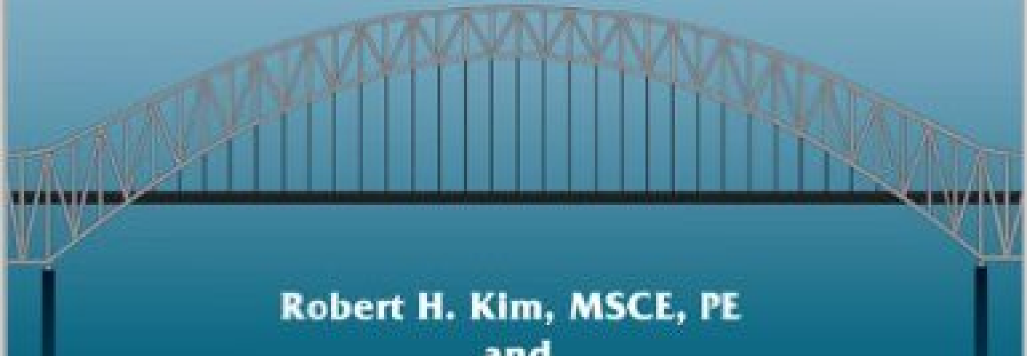
Bridge Design

for the
Civil and Structural Professional Engineering Exams

Second Edition



Robert H. Kim, MSCE, PE

and

Jai B. Kim, PhD, PE

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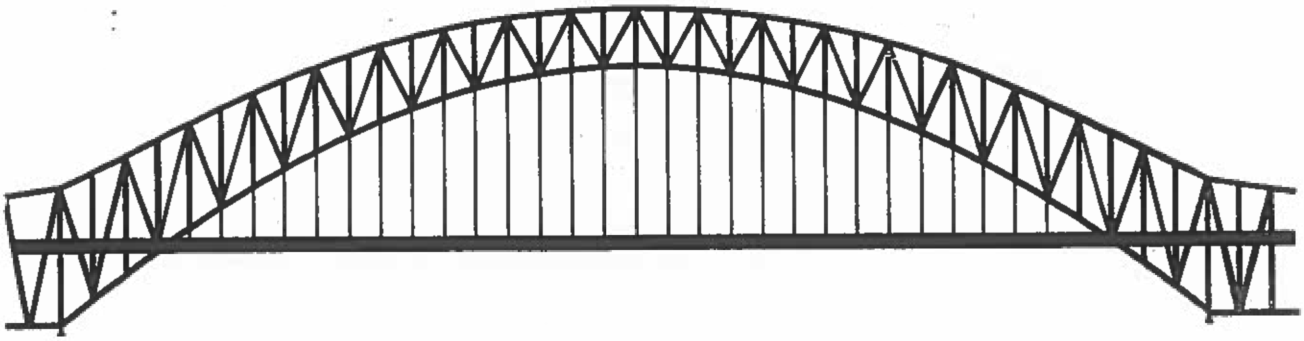

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BRIDGE DESIGN FOR THE CIVIL AND STRUCTURAL PROFESSIONAL ENGINEERING EXAMS
Second Edition

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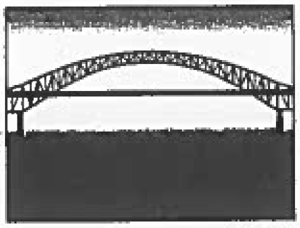
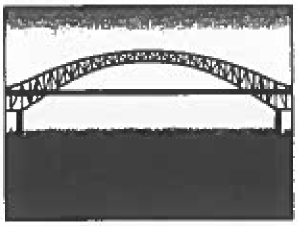


Table of Contents



Preface

This book is intended to serve both as a study reference for practicing engineers and engineering faculty preparing for the civil and structural professional engineering examinations, and as a classroom text for civil engineering seniors and graduate students.

It is essential that readers of this material have access to the latest edition of the *Standard Specifications for Highway Bridges* by the American Association of State Highway and Transportation Officials (AASHTO). The current version is the 16th edition (1996), with the 1997, 1998, and 1999 *Interim Revisions*. Another necessary reference is the AASHTO *Manual for Condition Evaluation of Bridges*, 1994, with the 1995, 1996, 1998, and 2000 *Interim Revisions*.

This book includes five comprehensive design examples that can quickly acquaint the reader with current bridge design practices in the United States. Work the design examples using the AASHTO references. When you have mastered the basic design principles in the examples, work the practice problems that follow to become familiar with the content and level of difficulty you are likely to encounter on the PE exam.

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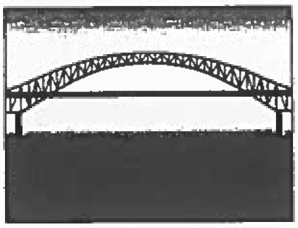

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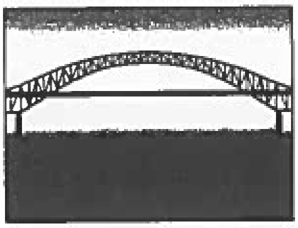
We are also grateful to Mrs. Yung J. Kim, mother of Robert and wife of Jai, who encouraged us continually throughout the writing of this book.



How to Use This Book

This book presents bridge design principles from the *Standard Specifications for Highway Bridges* (commonly known as the AASHTO Blue Book) and the *Manual for Condition Evaluation of Bridges*, by the American Association of State Highway and Transportation Officials. Five example problems follow, which apply AASHTO principles to the design of the major components of a simple-span highway bridge. Each step in the design of these components contains references to the applicable section of the Blue Book. Look up and study these references in the Blue Book, carefully follow the example problems, and then try to solve them on your own, without referring to the given solutions.

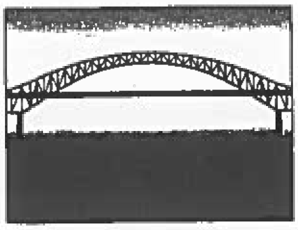
The example problems are followed by two practice problems that represent the content and level of difficulty of a typical bridge design problem on the NCEES PE examinations. When you feel comfortable with the design principles taught by the example problems, work these practice problems under exam conditions, both to test yourself on application of the principles presented and to practice test-taking under the time and materials constraints enforced during the PE exam.

Since the civil and structural examinations are open-book tests, mark this book, the Blue Book, and any other references you intend to use as much as necessary, and tab pages with critical information such as tables and commonly used equations before the exam.* Become as familiar with both books as possible. Preparation and organization are as much a key to passing the exams as knowledge.

When you take the exam, read the questions and determine which of them you have the best chance of answering quickly and completely before starting to solve any problems. Then answer them in order from easiest to most difficult. Remember that the test is timed; if you find yourself spending too much time on one question, go on to another and return to it only if you have time. If you finish early, don't leave; use the extra time to check your calculations and answers.

Good luck on the exam!

*Note that some states don't allow removeable tabs in books used during the exam. Check with your state board, or use permanent tabs.



Nomenclature

Unless defined otherwise in the text, the following nomenclature is used in this book.

Symbol	Definition (units)
a	depth of equivalent rectangular stress block in concrete (in)
a_b	depth of equivalent rectangular stress block for balanced strain conditions (in)
A_B	area of basic beam section (in^2)
A_c	composite area (in^2)
A_s	area of tension reinforcement (in^2)
A'_s	area of compression reinforcement (in^2)
A^*_s	area of prestressing steel (in^2)
A_{st}	total area of longitudinal reinforcement (in^2)
A_v	area of shear reinforcement (in^2)
b	compression flange width (in)
b	width of compression face of member
b	width of flange of flanged member or width of rectangular member (in)
b'	width of a projecting flange element, angle, or stiffener (in)
b'	width of a web of a flanged member (in)
b_w	web width (in)
B	buoyancy (lbf)
B	width of footing (ft)
C	web buckling coefficient
C	capacity of member in the load rating of bridge
CF	centrifugal force (lbf)
d	depth of beam or girder (in)
d	distance from extreme compression fiber to centroid of tension reinforcement (in)[1]
d	distance from extreme compressive fiber to centroid of the prestressing force (in)
d'	distance from extreme compression fiber to centroid of compression reinforcement (in)
D	clear distance between flanges (in)
D	dead load (lbf)
e_B	eccentricity of load in the B direction measured from centroid of footing (ft)
E	earth pressure (lbf/ft^2)
E	width of slab over which a wheel load is distributed (in)
E_c	modulus of elasticity of concrete (lbf/in^2)
EQ	earthquake
EQ	equivalent static horizontal force applied at the center of gravity of the structure (lbf)
E_s	modulus of elasticity of steel (lbf/in^2)
f	coefficient of friction between footing base and soil
f_c	compressive stress in concrete at service loads (lbf/in^2)
f'_c	compressive strength of concrete at 28 days (lbf/in^2)
f_{ci}	temporary stress before losses due to creep and shrinkage (lbf/in^2)

[1] For computing horizontal shear strength of composite members, d shall be the distance from the extreme compression fiber to the centroid of tension reinforcement for the entire composite section (in).

Symbol	Definition (units)
f'_{ci}	compressive strength of concrete at time of initial prestress (lbf/in^2)
f'_{cg}	compressive strength of concrete at 28 days for prestress I-beams
f_{cs}	compressive strength of concrete after losses (lbf/in^2)
f'_{cs}	compressive strength of concrete at 28 days for roadway slab
f_{pc}	compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads[2] (lbf/in^2)
f_{pe}	compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (lbf/in^2)
f_r	modulus of rupture of concrete (AASHTO Sec. 8.15.2 or Sec. 9.15.2.3) (lbf/in^2)
f'_s	ultimate stress of prestressing steel (lbf/in^2)
f'_s	stress in compression reinforcement at balanced condition (lbf/in^2)
f_{se}	effective steel prestress after losses
f_{si}	allowable stress in prestressing steel (lbf/in^2)
f^*_{su}	stress in prestressing steel at ultimate load (lbf/in^2)
f_{sy}	yield stress of non-prestressed conventional reinforcement in tension (lbf/in^2)
f_{ti}	tensile strength of concrete at time of initial prestress (lbf/in^2)
f_{ts}	tensile strength of concrete after losses (lbf/in^2)
FS_O	factor of safety against overturning
FS_S	factor of safety against sliding
FWS	future wearing surface
F_y	specified minimum yield point of steel (lbf/in^2)
g	centroid of prestressing strand pattern (in)
H	horizontal loads (lbf)

[2]In a composite member, f_{pc} is resultant compressive stress at centroid of composite section.

Symbol	Definition (units)
I	live load impact (lbf)
I	moment of inertia (in^4)
I_B	moment of inertia about the centroid of the basic beam section (in^4)
I_c	composite section moment of inertia (in^4)
ICE	ice pressure
I_g	moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement (in^4)
k	buckling coefficient
L	live load (lbf)
L	loaded length of span (ft)
L_b	unbraced length (ft)
LF	longitudinal force from live load (lbf)
LLM	maximum live load moment (in-lbf)
L_S	lateral pressure from 2 ft of soil for live load surcharge
M	computed moment capacity (in-lbf)
M_1	moments at the ends of a member* (in-lbf)
M_b	nominal moment strength of a section at balanced strain conditions*
M_{cr}	cracking moment*
M_{cr}	moment causing flexural cracking at section due to externally applied loads (AASHTO Sec. 8.13.3)*
M_D	dead load moment (maximum)*
M_D	moment due to slab weight per beam*
$M_{d/nc}$	noncomposite dead load moment*
M_f	factored moment at midspan*
M_H	moment due to horizontal loads
M_L	live load moment (maximum)*
M_L	moment at the ends of a member
M_{L+1}	live load moment multiplied by wheel load distribution factor and impact factor*
M_{max}	maximum factored moment at section due to applied loads*
M_n	nominal moment strength or capacity of a section*

Symbol	Definition (units)
M_0	moment due to beam weight*
M_s	moment due to superimposed dead load per beam*
M_u	factored design moment at section $\leq \phi M_n^*$
M_u	maximum design moment*
M_v	moment due to vertical loads
n	ratio of modulus of elasticity of one concrete strength to that of another concrete strength
n	ratio of modulus of elasticity of steel to that of concrete
N	group number for load combinations (AASHTO Table 3.22.1A)
P	load on one rear wheel of truck
P_{15}	12,000 lbf for HS loading
P_{20}	16,000 lbf for H_2O loading
P_b	nominal axial load strength at balanced strain conditions (lbf)**
P_e	effective prestress force after losses**
P_i	initial prestressing force**
P_{L+I}	wheel load plus impact**
P_n	nominal axial load strength at given eccentricity**
P_o	nominal axial load strength at zero eccentricity**
P_u	factored axial load at given eccentricity**
P_u	maximum axial compression capacity**
q	effective pressure at base of footing (k/ft^2)
r_y	radius of gyration with respect to the y-y axis (in)
R	reaction (lbf)
R	rib shortening force (lbf)
RF	rating factor for the load carrying capacity
s	longitudinal spacing of shear or web reinforcement (in)
S	section modulus (in^3)
S	shrinkage force (lbf)
S	average spacing of beams (in)

Symbol	Definition (units)
S	span length (in)
S_b, S_t	non-composite section moduli (in^3)
S_{bc}, S_{tc}	section moduli of composite beam section (in^3)
S_c, S_{bc}	composite section modulus where the tensile stress is caused by externally applied loads (in^3)
SF	stream flow pressure (lbf/in^2)
t	slab thickness (in)
t_f	thickness of the flange (in)
t_w	web thickness (in)
T	temperature force (lbf)
T_f	temperature force due to friction at bearing (lbf)
V	design shear force at section (lbf)
V	shearing force (AASHTO 10.48.5.3, 10.48.8)**
V	variable spacing of truck axles (AASHTO Figure 3.7.3A) (in)
V	vertical load**
V_c	nominal shear strength provided by concrete**
V_{ci}	nominal shear strength of concrete when diagonal cracking results from combined shear and moment**
V_D	dead load shear (maximum)**
V_f	factored shear at mid-span or at support**
V_i	factored shear force at section due to applied loads occurring simultaneously with M_{max}**
V_{L+I}	live load shear with wheel load distribution factor and impact factor
V_p	shear yielding of the web
V_s	nominal shear strength provided by shear reinforcement
V_u	maximum shear force factored
w_c	weight of concrete (lbf/ft^3)
W	combined weight on the first two axles of a standard HS Truck (AASHTO Figure 3.7.7A)

Symbol	Definition (units)
W	dead weight (beam, slab, or superimposed)
W	wind load on structure
W	weight of nominal truck in tons when used in the bridge load capacity rating
W_{c+P}	dead weight of curb and parapet
W_D	dead weight of slab
WL	wind load on live load
W_s	superimposed dead load
W_{sub}	wind load on substructure
W_{super}	wind load on superstructure
W_{up}	wind load upward causing the uplift
$\overline{x}$	distance of the resultant soil pressure from the footing toe
X	distance from load to point of support
y_t, y_b	distance from centroidal axis of beam gross section (neglecting reinforcement) to top and bottom fibers, respectively
y'_t, y'_b	y_t and y_b for composite beam gross section
⋮	
Z	reduction for ductility and risk assessment
Z	plastic section modulus (in^3)
β	(with appropriate script) coefficient applied to actual loads for service load and load factor designs (AASHTO Table 3.22.1A)
β_1	factor for concrete strength
γ	load factor (AASHTO Table 3.22.1A)
γ^*	factor for type of prestressing steel
μ	Poisson's ratio (AASHTO Article 3.23.4.3)
ρ	ratio of tension reinforcement
ρ'	ratio of compression reinforcement
ρ^*	A_s^*/bd, ratio of prestressing steel
ρ_b	reinforcement ratio at balanced strain conditions (AASHTO Sec. 8.16.3.2.2)
$\rho_{\max}$	maximum permitted reinforcement ratio (typically $0.75\rho_b$)
ϕ	strength reduction factor

Bridge Design

This book on highway bridges is based on the 16th Edition of the *Standard Specifications for Highway Bridges* by the American Association of State Highway and Transportation Officials (AASHTO), 1996, with the 1997, 1998, and 1999 Interim Revisions. Because AASHTO specifications are periodically revised, the reader should always refer to the most current edition. Another AASHTO reference used is the *Manual for Condition Evaluation of Bridges*, 1994, with the 1995, 1996, 1998, and 2000 Interim Revisions.

1. Loads

[AASHTO Section 3]

The bridge structure is designed to carry the following loads and forces: dead load; live load; impact or dynamic effect of the live load; wind loads; and other forces, when they exist, such as longitudinal forces, centrifugal forces, thermal forces, earth pressure, buoyancy, shrinkage stresses, rib shortening, erection stresses, ice and current pressure, and earthquake stresses.

Bridge members are proportioned either with reference to service loads and allowable stresses as provided in Service Load Design (Allowable Stress Design) or, alternatively, with reference to load factors and factored strength as provided in Strength Design (Load Factor Design).

Load factor design is a method of proportioning structural members for multiples of the design loads. To ensure serviceability and durability, consideration is given to the control of permanent deformations under overloads, to the fatigue characteristics under service loadings, and to the control of live load deflections under service loadings (see AASHTO 10.42).

For design purposes, the service loads are taken as the dead, live, and impact loadings (see AASHTO 10.43.2).

Service live loads are vehicles that may operate on a highway legally without special load permit (see AASHTO 10.43.1).

Overloads are the live loads that can be allowed on a structure on infrequent occasions without causing pemanent damage. For design purposes, the maximum overload is taken as 5(L + I)/3 (see AASHTO 10.43.2).

2. Combination of Loads

[AASHTO Section 3.22]

The load Groups I to X represent various combinations of loads and forces to which a structure may be subjected. Each component of the structure, or the foundation on which it rests, shall be proportioned to safely withstand all group combinations of forces and loads that are applicable. Group loading combinations for Service Load Design and Load Factor Design are given as follows. For an overload and permit loading, the load is applied in Group IB. For all loadings less than H 20, Group IA is used (AASHTO Sec. 3.22.5).

$$\begin{aligned}\text{Group } N = \gamma(&\beta_D D + \beta_L(L+I) + \beta_C \text{CF} + \beta_E E \\ &+ \beta_B B + \beta_S \text{SF} + \beta_W W + \beta_{\text{WL}} \text{WL} \\ &+ \beta_L \text{LF} + \beta_R(R+S+T) \\ &+ \beta_{\text{EQ}} \text{EQ} + \beta_{\text{ICE}} \text{ICE})\end{aligned}$$

[AASHTO Eq. 3-10]

N = group number
γ = load factor (see AASHTO Table 3.22.1A)
β = coefficient (see AASHTO Table 3.22.1A)
D = dead load
L = live load
I = live load impact
E = earth pressure
B = buoyancy
W = wind load on structure
WL = wind load on live load
LF = longitudinal force from live load
CF = centrifugal force
R = rib shortening
S = shrinkage
T = temperature
EQ = earthquake
SF = stream flow pressure
ICE = ice pressure

Table 1 (AASHTO Table 3.22.1A Table of Coefficients γ and β)

	col. no.	1	2	3	3A	4	5	6	7	8	9	10	11	12	13	14	
			β factors														
	group	γ	D	$(L+I)_n$	$(L+I)_p$	CF	E	B	SF	W	WL	LF	$R+S+T$	EQ	ICE	%	
service load	I	1.0	1	1	0	1	β_E	1	1	0	0	0	0	0	0	100	
service load	IA	1.0	1	2	0	0	0	0	0	0	0	0	0	0	0	150	
service load	IB	1.0	1	0	1	1	β_E	1	1	0	0	0	0	0	0	2	
service load	II	1.0	1	0	0	0	1	1	1	1	0	0	0	0	0	125	
service load	III	1.0	1	1	0	1	β_E	1	1	0.3	1	1	0	0	0	125	
service load	IV	1.0	1	1	0	1	β_E	1	1	0	0	0	1	0	0	125	
service load	V	1.0	1	0	0	0	1	1	1	1	0	0	1	0	0	140	
service load	VI	1.0	1	1	0	1	β_E	1	1	0.3	1	1	1	0	0	140	
service load	VII	1.0	1	0	0	0	1	1	1	0	0	0	0	1	0	133	
service load	VIII	1.0	1	1	0	1	1	1	1	0	0	0	0	0	1	140	
service load	IX	1.0	1	0	0	0	1	1	1	1	0	0	0	0	1	150	
service load	X	1.0	1	1	0	0	β_E	0	0	0	0	0	0	0	0	100	culvert
load factor design	I	1.3	β_D	1.67[1]	0	1.0	β_E	1	1	0	0	0	0	0	0	not applicable	
load factor design	IA	1.3	β_D	2.20	0	0	0	0	0	0	0	0	0	0	0	not applicable	
load factor design	IB	1.3	β_D	0	1	1.0	β_E	1	1	0	0	0	0	0	0	not applicable	
load factor design	II	1.3	β_D	0	0	0	β_E	1	1	1	0	0	0	0	0	not applicable	
load factor design	III	1.3	β_D	1	0	1	β_E	1	1	0.3	1	1	0	0	0	not applicable	
load factor design	IV	1.3	β_D	1	0	1	β_E	1	1	0	0	0	1	0	0	not applicable	
load factor design	V	1.25	β_D	0	0	0	β_E	1	1	1	0	0	1	0	0	not applicable	
load factor design	VI	1.25	β_D	1	0	1	β_E	1	1	0.3	1	1	1	0	0	not applicable	
load factor design	VII	1.3	β_D	0	0	0	β_E	1	1	0	0	0	0	1	0	not applicable	
load factor design	VIII	1.3	β_D	1	0	1	β_E	1	1	0	0	0	0	0	1	not applicable	
load factor design	IX	1.20	β_D	0	0	0	β_E	1	1	1	0	0	0	0	1	not applicable	
load factor design	X	1.30	1	1.67	0	0	β_E	0	0	0	0	0	0	0	0		culvert

$(L + I)_n$—live load plus impact for AASHTO highway H or HS loading

$(L + I)_p$—live load plus impact consistent with the overload criteria of the operation agency

[1]1.25 may be used for design of outside roadway beam when a combination of sidewalk live load as well as traffic live load plus impact governs the design, but the capacity of the section should not be less than required for highway traffic live load only using a beta factor of 1.67. 1.00 may be used for design of deck slab with combination of loads as described in AASHTO Sec. 3.24.2.2.

[2]Note:

$$\text{percentage} = \frac{\text{maximum unit stress (operating rating)}}{\text{allowable basic unit stress}} \times 100$$

For service load design:

% (Column 14) Percentage of Basic Unit Stress

No increase in allowable unit stresses shall be permitted for members or connections carrying wind loads only.

β_E = 1.00 for vertical and lateral loads on all other structures.

For culvert loading specifications, see AASHTO Sec. 6.2.

β_E = 1.0 and 0.5 for lateral loads on rigid frames (check both loadings to see which one governs). See AASHTO Sec. 3.20.

For load factor design,

β_E = 1.3 for lateral earth pressure for retaining walls and rigid frames excluding rigid culverts. For lateral at-rest earth pressures, β_E = 1.15.

β_E = 0.5 for lateral earth pressure when checking positive moments in rigid frames. This complies with AASHTO Sec. 3.20.

β_E = 1.0 for vertical earth pressure

β_D = 0.75 when checking member for minimum axial load and maximum moment or maximum eccentricity .... } For column design

β_D = 1.0 when checking member for maximum axial load and minimum moment .... } For column design

β_D = 1.0 for flexural and tension members

β_E = 1.0 for rigid culverts

β_E = 1.5 for flexible culverts

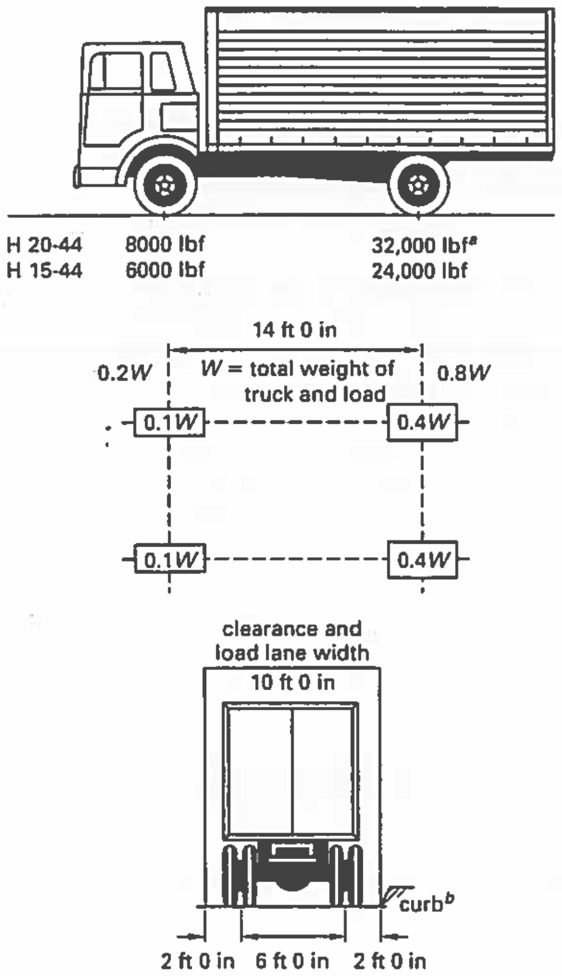
For Group X loading (culverts) the β_E factor shall be applied to vertical and horizontal loads.

3. Highway Live Loads

[AASHTO Section 3.7]

A. Standard Truck and Lane Loads (Figs. 1, 2, and 3)

The highway live loadings on bridges consist of standard trucks or lane loads that are equivalent to truck trains. Two systems of loading are provided: the H loadings represent a two-axle truck and the HS loadings represent a two-axle tractor plus a single-axle semi-trailer. As shown in the figures, the number after the H or HS indicates the gross weight in tons of the truck or tractor. The gross weight is divided between the front and rear axles. The affix "...44" refers to 1944, the year that the loadings were adopted.



[a]In the design of timber floors and orthotropic steel decks (excluding transverse beams) for H 20 loading, one axle load of 24,000 lbf or two axle loads of 16,000 lbf each, spaced 4 ft apart, may be used, whichever produces the greater stress, instead of the 32,000 lbf axle shown.
[b]For slab design, the centerline of wheels shall be assumed to be 1 ft from the face of the curb (see AASHTO Sec. 3.24.2).

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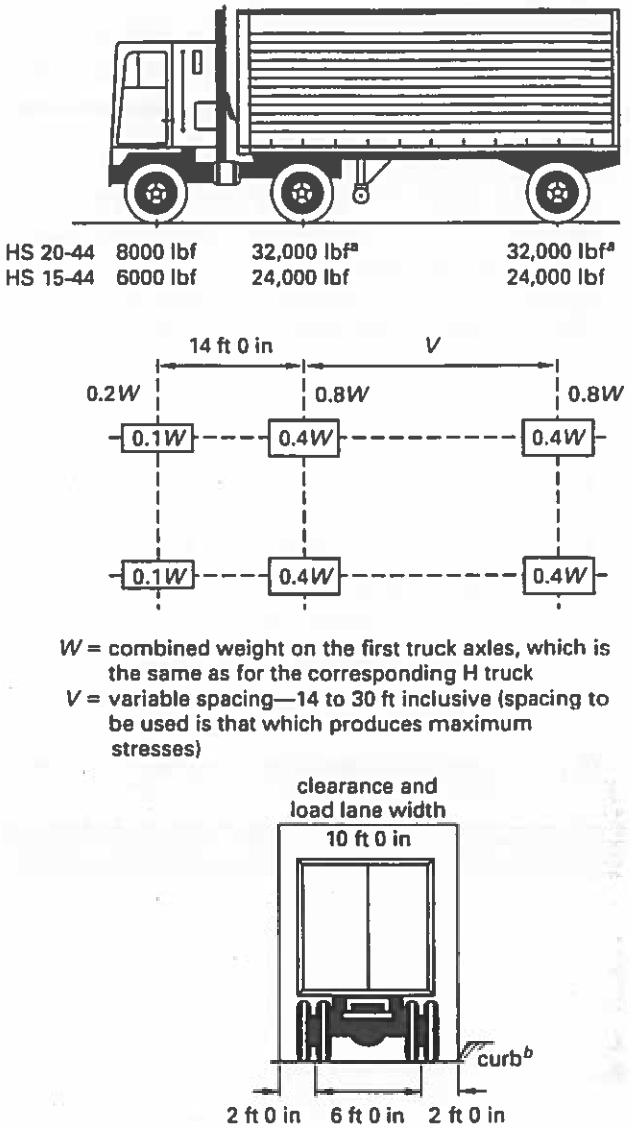
Figure 1 Standard H Trucks (AASHTO Fig. 3.7.6A)



[a]In the design of timber floors and orthotropic steel decks (excluding transverse beams) for H 20 loading, one axle load of 24,000 lbf or two axle loads of 16,000 lbf each, spaced 4 ft apart, may be used, whichever produces the greater stress, instead of the 32,000 lbf axle shown.
[b]For slab design, the centerline of wheels shall be assumed to be 1 ft from the face of the curb (see AASHTO Sec. 3.24.2).

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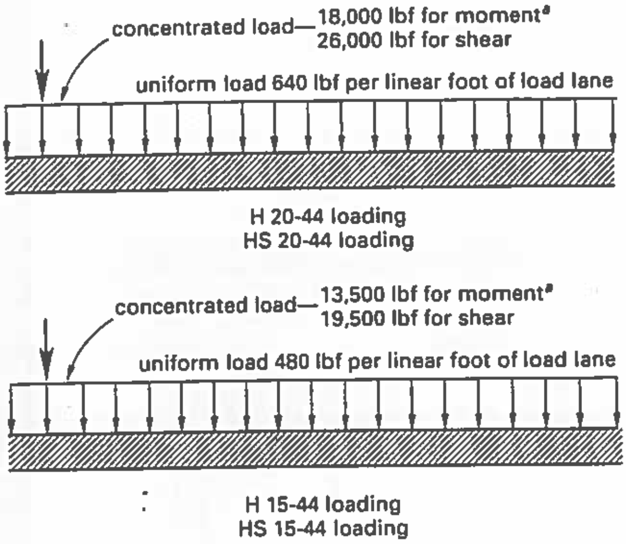
Figure 2 Standard HS Trucks (AASHTO Fig. 3.7.7A)

Lane loadings shown in Fig. 3 are used when they produce greater stress than the corresponding truck loadings. In general, the lane loadings govern the design of longer-span bridges. For example, in designing for bending moments, these spans exceed 56 ft for the H loadings and 140 ft for the HS loadings.

The lane loads are defined in such a way as to give a simpler method of calculating bending moments and shears than the method based on truck wheel loads and the truck train loadings shown in Appendix B of AASHTO Truck Train and Equivalent Loadings. Thus, the lane loads are the equivalent of the truck train loadings.

Each lane load consists of a uniform load per linear foot of traffic lane combined with a single concentrated load (or two concentrated loads in the case of continuous spans) placed on the span so as to produce maximum stress. The concentrated load and uniform load are considered uniformly distributed over a 10 ft width on a line normal to the centerline of the lane.



[a]For the loading of continuous spans involving lane loading, refer to AASHTO Sec. 3.11.3, which provides for an additional concentrated load.

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Figure 3 Lane Loading (AASHTO Fig. 3.7.6B)

B. Other Roadway Loadings

The bridge may be required to carry military vehicles or other extraordinary vehicles.

C. Sidewalk Loadings

Sidewalk floors, stringers, and their immediate supports shall be designed for a live load of at least 85 psf of sidewalk area.

4. Impact

[AASHTO Section 3.8]

Live loads are increased for those structural elements in Group A. Group A includes (a) superstructures; (b) piers, excluding footings and those portions below the ground line; and (c) the portions above the ground line of concrete or steel piles that support a superstructure. Impact allowances are not applied to those items in Group B. Group B includes (a) abutments, retaining walls, and piles except as specified in (c) above; (b) foundation pressures and footings; (c) timber structures; (d) sidewalk loads; and (e) culverts and structures having 3 ft or more of cover.

The impact formula is expressed as a fraction of the live load stress.

$$I = \frac{50}{L + 125}$$

I = impact fraction (maximum 30%)

L = length in feet of the portion of the span that is loaded to produce the maximum stress in the member (for the loaded length, L, see AASHTO Sec. 3.8.2.2)

5. Application of Live Loads

[AASHTO Section 3.11]

The following provisions are for selected AASHTO loading.

(1) The lane loading or standard truck loading shall be assumed to occupy a width of 10 ft.

(2) Each 10 ft lane loading or single standard truck shall be considered as a unit.

(3) Where maximum stresses are produced in any member by loading any number of traffic lanes simultaneously, the following percentages of the resultant live load stresses shall be used in view of the improbability of coincident maximum loading.

one or two lanes:	100%
three lanes:	90%
four or more lanes:	75%

6. Distribution of Loads

[AASHTO Section 3.23]

A. Distribution of Loads to Stringers, Longitudinal Beams, and Floor Beams

[AASHTO Section 3.23; Table 3.23.1]

Position of Loads for Shear

In calculating end shears and end reactions in transverse floor beams and longitudinal beams and stringers, no longitudinal distribution of the wheel load shall be assumed for the wheel or axle load adjacent to the transverse floor beams or the end of the longitudinal beam or stringers at which the stress is being determined. Lateral distribution of the wheel load at the ends of the beams or stringers shall be that produced by assuming that the flooring acts as a simple span between stringers or beams. For loads in other positions on the span, the distribution for shear shall be determined by the method prescribed for moment.

Bending Moments in Stringers and Longitudinal Beams

In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed. The lateral distribution shall be determined as follows.

(1) *Interior Stringers and Beams*
When a concentrated load, that is, a wheel load, is placed on a bridge deck or slab, the load is distributed over an area larger than the actual contact area. Because of the stiffness of the deck or slab, the adjacent longitudinal girders will share the concentrated load. However, no distribution is assumed in the direction of the span of the bridge.

AASHTO Tables 3.23.1 and 3.23.3.1 describe the distribution of wheel loads in longitudinal beams and transverse beams. Truck loadings are moved across the bridge, and as they move they generate changing moments, shears, and reactions in the bridge members. All of the loads described above are loads that occupy one traffic lane. It is necessary to apportion these loads into the deck slabs, girders, truss members, and so on, according to the distribution methods provided in the AASHTO tables mentioned previously.

Slabs are loaded by individual wheels. Bridge girders, stringers, and some floor beams are loaded by lines of wheel loads that roll along the deck. A wheel line is half of a truck load or half of one lane load. The number of wheel lines carried by each girder depends upon the girder spacing and the type of girder. For concrete decks, typical interior girder distributions of the wheel lines in longitudinal girders or beams are as shown in Table 2.

(2) *Outside Stringers*
The dead load supported by the outside roadway stringer or beam shall be that portion of the floor slab carried by the stringer or beam. Curbs, railings, and wearing surface, if placed after the slab has cured, may be distributed equally to all roadway stringers or beams.

The live load bending moment for outside roadway stringers or beams shall be determined by applying to the stringer or beam the reaction of the wheel load obtained by assuming that the flooring acts as a simple span between stringers or beams.

B. Bending Moments in Floor Beams (Transverse)

[AASHTO Section 3.23.3; Table 3.23.3.1]

In calculating bending moments in floor beams, no transverse distribution of the wheel loads shall be assumed. If the longitudinal stringers are omitted and the floor is supported directly on floor beams, the floor beams shall be designed for loads determined in accordance with AASHTO Table 3.23.3.1. For example, if the concrete floor is supported directly on floor beams, the fraction of wheel load allotted to each floor beam is $S/6$, where S equals the average spacing of beams in feet. If S exceeds 6 ft, the load on the beam shall be the reaction of the wheel loads, under the assumption that the flooring between beams acts as a simple beam.

Table 2 Distribution of Wheel Loads in Longitudinal Beams [AASHTO Table 3.23.1]

type of floor	one traffic lane, fraction of a wheel (line) load to each longitudinal girder[1]	two or more traffic lanes, fraction of a wheel (line) load to each longitudinal girder[1]
concrete slab:		
on steel I-beam stringers and prestressed concrete girders	$S/7.0$ (S max = 10 ft)	$S/5.5$ (S max = 14 ft)
on concrete T-beams	$S/6.5$ (S max = 6 ft)	$S/6.0$ (S max = 10 ft)
on concrete box girders	$S/8.0$ (S max = 12 ft)	$S/7.0$ (S max = 16 ft)
6 in or more thick glued-laminated floor panels:		
on glued-laminated stringers	$S/6.0$ (S max = 6 ft)	$S/5.0$ (S max = 7.5 ft)
on steel stringers	$S/5.25$ (S max = 5.5 ft)	$S/4.5$ (S max = 7.0 ft)

S = average stringer (or girder) spacing in feet

[1]If S exceeds S_{max}, the load on each stringer shall be the reaction of the wheel loads, assuming the flooring between the stringers acts as a simple beam.

C. Distribution of Loads and Design of Concrete Slabs

[AASHTO Section 3.24]

Span Lengths (AASHTO Sections 3.24.1 and 8.8)

For simple spans, the span length, S, shall be the distance center-to-center of supports but need not exceed clear span plus thickness of slab. The following effective span lengths, S, shall be used in calculating the distribution of loads and bending moments for slabs continuous over more than two supports.

Slabs monolithic with beam (without haunches):
S = clear span

Slabs supported on steel stringers:
S = distance between edges of flanges plus one-half the stringer flange width

Slabs supported on timber stringers:
S = clear span plus one-half the thickness of the stringer

Edge Distance of Wheel Loads (AASHTO Section 3.24.2)

In designing slabs, the centerline of the wheel load shall be assumed to be 1 ft from the face of the curb. If curbs or sidewalks are not used, the wheel load shall be 1 ft from the face of the rail.

Bending Moment (AASHTO Section 3.24.3)

Bending moment per foot width of slab shall be calculated according to methods given under Cases A and B below, unless more exact methods are used that consider tire contact area (see AASHTO Sec. 3.30). In both cases,

S = effective span length as defined under Span Lengths above, in ft
E = width of slab over which a wheel load is distributed, in ft
P = load on one rear wheel of truck (P_{15} or P_{20})
P_{15} = 12,000 lbf for H 15 loading
P_{20} = 16,000 lbf for H 20 loading

Case A—Main Reinforcement Perpendicular to Traffic (spans of 2 ft to 24 ft inclusive). The live load moment for simple spans shall be determined by the following formulas (impact not included).

For HS 20 loading,

$$\left(\frac{S+2}{32}\right)P_{20} = \text{moment in foot-pounds per foot-width of slab}$$

For HS 15 loading,

$$\left(\frac{S+2}{32}\right)P_{15} = \text{moment in foot-pounds per foot-width of slab}$$

In slabs continuous over three or more supports, a continuity factor of 0.8 shall be applied to the above formulas for both positive and negative moments.

Case B—Main Reinforcement Parallel to Traffic. For wheel loads, the distribution width, E, shall be $4 + 0.06S$ but shall not exceed 7.0 ft. Lane loads are distributed over a width of $2E$. Longitudinally reinforced slabs shall be designed for the appropriate HS loading.

For simple spans, the maximum live load moment (LLM) per foot width of slab, without impact, is closely approximated by the following formulas.

For HS 20 loading,
Spans up to and including 50 ft: LLM = $900S$ ft-lbf
Spans 50 ft to 100 ft:
LLM = $(1000)(1.3S - 20.0)$ ft-lbf

For HS 15 loading,
Use three-fourths of the values obtained from the formulas for HS 20 loading.

Moments in continuous spans shall be determined by suitable analysis using the truck or appropriate lane loading.

Shear and Bond (AASHTO Section 3.24.4)

Slabs designed for bending moment in accordance with AASHTO Sec. 3.24.3 shall be considered satisfactory in bond and shear.

Longitudinal Edge Beams (AASHTO Section 3.24.8)

Edge beams shall be provided for all slabs having main reinforcement parallel to traffic. The beam may consist of a slab section additionally reinforced, a beam integral with and deeper than the slab, or an integral reinforced section of slab and curb.

The edge beam of a simple span shall be designed to resist a live load moment of $0.10PS$, where

P = wheel load in lbf, P_{15} or P_{20}
S = span length, ft

For continuous spans, the moment may be reduced by 20% unless a greater reduction results from a more exact analysis.

Distribution Reinforcement (AASHTO Section 3.24.10)

To provide for the lateral distribution of the concentrated live loads, reinforcement shall be placed in the bottoms of all slabs transverse to the main steel reinforcement, except culvert or bridge slabs where the depth of fill over the slab exceeds 2 ft.

The amount of distribution reinforcement shall be the percentage of the main reinforcement steel required for positive moment as given by the following formulas.

For main reinforcement parallel to traffic,

$$\text{percentage} = \frac{100}{\sqrt{S}} \quad [\text{maximum} = 50\%]$$

For main reinforcement perpendicular to traffic,

$$\text{percentage} = \frac{220}{\sqrt{S}} \quad [\text{maximum} = 67\%]$$

S = the effective span length in ft

7. Other Loads

Provisions for other loads are found in AASHTO beginning with Section 3.9. Some of these provisions are listed in Table 3.

Table 3 Selected AASHTO Load Provisions

load	AASHTO section
longitudinal forces	3.9
centrifugal forces	3.10
sidewalk, curb, and railing loading	3.14
wind loads	3.15
thermal forces	3.16
uplift	3.17
forces from stream current, floating ice, and drift conditions	3.18
buoyancy	3.19
earth pressure	3.20
earthquakes	3.21
maximum moments, shears, and reactions—simple spans, one lane	Appendix A[a]
truck/train and equivalent loadings	Appendix B[a]

[a]Duplicated at the end of this book.

8. Load Rating

The load rating is based on the *Manual for Condition Evaluation of Bridges*, 1994, Second Edition, as revised by the 1995 to 2000 Interim Revisions by the American Association of State Highway and Transportation Officials (AASHTO).

Bridge load rating calculations provide a basis for determining the safe load capacity of a bridge based on existing structural conditions. Each bridge should be load rated at two levels, Inventory and Operating levels.

The Inventory rating level generally corresponds to the customary design level of stresses, but reflects the existing bridge and material conditions with regard to deterioration and loss of section. Inventory level load ratings allow comparisions with the capacity of new structures and, therefore, result in a live load that can safely utilize an existing structure for an indefinite period of time.

The Operating rating level corresponds to the maximum level of stresses. Thus, load ratings describe the maximum permissible live load to which the structure may be subjected.

Rating methods can be either the Allowable Stress method or the Load Factor method. In the Allowable Stress (or "Working Stress") method, the actual loadings are combined to produce a maximum stress in a member that is not to exceed the allowable or working stress. The Load Factor method analyzes a structure subject to multiples of the actual loads (factored loads). The load rating is determined such that the effects of the factored loads do not exceed the strength of members.

The bridge load rating of the bridge member in tons, RT, is determined as follows.

$$\text{RT} = \text{bridge member rating in tons} = (\text{RF})(W)$$

The load rating, RT, of a bridge is controlled by the member with the lowest load rating in tons.

W = weight (tons) of nominal truck used in determining the live load effect, L.

$\text{RF} = \dfrac{C - A_1 D}{A_2 L(1+I)}$ = rating factor for the live load carrying capacity.

C = capacity of the member (see the Manual section 6.6). The capacity, C, depends on the rating level desired, with the higher value for C used for the Operating level (see the Manual section 6.6.2) in the Allowable Stress method. In the Load Factor method, the nominal capacity, C, is the same regardless of the rating level desired (see the Manual section 6.6.3).

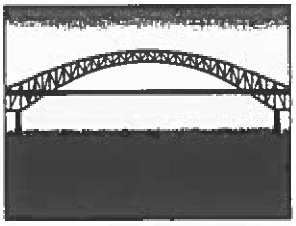
D = dead load effect on the member (see the Manual section 6.7.1).

L = live load effect on the member (see the Manual section 6.7.2).

I = impact factor to be used with the live load effect (see the Manual section 6.7.4).

A_1 = factor for dead load (A_1 = 1.0 for the Allowable Stress Method; and A_1 = 1.3 for the Load Factor Method).

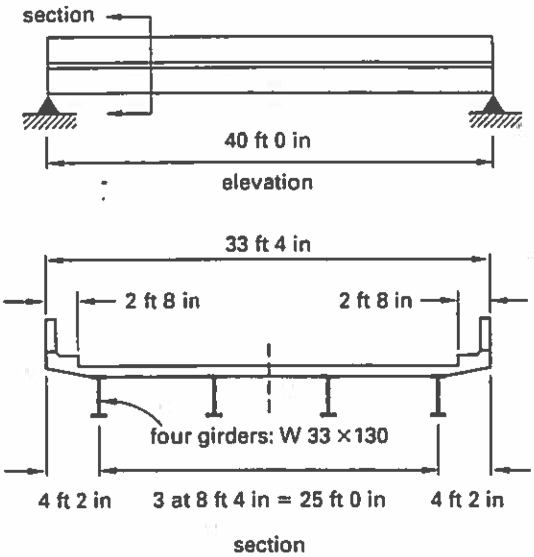
A_2 = factor for live load (A_2 = 1.0 for the Allowable Stress Method; for the Load Factor Method, A_2 = 2.1 for Inventory level, and A_2 = 1.3 for Operating level).



Design Examples

Design Example 1: Longitudinal Steel Girder

A simple span noncomposite steel girder bridge with a span of 40 ft is shown.

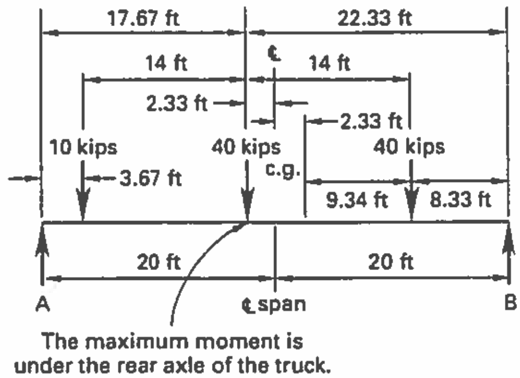


The overall width of the bridge is 33 ft, 4 in. The clear (roadway) width is 28 ft, 0 in. The roadway is a concrete slab 7 in thick supported by four W33 × 130 A36 steel girders that are spaced at 8 ft, 4 in apart. The compression flange is continuously supported by the concrete slab, and additional bracing is provided at the ends and at midspan. Noncomposite construction is assumed. Dead load per longitudinal girder (stringer) is 1.034 kips/ft for slab, stringer, curbs, railings, and beam details.

Review the longitudinal girder for adequacy against maximum live load, dead load, and shear load. Use AASHTO HS 25 loading.

Solution:

Step 1: Determine the maximum live load moment.



HS 25 loading = (1.25)(HS 20 loading)

The reaction at A is

$$\frac{(8.34 \text{ ft} + 22.33 \text{ ft})(40 \text{ kips})}{40 \text{ ft}} + \frac{(36.33 \text{ ft})(10 \text{ kips})}{40 \text{ ft}}$$

$$= 39.75 \text{ kips}$$

The live load moment is

$$M_L = (39.76 \text{ kips})(17.67 \text{ ft}) - (10.0 \text{ kips})(14 \text{ ft})$$
$$= 562.38 \text{ ft-kips}$$

From the table of maximum moments in AASHTO App. A (see at the end of this book), the moment for HS 25 loading is

$$(449.8 \text{ ft-kips})(1.25) = 562.25 \text{ ft-kips}$$

AASHTO [3.12] The live loading is not reduced because the bridge is loaded with two lanes only.

[3.23.2] The live load distribution factor is

$$\frac{S}{5.5} \text{ wheel load } \left(= \frac{S}{11} \text{ lane}\right)$$

$$= \frac{8.33 \text{ ft}}{5.5} = 1.515 \text{ wheel load}$$

S = average stringer spacing in feet

wheel load = 2 wheel lines

(Note that wheel line = wheels corresponding to one-half width of truck longitudinally or one-half of lane.)

The live load impact is

AASHTO [3.8.2]

$$I = \frac{50}{L + 125} = \frac{50}{40\text{ ft} + 125} = 0.303$$

[30% maximum allowed]

The maximum design live load moment per stringer is

$$M_{L+I} = M_L$$
$$\times \text{(distribution factor per stringer)}$$
$$\times \text{(impact factor)}$$
$$= (562.25\text{ ft-kips})\left(\frac{1.515}{2\text{ wheel lines}}\right)$$
$$\times (1.30)$$
$$= 553.68\text{ ft-kips}$$

Step 2: Determine the maximum dead load moment.

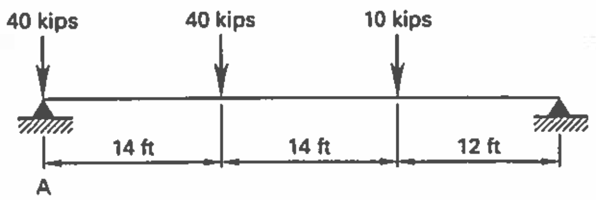
$$M_D = \frac{W_D L^2}{8} = \frac{\left(1.034\ \frac{\text{kips}}{\text{ft}}\right)(40\text{ ft})^2}{8}$$
$$= 206.8\text{ ft-kips}$$

Step 3: Determine the maximum shears.

The maximum dead load shear at span ends is

$$V_D = \frac{W_D L}{2} = \frac{(1.034\text{ ft-kips})(40\text{ ft})}{2}$$
$$= 20.7\text{ kips}$$

For HS 25 loading, the maximum live load shear is as follows.



The reaction at A is

$$40\text{ kips} + \frac{(10\text{ kips})(12\text{ ft}) + (40\text{ kips})(26\text{ ft})}{40\text{ ft}}$$
$$= 69.0\text{ kips}$$

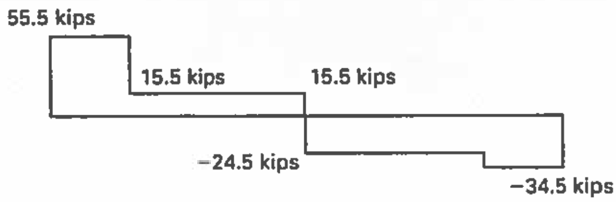
$$V_{L+I} = (69.0\text{ kips})\left(\frac{1.515}{2\text{ wheel lines}}\right)(1.30)$$
$$= 67.9\text{ kips}$$

The maximum shear at midspan is $V_D = 0.0$.

From the influence line analysis, the maximum live shear at midspan occurs with a 40 kip load at midspan. xfignoCEBRe01d

The reaction at A is

$$\frac{(10\text{ kips})(6\text{ ft}) + 40\text{ kips} + (20\text{ ft} + 34\text{ ft})}{40\text{ ft}}$$
$$= 55.5\text{ kips}$$



$$V_{L+I} = (-24.5\text{ kips})\left(\frac{1.515}{2\text{ wheel lines}}\right)(1.30)$$
$$= -24.1\text{ kips}$$

Step 4: Use the loads determined to perform the load factor design.

AASHTO [3.22.1 & Table 3.22.1A]

The combination of dead load (D) plus live load and impact ($L+I$) provides for the following load combination.

$$\text{Group I} = \gamma(\beta_D D + \beta_L(L+I))$$

The factored maximum moment is

$$M_f = \gamma(\beta_D M_D + \beta_L M_{L+I})$$

$\gamma = 1.3$ for load factor

$\beta_D = 1.0$ for flexural and tension members

$\beta_L = 1.67$ for live load plus impact for H or HS loading

$$M_f = (1.3)((1.0)(206.8\text{ ft-kips})$$
$$+ (1.67)(553.68\text{ ft-kips}))$$
$$= 1470.88\text{ ft-kips}$$

The factored shear at midspan is

$$V_f = \gamma(\beta_D V_D + \beta_L V_{L+I})$$
$$= (1.3)((1.0)(0.0 \text{ kips}) + (1.67)(-24.1 \text{ kips}))$$
$$= -52.32 \text{ kips}$$

The factored shear at support is

$$V_f = (1.3)((1.0)(20.7 \text{ kips}) + (1.67)(67.9 \text{ kips}))$$
$$= 174.32 \text{ kips}$$

Step 5: Use the section properties for W 33 × 130 (compact section) listed below to determine the moment adequacy at midspan. (Refer to the American Iron and Steel Construction Manual.)

$d = 33.09$ in	$F_y = 36$ kips/in^2 (A36 steel)
$b = b_f = 11.51$ in	$S_x = 406$ in^3
$t = t_f = 0.855$ in	$Z_x = 467$ in^3
$t_w = 0.580$ in	$r_y = 2.39$ in

Check local buckling criteria for projecting compression flange elements for the compact section requirement.

AASHTO [10.48.1.1(a)]

$$\frac{b'}{t} \leq \frac{2055}{\sqrt{F_y}}$$

$$b' = \frac{b - t_w}{2} = \frac{11.51 \text{ in} - 0.580 \text{ in}}{2} = 5.47 \text{ in}$$

$$\frac{b'}{t} = \frac{5.47 \text{ in}}{0.855 \text{ in}} = 6.40$$

$$\frac{2055}{\sqrt{F_y}} = \frac{2055}{\sqrt{36{,}000 \frac{\text{lbf}}{\text{in}^2}}} = 10.83$$

[> 6.40, so OK]

Check local buckling criteria for web thickness for compact section requirements.

AASHTO [10.48.1.1(b)]

$$\frac{D}{t_w} \leq \frac{19{,}230}{\sqrt{F_y}}$$

The clear distance between flanges is

$$D = 33.09 \text{ in} - (0.855 \text{ in})(2) = 31.38 \text{ in}$$

$$\frac{D}{t_w} = \frac{31.38 \text{ in}}{0.580 \text{ in}} = 54.1$$

$$\frac{19{,}230}{\sqrt{F_y}} = \frac{19{,}230}{\sqrt{36{,}000 \frac{\text{lbf}}{\text{in}^2}}} = 101.35$$

[> 54.1, so OK]

Check lateral bracing criteria for compression flange for a compact section.

AASHTO [10.48.1.1(c)]

$$\frac{L_b}{r_y} \leq \frac{\left(3.6 - (22)\left(\frac{M_l}{M_u}\right)\right)(10^6)}{F_y}$$

AASHTO [Eq. 10-92 & 10.48.1]

$M_u = F_y Z_x$ = the maximum moment strength. M_l = the smaller moment at the end of the unbraced length of the member. L_b = the unbraced length.

Since $L_b = 0$ (i.e., concrete composite deck), this criterion is automatically satisfied.

Step 6: Check moment adequacy.

The maximum moment capacity is

$$M_u = F_y Z_x = \left(36 \frac{\text{kips}}{\text{in}^2}\right)(467 \text{ in}^3)$$
$$= (16{,}812 \text{ in-kips})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$
$$= 1401 \text{ ft-kips}$$

Since M_u is less than M_f (1470.88 kt-kips), the stringer is not adequate in moment.

Step 7: Check shear adequacy at midspan.

AASHTO [10.48.8.1]

V_u, shear capacity, is equal to CV_p.

$$V_p = 0.58 F_y D t_w$$
$$= (0.58)\left(36 \frac{\text{kips}}{\text{in}^2}\right)(31.38 \text{ in})(0.58 \text{ in})$$
$$= 380.02 \text{ kips}$$

The web buckling coefficient, C, is

$$1.0 \text{ for } \frac{D}{t_w} < \frac{6000\sqrt{k}}{\sqrt{F_y}}$$

k (the buckling coefficient) for unstiffened beam is taken as 5.

$$\frac{D}{t_w} = \frac{31.38 \text{ in}}{0.58 \text{ in}} = 54.1$$

$$\frac{6000\sqrt{5.0}}{\sqrt{36{,}000 \frac{\text{lbf}}{\text{in}^2}}} = 70.7$$

$$V_u = CV_p = (1.0)(380.02 \text{ kips})$$
$$= 380.02 \text{ kips}$$

Since V_u is greater than V_f (−52.32 kips), the stringer is adequate in shear at midspan.

Step 8: Check shear adequacy at the support.

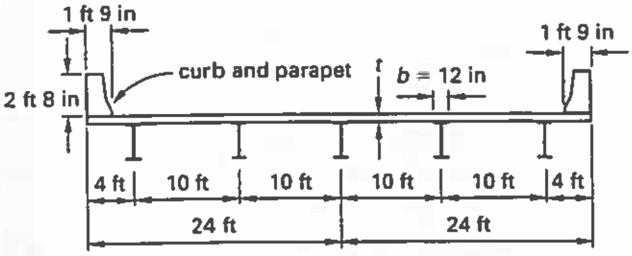
The shear capacity, V_u, is 380.02 kips.

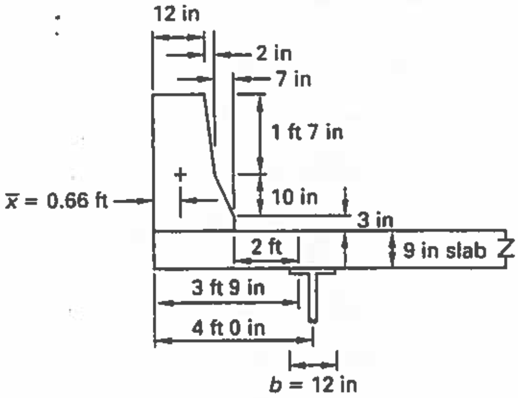
The factored shear at support, V_f, was calculated as 174.32 kips.

Since $V_u > V_f$ (174.32 kips), the stringer is adequate in shear at the support.

Design Example 2: Reinforced Concrete Slab

The cast-in-place concrete deck for a simple span composite bridge is continuous across five steel girders as shown.





The overall width of the bridge is 48 ft, 0 in. The clear (roadway) width is 44 ft, 6 in. The roadway is a concrete slab 9 in thick, with a concrete strength of $f'_c = 4.5$ kips/in^2 and steel reinforcement equal to $F_y = 60.0$ kips/in^2. The top flange width of the steel girders spaced 10 ft apart is 12 in. The future wearing surface (FWS) is 0.03 kips/ft^2. Design and review the reinforced concrete slab. Use HS 25 loading.

Solution:

Step 1: Determine the effective slab span length and minimum thickness of the slab.

The effective slab span length, S, is the clear span plus one-half the stringer top flange width.

AASHTO [3.24.1.2(b)]

$$S = 9 \text{ ft} + 0.5 \text{ ft} = 9.5 \text{ ft}$$

AASHTO [8.9.2] states that the minimum depths in Table 8.9.2 are recommended unless computation of deflection indicates that lesser depths may be used without adverse effects.

AASHTO [Table 8.9.2]

$$t_{\min} = \frac{S+10}{30} = \left(\frac{9.5 \text{ ft} + 10 \text{ ft}}{30}\right)\left(12 \ \frac{\text{in}}{\text{ft}}\right)$$
$$= 7.8 \text{ in}$$

The assumed slab thickness is $t = 7.8$ in + 0.5 in for integral wearing surface.

Use $t = 9.0$ in.

Step 2: Determine factored loads.

Group loading combinations for load factor design are

Table 3.22.1A

$$\text{Group I} = \gamma(\beta_D D + \beta_L (L+I))$$
$$= (1.30)\Big[((1.0)D + (1.67)(L+I))\Big]$$
$$= 1.30D + (2.17)(L+I)$$

Determine the factored dead loads.

$$W_D = (1.30)(\text{deck slab} + \text{FWS})$$
$$= (1.30)\left((9 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(0.150 \ \frac{\text{kip}}{\text{ft}^3}\right) + 0.03 \ \frac{\text{kip}}{\text{ft}^2}\right)$$
$$= 0.185 \text{ kip/ft}^2 \text{ (0.185 kip/ft per foot of width of slab)}$$

$$W_{c+p} = (1.30)(\text{curb and parapet})$$
$$= (1.30)(3.37 \text{ ft}^2)\left(0.150 \ \frac{\text{kip}}{\text{ft}^3}\right)$$
$$= 0.657 \text{ kip/ft of bridge span}$$

Determine the factored live plus impact loads. The 20 kip wheel load from the HS 25 loading will govern the design of the deck slab.

AASHTO [3.8.2]

The live load impact is

$$I = \frac{50}{L+125}$$
$$= \frac{50}{10 \text{ ft} + 125} = 0.37 \quad \left[\begin{array}{c}0.30 \text{ maximum} \\ \text{allowed}\end{array}\right]$$

The factored wheel plus impact load is

$$P_{L+I} = (2.17)(L+I)$$
$$= (2.17)\big(20 \text{ kips} + (0.30)(20 \text{ kips})\big)$$
$$= 56.4 \text{ kips}$$

Step 3: Analyze for factored moment.

For continuous spans, the factored positive and negative dead load moments are assumed to be

$$M_D = \frac{W_D S^2}{10} = \frac{\left(0.185 \ \frac{\text{kip}}{\text{ft}^2}\right)(9.5 \text{ ft})^2}{10}$$
$$= 1.67 \text{ ft-kips/ft width of slab}$$

AASHTO [3.24.3.1] In slabs continuous over three or more supports, a continuity factor of 0.8 is applicable. The factored positive and negative live load plus impact moments are

AASHTO [3.24.3.1 & Eq. 3-15]

$$M_{L+I} = (0.80)\left(\frac{S+2}{32}\right)P_{L+I}$$
$$= (0.8)\left(\frac{9.5 \text{ ft} + 2 \text{ ft}}{32}\right)(56.4 \text{ kips})$$
$$= 16.22 \text{ ft-kips/ft width of slab}$$

The total factored positive and negative moments are

$$M_u = M_D + M_{L+I}$$
$$= 1.67 \text{ ft-kips} + 16.22 \text{ ft-kips}$$
$$= 17.89 \text{ ft-kips/ft width of slab}$$

AASHTO [3.24.5] For cantilever spans, the factored negative dead load moment is

$$M_D = \frac{W_D S^2}{2} + W_{c+p}L$$

AASHTO [3.24.1.2]

$$S = 4 \text{ ft} - \frac{0.5 \text{ ft}}{2} = 3.75 \text{ ft}$$

[refer to the typical section and curb/parapet cross section]

$$L = 3.75 \text{ ft} - 0.66 \text{ ft} = 3.09 \text{ ft}$$

$$M_D = \frac{\left(0.185 \ \frac{\text{kips}}{\text{ft}^2}\right)(3.75 \text{ ft})^2}{2} + \left(0.657 \ \frac{\text{kip}}{\text{ft}}\right)(3.09 \text{ ft})$$
$$= 3.33 \text{ ft-kips/ft width of slab}$$

AASHTO [3.24.2.1] The centerline of the wheel will be placed 1 ft from the face of the curb.

[3.24.5] Each wheel on the slab perpendicular to traffic is distributed over a width of

[Eq. 3-17]

$$E = 0.8X + 3.75 \text{ ft}$$
$$= (0.8)(1.0 \text{ ft}) + 3.75 \text{ ft}$$
$$= 4.55 \text{ ft}$$

X = distance in feet from wheel load to point of support

$$X = 3.75 \text{ ft} - 2.75 \text{ ft} = 1.0 \text{ ft}$$

AASHTO [3.24.5.1] The factored negative wheel load plus impact moment is

$$M_{L+I} = P_{L+I}\left(\frac{X}{E}\right)$$
$$= (56.4 \text{ kips})\left(\frac{1.00 \text{ ft}}{4.55 \text{ ft}}\right)$$
$$= 12.40 \text{ ft-kips/ft width of slab}$$

The total factored negative moment is

$$M_u = M_D + M_{L+I}$$
$$= 3.33 \text{ ft-kips} + 12.40 \text{ ft-kips}$$
$$= 15.73 \text{ ft-kips/ft width of slab}$$

Step 4: Design for moment.

The compressive strength of the concrete at 28 days is $f'_c = 4500 \text{ lbf/in}^2$. The specified minimum yield point of the steel is $F_y = 60{,}000 \text{ lbf/in}^2$. Determine the maximum and minimum steel reinforcement needed.

AASHTO [8.16.3.1 & 8.16.3.2] The maximum ratio of tension reinforcement is

[8.16.3.1.1]

$$\rho_{max} = 0.75\rho_b = (0.75)(0.0311) = 0.0233$$

[8.16.2.7]

$$\beta_1 = 0.85 - 0.025 = 0.825$$

Since $f'_c = 4500 \text{ lbf/in}^2$,

[8.16.3.2.2]

$$\rho_b = \left(\frac{0.85\beta_1 f'_c}{F_y}\right)\left(\frac{87{,}000}{87{,}000 + F_y}\right)$$
$$= \left(\frac{(0.85)(0.825)\left(4500 \ \frac{\text{lbf}}{\text{in}^2}\right)}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}\right) \times \left(\frac{87{,}000}{87{,}000 + 60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}\right)$$
$$= 0.0311$$

Determine the areas of positive and negative steel.

AASHTO [8.16.3.2 & Eq. 8-16]

$$\phi M_n = \phi A_s F_y \left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s F_y}{0.85 f'_c b} = \frac{A_s \left(60 \ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(4.5 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})}$$

$$= 1.307 A_s$$

Assume No. 6 steel rebar.

$$d = 9 \text{ in} - 0.5 \text{ in for integral wearing surface} - 2.0 \text{ in for cover} - 0.38 \text{ in} = 6.12 \text{ in}$$

[8.16.1.2] $\phi = 0.9$ for flexure

$M_u = \phi M_n$ for continuous spans

$$\left(17.89 \ \frac{\text{ft-kips}}{\text{ft}}\right)\left(12 \ \frac{\text{in}}{\text{ft}}\right) = 0.9 A_s \left(60 \ \frac{\text{kips}}{\text{in}^2}\right) \times \left(6.12 \text{ in} - \left(\frac{1.307}{2}\right) A_s\right)$$

$$A_s^2 - 9.36 A_s + 6.08 = 0$$

$$A_s = \frac{9.36 - \sqrt{(9.36)^2 - (4)(6.08)}}{2}$$

$$= 0.70 \text{ in}^2/\text{ft of slab width}$$

Use #6 @ 7½ in ($A_s = 0.71 \text{ in}^2/\text{ft}$).

$$\rho = \frac{A_s}{bd} = \frac{0.71 \text{ in}^2}{(12 \text{ in})(6.12 \text{ in})} = 0.0097$$

[$< \rho_{max}$, so OK]

Check moment capacity.

$$a = \frac{A_s F_y}{0.85 f'_c b} = \frac{(0.71 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(4.5 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})}$$

$$= 0.928 \text{ in}$$

$$\frac{a}{2} = 0.464 \text{ in}$$

$$\phi M_n = (0.9)(0.71 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right) \times (6.12 \text{ in} - 0.464 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$= 18.08 \text{ ft-kips/ft of slab width}$$

$$M_u = 17.89 \text{ ft-kips/ft of slab width}$$

[$< \phi M_n$, so OK]

Check minimum steel.

AASHTO [8.17.1] In a flexural member where tension reinforcement is required by analysis, the minimum reinforcement provided shall be adequate to develop a moment capacity at least 1.2 times the cracking moment.

[Eq. 8-62] $\phi M_n \geq 1.2 M_{cr}$

[8.13.3 & Eq. 8-2] $M_{cr} = \dfrac{f_r I_g}{y_t}$

[8.15.2.1.1] f_r = modulus of rupture $= 7.5\sqrt{f'_c}$ for normal weight concrete

$f'_c = 4500 \text{ lbf/in}^2$

[8.1.2] I_g = moment of inertia of gross section about centroidal axis, neglecting reinforcement

y_t = distance from centroidal axis to extreme fiber in tension

[Eq. 8-62] $\phi M_n \geq 1.2 M_{cr}$

[8.13.3; 8.15.2]

$$M_{cr} = \frac{f_r I_g}{y_t} = \left(\frac{7.5\sqrt{f'_c}\left(\frac{(12 \text{ in})(9 \text{ in})^3}{12}\right)}{4.5 \text{ in}}\right) \times \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{1 \text{ kip}}{1000 \text{ lbf}}\right)$$

$$= 6.79 \text{ ft-kips/ft}$$

$$1.2 M_{cr} = (1.2)(6.79 \text{ ft-kips}) = 8.15 \text{ ft-kips/ft}$$

[$\leq \phi M_n$ (18.08 ft-kips/ft), so OK]

AASHTO [3.24.10] *Distribution Reinforcement*

Reinforcement transverse to the main steel reinforcement (which is perpendicular to traffic) is placed in the bottom of all slabs. The amount shall be a percentage of the main reinforcement required as determined in the following formula.

[3.24.10.2 & Eq. 3-22] The percentage is $220/\sqrt{S}$, with a maximum of 67%.

$$\frac{220}{\sqrt{9.5\ \text{ft}}} = 71.4\% \quad [67\%\ \text{maximum allowed}]$$

$$A_s = (0.67)(0.70\ \text{in}^2)$$
$$= 0.47\ \text{in}^2/\text{ft}$$

Use #6 @ 9 in (A_s = 0.59 in^2/ft) in the bottom and perpendicular to the main reinforcement in the middle half of the slab span.

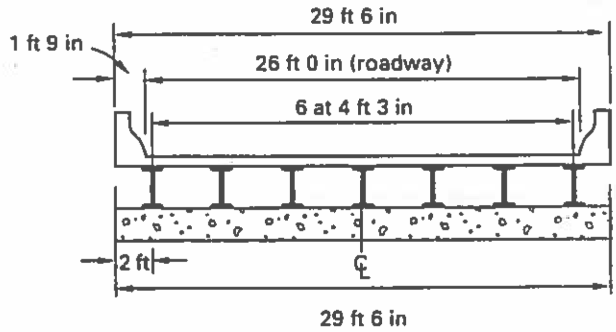
AASHTO [3.24.10.3] 50% of the specified distribution reinforcement is used in the outer quarters of the slab span.

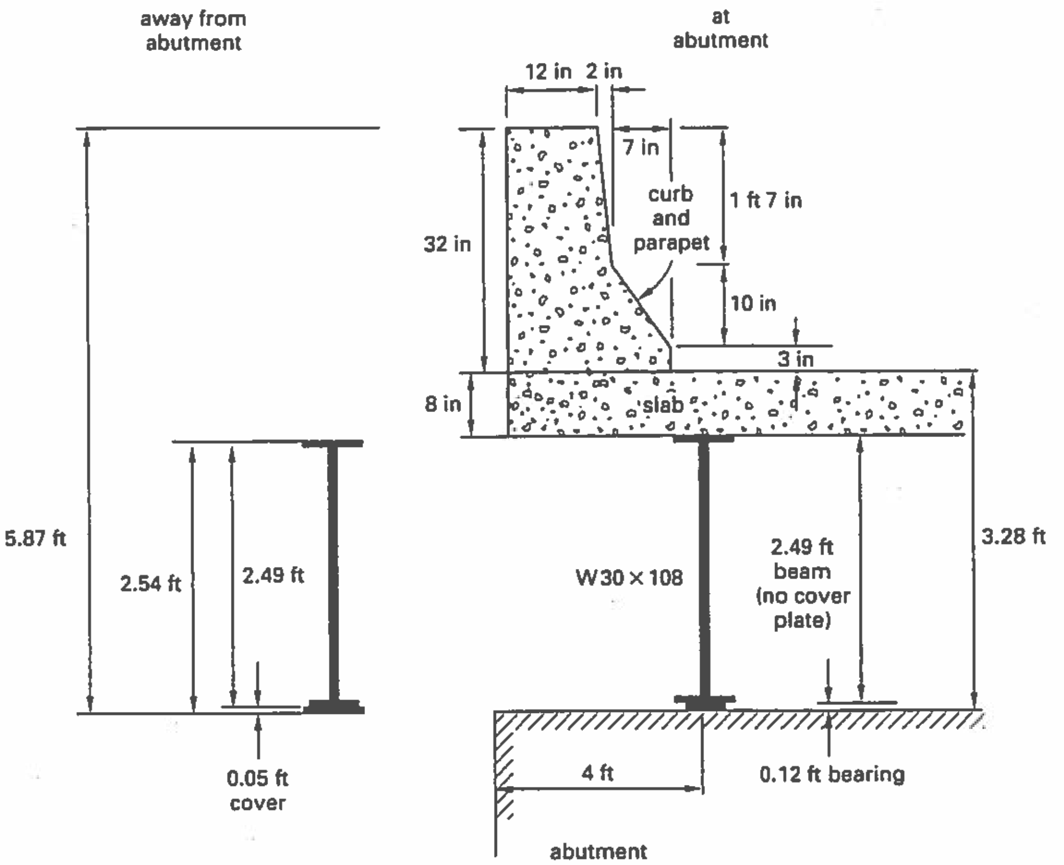
Step 5: Design for shear and bond.

[3.24.4] Slabs designed for bending moment in accordance with AASHTO Sec. 3.24.3 (wheel loads) are considered satisfactory in bond and shear.

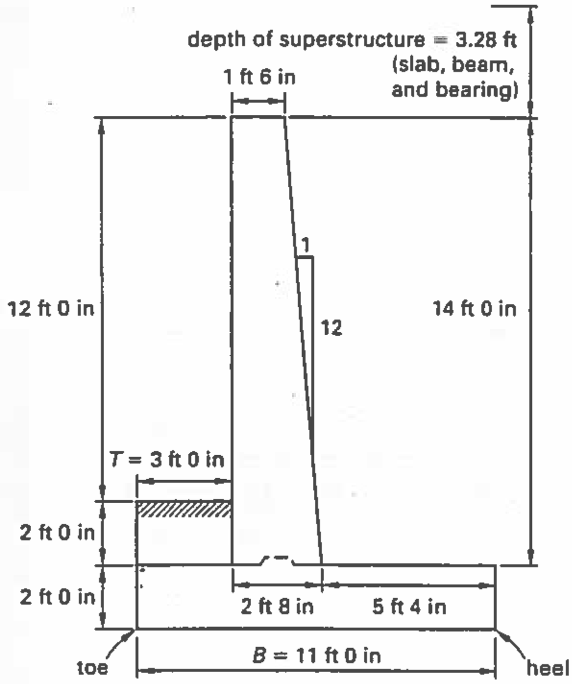
Design Example 3: Reinforced Concrete Abutment and Footing for Overpass Structure

A two-lane bridge for an HS 25 loading is supported by seven W 30 × 108 steel beams with a 14 in × 9/16 in cover plate and an 8 in × 29.5 ft wide (26.0 ft roadway width) concrete slab. The future wearing surface is 0.030 kips/ft^2. The concrete has an f'_c of 3.0 kips/in^2. The F_y of the reinforcing steel is 60.0 kips/in^2. The superstructure span is 60.2 ft center to center on bearings (the overall beam length is 61.70 ft). It is 3.28 ft high including the concrete slab, steel beams, and 0.12 ft bearing, and is attached to the top of the stem with a hinged support that transmits horizontal and vertical load but not moment.





The abutment has a total height of 16 ft and a length of 29.5 ft including 3.0 ft cheek walls, and it has a spread footing and no backwall. It is placed on a gravel and sand soil with a safe bearing capacity of 8 kips/ft^2. The density of the compacted earth fill is 120 lbf/ft^3, and the lateral earth pressure (equivalent fluid weight) is 35 lbf/ft^3. Assumed dimensions of the abutment are shown. A live surcharge of 2 ft of soil is placed on the bridge approach (AASHTO 3.20.3 and 5.5.2).



Analyze the structure for stability, design the footing and abutment system, and conduct a bearing check of the footing.

Solution:

Step 1: Determine loadings.

AASHTO [3.8.1.2; 3.20; 5.5; 7.5.2]

The loadings that must be considered in this example are dead load, live load, earth pressure, wind load on structure, wind load on live load, longitudinal force from live load, and longitudinal friction force due to temperature. There will be no buoyancy, stream flow pressure, or ice pressure loadings for this overpass structure. Impact loadings are not considered for abutments. Centrifugal forces do not exist for this straight structure. Earthquake loading is not considered.

AASHTO [3.22.2; 7.5.2.1; 5.5.5]

Service loads are used in determining if the abutment is safe against overturning about the toe of the footing, against sliding on the footing base, and against crushing of the foundation material at the point of maximum pressure.

AASHTO [3.22.1; Table 3.22.1A; and footnotes]

AASHTO load combinations for service load design for abutment and footing are given by

$$\text{Group N} = \gamma[(\beta_D D) + (\beta_L L) + (\beta_E E) + (\beta_W W) + (\beta_{WL}\text{WL}) + (\beta_L \text{LF}) + (\beta_R T)]$$

$$\begin{aligned}
\text{Group I} &= (1.0)(1D + 1L + 1E)\\
\text{Group II} &= (1.0)(1D + 1E + 1W)\\
\text{Group III} &= (1.0)(1D + 1L + 1E + 0.3W + 1\text{WL} + 1\text{LF})\\
\text{Group IV} &= (1.0)(1D + 1L + 1E + 1T)\\
\text{Group V} &= (1.0)(1D + 1E + 1W + 1T)\\
\text{Group VI} &= (1.0)(1D + 1L + 1E + 0.3W + 1\text{WL} + 1\text{LF} + 1T)\\
\text{Group VII} &= (1.0)(1D + 1E)\\
\text{Group VIII} &= (1.0)(1D + 1L + 1E)\\
\text{Group IX} &= (1.0)(1D + 1E + 1W)
\end{aligned}$$

Group VII loading is not as critical as Group I loading. Group VIII loading is the same as Group I loading, and Group IX loading is the same as Group II loading. Therefore, Group I, II, III, IV, V, and VI loadings will be the only ones considered in checking for stability and bearing pressure for the abutment.

AASHTO [3.22.3; 5.5.6]

Factored loads are used for designing structural members using the load factor concept.

Group loading combinations for load factor design are given by

AASHTO [3.22.1; Table 3.22.1A; and footnotes]

$$\text{Group N} = \gamma(\beta_D D + \beta_L L + \beta_E E + \beta_W W + \beta_{WL}\text{WL} + \beta_L\ \text{LF} + \beta_R T)$$

$$\begin{aligned}
\text{Group I} &= (1.3)((0.75 \text{ or } 1)D + 1.67L + 1E)\\
\text{Group II} &= (1.3)((0.75 \text{ or } 1)D + 1E + 1W)\\
\text{Group III} &= (1.3)((0.75 \text{ or } 1)D + 1L + 1E + 0.3W + 1\text{WL} + 1\text{LF})\\
\text{Group IV} &= (1.3)((0.75 \text{ or } 1)D + 1L + 1E + 1T)\\
\text{Group V} &= (1.25)((0.75 \text{ or } 1)D + 1E + 1W + 1T)\\
\text{Group VI} &= (1.25)((0.75 \text{ or } 1)D + 1L + 1E + 0.3W + 1\text{WL} + 1\text{LF} + 1T)
\end{aligned}$$

Note that in the equations for Groups I through VI, β_D depends on the magnitude of axial load and moment used for column design. Use 0.75 when checking the member for minimum axial load and maximum moment. Use 1.0 when checking the member for maximum axial load and minimum moment. For flexural and tension members, β_D is 1.0.

Group I, II, III, IV, V, and VI loadings will be the only ones considered for designing structural members by the load factor design.

Determine the total dead load weight of the superstructure on the abutment by calculating the dead load weights of the individual components and then adding them.

For seven beams (W 30 × 108),

$$(61.70 \text{ ft})\left(0.108\ \frac{\text{kips}}{\text{ft}}\right)\left(\frac{7}{2}\right)$$

$$= 23.32 \text{ kips}$$

For seven cover plates (14 in × 9/16 in), assuming the weight of steel is 0.49 kip/ft^3,

$$(14 \text{ in})\left(\frac{9}{16} \text{ in}\right)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{1 \text{ ft}}{12 \text{ ft}}\right)\left(0.49\ \frac{\text{kip}}{\text{ft}^3}\right)$$

$$= 0.0268 \text{ kip/ft}$$

$$(60.2 \text{ ft} - 2.0 \text{ ft})\left(0.0268\ \frac{\text{kip}}{\text{ft}}\right)\left(\frac{7}{2}\right)$$

$$= 5.46 \text{ kips}$$

For the end block and diaphragm,

$$(29.5 \text{ ft})(2.75 \text{ ft})(1.50 \text{ ft})\left(0.150\ \frac{\text{kip}}{\text{ft}^3}\right)$$

$$= 18.25 \text{ kips}$$

For the 29.5 ft × 8 in slab,

$$(61.70 \text{ ft})(29.50 \text{ ft})(0.667 \text{ ft})\left(\frac{0.150\ \frac{\text{kip}}{\text{ft}^3}}{2}\right)$$

$$= 91.05 \text{ kips}$$

For the curb and parapets,

$$(61.70 \text{ ft})(3.37 \text{ ft}^2)\left(0.150\ \frac{\text{kip}}{\text{ft}^3}\right) = 31.19 \text{ kips}$$

For a 30 lbf/ft^2 future wearing surface,

$$(61.70 \text{ ft})(26.00 \text{ ft})\left(\frac{(0.030\ \frac{\text{kip}}{\text{ft}^2}}{2}\right)$$

$$= 24.06 \text{ kips}$$

The total superstructure weight on the abutment is 193.33 kips.

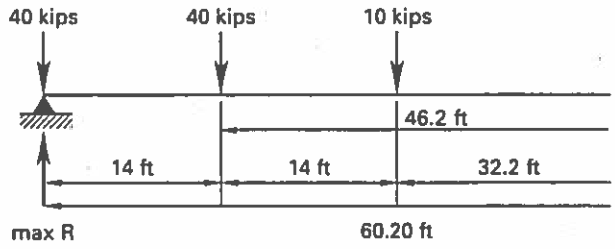
The dead load weight per foot of abutment is

$$D = \frac{193.33 \text{ kips}}{29.50 \text{ ft}} = 6.55 \text{ kips/ft}$$

Determine the live load weight on the superstructure. Use HS 25 loading.

AASHTO [Fig. 3.7.7A]

HS 25 truck (= HS 20 loading × 1.25)

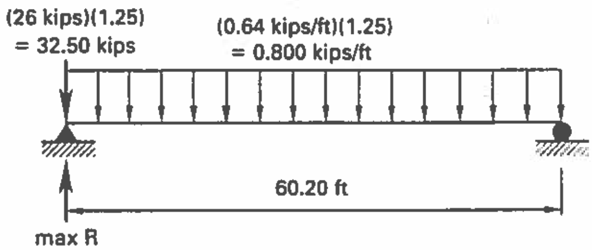


$$\text{max } R = 40 \text{ kips} + \left(\frac{46.2 \text{ ft}}{60.2 \text{ ft}}\right)(40 \text{ kips}) + \left(\frac{32.2 \text{ ft}}{60.2 \text{ ft}}\right)(10 \text{ kips})$$

$$= 76.05 \text{ kips/lane}$$

AASHTO [Fig. 3.7.6B]

For an HS 25 lane loading (simulate the truck train loading),



$$\text{max } R = 32.50 \text{ kips} + \frac{\left(0.800\ \frac{\text{kips}}{\text{ft}}\right)(60.20 \text{ ft})}{2} = 56.58 \text{ kips/lane}$$

HS 25 truck loading controls the maximum reaction, which is 76.05 kips/lane.

The maximum live load per foot of abutment is

$$L = \frac{(2 \text{ lanes})\left(76.05\ \frac{\text{kips}}{\text{lane}}\right)}{29.50 \text{ ft}} = 5.16 \text{ kips/ft}$$

Determine the lateral earth pressure.

AASHTO [5.5.2]

An equivalent fluid weight of 35 lbf/ft^3 is assumed for determining the lateral earth pressure.

[7.5.2.1]

The effect of passive pressure due to soil in front of the abutment will be neglected.

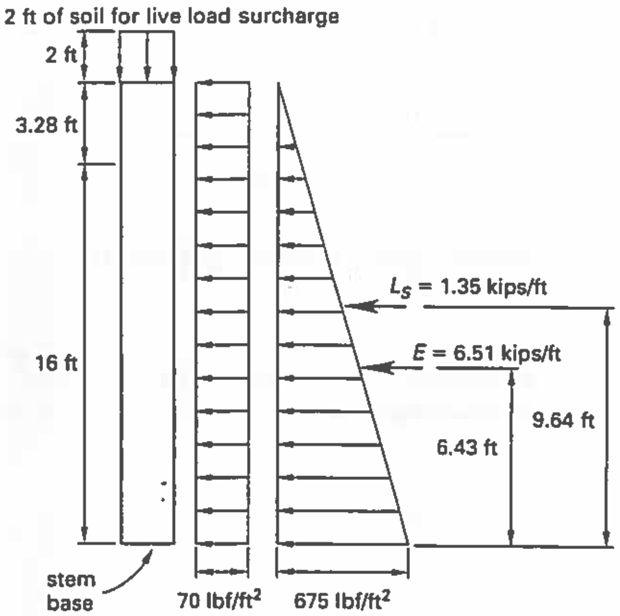
|3.20.3 & 5.5.2| A live load surcharge pressure equal to 2 ft of earth will be added to the approach.

$$(2\text{ ft})\left(35\ \frac{\text{lbf}}{\text{ft}^3}\right) = 70\text{ lbf/ft}^2$$

The lateral pressure due to the earth backfill is

$$(3.28\text{ ft} + 16\text{ ft})\left(35\ \frac{\text{lbf}}{\text{ft}^3}\right) = 675\text{ lbf/ft}^2$$

The load due to earth and live load surcharge is as follows.



$$L_S = \left(0.070\ \frac{\text{kip}}{\text{ft}^2}\right)(19.28\text{ ft}) = 1.35\text{ kips/ft}$$

$$E = \left(\frac{1}{2}\right)\left(0.675\ \frac{\text{kip}}{\text{ft}^2}\right)(19.28\text{ ft})$$
$$= 6.51\text{ kips/ft}$$

Determine other forces acting on the abutment.

AASHTO |3.15.2.1.3| W_{super} = wind load on the superstructure

|3.15.2.1.2; 3.15.2.1.3| WL = wind load on live load

|3.15.3| W_{up} = wind upward force (i.e., overturning tendency)

|3.9| LF_H = longitudinal force from the live load—horizontal

LF_V = longitudinal force from the live load—vertical

|3.16| T_f = longitudinal force due to temperature/friction

|3.15.2.2| W_{sub} = wind load directly on the substructure

Note: The transverse wind loads in the plane of abutment stem length are neglected since the abutment is long (i.e., 29.5 ft).

AASHTO |3.15.2.1.3| Determine the wind load, W_{super}, on the superstructure transmitted to the substructure. For the usual girder having span lengths less than 125 ft, the transverse wind loading on the superstructure can be taken as 50 lbf/ft², and the longitudinal wind loading can be taken as 12 lbf/ft².

The transverse wind loading in the plane of the stem will be neglected in this example since the abutment is so long (i.e., 29.5 ft).

The projected area of the superstructure is

$$\text{beam and cover} = 29.87\text{ in} + \frac{9}{16}\text{ in}$$
$$= 30.4\text{ in}$$

The height exposed to the wind, including the slab and parapet, is

$$(30.4 + 8\text{ in slab} + 32\text{ in curb and parapet})$$
$$\times\left(\frac{1\text{ ft}}{12\text{ in}}\right) = 5.87\text{ ft}$$

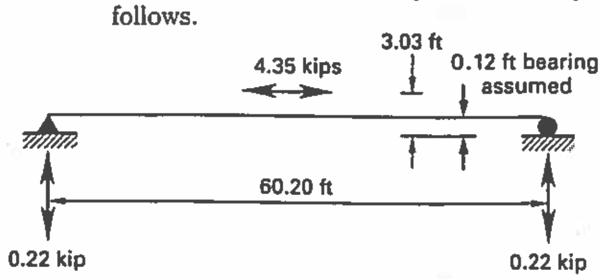
The distance of the center of gravity of the superstructure area above the top of the abutment is

$$\frac{5.87\text{ ft}}{2} - 0.05\text{ ft} + 0.12\text{ ft bearing} = 3.03\text{ ft}$$

The longitudinal wind loading is

$$(61.70\text{ ft})(5.87\text{ ft})\left(0.012\ \frac{\text{kip}}{\text{ft}^2}\right)$$
$$= 4.35\text{ kips}$$

This longitudinal force must be transmitted to the substructure through the bearings as follows.



The longitudinal (horizontal) wind loading at the top of the abutment is

$$W_{super(H)} = \frac{4.35 \text{ kips}}{29.5 \text{ ft abutment width}}$$
$$= 0.15 \text{ kip/ft width}$$

The vertical wind loading at the top of the abutment is

$$\frac{(4.35 \text{ kips})(3.03 \text{ ft})}{60.2 \text{ ft}} = 0.22 \text{ kip}$$

$$W_{super(V)} = \frac{0.22 \text{ kip}}{29.5 \text{ ft abutment width}}$$
$$= 0.007 \text{ kip/ft} \quad \text{[negligible]}$$

AASHTO [3.15.2.2]

Determine the wind load directly on the substructure, W_{sub}.

A wind loading of 40 lbf/ft^2 will act perpendicular to the exposed stem of the abutment. The horizontal wind loading at 10 ft (12 ft/2 + 2 ft + 2 ft) above the base of the abutment footing is

$$W_{sub(H)} = \left(0.040 \frac{\text{kip}}{\text{ft}^2}\right)(12 \text{ ft})$$
$$= 0.48 \text{ kip/ft}$$

12 ft = exposed stem height

AASHTO [3.15.3]

Determine the upward wind load, W_{up}.

The upward force will be 20 lbf/ft^2 of horizontal area for Group II and V combinations and 6 lbf/ft^2 for Group III and Group VI combinations. (The quarter point provision is ignored.)

For Groups II and V, the uplift is

$$W_{up} = \frac{(61.70 \text{ ft})(26.5 \text{ ft})\left(0.020 \frac{\text{kip}}{\text{ft}^2}\right)}{(2)(29.5 \text{ ft})}$$
$$= 0.55 \text{ kip/ft}$$

61.70 ft = overall beam length

26.5 ft = roadway deck width plus 3 in curbs assumed

29.5 ft = slab width = abutment width

For Groups III and VI, the uplift is

$$W_{up} = \frac{(61.70 \text{ ft})(26.5 \text{ ft})\left(0.006 \frac{\text{kip}}{\text{ft}^2}\right)}{(2)(29.5 \text{ ft})}$$
$$= 0.17 \text{ kip/ft}$$

AASHTO [3.15.2.1.2 & 3.15.2.1.3]

Determine the wind load, WL, transmitted to the substructure by the wind load on the moving live load.

The longitudinal wind loading on the live load is taken as 40 lbf/ft and acts at a point 6 ft above the deck.

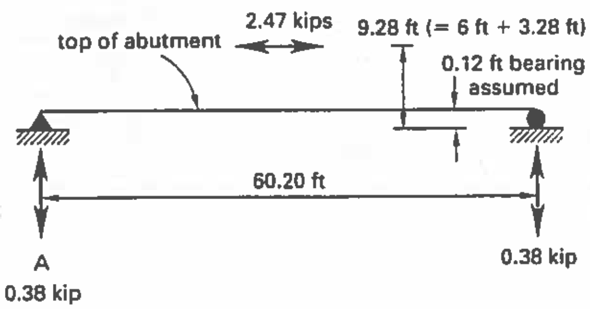
The longitudinal wind load is

$$(61.70 \text{ ft})\left(0.040 \frac{\text{kip}}{\text{ft}}\right) = 2.47 \text{ kips}$$

This longitudinal force is transmitted to the substructure through the bearings as follows.

The reaction at A is

$$\frac{(2.47 \text{ kips})(9.28 \text{ ft})}{60.2 \text{ ft}} = 0.38 \text{ kip}$$



The horizontal wind loading at the top of the abutment is

$$WL_H = \frac{2.47 \text{ kips}}{29.5 \text{ ft}} = 0.08 \text{ kip/ft}$$

The vertical wind loading at the top of the abutment is

$$WL_V = \frac{0.38 \text{ kip}}{29.5 \text{ ft}} = 0.01 \text{ kip/ft width}$$

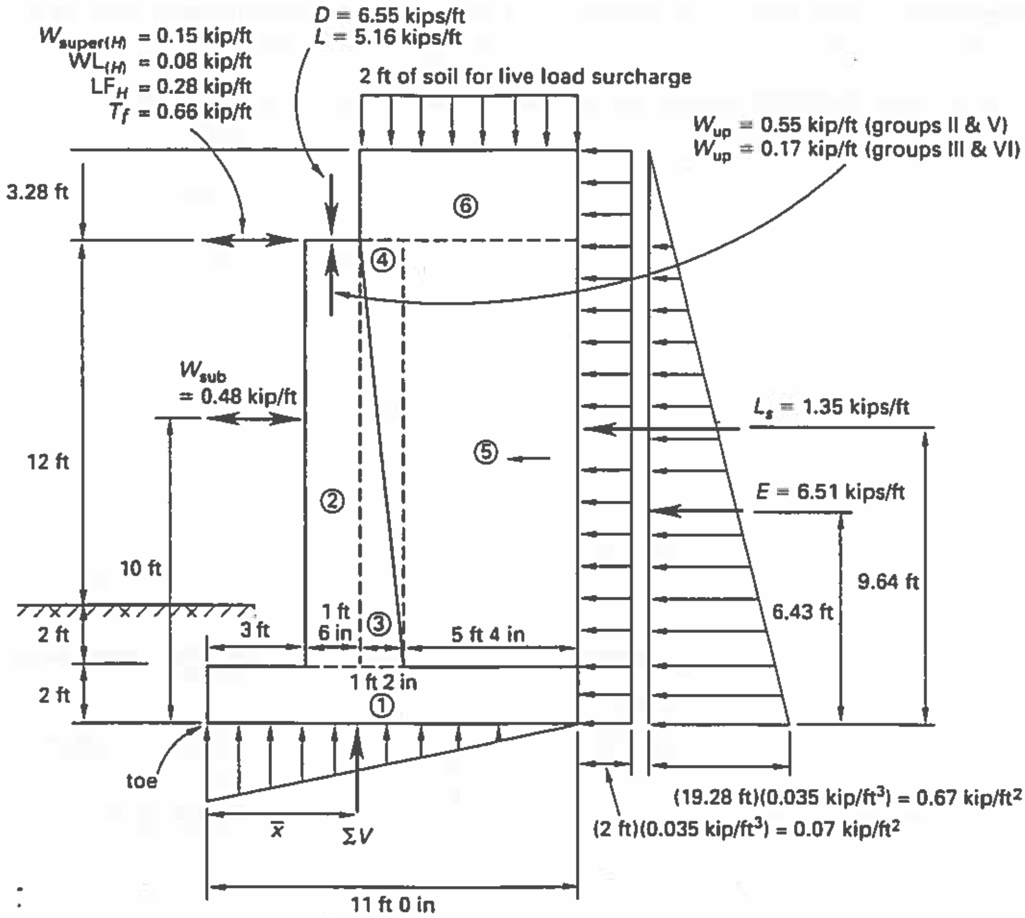
[negligible]

AASHTO [Fig. 3.7.6B & 3.9]

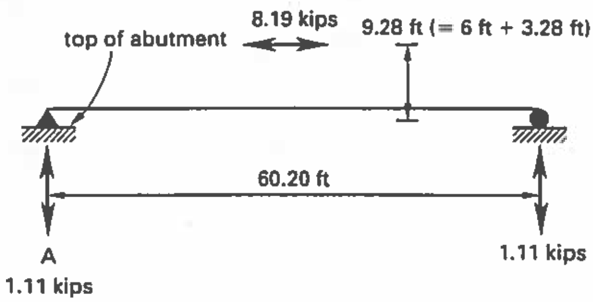
A longitudinal force of 5% of the live load in all lanes is located 6 ft above the floor slab.

For HS 25 loading, the longitudinal force is

$$(2 \text{ lanes})\left[(61.70 \text{ ft})\left(0.64 \frac{\text{kip}}{\text{ft}}\right)(1.25) + (26 \text{ kips})(1.25)\right](0.05) = 8.19 \text{ kips}$$



This longitudinal force is transmitted to the substructure through the bearings as follows.



The horizontal force at the top of the abutment is

$$LF_H = \frac{8.19 \text{ kips}}{29.5 \text{ ft}} = 0.28 \text{ kip/ft}$$

The reaction at A is

$$\frac{(8.19 \text{ kips})(9.28 \text{ ft})}{60.2 \text{ ft}} = 1.26 \text{ kips}$$

The vertical force at the top of the abutment is

$$LF_V = \frac{1.26 \text{ kips}}{29.5 \text{ ft}} = 0.042 \text{ kip/ft width} \quad \text{[negligible]}$$

AASHTO [3.16] Determine the temperature force, T_f, due to friction.

The longitudinal force due to friction at expansion bearings, which is transmitted to both abutments through the superstructure, is assumed to be 10% (coefficient of friction) of the dead load reaction.

The friction force is

$$T_f = (0.10)\left(6.55 \frac{\text{kips}}{\text{ft}}\right) = 0.66 \text{ kip/ft}$$

The service loads and moments due to the loads are summarized in Table 4.

Step 2: Perform a stability analysis and bearing pressure check (service load design).

AASHTO [3.3.6] The density of the normal weight concrete is 150 lbf/ft^3. The density of the compacted earth fill is 120 lbf/ft^3. To compensate for incidental field adjustments in the location of bearings, a 2 in longitudinal eccentricity from the theoretical centerline of bearing will be used.

L_S = lateral pressure from 2 ft of soil for live load surcharge

$$= \left((2\ \text{ft})\left(0.035\ \frac{\text{kip}}{\text{ft}^3}\right)\right)(19.28\ \text{ft})$$

$$= 1.35\ \text{kips/ft}$$

E = lateral earth pressure

$$= \left((19.28\ \text{ft})\left(0.035\ \frac{\text{kip}}{\text{ft}^3}\right)(19.28\ \text{ft})\left(\frac{1}{2}\right)\right)$$

$$= 6.51\ \text{kips/ft}$$

To create the maximum pressure under the toe and the minimum pressure under the heel of the footing, an eccentricity of 2 in to the left for vertical forces will be used.

Check against sliding and overturning.

The factor of safety against sliding is

AASHTO [Table 5.5.2B]

$$\text{FS}_S = \frac{f\Sigma V}{\Sigma H} = \frac{0.60\Sigma V}{\Sigma H} \geq 1.5$$

f is an assumed friction factor of 0.60.

[5.5.5] The minimum factor of safety against sliding is 1.50.

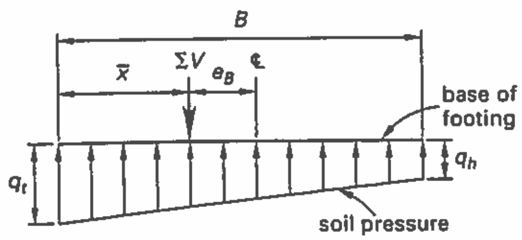
The factor of safety against overturning is

$$\text{FS}_0 = \frac{\Sigma M_{V,\text{toe}}}{\Sigma M_H} \geq 2.0$$

[5.5.5] The minimum factor of safety against overturning is 2.00 for footings set on soil.

The location of the resultant soil pressure from the toe is given by

$$\bar{x} = \frac{\Sigma M_{V,\text{toe}} - \Sigma M_H}{\Sigma V}$$



$$q_t = \frac{\Sigma V}{B} + \frac{(\Sigma V)e_B\left(\frac{B}{2}\right)}{\frac{B^3}{12}} = \left(\frac{\Sigma V}{B}\right)\left(1 + \frac{6e_B}{B}\right)$$

The eccentricity of the resultant soil pressure is given by

$$e_B = \frac{B}{2} - \bar{x}$$

[Fig. 4.4.7.1.1.1C] For $e_B < B/6$, the pressure under the footing is given by

$$q_t = \left(\frac{\Sigma V}{B}\right)\left(1 + \frac{6e_B}{B}\right) \quad \text{[at toe]}$$

$$q_h = \left(\frac{\Sigma V}{B}\right)\left(1 - \frac{6e_B}{B}\right) \quad \text{[at heel]}$$

AASHTO [5.5.5 & 4.4.7] For a footing set on soil, the location of the resultant soil pressure should be within the middle one-third of the base (i.e., $e_B < B/6$). $B/6$ in this problem is 11 ft/6 and is equal to 1.83 ft.

The safe bearing capacity of the gravel-sand foundation material is assumed to be 8 kips/ft^2.

Table 4 summarizes service loads and service load moments at the toe of the footing.

The stability and bearing pressure check is summarized in Table 5.

Also note that in this problem, Group VII loading is not as critical as Group I loading, Group VIII loading is the same as Group II loading, and Group IX loading is the same as Group II loading.

The minimum values of the factors of safety against sliding and overturning from Table 5 are

$\text{FS}_{S,\text{min}} = 2.01$ from Group V loading

[> 1.50, so OK]

$\text{FS}_{0,\text{min}} = 2.60$ from Group VI loading

[> 2.00, so OK]

Table 4 Summary of Service Loads and Moments (at toe of footing, M_V and M_H)

Vertical Loads					
	item number	items	V (kips/ft)	lever arm from toe (ft)	$M_{V,\text{toe}}$ (ft-kips/ft)
concrete	1	D(11.00 ft × 2.00 ft × 0.150 kcf)	3.30	5.50	18.15
concrete	2	D(14.00 ft × 1.50 ft × 0.150 kcf)	3.15	3.75	11.81
concrete	3	D(1/2 × 14.00 ft × 1.17 ft × 0.150 kcf)	1.23	4.89	6.01
soil	4	E(1/2 × 14.00 ft × 1.17 ft × 0.120 kcf)	0.98	5.28	5.17
soil	5	E(14.00 ft × 5.33 ft × 0.120 kcf)	8.95	8.33	74.55
soil	6	E(6.50 ft × 3.28 ft × 0.120 kcf)	2.56	7.75	19.84
live load surcharge	7	L_s(6.50 ft × 2.00 ft × 0.120 kcf)	1.56	7.75	12.09
superstructure	8	D	6.55	3.58	23.45
live load	9	L	5.16	3.58	18.47
wind upward	10	W_{up} (Groups II & V)	−0.55	3.58	−1.97
		$0.3W_{\text{up}}$ (Groups III & VI)	−0.05	3.58	−0.18
Horizontal Loads					
	item number	items	H (kips/ft)	lever arm from footing base (ft)	M_H (ft-kips/ft)
	11	L_S (live load surcharge)	1.35	9.64	13.01
	12	E	6.51	6.43	41.86
	13	LF	0.24	16.00	3.84
	14	T_f	0.66	16.00	10.56
	15	WL_H	0.08	16.00	1.28
	16	$W_{\text{super}(H)}$(Groups II & V)	0.15	16.00	2.40
	17	$0.3W_{\text{super}(H)}$(Groups III & VI)	0.045	16.00	0.72
	18	W_{sub}(Groups II & V)	0.48	10.00	4.80
	19	$0.3W_{\text{sub}}$(Groups III & VI)	0.144	10.00	1.44

Table 5 Stability and Bearing Pressures

group loading[a]	ΣV (kips)	ΣH (kips)	$\Sigma M_{V,\text{toe}}$ (ft-kips/ft)	ΣM_H (ft-kips/ft)	FS_S	FS_O	$\bar{x}$ (ft)	e_B (ft)	q_t (kips/ft^3)	q_h (kips/ft^3)
I	33.44	7.86	189.54	54.87	2.55	3.45	4.03	1.47	5.48	0.60
II	26.17	7.14	157.01	49.06	2.20	3.20	4.12	1.38	4.16	0.59
III	33.39	8.37	189.36	62.15	2.39	3.05	3.81	1.69	5.81	0.24
IV	33.44	8.52	189.54	65.43	2.35	2.90	3.71	1.79	6.01	0.07
V	26.17	7.80	157.01	59.62	2.01	2.63	3.72	1.78	4.69	0.07
VI	33.27	9.03	188.93	72.71	2.21	2.60	3.49	2.01	6.36	–

[a] Refer to Service Load Design in AASHTO, Table 3.22.1A.

The eccentricity of the resultant soil pressure, ΣV, is

$$e_{B,\max} = 2.01 \text{ ft from Group VI loading}$$

$$[> 1.83 \text{ ft} = (B/6 = 11 \text{ ft}/6)]$$

(The resultant soil pressure is 0.18 ft in outside that middle third for Group VI loading, but 2.01 ft is considered acceptable in this problem.)

Check soil pressure.

$$q_{t,\max} = 6.36 \text{ kips/ft}^2 \text{ from Group VI loading}$$

$$\left[\begin{array}{l} < \text{safe bearing capacity} \\ (= 8.00 \text{ kips/ft}^2), \text{ so OK} \end{array}\right]$$

Step 3: Analyze and design for footing (load factor design).

AASHTO [3.22] To get factored loads, multiply the service loads by the following factors for the various group loading combinations.

AASHTO [Table 3.22.1A Footnotes]

$\beta_E = 1.0$ for vertical earth pressure
$\beta_E = 1.3$ for lateral earth pressure
$\beta_D = 1.0$ for flexural members

AASHTO [Table 3.22.1A]

D and E_{vertical}: (1.30)(1.0) (Groups I, II, III IV) or (1.25)(1.0) (Groups V, VI)

L and L_S: (1.30)(1.67) (Group I) or (1.30)(1.0) (Groups III, IV) or (1.25)(1.0) (Group VI)

E_{lateral}: (1.30)(1.30) (Groups I, II, III, IV) or (1.25)(1.30) (Groups V, VI)

W: (1.30)(1.0) (Group II) or (1.30)(0.30) (Group III) or (1.25)(1.0) (Group V) or (1.25)(0.3) (Group VI)

WL_H: (1.30)(1.0) (Group III) or (1.25)(1.0) (Group VI) for horizontal wind force transmitted from live load

LF: (1.30)(1.0) (Group III) or (1.25)(1.0) (Group VI) for longitudinal force from live load

T_f: (1.30)(1.0) (Group IV) or (1.25)(1.0) (Groups V, VI) for temperature force due to friction at bearing

where

E = earth pressure: vertical or lateral

L_S = live load surcharge earth pressure equal to 2 ft of earth at the approach

W_{sub} = horizontal wind force acting on the abutment stem

$W_{\text{super}(H)}$ = horizontal wind force transmitted from superstructure

W_{up} = upward wind force (vertical)

D = dead load from superstructure

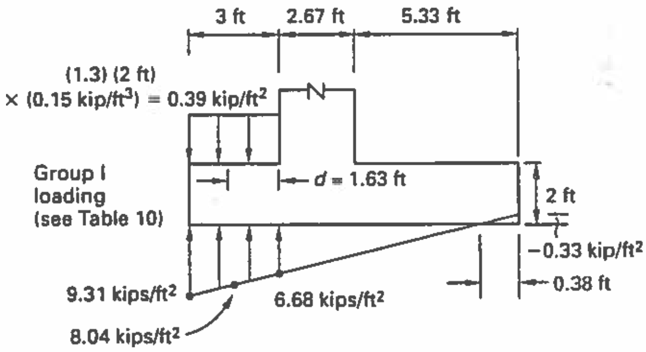
L = live load

Factored loads and moments resulting from the factored loads are given in Tables 6–9.

Determine the factored shear and moment.

The factored bearing pressures under the footing at toe (q_t) and heel (q_h) for the various group loading combinations are summarized in Table 10.

Group I loading is critical for the toe of the footing. The factored shear and moment at the front of the stem for this loading are (assume $d = 19.5$ in or 1.63 ft)



$$V_u = \left(\frac{9.31 \frac{\text{kips}}{\text{ft}^2} + 8.04 \frac{\text{kips}}{\text{ft}^2}}{2} - 0.39 \frac{\text{kips}}{\text{ft}^2} \right) \times (3.00 \text{ ft} - 1.63 \text{ ft})$$

$$= 11.35 \text{ kips/ft (at a distance } d \text{ from the face of the stem)}$$

$$M_u = \left(6.68 \frac{\text{kips}}{\text{ft}^2} - 0.39 \frac{\text{kips}}{\text{ft}^2}\right) \times (3.00 \text{ ft})(1.50 \text{ ft}) + \left(\frac{1}{2}\right)\left(9.31 \frac{\text{kips}}{\text{ft}^2} - 6.68 \frac{\text{kips}}{\text{ft}^2}\right) \times (3.00 \text{ ft})(2.00 \text{ ft})$$

$$= 36.20 \text{ ft-kips/ft}$$

Table 6 Factored Vertical Loads

item numbers	item	service V kips/ft	lever arm from toe (ft)	factored V (kips/ft) I	II	III	IV	V	VI
1	$D_{concrete}$	3.30	5.50	4.29	4.29	4.29	4.29	4.13	4.13
2	$D_{concrete}$	3.15	3.75	4.10	4.10	4.10	4.10	3.94	3.94
3	$D_{concrete}$	1.23	4.89	1.60	1.60	1.60	1.60	1.54	1.54
4	$E_{vertical}$	0.98	5.28	1.27	1.27	1.27	1.27	1.23	1.23
5	$E_{vertical}$	8.95	8.33	11.64	11.64	11.64	11.64	11.19	11.19
6	$E_{vertical}$	2.56	7.75	3.33	3.33	3.33	3.33	3.20	3.20
7	LS^a (surcharge)	1.56	7.75	3.39		2.03	2.03		1.95
8	$D_{superstructure}$	6.55	3.58	8.52	8.52	8.52	8.52	8.19	8.19
9	$L_{live\ load}$	5.16	3.58	11.20		6.71	6.71		6.45
10	W_{up}	−0.55	3.58		−0.72	−0.06		−0.69	−0.06
			ΣV	49.34	34.03	43.43	43.49	32.73	41.76

[a]Treated as a live load

Table 7 Moments Resulting from Factored Vertical Loads

item numbers	item	factored $M_{V,toe}$ (ft-kips/ft) I	II	III	IV	V	VI
1	$D_{concrete}$	23.60	23.60	23.60	23.60	22.72	22.72
2	$D_{concrete}$	15.38	15.38	15.38	15.38	14.78	14.78
3	$D_{concrete}$	7.82	7.82	7.82	7.82	7.53	7.53
4	$E_{vertical}$	6.71	6.71	6.71	6.71	6.49	6.49
5	$E_{vertical}$	96.96	96.96	96.96	96.96	93.21	93.21
6	$E_{vertical}$	25.81	25.81	25.81	25.81	24.80	24.80
7	LS (surcharge)	26.27		15.73	15.73		15.11
8	D	30.50	30.50	30.50	30.50	29.32	29.32
9	L	40.10		24.02	24.02		23.09
10	W_{up}		−2.58	−0.21		−2.47	−0.21
	$\Sigma M_{V,toe}$	273.15	204.20	246.32	246.53	196.38	236.84

Table 8 Factored Horizontal Loads

item numbers	item	service H (kips/ft)	lever arm (ft from footing base)	factored H (kips/ft) I	II	III	IV	V	VI
11	LS^a (surcharge)	1.35	9.64	2.93		1.76	1.76		1.69
12	$E_{lateral}$	6.51	6.43	11.00	11.00	11.00	11.00	10.58	10.58
13	LF	0.28	16.00			0.36			0.35
14	T_f	0.66	16.00				0.86	0.83	0.83
15	WL_H	0.08	16.00			0.10			0.10
16	$W_{super(H)}$	0.15	16.00		0.20	0.06		0.19	0.06
18	W_{sub}	0.48	10.00		0.62	0.19		0.60	0.18
			ΣH	13.93	11.82	13.47	13.62	12.20	13.79

[a]Treated as a live load

The eccentricity of the resultant soil pressure, ΣV, is

$$e_{B,\max} = 2.01 \text{ ft from Group VI loading}$$

$$[> 1.83 \text{ ft} = (B/6 = 11 \text{ ft}/6)]$$

(The resultant soil pressure is 0.18 ft in outside that middle third for Group VI loading, but 2.01 ft is considered acceptable in this problem.)

Check soil pressure.

$$q_{t,\max} = 6.36 \text{ kips/ft}^2 \text{ from Group VI loading}$$

$$\left[\begin{array}{l} < \text{safe bearing capacity} \\ (= 8.00 \text{ kips/ft}^2)\text{, so OK} \end{array}\right]$$

Step 3: Analyze and design for footing (load factor design).

AASHTO [3.22] To get factored loads, multiply the service loads by the following factors for the various group loading combinations.

AASHTO [Table 3.22.1A Footnotes]

$\beta_E = 1.0$ for vertical earth pressure

$\beta_E = 1.3$ for lateral earth pressure

$\beta_D = 1.0$ for flexural members

AASHTO [Table 3.22.1A]

D and E_{vertical}: (1.30)(1.0) (Groups I, II, III IV) or (1.25)(1.0) (Groups V, VI)

L and L_S: (1.30)(1.67) (Group I) or (1.30)(1.0) (Groups III, IV) or (1.25)(1.0) (Group VI)

E_{lateral}: (1.30)(1.30) (Groups I, II, III, IV) or (1.25)(1.30) (Groups V, VI)

W: (1.30)(1.0) (Group II) or (1.30)(0.30) (Group III) or (1.25)(1.0) (Group V) or (1.25)(0.3) (Group VI)

WL_H: (1.30)(1.0) (Group III) or (1.25)(1.0) (Group VI) for horizontal wind force transmitted from live load

LF: (1.30)(1.0) (Group III) or (1.25)(1.0) (Group VI) for longitudinal force from live load

T_f: (1.30)(1.0) (Group IV) or (1.25)(1.0) (Groups V, VI) for temperature force due to friction at bearing

where

E = earth pressure: vertical or lateral

L_S = live load surcharge earth pressure equal to 2 ft of earth at the approach

W_{sub} = horizontal wind force acting on the abutment stem

$W_{\text{super}(H)}$ = horizontal wind force transmitted from superstructure

W_{up} = upward wind force (vertical)

D = dead load from superstructure

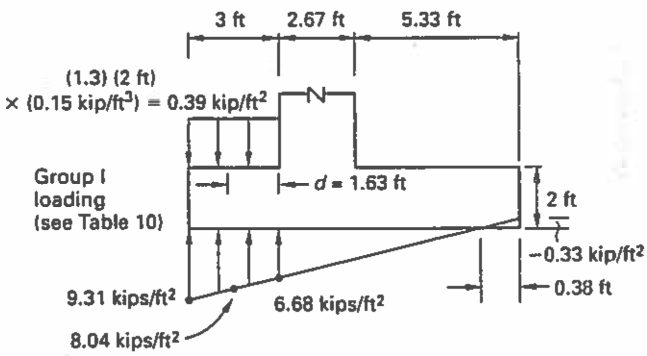
L = live load

Factored loads and moments resulting from the factored loads are given in Tables 6–9.

Determine the factored shear and moment.

The factored bearing pressures under the footing at toe (q_t) and heel (q_h) for the various group loading combinations are summarized in Table 10.

Group I loading is critical for the toe of the footing. The factored shear and moment at the front of the stem for this loading are (assume d = 19.5 in or 1.63 ft)



$$V_u = \left(\frac{9.31 \dfrac{\text{kips}}{\text{ft}^2} + 8.04 \dfrac{\text{kips}}{\text{ft}^2}}{2} - 0.39 \frac{\text{kips}}{\text{ft}^2}\right) \times (3.00 \text{ ft} - 1.63 \text{ ft})$$

$$= 11.35 \text{ kips/ft (at a distance } d \text{ from the face of the stem)}$$

$$M_u = \left(6.68 \frac{\text{kips}}{\text{ft}^2} - 0.39 \frac{\text{kips}}{\text{ft}^2}\right) \times (3.00 \text{ ft})(1.50 \text{ ft}) + \left(\frac{1}{2}\right)\left(9.31 \frac{\text{kips}}{\text{ft}^2} - 6.68 \frac{\text{kips}}{\text{ft}^2}\right) \times (3.00 \text{ ft})(2.00 \text{ ft})$$

$$= 36.20 \text{ ft-kips/ft}$$

Table 6 Factored Vertical Loads

item numbers	item	service V kips/ft	lever arm from toe (ft)	factored V (kips/ft)					
				I	II	III	IV	V	VI
1	$D_{concrete}$	3.30	5.50	4.29	4.29	4.29	4.29	4.13	4.13
2	$D_{concrete}$	3.15	3.75	4.10	4.10	4.10	4.10	3.94	3.94
3	$D_{concrete}$	1.23	4.89	1.60	1.60	1.60	1.60	1.54	1.54
4	$E_{vertical}$	0.98	5.28	1.27	1.27	1.27	1.27	1.23	1.23
5	$E_{vertical}$	8.95	8.33	11.64	11.64	11.64	11.64	11.19	11.19
6	$E_{vertical}$	2.56	7.75	3.33	3.33	3.33	3.33	3.20	3.20
7	LS^a (surcharge)	1.56	7.75	3.39		2.03	2.03		1.95
8	$D_{superstructure}$	6.55	3.58	8.52	8.52	8.52	8.52	8.19	8.19
9	$L_{live\ load}$	5.16	3.58	11.20		6.71	6.71		6.45
10	W_{up}	−0.55	3.58		−0.72	−0.06		−0.69	−0.06
			ΣV	49.34	34.03	43.43	43.49	32.73	41.76

[a]Treated as a live load

Table 7 Moments Resulting from Factored Vertical Loads

item numbers	item	factored $M_{V,toe}$ (ft-kips/ft)					
		I	II	III	IV	V	VI
1	$D_{concrete}$	23.60	23.60	23.60	23.60	22.72	22.72
2	$D_{concrete}$	15.38	15.38	15.38	15.38	14.78	14.78
3	$D_{concrete}$	7.82	7.82	7.82	7.82	7.53	7.53
4	$E_{vertical}$	6.71	6.71	6.71	6.71	6.49	6.49
5	$E_{vertical}$	96.96	96.96	96.96	96.96	93.21	93.21
6	$E_{vertical}$	25.81	25.81	25.81	25.81	24.80	24.80
7	LS (surcharge)	26.27		15.73	15.73		15.11
8	D	30.50	30.50	30.50	30.50	29.32	29.32
9	L	40.10		24.02	24.02		23.09
10	W_{up}		−2.58	−0.21		−2.47	−0.21
	$\Sigma M_{V,toe}$	273.15	204.20	246.32	246.53	196.38	236.84

Table 8 Factored Horizontal Loads

item numbers	item	service H (kips/ft)	lever arm (ft from footing base)	factored H (kips/ft)					
				I	II	III	IV	V	VI
11	LS^a (surcharge)	1.35	9.64	2.93		1.76	1.76		1.69
12	$E_{lateral}$	6.51	6.43	11.00	11.00	11.00	11.00	10.58	10.58
13	LF	0.28	16.00			0.36			0.35
14	T_f	0.66	16.00				0.86	0.83	0.83
15	WL_H	0.08	16.00			0.10			0.10
16	$W_{super(H)}$	0.15	16.00		0.20	0.06		0.19	0.06
18	W_{sub}	0.48	10.00		0.62	0.19		0.60	0.18
			ΣH	13.93	11.82	13.47	13.62	12.20	13.79

[a]Treated as a live load

Table 9 Moments Resulting from Factored Horizontal Loads

item numbers	item	factored M_H (ft-kips/ft) I	II	III	IV	V	VI
11	L_S (surcharge)	28.25		16.97	16.97		16.29
12	E_{lateral}	70.73	70.73	70.73	70.73	68.03	68.03
13	LF			5.76			5.60
14	T_f				13.76	13.28	13.28
15	WL_H			1.60			1.60
16	$W_{\text{super}(H)}$		3.20	0.96		3.04	0.96
18	W_{sub}		6.20	1.90		6.00	1.80
	$\Sigma M_{\text{toe }H}$	98.98	80.13	97.92	101.46	90.35	107.56

Table 10 Factored Bearing Pressures

group loading	ΣV (kips)	ΣH (kips)	$\Sigma M_{V,\text{toe}}$ (kips/ft)	ΣM_H (kips/ft)	$\overline{x}$ (ft)	e_B (ft)	q_l (kips/ft^2)	q_h (kips/ft^2)
I	49.34	13.93	273.15	98.98	3.53	1.97	**9.31**	**−0.33**
II	34.03	11.82	204.20	80.13	3.65	1.85	6.22	−0.03
III	43.28	13.42	246.32	97.92	3.43	2.07	8.38	−0.51
IV	43.49	13.62	246.53	101.46	3.33	2.17	8.63	−0.73
V	32.73	12.20	196.38	90.35	3.24	2.26	6.64	−0.68
VI	41.61	13.74	236.84	107.56	3.11	2.39	**8.71**	**−1.15**

Design the footing using strength criteria for concrete and steel of $f'_c = 3000$ lbf/in^2 and $F_y = 60{,}000$ lbf/in^2.

AASHTO [8.16.2.7; 8.16.3]

Calculate maximum and minimum steel reinforcement.

$$\rho_b = \left(\frac{0.85\beta_1 f'_c}{F_y}\right)\left(\frac{87{,}000}{87{,}000 + F_y}\right)$$

$$= \left(\frac{(0.85)(0.85)\left(3000\ \frac{\text{lbf}}{\text{in}^2}\right)}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right) \times \left(\frac{87{,}000}{87{,}000 + 60{,}000\ \frac{\text{lbf}}{\text{in}^2}}\right)$$

$$= 0.0214$$

$$\rho_{\max} = 0.75\rho_b = (0.75)(0.0214) = 0.0160$$

Design reinforcement for the toe.

It will be safe and conservative to neglect compression reinforcement in the design calculation.

For a singly reinforced section, the ultimate moment is

$$\phi M_n = \phi A_s F_y \left(d - \frac{a}{2}\right)$$

[8.16.1.2.2]

$\phi = 0.90$ for flexure

$$a = \frac{A_s F_y}{0.85 f'_c b}$$

AASHTO [8.22.1]

Use $d = 24.0$ in $-$ 4.0 in cover $-$ 0.5 in for 1/2 diameter of steel assumed = 19.5 in.

$$a = \frac{A_s\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(12\ \text{in})} = 1.960A_s$$

$$\frac{a}{2} = 0.980A_s$$

$M_u = \phi M_n$

$(36.20 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$

$= (0.9A_s)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)(19.5 \text{ in} - 0.980A_s)$

$A_s^2 - 19.90A_s + 8.21 = 0$

$A_s = 0.42 \text{ in}^2$

$\rho = \frac{A_s}{bd} = \frac{0.42 \text{ in}^2}{(12 \text{ in})(19.5 \text{ in})}$

$= 0.0018 \quad [< \rho_{max} \ (= 0.0160), \text{ so OK}]$

Try #7 @ 16 in, $A_s = 0.45 \text{ in}^2$.

Check minimum steel.

AASHTO [8.17.1]

The minimum reinforcement provided shall be adequate to develop a factored moment capacity at least 1.2 times the cracking moment, unless the area of reinforcement provided is at least one-third greater than that required by analysis.

$\phi M_n \geq 1.2M_{cr}$

$a = \frac{(0.45 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})} = 0.882 \text{ in}$

$\frac{a}{2} = 0.441 \text{ in}$

$\phi M_n = (0.9)(0.45 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)$

$\times \ (19.50 \text{ in} - 0.441 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$

$= 38.59 \text{ ft-kips/ft width}$

The cracking moment, M_{cr}, is determined by the transformed area method.

$M_{cr} = \frac{f_r I}{c}$

$I = I_{cg}$

$f_r = 7.5\sqrt{f'_c}$

$c = y_b$

AASHTO [8.7.1 & 8.7.2]

$E_c = 33w_c^{1.5}\sqrt{f'_c}$

$= (33)\left(145 \ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5}\left(\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}\right)$

$= 3{,}160{,}000 \text{ lbf/in}^2$

$E_s = 29{,}000{,}000 \text{ lbf/in}^2$

$n = \frac{29{,}000{,}000 \ \frac{\text{lbf}}{\text{in}^2}}{3{,}160{,}000 \ \frac{\text{lbf}}{\text{in}^2}} = 9.19 \quad [\text{round to } n = 9]$

The transformed area is

$(n-1)A_s = (9-1)(0.45 \text{ in}^2) = 3.60 \text{ in}^2$

$(24 \text{ in})(12 \text{ in}) + 3.60 \text{ in}^2 = 288.0 \text{ in}^2 + 3.60 \text{ in}^2$

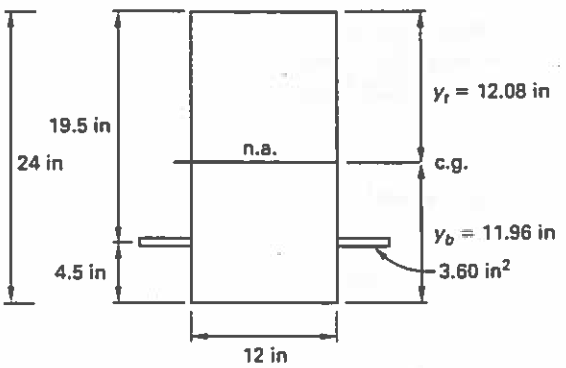
$= 291.6 \text{ in}^2$

Taking moments about the bottom and solving the equation,

$(291.60 \text{ in}^2)y_b = (288 \text{ in}^2)(12 \text{ in})$

$+ (3.60 \text{ in}^2)(4.5 \text{ in})$

$y_b = 11.96 \text{ in from bottom}$



The moment of inertia about the neutral axis is

$I_{cg} = \left(\frac{1}{12}\right)(12 \text{ in})(24 \text{ in})^3$

$+ (288 \text{ in}^2)(12.0 \text{ in} - 11.96 \text{ in})^2$

$+ (3.60 \text{ in}^2)(11.96 \text{ in} - 4.5 \text{ in})^2$

$= 14{,}025 \text{ in}^4$

AASHTO [8.15.2.1.1]

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}} = 411\ \text{lbf/in}^2$$

$$M_{cr} = \frac{f_r I}{c}$$

$$= \left(\frac{\left(0.411\ \frac{\text{kips}}{\text{in}^2}\right)(14{,}000\ \text{in}^4)}{11.96\ \text{in}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 40.29\ \text{ft-kips/ft}$$

$$1.2M_{cr} = 48.35\ \text{ft-kips}$$

$$\left[\begin{array}{c} > \phi M_n\ (= 38.59\ \text{ft-kips}), \\ \text{so no good} \end{array}\right]$$

Increase the amount of steel.

Try #8 @ 15 in, $A_s = 0.62\ \text{in}^2$.

$$a = \frac{(0.62\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(12\ \text{in})} = 1.216\ \text{in}$$

$$\frac{a}{2} = 0.608\ \text{in}$$

$$\phi M_n = (0.9)(0.62\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)$$

$$\times\ (19.50\ \text{in} - 0.61\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 52.70\ \text{ft-kips/ft}$$

The transformed area is

$$(n-1)A_s = (9-1)(0.62\ \text{in}^2) = 4.96\ \text{in}^2$$

$$288.0\ \text{in}^2 + 4.96\ \text{in}^2 = 292.96\ \text{in}^2$$

Taking moments about the bottom and solving the equation,

$$(292.96\ \text{in}^2)y_b = (288\ \text{in}^2)(12\ \text{in}) + (4.96\ \text{in}^2)(4.50\ \text{in})$$

$$y_b = 11.87\ \text{in}$$

$$I_{cg} = \left(\frac{1}{12}\right)(12\ \text{in})(24\ \text{in})^3 + (288\ \text{in}^2)(0.13\ \text{in})^2 + (4.96\ \text{in}^2)(7.37\ \text{in})^2$$

$$= 14{,}100\ \text{in}^4$$

$$M_{cr} = \left(\frac{\left(0.411\ \frac{\text{kips}}{\text{in}^2}\right)(14{,}100\ \text{in}^4)}{11.87\ \text{in}}\right) \times \left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 40.68\ \text{ft-kips/ft width}$$

$$1.2M_{cr} = 48.82\ \text{ft-kips}$$

$$\left[\begin{array}{c} < \phi M_n\ (= 52.70\ \text{ft-kips}), \\ \text{so OK} \end{array}\right]$$

Use #8 @ 15 in.

AASHTO [Eq. 8-49]

Check shear at a distance d from the face of the stem. The ultimate shear capacity without shear reinforcement is

$$\phi V_c = \phi\left(2\sqrt{f'_c}\right)b_w d$$

[8.16.1.2.2]

$$\phi V_c = (0.85)\left(2\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\right)(12\ \text{in})(19.5\ \text{in})$$

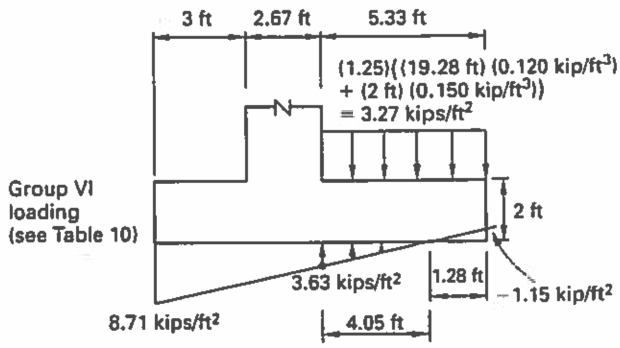
$$= 21{,}800\ \text{lbf}$$

$$= 21.80\ \text{kips}$$

[> V_u from Group I loading (= 11.35 kips at a distance d from the face of the stem), so OK]

Design reinforcement for the heel.

Group VI loading is critical for the heel of the footing. The factored shear and moment at the back of the stem for this loading are as follows.



$$V_u = \left(3.27\ \frac{\text{kips}}{\text{ft}^2}\right)(5.33\ \text{ft}) - \left(\frac{1}{2}\right)\left(3.63\ \frac{\text{kips}}{\text{ft}^2}\right)(4.05\ \text{ft})$$

$$= 10.08\ \text{kips/ft}$$

$$M_u = \left(3.27\ \frac{\text{kips}}{\text{ft}^2}\right)(4.05\ \text{ft})(2.67\ \text{ft})$$

$$-\left(\frac{1}{2}\right)\left(3.63\ \frac{\text{kips}}{\text{ft}^2}\right)(4.05\ \text{ft})\left(\frac{4.05\ \text{ft}}{3}\right)$$

$$= 36.61\ \text{ft-kips/ft}$$

AASHTO [8.22.1]

For the heel steel design, use $d = 24.0$ in − 3.0 cover − 0.5 in for 1/2 diameter of steel assumed = 20.5 in

$$a = \frac{A_s\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(12\ \text{in})} = 1.96A_s$$

$$\frac{a}{2} = 0.980A_s$$

The moment in the heel at the back of the stem from Group VI loading was 36.61 ft-kips.

$$M_u = \phi M_n = \phi A_s F_y\left(d - \frac{a}{2}\right)$$

$$(36.61\ \text{ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)$$

$$= 0.9A_s\left(60\ \frac{\text{kips}}{\text{in}^2}\right)(20.5\ \text{in} - 0.980A_s)$$

$$A_s^2 - 20.92A_s + 8.30 = 0$$

$$A_s = 0.40\ \text{in}^2/\text{ft}$$

$$\rho = \frac{0.40\ \text{in}^2}{(12\ \text{in})(20.5\ \text{in})} = 0.0016$$

[$< \rho_{max}$ (= 0.0160), so OK]

Try #8 @ 15 in, $A_s = 0.62\ \text{in}^2$.

Check minimum steel.

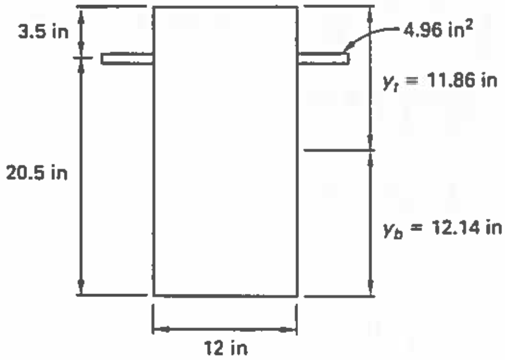
$$a = \frac{(0.62\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(12\ \text{in})} = 1.216\ \text{in}$$

$$\frac{a}{2} = 0.608\ \text{in}$$

$$\phi M_n = (0.9)(0.62\ \text{in}^2)\left(60\ \frac{\text{kips}}{\text{in}^2}\right)$$

$$\times\ (20.50\ \text{in} - 0.608\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 55.49\ \text{ft-kips/ft}$$



The transformed area is

$$(n-1)A_s = (9-1)(0.62\ \text{in}^2) = 4.96\ \text{in}^2$$

$$(24\ \text{in})(12\ \text{in}) + 4.96\ \text{in}^2 = 292.96\ \text{in}^2$$

Taking moments about the top and solving the equation,

$$(292.96\ \text{in}^2)y_t = (288\ \text{in}^2)(12\ \text{in}) + (4.96\ \text{in}^2)(3.50\ \text{in})$$

$$y_t = 11.86\ \text{in}$$

$$I_{cg} = \left(\frac{1}{12}\right)(12\ \text{in})(24\ \text{in})^3 + (288\ \text{in}^2)(0.14\ \text{in})^2 + (4.96\ \text{in}^2)(8.36\ \text{in})^2$$

$$= 14{,}180\ \text{in}^4$$

$$M_{cr} = \left(\frac{\left(0.411\ \frac{\text{kips}}{\text{in}^2}\right)(14{,}180\ \text{in}^4)}{11.86\ \text{in}}\right) \times \left(\frac{1\ \text{ft}}{12\ \text{in}}\right)$$

$$= 40.95\ \text{ft-kips/ft width}$$

$$1.2M_{cr} = 49.14\ \text{ft-kips}$$

[$< \phi M_n$ (= 55.49 ft-kips), so OK]

Use #8 @ 15 in.

Check shear in the heel at the back of the stem.

[8.16.1.2.2] $\phi V_c = (2)\left(2\sqrt{f'_c}\right) b_w d$

$\phi = 0.85$ for shear

$$\phi V_c = (0.85)\left(2\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\right)(12\ \text{in})(20.5\ \text{in})$$

$$= 22{,}900\ \text{lbf}$$

$$= 22.90\ \text{kips/ft width}$$

$\left[\begin{array}{c} > V_u \text{ at the back of the stem from} \\ \text{Group VI loading } (= 10.08 \text{ kips/ft}), \text{ so OK} \end{array}\right]$

Step 4a: Analyze for stem (Load Factor Design).

The stem will be designed for combined axial load and bending. To compensate for incidental field adjustments in the location of bearings for vertical loads, a 2 in longitudinal eccentricity from the theoretical centerline of bearing will be used. This 2 in eccentricity will produce the maximum or minimum moment at the stem base.

The stem will be designed only at its base in this example problem.

The following figure is a summary of service loads on the stem.

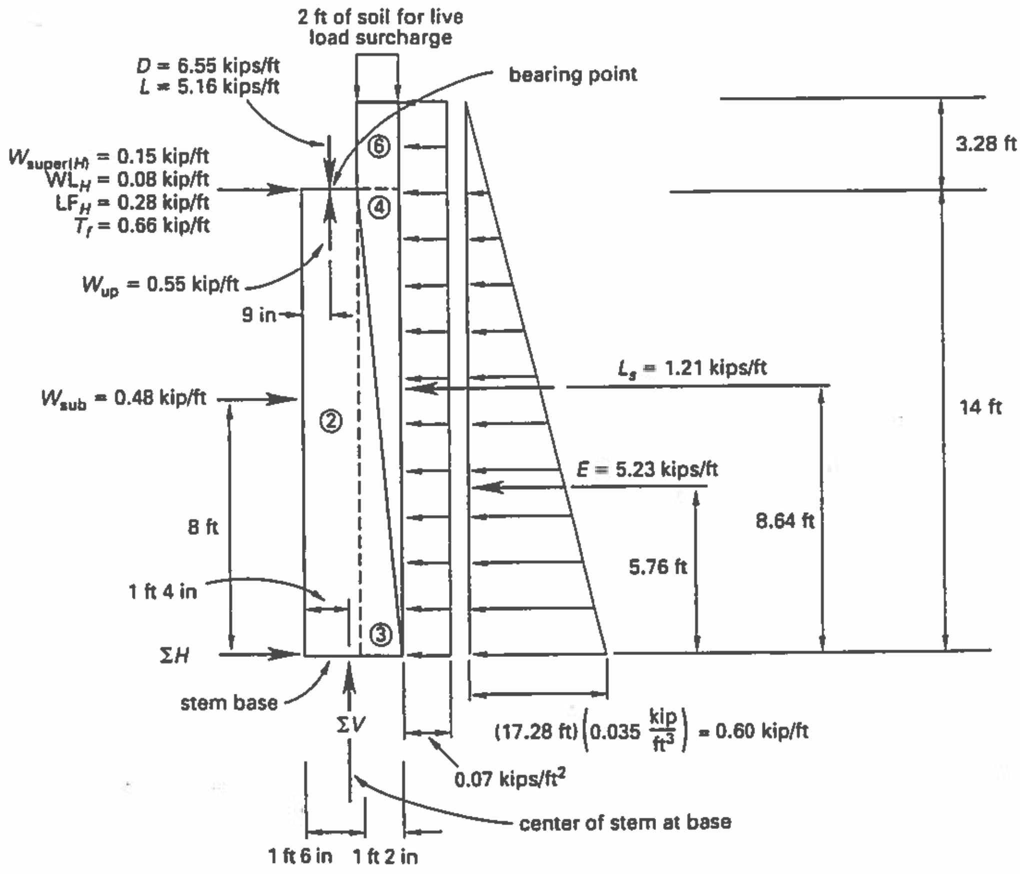
Step 4a(i): Determine the ***minimum axial load and maximum moment*** with a 2 in eccentricity toward the stem front face from the bearing point (Tables 11, 12, and 13).

To get factored loads (load factor design), the service loads will be multiplied by the following factors for the various group loading combinations.

AASHTO [Table 3.22.1A]

$\beta_D = 0.75$ when designing members for minimum axial load and maximum moment, and $\beta_E = 1.30$ for lateral earth pressure.

D and E_{vertical}:	(1.3)(0.75) (Groups I, II, III, IV) or (1.25)(0.75) (Groups V, VI)
L and L_s:	(1.3)(1.67) (Group I) or (1.3)(1) (Groups III, IV) or (1.25)(1) (Group VI)
E_{lateral}:	(1.3)(1.30) (Groups I, II, III, IV) or (1.25)(1.3) (Groups V, VI)
W:	(1.30)(1) (Group II) or (1.3)(0.30) (Group III) or (1.25)(1) (Group V) or (1.3)(0.30) (Group VI)
WL:	(1.30)(1) (Group III) or (1.25)(1) (Group VI)
LF:	(1.30)(1) (Group III) or (1.25)(1) (Group VI)
T:	(1.30)(1) (Group IV) or (1.25)(1) (Groups V, VI)



summary of service loads

Step 4a(ii): The *minimum* axial load and corresponding moment (Group V loading) are

$$\Sigma V = P_u = 10.94 \text{ kips/ft (see Table 11)}$$

$$M_u = 72.16 \text{ ft-kips/ft (see Table 13)}$$

The *maximum* moment and corresponding axial load (Group VI loading) are

$$M_u = 90.57 \text{ ft-kips (see Table 13)}$$

$$\Sigma V = P_u = 18.22 \text{ kips/ft (see Table 11)}$$

The maximum shear (Group VI loading) is

$$\Sigma H = V_u = 11.50 \text{ kips/ft (see Table 12)}$$

Determine the ***maximum axial load and minimum moment*** with a 2 in eccentricity toward the stem rear face from the bearing point (Tables 14, 15, and 16).

To get factored loads (load factor design), the service loads will be multiplied by the following factors for the various group loading combinations.

Table 11 Factored Vertical Loads[a]

item number	item	service V (kips/ft)	lever arm (ft)[b]	factored V (kips/ft) I	II	III	IV	V	VI
2	D_{concrete}	3.15	0.58	3.09	3.09	3.09	3.09	2.96	2.96
3	D_{concrete}	1.23	−0.55	1.21	1.21	1.21	1.21	1.16	1.16
4	E_{vertical}	0.98	−0.94	0.96	0.96	0.96	0.96	0.92	0.92
6	E_{vertical}	0.46	−0.75	0.45	0.45	0.45	0.45	0.43	0.43
7	L_s (surcharge)	0.28[c]	−0.75	0.61		0.36	0.36		0.35
8	$D_{\text{superstructure}}$	6.55	0.75[d]	6.42	6.42	6.42	6.42	6.16	6.16
9	$L_{\text{live load}}$	5.16	0.75[d]	11.20		6.71	6.71		6.45
10	W_{up}	−0.55	0.75[d]		−0.72	−0.21		−0.69	−0.21
			ΣV	23.94	11.41	18.99	19.20	**10.94**	**18.22**

[a]member for minimum axial load and maximum moment.
[b]from the center of stem base.
[c]$(1 \text{ ft } 2 \text{ in})(2 \text{ ft})\left(0.12 \ \frac{\text{kip}}{\text{ft}^3}\right)$.
[d]$\left(1 \text{ ft } 4 \text{ in} - (9 \text{ in} - 2 \text{ in})\right) = 9 \text{ in} = 0.75 \text{ ft}$

Table 12 Factored Horizontal Loads[a]

item number	item	service H (kips/ft)	lever arm (ft)[b]	factored H (kips/ft) I	II	III	IV	V	VI
11	L_s (surcharge)	1.21	8.64	2.63		1.57	1.57		1.51
12	E_{lateral}	5.23	5.76	8.84	8.84	8.84	8.84	8.52	8.52
13	LF	0.28	14.00			0.36			0.35
14	T_f	0.66	14.00				0.86	0.83	0.83
15	WL_H	0.08	14.00			0.10			0.10
16	$W_{\text{super}(H)}$	0.15	14.00		0.20	0.06		0.19	0.06
18	W_{sub}	0.48	8.00		0.62	0.19		0.60	0.18
			ΣH	11.47	9.66	11.12	11.27	10.14	**11.55**

[a]member for minimum axial load and maximum moment.
[b]from the stem base

Table 13 Factored Moments at Stem Base Resulting from Vertical and Horizontal Loads[a]

item number	item	factored M_{stem} (ft-kips/ft) I	II	III	IV	V	VI
2	$D_{concrete}$	1.79	1.79	1.79	1.79	1.72	1.72
3	$D_{concrete}$	−0.67	−0.67	−0.67	−0.67	−0.64	−0.64
4	$E_{vertical}$	−0.90	−0.90	−0.90	−0.90	−0.86	−0.86
6	$E_{vertical}$	−0.34	−0.34	−0.34	−0.34	−0.32	−0.32
7	L_s (surcharge)	−0.46		−0.27	−0.27		−0.26
8	$D_{superstructure}$	4.82	4.82	4.82	4.82	4.62	4.62
9	$L_{live\ load}$	8.40		5.03	5.03		4.84
10	W_{up}		−0.54	−0.16		−0.52	−0.16
11	L_s (surcharge)	22.72		13.56	13.56		13.05
12	E	50.92	50.92	50.92	50.92	49.08	49.08
13	LF			5.04			4.90
14	T_f				12.04	11.62	11.62
15	WL_H			1.40			1.40
16	$W_{super(H)}$		2.80	0.84		2.66	0.84
18	W_{sub}		4.96	1.52		4.80	1.44
	ΣM_{stem}	86.28	62.84	82.58	85.98	72.16	91.27

[a]member for minimum axial load and maximum moment

AASHTO [Table 3.22.1A]

$\beta_D = 1.00$ when designing members for maximum axial load and minimum moment, $\beta_E = 1.00$ for vertical earth pressure, and $\beta_E = 1.3$ for lateral earth pressure.

D and $E_{vertical}$: 1.30 (Groups I, II, III, IV) or 1.25 (Groups V, VI)

L and L_s: (1.3)(1.67) (Group I) or 1.30 (Groups III, IV) or 1.25 (Group VI)

$E_{lateral}$: (1.3)(1.3) (Groups I, II, III, IV) or (1.3)(1.25) (Groups V, VI)

W: 1.30 (Group II) or (1.25)(0.3) (Group III) or 1.25 (Group V) or (1.25)(0.3) (Group VI)

WL: 1.30 (Group III) or 1.25 (Group VI)

LF: 1.30 (Group III) or 1.25 (Group VI)

T: 1.30 (Group IV) or 1.25 (Groups V, VI)

The maximum axial load and corresponding moment (Group I loading) are

$\Sigma V = P_u = 27.90$ kips/ft (see Table 14)

$M_u = 81.32$ ft-kips/ft (see Table 16)

The minimum moment and corresponding axial load (Group V loading) are

$M_u = 32.99$ ft-kips/ft (see Table 16)

$\Sigma V = P_u = 14.79$ kips/ft (see Table 14)

The maximum shear (Group I loading) is

$\Sigma H = V_u = 11.47$ kips/ft (see Table 15)

Step 4b: Design for stem (Load Factor Design)

Design the stem using the same strength criteria for materials used for the toe and heel, $f'_c = 3000\ \text{lbf/in}^2$ and $F_y = 60{,}000\ \text{lbf/in}^2$.

Stem reinforcing will be designed for bending only as a singly reinforced beam. The section will then be checked for combined axial force and bending, neglecting the front face reinforcing steel.

Use $d = 32.0$ in − 3.0 in cover − 0.5 in for 1/2 steel assumed = 28.5 in

$$d'' = \frac{32.0\ \text{in}}{2} - 3.5\ \text{in} = 12.5\ \text{in}$$

$$a = \frac{A_s F_y}{0.85 f'_c b}$$

$$= \frac{A_s\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(12\ \text{in})} = 1.96 A_s$$

$$\frac{a}{2} = 0.980 A_s$$

Table 14 Factored Vertical Loads[a]

item number	item	service V (kips/ft)	lever arm (ft)[b]	factored V (kips/ft) I	II	III	IV	V	VI
2	$D_{concrete}$	3.15	0.58	4.10	4.10	4.10	4.10	3.94	3.94
3	$D_{concrete}$	1.23	−0.55	1.60	1.60	1.60	1.60	1.54	1.54
4	$E_{vertical}$	0.98	−0.94	1.27	1.27	1.27	1.27	1.23	1.23
6	$E_{vertical}$	0.46	−0.75	0.60	0.60	0.60	0.60	0.58	0.58
7	L_s (surcharge)	0.28	−0.75	0.61		0.36	0.36		0.35
8	$D_{superstructure}$	6.55	0.42[c]	8.52	8.52	8.52	8.52	8.19	8.19
9	$L_{live\ load}$	5.16	0.42[c]	11.20		6.71	6.71		6.45
10	W_{up}	−0.55	0.42[c]		−0.72	−0.21		−0.69	−0.21
			ΣV	**27.90**	15.37	22.95	23.16	**14.79**	22.07

[a]member for maximum axial load and minimum moment. [b]from the center of stem base.
[c]$\big(1\ \text{ft}\ 4\ \text{in} - (9\ \text{in} + 2\ \text{in})\big) = 5\ \text{in} = 0.42\ \text{ft}$.

Table 15 Factored Horizontal Loads[a]

item number	item	service H (kips/ft)	lever arm (ft)[b]	factored H (kips/ft) I	II	III	IV	V	VI
11	L_s (surcharge)	1.21	8.64	2.63		1.57	1.57		1.51
12	$E_{lateral}$	5.23	5.76	8.84	8.84	8.84	8.84	8.52	8.52
13	LF	−0.28	14.00			−0.36			−0.35
14	T_f	−0.66	14.00				−0.86	−0.83	−0.83
15	WL_H	−0.08	14.00			−0.10			−0.10
16	$W_{super(H)}$	−0.15	14.00		−0.20	−0.06		−0.19	−0.06
18	W_{sub}	−0.48	8.00		−0.62	−0.19		−0.60	−0.18
			ΣH	**11.47**	8.02	9.70	9.55	6.90	8.51

[a]member for maximum axial load and minimum moment. [b]from the stem base.

Table 16 Factored Moments at Stem Base Resulting from Vertical and Horizontal Loads[a]

item number	item	factored M_{stem} (ft-kips/ft) I	II	III	IV	V	VI
2	$D_{concrete}$	2.38	2.38	2.38	2.38	2.29	2.29
3	$D_{concrete}$	−0.88	−0.88	−0.88	−0.88	−0.85	−0.85
4	$E_{vertical}$	−1.19	−1.19	−1.19	−1.19	−1.16	−1.16
6	$E_{vertical}$	−0.45	−0.45	−0.45	−0.45	−0.44	−0.44
7	L_s (surcharge)	−0.46		−0.27	−0.27		−0.26
8	$D_{superstructure}$	3.58	3.58	3.58	3.58	3.44	3.44
9	$L_{live\ load}$	4.70		2.82	2.82		2.71
10	W_{up}		−0.30	−0.09		−0.29	−0.09
11	L_s (surcharge)	22.72		13.56	13.56		13.05
12	$E_{lateral}$	50.92	50.92	50.92	50.92	49.08	49.08
13	LF			−5.04			−4.90
14	T_f				−12.04	−11.62	−11.62
15	WL_H			−1.40			−1.40
16	$W_{super(H)}$		−2.80	−0.84		−2.66	−0.84
18	W_{sub}		−4.96	−1.52		−4.80	−1.44
	ΣM_{stem}	**81.32**	46.30	61.58	58.43	**32.99**	47.57

[a]member for maximum axial load and minimum moment

For the maximum moment, Group VI loading (see Table 13) is

$$M_u = (91.27 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$$

$$\phi M_n = (0.9A_s)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right) \times (28.5 \text{ in} - 0.980A_s)$$

$$M_u = \phi M_n$$

$$A_s^2 - 29.08A_s + 20.54 = 0$$

$$A_s = 0.72 \text{ in}^2/\text{ft for bending}$$

Try #6 @ 7 in ($A_s = 0.76 \text{ in}^2$).

$$\rho = \frac{0.76 \text{ in}^2}{(12 \text{ in})(28.5 \text{ in})} = 0.0022$$

[< ρ_{max} (= 0.016), so OK]

Use #6 @ 7 in ($A_s = 0.76 \text{ in}^2/\text{ft}$).

Check for combined axial force and bending. The AASHTO equations for pure compression and balanced conditions neglect the slight change of location for the plastic centroid when compression steel is used. The footing and wingwalls brace the abutment stem, so slenderness effects will not have to be considered.

For pure compression (assume $A_s = A_{st}$),

AASHTO [8.16.4.2.1; Eq. 8-31; Eq. 8-30]

$$\phi P_0 = \phi((0.85f'_c)(A_g - A_{st}) + (A_{st}F_y))$$
$$= (0.70)\left[(0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right) \times [(12 \text{ in})(32 \text{ in}) - 0.76 \text{ in}^2] + (0.76 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\right]$$
$$= 716 \text{ kips/ft}$$

$$\phi P_{n(\max)} = (0.80)\left(716 \ \frac{\text{kips}}{\text{ft}}\right) = 573 \text{ kips/ft}$$

(0.80 is the minimum eccentricity for a tied column.)

For balanced conditions,

[8.16.4.2.3; Eq. 8-32; Eq. 8-33]

$$\phi P_b = \phi(0.85f'_c ba_b + A'_s f'_s - A_s F_y)$$

$$\phi M_b = \left(0.85f'_c ba_b\left(d - d'' - \frac{a_b}{2}\right) + A'_s f'_s(d - d' - d'') + A_s F_y d''\right)$$

[Eq. 8-34]

$$a_b = \left(\frac{87{,}000}{87{,}000 + F_y}\right)\beta_1 d$$

[Eq. 8-35]

$$f'_s = (87{,}000) \times \left(1 - \left(\frac{d'}{d}\right)\left(\frac{87{,}000 + F_y}{87{,}000}\right)\right) \le F_y$$

$$A_s = 0.76 \text{ in}^2 \quad \text{[\#6 @ 7 in]}$$

$$A'_s = f'_s = 0 \quad \left[\begin{array}{c}\text{assuming a singly}\\ \text{reinforced stem section}\end{array}\right]$$

[Eq. 8-34]

$$a_b = \left(\frac{87{,}000}{87{,}000 + 60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}\right)(0.85)(28.5 \text{ in})$$
$$= 14.34 \text{ in}$$

[Eq. 8-32]

$$\phi P_b = (0.70)\left((0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in}) \times (14.34 \text{ in}) - (0.76 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\right)$$
$$= 275 \text{ kips/ft}$$

[Eq. 8-33]

$$\phi M_b = (0.70)\left((0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})(14.34 \text{ in}) \times \left(28.50 \text{ in} - 12.50 \text{ in} - \frac{14.34 \text{ in}}{2}\right) + (0.76 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)(12.50 \text{ in})\right)$$
$$= 3111 \text{ in-kips/ft} = 259 \text{ ft-kips/ft}$$

[8.16.4.2.2 Eq. 8-17]

For pure bending,

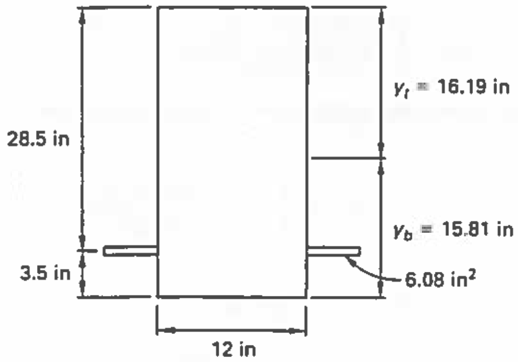
$$a = \frac{A_s F_y}{0.85f'_c b} = \frac{(0.76 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})} = 1.49 \text{ in}$$

$$\frac{a}{2} = 0.75 \text{ in}$$

[Eq. 8-16]

$$\phi M_n = (0.90)(0.76 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right) \times (28.50 \text{ in} - 0.75 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$
$$= 94.9 \text{ ft-kips/ft}$$

Check minimum steel for bending.



The transformed area is

$(n-1)A_s = (9-1)(0.76\text{ in}^2) = 6.08\text{ in}^2$

$(32\text{ in})(12\text{ in}) + 6.08\text{ in}^2 = 390.08\text{ in}^2$

Taking moments about the bottom and solving the equation,

$$(390.08\text{ in}^2)y_b = (384\text{ in}^2)(16\text{ in}) + (6.08\text{ in}^2) \times (3.50\text{ in})$$

$$y_b = 15.81\text{ in}$$

$$I_{cg} = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(12\text{ in})(32\text{ in})^3 + (384\text{ in}^2)(0.19\text{ in})^2 + (6.08\text{ in})(12.31\text{ in})^2 = 33{,}703\text{ in}^4$$

$I = I_{cg}$

$c = y_b$

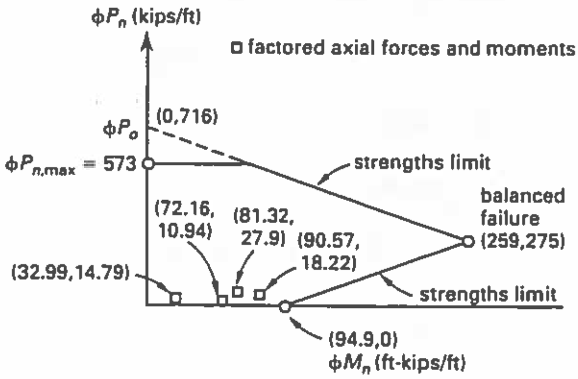
$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{3000}\ \frac{\text{lbf}}{\text{in}^2} = 411\text{ lbf/in}^2$$

$$M_{cr} = \frac{f_r I}{c} = \left(\frac{\left(0.411\ \frac{\text{kip}}{\text{in}^2}\right)(33{,}703\text{ in}^4)}{15.81\text{ in}}\right) \times \left(\frac{1\text{ ft}}{12\text{ in}}\right) = 73.0\text{ ft-kips/ft}$$

$1.2M_{cr} = 87.60$ ft-kips/ft

[$< \phi M_n (= 94.9$ ft-kips), so OK]

The following interaction diagram shows strengths and factored axial forces and moments in the stem. All four cases are indicated in square points, and they are within the strength limits in the interaction diagram.



stem design interaction cure

Use #6 @ 7 in (determined for the minimum axial load and maximum moment case) for the rear face of the stem.

The factored axial force and moment points fall inside the strength curve on the interaction diagram (see the square points).

Check for shear in the stem.

The ultimate shear capacity without shear reinforcement is

$$\phi V_c = \phi\left(2\sqrt{f'_c}\right)b_w d = (0.85)\left(2\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\right)(12\text{ in})(28.5\text{ in}) = 31{,}840\ \frac{\text{lbf}}{\text{ft}} = 31.84\text{ kips/ft}$$

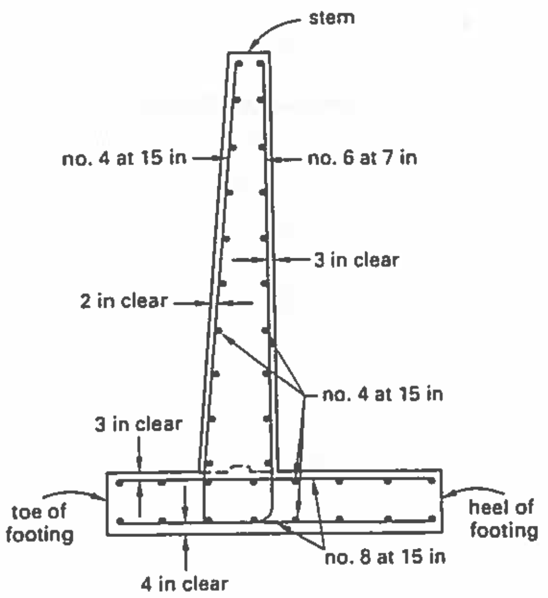
[$> \Sigma H$ for Group VI loading (= 11.50 kips/ft, see Table 12), so OK]

Determine temperature and shrinkage reinforcement.

AASHTO [8.20.1] Reinforcement should be provided near exposed surfaces not otherwise reinforced.

The area provided shall be A_s (temperature and shrinkage) = 0.125 in²/ft in each direction.

Use #4 @ 15 in ($A_s = 0.16\text{ in}^2$) or #4 @ 18 in ($A_s = 0.13\text{ in}^2$).



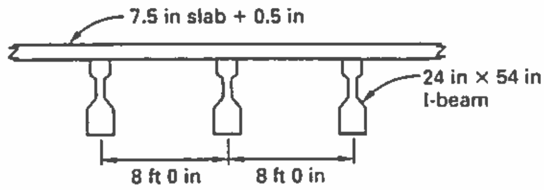
The toe reinforcement will be extended into the bottom of the heel to handle construction stresses without backfill in place. The heel reinforcement will also be extended into the top of the toe. Some reinforcement in the back of the stem could be terminated, as long as it is extended at least a distance equal to the effective depth of the section or 15 bar diameters (whichever is greater) but not less than 1 ft beyond the point at which computations indicate that it is no longer needed to resist stress.

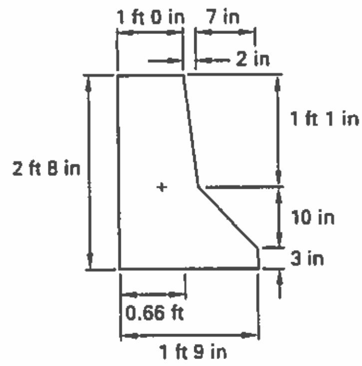
AASHTO [7.5.6] Wingwalls shall be of sufficient length to retain the roadway embankment and shall be designed as retaining walls.

Design Example 4: Interior Prestressed Concrete I-Beam

A bridge has a simply supported span 80 ft long and a cross section as shown. A composite deck and a standard curb/parapet are used. There are six prestressed 24 in × 54 in I-beams with a compressive strength of $f'_{ci} = 5.5$ kips/in^2 at initial prestress and $f'_{cg} = 6.5$ kips/in^2 at 28 days. The roadway is an 8 in slab (including a ½ in integral wearing surface), four lanes wide, with a compressive strength of $f'_{cs} = 4.5$ kips/in^2 at 28 days. The dead load of the future wearing surface is 0.030 kip/ft^2. The dead load of the curb/parapet is 0.506 kips/ft. Prestressing is to be provided by ½ in strand (seven-wire) steel with an area of $A^*_s = 0.153$ in^2 and ultimate stress of prestressing steel (stress relieved) of $f'_s = 270.0$ kips/in^2.

Design the prestressed concrete I-beam for HS 25 loading by the load factor design method.





The following data apply.

Six prestressed I-beam girders

$f'_{cg} = 6500$ lbf/in^2

$f'_{ci} = 5500$ lbf/in^2 at initial prestress

$f'_s = 270{,}000$ lbf/in^2

$A^*_s = 0.153$ in^2

$$E_{cg} = (W_c)^{1.5} 33\sqrt{f'_{cg}}$$

$$= \left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5} 33\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 4.89 \times 10^6\ \text{lbf/in}^2$$

AASHTO [9.15.2] Allowable concrete stresses for I-beam girders

[9.15.2.1; 9.15.2.2] Compression (pretensioned members) before losses due to creep and shrinkage,

$$f_{ci} = 0.60 f'_{ci}$$

$$= 0.60\left(5500\ \frac{\text{lbf}}{\text{in}^2}\right) = 3300\ \text{psi}$$

Tension (with no bonded reinforcement) before losses due to creep and shrinkage,

$$f_{ti} = 200\ \frac{\text{lbf}}{\text{in}^2} \text{ or } 3\sqrt{f'_{ci}}$$

(With bonded reinforcement, this value will be used to be conservative.)

[9.15.2.2] Compressive stress after losses,

$$f_{cs} = 0.40 f'_{cg} = 0.40\left(6500\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 2600\ \text{lbf/in}^2$$

Tension after losses,

$$f_{ts} = 6\sqrt{f'_{cg}} = 6\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 483.7\ \text{lbf/in}^2$$

For severe corrosive exposure conditions,

$$f_{ts} = 3\sqrt{f'_{cg}} = 241.87\ \text{lbf/in}^2$$

AASHTO [9.15.1] f_{si} = allowable stress in prestressing steel (stress-relieved strands)

$$= 0.70 f'_s$$

Slab concrete,

$$f'_{cs} = 4500\ \text{lbf/in}^2$$

$$E_{cs} = (W_c)^{1.5} 33\sqrt{f'_{cs}}$$

$$= \left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5} 33\sqrt{4500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 4.067 \times 10^6\ \text{lbf/in}^2$$

Solution:

Group loading combinations for allowable stress design at service conditions are given by

$$\text{Group I} = \gamma(\beta_D D + \beta_L(L+I))$$
$$= (1.0)(1D + (1)(L+I))$$

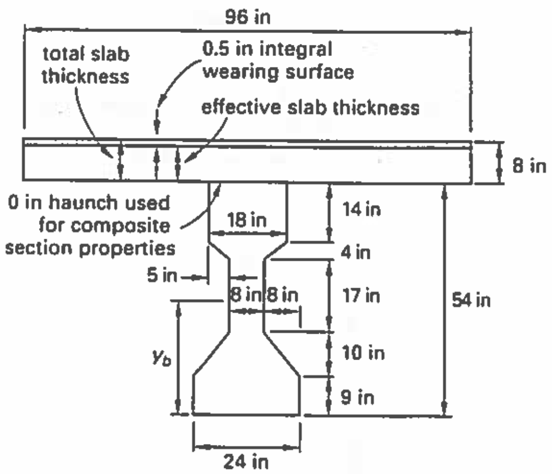
Group loading combinations for load factor design at factored conditions are given by

$$\text{Group I} = \gamma(\beta_D D + \beta_L(L+I))$$
$$= (1.3)(1D + 1.67(L+I))$$
$$= 1.3D + 2.17(L+I)$$

To get factored loads when using HS 25 design loads, multiply dead load by 1.30 and multiply live plus impact loads by 2.17 $(= (1.30)(1.67))$.

Determine the beam dead load.

A 24 in × 54 in I-beam is assumed. The weight of the beam is calculated using the dimensions shown.



The beam weight is

$$W = (816\ \text{in}^2)\left(\frac{1\ \text{ft}^2}{(12\ \text{in})(12\ \text{in})}\right)\left(0.15\ \frac{\text{kip}}{\text{ft}^3}\right)$$
$$= 0.85\ \text{kip/ft}$$

The dead load weight of the slab is

$$W_D = (8\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)\left(0.15\ \frac{\text{kip}}{\text{ft}^3}\right)(8\ \text{ft})$$
$$= 0.80\ \text{kip/ft}$$

AASHTO [Fig. 2.7.4B] The superimposed dead load, W_s, consists of the parapet/curb loads, distributed equally to the 6 beams, plus the load of the future wearing surface.

$$W_s = \frac{\left(0.506\ \frac{\text{kip}}{\text{ft}}\right)(2)}{6\ \text{beams}} + (8\ \text{ft})\left(0.03\ \frac{\text{kip}}{\text{ft}^2}\right)$$
$$= 0.409\ \text{kip/ft}$$

The parapet/curb weight is 0.506 kip/ft.

Determine the live load distribution.

[3.12.1] The reduction in load intensity for four or more traffic lanes loaded simultaneously is 75%. The transverse distribution of wheel loads for beam design for an interior beam is

[Table 3.23.1] [3.12.1; 3.12.2]

$$\frac{S}{5.5} = \left(\frac{8\ \text{ft}}{5.5}\right) = 1.45$$

This distribution factor will be applied for HS 25 loading.

Determine the impact load.

The impact fraction is

[3.8.2] $$I = \frac{50}{L + 125} = \frac{50}{80\ \text{ft} + 125} = 0.244$$

Determine service loads moment and shear.

The moment at midspan due to the weight of the beam is

$$\begin{aligned} M_O &= \left(\frac{1}{8}\right) W L^2 \\ &= \left(\frac{1}{8}\right)\left(0.850\ \frac{\text{kip}}{\text{ft}}\right)(80\ \text{ft})^2 \\ &= 680.00\ \text{ft-kips} \end{aligned}$$

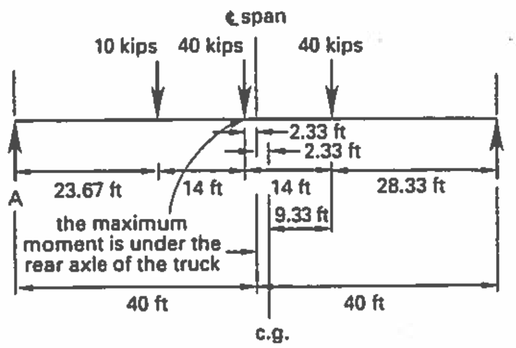
The moment at midspan due to the slab dead load per beam is

$$\begin{aligned} M_D &= \left(\frac{1}{8}\right) W_D L^2 \\ &= \left(\frac{1}{8}\right)\left(0.80\ \frac{\text{kip}}{\text{ft}}\right)(80\ \text{ft})^2 \\ &= 640.00\ \text{ft-kips} \end{aligned}$$

The moment at midspan due to the superimposed dead load per beam is

$$\begin{aligned} M_S &= \left(\frac{1}{8}\right) W_S L^2 \\ &= \left(\frac{1}{8}\right)\left(0.409\ \frac{\text{kip}}{\text{ft}}\right)(80\ \text{ft})^2 \\ &= 327.20\ \text{ft-kips} \end{aligned}$$

The maximum live load moment due to the HS 25 truck can be determined through the influence line method.



The live load reaction at A is

$$\begin{aligned} &\frac{(40\ \text{kips})(28.33\ \text{ft} + 42.33\ \text{ft})}{80\ \text{ft}} \\ &= \quad + \frac{(10\ \text{kips})(56.33\ \text{ft})}{80\ \text{ft}} = 42.37\ \text{kips} \end{aligned}$$

The live load moment, M_L, is

$$\begin{aligned} &(42.37\ \text{kips})(14.0\ \text{ft} + 23.67\ \text{ft}) - (10.0\ \text{kips})(14.0\ \text{ft}) \\ &\quad = 1456.07\ \text{ft-kips for all wheels} \end{aligned}$$

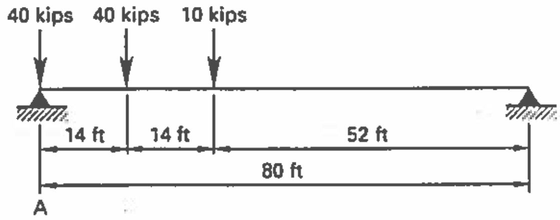
The maximum live load moment with impact for each stringer (or beam) is

$$\begin{aligned} M_{L+I} &= \left(\frac{1456.07\ \text{ft-kips}}{2\ \text{wheel lines}}\right)(1.45)(1.244) \\ &= 1313.23\ \text{ft-kips} \end{aligned}$$

The maximum dead load shear is

$$\begin{aligned} V_D &= \left(0.85\ \frac{\text{kip}}{\text{ft}} + 0.80\ \frac{\text{kip}}{\text{ft}} + 0.409\ \frac{\text{kip}}{\text{ft}}\right)(40\ \text{ft}) \\ &= 82.36\ \text{kips at each support bearing} \end{aligned}$$

The maximum live load shear due to the HS 25 truck is



The live load reaction at A is

$$\begin{aligned} &40\ \text{kips} + \frac{(40\ \text{kips})(52\ \text{ft} + 14\ \text{ft}) + (10\ \text{kips})(52\ \text{ft})}{80\ \text{ft}} \\ &= 79.5\ \text{kips} \end{aligned}$$

The maximum live load shear with impact for each stringer (or beam) is

$$\begin{aligned} V_{L+I} &= \left(\frac{79.5\ \text{kips}}{2\ \text{wheel lines}}\right)(1.45)(1.244) \\ &= 71.70\ \text{kips} \end{aligned}$$

Calculate factored moment and shear.

$$\begin{aligned} M_u &= (1.30)(1D + (1.67)(L + I)) \\ &= (1.30)((680.0\ \text{ft-kips} + 640.0\ \text{ft-kips} \\ &\quad + 327.2\ \text{ft-kips}) + (1.67)(988.10\ \text{ft-kips})) \\ &= 4286.5\ \text{ft-kips} \\ V_u &= (1.30)(1D + (1.67)(L + I)) \\ &= (1.30)(82.36\ \text{kips} + (1.67)(53.95\ \text{kips})) \\ &= 224.2\ \text{kips} \end{aligned}$$

Determine the allowable stresses on the beam. The strengths of the materials used are

slab concrete: $f'_{cs} = 4500\ \text{lbf/in}^2$

prestressed concrete I-beam: $f'_{cg} = 6500\ \text{lbf/in}^2$

prestressed strands: $f'_s = 270\ \text{kips/in}^2$ stress relieved, ½ in diameter strand

Calculate the concrete's modulus of elasticity in order to determine the ratio of transformation.

AASHTO [Eq. 10-68]

$$E_c = w_c^{1.5} 33\sqrt{f'_c}$$

$w_c = 150\ \text{lbf/ft}^3$ for normal concrete

$$E_{cs} = (33)(150)^{1.5}\sqrt{4500\ \frac{\text{lbf}}{\text{in}^2}} = 4.067 \times 10^6\ \text{lbf/in}^2 \text{ for slab concrete}$$

$$E_{cg} = (33)(150)^{1.5}\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}} = 4.89 \times 10^6\ \text{lbf/in}^2 \text{ for I-beam girder}$$

$$n = \frac{E_{cg}}{E_{cs}} = \frac{4.89}{4.067} = 1.2$$

The properties of the basic beam section are as follows.

$A_B = 816\ \text{in}^2$
$I_B = 255{,}194\ \text{in}^4$
$S_b = 10{,}083\ \text{in}^3$
$S_t = 8895\ \text{in}^3$
$y_b = 25.31\ \text{in}$
$y_t = 28.69\ \text{in}$ [54 in − y_b (= 25.31 in)]

Determine the composite properties as shown.

AASHTO [9.8.3.2]

The effective flange width is the lesser of

$$(80\ \text{ft})\left(\frac{1}{4}\ \text{span}\right) = \frac{(80\ \text{ft})\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)}{4} = 240\ \text{in}$$

or

$$(6)(8\ \text{in} - 0.5\ \text{in})(2) + 18\ \text{in} = 108\ \text{in}$$

or

center-to-center beam spacing = 96 in

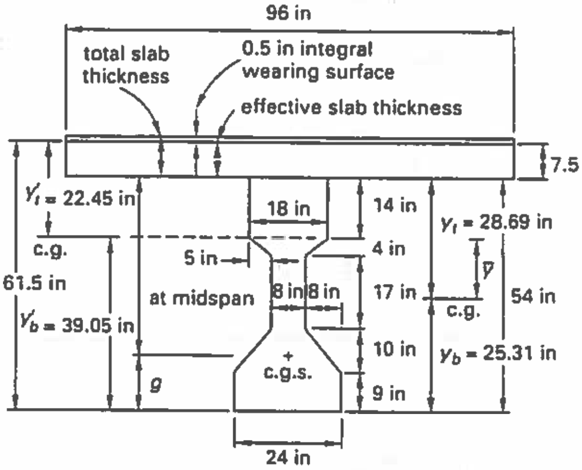
From the above, 96 in governs.

Use the transformation ratio ($n = 1.2$) to compute composite section properties by summing moments of areas about the centroid of the basic beam section.

$$A_c = A_B + \text{flange area} = 816\ \text{in}^2 + \left(\frac{96\ \text{in}}{1.2}\right)(7.5\ \text{in}) = 816\ \text{in}^2 + 600\ \text{in}^2 = 1416\ \text{in}^2$$

$$(1416\ \text{in}^2)\,\overline{y} = (600\ \text{in}^2)\left(y_t + \frac{7.5\ \text{in}}{2}\right) = (600\ \text{in}^2)(28.69\ \text{in} + 3.75\ \text{in}) = 19{,}464\ \text{in}^3$$

$\overline{y} = 13.74$ in from the centroid of the basic beam section



Thus, for the composite section,

$$y'_b = y_b + \overline{y} = 25.31\ \text{in} + 13.74\ \text{in} = 39.05\ \text{in}$$

$$y'_t = (54\ \text{in} + 7.5\ \text{in}) - y'_b = 22.45\ \text{in}$$

The composite moment of inertia about the centroid of the composite section, c.g.c., is

$$I_c = 255{,}194\ \text{in}^4 + (816\ \text{in}^2)(13.74\ \text{in})^2 + \frac{\left(\frac{96\ \text{in}}{1.2}\right)(7.5\ \text{in})^3}{12} + (600\ \text{in}^2)(22.45\ \text{in} - 3.75\ \text{in})^2 = 621{,}871.2\ \text{in}^4$$

The section modulus of the composite section for the bottom extreme fiber is

$$S_{bc} = \frac{I_c}{y'_b} = \frac{621{,}871.2\ \text{in}^4}{39.05\ \text{in}} = 15{,}925\ \text{in}^3$$

For the top extreme fiber,

$$S_{tc} = \frac{I_c}{y'_t} = \frac{621{,}871.2\ \text{in}^4}{22.45\ \text{in}} = 27{,}700\ \text{in}^3$$

Calculate prestress force and eccentricity.

The temporary stress limits before losses due to creep and shrinkage are as follows.

AASHTO [9.15.2]

For compression,

$$f_{ci} = 0.60 f'_{ci}$$

$$= (0.60)\left(5500\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 3300\ \text{lbf/in}^2$$

For tension,

$$f_i = 7.5\sqrt{f'_{ci}}$$

$$= 7.5\sqrt{5500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 556\ \text{lbf/in}^2$$

AASHTO [9.15.2.2]

Allowable stresses after losses are as follows.

For compression,

$$f_{cs} = 0.4 f'_{cg} = (0.4)\left(6500\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 2600\ \text{lbf/in}^2$$

For tension,[1]

$$f_{ts} = 3\sqrt{f'_{cg}} = 3\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 242\ \text{lbf/in}^2$$

The theoretical prestressing force to be selected must satisfy two conditions simultaneously:

(1) The allowable tensile stresses may not be exceeded in the top of the beam at the center of bearing at the time of prestress transfer.

(2) The allowable tensile stresses may not be exceeded in the bottom of the beam at midspan under final conditions (i.e., prestress after losses minus all loads).

AASHTO [9.16.2 & 9.16.2.2]

A 22.8% loss of prestress force is assumed in the straight tendons. It will be assumed that the critical stresses are the initial tensile stress at the top of the beam at the bearing and the final tensile stress at the bottom of the beam at midspan.

[1] Bonded reinforcement for severe corrosive exposure conditions

For initial tensile stress at the top of beam at bearing,

$$f_{ti} = -\frac{P_i}{A_B} + \frac{P_i e}{S_t}$$

Substituting values for the stresses and section properties gives

$$300\ \frac{\text{lbf}}{\text{in}^2} = -\frac{P_i}{816\ \text{in}^2} + \frac{P_i e}{8895\ \text{in}^3} \qquad \text{[I]}$$

For final tensile stress at the bottom of the beam at midspan,

$$f_{ts} = -\frac{P_e}{A_B} - \frac{P_e e}{S_b} + \frac{M_{0+D}}{S_b} + \frac{M_s + M_{L+I}}{S_{bc}}$$

$$242\ \frac{\text{lbf}}{\text{in}^2} = -\frac{0.772 P_i}{816\ \text{in}^2} - \frac{0.772 P_i e}{10{,}083\ \text{in}^3}$$
$$+ \left(\frac{680.0\ \text{ft-kips} + 640.00\ \text{ft-kips}}{10{,}083\ \text{in}^3}\right)$$
$$\times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$
$$+ \left(\frac{327.20\ \text{ft-kips} + 1313.23\ \text{ft-kips}}{15{,}925\ \text{in}^3}\right)$$
$$\times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right) \qquad \text{[II]}$$

Solving Eqs. I and II simultaneously gives

$P_i = 1{,}186{,}900$ lbf

$e = 13.15$ in

AASHTO [9.15.1]

The amount of force taken by one ½ in, seven-wire strand at 70% of ultimate stress f'_s is

$$F = A^*_s f_{si}$$

$$f_{si} = 0.70 f'_s$$

$$F = (0.153\ \text{in}^2)(0.70)\left(270{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 28{,}920\ \text{lbf}$$

The number of strands required is

$$N = \frac{1{,}186{,}900\ \text{lbf}}{28{,}920\ \text{lbf}} = 41.04 \quad \text{[use 42 strands]}$$

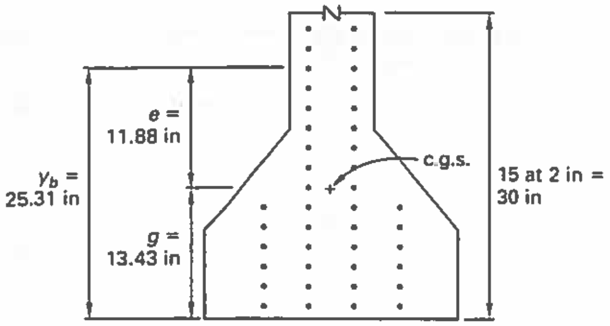
For 42 strands, the initial prestressing force is

$$P_i = (28{,}920\ \text{lbf})(42) = 1{,}214{,}600\ \text{lbf}$$

The eccentricity from Eq. I using P_i = 1,214,600 lbf is $e = 13.10$ in.

The eccentricity from Eq. II using P_i = 1,214,600 lbf is $e = 12.56$ in.

Try the following strand pattern.



This strand pattern gives a centroid g of 13.43 in and an eccentricity of

$$e = y_b - g$$
$$= 25.31 \text{ in } (= y_b) - 13.43 \text{ in } (= g)$$
$$= 11.88 \text{ in}$$

Check the assumed prestress loss.

The loss of prestress will be determined using the modified Bureau of Public Roads formula.

Total loss of prestress is

$$6000 \ \frac{\text{lbf}}{\text{in}^2} + 16 f_{cs} + 0.04 f_{si}$$

$$f_{si} = \text{allowable stress in pressing steel}$$
$$= 0.70 f'_s$$
$$= (0.70)\left(270{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)$$
$$= 189{,}000 \text{ lbf/in}^2$$

$$f_{cs} = \text{compressive strength of concrete after losses}$$
$$= \frac{P_i}{A_B} + \frac{P_i e^2 - M_0 e}{I_B}$$

$$\left[\begin{array}{c} M_0 = \text{moment at midspan} \\ \text{due to beam weight} \end{array}\right]$$

$$= \frac{1{,}214{,}600 \text{ lbf}}{816 \text{ in}^2} + \frac{(1{,}214{,}600 \text{ lbf})(11.88 \text{ in})^2 - (680.0 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\times\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)(11.88 \text{ in})}{255{,}194 \text{ in}^4}$$
$$= 1780 \text{ lbf/in}^2$$

Total loss of prestress is

$$6000 \ \frac{\text{lbf}}{\text{in}^2} + (16)\left(1780 \ \frac{\text{lbf}}{\text{in}^2}\right) + (0.04)\left(189{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right) = 42{,}040 \text{ lbf/in}^2$$

AASHTO [9.16.2.2; Table 9.16.2.2]

Estimate of total losses may be 45,000 lbf/in^2 with $f'_c = 5000$ lbf/in^2.

The percentage loss is

$$\left(\frac{42{,}040}{189{,}000}\right)(100) = 22.2\%$$

$$\left[\begin{array}{c} < 22.8\% \text{ assumed; therefore,} \\ \text{minimum loss of } 22.8\% \\ \text{assumed is conservative.} \end{array}\right]$$

Check stresses. ("–" indicates compressive stresses.)

The effective prestress force after losses is

$$P_e = (1.00 - 0.228)(1{,}214{,}600 \text{ lbf})$$
$$= 937{,}700 \text{ lbf}$$

Determine the critical stresses at the support and midspan.

Initial stresses before losses at support for the basic beam section are

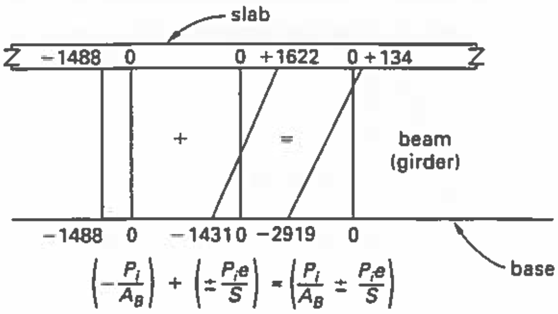
$$\frac{P_i}{A_B} = \frac{1{,}214{,}600 \text{ lbf}}{816 \text{ in}^2} = -1488 \text{ lbf/in}^2 \quad \text{(compression)}$$

$$\frac{P_i e}{S_t} = \frac{(1{,}214{,}600 \text{ lbf})(11.88 \text{ in})}{8895 \text{ in}^3}$$
$$= 1622 \text{ lbf/in}^2 \quad \text{(tension)}$$

$$\frac{P_i e}{S_b} = \frac{(1{,}214{,}500 \text{ lbf})(11.88 \text{ in})}{10{,}083 \text{ in}^3}$$
$$= -1431 \text{ lbf/in}^2 \quad \text{(compression)}$$

Check the allowable stresses ("+" indicates tensile stresses).

The initial stresses before losses are



note: units for stresses are in lbf/in²

AASHTO [9.15.2.1; 9.15.2.2]

For compression at the beam base, allowable concrete stress = f_{ci}

$$f_{ci} = 0.6f'_{ci}$$

$$= 0.6\left(5500\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 3300\ \text{lbf/in}^2 \left[\begin{array}{c} > |2919\ \text{lbf/in}^2| \\ (= -1488 - 1431),\ \text{so OK} \end{array}\right]$$

[9.15.2.1; 9.15.2.2]

For tension, to be conservative, allowable concrete stress = f_{ti}

$$f_{ti} = 200\ \text{lbf/in}^2 \left[\begin{array}{c} > 134\ \text{lbf/in}^2 \\ (= -1488 + 1622),\ \text{so OK} \end{array}\right]$$

or $f_{ti} = 3\sqrt{f'_{ci}} = 3\sqrt{5500} = 222.5\ \text{lbf/in}^2$

Final stresses after losses at midspan are as follows.

The stress in the basic beam section at the girder base is

$$-\frac{P_e}{A_B} - \frac{P_e e}{S_b}$$

$$= \frac{-937{,}700\ \text{lbf}}{816\ \text{in}^2} - \frac{(937{,}700\ \text{lbf})(11.88\ \text{in})}{10{,}083\ \text{in}^3}$$

$$= -1149\ \frac{\text{lbf}}{\text{in}^2} - 1105\ \frac{\text{lbf}}{\text{in}^2}$$

$$= -2254\ \text{lbf/in}^2 \quad \text{(compression)}$$

The stress at the top of the girder is

$$f = -\frac{P_e}{A_B} + \frac{P_e e}{S_t}$$

$$= -1149\ \frac{\text{lbf}}{\text{in}^2} + \frac{(937{,}700\ \text{lbf})(11.88\ \text{in})}{8895\ \text{in}^3}$$

$$= +103\ \text{lbf/in}^2 \quad \text{(tension)}$$

Due to the beam and slab weight, the stress at the top of the girder is

$$-\frac{M_{0+D}}{S_t} = \left(-\frac{680.0\ \text{ft-kips} + 640.0\ \text{ft-kips}}{8895\ \text{in}^3}\right) \times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$

$$= -1781\ \text{lbf/in}^2 \quad \text{(compression)}$$

The stress at the girder base is

$$+\frac{M_{0+D}}{S_b} = \left(\frac{680.0\ \text{ft-kips} + 640.0\ \text{ft-kips}}{10{,}083\ \text{in}^3}\right) \times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$

$$= 1571\ \text{lbf/in}^2 \quad \text{(tension)}$$

Due to the superimposed dead load (parapet and curb) plus the live load, the stress in the composite section at slab top is

$$f = -\frac{M_s + M_{L+I}}{S_{tc}}$$

$$= \left(-\frac{327.2\ \text{ft-kips} + 1313.23\ \text{ft-kips}}{27{,}700\ \text{in}^3}\right) \times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$

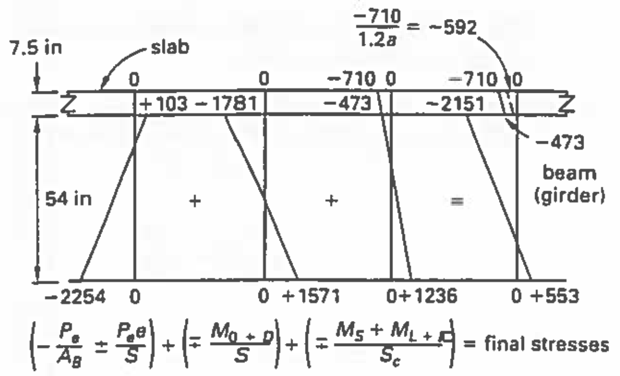
$$= -710\ \text{lbf/in}^2 \quad \text{(compression)}$$

The stress in the composite section at the girder base is

$$f = +\frac{M_s + M_{L+I}}{S_{bc}}$$

$$= \left(+\frac{327.2\ \text{ft-kips} + 1313.23\ \text{ft-kips}}{15{,}925\ \text{in}^3}\right) \times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$

$$= 1236\ \text{lbf/in}^2$$



note: units for stresses are in lbf/in^2

a: $E_{cg}/E_{cs} = 4.89/4.067 = 1.2$

Final Stresses After Losses at Midspan

Check the allowable stresses in the girder after losses.

AASHTO [9.15.2.2] For compression,

$$f_{cs} = 0.4f'_{cg} = 0.4\left(5500\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 2600\ \text{lbf/in}^2 \quad [> 2151\ \text{lbf/in}^2,\ \text{so OK}]$$

[9.15.2.2] For tension,

$$f_{ts} = 6\sqrt{f'_{cg}} = 6\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}} = 483.7\ \frac{\text{lbf}}{\text{in}^2}$$

$$= 483.7\ \text{lbf/in}^2 \quad [< 553\ \text{lbf/in}^2,\ \text{so no good}]$$

Check the stresses in the slab.

AASHTO [8.15.2.1.1] f_{cs} = allowable compressive stress in concrete (slab)

$$= 0.40f'_{cs} = (0.40)\left(4500\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 1800\ \text{lbf/in}^2$$

$$f_{\text{slab}} = \frac{M_s + M_{L+I}}{S_{tc}}$$

$$= \left(\frac{327.2\ \text{ft-kips} + 1313.23\ \text{ft-kips}}{27{,}700\ \text{in}^3}\right) \times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$

$$= 710\ \text{lbf/in}^2\ \text{(compression)}$$

$$[<1800\ \text{lbf/in}^2,\ \text{so OK}]$$

Begin checking moment capacity by load factor design. First determine whether the beam section is flanged or rectangular.

AASHTO [9.17.2]

$$a = \frac{A_s^* f_{su}^*}{0.85 f_{c,\text{slab}} b}$$

$$f'_{c,\text{slab}} = f'_{cs} = 4500\ \text{lbf/in}^2$$

[9.17.4.1] f_{su}^* = average stress in prestressing steel at ultimate load

$$f_{su}^* = f'_s\left(1 - \left(\frac{\gamma^*}{\beta_1}\right)\left(\frac{\rho^* f'_s}{f'_c}\right)\right)$$

[9.1.2] $\gamma^* = 0.40$ for stress relieved steel

[8.16.2.7]

$$\beta_1 = 0.85 - (0.05)\left(\frac{500}{1000}\right) = 0.825$$

$$f'_s = 270{,}000\ \text{lbf/in}^2$$

$$\rho^* = \frac{A_s^*}{bd} = \frac{(42)(0.153\ \text{in}^2)}{(96\ \text{in})(61.5\ \text{in} - 13.43\ \text{in})}$$

$$= 0.001393$$

$$f_{su}^* = \left(270{,}000\ \frac{\text{lbf}}{\text{in}^2}\right) \times \left(1 - \frac{\left(\frac{0.40}{0.825}\right)(0.001393) \times \left(270{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)}{4500\ \frac{\text{lbf}}{\text{in}^2}}\right)$$

$$= 259{,}059\ \text{lbf/in}^2$$

$$a = \frac{(42\ \text{strands})(0.153\ \text{in}^2)\left(259{,}059\ \frac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(4500\ \frac{\text{lbf}}{\text{in}^2}\right)(96\ \text{in})}$$

$$= 4.53\ \text{in}$$

Since $a < 7.5$ in, the effective thickness of the slab, the beam is rectangular.

The moment capacity by the load factor design is

AASHTO [9.1.7.2]

$$\phi M_n = \phi\left(A_s^* f_{su}^* d\left(1 - (0.6)\left(\frac{p^* f_{su}^*}{f'_c}\right)\right)\right)$$

[9.14] $\phi = 1.00$ for flexure in prestressed concrete members for load factor design.

$$\phi M_n = (1.0)\Bigg[(42\text{ strands})(0.153\text{ in}^2)\left(259{,}059\ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$\times\ (61.5\text{ in} - 13.43\text{ in})$$

$$\times\left(1-(0.6)\left[\frac{(0.001393)\left(259{,}059\ \frac{\text{lbf}}{\text{in}^2}\right)}{4500\ \frac{\text{lbf}}{\text{in}^2}}\right]\right)\Bigg]$$

$$= (1.0)\left((8.0\times10^7)(0.952)\right)$$

$$= (7.61\times10^7\text{ in-lbf})\left(\frac{1\text{ kip}}{1000\text{ lbf}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right)$$

$$= 6342.0\text{ ft-kips}$$

Determine the design moment by the load factor method (Group I loading).

$$M_u = 1.3D + (2.17)(L+I)$$

$$= (1.3)(M_0+M_D+M_S)+(2.17)(M_{L+I})$$

$$= (1.3)(680.0\text{ ft-kips} + 640.0\text{ ft-kips} + 327.2\text{ ft-kips}) + ((2.17)(988.10\text{ ft-kips}))$$

$$= 4285.5\text{ ft-kips}$$

$$[< \phi M_n\ (= 6342.0\text{ ft-kips}),\ \text{so OK}]$$

AASHTO [9.18.1] The prestressing steel's reinforcement index cannot exceed $0.36\beta_1$.

$$\frac{p^* f^*_{su}}{f'_c} = \frac{(0.001393)\left(259{,}059\ \frac{\text{lbf}}{\text{in}^2}\right)}{4500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.080\quad\left[< 0.36\beta_1\ \left(= (0.36)(0.825) = 0.297\right),\ \text{so OK}\right]$$

Determine the minimum amount of prestressed steel that will be necessary.

[9.18.2] The total amount of prestressed reinforcement shall be adequate to develop an ultimate moment at the critical section of at least 1.2 times the cracking moment M^*_{cr}.

$$\phi M_n \geq 1.2 M^*_{cr}$$

$$\phi M_n = 6342\text{ ft-kips}$$

$$M^*_{cr} = (f_r + f_{pe})S_{bc} - M_{d/nc}\left(\frac{S_c}{S_b} - 1\right)$$

$M_{d/nc}$ = non-composite dead load moment

$$= M_0 + M_D + M_S$$

$S_{bc} = S_c$ = composite section modulus where the tensile stress is caused by externally applied loads

S_b = non-composite section modulus where the tensile stress is caused by externally applied loads

[9.15.2.3] $$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 604.67\text{ lbf/in}^2$$

$$f_{pe} = \frac{P_{\text{effect}}}{A_b} + \frac{P_{\text{effect}}e}{S_b} - \frac{M_0}{S_b}$$

P_{effect} = effective prestressing force with 22.8% loss

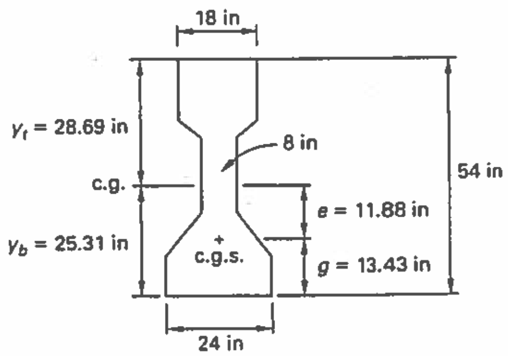
M_0 = moment at midspan due to beam weight

$$P_{\text{effect}} = P_i(0.772) = (0.70 f'_s A^*_s)(0.772)$$

$$= (0.70)\left(270\ \frac{\text{kips}}{\text{in}^2}\right)(0.153\text{ in}^2)$$

$$\times\ (42\text{ strands})(0.772)$$

$$= 937.60\text{ kips}$$



The basic beam section is

$$A_B = 816\text{ in}^2$$

$$e = 11.88\text{ in}$$

S_b = section modulus of the basic beam for the bottom fiber = 10,083 in^3

S_{bc} = section modulus of the composite section for the bottom fiber = 15,925 in^3

$$f_{pe} = \frac{937.60 \text{ kips}}{816 \text{ in}^2}$$

$$+ \frac{(937.60 \text{ kips})(11.88 \text{ in})}{10{,}083 \text{ in}^3}$$

$$- \left(\frac{680 \text{ ft-kips}}{10{,}083 \text{ in}^3}\right)\left(12 \frac{\text{in}}{\text{ft}}\right)$$

$$= 1.15 \frac{\text{kips}}{\text{in}^2} + 1.10 \frac{\text{kips}}{\text{in}^2} - 0.81 \frac{\text{kips}}{\text{in}^2}$$

$$= 1.44 \text{ kips/in}^2$$

$M_{d/nc}$ = non-composite dead load moment

$$= M_0 + M_D + M_s$$

$$= 680 \text{ ft-kips} + 640 \text{ ft-kips}$$

$$+ 327.20 \text{ ft-kips}$$

$$= 1647.2 \text{ ft-kips}$$

$$S_c = S_{bc} = 15{,}925 \text{ in}^3$$

$$S_b = 10{,}083 \text{ in}^3$$

$$M_{cr} = \left(0.6047 \frac{\text{kips}}{\text{in}^2} + 1.44 \frac{\text{kips}}{\text{in}^2}\right)$$

$$\times (15{,}925 \text{ in}^3)\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$$

$$- (1647.2 \text{ ft-kips})\left(\frac{15{,}925 \text{ in}^3}{10{,}083 \text{ in}^3} - 1\right)$$

$$= 2713.49 \text{ ft-kips} - 954.37 \text{ ft-kips}$$

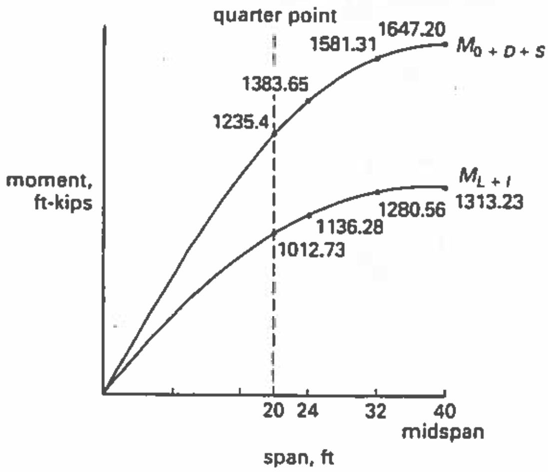
$$= 1759.12 \text{ ft-kips}$$

$$1.2M_{cr} = (1.2)(1759.12 \text{ ft-kips})$$

$$= 2110.94 \text{ ft-kips}$$

[< ϕM_n (= 6342 ft-kips), so OK]

The envelope of maximum moment (service load) is



Design for shear.

AASHTO [9.20] Shear design for prestressed beams with straight fully bonded strands will be in accordance with Article 9.20 of the AASHTO Specifications.

AASHTO [Eq. 9-26] $V_u \leq \phi(V_c + V_s)$

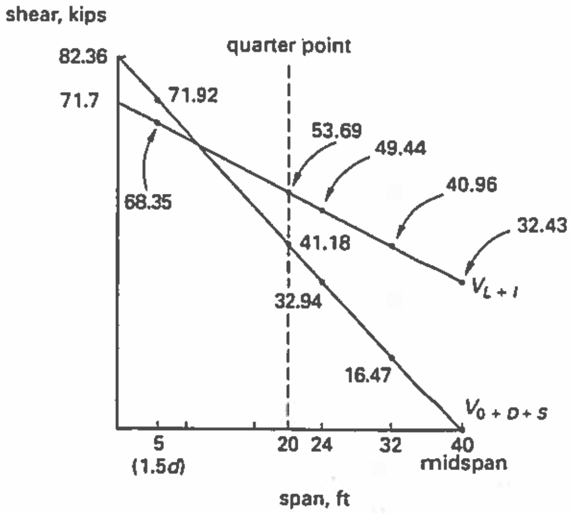
The shear stress at support is

$$V_{L+I} = \left(\frac{79.5 \text{ kips}}{2 \text{ wheel lines}}\right)(1.45)(1.244)$$

$$= 71.7 \text{ kips}$$

$$V_{0+D+S} = \left(0.85 \frac{\text{kip}}{\text{ft}} + 0.80 \frac{\text{kip}}{\text{ft}} + 0.409 \frac{\text{kip}}{\text{ft}}\right)(40 \text{ ft}) = 82.36 \text{ kips}$$

The envelope of maximum shear (service load) is



Calculate shear stress at the quarter point.

AASHTO [9.20.2.1 & 9.20.2.2]

The shear strength of concrete, V_c, shall be the lesser of V_{ci} or V_{cw}.

$$V_{ci} = 0.6\sqrt{f'_c}b'd + V_d + \frac{V_i M_{cr}}{M_{max}}$$

b' = width of beam web = 8 in

d = $y_t + e = 28.69 + 11.88 = 40.57$ in

$f'_c = f'_{cg} = 6500$ lbf/in^2

$$0.6\sqrt{f'_{cg}}b'd = \left(0.6\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}\right)(8\text{ in})(40.57) \times \left(\frac{1}{1000\ \frac{\text{lbf}}{\text{kip}}}\right)$$

$$= 15.62\text{ kips}$$

From the envelope curves of maximum shears at the quarter point, the shear force due to the unfactored dead load, V_d, is 41.18 kips.

The factored shear force due to externally applied loads occurring simultaneously with M_{max} is

$$V_i = (1.3)\big((1.67)(L+I)\big)$$
$$= (1.3)\big((1.67)(53.69\text{ kips})\big)$$
$$= 116.56\text{ kips}$$

$$M_{cr} = \left(\frac{I_c}{y_b}\right)\left(6\sqrt{f'_{cg}} + f_{pe} - f_d\right)$$
$$= \left(\frac{621{,}871.2\text{ in}^4}{39.05\text{ in}}\right) \times \left(6\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}} + f_{pe} - f_d\right)$$

Moments at quarter point are ($W_0 + W_D = WL$). For beam weight (W) and slab weight (W_D),

$$M_{0+D} = \left(\frac{\text{WL}}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{\text{WL}}{4}\right)\left(\frac{L}{8}\right)$$
$$= \frac{\text{WL}^2}{8} - \frac{\text{WL}^2}{32}$$
$$= \frac{3(\text{WL})^2}{32}$$

$$M_{0+D} = \left(\frac{3}{32}\right)\left(0.85\ \frac{\text{kip}}{\text{ft}} + 0.80\ \frac{\text{kip}}{\text{ft}}\right)(80\text{ ft})^2$$
$$= 990.0\text{ ft-kips}$$

For the superimposed dead load, W_S (parapet/curb),

$$M_s = \left(\frac{3}{32}\right)(W_S)^2$$
$$= \left(\frac{3}{32}\right)\left(0.409\ \frac{\text{kip}}{\text{ft}}\right)(80\text{ ft})^2$$
$$= 245.4\text{ ft-kips}$$

f_{pe} = compressive stress in concrete due to effective prestress only

$= 1.44$ kips/in^2

The stress due to the unfactored dead load at the quarter point in the beam span is

$$f_d = \frac{M_{0+D}}{S_b} + \frac{M_s}{S_{bc}}$$
$$= \left(\frac{990.0\text{ ft-kips}}{10{,}083\text{ in}^3}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right) + \left(\frac{245.4\text{ ft-kips}}{15{,}925\text{ in}^3}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)$$
$$= 1.3628\text{ kips/in}^2$$

$$M_{cr} = \left(\frac{621{,}871.2\text{ in}^4}{39.05\text{ in}}\right)\left(\left(6\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}\right) \times \left(\frac{1\text{ kip}}{1000\text{ lbf}}\right) + 1.44\ \frac{\text{kips}}{\text{in}^2} - 1.3628\ \frac{\text{kips}}{\text{in}^2}\right) \times \left(\frac{1\text{ ft}}{12\text{ in}}\right)$$
$$= 744.41\text{ ft-kips}$$

From the envelope curve of maximum moment, the maximum factored moment due to the externally applied load at quarter point is

$$M_{max} = M_{L+I}$$
$$M_{L+I} = (1.3)\big((1.67)(L+I)\big)$$
$$= (1.3)\big((1.67)(1012.73\text{ ft-kips})\big)$$

[at the quarter point]

$$= 2198.6\text{ ft-kips}$$

$V_c = V_{ci}$

$b' = 8$ in

$d = 40.57$ in

$f'_c = f'_{cg} = 6500$ lbf/in^2

AASHTO [9.20.2.1]

$$V_{ci} = 0.6\sqrt{f'_c}b'd + V_d + \frac{V_i M_{cr}}{M_{max}}$$

$$= 15.70 \text{ kips} + 41.18 \text{ kips} + \frac{(116.56 \text{ kips})(744.41 \text{ ft-kips})}{2198.6 \text{ ft-kips}}$$

$$= 96.34 \text{ kips}$$

Thus, $V_c = V_{ci} = 96.34$ kips.

[9.20.2.2]

$$V_{c,\text{min}} = 1.7\sqrt{f'_c}b'd$$

$$= \left(1.7\sqrt{6500 \frac{\text{lbf}}{\text{in}^2}}\right)\left(\frac{1}{1000 \frac{\text{lbf}}{\text{kip}}}\right) \times (8 \text{ in})(40.57 \text{ in})$$

$$= 44.48 \text{ kips}$$

[9.20.2.2] $d \geq 0.8h = (0.8)(54 \text{ in}) = 43.2$ in

(Note that actual $d = 40.37$ in. d need not be less than $0.8h$. This is close, so it is OK.)

V_{cw} (nominal shear strength of concrete when diagonal cracking results from excessive principal tensile stress in web) is not normally reviewed.

[Eq. 9-26] $V_U \leq \phi(V_c + V_s)$ [V_s = shear strength provided by web reinforcement]

[9.20.1.3]

$$V_U = (1.3)\big(1D + (1.67)(L + I)\big)$$

$$= (1.3)\big(41.18 \text{ kips} + (1.67)(53.69 \text{ kips})\big)$$

$$= 170.09 \text{ kips}$$

Use the envelope curve of maximum shears to determine the maximum factored shear force, V_u.

[9.14] For load factor design, $\phi = 0.90$ for shear.

$$\frac{V_u}{\phi} = \frac{170.00 \text{ kips}}{0.90} = 188.99 \text{ kips}$$

$$V_s = \frac{V_u}{\phi} - V_c = 188.99 \text{ kips} - 96.34 \text{ kips}$$

$$= 92.65 \text{ kips}$$

AASHTO [9.20.3.1] Check that $V_s \leq 8\sqrt{f'_{cg}}b'd$

$$8\sqrt{f'_c}b'd = 8\sqrt{6500 \frac{\text{lbf}}{\text{in}^2}}\left(\frac{1}{1000 \frac{\text{lbf}}{\text{kip}}}\right) \times (8 \text{ in})(40.57 \text{ in})$$

$$= 209.33 \text{ kips}$$

$V_s = 92.65$ kips < 209.33 kips [OK]

For Grade 60 No. 4 vertical stirrups, the spacing required is given by

[Eq. 9-30]

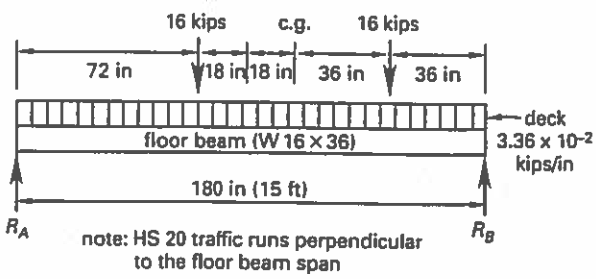
$$s = \frac{A_v f_{sy} d}{V_s} = \frac{(0.40 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)(40.57 \text{ in})}{60.66 \text{ kips}}$$

$$= 15.97 \text{ in}$$

Design Example 5: Load Rating of Floor Beam

A floor beam supported by hangers at both ends with a span of 180 in (15 ft) is shown. the floor beam is a W 16×36 section ($S_{xx} = 56.5$ in^3), and supports the deck dead load of 33.6 lbf/in and an HS 20 live load. The type of steel used is unknown. The date built is also unknown, but is assumed to be after 1963 based on field inspection of the floor beam. Section 6 of the AASHTO *Manual for Condition Evaluation of Bridges* is to be used for the floor beam load rating calculations by the Allowable Stress rating method.

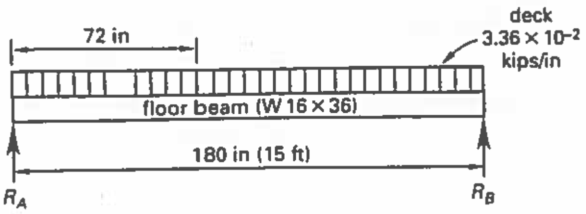
Determine the load rating of the floor beam in tons for an HS 20 loading.



Maximum live load moment occurs at 72 in.

$$R_A = \frac{16 \text{ kips}(36 \text{ in} + 108 \text{ in})}{180 \text{ in}} = 12.8 \text{ kips}$$

$$M_{LL} = 12.8 \text{ kips}(72 \text{ in}) = 921.6 \text{ kip-in}$$



Dead load moment at 72 in,

$$\text{total dead loads} = 3.36 \times 10^{-2}\ \frac{\text{kips}}{\text{in}} + 3.6 \times 10^{-2}\ \frac{\text{kips}}{\text{in}} = 6.96 \times 10^{-2}\ \text{kips/in}$$

$$R_A = \frac{\left(6.96 \times 10^{-2}\ \frac{\text{kips}}{\text{in}}\right)(180\ \text{in})}{2} = 6.26\ \text{kips}$$

$$M_{DL} = (6.26\ \text{kips})(72\ \text{in}) - \left(6.96 \times 10^{-2}\ \frac{\text{kips}}{\text{in}}\right)(72\ \text{in})\left(\frac{72\ \text{in}}{2}\right) = 270.3\ \text{kips-in}$$

From the Manual table [6.6.2.1-1 and 6.6.2.1-2], for the floor beam stéel (date built-steel unknown, after 1963), F_y is 36,000 lbf/in^2. The Inventory allowable stress for bending is $F_b = 20,000$ lbf/in^2; and Operating allowable stress for bending is $F_b = 27,000$ lbf/in^2. Therefore, the allowable bending moment for the Inventory rating is

$$M_{\text{all,inv}} = (F_b)(S_{xx}) = \left(20.0\ \frac{\text{kips}}{\text{in}^2}\right)(56.5\ \text{in}^3) = 1130.0\ \text{kips-in}$$

The allowable bending moment for the Operating rating is

$$M_{\text{all,opr}} = \left(27.0\ \frac{\text{kips}}{\text{in}^2}\right)(56.5\ \text{in}^3) = 1525.5\ \text{kips-in}$$

The load rating for the floor beam is determined by the Manual equations (6-1a) and (6-1b).

The Manual Eq. (6-1b)

RT = member load rating in tons = (RF)W

AASHTO [3.8.2]

The live load impact is

$$I = \frac{50}{L+125} = \frac{50}{15\ \text{ft} + 125} = 0.357$$

[30% maximum allowed]

The rating factor, RF, for the live-load carrying capacity is

The Manual Eq. (6-1a)

$$\text{RF} = \frac{C - A_1 D}{A_2 L(1+I)}$$

For the Inventory rating level,

$$\text{RF} = \frac{M_{\text{all,inv}} - (1)(M_{DL})}{(1)(M_{LL}(1+0.30)} = \frac{1130.0\ \text{kips-in} - (1)(270.3\ \text{kips-in})}{(1)(921.6\ \text{kips-in})(1.30)} = 0.72$$

Therefore, the Inventory load rating for HS 20 loading is

$$\text{RT} = (\text{RF})W = (0.72)(36\ \text{tons}) = 25.9\ \text{tons}$$

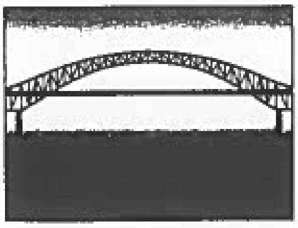
For the Operating rating level,

$$\text{RF} = \frac{M_{\text{all,opr}} - (1)(M_{DL})}{(1)(M_{LL})(1+0.30)} = \frac{1525.2\ \text{kips-in} - (1)(270.3\ \text{kips-in})}{(1)(921.6\ \text{kips})(1.30)} = 1.05\quad [\text{use } 1.0]$$

Therefore, the Operating load rating for HS 20 loading is

$$\text{RT} = (\text{RF})W$$

$$(1.0)(36\ \text{tons}) = 36\ \text{tons}$$



Practice Problems

Practice Problem 1: Prestressed Concrete Girder

An interior prestressed concrete girder (or beams) for a two-lane simply supported highway bridge with an 80 ft span is to be designed. The five girders are spaced at 7 ft 6 in. Design load is AASHTO HS 20-44. The following information applies.

- Superimposed dead load (parapet/curb/future wearing surface) is 427 lbf/ft.
- The slab has an integral wearing surface of 0.50 in.
- Group I load combination is to be used for load factor design.
- $f'_{cg} = 6500$ lbf/in^2 for prestressed I-beam.
- $f'_{ci} = 5500$ lbf/in^2 at the time of initial prestress.
- $f'_{cs} = 4500$ lbf/in^2 for 8 in slab (7.5 in plus 0.5 in integral wearing surface).
- $E_S = 28 \times 10^6$ lbf/in^2 for prestressing steel [AASHTO 9.16.2.1.2].
- Steel reinforcement consists of 44 @ ½ in diameter strands (seven-wire) or $A_s^* = (0.153 \text{ in}^2)(44) = 6.732 \text{ in}^2$.
- $f'_s = 270{,}000$ lbf/in^2 for low-relaxation prestress steel.
- The basic beam section properties are
  $A_B = 762 \text{ in}^2$
  $y_b = 23.38 \text{ in}$
  $I_B = 212{,}450 \text{ in}^4$
  $S_t = 7692 \text{ in}^3$
  $S_b = 9087 \text{ in}^3$

Using the 1996 AASHTO Bridge Specifications,

(1) Determine the composite section properties.
(2) Determine the factored design moment at midspan, M_u.
(3) Determine the girder moment capacity, ϕM_n.
(4) Determine the concrete stresses at midspan at release of prestress.
(5) Determine the final concrete stresses after all losses (except friction) at midspan.

Solution:

Calculate dead loads. The beam weight is

$$W = (762 \text{ in}^2)\left(\frac{1 \text{ ft}^2}{(12 \text{ in})(12 \text{ in})}\right)\left(0.15 \frac{\text{kip}}{\text{ft}^3}\right) = 0.79 \text{ kip/ft}$$

The slab dead load is

$$W_D = (8 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(0.15 \frac{\text{kip}}{\text{ft}^3}\right)(7.5 \text{ ft}) = 0.75 \text{ kip/ft}$$

The superimposed dead load, W_s, for the parapet/curb/future wearing surfaces is given as 0.427 kip/ft.

Calculate dead load moments at midspan. The beam weight moment is

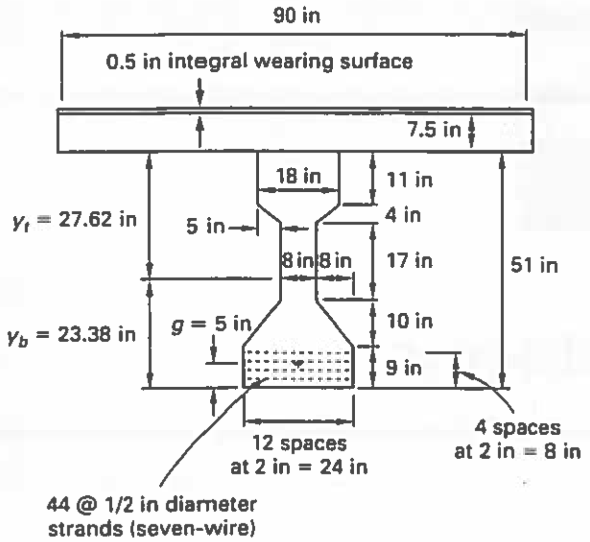
$$M_0 = \left(\frac{1}{8}\right) W L^2 = \left(\frac{1}{8}\right)\left(0.79 \frac{\text{kip}}{\text{ft}}\right)(80 \text{ ft})^2 = 632.0 \text{ ft-kips}$$

The slab dead load moment is

$$M_D = \left(\frac{1}{8}\right) W_D L^2 = \left(\frac{1}{8}\right)\left(0.75 \frac{\text{kip}}{\text{ft}}\right)(80 \text{ ft})^2 = 600.0 \text{ ft-kips}$$

The superimposed dead load moment is

$$M_s = \left(\frac{1}{8}\right) W_s L^2 = \left(\frac{1}{8}\right)\left(0.427 \frac{\text{kip}}{\text{ft}}\right)(80 \text{ ft})^2 = 341.6 \text{ ft-kips}$$



The maximum dead load shear is

$$V_D = \left(0.79\ \frac{\text{kip}}{\text{ft}} + 0.75\ \frac{\text{kip}}{\text{ft}} + 0.427\ \frac{\text{kip}}{\text{ft}}\right)(40\ \text{ft})$$
$$= 78.68 \text{ kips at support bearing}$$

Step 1: Determine the composite section properties.

AASHTO [9.8.1.1; 8.10.1; 9.8.3.2]

The effective width of the slab flange is the lesser of

$$\left(\frac{1}{4}\right)(\text{span}) = \left(\frac{1}{4}\right)(80\ \text{ft})\left(12\ \frac{\text{in}}{\text{ft}}\right)$$
$$= 240\ \text{in}$$

or

$$(12)(\text{slab thickness}) + \text{beam flange width}$$
$$= (12)(8\ \text{in}) + 18\ \text{in} = 114\ \text{in}$$

or

girder (beam) spacing = 90 in [governs]

$$E_C = w_c^{1.5} 33\sqrt{f'_c}$$

For normal concrete, $w_c = 150\ \text{lbf/ft}^3$.

$$E_{cg} = \left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5} 33\sqrt{6500\ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 4.89 \times 10^6\ \text{lbf/in}^2 \text{ for I-beam}$$

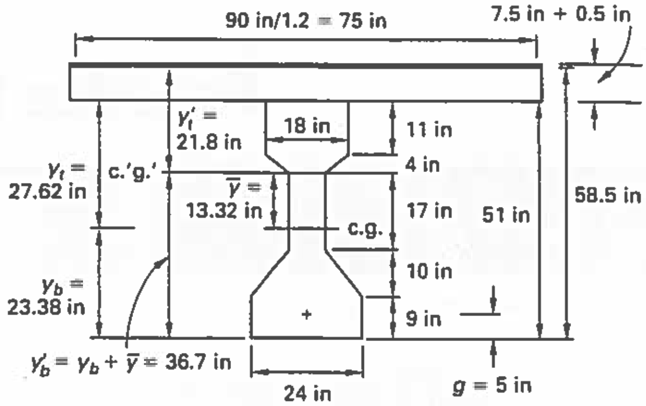
$$E_{cs} = \left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)^{1.5} 33\sqrt{4500\ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 4.07 \times 10^6\ \text{lbf/in}^2 \text{ for slab}$$

The modular ratio is

$$n = \frac{E_{cg}}{E_{cs}} = \frac{4.89}{4.07} = 1.2$$

The transformed area of the slab is

$$\frac{(7.5\ \text{in})(90\ \text{in})}{1.2} = 562.5\ \text{in}^2$$



The area of the composite section is

$$A_C = A_B + \text{transformed slab area}$$
$$= 762\ \text{in}^2 + 562.5\ \text{in}^2$$
$$= 1324.5\ \text{in}^2$$

From the centroid of the basic beam section,

$$(1324.5\ \text{in}^2)\bar{y} = (562.5\ \text{in}^2)\left(y_t + \left(\frac{7.5\ \text{in}}{2}\right)\right)$$
$$= (562.5\ \text{in}^2)(27.62\ \text{in} + 3.75\ \text{in})$$
$$= 17{,}646\ \text{in}^3$$

$\bar{y} = 13.32$ in from the centroid of the basic beam section

$$y'_b = y_b + \bar{y} = 23.38\ \text{in} + 13.32\ \text{in} = 36.7\ \text{in}$$
$$y'_t = (51\ \text{in} + 7.5\ \text{in}) - y'_b = 21.8\ \text{in}$$

The composite moment of inertia is

$$I_C = I_B + A_B\bar{y}^2 + \frac{(75\ \text{in})(7.5\ \text{in})^3}{12}$$
$$+ (562.5\ \text{in}^2)\left(y'_t - \frac{7.5\ \text{in}}{2}\right)^2$$
$$= 212{,}450\ \text{in}^4 + (762\ \text{in}^2)(13.32\ \text{in})^2$$
$$+ 2636.7\ \text{in}^4$$
$$+ (562.5\ \text{in}^2)(21.8\ \text{in} - 3.75\ \text{in})^2$$
$$= 533{,}546.5\ \text{in}^4$$

The composite section moduli are as follows.

For the bottom extreme fiber,

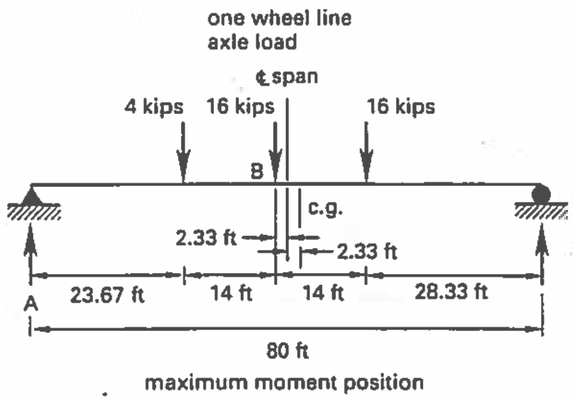
$$S_{bc} = \frac{I_C}{y'_b} = \frac{533{,}546.5 \text{ in}^4}{36.7 \text{ in}} = 14{,}538.0 \text{ in}^3$$

For the top extreme fiber (slab top),

$$S_{tc} = \frac{I_C}{y'_t} = \frac{533{,}546.5 \text{ in}^4}{21.8 \text{ in}} = 24{,}474.6 \text{ in}^3$$

Step 2: Determine the factored maximum moment and maximum shear.

Use AASHTO HS 20-44 live load.



The reaction at A is

$$\frac{(16 \text{ kips})(28.33 \text{ ft} + 42.33 \text{ ft})}{80 \text{ ft}} + \frac{(4 \text{ kips})(56.33 \text{ ft})}{80 \text{ ft}}$$
$$= 16.95 \text{ kips}$$

The moment at B is

$$M_L = (16.95 \text{ kips})(23.67 \text{ ft} + 14 \text{ ft}) - (4 \text{ kips})(14 \text{ ft})$$
$$= 582.5 \text{ ft-kips for one wheel line}$$

The transverse distribution factor of wheel live loads for an interior beam is

AASHTO [Table 3.23.1]

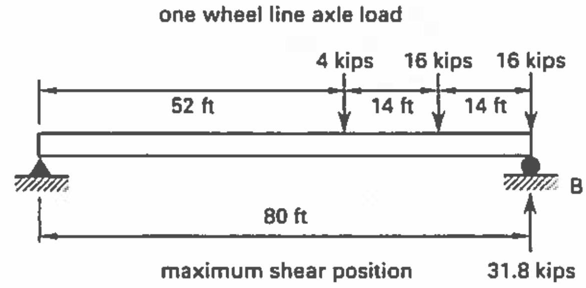
$$\frac{S}{5.5} = \frac{7.5 \text{ ft}}{5.5} = 1.36$$

[3.8.2] The impact factor is

$$I = \frac{50}{L + 125} = \frac{50}{80 \text{ ft} + 125} = 0.244$$

The maximum live load moment with impact for each girder (beam) is

$$M_{L+I} = (582.5 \text{ ft-kips})(1.36)(1.244)$$
$$= 985.5 \text{ ft-kips}$$



AASHTO [Appendix A] Maximum live load moment and shear for HS 20-44 given in AASHTO App. A are 1164.9 ft-kips and 63.6 kips, respectively.

Thus, the maximum moment per wheel line is (1164.9 ft-kips)/2 = 582.5 ft-kips (which agrees with the calculated value M_L above).

The maximum live load shear per wheel line with impact for each girder (beam) is

$$V_{L+I} = \left(\frac{63.6 \text{ kips}}{2}\right)(1.36)(1.244)$$
$$= 53.8 \text{ kips}$$

AASHTO [Table 3.22.1A] For load factor design values,

$$\text{Group I} = \gamma(\beta_D D + \beta_L(L + I))$$
$$= (1.30)(1.0D + (1.67)(L + I))$$

The factored design moment is

$$M_u = (1.3)(632.0 \text{ ft-kips} + 600.0 \text{ ft-kips} + 341.6 \text{ ft-kips}) + (1.67)(985.5 \text{ ft-kips}))$$
$$= 4191.7 \text{ ft-kips}$$

The factored shear at support is

$$V_u = (1.3)(78.68 \text{ kips} + (1.67)(53.8 \text{ kips}))$$
$$= 219.1 \text{ kips}$$

Step 3: Determine the girder moment capacity, ϕM_n.

Determine whether the beam is a flanged or rectangular section.

AASHTO [9.17.2]

$$a = \frac{A_S^* f_{su}^*}{0.85 f'_{c,\text{slab}} b} \quad [f'_{c,\text{slab}} = 4500 \text{ lbf/in}]$$

Using $g = 5$ in,

$$d = 51 \text{ in} + 7.5 \text{ in} - 5 \text{ in} = 53.5 \text{ in}$$

$$\rho^* = \frac{A_s^*}{bd} = \frac{6.732 \text{ in}^2}{(90 \text{ in})(53.5 \text{ in})} = 0.0014$$

f^*_{su} = stress in prestressing steel at ultimate load

[9.17.4.1] $$f^*_{su} = f'_s\left(1 - \left(\frac{\gamma^*}{\beta_1}\right)\left(\frac{\rho^* f'_s}{f'_c}\right)\right)$$

[9.1.2] $\gamma^* = 0.28$ for low-relaxation prestressing steel

$$\beta_1 = 0.85 - (0.05)\left(\frac{4500 - 4000}{1000}\right)$$

$$= 0.825 \quad [\text{for } f'_{c,\text{slab}} = 4.5 \text{ kips/in}^2]$$

$$f'_s = 270 \text{ kips/in}^2$$

$$f^*_{su} = \left(270\ \frac{\text{kips}}{\text{in}^2}\right)\left(1 - \left(\frac{0.28}{0.825}\right)(0.0014)\right.$$

$$\left.\times\left(\frac{270\ \frac{\text{kips}}{\text{in}^2}}{4.5\ \frac{\text{kips}}{\text{in}^2}}\right)\right)$$

$$\doteq 262.3 \text{ kips/in}^2$$

$$a = \frac{(6.732 \text{ in}^2)\left(262.3\ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(4.5\ \frac{\text{kips}}{\text{in}^2}\right)(90 \text{ in})} = 5.13 \text{ in}$$

Since 5.13 in is less than the 7.5 in, the section is considered rectangular.

[9.18.1] Check the maximum prestressing steel.

The maximum prestressing steel index is

$$\frac{\rho^* f^*_{su}}{f'_c} = \frac{(0.0014)\left(262.3\ \frac{\text{kips}}{\text{in}^2}\right)}{4.5\ \frac{\text{kips}}{\text{in}^2}}$$

$$= 0.0816 \left[< 0.36\beta_1 \ (= (0.36)(0.825) = 0.297), \text{ so OK}\right]$$

AASHTO [9.14] The girder moment capacity is

$\phi = 1.0$ for factory produced member

[Eq. 9-13] $$\phi M_n = \phi\left(A^*_s f^*_{su} d\left(1 - (0.6)\left(\frac{\rho^* f^*_{su}}{f'_c}\right)\right)\right)$$

$$= (1.0)\left((6.732 \text{ in}^2)\left(262.3\ \frac{\text{kips}}{\text{in}^2}\right)\right.$$

$$\times (53.5 \text{ in})(1 - 0.6)(0.0014)$$

$$\left.\times\left(\frac{262.3\ \frac{\text{kips}}{\text{in}^2}}{4.5\ \frac{\text{kips}}{\text{in}^2}}\right)\right)$$

$$= 89{,}844.9 \text{ in-kips or } 7487.1 \text{ ft-kips}$$

Step 4: Determine concrete stresses at midspan at release of prestress.

[9.15.2.1] Temporary allowable concrete stresses before losses due to creep and shrinkage are as follows (i.e., at time of initial prestress).

For compression,

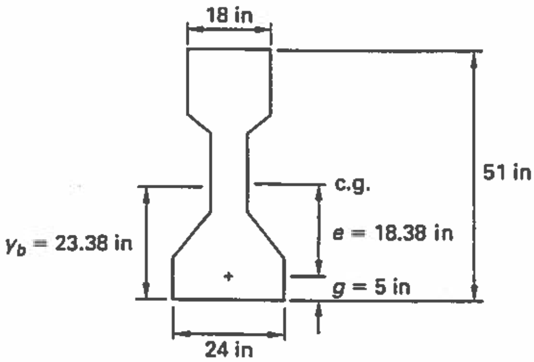
$$f_{ci} = 0.60 f'_{ci} = (0.6)\left(5.5\ \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 3.3 \text{ kips/in}^2$$

For tension,

$$f_{ti} = 0.556 \text{ kip/in}^2$$

(AASHTO does not specify, although f_{ti} can be $7.5\sqrt{f_{ci}} = 0.556$ kip/in^2.



[9.16.2.1.2] The reduced tendon stress immediately after transfer due to elastic shortening and tendon relaxation can be taken as $0.69 f'_s$ for low-relaxation prestressing steel.

[9.16.2.1] Thus, the reduced prestress force after transfer is

$$P = 0.69 f_s' A_s^*$$
$$= (0.69)\left(270\ \frac{\text{kips}}{\text{in}^2}\right)(6.732\ \text{in}^2)$$
$$= 1254.2\ \text{kips}$$

Concrete stress at the top fiber is

$$f_t = -\frac{P}{A_B} + \frac{Pe}{S_t} - \frac{M_0}{S_t}$$
$$= -\frac{1254.2\ \text{kips}}{762\ \text{in}^2} + \frac{(1254.2\ \text{kips})(18.38\ \text{in})}{7692\ \text{in}^3} - \frac{(632\ \text{ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{7692\ \text{in}^3}$$
$$= +0.36\ \text{kip/in (tension)}$$

$$\left[\begin{array}{c} > f_{ti}\ (= 0.556\ \text{kips/in}^2), \\ \text{so OK} \end{array}\right]$$

Concrete stress at the bottom fiber is

$$f_b = -\frac{P}{A_B} - \frac{Pe}{S_b} + \frac{M_0}{S_b}$$
$$= -\frac{1254.2\ \text{kips}}{762\ \text{in}^2} - \frac{(1254.2)(18.38\ \text{in})}{9087\ \text{in}^3} + \frac{(632\ \text{ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{9087\ \text{in}^3}$$
$$= -3.35\ \text{kips/in}^2\ \text{(compression)}$$

$$\left[\begin{array}{c} < f_{ci}\ (= 3.3\ \text{kips/in}^2); \\ \text{close, so OK} \end{array}\right]$$

Step 5: Determine final concrete stresses at midspan after all losses (except friction).

AASHTO [9.16.2.2; Table 9.16.2.2] Estimate losses. Prestress losses are estimated at 45,000 lbf/in² for $f_c' = 5000$ lbf/in². Thus, the effective steel prestress after losses is

$$f_{se} = f_{si} - 45{,}000\ \frac{\text{lbf}}{\text{in}^2}$$

[9.15.1] $f_{si} = 0.75 f_s'$ [low relaxation strands]

$$= (0.75)\left(270\ \frac{\text{kips}}{\text{in}^2}\right) = 202.5\ \text{kips/in}^2$$

$$f_{se} = 202{,}500\ \frac{\text{lbf}}{\text{in}^2} - 45{,}000\ \frac{\text{lbf}}{\text{in}^2}$$
$$= 157{,}500\ \text{lbf/in}^2$$

The effective prestress after loss is

$$P_e = f_{se} A_s^* = \left(157{,}500\ \frac{\text{lbf}}{\text{in}^2}\right)(6.732\ \text{in}^2)$$
$$= 1{,}060{,}290\ \text{lbf}$$

For the basic beam section at the beam's bottom fiber,

$$-\frac{P_e}{A_B} - \frac{P_e e}{S_b} = -\frac{1{,}060{,}290\ \text{lbf}}{762\ \text{in}^2} - \frac{(1{,}060{,}290\ \text{lbf})(18.38\ \text{in})}{9087\ \text{in}^3}$$
$$= -3536\ \text{lbf/in}^2$$

["–" indicates compression]

$$+\frac{M_0 + M_D}{S_b} = \left(\frac{632.0\ \text{ft-kips} + 600.0\ \text{ft-kips}}{9087\ \text{in}^3}\right) \times \left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)$$
$$= +1627\ \text{lbf/in}^2$$

Adding compressive and tensile prestresses (−3536 lbf/in² + 1627 lbf/in²) gives the net compressive stress of −1909 lbf/in².

At the basic beam's top fiber,

$$-\frac{P_e}{A_B} + \frac{P_e e}{S_t} = -\frac{1{,}060{,}290\ \text{lbf}}{762\ \text{in}^2} + \frac{(1{,}060{,}290\ \text{lbf})(18.38\ \text{in})}{7692\ \text{in}^3}$$
$$= +1142\ \text{lbf/in}^2$$

$$-\frac{M_0 + M_D}{S_t} = -\left(\frac{(632\ \text{ft-kips}) + (600\ \text{ft-kips})}{7692\ \text{in}^3}\right) \times \left(\left(12\ \frac{\text{in}}{\text{ft}}\right)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)\right)$$
$$= -1922\ \text{lbf/in}^2$$

Adding compressive and tensile prestresses (1142 lbf/in² − 1922 lbf/in²) gives the net compressive stress of −780 lbf/in².

For the composite section at the beam base,

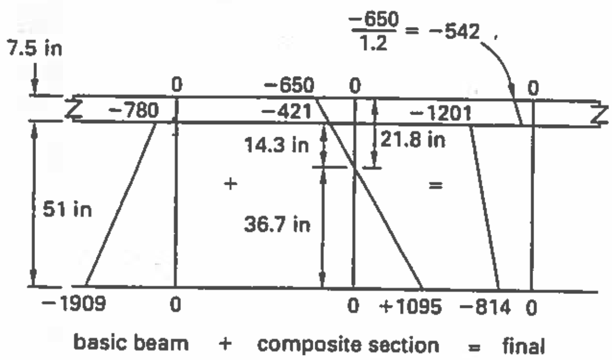
$$+\frac{M_s + M_{L+I}}{S_{bc}} = \left(\frac{341.6 \text{ ft-kips} + 985.5 \text{ ft-kips}}{14{,}538.0 \text{ in}^3}\right) \times \left(\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)\right)$$

$$= +1095 \text{ lbf/in}^2$$

At the slab top,

$$-\frac{M_s + M_{L+I}}{S_{tc}} = \left(-\frac{341.6 \text{ ft-kips} + 985.5 \text{ ft-kips}}{24{,}474.6 \text{ in}^3}\right) \times \left(\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)\right)$$

$$= -650 \text{ lbf/in}^2$$



(note: units for stresses are in lbf/in²)

Final stresses after losses

AASHTO [9.15.2.2]

Check that the allowable concrete stresses at service load after losses have occurred.

For compression,

$$f_{cs} = 0.40 f'_{cg} = (0.4)\left(6500 \ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= -2600 \text{ lbf/in}^2 \quad \left[> -1201 \text{ lbf/in}^2, \text{ so OK}\right]$$

For tension,

$$f_{ts} = 6\sqrt{f'_{cg}} = 6\sqrt{6500 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 484 \text{ lbf/in}^2 \quad [> -814 \text{ lbf/in}^2, \text{ so OK}]$$

["–" indicates compression]

Practice Problem 2: Center Pier

The center pier of a two-span continuous bridge is shown below. The pier is supported by twelve HP piles with an allowable bearing capacity of 120 kips each. Each span of the bridge is 100 ft long. The following data apply.

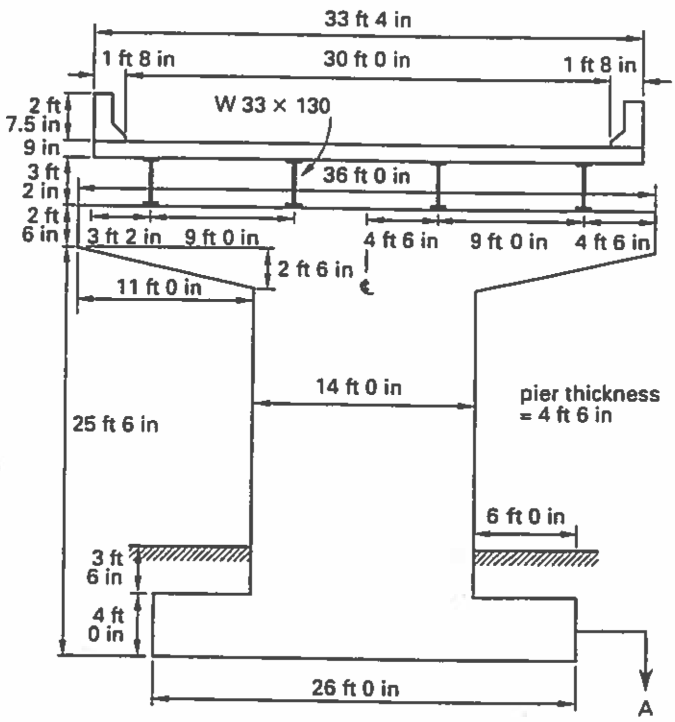
- AASHTO HS 20-44 loading and group loading combinations I and II
- There is no skew.
- The unit weight of the soil is 120 lbf/ft³.
- Parapet/curb weights are (506 lbf/ft)(2 sides) = 1012 lbf/ft

(1) Determine the dead loads at the base of the footing.

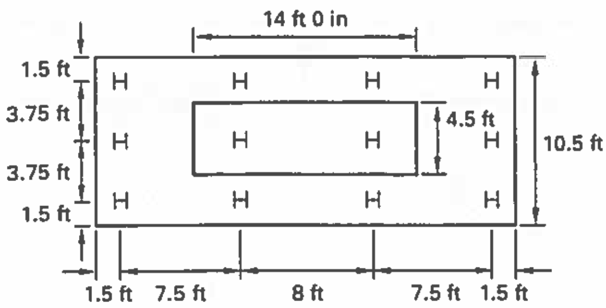
(2) Determine the live loadings.

(3) Determine the maximum pile forces per pile for Group I and Group II combinations.

Step 1: Determine the dead loads at the base of the footing.



pier elevation



Sec. A-A pile layout plan

Solution:
Calculate dead loads for the superstructure for the interior stringer.

For the slab,

$$\left((9\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\right)(9\text{ ft})(100\text{ ft})\left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)$$
$$= 101{,}250\text{ lbf}$$

For the W 33 × 130 stringer,

$$\left(130\ \frac{\text{lbf}}{\text{ft}}\right)(100\text{ ft}) = 13{,}000\text{ lbf}$$

For the parapets/curbs,

AASHTO [3.23.2.3]

$$\left(506\ \frac{\text{lbf}}{\text{ft}}\right)(2)(100\text{ ft})\left(\frac{1}{4}\text{ stringers}\right)$$
$$= 25{,}300\text{ lbf}$$

The total dead load for the interior stringer is

$$101{,}250\text{ lbf} + 13{,}000\text{ lbf} + 25{,}300\text{ lbf} = 139{,}550\text{ lbf}$$

AASHTO [3.23.2.3] Calculate the dead load for the exterior stringer by the same method.

For the slab,

$$\left((9\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)\right)(4.5\text{ ft} + 3.167\text{ ft})(100\text{ ft})\left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)$$
$$= 86{,}254\text{ lbf}$$

Dead loads for the stringer and parapets/curbs are the same as for the interior stringer.

$$\text{parapets/curbs} = 25{,}300\text{ lbf}$$
$$\text{stringer} = 13{,}000\text{ lbf}$$

The total dead load for the exterior stringer is

$$86{,}254\text{ lbf} + 25{,}300\text{ lbf} + 13{,}000\text{ lbf} = 124{,}554\text{ lbf}$$

Add the dead loads for the interior and exterior stringers to obtain the total dead load for the superstructure.

$$(139{,}550\text{ lbf})(2\text{ interior stringers}) + (124{,}554\text{ lbf})(2\text{ exterior stringers})$$
$$= 528{,}208\text{ lbf} = 528.2\text{ kips}$$

Calculate dead loads for the pier, footing, and soil above the footing.

For the pier,

$$\Big((36\text{ ft})(2.5\text{ ft}) + (11\text{ ft})(2.5\text{ ft}) + (21.5\text{ ft})(14\text{ ft})\Big) \times (4.5\text{ ft})\left(0.15\ \frac{\text{kip}}{\text{ft}^3}\right)$$
$$= 282.5\text{ kips}$$

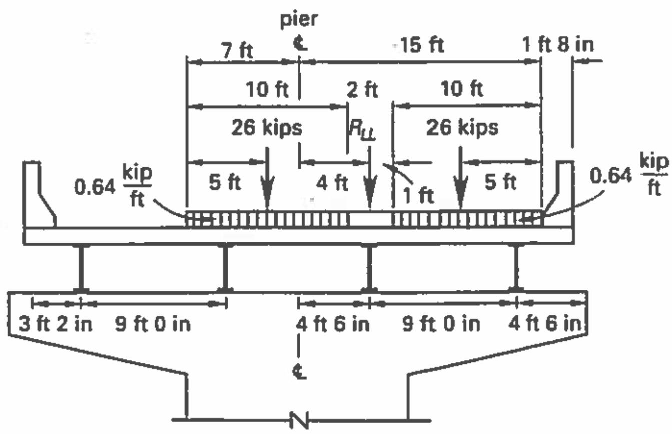
For the footing,

$$(26\text{ ft})(4\text{ ft})(10.5\text{ ft})\left(0.15\ \frac{\text{kip}}{\text{ft}^3}\right) = 163.8\text{ kips}$$

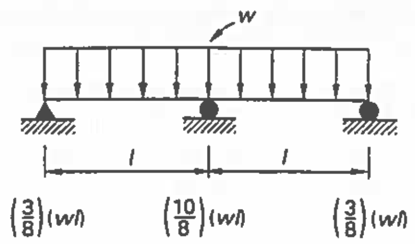
For the soil above the footing,

$$\big((26\text{ ft})(10.5\text{ ft}) - (14\text{ ft})(4.5\text{ ft})\big)(3.5\text{ ft})\left(0.12\ \frac{\text{kip}}{\text{ft}^3}\right)$$
$$= 88.2\text{ kips}$$

Step 2: Determine live loads using AASHTO HS 20-44 lane loading



Recall that the reaction of the interior support for a two-span continuous beam with two equal spans with uniform loads w on both spans is $(10/8)(wl)$.



AASHTO [Figure 3.7.6.B; 3.7.1.2; 3.8.1.2]

The resultant R_{LL} on the center pier is

$$\left(\left(0.64\ \frac{\text{kip}}{\text{ft}}\right)(100\ \text{ft})\left(\frac{10}{8}\right) + 26\ \text{kips}\right) \times (2\ \text{lanes}) = 212\ \text{kips}$$

Impact is not included for piles below groundlines.

Step 3: Determine maximum pile forces for Group I load combination.

$$P_D = \gamma(D + (L + I) + \beta_E E)$$

Factored values for Group I loading are

$$\gamma = 1.0$$
$$I = 0.0$$
$$\beta_E = 1.0$$

$$\begin{aligned} P_D = \gamma(D + 1.0E) &= (1.0)((528.2\ \text{kips} \\ &\quad + 282.5\ \text{kips} + 163.8\ \text{kips}) \\ &\quad + (1.0)(88.2\ \text{kips})) \\ &= 1062.7\ \text{kips} \end{aligned}$$

Thus, $P_D = 1062.7$ kips.

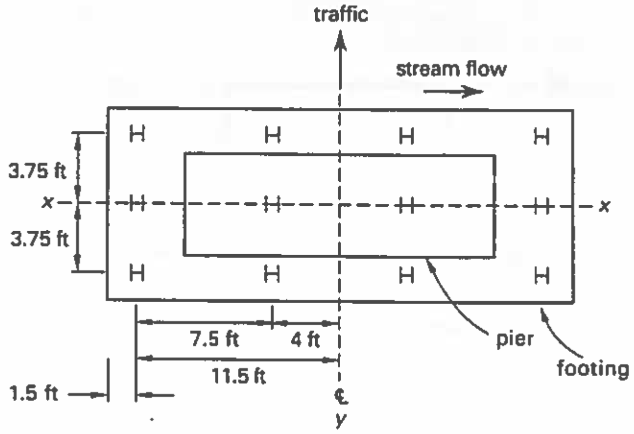
$\gamma L = 1.0L = R_{LL} = 212$ kips

The pile forces are

$$e_x = 4.0\ \text{ft}$$
$$M_{yy} = R_{LL}(e_x) = (212\ \text{kips})(4.0\ \text{ft}) = 848.0\ \text{ft-kips}$$
$$S_{yy} = \left(\frac{(3)(11.5\ \text{ft})^2 + (3)(4\ \text{ft})^2}{11.5\ \text{ft}}\right)(2) = 77.35\ \text{pile-ft}$$
$$S_{xx} = \left(\frac{(4)(3.75\ \text{ft})^2}{3.75\ \text{ft}}\right)(2) = 30\ \text{pile-ft}$$

The maximum force per pile is

$$\begin{aligned} P_D + R_{LL} + \frac{M_{yy}}{S_{yy}} &= \frac{1062.7\ \text{kips}}{12\ \text{piles}} + \frac{212\ \text{kips}}{12\ \text{piles}} + \frac{848.0\ \text{ft-kips}}{77.35\ \text{pile-ft}} \\ &= 88.6\ \text{kips} + 17.6\ \text{kips} + 11.0\ \text{kips} \\ &= 117.2\ \text{kips} \quad [< 120\ \text{kips, so OK}] \end{aligned}$$



The minimum force per pile is

$$\begin{aligned} \frac{P}{A} + \frac{R_{LL}}{A} - \frac{M_{yy}}{S_{yy}} &= 88.6\ \text{kips} + 17.6\ \text{kips} - 11.0\ \text{kips} \\ &= 95.2\ \text{kips} \quad [< 120\ \text{kips, so OK}] \end{aligned}$$

Step 4: Determine the maximum forces per pile for Group II load combination.

Factored values for Group II loading are

$$P_D = \gamma(\beta_D D + \beta_E E + \beta_W W)$$

Using $\beta = 1$ in all cases,

$$P_D = \gamma(D + E) = 1062.7\ \text{kips} \quad \left[= P_D \text{ in Group I loading case}\right]$$

AASHTO [3.15.2.1.3]

W = wind load on structure

Step 4a: Determine the wind load on the superstructure, W_{super}, transmitted to the substructure with a transverse wind load of 50 lbf/ft^2 and a longitudinal wind load of 12 lbf/ft^2.

The projected height of the superstructure above the pier top is

$$(38\ \text{in} + 9\ \text{in} + 31.5\ \text{in})\left(\frac{1\ \text{ft}}{12\ \text{in}}\right) = 6.54\ \text{ft from the pier top}$$

The centroid of the superstructure area above the pier top is (6.54 ft/2) + 0.12 ft (bearing assumed) = 3.39 ft.

The longitudinal wind loading on the superstructure is

$$\text{(beam length)(superstructure height)} \times \text{(longitudinal wind load)} = (101.0\ \text{ft})(6.54\ \text{ft})\left(0.012\ \frac{\text{kip}}{\text{ft}^2}\right) = 7.93\ \text{kips}$$

The transverse wind loading on the superstructure is

$$(101.1\ \text{ft})(6.54\ \text{ft})\left(0.05\ \frac{\text{kip}}{\text{ft}^2}\right) = 33.0\ \text{kips}$$

Step 4b: [3.15.2.2] Next, determine the wind load, W_{sub}, applied directly on the substructure above the groundline. Both longitudinal and transverse wind loads are 40 lbf/ft^2.

The exposed area in a longitudinal direction (into the front face) is

$$(36\ \text{ft})(2.5\ \text{ft}) + (11\ \text{ft})(2.5\ \text{ft}) + (18\ \text{ft})(14\ \text{ft}) = 369.5\ \text{ft}^2$$

The centroid of the exposed area is

$$\frac{(36\ \text{ft})(2.5\ \text{ft})(1.25\ \text{ft}) + (2.5\ \text{ft})(11\ \text{ft})(3.33\ \text{ft}) + (18\ \text{ft})(14\ \text{ft})(11.5\ \text{ft})}{369.5\ \text{ft}^2}$$

$= 8.4$ ft from the top of the pier

The longitudinal wind load is

$$\left(0.04\ \frac{\text{kip}}{\text{ft}^2}\right)(369.5\ \text{ft}^2) = 14.78\ \text{kips}$$

The exposed area in a transverse direction (into the pier side face) is

$$(4.5\ \text{ft})(20.5\ \text{ft}) = 92.25\ \text{ft}^2$$

The centroid of the exposed area is

$$\frac{20.5\ \text{ft}}{2} = 10.25\ \text{ft from the pier top}$$

The transverse wind load is

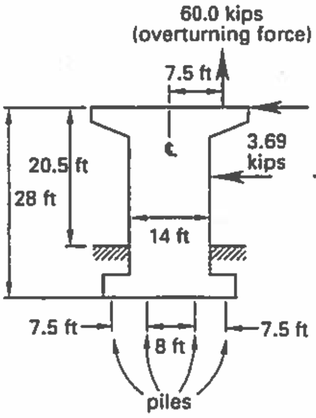
$$\left(0.04\ \frac{\text{kip}}{\text{ft}^2}\right)(92.25\ \text{ft}^2) = 3.69\ \text{kips}$$

Step 4c: [3.15.3] The wind overturning force acting upward at the quarter point of the transverse superstructure width (20 lbf/ft^2 of deck and sidewalk plan area) is

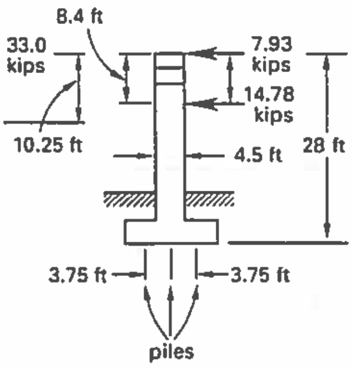
$$\text{(span length)(slab width)}\left(0.02\ \frac{\text{lbf}}{\text{ft}^2}\right) = (100\ \text{ft})(30\ \text{ft})\left(0.02\ \frac{\text{lbf}}{\text{ft}^2}\right) = 60.0\ \text{kips}$$

The quarter point is

$$\frac{30.0\ \text{ft}}{4} = 7.5\ \text{ft from the centerline of the roadway (or center of the pier)}$$



transverse wind loading



longitudinal wind loading

Step 4d: Determine the forces per pile.

In the transverse direction,

$$\begin{aligned} M_{yy} &= (60.0\ \text{kips})(7.5\ \text{ft}) + (33.0\ \text{kips})(28\ \text{ft}) \\ &\quad + (3.69\ \text{kips})(28\ \text{ft} - 10.25\ \text{ft}) \\ &= 1439.5\ \text{ft-kips} \end{aligned}$$

The maximum force per pile is

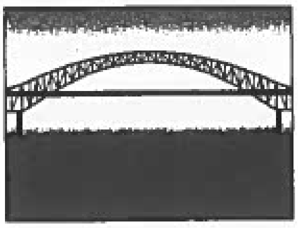
$$\begin{aligned} \frac{P_D - 60.0\ \text{kips}}{A} + \frac{M_{yy}}{S_{yy}} &= \frac{1062.7\ \text{kips} - 60.0\ \text{kips}}{12\ \text{piles}} + \frac{1439.5\ \text{ft-kips}}{77.35\ \text{pile-ft}} \\ &= 83.5\ \text{kips} + 18.6\ \text{kips} \\ &= 102.1\ \text{kips} \quad [< 120\ \text{kips, so OK}] \end{aligned}$$

In the longitudinal direction,

$$M_{xx} = (7.93 \text{ kips})(28 \text{ ft}) + (14.78 \text{ kips})(28 \text{ ft} - 8.4 \text{ ft}) = 511.8 \text{ ft-kips}$$

The maximum force per pile is

$$\frac{P_D}{A} + \frac{M_{xx}}{S_{xx}} = \frac{1062.7 \text{ kips}}{12 \text{ piles}} + \frac{511.8 \text{ ft-kips}}{30 \text{ pile-ft}} = 88.56 \text{ kips} + 17.06 \text{ kips} = 105.62 \text{ kips} \quad [< 120 \text{ kips, so OK}]$$



Appendices

Appendix A
AASHTO Live Load Tables

Loading—H 15-44

Table of Maximum Moments, Shears, and Reactions
(Simple Spans, One Lane)

Spans in feet; moments in thousands of foot-pounds; shears and reactions in thousands of pounds. These values are subject to specification reduction for loading of multiple lanes. Impact not included.

span	moment	end shear and end reaction[a]	span	moment	end shear and end reaction[a]
1	6.0[b]	24.0[b]	42	274.4[b]	29.6
2	12.0[b]	24.0[b]	44	289.3[b]	30.1
3	18.0[b]	24.0[b]	46	304.4[b]	30.5
4	24.0[b]	24.0[b]	48	319.2[b]	31.0
5	30.0[b]	24.0[b]	50	334.2[b]	31.5
6	36.0[b]	24.0[b]	52	349.1[b]	32.0
7	42.0[b]	24.0[b]	54	364.1[b]	32.5
8	48.0[b]	24.0[b]	56	379.1[b]	32.9
9	54.0[b]	24.0[b]	58	397.6	33.4
10	60.0[b]	24.0[b]	60	418.5	33.9
11	66.0[b]	24.0[b]	62	439.9	34.4
12	72.0[b]	24.0[b]	64	461.8	34.9
13	78.0[b]	24.0[b]	66	484.1	35.3
14	84.0[b]	24.0[b]	68	506.9	35.8
15	90.0[b]	24.0[b]	70	530.3	36.3
16	96.0[b]	24.8[b]	75	590.6	37.5
17	102.0[b]	25.1[b]	80	654.0	38.7
18	108.0[b]	25.3[b]	85	720.4	39.9
19	114.0[b]	25.6[b]	90	789.8	41.1
20	120.0[b]	25.8[b]	95	862.1	42.3
21	126.0[b]	26.0[b]	100	937.5	43.5
22	132.0[b]	26.2[b]	110	1097.3	45.9
23	138.0[b]	26.3[b]	120	1269.0	48.3
24	144.0[b]	26.5[b]	130	1452.8	50.7
25	150.0[b]	26.6[b]	140	1648.5	53.1

(continued)

[a] Concentrated load is considered placed at the support. Loads used are those stipulated for shear.
[b] Maximum value determined by Standard Truck Loading. Otherwise the Standard Lane Loading governs.

Appendix A *(continued)*
Loading—H 15-44

Table of Maximum Moments, Shears, and Reactions
(Simple Spans, One Lane)

span	moment	end shear and end reaction[a]	span	moment	end shear and end reaction[a]
26	156.0[b]	26.8[b]	150	1856.3	55.5
27	162.7[b]	26.9[b]	160	2076.0	57.9
28	170.1[b]	27.0[b]	170	2307.8	60.3
29	177.5[b]	27.1[b]	180	2551.5	62.7
30	185.0[b]	27.2[b]	190	2807.3	65.1
31	192.4[b]	27.3[b]	200	3075.0	67.5
32	199.8[b]	27.4[b]	220	3646.5	72.3
33	207.3[b]	27.5	240	4266.0	77.1
34	214.7[b]	27.7	260	4933.5	81.9
35	222.2[b]	27.9	280	5649.0	86.7
36	229.6[b]	28.1	300	6412.5	91.5
37	237.1[b]	28.4			
38	244.5[b]	28.6			
39	252.0[b]	28.9			
40	259.5[b]	29.1			

[a]Concentrated load is considered placed at the support. Loads used are those stipulated for shear.
[b]Maximum value determined by Standard Truck Loading. Otherwise the Standard Lane Loading governs.

Appendix A *(continued)*
Loading—HS 15-44

Table of Maximum Moments, Shears, and Reactions
(Simple Spans, One Lane)

Spans in feet; moments in thousands of foot-pounds; shears and reactions in thousands of pounds. These values are subject to specification reduction for loading of multiple lanes. Impact not included.

span	moment	end shear and end reaction[a]	span	moment	end shear and end reaction[a]
1	6.0[b]	24.0[b]	42	364.0[b]	42.0[b]
2	12.0[b]	24.0[b]	44	390.7[b]	42.5[b]
3	18.0[b]	24.0[b]	46	417.4[b]	43.0[b]
4	24.0[b]	24.0[b]	48	444.1[b]	43.5[b]
5	30.0[b]	24.0[b]	50	470.9[b]	43.9[b]
6	36.0[b]	24.0[b]	52	497.7[b]	44.3[b]
7	42.0[b]	24.0[b]	54	524.5[b]	44.7[b]
8	48.0[b]	24.0[b]	56	551.3[b]	45.0[b]
9	54.0[b]	24.0[b]	58	578.1[b]	45.3[b]
10	60.0[b]	24.0[b]	60	604.9[b]	45.6[b]
11	66.0[b]	24.0[b]	62	631.8[b]	45.9[b]
12	72.0[b]	24.0[b]	64	658.6[b]	46.1[b]
13	78.0[b]	24.0[b]	66	685.5[b]	46.4[b]
14	84.0[b]	24.0[b]	68	712.3[b]	46.6[b]
15	90.0[b]	25.6[b]	70	739.2[b]	46.8[b]
16	96.0[b]	27.0[b]	75	806.3[b]	47.3[b]
17	102.0[b]	28.2[b]	80	873.7[b]	47.7[b]
18	108.0[b]	29.3[b]	85	941.0[b]	48.1[b]
19	114.0[b]	30.3[b]	90	1008.3[b]	48.4[b]
20	120.0[b]	31.2[b]	95	1074.9[b]	48.7[b]
21	126.0[b]	32.0[b]	100	1143.0[b]	49.0[b]
22	132.0[b]	32.7[b]	110	1277.7[b]	49.4[b]
23	138.0[b]	33.4[b]	120	1412.5[b]	49.8[b]
24	144.0[b]	34.0[b]	130	1547.3[b]	50.7
25	155.5[b]	34.6[b]	140	1682.1[b]	53.1
26	166.6[b]	35.1[b]	150	1856.3	55.5
27	177.8[b]	35.6[b]	160	2076.0	57.9
28	189.0[b]	36.0[b]	170	2307.8	60.3
29	200.3[b]	36.6[b]	180	2551.5	82.7
30	211.6[b]	37.2[b]	190	2807.3	65.1
31	223.0[b]	37.7[b]	200	3075.0	67.5
32	234.4[b]	38.3[b]	220	3646.5	72.3
33	245.8[b]	38.7[b]	240	4266.0	77.1
34	257.7[b]	39.2[b]	260	4933.5	81.9
35	270.9[b]	39.6[b]	280	5649.0	86.7
36	284.2[b]	40.0[b]	300	6412.5	91.5
37	297.5[b]	40.4[b]			
38	310.7[b]	40.7[b]			
39	324.0[b]	41.1[b]			
40	337.4[b]	41.4[b]			

[a]Concentrated load is considered placed at the support. Loads used are those stipulated for shear.
[b]Maximum value determined by Standard Truck Loading. Otherwise the Standard Lane Loading governs.

Appendix A *(continued)*
Loading—H 20-44

Table of Maximum Moments, Shears, and Reactions
(Simple Spans, One Lane)

Spans in feet; moments in thousands of foot-pounds; shears and reactions in thousands of pounds. These values are subject to specification reduction for loading of multiple lanes. Impact not included.

span	moment	end shear and end reaction[a]	span	moment	end shear and end reaction[a]
1	8.0[b]	32.0[b]	42	365.9[b]	39.4
2	16.0[b]	32.0[b]	44	385.8[b]	40.1
3	24.0[b]	32.0[b]	46	405.7[b]	40.7
4	32.0[b]	32.0[b]	48	425.6[b]	41.4
5	40.0[b]	32.0[b]	50	445.6[b]	42.0
6	48.0[b]	32.0[b]	52	465.5[b]	42.6
7	56.0[b]	32.0[b]	54	485.5[b]	43.3
8	64.0[b]	32.0[b]	56	505.4[b]	43.9
9	72.0[b]	32.0[b]	58	530.1	44.6
10	80.0[b]	32.0[b]	60	558.0	45.2
11	88.0[b]	32.0[b]	62	586.5	45.8
12	96.0[b]	32.0[b]	64	615.7	46.5
13	104.0[b]	32.0[b]	66	645.5	47.1
14	112.0[b]	32.0[b]	68	675.9	47.8
15	120.0[b]	32.5[b]	70	707.0	48.4
16	128.0[b]	33.0[b]	75	787.5	50.0
17	136.0[b]	33.4[b]	80	872.0	51.6
18	144.0[b]	33.8[b]	85	960.5	53.2
19	152.0[b]	34.1[b]	90	1053.0	54.8
20	160.0[b]	34.4[b]	95	1149.5	56.4
21	168.0[b]	34.7[b]	100	1250.0	58.0
22	176.0[b]	34.9[b]	110	1463.0	61.2
23	184.0[b]	35.1[b]	120	1692.0	64.4
24	192.0[b]	35.3[b]	130	1937.0	67.6
25	200.0[b]	35.5[b]	140	2198.0	70.8
26	208.0[b]	35.7[b]	150	2475.0	74.0
27	216.9[b]	35.9[b]	160	2768.0	77.2
28	226.8[b]	36.0[b]	170	3077.0	80.4
29	236.7[b]	36.1[b]	180	3402.0	83.6
30	246.6[b]	36.3[b]	190	3743.0	86.8
31	256.5[b]	36.4[b]	200	4100.0	90.0
32	266.5[b]	36.5[b]	220	4862.0	96.4
33	276.4[b]	36.6[b]	240	5688.0	102.8
34	286.3[b]	36.9	260	6578.0	109.2
35	296.2[b]	37.2	280	7532.0	115.6
36	306.2[b]	37.5	300	8550.0	122.0
37	316.1[b]	37.8			
38	326.1[b]	38.2			
39	336.0[b]	38.5			
40	346.0[b]	38.8			

[a]Concentrated load is considered placed at the support. Loads used are those stipulated for shear.
[b]Maximum value determined by Standard Truck Loading. Otherwise the Standard Lane Loading governs.

Appendix A *(continued)*
Loading—HS 20-44

Table of Maximum Moments, Shears, and Reactions
(Simple Spans, One Lane)

Spans in feet; moments in thousands of foot-pounds; shears and reactions in thousands of pounds. These values are subject to specification reduction for loading of multiple lanes. Impact not included.

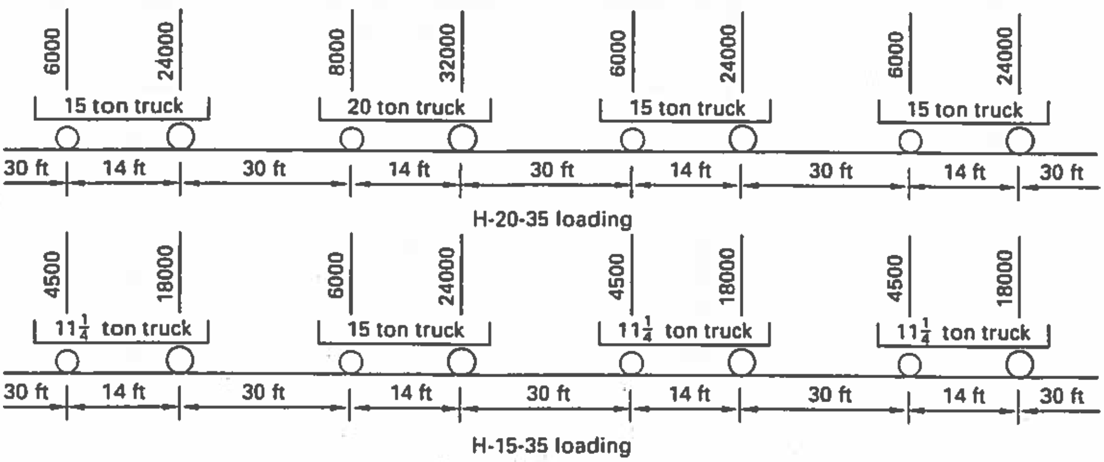
span	moment	end shear and end reaction[a]	span	moment	end shear and end reaction[a]
1	8.0[b]	32.0[b]	42	485.3[b]	56.0[b]
2	16.0[b]	32.0[b]	44	520.9[b]	56.7[b]
3	24.0[b]	32.0[b]	46	556.5[b]	57.3[b]
4	32.0[b]	32.0[b]	48	592.1[b]	58.0[b]
5	40.0[b]	32.0[b]	50	627.9[b]	58.5[b]
6	48.0[b]	32.0[b]	52	663.6[b]	59.1[b]
7	56.0[b]	32.0[b]	54	699.3[b]	59.6[b]
8	64.0[b]	32.0[b]	56	735.1[b]	60.0[b]
9	72.0[b]	32.0[b]	58	770.8[b]	60.4[b]
10	80.0[b]	32.0[b]	60	806.5[b]	60.8[b]
11	88.0[b]	32.0[b]	62	842.4[b]	61.2[b]
12	96.0[b]	32.0[b]	64	878.1[b]	61.5[b]
13	104.0[b]	32.0[b]	66	914.0[b]	61.9[b]
14	112.0[b]	32.0[b]	68	949.7[b]	62.1[b]
15	120.0[b]	34.1[b]	70	985.6[b]	62.4[b]
16	128.0[b]	36.0[b]	75	1075.1[b]	63.1[b]
17	136.0[b]	37.7[b]	80	1164.9[b]	63.6[b]
18	144.0[b]	39.1[b]	85	1254.7[b]	64.1[b]
19	152.0[b]	40.4[b]	90	1344.4[b]	64.5[b]
20	160.0[b]	41.6[b]	95	1434.1[b]	64.9[b]
21	168.0[b]	42.7[b]	100	1524.0[b]	65.3[b]
22	176.0[b]	43.6[b]	110	1703.6[b]	65.9[b]
23	184.0[b]	44.5[b]	120	1883.3[b]	66.4[b]
24	192.7[b]	45.3[b]	130	2063.1[b]	67.6
25	207.4[b]	46.1[b]	140	2242.8[b]	70.8
26	222.2[b]	46.8[b]	150	2475.1	74.0
27	237.0[b]	47.4[b]	160	2768.0	77.2
28	252.0[b]	48.0[b]	170	3077.1	80.4
29	267.0[b]	48.8[b]	180	3402.1	83.6
30	282.1[b]	49.6[b]	190	3743.1	86.8
31	297.3[b]	50.3[b]	200	4100.0	90.0
32	312.5[b]	51.0[b]	220	4862.0	96.4
33	327.8[b]	51.6[b]	240	5688.0	102.8
34	343.5[b]	52.2[b]	260	6578.0	109.2
35	361.2[b]	52.8[b]	280	7532.0	115.6
36	378.9[b]	53.3[b]	300	8550.0	122.0
37	396.6[b]	53.8[b]			
38	414.3[b]	54.3[b]			
39	432.1[b]	54.8[b]			
40	449.8[b]	55.2[b]			

[a] Concentrated load is considered placed at the support. Loads used are those stipulated for shear.

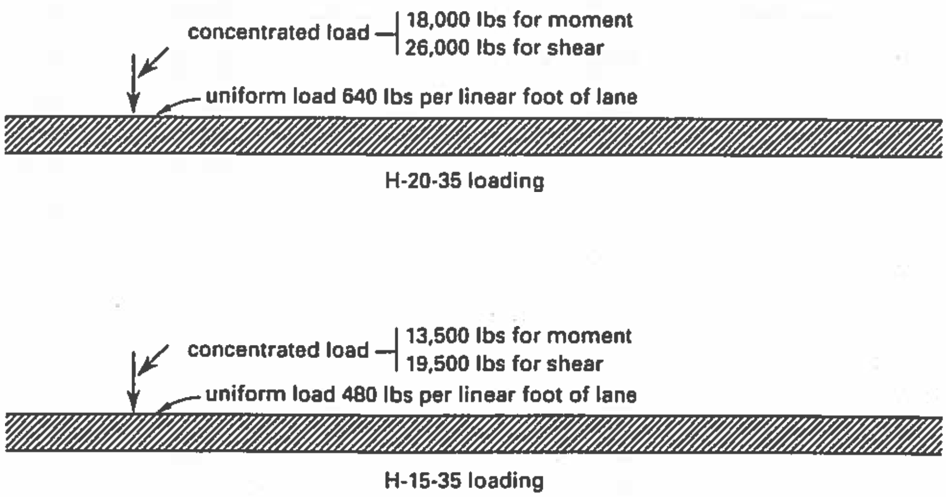
[b] Maximum value determined by Standard Truck Loading. Otherwise the Standard Lane Loading governs.

Appendix B
AASHTO Truck Train Loadings

Truck Train and Equivalent Loadings—1935 Specifications
American Association of State Highway Officials



Truck Train Loading



EQUIVALENT LOADING
lane width 10 feet