
Viscosity, dynamic	1 poise	0·1 Ns/m^2
	1 lb/ft h	0·413 38 mNs/m^2
	1 lb/ft s	1·488 2 Ns/m^2
Viscosity, kinematic	1 stoke	10^{-4} m^2/s
	1 ft^2/h	0·258 06 cm^2/s

1.4. CALORIFIC VALUES (CV)

The calorific value of any substance is defined as the heat available when unit weight is burned completely under specified conditions and the products of combustion cooled down to the standard temperature of 60°F or 15°C.

In theory the whole of the heating value of a fuel can be profitably used. If the fuel contains moisture and/or hydrogen it can be argued that this does not condense in industrial plant and the latent heat should not be included in the calorific value of the fuel.

For solid or liquid fuels the gross or higher calorific value includes the heat given up by the condensation and cooling of any water present in the products of combustion. For a gas, it is defined as the number of heat units liberated by the combustion in air of unit volume of gas saturated with water vapour, after the products of combustion have cooled down to 15°C.

The net or lower calorific value can be obtained by deducting from the gross calorific value the latent heat given up by the condensation and cooling of any water present in the products of combustion, including water originally present in the fuel.

For this purpose the latent heat of condensation of steam at 60°F is taken as 1055 Btu/lb. At 15°C this is equivalent to 2454 kJ/kg.

(*Note*. Confusion often arises as to the value of 1055 for the latent heat of cooling and condensation of steam at 60°F. If the latent heat of condensation of steam at 212°F is determined it is found to be 970·1 Btu/lb. The sensible heat in cooling from 212°F to 60°F can readily be calculated and = 1 lb × 1 (specific heat water) × (212 − 60) = 152 Btu. The total heat of condensation and cooling = 970·1 + 152 = 1152·1 Btu/lb. This value is not obtained in practice, and a figure of 1055 is more realistic, as it takes into account the losses occurring in plant operation.)

For solid and liquid fuels (per lb):

$$\text{net CV} = \text{gross CV} - 1055 \times M,$$

where M = weight of moisture (lb) in products of combustion per pound of fuel.

$$\text{net CV} = \text{gross CV} - 2454 \times M',$$

where M' = weight of moisture (kg) in the products of combustion per kg of fuel.

For a gas:

$$\text{net CV} = \text{gross CV} - 50V,$$

where V = volume of hydrogen in 1 ft^3 of saturated gas at 30 in.Hg pressure and at 60°F (as a fraction of a cubic foot).

1.5. DETERMINATION OF CALORIFIC VALUES

It is possible to arrive at calorific values for coals by using formulae based upon proximate analysis (ash, moisture, volatile matter, fixed carbon) and ultimate analysis (carbon, hydrogen, sulphur, oxygen, nitrogen). The Gouthal formula based upon proximate analysis gives

$$\text{CV gross, Btu/lb} = 147{\cdot}6\,\text{C} + aV,$$

where C is the percentage of fixed carbon, V the volatiles, and a a factor depending on the yield of volatiles calculated on the pure coal.

V	1–4	10	15	20	25	30	35	40
a	270	261	210·6	196·2	185·4	176·4	171	144

Unfortunately, the results are not in very close agreement to those experimentally determined, but are useful where no other data are available.

The Dulong formula based upon ultimate analysis gives:

$$\text{CV gross, Btu/lb} = 145{\cdot}4\,\text{C} + 620{\cdot}3\,\text{H}(\text{H} - \text{O}/8) + 40{\cdot}5\,\text{S},$$

where C, H, O, and S are the percentages of carbon, hydrogen, oxygen, and sulphur in the coal. Over a range of coals where the carbon contents lie between 78 and 86%, this formula can be expected to give results within 150 Btu of those obtained by direct determination. With fuels containing over 90% carbon the formula ceases to be accurate; also when the oxygen content exceeds 7%.

For petroleum oils, the US Bureau of Mines gives the formula

$$\text{CV gross, Btu/lb} = 22{,}320 - 3780d^2,$$

where d = SG at 60°F/60°F.

The only really satisfactory method for the direct determination of the calorific value of solid and liquid fuels is the use of the high-pressure bomb calorimeter, in which the fuel is burned in oxygen under pressure in a "bomb" immersed in water, the temperature of which is measured. This measures the gross calorific value of the fuel at constant volume.

When fuel is burned in a bomb, the heat equivalent of the work which would be done by the atmosphere on the products of combustion, if unit quantity of the fuel were burned at constant pressure, may be calculated and added to the gross calorific value at constant volume. The deviation for fuels low in hydrogen (most British coals) is less than 0·1 %, which is well within the experimental error of the determination.

Since fuel gases consist of mechanical mixtures of a comparatively limited number of constituents, their calorific values may be calculated with reasonable accuracy provided the composition of the mixture and the heats of combustion of the constituents are known. Typical heats of combustion of the more important gases are given in Table 6.

The main source of error in calculating calorific values is the uncertainty of the nature of the unsaturated hydrocarbons and the complexity of the saturated hydrocarbons. Calculated values for coal gas, producer gas, and blue water gas agree to within 2 or 3 % of the experimentally determined values.

TABLE 6. CALORIFIC VALUES OF ELEMENTS AND SIMPLE GASEOUS COMPONENTS

	CV (Btu/ft^3)			MJ/m^3 (NTP gross)	CV (Btu/lb gross)	CV (kJ/kg gross)
	NTP gross	STP gross	STP net			
Carbon monoxide (CO)	341	318	318	12·7	4,390	10,320
Hydrogen (H_2)	343	320	270	12·8	61,340	142,000
Methane (CH_4)	1067	995	895	39·7		
Ethane (C_2H_6)	1855	1730	1580	69·0		
Propane (C_3H_8)	2660	2480	2280	84·8		
Ethylene (C_2H_4)	1673	1580	1460	62·2		
Propylene (C_3H_6)	2467	2300	2150	91·0		
Hydrogen sulphide (H_2S)	703				7,330	17,080
Carbon (to CO)					4,350	10,135
Carbon (to CO_2)					14,590	33,995
Sulphur (to SO_2)					3,930	9,156
Sulphur (to SO_3)					5,780	13,465

The usual method of direct determination is to burn the gas at a constant rate, in a vessel, under such conditions that the heat produced is absorbed in water which flows at a known constant rate. Knowing the volume of gas burned in a given time, the volume and therefore the weight of water passed, and the average rise in temperature of the water, the calorific value of the gas may be obtained.

The specification for testing the calorific value of town gas is laid down in detail in *General Notification of the Gas Referees* (HMSO). A detailed account of the apparatus and method is given in this publication. It is possible to determine directly the net calorific value of the gas when using the Boy's calorimeter.

1.6. STATISTICAL ANALYSIS

Fuel technologists are concerned with the sampling and analysis of materials and hence a knowledge of elementary statistical techniques is essential for the correct interpretation of collected data. It is also of considerable importance in analysing the results of plant trials.

Populations and samples

A population is the total number of discrete units in a bulk, and samples are a limited number of such units. It is convenient mathematically to take N samples from a population K. Numerical characteristics of a population are called *parameters*; those of selected samples, *statistics*, which may be practically measured and examined and which give an estimate of the parameters of the whole.

Random selection

When every unit in a population K has the same probability of selection, viz. $1/K$, the selection is referred to as random. In selecting randomly all prejudice or bias must be avoided; this is often difficult. A representative sample is often best obtained by means of combining small increments obtained by means of a systematic technique. This is because "random" or "spot" selection is open to the effect of prejudice on the part of the sampler and of segregation in the population.

Average or arithmetic mean

It can be proved mathematically that the average or mean result of observations is the most probable value of a population.

$$\mu = \frac{x_1+x_2 \ldots x_K}{K} = \frac{\Sigma x_K}{K}.$$

For a limited number of observations, randomly selected (samples), an unbiased estimate of μ is the mean of the samples

$$\bar{x} = \frac{\Sigma K_N}{N}.$$

The median is the middle of a series of results placed in ascending order.

The difference between any results and the mean of a series is the deviation of the result, $x - \bar{x} = d$. The standard deviation is defined

$$\sigma = \sqrt{\left(\frac{\Sigma d^2}{K}\right)}$$

and $$d = x_N - \mu,$$

where K is the number of measurements.

$$\sigma^2 = \frac{\Sigma d^2}{K} = \text{variance } V.$$

The variance V is linearly additive provided that the sources of variants are independent.

The equivalent parameters for samples are

$$S' = \sqrt{\left(\frac{\Sigma d^2}{N}\right)} = \sqrt{\left[\frac{\Sigma(x_N-\bar{x})^2}{N}\right]}.$$

However, this is a biased estimate of σ unless $\bar{x}$ is identical with μ. This need not be true because $\bar{x}$ is only an estimate of μ. The best estimate S^2 that can be made is

$$\sigma^2 = \frac{N}{N-1} \times S'^2;$$

thus
$$S = \sqrt{\left(\frac{\Sigma d^2}{N-1}\right)}.$$

The term $N-1$ is the number of degrees of freedom of the system. In most examples there is only one constraint. This is arrived at when, in a series of results, the mean is known, $N-1$ of the results could have any value, but the Nth term would have its value defined by the actual values of the $N-1$ and that of the mean.

Frequency distribution

If a whole population is examined for a particular property a plot of values of the measurements of this property against the number of times a value occurs (frequency) will take at least approximately one of the following shapes: the Gaussian, the binominal, or Poisson distribution.

The Gaussian or normal distribution is the most common and takes the shape illustrated in Fig. 1.1. The curve obeys the equation

$$\frac{h}{\sqrt{\pi}} = \exp\left[-h^2(x-\mu)^2\right],$$

where $h^2 = \frac{1}{2}\sigma^2$, σ is the standard deviation, and h is the precision index.

In the Gaussian expression, the probability that an observation falls between x_1 and x_2 is given by

$$P = \frac{h}{\sqrt{\pi}} \int_{x_1}^{x_2} \{\exp\left[-h^2(x-\mu)^2\right]\}\, dx.$$

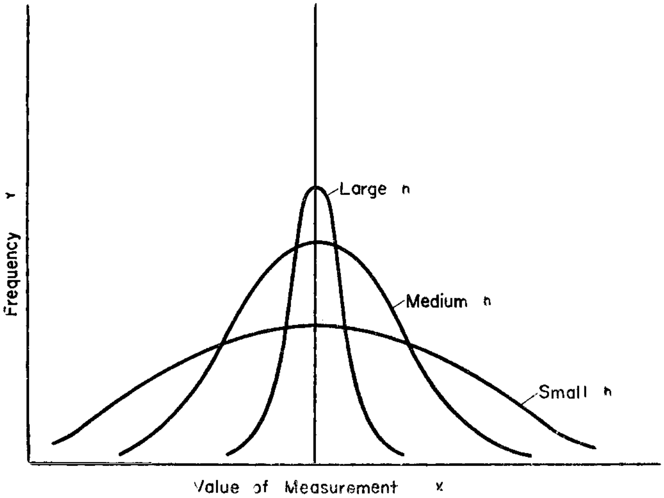

FIG. 1.1. Normal or Gaussian distribution curve

The term $\sqrt{(2)}\,h(x-\mu)$, viz. $\dfrac{x-\mu}{\sigma} = \dfrac{d}{\sigma}$ is generally referred to as C the normal deviate.

Thus

$$P = \frac{1}{\sqrt{(2\pi)}} \int_{C_1}^{C_2} [\exp(-\tfrac{1}{2}C^2)]\,dC.$$

This equation can be expressed as the difference of two integrals, and the results, called error function, is given:

$$\frac{1}{2}\operatorname{erf}(C\sqrt{2}) = \frac{1}{\sqrt{\pi}} \int_{0}^{C\sqrt{2}} \exp(-C^2)\,dC.$$

This function has been solved, and from the resulting tabulated data any value of C can be found for a given probability and vice versa.

When $C = 1, 2,$ and 3 the probability of results falling within

$\bar{x} \pm S$, $\bar{x} \pm 2S$, and $\bar{x} \pm 3S$, are 68·3, 95·4, and 99·7% respectively. Thus 95·4% of the population has values which lie between $\bar{x} \pm 2S$.

Significance of results

It is often required to determine whether a particular analytical result from one laboratory, or by a new technique, is being reproduced correctly. To estimate this significance tests are applied. The method is to adopt a null hypothesis, that is, assume there is no real difference, and then calculate the probability of the differences actually obtained having arisen by pure chance variation. If this probability is high then it can be concluded that no significant difference between the observed and true result attains.

In order to apply significance tests it will be necessary to obtain the standard error of the mean.

$$\text{Standard error of mean } S_x = \frac{\text{standard deviation}}{\sqrt{N}}.$$

The analogous term for variance $= S^2/N$.

The corresponding estimate then becomes

$$t = \frac{\bar{x} - \mu}{S_x} = \frac{(\bar{x} - \mu)\sqrt{N}}{S}.$$

Tables are available in which the probability of t exceeding certain limits is given. Another useful test is based upon the F (Fisher) distribution. Provided that two series of samples are taken from the same population their variances are determined independently.

$$F = \frac{S_1^2}{S_2^2}.$$

Here the probability of F exceeding a certain value depends only upon the number of degrees of freedom for the two series. It can be decided whether the variances of the two series of results are likely to be equal. Tables of F values are available in standard statistical works. The application of such tests is best illustrated by reference to an example.

EXAMPLE 1.1

It is required to assess the efficiency of the staff of two coke-oven laboratories for the determination of volatile matter in coal. Samples of coal containing 24·1 % volatile matter were sent to laboratories 1 and 2 and the results obtained are given below.

Sample no.	1	2	3	4	5	6	7	8	9	10
Lab. 1	25·0	24·7	23·5	25·2	26·0	24·8	26·1	25·3	25·7	22·0
Lab. 2	24·7	24·2	24·9	25·1	23·8	25·0	24·4	24·7	24·0	24·8

TABLE 6A

No.	Lab. 1	$(x-\bar{x})$	$(x-\bar{x})^2$	Lab. 2	$(x-\bar{x})$	$(x-\bar{x})^2$
1	25·0	+0·17	0·0289	24·7	+0·14	0·0196
2	24·7	−0·13	0·0169	24·2	−0·36	0·1296
3	23·5	−1·33	1·769	24·9	+0·34	0·1156
4	25·2	+0·37	0·1369	25·1	+0·64	0·4096
5	26·0	+1·17	1·3689	23·8	−0·76	0·5776
6	24·8	+0·03	0·0009	25·0	+0·44	0·1936
7	26·1	+1·27	1·6139	24·4	−0·16	0·0256
8	25·3	+0·53	0·2809	24·7	+0·14	0·0196
9	25·7	+0·87	0·7569	24·0	+0·56	0·3136
10	22·0	−2·83	8·0089	24·8	−0·24	0·0576

$\bar{x} = 24{\cdot}83$	$\bar{x} = 24{\cdot}56$
$\Sigma d^2 = 13{\cdot}982$	$\Sigma d^2 = 1{\cdot}862$
standard deviation	standard deviation
$\sqrt{\dfrac{\Sigma d^2}{N-1}}$	$\sqrt{\dfrac{\Sigma d^2}{N-1}}$
$S = 1{\cdot}247$	$S = 0{\cdot}4528$

The first stage is the preparation of tables of $(x-\bar{x})$ and $(x-\bar{x})^2$ for every result. This has been done and is set out in Table 6A.

In evaluating Σd^2 the mechanical operation becomes tedious when N is large, but the formula

$$\Sigma d^2 = \Sigma(x-L)^2 - \frac{[\Sigma(x-L)]^2}{N}$$

may be used. L is any convenient round number near the mean. The use of this formula is illustrated in Table 6B, taking $L = 25{\cdot}0$.

TABLE 6B

No.	Result	$(x-L)$	$(x-L)^2$
1	25·0	0	0
2	24·7	−0·3	0·09
3	23·5	−1·5	2·25
4	25·2	+0·2	0·04
5	26·0	+1·0	1·00
6	24·8	−0·2	0·04
7	26·1	+1·1	1·21
8	25·3	+0·3	0·09
9	25·7	+0·7	0·49
10	22·0	−3·0	9·00
		−1·7	14·21

$$\Sigma d^2 = 14{\cdot}21 - \frac{2{\cdot}89}{10} = 13{\cdot}9$$

The significance of these results can be determined by application of the t and F tests referred to previously.

	Lab. 1	Lab. 2
S_x	$\dfrac{1{\cdot}247}{\sqrt{10}}$ 0·3942	$\dfrac{0{\cdot}4528}{\sqrt{10}}$ 0·1472
$t = \dfrac{\bar{x}-\mu}{S_x}$	$\dfrac{24{\cdot}83-24{\cdot}10}{0{\cdot}3942}$ 1·799	$\dfrac{24{\cdot}56-24{\cdot}10}{0{\cdot}1472}$ 3·125

From tables of t versus degrees of freedom, P for Lab. 1 = 0·1 and for Lab. 2 = 0·01 (degrees of freedom = $N-1$). This means that the probability of such a difference occurring by pure chance variation for Lab. 1 is approximately 10%; for Lab. 2 the probability of there being a difference of 0·46 between the mean and the true result is only 1%. This suggests that in Lab. 1 the results are not significant, and the null hypothesis assumed is correct. With Lab. 2 the null hypothesis is not correct and therefore a real bias does exist in the results.

$P = 0{\cdot}05$ corresponds to a significant result;

$P = 0{\cdot}01$ corresponds to a highly significant result.

The corresponding F value is

$$F = \frac{13{\cdot}98}{9}\bigg/\frac{1{\cdot}86}{9} = 7{\cdot}5.$$

From tables of F values, with 9 degrees of freedom for each, then the probability is less than 0·01. Thus the difference in the variability of the two sets of results is highly significant. This test does not require the actual known results to be given (24·1%). The results of this test indicate that the probability of obtaining by pure chance variation equalities of the two variances is less than 1%. If they are from the same population they should be equal and this can only mean that the workers from the two laboratories have different characteristics.

EXAMPLE 1.2

In case of dispute, joint sampling is carried out by the two parties and it is important to establish whether any difference in results is significant. If it is assumed that the results in Example 1.1 were obtained by a joint sampling of the same consignment, then the two mean values may be compared by significance tests.

$$S_1^2 = \frac{\Sigma(x_1-\bar{x}_1)^2}{N_1-1},$$

$$S_2^2 = \frac{\Sigma(x_2-\bar{x}_2)^2}{N_2-1}.$$

The combined estimate of σ for both samples

$$S^2 = \frac{\Sigma(x_1-\bar{x}_1)^2+\Sigma(x_2-\bar{x}_2)^2}{N_1+N_2-2} = \text{variance}.$$

To find if there is any difference between $\bar{x}_1$ and $\bar{x}_2$ the t value is calculated.

$$t = \frac{\bar{x}_1-\bar{x}_2}{S}\sqrt{\left(\frac{N_1\times N_2}{N_1+N_2}\right)}.$$

In this particular example

$$S = \sqrt{\left(\frac{13{\cdot}98+1{\cdot}862}{10+10-2}\right)},$$

$$t = \frac{24{\cdot}83-24{\cdot}56}{0{\cdot}938}\sqrt{\left(\frac{10\times 10}{10+10}\right)},$$

$$t = \frac{0{\cdot}27\sqrt{5}}{0{\cdot}938} = 0{\cdot}64.$$

From t tables $P > 0{\cdot}5$ [degrees of freedom $(N_1+N_2-2) = 18$]. Thus it can be concluded that the probability of obtaining such a difference between the means is high from pure chance considerations alone. The fact that such a difference has been obtained is not significant, and each laboratory has carried out the analysis with similar accuracy.

BIBLIOGRAPHY

FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.

HIMUS, G. W., *Fuel Testing*, 3rd edn., Leonard Hill, 1953.

National Conference "The Change to the International System (SI) Units for Energy", Church House, Westminster, London, 31 October–1 November 1968.

PANTONY, D. A., *Design of Experiment*, Royal Institute of Chemistry, Monograph No. 2, 1961.

PRATT, A. D., *Principles of Combustion in the Steam Boiler Furnace*, Babcock & Wilcox Ltd., 1936.

SNEEDEN, J. B. O., *Applied Heat for Engineers*, Blackie & Sons Ltd., 1953.

SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.

WILLIAMS, D. A. and JONES, J., *Liquid Fuels*, Pergamon Press, 1963.

The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 2

Combustion Calculations

2.1. INTRODUCTION

As discussed in Chapter 1, it is the exothermic reactions which occur when carbon, hydrogen, and, to a lesser extent, sulphur, are burned that release the energy available in the fuel.

A study of the reaction kinetics reveals that most combustion reactions occur via chain mechanisms. Whilst individual reactions are fast, the whole sequence requires a finite time and in these cases combustion cannot occur instantaneously. Where the reaction approaches spontaneity, the result is an explosion. Every fuel has to be raised to a certain temperature, and maintained at or above that temperature for combustion to be completed. In the combustion of complex materials such as coals and heavy oils which proceed by chain mechanism, energy is absorbed in breaking the complex molecules down to simpler hydrocarbons and carbon monoxide. Unless the reaction chamber is maintained at a sufficiently high temperature, it is possible for the reacting materials to be chilled below the temperature at which combustion can proceed. This gives rise to loss of energy as unburned gases and soot. This is also a particular problem when burning highly radiating fuels such as pulverized coal, where the rate of heat loss is so high that extinction of the flame may result.

In order to obtain rapid and complete combustion both fuel and oxygen supply (air) have to be intimately mixed. This is not difficult to achieve with those gaseous fuels where premixing of nearly, or stoichiometric, amounts of fuel and air can be achieved before combustion begins. This cannot be achieved with coal, oil, and with some gaseous fuels. Although the design of appliances to promote turbulence can improve the completeness of combustion in a given time, in industrial practice often the only effective means of ensuring complete combustion is to use excess air. The amount used plays an important role in the efficient use of fuels. Insufficient air results in incomplete combustion and loss of heating value, whereas too much air leads to excessive loss of sensible heat in the combustion products. In each combustion reaction appliance there is an optimum percentage of excess air where the combined losses due to insufficient combustion and sensible heat will be a minimum.

2.2. CALCULATION OF MINIMUM (THEORETICAL) AIR FOR COMPLETE COMBUSTION

Atmospheric air is a mechanical mixture of oxygen, nitrogen, and small amounts of carbon dioxide, water vapour, argon, and other noble gases. For engineering purposes the carbon dioxide and noble gases are included with the nitrogen, and the values used in industrial practice for the composition of dry air are as in Table 7.

TABLE 7. COMPOSITION OF DRY AIR

	Volume (%)	Weight (%)
Oxygen	21	23·2 (23)
Nitrogen	79	76·8 (77)

Equal volumes of gases under the same conditions of temperature and pressure contain the same number of molecules (Avogadro's hypothesis). It follows from this hypothesis that because 2 molecules of hydrogen and 1 molecule of oxygen combine for complete combustion, 2 volumes of hydrogen will combine with 1 volume of oxygen and the product will be 2 volumes of water:

$$2\,H_2+O_2 = 2\,H_2O.$$

At NTP the molecular weight in grams of an ideal gas occupies 22·4 l. and the molecular weight in kilograms occupies 22·414 m^3. Similarly, the molecular weight in pounds occupies 359 ft^3. At STP (saturated) the molecular weight in pounds occupies 385 ft^3. At STP (not saturated) the molecular weight in pounds occupies 379 ft^3.

Since the compositions of gaseous fuels are expressed as percentages by volume, combustion calculations are simplified as there is no need to convert percentages by weight into pound (kg) molecules. The percentage composition of a gas, if given on a volume basis, also holds on a kilogram-molecule basis.

The following example of calculating the theoretical air requirement for a coke-oven gas illustrates the simplicity of the calculation.

EXAMPLE 2.1

A gas of the following volumetric composition: O_2, 0·4; CO_2, 2·0; C_3H_6, 2·6; CO, 7·4; H_2, 54·0; CH_4, 28·0; N_2, 5·6%, is burned with air.

The relevant combustion equations are:

$$\begin{aligned} 2\,C_2H_6+7\,O_2 &= 4\,CO_2+6\,H_2O;\\ 2\,CO+O_2 &= 2\,CO_2;\\ CH_4+2\,O_2 &= CO_2+2\,H_2O;\\ 2\,H_2+O_2 &= 2\,H_2O. \end{aligned}$$

These equations represent the stoichiometric quantities for complete oxidation of the combustible constituents in the gas. Hence the oxygen requirement for combustion of 100 m^3 of gas

$$= 9·1+3·7+56+27 = 95·8\ m^3.$$

There is already 0·4 m^3 of oxygen in the gas, hence the oxygen needed to be supplied by an external source (air)

$$= 95·8-0·4 = 95·4\ m^3 \text{ at NTP.}$$

The air needed for combustion per cubic metre of gas

$$= \frac{95{\cdot}4\times 100}{21\times 100} = 4{\cdot}54 \text{ m}^3. \qquad \textit{(Answer)}$$

(O_2 = 21% by volume of air.)

Solid fuels must be converted to the as-fired basis. The method of calculation is best illustrated by reference to another example.

EXAMPLE 2.2

A coal of the following analysis on the dry, ash-free basis: C, 89·3; H, 5·0; N, 1·5; S, 0·8; O, 3·4% by weight is used with 8·0% ash and 7·0% moisture. Gross CV 15,650 Btu/lb (36,000 kJ/kg).

It is necessary to have the analysis on a molecular basis because combustion formulae are best calculated on this basis. First, convert the analysis from percentage weight to a pound molecule basis by dividing the mass percentage by the molecular weight. Thus for 100 kg of d.a.f. coal:

$$\text{C} = \frac{89{\cdot}3}{12} = 7{\cdot}4 \text{ kg-atom}; \qquad \text{S} = \frac{0{\cdot}8}{32} = 0{\cdot}025 \text{ kg-atom};$$

$$H_2 = \frac{5{\cdot}0}{2} = 2{\cdot}5 \text{ kg-mol}; \qquad O_2 = \frac{3{\cdot}4}{32} = 0{\cdot}106 \text{ kg-mol};$$

$$N_2 = \frac{1{\cdot}5}{28} = 0{\cdot}053 \text{ kg-mol}.$$

It must be remembered that in coal, these elements do not, in fact, exist as molecular nitrogen, oxygen, hydrogen, carbon, and sulphur. The relevant combustion equations are:

$C+O_2 = CO_2$; O_2 requirement = 7·4 kg-mol;

$S+O_2 = SO_2$; O_2 requirement = 0·025 kg-mol;

$2\,H_2+O_2 = 2\,H_2O$; O_2 requirement = 1·25 kg-mol.

Oxygen required for combustion

$$= 7{\cdot}4+1{\cdot}25+0{\cdot}025-0{\cdot}106 = 8{\cdot}569 \text{ kg-mol}.$$

The oxygen in the fuel is considered to be liberated and available for combustion purposes when the coal molecule is broken down.

$$\text{Air requirement} = \frac{100 \times 8{\cdot}569}{21} = 40{\cdot}82 \text{ kg-mol}$$

$$= 40{\cdot}82 \times 22{\cdot}4 = 914{\cdot}7 \text{ m}^3 \text{ at NTP.}$$

Hence 1 kg of a d.a.f. coal would require 9·147 m^3 of air for combustion (measured at NTP).

Alternatively, if the weight of air for combustion is required then it may be obtained as follows:

$$O_2 \text{ requirement} = 8{\cdot}569 \text{ kg-mol.}$$

By definition, 1 kg-mol = molecular weight in kilograms.
Hence oxygen requirement = 8·569×32 kg-wt.
Therefore if oxygen supplied as air, then weight of air

$$= \frac{8{\cdot}569 \times 32}{23} = 1196{\cdot}5 \text{ kg-wt. (composition air } 23\% \; O_2\text{).}$$

Weight of air required to burn 1 kg of a d.a.f coal

$$= 11{\cdot}965 \text{ kg-wt.} \quad \textit{(Answer.)}$$

The volume of air required for "as-fired" coal will not be as large as for d.a.f fuel because the quantity of combustible material contained in 1 kg of fuel will not be so high.

$$\text{Volume of air} = \frac{9{\cdot}147 \times (100-7-8)}{100}$$

$$= 7{\cdot}77 \text{ m}^3 \text{ at NTP.} \quad \textit{(Answer.)}$$

2.3. CALCULATION OF QUANTITY AND ANALYSIS OF FLUE GAS

Most gases are analysed on a dry basis using analytical instruments similar in principle to the Orsat, any water in the gas being condensed out before analysis. Although water from the combustion of hydrogen does not appear in the gas analysis, nitrogen from the combustion air does remain in the dry gas.

EXAMPLE 2.3

Calculate the analysis of the dry gas from combustion of the as-fired coal used in Example 2.2.

The flue or waste gas from the combustion of 1 kg of coal will be composed of:

$$N_2 \text{ from coal} = \frac{0{\cdot}053 \times 85 \times 22{\cdot}4}{100 \times 100} = 0{\cdot}01 \text{ m}^3.$$

(It must be remembered that 0·053 kg-mol of nitrogen are present in 100 kg d.a.f. coal.)

$$N_2 \text{ from combustion air} = \frac{7{\cdot}77 \times 79}{100} = 6{\cdot}1 \text{ m}^3.$$

$$SO_2 = \frac{0{\cdot}025 \times 85 \times 22{\cdot}4}{100 \times 100} = 0{\cdot}005 \text{ m}^3.$$

$$CO_2 = \frac{7{\cdot}4 \times 85 \times 22{\cdot}4}{100 \times 100} = 1{\cdot}4 \text{ m}^3.$$

Hence volumetric analysis at NTP:

$$CO_2 = \frac{1{\cdot}4}{1{\cdot}4 + 0{\cdot}005 + 0{\cdot}01 + 6{\cdot}08} \times 100 = 18{\cdot}7\%.$$

$$SO_2 = \frac{0{\cdot}005}{1{\cdot}4 + 0{\cdot}005 + 0{\cdot}01 + 6{\cdot}08} \times 100 = 0{\cdot}07\%.$$

$$N_2 = \frac{(0{\cdot}01 + 6{\cdot}08)}{1{\cdot}4 + 0{\cdot}005 + 0{\cdot}01 + 6{\cdot}08} \times 100 = 81{\cdot}3\%.$$

Although analyses are normally reported on a dry basis it is often necessary to estimate the total volume of gases passing through the flue system, chimney, or fan. To do this the total volume at NTP must be calculated and then the gas laws applied to obtain the volume at the required conditions. The wet flue gas will be composed of:

Water vapour from moisture present in coal = 7 kg-wt.

$$= \frac{7}{18}\text{ kg-mol} = \frac{7 \times 22{\cdot}4}{18 \times 100} = 0{\cdot}087 \text{ m}^3 \text{ NTP}.$$

Water vapour from combustion of hydrogen in coal

$$= \frac{2{\cdot}5 \times 85 \times 22{\cdot}4}{100 \times 100} = 0{\cdot}5 \text{ m}^3.$$

N_2 from combustion air = 6·08 m^3.
N_2 from coal = 0·01 m^3.
SO_2 = 0·005 m^3.
CO_2 = 1·4 m^3.

(*Note*. When burning heavy oil atomized by steam, the steam used must be included in this calculation.)

Total volume of wet gas at NTP = 8·082 m^3. Volumetric analysis:

$$N_2 = \frac{(6{\cdot}08 + 0{\cdot}01)}{8{\cdot}082} \times 100 = 75{\cdot}3\%.$$

$$CO_2 = \frac{1{\cdot}4}{8{\cdot}082} \times 100 = 17{\cdot}4\%.$$

$$SO_2 = \frac{0{\cdot}005}{8{\cdot}082} \times 100 = 0{\cdot}06\%.$$

$$H_2O = \frac{(0{\cdot}5 + 0{\cdot}087)}{8{\cdot}082} \times 100 = 7{\cdot}2\%.$$

2.4. CALCULATION OF NET CALORIFIC VALUE OF AS-FIRED COAL

EXAMPLE 2.4

Calculate the net calorific value of the as-fired coal used in Example 2.2.

$$\text{Gross CV as-fired} = \frac{36{,}000 \times 85}{100} = 30{,}700 \text{ kJ/kg}.$$

Net CV = gross CV − 2454 × weight of water in products of combustion.

Weight of water from combustion of hydrogen

$$= \frac{2{\cdot}5 \times 18 \times 85}{100 \times 100} = 0{\cdot}38 \text{ kg-wt.}$$

Weight of water from coal as moisture = 0·07 kg-wt.

$$\text{Net CV} = 30{,}700 - (2454 \times 0{\cdot}45)$$
$$= 30{,}700 - 1104 = 29{,}596 \text{ kJ/kg.} \quad \textit{(Answer.)}$$

2.5. CALCULATION OF EXCESS AIR USED FROM FLUE GAS ANALYSIS

Whenever gas analysis is carried out, the full analysis should always be completed if possible. Many operators merely record the percentage carbon dioxide and this gives no indication of any loss of heat due to carbon monoxide. Carbon dioxide is often present together with oxygen because the gases may have been chilled below their ignition temperature before combustion was completed, or mixing was inadequate or the combustion time was too short. Also, oxygen may be present due to ingress of air by leakage after the combustion zone.

The oxygen content is therefore a better indication of combustion conditions. In many plants, carbon dioxide is evolved as part of the process, as in cement and doloma manufacture, and hence estimation of this gas alone would be of little value in assessing combustion efficiency. Most industrial equipment has automatic carbon dioxide and oxygen recorders.

The simplest approach to the calculation of excess air is to draw a graph of carbon dioxide and oxygen versus excess air and then it is possible to read off directly the required value. The graphs drawn in Figs. 2.1 and 2.2 illustrate how the increase of excess air from 0 to 100% is related to the carbon dioxide and oxygen values in the dry flue gas. Similar graphs can be constructed on the wet gas analysis if required. The calculations upon which these are based are given below, using a coal of the following specification: Coal, d.a.f.; C, 89·3; H, 5·0; N, 1·5; S, 0·8; O, 3·4%. As used, the coal contains 8% ash and 7% moisture.

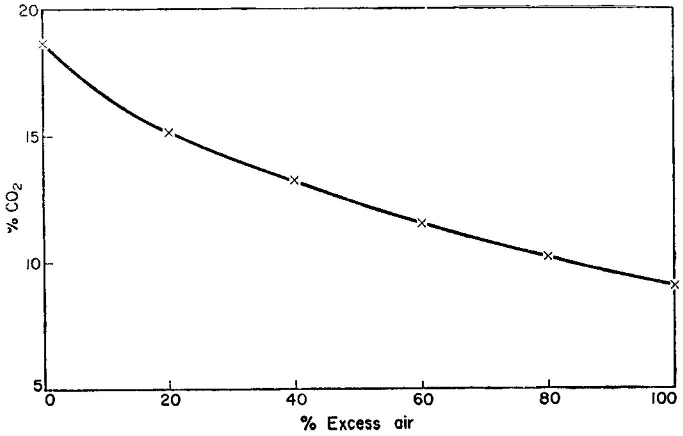

FIG. 2.1. Variation of carbon dioxide with excess air

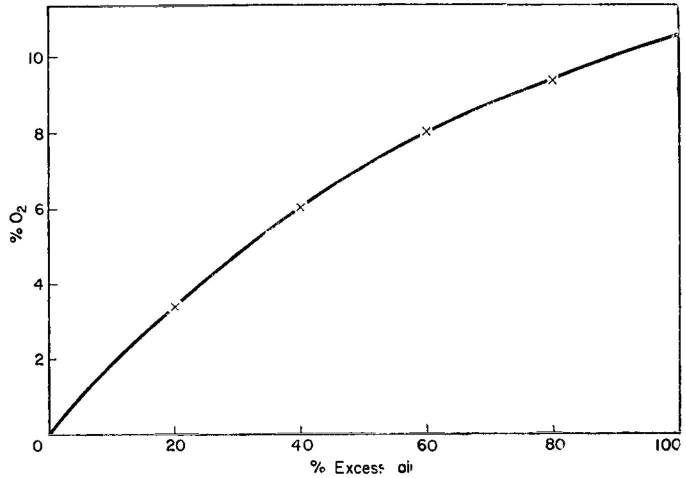

FIG. 2.2. Variation of oxygen with excess air

Theoretical air required to burn 1 kg of as-fired coal is 7·77 m^3. This represents 0% excess air, and the oxygen value will also be zero. The percentage carbon dioxide is given by the expression:

$$CO_2 = \frac{\text{volume of carbon dioxide}}{\text{total volume of flue gas}} \times 100$$

$$= \frac{1{\cdot}4}{7{\cdot}5} \times 100 = 18{\cdot}7\%.$$

With 20% excess air the denominator is increased by 20% of the theoretical air. (Excess air is expressed as a percentage in excess of the theoretical requirement.)

With 20% excess air:

$$CO_2 = \frac{1{\cdot}4}{9{\cdot}05} \times 100 = 15{\cdot}4\%.$$

$$O_2 = \frac{\text{volume of oxygen}}{\text{total volume}} \times 100.$$

Since oxygen is 21% of air by volume, the volume of O_2

$$= \frac{21}{100} \times 1{\cdot}55 = 0{\cdot}326 \text{ m}^3.$$

$$\text{Hence } O_2 \text{ percentage} = \frac{0{\cdot}326}{9{\cdot}05} \times 100 = 3{\cdot}5\%.$$

With 40% excess air:

$$\text{Volume of combustion products} = 7{\cdot}5 + \frac{40}{100} \times 7{\cdot}8$$

$$= 10{\cdot}62 \text{ m}^3.$$

$$CO_2 = \frac{1{\cdot}4}{10{\cdot}62} \times 100 = 13{\cdot}2\%.$$

$$O_2 \text{ volume} = \frac{21}{100} \times 3{\cdot}1 = 0{\cdot}65 \text{ m}^3.$$

$$O_2 = \frac{0{\cdot}65}{10{\cdot}62} \times 100 = 6{\cdot}0\%.$$

Percentages of carbon dioxide and oxygen for excess air values of 60, 80, and 100% are calculated in a similar manner.

(*Note*. Some operators add the volume of sulphur dioxide produced to that of the carbon dioxide and call the total combined value carbon dioxide. This is because in carbon dioxide detectors which rely upon absorption by alkali, the sulphur dioxide is absorbed and will be recorded as carbon dioxide. Hence the carbon dioxide figures will be slightly higher than those recorded above.)

An approximate but sufficiently accurate determination can be made by the use of It diagrams (Spiers) which are graphical solutions to combustion problems. These depend on the facts that the enthalpies of equal volumes of industrial gases can be used as a measure of temperature and that there is a statistical relationship between the net CV's of industrial fuels, their air requirements, and the volume of gases produced.

EXAMPLE 2.5

Calculate the percentage of excess air in the flue gases from combustion of the same coal as in Example 2.2 given that the carbon dioxide content of the dry flue gas is 12%.

The products of combustion of 1 kg of coal with air are:

N_2 from theoretical air = 6·1 m^3.
N_2 from coal = 0·01 m^3.
SO_2 = 0·005 m^3.
CO_2 = 1·4 m^3.
Excess air = a m^3.

In this case it is assumed that the 12% CO_2 includes any sulphur dioxide.

$$\% \, CO_2 = \frac{1{\cdot}405(CO_2+SO_2)}{1{\cdot}4+6{\cdot}1+0{\cdot}01+0{\cdot}005+a} \times 100.$$

$a = 4{\cdot}2\ m^3$ excess air.

$$\% \text{ Excess air} = \frac{\text{excess air}}{\text{theoretical air}} \times 100$$

$$= \frac{4{\cdot}2}{7{\cdot}77} \times 100 = 54\%. \quad \textit{(Answer.)}$$

[*Note*. The nitrogen from the coal has been included in this calculation, but where the nitrogen in the fuel is small (less than 3%) its contribution to the flue gas can be safely ignored.]

Provided combustion is complete, all the carbon in the coal may be assumed to appear as carbon in the combustion or gasification products. Where combustion is incomplete this approach may still be used provided the amount of carbon which is gasified is known.

100 kg coal contain $7{\cdot}44 \times 0{\cdot}85 = 6{\cdot}32$ kg-atom carbon.

100 kg-mol of gas may be equated to 100 volumes for analysis (Avogadro's hypothesis), so 100 kg-mol of gas contain 12·0 kg-atom of carbon.

$$\text{Thus}\quad 12{\cdot}0 = \frac{6{\cdot}32}{\text{total volume kg-mol}} \times 100.$$

$$\text{Volume gas} = 52{\cdot}5 \text{ kg-mol}$$

$$= 52{\cdot}5 \times 22{\cdot}4 = 1176 \text{ m}^3 \text{ at NTP.}$$

Volume from 1 kg coal = 11·76 m^3 at NTP.
The combustion products from 1 kg of coal are:

N_2 (theoretical air+coal) = 6·11 m^3.
SO_2 = 0·005 m^3.
CO_2 = 1·4 m^3.
Excess air = a m^3.

Therefore excess air = $11{\cdot}76 - 7{\cdot}515 = a = 4{\cdot}245$ m^3.

$$\% \text{ excess air} = \frac{4{\cdot}245}{7{\cdot}77} \times 100 = 54\%. \qquad \textit{(Answer.)}$$

EXAMPLE 2.6

In this calculation it is assumed that the dry flue gas contains 8% oxygen.

Products of combustion of 1 kg of as-fired coal:

N_2 from theoretical air $= 6{\cdot}1$ m^3.

N_2 from coal $= 0{\cdot}01$ m^3.

SO_2 $= 0{\cdot}005$ m^3.

CO_2 $= 1{\cdot}4$ m^3.

Excess air $= a$ m^3.

$$\%\ O_2 = 8 = \frac{0{\cdot}21\ a}{6{\cdot}1+0{\cdot}01+0{\cdot}005+1{\cdot}4+a}\times 100.$$

$$a = 4{\cdot}64 \text{ m}^3.$$

$$\%\text{ excess air} = \frac{4{\cdot}64}{7{\cdot}8}\times 100 = 60\%. \quad \textit{(Answer.)}$$

EXAMPLE 2.7

Where the complete flue gas analysis is known, it is possible to use the nitrogen and oxygen values to arrive at the excess air percentage. This method relies upon the assumption that all the nitrogen in the gas has come from combustion air and excess air. The inaccuracy resulting from this assumption is eliminated if the nitrogen in the fuel is known, and this method is then as accurate as any other.

Assume that the coal previously referred to is burned with air and gives rise to flue gas of the following volumetric composition (dry basis): O_2, 4·2; CO_2, 15·0; and N_2 80·8%. (There is no need to know the analysis of the fuel provided that the nitrogen content is low, less than 3%.)

The oxygen represents excess air within the system, therefore for 100 m^3 of flue gas the total excess air

$$= 4{\cdot}2+\frac{79}{21}\times 4{\cdot}2$$

$$= 4{\cdot}2+15{\cdot}8 \text{ m}^3.$$

The nitrogen associated with the excess air $= 15{\cdot}8$ m^3.
Therefore nitrogen from air burned $= 80{\cdot}8-15{\cdot}8 = 65{\cdot}0$ m^3.

Therefore excess air percentage

$$= \frac{\text{volume excess air}}{\text{theoretical air}} \times 100 = \frac{0{\cdot}79 \times \text{excess air}}{0{\cdot}79 \times \text{theoretical air}} \times 100$$

$$= \frac{15{\cdot}8}{65{\cdot}0} \times 100 = 24{\cdot}3\%. \quad \textit{(Answer.)}$$

The advantage of this method is that provided the nitrogen content of the fuel is low (most solid and liquid fuels), and provided no ingress of air has occurred at the sampling point, then a full analysis of combustion gases enables the percentage excess air to be calculated without a knowledge of the fuel analysis.

2.6. INCOMPLETE COMBUSTION

Assume that the coal described in Example 2.2 is burnt in air. If the oxygen content of the dry flue gas is plotted against the carbon dioxide percentage a straight-line graph results (Fig. 2.3). In practice, combustion of solid fuel is rarely complete, since there will be loss of carbon in the ashes, as "smoke" and possibly as carbon monoxide in the flue gases. Formation of carbon monoxide results in a loss of heating value equiva-

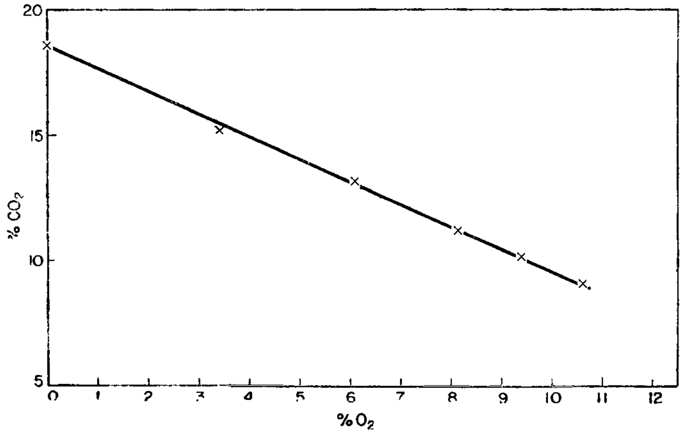

FIG. 2.3. Graph of oxygen percentage versus carbon dioxide percentage

lent to about two-thirds of that resulting from combustion to carbon dioxide. The loss due to smoke is almost impossible to determine directly.

Figure 2.3 can be used directly to determine if the combustion has been complete. If the oxygen in the gas is 5·4% then the carbon dioxide content should be 13·2%. If the actual carbon dioxide content is less than this then incomplete combustion is indicated. If the gas analysis is reconstituted so that the carbon monoxide present is converted to carbon dioxide, the oxygen diminished by the amount required to burn the carbon monoxide, and the percentage of each constituent recalculated then it is possible to determine whether any loss of carbon as "smoke" or soot has taken place.

EXAMPLE 2.8

Coal, d.a.f. analysis: C, 89·3; H, 5·0; N, 1·5; S, 0·8; O, 3·4% by weight, and as-fired containing 8·0% ash and 7·0% moisture. The coal on combustion gives rise to a gas of the following volumetric composition (dry basis): O_2, 6·3; CO_2, 11·1; CO, 2·0; N_2, 80·6%.

It is desired to determine if any actual loss of carbon has occurred.

The reconstituted analysis of 100 m^3 flue gas will be:

$$CO_2 = 11{\cdot}1+2{\cdot}0 \text{ (from CO)} = 13{\cdot}1 \text{ m}^3.$$

$$O_2 = 5{\cdot}3 \text{ (less } 1{\cdot}0 \text{ to burn CO).}$$

$$N_2 = 80{\cdot}6 \text{ m}^3.$$

$$\%\, O_2 = \frac{5{\cdot}3}{80{\cdot}6+5{\cdot}3+13{\cdot}1}\times 100 = 5{\cdot}4\%.$$

$$\%\, CO_2 = \frac{13{\cdot}1}{80{\cdot}6+5{\cdot}3+13{\cdot}1}\times 100 = 13{\cdot}2\%.$$

From Fig. 2.3 it can be seen that there has been no actual loss of carbon from the system.

EXAMPLE 2.9

The same coal on combustion gives rise to smoke and it is desired to estimate the amount of carbon lost in this manner.

Assume that the volumetric dry flue gas analysis is: CO_2, 12·0; CO, 1·0; N_2, 80·5; O_2, 6·5%.

Assuming that 100 kg of d.a.f coal is burned,

$$C = \frac{89{\cdot}3}{12} = 7{\cdot}4 \text{ kg-atom}; \quad S = \frac{0{\cdot}8}{32} = 0{\cdot}025 \text{ kg-atom};$$

$$O_2 = \frac{3{\cdot}4}{32} = 0{\cdot}106 \text{ kg-mol}; \quad H_2 = \frac{5{\cdot}0}{2} = 2{\cdot}5 \text{ kg-mol};$$

$$N_2 = \frac{1{\cdot}5}{28} = 0{\cdot}05 \text{ kg-mol.}$$

The combustion equations are:

$$C+O_2 = CO_2; \quad 2\,H_2+O_2 = 2\,H_2O; \quad S+O_2 = SO_2.$$

$$\text{The oxygen requirement} = 7{\cdot}4+0{\cdot}025+1{\cdot}25-0{\cdot}106$$
$$= 8{\cdot}57 \text{ kg-mol.}$$

$$\text{Air requirement} = \frac{8{\cdot}57\times 100}{21} = 40{\cdot}81 \text{ kg-mol.}$$

Therefore nitrogen = 40·81 − 8·57 = 32·24 kg-mol.
Nitrogen associated with as-fired coal

$$= \frac{32{\cdot}24\times(100-7-8)}{100} = 27{\cdot}4 \text{ kg-mol.}$$

Carbon in as-fired coal = 7·4 × 0·85 = 6·3 kg-mol.
Rewriting the flue gas analysis, from 100 m^3 gas:

CO_2 = 12·0
CO = 1·0
O_2 to burn CO = 0·5
Excess air = 28·56 $\left(\text{remaining } O_2+\frac{79}{21}\times O_2\right)$
N_2 = 57·94 from air burned
100·0

In this example reconstituting the analysis does not cause the carbon dioxide value to coincide with the oxygen value (Fig. 2.3). Hence there must be some actual loss of carbon out of the system. This loss can be estimated in the following way.

If X kg-mol of carbon remain unburned (either as soot or in ashes) then the oxygen required is less and becomes $6{\cdot}3-X$ kg-mol (6·3 kg-mol is the theoretical C value in the as-fired coal). This quantity of oxygen is associated with $27{\cdot}4-3{\cdot}76$ kg-mol of nitrogen and the carbon in the gases is $6{\cdot}3-X$ kg-atom ($3{\cdot}76 = 79/21$). From the reconstituted analysis 13·0 kg-atom of carbon are associated with 57·94 kg-mol of nitrogen from the air actually burned.

Therefore $$\frac{6{\cdot}3-X}{13{\cdot}0}=\frac{27{\cdot}4-3{\cdot}76X}{57{\cdot}94},$$

i.e. $$\frac{\text{kg-atom carbon burned}}{\text{kg-atom carbon in gas}}$$

$$=\frac{\text{nitrogen associated with carbon burned}}{\text{nitrogen from air burned}}.$$

Therefore $X = 1{\cdot}0$ kg-atom/100 kg as-fired coal.

$$\text{Loss per kg coal} = \frac{1{\cdot}0\times 12}{100} = 0{\cdot}12\text{ kg.}\quad \textit{(Answer.)}$$

Now that the loss of carbon has been estimated a carbon balance can be applied with confidence. The total dry flue gas is given by:

$$\%\text{ C in flue gas} = \frac{\text{actual carbon}}{\text{total flue gas}}\times 100$$

$$13{\cdot}0 = \frac{(6{\cdot}3-1{\cdot}0)}{\text{total}}\times 100.$$

Total flue gas = 41·8 kg-mol/100 kg coal as-fired.

$$N_2\text{ in flue gas} = \frac{41{\cdot}8\times 80{\cdot}5}{100} = 33{\cdot}65\text{ kg-mol,}$$

i.e. total flue gas × % N_2 in flue gas.

N_2 from excess air = 33·65 − 27·40 = 6·15 kg-mol.

$$\% \text{ excess air} = \frac{6{\cdot}15}{27{\cdot}38 \text{ (less nitrogen in fuel)}} \times 100$$

$$= 22{\cdot}4\%. \quad \textit{(Answer.)}$$

2.7. LOSS OF HEAT IN FLUE GASES

It is normally required to estimate the loss of sensible heat, although sometimes the total loss of heat is required.

Where combustion is complete the total wet gases must be calculated and from a knowledge of the mean specific heats and the temperature of the gases leaving the system the loss due to the sensible heat content may be calculated.

Example 2.10

A coal tar has the following analysis by weight: C, 90·0; H, 5·9; S, 0·4; N, 1·0; O, 2·7%. Assume that it is burned with 20% excess air and that 0·5 lb of steam are used per pound of fuel for atomization purposes.

Calculate the heat loss per pound of fuel burned when the gases leave the combustion chamber at 560°F.

The first step involves the calculation of the total quantity of combustion products obtained from 1 lb of fuel. The steam for atomization purposes must be included in this calculation, because it is the total wet flue gas that is required.

Two approaches will be adopted, the first involves calculation of the total volume of combustion products measured at NTP. The second involves calculation of the total weight of combustion products. In the first case the specific heats will need to be in volume units, and in the second on a mass basis.

The total volume of products of combustion from 100 lb of tar is calculated.

$$C = \frac{90}{12} = 7{\cdot}5 \text{ lb-atom}; \qquad H_2 = \frac{5{\cdot}9}{2} = 2{\cdot}95 \text{ lb-mol};$$

$$O_2 = \frac{2{\cdot}7}{32} = 0{\cdot}084 \text{ lb-mol}; \qquad S = \frac{0{\cdot}4}{32} = 0{\cdot}012 \text{ lb-mol};$$

$$N_2 = \frac{1{\cdot}0}{28} = 0{\cdot}035 \text{ lb-mol}.$$

The relevant combustion reactions are:

$$C+O_2 = CO_2; \qquad 2\,H_2+O_2 = 2\,H_2O; \qquad S+O_2 = SO_2.$$

$$\text{Oxygen for combustion} = 7{\cdot}5+1{\cdot}475+0{\cdot}012-0{\cdot}084 = 8{\cdot}903 \text{ lb-mol}.$$

$$\text{Theoretical air} = \frac{8{\cdot}903\times100\times359}{21} = 15{,}220 \text{ ft}^3 \text{ at NTP}.$$

$$\text{Volume/lb tar} = 152{\cdot}2 \text{ ft}^3 \text{ NTP}.$$

The products of combustion from 1 lb of tar are:

N_2 from theoretical air $= 152{\cdot}2\times0{\cdot}79 = 120{\cdot}34$ ft^3.

Since excess air = 20%,

$$O_2 = \frac{152{\cdot}2\times20\times21}{100\times100} = 6{\cdot}4 \text{ ft}^3.$$

N_2 from excess air $= 24{\cdot}0$ ft^3.

$$N_2 \text{ from tar} = \frac{0{\cdot}035\times359}{100} = 0{\cdot}13 \text{ ft}^3.$$

$$SO_2 = \frac{0{\cdot}012\times359}{100} = 0{\cdot}04 \text{ ft}^3.$$

Water vapour from combustion of H_2

$$= \frac{2{\cdot}95\times359}{100} = 10{\cdot}59 \text{ ft}^3.$$

Since 0·5 lb steam is used per pound oil, then volume of steam from this source

$$= \frac{0{\cdot}5 \times 359}{18} = 9{\cdot}99 \text{ ft}^3.$$

$$CO_2 = \frac{7{\cdot}5 \times 359}{100} = 26{\cdot}93 \text{ ft}^3.$$

The combustion products consist of:

$N_2 = 120{\cdot}24 + 24{\cdot}0 + 0{\cdot}13 = 144{\cdot}27 \text{ ft}^3.$

$O_2 = 6{\cdot}4 \text{ ft}^3.$

$CO_2 + SO_2 = 26{\cdot}93 + 0{\cdot}04 = 26{\cdot}97 \text{ ft}^3.$

H_2O vapour $= 10{\cdot}59 + 9{\cdot}99 = 20{\cdot}58 \text{ ft}^3.$

Sensible heat loss = volume $\times MC_p \times \Delta T$.

The mean specific heats between 32°F and 560°F are obtained from tables (Spiers) and are:

$N_2 = 0{\cdot}02$ Btu/ft^3 °F at NTP.

$CO_2 = 0{\cdot}0288$ Btu/ft^3 °F at NTP.

$O_2 = 0{\cdot}0104$ Btu/ft^3 °F at NTP.

$H_2O = 0{\cdot}0231$ Btu/ft^3 °F at NTP.

Thus sensible heat loss

$$\begin{aligned} &= 144{\cdot}27 \times 0{\cdot}02 \times (560-32) + 6{\cdot}4 \times 0{\cdot}104 \times (560-32) \\ &\quad + 26{\cdot}97 \times 0{\cdot}028 \times (560-32) + 20{\cdot}58 \times 0{\cdot}0231 \\ &\quad \times (560-32) \\ &= 1524 + 68{\cdot}93 + 398{\cdot}7 + 250 \text{ Btu} \\ &= 2243 \text{ Btu/lb tar burned.} \quad \textit{(Answer.)} \end{aligned}$$

The calculation would be greatly simplified if the mean specific heat of the mixture was known from tables or by experiment.

The weight of products of combustion from 1 lb tar are found in the following way:

O_2 required for combustion $= 0{\cdot}089$ lb-mol

$= 0{\cdot}089 \times 32$ lb-wt.

$$\text{Air requirement} = \frac{0{\cdot}089 \times 32 \times 100}{23} = 12{\cdot}39 \text{ lb-wt.}$$

N_2 from theoretical air $= 12{\cdot}39 \times 0{\cdot}77 = 9{\cdot}54$ lb.

$$N_2 \text{ from fuel} = \frac{0{\cdot}035 \times 28}{100} = 0{\cdot}01 \text{ lb.}$$

Since excess air $= 20\%$,

N_2 from excess air $= 12{\cdot}39 \times 0{\cdot}20 \times 0{\cdot}77 = 1{\cdot}909$ lb.
O_2 content $= 0{\cdot}57$ lb.
Steam for atomizing $= 0{\cdot}5$ lb.
Steam from combustion $H_2 = 0{\cdot}295 \times 18 = 0{\cdot}531$ lb.
$SO_2 = 0{\cdot}00012 \times 64 = 0{\cdot}0008$ lb.
$CO_2 = 0{\cdot}075 \times 44 = 3{\cdot}3$ lb.

The combustion products consist of:

$N_2 = 9{\cdot}54 + 0{\cdot}01 + 1{\cdot}909 = 11{\cdot}459$ lb.
$O_2 = 0{\cdot}57$ lb.
$H_2O = 0{\cdot}5 + 0{\cdot}531 = 1{\cdot}031$ lb.
$CO_2 + SO_2 = 3{\cdot}3 + 0{\cdot}0008 = 3{\cdot}3008$ lb.

Sensible heat loss $= \text{mass} \times MC_p \times \Delta T$.

The mean specific heats between 32°F and 560°F are obtained from tables (Spiers), and are:

$O_2 = 0{\cdot}23$ Btu/lb °F. $H_2O = 0{\cdot}462$ Btu/lb °F.
$N_2 = 0{\cdot}252$ Btu/lb °F. $CO_2 = 0{\cdot}236$ Btu/lb °F.

Total sensible heat loss

$$\begin{aligned} &= 11{\cdot}46 \times 0{\cdot}251 \times (560-32) + 0{\cdot}57 \times 0{\cdot}23 \times (560-32) \\ &\quad + 1{\cdot}03 \times 0{\cdot}462 \times (560-32) + 3{\cdot}3 \times 0{\cdot}236 \times (560-32) \\ &= 1524 + 69{\cdot}2 + 251 + 399 \\ &= 2243 \text{ Btu/lb fuel burned.} \end{aligned}$$

(Answer.)

The total heat loss will be composed of the loss in the dry products of combustion plus the loss due to water vapour.

Loss due to the dry products of combustion:

$$\begin{array}{lrl} N_2 & = 1524 & \text{Btu} \\ CO_2 & = 400 & \text{Btu} \\ O_2 & = 69 & \text{Btu} \\ \hline & 1993 & \text{Btu} \end{array}$$

The loss due to water vapour is the total heat of the water vapour in the flue gas at its partial pressure and temperature minus its heat content as water at room temperature.

The partial pressure of water vapour

$$= 14{\cdot}7 \times \frac{\text{volume of moisture at NTP}}{\text{total volume of flue gases at NTP}} \text{ lb/in}^2 \text{ abs.}$$

1 atm pressure is 14·7 lb/in^2 abs.

$$\text{Hence partial pressure} = 14{\cdot}7 \times \frac{20{\cdot}58}{198{\cdot}2} = 1{\cdot}52 \text{ lb/in}^2 \text{ abs.}$$

The total heat of superheated steam at 1·52 lb/in^2 abs. pressure and at 560°F may be obtained from *Steam Tables*, and for this example is 1228 Btu/lb.

The weight of steam in the combustion products has been shown to be 1·03 lb/lb tar burned.

Hence heat loss = 1·03 × 1228 = 1369 Btu. *(Answer.)*

An alternative solution is as follows:

If it is assumed that the mean specific heat of steam is 0·475 Btu/lb °F in the range 212–560 °F, and the latent heat of condensing steam at 212°F to be 970·1 Btu/lb, then heat loss is:

$$= 1{\cdot}03 \times 970{\cdot}1 + 1{\cdot}03 \times 0{\cdot}475 \times (560-32) + 1 \times (212-32)$$
$$\times 1{\cdot}03 = 997{\cdot}07 + 171{\cdot}8 + 185{\cdot}4$$
$$= 1354 \text{ Btu.} \quad \textit{(Answer.)}$$

Another slightly different approach would be to use the latent heat of condensation and cooling of steam to be 1055

Btu/lb, and this cuts out the separate calculation of latent heat of condensation of steam and sensible heat of the water. This value will not be the same as the other, but is probably a more realistic figure.

2.8. ADDITIONAL WORKED COMBUSTION CALCULATIONS

EXAMPLE 2.11

The percentage volumetric analysis of a producer gas is as follows: H_2, 15; CO, 22; CO_2, 8; N_2, 51 %. If 20% excess air is supplied and complete combustion attained, find the percentage composition of the products of combustion.

The appropriate combustion equations are:

$$2\,H_2+O_2 = 2\,H_2O; \qquad 2\,CO+O_2 = 2\,CO_2;$$

$$CH_4+2\,O_2 = CO_2+2\,H_2O.$$

(a) Calculation of theoretical air for combustion of 100 m³ gas at NTP.

$$\begin{aligned} O_2 \text{ to burn } H_2 &= 7{\cdot}5 \text{ m}^3 \\ O_2 \text{ to burn CO} &= 11{\cdot}0 \text{ m}^3 \\ O_2 \text{ to burn } CH_4 &= \underline{8{\cdot}0 \text{ m}^3} \\ &\ \underline{26{\cdot}5 \text{ m}^3} \end{aligned}$$

Since oxygen = 21 % by volume of air, then theoretical air required

$$= \frac{26{\cdot}5\times100}{21} = 126{\cdot}2 \text{ m}^3.$$

$$\text{Actual air used} = 126{\cdot}2\times\frac{120}{100} = 151{\cdot}4 \text{ m}^3.$$

(b) Composition of products of combustion:

N_2 from original fuel = 51·0 m^3
N_2 from theoretical air = $126{\cdot}2\times0{\cdot}79$ = 99·7 m^3
N_2 from excess air = $126{\cdot}2\times0{\cdot}20\times0{\cdot}79$ = 19·9 m^3

Total N_2 = 170·6 m^3

O_2 from excess air = $126{\cdot}2\times0{\cdot}20\times0{\cdot}21$ = 5·3 m^3
CO_2 in fuel = 8·0 m^3
CO_2 from combustion of CO = 22·0 m^3
CO_2 from combustion of CH_4 = 4·0 m^3

Total CO_2 = 34·0 m^3

H_2O from combustion of H_2 = 15·0 m^3
H_2O from combustion of CH_4 = 8·0 m^3

Total H_2O = 23·0 m^3

Total volume combustion products = 232·9 m^3

Analysis of products of combustion:

$$N_2 = \frac{170{\cdot}9\times100}{232{\cdot}9} = 73{\cdot}2\%.$$

$$O_2 = \frac{5{\cdot}3\times100}{232{\cdot}9} = 2{\cdot}2\%.$$

$$CO_2 = \frac{34\times100}{232{\cdot}9} = 14{\cdot}6\%.$$

$$H_2O = \frac{23{\cdot}0\times100}{232{\cdot}9} = 10{\cdot}0\%. \quad \textit{(Answer.)}$$

EXAMPLE 2.12

Blast furnace gas having the analysis by volume, dry, of CO_2, 12·5; CO, 25·4; H_2, 3·5; N_2, 58·6% is burned in a furnace. Calculate (a) the percentage excess air when the dry products of combustion contain 3·2% O_2, (b) the percentage excess air when the dry flue gas contains 19·7% CO_2, 4·6% O_2, and 75·7% N_2.

Calculation of the theoretical air for combustion of 100 m^3 of gas at NTP.

The relevant combustion equations are:

$$2\,H_2+O_2 = 2\,H_2O; \qquad 2\,CO+O_2 = 2CO_2.$$

$$\begin{aligned} O_2 \text{ to burn } H_2 &= 1{\cdot}75 \text{ m}^3\\ O_2 \text{ to burn } CO &= \underline{12{\cdot}7} \text{ m}^3\\ & \underline{14{\cdot}45} \text{ m}^3 \end{aligned}$$

Stoichiometric air requirement

$$= \frac{14{\cdot}45\times 100}{21} = 69 \text{ m}^3 \text{ at NTP.}$$

(a) When dry products of combustion contain 3·2% oxygen.

The dry products of combustion contain:

N_2 from fuel = 58·6 m^3.

N_2 from theoretical air = 69×0·79 = 54·45 m^3.

CO_2 in fuel = 12·5 m^3.

CO_2 from combustion of CO = 25·4 m^3.

Excess air = a m^3.

$$\text{Hence } \%\, O_2 = 3{\cdot}2 = \frac{\text{volume } O_2}{a+150{\cdot}95}\times 100.$$

$$3{\cdot}2a+3{\cdot}2\times 150{\cdot}95 = 21a.$$

$$a = \frac{3{\cdot}2\times 150{\cdot}95}{17{\cdot}8} = 27{\cdot}14 \text{ m}^3.$$

Therefore excess air $= \dfrac{27{\cdot}14}{69{\cdot}0}\times 100 = 39\%$. *(Answer.)*

(b) When analysis of flue gas is O_2, 4·6; CO_2, 19·7; N_2, 75·7%.

This calculation can be carried out in an identical manner to that in part (a) but a simpler calculation can be based on carbon dioxide content.

(*Note.* The method illustrated in Example 2.7, using ratios of nitrogen contents cannot be used in this case since all the nitrogen in the combustion products does not arise wholly from combustion air.)

$$\%\,CO_2 = 19{\cdot}7 = \frac{\text{volume } CO_2}{\text{total volume}} \times 100$$

$$= \frac{37{\cdot}9}{a+150{\cdot}95} \times 100.$$

$$19{\cdot}7a + 19{\cdot}7 \times 150{\cdot}95 = 3790.$$

$$a = \frac{3790-(19{\cdot}7\times 150{\cdot}95)}{19{\cdot}7}$$

$$= 41{\cdot}41 \text{ m}^3 \text{ at NTP.}$$

Alternatively,

$$\%\,O_2 = \frac{0{\cdot}21a}{a+150{\cdot}95} \times 100 = 4{\cdot}5.$$

$$4{\cdot}5a + 4{\cdot}5 \times 150{\cdot}95 = 21a.$$

$$a = \frac{4{\cdot}6 \times 150{\cdot}95}{16{\cdot}5} = 41{\cdot}4 \text{ m}^3 \text{ at NTP.}$$

$$\%\text{ excess air} = \frac{41{\cdot}4}{69{\cdot}0} \times 100 = 60\%. \qquad (\textit{Answer.})$$

EXAMPLE 2.13

Calculate (a) the volume of air at NTP theoretically required to burn 1 kg of coal of the composition given below, and (b) the weight of air required to burn the same weight of coal. Coal C, 75; H, 5; O, 8; S, 2; ash and moisture 10% by weight.

(a) Theoretical volume of air at NTP.

The appropriate combustion equations are:

$$C+O_2 = CO_2; \qquad 2\,H_2+O_2 = 2\,H_2O; \qquad S+O_2 = SO_2.$$

When calculating the theoretical volume of air required, starting from molecular equations it is first necessary to convert the analysis from a weight basis to a kilogram-molecule basis, using a basic weight of 100 kg of coal.

$$C = \frac{75}{12} = 6{\cdot}25 \text{ kg-atom}; \qquad H_2 = \frac{5{\cdot}0}{2} = 2{\cdot}5 \text{ kg-mol};$$

$$O_2 = \frac{8}{32} = 0{\cdot}25 \text{ kg-mol}; \qquad S = \frac{2{\cdot}0}{32} = 0{\cdot}06 \text{ kg-atom}.$$

Therefore oxygen requirement $= 6{\cdot}25+1{\cdot}25+0{\cdot}06 = 7{\cdot}56$ kg-mol.

There is some oxygen present in the fuel, hence oxygen needed to be supplied

$$= 7{\cdot}56-0{\cdot}25 = 7{\cdot}31 \text{ kg-mol} = 7{\cdot}31\times 22{\cdot}4 \text{ m}^3 \text{ at NTP}.$$

$$\text{Air required} = \frac{7{\cdot}31\times 22{\cdot}4\times 100}{21} = 773 \text{ m}^3.$$

Air required to burn 1 kg of coal completely

$= 7{\cdot}73$ m^3 at NTP. (*Answer* (a).)

(b) Theoretical air requirement (mass).

Oxygen requirement $= 7{\cdot}31$ kg-mol $= 7{\cdot}31\times 32$ kg-wt.

$$\text{Therefore weight of air} = \frac{7{\cdot}31\times 32\times 100}{23} = 1017 \text{ kg}.$$

Weight of air per pound of coal burned

$= 10{\cdot}17$ kg. (*Answer* (b).)

EXAMPLE 2.14

A coal containing 79·9% C, 5·0% H, 4·3% O, 1·1% S, and 1·9% N is used as fuel in a brick kiln.

(a) Calculate the theoretical volume of air at NTP required for the complete combustion of 1 kg of this coal.

(b) Determine the volume of excess air used when the dry flue gas analysis contains 11·0% CO_2, either by direct calculation or by plotting a graph of excess air (0–100%) versus carbon dioxide (Orsat value) of the flue gas.

[HNC Metallurgy, Swansea College of Technology.]

The relevant combustion equations are:

$$C+O_2 = CO_2; \quad S+O_2 = SO_2; \quad 2\,H_2+O_2 = 2\,H_2O.$$

100 kg of coal will contain:

$$C = \frac{79{\cdot}9}{12} = 6{\cdot}658 \text{ kg-atom}; \qquad O_2 = \frac{4{\cdot}3}{32} = 0{\cdot}134 \text{ kg-mol};$$

$$S = \frac{1{\cdot}1}{32} = 0{\cdot}034 \text{ kg-mol}; \qquad N_2 = \frac{1{\cdot}9}{28} = 0{\cdot}07 \text{ kg-mol};$$

$$H_2 = \frac{5{\cdot}0}{2} = 2{\cdot}5 \text{ kg-mol}.$$

Oxygen to be supplied $= 6{\cdot}658+0{\cdot}034+1{\cdot}25-0{\cdot}134 = 7{\cdot}808$ kg-mol.

$$\text{Air requirement} = \frac{7{\cdot}81\times 22{\cdot}4\times 100}{21\times 100}$$

$$= 8{\cdot}35 \text{ m}^3 \text{ at NTP/kg coal.}$$ (*Answer* (a).)

The composition of the dry flue gas from combustion of 1 kg of coal will be:

$$N_2 \text{ from coal} = \frac{0{\cdot}07\times 22{\cdot}4}{100} = 0{\cdot}016 \text{ m}^3.$$

$$N_2 \text{ from theoretical air} = 8{\cdot}35\times 0{\cdot}79 = 6{\cdot}6 \text{ m}^3.$$

$$CO_2 \text{ (S added to C)} = \frac{6{\cdot}682\times 22{\cdot}4}{100} = 1{\cdot}5 \text{ m}^3.$$

$$\text{Excess air} = a \text{ m}^3.$$

Method A

$$\%CO_2, \text{ zero excess air} = \frac{1{\cdot}5}{8{\cdot}116}\times 100 = 18{\cdot}5\ \%.$$

$$\text{With } 20\% \text{ excess air} = \frac{1{\cdot}5}{8{\cdot}116+1{\cdot}67}\times 100 = 15{\cdot}36\%.$$

With 40% excess air $= \frac{1{\cdot}5}{8{\cdot}116+3{\cdot}34}\times 100 = 13{\cdot}2\ \%.$

With 60% excess air $= \frac{1{\cdot}5}{8{\cdot}116+5{\cdot}01}\times 100 = 11{\cdot}45\%.$

With 80% excess air $= \frac{1{\cdot}5}{8{\cdot}116+6{\cdot}68}\times 100 = 10{\cdot}15\%.$

With 100% excess air $= \frac{1{\cdot}5}{8{\cdot}116+8{\cdot}35}\times 100 = 9{\cdot}11\%.$

From a graph constructed using these values, when carbon dioxide content = 11·0%

excess air = 66%. (*Answer* (b).)

Method B

$$\%\ CO_2 = \frac{\text{volume } CO_2}{\text{total volume dry gases}}\times 100.$$

$$11{\cdot}0 = \frac{1{\cdot}5}{8{\cdot}116+a}\times 100.$$

$$a = 5{\cdot}51 \text{ m}^3.$$

$$\%\text{ excess air} = \frac{5{\cdot}51}{8{\cdot}35}\times 100 = 66\%.$$ (*Answer* (b).)

Method C

Assuming that all of the carbon in the coal enters the gas a carbon balance may be carried out. If the sulphur is added to the carbon then from 1 kg of coal the total volume of dry flue gas is given by

$$\frac{100\times 6{\cdot}682\times 22{\cdot}4}{100\times 11{\cdot}0} = 13{\cdot}61 \text{ m}^3 \text{ at NTP}.$$

$$\text{Excess air} = 13{\cdot}61-8{\cdot}116 = 5{\cdot}494 \text{ m}^3.$$

% excess air = 66%. (*Answer* (b).)

EXAMPLE 2.15

A fuel oil containing 84% C, 12% H, 1·5% O, and 2·5% S and having a gross CV of 42,000 kJ/kg is burned to completion in a metallurgical furnace. Determine (i) the theoretical volume of air at NTP required to burn 1 kg of fuel to completion, (ii) the percentage of the net CV of the fuel which escapes as sensible heat in the total flue gas when 50% excess air is used and the gases leave the furnace at 535°C. Mean specific heat flue gases = 1·476 kJ/m³ °C at NTP.

[HNC Metallurgy, Swansea College of Technology.]

Taking a unit weight of 100 kg of fuel the following is the composition in kg-mol:

$$C = \frac{84}{12} = 7{\cdot}0 \text{ kg-atom}; \qquad H_2 = \frac{12{\cdot}0}{2} = 6{\cdot}0 \text{ kg-mol};$$

$$S = \frac{2{\cdot}5}{32} = 0{\cdot}078 \text{ kg-atom}; \qquad O_2 = \frac{1{\cdot}5}{32} = 0{\cdot}047 \text{ kg-mol}.$$

The combustion equations are:

$$C+O_2 = CO_2; \quad 2H_2+O_2 = 2H_2O; \quad S+O_2 = SO_2.$$

Oxygen requirement $= 7{\cdot}0+3{\cdot}0+0{\cdot}078-0{\cdot}047 = 10{\cdot}031$ kg-mol.

$$\text{Air requirement} = \frac{10{\cdot}031\times 22{\cdot}4\times 100}{21} = 1073{\cdot}3 \text{ m}^3 \text{ at NTP.}$$

Air/kg fuel = 10·73 m³ at NTP. (*Answer* (i).)

The waste gas from combustion of 1 kg oil:

N_2 from theoretical air $= 10{\cdot}73 \times 0{\cdot}79 = 8{\cdot}48 \text{ m}^3$
CO_2 $= 7{\cdot}0 \times 0{\cdot}224 = 1{\cdot}57 \text{ m}^3$
SO_2 $= 0{\cdot}078 \times 0{\cdot}224 = 0{\cdot}02 \text{ m}^3$
H_2O $= 6{\cdot}0 \times 0{\cdot}224 = 1{\cdot}34 \text{ m}^3$
Excess air $= 10{\cdot}73 \times 0{\cdot}5 = 5{\cdot}36 \text{ m}^3$
$16{\cdot}77 \text{ m}^3$

Sensible heat loss = $16{\cdot}77 \times 1{\cdot}476 \times (535-15) = 12{,}860$ kJ.
Net CV = $42{,}000-(2454 \times$ wt. water in kg),
Weight of water = $0{\cdot}06 \times 18 = 1{\cdot}08$ kg (kg-mol×mol. wt.).
Net CV = $42{,}000-(2454 \times 1{\cdot}08) = 39{,}350$ kJ/kg.

$$\% \text{ required} = \frac{12{,}860}{39{,}350} \times 100 = 33\%. \quad (\textit{Answer} \text{ (ii).})$$

EXAMPLE 2.16

A coal containing C, 67·9; H, 4·4; S, 0·8; N, 1·6; O, 7·9; ash, 4·5; water, 12·9% as-fired and having a gross CV of 28,000 kJ/kg is burned in a combustion appliance. The products of combustion issue at a temperature of 545°C, and the dry gas analysis is CO_2, 14·5%; O_2, 4·7%; remainder N_2.

(i) Determine the volume of air used for combustion.

(ii) Determine the sensible heat in the dry products of combustion expressed as a percentage of the net CV of the coal.

(iii) Determine the heat loss in the combustion of hydrogen, and in the moisture in the coal, as kJ/kg coal as-fired.

Specific heat of dry gases = 1·341 kJ/m^3 °C at NTP.
Specific heat of steam = 1·92 kJ/kg °C.
Heat of condensation of moisture at 15°C = 2454 kJ/kg.
Ambient temperature = 15°C.

[City and Guilds Advanced, Fuel Plant Technology.]

Taking a unit weight of 100 kg coal the following is the composition in kg-mol:

$$C = \frac{67{\cdot}9}{12} = 5{\cdot}66 \text{ kg-atom}; \quad H_2 = \frac{4{\cdot}4}{2} = 2{\cdot}2 \text{ kg-mol};$$

$$S = \frac{0{\cdot}8}{32} = 0{\cdot}025 \text{ kg-atom}; \quad N_2 = \frac{1{\cdot}6}{28} = 0{\cdot}058 \text{ kg-mol};$$

$$O_2 = \frac{7{\cdot}9}{32} = 0{\cdot}25 \text{ kg-mol}.$$

The combustion equations are:

$$C+O_2 = CO_2; \quad 2\,H_2+O_2 = 2\,H_2O; \quad S+O_2 = SO_2.$$

Oxygen requirement $= 5{\cdot}66+1{\cdot}1+0{\cdot}025-0{\cdot}25 = 6{\cdot}535$ kg-mol.

$$\text{Air requirement} = \frac{6{\cdot}535\times 100\times 22{\cdot}41}{21} = 700 \text{ m}^3 \text{ at NTP.}$$

The composition of the dry flue gas from combustion of 1 kg of coal is:

N_2 from theoretical air $= 7{\cdot}0 \times 0{\cdot}79 = 5{\cdot}53$ m^3.
N_2 from coal $= 0{\cdot}058\times 0{\cdot}224 = 0{\cdot}013$ m^3.
CO_2 $= 5{\cdot}66 \times 0{\cdot}224 = 1{\cdot}27$ m^3.
SO_2 $= 0{\cdot}025\times 0{\cdot}224 = 0{\cdot}006$ m^3.
Excess air $= a$ m^3.

$$\%\, CO_2 = 14{\cdot}5 = \frac{\text{volume } CO_2}{\text{total volume}}\times 100$$

$$= \frac{1{\cdot}27}{6{\cdot}819+a}\times 100$$

$$14{\cdot}5\times 6{\cdot}819+14{\cdot}5\times a = 1{\cdot}27\times 100.$$

$$a = \frac{127-(6{\cdot}82\times 14{\cdot}5)}{14{\cdot}5} = 1{\cdot}96 \text{ m}^3 \text{ at NTP.}$$

Actual air used per kilogram coal $= 7{\cdot}0+1{\cdot}96 = 8{\cdot}96$ m^3 at NTP. (*Answer* (i).)

The sensible heat loss per kilogram of coal
$= \text{volume of dry products of combustion} \times MC_p\times \Delta T$
$= (1{\cdot}96+6{\cdot}819)\times 1{\cdot}341\times (545-15)$
$= 8{\cdot}78\times 1{\cdot}341\times 530 = 6240$ kJ.

Net CV = gross CV $-$(2454$\times$wt. water).

$$\text{Weight of water} = \frac{12{\cdot}9}{100} = 0{\cdot}129 \text{ kg as moisture in coal.}$$

$$\text{Water from combustion of } H_2 = \frac{2{\cdot}2\times 18}{100} = 0{\cdot}2958 \text{ kg.}$$

Total water $= 0{\cdot}525$ kg.
Net CV $= 28{,}000-(2454\times 0{\cdot}525) = 26{,}712$ kJ/kg.

Sensible heat loss as percentage of net calorific value of fuel

$$= \frac{6240}{26,712} \times 100 = 23{\cdot}3\%. \quad (\textit{Answer} \text{ (ii).})$$

The heat loss due to moisture and combustion of hydrogen will be the sum of that due to the cooling of steam to 100°C, and the latent heat of condensation and the cooling of water to 15°C.

Heat lost per kg of coal

$$= 0{\cdot}525 \times (545-100) \times 1{\cdot}92 + 0{\cdot}525 \times 2454$$

$$= 347{\cdot}7 + 1288{\cdot}3 = 1636 \text{ kJ}. \quad (\textit{Answer} \text{ (iii).})$$

EXAMPLE 2.17

A petroleum oil containing C, 85·4; H, 11·2; S, 3·4%, having a gross CV of 18,250 Btu/lb and SG of 0·95 at 15·5°C, is atomized using 3 lb of steam per gallon of oil. It is burned completely with air in a furnace plant giving exhaust gases at a temperature of 842°F with a carbon dioxide content of 10·5%. Determine (i) the volumetric composition of the wet products of combustion in respect of O_2, N_2, H_2O, SO_2, and CO_2, (ii) the sensible heat above 60°F in the exhaust gases, expressed as a percentage of the gross calorific value of the fuel.

Mean specific heat products of combustion $= 0{\cdot}02$ Btu/°F ft^3 at 60°F and 30 in.Hg.

[City and Guilds Advanced, Metallurgy, Section J, Fuels.]

The lb-mol composition of 100 lb of fuel oil is as follows:

$$C = \frac{85{\cdot}4}{12} = 7{\cdot}12 \text{ lb-atom}; \qquad H_2 = \frac{11{\cdot}2}{2} = 5{\cdot}6 \text{ lb-mol};$$

$$S = \frac{3{\cdot}4}{32} = 0{\cdot}106 \text{ lb-atom}.$$

The combustion equations are:

$$C + O_2 = CO_2; \qquad S + O_2 = SO_2; \qquad 2\,H_2 + O_2 = 2\,H_2O.$$

Oxygen requirement $= 7{\cdot}12+2{\cdot}8+0{\cdot}106 = 10{\cdot}026$ lb-mol.

$$\text{Air requirement} = \frac{10{\cdot}026\times 379\times 100}{21} = 18{,}090\ \text{ft}^3.$$

Air/lb oil $= 180{\cdot}9$ ft^3 at STP.

The products of combustion from 1 lb oil are:

N_2 from theoretical air $= 180{\cdot}9\times 0{\cdot}79 = 143$ ft^3.
CO_2 $= 7{\cdot}12 \times 3{\cdot}79 = 27{\cdot}0$ ft^3.
SO_2 $= 0{\cdot}106\times 3{\cdot}79 = 0{\cdot}4$ ft^3.
Moisture from H_2 $= 5{\cdot}6 \times 3{\cdot}79 = 21{\cdot}22$ ft^3.

Moisture from atomizing steam = 3 lb/gal oil.
1 gal = 10 lb (water) therefore 1 gal oil $= 10\times 0{\cdot}95$ lb.

$$\text{Steam/lb oil} = \frac{3}{9{\cdot}5}\ \text{lb-wt.}$$

Moisture from atomizing steam

$$= \frac{\left(\dfrac{3}{9{\cdot}5}\right)\times 379}{18} = 6{\cdot}648\ \text{ft}^3.$$

Excess air $= a$ ft^3.

$$\%\ CO_2 = 10{\cdot}5 = \frac{27{\cdot}0}{27{\cdot}0+143{\cdot}0+0{\cdot}4+21{\cdot}2+6{\cdot}65+a}\times 100.$$

$$10{\cdot}5\times 198{\cdot}25+10{\cdot}5\times a = 2700.$$

$$a = \frac{2700-(198{\cdot}25\times 10{\cdot}5)}{10{\cdot}5} = 257{\cdot}1-198{\cdot}25.$$

$$a = 58{\cdot}85\ \text{ft}^3\ \text{at STP}.$$

Final composition, including excess air:

$N_2 = 143{\cdot}0+58{\cdot}85\times 0{\cdot}79 = 189{\cdot}5$ ft^3
CO_2 $= 27{\cdot}0$ ft^3
SO_2 $= 0{\cdot}4$ ft^3
H_2O $= 27{\cdot}9$ ft^3
O_2 $= 12{\cdot}4$ ft^3

257·2 ft^3

$$O_2 = \frac{12{\cdot}4}{257{\cdot}2} \times 100 = 4{\cdot}8\%$$

$$N_2 = \frac{189{\cdot}5}{257{\cdot}2} \times 100 = 73{\cdot}7\%$$

$$H_2O = \frac{27{\cdot}9}{257{\cdot}2} \times 100 = 10{\cdot}8\%$$

$$SO_2 = \frac{0{\cdot}4}{257{\cdot}2} \times 100 = 0{\cdot}2\%$$

$$CO_2 = \frac{27{\cdot}0}{257{\cdot}2} \times 100 = \underline{10{\cdot}5\%}$$

$$\underline{100{\cdot}0\%} \quad (\textit{Answer}\ \text{(i).)}$$

Sensible heat lost in products of combustion

$$= 257{\cdot}2 \times 0{\cdot}02 \times (842-60) = 4022 \text{ Btu.}$$

$$\text{As a percentage of gross CV} = \frac{4022}{18{,}250} \times 100 = 22\%.$$

(*Answer* (ii).)

EXAMPLE 2.18

Compare the efficiency of combustion of the following fuels under conditions in which combustion is complete with the necessary stoichiometric volume of air. The products leave the furnace system at a temperature of 1020°F, and the ambient temperature is 60°F. Discuss the relevance of the results of these calculations and given properties of the fuels to their suitability for combustion purposes.

(a) A bituminous coal containing as-used: C, 75·9; H, 4·25; N, 1·27; S, 0·68; O, 2·89; ash, 10·0; H_2O, 5·0% by weight. The gross CV is 13,000 Btu/lb. The volatile matter is 30% on the dry, ash-free fuel, and the BS swelling number is 8.

(b) A fuel oil having a viscosity of 950 sec Redwood No. 1 with C, 85·3; H, 11·2; S, 3·5% by weight. The gross CV is 18,230 Btu/lb.

The mean specific heats of the combustion products are respectively for (a) 0·253 Btu/lb °F and for (b) 0·258 Btu/lb °F.

The latent heat of condensing steam in the gases is 1055 Btu/lb. Composition of air may be taken as $O_2/N_2 = 23/77$ by weight. [Institution of Metallurgists, Old Regulations, Fuels.]

(a) Taking 100 lb of coal and converting to lb-mol:

$$C = \frac{75{\cdot}9}{12} = 6{\cdot}32 \text{ lb-atom}; \qquad H_2 = \frac{4{\cdot}25}{2} = 2{\cdot}125 \text{ lb-mol};$$

$$N_2 = \frac{1{\cdot}27}{28} = 0{\cdot}045 \text{ lb-mol}; \qquad S = \frac{0{\cdot}68}{32} = 0{\cdot}021 \text{ lb-atom};$$

$$O_2 = \frac{2{\cdot}89}{32} = 0{\cdot}090 \text{ lb-mol}.$$

The combustion equations are:

$$C+O_2 = CO_2; \quad S+O_2 = SO_2; \quad 2\,H_2+O_2 = 2\,H_2O.$$

Oxygen requirement

$$= 6{\cdot}32+0{\cdot}021+1{\cdot}0625-0{\cdot}09 = 7{\cdot}314 \text{ lb-mol}$$

$$= 7{\cdot}314\times 32 \text{ lb-wt}.$$

$$\text{Air/lb fuel} = \frac{7{\cdot}314\times 32\times 100}{23\times 100} = 10{\cdot}17 \text{ lb}.$$

The products of combustion from 1 lb of fuel are:

$CO_2 = 6{\cdot}32\times 0{\cdot}44$	=	2·78 lb
$SO_2 = 0{\cdot}021\times 0{\cdot}64$	=	0·013 lb
N_2 in fuel $= 0{\cdot}045\times 0{\cdot}28$	=	0·0127 lb
N_2 from theoretical air $= 10{\cdot}17\times 0{\cdot}77$	=	7·83 lb
H_2O as moisture	=	0·05 lb
H_2O from combustion of $H_2 = 2{\cdot}125\times 0{\cdot}18$	=	0·38 lb
Total products of combustion	=	11·07 lb

Alternatively:

The weight of products of combustion

$$= \text{weight of coal} - \text{ash} + \text{weight of air used}$$

$$= 1{\cdot}0 - 0{\cdot}10 + 10{\cdot}17 = 11{\cdot}07 \text{ lb.}$$

$$\text{Net CV} = \text{gross CV} - (1055 \times \text{weight water})$$

$$= 13{,}300 - (1055 \times (0{\cdot}05 + 0{\cdot}38))$$

$$= 13{,}300 - 453{\cdot}65 = 12{,}846 \text{ Btu/lb.}$$

The heat carried away in combustion products from 1 lb of coal

$$= 11{\cdot}07 \times 0{\cdot}253 \times (1020 - 60) = 2687 \text{ Btu.}$$

The percentage of the gross potential heat available in the combustion chamber

$$= \frac{(12{,}846 - 2687)}{13{,}300} \times 100 = 76{\cdot}4\%.$$

(b) If 100 lb of oil is converted to lb-mol:

$$C = \frac{85{\cdot}3}{12} = 7{\cdot}1 \text{ lb-atom}; \quad H_2 = \frac{11{\cdot}2}{2} = 5{\cdot}6 \text{ lb-mol};$$

$$S = \frac{3{\cdot}5}{32} = 0{\cdot}11 \text{ lb-atom.}$$

$$\text{Oxygen requirement} = 7{\cdot}1 + 2{\cdot}8 + 0{\cdot}11 = 10{\cdot}01 \text{ lb-mol}$$

$$= 10{\cdot}01 \times 32 \text{ lb-wt.}$$

$$\text{Air required per pound oil} = \frac{10{\cdot}01 \times 32 \times 100}{23 \times 100} = 13{\cdot}9 \text{ lb.}$$

Weight of combustion products = weight oil + weight air used = $1{\cdot}0 + 13{\cdot}9 = 14{\cdot}9$ lb.

$$\text{Heat lost} = 14{\cdot}9 \times 0{\cdot}258 \times (1020 - 60) = 3691 \text{ Btu.}$$

$$\text{Net CV} = \text{gross CV} - (1055 \times (0{\cdot}056 \times 18))$$

$$= 18{,}230 - (1055 \times 1{\cdot}008) = 18{,}230 - 1063{\cdot}4$$

$$= 17{,}166 \text{ Btu/lb.}$$

The percentage of the gross potential heat available in the combustion chamber $= \dfrac{(17{,}166-3691)}{18{,}230} \times 100 = 73{\cdot}8\%$.

Therefore it can be concluded that the coal fuel is more efficient. *(Answer.)*

EXAMPLE 2.19

In a petroleum refinery the steam boiler plant has an overall thermal efficiency on gross calorific value of the fuel of 85% and an output of 200,000 lb of steam per hour. The steam is generated at a pressure of 250 lb/in^2 g, and superheated to 450°F from feed at 120°F.

Refinery gas of the following volumetric composition is used to fire the plant: N_2, 9; H_2, 21; CO, 1; CO_2 1; H_2S, 2; CH_4, 15; C_2H_6, 15%; hydrocarbons as C_3H_6, 36%.

From the information given below, (i) calculate the quantity of gas necessary to fire the plant. (ii) If the exit gas temperature from the plant is 400°F calculate the capacity of the induced draught fan in cubic feet per minute assuming the gas is burned with 20% excess air. Calorific value at NTP, gross Btu/ft^3: CO, 340; CH_4, 1000; H_2, 340; C_2H_6, 1930; H_2S, 703; C_3H_6, 2300.

[City and Guilds Advanced, Liquid Fuels.]

The combustion equations are:

$$2CO+O_2 = 2CO_2; \qquad 2H_2S+2O_2 = 2H_2O+SO_2;$$

$$CH_4+2O_2 = CO_2+2H_2O; \qquad 2C_2H_6+7O_2 = 4CO_2+6H_2O;$$

$$2H_2+O_2 = 2H_2O; \qquad 2C_3H_6+9O_2 = 6CO_2+6\,H_2O.$$

The calorific value of a cubic foot of gas at NTP:

$$CO = \frac{1\times 340}{100} = 3{\cdot}4 \text{ Btu}$$

$$H_2 = 21\times 3{\cdot}40 = 71{\cdot}4 \text{ Btu}$$

$$CH_4 = 15\times 10{\cdot}0 = 150{\cdot}0 \text{ Btu}$$

$$\begin{aligned} C_2H_6 &= 15\times 19{\cdot}3 &&= 289{\cdot}5 \text{ Btu} \\ C_3H_6 &= 36\times 23{\cdot}0 &&= 828{\cdot}0 \text{ Btu} \\ H_2S &= 2\times 7{\cdot}03 &&= 14{\cdot}1 \text{ Btu} \\ & && \overline{\text{CV} = 1356{\cdot}4 \text{ Btu}} \end{aligned}$$

(i) The heat required to raise steam, from *Steam Tables*, is:

1 lb steam at 260 lb/in^2 g and superheated to 450°F = 1230 Btu heat in steam per hour = 1230×200,000 Btu.
Heat in feedwater = 200,000×(120 − 32) Btu.
Therefore heat to be supplied by fuel for steam raising

$$= 200{,}000\times(1230-88) = 200{,}000\times 1142 \text{ Btu.}$$

Heat gained by steam = heat evolved by burning gas.

$$\text{Heat evolved by gas} \times\frac{85}{100} = 200{,}000\times 1142.$$

$$\text{Heat evolved by gas} = \frac{200{,}000\times 1142\times 100}{85} \text{ Btu.}$$

Heat evolved = 268,700,000 Btu.

CV gas = 1356·4 Btu/ft^3.

$$\text{Therefore volume gas required} = \frac{268{,}700{,}000}{1356{\cdot}4} = 198{,}100 \text{ ft}^3$$

at NTP. (*Answer* (i).)

Volume per minute = 3302 ft^3.

(ii) The induced fan capacity is found by calculating the total volume of wet flue gases at the furnace exit temperature.

The theoretical oxygen requirement for 100 ft^3 gas at NTP

$$= 2{\cdot}0+0{\cdot}5+10{\cdot}5+30{\cdot}0+52{\cdot}5+163{\cdot}0 = 258{\cdot}5 \text{ ft}^3.$$

$$\text{Theoretical air requirement} = \frac{258{\cdot}5\times 100}{21} = 1231 \text{ ft}^3.$$

Products of combustion with 20% excess air from 100 ft^3 gas:

N_2 from gas = 9·0 ft^3
N_2 from theoretical air = 1231×0·79 = 972·5 ft^3
Excess air = 1231×0·20 = 247·8 ft^3
CO_2 from gas = 1·0 ft^3
CO_2 from combustion of CO = 1·0 ft^3
CO_2 from combustion of CH_4 = 15·0 ft^3
CO_2 from combustion of C_2H_6 = 30·0 ft^3
CO_2 from combustion of C_3H_6 = 108·0 ft^3
SO_2 from combustion of H_2S = 2·0 ft^3
H_2O from combustion of H_2 = 21·0 ft^3
H_2O from combustion of CH_4 = 30·0 ft^3
H_2O from combustion of H_2S = 2·0 ft^3
H_2O from combustion of C_2H_6 = 45·0 ft^3
H_2O from combustion of C_3H_6 = 108·0 ft^3

Total gas volume = 1592·3 ft^3

The volume of gas burned per hour = 198,100 ft^3 at NTP. Therefore the capacity of the induced draught fan at NTP

$$= \frac{198{,}100 \times 1592{\cdot}3}{60 \times 100} = 52{,}550 \text{ ft}^3/\text{min.}$$

It is assumed that the gas enters at 32°F. Since the combustion gases leave at 400°F, the capacity of the induced draught fan is obtained by application of the gas laws:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

$$\frac{52{,}550}{460+32} = \frac{V}{460+400}.$$

$$V = \frac{52{,}550 \times 860}{492} = 87{,}000 \text{ ft}^3/\text{min.} \qquad (\textit{Answer} \text{ (ii).})$$

EXAMPLE 2.20

A coal-tar fuel containing 89·9% C, 6·0% H, 0·4% S, 1·1% N, and 2·6% O, of gross CV 37,750 kJ/kg, and SG 1·15 is burned with air as the atomizing agent.

Sketch and describe a burner suitable for burning the oil at a rate of 50 kg/hr. Indicate briefly the essential provisions for supplying the oil to the burner in a condition suitable for atomization. Assuming that 15% excess air is used, and the combustion is complete, calculate:

(a) the volume of air at NTP supplied per hour;

(b) the volume of the resulting products of combustion at a temperature of 500°C.

[Institution of Metallurgists, Part IV, AIM, New Regulations, specimen question.]

If 100 kg of tar is converted to kg-mol, the composition is:

$$C = \frac{89 \cdot 9}{12} = 7 \cdot 5 \text{ kg-atom}; \qquad H_2 = \frac{6 \cdot 0}{2} = 3 \cdot 0 \text{ kg-mol};$$

$$S = \frac{0 \cdot 4}{32} = 0 \cdot 012 \text{ kg-atom}; \qquad O_2 = \frac{2 \cdot 6}{32} = 0 \cdot 08 \text{ kg-mol}.$$

The combustion equations are:

$$C + O_2 = CO_2; \qquad S + O_2 = SO_2; \qquad 2\,H_2 + O_2 = 2\,H_2O.$$

$$\text{Oxygen requirement} = 7 \cdot 5 + 1 \cdot 5 + 0 \cdot 012 - 0 \cdot 08 = 8 \cdot 93 \text{ kg-mol}.$$

$$\text{Theoretical air} = \frac{8 \cdot 93 \times 22 \cdot 4 \times 100}{21 \times 100} = 9 \cdot 56 \text{ m}^3 \text{ at NTP/kg tar}.$$

$$\text{Air} = \left(9 \cdot 56 + \frac{9 \cdot 56 \times 15}{100}\right) \times 50$$

$$= 550 \text{ m}^3 \text{ NTP}. \quad (\textit{Answer} \text{ (a).})$$

The products of combustion from 1 kg tar

$$= (7{\cdot}5+3{\cdot}0+0{\cdot}012)\times 0{\cdot}224+9{\cdot}56\times 0{\cdot}79$$
$$+9{\cdot}56\times 0{\cdot}15 = 11{\cdot}34 \text{ m}^3.$$

Volume in 1 hr = $11{\cdot}34\times 50 = 567$ m^3 at NTP.
At 500°C:

$$\frac{567}{273} = \frac{V}{773}.$$

Volume required = 1605 m^3. (*Answer* (b).)

BIBLIOGRAPHY

CALLENDAR, G. S. and EGERTON, A. C., *The 1939 Callendar Steam Tables*, Edward Arnold, 1944.

FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.

HAYES, A. E. J., *Applied Thermodynamics*, Pergamon Press, 1963.

HIMUS, G. W., *Fuel Testing*, 3rd edn., Leonard Hill, 1953.

PRATT, A. D., *Principles of Combustion in the Steam Boiler Furnace*, Babcock & Wilcox Ltd., 1936.

ROGERS, G. F. C. and MAYHEW, J. R., *Engineering Thermodynamics, Work and Heat Transfer*, Longmans Green & Co. Ltd., 1959.

SMITH, H. J. and HARRIS, J. W., *Worked Examples in Engineering Thermodynamics*, 2nd edn., MacDonald & Co. Ltd., 1963.

SNEDEEN, J. B. O., *Applied Heat for Engineers*, Blackie & Sons Ltd., 1963.

SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.

IHVE Guide, 1965, 3rd edn., Institution of Heating and Ventilating Engineers, London.

The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 3

Gasification Calculations

3.1. PRODUCER GAS CALCULATIONS

When coals or oils are gasified to produce towns gas or gas for any other purpose it is often necessary to calculate the volume of gas generated per unit weight of fuel, the air and steam requirements, and often the total heat contained in the gas.

EXAMPLE 3.1

A coal is used to manufacture producer gas. Its analysis on the dry basis is: C, 78·0; H, 5·4; N, 1·4; S, 1·0; O, 10·0; ash, 4·2% by weight. The coal as charged to the producer plant has a moisture content of 5%. The blast saturation temperature is 50°C. Analysis of the dry gas: CO_2, 4·40; CO, 28·10; H_2, 15·45; CH_4, 3·00; N_2, 49·05% by volume. The temperature of the hot gases leaving the producer plant is 205°C, and have a mean specific heat of 1·341 kJ/m^3 °C NTP.

Calculate (a) the volume of gas generated per kilogram coal charged, (b) the volume of air supplied to gasify a kg coal, (c) the steam used in the blast, per kilogram coal gasified, and (d) the heat content of the producer gas per kg coal gasified.

It is necessary to know the amount of carbon discharged with the ashes, the composition and quantity of any tar produced, so that a carbon balance may be applied. It is also necessary to know the amount of ammonia liquor produced so that a hydrogen balance can be carried out in order to arrive at the percentage of steam decomposed. For the sake of this particular calculation it is assumed that 10% of the carbon charged to the producer appears as tar fog and carbonaceous matter in ashes and that 40% of the nitrogen charged appears as ammonia.

(a) Taking 100 kg of coal the kg-mol composition is:

$$C = \frac{78{\cdot}0 \times 95}{12 \times 100} = 6{\cdot}18 \text{ kg-atom} \quad \text{(moisture 5\%)}.$$

Thus the carbon appearing in the gas

$$= \frac{6{\cdot}18 \times 90}{100} = 5{\cdot}56 \text{ kg-atom} \quad \text{(10\% lost)}.$$

If 100 kg-mol of producer gas is analysed:

Carbon content $= 4{\cdot}4 + 28{\cdot}10 + 3{\cdot}0 = 35{\cdot}5$ kg-atom.

$$\text{For the producer gas, } \%\,C = \frac{\text{amount carbon}}{\text{total gas}} \times 100.$$

$$35{\cdot}5 = \frac{5{\cdot}56}{\text{total gas volume kg-mol}} \times 100.$$

Gas volume = 15·6 kg-mol.

Gas made per kilogram of coal charged to the producer

$$= \frac{15{\cdot}6 \times 22{\cdot}4}{100} = 3{\cdot}49 \text{ m}^3 \text{ at NTP.}$$ (*Answer* (a).)

(b) The air supplied for gasification purposes is conveniently arrived at by means of a nitrogen balance. If a nitrogen balance is based upon unit weight of 100 kg-mol gas:

$$N_2 \text{ appearing in gas} = \frac{15{\cdot}6 \times 49{\cdot}05}{100} = 7{\cdot}66 \text{ kg-mol.}$$

$$N_2 \text{ from fuel} = \frac{1{\cdot}4 \times 95 \times 60}{28 \times 100 \times 100} = 0{\cdot}03 \text{ kg-mol.}$$

(40% nitrogen is lost in ammonia liquor.)

Hence N_2 supplied as air $= 7{\cdot}66-0{\cdot}03 = 7{\cdot}63$ kg-mol.

$$\text{Air required} = \frac{7{\cdot}63\times100\times22{\cdot}4}{79} = 216{\cdot}2\ \text{m}^3 \text{ at NTP.}$$

Air/kg coal charged $= 2{\cdot}16$ m^3 at NTP. (*Answer* (b).)

(c) The steam used in the blast is obtained by construction of a hydrogen balance.

From the gas laws, $PV = RT$, where R is the universal gas constant for 1 mol, and $PV = nRT$ applies to n mol of an ideal gas. For mixtures, $PV = (n_1+n_2+n_3+\ldots)RT$.

The partial pressure of each gas is given by

$$P_1V = n_1RT \quad \text{and} \quad P_2V = n_2RT,$$

hence

$$\frac{P_1}{P_2} = \frac{n_1}{n_2}.$$

If air is saturated with steam at 50°C then the vapour pressure of water at 50°C $= 92{\cdot}55$ mmHg (Spiers). If the total pressure $= 760$ mmHg, then the partial pressure of the dry air $= 760-92{\cdot}55 = 667{\cdot}45$ mmHg. Thus the water vapour per mol of dry air $= \dfrac{92{\cdot}55}{667{\cdot}45} = 0{\cdot}133$ kg-mol.

The nitrogen required for gasification purposes was shown to be 7·63 kg-mol, therefore the air required

$$= \frac{7{\cdot}63\times100}{79} = 9{\cdot}65 \text{ kg-mol.}$$

Thus the total steam consumption per kilogram of coal

$$= \frac{0{\cdot}133\times9{\cdot}65\times18}{100} \text{ (mol. wt. water = 18)}$$

$$= 0{\cdot}22 \text{ kg} \quad (\textit{Answer} \text{ (c).)}$$

It should be noted that when steam is mixed with air under adiabatic conditions to produce a saturated mixture, some condensation occurs because the increase in enthalpy of the air is usually greater than the drop in the enthalpy of the steam.

Thus if a dry saturated mixture of air and steam is required, either the air has to be preheated or the steam superheated. If this is not the case then a mixture of saturated air and droplets of liquid water is obtained.

In practice the blast is usually preheated before introduction into the producer. There will also be an indeterminate amount of moisture introduced via any water seal.

It is often necessary to calculate the percentage of the steam decomposed in the blast. This is arrived at by construction of a hydrogen balance (per 100 kg coal).

$$H_2 \text{ supplied to producer in coal} = \frac{5{\cdot}4}{2} \times \frac{95}{100}$$

$$= 2{\cdot}57 \text{ kg-mol.}$$

$$H_2 \text{ supplied as moisture} = \frac{5{\cdot}0}{18} = 0{\cdot}28 \text{ kg-mol.}$$

$$H_2 \text{ in steam in blast} = 0{\cdot}133 \times 9{\cdot}65 = 1{\cdot}28 \text{ kg-mol.}$$

$$\text{Total} = 2{\cdot}57 + 0{\cdot}28 + 1{\cdot}28 = 4{\cdot}13 \text{ kg-mol.}$$

H_2 appearing in the gas as combustibles

$= 15{\cdot}45\%$ as H_2 and $6{\cdot}0\%$ as CH_4 ($2\,H_2$)

$= 21{\cdot}45$ kg-mol/100 kg-mol gas.

Actual H_2 in gas as combustibles $=$ total × percentage

$$= \frac{15{\cdot}6 \times 21{\cdot}45}{100} = 3{\cdot}34 \text{ kg-mol.}$$

$$H_2 \text{ converted to } NH_3 = \frac{1{\cdot}4 \times 95 \times 40 \times 3{\cdot}0}{28 \times 100 \times 100} = 0{\cdot}06 \text{ kg-mol.}$$

(1 N_2 goes to 2 NH_3, i.e. 1 N_2 combines with 3 H_2.)

$$H_2 \text{ leaving as steam} = 4{\cdot}13 - (0{\cdot}06 + 3{\cdot}34) = 0{\cdot}73 \text{ kg-mol.}$$

$$\text{Steam equivalent of oxygen in coal} = \frac{95 \times 10{\cdot}0 \times 2}{100 \times 32}$$

$$= 0{\cdot}62 \text{ kg-mol.}$$

(For the sake of such calculations it is assumed that the oxygen in the coal combines with its quota of hydrogen to form steam, although the validity of this argument is questionable.)

Steam as moisture in coal = 0·28 kg-mol.
Steam remaining undecomposed = 0·73 − (0·28 + 0·62)
= −ve value or zero.

In this particular example, all the steam in the blast has been decomposed.

(d) The total heat content of the gas is made up of sensible heat in the hot gases and potential heat which is realized when the gas is burned.

Taking the heat content per kilogram of coal gasified:

Potential heat as CO = $3{\cdot}49 \times 12{\cdot}7 \times 0{\cdot}281 = 12{\cdot}46$ kJ
Potential heat as H_2 = $3{\cdot}49 \times 12{\cdot}8 \times 0{\cdot}1545 = 7{\cdot}0$ kJ
Potential heat as CH_4 = $3{\cdot}49 \times 39{\cdot}7 \times 0{\cdot}03 = 4{\cdot}16$ kJ
Sensible heat in the dry gases above 15°C
= $3{\cdot}49 \times 1{\cdot}341 \times 190 = 889$ kJ (mst).

(CVs taken as typical values from Table 6.)

The heat in the steam may be calculated by first determining the partial pressure of the steam.

$$\text{Partial pressure} = 14{\cdot}7 \times \frac{0{\cdot}73}{0{\cdot}73 + 15{\cdot}6} = 0{\cdot}7 \text{ lb/in}^2 \text{ abs.}$$

The total heat in the steam at this pressure and at 205°C may be obtained from *Steam Tables*.

Alternatively, an estimate can be obtained in the following way. If the specific heat of steam in the range required is taken to be 1·93 kJ/kg °C, and the latent heat of condensation and cooling of steam from 100° to 15°C taken as 2454 kJ/kg, then the heat in steam (0·73 kg-mol/100 kg coal)

$$= \frac{2454 \times 0{\cdot}73 \times 18}{100} + \frac{0{\cdot}73 \times 18 \times 1{\cdot}93(205 - 100)}{100}$$

$$= 322{\cdot}5 + 27{\cdot}26 = 349{\cdot}76 \text{ kJ.}$$

Thus the total heat in producer gas obtained from gasification 1 kg of coal = 350 + 889 = 1239 kJ. (*Answer* (d).)

3.2. ADDITIONAL EXAMPLES

EXAMPLE 3.2

Light petroleum distillate is gasified at 20 atm pressure in the UK Gas Councils Two-stage Autothermic Process. The table given below indicates the analysis of the gas at various stages of the operation. If the naphtha is assumed to be C_7H_{15}, calculate (a) the steam used for reaction, (b) the air requirement for the second stage, and (c) the final volume of dry gas (per kg of naphtha used).

(1) Volume (%)	(2) After reaction with steam at 500°C	(3) After air addition and reaction at 675°C	(4) After conversion of carbon monoxide	(5) After removal of carbon dioxide and drying
CO_2	13·9	11·8	16·0	1·0
CO	1·3	6·0	1·8	2·9
H_2	12·9	23·7	27·9	44·6
CH_4	36·2	22·0	22·0	35·0
N_2	—	10·3	10·3	10·5
H_2O	35·7	26·2	22·0	—

From the weight of carbon in kg-mol of C_7H_{15} = 84 kg and a calculation of the carbon and hydrogen contents of 100 kg-mol of gas, utilizing the analysis given in column 2 above.

mol. wt.	kg	kg carbon	kg hydrogen
$CO_2 = 12+32$	$= 44\times 13{\cdot}9$	$12\times 13{\cdot}9$	
$CO = 12+16$	$= 28\times 1{\cdot}3$	$12\times 1{\cdot}3$	
$CH_4 = 12+4$	$= 16\times 36{\cdot}2$	$12\times 36{\cdot}3$	$4\times 36{\cdot}2$
$H_2 = 2$	$= 2\times 12{\cdot}9$		$2\times 12{\cdot}9$
$H_2O = 2+16$	$= 18\times 25{\cdot}7$		$2\times 35{\cdot}7$

$$\% C = \frac{(166{\cdot}8+15{\cdot}6+434{\cdot}4)}{1904{\cdot}6}\times 100$$

$$= 32\%.$$

$$\% H_2 = \frac{(144{\cdot}8+25{\cdot}8+71{\cdot}4)}{1904{\cdot}6}\times 100$$

$$= 13{\cdot}3\%.$$

(a) $$\% C = 32 = \frac{84}{\text{total mass gas}}\times 100.$$

Mass of gas = 264·3 kg.
Mass of H_2 in 99 kg naphtha = 15 kg.
% H_2 in gas = 13·3.

$$\text{Mass of } H_2 \text{ in gas} = \frac{264{\cdot}3\times 13{\cdot}3}{100} = 34{\cdot}97 \text{ kg.}$$

H_2 from steam = 34·97 − 15 = 19·97 kg.

$$H_2 \text{ from 1 kg naphtha} = \frac{19{\cdot}97}{99} \text{ kg.}$$

$$\text{Steam used} = \frac{19{\cdot}97\times 9}{99} = 1{\cdot}8 \text{ kg/kg naphtha.}$$ *(Answer.)*

(b) The air required for the second stage is calculated using the data from column 3. This must be arrived at by means of a nitrogen balance.

$$\% N_2 \text{ in gas} = \frac{10{\cdot}3\times 28\times 100}{1847} = 15{\cdot}6\%.$$

$$\text{Weight of } N_2 \text{ from 1 kg naphtha} = \frac{323\times 15{\cdot}6}{99\times 100} \text{ kg}$$

(323 = mass gas from a carbon balance of column 3).

$$\text{Mass of air required} = \frac{323\times 15{\cdot}6\times 100}{99\times 100\times 77} = 0{\cdot}66 \text{ kg.}$$

(Answer.)

(c) The volume of gas generated is obtained by utilizing the data from column 5.

99 kg naphtha $= \frac{84}{12} = 7$ kg-mol C.

C in gas $= 1{\cdot}0\,(CO_2)+2{\cdot}9\,(CO)+35{\cdot}0\,(CH_4) = 38{\cdot}9$.

$$38{\cdot}9 = \frac{7}{\text{total volume kg-mol}} \times 100.$$

$$\text{Volume} = \frac{7\times 100\times 22{\cdot}4}{38{\cdot}9\times 99} = 4{\cdot}08 \text{ m}^3 \text{ at NTP/kg naphtha.}$$

(Answer.)

EXAMPLE 3.3

A coke containing on the dry, ash-free basis 94% C, and when used 10% moisture and 4% ash, is gasified in a producer gas plant giving a gas as analysed of the following volumetric composition: CO_2, 5·2; CO, 25·1; H_2, 12·3; CH_4, 0·4%; remainder N_2.

Calculate (a) the volume of gas generated per kilogram of coke charged to the producer; (b) the volume of air required for gasification of each kilogram of coke (assume nitrogen content negligible); and (c) the theoretical volume of air at NTP required to burn the producer gas obtained from gasification of 1 kg of coke.

[HNC Metallurgy, Swansea College of Technology.]

The C content of 100 kg of coke and 100 kg-mol of gas is determined.

$$\text{C charged} = \frac{94\times 86}{12\times 100} \text{ kg-mol.}$$

$$\% \text{ C in gas (in kg-mol)} = 5{\cdot}2\ (CO_2)+25{\cdot}1\,(CO) +0{\cdot}4\ (CH_4) = 30{\cdot}7\%.$$

$$30{\cdot}7 = \frac{6{\cdot}736}{\text{total gas volume}} \times 100.$$

$$\text{Gas volume} = \frac{6{\cdot}736\times 100}{30{\cdot}7} = 21{\cdot}95 \text{ kg-mol.}$$

Total gas = $21{\cdot}95\times 22{\cdot}4$ m^3 per 100 kg coke.

$$\text{(a) Gas made per kg of coke} = \frac{21{\cdot}95\times 22{\cdot}4}{100} = 4{\cdot}92 \text{ m}^3 \text{ at NTP.}$$

(Answer.)

(b) 100 kg-mol of gas contain

$$100-(5{\cdot}2+25{\cdot}1+12{\cdot}3+0{\cdot}4) = 57{\cdot}0 \text{ kg-mol } N_2.$$

The number of kg-mol N_2 in the gas made from gasification of 1 kg of coke $= \dfrac{21{\cdot}95\times 57}{100\times 100} = 12{\cdot}51$ kg-mol.

This nitrogen must have come from the air supplied, hence:

$$\text{Volume of air used} = \frac{12{\cdot}51\times 100\times 22{\cdot}4}{79\times 100} = 3{\cdot}55 \text{ m}^3 \text{ at NTP.}$$

(Answer.)

(c) Since 1 kg of coke gives 4·92 m^3 gas, the gas composition obtained by gasification of 1 kg of coke is:

$$CO_2 = \frac{5{\cdot}2\times 4{\cdot}92}{100} \text{ m}^3.$$

$$CO = \frac{25{\cdot}1\times 4{\cdot}92}{100} = 1{\cdot}235 \text{ m}^3.$$

$$H_2 = \frac{12{\cdot}3\times 4{\cdot}92}{100} = 0{\cdot}605 \text{ m}^3.$$

$$CH_4 = \frac{0{\cdot}4\times 4{\cdot}92}{100} = 0{\cdot}02 \text{ m}^3.$$

$$N_2 = \frac{57\times 4{\cdot}92}{100} \text{ m}^3.$$

The combustion reactions are:

$$2CO+O_2 = 2CO_2; \qquad 2H_2+O_2 = 2H_2O;$$

$$CH_4+2O_2 = CO_2+2H_2O.$$

Volume of O_2 required per kilogram

$$= \left[\frac{1{\cdot}235}{2}+\frac{0{\cdot}605}{2}+(2\times 0{\cdot}02)\right] = 0{\cdot}96 \text{ m}^3.$$

$$\text{Air required} = \frac{0{\cdot}96\times 100}{21} = 4{\cdot}6 \text{ m}^3 \text{ at NTP.}$$ *(Answer.)*

EXAMPLE 3.4

From the data given below, calculate the cubic feet of dry gas at 15°C and 1013 mbar per kg of dry coal charged to a gas producer.

Coal analysis, dry basis (%)		Producer gas analysis, dry (%)	
C	76·0	CO_2	7·0
O	6·0	O_2	0·7
H	5·0	CO	20·3
N	1·5	H_2	12·5
S	1·5	C_2H_4	0·5
Ash	10·0	CH_4	3·0
		N_2	56·0

Clinker analysis, dry basis, C, 15·0%; ash 85·0%.

[City and Guilds Advanced, Solid Fuels.]

From the analysis, the carbon content of 100 kg of dry coal is 76 kg which converted to kg-atom $= \frac{76{\cdot}0}{12} = 6{\cdot}33$ kg-atom.

100 kg of coal when gasified gives rise to 10·0 kg ash; but the clinker produced contains 15% C.

$$10 \text{ kg ash} = \frac{10\times 100}{85} = 11{\cdot}76 \text{ kg clinker.}$$

C lost per 100 kg coal charged:

$$15\% = \frac{1{\cdot}76}{11{\cdot}176}\times 100.$$

$$\text{Actual amount} = 1{\cdot}76 \text{ kg} = \frac{1{\cdot}76}{12} = 0{\cdot}146 \text{ kg-atom.}$$

$$\text{Actual C gasified} = 6{\cdot}33 - 0{\cdot}15 = 6{\cdot}18 \text{ kg-atom.}$$

The C content of 100 kg-mol gas can be calculated as follows:

$$\text{C in kg-atom} = 7{\cdot}0\,(CO_2) + 20{\cdot}3\,(CO) + 3{\cdot}0\,(CH_4)$$
$$+ 1{\cdot}0\,(C_2H_4) = 31{\cdot}3 \text{ kg-atom.}$$

$$\%\,C = 31{\cdot}3 = \frac{6{\cdot}18}{\text{gas volume kg-mol}} \times 100.$$

$$\text{Gas volume} = \frac{6{\cdot}18 \times 100}{31{\cdot}3} = 19{\cdot}74 \text{ kg-mol.}$$

$$\text{Vol/kg dry coal} = \frac{19{\cdot}74 \times 22{\cdot}4}{100} = 4{\cdot}42 \text{ m}^3 \text{ at NTP.}$$

(Answer.)

EXAMPLE 3.5

A coal containing C, 76·0; H, 5·0; N, 1·6; S, 1·4; ash, 6·0; H_2O, 5·0% is gasified in an air-steam blast. The rate of gasification is 25 cwt/hr and the composition of the gas as analysed is CO_2, 4·8; CO, 27·5; CH_4, 2·2; H_2, 12·5%; remainder N_2. Ten per cent of the coal substance may be regarded as lost in tar and partially carbonized dust.

(a) Calculate the volume of air and the weight of steam required to be supplied per hour for gasification purposes.

(b) Indicate the operative factors which govern the relationship between the air–steam ratio in the blast and the most efficient composition of the resulting gas.

[Institution of Metallurgists, Old Regulations, Fuels.]

The C content of 100 lb coal is calculated from the analysis.

$$C = \frac{76{\cdot}0}{12} = 6{\cdot}33 \text{ lb-atom.}$$

C lost = 0·633 lb-atom; therefore C gasified = 5·7 lb-atom.

The C content of 100 lb-mol gas calculated from the analysis is:

$$\text{C in gas in lb-mol} = 4{\cdot}8\ (CO_2) + 27{\cdot}5\ (CO) + 2{\cdot}2\ (CH_4)$$

$$= 34{\cdot}5 \text{ lb-mol} = \%\ \text{C}.$$

$$34{\cdot}5 = \frac{5{\cdot}7}{\text{vol in lb-mol}} \times 100.$$

Volume of gas = 16·53 lb-mol.

$$\text{Vol/hr} = \frac{16{\cdot}53 \times 379 \times 112 \times 25}{100}$$

$$= 175{,}300 \text{ ft}^3 \text{ at STP.}$$

$$\%\ N_2 \text{ in gas} = 100 - (27{\cdot}5 + 4{\cdot}8 + 3{\cdot}2 + 12{\cdot}5) = 52{\cdot}0.$$

$$\text{Volume of } N_2 \text{ in gas} = \frac{52{\cdot}0 \times 175{,}300}{100} = 91{,}160 \text{ ft}^3 \text{ at STP.}$$

$$N_2 \text{ from coal} = \frac{1{\cdot}6}{28} \text{ lb-mol/100 lb coal.}$$

$$N_2 \text{ from 25 cwt} = \frac{1{\cdot}6 \times 112 \times 25 \times 379}{28 \times 100}$$

$$= 606{\cdot}1 \text{ ft}^3 \text{ at STP.}$$

$$N_2 \text{ supplied by air} = 91{,}160 - 606 = 90{,}554 \text{ ft}^3 \text{ at STP.}$$

$$\text{Air supplied} = \frac{90{,}554 \times 100}{79} = 114{,}625 \text{ ft}^3 \text{ at STP.}$$

(Answer.)

H_2 supplied in 100 lb coal = $\frac{5}{2}$ = 2·5 lb-mol.

Amount gasified = 2·5 − 0·25 (10% loss) = 2·25 lb-mol.

In 1 hr H_2 from coal substance entering gas

$$= \frac{2{\cdot}25 \times 112 \times 25}{100} = 62{\cdot}99 \text{ lb-mol.}$$

From 100 lb coal there will be $\frac{5}{18}$ lb-mol water.

$$H_2 \text{ entering gas from water} = \frac{0{\cdot}277 \times 112 \times 25}{100}$$

$$= 7{\cdot}756 \text{ lb-mol.}$$

$$H_2 \text{ entering gas from fuel} = 62{\cdot}99 + 7{\cdot}756$$

$$= 70{\cdot}746 \text{ lb-mol.}$$

H_2 to be supplied via steam will be the H_2 in the gas less this amount.

The H_2 content of 100 lb-mol gas is calculated from the analysis.

$$H_2 \text{ in lb-mol} = 12{\cdot}5\,(H_2) + 4{\cdot}4\,(CH_4) = 16{\cdot}9.$$

$$\%\,H_2 = 16{\cdot}9.$$

$$\text{Total } H_2 \text{ in gas} = \frac{16{\cdot}9 \times 16{\cdot}53 \times 112 \times 25}{100 \times 100}$$

$$= 78{\cdot}16 \text{ lb-mol.}$$

H_2 to be supplied by steam $= 78{\cdot}16 - 70{\cdot}75 = 7{\cdot}41$ lb-mol.

H_2O required $= 7{\cdot}41 \times 18 = 133$ lb/hr. *(Answer.)*

EXAMPLE 3.6

Coke gasified in a producer by means of an air-steam blast provides a gas of analysis: CO_2, 4·8; CO, 27·3; H_2, 14·0; CH_4, 0·4%; remainder N_2. The gas is burnt with 15% excess air in a furnace which is equipped with a metallic recuperator, the waste gases entering at 950°C.

Determine (i) the volume of the products of combustion per ton of carbon gasified; (ii) the composition of the products if 80% of the steam used is decomposed in the producer; and (iii) the temperature of the preheated air leaving the recuperator, assuming a recuperator efficiency of 40%.

Mean volumetric specific heats, taken as 0·0215 Btu/°F ft^3 at NTP for waste gas, 0·02 for air.

[City and Guilds Advanced, Fuel Plant Technology.]

The C content of 100 lb-mol of gas from the analysis is:

$$\text{C in lb-mol} = 4{\cdot}8\,(CO_2)+27{\cdot}3\,(CO)+0{\cdot}4\,(CH_4) = 32{\cdot}5.$$

The volume of gas generated per ton of C gasified is obtained by a carbon balance:

$$32{\cdot}5 = \frac{2240}{12}\times 100 \text{ volume gas in lb-mol.}$$

Gas volume = 575·4×359 = 206,200 ft^3 at NTP.

The combustion equations are:

$$2\,CO+O_2 = 2\,CO_2; \qquad 2\,H_2 + O_2 = 2\,H_2O;$$
$$CH_4+2\,O_2 = CO_2+2\,H_2O.$$

Oxygen required to burn 100 ft^3 of gas at NTP

$$= 13{\cdot}65+7{\cdot}0+0{\cdot}8 = 21{\cdot}45 \text{ ft}^3.$$

$$\text{Theoretical air} = \frac{21{\cdot}45\times 100}{21} = 102{\cdot}14 \text{ ft}^3 \text{ at NTP.}$$

The products of combustion when 100 ft^3 of gas is burned with 15% excess air are:

Excess air	= 15·32 ft^3
N_2 from theoretical air	= 80·69 ft^3
N_2 in gas	= 53·5 ft^3
CO_2 in gas	= 4·8 ft^3
CO_2 from CO combustion	= 27·3 ft^3
CO_2 from CH_4 combustion	= 0·4 ft^3
H_2O from H_2 combustion	= 14·0 ft^3
H_2O from CH_4 combustion	= 0·8 ft^3
Total	= 196·81 ft^3

Therefore the volume produced when 206,200 ft^3 of gas burnt

$$= \frac{206{,}200\times 196{\cdot}81}{100} = 405{,}800 \text{ ft}^3 \text{ at NTP.} \quad (\textit{Answer} \text{ (i).})$$

If 80% of the steam is decomposed then it is combined either as methane or hydrogen in the gas. It is first necessary to determine the hydrogen content of the gas so as to arrive at the 20% not recorded in the analysis.

H_2 in lb-mol = 14·0+0·8 = 14·8 lb-mol per 100 lb-mol of gas.

$$100\% = \frac{14{\cdot}8 \times 100}{80} = 18{\cdot}5 \text{ lb-mol}$$

(1 mol H_2 = 1 mol H_2O).

Water undecomposed = 18·5 − 14·8 = 3·7 lb-mol.

100 lb-mol gas give rise to:

H_2O = 14·8+3·7 = 18·5 lb-mol.

CO_2 = 4·8+27·3+0·4 = 32·5 lb-mol.

O_2 = 3·22 lb-mol (from excess air).

N_2 = 80·69+12·10 (excess air N_2)+53·5

= 146·29 lb-mol.

Total = 200·51 lb-mol.

$$O_2 = \frac{3{\cdot}22 \times 100}{200{\cdot}51} = 1{\cdot}38\%.$$

$$N_2 = \frac{146{\cdot}29 \times 100}{200{\cdot}51} = 72{\cdot}7\ \%.$$

$$CO_2 = \frac{32{\cdot}5 \times 100}{200{\cdot}51} = 16{\cdot}05\%.$$

$$H_2O = \frac{18{\cdot}5 \times 100}{200{\cdot}51} = 10{\cdot}05\%. \quad (\textit{Answer}\ \text{(ii).})$$

For every 100 ft^3 of dry producer gas there will be produced 196·8 ft^3 of products of combustion. Associated with this there will be 3·7 ft^3 undecomposed steam.

Total = 196·81+3·7 = 200·51 ft^3.

100 ft^3 producer gas requires 102·14+15·32 ft^3 air

= 117·46 ft^3 air at NTP.

950°C = 1742°F. (It is necessary to convert because MC_p units are in Btu/ft^3 °F units.)

Heat gained by the air = heat lost by waste gases.
Assuming a datum level of 32°F:

$$200{\cdot}5\times(1742-32)\times0{\cdot}0215\times0{\cdot}4 \text{ (heat lost by flue gases)}$$
$$= 117{\cdot}46\times(T-32)\times0{\cdot}02$$
$$T-32 = 1254°\text{F}.$$

Temperature preheated air leaving recuperator = T = 1286°F. (*Answer* (iii).)

BIBLIOGRAPHY

CALLENDAR, G. S. and EGERTON, A. C., *The 1939 Callendar Steam Tables*, Edward Arnold, 1944.
FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.
HAYES, A. E. J., *Applied Thermodynamics*, Pergamon Press, 1963.
HIMUS, G. W., *Fuel Testing*, 3rd edn., Leonard Hill, 1953.
SMITH, H. J. and HARRIS, J. W., *Worked Examples in Engineering Thermodynamics*, 2nd edn., MacDonald, 1963.
SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.
Gas Making, British Petroleum, London, 1965.
The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 4

Flame Temperature

4.1. THEORETICAL FLAME TEMPERATURE

Knowledge of this is useful in that it enables comparison to be made between different fuels. An accurate calculation is very difficult because it is necessary to know the specific heats of the products of combustions, and these vary with temperature.

Theoretical flame temperature is calculated on the assumption that no heat is radiated from the flame, that the heat is liberated instantaneously, heating up the products of combustion, and that no dissociation of any gas occurs. (Theoretical flame temperature – room temperature) × mean specific heat waste gases × volume products of combustion = net CV gas + any sensible heat in fuel + any sensible heat in combustion air.

Such calculations are best explained by means of an example.

EXAMPLE 4.1

A gas has the following volumetric composition: O_2, 0·4; CO_2, 2·0; C_2H_6, 2·6; CO, 7·4; H_2, 54·0; CH_4, 28·0; N_2, 5·6%. Net CV = 17·163 MJ/m^3 at NTP. It is burned (dry) with stoichiometric air and it is assumed that no dissociation occurs and that the air and gas are initially at 0°C.

Calculate the theoretical flame temperature.

MC_p products of combustion:

2200°C, 1·678; 1800°C, 1·543; 2000°C, 1·610 kJ/m^3 °C at NTP.

The combustion equations are:

$$2\,C_2H_6+7\,O_2 = 4\,CO_2+6\,H_2O; \quad 2\,CO+O_2 = 2\,CO_2;$$
$$CH_4+2\,O_2 = CO_2+2\,H_2O; \quad 2\,H_2+O_2 = 2\,H_2O.$$

Oxygen required for combustion of 100 m^3 of gas

$$= 9{\cdot}1+3{\cdot}7+56{\cdot}0+27{\cdot}0-0{\cdot}4 = 95{\cdot}4 \text{ m}^3.$$

$$\text{Air required} = \frac{95{\cdot}4\times 100}{21} = 454 \text{ m}^3.$$

The products of combustion with stoichiometric air:

$$\begin{array}{lll}
N_2 & = 454\times 0{\cdot}79 = 358{\cdot}7+5{\cdot}6 & = 364{\cdot}3 \text{ m}^3 \\
CO_2 & = 7{\cdot}4+2{\cdot}0+5{\cdot}2+28{\cdot}0 & = 42{\cdot}6 \text{ m}^3 \\
\text{Water vapour} & = 54{\cdot}0+56{\cdot}0+7{\cdot}8 & = 117{\cdot}8 \text{ m}^3 \\
\hline
\quad\text{Total} & & 524{\cdot}7 \text{ m}^3
\end{array}$$

$$CO_2 = \frac{42{\cdot}6\times 100}{524{\cdot}7} = 8{\cdot}12\%.$$

$$H_2O = \frac{117{\cdot}8\times 100}{524{\cdot}7} = 22{\cdot}45\%.$$

$$N_2 = \frac{364{\cdot}3\times 100}{524{\cdot}7} = 69{\cdot}43\%.$$

The percentage analysis is used to find the MC_p values from tables.

$$(\text{Flame temperature}-0) = \frac{17{,}163\times 100}{524{\cdot}7\times MC_p}.$$

The mean specific heat varies with temperature so that it is first necessary to assume a temperature and use the value of MC_p at this temperature. If the temperature calculated is very different from the assumed value, then another value is assumed and the calculation repeated until the calculated and assumed values agree sufficiently closely.

Assuming $T = 2200°\text{C}$

$$T-0 = \frac{17{,}163\times 100}{524{\cdot}7\times 1{\cdot}678} = 2015°\text{C}.$$

Assuming $T = 1800°\text{C}$

$$T = \frac{17{,}163\times 100}{524{\cdot}7\times 1{\cdot}543} = 2190°\text{C}.$$

Assuming $T = 2000°\text{C}$

$$T = \frac{17{,}163\times 100}{524{\cdot}7\times 1{\cdot}610} = 2100°\text{C}.$$

It is clear that the theoretical flame temperature lies between 2100° and 2000°C. This will be considered of sufficient accuracy in this particular calculation.

4.2. DISSOCIATION OF GASES AND FLAME TEMPERATURE

During combustion, an equilibrium is set up between the fuel gases, oxygen, and the products of combustion, carbon dioxide and water vapour. At high temperature, dissociation of carbon dioxide into carbon monoxide and oxygen, and water vapour into hydrogen and oxygen occurs. These reactions are important above 1600°C. At very high temperatures other dissociation reactions occur.

It is necessary when making calculations of flame temperature to assume that equilibrium is attained at the temperature concerned. This being fully justified because equilibrium conditions are very rapidly attained at high temperatures.

In the general case of waste gas of the following composition, CO_2, $a\%$; H_2O, $b\%$; O_2, $c\%$; and N_2, $100-(a+b+c)\%$, if x and y are the fractions of carbon dioxide and water vapour that remain undissociated per molecule, then from 100 ft^3 waste gas the new products of combustions will become:

Gas	Molecules
CO_2	ax
CO	$a(1-x)$
H_2O	by
H_2	$b(1-y)$
O_2	$\frac{a}{2}(1-x)+\frac{b}{2}(1-y)+c = m-100+c$
N_2	$= 100-a-b-c$
Total	$= m$

If it is assumed that combustion takes place at constant pressure (P atm), the partial pressure of the gases partaking in equilibria will be:

$$CO_2 \qquad P_{CO_2} = \frac{ax}{m} \cdot P.$$

$$CO \qquad P_{CO} = \frac{a(1-x)}{m} \cdot P.$$

$$O_2 \qquad P_{O_2} = \frac{m-100+c}{m} \cdot P.$$

$$H_2O \qquad P_{H_2O} = \frac{by}{m} \cdot P.$$

$$H_2 \qquad P_{H_2} = \frac{b(1-y)}{m} \cdot P.$$

For the reaction:

$$CO_2 \rightleftharpoons CO+\tfrac{1}{2}O_2$$

$$K_{P_{CO_2}} = \frac{P_{CO_2}}{P_{CO}\times P_{O_2}^{1/2}} = \frac{x}{1-x}\sqrt{\left(\frac{m}{m-100+c}\right)}\sqrt{\left(\frac{1}{P}\right)}.$$

For the reaction:

$$H_2O \rightleftharpoons H_2+\tfrac{1}{2}O_2$$

$$K_{p_{H_2O}} = \frac{p_{H_2O}}{p_{H_2}\times p_{O_2}^{1/2}} = \frac{y}{1-y}\sqrt{\left(\frac{m}{m-100+c}\right)}\sqrt{\left(\frac{1}{P}\right)}.$$

In a mixture of steam and carbon dioxide both reactions coexist and the partial pressures of the various constituents will be adjusted so that equilibrium constants in the mixtures are the same as in the separate systems. Simultaneously with the above two processes the water–gas reaction

$$CO_2+H_2 \rightleftharpoons H_2O+CO$$

must also attain equilibrium, and for the water–gas reaction

$$K_{w\cdot g} = \frac{p_{H_2O}\times p_{CO}}{p_{CO_2}\times p_{H_2}} = \frac{K_{p_{H_2O}}}{K_{p_{CO_2}}}.$$

Since all the equations contain two unknowns (x and y), even though the equilibrium constants at a particular temperature can be obtained from consideration of the individual systems the determination of x and y has to be carried out by trial and error. A graphical method has been developed by Underwood (Spiers).

EXAMPLE 4.2

To illustrate the effect of dissociation on flame temperature, consider the combustion of hydrogen with stoichiometric air, both gas and air assumed dry and supplied at 0°C. Net CV H_2, 10·16 MJ/m^3.

Mean specific heats products of combustion: 2200°C, 1·657; 2100°C, 1·651; 2000°C, 1·643; kJ/m^3 °C at NTP.

The theoretical flame temperature is calculated assuming no dissociation.

$$2\,H_2+O_2 = 2\,H_2O.$$

From this equation it is evident that 2 m^3 of hydrogen requires 1 m^3 of oxygen for combustion. Therefore the air required

$$= \frac{1\times 100}{21} = 4{\cdot}76 \text{ m}^3.$$

Air/m^3 hydrogen = 2·38 m^3.

The final gas composition after combustion is 1·88 m^3 nitrogen, 1·0 m^3 water vapour. The composition of the flue gas

$$= 65{\cdot}3\%\ N_2, \quad 34{\cdot}7\%\ H_2O.$$

Theoretical flame temperature:

$$T \quad = \frac{10{\cdot}16\times 1000}{2{\cdot}88\times MC_p}.$$

Assuming T = 2200°C,

$$T-0 = \frac{10{\cdot}16\times 1000}{2{\cdot}88\times 1{\cdot}657} = 2125^\circ\text{C}.$$

Assuming T = 2100°C,

$$T \quad = \frac{10{\cdot}16\times 1000}{2{\cdot}88\times 1{\cdot}651} = 2136^\circ\text{C}.$$

This is sufficiently close to the assumed value.

At this temperature steam is dissociated to the extent of approximately 4% (Spiers). Thus the actual heat developed will be less, and the products of combustion greater.

$$H_2O \rightleftharpoons H_2 + \tfrac{1}{2}O_2.$$

Under these conditions the flame temperature will be:

$$T \quad = \frac{10{,}160\times 0{\cdot}96}{2{\cdot}9\times MC_p}.$$

Assuming T = 2000°C,

$$T \quad = \frac{10{,}160\times 0{\cdot}96}{2{\cdot}9\times 1{\cdot}643} = 2050^\circ\text{C}.$$

This is sufficiently close to the assumed value.

The effect of dissociation has been to reduce the flame temperature from 2100° to 2000°C.

In practice the temperature is still further reduced by radiation from the flame. In making flame temperature calculations it is generally assumed that 10% of the heat is lost in this way. The maximum flame temperature is achieved when using slightly less than stoichiometric air.

4.3. ADDITIONAL FLAME TEMPERATURE EXAMPLES

EXAMPLE 4.3

Discuss the effects of dissociation on flame temperature. A producer gas of composition: CO_2, 5; CO, 29; H_2, 12; N_2, 54% is burned with theoretical amount of air. Assuming hypothetically that 18% of the carbon dioxide and 5% of the steam is dissociated, what would be the final waste gas composition? Calculate the equilibrium constant for the water–gas reaction. [City and Guilds Advanced, Gaseous Fuels.]

The combustion reactions are:

$$2\,CO+O_2 = 2\,CO_2; \qquad 2\,H_2+O_2 = 2\,H_2O.$$

Oxygen required per 100 m^3 gas = 14·5+6·0 = 20·5 m^3.

$$\text{Air required} = \frac{20{\cdot}5\times 100}{21} = 97{\cdot}5\ m^3.$$

The composition of the waste gas from burning 100 m^3 gas with no dissociation is:

N_2 from stoichiometric air	=	77·5 m^3
N_2 present in gas	=	54·0 m^3
Total N_2		131·5 m^3

CO_2 = 5·0+29·0 = 34·0 m^3.

H_2O = 12·0 m^3.

The dissociation reactions are:

$$2\,CO_2 \rightleftharpoons 2\,CO+O_2,$$
$$2\,H_2O \rightleftharpoons 2\,H_2+O_2.$$

The composition of the gas after dissociation is:

N_2 = 131·5 m^3.
CO_2 = 34·0×0·82 = 27·88 m^3.
CO from CO_2 dissociation = 6·12 m^3.
O_2 from dissociation = 3·06 m^3.
H_2O = 12×0·05 = 11·40 m^3.
O_2 from H_2O dissociation = 0·30 m^3.
H_2 from H_2O dissociation = 0·60 m^3.

The Total Gas Composition. *(Answer.)*

	Volume (m^3)	(%)
N_2	131·5	72·69
CO_2	27·88	15·41
CO	6·12	3·38
H_2O	11·4	6·30
H_2	0·6	0·33
O_2	3·36	1·89
	180·86	

The water–gas reaction:

$$CO + H_2O \rightleftharpoons CO_2 + H_2.$$

$$K_P = \frac{P_{CO_2} \times P_{H_2}}{P_{CO} \times P_{H_2O}} = \frac{15{\cdot}41 \times 0{\cdot}33}{3{\cdot}38 \times 6{\cdot}30} = 0{\cdot}24. \quad \textit{(Answer.)}$$

It would have been equally correct if the reciprocal of this had been calculated, viz. 1/0·24 = 4·2.

EXAMPLE 4.4

Discuss the factors you would consider in the selection of a suitable fuel for an open-hearth steel furnace. Calculate the theoretical flame temperature attained, assuming no dissociation of combustion products when burning the following:

(i) a fuel oil of composition 87·5% C, 12·5% H, with 25% excess air;

(ii) blast furnace gas containing 28% CO, 2% H_2, 70% N_2, with 20% excess air;

(iii) coke oven gas containing 8% CO, 52% H_2, 30% CH_4, 10% N_2, with 20% excess air.

Comment on the results and suggest ways by which the flame temperature might be increased.

Mean specific heats between 0°C and flame temperature of products of combustion: 0·022 Btu/ft^3 °F at NTP.

Net CVs: C, 14,500 Btu/lb; H, 52,000 Btu/lb.

CO, 340 Btu/ft^3 at NTP; H_2, 290 Btu/ft^3 at NTP; CH_4, 1000 Btu/ft^3 at NTP.

[Institution of Metallurgists, Part IV, Furnace Technology, specimen question.]

The carbon content of 100 lb of fuel is calculated in lb-mol.

$$C = \frac{87{\cdot}5}{12} = 7{\cdot}3 \text{ lb-atom}; \qquad H_2 = \frac{12{\cdot}5}{2} = 6{\cdot}25 \text{ lb-mol.}$$

The combustion equations are:

$$C+O_2 = CO_2; \qquad 2\,H_2+O_2 = 2\,H_2O.$$

Oxygen requirement = 7·3+3·13 = 10·43 lb-mol.

$$\text{Theoretical air} = \frac{10{\cdot}43\times100\times359}{21} = 17{,}840 \text{ ft}^3.$$

Air/lb fuel = 178·4 ft^3 at NTP.

The volume of the products of combustion from 1 lb oil:

N_2 from theoretical air = 178·4×0·79 = 140·9 ft^3
Excess air = 178·4 ×0·25 = 44·6 ft^3
CO_2 = 7·3 ×3·59 = 27·3 ft^3
H_2O = 6·25×3·59 = 22·4 ft^3
235·2 ft^3

The calorific value of the fuel is found from the analysis

$$\begin{aligned}\text{Net CV} &= 0{\cdot}875\times14{,}500+0{\cdot}0125\times52{,}000\\ &= 12{,}690+6500 = 19{,}190 \text{ Btu/lb.}\end{aligned}$$

This is equal to the heat in the flame gases.

$$19{,}190 = 235{\cdot}2 \times 0{\cdot}022 \times (T-32)$$

$$T-32 = \frac{19{,}190}{235{\cdot}2 \times 0{\cdot}022} = 4078°\text{F}.$$

$$T = 4100°\text{F}. \quad (\textit{Answer}\ \text{(i).})$$

For the blast furnace gas the combustion equations are:

$$2\,CO + O_2 = 2\,CO_2; \qquad 2\,H_2 + O_2 = 2\,H_2O;$$

$$CH_4 + 2\,O_2 = CO_2 + 2\,H_2O$$

The oxygen required for combustion of 100 ft^3 blast furnace gas

$$= 14 + 1 = 15{\cdot}0 \text{ ft}^3.$$

$$\text{Theoretical air} = \frac{15 \times 100}{21} = 71{\cdot}4 \text{ ft}^3.$$

The volume of the products of combustion with 20% excess air:

N_2 from theoretical air $= 71{\cdot}4 \times 0{\cdot}79$	=	56·3 ft^3
Excess air $= 71{\cdot}4 \times 0{\cdot}20$	=	14·3 ft^3
N_2 in fuel	=	70·0 ft^3
CO_2	=	28·0 ft^3
H_2O	=	2·0 ft^3
		170·6 ft^3

The CV of blast furnace gas $= 340 \times 28 + 290 \times 2 = 10{,}100$ Btu.

$$\text{CV/ft}^3 \text{ gas at NTP} = 101 \text{ Btu}.$$

This is equal to the heat in the flame gases.

$$101 = 170{\cdot}6 \times 0{\cdot}022 \times (T-32).$$

$$T-32 = \frac{101}{170{\cdot}6 \times 0{\cdot}022} = 2692°\text{F}.$$

$$T = 2724\,°\text{F}. \quad (\textit{Answer}\ \text{(ii).})$$

For combustion of 100 ft^3 of coke oven gas:

$$\text{Oxygen requirement} = 4+26+60 = 90\ \text{ft}^3.$$

$$\text{Theoretical air} = \frac{90\times 100}{21} = 428\ \text{ft}^3.$$

The products of combustion are:

$$\begin{array}{llll}
N_2 \text{ present in gas} & & = & 10\ \text{ft}^3 \\
N_2 \text{ in theoretical air} & = 428\times 0{\cdot}79 & = & 339{\cdot}2\ \text{ft}^3 \\
CO_2 & = 8+30 & = & 38{\cdot}0\ \text{ft}^3 \\
H_2O & = 52+60 & = & 112{\cdot}0\ \text{ft}^3 \\
\text{Excess air} & = 428{\cdot}0\times 0{\cdot}20 & = & \underline{85{\cdot}6\ \text{ft}^3} \\
 & & & \underline{584{\cdot}8\ \text{ft}^3}
\end{array}$$

$$\begin{aligned}
\text{The CV of coke oven gas} &= 8\times 340+52\times 290+30\times 1000 \\
&= 48{,}800\ \text{Btu}. \\
\text{CV/ft}^3 \text{ at NTP} &= 488{\cdot}0\ \text{Btu}. \\
488{,}000 &= 584{\cdot}8\times 0{\cdot}022\times (T-32) \\
T &= 3833^\circ\text{F}. \quad (\textit{Answer}\ \text{(iii).})
\end{aligned}$$

BIBLIOGRAPHY

FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.

HIMUS, G. W., *Fuel Testing*, 3rd edn., L. Hill, 1953.

LUPTON, H. P., *Industrial Gas Engineering*, North Western Gas Board, (Walter King Ltd., London), 1960.

SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.

The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 5

Heat Transfer

5.1. INTRODUCTION

If a temperature gradient exists between two points in a system, heat will flow between them by one of three possible ways.

(1) By electromagnetic waves—radiation.

(2) By either natural convection (arising from density changes caused by temperature gradients) or by forced convection which arises from eddy current formation accompanying turbulent flow.

(3) By conduction which transfers energy from one molecule to another without any actual mixing occurring.

There is an important analogy between the flow of electrical and heat energy, the analogy is:

Rate of flow of electrical energy:	Rate of heat flow.
Potential gradient:	Temperature gradient.
Electrical resistance:	Thermal resistance.

Just as in an electrical circuit the current flows by virtue of the potential difference between two points, so heat flows by virtue of the temperature difference between two points.

For resistances in series $R = r_1+r_2+r_3+ \ldots.$

For resistances in parallel $1/R = 1/r_1+1/r_2+1/r_3 \ldots.$

For all three modes of heat transfer a simple equation of the following type is useful:

Q (rate of heat transfer) $= hA\,\Delta T$, where A = area; ΔT the temperature difference; h the heat transfer coefficient.

$$\text{Thus,} \qquad Q = \frac{\Delta T}{1/hA} = \frac{\text{driving force}}{\text{thermal resistance}}$$

($I = E/R$ in electrical circuits).

5.2. HEAT TRANSFER BY RADIATION

When radiation falls upon a body then it can be (i) absorbed, (ii) reflected, or (iii) transmitted.

Hence $a+r+t = 1$ (a = fraction absorbed, r = fraction reflected, and t = fraction transmitted).

For most solid bodies, the amount transmitted can be regarded as negligible, so $t = 0$.

Hence, for opaque bodies, $a+r = 1$.

Black body radiation

A black body is a material which absorbs all of the radiant energy falling upon it ($a = 1$). The total energy which is emitted per unit time per unit surface area of such a body is proportional to the fourth power of its absolute temperature and this relationship is known as the Stefan–Boltzmann law.

$$Q \text{ proportional } T^4.$$

$$Q = \sigma T^4.$$

$$\begin{aligned} \sigma &= \text{Stefan–Boltzmann constant} \\ &= 1{\cdot}713\times 10^{-9} \text{ Btu/ft}^2 \text{ hr R}^4 \\ &= 1{\cdot}01\times 10^{-8} \text{ lb cal/ft}^2 \text{ hr K}^4 \\ &= 5{\cdot}6697\times 10^{-8} \text{ W/m}^2 \text{ K}^4. \end{aligned}$$

However, the energy radiated is non-uniform. It is not radiated with equal intensity in all directions and the energy is not distributed uniformly over the whole range of wavelengths.

Lambert's radiation law (Fig. 5.1)

Radiation from a surface in a direction α to the surface varies as the cosine of the angle θ between the direction of radiation and the normal to the surface. If q_n is the radiation normal to the surface the radiation in a given direction = $q_n \cos \theta$.

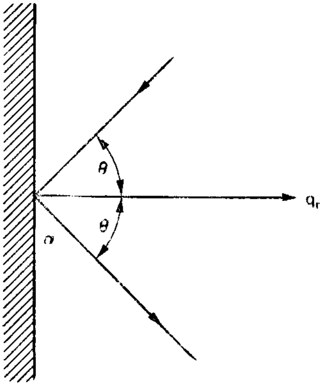

FIG. 5.1. Lambert's radiation law

Inverse square law

The intensity of radiation from a point source of radiant energy decreases with the square of the distance from the point source.

Spatial distribution of energy (Fig. 5.2)

Consider a small element area dA, radiating in a hemisphere of radius r. The radiation will be supposed to fall on areas dA_1 and dA_2 on the surface of the hemisphere. Let I_n be the intensity of radiation per unit area per unit solid angle normal to dA.

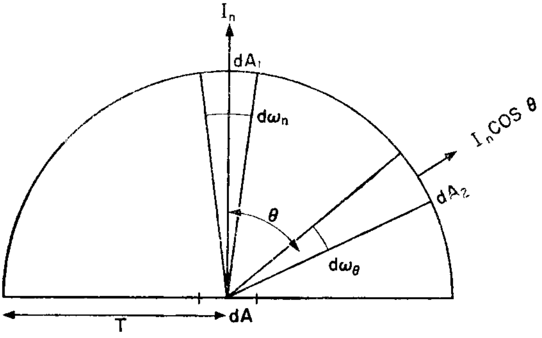

FIG. 5.2. Spatial distribution of radiation

Solid angle subtended by whole hemisphere at

$$dA = \frac{2\pi r^2}{r^2} = 2\pi \text{ steradians.}$$

Solid angle subtended by $dA_1 = d\omega_n = \dfrac{dA_1}{r^2}$.

Solid angle subtended by $dA_2 = d\omega_\theta = \dfrac{dA_2}{r^2}$.

Rate of heat flow through $dA_1 = dQ_n$,

$$dQ_n = I_n \, d\omega_n \, dA \qquad \text{(per unit time).}$$

Similarly,

rate of heat flow through $dA_2 = dQ_\theta$,

$$dQ_\theta = I_\theta \, d\omega_\theta \, dA \qquad \text{(per unit time)}$$

(I_θ = intensity radiation at an angle θ to normal).

$$\begin{aligned} dQ_\theta &= I_\theta \, d\omega_\theta \, dA \\ &= I_n \cos\theta \, d\omega_\theta \, dA, \\ d\omega_\theta &= \frac{2\pi r \sin\theta (r \, d\theta)}{r^2}. \end{aligned}$$

Total radiation passing through the hemisphere in unit time,

$$\int dQ = I_n\, 2\pi\, dA \int_0^{\pi/2} \sin\theta \cos\theta\, d\theta$$

$$= I_n\, 2\pi\, dA \int_0^{\pi/2} \tfrac{1}{2} \sin 2\theta\, d\theta,$$

$$Q = I_n\, \pi\, dA.$$

From the Stefan–Boltzmann radiation law

$$Q = \sigma T^4,$$

$$dA\sigma\, T^4 = I_n \pi\, dA$$

and

$$I_n = \frac{\sigma T^4}{\pi}.$$

Distribution of energy with wavelength (Fig. 5.3)

Let f = frequency of the radiation; λ = wavelength; c = velocity of light.

For radiation *in vacuo* $f\lambda = c$. This can be regarded for practical purposes to hold in air and furnace atmospheres.

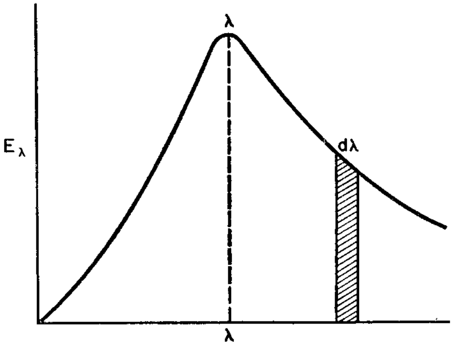

FIG. 5.3. Distribution of radiant energy with wavelength

The total energy radiated per unit time per unit area from a black surface within waveband $d\lambda$ is given by:

$$dQ = E_\lambda \, d\lambda.$$

The spectral distribution is given by Planck's law

$$E_\lambda = \frac{c_1 \lambda^{-5}}{\exp(c_2/\lambda T) - 1},$$

where c_1 = constant = $3{\cdot}741\,5 \times 10^{-16}$ W/m²; c_2 = constant = $1{\cdot}438\,79 \times 10^{-2}$ m K.

The total radiant energy per unit time per unit area for a black body will be the total area under the curve in Fig. 5.3.

$$\int_0^\infty E_\lambda \, d\lambda = \sigma T^4.$$

Wien's law

Wien found that the wavelength λ_m at which the maximum energy is radiated was inversely proportional to the absolute temperature, and with wavelength in metres and temperature degrees Kelvin.

$$K\lambda_m = 2{\cdot}897\,8 \times 10^{-3}.$$

Kirchhoff's radiation law and grey body radiation

The capacity of a body to absorb radiation varies with the wavelength of the incident radiation and angle of incidence. It is found that Q = function (λ, θ, T).

The total hemispherical emissivity e is defined as the ratio of the energy Q emitted by a surface to the total energy Q_b emitted by a black surface at the same temperature.

$$e = (Q/Q_b)_T\,.$$

Emissivity, $e = f(\lambda, \theta, T)$.

Selective emitter

This is defined as a material whose emissivity varies with either λ, θ, or T.

Kirchhoff's law states:

> The monochromatic emissivity of a surface 1 at T_1 is equal to its monochromatic absorptivity received from surface 2 at the same temperature.

$$e = a \quad \text{and} \quad e/a = 1.$$

Grey body

A body whose monochromatic emissivity does not depend upon λ, θ, or T is defined as a grey body. At any particular temperature it always radiates the same proportion of energy as a black body.

Interchange of radiation between two plane non-parallel surfaces (Fig. 5.4)

Let dA_2 subtend solid angle $d\omega_1$ such that

$$d\omega_1 = \frac{dA_2 \cos \theta_2}{X^2} \quad \text{at} \quad dA_1.$$

The intensity of radiation from dA_1 in the X direction $=$ $I_1 = I_{n_1} \cos \theta_1$.

Therefore the incident radiation upon and absorbed by dA_2,

$$dQ_{1 \to 2} = \frac{I_{n_1} \cos \theta_1 \cos \theta_2 \, dA_1 \, dA_2}{X^2}.$$

dA_1 subtends solid angle $d\omega_2$ such that

$$d\omega_2 = \frac{dA_1 \cos \theta_1}{X^2} \quad \text{at} \quad dA_2.$$

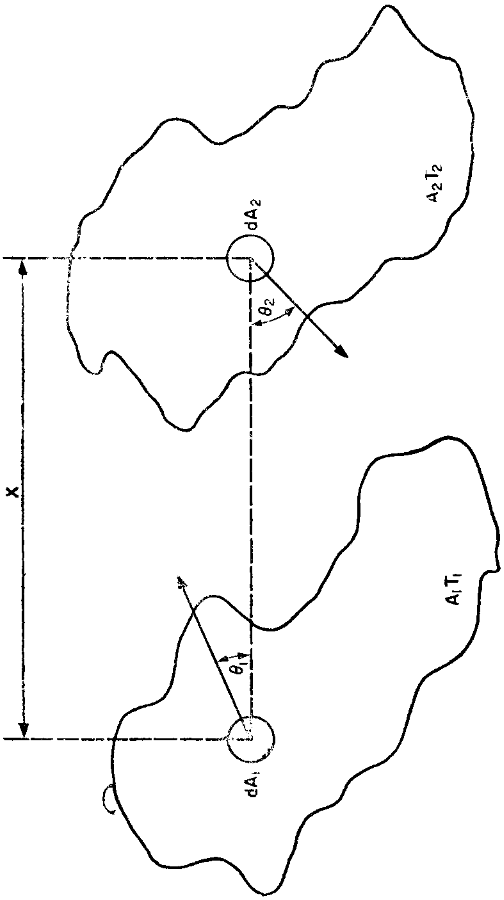

FIG. 5.4. Interchange of radiation between non-parallel surfaces

Therefore radiation incident upon dA_1 from dA_2,

$$dQ_{2\to 1} = I_2\, d\omega_2\, dA_2$$

$$= \frac{I_{n_2} \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{X^2}$$

since $$I_n = \frac{\sigma T^4}{\pi}.$$

The interchange of energy $dQ(A_1 - A_2)$

$$= dQ_{1\to 2} - dQ_{2\to 1}$$

$$= \frac{\sigma \cos\theta_1 \cos\theta_2\, dA_1\, dA_2 (T_2^4 - T_1^4)}{\pi X^2}.$$

Such problems have to be solved by evaluating the double integral in this equation.

Where the surfaces are small compared with the distance apart, reasonably flat, and uniform in temperature, an approximate solution can be obtained without integration by substituting areas A_1 and A_2 in the above equation, together with the appropriate values of θ_1 and θ_2.

For two small areas, parallel and coaxial with each other, $\cos\theta_1$ and $\cos\theta_2$ can be approximated to 1, and if both areas are small the areas can be substituted directly into the general equation.

If one of the areas is small and the other much larger (radiation from a small aperture in a furnace falling on an object) then an approximation can be made by subdividing the receiving disc into elemental areas. The energy falling upon each is then calculated and the sum total found by a single integration.

Instead of calculating the energy falling on the disc it is easier to calculate the energy falling on the cap of a sphere seen within the angle 2ϕ (Fig. 5.5).

Consider a strip on the sphere at an angle θ_1, subtended by an angle $d\theta_1$ at dA_1.

The area of the strip

$$dA_2 = 2\pi r \sin\theta_1 (r\, d\theta_1) = 2\pi r^2 \sin\theta_1\, d\theta_1.$$

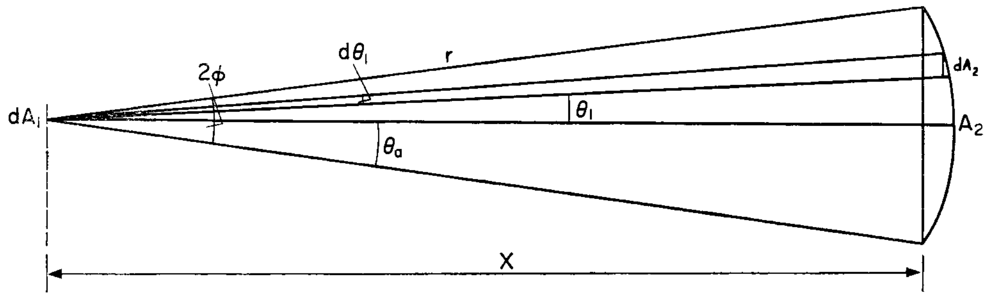

FIG. 5.5. Radiation exchange from small opening to larger area

The energy falling on this strip

$$dQ_{1\rightarrow 2} = \frac{\sigma \cos\theta_1 \cos 0 A_1 2\pi r^2 \sin\theta_1\, d\theta_1 T_1^4}{\pi r^2}$$

$$= \sigma A_1 T_1^4\, 2 \sin\theta_1 \cos\theta_1\, d\theta_1$$

$$= \sigma A_1 T_1^4 \sin 2\theta_1\, d\theta_1\,.$$

The total radiant energy falling on the cap of the sphere and therefore the disc

$$Q_{1\rightarrow 2} = \int_0^{\theta_a} \sigma A_1 T_1^4 \sin 2\theta_1\, d\theta_1$$

$$= -\sigma A_1 T_1^4 \left(\tfrac{1}{2}\cos\theta_1\right)_0^{\theta a}.$$

Where this approximation is not possible, then the equation may be written in the form:

$$dQ_{1\rightarrow 2} = \frac{\sigma T_1^4\, dA_1 \cos\theta_1 \cos\theta_2\, dA_2}{\pi X^2}$$

$$= \sigma T_1^4\, dA_1\, dF_{1\rightarrow 2}\,.$$

$F_{1\rightarrow 2}$ is called the geometric or configuration factor of the surface A_2 with respect to dA_1, and is found by a single integration of $dF_{1\rightarrow 2}$ over the area A_2.

$$F_{1\rightarrow 2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_2}{\pi X^2}\,.$$

If A_2 occupies the whole field of A_1 then $F_{1\rightarrow 2} = 1$. It is a measure of how much the field of view of dA_1 is occupied by A_2.

Radiation exchange between a black body and a black enclosure (Fig. 5.6)

Consider an element dA_2 on A_2; the geometric factor A_1 with respect to A_2 is less than unity because A_1 does not occupy the whole field of view of dA_2.

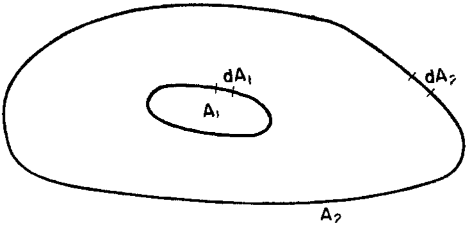

FIG. 5.6. Radiation exchange between black body and black enclosure

Also consider an element dA_1 on A_1; the geometric factor of A_2 with respect to $A_1 = 1$ because the whole field of view of dA_1 is occupied by A_2.

$$Q = \sigma(T_1^4 - T_2^4)A_1.$$

If the enclosure is not black but grey, provided it is large compared with A_1 the above equation still holds. The radiation emitted by $A_1 = \sigma e_1 T_1^4 A_1$ and all of this is absorbed by the enclosure even though it is not black. This is because any reflected radiation is eventually absorbed by A_2 after multiple reflections, and only a negligible amount falls on A_1.

Radiation exchange between two grey bodies

Two parallel plates (Fig. 5.7) (geometric factor = 1)

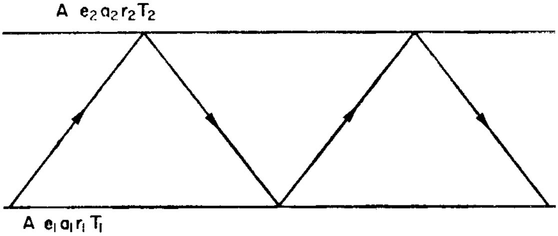

FIG. 5.7. Radiation exchange between parallel surfaces

Radiation emitted from surface

$1 = e_1\sigma T_1^4 A.$

The amount absorbed by surface

$2 = a_2(e_1\sigma T_1^4 A) = e_1 e_2 \sigma T_1^4 A (a_1 = e_1;\ a_2 = e_2).$

The amount reflected by surface

$2 = r_2(e_1\sigma T_1^4 A).$

Surface 1 absorbs amount

$= a_1 r_2(e_1\sigma T_1^4 A).$

It reflects amount

$= r_1 r_2(e_1\sigma T_1^4 A)$, etc.

The net amount of energy leaving surface 1

$$Q_1 = e_1 e_2 \sigma T_1^4 A[1+(r_1r_2)+(r_1r_2)^2+\ \ldots].$$

Similarly the amount of energy radiated from surface 2

$$Q_2 = e_1 e_2 \sigma T_2^4 A[1+(r_1r_2)+(r_1r_2)^2+\ \ldots].$$

Thus the interchange of energy

$$Q_2 - Q_1 = e_1 e_2 \sigma (T_2^4 - T_1^4) A[1+(r_1r_2)+(r_1r_2)^2+\ \ldots].$$

The sum of the series in [ ] $= \dfrac{1}{1-r_1r_2}$.

Thus $Q_2 - Q_1 = \dfrac{e_1e_2}{1-r_1r_2}\sigma(T_2^4-T_1^4)A.$

However, $r = 1-a = 1-e,$

$$Q_2 - Q_1 = \frac{e_1e_2\sigma}{1-(1-e_1)(1-e_2)}(T_2^4-T_1^4)A$$

$$= \frac{e_1e_2\sigma}{e_2+e_1-e_1e_2}(T_2^4-T_1^4)A.$$

Similarly, it can be shown that for twin concentric cylinders, of radii R_1 and R_2 (R_1 on the outside at temperature T_1),

$$Q_2 - Q_1 = \frac{e_1e_2\sigma}{e_2+e_1(1-e_2)(R_2/R_1)}(T_2^4-T_1^4)A.$$

Heat transfer coefficient by radiation

Earlier it was shown that $Q = hA\Delta T$.

Therefore $h_r A(T_1 - T_2) = e\sigma(T_1^4 - T_2^4)A$

$$h_r = \frac{e\sigma(T_1^4 - T_2^4)}{(T_1 - T_2)}$$

$$= \frac{e\sigma(T_1 - T_2)(T_1 + T_2)(T_1^2 + T_2^2)}{T_1 - T_2}.$$

$$h_r = e\sigma(T_1 + T_2)(T_1^2 + T_2^2).$$

5.3. GAS RADIATION

Most of the simple monatomic and diatomic gases are transparent to thermal radiation, but some gases, such as water vapour, carbon dioxide, and hydrocarbons, absorb radiation at certain frequencies.

If I is the intensity of radiation falling upon a gas of layer thickness t, then the intensity I_0, after having passed through the gas, is given by the expression

$$I = I_0 e^{-mt},$$

where m is proportional to the partial pressure of the gas at one wavelength.

Absorption = function $P_g \times L$, where P_g is the partial pressure of the gas and L the equivalent thickness of the gas layer.

$$L = \frac{3{\cdot}4 \times \text{gas volume}}{\text{area of the retaining walls}}.$$

Heat exchange in a combustion chamber containing non-luminous gases proceeds as follows:

(1) gas radiation to the walls which depends upon the quantity of radiating gases (the concentration of carbon dioxide and water, gas layer thickness, and pressure and temperature of gases);

(2) a small fraction of this heat is absorbed by the walls; most is reflected back into the furnace body;

(3) if there are temperature gradients within the furnace there

will be heat exchange between walls; only a small proportion of this heat is absorbed by the gases.

A method of estimating the effect of gas radiation, after Hottel, involves evaluation of the gas emissivity coefficients and then applying them to a modified fourth-power law correcting for the influence of gas composition and for gas layer thickness.

$$Q/\text{hr} = 0{\cdot}01713 \times e'_s \left[e'_G \left(\frac{T_G}{100} \right)^4 - a'_G \left(\frac{T_s}{100} \right)^4 \right] \text{ Btu/ft}^2.$$

The steps in such a calculation (see *Efficient Use of Fuel*) are:

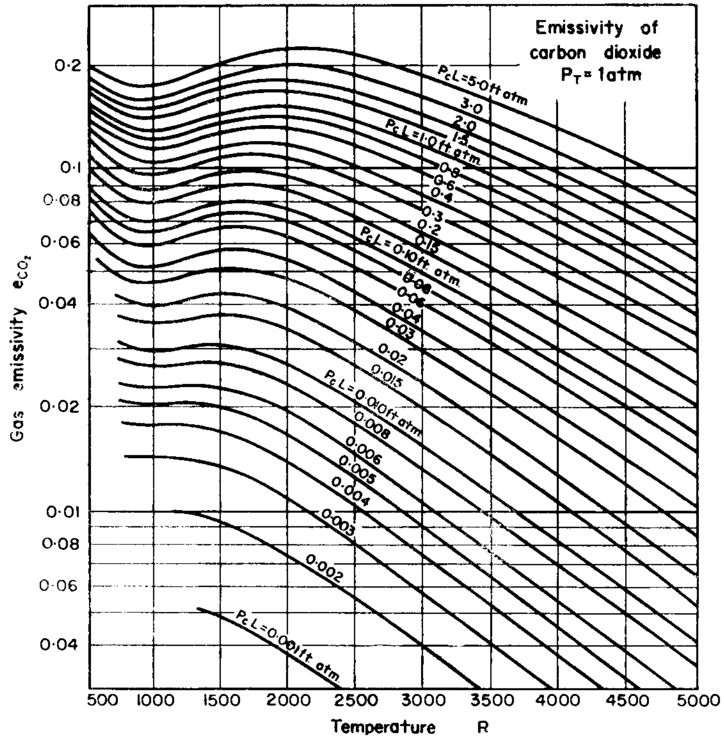

FIG. 5.8. Emissivity values of carbon dioxide for various values of P_cL. (Reproduced by permission from *Heat Transmission*, 3rd edn., by W. H. McAdams *et al.*, McGraw-Hill.)

(1) Estimation of partial pressures of carbon dioxide and water from the gas analysis.

(2) Calculation of "equivalent thickness" of gas layer.

(3) Evaluation of partial pressure × "equivalent thickness" for carbon dioxide (P_cL) and water vapour (P_wL) respectively.

(4) The values of the emissivity of carbon dioxide and water vapour are then obtained from the accompanying graphs (Figs. 5.8 and Fig. 5.9).

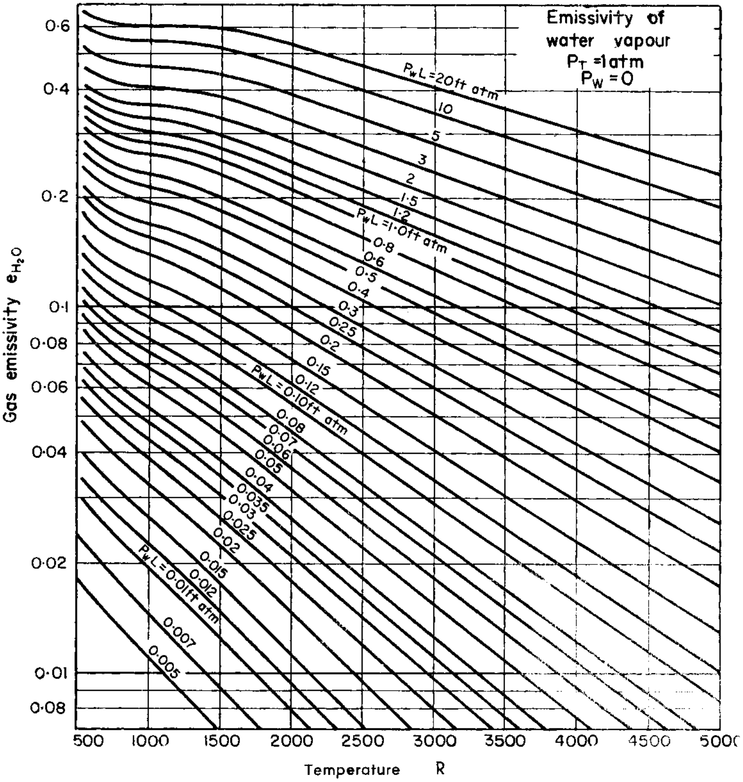

FIG. 5.9A. Emissivity values of water vapour for various values P_wL. (Reproduced by permission from *Heat Transmission*, 3rd edn., by W. H. McAdams *et al.*, McGraw-Hill.)

(5) The value of e_{H_2O} obtained from Fig. 5.9A cannot be used directly because the emissivity of water is also dependent upon concentration. Thus $e'_{H_2O} = e_{H_2O} \times C_w$ (Fig. 5.9B).

(6) The values of e_{CO_2} and e'_{H_2O} have to be further corrected for radiation absorbed by the gas. This is obtained from Fig. 5.10.

$$e'_G = e_{CO_2} + e'_{H_2O} - \Delta a \qquad (\Delta e = \Delta a).$$

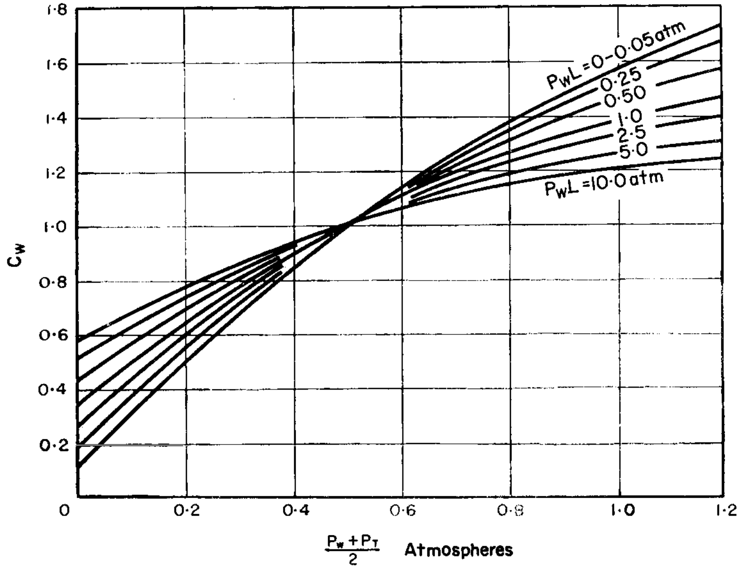

FIG. 5.9B. Correction factor for converting emissivity of water vapour to values of P_w and P_T other than 0 and 1 atm respectively.
(Reproduced by permission from *Heat Transmission*, 3rd edn., by W. H. McAdams *et al.*, McGraw-Hill.)

(7) Using the temperature of the walls of the furnace the absorptivity of the carbon dioxide and water vapour can be obtained from Figs. 5.8 and 5.9.

Thus $a_{gas} = a_{CO_2} + a_{H_2O}$

$$a'_G = a_{CO_2} + a'_{H_2O} - \Delta a \qquad (a'_{H_2O} = a_{H_2O} \times C_w).$$

This gives values of sufficient accuracy for most applications where the gas temperature (T_G) is considerably higher than the walls of the furnace (T_s). However, if this is not the case, then the absorptivities so found have to be corrected by multiplication of P_cL and P_wL by $\left(\frac{T_s}{T_G}\right)$; and then by an empirical factor $\left(\frac{T_G}{T_s}\right)^{0.65}$.

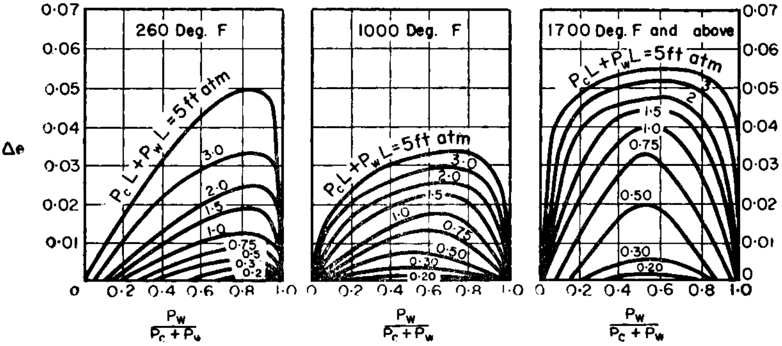

FIG. 5.10. Chart for evaluating the absorption of gas radiation by the gas itself.

(Reproduced by permission from *Heat Transmission*, 3rd edn. by W. H. McAdams *et al.*, McGraw-Hill.)

$$\text{Thus } P_cL' = \left[P_cL \times \frac{T_s}{T_G}\right]\left(\frac{T_G}{T_s}\right)^{0.65}$$

$$P_wL' = \left[P_wL \times \frac{T_s}{T_G}\right]\left(\frac{T_G}{T_s}\right)^{0.65}$$

These values are then used to find the emissivities of carbon dioxide and water vapour from Figs. 5.8 and 5.9.

(8) If the walls of the furnace have an emissivity less than 1 (non-black body) then e_s' must be introduced into the equation. The effective emissivity of the surface will lie between that of a black body and the surface, because most of the radiation from

the walls passes through the gas layer and has further opportunity for absorption at another surface.

Thus $e_s' = \frac{1}{2}(e_s + 1)$, where e_s = emissivity walls.

5.4. HEAT TRANSFER BY CONVECTION

In this case heat transfer is attributable to macroscopic motion of the fluid and is thus confined to liquids and gases. In natural convection it is caused by differences in density resulting from temperature gradients in the fluid. Forced convection is caused by eddy currents in fluids under turbulent flow conditions.

It was shown in the case of heat transfer by radiation that the rate of heat transfer per hour Q could be represented by the expression: $Q = UA\Delta T$.

To calculate Q, or the area necessary to achieve a given quantity of heat transfer in a heat exhanger U, the overall heat transfer coefficient must be evaluated. To determine this, the individual film coefficients have to be calculated or determined experimentally. So many factors govern the value of h_c, the convective heat transfer coefficient, that it is almost impossible to determine their individual effects by direct experimental methods.

Dimensionless groups have been used with considerable success in arriving at equations which allow h_c to be determined. Dimensional analysis has also been used with considerable success in the problem of convective heat transfer.

Application of dimensional analysis to convective heat transfer

The heat transfer (q) per unit area per unit time depends upon a number of variables. Over the range considered it is assumed that the relation between the heat flow q and the variables can be expressed by a simple power series:

$$q \propto \left[V^{x_1} \cdot L^{x_2} \cdot \Delta T^{x_3} \cdot \mu^{x_4} \cdot k^{x_5} \cdot \varrho^{x_6} \cdot C_p^{x_7} \cdot (\beta g)^{x_8}\right].$$

	Dimensions
q heat transferred per unit area per unit time	$QL^{-2}T^{-1}$
V velocity	LT^{-1}
L linear dimension	L
ϱ density	ML^{-3}
μ viscosity	$ML^{-1}T^{-1}$
k thermal conductivity	$QT^{-1}L^{-1}\theta^{-1}$
C_p specific heat at constant pressure	$QM^{-1}\theta^{-1}$
ΔT temperature difference	θ
(βg) coefficient of thermal expansion	
x gravitational constant (buoyancy factor)	$LT^{-2}\theta^{-1}$

The dimensions used are length (L), mass (M), time (T), temperature (θ), and heat Q.

If indices on both sides of the equation are equated:

$$\begin{array}{lrl}
\text{in length} & -2 = & x_1+x_2-x_4-x_5-3x_6+x_8; \\
\text{in time} & -1 = & -x_1-x_4-x_5-2x_8; \\
\text{in mass} & 0 = & x_4+x_6-x_7; \\
\text{in heat} & 1 = & x_5+x_7; \\
\text{in temperature} & 0 = & x_3-x_5-x_7-x_8.
\end{array}$$

These equations can all be solved in terms of x_1, x_7, and x_8 to include only one of the variables, V, C_p, and βg.

$$\begin{aligned}
x_6 &= x_1+2x_8, \\
x_5 &= 1-x_7, \\
x_4 &= -x_1+x_7-2x_8, \\
x_3 &= 1+x_8, \\
x_2 &= -1+x_1+3x_8.
\end{aligned}$$

Thus

$$q = CV^{x_1}L^{-1+x_1+3x_8}\,\Delta T^{1+x_8}\mu^{-x_1+x_7-2x_8}k^{1-x_7}\,\varrho^{x_1+2x_8}C_p^{x_7}(\beta g)^{x_8}$$

or

$$q = \frac{Ck\,\Delta T}{L}\left(\frac{LV\varrho}{\mu}\right)^{x_1}\left(\frac{C_p\mu}{k}\right)^{x_7}\left(\frac{\beta g\,\Delta TL^3\varrho^2}{\mu^2}\right)^{x_8}.$$

Since $q = h\,\Delta T \quad (A = 1)$,

$$\frac{qL}{\Delta Tk} = \left(\frac{hL}{k}\right) = C\left(\frac{LV\varrho}{\mu}\right)^{x_1}\left(\frac{C_p\mu}{k}\right)^{x_7}\left(\frac{\beta g\,\Delta TL^3\varrho^2}{\mu^2}\right)^{x_8}.$$

These four groups are dimensionless and are known as

$\frac{hL}{k}$ = Nusselt group (Nu),

$\frac{LV\varrho}{\mu}$ = Reynolds group (Re),

$\frac{C_p\mu}{k}$ = Prandtl group (Pr),

$\frac{\beta g\,\Delta TL^3\varrho^2}{\mu^2}$ = Grashof group (Gr).

For natural convection:

Nu = function ($PrGr$).

For forced convection:

Nu = function ($RePr$).

For gases the Prandtl group is constant over a wide range of temperatures and pressures.

Convective heat transfer by forced convection

$Nu = f(RePr)$.

In tubes

For values of Reynolds numbers greater than 4000, and for liquids of not more than twice the viscosity of water

$$Nu = 0{\cdot}023\ Re^{0{\cdot}8}Pr^{0{\cdot}4}.$$

With diatomic gases, $Pr = 0{\cdot}74$, and $Nu = 0{\cdot}02\ Re^{0{\cdot}8}$.

Where the flow in tubes is stream-line† heat transfer can be considered to take place solely by conduction. In the case of a fluid flowing in a tube the velocity distribution is parabolic. If heat is supplied from the outside the velocity distribution is altered and the heat transfer becomes more complex. For gases the empirical relationship

$$Nu = 1{\cdot}36\,[RePr(d/L)]^{0{\cdot}4}$$

has been found to be valid (d = tube diameter; L = length of tube).

Forced convection outside tubes

For flow across single cylinders:

$Nu = 0{\cdot}26\, Re_{\max}^{0{\cdot}6} Pr^{0{\cdot}3}$ for Reynolds numbers between 1000 and 100,000.

For gases this reduces to $Nu = 0{\cdot}24\, Re_{\max}^{0{\cdot}6}$.

$Re_{\max}$ is used because the velocity distribution around a cylinder is non-uniform.

For flow at right angles to bundles of tubes such as occur in many heat exchangers $Nu = 0{\cdot}33\, C\, Re_{\max}^{0{\cdot}6} Pr^{0{\cdot}3}$. The values of C depend upon the geometric arrangement of the tubes, i.e. in-line or staggered.

Flow in an annulus between concentric tubes

Such heat exchangers are widely used because of their simplicity of construction and operation. They can operate with turbulent conditions (desirable for high rates of heat transfer) where the flow rates are low.

A difficulty in obtaining values of h is in selecting the best diameter. One such relationship gives the outside film coefficient

$$\frac{hd_e}{k}\left(\frac{\mu_s}{\mu}\right)^{0{\cdot}14} = 0{\cdot}027\left(\frac{d_e V \varrho}{\mu}\right)^{0{\cdot}8} Pr^{0{\cdot}33},$$

where μ = viscosity at mean bulk temperature; μ_s = viscosity at surface of wall; $d_e = d_2 - d_1$.

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† See Chapter 6.

Simplified equation for convective heat transfer to air

For stream-line flow and turbulent flow from walls and pipes to air q = constant $(\Delta T)^{1\cdot25}$. The constant varies with dimensional configuration and depends on whether flow is stream-line or turbulent.

Heat transfer in the condensation of vapours

Film condensation occurs when the condensate wets the solid surface; it forms a continuous film and all the heat is then transferred by conduction through the film.

Drop condensation occurs when the condensed liquid does not wet the surface; the drops of condensate roll off once they have reached a certain size. Much higher rates of heat transfer are thus obtained.

In most industrial plant condensation is usually mixed. Droplet condensation can be promoted if the condensing liquid is coated with a monomolecular layer of certain materials.

Film condensation

(a) Horizontal tubes: when condensation occurs in a vertical line of N tubes the heat transfer coefficient between vapour and tube surface is given by

$$h = 0\cdot72\left(\frac{k^3\varrho^2 g\lambda}{N d\mu\theta}\right)^{0\cdot25},$$

where θ = temperature difference between the vapour and condensing surface; λ = latent heat of condensation fluid.

(b) Vertical tubes:

(i) Stream-line flow $h = 1\cdot13\left(\frac{k^3\varrho^2 g\lambda}{LV\theta}\right)^{0\cdot25}$.

(ii) Turbulent flow $h = 7\cdot7\times10^{-3}Re^{0\cdot4}\left(\frac{k^3\varrho^2 g}{\mu^2}\right)^{0\cdot333}$

(*Re* greater than 2000).

For steam condensing as a film at atmospheric pressure,

$$h = 3100 \cdot d^{-0 \cdot 25} \theta^{-0 \cdot 333} \qquad \text{(FPH units).}$$

For a single horizontal tube, and for vertical tubes,

$$h = 4000 L^{-0 \cdot 25} \theta^{-0 \cdot 333} \qquad \text{(FPH units).}$$

Various attempts have been made to evaluate the heat transfer coefficient from the physical properties of the system. When a fluid flows under turbulent conditions over a surface three regions (Fig. 5.11) may be observed.

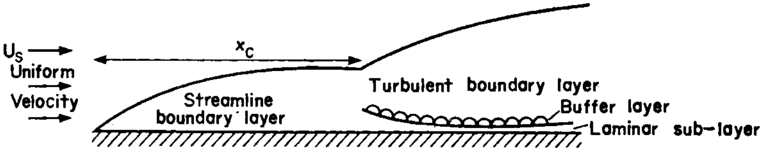


FIG. 5.11. Development of boundary layer.
(Reproduced from *Chemical Engineering*, vol. 1, J. M. Coulson and J. F. Richardson, Pergamon Press, 2nd revised edn., 1964)

(1) At the surface, the laminar sub-layer, in which the only motion at right angles to the surface is due to molecular diffusion.

(2) Next to the surface, a buffer layer, in which both molecular diffusion and eddy motion are of equal importance.

(3) In the major portion of the fluid, there is a turbulent region in which eddy motion is the most important.

The simple Reynolds analogy

For forced convection $Nu = C\, Re^a Pr^b$.

Since heat transfer and momentum transfer are both related to the Reynolds group a number of expressions have been developed to show a direct relationship between heat transfer and the distribution of momentum in a moving stream.

The first relationship is known as the Reynolds analogy and shows

$$\frac{h}{C_p \varrho U_s} \propto Re^{-0\cdot25},$$

where U_s = velocity at surface.

For turbulent flow in pipes

$$\frac{h}{C_p \varrho U_s} = 0\cdot032\, Re^{-0\cdot25}.$$

In this analysis the turbulent nature of the fluid is assumed to exist all the way to the surface and no allowance is made for any variation of the physical properties of the fluid.

Taylor and Prandtl modification (in pipes)

In this case, with turbulent flow it was assumed that heat passed directly from the turbulent region to the laminar sub-layer and the buffer layer was ignored. For turbulent flow,

$$\frac{h}{C_p U \varrho} = 0\cdot032\, Re^{-0\cdot25}[1+2\cdot0\, Re^{-0\cdot125}(Pr-1)]^{-1}.$$

The universal velocity profile

Where the effect of the buffer layer is taken into consideration, then for turbulent flow,

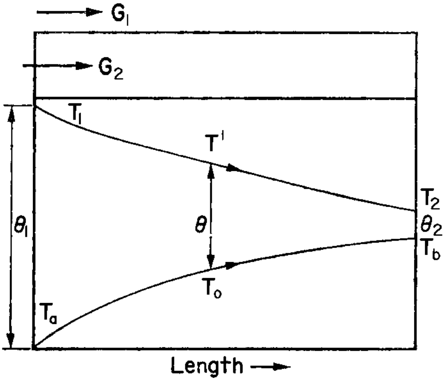
$$\frac{h}{C_p U \varrho} = \frac{0\cdot032\, Re^{-0\cdot25}}{1+0\cdot82\, Re^{-0\cdot125}[(Pr-1)+\ln(5/6Pr+1/6)]}.$$

For more detailed results, see Coulson and Richardson.

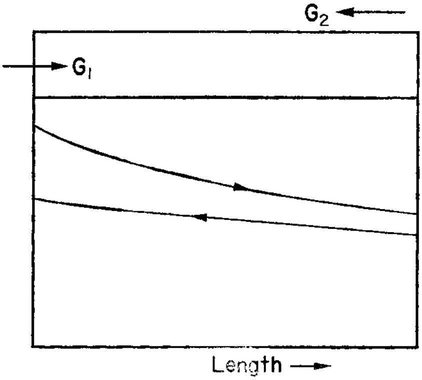
5.5. HEAT EXCHANGER PROBLEMS

In most heat exchangers the temperature of at least one of the fluids is continually changing as it passes through the exchanger so that the temperature varies over its length (Fig. 5.12). It is therefore necessary to find an expression for the average temperature difference θ_m to be used in the general equation

$$\underset{\sim}{Q} = UA\theta_m.$$



Parallel flow



Counter current flow

FIG. 5.12. Temperature gradients in heat exchangers

Parallel flow heat exchanger

Suppose that the outside stream of fluid which has a specific heat of C_{p_1} flows at a rate G_1 with temperature changes as indicated.

Suppose also that the inside stream of fluid which has a specific heat of C_{p_2} flows at a mass of rate G_2, also with temperature changes as indicated.

Over a small element of area dA where the temperatures are T_0 and T' respectively

$$\theta = T' - T_0, \quad \text{and} \quad d\theta = dT' - dT_0 .$$

The heat given out by the hot stream

$$= dQ = -G_1 C_{p_1}\, dT' .$$

The heat taken up by the cold stream

$$= dQ = G_2 C_{p_2}\, dT_0 .$$

Thus $d\theta = \dfrac{-dQ}{G_1 C_{p_1}} - \dfrac{dQ}{G_2 C_{p_2}} = -dQ\left(\dfrac{1}{G_1 C_{p_1}} + \dfrac{1}{G_2 C_{p_2}}\right) = -dQ \cdot H.$

However, $$QH = \theta_1 - \theta_2 \ldots \qquad (1)$$

and $$U\, dA\theta = dQ = -\frac{d\theta}{H},$$

$$HU \int_0^A dA = -\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta},$$

$$HUA = -\log_e \frac{\theta_2}{\theta_1} .$$

Thus $$H = \frac{\log_e (\theta_1/\theta_2)}{UA} \ldots \qquad (2)$$

From eqn. (1)

$$\theta_1 - \theta_2 = QH \quad \text{and} \quad Q = \frac{\theta_1 - \theta_2}{H} .$$

By definition, $$Q = UA\theta_m .$$

Thus $$Q = \frac{(\theta_1 - \theta_2)UA}{\log_e (\theta_1/\theta_2)} \quad \text{from eqn. (2).}$$

Hence $$\theta_m = \frac{\theta_1 - \theta_2}{\log_e (\theta_1/\theta_2)} .$$

θ_m is known as the logarithmic mean temperature difference. The logarithmic mean temperature difference is applicable to both simple parallel-flow and counter-current flow heat exchangers.

For a given set of terminal temperatures the logarithmic mean temperature difference for parallel flow is always less than for counter-current flow unless the temperature of one fluid remains constant.

For multipass and cross-flow heat exhangers, the true mean temperature difference is less than for counter-current flow and greater than parallel flow. For such heat exchangers the logarithmic mean temperature difference applicable to a counter-current exchanger working between the same terminal temperatures must be multiplied by a factor F which depends upon two functions A and B which have values of:

$$A = \frac{\text{temp. of tube fluid leaving} - \text{temp. of tube fluid entering}}{\text{temp. of shell fluid entering} - \text{temp. of tube fluid entering}},$$

$$B = \frac{\text{temp. of shell fluid entering} - \text{temp. of shell fluid leaving}}{\text{temp. of tube fluid leaving} - \text{temp. of tube fluid entering}}.$$

A more detailed treatment is given in Spiers *(Technical Data on Fuel)*.

5.6. STEADY-STATE HEAT TRANSFER BY CONDUCTION

In a solid the flow of heat by conduction is brought about by the transfer of vibrational energy from one molecule to another, while in liquids it is the result of the transfer of kinetic energy.

Steady-state heat flow in slabs, where the area is large compared to its thickness

The quantity of heat conducted through a body in a given time is related by Fourier's law to the temperature difference and the dimensions in the following ways:

(1) it is directly proportional to the difference in temperature between hot and cold faces;

(2) it is inversely proportional to the thickness of the material through which heat is passing;

(3) it is directly proportional to the total area of the surfaces normal to the direction of heat flow through which heat is passing.

$$\text{Rate of heat flow} \propto -A\frac{\Delta T}{\Delta X} = kA-\frac{\Delta T}{\Delta X}$$

(X = thickness of slab).

The value of k is called the coefficient of thermal conductivity of the material.

Where the thickness and temperature differences are small,

$$\frac{dQ}{dt} = -kA\frac{dT}{dX}.$$

The negative sign indicates that the temperature gradient is in the opposite direction to the flow of heat.

The above equation can also be written in the form

$$Q/\text{hr} = \frac{\text{driving force}}{\text{thermal resistance}} = \frac{\Delta T}{X/(kA)}.$$

Units of k in common use are:

k = lb-cal/hr ft^2 °C/ft (in.) = W/m K = J/m s °C
= Btu/hr ft^2 °F/ft (in.).

(*Note.* Units of X must correspond to the last unit in k.) Thermal conductivity depends upon the material, and the value varies with temperature (Fig. 5.13).

Composite walls

Cross-sectional area of wall = A, rate of heat flow = Q,

$$T_1-T_2 = \left(\frac{x_1}{k_1A}\right)Q,$$

$$T_2-T_3 = \left(\frac{x_2}{k_2A}\right)Q,$$

$$T_3-T_4 = \left(\frac{x_3}{k_3A}\right)Q.$$

Adding these three terms,

$$T_1 - T_4 = \left(\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} + \frac{x_3}{k_3 A}\right) Q,$$

$$Q = \frac{T_1 - T_2}{x/(kA)} = \frac{\text{driving force}}{\text{thermal resistance}} = \frac{T_1 - T_4}{\Sigma x/(kA)}.$$

It is also worth noting that

$$\frac{\text{temperature drop over } x_1}{\text{total temperature drop}} = \frac{\text{thermal resistance } x_1}{\text{total thermal resistance}}.$$

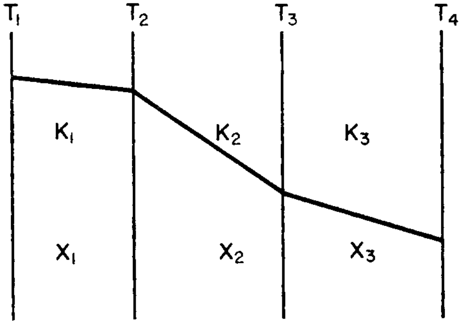


FIG. 5.13. Conduction in multiple walls

Calculation of heat flow through wall of a cylindrical tube (Fig. 5.14)

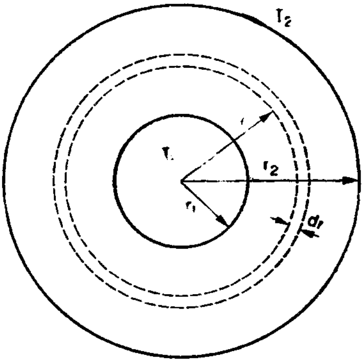


FIG. 5.14. Conduction of heat in cylinders

Let the internal and external radii be r_1 and r_2 and let the steady-state temperature of the inside and outside of the tube be T_1 and T_2; let T_1 be the greater. Let L be the length of the tube.

Consider a thin cylindrical element of radii r and $r+dr$.

Since this is very thin the heat flow across the surfaces can be regarded as normal and so it can be treated as a thin, parallel-sided slab. Thus

$$\frac{dQ}{dt} = -kA\,\frac{dT}{dr}.$$

Since $\quad A = 2\pi rL,$

$$\frac{dQ}{dt} = -k2\pi rL\frac{dT}{dr} = -2\pi kLr\,\frac{dT}{dr}. \tag{1}$$

Since there are steady-state conditions, dQ/dt must be the same for all values of r, and both k and L are constants, hence $r(dT/dr)$ must also be a constant.

$$r\frac{dT}{dr} = a; \qquad dT = a\,\frac{dr}{r};$$

$$\int_{T_1}^{T_2} dT = a\int_{r_1}^{r_2}\frac{dr}{r}$$

$$(T_2-T_1) = a\left(\log_e\frac{r_2}{r_1}\right).$$

Substituting in eqn. (1)

$$\frac{dQ}{dt} = \frac{2\pi kL(T_1-T_2)}{\log_e\dfrac{r_2}{r_1}}$$

$$= \frac{\text{driving force}}{\text{thermal resistance}} = \frac{T_1-T_2}{\dfrac{(r_2-r_1)}{k\cdot 2\pi r_m L}}.$$

$$A = 2\pi r_m L,$$

where r_m = logarithmic mean radius $= \dfrac{r_2 - r_1}{2{\cdot}303 \log_{10} \dfrac{r_2}{r_1}}$.

For thick-walled tubes r_m must be evaluated to calculate the heat transfer rate. It is not always convenient to use r_m and for thin-walled tubes r_a the arithmetic mean radius may be used: $r_a = \dfrac{r_1 + r_2}{2}$.

r_a will give an accuracy within 10% if $\dfrac{r_2}{r_1} < 3{\cdot}2$.

r_a will give an accuracy within 1% if $\dfrac{r_2}{r_1} < 1{\cdot}5$.

Economic thickness of lagging

$$\text{Lagging thickness} = r_2 - r_1 = r_1\left(\frac{r_2}{r_1} - 1\right)$$

$$= r_1(r-1), \text{ where } r = \frac{r_2}{r_1} \text{ mm.}$$

Log mean radius lagging

$$= \frac{r_2 - r_1}{\log_e (r_2/r_1)} = \frac{r_1(r-1)}{1000 \log_e r} \text{ m.}$$

Area over which heat is being lost for 1 m length pipe

$$= \frac{2\pi r\,(r-1)}{1000 \log_e r} \times 1 \text{ m}^2.$$

Heat loss per unit time

$$= \frac{\text{driving force}}{\text{thermal resistance}}$$

$$= \frac{\Delta T/[r_1(r-1)]}{[k2\pi r_1(r-1)]/1000 \log_e r} = \frac{\Delta T k \pi}{1000 \log_e r}.$$

Let B = annual cost in new pence of a heat loss of 1 W through the lagging when the pipe is in service,

A = initial cost of lagging per cubic metre also in new pence, and

P = the annual maintenance and capital charges as a fraction of the initial cost.

Then annual heat lost $= \dfrac{\pi\, \Delta TkB}{1000 \log_e r}$.

Volume of lagging $= \dfrac{\pi(r_2^2 - r_1^2)1}{10^6} = \dfrac{\pi r_1^2(r^2-1)}{10^6}$.

Annual charges $= \dfrac{\pi r_1^2(r^2-1)AP}{10^6}$.

Hence total cost $= \dfrac{\pi\, \Delta TkB}{10^3 \log_e r} + \dfrac{\pi APr_1^2(r^2-1)}{10^6} = C$.

The cost is a minimum when $dC/dr = 0$.

$$\frac{dC}{dr} = -\frac{\pi\, \Delta TkB}{10^3 r(\log_e r)^2} + \frac{APr_1^2 \times 2r}{10^6}.$$

Equating to zero and transposing,

$$\frac{\pi\, \Delta TkB}{10^3 r(\log_e r)^2} = \frac{2\pi r_1^2 rAP}{10^6},$$

$$r^2(\log_e r)^2 = \frac{1000\, \Delta TkB}{r_1^2 AP},$$

$$r(\log_e r) = \frac{10}{r_1}\left[\frac{10\, \Delta TkB}{AP}\right]^{1/2}.$$

Economic thickness of lagging $= r_1(r-1)$ mm

$$= \frac{10}{r_1}\left[\frac{10\, \Delta TkB}{AP}\right]^{1/2}.$$

ΔT is the temperature drop over the lagging surface.

For a lagged furnace wall, per square metre of wall the volume for thickness t mm $= \frac{t}{10^3}$ m^3.

Annual charges $= \frac{APt}{10^3}$ new pence.

Heat lost per second by conduction $= \frac{\Delta Tk1}{t}$.

Annual cost $= \frac{\Delta TkB}{t}$ new pence.

Total cost $= \frac{APt}{10^3} + \frac{\Delta TkB}{t} = C$.

$$\frac{dC}{dt} = \frac{AP}{10^3} - \frac{\Delta TkB}{t^2}.$$

Equating to zero, and solving for t,

$$t^2 = \frac{10^3 \Delta TkB}{AP},$$

$$t = 10\left[\frac{10\,\Delta TkB}{AP}\right]^{1/2},$$

t = economic thickness,

where ΔT is temperature drop over lagging and k = thermal conductivity W/m °C.

Calculation of heat flow through the walls of a sphere

$$Q/hr = -kA\frac{dT}{dr}.$$

For a sphere of radius r_1 and thickness dr,

$$\frac{dQ}{dt} = -k4\pi r^2 \frac{dT}{dr}$$

$$= -4\pi k \, dT \frac{r^2}{dr},$$

$$\int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{4\pi k}{dQ} \int_{T_1}^{T_2} dT,$$

r_1 = inner radius, r_2 = outer radius, T_2 = outer temperature, T_1 = inner temperature.

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{4\pi k(T_2 - T_1)}{Q},$$

$$Q = \frac{4\pi k(T_2 - T_1) r_1 r_2}{r_2 - r_1}.$$

Calculation of the rate of solidification of a metal in casting (analogous to thickening of ice on a pond)

$$\frac{dQ}{dt} = -kA \frac{\Delta T}{x}.$$

If it is assumed that ΔT is constant (i.e. there is a large body of metal, in proportion to the amount solidifying):

$$\frac{dQ}{dt} = LA\varrho \frac{dx}{dt}$$

(L = latent heat metal per unit mass; σ = density metal),

$$\frac{dx}{dt} = \text{rate of thickening.}$$

Thus
$$-kA \frac{\Delta T}{x} = L\varrho A \frac{dx}{dt},$$

$$\int_0^t dt = \frac{L\varrho}{k\,\Delta T} \int_0^x x\,dx; \qquad t = \frac{L\varrho x^2}{2k\,\Delta T}.$$

Thus $x \propto \sqrt{t}$, i.e. the rate of solidification is proportional to $\sqrt{t}$.

5.7. SOME SPECIAL APPLICATIONS OF HEAT TRANSFER APPLICABLE TO FURNACE TECHNOLOGY

Heat flow through a cooling fin

When heat has to be transferred from a metal surface to a fluid, the effective area for heat transfer can be increased by fins projecting from the surface. Most of the effective resistance is in the fluid boundary adjacent to the wall, and this is greatly reduced by increasing the surface area; the increase in resistance due to the extra metal can be neglected.

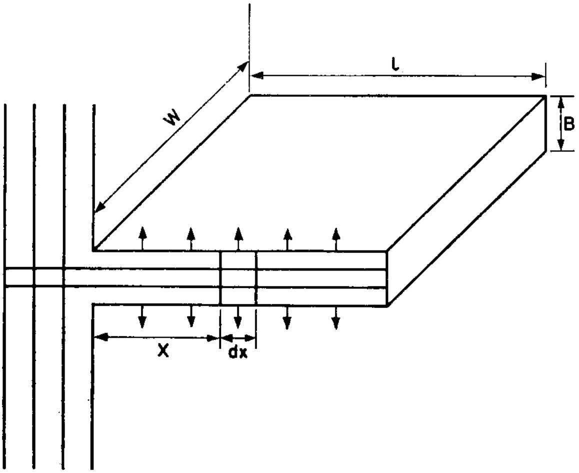


FIG. 5.15. Heat conduction in simple fin

In the case of a simple rectangular fin projecting from a flat surface (Fig. 5.15), if the fin is very thin then it can be assumed that the lines of heat flow are parallel to the flanks of the fin, and heat transfer can be reduced to a one-dimensional problem.

The heat flow per unit time into an element at distance X from the base of the fin and of thickness dX is given by the relationship

$$Q_X = -kA\frac{dT}{dX} = -k(WB)\frac{dT}{dX}. \tag{1}$$

Thus the heat leaving the elemental section through $X+dX$ from the base of the fin

$$= Q_{X+dX} = Q_X + \frac{dQ}{dX}dX \tag{2}$$

$$= -k(WB)\frac{dT}{dX} - k(WB)\frac{d^2T}{dX^2}dX \tag{3}$$

The net outflow to the surrounding air at temperature T_a from the element

$$\begin{aligned} dX = dQ &= Q_X - Q_{(X+dX)} \\ &= k(WB)\frac{d^2T}{dX^2}dX. \end{aligned} \tag{4}$$

If it is assumed that the combined heat transfer coefficients by convection and radiation remain reasonably constant over the whole area of the fin, then $h = (h_r + h_c) = \text{constant}$.

$$\begin{aligned} dQ &= hA\Delta T \\ &= h2(W+B)\,dX(T-T_a), \end{aligned}$$

where T = surface fin temperature.

If the fin is very thin, i.e. B is small compared with L,

$$dQ = 2Wh(T-T_a)\,dX. \tag{5}$$

Hence eqn. (4) equals eqn. (5), and thus

$$\frac{d^2(T-T_a)}{dX^2} = \frac{d^2\Delta T}{dX^2} = \frac{2h}{kB}\Delta T. \tag{6}$$

The general solution to this differential equation is

$$\Delta T = M\,e^{mX} + N\,e^{-mX}, \tag{7}$$

$$m = \left(\frac{2h}{kB}\right)^{1/2}.$$

The values of the constants of integration M and N have to be evaluated by consideration of the boundary conditions.

At the base of the fin ($X = 0$) the fin temperature $= T_0$. Therefore

$$\Delta T_0 = (T_0 - T_a) = M + N. \tag{8}$$

The heat lost from the tip of the fin can be regarded as very small and regarded as zero. Thus at the tip of the fin, where $X = L$,

$$Q_L = -kLB\left(\frac{d\,\Delta T}{dX}\right)_{(X=L)} = 0. \tag{9}$$

Therefore
$$\left(\frac{d\,\Delta T}{dX}\right)_{(X=L)} = 0. \tag{10}$$

If eqn. (7) is differentiated with respect to X, when $X = L$,

$$\frac{d\,\Delta T}{dX_{(X=L)}} = 0 = Mm\,e^{mL} - Nm\,e^{-mL}. \tag{11}$$

Thus
$$M = \Delta T_0\left[\frac{e^{-mL}}{e^{mL}+e^{-mL}}\right]$$

and
$$N = \Delta T_0\left[\frac{e^{mL}}{e^{mL}+e^{-mL}}\right].$$

Substituting these values into eqn. (7)

$$\Delta T = \Delta T_0\left[\frac{e^{m(L-X)}+e^{-m(L-X)}}{e^{mL}+e^{-mL}}\right]$$
$$= \Delta T_0\left[\frac{\cosh m(L-X)}{\cosh mL}\right],$$

and this is the temperature profile along the fin.

For steady-state conduction, the heat flow from the surface of the fin is equal to the heat flow through the base of the fin.

$$Q = -kWB\left(\frac{d\,\Delta T}{dX}\right)_{(X=0)}$$
$$= mkWB\,\Delta T_0\left[\frac{\sinh m(L-X)}{\cosh mL}\right]_{(X=0)}.$$
$$Q = mkWB\,\Delta T_0 \tanh mL.$$

The determination of the temperature distribution in a hollow rod fixed at one end to a plate and projecting into a hot fluid is a similar problem.

An example is provided by a thermometer pocket in a pipe. The thermometer indicates a temperature T, somewhere between T_F and T_0. (T_F = fluid temperature and T_0 is pipe temperature.)

$$T-T_F = (T_0-T_F)\left[\frac{\cosh m(L-L)}{\cosh mL}\right].$$

The temperature at the end of the pocket is obtained when $X = L$.

$$T-T_F = \frac{(T_0-T_F)1}{\cosh mL}.$$

Thus
$$T = \frac{(T_0-T_F)}{\cosh mL}+T_F.$$

In this particular case it may be shown that

$$m = \left[\frac{h\times\text{circumference rod}}{k\times\text{solid cross-sectional area}}\right]^{1/2}.$$

From the above expression it is clear that the minimum error in reading the fluid temperature is when $T = T_F$, thus

$$\frac{T_0-T_F}{\cosh mL} = 0.$$

This can be achieved by:

(1) lagging the thermometer pocket so that T_0 more closely approaches T_F;

(2) making either m or L greater: (i) m can be increased by using a thinner tube or material of lower thermal conductivity (within limits), (ii) L can be increased by inclining the thermometer pocket, and letting it project more than half-way into the fluid stream.

Thermocouple in a gas stream

If a thermocouple is inserted into a gas stream the temperature recorded is intermediate between the fluid temperature T_F and the temperature of the walls of the conveying pipe T_w.

The exact value of the thermocouple temperature T is obtained by a balance of heat transfers such that

$$h_c A(T_F - T) = h_r A(T - T_w),$$

where h_c and h_r are the respective heat transfer coefficients by convection and radiation.

Thus $$T_F = T + (T - T_w)\frac{h_r}{h_c}.$$

From this expression it is evident that the most accurate value of T_F will be obtained when $T = T_F$.

Thus $$(T - T_w)\frac{h_r}{h_c} = 0.$$

These conditions can be satisfied by:

(1) lagging the pipe so that T more nearly approaches T_w;

(2) increasing the convective heat transfer coefficient by (i) using thinner wires, (ii) increasing the velocity of flow (suction pyrometer);

(3) by decreasing h_r by having a radiation shield around the thermocouple (suction pyrometer).

5.8. UNSTEADY-STATE HEAT TRANSFER

This is important in calculating the time taken for furnace walls to heat up or cool down, and in evaluating the time taken for ingots to reach soaking temperature when admitted to a furnace.

The problem is best approached by taking a volume element

dX, dY, dZ with edges parallel to the coordinates shown in Fig. 5.16.

Let the temperature at the point X, Y, Z be T and at the point $X+dX$, $Y+dY$, $Z+dZ$ be $T+dT$.

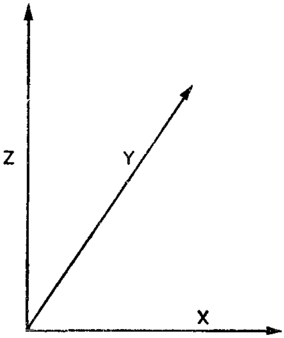


FIG. 5.16.

Then the rate of conduction of heat through the element

$$= -k\,dY\,dZ\left(\frac{\partial T}{\partial X}\right)_{YZ} \quad \text{in the } X\text{-direction,}$$

$$= -k\,dZ\,dX\left(\frac{\partial T}{\partial Y}\right)_{ZX} \quad \text{in the } Y\text{-direction,}$$

$$= -k\,dX\,dY\left(\frac{\partial T}{\partial Z}\right)_{XY} \quad \text{in the } Z\text{-direction.}$$

The rate of change of heat content of the element is the loss of heat flow from XYZ to $(X+dX)(Y+dY)(Z+dZ)$.

This is given by

$$k\,dY\,dZ\left(\frac{\partial^2 T}{\partial X^2}\right)_{YZ} dX + k\,dZ\,dX\left(\frac{\partial^2 T}{\partial Y^2}\right)_{ZX} dY$$

$$+k\,dX\,dY\left(\frac{\partial^2 T}{\partial Z^2}\right)_{XY} dZ$$

$$= k\,dX\,dY\,dZ\left[\left(\frac{\partial^2 T}{\partial X^2}\right)_{YZ} + \left(\frac{\partial^2 T}{\partial Y^2}\right)_{ZX} + \left(\frac{\partial^2 T}{\partial Z^2}\right)\quad\right].$$

However, the rate of heat gain is also equal to the heat capacity multiplied by the rate of temperature increase

$$= C_p \varrho \, dX \, dY \, dZ \frac{\partial T}{\partial t}.$$

Thus

$$C_p \varrho \, dX \, dY \, dZ \frac{\partial T}{\partial t}$$

$$= k \, dX \, dY \, dZ \left[\left(\frac{\partial^2 T}{\partial X^2} \right)_{YZ} + \left(\frac{\partial^2 T}{\partial Y^2} \right)_{ZX} + \left(\frac{\partial^2 T}{\partial Z^2} \right)_{XY} \right],$$

$$\frac{\partial T}{\partial t} = \frac{k}{C_p \varrho} \left[\left(\frac{\partial^2 T}{\partial X^2} \right)_{YZ} + \left(\frac{\partial^2 T}{\partial Y^2} \right)_{ZX} + \left(\frac{\partial^2 T}{\partial Z^2} \right)_{XY} \right],$$

$\dfrac{k}{C_p \varrho}$ = thermal diffusivity of the material.

The above final equation cannot be solved directly since temperature is a function of both distance and time.

Use of the Laplace transform

The Laplace transform involves transforming the equation so that only the transform of T with respect to time is used in place of T. The equation then involves only the Laplace transform θ and distance X. The Laplace transform is defined as

$$\theta = \int_0^\infty T \mathrm{e}^{-pt} \, dt, \tag{1}$$

where p is a parameter; θ is now a function of T and t, but no longer of X.

The basic differential equation to be solved is

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial X^2} \qquad (D = \text{thermal diffusivity}),$$

then this becomes

$$\frac{\partial^2 T}{\partial X^2} = \frac{\partial^2 \theta}{\partial X^2}$$

$$\frac{\partial \theta}{\partial t} = \int_0^\infty \frac{\partial T}{\partial t} \mathrm{e}^{-pt}\, dt \qquad \text{[from eqn. (1)]},$$

$$\frac{\partial \theta}{\partial t} = \left[T\mathrm{e}^{-pt}\right]_0^\infty + p\int_0^\infty \mathrm{e}^{-pt}\, T\, dt = -T_{t=0} + p\theta.$$

Since $\dfrac{\partial \theta}{\partial t} = D\dfrac{\partial^2 T}{\partial X^2}$,

$$p\theta - T_{t=0} = D\frac{\partial^2 \theta}{\partial X^2}.$$

Thus
$$\frac{\partial^2 \theta}{\partial X^2} - \frac{p\theta}{D} - \frac{T_{t=0}}{D} = 0. \tag{2}$$

If the temperature is *initially* constant, $T_{t=0}$ = constant. The above equation is of standard form, viz.

$$\frac{\partial^2 X}{\partial Y^2} - AX + K = 0,$$

and a solution can be obtained by the following method.

Let
$$\frac{\partial^2 \theta}{\partial X^2} - \frac{p\theta}{D} = 0. \tag{2a}$$

Initially, try
$$\theta = \mathrm{e}^{mX},$$

thus
$$\frac{\partial \theta}{\partial X} = m\,\mathrm{e}^{mX},$$

so
$$\frac{\partial^2 \theta}{\partial X^2} = m^2\,\mathrm{e}^{mX}.$$

This satisfies eqn. (2a) if and only if

$$m^2 \, e^{mX} - \frac{p \, e^{mX}}{D} = 0.$$

Therefore $m = \pm\sqrt{\left(\frac{p}{D}\right)}$.

The two solutions are thus

$$A \, e^{+\sqrt{(p/D)}X} \quad \text{and} \quad B \, e^{-\sqrt{(p/D)}X}.$$

Since the third term in eqn. (2) is a constant, then it will be assumed that $\theta = \alpha$ (constant).

This satisfies eqn. (2) if

$$-\frac{p}{D}\alpha = K = -\frac{T_{t=0}}{D}.$$

Therefore

$$\alpha = \frac{T_{t=0}}{p}.$$

Hence the general solution to eqn. (2) is

$$\theta = A \, e^{\sqrt{(p/D)}X} + B \, e^{-\sqrt{(p/D)}X} + T_{t=0}p^{-1}. \tag{3}$$

Thus

$$\frac{\partial\theta}{\partial X} = A\frac{p}{D} \, e^{\sqrt{(p/D)}X} - B\frac{p}{D} \, e^{-\sqrt{(p/D)}X}. \tag{4}$$

The temperature T which corresponds to this transform θ may be found by reference to the Laplace transform, tables of which may be found in Carslaw and Jaegar. Equation (4) contains two constants, A and B, and these have to be found using the boundary conditions applicable to the particular problem under investigation because they will in general involve the parameter p. This can only be carried out for a comparatively simple mathematical model, such as the unidirectional flow of heat through a body with plane parallel surfaces. If the distance apart is L, the heat flow at right angles to the surfaces, and the temperature initially constant at zero throughout the material, then the boundary conditions applicable will be:

$$t = 0; \qquad T = 0.$$

$$t > 0; \qquad T = T' \quad \text{when} \quad X = 0.$$

$$t > 0; \qquad \frac{\partial T}{\partial X} = 0 \quad \text{when} \quad X = L.$$

The exact mathematical solution of problems of unsteady heat transfer are difficult and often impossible. Because of this, other methods have been evolved.

In the above case it can be shown (Coulson and Richardson) that

$$T = \sum_{N=0}^{N=\infty} (-1)^N T' \left[\operatorname{erfc} \frac{2LN+X}{2\sqrt{(D\cdot T)}} + \operatorname{erfc} \frac{2(N+1)L-X}{2\sqrt{(D\cdot t)}} \right].$$

Values of erfc X are found in mathematical tables such as are included in Carslaw and Jaegar.

The finite difference method (Schmidt's method)

The variation of temperature T in any particular direction can be expressed by means of a power series:

$$T = (a_0 + a_1X + a_2X^2 + a_3X^3 \ldots) \text{ in the } X\text{-direction.}$$

By using Maclaurin's theorem, T can be expressed at any distance from point O in terms of the differential coefficients of $T = f(X)$ at $X = 0$.

$$T_n = an$$

$$\left(\frac{\partial T}{\partial X}\right)_0 = 1\times a_1, \qquad \text{therefore} \quad a_1 = \left(\frac{\partial T}{\partial X}\right)_0,$$

$$\left(\frac{\partial^2 T}{\partial X^2}\right) = 1\times 2\times a_2, \qquad \text{therefore} \quad a_2 = \left(\frac{\partial^2 T}{\partial X^2}\right)_{1\times 2},$$

$$\left(\frac{\partial^3 T}{\partial X^3}\right) = 1\times 2\times 3\times a_3, \qquad \text{therefore} \quad a_3 = \left(\frac{\partial^3 T}{\partial X^3}\right)_{1\times 2\times 3}.$$

Hence the variation of T in the X-direction is given by the expression

$$T = T_0 + \left(\frac{\partial T}{\partial X}\right)\frac{X}{1} + \left(\frac{\partial^2 T}{\partial X^2}\right)\frac{X^2}{1\times 2} + \left(\frac{\partial^3 T}{\partial X^3}\right)\frac{X^3}{1\times 2\times 3} \cdots.$$

The values of temperature at distance $X = \Delta X$ and $X = -\Delta X$ from the point of origin (equal distances in either direction) are given by (Fig. 5.17):

$$T_1 = T_0 + \left(\frac{\partial T}{\partial X}\right)_0 \frac{\Delta X}{1} + \left(\frac{\partial^2 T}{\partial X^2}\right)_0 \frac{(\Delta X)^2}{1\times 2} + \left(\frac{\partial^3 T}{\partial X^3}\right)_0 \frac{(\Delta X)^3}{1\times 2\times 3} \cdots,$$

$$T_3 = T_0 - \left(\frac{\partial T}{\partial X}\right)_0 \frac{(\Delta X)}{1} + \left(\frac{\partial^2 T}{\partial X^2}\right)_0 \frac{(\Delta X)^2}{1\times 2} - \left(\frac{\partial^3 T}{\partial X^3}\right)_0 \frac{(\Delta X)^3}{1\times 2\times 3} \cdots.$$

Neglecting fourth-power and higher terms,

$$T_1 + T_3 = 2T_0 + (\Delta X)^2 \left(\frac{\partial^2 T}{\partial X^2}\right)_0,$$

$$\left(\frac{\partial^2 T}{\partial X^2}\right)_0 = \frac{T_1 + T_3 - T_0}{(\Delta X)^2}.$$

Similarly, the value of $(\partial T/\partial t)_0$ can be represented by a difference equation

$$\left(\frac{\partial T}{\partial t}\right)_0 = \frac{T_4 - T_0}{\Delta t}.$$

It was shown earlier that

$$\frac{\partial T}{\partial t} = D \cdot \frac{\partial^2 T}{\partial X^2} \quad \text{in the } X\text{-direction.}$$

Thus
$$\frac{T_4 - T_0}{\Delta t} = \frac{D(T_1 + T_3 - 2T_0)}{(\Delta X)^2},$$

$$T_4 - T_0 = \frac{2D\,\Delta t}{(\Delta X)^2}\left[\frac{T_1 + T_3}{2} - T_0\right] \qquad \text{(Fig. 5.18),}$$

$$\Delta T = \frac{T_{X=1} + T_{X=3}}{2} - T_{X=0}.$$

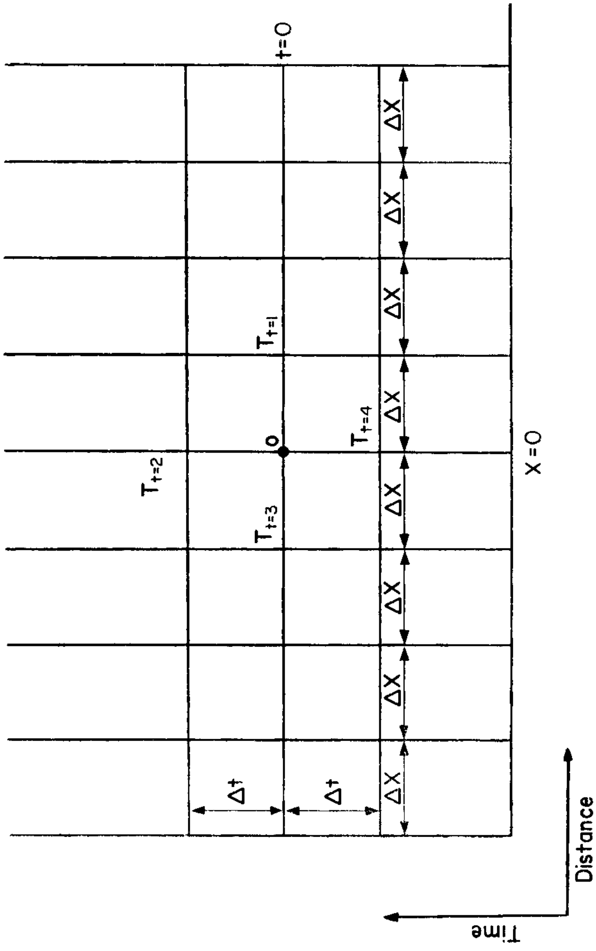


FIG. 5.17. Temperature distribution in solid body in distance and time

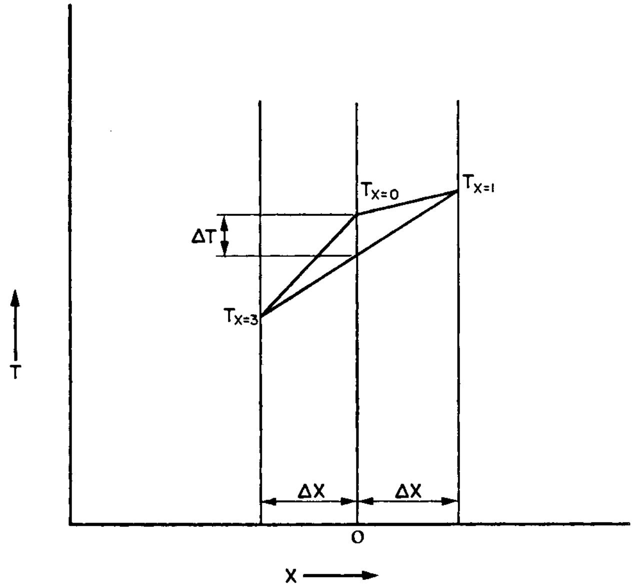


FIG. 5.18.

If
$$\frac{2D\cdot\varDelta t}{(\varDelta X)^2} = 1 \quad \text{or} \quad \varDelta t = \frac{(\varDelta X)^2}{2D}, \tag{5}$$

then
$$\varDelta T = T_{t=4} - T_{t=0}.$$

This is the increment of temperature at section X_0 during the time interval $\varDelta t = (T_4 - T_0)$.

Equation (5) can alway be satisfied by suitable selection of $\varDelta X$ and $\varDelta t$ using the thermal diffusivity of the material.

Values of T at time $t+\varDelta t$ at other sections of X can then be found.

In the case of a furnace wall, initially at ambient temperature, one face being suddenly raised and maintained at high temperature, this method will allow the temperature distribution at successive time intervals to be given by

$$\varDelta t = \frac{(\varDelta X)^2}{2D}.$$

To obtain the temperature at the junction of two planes within the wall at the end of any time interval a straight line is drawn connecting the temperatures of the two adjacent planes at the beginning of the interval.

Solution of unsteady heat transfer by the use of dimensionless groups

For a body of characteristic dimension L, initially at uniform temperature T_0 exposed suddenly to surroundings at T', the temperature T at any given time t at distance X from the central plane can be combined with the other variables to give the dimensionless groups:

For slabs	For cylinders or spheres
$Y = \dfrac{T' - T}{T' - T_0}$	$\dfrac{T' - T}{T' - T_0}$
$N = \dfrac{X}{L}$	$\dfrac{r}{R}$
$X = \dfrac{Dt}{L^2}$	$\dfrac{Dt}{R^2}$
$M = \dfrac{k}{hL}$	$\dfrac{k}{hR}$

h = heat transfer coefficient from surface.
L = semi-thickness of slab.
X = distance from centre of slab.
r = distance from axis of cylinder or centre of sphere.
D = thermal diffusivity material.
t = time.
R = radius of cylinder or sphere.

Curves connecting these groups have been drawn by a number of workers. They are reproduced as Figs. 5.19, 5.20, and

5.21. The numerical values of three of these groups must be found, and the fourth will then involve only one unknown which can be determined from the appropriate graph. Where there is no barrier to heat transfer at the surface, *h* becomes infinitely large and the fourth group becomes numerically equal to zero.

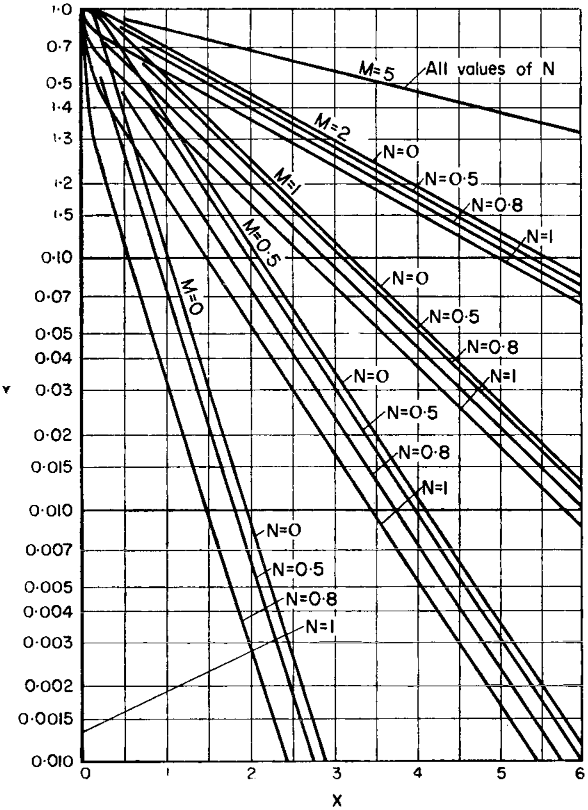


FIG. 5.19. Unsteady heat flow in slabs.
(Reproduced by permission from *Technical Data on Fuel*, 6th edn., 1961, edited by H. M. Spiers; British National Committee, World Power Conference; from an original of Special Report 14 (1936), Iron and Steel Institute.)

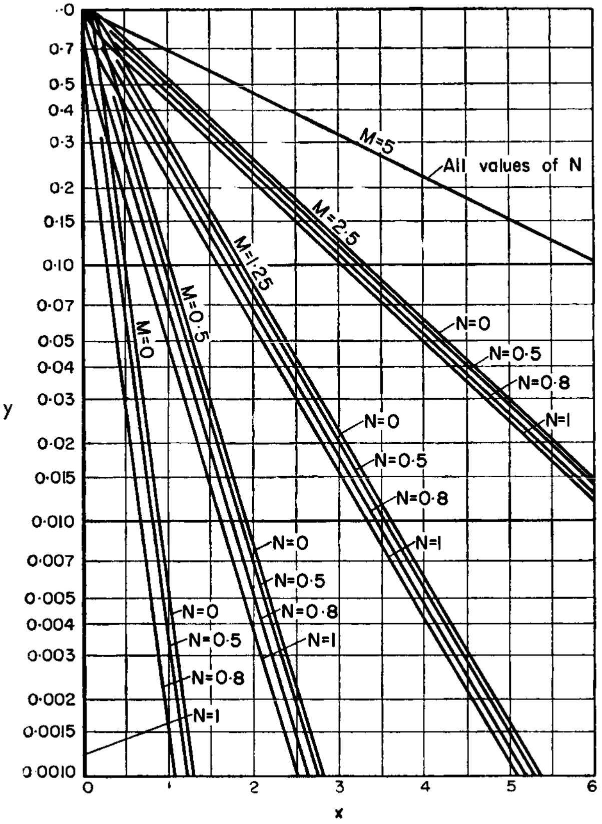


FIG. 5.20. Unsteady heat flow in cylinders.

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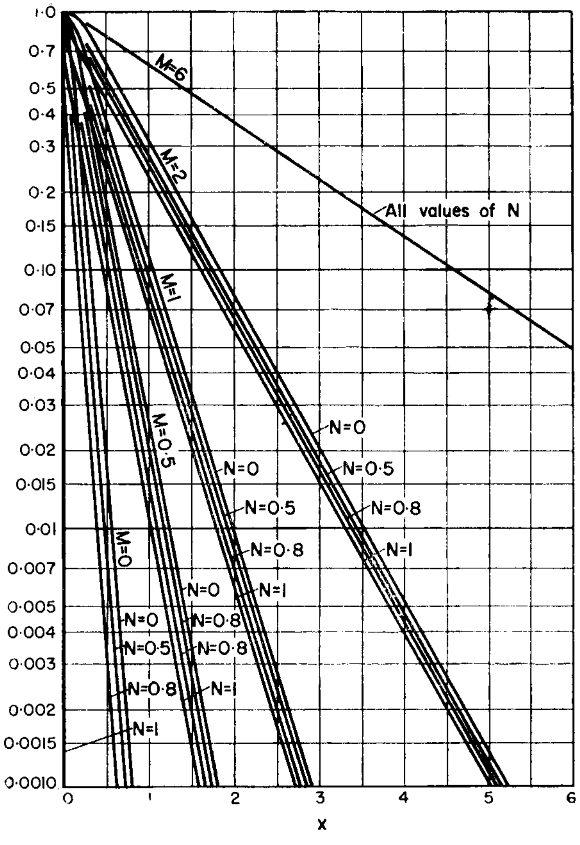

FIG. 5.21. Unsteady heat flow in spheres.

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5.9. HEAT LOST DURING A TEMPORARY SHUTDOWN OF A FURNACE

The thermal storage in the refractory walls can be found from the mean temperature of the brickwork.

T_1 = hot face temperature.
T_2 = cold face temperature.
d = thickness of refractory brickwork.
M = mass per m^2 = volume × density.

Heat stored $= M \times C_p \times \Delta T.$

$$Q_s = (d \times 1) \times \varrho \times C_p \times \left[\frac{(T_1+T_2)}{2} - T_{\text{ambient}}\right] J$$

$$\varrho = \text{density kg m}^{-3}$$

$$C_p = \text{specific heat J kg}^{-1}\ {}^{\circ}\text{C}^{-1}$$

When a furnace is shut off the heat loss per unit time will initially be the same as under steady-state conditions. This is given by

$$Q = \frac{\text{driving force}}{\text{thermal resistance}} \text{ J s}^{-1}.$$

After an infinite period the heat loss will be that which was originally stored in the brickwork.

If the rate of heat dissipation is assumed to continue at the initial rate then the time in which all the heat stored is lost, $t = Q_s/Q$.

The total heat lost in the shutdown period

$$Q_0 = Q_s[1 - e^{-t_0/t}] \text{ J m}^{-2},$$

where t_0 is the shutdown period.

5.10. HEAT TRANSFER CALCULATIONS

EXAMPLE 5.1

Calculate the time taken for a mild steel slab 0·6 m thick to attain a temperature of 1000°C when placed in a furnace at a temperature of 1200°C. It can be assumed that there is no barrier to heat transfer at the surface of the metal ($h = \infty$). Initial ambient temperature = 20°C; mean specific heat over the temperature range considered = 0·65 kJ/kg °C; thermal conductivity = 27·634 W/m °C, density = $7{\cdot}7 \times 10^3$ kg/m^3.

Method 1. By direct calculation (Fig. 5.22)

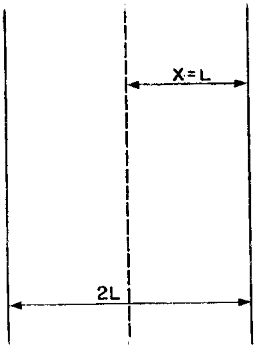

FIG. 5.22. Heat transfer in semi-infinite slabs

The appropriate equation to use is

$$T = \sum_{N=0}^{N=\infty} (-1)^N T' \left[\operatorname{erfc} \frac{2LN+X}{2\sqrt{(Dt)}} + \operatorname{erfc} \frac{2(N+1)L-X}{2\sqrt{(Dt)}}\right]$$

$$= \sum_{N=0}^{N=\infty} (-1)^N T' 2 \operatorname{erfc} \frac{(2N+1)L}{2\sqrt{(Dt)}} \qquad \text{(when } X = L\text{)}.$$

The first step is to make the initial temperature throughout equivalent to zero.

$$T = 1000-20 = 980°\text{C}.$$

$$T' = 1200-20 = 1180°\text{C}.$$

Therefore $$\frac{T}{T'} = \frac{980}{1180} = 0{\cdot}83.$$

The thermal diffusivity

$$D = \frac{k}{C_p\varrho} = \frac{27{\cdot}634}{0{\cdot}65\times 10^3\times 7{\cdot}7\times 10^3} = 5{\cdot}5\times 10^{-6}\ \text{m}^2\,\text{s}^{-1}.$$

Therefore $$\sqrt{D} = 0{\cdot}002352.$$

Thus $$\frac{0{\cdot}83}{2} = 0{\cdot}415 = \left[\operatorname{erfc}\frac{0{\cdot}3(2N+1)}{2\times 0{\cdot}002352t^{1/2}}\right]$$

Error-function X-values may be obtained from mathematical tables, such are included in Carslaw and Jaegar, and for the value of X given above $= 0{\cdot}591$.

Thus $0{\cdot}591 = 63{\cdot}73\ t^{-1/2}$ (when $N = 0$) as a first approximation.

Therefore $$t^{1/2} = \frac{63{\cdot}72}{0{\cdot}591}$$

$$t = 11{,}600\ \text{sec.}$$ *(Answer.)*

Method 2. By the use of dimensionless groups

$$\frac{T'-T}{T'-T_0} = \frac{1200-1000}{1200-20} = \frac{200}{1180} = 0{\cdot}17,$$

$$\frac{X}{L} = 0,$$

$$\frac{k}{hL} = 0,$$

$$\frac{Dt}{L^2} = \frac{0{\cdot}0000055\times t}{0{\cdot}3^2}.$$

Using the appropriate curves,

$$0{\cdot}75 = \frac{0{\cdot}0000055 \times t}{0{\cdot}3^2},$$

$$t = 12{,}200 \text{ sec.} \quad \textit{(Answer.)}$$

Method 3. By Schmidt's graphical method

The temperature at the end of each time interval is indicated by the number on Fig. 5.23.

At 0·3 m distance, the temperature gradient must be zero (at the central plane). Therefore the line connecting the temperature at the 0·24 and 0·36 m mark must be horizontal.

Therefore Δt time intervals = between 38 and 39 = 38·5.

$$\Delta t = \frac{(\Delta X)^2}{2D} = \frac{0{\cdot}0036}{2 \times D} = \frac{0{\cdot}0018}{0{\cdot}0000055} = 330 \text{ sec.}$$

Diffusivity $D = 5{\cdot}5 \times 10^{-6}$.

Therefore time = 330×38·5 = 11,550 sec. *(Answer.)*

EXAMPLE 5.2

An ingot of temperature 1000°C is placed inside a furnace, the walls of which are maintained at 1500°C. The emissivity of the ingot = 0·75, and it behaves as a grey body while the walls of the furnace behave as a black body.

Calculate the temperature indicated by a total radiation pyrometer when sighted upon the ingot.

The observed temperature as indicated by the instrument will be higher than the actual temperature due to reflection from the walls of the furnace.

Hence heat transferred from ingot to instrument

$$= \sigma e\left(T^4_{\text{ingot}} - T^4_{\text{air}}\right) + \sigma r\left(T^4_{\text{wall}} - T^4_{\text{air}}\right).$$

Heat transferred $= \sigma e T^4_{\text{ingot}} + \sigma r T^4_{\text{wall}}$.
(T^4_{air} is small compared with the wall and ingot temperatures and can be neglected.)

This must be equal to $= \sigma T^4_{\text{observed}}$.

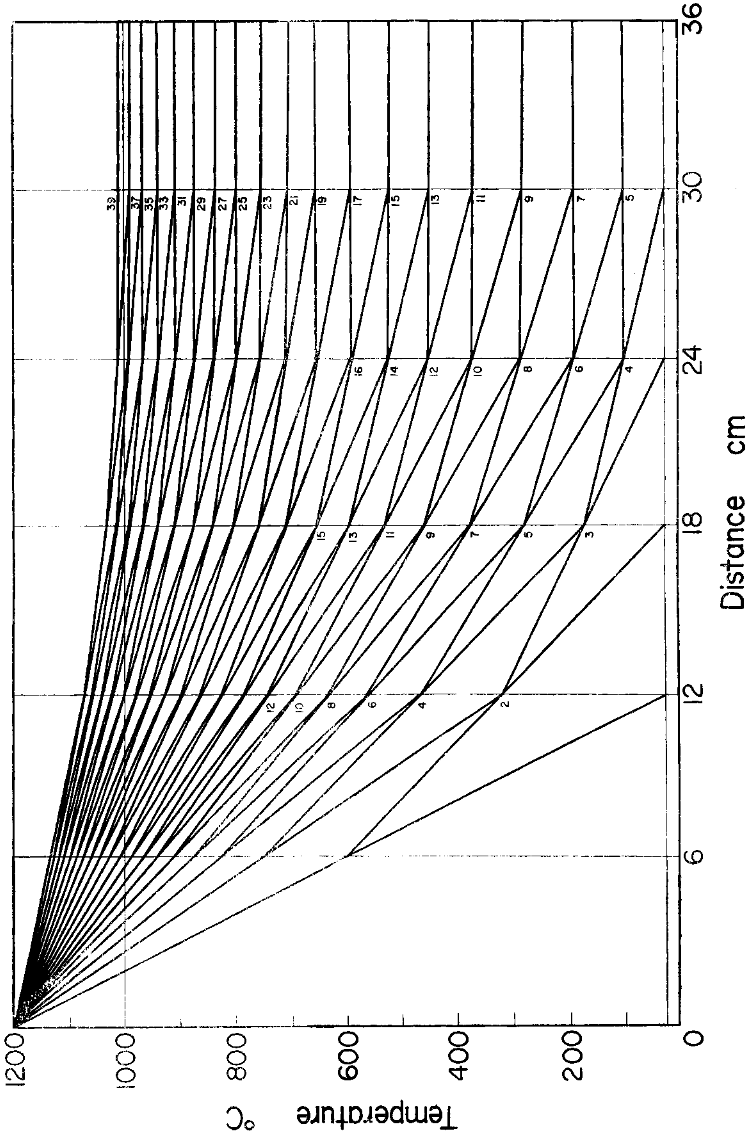

FIG. 5.23. Schmidt's method for unsteady heat transfer

Thus $T^4_{\text{observed}} = eT^4_{\text{ingot}} + rT^4_{\text{wall}}$.

$$T^4_{\text{observed}} = 0{\cdot}75 \times 1273^4 + 0{\cdot}25 \times 1773^4.$$

$$T_{\text{observed}} = 1450 \text{ K}.$$

Temperature indicated = 1177°C. *(Answer.)*

EXAMPLE 5.3

State the Stefan–Boltzmann law of radiation, and define "black-body" and "grey-body" conditions. Calculate the radiant heat transfer per metre run of pipe from a 150 mm diameter horizontal steam pipe carrying steam at 220°C across a room of average temperature 20°C. The surface temperature of the pipe may be taken to be the same as the steam, the emissivity of the pipe surface 0·8, and grey-body conditions applicable.

What other mode of heat transfer would apply in this case? Stefan–Boltzmann constant = $5{\cdot}67 \times 10^{-8}$ W/m² K⁴.

[HNC Metallurgy, Swansea College of Technology.]

$Q/s = 5{\cdot}67 \times 10^{-8} \times (473^4 - 293^4)$ J/m² = 1·936 kJ/m².

1 m run of pipe = $\pi \times 0{\cdot}150 \times 1$ m² in area = 0·471 m².

Loss per metre run pipe = $0{\cdot}471 \times 1{\cdot}936$ = 911·85 J/s.

(Answer.)

EXAMPLE 5.4

Calculate the heat loss per m² per unit time through a wall consisting of 225 mm of firebrick of thermal conductivity 1·442 W/m s °C with a backing of 100 mm of insulating material of thermal conductivity 0·144 W/m s °C when the hot temperature is 1500°C and the ambient temperature is 20°C.

The external surface coefficient of heat transfer can be taken as 14·195 W/m² °C.

[HNC, part II, Institution of Metallurgists, specimen question but with units changed.]

$$Q/s \text{ per m}^2 = \frac{\text{driving force}}{\text{thermal resistance}}$$

$$= (1500-20)\bigg/\frac{0{\cdot}255}{1{\cdot}442}+\frac{0{\cdot}10}{0{\cdot}144}+\frac{1}{14{\cdot}195}\,.$$

$$Q/s = \frac{1480}{0{\cdot}1776+0{\cdot}693+0{\cdot}07042} = \frac{1480}{0{\cdot}9410}$$

$$= 1573 \text{ J/s.} \quad \textit{(Answer.)}$$

EXAMPLE 5.5

A furnace is constructed of 9 in. of magnesite brick of mean thermal conductivity 15 lb-cal/ft^2 hr °C/in. It is desired to reduce the outside temperature to 400°C by using calcined diatomite insulating material of mean thermal conductivity 1·3 lb-cal/ft^2 hr °C/in. as an additional outer layer.

(a) What thickness of material will be necessary?

(b) From your knowledge of refractory materials and the temperature of the interface would this material be suitable?

The heat transfer coefficient from the outer surface to air at 20°C may be taken as 2·0 lb-cal/ft^2 hr °C, and the inner wall temperature 1500°C (Fig. 5.24).

[HND II, Metallurgy, Swansea College of Technology.]

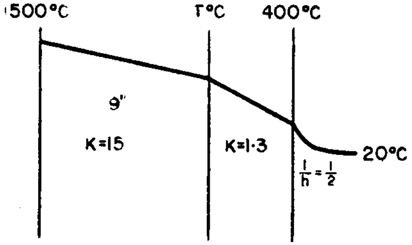

FIG. 5.24.

$$Q/\text{hr} = \frac{\text{driving force}}{\text{thermal resistance}}$$

$$= (1500-20)\Big/\frac{9}{15\times 1}+\frac{X}{1{\cdot}3\times 1}+\frac{1}{2{\cdot}0\times 1}$$

$$= (400-20)\Big/\frac{1}{2{\cdot}0\times 1} \qquad \text{(steady-state conditions)}$$

$$= 380\times 2 = 760 \text{ Btu/ft}^2.$$

$$760 = \frac{1480}{0{\cdot}6+(X/1{\cdot}3)+0{\cdot}5}$$

$$1480 = 760\times 1{\cdot}1+\frac{760{\cdot}X}{1{\cdot}3}$$

$$X = 1{\cdot}09 \text{ in.} \quad (\textit{Answer } \text{(a).})$$

$$Q/\text{ft}^2\text{ hr} = 760 = \frac{T_{\text{interface}}-400}{1{\cdot}09/(1{\cdot}3\times 1)}.$$

$$\Delta T = \frac{760\times 1{\cdot}09}{1{\cdot}3},$$

$$\Delta T = 637{\cdot}2^\circ\text{C},$$

$$T_{\text{interface}} = 637{\cdot}2+400 = 1037{\cdot}2^\circ\text{C}.$$

This temperature would normally be outside the range for using diatomite insulation. (*Answer* (b).)

EXAMPLE 5.6

Indicate what is meant by the terms (a) counter-current, and (b) parallel-flow heat exchange in recuperators.

A counter-current heat exchanger is required to heat 2500 lb of air per hour from 68°F to 1020°F. The heating gases enter the heat exchanger at 1660°F and at the rate of 3000 lb/hr. Calculate:

(a) the temperature at which the heated gases leave the heat exchanger;

(b) the approximate area of the surface of the heat exchanger given the overall heat transfer coefficient U is 5 Btu/ft^2 hr °F.

Mean specific heats: of air, 0·249, of heating gases, 0·36 Btu/lb °F (Fig. 5.25).

[HNC Metallurgy, Swansea College of Technology.]

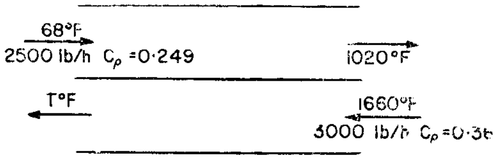

FIG. 5.25.

(a) Heat gained = heat lost.

$$2500 \times 0{\cdot}249 \times (1020-68) = (1660-T) \times 3000 \times 0{\cdot}36.$$

$$T = 1111{\cdot}3\text{°F}. \qquad (\textit{Answer}\ \text{(a)}.)$$

(b) $$Q = UA\theta_m.$$

θ_m = logarithmic mean temperature difference.

$$\theta_m = \frac{\theta_1-\theta_2}{2{\cdot}303 \log_{10}(\theta_1/\theta_2)}$$

$$\theta_1 = 1111{\cdot}3-68 = 1043{\cdot}3\text{°F}.$$

$$\theta_2 = 1660-1020 = 640\text{°F}.$$

$$\theta_m = \frac{1043{\cdot}3-640}{2{\cdot}303 \log_{10}(1043{\cdot}3/640)} = 825{\cdot}6\ \text{°F}.$$

$$\begin{aligned} Q/\text{hr} &= 2500 \times 0{\cdot}249 \times 953 \\ &= 3000 \times 0{\cdot}36 \times 648{\cdot}7 \\ &= 592{,}600\ \text{Btu}. \end{aligned}$$

$$592{,}600 = 5 \times A \times 825{\cdot}6.$$

$$\text{Area of heat exchanger surface} = \frac{592{,}600}{5\times 825{\cdot}6}$$

$$= 143{\cdot}5\ \text{ft}^2. \qquad (\textit{Answer}\ \text{(b).})$$

EXAMPLE 5.7

A horizontal pipe, of thermal conductivity 44·64 W/m °C is lagged with a 50 mm thickness of 85% magnesia insulation of thermal conductivity 0·0576 W/m °C. If the pipe is 150 mm internal diameter, 180 mm outside diameter, calculate the heat loss per hour per metre run of pipe when the mean temperature of the gases flowing in the pipe is 260°C. The ambient temperature of the air may be taken as 20°C, and the combined radiation and convective external heat transfer coefficient from the outside surface to air at 20°C may be taken as 19·8 W/m² °C. The inside film coefficient may be taken as 22·7 W/m² °C.

Estimate the external surface temperature of the lagging. [Fuel and Combustion Engineering, Swansea College of Technology.]

For the metal surface, since r_2/r_1 is less than 1·5, it will be of sufficient accuracy to use the arithmetic mean radius

$$r_a = \frac{75+90}{2} = 82{\cdot}5\ \text{mm}.$$

For the insulating layer r_m the logarithmic mean radius must be used.

$$r_m = \frac{r_2-r_1}{2{\cdot}303\log(r_2/r_1)} = \frac{140-90}{2{\cdot}303\log(140/90)} = 113\ \text{mm}.$$

$$\text{Heat lost per unit time} = \frac{\text{driving force}}{\text{thermal resistance}}$$

$$= 260-20\Big/\frac{1}{22{\cdot}7\times\pi\times 0{\cdot}15}+\frac{0{\cdot}15}{44{\cdot}64\times\pi\times 0{\cdot}165}$$

$$+\frac{0{\cdot}050}{0{\cdot}0576\times\pi\times 0{\cdot}226}+\frac{1}{19{\cdot}8\times\pi\times 0{\cdot}28}$$

$$= \frac{240}{0{\cdot}0935+0{\cdot}0006+1{\cdot}2225+0{\cdot}057} = \frac{240}{2{\cdot}373}$$

$$= 101 \text{ W/s m run pipe.}$$

$$\frac{\text{temperature drop over surface to air}}{\text{total temperature drop}}$$

$$= \frac{\text{thermal resistance surface to air}}{\text{total thermal resistance}}.$$

Thus

$$\Delta T = \frac{240 \times 0{\cdot}057}{2{\cdot}373}.$$

$$\Delta T = 5{\cdot}806^\circ\text{C}.$$

Surface temperature $= 20+5{\cdot}8 = 25{\cdot}8^\circ$C. *(Answer.)*

Alternatively,

$$Q/s = \frac{T \text{ surface to air}}{\text{resistance air film}}$$

$$101 = \frac{T}{0{\cdot}057}$$

$$T = 5{\cdot}8^\circ\text{C}.$$

EXAMPLE 5.8

(i) Explain carefully the advantages of counter-current flow operation over parallel-flow operation in heat exchangers. Indicate in either case the data required to establish the relationship between the temperature of the heat exchanging fluids.

(ii) Calculate the overall coefficient of heat transfer across a metallic partition 60 mm thick when the film heat transfer coefficients are respectively 22·68 and 56·7 W/m^2 °C. The thermal conductivity of the partition is 20·9 W/m °C.

[Institution of Metallurgists, Old Regulations, Fuels.]

$$\frac{1}{U} = \frac{1}{h_1} + \frac{x}{k} + \frac{1}{h_2}$$

$$= \frac{1}{22 \cdot 68} + \frac{0 \cdot 06}{20 \cdot 9} + \frac{1}{56 \cdot 7}$$

$$\frac{1}{U} = 0 \cdot 0441 + 0 \cdot 002865 + 0 \cdot 01764 = 0 \cdot 06457.$$

$$U = 15 \cdot 48 \text{ W/m °C.} \qquad \textit{(Answer)}$$

EXAMPLE 5.9

An electrically heated furnace of internal diameter 6 ft 6 in. is lined with a composite lining of 9 in. firebrick backed by 3 in. diatomaceous insulation, the heating elements being suspended vertically and not recessed into the firebrick. The coefficient of heat transfer from the external surface to the surroundings is 2·3 Btu/ft^2 hr °F and the properties of the firebrick and diatomaceous insulation are as follows:

Firebrick: thermal conductivity 7·5 Btu/ft^2 hr °F/in; density 120 lb/ft^3.

Diatomaceous insulation: thermal conductivity 1·5 Btu/ft^2 hr °F/in.; density 30 lb/ft^3.

For a furnace temperature of 1000°C calculate:

(i) the temperature of the firebrick–diatomaceous insulation interface;

(ii) the steady-state heat loss per unit height of furnace from the external surface to the surroundings, ignoring end effects. The temperature of the surroundings is 20°C.

$$\log_e = 2 \cdot 303 \log_{10}.$$

[Institution of Metallurgists, Part IV, AIM, Furnace Technology.]

$$Q/\text{hr} = \frac{\text{driving force}}{\text{thermal resistance}}$$

$$= \frac{(1000-20) \times 1 \cdot 8}{\text{thermal resistance}}.$$

The question is slightly ambiguous, but for this calculation lining is taken to mean on the inside of the furnace. The arrangement of bricks and insulation must therefore be as in Fig. 5.26.

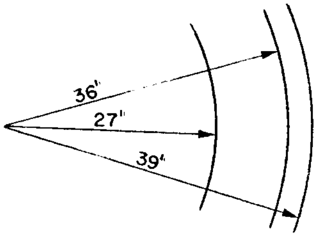

FIG. 5.26.

For firebrick

$$\frac{r_2}{r_1} = \frac{36}{27} = 1{\cdot}3.$$

For diatomite

$$\frac{r_2}{r_1} = 1{\cdot}1.$$

Therefore to calculate the area over which heat is being lost it will be sufficiently accurate to use the arithmetic mean radii in each case.

For firebrick,

$$\text{thermal resistance} = \frac{X}{2\pi r_a k 1}$$

$$= \frac{9}{2\pi \frac{31{\cdot}5}{12} 1} = 0{\cdot}0726.$$

For diatomaceous lining, thermal resistance

$$= \frac{3}{2\pi \frac{37{\cdot}5}{12} 1} = 0{\cdot}103.$$

Therefore

$$\text{(ii)}\quad Q/\text{hr} - \text{unit height} = \frac{(1000-20)\times 1{\cdot}8}{0{\cdot}073+0{\cdot}103+0{\cdot}021}$$

$$= \frac{980\times 1{\cdot}8}{0{\cdot}197} = 8954 \text{ Btu/hr} - \text{ unit height.} \quad \textit{(Answer.)}$$

$$9854 = \frac{(1000-T)\times 1{\cdot}8}{0{\cdot}073}$$

$$T = 1000-363{\cdot}1$$

$$= 636{\cdot}9°\text{F} \quad \text{the temperature of the interface required.}$$

EXAMPLE 5.10

(i) Determine the surface heat loss per foot run of an insulated pipe, external diameter 6 in. The thickness of the insulation is $2\frac{1}{2}$ in. The thermal conductivity of the insulation has a mean value of 0·44 Btu/ft^2 hr °F/in. The pipe surface is maintained at a temperature of 480°F, and the ambient temperature is 68°F. The coefficient of the external surface conductance of the insulation is 2·5 Btu/ft^2 hr °F (Fig. 5.27).

(ii) Write down but do not evaluate a numerical expression to determine the combined surface conductance due to convection and radiation to the atmosphere of a horizontal metal surface facing upwards, given the following data: area, 100 ft^2; temperature, 300°F; convection constant, 0·39; emissivity of surface, 0·82; ambient temperature, 70°F.

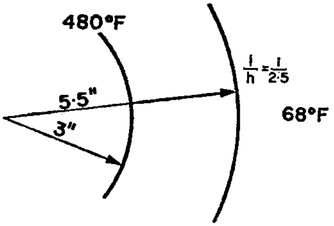

FIG. 5.27.

Stefan–Boltzmann constant = $1{\cdot}72\times10^{-9}$ Btu/ft² hr R⁴.
[City and Guilds Advanced, Fuel Plant Technology.]

$$Q/\text{hr}-\text{ft run} = \frac{\text{driving force}}{\text{thermal resistance}}.$$

$$Q/\text{hr} = (480-68)\Bigg/\left(\frac{2{\cdot}5}{0{\cdot}44\times2\times\pi\times r_m}+\frac{1}{2{\cdot}5\times2\times\pi\times\frac{5{\cdot}5}{12}\times1}\right)$$

$$r_m = \frac{5{\cdot}5-3}{2{\cdot}303\log_{10}\frac{5{\cdot}5}{3}}$$

$$= 4{\cdot}135 \text{ in.}$$

$$Q/\text{hr} = 412\Bigg/\left(\frac{2{\cdot}5}{4{\cdot}4\times2\times\pi\times\frac{4{\cdot}135}{12}}+0{\cdot}139\right)$$

$$= \frac{412}{2{\cdot}624+0{\cdot}139}.$$

Q/hr = 150 Btu/ft run. *(Answer.)*

(ii) $Q/\text{hr radiation} = 1{\cdot}72\times10^{-9}\times100\times0{\cdot}82\times(760^4-530^4)$.

$Q/\text{hr convection} = 0{\cdot}39\times100\times(300-70)^{1{\cdot}25}$.

EXAMPLE 5.11

A parallel-flow heat exchanger constructed of metallic tubes is used to preheat air for combustion by means of the waste heat in the products of combustion of a gaseous fuel. The waste gases enter the heat exchanger at a temperature of 1470°F. The air enters at 70°F, and is to be preheated to a temperature of 720°F (Fig. 5.28). The preheated air is used to burn the gas, the products from which enter the heat exchanger. The air–fuel gas ratio is 4·57, and the products of combustion occupy 5·27 vol. per volume of gas burned. The volume of excess air used in combustion is 10% of theoretical requirement. The mean specific heats of the waste gases and the air are respectively 0·0215 and 0·0197 ft³ °F at NTP. The fuel gas is burnt at the rate of 20,000 ft³/hr at NTP.

Determine (i) the temperature of the waste gases leaving the heat exchanger; (ii) the area of heat exchange surface required if the mean value of the heat transfer coefficients for the inner and outer surfaces of the tube elements are respectively 7 and 5 Btu/ft² hr °F (neglect additional air leakage or loss and external surface loss from the exchanger).

$$\log_e = 2{\cdot}303 \log_{10}.$$

[City and Guilds Advanced, Fuel Plant Technology.]

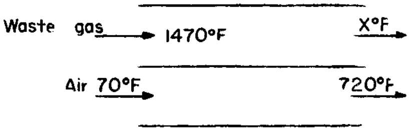

FIG. 5.28.

$$\frac{\text{Air}}{\text{Fuel}} = 4{\cdot}57.$$

$$\frac{\text{Products of combustion}}{\text{volume gas burnt}} = 5{\cdot}27 \text{ vol./vol.}$$

For each hour of operation

Gas burnt = 20,000 ft³.

Air to burn = 20,000×4·57 = 91,400 ft³.

Excess air = 10% = 9140 ft³.

Total air required = 91,400+9140 = 100,540 ft³.

Products of combustion = 5·27×20,000 = 105,400 ft³.

Heat gained by air = heat lost by waste gases:

$$100{,}540\times0{\cdot}0197\times(720-70) = 105{,}400\times0{\cdot}0215\times(1470-T).$$

$$T = 598{\cdot}2^\circ\text{F}. \quad (\textit{Answer}\ \text{(i).})$$

$$Q/\text{hr} = UA\theta_m,$$

$$\theta_m = \frac{\theta_1-\theta_2}{2{\cdot}303\log_{10}(\theta_1/\theta_2)},$$

$$\theta_1 = 1470-70 = 1400^\circ\text{F},$$

$$\theta_2 = 720-598{\cdot}2 = 121{\cdot}8^\circ\text{F},$$

$$\theta_m = \frac{1400-121{\cdot}8}{2{\cdot}303\log_{10}(1400/121{\cdot}8)}$$

$$= 523{\cdot}0^\circ\text{F}.$$

$$\frac{1}{U} = \frac{1}{h_1}+\frac{1}{h_2}$$

$$= \frac{1}{7}+\frac{1}{5}$$

$$U = 2{\cdot}91\ \text{Btu/ft}^2\ \text{hr}\ ^\circ\text{F}.$$

$$Q/\text{hr} = 100{,}540\times0{\cdot}0197\times650 = 1{,}286{,}000\ \text{Btu}$$

$$= 2{\cdot}91\times A\times523.$$

$$\text{Area} = 845{\cdot}7\ \text{ft}^2. \quad (\textit{Answer}\ \text{(ii).})$$

EXAMPLE 5.12

A 3 in. (o.d.) pipe carries high-pressure hot water at a temperature of 300°F. Determine the heat loss from 1 ft run of pipe for a period of 3000 hr (a) when the pipe is unlagged and (b) when the pipe is lagged with a 2 in. thickness of insulation having a thermal conductivity of 0·04 Btu/ft^2 hr °F/ft. The combined convective and radiative surface coefficients may be taken as 2·5 Btu/ft^2 hr °F. Ambient temperature 60°F.

[Institution of Metallurgists, Old Regulations, Fuels.]

Assume that the temperature inside pipe = 300°F. Since it is thin walled, the outside temperature can also be taken as 300°F.

$$Q/\text{hr} = \frac{\text{driving force}}{\text{thermal resistance}} = \frac{300-60}{(1/2{\cdot}5A)}$$

$$A = 2\times\pi\times\frac{1{\cdot}5}{12}\times 1 \quad \text{for 1 ft run.}$$

(a) For 3000 hr, $Q = \dfrac{(300-60)\times 3000\times 2{\cdot}5\times\pi\times 3}{12}$

$= 1{,}413{,}600$ Btu. (*Answer* (a).)

(b) With lagging

$$Q = (300-60)\times 3000 \Bigg/ \left(\frac{2/12}{0{\cdot}04\times 2\times\pi\times r_m\times 1} + \frac{1}{2{\cdot}5\times\pi\times(7/12)\times 1}\right)$$

$$r_m = \frac{3{\cdot}5-1{\cdot}5}{2{\cdot}303\log_{10}(3{\cdot}5/1{\cdot}5)} = 2{\cdot}36 \text{ in.}$$

$$Q = \frac{240\times 3000}{2{\cdot}741+0{\cdot}306}$$

$= 185{,}400$ Btu. (*Answer* (b).)

EXAMPLE 5.13

A steel pipe of 9 in. diameter and 0·375 in. wall thickness is covered with 2 in. lagging. The pipe carries steam at 500°F, and the surroundings are at 80°F. The heat transfer coefficient inside the pipe is 2500 Btu/ft^2 hr °F, and from the outer surface of the lagging 1·6 Btu/ft^2 hr °F. The thermal conductivity of steel is 25 Btu/ft hr °F and of the insulation 0·05 Btu/ft hr °F.

Calculate the surface temperature of the lagging (Fig. 5.29). [City and Guilds Advanced, Fuel Engineering.]

$r_1 = 4{\cdot}5$ in. $r_2 = 4{\cdot}875$ in. $r_3 = 6{\cdot}875$ in.

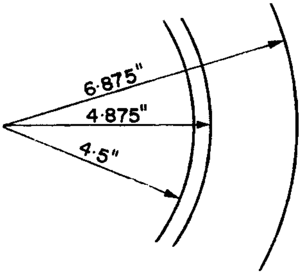

FIG. 5.29.

Thus r_2/r_1 and r_3/r_2 are both less than 1·5 and the arithmetic mean radii may be used to calculate areas.

For metal wall $\qquad r_a = \dfrac{4{\cdot}5+4{\cdot}875}{2} = 4{\cdot}687$ in.

For lagging $\qquad r_a = \dfrac{6{\cdot}875+4{\cdot}875}{2} = 5{\cdot}875$ in.

$$Q/\text{hr} = \frac{\text{driving force}}{\text{thermal resistance}}$$

$$(500-80)\Big/\left[\frac{1}{2500\pi(9/12)}+\frac{0{\cdot}375/12}{25\pi 2(4{\cdot}687/12)}+\frac{2/12}{0{\cdot}05\pi 2(5{\cdot}875/12)}\right.$$

$$\left.+\frac{1}{1{\cdot}6\pi 2(6{\cdot}875/12)}\right] = \frac{420}{0{\cdot}00017+0{\cdot}00012+1{\cdot}083+0{\cdot}173}$$

$$= \frac{420}{1{\cdot}256}\,.$$

It is evident that the first two terms may be neglected.

$$Q/\text{hr} = 334{\cdot}5 \text{ Btu/ft run.}$$

$$Q/\text{hr} = 334{\cdot}5 = \frac{T_{\text{surface}}-80}{0{\cdot}173}\,.$$

$$\Delta T = 57{\cdot}8\,^\circ\text{F}.$$

Therefore temperature surface = 57·8+80 = 137·8°F. *(Answer.)*

EXAMPLE 5.14

Give an account of the application of dimensionless groups in the problems of convective heat transfer.

(i) Determine the average film coefficient of heat transfer in the case of a gas flowing at a mass rate of 650 kg/hr through a parallel nest of fifty tubes, each 25 mm i.d., immersed in a medium at 235 °C, the gas entering at 25°C.

(ii) Show how the overall heat transfer coefficient and further information required is used to predict the performance of the recuperator.

$$Nu = 0{\cdot}023\, Re^{0{\cdot}8}\, Pr^{0{\cdot}33}.$$

Specific heat at constant pressure = 1·06 kJ/kg °C.
Thermal conductivity = 0·0277 W/m °C.
Density = 1·121 kg/m^3.
Viscosity = 0·0227 mN s/m^2.

[City and Guilds Advanced, Fuel Plant Technology; units changed.]

$$\text{Mass rate per tube} = \frac{650}{50\times 3600} = \text{kg/s}.$$

$$\text{Mass rate of flow } G = UA\varrho,$$

$$G = \frac{U\pi d^2\varrho}{4},$$

$$\frac{4G}{\pi d} = U d\varrho.$$

$$Re = \frac{U d\varrho}{\mu} = \frac{4G}{\pi d\mu},$$

$$(Re)^{0{\cdot}8} = \left(\frac{4\times 0{\cdot}00361}{\pi\times 0{\cdot}025\times 0{\cdot}0227\times 10^{-3}}\right)^{0{\cdot}8}$$

$$= 1345.$$

$$(Pr)^{0\cdot33} = \left(\frac{C_p\mu}{k}\right)^{0\cdot33}$$

$$= \left(\frac{1\cdot06\times1000\times0\cdot0227\times10^{-3}}{0\cdot0277}\right)^{0\cdot33}$$

$$= 0\cdot8837.$$

$$\frac{hd}{k} = 0\cdot023\times1345\times0\cdot8837,$$

$$h = \frac{0\cdot023\times1345\times0\cdot8837\times0\cdot0277}{0\cdot025},$$

$$h = 30\cdot16 \text{ W/m °C.} \qquad \textit{(Answer.)}$$

EXAMPLE 5.15

A building is to be heated by means of radiators connected to a two-pipe riser LPHW system. It is required that the inside temperature be maintained at 60°F when the outside temperature is 30°F.

(a) Calculate the heat loss from the building from the following details:

Window area 600 ft^2 U value	1·0 Btu/ft^2 hr °F.
Wall area 1400 ft^2 U value	0·3 Btu/ft^2 hr °F.
Roof area 2400 ft^2 U value	0·33 Btu/ft^2 hr °F.
Floor area 1800 ft^2 U value	0·1 Btu/ft^2 hr °F.

Volume of building	30,000 ft^3.
Air change	2 per hr.
Density of air	0·076 lb/ft^3.
Specific heat of air	0·24 Btu/lb °F.

(b) Determine the amount of heating surface required if:

Boiler flow temperature = 180°F.
Boiler return temperature = 160°F.
Temperature drop through radiators = 20°F.
Radiator emission = 1·6 Btu/ft^2 hr °F.
Ignore pipe emission.

[Institution of Heating and Ventilating Engineers, Section B, Principles of Heating, Combustion and Air Conditioning.]

$$Q/\text{hr} = \frac{\text{driving force}}{\text{thermal resistance}} = \frac{\Delta T}{1/UA},$$

$$Q/\text{hr} = \Delta TUA.$$

$$\begin{aligned}
\text{Window loss} &= 30\times 1 \times 600 = 18{,}000 \text{ Btu}\\
\text{Roof loss} &= 30\times 0{\cdot}33\times 2400 = 23{,}760 \text{ Btu}\\
\text{Wall loss} &= 30\times 0{\cdot}3 \times 1400 = 12{,}600 \text{ Btu}\\
\text{Floor loss} &= 30\times 0{\cdot}1 \times 1800 = \underline{5{,}400} \text{ Btu}\\
& \qquad\qquad\qquad\qquad\quad \underline{59{,}760} \text{ Btu}
\end{aligned}$$

Weight of air removed per hour $= 30{,}000\times 2\times 0{\cdot}076$ lb.

Heat lost by air $= 30{,}000\times 2\times 0{\cdot}076\times 0{\cdot}24\times 30$ (mst)
$= 32{,}832$ Btu.

(a) Total heat loss per hour = 92,592 Btu. *(Answer.)*

Emission from radiators $= 1{\cdot}6\times 20 = 32$ Btu/ft^2 hr.

(b) Heating surface required $= \dfrac{92{,}592}{32} = 2893$ ft^2. *(Answer.)*

EXAMPLE 5.16

Describe the characteristics of the materials used in the construction of an arched furnace.

A furnace wall 345 mm thick is found to have an external temperature of 230°C when the inside temperature is 1100°C. Calculate:

(a) the heat loss under these conditions;

(b) the thickness of the insulation required to reduce the heat loss to 800 J/m^2 s and the external temperature to 80°C;

(c) the temperature of the interface.

Assume that the conductivities for firebrick and insulation are 1·44 W/m °C and 0·17 W/m °C respectively.

[Institution of Heating and Ventilating Engineers, Section C, Group V, Combustion Engineering; units changed.]

$$Q/\mathrm{s} = \frac{\text{driving force}}{\text{thermal resistance}}.$$

$$Q/\mathrm{s}-\mathrm{m}^2 = \frac{1100-230}{0{\cdot}345/1{\cdot}44} = \frac{870}{0{\cdot}24}$$

$$= 3625\ \mathrm{W}. \quad (\textit{Answer.})$$

$$800 = \frac{1100-80}{(0{\cdot}345/1{\cdot}44)+(x/0{\cdot}17)}$$

$$800 = \frac{1020}{0{\cdot}24+(x/0{\cdot}17)}$$

$$1020 = 800\times 0{\cdot}24+\frac{800x}{0{\cdot}27}$$

$$x = 0{\cdot}176\ \mathrm{m} = 176\ \mathrm{mm}. \quad (\textit{Answer.})$$

$$800 = \frac{1100-T}{0{\cdot}24} = 192^\circ\mathrm{C}.$$

$$800 = \frac{T-80}{(0{\cdot}176/0{\cdot}17)} = 828^\circ\mathrm{C}.$$

Interface temperature $T = 908^\circ\mathrm{C}$. *(Answer).*

BIBLIOGRAPHY

CARSLAW, H. S. and JAEGAR, J. C., *Conduction of Heat in Solids*, Oxford University Press, 1947.

COULSON, J. M. and RICHARDSON, J. F., *Chemical Engineering*, vol. 1, Pergamon Press, 1957.

FISHENDON, M. and SAUNDERS, O. A., *An Introduction to Heat Transfer*, Oxford University Press, 1950.

FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.

HAYES, A. E. J., *Applied Thermodynamics*, Pergamon Press, 1963.

LUPTON, H. P., *Industrial Gas Engineering*, North Western Gas Board (Walter King Ltd., London), 1960.

LYLE, O., *The Efficient Use of Steam*, HMSO, 1947.
MCADAMS, E. H., *Heat Transmission*, 2nd edn., McGraw-Hill, 1942.
ROGERS, G. F. C. and MAYHEW, J. R., *Engineering Thermodynamics, Work and Heat Transfer*, Longmans, Green & Co. Ltd., 1959.
SMITH, H. J. and HARRIS, J. W., *Worked Examples in Engineering Thermodynamics*, 2nd edn., Macdonald & Co. Ltd., 1963.
SNEEDEN, J. B. O., *Applied Heat for Engineers*, Blackie & Sons Ltd., 1953.
SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.
THRING, M. W., *The Science of Flames and Furnaces*, 2nd edn., Chapman & Hall Ltd., London, 1962.
The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 6

Heat Balances

6.1. INTRODUCTION

A heat balance is a useful guide to the assessment of the fuel efficiency of a furnace or boiler installation. The performance of a heating unit can be expressed in a number of different ways. For example, a boiler rating may be expressed as pounds of fuel used per pound of steam raised, or as in open-hearth steel melting practice, as pounds of fuel per ton of steel produced. This method is useful when comparing different appliances under exactly the same conditions or when employing different furnaces performing the same operation, but under different conditions it is necessary to refer all performance data to some standard. The standard generally used is the ratio of weight of fuel which is actually used to carry out the operation, to that actually used in industrial practice. Alternatively, the quantity of heat theoretically required to perform any particular operation must be fixed thermodynamically and this can be compared with the actual energy used in practice. For example, 1 lb of water at 212°F and at 1 atm pressure requires a minimum of 970·1 Btu to produce 1 lb of steam at the same temperature and pressure. Thus the value of 970·1 Btu may be compared with the energy actually used to produce 1 lb of steam under these conditions, whatever form of energy is used. These methods of assessment are useful, and give an index of plant efficiency but they do not show where inefficiencies, if any, occur. This is best shown by conducting a detailed breakdown of the total energy entering and leaving the system in the form of a heat balance. Heat balances are constructed on the basis of some unit, which may be one hour or the time necessary to complete one cycle of operation, or upon a unit of material manufactured or processed.

A heat balance will consist essentially of the following information.

Input

(a) Chemical energy of the fuel. Here it must be decided whether to use the net or gross calorific value of the fuel. If the gross value is used then unless the latent heat of condensation of steam can be utilized the efficiency of the process will be lower than if the net value is used.

(b) The sensible heat of the fuel above some datum level, usually taken as room temperature or STP.

(c) The sensible heat of the combustion medium, also at STP.

(d) The sensible heat of the material being processed, also at STP. In boilers charged with fuel at room temperature this will be small, but in steel-melting furnaces receiving hot metal charges this energy represents a significant contribution to the input.

(e) Electrical or mechanical energy supplied to such auxiliaries as pumps and fans, to the grate in chain grate stokers, and to the compressor in an air atomized liquid fuel.

(f) The heat evolved during any exothermic reactions.

Output

(a) The total heat content of the material which leaves the appliance. This includes both sensible and latent heat. Once again it is taken above an arbitrary datum level (STP).

(b) The total heat content of all the combustion products. If waste gases are used to preheat air and/or fuel, their heat content must be taken at the final point before discharge into the atmosphere. This will include sensible heat of the gases; any undeveloped heat such as the presence of unburned carbon, carbon monoxide, hydrogen, etc., leaving the system. It also normally includes the latent heat in the steam. If, however, the net calorific value of the fuel is used, then the latent heat from the combustion of hydrogen in the fuel should not be debited here, as it has already been considered "not available".

(c) Heat losses from the structure by conduction through the walls and then by convection and radiation to surroundings.

(d) Heat which is stored in the structure of the furnace.

(e) The heat absorbed in any endothermic reactions.

(f) Losses in cinder when burning solid fuel, cooling water losses such as occur when cooling water is used to prolong the life of refractories. Losses by radiation through open doors, and in bogies, etc., in conveyor-type furnaces.

The actual construction of a heat balance for any plant is difficult, and invariably many assumptions have to be made. Most balances are constructed by dividing the plant into sections and carrying out a detailed analysis of the heat requirements of each section.

6.2. SANKEY DIAGRAMS

The Sankey diagram is a graphical representation of a heat balance which indicates clearly the energy distribution. Sankey diagrams are based upon the assumption that it is possible to make a complete heat balance for all stages of a process; the heating medium arrives and leaves at each stage with a known energy content. A known amount of heat is given to the material being processed, and a known amount to the furnace structure. An outline diagram is then made of the system showing the various parts and their interconnection.

The flow of energy is then represented in the diagram as a stream flowing through the system. If the units of energy are expressed in terms of pounds of coal, cubic feet of gas, or gallons of oil per unit weight of product, then two furnaces producing the same product can be readily compared.

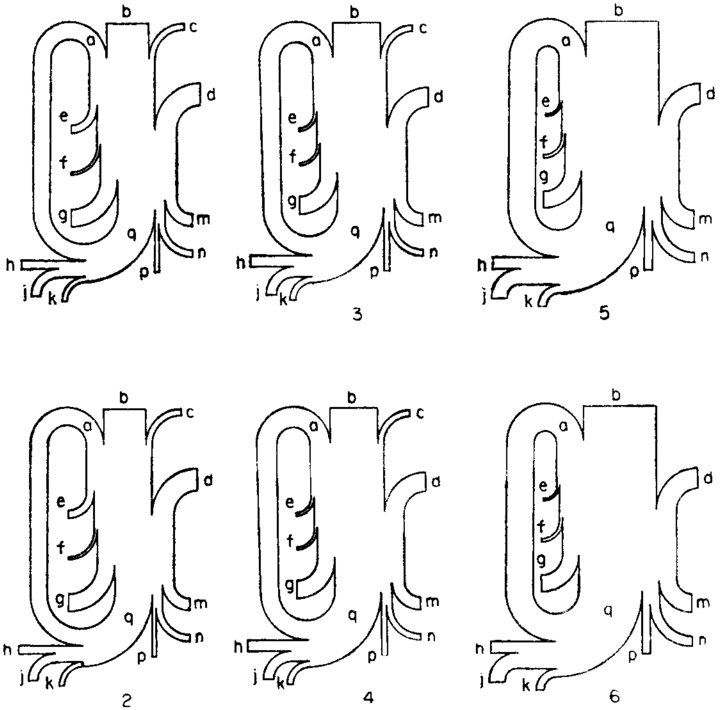

FIG. 6.1. Sankey diagrams for Table 9

The heat balance and Sankey diagrams for the production of steel by the open-hearth process are given in Tables 8, 9, and 10 and in Fig. 6.1; a heat balance and relevant Sankey diagram is included in Table 11 and in Fig. 6.2 for the production of iron in a blast furnace. These two examples clearly indicate the usefulness of both heat balances and Sankey diagrams.

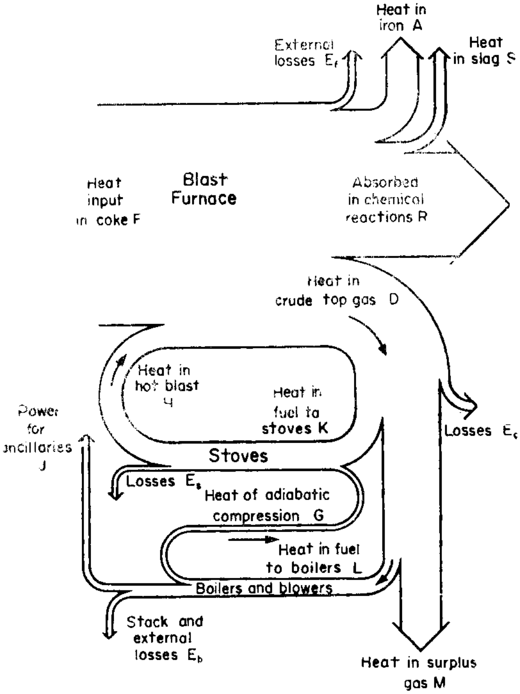

FIG. 6.2. Sankey diagrams for Table 11
(Fig. 6.2 and Table 11 reproduced from Leckie and Waring, *J. Inst. Fuel*, 1961, p. 414.)

TABLE 8. ELECTRICAL ENERGY CONSUMED PER CHARGE PER HOUR

	kW	Btu × 10^6
Waste gas fan	150	0·511
Air fan	15	0·051
Cooling water for oxygen guns	58	0·200
Other cooling water	54	0·185
Chargers, cranes, etc.	small	

TABLE 9. Btu$\times 10^6$/ton STEEL PRODUCED FOR THE PERIOD START CHARGE TO TAP

	60% hot metal		40% hot metal		Cold charge	
	without O_2	with O_2	without O_2	with O_2	without O_2	with O_2
Furnace laboratory						
In fuel	2·87	2·18	3·34	2·34	4·84	4·53
Air preheat	1·21	0·75	1·35	0·79	1·81	1·45
Hot metal	0·46	0·46	0·32	0·32	nil	nil
Bath reactions						
Fe FeO burnt	0·20	0·24	0·22	0·26	0·31	0·48
C CO	0·21	0·21	0·16	0·16	0·18	0·18
Si SiO_2	0·18	0·18	0·13	0·13	0·15	0·15
P P_2O_5	0·04	0·04	0·03	0·03	0·04	0·04
Mn MnO	0·06	0·06	0·05	0·05	0·04	0·04
Fe–FeO slag	0·14	0·14	0·14	0·14	0·14	0·14
$(CaO)_3P_2O_5$	0·03	0·03	0·02	0·02	0·03	0·03
$(CaO)_2SiO_2$	0·03	0·03	0·02	0·02	0·02	0·02
CO–CO_2	0·57	0·57	0·41	0·41	0·46	0·46
Burner steam	0·03	0·01	0·03	0·01	0·05	0·02
	6·03	4·90	6·22	4·68	8·07	7·54
Out steel	1·32	1·32	1·32	1·32	1·32	1·32
Slag	0·21	0·21	0·20	0·20	0·21	0·21
Waste gas	2·56	1·76	2·98	1·81	4·12	3·42
Water cooling	0·34	0·29	0·39	0·30	0·53	0·55
Surface loss	0·78	0·56	0·88	0·59	1·18	1·08
Losses to boxes and radiation	0·06	0·04	0·07	0·04	0·18	0·18
Reactions Fe_2O_3–Fe	0·43	0·33	0·12	0·06	0·15	0·15
$CaCO_3$–CaO	0·07	0·05	0·07	0·07	nil	nil
Unaccounted	0·26	0·34	0·19	0·29	0·38	0·63
	6·03	4·90	6·22	4·68	8·07	7·54
Waste-heat boilers						
In waste gas entering	1·26	0·88	1·41	0·94	1·90	1·73
Out gas leaving	0·61	0·43	0·68	0·46	0·93	0·86
Steam raised	0·61	0·45	0·72	0·48	0·98	0·89

TABLE 10. (Legend Sankey diagram Fig. 6.1.) Btu×10^6/ton STEEL PRODUCED FOR THE PERIOD START CHARGE TO TAP

		60% hot metal		40% hot metal		Cold charge	
		without O_2	with O_2	without O_2	with O_2	without O_2	with O_2
	Diagram number	1	2	3	4	5	6
a	Air preheat	1·21	0·75	1·35	0·75	1·81	1·45
b	Fuel and steam	2·90	2·19	3·37	2·35	4·89	4·55
c	Hot metal	0·46	0·46	0·32	0·32	—	—
d	Bath reactions	1·46	1·50	1·18	1·22	1·37	1·54
e	Decomposition of ore and stone	0·50	0·38	0·19	0·13	0·15	0·15
f	Heat in slag	0·21	0·21	0·20	0·20	0·21	0·21
g	Heat in steel	1·32	1·32	1·32	1·32	1·32	1·32
h	Steam raised	0·63	0·45	0·72	0·48	0·98	0·89
j	Waste gas leaving boiler	0·61	0·43	0·68	0·46	0·93	0·86
k	Heat loss from regenerator flues and boiler (by difference)	0·11	0·13	0·23	0·12	0·40	0·22
m	Surface losses and radiation	0·84	0·60	0·95	0·63	1·36	1·26
n	Water cooling losses	0·34	0·29	0·39	0·30	0·53	0·55
p	Losses unaccounted	0·26	0·41	0·19	0·29	0·38	0·63
q	Waste gas leaving furnace	2·56	1·76	2·98	1·81	4·12	3·42

(Tables 8, 9, and 10, and Fig. 6.1 reproduced from Mayorcas and McGregor, *J. Inst. Fuel*, 1961, p. 153.)

TABLE 11. HEAT BALANCE AND SUMMARY OF MAIN QUANTITIES FOR VARIOUS BURDEN WEIGHTS, FOR ONE TON OF IRON

Item shown on Fig. 6.2	Burden (cwt)	High-phosphorus iron				
		35	40	45	50	55
	Calculated coke consumption (cwt)	11·2	12·5	13·7	14·7	16·0
	Heat balance of furnace stack	Therms	Therms	Therms	Therms	Therms
F	Heat input in coke	155·2	172·5	189·2	203·7	221·2
H	Heat in hot blast	17·9	20·4	22·9	25·1	27·6
	Total input	173·1	192·9	212·1	228·8	248·8
	Heat in iron-sensible heat	12·5	12·5	12·5	12·5	12·5
	CV. of carbon therein	11·4	11·4	11·4	11·4	11·4
S	Heat in slag	8·1	12·8	17·6	22·3	27·1
R	Absorbed in reactions	71·9	71·7	71·9	71·9	71·9
D	Heat in top gas CV.	59·1	73·1	86·7	97·9	112·1
	sensible heat	5·0	5·6	6·3	6·9	7·5
E_f	External losses (approx.)	5·1	5·6	5·7	5·9	6·3
	Total out	173·1	192·9	212·1	228·9	248·8
	Heat balance of stoves					
K	Fuel gas	19·0	21·7	24·3	26·7	29·4
G	Adiabatic compression	1·8	2·0	2·2	2·5	2·7
	Total in	20·8	23·7	26·5	29·2	32·1
H	Heat in hot blast	17·9	20·4	22·9	25·1	27·6
E_s	Stack and external losses	2·9	3·3	3·6	4·1	4·5
	Total out	20·8	23·7	26·5	29·2	32·1

Table 11 (cont.)

Item shown on Fig. 6.2	Burden (cwt)	High-phosphorus iron				
		35	40	45	50	55
	Calculated coke consumption (cwt)	11·2	12·5	13·7	14·7	16·0
	Heat balance, boilers and blowers					
L	Heat required-blowing*	7·2	8·2	9·2	10·1	11·1
	ancillaries†	1·4	1·6	1·8	1·9	2·1
	Total in	8·6	9·8	11·0	12·0	13·2
G	Adiabatic compression	1·8	2·0	2·2	2·5	2·7
J	Ancillaries, as electricity†	0·4	0·4	0·4	0·5	0·5
E_b	Losses	6·4	7·4	8·4	9·0	10·0
	Total out	8·6	9·8	11·0	12·0	13·2
	Blast-furnace gas balance					
D	Total heat in gas	64·1	78·7	93·0	104·8	119·6
E_g	Losses in system, including sensible heat	8·0	9·3	10·6	11·8	13·1
K	To stoves	19·0	21·7	24·3	26·7	29·4
L	To boilers*	8·6	9·8	11·0	12·0	13·2
M	Surplus*	28·5	37·9	47·1	54·3	63·9
		64·1	78·7	93·0	104·8	119·6
Net energy used (item F – item M)		126·7	134·6	142·1	149·4	157·3

* The heat quantity L is that required for operating the furnace. In practice some of the spare gas M is likely to be burnt at the boilers so that its energy equivalent appears as process steam or electricity available for other departments of the works.

† The mean value of 13 kW/ton, referred to in the text (equivalent to 1·8 therms/ton fuel input to boilers at 25% efficiency of conversion), is taken as applicable to the mean burden weight. Heavier burdens will mean more use of power for hoists, slag handling, and so forth per ton of iron and this has been varied as ±20% at the two ends of the burden scale studied, rather less than the variation in burden weight.

Item shown on Fig. 6.2	Burden (cwt)	Low-phosphorus iron				
		35	40	45	50	55
	Calculated coke consumption (cwt)	10·9	12·1	13·3	14·3	15·5
	Heat balance of furnace stack	Therms	Therms	Therms	Therms	Therms
F	Heat input in coke	150·5	167·3	183·7	198·0	215·0
H	Heat in hot blast	17·0	19·5	21·9	24·1	26·6
	Total input	167·5	186·8	205·6	222·1	241·6
	{Heat in iron-sensible heat	12·5	12·5	12·5	12·5	12·5
	{CV. of carbon therein	13·0	13·0	13·0	13·0	13·0
S	Heat in slag	8·3	13·1	17·8	22·6	27·3
R	Absorbed in reactions	67·6	67·6	67·6	67·6	67·6
D	{Heat in top gas CV.	56·5	70·1	83·2	94·1	107·9
	{sensible heat	4·7	5·4	6·0	6·6	7·2
E_f	External losses (approx.)	4·6	5·1	5·5	5·7	6·1
	Total out	167·8	167·5	205·6	222·1	241·6
	Heat balance of stoves					
K	Fuel gas	18·1	20·7	23·3	25·6	28·3
G	Adiabatic compression	1·7	1·9	2·1	2·4	2·6
	Total in	19·8	22·6	25·4	28·0	30·9
H	Heat in hot blast	17·0	19·5	21·9	24·1	26·6
E_s	Stack and external losses	2·8	3·1	3·5	3·9	4·3
	Total out	19·8	22·6	25·4	28·0	30·9

Table 11 (cont.)

	Heat balance, boilers and blowers										
L	Heat required-blowing*	6·8		7·8		8·8		9·7		10·7	
	ancillaries†	1·4		1·6		1·8		1·9		2·1	
	Total in		8·2		9·4		10·6		11·6		12·8
G	Adiabatic compression	1·7		1·9		2·1		2·4		2·6	
J	Ancillaries, as electricity†	0·4		0·4		0·4		0·5		0·5	
E_b	Losses	6·1		7·1		8·1		8·7		9·7	
	Total out		8·2		9·4		10·6		11·6		12·8
	Blast-furnace gas balance										
D	Total heat in gas		61·2		75·5		89·2		100·7		115·1
E_g	Losses in system, including sensible heat	7·5		8·9		10·2		11·3		12·6	
K	To stoves	18·1		20·7		23·3		25·6		28·3	
L	To boilers*	8·2		9·4		10·6		11·6		12·8	
M	Surplus*	27·4		36·5		45·1		52·2		61·4	
			61·2		75·5		89·2		100·7		115·1
Net energy used (item F – item N)			123·1		130·8		138·6		145·8		153·6

* The heat quantity L is that required for operating the furnace. In practice some of the spare gas M is likely to be burnt at the boilers so that its energy equivalent appears as process steam or electricity available for other departments of the works.

† The mean value of 13 kW/ton, referred to in the text (equivalent to 1·8 therms/ton fuel input to boilers at 25% efficiency of conversion), is taken as applicable to the mean burden weight. Heavier burdens will mean more use of power for hoists, slag handling, and so forth per ton of iron and this has been varied as ±20% at the two ends of the burden scale studied, rather less than the variation in burden weight.

6.3. EXAMPLES

EXAMPLE 6.1

Determine the heat balance of a boiler installation, given the following data:

Coal composition, ash free basis; C, 88·5; H, 4·7; N, 1·5; O, 4·6%. As-used basis: moisture, 13·6; ash, 14·9%; gross CV, 10,880 Btu/lb. Fuel fired, 903 lb/hr. Actual evaporation, 5754 lb/hr. Steam: pressure, 81 lb/in^2 g, final temperature 325°F. Flue gases: temperature leaving boiler, 617°F; analysis, CO_2, 9·82; O_2, 9·48%.

Ashes and clinker, 197 lb/hr; percentage combustibles, 37·6. Grit emission: percentage of fuel fired, 2·88; percentage combustibles in grit, 51·2. Discuss the results.

Mean volumetric specific heat of products of combustion 0·205 Btu/°F ft^3 at NTP.

[City and Guilds Advanced, Fuel Plant Technology.]

The carbon content of 100 lb of coal is calculated from the analysis.

$$C = \frac{88{\cdot}5}{12} = 7{\cdot}37 \text{ lb-mol},$$

$$H_2 = \frac{4{\cdot}7}{2} = 2{\cdot}35 \text{ lb-mol},$$

$$\%\, CO_2 = 9{\cdot}82 = \frac{\text{amount carbon}}{\text{total}} \times 100.$$

Carbon used in as-fired coal

$$= \frac{\text{amount} \times (100 - 13{\cdot}6 - 14{\cdot}9)}{100}.$$

Total volume dry flue gas

$$= \frac{\text{amount} \times 0{\cdot}715 \times 359}{9{\cdot}82} \text{ ft}^3.$$

Carbon gasified:

Carbon available as-fired $= 7{\cdot}37 \times 0{\cdot}715 = 5{\cdot}27$ lb-mol/100 lb coal fed.

Feed/hr = 903 lb.

$$\text{Carbon available per 903 lb} = \frac{903 \times 5{\cdot}27}{100} = 47{\cdot}58 \text{ lb-mol.}$$

Carbon in ashes and clinker

$$= \frac{197 \times 0{\cdot}376 \times 0{\cdot}885}{12} \; (88{\cdot}5\% = \text{C in coal}) \text{ lb-mol.}$$

$$= 5{\cdot}46 \text{ lb-mol.}$$

$$\text{Carbon lost in grit} = \frac{903 \times 2{\cdot}88 \times 51{\cdot}2 \times 0{\cdot}885}{12 \times 100 \times 100}$$

$$= 0{\cdot}982 \text{ lb-mol.}$$

$$\text{Carbon gasified per hour} = 47{\cdot}58 - (5{\cdot}46 + 0{\cdot}98)$$

$$= 41{\cdot}4 \text{ lb-mol.}$$

$$\text{Volume of dry flue gas} = \frac{41{\cdot}4 \times 100 \times 359}{9{\cdot}82}$$

$$= 150{,}400 \text{ ft}^3 \text{ at NTP.}$$

To this must be added the moisture present as such and from the combustion of hydrogen.

$$2\,H_2 + O_2 = 2\,H_2O.$$

In 100 lb as-fired coal, moisture $= 2{\cdot}35 \times 0{\cdot}715$ lb-mol.

$$\text{Per hour} = \frac{2{\cdot}35 \times 0{\cdot}715 \times 359 \times 903}{100} = 5448 \text{ ft}^3.$$

Moisture present as such per hour = 13·6 lb/100 lb

$$= \frac{13{\cdot}6}{18} \text{ lb-mol/100 lb}$$

$$= \frac{13{\cdot}6 \times 903 \times 359}{18 \times 100} \text{ ft}^3 \text{ at NTP}$$

$$= 2449 \text{ ft}^3 \text{ at NTP.}$$

Total volume products of combustion

$$= 150{,}400 + 5448 + 2449 \text{ ft}^3 \text{ at NTP.}$$

TABLE 12. PER HOUR

Heat available	Heat in steam	Heat losses
= 10,880 × 903 = 9,824,000 Btu	From steam tables = 1185 × 5754 = 6,819,000 Btu	As sensible heat = 158,297 × 0·0205 × (617 − 60) = 1,772,000 Btu As "unburned coal" = 197 × 0·376 × 10,880 = 806,000 Btu As "unburned fuel in grit" = 903 × 0·0288 × 0·512 × 10,880 = 144,900 Btu Total loss = 2,722,900 Btu

Heat input = 9,824,000 Btu.
Heat losses + heat in steam = 9,541,900 Btu.

EXAMPLE 6.2

A furnace hearth of dimensions 2 m × 4·5 m is used intermittently for firing a charge of stock to a minimum temperature of 920°C. The heating time is 8 h. When the stock is being heated with an average hearth loading of 58 kg/m^2 of hearth per hour the gaseous fuel is burned at the rate of 77 m^3/h at NTP. The composition of the gaseous fuel is CO, 10; CH_4, 24; H_2, 50; C_2H_4, 3%, remainder N_2. The temperature of the offtake gases when the charge is withdrawn is 700°C, the furnace being cold when started up. The average content of carbon dioxide of the dry exhaust gases is 10%.

Determine a heat balance of the furnace given the following data:

Mean specific heat of stock = 0·7 kJ/kg °C.
Mean volumetric specific heat of the exhaust gases = 1·34 kJ/m^3 °C at NTP.
Temperature of shop atmosphere = 20°C.
Gross CV of combustible gases, respectively = 12·7, 39·7, 12·8, 62·2 MJ/m^3 at NTP.
[Institution of Metallurgists, Old Regulations, Fuels; units changed.]

Heat taken up by the metal over 8 h = mass × specific heat × temperature range.

$$\text{Mass metal heated} = 58\times 8\times 4{\cdot}5\times 2 = 4176 \text{ kg.}$$

$$\text{Heat to metal} = 4176\times 0{\cdot}7\times(920-20) = 2630{\cdot}7 \text{ MJ.}$$

$$\begin{aligned}\text{Calorific value of gas} &= 0{\cdot}24\times 39{\cdot}7+0{\cdot}5\times 12{\cdot}8+0{\cdot}10\times\\ &\quad\times 12{\cdot}7+62{\cdot}2\times 0{\cdot}03\\ &= 9{\cdot}528+6{\cdot}4+0{\cdot}127+1{\cdot}866\\ &= 17{\cdot}921 \text{ MJ/m}^3.\end{aligned}$$

The combustion products from 100 m^3 are found:

$$CH_4+2\,O_2 = CO_2+2\,H_2O;$$
$$2\,H_2+O_2 = 2\,H_2O; \qquad 2\,CO+O_2 = 2\,CO_2;$$
$$C_2H_4+3\,O_2 = 2\,CO_2+2\,H_2O.$$

$$\text{Theoretical oxygen} = 48+25+5+9 = 87 \text{ m}^3.$$

$$\text{Air} = \frac{87\times 100}{21} = 424 \text{ m}^3.$$

$$\%\,CO_2 = \frac{\text{volume } CO_2}{\text{total volume}}\times 100.$$

$$CO_2 = 6+24+10 = 40 \text{ m}^3.$$

$$N_2 = 337(TA)+13 = 350 \text{ m}^3.$$

Excess air $= a$ m^3

$$10{\cdot}0 = \frac{40}{390+a}\times 100.$$

$$a = 10 \text{ m}^3 \text{ at NTP}.$$

Volume of $H_2O = 6+48+54 = 104$ m^3.

Total volume of flue gases per 100 m^3 gas burned

$$= 390+10+104 = 504 \text{ m}^3.$$

Volume products of combustion during 8 h period

$$= \frac{504\times 77\times 8}{100} = 3105 \text{ m}^3.$$

Heat lost $= 3105\times 1{\cdot}34\times(700-20) = 2830$ MJ.

Heat input	Heat output
Heat in gas $= 17{\cdot}92\times 77\times 8$ $= 11{,}026$ MJ	Heat to stock = 2630 MJ Heat lost in flue gases = 2830 MJ

EXAMPLE 6.3

Give an account of the laws of heat transmission applicable to the fire tubes of a waste heat boiler, citing the type of equations applicable to the calculation of the heat transmission and the pressure drop in the tubes.

In a trial of a waste heat boiler with superheater the following data relating to its average performance were obtained.

Actual rate evaporation, 4000 lb/hr.
Steam pressure 180 lb/in^2 g.
Rate of flow of furnace gases, 50,000 lb/hr.
Temperature of gases at inlet to boiler, 850°F.

Temperature of gases at outlet to boiler, 460°F.
Temperature of superheat of steam, 530°F.
Temperature of feed water, 190°F.

Derive a heat balance of the plant and discuss its implications.

Mean specific heat of the gases, 0·32 Btu/lb °F.

[City and Guilds Advanced, Fuel Plant Technology.]

Actual evaporation = 4000 lb/hr.

Steam pressure = 180 lb/in^2 g.

Heat in steam = 4000×1280 = 5,120,000 Btu.

Heat in feed water = $4000 \times (190-32) \times 1$ = 632,000 Btu.

Heat in gases = $50{,}000 \times 0{\cdot}32 \times (850-460)$ = 6,240,000 Btu.

Heat input	Heat output
Heat abstracted from gases = 6,240,000 Btu Heat in feed water = 632,000 Btu Total = 6,872,000 Btu	Heat in steam = 5,120,000 Btu

EXAMPLE 6.4

In commissioning an oil-fired continuous recuperative furnace, tests were carried out to determine its performance. The effective hearth was 9 m wide and 12 m long and the furnace consumed 20·3 l/s of oil when heating cold steel billets to rolling temperature at a rate of 250 kg/m^2 of hearth per hour.

The heat absorbed by the charge was 1050 kJ/kg and the gross and net CVs of the oil were 44·04 and 41·24 MJ/kg respectively; its specific gravity was 0·95.

The sensible heat content of the combustion products discharged from the furnace was 15·145 MJ/kg of oil burned; of this 3·495 MJ were usefully returned from the recuperator to the furnace in supplying preheated air to the burners. From the above data calculate:

(i) the total hourly rate of loss of heat from the furnace chamber;

(ii) the overall thermal efficiency of the furnace;

(iii) the increase in oil consumption necessary if the recuperator had been bypassed and no waste heat recovered (all other conditions stated above);

(iv) the effect of the recuperator on the cost of oil per tonne of steel heated, assuming the price of oil to be 1 p/l.

[Institution of Metallurgists, Part IV, Furnace Technology; units changed.]

Assuming that the furnace is in operation for 1 h:

Effective hearth area $= 9 \times 12 = 108$ m^2.

Weight of billets heated $= 250 \times 108 = 27{,}000$ kg.

Heat absorbed by charge $= 27{,}000 \times 1050 = 28{,}350{,}000$ kJ.

Oil consumed $= 20{\cdot}3$ l/s.

Oil consumed per h $= 20{\cdot}3 \times 3600 \times 10^{-3} \times 1000 \times 0{\cdot}95$ kg/h (density of water $= 1000$ kg/m^3).

Oil consumed per h $= 694{\cdot}26$ kg/h.

Heat released by fuel $= 41{\cdot}24 \times 694{\cdot}26 = 28{,}631{,}000$ kJ.

The heat available in the furnace will be the net calorific value of the fuel plus any sensible heat in the oil and preheated air less the heat carried away in combustion products and radiation losses.

Sensible heat lost in combustion products

$$= (15{\cdot}145 - 3{\cdot}495) \times 694{\cdot}26 = 8{,}088{,}169 \text{ kJ}.$$

Heat returned to furnace in preheated air $= 3{\cdot}495 \times 694{\cdot}26 = 2{,}426{,}440$ kJ.

In 1 hour:

Heat input to furnace	Heat output
Heat in fuel = 28,631,000 kJ	Heat to charge = 28,350,000 kJ
Heat as sensible heat in air = 2,426,440 kJ	Heat lost as sensible heat = 8,088,169 kJ
Total = 31,057,440 kJ	Total = 36,438,169 kJ

It would appear from these figures that energy may be "created". However, a more logical explanation is that the data given have not been complete, and there is probably an exothermic heat release in the furnace.

Hourly rate of heat loss = 8088 MJ. (*Answer* (i).)

Efficiency is a most difficult parameter to define for metallurgical furnaces. Overall thermal efficiency could be expressed as the heat entering the charge as a percentage of the gross potential heat in the fuel.

In 1 h heat to charge $= 694{\cdot}26 \times 44{\cdot}04 = 30{,}549$ MJ.

$$\text{Efficiency} = \frac{28{,}350}{30{,}549} \times 100 = 92\%. \quad (\textit{Answer}\ \text{(ii).})$$

Heat saved as a result of recuperation = 2426·4 MJ.
Net CV = 41·24 MJ/kg.

$$\text{Increase in oil consumption} = \frac{2426{\cdot}4}{41{\cdot}24} = 58\ \text{kg}.$$

(*Answer* (iii).)

Since oil costs 1 new penny per litre then the saving

$$= 1 \times 10^{-3} \times 10^{3} \times 0{\cdot}95 = 0{\cdot}95\ \text{new pence/kg oil}$$

$$= \frac{0{\cdot}95 \times 58 \times 10^{3}}{27{,}000}\ \text{new pence per tonne}$$

= 2·1 new pence per tonne of metal heated. (*Answer*(iv).)

BIBLIOGRAPHY

FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.

HAYES, A. E. J., *Applied Thermodynamics*, Pergamon Press, 1963.

HIMUS, G. W., *The Elements of Fuel Technology*, 2nd edn., L. Hill, 1958.

LECKIE, A. H. and WARING, F. L., Fuel requirements in blast-furnace operation, *J. Inst. Fuel*, 1961, p. 414.

LYLE, O., *The Efficient Use of Steam*, HMSO, 1947.

MAYORCAS, R. and MCGREGOR, I. H., Fuel and energy required for steelmaking in open-hearth furnace, *J. Inst. Fuel*, 1961, p. 153.

SMITH, H. J., and HARRIS, J. W., *Worked Examples in Engineering Thermodynamics*, 2nd edn., MacDonald & Co., Ltd., 1963.

SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference, 1962.

THRING, M. W., *The Science of Flames and Furnaces*, 2nd edn., Chapman & Hall Ltd., London, 1962.

The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 7

Furnace Aerodynamics

7.1. INTRODUCTION

In the chapter on heat transfer reference was made to the analogy of heat flow with the flow of electrical current in a circuit. In a similar way an analogy can be made for the flow of gases in a furnace.

Frictional resistance due to walls and changes of section	Electrical resistance
The atmosphere	Earth
Furnace flues	Electrical circuit
Volume of gases flowing	Quantity of electric current flowing
Pressure inside furnace relative to atmosphere	Potential relative to earth
Aeromotive force generators; chimneys, fans, etc.	Batteries, electric generators

This analogy is very useful in understanding many problems relating to furnace aerodynamics.

7.2. FRICTIONAL RESISTANCE TO FLOW

A fluid is unable to support a shearing stress and, if such a force is applied to a liquid, layers in the fluid move with respect to one another. This results in a velocity gradient at right angles to the direction of flow.

If the velocity is V at a distance X from the solid surface, the velocity gradient is dV/dX. If the flow is steady there is no acceleration and an equal and opposite drag force will be exerted. This drag is a force per unit area of the layer, and is equal to $\lambda(dV/dX)$; λ = constant for each fluid and is referred to as the coefficient of viscosity of the fluid.

$$\lambda = \frac{(\text{force})\times L}{(\text{area})(\text{velocity})}.$$

$$\lambda = ML^{-1}T^{-1}.$$

The CGS unit of dynamic viscosity is referred to as the poise. The ratio of dynamic viscosity to density is known as kinematic viscosity. The kinematic viscosity in CGS units is known as the Stoke. The name Poiseuille (Pl) has been given to the SI unit of dynamic viscosity (Ns/m^2). [10^{-3} Ns/m^2 = 1 cPoise and 10^{-6} m^2/s = 1 cStoke.]

Stream-line and turbulent flow

Stream-line flow can be defined as the flow of fluid such that each path in the fluid is steady, there is no motion at right angles to the direction of flow. Turbulent flow occurs when the motion consists of changing eddies, i.e. there is bulk motion at right angles to the direction of flow.

Sir Osborne Reynolds found that the velocity at which the flow changed from stream-line to turbulent flow depended upon the fluid density, fluid viscosity, and diameter of the tube in which flow was taking place.

$$\begin{aligned}(\text{velocity}) &= k\times(\text{density})^a\times(\text{viscosity})^b\times(\text{diameter})^c,\\ \mathrm{LT}^{-1} &= k\times M^aL^{-3a}\times M^bL^{-b}T^{-b}\times L^c,\\ \mathrm{LT}^{-1} &= kM^{a+b}L^{c-b-3a}T^{-b}.\end{aligned}$$

Using the technique of dimensional analysis

$$\begin{aligned}a+b &= 0,\\ c-b-3a &= 1,\\ b &= 1.\end{aligned}$$

Thus $\quad a = -1, \quad b = 1, \quad c = -1.$

$$\text{Thus critical velocity} = \frac{k \times \text{viscosity}}{\text{density} \times \text{diameter}}.$$

$$k = \frac{\text{velocity} \times \text{density} \times \text{diameter}}{\text{viscosity}}$$

$$= \text{Reynolds number } (Re).$$

It was found that below values of 2000 the flow was stream-line, and above about 2500 it was turbulent.

Bernoulli's theorem

When an incompressible fluid is in steady flow along definite stream-lines the work done by pressure P per unit volume of fluid in crossing any given cross-section of a stream-tube bounded by stream-lines is P. If the pressure varies, then an equivalent amount of energy must be added to or removed from the fluid. If the velocity of flow is u, then the kinetic energy of the fluid per unit volume $= \frac{1}{2}\varrho u^2$; if the height is H above a fixed reference level then the potential energy $= g\varrho H$ per unit volume.

If the viscous forces are negligible, the principle of conservation of energy shows that

$$P + \tfrac{1}{2}\varrho u^2 + g\varrho H = \text{constant}.$$

Per unit mass

$$\frac{P}{\varrho} + \frac{u^2}{2} + gH = \text{constant}$$

$$= PV + \tfrac{1}{2}u^2 + gH \quad (V = \text{specific volume, i.e. the volume occupied by unit mass} = 1/\varrho).$$

For unit mass of fluid passing from point 1 to point 2 in a stream-tube then $P_1V + \frac{1}{2}u_1^2 + gH_1 = P_2V + \frac{1}{2}u_2^2 + gH_2$, where there is no frictional resistance to flow.

If the frictional resistance to flow is F per unit mass of fluid and the work done by the fluid in going from point 1 to point 2 = W/unit mass then:

$$P_1V - P_2V + \tfrac{1}{2}u_1^2 - \tfrac{1}{2}u_2^2 + gH_1 - gH_2 + W + F = 0.$$

Flow in a circular pipe

$$\text{Average flow velocity} = \frac{\text{volumetric flow rate}}{\text{cross-sectional area}}.$$

If the fluid is flowing from point 1 to point 2 in a system, then provided there is no loss or gain of fluid between these two points: $u_1A_1\varrho_1 = u_2A_2\varrho_2$.

For an incompressible fluid $\varrho_1 = \varrho_2$.

Thus $u_1A_1 = u_2A_2$.

For turbulent flow the velocity at points 1 and 2 is equal to the average velocity. For stream-line flow the velocity profile in a circular pipe is parabolic, and u_{average} varies with distance from the pipe axis.

In order to use the average velocity in the general equation a correcting factor must be introduced.

$$u^2_{\text{average}} = u^2\alpha.$$

For incompressible fluids:

$$V(P_1 - P_2) + \left(\frac{u_1^2}{2\alpha_1} - \frac{u_2^2}{2\alpha_2}\right) + g(H_1 - H_2) + W + F = 0.$$

For a compressible fluid (gases) the pressure change cannot be represented by $V(P_1 - P_2)$ but instead the integral $V\int_{P_2}^{P_1} dP$ must be evaluated for the particular gas conditions.

7.3. UNITS AND DIMENSIONS

The above equation represents the fundamental energy expression for fluid flow per unit mass of fluid. There is often considerable confusion over the units employed, and for this reason they are represented below and in Table 13.

TABLE 13. UNITS

Quantity	SI	CGS	FPS	FPS Engineering	Dimensions
Force	Newton	dyn	poundal	pound weight	$M\ L\ T^{-2}$
Energy	Joule	erg	ft-pdl	ft-lb	$M\ L^2\ T^{-2}$
Pressure	Newtons/m^2	dyn/cm^2	pdl/ft^2	lbf/ft^2	$M\ L^{-1}\ T^{-2}$
Power	Watt (Joules/sec)	erg/sec	ft-pdl/sec	ft-lb/sec	$M\ L^2\ T^{-3}$

CGS units

The unit of force is that force which will give a mass of 1 g an acceleration of 1 cm/sec/sec. This is referred to as the dyn.

1 dyn = 1 g cm sec^{-2}.

FPS units

The unit of force is the poundal.

1 pdl = 1 lb mass ft sec^{-2}.

British Engineering units

Here the unit of force is based on the slug, which is the mass which gives an acceleration of 1 ft/sec/sec when acted upon by a force of 1 lb-wt.

1 slug = 1 lbf ft^{-1} sec^2.
1 lb-wt = 32·2 pdl.
1 slug = 32·2 lb mass.

In SI the unit of force is the Newton which is that force which will give a mass of 1 kg an acceleration of 1 m/s/s.

$$N = kg\ m/s^2.$$

The fundamental energy equation per unit weight of fluid is:

$$\frac{\Delta u^2}{2\alpha g}+\frac{V}{g}\int_{P_2}^{P_1} dP+\Delta H+\frac{W}{g}+\frac{F}{g}=0.$$

Each of these terms has the dimensions of length (height) and thus:

$\frac{\Delta u^2}{2\alpha g}$ = velocity head.

$\frac{F}{g}$ = friction head (i.e. head lost due to friction).

Although the custom of giving the fundamental energy equation in the form of per unit weight of fluid is convenient because of the "velocity head" concept, in SI it will probably be better to use unit mass.

For an incompressible fluid, Bernoulli's equation gives per unit mass,

$$\frac{u^2}{2}+PV+gH=0,$$

$g = \mathrm{m/s^2}$, $V = \mathrm{m^3/kg}$, $P = \mathrm{N/m^2}$, $H = \mathrm{m}$, $u = \mathrm{m/s}$.

This equation is an energy equation because the various parts reduce to J/kg (specific energy) and $\mathrm{m^2/s^2}$ (kinetic energy).

$$PV = \mathrm{N/m^2\ m^3/kg = N\ m/kg = J/kg}$$
$$Hg = \mathrm{m\ m/s^2 = (m/s)^2}$$

The SI unit of pressure $\mathrm{N/m^2}$ is very small ($1\ \mathrm{N/m^2} = 1{\cdot}5\times\times 10^{-4}\ \mathrm{lb/in^2}$) and it has been suggested that the bar ($10^5\ \mathrm{N/m^2}$) should be used as the unit of pressure. This results in a unit that is far too large for many applications and another suggestion is that $\mathrm{N/m^2}$ be termed Pascal (Pa) and the kPa would be of appropriate size.

$1\ \mathrm{kN/m^2} = 0{\cdot}102\ \mathrm{m\ H_2O} = 102\ \mathrm{mm\ H_2O} = 7{\cdot}506\ \mathrm{mmHg}$.

7.4. PRESSURE DROP DUE TO FRICTION

For a horizontal pipe of constant cross-sectional area and with no work done on or by the fluid other than in overcoming friction:

$$\frac{V}{g}(P_1-P_2)+\frac{F}{g}=0.$$

Therefore

$$(P_2-P_1)=\frac{F}{V}=h\varrho g=\Delta P_{\text{friction}}.$$

Assuming that the fluid is flowing in a horizontal pipe of constant cross-sectional area and that the frictional resistance per unit surface area of pipe $=f$ (force/area $= \mathrm{ML^{-1}T^{-2}}$).

Resistance to flow = frictional resistance to flow.

If the pressure drop over a small elemental length $dl = \Delta P_f$ then:

$$\Delta P_f \times \pi r^2 = f\times 2\times\pi\times r\times dl,$$

$$\Delta P_f = \frac{2f\,dl}{r} = 2\left(\frac{f}{\varrho u^2}\right)\frac{dl\,\varrho u^2}{r},$$

$$\Delta P_f = 4\left(\frac{f}{\varrho u^2}\right)\frac{dl\,\varrho u^2}{d}.$$

For an incompressible fluid, and a horizontal pipe of uniform cross-sectional area:

$$\Delta P_f = 4\left(\frac{f}{\varrho u^2}\right)\frac{l\varrho u^2}{d}$$ (N/m² or pdl/ft² FPS units).

$$\Delta P_{\text{friction}} = 4\left(\frac{f}{\varrho u^2}\right)\frac{l\varrho u^2}{dg}$$ lb/ft² or kg/m² ($g = 32{\cdot}17$ ft sec^{-2}).

$$\Delta P_f = 8\left(\frac{f}{\varrho u^2}\right)\frac{lu^2}{d2g}$$ m/ft head lost due to friction.

$$\Delta P_{\text{friction}} = 8\left(\frac{f}{\varrho u^2}\right)\frac{l\varrho u^2}{d2g}$$ lb/ft² or kg/m².

$$\Delta P_{\text{friction}} = \text{friction factor}\times\frac{l\varrho u^2}{M2g}.$$

This latter expression is known as Fanning's equation; M is known as the hydraulic mean depth.

$$\Delta P_{\text{friction}} = F' \times \frac{4l}{d} \times \frac{\varrho u^2}{2g}$$

for circular pipe which is another form of Fanning's equation.

For turbulent flow in non-circular ducts the hydraulic mean diameter may be used instead of the pipe diameter and then the formulae developed for pipes may be used without introducing large errors into the calculation.

Hydraulic mean diameter

$$= \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}} = D_m.$$

For a duct of rectangular cross-section $a \times b$,

$$D_m = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}.$$

Sometimes the value hydraulic mean diameter is replaced by the term "hydraulic mean depth",

$$M = \frac{\text{cross-sectional area}}{\text{wetted perimeter}}.$$

Estimation of pressure drop for practical systems

Stanton and Pannell measured pressure drop due to friction for various pipes, fluids, and pipe surface conditions. Their results were expressed by plotting $(f/\varrho u^2)$ versus Reynolds number. The work was extended by many other workers, notably Moody. He plotted $(f/\varrho u^2)$ (or functions of it) versus Reynolds number and relative surface roughness, which is the dimensionless group e/d. These results are represented graphically in Figs. 7.1 and 7.2 and are of great value in pressure-drop estimations relative to friction.

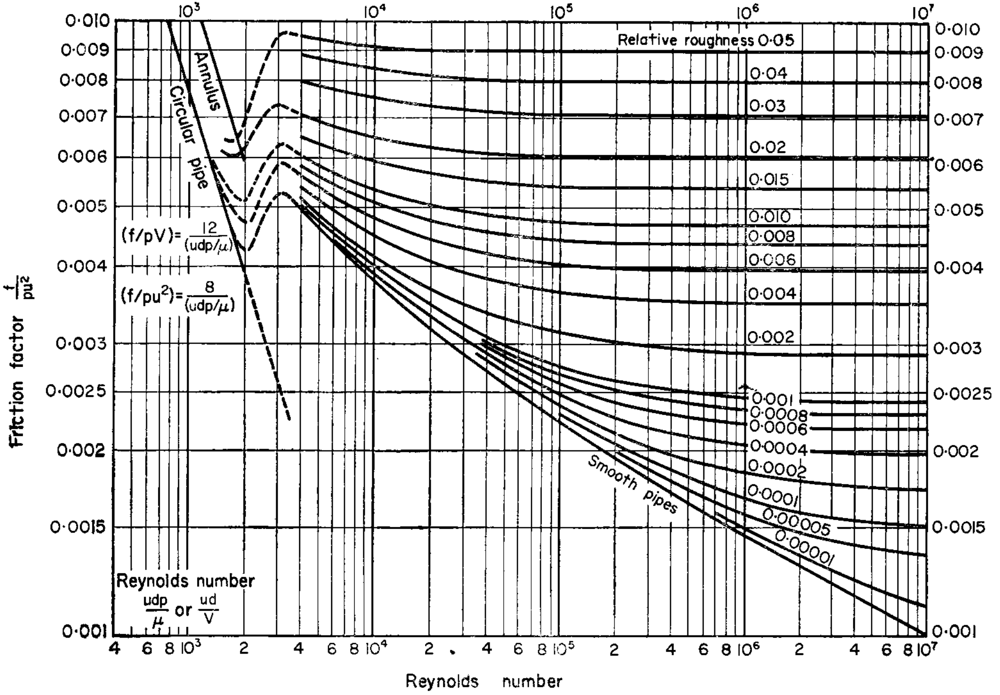

FIG. 7.1. Friction factor versus Reynolds number with appropriate roughness values.

(Reproduced by permission from *Technical Data on Fuel*, 6th edn., 1961, edited by H. M. Spiers, British National Committee, World Power Conference; from an original by British Petroleum Co. Ltd.)

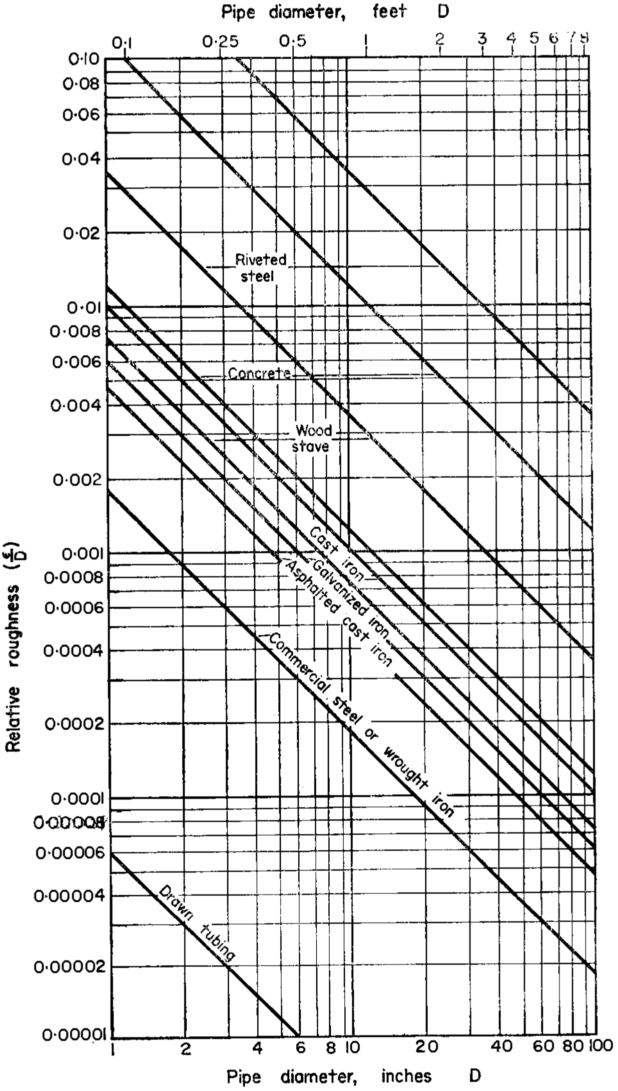

FIG. 7.2. Relative roughness for different pipes.

(Reproduced by permission from *Technical Data on Fuel*, 6th edn., 1961, edited by H. M. Spiers, British National Committee, World Power Conference; from a paper by Moody, published in *Trans. Amer. Soc. Mech. Eng.*, 1944, vol. 66, p. 671.)

It is often required to estimate pressure drop in a pipe for a given flow rate for an incompressible fluid; alternatively, it is often required to estimate flow rate when a given pressure difference exists.

In the first case it is possible to use the value of Reynolds number and the appropriate graph (Fig. 7.1) to obtain the required value. In the second case it is not possible to obtain the friction factor directly because it requires a knowledge of Reynolds number which is itself a function of flow velocity.

One method of solution is by a process of trial and error. The value of the friction factor is estimated, and then the Reynolds number appropriate to the flow conditions found. The flow velocity is obtained from the estimated value of Reynolds number. A value for pressure drop can then be found using this estimated flow velocity and the physical constants of the system. If the original estimate of friction factor, and hence Reynolds number, has been sufficiently accurate, the two values of pressure drop (actual and calculated) are in close agreement. The process is repeated until the two values agree sufficiently closely.

An alternative method given in Coulson and Richardson uses a plot of Reynolds number versus friction factor multiplied by (Reynolds number)^2, viz. $(f/\varrho u^2)\times(Re)^2$.

This latter expression does not include the velocity term and hence the Reynolds number can be obtained and the velocity can be calculated.

$$\left(\frac{f}{\varrho u^2}\right)\times\left(\frac{u^2d^2\varrho^2}{\mu^2}\right)=\left(\frac{fd^2\varrho}{\mu^2}\right)$$

$$\Delta P_{\text{friction}}=4f\frac{l}{d}$$

$$f=\frac{\Delta Pd}{4l}.$$

Thus,
$$\frac{fd^2\varrho}{\mu^2}=\frac{\Delta Pd^3}{4l\mu^2}.$$

Provided the pressure drop and physical properties of the system are known, Reynolds number can be obtained and the flow velocity found. Such a graph is reproduced in Fig. 7.3.

7.5. INCIDENTAL PRESSURE LOSSES

The equations for determination of pressure drop in a pipe apply only for the pressure drop in a straight pipe. In a complete pipe system there are additional energy losses such as occur at (i) bends, elbows, and other pipe fittings, and (ii) losses when the fluid enters or leaves the pipe system.

There are two methods whereby these additional losses can be taken into consideration.

The velocity head method

$$\Delta P_{\text{friction}} = 8\left(\frac{f}{\varrho u^2}\right)\frac{l}{d}\,\frac{\varrho u^2}{2g} \qquad \text{(lb/ft}^2\text{ FPS system).}$$

Replacing $2(f/\varrho u^2)$ by friction factor F', and d by M (Fannings equation)

$$\Delta P_{\text{straight pipe}} = F' \times \frac{l}{M}\,\frac{\varrho u^2}{2g}$$

$$= 4F'\frac{l}{d}\,\frac{\varrho u^2}{2g} \qquad \text{for circular pipes.}$$

In this method, ΣS, the total number of velocity pressure heads lost in the pipe system, is evaluated and multiplied by the velocity head $\varrho u^2/2g$. For a sudden enlargement from one section to another it can be shown that $S = (1-(a/A))^2$. For a sudden contraction $S = 0{\cdot}5(1-(a/A))$. The area of the small pipe $= a$, and the larger A. Tables have been compiled for various pipe fittings (Spiers, *Technical Data on Fuel*, also *Efficient Use of Fuel*).

The equivalent length of pipe method

Some workers prefer to use a new length of pipe l_e to be added to the value of l in the general equation:

$$\Delta P_{\text{friction}} = \frac{F'(l+l_e)\varrho u^2}{M2g} \qquad \text{(kg/m}^2\text{ SI and lb/ft}^2\text{ FPS system).}$$

Losses in stream-line flow due to changes in section are much less important than in turbulent flow because of the relatively small value of the velocity head. However, it can be important in short lengths of pipes.

7.6. FLOW OF COMPRESSIBLE FLUIDS IN PIPES

For a small change in a flowing fluid the energy balance can be written:

$$d\left(\frac{u^2}{2\alpha}\right)+g\,dH+V\,dP+\delta F+\delta W = 0.$$

With compressible fluids V the specific volume, and hence ϱ, u, and P vary along the length of pipe. Since the mass rate of flow must, however, remain constant at all sections the variables must be expressed in terms of mass rate of flow G,

$$G = uA\varrho = \frac{uA}{V},$$

equivalent to

$$\Delta P_{\text{friction}} = \frac{F'\,dl\varrho u^2}{M2} \qquad \text{(N/m}^2\text{ SI and pdl/ft}^2\text{ FPS),}$$

$$\delta F = \frac{F'\,dl\varrho u^2}{M\varrho 2} \qquad \text{(watts or J/s SI; ft-pdl per unit mass fluid FPS system).}$$

In the case of turbulent flow of a compressible fluid over a small element of section when no work is done on or by the fluid:

$$d\left(\frac{u^2}{2}\right)+V\,dP+g\,dH+\frac{F'\,dlu^2}{M2}=0,$$

$$\left(\frac{G}{A}\right)^2\frac{V^2}{2}+V\,dP+g\,dH+\frac{F'\,dlu^2}{2M}=0.$$

For horizontal pipes:

$$\left(\frac{G}{A}\right)^2 V\,dV+V\,dP+\frac{F'\,dlu^2}{2M}=0.$$

Over a length of pipe l:

$$\left(\frac{G}{A}\right)^2 V\,dV+V\,dP+\frac{F'l}{2M}\left(\frac{G}{A}\right)^2 V^2=0.$$

Dividing throughout by V^2:

$$\left(\frac{G}{A}\right)^2\frac{dV}{V}+\frac{dP}{V}+\frac{F'l}{2M}\left(\frac{G}{A}\right)^2\frac{V^2}{V^2}=0.$$

It is only possible to evaluate the integrals for known conditions of flow. In the particular case of isothermal flow:

$$\int_{P_1}^{P_2}\frac{dP}{V}=\frac{1}{P_1V_1}\int_{P_1}^{P_2}P\,dP=\frac{P_2^2-P_1^2}{2P_1V_1}=\frac{(P_2-P_1)(P_2+P_1)}{2P_1V_1}.$$

(By application of the gas laws $P_1V_1=P_2V_2=PV=RT$ $=$ constant,

$$V=\frac{P_1V_1}{P}.\Big)$$

Thus

$$\left(\frac{G}{A}\right)^2\log_e\frac{V_2}{V_1}+\frac{P_2^2-P_1^2}{2P_1V_1}+\frac{F'l}{2M}\left(\frac{G}{A}\right)^2=0.$$

If V_m is defined as the average or mean specific volume in the pipe such that $P_1V_1 = \frac{1}{2}(P_1+P_2)V_m$,

$$\left(\frac{G}{A}\right)^2 \log_e \frac{V_2}{V_1} + \frac{(P_2-P_1)(P_2+P_1)}{2P_1V_1} + \frac{F'l}{2M}\left(\frac{G}{A}\right)^2 = 0,$$

$$\left(\frac{G}{A}\right)^2 \log_e \frac{V_2}{V_1} + \frac{(P_2-P_1)}{V_m} + \frac{F'l}{2M}\left(\frac{G}{A}\right)^2 = 0.$$

The above equation is the general expression for the energy loss for a compressible fluid flowing under isothermal flow conditions from point 1 to point 2, and the pressure drop

$$(P_1-P_2) = \frac{F'l}{2M}\left(\frac{G}{A}\right)^2 V_m + \left(\frac{G}{A}\right)^2 \log_e \frac{V_2}{V_1} V_m.$$

For small pressure drops, that is less than 10% of the total pressure, the second term can be neglected, and

$$P_1-P_2 = \frac{F'l}{2M}\left(\frac{G}{A}\right)^2 V_m$$

$$= \frac{F'lu_m^2\varrho_m^2V_m}{2M}.$$

$$P_1-P_2 = \frac{F'lu_m^2V_m}{2MV_m^2}.$$

$$P_1-P_2 = \frac{F'lu_m^2}{2MV_m}$$

$$= \frac{F'l\varrho_m u_m^2}{2M}.$$

$$P_1-P_2 = 8\left(\frac{f}{\varrho u^2}\right)\frac{l}{d}\,\frac{\varrho_m u_m^2}{2g} \text{ kg/m}^2 \text{ SI (lb/ft}^2 \text{ FPS system).}$$

7.7. FLOW OF GASES IN FURNACES

The flow of gases in furnaces is similar in many respects to flow in pipes. The pressure drop in furnaces is due mainly to the numerous bends and changes in section, although frictional and buoyancy effects are also important. Hence the pressure drop is preferably calculated in terms of the "velocity head" method, rather than the "equivalent length". Also, because the pressure changes involved are small in relation to absolute pressure, the compressibility of the gases is usually neglected. The total pressure in a furnace is made up of:

(a) buoyancy changes;

(b) wall friction;

(c) changes of section and eddying at bends.

Pressure differences due to buoyancy effects

$$\text{Pressure drop} = \Delta P_B = H \times (\varrho_a - \varrho_g),$$

where ϱ_a = density of surrounding atmospheric air; ϱ_g = density of gas flowing.

Both of these densities are measured at their respective temperatures and pressures, with due regard to the presence of any water vapour. A mean value of the different pressures and temperatures is normally used.

If the specific gravity of the gas relative to air = SG, and H is the difference in levels:

$$\Delta P_B = H(\varrho_a - \varrho_g) = H\varrho_a(1 - \text{SG}).$$

If ϱ is in kg/m³ and H in m; ΔP_B = kg/m².

If the density of air at 0°C = 1·3 kg/m³, then

$$\varrho_a = \frac{1{\cdot}3 \times 273}{273 + t_{\text{air}}\,^{\circ}\text{C}}; \qquad \varrho_g = \frac{1{\cdot}3 \times \text{SG} \times 273}{273 + t_{\text{gas}}\,^{\circ}\text{C}}.$$

ΔP_B is the static pressure difference for a chimney, and can be used to determine the height of the chimney (H m). This is not the available draught for velocities greater than 5 m/s, because of the loss of energy due to wall friction and as the kinetic energy in the moving gases.

Losses due to wall friction

Loss due to friction

$$\Delta P_F = 8\left(\frac{f}{\varrho u^2}\right)\frac{l}{d}\,\frac{\varrho u^2}{2g}$$

$$= \frac{F'l\varrho u^2}{M2g}\ \text{lb/ft}^2 \text{ or kg/m}^2$$

$$= F'\frac{l}{M}\,\frac{\varrho u^2}{2}\ \text{N/m}^2.$$

(*Note.* The density ϱ and velocity u must be measured at the mean temperature of the gases flowing in the chimney. F' is sometimes referred to as the friction factor, although some operators prefer to use other values, i.e. $F'/2$.)

Loss due to kinetic energy of the gases

Loss due to kinetic energy

$$= \frac{\varrho u^2}{2g}\ \text{lb/ft}^2 \text{ or kg/m}^2$$

$$= \frac{\varrho u^2}{2}\ \text{N/m}^2.$$

Thus the reduction in static draught

$$= \frac{\varrho u^2}{2g}(1+F'l/M)\ \text{lb/ft}^2 \text{ or kg/m}^2.$$

For circular chimneys $F'/4$ has been given the value of 0·05 *(Efficient Use of Fuel)* and 0·09 (Reber, *Technical Data on Fuel*, ed. Spiers). Since 1 in. w.g. = 5·2 lb/ft², the effective draught of a chimney is given by

$$\text{draught in. w.g.} = \frac{\varrho u^2}{2g}\left(1+0{\cdot}09\,\frac{H}{D}\right)\Big/5{\cdot}2,$$

where H = height of chimney in feet, and D = chimney diameter in feet.

(*Note.* Once again the values of ϱ and u must be taken at the mean temperature of the gases flowing in the chimney.)

Pressure drop due to wall friction

This is similar to the formula used for chimneys, the frictional pressure drop being given by:

$$\Delta P_{\text{friction}} = 8\left(\frac{f}{\varrho u^2}\right)\frac{l\varrho u^2}{d2g}$$

$$= \frac{F' l \varrho u^2}{M2g}\ \text{lb/ft}^2 \text{ or kg/m}^2.$$

Pressure drop due to changes in section, and eddying at bends

$$\Delta P = S' \times \frac{\varrho u^2}{2g}\ \text{lb/ft}^2 \text{ or kg/m}^2.$$

The values of ϱ and u are in lb/ft³ or kg/m³ and ft/sec or m/s measured at the mean temperature and pressure of the gases flowing in the furnace. S' is the number of velocity heads lost due to each section, fitting, or effect. Values of S' for furnaces have been calculated and a comprehensive table is given in *Efficient Use of Fuel.*

7.8. FLOW MEASUREMENT

Instruments which measure the rate of flow of fluids often rely upon changes in pressure to give an indication of flow velocity.

Fluid pressure

In a stationary fluid the pressure at any point is equal in all directions and this is referred to as static pressure. In a moving fluid this static pressure will be exerted in any plane which is parallel to the flow direction. In such a moving system the pressure exerted at right angles to the direction of flow will be equal to the static pressure at that point plus the pressure equivalent of the kinetic energy which the fluid would give up if brought to rest at that point (Bernouilli's theorem). Bernouilli's theorem can be used to find the values of both static pressure and the pressure equivalent of the kinetic energy term.

For a stationary fluid, then,

$$g\,dH + V\,dP = 0.$$

For an incompressible fluid, then this equation may be integrated directly:

$$V(P_2 - P_1) = -gH,$$

$$P_1 - P_2 = \Delta P = \frac{gH}{V} = g\varrho H \text{ pdl/ft}^2.$$

For a compressible fluid it is necessary to know the conditions of flow. In the case of isothermal flow, then

$$V\int_{P_1}^{P_2} dP = P_1V_1 \log_e \frac{P_2}{P_1} = \frac{RT}{M} \log_e \frac{P_2}{P_1}$$

(for unit mass of gas $PV = RT/M$; M = molecular weight).

When the static pressure in a moving fluid has to be measured it is essential that the measuring device be parallel to the direction in which the fluid is moving; also there must be no projections into the stream of fluid. These conditions are necessary to ensure that none of the kinetic energy of the fluid is inadvertently measured with the static pressure. A device which measures static pressure in a tube is known as a Piezometer tube. Static pressure should always be measured well away from any changes of section or pipe fittings; fifty pipe diameters is the accepted distance from such obstructions to avoid eddy formation.

Measurement of pressure

The simplest instrument is the manometer. This consists of a simple U-tube, one end connected to the fluid at point 1 and the other connected to point 2 in the system; the differential pressure between these two points is obtained from a simple balance of pressures in the two limbs. If the pressures at the two points are respectively P_1 and P_2, the fluid density ϱ_F, the differential height of the immiscible fluid in the U-tube h m, and its density ϱ, then if P_1 is greater than P_2, $(P_1-P_2) = \Delta P = h(\varrho-\varrho_F)g$.

If the fluid whose pressure is to be determined is a light gas, and the fluid in the U-tube a heavy liquid then the above expression can be simplified as: $\Delta P = h\varrho g$ with little error. A variation of the simple manometer is the inclined manometer. This increases the displacement of the liquid in the U-tube and hence allows a more accurate reading to be obtained. If the displacement in the tube is lm, and the angle between the inclined limb and the horizontal $= \theta$:

$$\Delta P = l \cos \theta \varrho g \qquad (\varrho = \text{density liquid in manometer}).$$

Measurement of static pressure plus pressure equivalent of kinetic energy fluid

Bernoulli's equation can be used to obtain the required values. If fluid flows between two sections in a pipe system such that it is brought to rest at section 2, then all of the kinetic energy is converted into pressure energy. If the two sections are sufficiently close that frictional resistance of the walls is negligible, and no work is done on or by the fluid in passing from section 1 to section 2,

$$\frac{\Delta u^2}{2\alpha}+g\,\Delta H+V\int_{P_2}^{P_1} dP = 0.$$

Most instruments which measure pressure are designed to operate in horizontal sections of the tube, thus for a horizontal pipe, for turbulent flow, and with an incompressible fluid (liquids):

$$\frac{u_1^2}{2}-\frac{u_2^2}{2} = V(P_2-P_1).$$

Since the fluid is stationary at section 2, $u_2 = 0$.

$$u_1^2 = 2V(P_1-P_2),$$
$$u_1 = \sqrt{[2V(P_2-P_1)]} = \sqrt{(2V\,\Delta P)},$$
$$u_1 = \sqrt{(2V\varrho gh)} \qquad (V = 1/\varrho),$$
$$u_1 = \sqrt{(2gh)}\ \text{m/s},$$

where h is the head in metres of fluid flowing in the pipe which is lost as a result of the fluid having been brought to rest. It is important to note that it is not the differential height obtained in a manometer which may be at a different pressure and temperature to the fluid flowing, and probably uses a totally different measuring fluid.

The pitot tube

This instrument consists of a pair of concentric tubes positioned parallel to the direction of flow. The open part of the inner tube measures total pressure. The annular part is sealed off at the head of the instrument, but is open to the fluid at pre-set positions to measure the static pressure. The difference between the two gives the pressure equivalent of the kinetic energy and hence allows the rate of flow to be determined. Actual dimensions are given in the appropriate British Standard (1042).

The head of the instrument must be aligned with the direction of flow and it is direct reading; calibration is unnecessary. It only measures a thin filament of the passing fluid and unless the velocity distribution in the pipe or furnace is known it cannot be used to measure mass rates of flow. It can, however, be used to find the velocity profile in a pipe or duct.

The orifice plate and venturi meter

In these instruments the fluid to be measured is accelerated by causing it to flow through a constriction such that the kinetic energy is increased and the corresponding change in the pressure is measured. The fluid is then allowed to expand back to its normal flow condition, but unfortunately not all of the pressure energy is recovered as kinetic energy.

Since the fluid is not brought to rest in these applications, then for a horizontal pipe, turbulent flow, and incompressible fluid flowing from section 1 (no constriction) to a constriction at section 2:

$$\frac{u_2^2}{2}-\frac{u_1^2}{2} = V(P_1-P_2).$$

The mass rates of flow at each section must be identical:

$$u_1A_1\varrho_1 = u_2A_2\varrho_2,$$

$$u_1 = \frac{u_2A_2}{A_1} \quad \text{for an incompressible fluid } (\varrho_1 = \varrho_2).$$

A_1 and A_2 refer to cross-sectional areas at section 1 and section 2 respectively.

In a venturi meter A_2 is the area of the throat. With an orifice plate meter A_2 is not the area of the orifice but the area of minimum cross-section at a point known as the *vena-contracta* (Fig. 7.4).

$$u_2^2 = 2V\Delta P + u_1^2,$$

$$u_2^2 = 2V\Delta P + u_2^2\frac{A_2^2}{A_1^2},$$

$$u_2^2\left(1 - \frac{A_2^2}{A_1^2}\right) = 2V\Delta P.$$

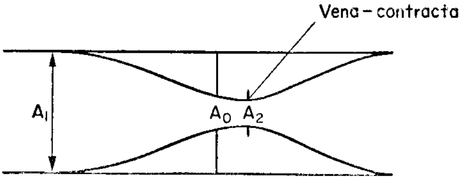

FIG. 7.4.

It is difficult to measure A_2, and since the value of the orifice area is known accurately, the ratio A_0/A_1 is also known accurately (A_0 = orifice area) and this value is used in the expression for u_2. The above equation will have to be multiplied by a correcting factor to allow this to be done. The factor also takes into consideration the effect of frictional losses as a result of the orifice and allows u_m, the average velocity at point 2 (now the orifice), to be found.

Thus $u_{2m} = C_D\sqrt{\left[\frac{2V\Delta P}{1-(A_0/A_1)^2}\right]}$ (C_D = coefficient of discharge).

The ratio (A_0/A_1) is often written as m, and so the equation becomes:

$$u_{2m} = C_D\sqrt{\left(\frac{2V\Delta P}{1-m^2}\right)}.$$

Also, the value $\sqrt{\left(\frac{1}{1-m^2}\right)}$ is often referred to as the velocity of approach factor and is incorporated into the general equation as

$$E = \sqrt{\left(\frac{1}{1-m^2}\right)},$$

$$u_{2m} = C_D E \sqrt{(2V \Delta P)},$$

$$= C_D E \sqrt{(2gh)}$$

(*h* is once again the head of fluid flowing which is equivalent to the kinetic energy term).

The most important feature in an orifice meter is the relationship between the size of the orifice and the diameter of the pipe in which it is fitted. The position and method of pressure tapping are important because the area of flow and the velocity of the fluid change in the region of the orifice.

The flow rate for a given pressure differential is less than the theoretical because of friction losses. These are allowed for in the coefficient of discharge but they are dependent upon Reynolds number, pipe roughness, exact shape of orifice, plate thickness, type of pressure tappings and the distance the meter is from any changes of section or obstructions. The orifice meter should be sited at least fifty pipe diameters from any likely obstructions to normal flow pattern. Relevant constructional details are given in the appropriate British Standard (1042).

With the venturi meter C_D is always higher than for the orifice plate because the fluid is accelerated slowly to the minimum cross-section, the angle of convergence being not greater than 20°. From the throat onwards the pipe diverges and this angle should not exceed 7°. Exact measurements are given in the British Standard.

The equation developed for the orifice plate may be used in this case. C_D is approximately 0·90–0·99, and the recovery of pressure energy is high.

Formulae developed for incompressible fluids can be used for compressible fluids at low values of *u* (less than 200 ft/sec or 60 m/s). At higher velocities they must be corrected as in the example below:

$$u_{2m} = C_D E \varepsilon \sqrt{(2gh)},$$

where ε = compressibility factor.

7.9. EXAMPLES

EXAMPLE 7.1

Establish the equations used for the calculation of pressure drop in a furnace flue system in terms of velocity head, buoyancy, and frictional resistance.

A furnace consuming 0·085 l./s of a fuel oil of SG 0·95 at 15·5°C is exhausted by means of a flue system comprising a duct 0·6 m×0·75 m, 120 m long, having at the front a sudden enlargement from a section 0·45 m×0·45 m and, at the other end, an enlargement to 0·9 m×0·9 m. The duct has bends producing a total pressure loss equivalent to the effect of 4·5 velocity heads. The frictional resistance of the walls may be taken as equivalent to 1 velocity head, and the volume of gases correspond to a volume of 11·35 m^3/kg fuel burned. The mean temperature of the gases in the duct is 250°C. The gases have a SG relative to air of 0·897.

Determine the pressure drop in the flue system. Density air = 1·39 kg/m^3 at NTP.

[City and Guilds Advanced, Fuel Plant Technology; units changed.]

$$\text{Bend losses} = 4{\cdot}5 \times \frac{\varrho u^2}{2g}.$$

$$\text{Frictional losses} = \frac{\varrho u^2}{2g}.$$

$$\begin{aligned} \text{Volume of gases} &= 11{\cdot}35 \text{ m}^3/\text{kg}. \\ \text{Vol./s} &= 11{\cdot}35 \times (0{\cdot}08525 \times 10^{-3} \text{ m}^3/\text{s}) \times 0{\cdot}95 \times 10^3 \\ &= 0{\cdot}922 \text{ m}^3/\text{s}. \end{aligned}$$

$$u\ \text{m/s} = \frac{0{\cdot}922}{0{\cdot}6\times0{\cdot}75} = 2{\cdot}05\ \text{m/s}.$$

$$u\ \text{at}\ 250^\circ\text{C} = 20{\cdot}5\times\tfrac{523}{273} = 3{\cdot}924\ \text{m/s}.$$

$$\varrho\ \text{at}\ 250^\circ\text{C} = 1{\cdot}39\times0{\cdot}897\times\tfrac{273}{523} = 0{\cdot}65\ \text{kg/m}^3.$$

Pressure drop for sudden enlargement $= (1-A_1/A_2)^2\dfrac{\varrho u^2}{2g}$.

$$\text{For entering the system} = \left(1-\frac{0{\cdot}45\times0{\cdot}45}{0{\cdot}75\times0{\cdot}6}\right)^2$$

$$= 0{\cdot}3025.$$

$$\text{For leaving the system} = \left(1-\frac{0{\cdot}45}{0{\cdot}81}\right)^2$$

$$= 0{\cdot}1936.$$

$$\text{Total losses} = 1+0{\cdot}3025+0{\cdot}1936+4{\cdot}5 = \text{velocity heads}$$

$$= \frac{5{\cdot}996\times0{\cdot}65\times3{\cdot}924^2}{2\times9{\cdot}807}\quad\text{kg/m}^2;$$

$$\Delta P = 3{\cdot}074\ \text{kg/m}^2.$$

$$= \frac{3{\cdot}074\times9{\cdot}807}{1000} = 0{\cdot}301\ \text{kPa}\ (3{\cdot}06\ \text{mm H}_2\text{O}).$$

EXAMPLE 7.2

Give an account of the scientific principles that govern the flow of hot gases in furnace ducts.

Determine the pressure drop in a flue of dimensions 2 ft 0 in.×2 ft 6 in., in which are flowing 126,000 lb/hr of waste gases having a density of 0·09 lb/ft^3 at NTP. The mean temperature of the gases is 500°C (932°F), and the resistance due to friction and changes of section is equivalent to 15 times the "velocity head" at the mean temperature. $g = 32{\cdot}17$ ft/sec^2; 1 in. w.g. = 5·2 lb/ft^2.

[Institution of Metallurgists, Old Regulations, Fuels.]

At 932°F $\varrho = \dfrac{0{\cdot}09 \times 492}{1392} = 0{\cdot}031\ \text{lb/ft}^3.$

$$\text{Mass rate flow} = \frac{126{,}000}{60 \times 60}\ \text{lb/sec}$$

$$= u \times 2 \times 2{\cdot}5 \times 0{\cdot}031 \quad (\text{at } 932°\text{F}).$$

$$u\ \text{ft/sec} = 306{\cdot}3\ \text{ft/sec at } 932°\text{F}.$$

$$\text{Pressure drop} = \frac{15 \times 0{\cdot}02975 \times 306{\cdot}3^2}{2 \times 32{\cdot}17 \times 5{\cdot}2}.$$

Pressure drop $= 12{\cdot}7$ in. w.g. *(Answer.)*

EXAMPLE 7.3

Explain the use of the Reynolds number in the determination of the frictional resistance of fluids flowing in circular and non-circular pipes.

Carbon dioxide flows in a circular pipe of 75 mm internal diameter at a rate of 1·31 kg/s at a temperature of 15°C. Determine:

(i) the Reynolds number for this condition of flow, the density of the gas being given as 1·86 kg/m^3 and the absolute viscosity $13{\cdot}9 \times 10^{-6}$ Ns/m^2;

(ii) the frictional coefficient corresponding to this value of Reynolds number for the pipe in question is 0·015. Determine the pressure drop for a length of straight pipe.

[City and Guilds Advanced, Fuel Plant Technology; units changed.]

Mass rate flow $G = uA\varrho$

$$1{\cdot}31 = \frac{u \times \pi \times d^2 \times \varrho}{4}.$$

$$u\,d\varrho = \frac{1{\cdot}31 \times 4}{\pi \times d} = \frac{1{\cdot}31 \times 4}{\pi \times 0{\cdot}075}.$$

$$\text{Reynolds number} = \frac{1{\cdot}31 \times 4}{\pi \times 0{\cdot}075 \times 0{\cdot}0139 \times 1000}$$

$$= 1{\cdot}6 \times 10^6. \quad (\textit{Answer}\ \text{(i).})$$

$$\text{Frictional drop} = \text{frictional factor} \times \frac{l}{d} \times \frac{\varrho u^2}{2} \quad \text{N/m}^2.$$

$$\text{Frictional drop} = \frac{0{\cdot}015 \times 100 \times 1{\cdot}86 \times u^2}{2 \times d} \quad \text{N/m}^2$$

$$\frac{u \times \pi \times 0{\cdot}075^2 \times 1{\cdot}86}{4} = 1{\cdot}31.$$

$$u = \frac{4 \times 1{\cdot}31}{\pi \times 1{\cdot}86 \times 0{\cdot}075^2} = 160 \text{ m/s}.$$

$$\text{Frictional pressure drop} = \frac{0{\cdot}015 \times 100 \times 160^2 \times 1{\cdot}86}{0{\cdot}075 \times 2}$$

$$= 0{\cdot}476 \times 10^6 \text{ N/m}^2$$

$$= 0{\cdot}476 \times 10^3 \text{ kN/m}^2$$

$$= 0{\cdot}476 \times 10^3 \text{ kPa}$$

= 476 kPa per 100 m length.
(*Answer* (ii).)

(1 kPa = 102 mm H_2O.)

EXAMPLE 7.4

A gas of density 0·249 kg/m^3, and rate flow 600 m^3/h is flowing at a temperature of 20°C and the pressure slightly above atmospheric, in 1000 m of straight circular main of 150 mm diameter (i.d.). Determine:

(i) the Reynolds number applicable to the flow conditions if the corresponding absolute viscosity is $2{\cdot}113 \times 10^{-5}$ Ns/m^2;

(ii) the pressure drop in appropriate units given that the friction factor is 0·0036 in an equation of flow expressed in terms of:

pressure drop

$$= 8 \times \text{friction factor} \times \frac{\text{length}}{\text{diameter}} \times \text{dynamic pressure};$$

(iii) explain the character of the additional data required if the main is not straight but has bends and other types of frictional resistances.

[Institution of Metallurgists, Old Regulations, Fuels; units changed.]

Mass rate flow $G = uA\varrho$.

$$u = \frac{G}{A\varrho} = \frac{600\times 4}{\pi\times 0{\cdot}15^2\times 3600} = 9{\cdot}88 \text{ m/s.}$$

$$\text{Reynolds number} = \frac{u\,d\varrho}{\mu} = \frac{9{\cdot}88\times 0{\cdot}15\times 0{\cdot}249}{2{\cdot}113\times 10^{-5}}$$

$= 16{,}670$. (*Answer* (i).)

$$\text{Pressure drop} = \frac{8\times 0{\cdot}0036\times 1000\times \varrho\times u^2}{0{\cdot}15\times 2}$$

$= 2338\ \text{N/m}^2$

$= 2{\cdot}338\ \text{kN/m}^2 = 2{\cdot}338$ kPa.

(*Answer* (ii).)

(1 kPa = 102 mm H_2O.)

EXAMPLE 7.5

Explain the following terms: (a) stream-line flow, (b) turbulent flow, (c) friction factor, and (d) dynamic mean diameter or hydraulic mean depth. What is the equivalent value of (d) for a pipe of rectangular cross-section?

Crude oil has to be pumped from a jetty at sea level to a tank farm 100 ft above sea level through 6 in. pipe 3 miles long. If the required flow is 7500 gal/hr and the viscosity of the oil is taken as $4{\cdot}5\times 10^{-3}$ lb-sec/ft², what is the theoretical HP required? Specific gravity oil = 0·90, assume friction factor $f = 0{\cdot}049/Re^{0{\cdot}2}$.

[City and Guilds, Advanced, Fuel Engineering.]

The following additional data will be required:

density water = 62·4 lb/ft³; 1 gal water weighs 10 lb
1 HP = 550 ft-lb/sec.

Fundamental energy equation per unit mass fluid

$$(P_1-P_2)V+gH+\frac{\Delta u^2}{2}+W+F = 0.$$

Since a liquid, then it is assumed incompressible, and since being pumped it is under turbulent flow.

The change in kinetic energy will be ignored since the pipe is so long, and will be small compared to the other terms in the equation. Also, since the tanks will be at the same atmospheric pressure at both levels, $P_1-P_2 = 0$. Thus: $gH+W+F = 0$.

$$\Delta P_{\text{friction}} = 4F'\times\frac{\text{length}}{\text{diameter}}\times\frac{\varrho u^2}{2}.$$

$$F' = \frac{0{\cdot}049}{Re^{0{\cdot}2}}.$$

$$Re = \frac{u\,d\varrho}{\mu}.$$

$$G \text{ mass rate flow} = uA\varrho = \frac{\pi\,d^2\varrho u}{4}.$$

$$\frac{4G}{\pi d} = u\,d\varrho.$$

$$\frac{4G}{\pi\,d\mu} = \frac{u\,d\varrho}{\mu}.$$

$$\text{Reynolds number} = \frac{7500\times10\times0{\cdot}9\times4}{3600\times0{\cdot}5\times4{\cdot}5\times10^{-3}} = 13{,}050.$$

$$F' \text{ friction factor} = \frac{0{\cdot}049}{(13{,}050)^{0{\cdot}2}} = 0{\cdot}007.$$

$$\text{Energy lost per unit mass as a result of friction} = \frac{\Delta P_F}{\varrho} = F$$

$$= \frac{0{\cdot}007\times5280\times3\times4\times u^2\times\varrho}{0{\cdot}5\times2\times\varrho}$$

Mass rate flow $G = uA\varrho$,

$$u = \frac{7500\times 10\times 0\cdot 9\times 4}{3600\times\pi\times 0\cdot 5^2\times 62\cdot 4\times 0\cdot 9}$$

$$u = 1\cdot 7 \text{ ft/sec.}$$

$$\text{Therefore } F = \frac{\Delta P_F}{\varrho} = \frac{4\times 0\cdot 007\times 5280\times 3\times 1\cdot 7^2\times\varrho}{0\cdot 5\times 2\times\varrho}$$

$$= 1400 \text{ ft-pdl.}$$

Therefore $-W = 100\times 32\cdot 2+1400 = 4620$ ft-pdl,

HP = rate of doing work

= work done per unit mass ×

× mass rate flow $= 4620\times uA\varrho$

$$= \frac{4620\times 1\cdot 7\times\pi\times 0\cdot 5^2\times 62\cdot 4\times 0\cdot 9}{4\times 32\cdot 2\times 550}$$

$= 3\cdot 9$. *(Answer.)*

EXAMPLE 7.6

Determine the available draught in a chimney of height 110 ft and internal diameter (D) 6 ft, given the following conditions:

Weight of gases passing up the chimney = 1613 lb/min.
Density gases, relative to air = 0·91.
Density air at 32°F and 30 in.Hg = 0·08071 lb/ft^3.
Mean temperature of gases in chimney = 300°F.
Mean temperature of atmosphere = 60°F.

Frictional loss in chimney in terms of one pressure head is given by $0\cdot 05\ H/D$; $g = 32\cdot 17$ ft/sec^2; 1 in. w.g. = 5·2/ft^2.

[Institution of Metallurgists, Old Regulations, Fuels.]

ϱ gas $= 0\cdot 08071\times 0\cdot 91$ at NTP.

Static draught $= H(\varrho_a-\varrho_g)$.

This is reduced by friction and kinetic energy.

$$\text{Loss due to friction} = \frac{0{\cdot}05H\varrho u^2}{D2g}$$

$$\varrho_a = \frac{0{\cdot}08071 \times 492}{520} = 0{\cdot}078 \text{ lb/ft}^3.$$

$$\varrho_g = \frac{0{\cdot}08071 \times 0{\cdot}91 \times 492}{760} = 0{\cdot}052 \text{ lb/ft}^3.$$

$$\text{Static draught} = 110\,(0{\cdot}078 - 0{\cdot}052) = 2{\cdot}86 \text{ lb/ft}^2.$$

$$\text{Reduction in draught} = \left(1 + 0{\cdot}05 \times \frac{110}{6}\right)\frac{\varrho u^2}{2g}$$

$$= \frac{1{\cdot}917 \times 0{\cdot}052 \times u^2}{2 \times 32{\cdot}17}.$$

$$\text{Rate flow} = \frac{1613}{60} \quad \text{lb/sec.}$$

$$u = \frac{1613}{60 \times \pi \times \frac{36}{4} \times 0{\cdot}052}$$

$$= 18{\cdot}4 \text{ ft/sec.}$$

$$\text{Reduction in draught} = \frac{1{\cdot}917 \times 0{\cdot}052 \times 18{\cdot}4^2}{2 \times 32{\cdot}17}$$

$$= 0{\cdot}532 \text{ lb/ft}^2.$$

$$\text{Effective draught} \quad = \frac{2{\cdot}86 - 0{\cdot}53^2}{5{\cdot}2} = 0{\cdot}45 \text{ in. w.g.}$$

EXAMPLE 7.7

Calculate the volume of clean producer gas at 15°C and 1013 mbar pressure passing per hour through a 0·45 m gas main in which there is an orifice plate of 90 mm diameter, if the pressure 0·45 m upstream from the orifice is 180 mm w.g. and the differential pressure between this point and a point 190 mm downstream from the orifice plate is 46 mm w.g., the barometric pressure is 989 mbar and the gas temperature is 60°C. Take the coefficient of discharge as 0·61 and the density of the producer gas at 15°C and 1013 mbar as 1·36 kg/m³.

[City and Guilds Advanced, Gaseous Fuels; units changed.]

$$U_{2m} = C_D E \sqrt{(2gh)}.$$

E velocity of approach factor $= \sqrt{\left(\frac{1}{1-m^2}\right)}$.

$m = A_0/A_1$

$= 90/450 = 0{\cdot}2.$

$E = 1{\cdot}021.$

Differential pressure = 46 mm w.g. = 0·046 m w.g.

$\varrho_{\text{water}} \times g \times h_{\text{water}} = \varrho_{\text{gas}} \times g \times h_{\text{gas}}.$

$$\varrho_{\text{gas}} = \frac{1{\cdot}36 \times 288 \times 1006}{333 \times 1013} = 1{\cdot}16 \text{ kg/m}^3 \text{ at } 60°\text{C}.$$

Pressure gas = 989 mbar + 17 mbar (180 mm H_2O)

= 1006 mbar.

$\varrho_{\text{water}} = 1000$ kg/m³ at 15°C.

$$h_{\text{gas}} = \frac{h_{\text{water}} \times \varrho_{\text{water}}}{\varrho_{\text{gas}}}$$

$$= \frac{0{\cdot}046 \times 1000}{1{\cdot}16} = 39{\cdot}6 \text{ m head.}$$

$$U_{2m} = 0{\cdot}61 \times 1{\cdot}021 \times \sqrt{(2 \times 9{\cdot}807 \times 39{\cdot}6)}$$

$= 17{\cdot}38$ m/s.

$$\text{Flow rate} = \frac{17{\cdot}38 \times \pi \times 0{\cdot}09^2 \times 3600}{4} \text{ m}^3\text{/h at } 60°\text{C}$$

= 378 m³/h at 989 mbar
+ 180 mm H_2O pressure.

Flow rate at 15°C and 1013 mbar pressure

$$= \frac{378 \times 288 \times 1006}{333 \times 1013}$$

$$= 340 \text{ m}^3/\text{h}. \quad (\textit{Answer.})$$

BIBLIOGRAPHY

COULSON, J. M. and RICHARDSON, J. F., *Chemical Engineering*, vol. 1, Pergamon Press, 1957.

FRANCIS, W., *Fuel Technology, a Summarised Manual*, Pergamon Press, 1965.

LYLE, O., *The Efficient Use of Steam*, HMSO, 1947.

SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.

THRING, M. W., *The Science of Flames and Furnaces*, 2nd edn., Chapman & Hall Ltd., London, 1962.

British Standards, 1042.

The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

CHAPTER 8

Furnace Atmospheres

8.1. INTRODUCTION

Metals and alloys have to be heated, either so that they can be more easily worked, or so that the properties can be modified by heat treatment. If this is carried out in a fuel-fired furnace, the carbon dioxide and excess oxygen will react and cause scaling losses. The proportion of loss from ingots and slabs is very small, but it can be high from shapes which are thin or of small cross-section, such as sheet, strip, rod, and wire. Removal of the scale by pickling is expensive, disposal of the waste liquors difficult, and the surface finish is impaired. In many cases the impairment of the surface and the changes in dimensions cannot be tolerated and means must be found to eliminate oxidation and pickling.

Heating in an atmosphere of controlled composition will remove both of these difficulties, but it is important to ensure that the analysis of the atmosphere is appropriate to the composition of the metal involved. Inert atmospheres of nitrogen or argon can be used for some purposes, but even nitrogen can react with ferrous alloys and argon is extremely expensive. Neither gas has any protective or remedial action in the event of temporary failure of supply. Controlled combustion of fuels containing carbon and hydrogen can result in the production of gases containing carbon monoxide, carbon dioxide, hydrogen, and water, as well as nitrogen if air is used for combustion. If copper is involved, a hydrogen–steam atmosphere containing 99% water reduces cuprous oxide at 800°C and a clean metal surface remains bright, but iron is oxidized to magnetite under the same conditions. Iron forms cementite on the surface if exposed to carbon monoxide at 950°C and is oxidized if exposed to carbon dioxide at the same temperature. For these reasons mixtures of hydrogen and water and carbon monoxide and carbon dioxide are used to prevent oxidation and change of carbon content where this is desired.

In certain cases, the carbon content of ferrous components may be raised in order to improve hardness and wear resistance. It is therefore essential to be able to calculate the composition of an atmosphere at any particular temperature and then to ascertain if any chemical reactions will take place. Various surface reactions may take place during heating and cooling in a controlled atmosphere but at the end of the treatment the metal must be left in the required state, free from any etching or other effects. In many cases no surface reactions must be allowed to take place and the rate of heating and cooling, together with the atmosphere, must be determined carefully.

8.2. REACTIONS WITHIN FURNACE

The following reactions within the furnace atmosphere and between atmosphere and charges are the most important:

$$\mathrm{C} + \mathrm{CO_2} \rightleftharpoons 2\,\mathrm{CO} \tag{1}$$

$$2\,\mathrm{H_2} + \mathrm{O_2} \rightleftharpoons 2\,\mathrm{H_2O} \tag{2}$$

$$2\,\mathrm{CO} + \mathrm{O_2} \rightleftharpoons 2\,\mathrm{CO_2} \tag{3}$$

$$\mathrm{H_2O} + \mathrm{CO} \rightleftharpoons \mathrm{H_2} + \mathrm{CO_2} \tag{4}$$

$$2\frac{X}{Y}M + \mathrm{O_2} \rightleftharpoons \frac{2}{Y}M_X\mathrm{O}_Y \tag{5}$$

$$\mathrm{C} + \mathrm{O_2} \rightleftharpoons \mathrm{CO_2} \tag{6}$$

$$2\,\mathrm{C} + \mathrm{O_2} \rightleftharpoons 2\,\mathrm{CO} \tag{7}$$

The thermodynamic relationship

$$\Delta G^0 = \Delta H - T \cdot \Delta S. \tag{8}$$

is also important for every one of these reactions. For reaction (5), if 1 mol of oxygen at 1 atm pressure reacts with metal, then

$$-\Delta G^0 = RT \log_e K_p \qquad (9)$$

$$K \text{ for reaction (5)} = \frac{[M_X O_Y]^{2/Y}}{[M]^{2X/Y} p\,O_2}.$$

For pure reactants, taking the activity of pure solid reactants and products as unity, $K = 1/p\,O_2$.

Thus $\Delta G^0 = RT \log_e p\,O_2$, where $p\,O_2$ is known as the oxygen potential of the system. (ΔG^0 = standard free energy change; T = absolute temperature; ΔS = entropy change; ΔH = enthalpy change; R = gas constant = 1·98 cal °C^{-1} mol^{-1}; [ ] = mol and pure materials.)

At equilibrium, this is equal to $p\,O_2$ for either the CO/CO_2, the H_2/H_2O, or the combined systems.

Consider the reaction

$$2CO + O_2 \rightleftharpoons 2CO_2.$$

$$K = \frac{p^2\,CO_2}{p^2\,CO \cdot p\,O_2}.$$

Thus $$p\,O_2 = \frac{p^2\,CO_2}{K \cdot p^2\,CO}.$$

At equilibrium, $$p\,O_2 = \text{antilog}\,\frac{\Delta G^0}{RT}.$$

A value of $p\,O_2$ can be calculated for each metal oxidation reaction, and it can be equated to the corresponding CO/CO_2 ratio. It can be seen that the metal will not be oxidized if $p\,O_2$ for the above reaction at a given temperature is less than $p\,O_2$ from the metal–oxygen reaction at the same temperature.

The system H_2/H_2O can be treated in a similar manner.

Production of atmospheres from carbon monoxide and hydrogen is expensive and in most cases atmospheres containing carbon monoxide, hydrogen, carbon dioxide, and water are prepared from cheaper sources such as producer gas, water gas, natural gas, and liquefied petroleum gases. In complicated systems the water–gas shift reaction must also attain its equilibrium:

$$CO_2+H_2 \rightleftharpoons CO+H_2O.$$

At temperatures above 907°C the tendency is for this reaction to proceed from right to left and vice versa.

It is possible to calculate the oxygen potential from the thermodynamic functions such as (8) and (9). However, if ΔG^0 for reaction of 1 mol of oxygen at 1 atm pressure is plotted as a function of temperature, curves known as Ellingham diagrams are obtained. Such a series of curves is given in Fig. 8.1.

The value of these graphs lies in that they indicate clearly the oxygen potential of any metal–metal oxide system. Those reactions with the greatest negative value of ΔG^0 involve the metals with the greatest affinity for oxygen. Thus, in theory, an element below another in the table will reduce the oxides of all elements placed above it. The graphs are constructed for 1 atm pressure and it can be seen that all the reactions can be represented by curves whose negative value of ΔG^0 decreases with rising temperature. Carbon, when oxidized to carbon monoxide, occupies a unique position in that with increasing temperature the negative value of ΔG^0 increases. This is because of the increase in ΔS in the general equation: $\Delta G^0 = \Delta H - T\Delta S$.

The oxidation of metals represents a decrease in the disorder of the system, i.e. it becomes more ordered as the oxygen gas phase vanishes. Since S is a measure of the disorder of a system, for the reaction $C+O_2 \rightleftharpoons CO_2$ there is only a small change, hence the line is almost horizontal. For the reaction $2\,C+O_2 \rightleftharpoons 2\,CO_2$, there is an increase in the disorder of the system.

This is the explanation why carbon can be described as the "key to extraction metallurgy", because if the temperature is raised sufficiently high then the curve for oxidation of carbon to carbon monoxide can be brought below the metal oxidation curve. Under these conditions the metal oxide will be reduced, but there may be a complication in certain cases due to carbide formation.

As most combustion gases contain both carbon dioxide and water it is necessary to consider their oxidizing effect; individually and in admixture with one another.

Consider the straightforward oxidation reaction

$$2\frac{X}{Y}M+O_2 \rightleftharpoons \frac{2}{Y}M_XO_Y,$$

$$\Delta G_T^0 = RT \log_e p\,O_2.$$

If $p\,O_2$ is made equal to 10^0, 10^{-1}, 10^{-2}, ... 10^{-n} $\Delta G_T = 0$, $-2{\cdot}303RT$, $-4{\cdot}606RT$, ... $-2{\cdot}303nRT$, and can be represented by a series of straight lines radiating from the origin

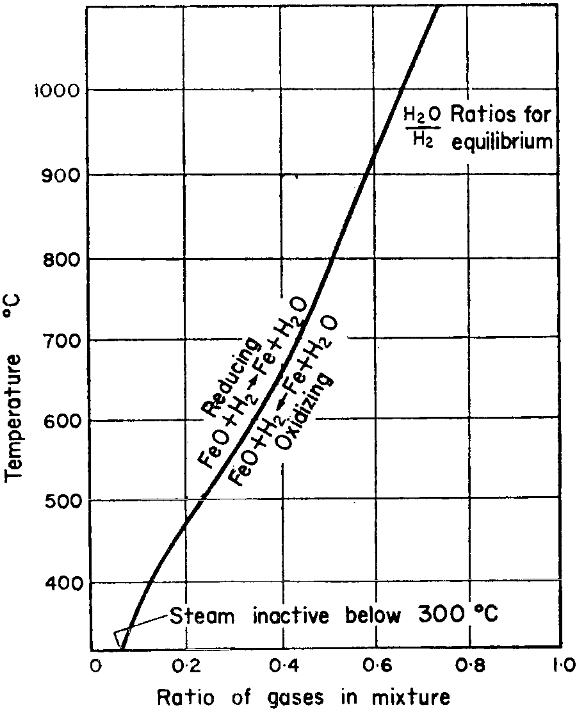

FIG. 8.2. Reduction of iron oxide by hydrogen.
(Reproduced from *Controlled Atmospheres for Heat Treatment of Metals* by Ivor Jenkins, Chapman & Hall, 1951.)

with slopes, 0, $-2{\cdot}303R$, $-4{\cdot}606R$, and $-2{\cdot}303nR$ respectively. Such lines are given in Fig. 8.1, and they allow the oxygen potential of any system to be obtained quickly. The diagram shows no more than can be calculated but it gives an immediate indication of the value of p O_2 at a particular temperature in equilibrium with a metal, carbon or hydrogen.

It is probable that the heat treatment of steel provides the greatest number of problems relating to oxidation losses, with a correspondingly extended use of controlled atmospheres. The graphs in Figs. 8.2 and 8.3 show the effect of H_2/H_2O and CO/CO_2 mixtures with varying temperature on the oxidation of iron. In addition to their individual effects it is important to consider the reactions:

$$CO_2+C \rightleftharpoons 2CO$$
$$CO_2+H_2 \rightleftharpoons CO+H_2O.$$

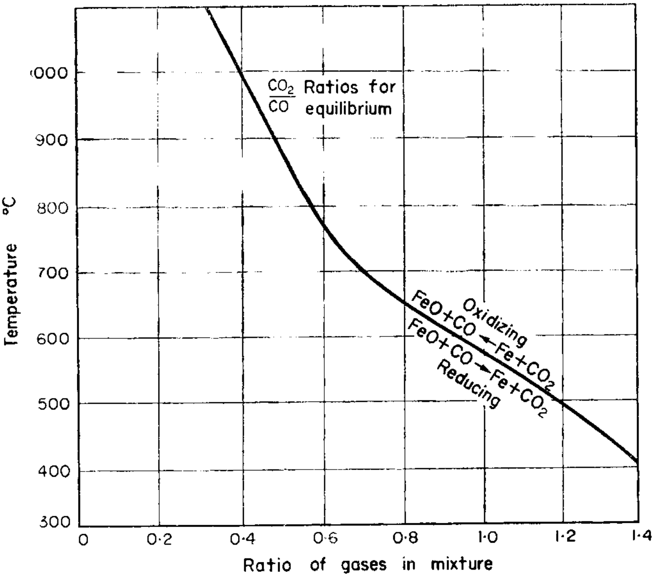

FIG. 8.3. Reduction of iron oxide by carbon monoxide. (Reproduced from *Controlled Atmospheres for Heat Treatment of Metals* by Ivor Jenkins, Chapman & Hall, 1951.)

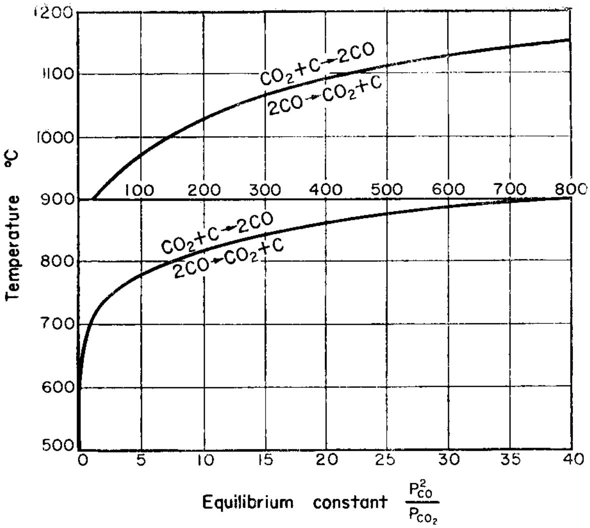

FIG. 8.4. Equilibrium relationship for the reaction CO_2 and $C \rightleftharpoons 2\,CO$. (Reproduced from *Controlled Atmospheres for Heat Treatment of Metals* by Ivor Jenkins, Chapman & Hall, 1951.)

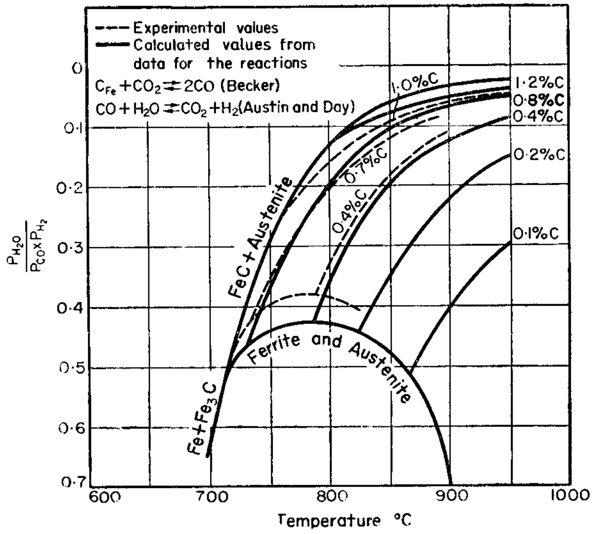

FIG. 8.5. Equilibrium for the system $C_{Fe}+H_2O \rightleftharpoons CO+H_2$. (Reproduced from *Controlled Atmospheres for Heat Treatment of Metals* by Ivor Jenkins, Chapman & Hall, 1951.)

The effect of these reactions is illustrated in Figs. 8.4 and 8.5. In the heat treatment of steel there is the added complication of carbon being present, and, since it is in most cases important that there is no change in the quantity of carbon present, the $CO/CO_2/C_{\text{in Fe}}$ system must be examined.

The variations in this case are between carburizing and decarburizing and are also shown graphically in Fig. 8.5.

8.3. CARBURIZING OF STEEL (FIG. 8.6)

Assume that the initial carbon content of the surface of the metal is zero, and that a constant carbon potential is available so that the concentration of diffusing carbon is a constant value C_0. If the depth of case is X after time t then the concentration gradient can be represented by a straight line, and if the increase in case depth in time dt is dX then the amount diffused per unit surface area of section is given by Fick's law:

$$\frac{DC_0\,dt}{X},$$

where D is the diffusivity in weight per unit area per unit time. The amount diffused per unit surface area in time dt will be the area of triangle $EAB = \frac{1}{2}C_0\,dX$.

Thus $$\frac{1}{2}C_0\,dX = D\frac{C_0}{X}\,dt.$$

Thus $$X^2 = 4Dt \quad (\text{if } X = 0 \quad \text{when} \quad t = 0).$$

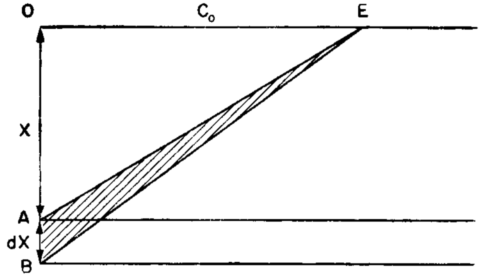

FIG. 8.6. Carburizing of steel

8.4. DECARBURIZING OF STEEL

The diagram (Fig. 8.7) represents the condition of a steel after decarburization has been in progress for a time t. The carbon concentration at the surface has been reduced to zero. C_1 represents the original concentration of carbon (dissolved and as carbide), and C_2 the maximum solid solubility of carbon in solution. It is assumed that carbide dissolves instantaneously to maintain C_2 at E, and the diffusion constant is likewise assumed constant (not strictly true).

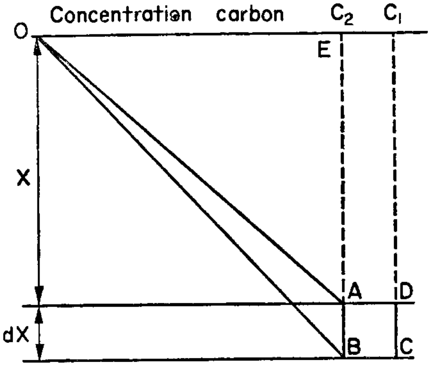

FIG. 8.7. Decarburizing of steel

In time dt carbon transferred from E to O is given by Fick's law:

$$D\frac{C_2}{X}\,dt.$$

The carbon transferred is given by sum of areas OAB and $ABCD$

$$= \tfrac{1}{2}C_2\,dX + (C_1 - C_2)\,dX.$$

Thus

$$D\frac{C_2}{X}\,dt = \tfrac{1}{2}C_2\,dX + (C_1 - C_2)\,dX,$$

$$X^2 = \frac{DC_2t}{C_1 - \frac{1}{2}C_2} + \text{constant}.$$

If $$X = 0 \quad \text{when} \quad t = 0,$$

$$X^2 = \frac{4DC_2t}{2C_1 - C_2}.$$

If W = weight loss per unit area of surface,

$$W = X(C_1 - \tfrac{1}{2}C_2).$$

Where $C_1 = C_2$, i.e. a carbon steel with no free cementite,

$$X^2 = 4Dt.$$

D varies considerably with temperature, and increases by a factor of approximately 4 between 925° and 1000°C. D is not constant and varies with both carbon content and alloying constituents. The equations derived are based upon the assumption that the process is diffusion controlled. It can also be controlled by either the rate of dissolution of free carbide or by the rate of reaction at the surface.

In selecting atmospheres for gas carburizing it is important to be able to control the uptake of carbon.

Carbon monoxide, methane, propane, and butane are the simplest carburizing gases. Carbon monoxide carburizes at a higher rate if hydrogen is present because of the equilibrium:

$$CO + H_2 \rightleftharpoons C_{Fe} + H_2O.$$

This and similar reactions can be controlled by adjustment of the dewpoint relative to carbon monoxide and hydrogen content. Carburization ceases when using a CO/H_2 atmosphere when 0·5% water is produced in the above reaction. Because of this large volumes of these gases would be required to achieve a reasonable depth of carburization. This can be overcome if hydrocarbon gases are added, and with methane the reaction is $CH_4 \rightleftharpoons C_{Fe} + 2\,H_2$.

Hydrocarbons are not used neat because they pyrolyse and form soot at concentrations just above that required to fully carburize steel. In the absence of catalysts the approximate decomposition temperatures for hydrocarbon gases are:

methane 680°C, ethane 490°C, propane 460°C, and butane 430°C. One volume of methane is equivalent to 100 volumes of 1 : 1 CO/H_2 mixture.

8.5. EXAMPLES

EXAMPLE 8.1

Copper powder has been compressed and is to be sintered at 900°C. A gas consisting of carbon monoxide and carbon dioxide is available. Calculate the composition which would just prevent oxidation at this temperature.

$$2\,Cu+O_2 = 2\,Cu_2O.$$

From Fig. 8.1, $p\,O_2 = 10^{-9}$ atm at 900°C. At this temperature CO/CO_2 ratio $= 1/10^4$ atm.

EXAMPLE 8.2

Derive an equation relating depth of decarburization X, at time t, for a structure containing C g/cm^3 of carbon but whose maximum solid solubility for carbon at the decarburizing temperature is S g/cm^3 and $S > C$. Assume a diffusivity constant D g/cm^2/sec, independent of concentration; a linear carbon gradient in decarburized layer; and sufficient excess of easily dissolved carbon to ensure that the decarburization is diffusion controlled.

Comment upon the validity of any assumptions and show how the equation applies, or may be modified to apply, to the practical cases of (a) white irons, (b) 0·6% carbon steel at 850°C, and (c) spheroidized steel of eutectoid composition at 700°C.

[Institution of Metallurgists, Part IV, Applied Metallurgy A.]

The first part of this question has been adequately answered in the text.

In answering the second part it is necessary to define the term "decarburization". In the text this was taken to be when the carbon content of the surface layer is reduced to zero.

However, metallurgically this is taken as when no carbon can be detected metallographically at room temperature. Although the solubility of carbon is low at room temperature it is not zero and the metallographic end point is reached when there is still carbon in solution.

In case (a) the quantity of carbon in the metal is above the maximum solubility and it is dispersed heterogeneously in the metal. The rate of decarburization is likely to be determined by the rate of dissolution of this dispersed carbon in the metal, rather than by the rate of diffusion of carbon to the surface.

In case (b) the carbon is completely in solution and the rate of decarburization is limited only by the rate of diffusion at this temperature. Since the concentration gradient is maintained at a maximum this will depend upon the diffusivity D.

In case (c) the solubility of carbon is very low, and the rate of diffusion will therefore be limited by the concentration gradient possible, rather than by the diffusivity at this temperature. The rate at which carbon dissolves is unlikely to be the rate determining factor.

EXAMPLE 8.3

Write a review of the mechanism of diffusion and discuss the significance of the Kirkendall effect.

A piece of steel containing 0·2% carbon is to be carburized at 930°C until the carbon content is raised to 0·6% at a depth of 0·050 cm below the surface. The carburizing gas holds the surface concentration at 1% carbon for all times (Fig. 8.8). If D is approximately $1{\cdot}4\times10^{-7}$ cm^2 sec^{-1} for all compositions, and given that the composition half-way between the initial and ultimate concentrations occurs when $X/\sqrt{(Dt)} = 0{\cdot}954$, calculate (a) the time required at the carburizing temperature of 930°C, (b) the temperature required to get a 0·6% carbon at a depth of 0·10 cm in the same time as 0·6% carbon was attained at a depth of 0·050 cm at 930°C. ($D_0 = 0{\cdot}25$ cm^2 sec^{-1}, $\Delta H = -34{,}500$ cal/mol, $R = 1{\cdot}987$ cal/deg mol.)

[University of Wales, Cardiff, Hons. Metallurgy.]

(a) The composition half-way between the initial and ultimate (maximum theoretical) concentrations occurs when :

$$\frac{X}{\sqrt{(Dt)}} = 0{\cdot}954,$$

i.e. composition of 0·6 % carbon occurs at $X/\sqrt{(Dt)} = 0{\cdot}954$.

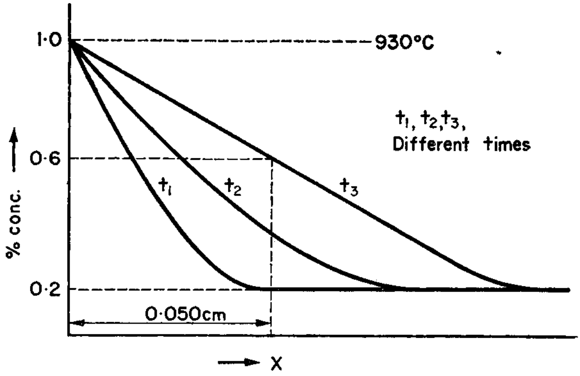

FIG. 8.8.

Therefore the time required to give 0·6 % carbon at a distance 0·050 cm is given by:

$$\frac{0{\cdot}050}{\sqrt{(1{\cdot}4\times10^{-7}\,t)}} = 0{\cdot}954.$$

Therefore
$$\sqrt{t} = \frac{0{\cdot}050}{0{\cdot}954\times\sqrt{(1{\cdot}4\times10^{-7})}}$$

$$t = \frac{0{\cdot}050^2}{0{\cdot}954^2\times1{\cdot}4\times10^{-7}}$$

$$t = 19{,}630 \text{ sec} = 5{\cdot}45 \text{ hr}.$$

(b) Assume that the expression for the half-way composition $X/\sqrt{(Dt)} = 0{\cdot}954$ still holds at the higher T_2. The 0·6 % carbon concentration now occurs at 0·10 cm in $1{\cdot}963\times 10^4$ sec.

Therefore $$\frac{0{\cdot}10}{\sqrt{(D_2\times 1{\cdot}963\times 10^4)}} = 0{\cdot}954,$$

$$\sqrt{D_2} = \frac{0{\cdot}10}{0{\cdot}954\times\sqrt{(1{\cdot}963\times 10^4)}}.$$

$$D_2 = \frac{10^{-6}}{0{\cdot}954^2\times 1{\cdot}963\times 10^4} = 5{\cdot}599\times 10^{-7}\ \text{cm}^2\,\text{sec}^{-1}.$$

In order to find T_2 the expression $D = D_0 \exp[-\Delta H/RT]$ must be used. This is the general temperature dependence of the diffusion constant.

Thus $$\log_e D = -\frac{\Delta H}{RT}+\log_e D_0,$$

$$\log_{10} D_2 = -\frac{\Delta H}{2{\cdot}303\,RT_2}+\log_{10} D_0.$$

However, $$D_2 = 5{\cdot}599\times 10^{-7}\ \text{cm}^2\,\text{sec}^{-1}.$$

Therefore $$\log(5{\cdot}599\times 10^{-7}) = \frac{34{,}500}{2{\cdot}303\times 1{\cdot}987\,T_2}+\log 0{\cdot}25,$$

$$-6{\cdot}2519 = -\frac{34{,}500}{2{\cdot}303\times 1{\cdot}987\times T_2}-0{\cdot}6021,$$

$$5{\cdot}6498 = \frac{34{,}500}{2{\cdot}303\times 1{\cdot}987\times T_2}.$$

Therefore $$T_2 = \frac{34{,}500}{2{\cdot}303\times 1{\cdot}987\times 5{\cdot}6498},$$

$$T_2 = 1337\ \text{K}$$

$$= 1064^\circ\text{C}.$$

Alternatively, if the value of D_0 had not been known the problem could have been solved:

$$\log_{10}(1{\cdot}4\times 10^{-7}) = \frac{-34{,}500}{2{\cdot}303R\,1203}+\log D_0$$

$$\log_{10}(5{\cdot}598\times 10^{-7}) = \frac{-34{,}500}{2{\cdot}303RT_2}+\log D_0$$

$$-6{\cdot}8539+6{\cdot}2520 = \frac{-34{,}500}{2{\cdot}303R}\left[\frac{1}{1230}-\frac{1}{T_2}\right]$$

$$-0{\cdot}6019 = -7500\left[\frac{T-1203}{1203T}\right]$$

$$T = 1330\text{ K} = T_2$$

$$T_2 = 1057\,^\circ\text{C},$$

which is in close approximation to the previously calculated value.

EXAMPLE 8.4

Producer gas, analysis CO_2, 5·2; CO, 25·1; H_2, 12·3; N_2, 57·4% by volume at NTP is fed into a furnace at 900°C. If the total pressure in the furnace is maintained at 1 atm pressure calculate the actual composition of the gas at this temperature and pressure.

(1) $C_{(s)}+\frac{1}{2}O_{2(g)} \rightleftharpoons CO_{(g)}\,\Delta G^0 = -28{,}100-20{\cdot}20T$ cal.

(2) $C_{(s)}+O_{2(g)} \rightleftharpoons CO_{2(g)}\,\Delta G^0 = -94{,}640-0{\cdot}05T$ cal.

(3) $H_{2(g)}+\frac{1}{2}O_{2(g)} \rightleftharpoons H_2O_{(g)}\,\Delta G^0 = -60{,}180+13{\cdot}93T$ cal.

(Thermodynamic data from Spiers.)

Hence at 900°C, 1173 K:

$$\Delta G^0_{(1)} = -51{,}795\text{ cal.}$$

$$\Delta G^0_{(2)} = -94{,}698\text{ cal.}$$

$$\Delta G^0_{(3)} = -43{,}895\text{ cal.}$$

In addition to reactions 1, 2 and 3 listed above, the appropriate water–gas equilibrium must also be set up.

$$CO_{(g)}+H_2O_{(g)} \rightleftharpoons CO_{2_{(g)}}+H_{2_{(g)}} \tag{4}$$

$$\Delta G^0_{(4)} = \Delta G^0_{(2)}-(\Delta G^0_{(1)}+\Delta G^0_{(3)}) = +722 \text{ cal.}$$

$$-\Delta G^0 = 2{\cdot}303R\ 1173 \log_{10} K_p.$$

$$\log_{10} K_p = \frac{-722}{4{\cdot}575\times 1173} = -0{\cdot}1346 \text{ for reaction (4)}$$

$$K_p = \text{antilog}-0{\cdot}1346$$

$$= \text{antilog } \bar{1}{\cdot}8654$$

$$K_p = 0{\cdot}7352. \tag{5}$$

The equilibrium constant for reaction (4) is given by

$$K_p = \frac{p\,CO_2\times p\,H_2}{p\,CO\times p\,H_2O} = 0{\cdot}7352 \tag{6}$$

assuming ideal behaviour of the gases.

Nitrogen in the atmosphere undergoes no change and since the total pressure is 1 atm,

$$p\,CO+p\,CO_2+p\,H_2+p\,H_2O = 1-0{\cdot}574(p\,N_2) = 0{\cdot}426 \text{ atm.} \tag{7}$$

By the law of conservation of matter, the number of atoms of carbon, hydrogen, and oxygen must remain the same at all temperatures and pressures. Since the number of atoms is directly related to partial pressure of the gases:

C atoms $= p\,CO+p\,CO_2 =$ constant.
H atoms $= 2p\,H_2+2p\,H_2O =$ constant.
O atoms $= p\,CO+2p\,CO_2+p\,H_2O =$ constant.

The original gas analysis gives:

$p\,CO = 0{\cdot}251$; C atoms $= 0{\cdot}251+0{\cdot}052 = 0{\cdot}303$.
$p\,CO_2 = 0{\cdot}052$; O atoms $= 0{\cdot}251+0{\cdot}104 = 0{\cdot}355$.
$p\,H_2 = 0{\cdot}123$; H atoms $= 0{\cdot}246$.

Therefore C atoms $= 1{\cdot}23$ H atoms.
C atoms $= 1{\cdot}17$ O atoms.
O atoms $= 1{\cdot}44$ H atoms.

It is necessary to use the above information to evaluate any three unknown expressions in eqn. (6) in terms of the fourth. In this way the partial pressures of all the constituents at 900°C can be found.

$$\text{Number of C atoms} = p\,CO + p\,CO_2$$

$$= 1{\cdot}23\ (2p\,H_2 + 2p\,H_2O)$$

$$p\,CO + p\,CO_2 = 2{\cdot}46p\,H_2 + 2{\cdot}46p\,H_2O.$$

If this is substituted in eqn. (7) then

$$2{\cdot}46p\,H_2 + 2{\cdot}46p\,H_2O + p\,H_2 + p\,H_2O = 0{\cdot}426.$$

$$3{\cdot}46p\,H_2 + 3{\cdot}46p\,H_2O = 0{\cdot}426.$$

$$p\,H_2 = 0{\cdot}123 - p\,H_2O. \qquad (8)$$

It is necessary to find expressions for $p\,CO$ and $p\,CO_2$ in terms of $p\,H_2O$.

$$\text{Number of O atoms} = p\,CO + 2p\,CO_2 + p\,H_2O.$$

$$\text{Number of O atoms} = 1{\cdot}44\ (2p\,H_2 + 2p\,H_2O).$$

$$p\,CO + p\,CO_2 = 1{\cdot}88p\,H_2O + 2{\cdot}88p\,H_2 - p\,CO_2.$$

Substituting in eqn. (7):

$$1{\cdot}88\,p\,H_2O + 2{\cdot}88\,p\,H_2 + p\,H_2 + p\,H_2O - p\,CO_2 = 0{\cdot}426;$$

$$2{\cdot}88\,p\,H_2O + 3{\cdot}88\,p\,H_2 - p\,CO_2 = 0{\cdot}426;$$

$$p\,CO_2 = 2{\cdot}88\,p\,H_2O + 3{\cdot}88\,p\,H_2 - 0{\cdot}426.$$

From eqn. (8), $H_2 = 0{\cdot}123 - p\,H_2O$.

Therefore

$$p\,CO_2 = 2{\cdot}88\,p\,H_2O + 3{\cdot}88\,(0{\cdot}123 - p\,H_2O) - 0{\cdot}426;$$

$$p\,CO_2 = 0{\cdot}051 - p\,H_2O. \qquad (9)$$

Substituting these values into eqn. (7):

$$p\,CO + (0{\cdot}051p\,CO_2 - p\,H_2O) + (0{\cdot}123p\,H_2 - p\,H_2O) + p\,H_2O = 0{\cdot}426;$$

$$p\,CO = 0{\cdot}252 + p\,H_2O. \qquad (10)$$

Therefore $$0{\cdot}7352 = \frac{(0{\cdot}051 - p\,H_2O)\,(0{\cdot}123 - p\,H_2O)}{(0{\cdot}252 + p\,H_2O)\,p\,H_2O},$$

$$0{\cdot}265p^2\,H_2O - 0{\cdot}359p\,H_2O + 0{\cdot}00627 = 0.$$

This is a simple quadratic equation.

$$p\,H_2O = \frac{0{\cdot}359 \mp \sqrt{[(0{\cdot}359)^2 - 4 \times 0{\cdot}265 \times 0{\cdot}00627]}}{2 \times 0{\cdot}265}$$

$$p\,H_2O = 0{\cdot}018 \text{ atm.}$$

Thus $$p\,CO_2 = 0{\cdot}051 - 0{\cdot}018 = 0{\cdot}033 \text{ atm.}$$

$$p\,CO = 0{\cdot}252 + 0{\cdot}018 = 0{\cdot}27 \text{ atm.}$$

$$p\,H_2 = 0{\cdot}123 - 0{\cdot}018 = 0{\cdot}105 \text{ atm.}$$

COMPOSITION GAS, PERCENTAGE VOLUME

	900°C	Original at NTP
N_2	57·4	57·4
CO	27·0	25·1
CO_2	3·3	5·2
H_2O	1·8	0·0
H_2	10·5	12·3
	100·0	100·0

The calculation just performed is rather tedious and with more variables in the initial gas composition it becomes very long. The equations are then best solved by the use of determinants. Also, the equilibrium constant for the water–gas reaction may be obtained directly from tables (Spiers) without the need to obtain it from thermochemical data.

BIBLIOGRAPHY

BODSWORTH, C. and APPLETON, A. S., *Problems in Applied Thermodynamics*, Longmans, Green & Co. Ltd., 1965.

GILCHRIST, J. D., *Extraction Metallurgy*, Pergamon Press, 1967.

HOPKINS, D. W., *Physical Chemistry of Metal Extraction*, J. Garnet Miller, London, 1954.

JENKINS, I., *Controlled Furnaces for Heat Treatment of Metals*, Chapman & Hall Ltd., 1951.

SPIERS, H. M. (Ed.), *Technical Data on Fuel*, 6th edn., London, British World Power Conference Committee, 1962.

Heat Treatment of Metals, Institution of Metallurgists, Refresher Course, 1962, Iliffe Books, London.

The Efficient Use of Fuel, 2nd edn., HMSO, 1958.

Additional Examples

1. (a) Sketch the Orsat flue gas testing apparatus. Describe its method of use assuming the fuel is principally carbon, together with a small percentage of hydrogen. (b) A coal has the following percentage analysis by weight, C, 85%; H, 5%; O, 4%; and ash, 6%.

Calculate (i) the theoretical weight of air supplied per unit weight fuel burnt, (ii) the volume of air supplied at NTP.

[Institution of Heating and Ventilating Engineers, Section B, Principles of Heating, Combustion, and Air Conditioning.]

Ans. (i) 11·4 lb/lb or 11·4 kg/kg. (ii) 141·3 ft^3/lb or 8·75 m^3/kg.

2. Calculate the percentage heat loss due to the heat in the flue gases when the following fuels are completely burned in the theoretical amount of air.

Solid fuel: C, 74%; H, 5%; CV, 12,000 Btu/lb or 27,900 kJ/kg.

Gaseous fuel CH_4, CV 1200 Btu/ft^3 (44·6 MJ/m^3). Assume flue temperatures of 600°F (315°C) and 500°F (260°C) respectively, and ambient temperature 60°F (15°C).

Mean specific heats at 60°F (15°C), CO_2, 0·0226 (1·516); H_2O, 0·0210 (1·409); N_2, 0·0184 (1·245); air, 0·0196 (1·316) Btu/ft^3 °F (kJ/m^3 °C).

[Institution of Heating and Ventilating Engineers, Section C, Group V, Combustion Engineering.]

Ans. (a) 11·4%. (b) 7·5%.

3. Calculate the composition of the flue gases when a fuel containing 76% C, 8% H, 9% moisture and 7% ash, is burnt with the theoretical amount of air.

If the carbon dioxide content of the dry flue gases is 10%, calculate the percentage of excess air and loss of heat when the flue temperature is 500°F (260°C), ambient temperature 60°F (15°C), and the calorific value of the fuel 14,000 Btu/lb (32,564 kJ/kg). Specific heats as in previous question.

[Institution of Heating and Ventilating Engineers, Section C, Group V, Combustion Engineering.]

Ans. Theoretical air = 142 ft^3/lb at NTP or 8·91 m^3/kg at NTP.
Excess air = 63%.

Sensible heat loss = 14·85% (as % CV fuel).

Analysis flue gases using stoichiometric air: CO_2, 15·0%; N_2, 74·3%; H_2O, 10·7%.

4. A petroleum oil having a gross CV of 18,250 Btu/lb (42,450 kJ/kg) and containing (by weight) 85·4% C, 11·2% H, 2·4% S, and 1·0% O is burned completely with air giving exhaust gases at a temperature of 545°F (285°C). Determine (a) the theoretical volume of air required to burn unit weight of oil to completion, at NTP; (b) the percentage excess air used when the dry flue gas on analysis shows 10% carbon dioxide on a volume basis; (c) the sensible heat above 60°F (15°C) in the total flue gases as a percentage of the net CV of the fuel when the dry flue gas contains 10% carbon dioxide.

[Fuel and Combustion Engineering, Swansea College of Technology.]

Ans. (a) 170 ft^3/lb or 10·6 m^3/kg. (b) 56%. (c) 15%.

5. A gas oil is used to melt a 10 ton charge of ferrous metal in a small furnace. Using the operational data given below calcuate the consumption of oil in gallons, at 15·5°C, and express this as (a) gallons per ton of charge and (b) gallons per hour of furnace operation (average).

Composition of oil: C, 85·7; H, 13·4; S, 0·9 weight %. Specific gravity oil at 15·5°C = 0·84 (referred to water at 15·5°C). Calorific value oil = 19,500 Btu/lb gross; 18,230 Btu/lb net. Maximum temperature reached by charge = 1922°F. Temperature of furnace interior = 202°F. Ambient temperature = 72°F. Mean temperature of wet gases leaving furnace = 1072°F. Mean specific heat of metal = 0·16 Btu/lb °F. Mean specific heat of products of combustion = 0·0218 Btu/ft^3 °F (measured at 32°F and 30 in.Hg). Total heat losses to storage and to surroundings during heat = 86·9 therms. Excess air = 25% over theoretical. Duration of heat = 10 hr.

1 gal water weighs 10 lb at 15·5°C.

[Institution of Metallurgists, Fuels, Old Regulations.]

Ans. (a) 13·2 gal/ton. (b) 13.2 gal/hr.

6. A heavy fuel oil contains C, 86·1; H, 11·8; S, 2·1%, and the gross calorific value of the fuel is 18,900 Btu/lb (44,100 kJ/kg). The oil temperature required at the burner is 190°F (88°C). Determine (a) the heat available in the combustion chamber (expressed as a percentage of the gross potential heat in the fuel) when the oil is completely burned and the hot waste gases leave the combustion chamber at a temperature of 600°C and contain 11·2% carbon dioxide as analysed; (b) the theoretical power required in kWh per unit weight of oil to raise the oil to the temperature of atomization; (c) the concentration of sulphur dioxide expressed as grains per cubic foot at NTP (or ppm at NTP). Mean specific heat products of combustion = 0·021 Btu/ft^3 at NTP (1·408 kJ/m^3 at NTP). Mean specific heat of fuel oil = 0·465 Btu/lb °F (1·946 kJ/kg °C).

1 kWh = 3413 Btu. 1 lb = 7000 gr. ppm = mg/l.
[Institution of Metallurgists, Fuels, Old Regulations.]

Ans. (a) 65%. (b) 0·018 kWh/lb or 0·0475 kWh/kg. (c) 1·16 gr/ft^3 or 2·87 ppm.

7. A fuel oil containing C, 84·5; H, 10·9; S, 2·3% and having a gross calorific value of 18,900 Btu/lb (44,100 kJ/kg) is burned in a combustion appliance, from which the gases are exhausted at a temperature of 660°F (350°C), the temperature of the surroundings being 60°F (15°C). The dry products of combustion contain 12·77% of (CO_2+SO_2) by volume. The total heat in the steam at 660°F (350°C) at the partial pressure of the steam in the products of combustion is 1112 Btu/lb from 32°F (2590 kJ/kg from 0°C).

Determine (i) the volume of air at NTP to burn unit weight of oil; (ii) the sensible heat (expressed as a percentage of the gross calorific value of the fuel) lost in (a) the wet products of combustion and (b) the steam contained in the waste gases. Volumetric specific heat of dry gases, mean for temperature range = 0·021 Btu/ft^3 at NTP (1·408 kJ/m^3 at NTP).
[City and Guilds Advanced, Metallurgy, Section J, Fuels.]

Ans. (i) 210 ft^3/lb or 10·5 m^3/kg. (ii) (a) 14·6%; (b) 5·8%.

8. Sketch the Orsat flue gas testing apparatus. Describe how you would determine the carbon monoxide, oxygen, and carbon dioxide contents of a flue gas.

Calculate the percentage excess air used when a solid fuel (small nitrogen content) is burned with air and gives rise to the following volumetric flue gas composition: CO_2, 11·0%; O_2, 8·5%; N_2, 80·5%.
[HNC Metallurgy, Swansea College of Technology.]

Ans. 66%.

9. A coal having the following dry, ash free analysis: C, 86·5; H, 5·4; O, 4·6; N, 2·2; S, 1·3% is used for firing a furnace.

(a) Determine the theoretical volume of air at NTP required for the complete combustion of unit weight of coal as fired if the coal contains 7·6% ash and 2·9% moisture.

(b) Determine the percentage of excess air per unit weight of dry coal used if the flue gas contains 6·1% oxygen as analysed.
[Fuel and Combustion Engineering, Swansea College of Technology.]

Ans. (a) 133·6 ft^3/lb or 8·95 m^3/kg. (b) 40%.

10. A fuel oil with the analysis on weight basis 87% C, 11% H, and 2% O is burned in a furnace and the dry products of combustion contain 3·3% O_2 on a volume basis. The gross calorific value of the fuel is 19,500 Btu/lb (45,400 kJ/kg) and the furnace operates with a flue gas temperature of 2300°F (1260°C).

Calculate (a) the stoichiometric air requirement at NTP; (b) the percentage excess air; (c) the sensible heat of the total flue gases expressed as a percentage of the net calorific value of the fuel.

Mean specific heat of flue gases = 0·022 Btu/ft^3 at NTP (°F) (1·474 kJ/m^3 at NTP (°C)).

[Institution of Metallurgists, Part II, specimen question.]

Ans. (a) 170 ft^3/lb or 10·6 m^3/kg. (b) 17·6%. (c) 55·4%.

11. The following figures were obtained for bulk densities of a number of samples from a batch of calcined dolomite.

Bulk density (g/ml): 2·75, 2·73, 2·65, 2·63, 2·52, 2·67, 2·29, 2·55, 2·63, 2·69, 2·56, 2·61, 2·55, 2·60, 2·65, 2·75, 2·55, 2·65, 2·66, 2·49, 2·08, 2·43, 2·58, 2·66, 2·61.

Assuming a normal distribution determine: (a) the average or arithmetic mean, (b) the standard deviation, (c) the range, (d) the coefficient of variation.

Comment upon the importance of sampling of refractory materials and discuss the significance of the above derivations.

[HNC Metallurgy, Swansea College of Technology.]

Ans. (a) 2·581 g/ml. (b) 0·146 g/ml. (c) 0·67 g/ml. (d) 5·6%.

12. A mixed fuel burner uses 200 ft^3 of towns gas measured at NTP together with 1 lb of fuel oil; 15% excess air is used in combustion. The air is pre-heated to a temperature of 200°F in a recuperator heated by the exhaust gases. Its efficiency is 31·1% in terms of the pre-heat in the air as a proportion of the heat in the gases entering the recuperator. The fuel oil has the composition: C, 90; H, 5·9; S, 0·4; N, 1·2%. Two hundred cubic feet of towns gas produces on theoretical combustion 948 ft^3 of products of combustion and requires 812 ft^3 air. The mean specific heats of the air and gaseous products are respectively 0·0194 and 0·02 Btu/ft^3 at NTP. Ambient temperature is 60°F.

If the gases leaving the recuperator are not to fall below 300°F, determine: (a) the temperature at which the exhaust gases should enter the recuperator; and (b) the logarithmic mean temperature difference of the hot and cold streams in a counterflow recuperator. What purpose does this criterion serve in design? Why should the exhaust gases not fall below 300°F on leaving the air heater?

[City and Guilds Advanced, Fuel Plant Technology.]

Ans. (a) 687°F. (b) 349°F.

13. A coke containing, on the dry, ash free basis, 95% C, and when used 2% moisture and 10% ash, is gasified by means of an air-steam blast, giving a gas as analysed of the volumetric composition: CO, 25·1; CO_2, 5·2; H_2, 12·3; CH_4, 0·4%; remainder nitrogen. The gross calorific value of the coke as used is 12,000 Btu/lb (27,900 kJ/kg).

Determine the volume of gas generated per ton (tonne) of coke gasified and thence the percentage of the gross potential heat in the fuel available as gas.

Gross CV for CO, 341 (12·7); H_2, 343 (12·76); CH_4, 1067 (39·7) Btu/ft^3 (MJ/m^3) at NTP.

[Institution of Metallurgists, Fuels, Old Regulations.]

Ans. Volume gas per ton = 182,000 ft^3 at NTP.
Volume gas per tonne = 5080 m^3 at NTP.
Required % = 83%.

14. A gas of volumetric percentage composition: CO_2, 21·8; CO, 34·7; H_2, 39·4; CH_4, 1·7; N_2, 0·9; H_2S, 1·5; is generated from a fuel containing as used; C, 54·2; H, 4·1; O, 13·9; N, 0·8; S, 3·3; ash, 15·3; moisture, 8·4% by weight. For the gasification process there are used 270 ft^3 (m^3) oxygen, 300 ft^3 (m^3) of steam, both measured at NTP, per thousand cubic feet (metres) of gas made. The yield of gas is stated to be 15·5 ft^3 (m^3) at NTP, per pound (kilogram) of fuel gasified.

Investigate the efficiency of utilization of the steam and the carbon. Mention a type of gasification plant to which such figures could be applicable and comment on the accuracy of the data.

[City and Guilds Advanced, Fuel Plant Technology.]

Ans. Efficiency carbon utilization = 52%.
Efficiency steam utilization = 50%.

15. "In many metal finishing operations the quality of the final product is largely influenced by its previous thermal history." Discuss the significance of this statement by making particular reference to the design and operation of furnaces used in the final heat treatment of ferrous and non-ferrous products.

An endothermic gas generator burns towns gas substoichiometrically and the product purge gas is used as a furnace atmosphere in an annealing operation. Calculate the percentage air deficiency if the gases have the following compositions and indicate the range of metals which could be annealed in such an atmosphere:

Towns gas: CO_2, 2; O_2, 1; N_2, 12; CO, 7; H_2, 46; CH_4, 30; C_3H_6, 2% by volume.

Purge gas: H_2, 18; CO, 10; CO_2, 3; H_2O, 6% by volume; balance nitrogen.

[Institution of Metallurgists, Part IV, AIM, Furnace Technology.]

Ans. Deficiency = 42%.

16. In a cyclic catalytic oil gasification plant of conventional design $13·57\times10^6$ ft^3 ($0·384\times10^6$ m^3) of gas is produced over a period of 22 days. For this purpose 55,480 gal (251,000 l.) of oil are gasified to produce the gas of the following average composition.

30 in.Hg and 60°F (1013 mbar and 15°C). CO_2, 11·0; O_2, 0·4; C_nH_m, 6·6; CO, 14·7; H_2, 48·9; C_2H_6, 0·6; CH_4, 14·7; N_2, 3.0% by volume.

For heating up the catalyst during the "blow", 4·6% of the make gas is used.

Neglecting any credits for tar or debits for steam, calculate with the information given below, the net gaseous thermal yield per gallon and the thermal efficiency of the gas-making process. Oil specific gravity = 0·963 at 60°F (15°C). CV = 18,260 Btu/lb gross (42,600 kJ/kg gross). CVs at 30 in.Hg and 60°F (1013 mbar and 15°C): CO and H_2 = 320 Btu/ft^3 (11·92 MJ/m^3); C_nH_m = 2300 Btu/ft^3 (85·20 MJ/m^3); C_2H_6 = 1700 Btu/ft^3 (63·2 MJ/m^3); CH_4 = 995 Btu/ft^3 (37·05 MJ/m^3).

[City and Guilds Advanced, Liquid Fuels.]

Ans. Thermal yield = 1·2×10^5 Btu/gal.
= 0·28×10^5 kJ/l.
Efficiency = 67%.

17. Two gases *A* and *B*, initially at 60°F (15°C), of compositions indicated, are burned with the theoretical quantity of air, also at 60°F (15°C). Assuming no dissociation and neglecting the sensible heat content of the fuel and air, calculate the theoretical flame temperature attained by each of these gases. Indicate which commercial gases have compositions approximating to *A* and to *B*. What effect would dissociation have on the flame temperature? Indicate other factors which might be expected to alter the calculated theoretical flame temperature. Comment upon why the gas with the lowest calorific value gives rise to the highest theoretical flame temperature.

	Gas *A* (% by volume)	Gas *B* (% by volume)
CO_2	4·7	22·0
O_2	0·0	0·4
$C_{2\cdot5}H_5$	0·0	2·6
CO	41·0	7·4
H_2	49·0	54·0
CH_4	0·8	28·0
N_2	4·5	6·2

Net CVs: CO, 318 (11·85); H_2, 270 (10·05); CH_4, 895 (33·35); $C_{2\cdot5}H_5$, 1600 (59·5); Btu/ft^3 (MJ/m^3) at 60°F and 30 in.Hg (15°C and 1013 mbar).

Mean specific heats of combustion products between 60°F (15°C) and temperature of combustion, for gas *A*, 0·028 (1·875), and for gas *B*, 0·026 (1·744) Btu/ft^3 °F (kJ/m^3 K).

[Fuel and Combustion Engineering, Swansea College of Technology.]

Ans. 3473°F (1910°C) and 3370°F (1850°C).

18. Coal is fired in a boiler furnace at the rate of 25 lb/hr (12·2 kg/hr) per ft^2 (m^2) of grate area. If the temperature of the fuel bed is 2500°F (1370°C) and the calorific value of the fuel 13,000 Btu/lb (30,200 kJ/kg) find the fraction of the heat radiated per square foot (metre) of grate area per hour when the walls are at (a) 340°F (171°C), and (b) 540°F (282°C). Take the emissivity to be 0·9 and grey body conditions to apply.

(Stefan–Boltzmann constant $= 1{\cdot}73\times10^{-9}$ Btu/hr ft^2 R^4
$= 5{\cdot}67\times10^{-8}$ W/m^2 K^4.)

[HNC Metallurgy, Swansea College of Technology.]

Ans. (a) 0·366. (b) 0·363.

19. The walls of a furnace are built of a 6 in. (152·4 mm) thickness of refractory material of thermal conductivity 0·50 lb-cal/hr ft^2 °C/ft (0·865 W/m °C). The surface temperatures of the inner and outer faces of the refractory are 1100° and 240°C respectively. The coefficient of heat transfer from the outer surface to the air, which is at 20°C, may be taken as 4·0 lb-cal/hr ft^2 °C (22·713 W/m^2 °C).

If a layer of insulating material 1 in. (25·4 mm) thick, of thermal conductivity 0·10 lb-cal/hr ft^2 °C/ft (0·1731 W/m^2 °C), is added onto the outside wall, what temperature will the face exposed to the air attain, and what is the saving in heat? Assume that the inner surface remains at 1100°C. The coefficient of heat transfer from the outer surface of the insulating material to air at 20°C may be taken as 3·3 lb-cal/hr ft^2 °C (18·74 W/m^2 °C).

[LRIC Fuel and Industrial Chemistry, Swansea College of Technology.]

Ans. (a) Saving in heat = 358 lb-cal/hr ft^2 = 2124 W/m^2. (b) Temperature = 173°C.

20. (a) Derive from first principles the following formula:

$$U = \frac{1}{R_{s1} + R_{s2}\dfrac{L}{K}},$$

where R_{s1} = inside surface resistance; R_{s2} = outside surface resistance; L = thickness material; K = conductivity.

(b) (i) Explain the phenomenon of pattern staining on a ceiling. (ii) It is desired to eliminate pattern staining in a bungalow ceiling comprising

3/4 in. plaster below 6×2 in. wood joists. Determine the required thickness of insulation between the joists, given: conductivity of plaster = 4·0 Btu/ft^2 hr °F; conductivity of wood = 1·0 Btu/ft^2 hr °F; conductivity of insulation = 0·25 Btu/ft^2 hr °F.

[Institution of Heating and Ventilating Engineers, Section B, Principles of Heating, Combustion, and Ventilation.]

Ans. (b) (ii) 1·5 in.

21. Develop from first principles, an expression for the radial heat flow through and from walls of a thick cylinder in terms of r_1 the inner radius, and r_2 the outer radius, T_1 the temperature of the inner surface, T_2 the temperature of the outer surface, and k the thermal conductivity of the material of the walls of the cylinder.

Calculate the heat loss per hour per square foot (metre) of pipe surface from a pipe of internal diameter 4 in. (100 mm), covered with 2 in. (50 mm) of insulation of thermal conductivity 2·5 Btu/ft^2 hr °F/in. (0·346 W/m^2 °C). The temperature of the inner surface is 400°F (205°C) and the outer surface temperature is 100°F (38°C).

[HNC Metallurgy, Swansea College of Technology.]

Ans. 216 Btu/ft^2 hr (102·5 W/m^2).

22. Explain carefully the difference between counter-flow and parallel-flow operation in heat exchangers.

In a counter-flow heater 6000 ft^3/min (2·8 m^3/s) of combustion products are used to pre-heat 5000 ft^3/min (2·33 m^3/s) of air. The hot gas enters the heat exchanger at 1470°F (799°C). The air enters at 60°F (15°C) and is heated to 1000°F (538°C).

(a) Determine the temperature at which hot gases leave the exchanger, neglecting external losses.

(b) Thence determine the value of the overall heat transfer coefficient if the area of the heat exchanging surface is 810 ft^2 (73 m^2).

Mean specific heat of gases, 0·0215 (1·44), of air 0·0197 (1·32) Btu/ft^3 °F (kJ/m^3 °C). All volumes measured at NTP.

[Fuel and Combustion Engineering, Swansea College of Technology.]

Ans. (a) 752°F (395°C). (b) 12 Btu/ft^2 hr °F (68 W/m^2 °C).

23. Write down the pertinent equations for the steady flow of heat through: (a) a composite wall of different materials in solid contact, the surface film coefficients being given for the hot and cold walls; and (b) the walls of a hollow cylinder whose internal and external walls are maintained at specific temperatures.

In the case of (a) compute the heat loss per square foot (metre) of exterior surface per hour given the following data. The wall consists of 4½ in. (114·4 mm) of material A, 9 in. (228·8 mm) of material B, and 4½ in. (114·4 mm) of material C.

The respective film coefficients at the hot and cold surfaces are 4·5 (25·54) and 2·5 Btu/ft^2 hr °F (14·2 W/m^2 °C). The respective thermal conductivities are 10·0 (1·443), 2·0 (0.289), and 0·6 Btu/ft^2 hr °F/in. (0·0867 W/m^2 °C). The temperature of the surroundings at the hot face is 2192°F (1200°C). The ambient temperature (cold face) is 68°F (20°C).

Dimensions and temperatures in (b) are as follows: Length of cylinder 5 ft (1·525 m); i.d. 0·75 ft (0·2285 m); o.d. 1·0 ft (0·305 m). Temperature inner surface 536°F (280°C). Drop of temperature across rim 1·8°F (1°C).

Calculate the heat flow through the walls of the cylinder if thermal conductivity k = 128·2 Btu ft^2 hr °F/ft (222 W/m °C).

[Institution of Metallurgists, Old Regulations, Fuels.]

Ans. (a) 163 Btu/ft^2 hr (515 W/m^2). (b) 25,430 Btu/hr (7448 W).

24. The total radiation energy, in Btu/hr emitted to the surroundings from the surface of a lagged steam pipe, is $12\times10^{-10}\times(T_1^4-T_2^4)\times A$, where A is the area of the outer surface of the lagging and T_1 and T_2 are the absolute temperatures in degrees Rankine of the outer surface and the surroundings respectively.

The heat lost by convection is $0{\cdot}39\times A\times(t_1-t_2)^{1\cdot25}$, where t_1 °F and t_2 °F are the temperatures of the outer surfaces of the lagging and the surrounding air.

If the pipe diameter is 6 in. and the thickness of the lagging is 2 in., calculate graphically or otherwise the temperature of the steam at which the heat lost by radiation will be equal to the heat lost by convection.

Mean thermal conductivity lagging 0·035 Btu/ft^2 hr °F/ft. Temperature surroundings 60°F.

[City and Guilds Advanced, Fuel Engineering.]

Ans. Surface temperature = 70°F; steam temperature = 155°F.

25. Outline the basic principles underlying the construction of a suction pyrometer.

Air at atmospheric pressure flows over a thermocouple junction with a gas velocity of 10 ft/sec (0·305 m/s). The temperature of the wall T_w is 1200 R (666 K), and the couple indicates a temperature of 1500 R (833 K) (T_p). The diameter of the couple junction is 0·0042 ft (0·00126 m) and its emissivity 0·2.

For flow across wires, the convective heat transfer coefficient can be found from: $Nu = 0{\cdot}5\ Re^{0\cdot5}$. Over the temperature range considered, the thermal conductivity of air = 0·0344 Btu/ft hr °F (0·0595 W/m °C) and the kinematic viscosity $9{\cdot}44\times10^{-4}$ ft^2/sec ($0{\cdot}85\times10^{-4}$ m^2/s).

Calculate the convective and radiative heat transfer coefficients, and thence the true gas temperature.

(Nu = Nusselt group, Re = Reynolds group, Stefan–Boltzmann constant = $1{\cdot}71\times10^{-9}$ Btu/ft^2 hr R^4 = $5{\cdot}67\times10^{-8}$ W/m^2 K^4.)

[Fuel and Combustion Engineering, Swansea College of Technology.]

Ans. h_c = 27·4 Btu/ft^2 hr °F (155·4 W/m^2 °C).
h_r = 3·41 Btu/ft^2 hr °F (19·33 W/m^2 °C).

Temperature gas = 1537 R (855 K).

26. A horizontal duct is 80 ft (24·37 m) long. Its shell is 3 ft 3 in. (991 mm) internal diameter, and is lined with 4½ in. (114·4 mm) of insulating material having a mean thermal conductivity of 0·08 Btu/ft^2 hr °F/ft (0·1384 W/m °C). The internal film coefficient of conductance is 4 Btu/ft^2 hr °F (22·712 W/m^2 °C); and the external surface coefficient is 3·5 (19·87 W/m^2 °C). The ambient temperature is 60°F (15°C).

Determine the heat loss, in therms (new therms) per hour from the surface of the duct when the mean temperature of the gases flowing in the duct is 482°F (250°C).

1 new therm = 100 MJ.

[City and Guilds Advanced, Metallurgy, Section J, Fuels.]

Ans. 0·69 therm or 0·73 new therm.

27. A radiant tube, in which gaseous fuel is burnt, is used to heat a furnace operating with a controlled atmosphere. The tube is cylindrical and has an internal diameter of 6 in. (152·4 mm) and an external diameter of 7 in. (177·8 mm). The external temperature of the tube is 1500°F (810°C) and the tube radiates to a surrounding body that is at a temperature of 1400°F (760°C). If the effective emissivity of the heat exchanging surfaces, external to the tube, is 0·9, and the thermal conductivity at the mean temperature of the metal of the tube is 151 Btu/ft^2 hr °F/in. (21·8 W/m °C), and the radiation constant is $1{\cdot}72\times10^{-9}$ Btu/ft^2 hr R^4 ($5{\cdot}67\times10^{-8}$ W/m^2 K^4), determine the temperature gradient in the wall of the tube.

Discuss the factors that may enter into the problem due to the geometrical relationships of the heat exchanging surfaces.

[Institution of Metallurgists, Old Regulations, Fuels.]

Ans. Gradient = 30·5°F/in. (0·67°C/mm).

28. Write a critical account of the heat transfer mechanism which operates during the cooling of a large steel ingot from the completion of casting the molten metal at 1650°C into a heavy cast-iron mould at 200°C until the metal is solid and the mould removed, and then during air cooling to ambient temperature.

An ingot, 0·5 m square in cross-section and 2 m high, is stripped of its mould when the surface temperature is 815°C. Calculate the rate at which heat is lost to the air and the surroundings at 15°C using the formula:

$H = 2{\cdot}7\times10^{-5}\,\theta^{1{\cdot}33}$ cal/cm^2 sec for the rate of heat loss for natural convection from vertical surfaces (one-third greater for horizontal surfaces); emissivity of ingot = 0·5; and the Stefan–Boltzmann constant $= 1{\cdot}37 \times 10^{-12}$ cal/cm^2 sec K^4.

[Institution of Metallurgists, Old Regulations, Fuels.]

Ans. Loss by convection = 867,440 cal/sec.
Loss by radiation = $4{\cdot}02 \times 10^6$ cal/sec.

29. Discuss the statement "Dimensionless groups have contributed significantly to the solution of problems involving heat transfer".

A steel slab at 20°C is introduced into a furnace which is at 1250°C. Determine the time which will elapse before the central plane of the slab will reach a temperature of 1200°C.

Slab thickness = 15 cm.
Thermal conductivity = 0·075 cal/cm^2 s °C/cm.
Density = 7·5 g/cm^3.

State any assumptions necessary in the calculation.

x	erfc x
0·4	0·5716
0·45	0·5245
0·5	0·4795

[Advanced Diploma, Applied Heat, Swansea College of Technology.]

Ans. 1080 sec.

30. A rectangular flue 1 ft 9 in. by 1 ft 9 in. (53·3 mm by 53·3 mm) in cross-section exhausts the products from a furnace at the rate of 144,000 ft^3/hr (0·113 m^3/s) measured at NTP. It is 150 ft (45·6 m) long, and there are four right-angled bends in a horizontal plane. The mean temperature of the gases in the flue is 300°C (573°F). The value of the frictional factor in the Fanning equation may be taken as 0·016, when using hydraulic mean depth. The loss in pressure-head in a right-angled bend may be taken as 1·5 times the velocity head.

(i) Determine the pressure loss in the flue, and the fan power expected to be required to exhaust the gases.

(ii) Indicate the principles involved in such a calculation if two vertical ducts are interposed into the system.

Density gases = 0·0825 lb/ft^3 (1·32 kg/m^3) at NTP. 1 HP = 33,000 ft lb/min. g = 32·2 ft/sec^2 (9·81 m/s^2); 1 in. w.g. = 5·2 lb/ft^2.

[City and Guilds Advanced, Fuel Plant Technology.]

Ans. 0·8 HP (59·6 W). 1·01 in. w.g. (249 N/m^2 = 25·4 mm H_2O).

31. Give an account, illustrated by sketches, of the provisions required for the supply of fuel oil to an industrial space heating unit, indicating the character of the test data required and safety precautions necessary.

Determine the pressure drop that would occur in 100 ft (100 m) length of 3 in. (76·2 mm) internal diameter pipe, carrying at a temperature of 100°F (38°C), 2000 gal/hr (2·52 l./s) of oil fuel, having the following properties: specific gravity and kinematic viscosity at 100°F (38°C), respectively 0·86 and 0·00023 ft²/sec (0·00002136 m²/s) (the Fanning equation in accepted symbols is given as

$$P = \frac{fL\varrho u^2}{M2g} \quad \text{and} \quad f = 16/Re.)$$

[City and Guilds Advanced, Fuel Plant Technology.]

Ans. 36·5 lb/ft² per 100 ft.
5550 N/m² per 100 m.
$f = 0{\cdot}008$.

32. (a) Outline the basic principles involved in the determination of pressure drop in furnace flues due separately to velocity head, friction, and buoyancy.

(b) Determine the static draught of a chimney 100 ft (100 m) high and having a mean temperature of the gases of 350°F (177°C).

(c) For the same derive the loss of draught due to velocity head and friction given the following data:

Internal diameter = 6 ft (1·83 m).
Mean velocity gases = 15 ft/sec (4·675 m/s).
Density gases = 0·082 lb/ft³ (1·312 kg/m³) at NTP.
Frictional coefficient = 0·026.
1 in. w.g. = 5·2 lb/ft²; 1 kPa = 102 mm H_2O.
Gravitational constant = 32·17 ft/sec² = 9·81 m/s².
Ambient temperature = 60°F (15°C).
Density air = 0·08071 lb/ft³ (1·292 kg/m³) at NTP.

[Institution of Metallurgists, Old Regulations, Fuels.]

Ans. (b) 0·5 in. w.g. (441 N/m² = 45 mm water). (c) 0·09 in. w.g. (58·08N/m² = 5·81 mm H_2O).

33. Explain the principles of the orifice gauge and show how these principles are applied in the practical case of measurement of fluid flow.

A saturated gas flows in a circular main of internal diameter 20 in. (508 mm). The flow of gas is measured by means of an orifice plate with corner tappings, the diameter of the orifice is 10·88 in. (276 mm). The gas at the upstream tapping has a density of 0·034 lb/ft³ (0·55 kg/m³). The differential pressure across the orifice is 2 in. w.g. (50·8 mm w.g. = 5 kPa).

(i) Determine the rate of flow in ft^3 (m^3)/hr for the conditions applying, given the following data: C, coefficient of discharge = 0·625; m, ratio of orifice area to pipe area = 0·2959; E, velocity of approach factor = 1·047.

(ii) What additional information is required in order to determine the number of therms passing per hour? 1 in. w.g. = 5·2 lb/ft^2; g = 32·17 ft/sec^2 = 9·81 m/s^2.

[City and Guilds Advanced, Fuel Plant Technology.]

Ans. 206,700 ft^3/hr (5790 m^3/h).

LOGARITHMS

$\log_{10} x$

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1·0	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
1·1	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
1·2	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
1·3	1139	1173	1206	1239	1271						3	7	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	6	10	13	16	19	22	25	29
1·4	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
1·5	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
1·6	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
1·7	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
1·8	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
1·9	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19

LOGARITHMS

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
2·0	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
2·1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
2·2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
2·3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
2·4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
2·5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
2·6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
2·7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
2·8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
2·9	4624	4639	4564	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
3·0	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
3·1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
3·2	5052	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
3·3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
3·4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
3·5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
3·6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
3·7	5682	4694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
3·8	5798	5809	5821	5832	5843	5855	5866	5277	5888	5899	1	2	3	5	6	7	8	9	10
3·9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10

LOGARITHMS

$\log_{10} x$

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
4·0	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
4·1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
4·2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
4·3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
4·4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
4·5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
4·6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
4·7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
4·8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
4·9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
5·0	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
5·1	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
5·2	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
5·3	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
5·4	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
5·5	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
5·6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
5·7	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
5·8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
5·9	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
6·0	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
6·1	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
6·2	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
6·3	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
6·4	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
6·5	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
6·6	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
6·7	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
6·8	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
6·9	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
7·0	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
7·1	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
7·2	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
7·3	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
7·4	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
7·5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
7·6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
7·7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
7·8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
7·9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5

LOGARITHMS

LOGARITHMS $\log_{10} x$

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
8·0	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
8·1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
8·2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
8·3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
8·4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
8·5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
8·6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
8·7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
8·8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
8·9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
9·0	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
9·1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
9·2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
9·3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
9·4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
9·5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
9·6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
9·7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
9·8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
9·9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS

ANTILOGARITHMS 10^x

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

ANTILOGARITHMS

ANTILOGARITHMS 10^x

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2023	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8

ANTILOGARITHMS

ANTILOGARITHMS 10^x

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6642	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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