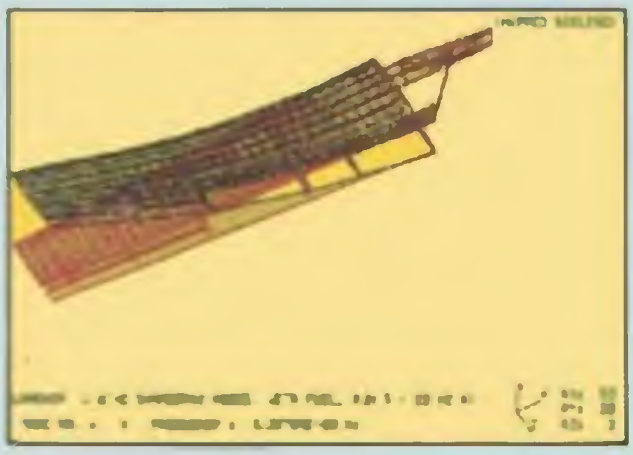
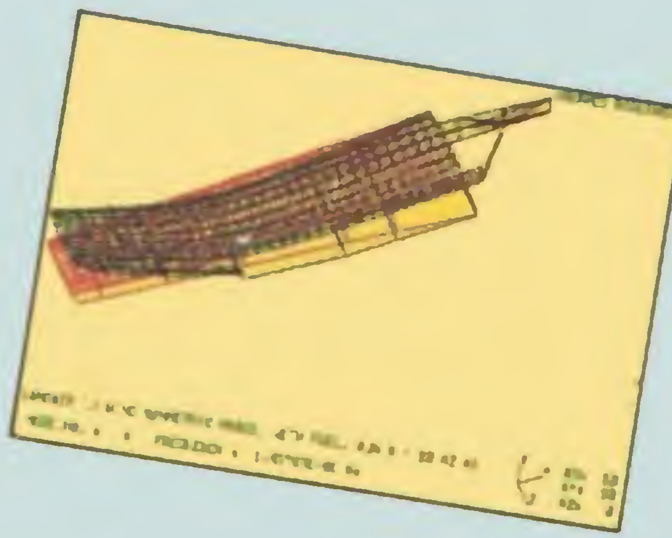
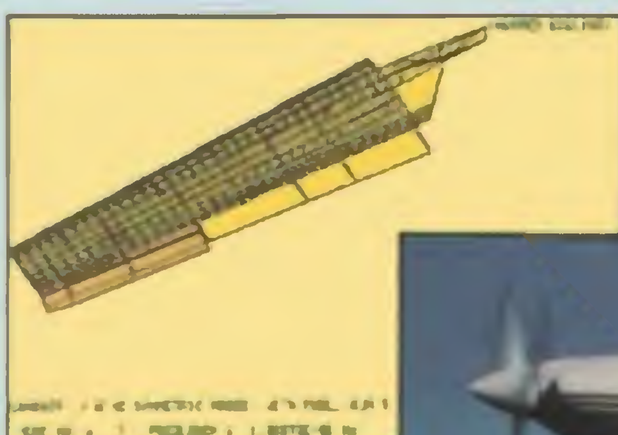
# MODERN AERODYNAMIC FLUTTER ANALYSIS

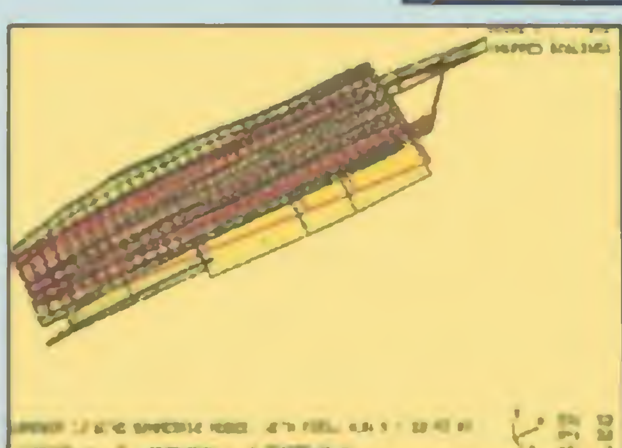
How to Determine Aircraft Critical Flutter Speeds using a 386-PC

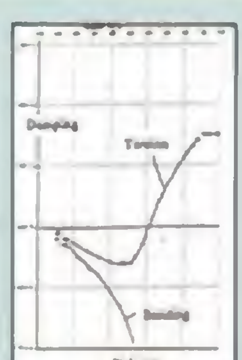
by Martin Hollmann

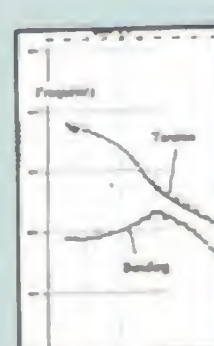










# Modern Aerodynamic Flutter Analysis

by Martin Hollmann, MSME

Published by Martin Hollmann
Printed in the United States of America
Monterey, California



AIRCRAFT DESIGNS, INC.
5 Harris Court, Building S
Monterey, California 93940

DISCLAIMER

M. HOLLMANN OR AIRCRAFT DESIGNS, INC. MAKES NO WARRANTY OR REPRESENTATIONS IN CONNECTION WITH SAF OR NISA386* OR ANY OTHER SOFTWARE PROGRAM OF AIRCRAFT DESIGNS, INC. EXPRESS OR IMPLIED, INCLUDING ANY IMPLIED WARRANTY OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE.

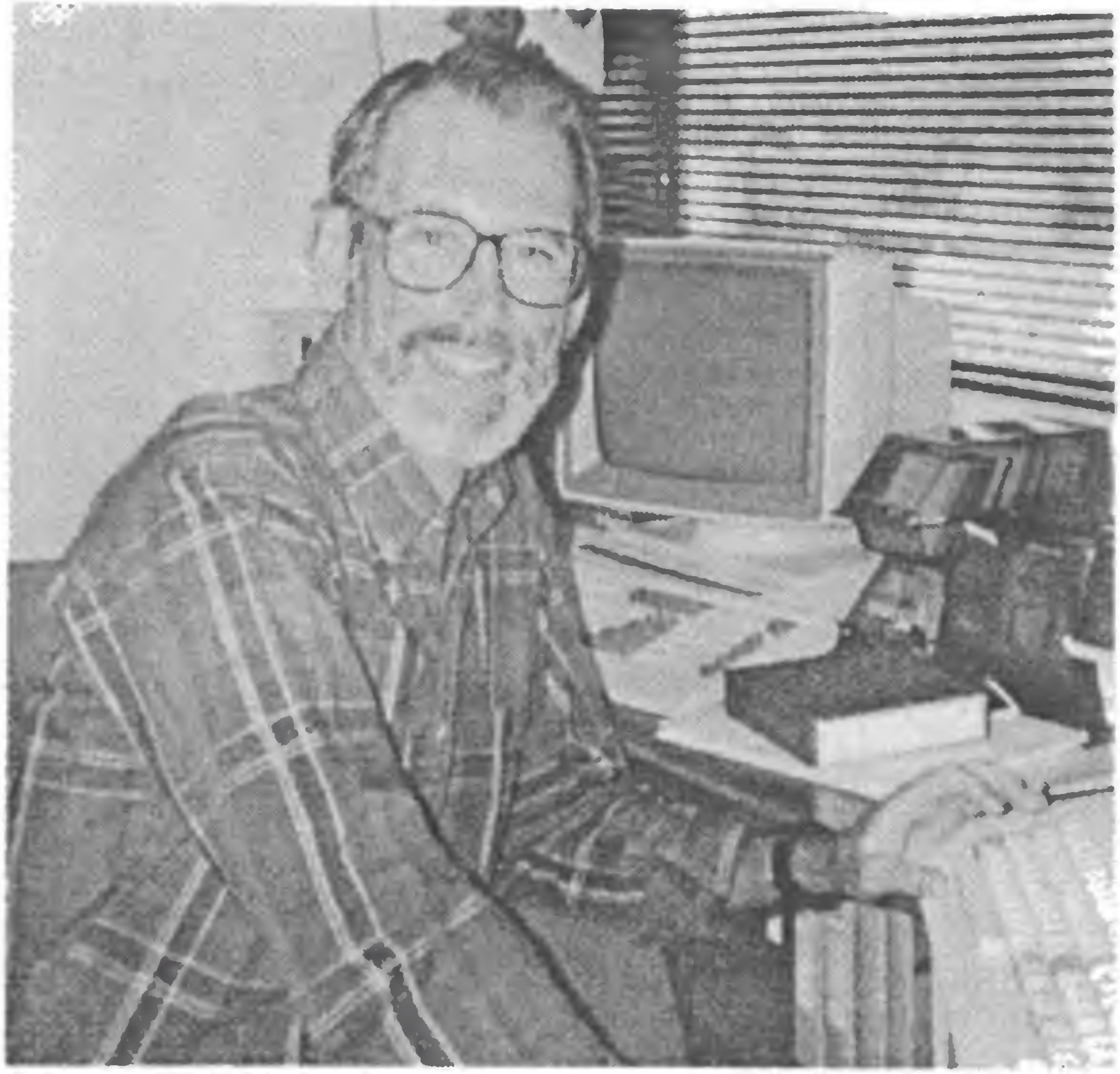
M. HOLLMANN OR AIRCRAFT DESIGNS, INC. ASSUMES NO RESPONSIBILITY FOR THE USE OF THE SAF PROGRAM OR NISA386 OR FOR THE ACCURACY OR VALIDITY OF ANY RESULTS OBTAINED. M. HOLLMANN OR AIRCRAFT DESIGNS, INC. SHALL NOT BE LIABLE FOR LOSS OF LIFE, INJURY, LOSS OF PROFIT, LOSS OF BUSINESS, OR OTHER FINANCIAL LOSS WHICH MAY BE CAUSED DIRECTLY OR INDIRECTLY BY THE SAF OR NISA386 SOFTWARE, WHEN USED FOR ANY PURPOSE OR USE, OR DUE TO ANY DEFECT OR DEFICIENCY THEREIN.

Any questions relating to the use or interpretation of SAF or NISA386 or their operation should be directed to:

Aircraft Designs, Inc. 25380 Boots Road, Monterey, CA 93940.

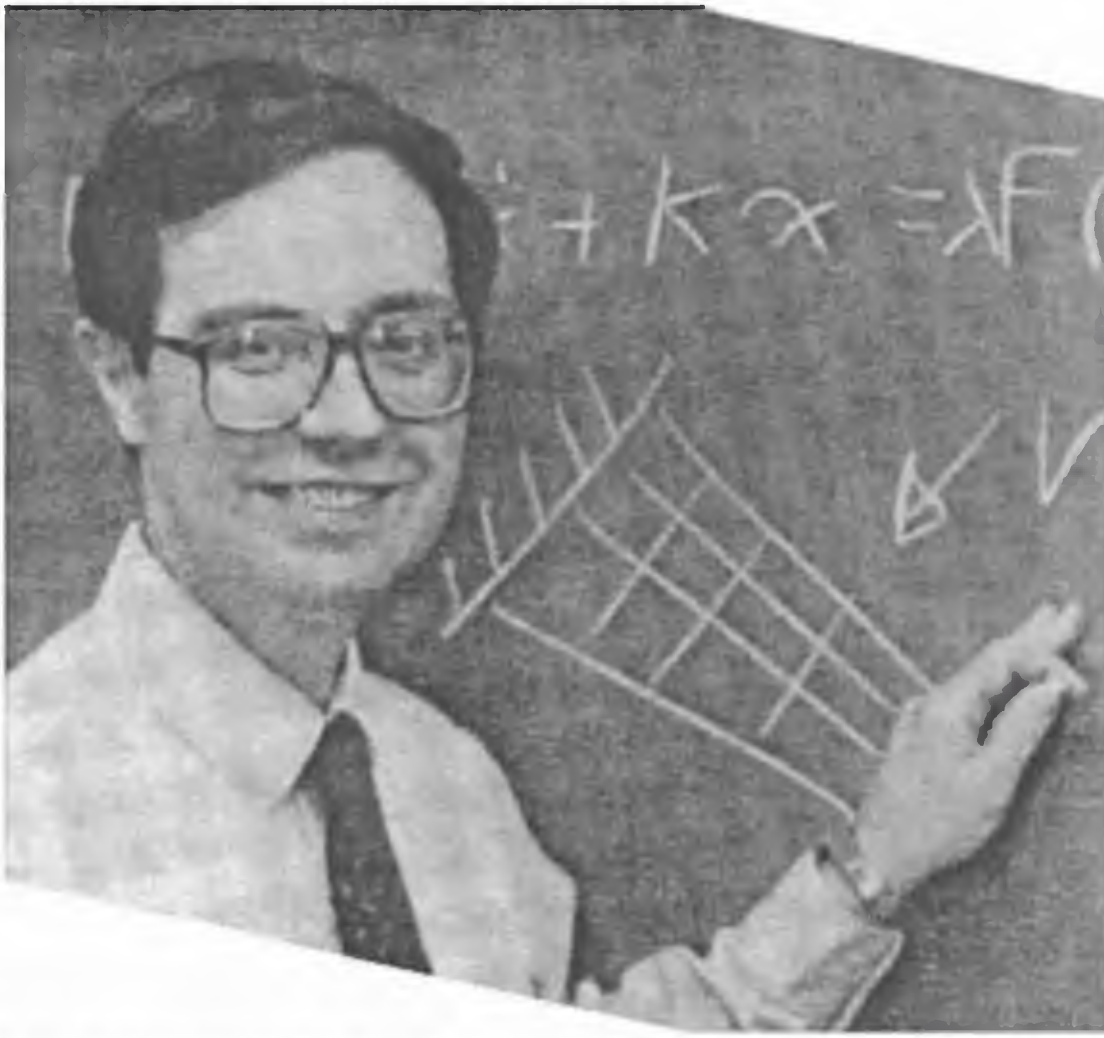
☎ (408) 649-6212.

* Trade Mark and software program of EMRC.



Dr. Sam McIntosh is one of the recognized world leaders in aeroelasticity and dynamics. Many of his students are working in the aerospace industry or universities throughout the world today.

Designer and builder of the Lancair IV is Mr. Lance Neibauer. The Lancair IV is the aircraft pictured on the cover and it is the example aircraft used in this book.



Dr. Ron Taylor edited SAF so that it would run on a PC.

Mr. Phillip Ho who compiled SAF to run on a PC. While working for EMRC Phillip Ho was responsible for all NISA386 development for the PC.

# *TABLE OF CONTENTS*

# *PREFACE*

Flutter is a phenomenon that is for most people somewhat difficult to envision and even more difficult to predict. We know that it can be catastrophic to both high performance and low speed aircraft. However, the severity of the problem is accentuated by high speed. I have talked to three friends that have encountered flutter in low speed aircraft; Walt Mooney during the test flights of the VP-2, Rick McWilliams in an old Ercoupe, and Pete Celliers in testing his GA-1 glider. I have never talked to anyone who had encountered high speed flutter and lived. Flutter is an important topic in flight safety, especially in the world of high performance aircraft.

There is one misconceptions which I would like to clarify. Many people think that if an aircraft is flown by a test pilot and the pilot has attempted to induce flutter by hitting the control stick at various flight speeds and no flutter is encountered, the aircraft is free from flutter. This is not the case. Reference 1 states that a pilot cannot induce flutter excitations in excess of 7 - 10 Hz. Most flutter speeds occur at frequencies well above these values as demonstrated in this report. The pilot will not be able to excite them. They are usually excited by a large shaker mounted in the aircraft or inadvertently when the aircraft encounters severe gust conditions at a critical flutter speed. And these critical flutter speeds depend on the weight of the aircraft, fuel level, altitude, and other factors.

With the development of the 386 personal computer, PC, and a computer code called Subsonic Aerodynamic Flutter, SAF, described herein, flutter analysis has become available and affordable to any engineer willing to take the time to learn. It is my hope that this information will serve to make aircraft safer to fly and more fun for everyone. This is especially important as aircraft achieve higher speeds and better performance. Composite materials are being used in many of these high performance and fuel efficient aircraft. These materials are softer than aluminum, and as such more prone to aeroelastic problems.

A number of people deserve a special thank you for making SAF and this information available. First and foremost is Dr. Sam McIntosh who is one of the world's leading experts in this field. Dr. McIntosh taught aeroelasticity at Stanford University for many years and many of the leading experts in the world today were students of Sam at one time. Sam and I performed the flutter analysis on the Lancair IV which is used as one of the examples in this book. Dr. Ronald Taylor from the University of Dayton was one of the originators of the FASTEX, Flutter Analysis with Shorter Turnaround and EXecution, program from which SAF evolved. Following is a summary of his experience.

Dr. Taylor received his B.S. degree in Physics and Mathematics from Wilmington College in 1967. He earned an M.S. degree in Applied Mathematics in 1971 and in 1979, received his Ph.D. in Mechanical Engineering from UD. A member of the American Institute of Aeronautics andAstronautics, Dr. Taylor is currently conducting research in nonlinear dynamics and optimization within the Aerospace Mechanics Division of the

University of Dayton Research Institute and has over 20 years of research experience. He has also developed expert systems to improve the utility of engineering analysis software. Prior to joining the University of Dayton in 1977, Dr. Taylor conducted structural design and analysis methods development for 10 years at the Air Force Flight Dynamics Laboratory. During this time he worked with Stanford University researchers on the development of the first general-purpose aeroelastic optimization code (SOAR).

Dr. Taylor has taught courses in Aerospace & Mechanical Engineering and Engineering Mechanics. These include Numerical Methods for Engineers, Mechanical Vibrations, and Computational Methods in Design.

Publications:

❑ Gwin, L. B. and R. F. Taylor, "A General Method for Flutter Optimization," AIAA Journal, Vol 11, No 12, December 1973, pp. 1613-1617.

❑ Wilkinson, K., E. Lerner, and R. F. Taylor, "Practical Design of Minimum Weight Aircraft Structures for Strength and Flutter Requirements," Journal of Aircraft, Vol 13, No 8, August 1976, pp. 614-624.

❑ Taylor, R. F., "Development of Stability Methods for Application to Nonlinear Aeroelastic Optimization," AFFDL-TR-29-3114, Wright-Patterson Air Force Base, Ohio, and Ph.D. Dissertation, Department of Mechanical Engineering, University of Dayton, Dayton, Ohio July 1979.

❑ Taylor, R. F., F. K. Bogner, and V. B. Venkayya, "An Optimality Criterion Approach to the Design of Damage Tolerant Structures," 23rd AIAA/ASME/CE/AHS Structures, Structural Dynamics and Materials Conference, New Orleans, Louisiana, May 1982.

❑ Taylor, R. F., "ASTROS-Advisor: A Prototype Expert System for Automated Structural Optimization," AFCEA Regional Conference, July, 1990.

My thanks to Mr. Terry Harris, Dr. Larry Huttsell, and Dr. Max Blair from the Flight Dynamics Laboratory at Wright Patterson Air Force Base who were invaluable in supplying information on how to use the FASTEX program and its history. Mr. Lance Neibauer for agreeing and paying for the flutter analysis of his Lancair IV. My special thanks to my good friend Mr. Phillip Ho who compiled SAF to run on the PC. And last but not least I am indebted to Mrs. Andrea Naime who typed and compiled this manual.

Without the dedication, hard work, and kind help of these individuals this information and SAF would not be available to you.

*Martin Hollmann*

# CHAPTER 1

## *BACKGROUND*

The problem of flutter has been with us since Otto Lilienthal and the Wright Brothers made their first historic flights. To avoid flutter problems, early aircraft used extensive external wire bracing to support and stiffen the lifting surfaces and control surfaces. Although the wire bracing stiffened the structure, it generated much drag causing early aircraft to fly slow. This slow speed and the stiff structure worked well to minimize flutter. Because of the success of this type of structure, some aircraft such as the Christian Eagle still use this type of structure today.

As aircraft flight speeds increased, it became apparent that analytic methods of predicting flutter were needed. The first papers on flutter appeared in 1924. In that year, Birnbaum published a paper titled "Das ebene Problem des schlagenden Flügels" in the Zeitschrift angewandte Mathematik und Mechanik, Volume 4, No. 4, page 277.

During World War II many high performance aircraft encountered flutter problems. Hand calculations using tables of aerodynamic coefficients were used as documented in Air Corps Technical Report No 4798 "Application of Three-Dimensional Flutter Theory to Aircraft Structures." See Reference 10. With the development of mainframe computers in the early 1970's, aerospace companies within the United States began developing computer programs capable of more accurately predicting flutter. One such program was developed by Grumman Aerospace Corporation in Bethpage, New York in 1973 and sponsored by the Air Force Flight Dynamics Laboratory at Wright Patterson Air Force Base in Ohio. This program was tilted FASTOP for Flutter And STrength Optimization Program. Reference 2, 3, 4, 5, 6, and 7. FASTOP consisted of two main programs. Each program is designed to perform successive analysis and resizing of structures in a single computer session. The first program titled SOP for Strength Optimization Program, focuses on basic aspects of structures for minimum weight based on strength criteria. It is a finite element analysis program that performs structural and dynamic analysis and calculates eigenvalues and mode shapes. It also prepares data for direct input to the second program which is called FOP. FOP stands for Flutter Optimization Program. FASTOP consists of *SLOP* and *FLOP* !!! No! I mean FASTOP is made up of SOP and FOP. FOP performs the flutter analysis and prepares data for SOP when a rerun is performed. Although FASTOP was designed to be user interactive, it was a big and cumbersome program to run. It also turned out that by far the greatest use of the code was to calculated flutter and divergence speeds. Its aeroelasticity capability has a number of state-of-the-art features which make it a highly versatile flutter analysis tool.

In 1980 the Air Force flight Dynamics Laboratory awarded a contract to the University of Dayton to streamline the FASTOP flutter analysis capability. The primary objective was to update the code so that it would operate only as an analysis tool without the optimization capability. In 1981 the code was reorganized by Dr. Ronald Taylor for faster execution times on a CYBER-175. The resulting code, FASTEX. FASTEX has since been extensively used in both government and industry for the analysis of advanced flight vehicles such as the X-29A. Reference 4.

In 1991 Aircraft Designs, Inc. contracted the University of Dayton to modify the FASTEX program so it could run on a PC and the program SAF for Subsonic Aerodynamic Flutter was born. The use of SAF is described in this book. Considerable changes were made to FASTEX including replacing the eigenvalue solver. The FASTEX eigenvalue solver ran in single precision which would not allow it to run on a PC which runs in double precision. The supersonic Mach-box routine was removed. And instead of a formated input format, a free format was incorporated. The free format makes it easy to use.

Mr. Phillip Ho was contracted to compile the program and make it compatible with the NISA386 finite element analysis code. Phillip also incorporated the free format capability.

Table 1. SAF Capabilities

- ❑ Doublet-lattice (Rodden) routine $\quad$ M = 0 to 0.9
- ❑ Maximum number of lifting surfaces for doublet-lattice routine = 30
- ❑ Maximum air densities = 10
- ❑ Maximum number of velocities = 20
- ❑ Maximum number of vibration modes = 20
- ❑ Maximum number of control surfaces on main surface, doublet-lattice routine = 5
- ❑ Maximum number of aerodynamic panels = 400
- ❑ P-K flutter analysis
- ❑ Pressure calculations only
- ❑ K flutter analysis
- ❑ Divergence analysis

It should be noted that other flutter analysis programs have been developed and are presently in use. Wright Patterson's Flight Dynamics Laboratory has developed and presently uses a program called ASTROS Version 8 which was developed from and is very similar to FASTEX. This code runs on work stations and mainframes such as a CRAY, VAX/VMS, IBM/MVS, IBM RS6000/Unix, SUN Sparcstation/Unix, CONVEX/Unix. A Unix operating system must be used. This program is only available to those who are working on military contracts. It is not available to the general public.

The MacNeal-Schwendler Corporation (MSC) leases the full blown MSC NASTRAN program which now runs on a 386 PC for $9,600 per year. Their aerodynamic code which is used in conjunction with ___ MSC NASTRAN leases for $1,200 per year. The pre and postprocesser for MSC NASTRAN is called MSCXL and leases for $2,400 per year. The total lease fee is $13,200 per year plus and an initial fee for installing the program on the PC of $2,000. However, the full blown MSC NASTRAN program with MSCXL for the 386 PC can be purchased for $34,500. A $305 monthly maintenance fee is required. The MSC aero code is the N5KA program which for all practical purposes is the same as the H7WC doublet lattice program used in SAF. SAF uses the H7WC routine which is also used in FASTEX and, with the exception of body interference panels, is the same as the N5KA program. MSC is located at 815 Colorado Blvd. Los Angeles, CA 09941.

The input data for SAF is identical to the FASTEX program. However, a free format is used in SAF instead of the formated input style used in FASTEX. The FASTEX format is a carryover from the card keypunch days.

Finite Element Analysis and Flutter Analysis go hand-in-hand and can now be performed on a low cost PC thanks to SAF. Some people prefer to call this flutter program "Save my Ass from Flutter."

Computer Technology in the past 20 years . . . . You have come a long way Baby!!!

The input data for the SAF program is generated by using a finite element analysis program such as ALGOR or NISA386. In the past I have used the NISA386 program because it was the only program that had a sandwich plate element. Therefore, the examples in this book are performed using the NISA386 program. However with time, other programs such as ALGOR have developed and presently incorporate composite material and sandwich elements. However, they are not as good as NISA and I still use NISA. Very few aircraft fea models are over 5,000 DOF and a 5,000 DOF NISA386 program can be purchased for $3,000. The NISA386 modulus that are required to perform stress and the eigenvalue calculations for flutter on a PC consist of DISPLAY, STATIC, and DYNAMIC. The ALGOR modules that perform the same analysis consist of Supercap (Part No. 450) $200, Stress/Vib/Mode Shape w/ViziCad Plus (100-3H) $2,100., and the Composite Processor Stress/Vib (112-3H) $700, for a total of $3,000. So the price is same. As such I presently recommend buying NISA386.

# CHAPTER 2

## *SYSTEM REQUIREMENTS*

Table 2 shows two systems. One is a minimum cost configuration with ALGOR and the other is a higher cost configuration with NISA386. Although many finite element programs can be used to generate modal data for SAF, the examples shown here use ALGOR and NISA386 finite element analysis codes. The ALGOR or NISA386 program is used to determine the eigenvalues and mode shapes. A minimum hardware requirement is a 386-16 MHz PC with at least a 40 MByte hard drive and 4 Mbytes of RAM and a math coprocessor. However, the power and low price of the 486-33 MHz PC by far out ways the small cost savings of the 386-16 MHz PC. A 486-33 MHz PC with a 120 MByte hard drive and a Super VGA monitor costs $1,500 and runs 10 times faster than the 386-16 PC.

Table 2. Cost Summary of Systems Required for Flutter Analysis

| HARDWARE | System 1 | System 2 |
|---|---|---|
| 486-33 MHz with SVGA Monitor, | | |
| 120 Mbytes H.D. 4 Mbyte of RAM | $1,500 | $1,500 |
| 4 Mbytes RAM chips 4 x $45 | 180 | 180 |
| Dot matrix printer | 200 | 200 |
| HP Paintjet | - | 500 |
| Hardware cost | $1,880 | $2,380 |
| **SOFTWARE** | **System 1** | **System 2** |
| ALGOR Part No's 450, 100-3H, 112-3H | $3,000 | - |
| NISA386 v. 91 Production | | |
| Display, Static, Dynamic | - | $6,000 |
| SAF, flutter analysis program | 650 | 650 |
| Modern Aircraft Flutter Analysis book | 28 | 28 |
| Software cost | $3,678 | $6,678 |
| Total cost | $5,558 | $9,058 |

Shipping and tax are not included and prices will change. The above data is provided to show a cost estimate that will help you make a decision on what system you want.

Shipping and tax are not included and prices will change. The above data is provided to show a cost estimate that will help you make a decision on what system you want. Programs will usually run from 5 to 6 times faster on the 486 PC compared with a 386-25 MHz PC. Performance is important since the finite element analysis eigenvalue calculations for the example Lancair IV wing with 3,500 degrees of freedom will take about 4.5 hours to run on a 386-25 MHz PC and 45 minutes on the 486-33 MHz PC. The flutter analysis of this wing only takes 10 minutes to run on the 486-33 MHz PC. However, a large flutter problem such as the fuselage and tail example of Section 7.3 takes 4.5 hours to run on a VAX II or a 386-25 MHz PC but will only take 45 minutes on the 486-33 PC. These programs are computer intensive and I highly recommend the 486-33 PC. The 386-25 PC will soon become obsolete.

The 486-33 PC that I am using costs $1,500 from MC Systems. Their address is 1310 Tully Road, Bldg. 103, San Jose, CA. ☎ (408) 293-9228.

Aircraft Designs, Inc. is a resaler for the ALGOR and NISA386 fea programs. These programs and SAF can be purchased directly from ADI.

# CHAPTER 3

## ANALYSIS PROCEDURE

The flutter analysis procedure that is used in this book consist of the following sequential steps. These steps are summarized in figure 1.

### 3.1 Finite Element Modeling (fem)

A wing, tail, or complete aircraft is modeled using a finite element analysis (fea) program. This fea determines the mode shapes and frequencies which are used by SAF. ALGOR and NISA386 are two fea programs that have composite material elements and they are good programs to solve dynamic (eigenvalue) problems. The structural fem is often used to verify the strength under statically applied loads which can include a buckling analysis of the skin panels. It is recommended that the fem be first used to solve the structural problem and then modified for the dynamic analysis. With only minor modifications to the fem static model input file, the dynamic analysis is performed to determine the eignevalues (natural frequencies) and modal displacements. These eigenvalues and modal displacements are used by SAF to predict the flutter speeds.

Most fem range in size from 3,000 to 6,000 degrees of freedom (DOF). ALGOR and NISA386 can solve problems up to 30,000 DOF. By-the-way, there are usually six DOF per node as listed below.

1. Translation in the x-axis
2. Translation in the y-axis
3. Translation in the z-axis
4. Rotation about the x-axis
5. Rotation about the y-axis
6. Rotation about the z-axis

ALGOR and NISA386 run on a 286, 386, or 486 PC.

It is important to set up this fea model accurately so that material stiffness and weights are realistically modeled. For flying surfaces with control surfaces, it is essential to accurately predict the control system stiffness. Control surfaces must also be accurately mass balanced about their hinge lines. A weight difference of 5 oz. on mass balance weights of the ailerons of the new Lancair SE4 reduced the flutter speed from 360 knots to 160 knots. Figure 1 shows the analysis sequence and how it ties in with designing and building the aircraft.

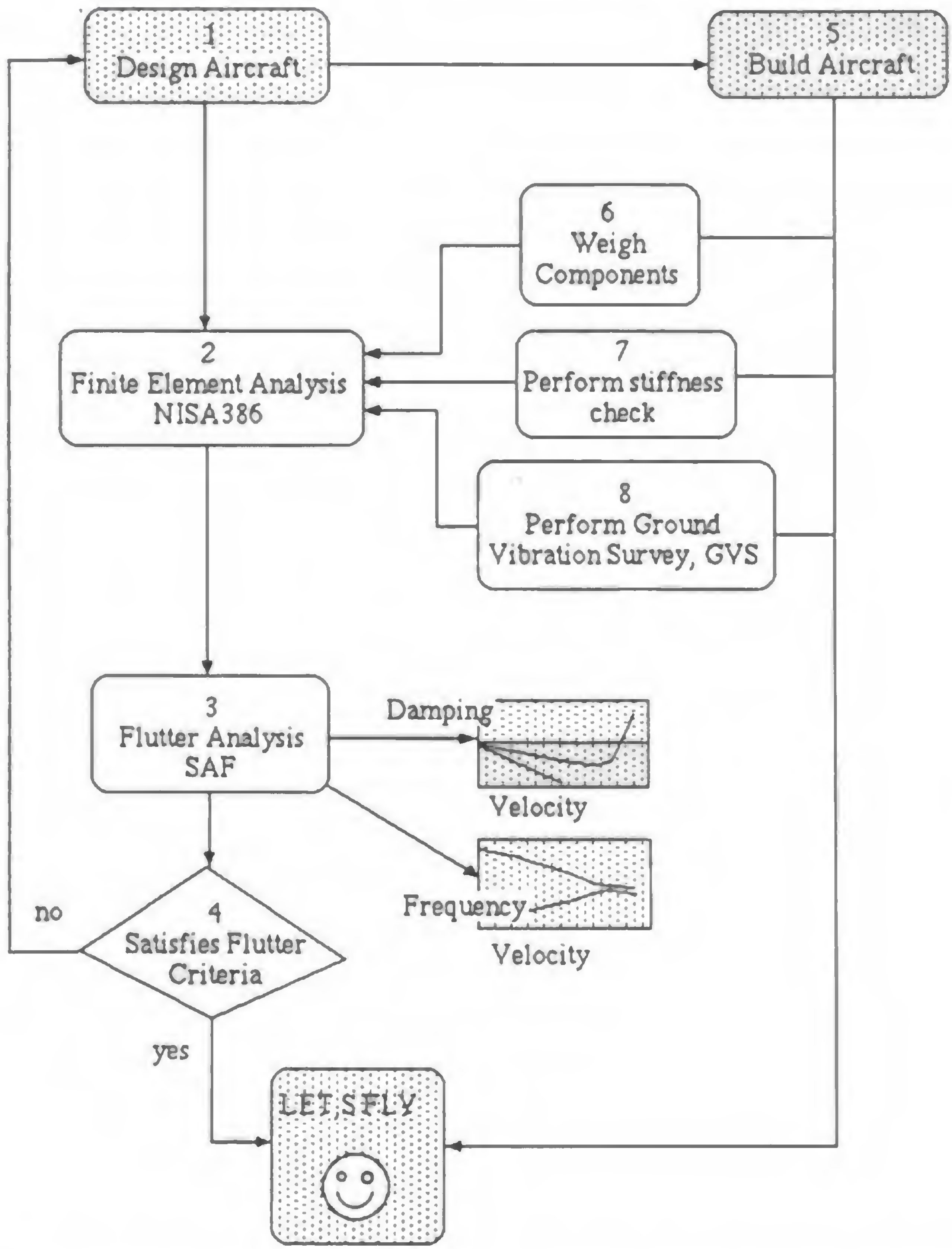



Figure 1. Flutter Analysis Flow Chart. The Numbers Show the Order of Sequence.

3.2 Verification of the Finite Element Model

Before the aircraft is built, a fea and flutter analysis is performed for the aircraft design. After the aircraft is built, the components of the aircraft are carefully weighed and the fem is modified to reflect the actual weight. Control system stiffness is measured by applying a force at the trailing edge of the control surface and measuring the deflection as the load is increased. Again, the fem is modified to reflect the actual, measured stiffness.

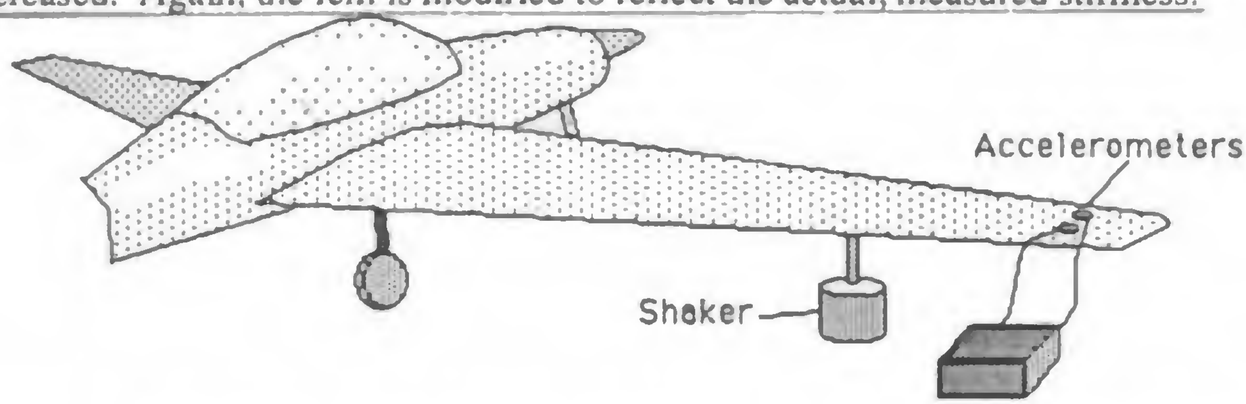



Figure 2. Typical set up for a ground vibration survey (gvs) to determine the 1st wing bending mode. Location of the accelerometers is determined from the fea mode shapes. It is desirable to locate the accelerometers at extreme model displacements and high accelerations.

3.3 Ground Vibration Survey (gvs)

With the aircraft sitting on its tires, accelerometers are placed at various locations and a vibrator (shaker) is used to excite the structure. The response (acceleration or velocity) of the accelerometers is observed on a cathode ray tube and the excitation frequency is recorded. Figure 2 shows a minimum set up for performing a gvs. The location of the accelerometers and the expected natural frequency is determined from the fea. The accelerometers are located at the maximum deflection points on the structure. As few as two accelerometers can be used if fea data is available. The accelerometers are bonded directly to the aircraft surface with M-Bond 200 adhesive form M-Line Accessories, Raleigh, N.C. After completing the gvs the accelerometers can be snapped off without damaging the paint

The accelerometer shown in figure 2 are positioned to determine the first wing bending mode. As the shaker frequency is increased closer to the natural frequency of the structure, the dynamic response of the accelerometers will increase. The frequency at which the maximum response is observed represents the natural frequency of that mode. A list to the equipment rented for the gvs on the Lancair IV is shown in table 3.

By meticulous mapping the dynamic response with the use of a multitude of accelerometers (12 to 30), the mode shapes and modal deformations can be recorded and directly used in SAF. This eliminates the need for a fea. Although this procedure has been used in the past, it is not recommended. Today a fea can be readily preformed and there is no reason why it should not be done. Today, fea is affordable and available to everyone.

The observed natural frequencies from the gvs are compared to the calculated natural frequencies and the fem is updated as required to get the frequency match.

Table 3. Rented Equipment Used for the GVS of the Lancair IV. This is a Bare Minimum Equipment List for GVS Testing.

| Instrument Used | | Rent cost/2 days |
|---|---|---|
| 1. | HP 3582 A Spectrum Analyzer, Dual Channel | $400 |
| 2. | MBPM Shaker, 50 lbf | 220 |
| 3. | 2 Endevco 2213C Accelerometers | 70 |
| | Calibration | 45 |
| 4. | 2 Endevco 2113 Charge Amplifiers | 150 |
| 5. | HP 3311 Oscillator | 40 |
| 6. | 125 VA Power Amplifier, Mod. No. 2120 MB | |
| | MB Electronics, New Haven, Conn. | |
| | HP 5384 A Frequency Counter | |

Rented from: DATACRAFT Inc. 13714 S. Normandie, Gardena, CA 90249-2696. Dave Johnson ☎ (213) 321-2320.

### 3.4 Flutter Analysis Using SAF

To run SAF on a 386 PC it will be necessary to open the CONFIG.SYS file and set files = 50. SAF opens and closes a minimum of 36 files during a run.

Prior to running SAF, an ASCII batch file using Qedit is set up using the procedure outlined in this book. The relative displacements from the NISA386 fea are used for the mode shapes in SAF. By-the-way, Qedit is included on the SAF disc. This is an excellent editor designed for setting up program files.

SAF generates plots showing the damping parameter, g, and the natural frequency, $\omega$, as a function of airspeed, V. The critical flutter speed is determined by the transition from negative to positive of the value g. Plots of $\omega$ vs. V are used to identify the modes participating most actively in the flutter. Tne active modes converge toward each other at the critical flutter speed. The flutter speed is also listed at the end of the output file which can be read using a program called LIST. LIST is also included on the SAF disc.

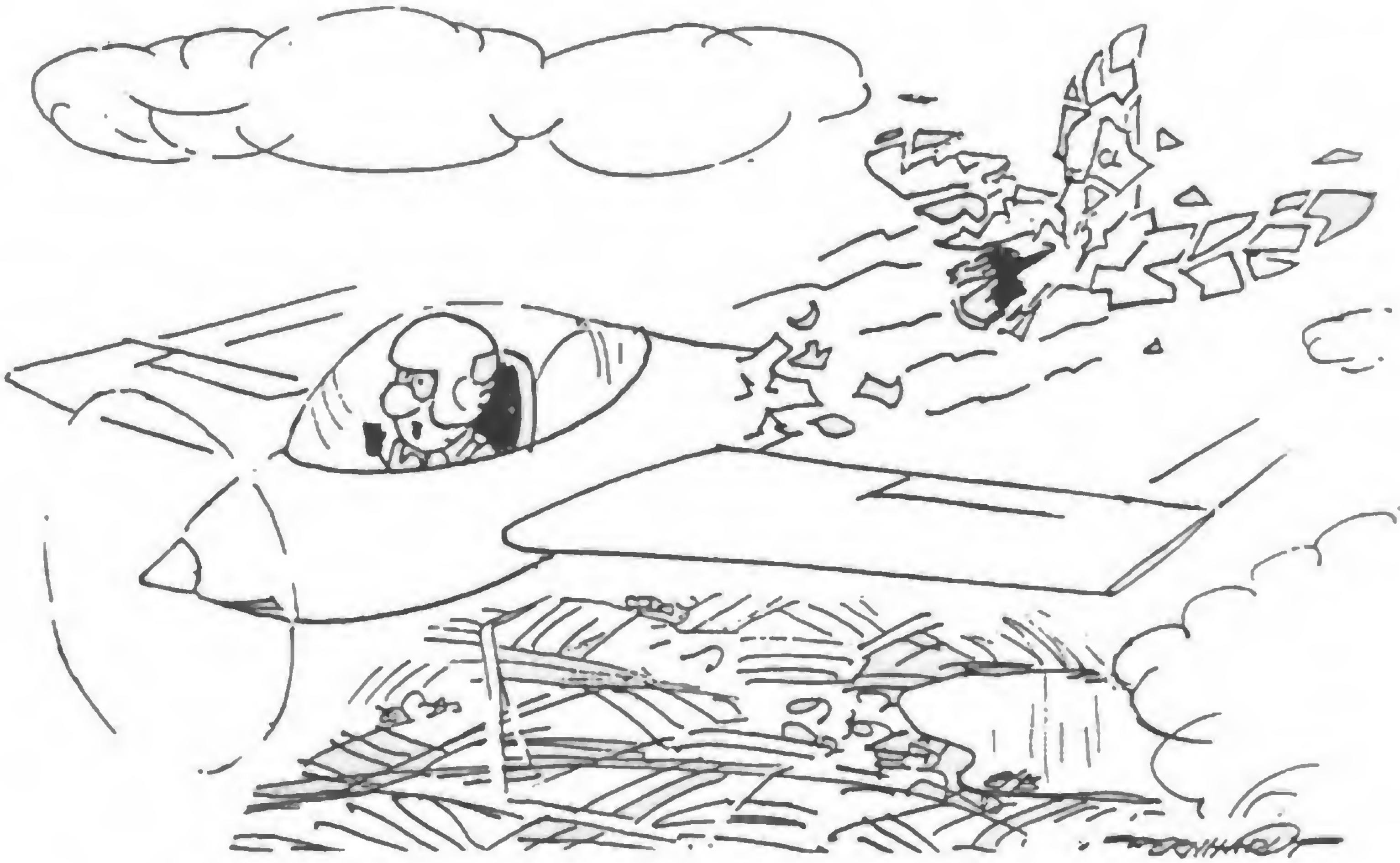
SAF also generates three binary files with a suffix GPH. These files are used by the NISA postprocessor called DISPLAY to show colored plots. The first file includes damping, g as a function of velocity. The second file contains frequency, and the third file contains the imaginary frequency. See figures 12 and 13 for the damping and frequency plots.

To display the files follow these instructions:

After evoking the NISA386 postprocesser by typing NISA386 and pressing return and following the cues, enter the command mode by typing *C* and pressing return. In the command mode, type *GXY* and press return. NISA DISPLAY will ask for a file name. Type in the file name that SAF has generated and press return. Make certain to include the suffix GPH. Once the file is loaded, type *PLOT* and press return and the graph will be displayed. If you have a color printer such as the HP Paintjet, a hard copy of the image on the screen is made by pressing the *f10* key. The graph can also be modified with the following commands.

| | |
|---|---|
| GBA | remove background color |
| GST | changes solid lines to dashed lines |
| GCO | select different line colors |
| GRN | select the graph scale ranges |

The flutter analysis is carried out for various altitudes, with and without fuel in the wing, and for symmetric and antisymmetric mode shapes. It is also good to perform a sensitivity study in which control surfaces are under balanced. The amount of tolerable under or over balance is of interest since flutter often occurs after a control surface has been painted without rebalance. With composite materials, the stiffness and hence the structural frequency changes with temperature. It would be advantages to determine the affect of temperature on frequency for these types of structure. As far as I know, todate no one has performed a study to show the changes in flutter speed for various temperatures for composite aircraft.



"I think we just ran into some flutter!"

# CHAPTER 4

## *FINITE ELEMENT ANALYSIS USING NISA*

The full blown NISA386 program, which costs $10,500, allows solving static, composite, and structural problems, dynamic problems (including shock spectrum, eigenvalue, transient dynamic response, frequency response, and random vibration), thermal analysis using NISA HEAT (which includes linear and nonlinear conduction, convection, radiation, and transient thermal analysis), and DISPLAY, which allows interactive pre and post processing of FEA data files. Loading options include, point forces and moments, distributed pressures and gravity, angular acceleration, velocity, thermal loading, and specified displacements are available.

To demonstrate the use of NISA, a simple curved plate is set up. The plate represents a composite sandwich panel on the upper surface of a wing and we wish to determine the first three natural frequencies. It is made up of two outer plies of 0.020 inch thick fiberglass (oriented at ± 45 degrees) and a 0.25-inch thick honeycomb core and a single 0.010-inch thick layer of fiberglass on the inside (also oriented at 45 degrees to the chord line). In setting up this problem, a 1/10 scaled drawing and nodal locations are measured in Cartesian coordinates. The input data is generated as an ASCII file using Qedit. Simply type *Q file name.*

- Toggle to the insert mode by pressing the *Ins* key.
- Now enter the lines of text file and hit return to go to each successive line.
- After the file has been entered, press *Ctrl + K* keys together and then the *X* key to save the file.
- Type *DIR/P*. You should see your file name among the other files indicating that your file has been saved.
- To reenter the file, simply type *Q file name* and press return.

The NISA data file for the plate problem is shown in table 4.

Once the program in table 4 has been created you can submit it for a run by invoking the Dynamic Program. Type NISA386 and press Return. A menu will appear. Press *2* for NISA programs and *5* for DYNAMIC. DYNAMIC will cue you to supply the input data file name and a name for the output file.

The program runs, and you are cued on the status of the job. If the program finds any mistakes, a *warning* or *fatal error* message is displayed. An output file is created in which you can see the cause of program termination. If a mistake is found, make the required corrections to the input file using Qedit.

Table 4. NISA Plate Program Listing and Description

```
ANALYSIS=EIGENVALUE ---------type of analysis
AUTO CONSTRAINT=ON --------- applies inplane rotation stiffness
FILE NAME=PLATE
SAVE FILE=26,27
EIGEN EXTRACTION=SUBSPACE, CONVENTIONAL
MASS FORMULATION=CONSISTENT
*TITLE ------------------------ group TITLE is always the title
 COMPOSITE SANDWICH PLATE          screen title
*ELTYPE
 1, 33, 1 ---------------------- element type 33, NKPT=33 , has 4
                                   nodes: NORD 1
*RCTABLE
 1, 4
 .02/// ---------------------     real constant has a thickness of .02
 2, 4
 .25/// --------------------      real constant has a thickness of .25
 3, 4
 .01/// ----------------------    real constant has a thickness of .01
*LAMANGLE ------------------      ply orientation group for composites
 1,4
45/// ------------------------     ply orientation at 45 degrees
2,4
0/// ------------------------- ply orientation at 0 degrees
*LAMSEQ
 3,0,1,2,3, 1,2,1, 1,2,1 ----------- composite laminate schedule using
                                   indices and schedules shown in
                                   materials group *MATERIAL and *RCTABLE
                                   and *LAMANGLE
*E1 -------------------------     element connectivity group
$$-4,5,4
1,7,6,1,2,$,1,1,0,4,1,1,2
*F1 --------------------------- nodal locations group
  -5,0,5,5,0,0,7.5
 1,0,$,-18,0,0 ----------------   node 1 is located at 18, 0, 0 in the x,y,z
                                   directions
  2,0,$,-13.5,1.8,0
  3,0,$,-9,3.04,0
  4,0,$,-4.5,3.76,0
  5,0,$,0,4,0
*MATERIAL -------------------     material properties group
** Fiberglass
EX,1,0,2.3E6
DENS,1,0,2E-4
```

```
EY,1,0,2.3E6
NUXY,1,0,.2
GXY,1,0,.6E6
DENS,2,0,6.7E-6
** Core
EX,2,0,0
GXZ,2,0,1E3
GYZ,2,0,1E3
*EIGCNTL
4,0,0,0,0,1000,10E-5
*EIGOUT- - - - - - - - - - - - - - - - - - - - -  output control card
  0,0,0,0,-1,0,1
*MODEOUT
1,4,1,1
*SPDISP- - - - - - - - - - - - - - - - - - - - -  specified constraints
 1,ALL,0,5,1  - - - - - - - - - - - - - - - - -  all edges fixed on nodes shown
 21,ALL,0,25,1
 6,ALL,0,16,5
 10,ALL,0,20,5
*ENDDATA - - - - - - - - - - - - - - - - - - -  end of program
```

---

Once the program run is completed, view the results in the output file or with the DISPLAY program.

To view the mode shapes, type *NISA386* and hit return. After NISA has loaded, type *2*. Then type *3* for Postprocessing. After DISPLAY is loaded type *C* to get into the command mode. In the command mode you are qued with a COMMAND > prompt.

You can open two binary output files by typing the letters BIN and pressing return. If the input file name is PLATE, see line 3 in table 4, the first binary file name is PLATE26.DAT, and the second binary file is PLATE27.DAT. Continue to enter data to view the output.

For those of you who want to learn how to use ALGOR or NISA and SAF I recommend attending one of my classes. I hold two day, hands on, fea and flutter classes on a regular basis. From the favorable response that I have received over the past five years for the fea classes, I can say that the class is well worth it.

The finite element model can be generated using NISA's powerful preprocessor. However, I prefer to use the old fashion method by using a drawing board. The geometry is layed out in a scale of 1/10 or larger and nodes and elements are located and numbered as shown in figure 3 for the Lancair IV wing.

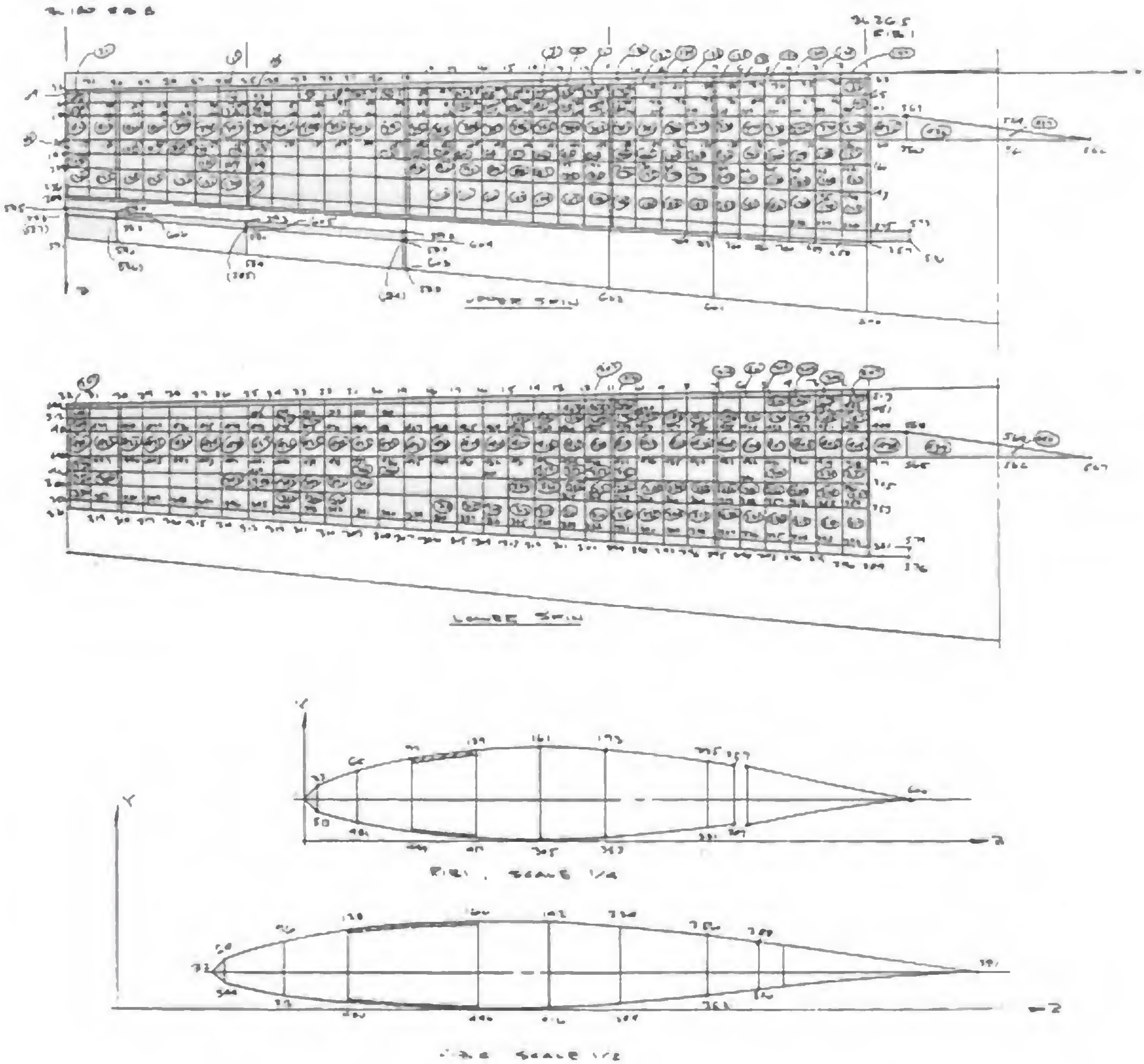


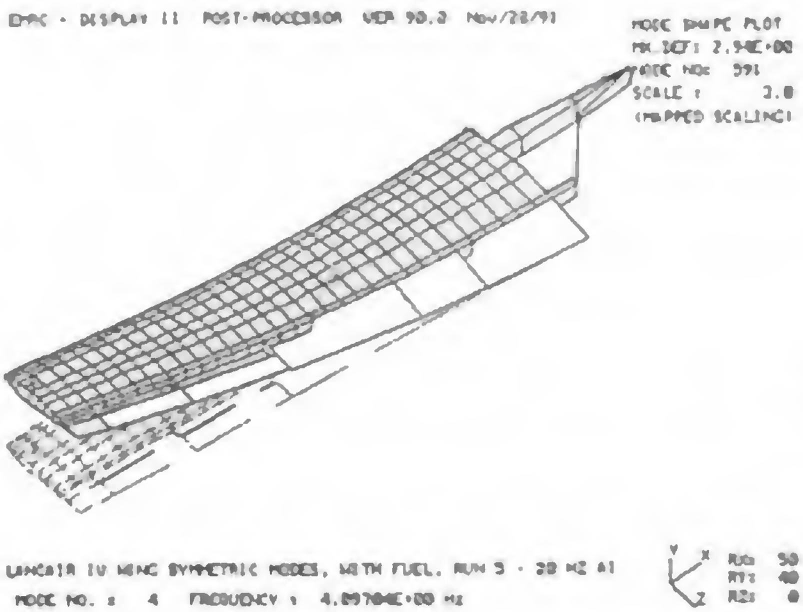
Figure 3. The Finite Element Model of the Lancair IV Wing Showing Nodes and Elements.

A node representing a lumped mass with the rotational inertias of the fuselage is located at the center of gravity of the aircraft. This lumped mass is attached to the wing spars by rigid beams. The motion of this node is restrained so that the symmetric modes and antisymmetric modes are simulated. For the symmetric modes the wing is constrained at this mass node in the fore and aft direction and in rotation about the aircraft vertical and longitudinal axis. Pitch and plung is allowed and these rigid body modes are included in the eigenvalue calculations. The rigid body modes affect most of the other frequencies and it is important to include them in the eigenvalue calculations. It is not important to include them in the flutter analysis. For the antisymmetric modes the mass node is constrained in the fore and aft direction and in rotation about the vertical axis. Rigid body roll and pitch modes are determined. The frequency for the rigid body modes is zero or very close to zero.

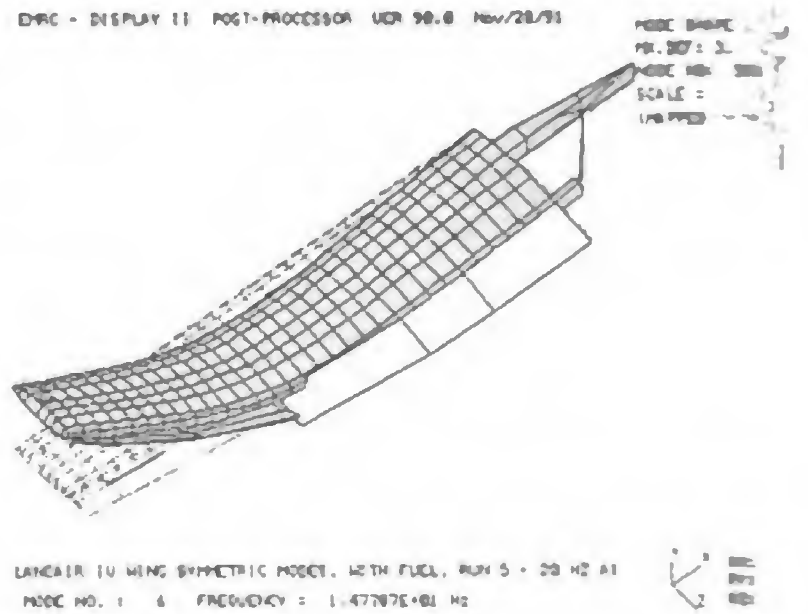
Appendix B shows the NISA input file for the Lancair IV wing and table 5 below displays a small section of the output file. Note that some of the frequencies are 0 or very small. These are the rigid body modes which we just pointed out and are included in the frequency calculations but do not need to be included in the flutter speed analysis. The rigid body modes primarily affect the fundamental mode frequencies. Mode shape plots from NISA are shown in figure 4.

Table 5. Some NISA Output File Data Showing Frequencies and Modal Displacements. Only the Displacements in UY are Used.

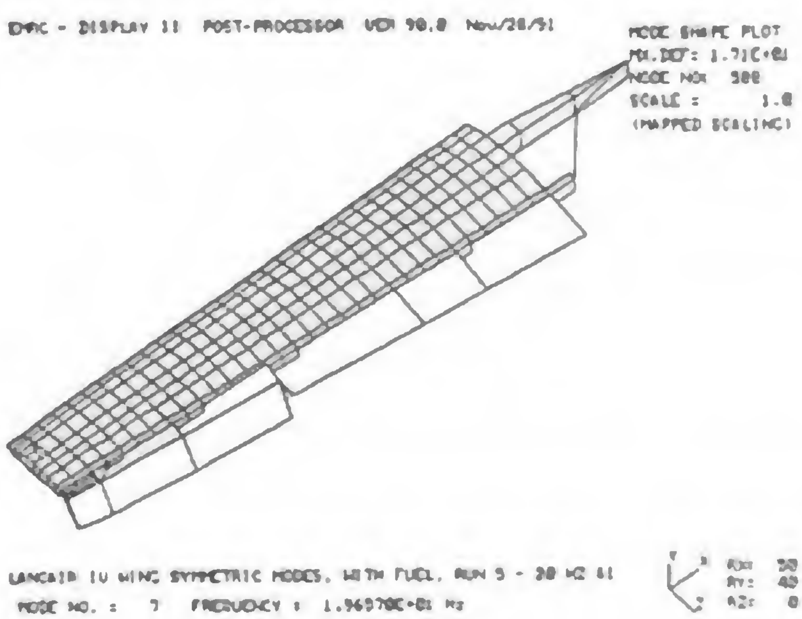
```
   LANCAIR IV WING SYMMETRIC MODES, WITH FUEL, RUN 5 - 20 HZ AILERON
****** E I G E N V A L U E   A N A L Y S I S ******
   MODE       ****** FREQUENCY ******          PERIOD       TOLERANCE
   NUMBER    (RAD/SEC)     (CYCLES/SEC)         (SEC)
       1   0.000000E+00   0.000000E+00   0.000000E+00   7.303898E-06
       2   0.000000E+00   0.000000E+00   0.000000E+00   7.114224E-06
       3   7.166098E-02   1.140520E-02   8.767931E+01   6.193055E-10
       4   2.574244E+01   4.097037E+00   2.440788E-01   1.421156E-06
       5   5.494445E+01   8.744681E+00   1.143552E-01   1.622891E-04
       6   9.285751E+01   1.477873E+01   6.766480E-02   1.277031E-05
       7   1.235088E+02   1.965704E+01   5.087236E-02   5.531483E-05
       8   1.716095E+02   2.731249E+01   3.661328E-02   8.065116E-04
       9   1.833188E+02   2.917609E+01   3.427464E-02   1.712057E-05
      10   2.082029E+02   3.313652E+01   3.017818E-02   6.902626E-04

   *** SUM OF EFFECTIVE MASSES OF ALL MODES  ***
      X-MASS         Y-MASS          Z-MASS           MI-X            MI-Y            MI-Z
   1.555585E+00  4.735672E+00  4.735672E+00  8.221017E+04  1.333158E+05  1.328261E+05
   LANCAIR IV WING SYMMETRIC MODES, WITH FUEL, RUN 5 - 20 HZ AILERON
  MODE NO.   4          FREQUENCY =  4.097037E+00 CYC/SEC
   ****** EIGENVECTOR ******
     NODE       UX              UY              UZ              ROTX            ROTY            ROTZ
       1    -4.47546E-03    -1.26035E-01    -3.36316E-04    -2.08872E-03     4.63132E-05    -4.44148E-03
       2    -4.79474E-03    -1.04085E-01     3.60019E-04    -2.39648E-03     4.44271E-05    -4.44425E-03
       3    -5.35603E-03    -8.05285E-02     8.87345E-04    -2.30687E-03    -6.80148E-05    -4.81107E-03

     604    -1.69036E-02     1.07585E+00     7.74933E-03     4.64410E-04    -7.09476E-04    -2.26512E-02
     605    -2.55016E-02     1.87608E+00     1.19971E-02     7.08165E-04    -7.37930E-04    -2.83494E-02
     606    -3.43916E-02     2.59710E+00     1.45384E-02    -2.69046E-03    -8.57118E-06    -2.90702E-02
     607     0.00000E+00    -1.54716E-01    -1.87172E-03     2.94045E-06     0.00000E+00     0.00000E+00
  *** EMRC NISA ***    -- Version 90.0 (08/20/90-80387/8Meg)          NOV/15/1991  21:39:16
```



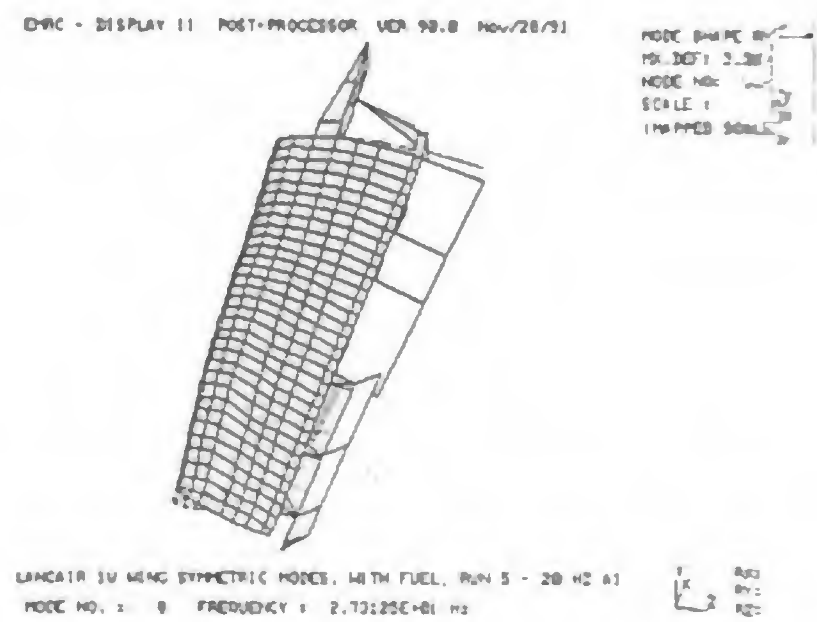

Mode 1. First wing bending




Mode 2. Second wing bending mode




Mode 3. Aileron flapping




Mode 4. Wing twisting

Figure 4. Mode Shapes from the NISA Run for the Lancair IV Wing. Symmetric Case, with 40 lbs of Fuel in Wing and 2800 pound/2 Fuselage Weight.

# CHAPTER 5

## *PREPARING INPUT DATA FOR SAF*

SAF can be used to model a complete aircraft. However, this is normally not done since the model simply becomes very large and difficult to handle. The common procedure is to set up a model of a wing as a single input file or an empennage assembly with aft fuselage as shown in figure 6. To investigate the symmetric and antisymmetric mode shapes, separate files are generated which include the modal deflections and frequencies for each case. Each model requires a definition of the configuration, mass distribution, modal deflections with corresponding natural frequencies.

### 5.1 Geometric Data

The geometry of the wing is entered by defining the corners of panels in cartesian coordinates. Each corner is defined by x, y, and z coordinates in inches. A simple wing is entered as single panel and more complex shapes are divided into several panels as shown in figure 5. Usually the center line of the fuselage defines the x axis of the global coordinated system. For multiple surfaced structures, multiple panels are used with local reference axis as shown in figure 6. A tail assembly showing multiple surfaces is shown in figure 6. Oh! Please don't confuse the coordinate system of the feam with that used in SAF. Remember that feam and the flutter model are two separate models.

### 5.2 Modal Data

Up to 20 mode shapes can be entered per SAF run. However usually less mode shapes are required. For example, for the Lancair IV wing only the first 5 modes were used. These modes consist of:

- First two wing bending modes
- The aileron flapping mode
- The wing torsional mode
- The third wing bending mode

The displacements in the vertical direction (normal to the lifting surface) are determined by the fea and entered into the SAF ASCII file for each mode. The displacement points are positioned along lines which are oriented along the longitudinal axis of the wing as shown in figure 7. Five lines are used in the example of figure 7. Three for the wing and two for the aileron are used. Modal data points along these lines are numbered sequentially starting inboard and going outboard along each line.

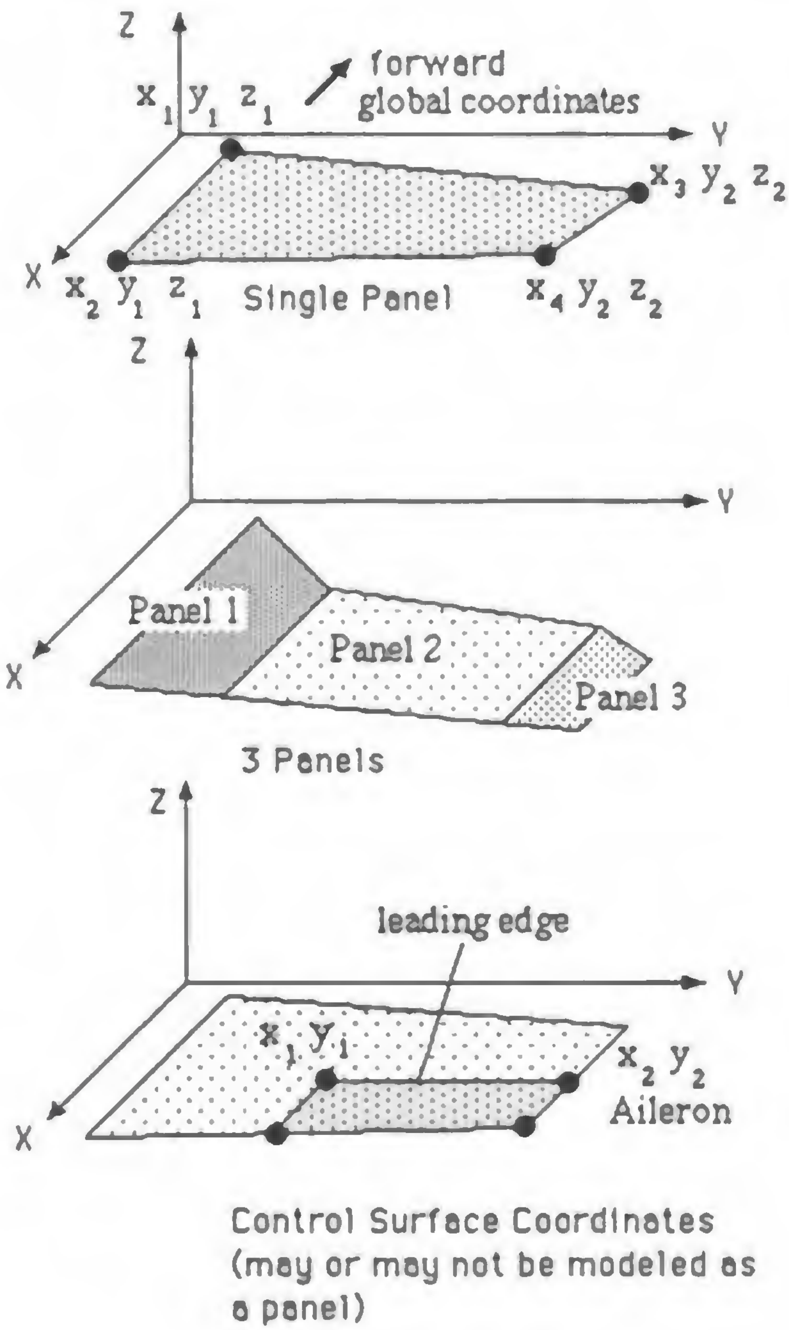



Figure 5. Wing Geometry is Defined by Cartesian Coordinates. Each panel is defined by the location of four corners as shown in the top drawing. Multiple panels are used for complex shapes as shown in the center drawing. And finally, a control surface is defined as shown in the bottem picture. It should be noted that the coordinate system used in the fea will most likely be different than the one used in the flutter analysis.

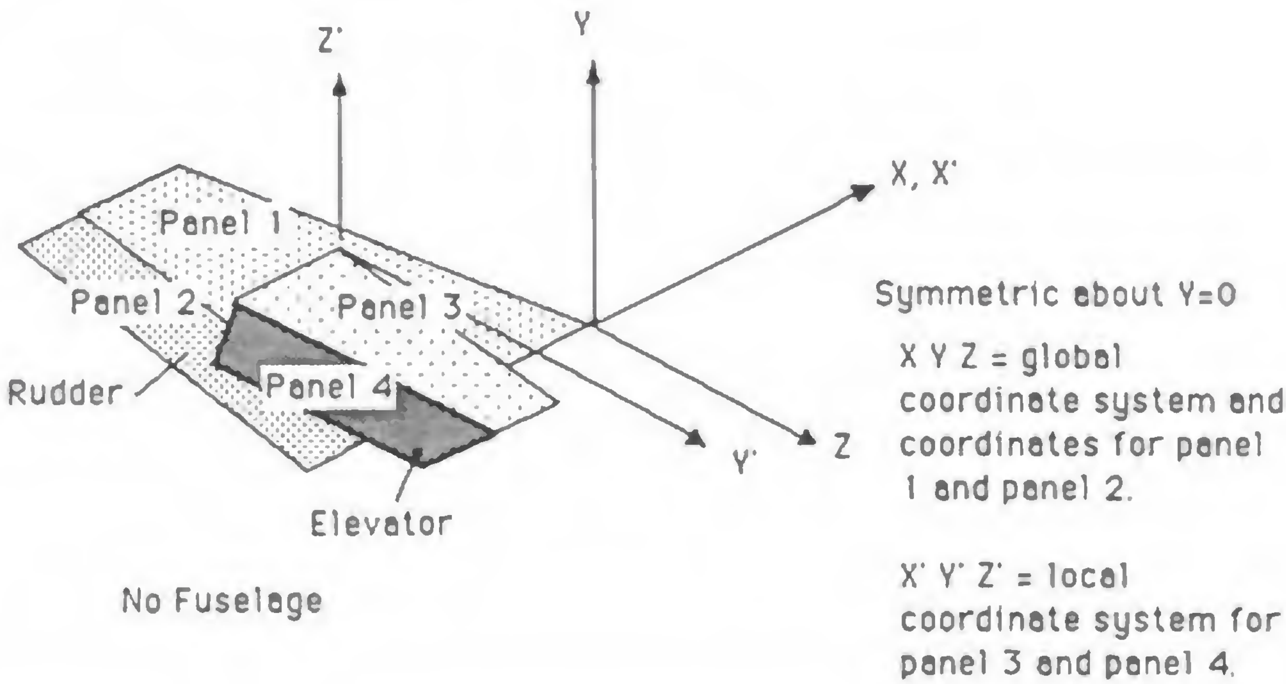



Figure 6. SAF Doublet-Lattice Model for Empennage. The above model is for the Wheeler Express. Only one half of the horizontal tail is modeled and the local coordinate system for the horizontal tail is rotated by 90 degress from the global coordinate system by setting NC in item 52 of Chapter 6 equal to 89.9.

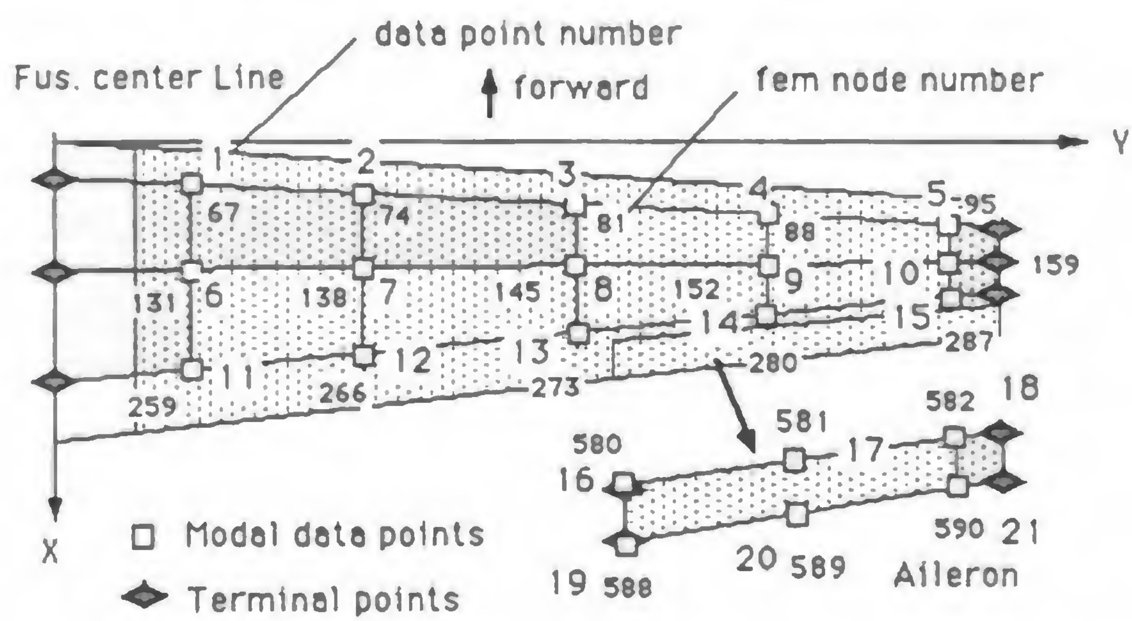
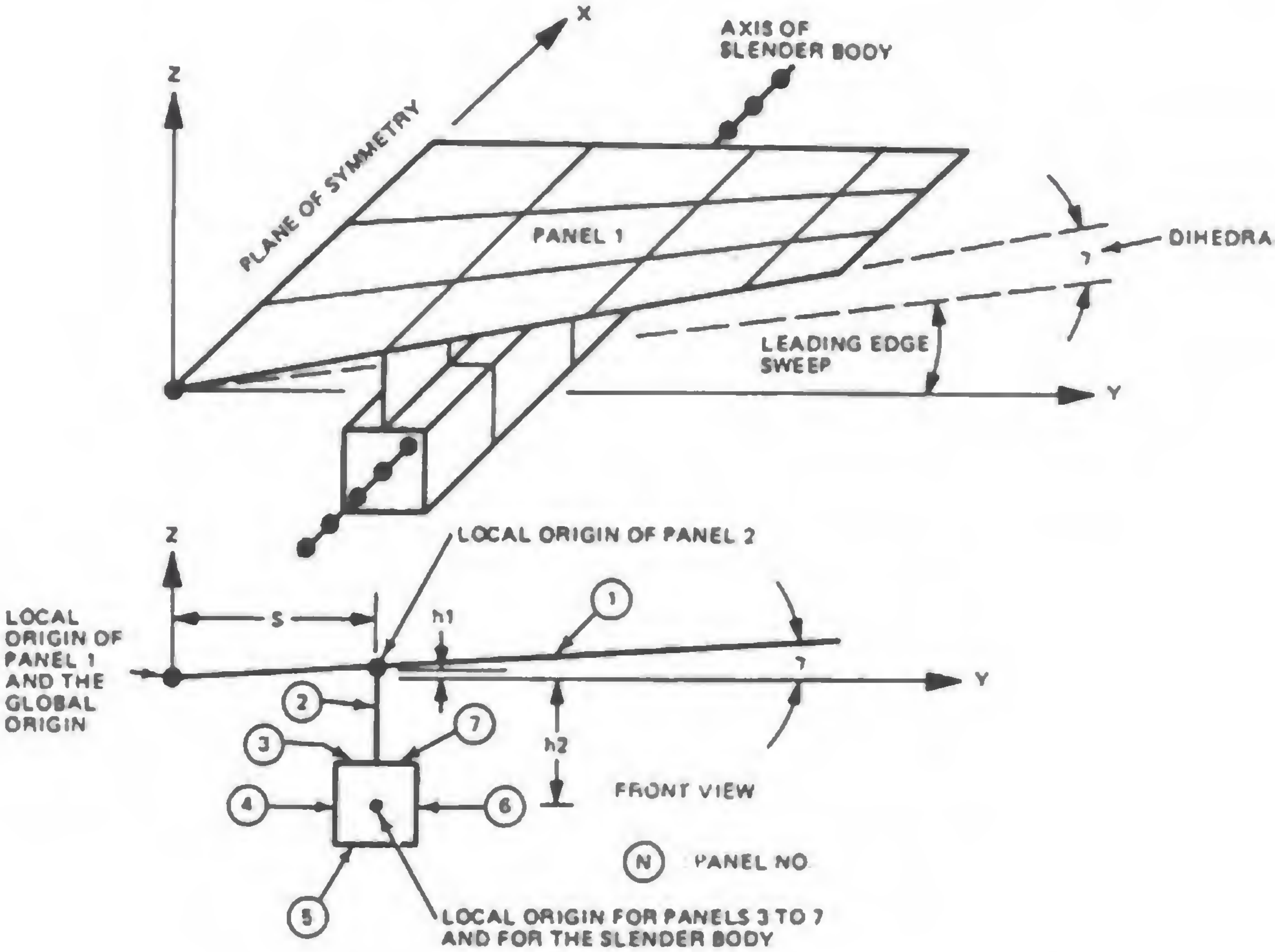



Figure 7. Modal data is entered along point located on lines. These lines run along the span of the wing as shown in the picture above. This example is for the Lancair IV Wing.




REFERENCE COORDINATES

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| XO(1) | = 0. | YO(1) | = 0. | ZO(1) | = 0. | GGMAS(1) | = γ |
| XO(2) | = 0 | YO(2) | = S | ZO(2) | = h1 | GGMAS(2) | = −90° |
| XO(3) | = 0. | YO(3) | = S | ZO(3) | = -h2 | GGMAS(3) | = 0 |
| XO(4) | = 0. | YO(4) | = S | ZO(4) | = -h2 | GGMAS(4) | = 0 |
| XO(5) | = 0. | YO(5) | = S | ZO(5) | = -h2 | GGMAS(5) | = 0 |
| XO(6) | = 0. | YO(6) | = S | ZO(6) | = -h2 | GGMAS(6) | = 0 |
| XO(7) | = 0 | YO(7) | = S | ZO(7) | = -h2 | GGMAS(7) | = 0 |
| XBO(1) | = 0. | YBO(1) | = S | ZBO(1) | = -h2 | | |

NOTE: INTERFERENCE PANELS ASSOCIATED WITH A BODY MUST HAVE THE SAME LOCAL ORIGIN AS THE AXIAL ELEMENTS OF THAT BODY. ALSO, THE BODY CAN ONLY BE TRANSLATED INTO THE GLOBAL SYSTEM, I.E. GGMAS(3) THROUGH GGMAS(7) = 0.

Figure 8. Modeling of Multipanels is Shown for an External Wing Store in the Figure Above. Panels may be Defined with Global or Local Coordinate Systems.

Start with the front line and move aft. Normal deflections in inches and pitch rotations in radians (rotation about the y-axis) can be entered. However, it is more convenient to use deflections only. SAF allows you to use both types of entries.

The SAF automated modal interpolation routine computes the modal deflections at the required downwash and lift points for any of the two available aerodynamic routines. The routine also permits the user to specify the discontinuous downwash associated with control surfaces attached to the main aerodynamic surface. The input data requirements associated with modal interpolation and control surface representation are illustrated in figures 5, 6, 7, and 8.

The straight lines along which the input modal data are prescribed must satisfy the requirements presented below.

- Each line must contain at least two points at which modal data are specified.
- No two lines may cross. However, two lines may intersect at a shared terminus. Three or more lines may not have a shared terminus.
- Two lines which share a common terminus must be numbered sequentially (inboard first). Also, the shared terminus must be the outboard terminus of the first line and the inboard terminus of the second line.
- Modal data must be specified at a shared terminus. But the data must be specified only once. The common point is counted as belonging to the first (inboard) line and is not counted as a point on the second (outboard) line.
- Lines must be numbered such that any imaginary streamwise section encounters lines in ascending order only.
- Except when the elastic axis representation is used, lines must be arranged such that any imaginary streamwise section encounters at least two lines.
- No streamwise lines are permitted.

## 5.3 Doublet-Lattice Elements

Each panel is divided into elements which are numbered starting at the inboard leading edged and going aft as shown in figure 9. It is desirable to use an element aspect ratio of one to two. The aspect ratio is defined as the chordwise length divided by the spanwise width. For multiple lifting surfaces it is essential that spanwise element strips are aligned on all surfaces in the spanwise direction. See figure 10.

## 5.4 Mass Matrix

The modal displacements calculated by NISA386 are mass normalized such that the mass is entered as a unit diagonal matrix multiplied by $12^2 = 144$. The value of 144 assures that the proper units for SAF. Only the in-lb-sec system can be used with SAF.

Only non-zero values on the diagonal of the mass matrix are entered. The order of the matrix is equal to the number of modes used in the analysis which is equal to the number of entries. Only the numbers on the diagonal are entered since all other numbers are zero.

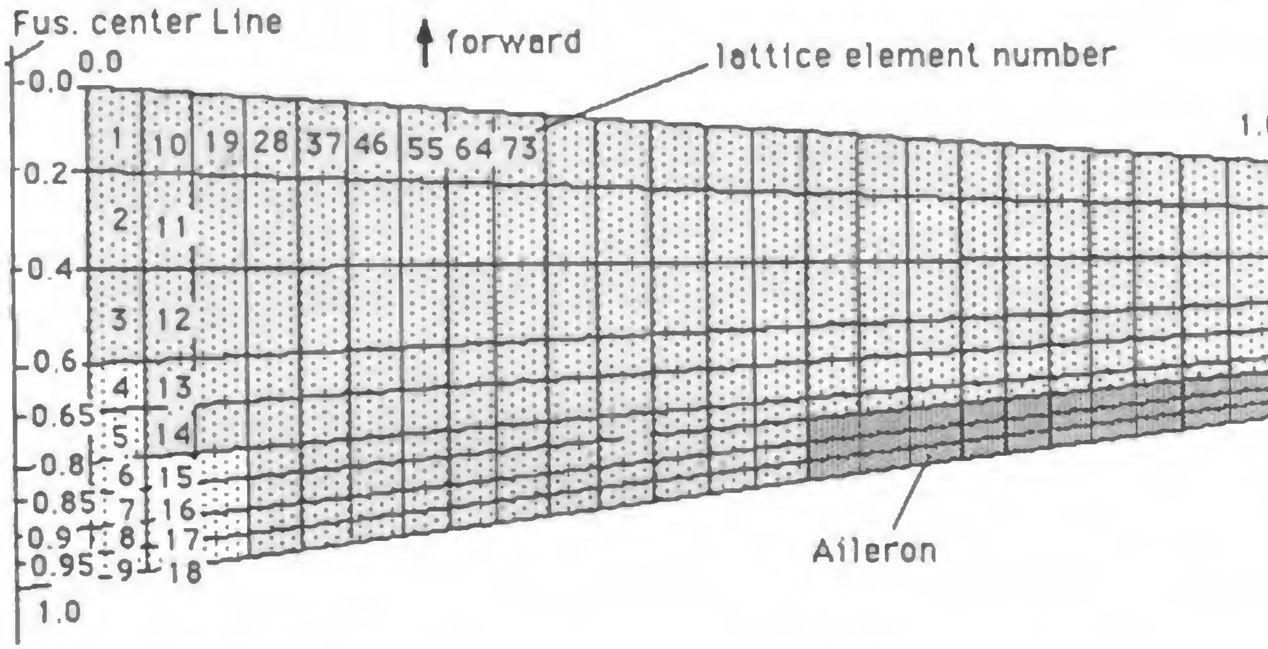



Figure 9. Doublet-Lattice Elements are Numbered Sequentially Starting at the Inboard Leading Edge and going Aft and then Outboard. The Example Above Contains 216 Elements. A Minimum of Three Chordwise Rows of Elements Should be Used for Control Surfaces such as the Aileron Above.

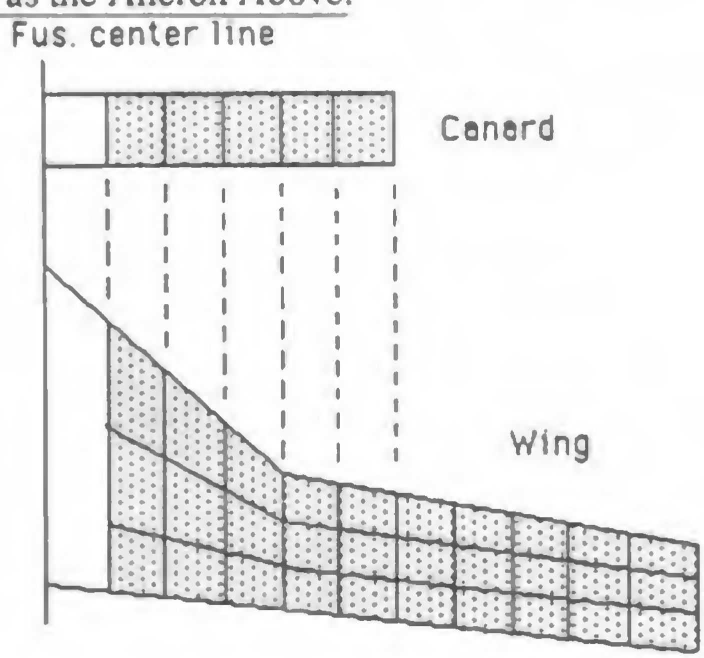



Figure 10. Lattice Elements for Multiple Surfaces must be Alinged in the Spanwise Direction as Shown Above. It is also Important to Aligne the Elements in the Chordwise Direction such as the Wing with Aileron Model shows in Figure 9 Above.

# CHAPTER 6

## *SAF INPUT FILE*

An ASCII file is created using Qedit. To make a file, simply type *Q file name* . Press the *Ins* key and using *Caps Lock* , type in the required data. The cursor location, row and column number are displayed at the top of the screen. With the free format used in SAF the colume that the data is entered is not important. However, all data entry must be separated by either a comma or a space.

**Caution:** Do not use tabs to input data since the Fortran code used in SAF does not recognize the tab keystroke.

### 6.1 Data Definition

Line 1 - 7. A number 1 is entered in the first entry of the first line. The next six lines are used to describe the input file. Such information as file title, size of problem such as the number of lattice elements and mode numbers, load such as fuel or no fuel in wing, symmetric or antisymmetric mode, and aircraft weight and airspeed are entered.

1. Line 8.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Example Value | -1, | 5, | 1, | 6, | 1, | 0, | 0, | 0, | 0, | 0 |

entry 1 — -1 = P-K flutter analysis
0 = Pressure calculations only
1 = K flutter analysis
2 = Divergence analysis

entry 2 — Number of vibration modes to be used in the analysis. 20 max.

entry 3 — Number of lifting surfaces. For doublet lattice the maximum number is 30. In the doublet lattice analysis, they aerodynamically interact, whereas in other analysis they do not.

entry 4 — Number of reduced velocities for which aerodynamic pressures and/or forces are to be computed. Forces will be interpolated when line 8, entry 1 = 1 and line 9, entry 3 = 1.

If line 8, entry 1 = -1 let this value = 6.
If line 8, entry 1 = 0 or 1 then this value must be equal to or less than 30.
If line 8, entry 1 = 2, let value = 1.

entry 5 — Number of air densities for which the flutter or divergence analysis will be run. Maximum number is 10.
If line 8, entry 1 = 0, let value = 0.

entry 6 = 1 — List aerodynamic forces, computed and interpolated, at the tested reduced frequencies of the generalized aerodynamic force interpolation.
= 0 — No display.

entry 7 = 1 — List calculated pressures.
= 0 — No display.

entry 8 = 1 — List lift and moment coefficients.
= 0 — No display.

entry 9 = 1 — Frequency independent additions to the aerodynamic matrix QBAR are to be read as data.
= 0 — No such additions are to be made.

entry 10 = 0 — No display.

2. Line 9.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Example Value | 1, | 0, | 1, | 0, | 0, | 1, | 0, | 0, | 0, | 0 |

entry 1 — Index of modal frequency to be used as a normalization factor in the flutter determinant. Use first non-zero modal frequency. Suggest value = 1.

entry 2 = 1 — Flutter determinant is formulated as the product of the inverse of the generalized stiffness matrix and the sum of the generalized mass and aerodynamic force matrices.
= 0 — Determinant is formulated as the product of the inverse of the sum of the generalized mass and aerodynamic matrices, and the stiffness matrix. Note that if zero-frequency modes are present in the analysis this value must be 0.
If line 8, entry 1 does not equal 1 or 2 let this value = 0.

entry 3 = 1 — Generalized aerodynamic force interpolation is used.
= 0 — Aerodynamic forces are directly computed at each reduced frequency.
If line 8, entry 1 = -1 let value = 1.
If line 8, entry 1 = 0 or 2 let value = 0.
If line 8, entry 1 = 1 let value = 0 or 1.

entry 4 = 0

amortiguamiento estructural

| | | |
|---|---|---|
| entry 5 | = 1 | Velocity scale in the flutter solution plots is in true airspeed. |
| | = 0 | Scale is in equivalent airspeed. |
| entry 6 | = 0 | No structural damping is added to complex stiffness matrix. |
| | = -1 | Different damping values are added to the complex stiffness matrix for each mode. |
| | = 1 | The same value of damping is added for all modes. |
| entry 7 | = 1 | Display the number of iterations required to obtain each root in the P-K flutter analysis. |
| | = 0 | No display. |
| | | If line 8, entry 1 does not equal - 1 let value = 0. |
| entry 8 | = 0 | The value of the root at the previous velocity is used as the root estimate. |
| entry 9 | = 1 | Order the roots after solution by the P-K flutter analysis. |
| | = 0 | No ordering. |
| | | If line 8, entry 1 = -1, value = 1. |
| entry 10 | = 1 | Display the root iterations in the P-K flutter analysis (line 8, entry 1 = -1) or display intermediate results of the K flutter analysis (line 8, entry 1 = 1). |
| | = 0 | No display. |

3. Line 10.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Example | 1, | 0, | 0, | 1, | 0, | 0, | 0, | 0, | 0, | 0 |

Value

| | | |
|---|---|---|
| entry 1 | = 1 | Subsonic doublet lattice (Rodden). |
| entry 2 | = 0 | Compute AIC arrays and place on an output data set to save for future as well as present use. |
| | = 1 | AIC arrays exist on an input data set and do no need to be recomputed. |
| entry 3 | = 1 | Display modal input data. |
| | = 0 | No display. |
| entry 4 | = 1 | Display interpolated modal data. |
| | = 0 | No display. |
| entry 5 | | Number of modal elimination cycles requested for the flutter analysis. Minimum number is 0. Maximum is 25. |
| entry 6 | | Number of stiffness variation cycles requested for the flutter analysis. Minimum number is 0. Maximum is 20. |
| entry 7 | | Index of the vibration mode whose stiffness is to be varied in the flutter analysis. If line 10, entry 6 = 0 let value= 0. |

| | | |
|---|---|---|
| entry 8 | = 1 | Display eigenvectors |
| | = 0 | No display. |
| | | If line 8, entry 1 = 0 or 2 let value = 0. |
| | | If line 8, entry 1 = -1 The eigenvectors for the critical flutter root in a user-chosen velocity interval is displayed. |
| | | If line 8, entry 1 = 1, the eigenvectors for all roots between user-chosen reduced velocities and real frequencies are displayed. |
| entry 9 | = 1 | Display physical vectors corresponding to the displayed modal eigenvectors. |
| | = 0 | No display. |
| entry 10 | = 1 | Display flutter determinant matrix in K flutter analysis. |
| | = 0 | No display. |
| | | If line 8, entry 1 = -1 or 0 let value = 0. |

4. Line 11.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Example | 0, | 0, | 0, | 0, | 0, | 0, | 0 |
| Value | | | | | | | |

| | | |
|---|---|---|
| entry 1 | = 1 | User will input changes to the generalized massed and the modal frequencies. |
| | = 0 | No changes. |
| entry 2 | = 1 | User will input revisions to the generalized stiffness matrix. |
| | = 0 | No revisions. |
| entry 3 | = 1 | Steady state analysis. |
| | = 0 | Oscillatory analysis. |
| | | If line 8, entry 1 = 2, let value = 1. |
| entry 4 | = 1 | User will input factors to scale the computed aerodynamic forces. |
| | = 0 | No factors. |
| entry 5 | = 0 | |
| entry 6 | = 0 | |
| entry 7 | = 1 | Display geometric data associated with doublet elements. |
| | = 0 | No display. |

5. Line 12.

| Entry | 1 |
|---|---|
| Example | 21 |
| Value | |

NC = Total number of modal degrees of freedom (DOF) used to define a mode shape for the flutter math-model. (Include all surfaces plus body)
DOF=Number of data points x DOF/node.
For translation only DOF/node = 1.
For translation and rotation DOF/node = 2.

6. Line 13. Maximum of 7 entries per line.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Example | -0.0722, | 0.222, | 0.7832, | 1.655, | 2.673, | -0.048, | 0.245 |
| Value | 0.810, | 1.687, | 2.698, | -0.0288, | 0.272, | 0.850, | 1.722 |
| | 2.729, | 1.0759, | -0.0072, | 2.5973, | -0.00713, | 1.915, | -0.00712 |

Repeat the following set of entries (seven values per card) for K=1 . . . NC

QZ(K,I) = Modal deformations for the i'th mode and k'th degree of freedom. These modes are ordered such that deformation on each surface and each body are input as a block. The blocks are input in the same order as the geometry data for the surfaces (and bodies) will be entered.

Within a block, deformations on the surface precede those on its control surface.
For a surface, the points at which deformations are specified lie on sets of lines oriented spanwise and sequenced forward to aft. On each such line, the points are ordered inboard to outboard. the deformations must be ordered correspondingly.
For a surface, elastic axis representation may be used along a line. At each point a displacement in inches and an accompanying pitch rotation in radians are input as consecutive deformations.
For a body, displacements along a center line are input, forward to aft.
An illustrative example showing the ordering for a lifting surface with a control surface is shown in figure 7.

7. One entry.

| Entry | 1 |
|---|---|
| Example | 2 |
| Value | |

NLINE = Number of lines to follow containing all non-zero elements of the generalized mass matrix.

8. Mass Matrix Maximum of 9 entries per line.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Example | 1, | 1, | 144.0, | 2, | 2, | 144.0, | 3, | 3, | 144.0 |
| Value | 4, | 4, | 144.0 | | | | | | |

Repeat the following item for n=1 . . . .NLINE and enter three groups of values per line.
I = Row index of generalized mass matrix.
J = entry index of generalized mass matrix.
WW(I,J) = Generalized mass matrix of each non-zero value, pounds.

9. Modal Frequencies. Enter seven values per line and repeat the following item for I=1 . . . .number of modes.

| Entry | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Example Value | 4.097, | 14.778, | 19.657, | 27.31, | 29.176 |

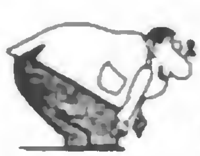
OMG (I) = Modal frequencies in proper sequence, Hz.

10. Omit this entry if line 8 entry 1 = -1. Ten entries per line. Repeat the following item for I = 1 . . . . line 8, entry 2.
IFLMD(I) = Indices of the modes to be used in the flutter analysis. Knowledge of the sequence and nature of the modes computed in the vibration analysis is needed to make an intelligent selection here of modes that will be important to the flutter mechanism.

11.

| Entry | 1 | 2 |
|---|---|---|
| Example Value | 19.25, | 0.4 |

BR = Reference semichord, inches. (Usually the mean geometric chord.)
FMACH = Free stream mach number.



12. Take a Peek. No entry. Based on the type of analysis chosen, the following items are selected.
For P-K flutter analysis (line 8, entry 1 = -1) skip to item 15.
For steady state pressures (line 8, entry 1 = 0) and (line 11, entry 3 = 1) omit items 13 to 47 and go to item 48. Otherwise for oscillatory analysis (line 11, entry 3 = 0) continue with next item.
For K flutter analysis (line 8, entry 1 = 1) continue with item 13.
For divergence analysis (line 8, entry 1 = 2) omit items 13 to 26. and go to item 27.

13. Enter 7 values per line.
VEC(I) = Reduced velocity to be used in the K-flutter analysis or pressure calculations.

14 Take a Peek. No entry. If generalized aerodynamic force interpolation is used (line 9, entry 3 = 1) go to item 15. If aerodynamic forces are directly computed at each reduced velocity (line 9, entry 3 = 0) go to item 17.

15 P-K flutter analysis parameters.

| Entry | 1 | 2 | 3 |
|---|---|---|---|
| Example | 20, | 50.0, | 24.0 |
| Value | | | |

NV = Number of velocities at which the analysis is to be performed initially, maximum value is 20.

V1 = Lowest velocity at which the analysis is to be performed, knots.

DV = Interval between initial velocities at which the analysis is performed, knots.

V1 and DV are in true airspeed. The analysis is initially done at a set of NV velocities given by V(I) = V(I-1) + CV, where V(1) = V1. The program detects undulations in the damping and frequency variations with velocity and, under certain conditions, calculates additional solutions at velocities given by V(J) = V(J-1) + DV/5. It is suggested that V1 be chosen to be at least 100 and that DV be less than 250.

16 Generalized Aerodynamic Force Interpolation Parameters.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Example | 0.02, | 0.50, | 1.0, | 5.0, | 10.0, | 15.0, | 50.0 |
| Value | | | | | | | |

TOL = Tolerance used for testing the goodness of fit of this interpolation.

RVBC = Six (I= 1,6) reference values of reduced velocity from which the EAS is (or known points of) the interpolation is derived.

A nominal value of TOL = 0.02 is recommended. The RVBC(I)'s should span the entire range of reduced velocities required for the flutter analysis. For the P-K flutter analysis, the following approximations should be used.

RVBC(1) is less than or equal to 1.69x 12 x VMIN /(BRxWMIN)

Where,

VMIN = V1, knots

VMAX = V1 + (NV-1)xDV, knots

WMAX and WMIN are the maximum and minimum modal frequencies in radians/second.

BR is the reference semichord in inches.

17 Take a Peek. No entry. If no changes are included in the generalized masses and modal frequencies (line 11, entry 1 = 0) skip to item 22. Otherwise (line 11, entry 1 = 1) continue with item 18 below.

18. One line.

| | |
|---|---|
| MADD | = Number of lines to follow containing changes to the generalized mass matrix. |
| IADO | = Number of lines with changes to the modal frequencies. |
| MSYM | = 0 If changes to mass matrix are symmetric (WW(I, J) = WW(J, I)). |
| | =1 If changes to mass matrix are not symmetric. |

19 Number of lines is MADD. Repeat the following item for K=1 . . . MADD.

| | |
|---|---|
| I | = Row index of the altered element of the generalized mass |
| J | = Column index of the altered element of the generalized mass. |
| WW(I, J) | = New value of the altered elements of the generalized mass. If MSYM = 0, specify only the upper triangular elements. |

20 Take a Peek. No entry. If the number of data lines with changes to the modal frequencies are 0 (IADO = 0) skip item 21 and go to item 22.

21 Number of lines is IADO. Repeat the following item for K = 1 . . . . IADO.

| | |
|---|---|
| I | = Index of the altered modal frequencies. |
| CMG(I) | = New value of the altered modal frequencies, Hz. |

AMORTIGUAMIENTO

22 Take a Peek. No entry. If no structural damping is added to the stiffness matrix (line 9, entry 6 = 0) go to item 26. If line 9, entry 6 = -1 or 1 continue with item 23.

If different structural damping values are added to the complex stiffness matrix in various modes (line 9, entry 6 = -1),skip to item 24.
If the same value of damping is added for all modes (line 9, entry 6 = 1) enter data for item 23 and omit items 24 and 25.

23 One line. AMORTIGUAMIENTO HISTÉRESIS
GDD = Hysteretic structural damping to be applied to all modes. The diagonal of the stiffness matrix will be scaled by (1+ *GDD). Now go to item 26.

24 One line.

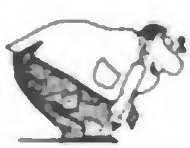
NCD = Number of individual modes for which structural damping will be specified.

25 Number of lines is NCD. Repeat the following item for K=1 ..... NCD

I = Mode index of hysteretic damping.
GDP(I) = Value of hysteretic damping applied to a mode.

26 One line.
The following item provides print-plot parameters for displaying flutter solutions - damping and frequency as functions of velocity.
GMAX = Maximum value of damping scale.
GMIN = Minimum value of damping scale, knots.
VMAX = Maximum value of velocity scale, knots.
FMAX = Maximum value of frequency scale, Hz.
27 Maximum of 7 entries per line. Maximum total number is 10 entries. Reference to sea level density. E.g. sea level is 1.0.

RHOR(I) = Density ratios.



28. Take a Peek. No entry. If user inputs factors to scale the computed aerodynamic forces (line 11, entry 4 = 1) enter data for items 29 through 33. Otherwise (line 11, entry 4 = 0) go to item 34.

29. One line.
NQWT = Number of lifting surfaces for which the generalized aerodynamic forces due to specified modes will be eliminated.
NGE = Number of lifting surfaces for which the generalized aerodynamic forces will be multiplied by a factor not equal to 1.

30. Take a Peek. No entry. If generalized aerodynamic forces are to be eliminated for a number of lifting surfaces (NQWT greater than zero) enter data for item 31. Otherwise NQWT = 0, skip item 31 and go to item 32.

31. Repeat the following entries for I=1 . . NQWT. Enter 10 values per line.

| | | |
|---|---|
| ISF | = Surface index. |
| NISF | = Number of modes for which the generalized aerodynamic force of surface ISF will be eliminated. |
| NQA(J) | J=1 . . . . . NISF, modal indices for generalized aerodynamic elimination. |

For the ISF surface, QBAR( K, NQA(J) ), QBAR( NQA(I), K), and QEAR( NQA(J) ) are to be eliminated.

32. Take a Peek No entry. If generalized aerodynamic forces are to be multiples of a factor other than one for a number of lifting surfaces (NQE greater than zero) enter data for item 33. Otherwise NQE = 0 skip to item 34

33. One line. Repeat the this item for J=1 . . . . . . .NQE

| | | |
|---|---|
| I | = Surface index |
| QWT(I) | = Weighting factor for the generalized aerodynamic force for the i'th surface. |

34. Take a Peek. No entry. If frequency-independent additions to the aerodynamic matrix are to be included (line 8, entry 9 = 1) enter data for items 35 and 36. Otherwise (line 8, entry 9 = 0) skip to item 37.

35. One line.

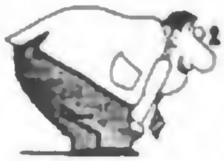
| | | |
|---|---|---|
| NADDF | = Number of lines to follow on which additions to the flutter determinant's aerodynamic matrix, QEAR, will be input. | |
| NSYM | = 1 | Specify all non-zero additions. |
| | = 0 | Additions are symmetric. Supply only the upper triangular array. |

36 Number of lines is NADDF. Repeat the following item for
K = 1 . . . . . . . NADDF

I = Row index of additions to the aerodynamic matrix.

J = column index of additions to the aerodynamic matrix.

DETAD(I,J) = Value of the additions to the flutter determinant's aerodynamic matrix. Specify both a real and imaginary part of the value. These additions are frequency independent additions to the aerodynamic matrix. QBAR + DETAD(REAL) / K**2 + IMAG * DETAD(IMAG)/ K, where K is the reduced frequency.



37 Take a Peek No entry. If revisions to the generalized stiffness are to be included (line 11, entry 2 = 1) enter data for the items 38 and 39, otherwise skip to item 40.

38 Number of lines is 1.

NADDS = Number of data cards to follow containing changes to the stiffness matrix.

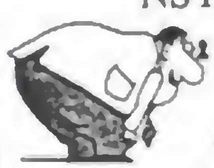
NSYM = 0 If changes are symmetric (B(I, J) = B(J, I).
= 1 If changes are not symmetric.

39 Repeat the following item for K = 1 . . . . . . . NADDS

I = Row index of new stiffness matrix component.

J = Column index of new stiffness matrix component.

B(I, J) = New values of the complex stiffness matrix. The new specified components replace the original components an original component still applies if a new value is not specified. Note that the values may be complex numbers. If NSYM = 0, only the upper triangular elements need be specified.



40 Take a Peek No entry. If stiffness variation is to be included in the flutter analysis (line 10, entry 6) is greater than 0 enter data for item 41. Otherwise skip to item 42.

41 Stiffness Variations. Repeat this item for I=1 . . . . . . .line 10, entry 6. After the initial flutter or divergence analysis is performed, additional (line 10 entry 6) analyses - up to a maximum of 20 - are run with the stiffness of mode (line 10, entry 7) varied by ratioing its modal frequency various selected amounts.

RATCM(I) = Desired ratios of modal frequencies.

42. Take a Peek. No entry. If modal elimination is to be included in the flutter analysis (line 10, entry 5 greater than 0), enter data for item 42. Otherwise omit item 43 and go to item 44.

43. Modal Elimination Data. Repeat this item for I=1 . . . . line 10, entry 5 value. A flutter or divergence analysis is performed using the modes selected in item 10, after which additional (line 10, entry 5) analyses - up to a maximum of 25 - are run with different selected combinations of modes deleted from the analysis at each re-run.

NOTIR = Number of modes to be eliminated.

NINZ(J, I) J=1 . . . . . .NOTIR, indices of the modes to be eliminated.

44. Take a Peek. No entry. If eigenvectors are not to be displayed (line 10 entry 8 = 0) or if divergence analysis is to be performed (line 8, entry 1 = 2), omit items 45 through 47 and go to item 48.

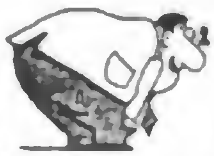
45. One line.

VA = Lower bound of the range over which eigenvectors are to be calculated.

VB = Upper bound of the range over which eigenvectors are to be calculated.

If line 8, entry 1 = -1, the range is velocity, knots.

If line 8, entry 1 = 1, the range is reduced velocity, V /(B x OMEGA).



46. Take a Peek. No entry. If P-K flutter analysis is to be performed (line 8, entry 1= -1) skip to item 48.

47. One line.

FLO = Lower bound of frequency range over which eigenvectors are to be displayed, Hz.

FHI = Upper bound of frequency range over which eigenvectors are to be displayed, Hz.

48. One line.

FL = Reference chord to be used in computing the total surface aerodynamic force coefficients, inches.

ACAP = Reference area to be used in computing the total surface aerodynamic force coefficients, sq.inches.

Usually the mean aerodynamic chord is used for FL and the total surface area is used for ACAP. You can set ACAP= 1.0.

49 One line.

| | | |
|---|---|---|
| NDELT | = 1 | Aerodynamics are symmetric about y = 0. |
| | =-1 | Aerodynamics are antisymmetric about y = 0. |
| | = 0 | No symmetry about y = 0 (single surface). |
| NP | = Total number of panels on all lifting surfaces and all optional interacting bodies. 50 max. Each surface is divided into major trapezoidal subdivisions called panels based upon geometrical discontinuities. See figures 5 and 6. The parallel edges are parallel to the free stream. | |
| NB | = Number of bodies that aerodynamically interact with the surfaces. Maximum number is 20. | |
| NCORE | = Size of the problem being solved = N x M. The variable N is the number of elements on the lifting surface and the bodies, N = sum from 1 to NP. The variable M is the number of modes (line 8, entry 2). Note that maximum value of N is 400. | |
| N3 | = 1 | Display pressure influence coefficients. |
| | = 0 | No display. |
| N4 | = 1 | Display influence coefficients relating downwash on lifting surfaces to body element pressures. |
| | = 0 | No display. |
| N7 | = 1 | Calculate pressures and generalized aerodynamic forces. |
| | = 0 | Cease computations after the influence coefficients are determined. |

50 One line for each panel.

<u>Repeat the items 50 through 54 for each panel for I=1 . . . . . NP.</u> See item 49.

The proper sequence is.

1. Vertical panels on the symmetry plane, Y = 0 such as a conventional vertical tail.
2. Panels on the other surfaces, such as the wing.
3. Body interference panels.

*The following coordinates are in the global (aircraft) system and indicate the position of the origin of the local coordinate system for each panel.*

X0(I) = x reference coordinate of i'th panel, inches.

Y0(I) = y reference coordinate of i'th panel, inches.

| | |
|---|---|
| Z0(I) | = z reference coordinate of i'th panel, inches. |
| GGMAS(I) | = Dihedral for the i'th panel, degrees. (zero for body interference panels) |

Modal interpolation is performed before these reference values are used to translate and rotate the panels. Consequently, the standard practice is to assign each panel on a particular surface the same reference values. See figure 5. Also, each interference panel associated with a particular body is assigned the same reference values.

51. One line.

| | |
|---|---|
| X1 | = x edge coordinate of the i'th panel,inches. |
| X2 | = x edge coordinate of the i'th panel, inches. |
| X3 | = x edge coordinate of the i'th panel, inches. |
| X4 | = x edge coordinate of the i'th panel, inches. |
| Y1 | = y edge coordinate of the i'th panel, inches. |
| Y2 | = y edge coordinate of the i'th panel, inches. |

The coordinates (X1, Y1) and (X2, Y1) are the leading and trailing inboard corners. The coordinates (X3, Y2) and (X4, Y2) are the leading and trailing outboard corners. See Figure 5. These coordinates are specified in the *local axis system* of each surface.

52. One line.

| | |
|---|---|
| Z1 | = Vertical coordinate of the inboard edge of i'th panel, inches. |
| Z2 | = Vertical coordinate of the outboard edge of i'th panel, inches. |
| NS | = Number of element boundaries in the spanwise direction. Maximum number is 50. (NS = 2 for each body interference panel) |
| NC | = Number of element boundaries in the chordwise direction. Maximum number is 50. |
| COEFF | = 0. |

The panel is divided into a number of smaller trapezoids, called elements, by lines of constant percent panel chord and of constant percent panel span. See Figures 9 and 10.

53. 6 values per line. Repeat for J=1 . . . . . . .NC

| | |
|---|---|
| TH(J) | = Chordwise element boundaries for the i'th panel in fraction of chord. Normally TH(1) = 0.0 and TH(NC) = 1.0 |

54. Enter 6 values per line. Repeat for j=1 . . . . . NS

| | |
|---|---|
| TAU(J) | = Spanwise element boundaries for the panel in fraction of span. Normally TAU (1) = 0.0 and TAU (NS) = 1.0 |

55 Take a Peek. No entry. If there are bodies that aerodynamically interact with the surfaces (NB greater than zero) enter data for items 56 through 59. Otherwise NB=0 omit these items and go to item 60.

56 One line per each body. Repeat the following four items for each body for I=1,. . . . . . . .,NB

A body has either vertical or lateral vibrations. To model a physical body having both degrees of freedom, two bodies must be input. All vertically vibrating bodies must be input before laterally vibrating bodies.

| | |
|---|---|
| XBO(I) | = x global reference coordinate for the i'th body, inches. |
| YBO(I) | = y global reference coordinate for the i'th body, inches. |
| ZBO(I) | = z global reference coordinate for the i'th body, inches. |

See figures 6 and 8. These data should agree with item 49 for the associated body interference panels.

57 One line for each body.

| | | |
|---|---|---|
| ZSC | = Local vertical coordinate of the i'th body axis, inches. | |
| YSC | = Local lateral coordinate of the i'th body axis, inches. | |
| NF | = The body is divided along its axis into a number of elements equal to (NF - 1). | |
| NZ | = 1 | Body is vibrating vertically. |
| | = 0 | Body is not vibrating vertically. |
| NY | = 1 | Body is vibrating laterally. |
| | = 0 | Body is not vibrating laterally. NZ must not equal NY. |
| COEFF | = 0 | |
| MRK(I, 1) | = Index of the first panel element on the interference panels associated with the i'th body. | |
| MRK(I, 2) | = Index of the last panel element on the interference panels associated with the i'th body. | |

58 One line. Repeat for J = 1 . . . . . .NF

| | |
|---|---|
| F(J) | = Streamwise (X) coordinates of the divisions of the i'th body, starting with body nose and proceeding aft, inches. *(in local coordinates.)* |

59 Repeat the following item for J = 1 . . . . . .NF

| | |
|---|---|
| RAD(J) | = Radii of the i'th body elements at the end points of divisions, inches. |

The j'th element of the body is a frustum of a right cone having base radii of RAD(J) and RAD(J+1).

60. One line.

NSTRIP = Total number of chordwise strips of elements on all panels. 50 max.
When line 8, entry 8 = 1, lift and moment coefficients are printed for these strips. When line 8, entry 8 = 0, the user may set NSTRIP =1, reducing the number of lines needed in item 61 to 1.
Do not set NSTRIP = 0.

| | | |
|---|---|---|
| NPR1 | = 1 | Print pressures. Use for debugging only. |
| | = 0 | No printout. |
| JSPECS | = 1 | Antisymmetric aerodynamics about z = 0. (Biplane or "Jet" effect). |
| | = -1 | Symmetric aerodynamics about z = 0. (Ground effect). |
| | = 0 | No symmetry. Plane Z = 0. |

NSV = Number of strips on all vertical panels lying on the symmetry plane. y = 0.

NBV = Number of elements on all vertical panels on the plane y = 0.

NYAW = 0, if NDELT = 1
= 1, if NDELT = -1
= 0 or 1, if NDELT = 0. See item 49.

61. Enter 6 groups of values per line (18 entries). Repeat for J=1 . . . . . .NSTRIP. See item 60.

| Entry | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Example Value | 1, | 9, | 0, | 10, | 18, | 0, | 19, | 27, | 0, | 28, | 36, | 0, | 37, | 45, | 0, | 46, | 54, | 0 |

If coefficients are not required (NSTRIP = 1) omit and skip to item 61.

LIM(J, 1) = Index of first element on each chordwise strip.
LIM(J, 2) = Index of last element on each chordwise strip.
LIM(J, 3) = 0

When line 8, entry 8 = 1, lift and moment coefficients for each strip are calculated by appropriate integrations chordwise from element LIM(J, 1) to LIM(J, 2).

62. One line with 3 entries. Repeat items 62 through 75 for each surface for IS=1. . . . . line 8, entry 3.

Primary Surface Data Associated with Modal Interpolation

| | | |
|---|---|---|
| KSURF | = T | This surface has one or more control surfaces with forward hinge lines. |
| | = F | This surface has no control surfaces. |

When a control surface is present, modal interpolation is done separately over the area aft of the control surface leading edge. Consequently, modes are discontinuous chordwise at the leading edge and spanwise at the control surface edges.

| | | |
|---|---|---|
| NBOXS | = Number of elements (doublet boxes) in this surface including control surfaces. | |
| NCS | = Number of control surfaces on primary surface. Maximum number is 5. | |

63 One line.

| | | |
|---|---|---|
| NLINES | = Number of lines on this surface along which modal data are input. 20 max. If NELAXS = 1 (see variable below) let NLINES = 1 | |
| NELAXS | = 1 | Translation and pitch rotation are prescribed at each input point. |
| | = 0 | Only translation is prescribed. |

NICH is the control word option for the type of extrapolation done in the chordwise direction, in interpolating modal data to the aerodynamics grid.

| | | |
|---|---|---|
| NICH | = 0 | Linear |
| | = 1 | Quadratic |
| | = 2 | Cubic |

NISP is the control word option for the type of extrapolation done in the spanwise direction, in interpolating modal data to the aerodynamics grid.

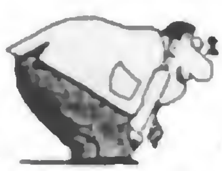
| | | |
|---|---|---|
| NISP | = 0 | Linear |
| | = 1 | Quadratic |
| | = 2 | Cubic |

64 One line for each surface. Repeat items 64 and 65 for I=1 . . NLINES. See figure 7.

| | |
|---|---|
| NGP(I) | = Number of points on the i'th line of primary surface at which the modal data are specified. 20 max. |
| XTERM1(I) | = x coordinate of the inboard terminus of the i'th line for the primary surface, inches. *(In local, not global, coordinates).* |
| YTERM1(I) | = y coordinate of the inboard terminus of the i'th line for the primary surface, inches. *(In local, not global, coordinates).* |
| XTERM2(I) | = x coordinate of the outboard terminus of the i'th line for the primary surface, inches. *(In local, not global, coordinates).* |
| YTERM2(I) | = y coordinate of the outboard terminus of the i'th line for the primary surface, inches. *(In local, not global, coordinates).* |

65. Repeat for J=1 ..... NGP(I). Maximum of 8 entries per line.

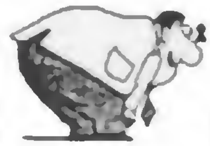
YGP(J,I) = Spanwise coordinates of the points along the i'th line at which input modal data are given, inches. *(In local coordinates)* start with the most inboard point and proceed outboard.



66. Take a Peek. No entry. If translation and pitch rotation are prescribed at each point (NELAXS = 1) enter data for item 67, otherwise (NELAXS = 0) omit item 67 and go to item 68.

67. One line for each primary surface.

DIST = An arbitrary chordwise distance for a primary surface from the given line to a reference line on which modal displacements are calculated, inches. Note that modal displacements are calculated by H1 = H0 + AO x DIST, where HO and AO are the displacement and rotation of a point on a given line and H1 is the displacement of the corresponding point on the new line. The given deformations HO and AO along a line are, thus, converted to displacements HO and H1 along two parallel lines and the modal interpolation is based on these. See figure 11.



68. Take a Peek. No entry. If a primary surface has one or more <u>control surfaces</u> with forward hinge lines (KSURF = T) enter data for the item 69. Otherwise (KSURF = F) omit item 69.

69. Number of lines is NCS. Repeat item for J=1 ......NCS
Starting inboard and proceeding outboard.

X1(J) = x coordinate of the inboard terminus of the j'th control surface leading edge, inches. *(In local coordinates)* .

Y1(J) = y coordinate of the inboard terminus of the j'th control surface leading edge, inches. *(In local coordinates).*

X2(J) = x coordinate of the outboard terminus of the j'th control surface leading edge, inches. *(In local coordinates).*

Y2(J) = y coordinate of the outboard terminus of the j'th control surface leading edge, inches. *(In local coordinates).*

70 Take a Peek. No entry. Control surface data for modal interpolation
If a primary surface has one or more control surfaces with forward hinge lines (KSURF=T) enter data for items 71 through 75. Otherwise (KSURF = F) go to item 76.

71 One line.
Items 71 through 75 are entered once and are applicable to all the control surfaces. The variable NLINES is the total for all control surfaces.

NLINES = Number of lines on all control surfaces along which modal data are input. 20 max.

NELAXS =1 Translation and pitch rotation are prescribed at each input point.
=0 Only translation is prescribed.

NICH Control word option for the type of extrapolation done in the chordwise direction, in interpolating modal data to the aerodynamics grid.
= 0 Linear
= 1 Quadratic
= 2 Cubic

NISP Control word option for the type of extrapolation done in the spanwise direction, in interpolating modal data to the aerodynamics grid.
= 0 Linear
= 1 Quadratic
= 2 Cubic

72 One line.
Modal data are prescribed, starting with the most forward, most inboard line and proceeding outboard and aft. Repeat the following two items for I=1 . . . . . NLINES

NGP(I) = Number of points on the i'th line of control surface at which the modal data are specified. 12 max.

XTERM1(I) = x coordinate of the inboard terminus of the i'th line for the control surface, inches. *(In local coordinates).*

YTERM1(I) = y coordinate of the inboard terminus of the i'th line for the control surface, inches. *(In local coordinates).*

XTERM2(I) = x coordinate of the outboard terminus of the i'th line for the control surface, inches. *(In local coordinates).*

YTERM2(I) = y coordinate of the outboard terminus of the i'th line for the control surface, inches. *(In local coordinates).*

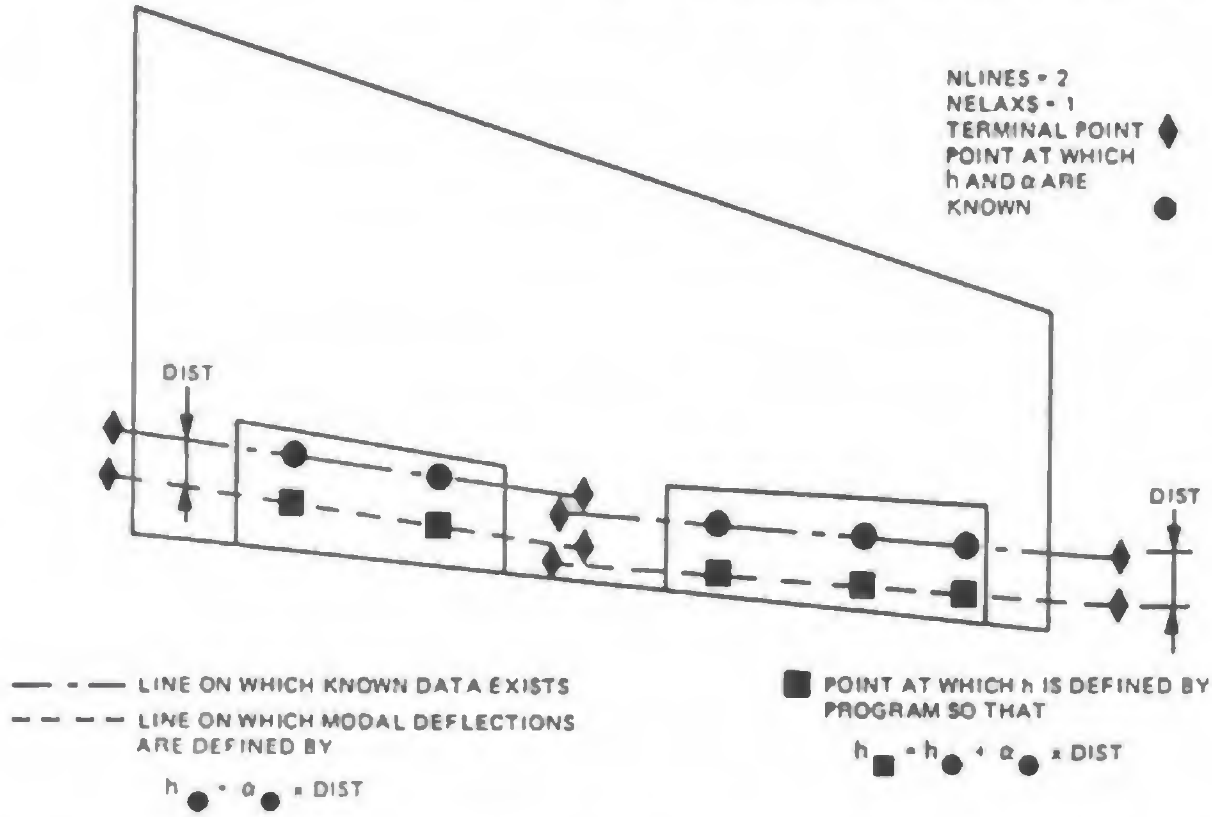



Figure 11. A Control Surface can be Modeled by One Data Line with Data Points and Pitch Rotations, α. If This Form of Data is Used, DIST must be Specified.

73. Maximum of 8 entries per line.

YGP(J,I) = Spanwise locations of the points along the i'th line at which input modal data are given, inches. *(in local coordinates)* .

74. Take a Peek. No entry. If translation and pitch rotation are prescribed at each point (NELAXS = 1) enter data for the item 75. Otherwise (NELAXS = 0) omit item 75.

75. One line for each control surface.

DIST = An arbitrary chordwise distance for a control surface from the given line to a reference line on which modal displacements are calculated, inches. Note that modal displacements are calculated by H1 = H0 + A0 x DIST, where H0 and A0 are the displacement and rotation of a point on a given line and H1 is the displacement of the corresponding point on the new line. The given deformations H0 and A0 along a line are, thus, converted to displacements H0 and H1 along two parallel lines and the modal interpolation is based on these. See figure 11.

76 Take a Peek. No entry. Body Surface Data Associated with Modal Interpolation. If bodies that aerodynamically interact with surfaces are included in the analysis (NB greater than zero) enter data for items 77 and 78. Otherwise (NB = 0) skip these items and go to item 79.

77 One line. Repeat the items 77 and 78 for each body for J=1 . . . .NB

| | |
|---|---|
| NGP | = Number of points on the j'th body axis at which modal data are prescribed. 20 max. |
| NSTRIP | = Number of interference panels (or strips) associated with the j'th body. Interference panels are allowed to be only one element wide (NS = 2). A particularly wide panel should be replaced with two or more panels. |
| IPANEL | = Index of the first such interference panel associated with the j'th body. |

78 6 entries per line. Repeat the for J = 1 . . . . . .NGP.

| | |
|---|---|
| XGP(J) | = Streamwise coordinates of each point at which modal data are prescribed, inches. *(In local coordinates).* |

79 One line.

| | | |
|---|---|---|
| KLUGLB | = 1 | Print global geometry. This is the geometry after transformations X0(I), Y0(I), Z0(I), and GGMAS(I) and XB0(J), YB0(J), and ZB0(J) have been applied. |
| | = 0 | No display. |

## 6.2 General Rules

1. Entries are separated by blank(s) or a comma. If a blank is used to separate entries, multiple blanks are allowed. If a comma is used, only one comma is needed. However, multiple commas can be used. For example:
   0,,1 ≠ 0,0,1. Instead 0,,1 = 0,1.

2. Blank lines are ignored.

3. ** indicates comment line which is ignored by SAF but useful to clarify data entry.
   Example:
   ** Modal data for 8 data points

4. If data entry is too large, SAF will display error message. Example: 0.16666666667 for a 10 column field will cause truncation.

5. If a line contains all zeros, at least one zero must be input. Otherwise the line is considered a blank line and will be ignored.

6. If there are five entries and the first four entries are zero and the fifth is a nonzero value, enter the data as follows.
   0,0,0,0,721

7. Do not use the tabkey during data entry. The Fortran code does not recognize tabkey entries and SAF will not execute an input file with a tabkey entry.

8. It is a good idea to model your first program by editing an existing example input file to conform to your problem.

# CHAPTER 7

## *EXAMPLES*

Now that we have an overview of how to solve a flutter analysis problem, lets put SAF to work. The first example is the Lancair IV wing which has been use as an example in the previous chapters.

### 7.1 Lancair IV Wing

Following is a listing of the complete SAF input file for the Lancair IV wing:

The first row is a reference for the item listed in Chapter 6. This entry is not included in the input file.

Table 6. SAF Input File for the Lancair IV Wing Example.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | | | | | | |
| | FLUTTER, LANCAIR IV WING | | | | | | |
| | 2800 LBS GROSS WITH 40 LBS FUEL | | | | | | |
| | SYMMETRIC CASE | | | | | | |
| | P-K METHOD | | | | | | |
| | 5 VIBRATION MODES | | | | | | |
| | MACH=0.4 | | | | | | |
| Item 1 | -1,5,1,6,1,0,0,0,0,0 | | | | | | |
| Item 2 | 1,0,1,0,0,1,0,0,1,0 | | | | | | |
| Item 3 | 1,0,0,1,0,0,0,0,0,0 | | | | | | |
| Item 4 | 0,0,0,0,0,0,0 | | | | | | |
| Item 5 | 21 | | | | | | |
| Item 6 | -0.0722 | 0.222 | 0.7832 | 1.655 | 2.673 | -0.048 | 0.245 |
| | 0.810 | 1.687 | 2.698 | -0.0288 | 0.272 | 0.850 | 1.722 |
| | 2.729 | 1.0759 | 1.876 | 2.597 | 1.115 | 1.915 | 2.632 |
| | -0.1948 | -0.8848 | -1.145 | 0.2125 | 2.905 | -0.2637 | -0.988 |
| | -1.198 | 0.1639 | 2.876 | -0.386 | -1.172 | -1.435 | 0.0323 |
| | 2.813 | -1.338 | 0.366 | 2.393 | -0.692 | 0.999 | 2.975 |
| | 0.00462 | -0.0252 | 0.04197 | 0.2055 | 0.2808 | -0.0258 | -0.0721 |
| | -0.0244 | 0.1234 | 0.1542 | -0.0895 | -0.164 | -0.1599 | -0.0343 |
| | -0.0276 | -0.1884 | -0.1159 | -0.1043 | -17.008 | -16.55 | -15.379 |
| | 0.456 | 0.800 | 1.650 | 1.961 | 0.952 | 0.0266 | 0.109 |
| | 0.5437 | 0.287 | -0.3094 | -0.880 | -1.161 | -1.297 | -1.600 |
| | -1.997 | -1.853 | -2.157 | -2.481 | -0.490 | -0.842 | -1.298 |
| | -0.3577 | -0.6083 | 0.08126 | 0.1432 | -2.323 | -0.3761 | -0.61 |
| | 0.2239 | 0.4176 | -2.264 | -0.5027 | -0.7807 | 0.5891 | 0.7297 |
| | -2.215 | 1.0369 | 0.488 | -1.734 | 1.8875 | 1.311 | -0.945 |
| Item 7 | 2 | | | | | | |
| Item 8 | 1 1 144.0 2 2 144.0 3 3 144.0 | | | | | | |
| | 4 4 144.0 5 5 144.0 | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Item 9 | 4.097 | 14.778 | 19.657 | 27.31 | 29.176 | | |
| Item 11 | 19.25 | 0.4 | | | | | |
| Item 15 | 20 | 50.0 | 24.0 | | | | |
| Item 16 | 0.02 | 0.50 | 1.0 | 5.0 | 10.0 | 15.0 | 50.0 |
| Item 23 | 0.03 | | | | | | |
| Item 26 | 0.5 | -0.5 | 440.0 | 20.0 | | | |
| Item 27 | 1.0 | | | | | | |
| Item 48 | 38.5 | 1.0 | | | | | |
| Item 49 | 1 1 | 0 1080 | 0 0 | 1 | | | |
| Item 50 | 0.0 | 0.0 | 0.0 | 0.0 | | | |
| Item 51 | 0.0 | 47.0 | 3.5 | 33.5 | 25.0 | 180.0 | |
| Item 52 | 0.0 | 0.0 | 25 10 | 0.0 | | | |
| Item 53 | 0.0 | 0.2 | 0.4 | 0.6 | 0.65 | 0.8 | |
| | 0.85 | 0.9 | 0.95 | 1.0 | | | |
| Item 54 | 0.0 | 0.0417 | 0.0833 | 0.125 | 0.1667 | 0.208 | |
| | 0.25 | 0.2917 | 0.3333 | 0.375 | 0.4167 | 0.458 | |
| | 0.5 | 0.542 | 0.583 | 0.625 | 0.667 | 0.708 | |
| | 0.75 | 0.792 | 0.833 | 0.875 | 0.917 | 0.958 | |
| | 1.0 | | | | | | |
| Item 60 | 24 0 0 0 0 0 | | | | | | |
| Item 61 | 1,9,0,10,18,0,19,27,0,28,36,0,37,45,0,46,54,0 | | | | | | |
| | 55,63,0,64,72,0,73,81,0,82,90,0,91,99,0,100,108,0 | | | | | | |
| | 109,117,0,118,126,0,127,135,0,136,144,0,145,153,0,154,162,0 | | | | | | |
| | 163,171,0,172,180,0,181,189,0,190,198,0,199,207,0,208,216,0 | | | | | | |
| Item 62 | T 216 1 | | | | | | |
| Item 63 | 3 0 1 1 | | | | | | |
| Item 64 | 5 4.0 | 0.0 | 6.0 | 180.0 | | | |
| Item 65 | 35.0 | 70.0 | 105.0 | 140.0 | 175.0 | | |
| Item 64 | 5 13.0 | 0.0 | 13.2 | 180.0 | | | |
| Item 65 | 35.0 | 70.0 | 105.0 | 140.0 | 175.0 | | |
| Item 64 | 5 35.0 | 0.0 | 24.0 | 180.0 | | | |
| Item 65 | 35.0 | 70.0 | 105.0 | 140.0 | 175.0 | | |
| Item 69 | 33.5 | 115.0 | 28.5 | 180.0 | | | |
| Item 71 | 2 0 1 1 | | | | | | |
| Item 72 | 3 33.5 | 115.0 | 28.5 | 180.0 | | | |
| Item 73 | 115.0 | 145.0 | 170.0 | | | | |
| Item 72 | 3 39.5 | 115.0 | 33.5 | 180.0 | | | |
| Item 73 | 115.0 | 145.0 | 170.0 | | | | |
| Item 79 | 0 | | | | | | |

On a 486 PC this problem will run in about 10 minutes. SAF will generate 4 files. One file will be an ASCII file showing all results and plots of the damping, frequency, and reduced frequency as a function of velocity. A portion of this output file is shown in table 7. The other 3 files are binary files which are used by the NISA386 for viewing the graphs in color. Figures 12 and 13 show two of the plots.

It is also interesting to compare the answer of SAF to other programs. Following is such a comparison for the flutter speeds of the Lancair IV wing calculated by three different methods. Sea level for case desribed in this book.

| | |
|---|---|
| SAF on a 486 PC | 411 mph |
| FASTEX on a VAX 8850 | 419 mph |
| McIntosh Stuctural Dynamics, Inc. on a 386 PC | 450 mph |

Table 7. SAF Output File for the Lancair IV Wing. The Numbers in the Plots Indicate the Mode Number. A Small Portion of the File is Shown. The Mode Shapes are Identified from the FEA Run. See Chapter 4. The Eigenvectors are Summarized:

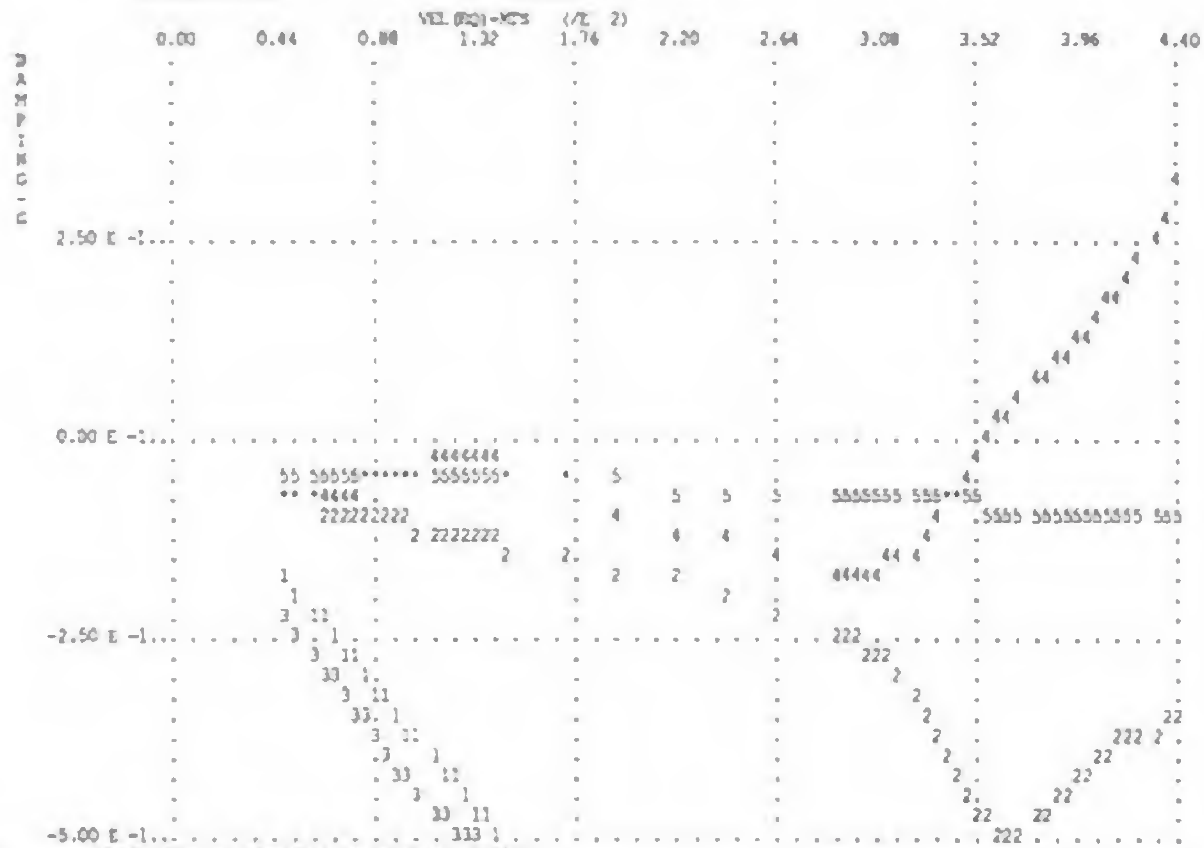
| MODE | FREQUENCY, Hz | DEFINITION |
|---|---|---|
| 1 | 4.097000E+00 | 1st Wing Bending Mode |
| 2 | 1.477800E+01 | 2nd Wing Bending Mode |
| 3 | 1.965700E+01 | Aileron Flapping |
| 4 | 2.731000E+01 | Wing Twisting |
| 5 | 2.917600E+01 | 3rd Wing Bending |

SEA LEVEL OUTPUT.

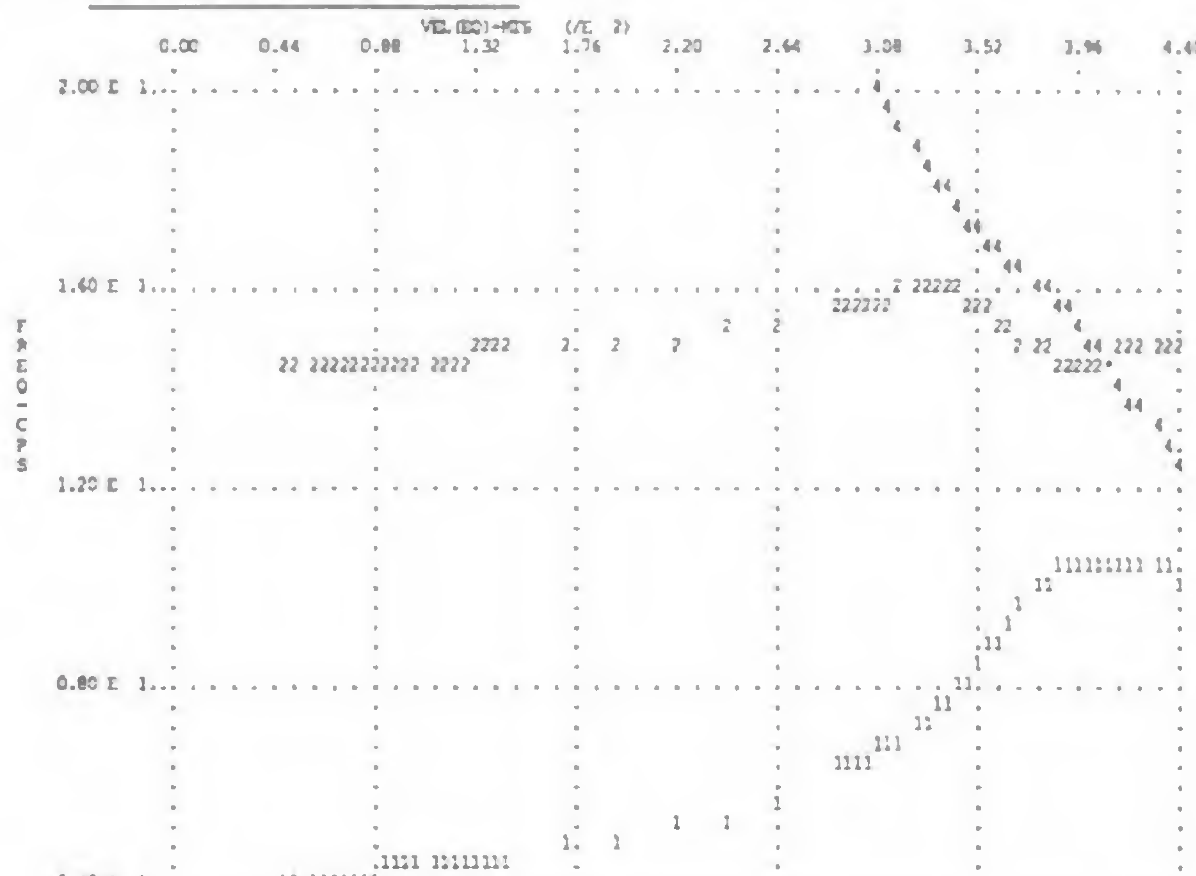
```
1                 RESULTS OF TESTS ON INTERPOLATION USING   4VALUES OF QRS
                 VALUES OF VBO USED FOR ANALYSIS
                  0.500      1.00      5.00      50.0

                 VALUE OF VBO TESTED =   10.0
                 ALLOWABLE DEVIATION FOR INTERPOLATION =  0.200E-01

                 DEVIATION FOR EUCLIDEAN NORM OF ARRAY =  0.250E-02

                 DEVIATION FOR EUCLIDEAN NORM OF DIAGONAL TERMS =  0.104E-02

                 DEVIATIONS FOR EUCLIDEAN NORM OF DIAGONAL AND OFF-DIAGONAL TERMS FOR VALUES OF
I AND J
                  I       J      DEVIATION
                  1       2       0.744E-02
                  2       3       0.375E-02
                  3       4       0.661E-02
                  4       5       0.830E-02

                 INTERPOLATION SATISFACTORY

             FLUTTER ANALYSIS USING THE P-K METHOD
             -------------------------------------
             DENSITY VARIATIONS
             ------------------
             DENSITY =    7.647400E-02,  LB/FT**3
             FREQUENCY VARIATIONS
             --------------------
             VARIATION IN FREQUENCY FOR MODE NO.  0
```

```
          GENERALIZED MASS, FREQUENCY, AND MODAL STIFFNESS
          ------------------------------------------------

          GENERALIZED MASS, LB
             1.440000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
1.440000E+02  0.000000E+00
             0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  1.440000E+02  0.000000E+00
0.000000E+00  0.000000E+00
             0.000000E+00  0.000000E+00  1.440000E+02  0.000000E+00  0.000000E+00  0.000000E+00
0.000000E+00  0.000000E+00
             1.440000E+02

           MODE  FREQUENCY      FREQUENCY
                 CYC/SEC        RAD/SEC
              1  4.097000E+00  2.574221E+01
              2  1.477800E+01  9.285289E+01
              3  1.965700E+01  1.235085E+02
              4  2.731000E+01  1.715938E+02
              5  2.917600E+01  1.833182E+02


          COMPLEX GENERALIZED MODAL STIFFNESS, (REAL, IMAG),_ LB/IN

             ( 9.5423E+04,  2.8627E+03)  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)
( 0.0000E+00,  0.0000E+00)
  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  ( 1.2415E+06,  3.7246E+04)  (
0.0000E+00,  0.0000E+00)
  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  (
0.0000E+00,  0.0000E+00)
  ( 2.1966E+06,  6.5899E+04)  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  (
0.0000E+00,  0.0000E+00)
  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  ( 4.2400E+06,  1.2720E+05)  (
0.0000E+00,  0.0000E+00)
  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  ( 0.0000E+00,  0.0000E+00)  (
0.0000E+00,  0.0000E+00)
  ( 4.8392E+06,  1.4518E+05)  (


          MODAL ELIMINATION VARIATIONS
          ----------------------------
          NUMBER OF MODES ELIMINATED ARE ZERO
1           VELOCITY, DAMPING, AND FREQUENCY VARIATIONS
          -------------------------------------------

                    VELOCITY, KNOTS           DAMPING          FREQUENCY
 MODE    NO      EQUIVALENT       TRUE          RATIO       CYC/SEC      RAD/SEC

    1     1      0.5000E+02  0.5000E+02 -0.1868E+00  0.4056E+01  0.2549E+02
    1     2      0.5480E+02  0.5480E+02 -0.2017E+00  0.4069E+01  0.2556E+02
    1     3      0.5960E+02  0.5960E+02 -0.2170E+00  0.4082E+01  0.2565E+02
    1     4      0.6440E+02  0.6440E+02 -0.2325E+00  0.4097E+01  0.2574E+02
    1     5      0.6920E+02  0.6920E+02 -0.2483E+00  0.4112E+01  0.2584E+02


 1           CRITICAL FLUTTER SPEED AND ASSOCIATED PARAMETERS
          ----------------------------------------------
          RHO= 7.6474003E-02  LBS/CU.FT.,  FOR  5  MODES
          VELOCITY(KNOTS)              DAMPING               FREQUENCY
        EQUIV.         TRUE             RATIO          C.P.S.          RAD./SEC.
         357.2537      357.2537         0.000013        16.8875         106.1078
```

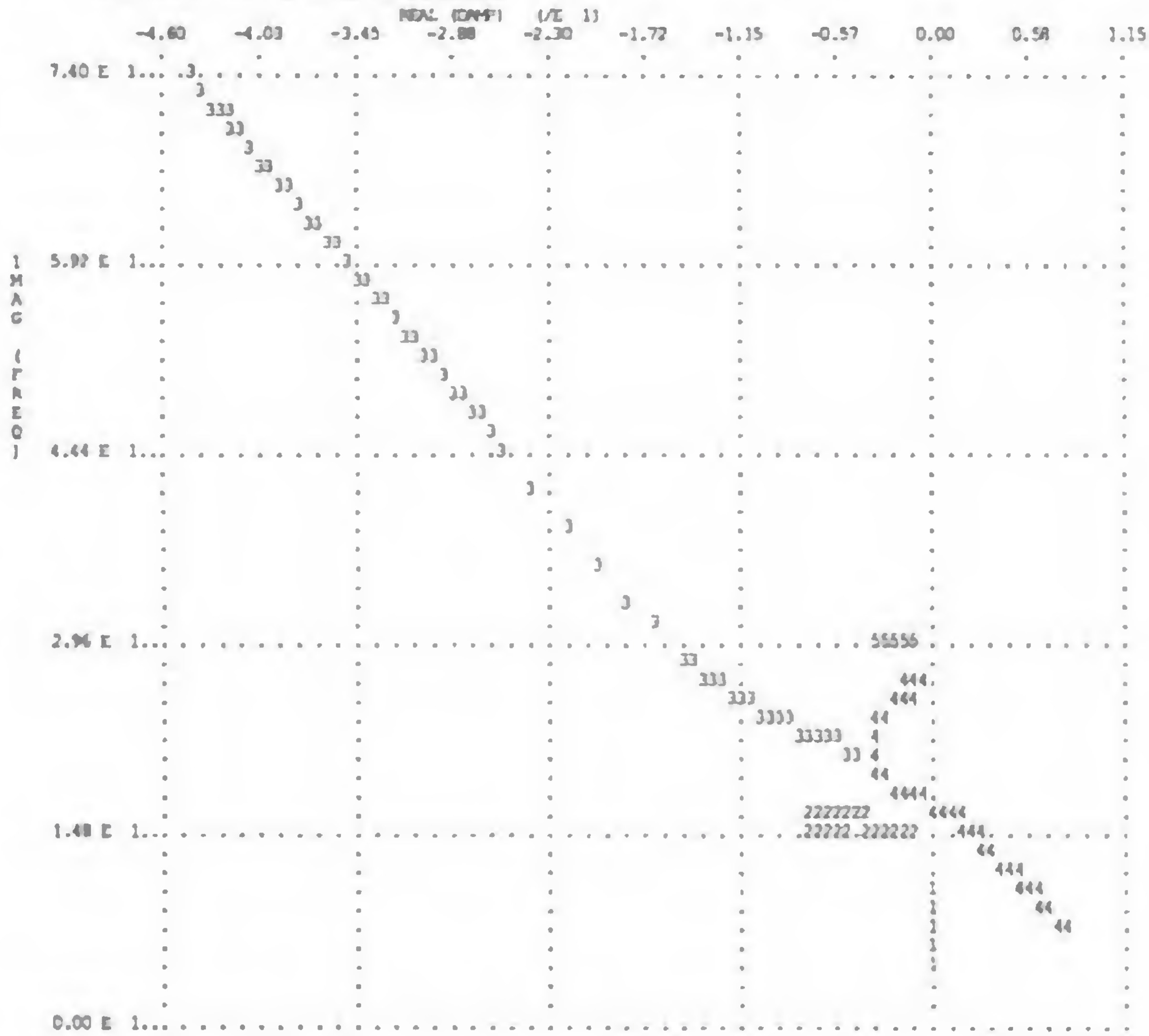
1 PRINT-PLOT OF DAMPING VS VELOCITY (EQUIV)




1 PRINT-PLOT OF FREQUENCY VS VELOCITY (EQUIV)



1 PRINT-PLOT OF ROOT LOCUS OF FLUTTER SOLUTION

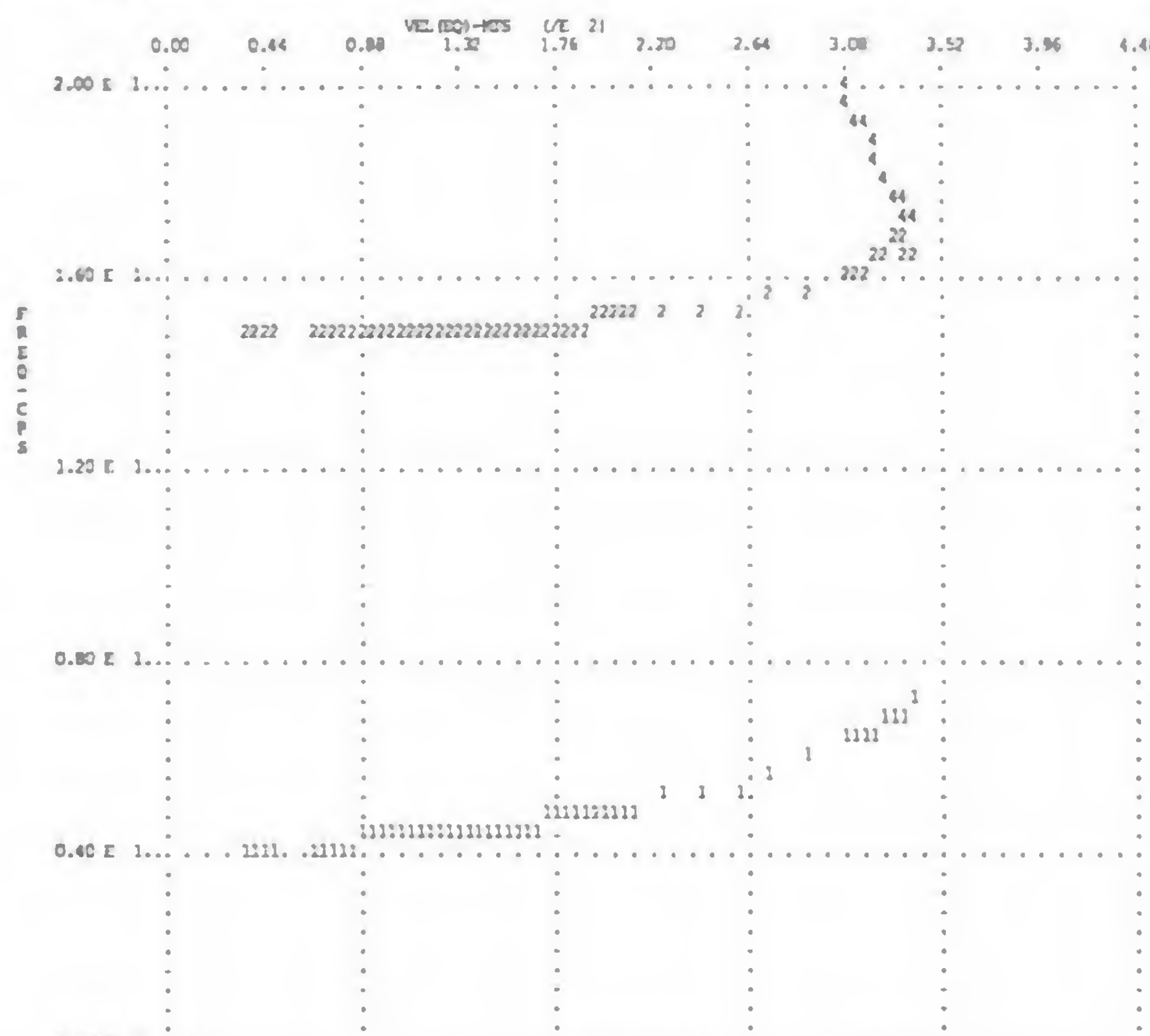



<u>25,000 FEET ALTITUDE.</u> Note the "Hump Back Mode" in which the the wing twisting mode (line 4) crosses the zero line in the damping curve and crosses back over until it goes back up and crosses at a steep slope at 340 knots. The flutter speed of 119 knots below is for the hump back mode. Because of its low damping value it does not represent a flutter problem.

```
1           CRITICAL FLUTTER SPEED AND ASSOCIATED PARAMETERS
            ---------------------------------------------

            RHO= 3.4336828E-02  LBS/CU.FT.,   FOR  5  MODES
            VELOCITY(KNOTS)              DAMPING              FREQUENCY
         EQUIV.          TRUE             RATIO         C.P.S.          RAD./SEC.

          119.3602       178.1297         0.000001        26.0879         163.9153
```

PRINT-PLOT OF DAMPING VS VELOCITY (EQUIV)


PRINT-PLOT OF FREQUENCY VS VELOCITY (EQUIV)



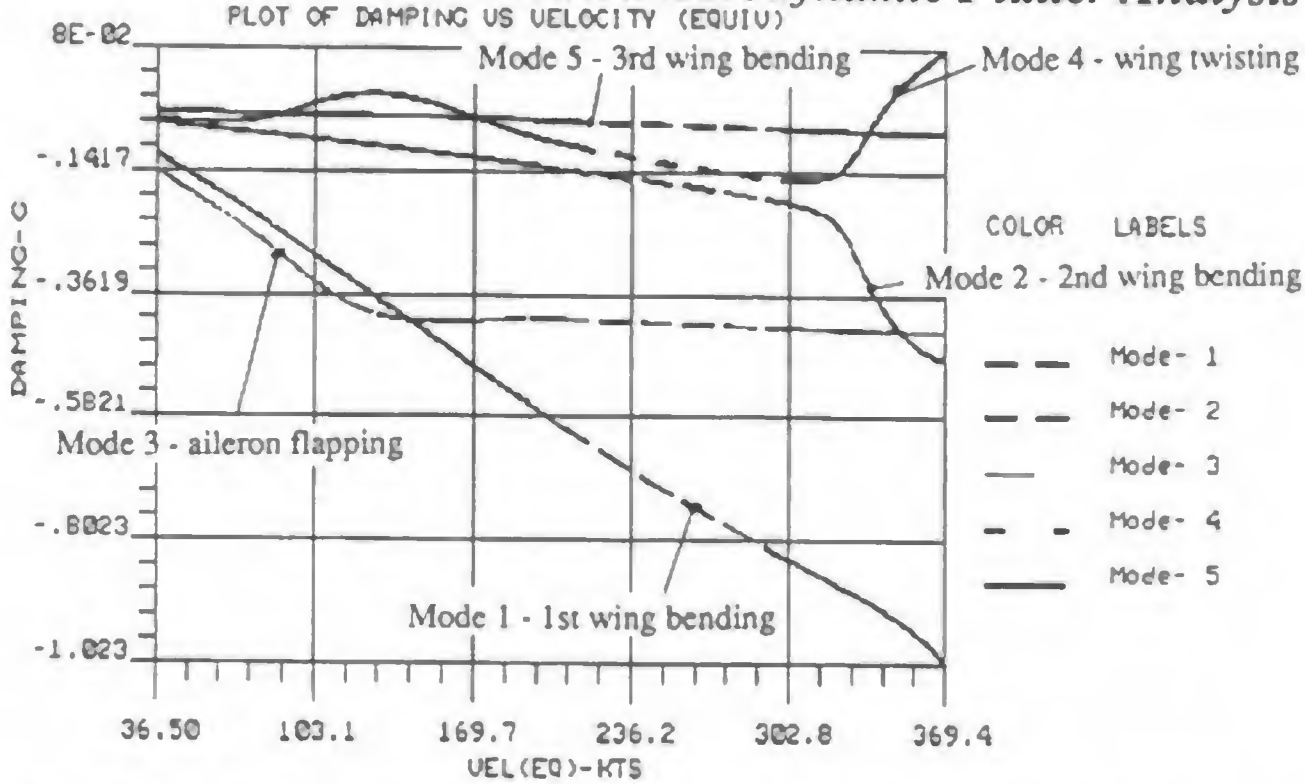



Figure 12. Damping vs. Velocity for the Lancair IV Wing. Mode 4 (Wing Twisting) Crosses 0 Damping at 357.25 knots or 411 mph. This is the Critical Flutter Speed at Sea Level. The Above Plot was Printed using the NISA386 Postprocessor and an HP Paintjet Printer.

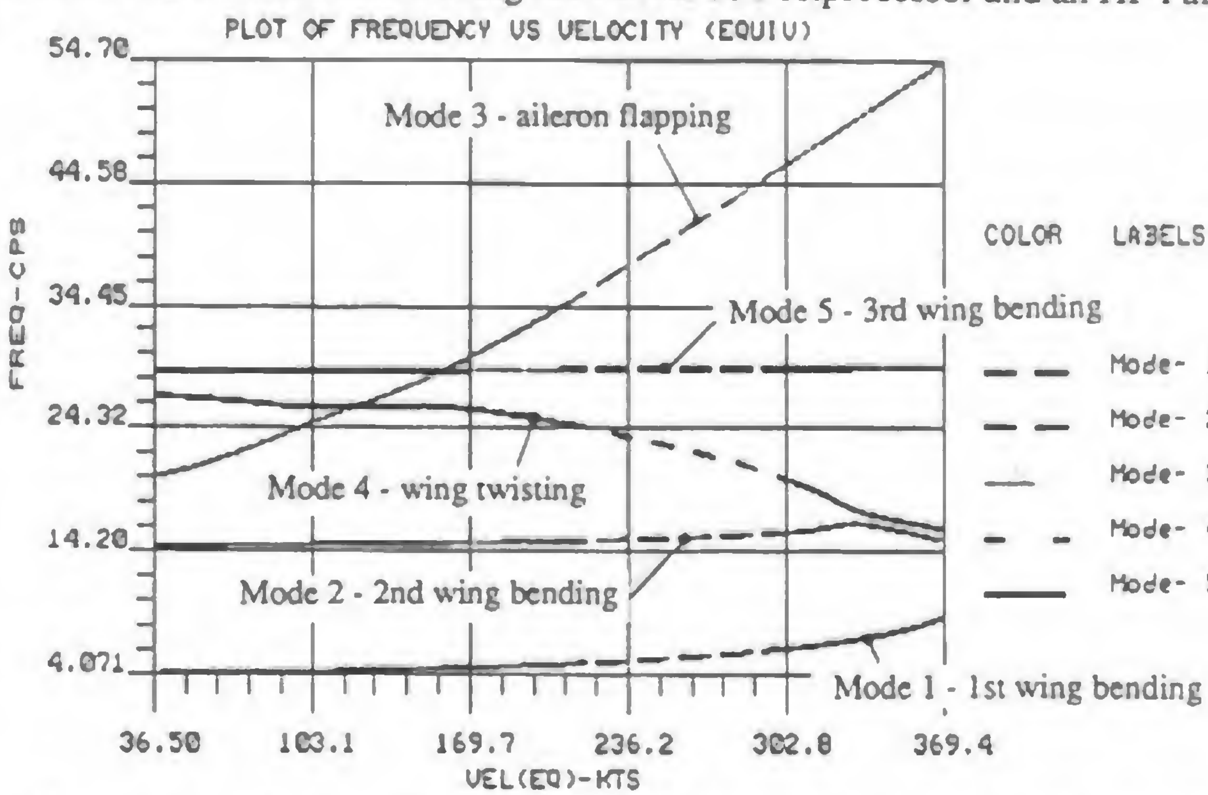



Figure 13. Frequency vs Velocity Show that Mode 4 and Mode 2 (2nd Wing Bending) Converge at 357.25 knots. It is the Coupling of these Two Modes that is Causing the Flutter.

## *Examples*

The flutter speed is calculated for various altitudes and the flutter boundary is plotted in figure 14 for the symmetric mode case.

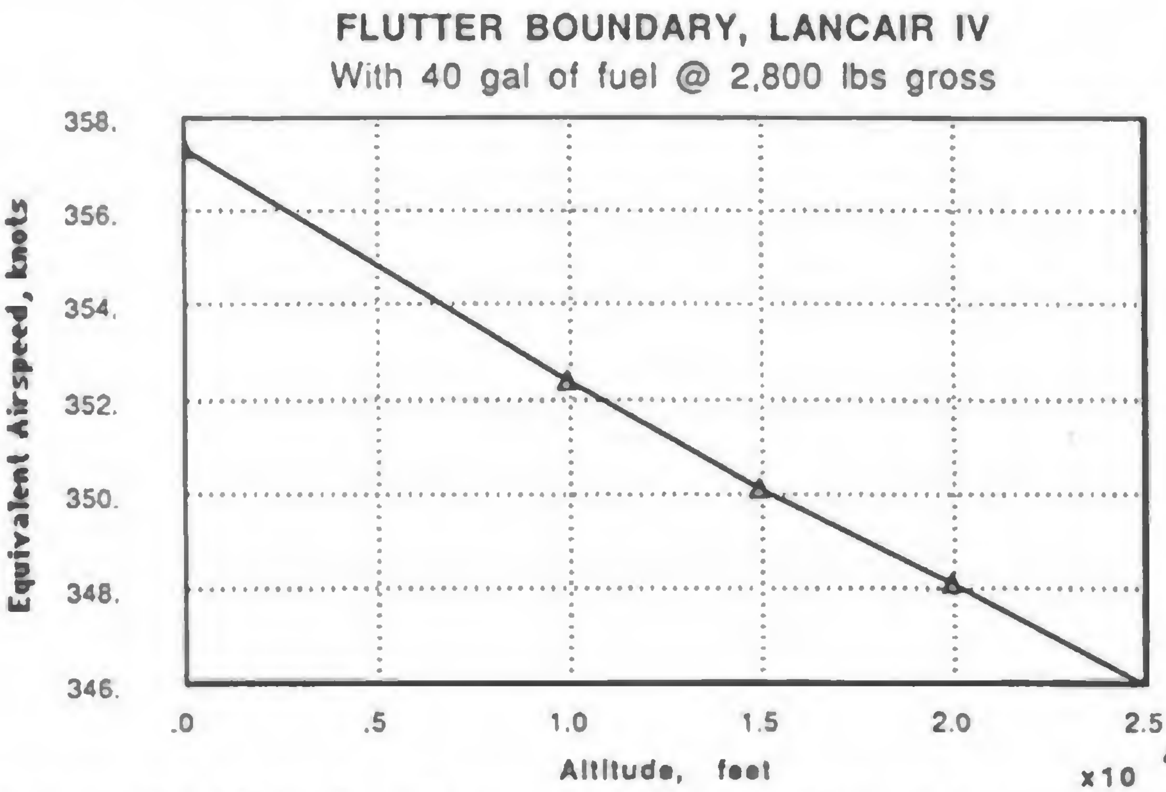



Figure 14. The Flutter Boundary for the Lancair IV Wing for the Symmetric Mode Case. This is the Most Severe Case for this Aircraft.

FAR 23.629(b) specifies that the lowest flutter speed of an aircraft should be above 1.2 x $V_D$. Quote, *"A rational analysis may be used to show that the airplane is free from flutter, control reversal, and divergence if the analysis shows freedom from flutter for all speeds up to 1.2$V_D$."* The International OSTIV, Germany, France have accepted the same criteria. See Reference 18.

If the flutter speed is less, the aircraft should be placarded to fly below $V_{DF}/1.2$. For the Lancair IV this would be 357.25 x 1.15/1.2 = 342 mph at sea level with full tanks of fuel. The following placard should be placed in the aircraft.

This aircraft should not be flown above the following *REDLINE* airspeeds:

| | |
|---|---|
| At sea level: | 342 mph indicated A/S or Mach 0.45 |
| At 10,000 ft: | 337 mph indicated A/S or Mach 0.53 |
| At 15,000 ft | 335 mph indicated A/S or Mach 0.58 |
| At 20,000 ft. | 333 mph indicated A/S or Mach 0.64 |
| At 25,000 ft. | 325 mph indicated A/S or Mach 0.70 (488 mph True A/S) |

SAF calculates both the equivalent airspeed and the true airspeed in knots. A definition of these airspeeds and other airspeeds is in order:

| | |
|---|---|
| Indicated airspeed | = gage reading. |
| Calibrated airspeed | = indicated A/S corrected for pitot tube location. |
| Equivalent airspeed | = calibrated A/S corrected for adiabatic compressible flow. |
| True airspeed | = equivalent airspeed corrected for air density from standard value at sea level. Increases with altitude when indicated stays the same. |
| Mach | = true airspeed/ velocity of sound. |
| A/S | = airspeed. |

## 7.2 The Goland Wing

The Goland wing is a hypothetical wing which was presented in an article titled "The Flutter of a Uniform Cantilever Wing" by Martin Goland in 1945. See Reference 19. The wing is modeled as one half span and cantilevered at the center of the fuselage. It has a semi span of 20 feet, a constant wing chord of 6.0 feet and a weight of 4 pounds/sq.ft. It is rectangular in shape (Hershey bar). The radius of gyration of the wing about the center of gravity is 25% chord from the leading edge. The Spanwise elastic axis of the wing is at the 33% chord, and the center of gravity is at the 43% chord. The 1st wing bending frequency is 50 Hz and the wing torsional frequency is 87 Hz. The stiffness of the wing is $31.7 \times 10^6$ lb ft$^2$ per slug and its torsional stiffness is $1.23 \times 10^6$ lb ft. per slug. I = 1.943 slugs ft$^2$ per ft. and m = 0.746 slugs per foot. Figure 15 shows the axis system and the geometry of the GOLAND wing model. This is a very simple wing with known flutter characteristics which have been calculated by hand and other means. It is used as a test case.

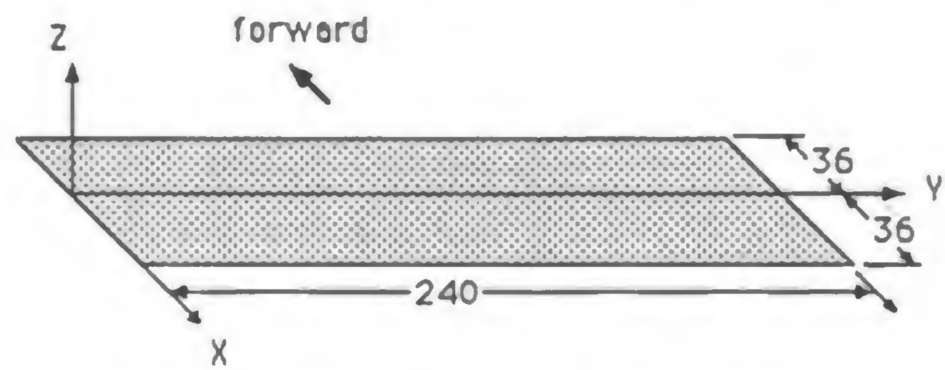



Figure 15. Geometery of the Goland Wing.

Table 8. The SAF Input File for the Goland Wing.

1

```
FLUTTER ANALYSIS PROGRAM FOR GOLAND WING
ONE PANEL WITH 120 ELEMENTS
NO CONTROL SURFACES, NO ANHEDRAL
P-K TYPE FLUTTER ANAYSIS
```

```
2 VIBRATION MODES
MACH=0.1
-1    2    1    6    1    0    0    0    0    0
3    0    1    0    0    0    0    0    1    0
1    0    0    0    0    0    0    0    0    0
0    0    0    0    0    0    0
72
.003680   0.006774  0.004552  -0.002558 -0.014143 -0.029814 -0.049199
-0.071946 -0.097723 -0.126218 -0.157137 -0.190206 -0.225173 -0.261801
-0.299877 -0.339205 -0.379608 -0.420931 -0.463037 -0.505808 -0.549148
-0.592977 -0.637236 -0.681889 -0.000694 -0.006072 -0.016398 -0.031243
-0.050195 -0.072864 -0.098879 -0.127888 -0.159559 -0.193579 -0.229655
-0.267513 -0.306899 -0.347580 -0.389339 -0.431981 -0.475332 -0.519233
-0.563549 -0.608162 -0.652975 -0.697909 -0.742905 -0.787926 -0.009442
-0.031764 -0.058297 -0.088612 -0.122298 -0.158965 -0.198240 -0.239773
-0.283231 -0.328301 -0.374690 -0.422125 -0.470352 -0.519136 -0.568262
-0.617535 -0.666779 -0.715837 -0.764573 -0.812870 -0.860629 -0.907773
-0.954243 -1.000000
.027259   0.081830  0.136308  0.190504  0.244236  0.297334  0.349634
.400980   0.451228  0.500239  0.547885  0.594045  0.638607  0.681469
.722535   0.761720  0.798947  0.834146  0.867258  0.898231  0.927021
.953596   0.977927  1.000000  0.000305  0.002671  0.007213  0.013742
.022079   0.032050  0.043493  0.056253  0.070184  0.085148  0.101016
.117669   0.134994  0.152887  0.171256  0.190013  0.209081  0.228391
.247884   0.267508  0.287219  0.306984  0.326777  0.346579  -0.053603
-0.155648 -0.250978 -0.339780 -0.422236 -0.498517 -0.568787 -0.633201
-0.691903 -0.745034 -0.792720 -0.835082 -0.872233 -0.904275 -0.931303
-0.953403 -0.970651 -0.983118 -0.990863 -0.993937 -0.992384 -0.986238
-0.975525 -0.960262
1
1    1 91.191071    2    2 63.531044
7.71     15.22
36.0       0.1
18      50.0      30.0
0.02      0.01       0.1       3.0       5.0       20.0      100.0
0.5      -0.5     500.0      20.0
1.0
72.0   17280.0
1    1    0  120    0    0    1
0.0       0.0       0.0       0.0
-36.0      36.0     -36.0      36.0       0.0      240.0
0.0       0.0  21  7           0.0
0.0 0.1666667 0.3333334       0.5 0.6666667 0.8333335
1.0
0.00      0.05      0.10      0.15      0.20      0.25
0.30      0.35      0.40      0.45      0.50      0.55
0.60      0.65      0.70      0.75      0.80      0.85
0.90      0.95      1.00
20    0    0    0    0    0
```

```
1,6,0,7,12,0,13,18,0,19,24,0,25,30,0,31,36,0
37,42,0,43,48,0,49,54,0,55,60,0,61,66,0,67,72,0
73,78,0,79 84,0,85,90,0,91,96,0,97,102,0,103,108,0
109,114,0,115,120,0
F  120    0
3    0    0    0
24      -36.0       -5.0       -36.0       245.0
5.0       15.0       25.0       35.0       45.0       55.0       65.0       75.0
85.0       95.0       105.0      115.0      125.0      135.0      145.0      155.0
165.0      175.0      185.0      195.0      205.0      215.0      225.0      235.0
24      -12.0       -5.0       -12.0       245.0
5.0       15.0       25.0       35.0       45.0       55.0       65.0       75.0
85.0       95.0       105.0      115.0      125.0      135.0      145.0      155.0
165.0      175.0      185.0      195.0      205.0      215.0      225.0      235.0
24       36.0       -5.0        36.0       245.0
5.0       15.0       25.0       35.0       45.0       55.0       65.0       75.0
85.0       95.0       105.0      115.0      125.0      135.0      145.0      155.0
165.0      175.0      185.0      195.0      205.0      215.0      225.0      235.0
0
```

Table 9 shows a comparison of the results from SAF to other methods. Close agreement between most techniques is realized. Flutter occurs when 1st wing bending mode couples with wing torsion as shown in figure 16.

Table 9. A Comparison of Critical Flutter Speeds Calculated by Various Methods.

| Method | Flutter Speed | Frequency |
|---|---|---|
| SAF | 306 knots | 10.65 Hz |
| ASTROS version 5 | 300 knots | 11.0 Hz |
| Goland Paper, Ref. 4. | 335 knots | 10.7 Hz |
| FLUT2D | 260 knots | 11.2 Hz |

Figure 16 shows the damping and frequency plots for the GOLAN wing as plotted by SAF. The flutter speed is determined as that speed at which the damping crosses from a negative value to a positive one.

## 7.3 Fuselage and Tail

Following is a SAF input file for a more complicated model. This model takes about 45 minutes to run on a 486-33 PC.

Table 10. SAF Fuselage and Tail Model.

```
1
FLUTTER MODEL WITH RIGID FUSELAGE AND RIGID ALL MOVEABLE TAIL,
SYMMETRIC FLUTTER ANALYSIS,
P-K METHOD, M=0.8,
2 RIGID BODY MODES, 2 ELASTIC MODES (EXTRAPOLATED OUT FOR
```

```
RIGID FUSELAGE AND TAIL) AND TWO CONTROL SURFACE MODES
NO ANHEDRAL FOR THE WING
-1    6    4    6    1    0
3    0    1    0    0    0    0    0    1    0
1    0    0    0    0    0    0    0    0    0
0    0    0    0    0    0    0
42
1.0         1.0         1.0         1.0         1.0         1.0         1.0
1.0         1.0         1.0         1.0         1.0         0.0         1.0
0.0         1.0         0.0         1.0         0.0         1.0         0.0
1.0         0.0         1.0         0.0         1.0         0.0         1.0
0.0         1.0         0.0         1.0         0.0         1.0         0.0
1.0         0.0         1.0         1.0         1.0         1.0         1.0
22.1        22.1        27.1        27.1        -41.96      -41.96      -41.96
-23.85      -23.85      -23.85      -23.85      -5.19       1.0         -1.926
1.0         -.2209      1.0         1.882       1.0         3.567       1.0
5.852       1.0         8.245       1.0         9.985       1.0         11.62
1.0         13.68       1.0         63.1        1.0         63.1        1.0
63.1        1.0         -96.9       -41.96      -23.85      13.1        63.1
0.23        0.415       0.25        0.452       -0.03061    -0.03061    -0.03061
-0.03944    -0.03944    -0.03944    -0.03944    -0.04617    .00005811   -0.01502
0.0007823   0.0158      0.001522    0.06081     0.002507    0.116       0.003623
0.2199      0.005533    0.3739      0.008089    0.5198      0.01037     0.6845
0.01267     0.8824      0.01247     -0.08183    -.0004875   -0.08183    -.0004875
-0.08183    -.0004875   -0.00382    -0.03061    -0.03944    -0.05746    -0.08183
-0.05       -0.09       -0.085      -0.155      -0.01259    -0.01259    -0.01259
-0.006099   -0.006099   -0.006099   -0.006099   0.008852    -.0007303   0.01916
-0.001519   0.02793     -0.002306   0.03988     -0.003518   0.05429     -0.005243
0.08011     -0.009005   0.1158      -0.01584    0.1515      -0.02501    0.1919
-0.03890    0.1576      -0.03963    .02507      0.0003584   0.02507     0.0003584
0.02507     0.0003584   -0.03288    -0.01259    -0.006099   0.007145    0.02507
3.812       2.877       8.812       7.877       0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         0.0         0.0         0.0
0.0         0.0         0.0         0.0         1.0         0.0         1.0
0.0         1.0         0.0         0.0         0.0         0.0         0.0
2
1    1    227.19      2    2    360217.0    3    3    10.023
4    4    3.2777      5    5    3.304       6    6    0.1
0.01        0.01        4.628       7.186       200.0       250.0
17.82       0.8
16     100.0      25.0
0.02        0.1         3.0         5.0         10.0        15.0        1350.0
0.3       -0.3        500.        20.
0.45
35.64       1970.32
1   15      1 2142                  1
0.0         0.0         0.0         0.0
155.655     164.447     156.655     164.039     40.8        48.9
0.0         0.0            4  4     0.0
0.0         0.3333      0.6667      1.0
```

```
0.0       0.3333    0.6667    1.0
0.0       0.0       0.0       0.0
75.0      121.083   121.083   121.083   11.1      17.0
0.0       0.0          3  5   0.0
0.0       0.25      0.5       0.75      1.0
0.0       0.5       1.0
0.0       0.0       0.0       0.0
121.083   166.5     121.083   166.0     11.1      17.0
0.0       0.0          3 12   0.0
0.0       0.0676    0.1352    0.2028    0.3513    0.4999
0.6484    0.7969    0.8477    0.8985    0.9492    1.0
0.0       0.5       1.0
0.0       0.0       0.0       0.0
121.083   166.0     133.584   164.447   17.0      40.8
0.0       0.0          7 12   0.0
0.0       0.0676    0.1352    0.2028    0.3513    0.4999
0.6484    0.7969    0.8477    0.8985    0.9492    1.0
0.0       0.1869    0.3738    0.5607    0.7476    0.9345
1.0
0.0       0.0       0.0       0.0
133.584   155.655   137.651   155.655   40.8      48.9
0.0       0.0          4  9   0.0
0.0       0.1863    0.3726    0.5589    0.7452    0.8089
0.8726    0.9363    1.0
0.0       0.3333    0.6667    1.0
0.0       0.0       0.0       0.0
137.651   164.039   144.73    163.33    48.9      63.0
0.0       0.0          5 12   0.0
0.0       0.0676    0.1352    0.2028    0.3513    0.4999
0.6484    0.7969    0.8477    0.8985    0.9492    1.0
0.0       0.25      0.5       0.75      1.0
0.0       0.0       0.0       0.0
144.73    163.33    146.42    163.16    63.0      66.376
0.0       0.0          3 12   0.0
0.0       0.0676    0.1352    0.2028    0.3513    0.4999
0.6484    0.7969    0.8477    0.8985    0.9492    1.0
0.0       0.5       1.0
0.0       0.0       0.0       0.0
183.341   207.1     188.32    210.254   11.1      17.0
-3.0      -3.0         3  5   0.0
0.0       0.25      0.5       0.75      1.0
0.0       0.5       1.0
0.0       0.0       0.0       0.0
188.32    210.254   207.751   222.551   17.0      40.8
-3.0      -3.0         7  5   0.0
0.0       0.25      0.5       0.75      1.0
0.0       0.1869    0.3738    0.5607    0.7476    0.9345
1.0
0.0       0.0       0.0       0.0
75.0      207.1     75.0      207.1     0.0       3.0
5.0       5.0          2 21   0.0
0.0       0.05      0.1       0.15      0.2       0.25
0.3       0.35      0.4       0.45      0.5       0.55
0.6       0.65      0.7       0.75      0.8       0.85
0.9       0.95      1.0
0.0       1.0
0.0       0.0       0.0       0.0
```

```
75.0      207.1     75.0      207.1     3.0       11.1
5.0       3.0          2 21   0.0
0.0       0.05      0.1       0.15      0.2       0.25
0.3       0.35      0.4       0.45      0.5       0.55
0.6       0.65      0.7       0.75      0.8       0.85
0.9       0.95      1.0
0.0       1.0
0.0       0.0       0.0       0.0
75.0      207.1     75.0      207.1     11.1      11.1
3.0       0.0          2 21   0.0
0.0       0.05      0.1       0.15      0.2       0.25
0.3       0.35      0.4       0.45      0.5       0.55
0.6       0.65      0.7       0.75      0.8       0.85
0.9       0.95      1.0
0.0       1.0
0.0       0.0       0.0       0.0
75.0      207.1     75.0      207.1     11.1      11.1
0.0       -3.0         2 21   0.0
0.0       0.05      0.1       0.15      0.2       0.25
0.3       0.35      0.4       0.45      0.5       0.55
0.6       0.65      0.7       0.75      0.8       0.85
0.9       0.95      1.0
0.0       1.0
0.0       0.0       0.0       0.0
75.0      207.1     75.0      207.1     11.1      3.0
-3.0      -5.0         2 21   0.0
0.0       0.05      0.1       0.15      0.2       0.25
0.3       0.35      0.4       0.45      0.5       0.55
0.6       0.65      0.7       0.75      0.8       0.85
0.9       0.95      1.0
0.0       1.0
0.0       0.0       0.0       0.0
75.0      207.1     75.0      207.1     3.0       0.0
-5.0      -5.0         2 21   0.0
0.0       0.05      0.1       0.15      0.2       0.25
0.3       0.35      0.4       0.45      0.5       0.55
0.6       0.65      0.7       0.75      0.8       0.85
0.9       0.95      1.0
0.0       1.0
0.0       0.0       0.0
0.0       0.0        11 1 0   0.0       228347
40.0      50.0      60.0      80.0      100.0     120.0
140.0    160.0     180.0     200.0      207.1
0.0       .1        .1        .1        .1        .1
1        .1        .1        .1        0.
36    0    0    0    0    0
1,3,4,6,7,9,10,13,14,17,18,28
29,39,40, 50,51,61,62,72,73,83,84,94
95,105,106,113,114,121,122,129,130,140,141,151
152,162,163,173,174,184,185,195,196,199,200,203
204,207,208,211,212,215,216,219,220,223,224,227
228,247,248,267,268,287,288,307,308,327,328,347
F    9    0
2    0    0    1
2 159.0    40.8       159.0     48.9
40.8      48.9
2 164.0    40.8       164.0     48.9
40.8      48.9
```

```
F    8    0
2    0    1    1
3 94.94    0.0       94.94     20.0
0.0       8.0       13.03
4 113.05   0.0       113.05    20.0
0.0       5.0       10.0      15.9
F  178    0
1    1    0    1
10 126.43   0.0       150.704   66.376
14.395    24.311    28.541    33.771    38.403    44.679    51.254    56.036
60.519    66.376
13.95
F   32    0
1    1    0    1
3,200.0,0.0,200.0,40.8
11.1      17.0      31.37
10.0
5    6    10
40.0      94.94     113.05    150.0     207.1
0
```

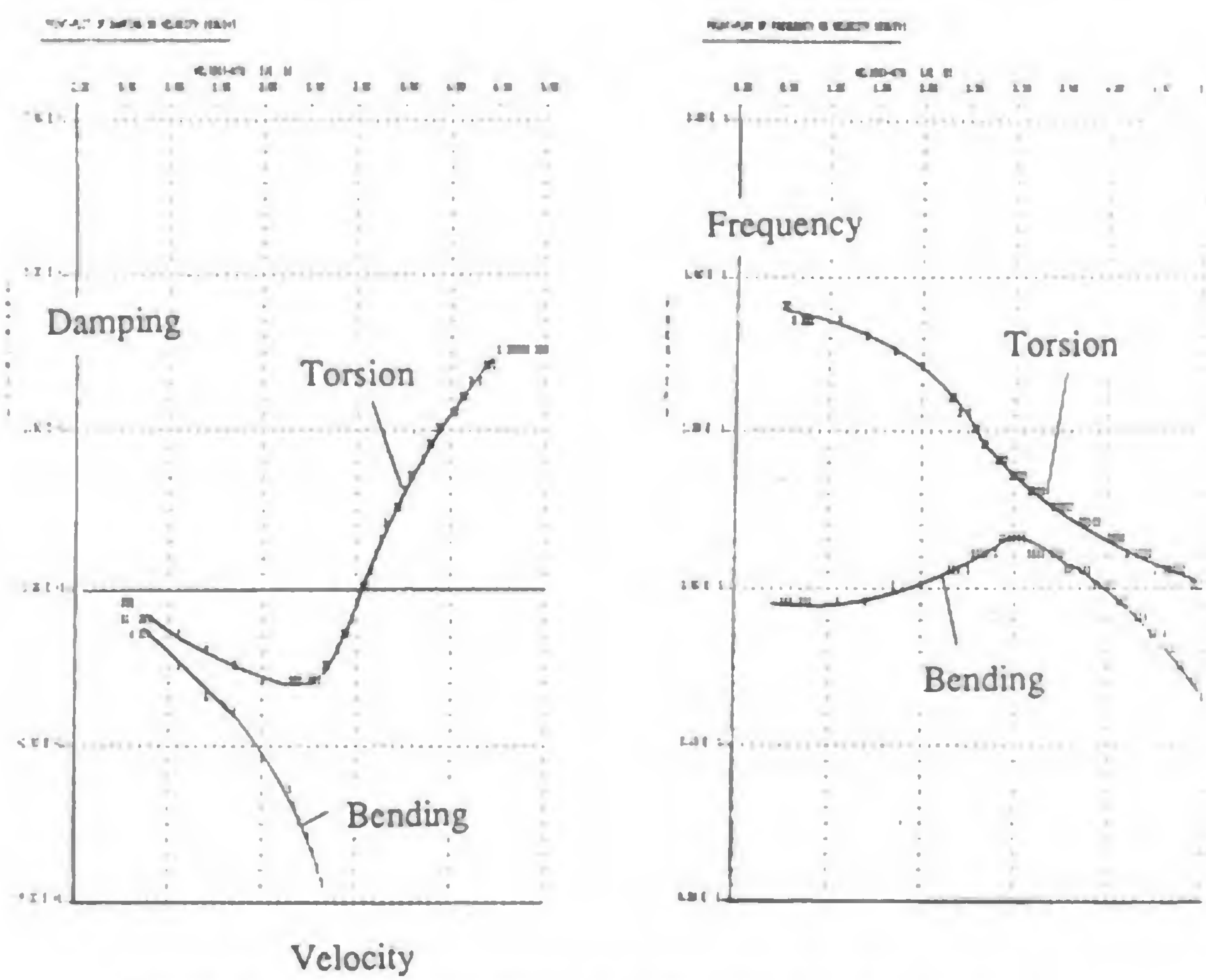



Figure 16. Damping and Frequency Plots for the Goland Wing.

# CHAPTER 8

## *WHAT IS FLUTTER?*

Flutter can be a problem which when detected in flight can be catastrophic. The method outlined in this book is used to determine the flutter speed before the actual flutter flight test is performed, the flutter mode and flutter speed can be predicted and approached with caution. Following is a simple explanation of this most intriguing phenomena.

I was seventeen and she was sixteen, tall, blonde and attractive. Every time I looked at her my heart would start pounding and I would start to sweat. The pounding of my hear was so loud that sometimes I was afraid someone might hear. This was my first encounter of heart *"Flutter"* . At least that is what I thought flutter was at that time. I now as I have become older I realize flutter in aircraft is something else. Maybe not as exciting but definitely worth learning about. So here goes.

Every structure has modes in which it will vibrate. If you envision a diving board with a diver on the end you also know that the diving board will bounce up and down if the diver drops down to sit on the end of the board. See figure 11. The bending of the diving board is the first natural frequency or fundamental mode. The inverse of the time it takes for the board to bend, down and up and down, through one cycle, is called the natural frequency or eigenvalue of the board. This eigenvalue is proportional to the square root of the stiffness divided by approximately one third the mass of the diving board and the mass of the diver. If the diver bounces up and down in rhythm to the natural frequency on the board, he can increase the amplitude of each subsequent cycle until the board has stored enough energy to throw him high into the air. This condition is known as resonance.

Flutter is very much like the diver on the diving board except it is not the diving board that is bouncing up and down but the wing, the fuselage, or the control surfaces. The diver is replaced by the air which excites the flexing of the aircraft structure. If a resonance condition occurs, the only thing that keeps the structure from failing is damping. In an aircraft, several types of damping are present. One is structural damping, which is normally very low but excellent for most composite structures. Most structures have a damping factor of 0.03 (3.0%). From the vibration tests performed on the all graphite/epoxy Starship a damping factor of 0.04 was realized. The other is aerodynamic damping which is usually very high. And the last is friction damping in the control system.

Let's look at a wing section to see how a wing can flutter. See figure 12 for a wing section as it is shown mechanically. Springs are used to represent translational and torsional stiffness. A gust load can cause the wing to bend up and the wing will twist about its neutral axis since the center of pressure for the gust load is ahead of the neutral axis of the wing. If the bending frequency of the wing matches its torsional frequency and no damping were present, resonance (flutter) will exist.

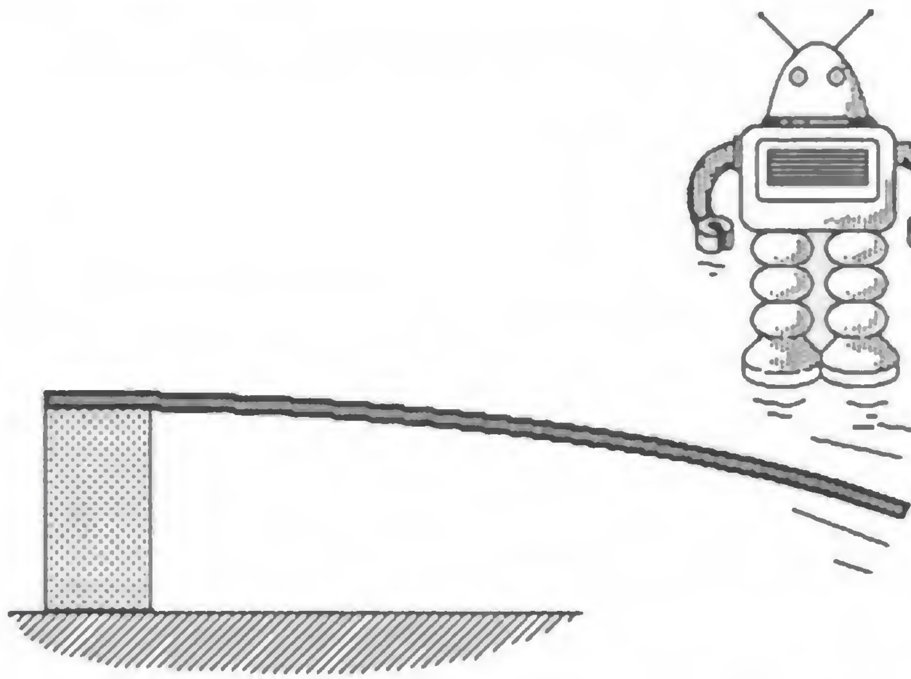


Figure 11. A Diver Bouncing on a Diving Board is Similar to a Wing of an Aircraft in Resonance with the Wind. For the Wing, the Diver is Replaced by the Airstream.

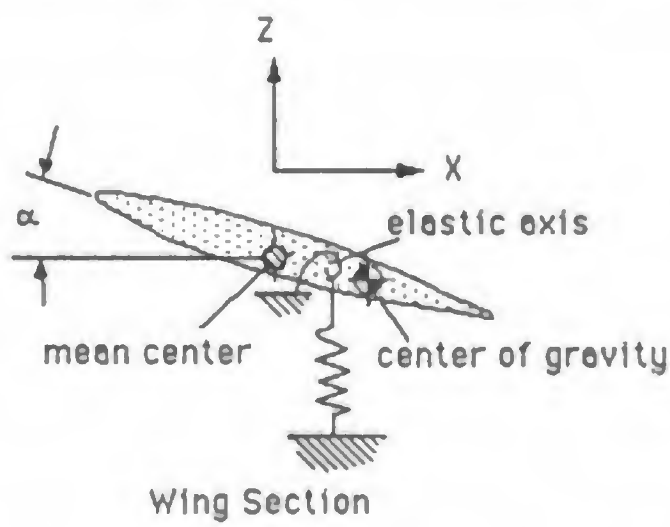



Figure 12. A Section of a Wing in which the Bending Stiffness is Shown by Linear Spring and the Torsional Stiffness is shown by a Torsion Spring. The Center of Gravity is Located Aft of the Elastic Axis such that the Wing will Twist as it Bends. Twisting of the Wing Changes the Airload. Coupling between the Wing Bending and Twisting (Airload) will cause Flutter at the Critical Flutter Speed.

Fortunately the first bending mode frequency for most aircraft wings is much lower than the torsional frequency and all wings have large amounts of aerodynamic damping for the first bending mode. For example, the bending frequency of the wing for the Lancair IV is 4.097 cycles per second or 4.097 Herz (Hz) and the torsional frequency is 27.31 Hz. See figure 4 for finite element model results showing the deflected mode shape of the Lancair IV wing superimposed on the non-deflected shape.

The airload acting on the aileron of a wing or any other control surface acts as a spring which tends to return the aileron to its natural float angle. This spring increases in stiffness as the airspeed increases and it together with the inertia of the aileron gives the aileron a natural frequency that increases with airspeed. See figure 13 and figure 13, mode 4 which is the aleron flapping mode for the Lancair IV wing. If the frequency of the aileron coincides with other frequencies such as the second wing bending mode frequency or the control linkage/aileron frequency, flutter can occur.

The empennage and fuselage also have a large number of natural frequencies that can couple and for high performance aircraft, wing modes can couple with empennage modes. For example, wing antisymmetric bending modes often couple with fuselage torsional modes. What also complicates the flutter problem is that reversible control tabs can couple with the main control surface modes. Control linkage play, mass distribution such as fuel in the wings, stick free and stick fixed, and material stiffness for composite materials affect the natural frequencies. See Appendix C.

The speeds at which flutter occurs is called the flutter speed, $V_{DF}$.

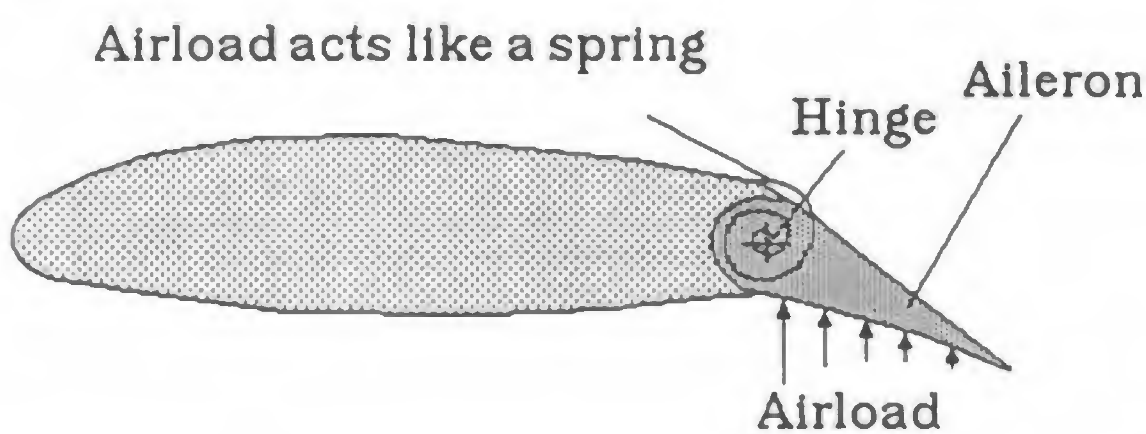



Figure 13. The Airload on the Aileron Acts as a Spring in which the Stiffness Increases with Increasing Dynamic Pressure, Airspeed.

# CHAPTER 9

## THEORY

A flutter analysis is performed by solving the matrix equation:

$$-\omega^2[GM]+(1+ig_0)[GK]-\omega^2[GQ]\{q\} = 0 \tag{1}$$

where $\omega$ is frequency, $g_0$ is the inherent structural damping assumed in the structure, $\{q\}$ is the complex eigenvector, $i=\sqrt{-1}$, [GM] is the generalized mass matrix, [GK] is the generalized stiffness matrix, and [GQ] is the complex generalized aerodynamic force matrix. The matrix [GQ] depends on the Mach number M and the reduced frequency $k=\omega b/V$, where b is a reference length and V is the flight speed. Equation (1) can be rewritten in standard eigenvalue form as,

$$[A]\{q\}= \lambda\{Q\} \tag{2}$$

with,

$$[A]=[\dot{G}\dot{K}]^{-1}([GM]+[GQ]) \tag{3}$$

and,

$$[\dot{G}\dot{K}]=(1+ig_0)[GK] \tag{4}$$

The eigenvalue $\lambda$ is written as,

$$\lambda=(1+ig)/\omega^2 \tag{5}$$

In the so-called V-g solution procedure, the Mach number and altitude are fixed, and Eq. (2) is solved for a series of values of reduced frequency k. Each solution produces a number of eigenvalues $\lambda$ equal to the order of the matrices (the number of modes used). From Eq. (5), associated values of the damping parameter g and the frequency $\omega$ are calculated. Plotting g vs. V produces a set of curves where the critical flutter instability is determined by the transition from negative to positive values of g. Plots of $\omega$ vs. V are used to identify the modes participating most actively in the flutter. A final step involves varying the altitude or the Mach number until the critical flutter speed matches that implied by the choice of altitude and Mach number. The matrices [GM] and [GK] are in all cases diagonal; since the mode shapes (eigenvector) are mass normalized when using NISA386, [GM] is simply the idem matrix and [GK] contained the squares of the natural frequencies along its diagonal.

9.1 Subsonic Doublet-Lattice H7WC Program. Reference 2.

For analyzing control surface configurations, multiple interfering surfaces and interfering surface-body configurations, the doublet-lattice program is considered to be one of the best methods and it is used in SAF. The formulation of this method starts by equating the wash normal to a harmonically oscillating surface to the lifting pressure:

$$w(P_j) = \frac{1}{8\pi} \iint_S C_p(P)\ K(P_i, P, k, M)\ d\xi\, d\sigma, \tag{6}$$

where,

- P is any point on the planform, the coordinates of which are $\xi, \eta, \zeta$
- $P_j$ is a $j^{th}$ point, the coordinates of which are x, y, z
- x, $\xi$ are streamwise coordinates
- y, $\eta$ are spanwise coordinates
- z, $\zeta$ are vertical coordinates
- $\sigma$ is the tangential spanwise coordinate

$$\frac{\overline{w}}{U} = \alpha + i\frac{k}{b_o}h$$

- w is the normal wash angle
- $C_p$ is the differential pressure coefficient
- K is the kernel function relating the normal wash at $P_j$ to the a unit pressure at P
- M is the Mach number
- $b_o$ is the reference semichord
- h is the vertical displacement of the surface
- $\alpha$ is the streamwise slope of the deformed surface
- k is the reduced frequency
- S is the surface area of all lifting surfaces included in the analysis.

If the surface is divided into J elements over which the pressure is assumed constant, Eq (6) becomes,

$$w(x,y,z) = \frac{1}{8\pi}\sum_{j=1}^{J} C_{p_j} \iint_{\text{Element } j} K\,(x-\xi, y-\eta, z-\zeta, \omega, M)\, d\xi\, d\sigma. \tag{7}$$

Next, the pressure is assumed to arise from a loaded line at the 1/4-chord of each element. As illustrated in figure 17, this is equivalent to an unsteady horseshoe vortex whose bound portion lies along the 1/4-chord of the element . The resulting expression is:

$$w(x,y,z) = \frac{1}{8\pi}\sum_{j=1}^{J} C_{p_j} \Delta\xi_j \int_{\text{Element } j} K\left(x - \frac{\xi_j}{4}, y-\eta, z-\zeta, \omega, M\right) d\sigma, \tag{8}$$

where $\Delta\xi_j$ is the length of the average chord of element J, and the integration is taken along the 1/4-chord line of the j$^{th}$ element.

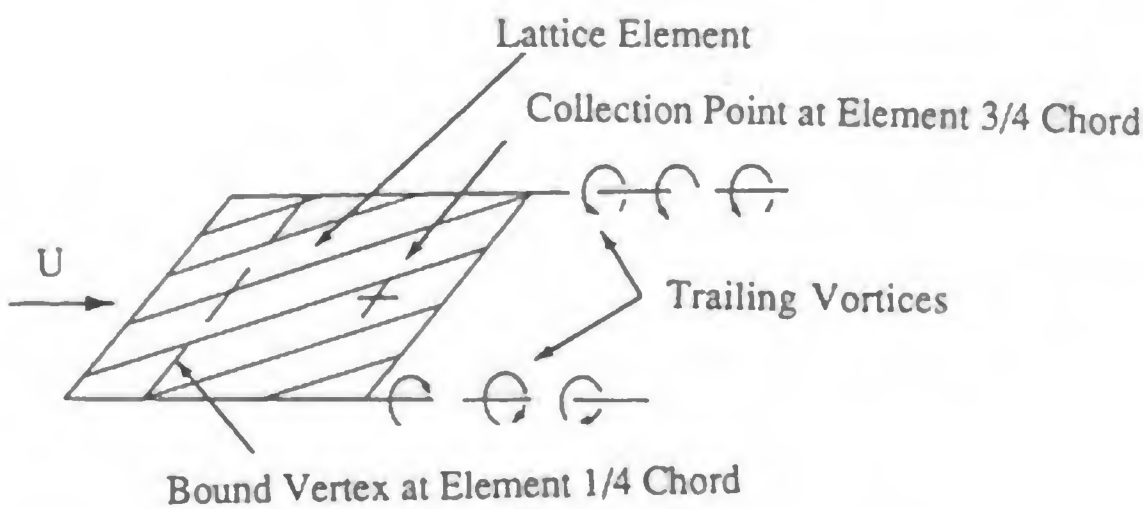



Figure 17. Horseshoe Vortex Element used in Doublet-Lattice Method.

If the downwash is calculated at points located at the 3/4-chord midspan of each of the J elements and Eq (8) is satisfied at each of these points, the following set of equations holds for i = 1 to J:

$$w_i = \frac{1}{8\pi}\sum_{j=1}^{J} C_{p_j}\,\Delta\xi_j \int_{\text{Element}\,j} K\left(x_i - \frac{\xi_j}{4},\, y_i - \eta,\, z_i - \zeta,\, \omega,\, M\right) d\sigma \tag{9}$$

This can be written in matrix form as:

$$\{w\} = [D]\,\{c_p\}, \tag{10}$$

where $D_{ij}$ relates the downwash at the $i^{th}$ point to the pressure over the $j^{th}$ element. By solving this set of linear equations, the pressure distribution over the surface is calculated.

Because the doublet lattice method is not explicitly fitting pressure functions on a planform, multiple aerodynamically interacting surfaces can be modelled by simply defining lattice elements on each surface. In this case, Equation (9) still holds, but J is now the number of elements on all surfaces.

To account for aerodynamic interaction between bodies and surfaces each body is modelled by axial doublets along the body axis and by panels of unsteady horseshoe vortex elements on the body surface in the vicinity of each lifting surface with which the body might interact. See Figure 18, for example. The strength of each axial doublet is calculated by slender body theory. The incremental downwash on the panels on the body surfaces and on the lifting surfaces caused by these axial doublets is then computed and subtracted from the prescribed downwash for these surfaces. Equation (10) becomes,

$$\left[\frac{D_{SS}\;D_{SI}}{D_{IS}\;D_{II}}\right]\left\{\frac{C_{p(S)}}{C_{p(I)}}\right\} = \left\{\frac{w_S}{w_I}\right\} - \left[\frac{F_{SB}}{F_{IB}}\right]\left\{C_{p_{(B)}}\right\}, \tag{11}$$

where,

| | |
|---|---|
| $C_{p(S)}$ | is the pressure coefficient distribution on the lifting surfaces |
| $C_{p(I)}$ | is the pressure coefficient distribution on the interacting body surfaces |
| $C_{p(B)}$ | is the pressure coefficient distribution on the slender-body axial elements |
| $w_S$ | is the downwash distribution prescribed on the lifting surfaces |
| $w_I$ | is the downwash distribution prescribed on the interacting body surfaces |
| $F_{SB}$ | is the downwash distribution on the lifting surfaces caused by unit pressure coefficients along the slender bodies |
| $F_{IB}$ | is the downwash distribution on the interacting body surfaces caused by unit pressure coefficients along the slender bodies |

$D_{SS}$ is the downwash distribution on the lifting surfaces caused by unit pressure coefficients on the lifting surfaces

$D_{SI}$ is the downwash distribution on the lifting surfaces caused by unit pressure coefficients on the interacting body surfaces

$D_{IS}$ is the downwash distribution on the interacting body surface caused by unit pressure coefficients on the lifting surfaces

$D_{II}$ is the downwash distribution on the interacting body surfaces caused by unit pressure coefficient on the interacting body surfaces.

Since $\left\{ C_{p_{(B)}} \right\}$ was calculated from the downwash and geometry of each body using slender-body theory, the only unknowns in this matrix equation are $\left\{ C_{p_{(S)}} \right\}$ and $\left\{ C_{p_{(I)}} \right\}$. This set of equations is solved for these pressure distributions by a standard linear system solution algorithm using Gaussian triangularization and back solution.

Generalized aerodynamic forces arising from lifting surface pressures are calculated by,

$$Q_{rs} = \frac{b^2 p}{2k^2} \sum_{j=1}^{J} h_r^{(j)} \, C_{p_s}^{(j)} \, A^{(j)} \tag{11}$$

where, here A(j) is the area of the j$^{th}$ element. This can be rewritten in matrix form for all N modes as:

$$[Q] = -\frac{b^2 p}{2k^2} [h] \, [A] \, [c_p] \tag{12}$$

where [A] is a diagonal matrix. When interacting bodies are present, three generalized forces are present:

$$[Q] = \left[ Q_S + Q_I + Q_B \right] \tag{13}$$

The first arises from the product of deformations, pressures and areas of the lifting surface elements; the second from the product of those of the interacting body surface elements; and the third from the produce to those of the body elements - appropriate areas in this last case are the products of the body element diameters and lengths.

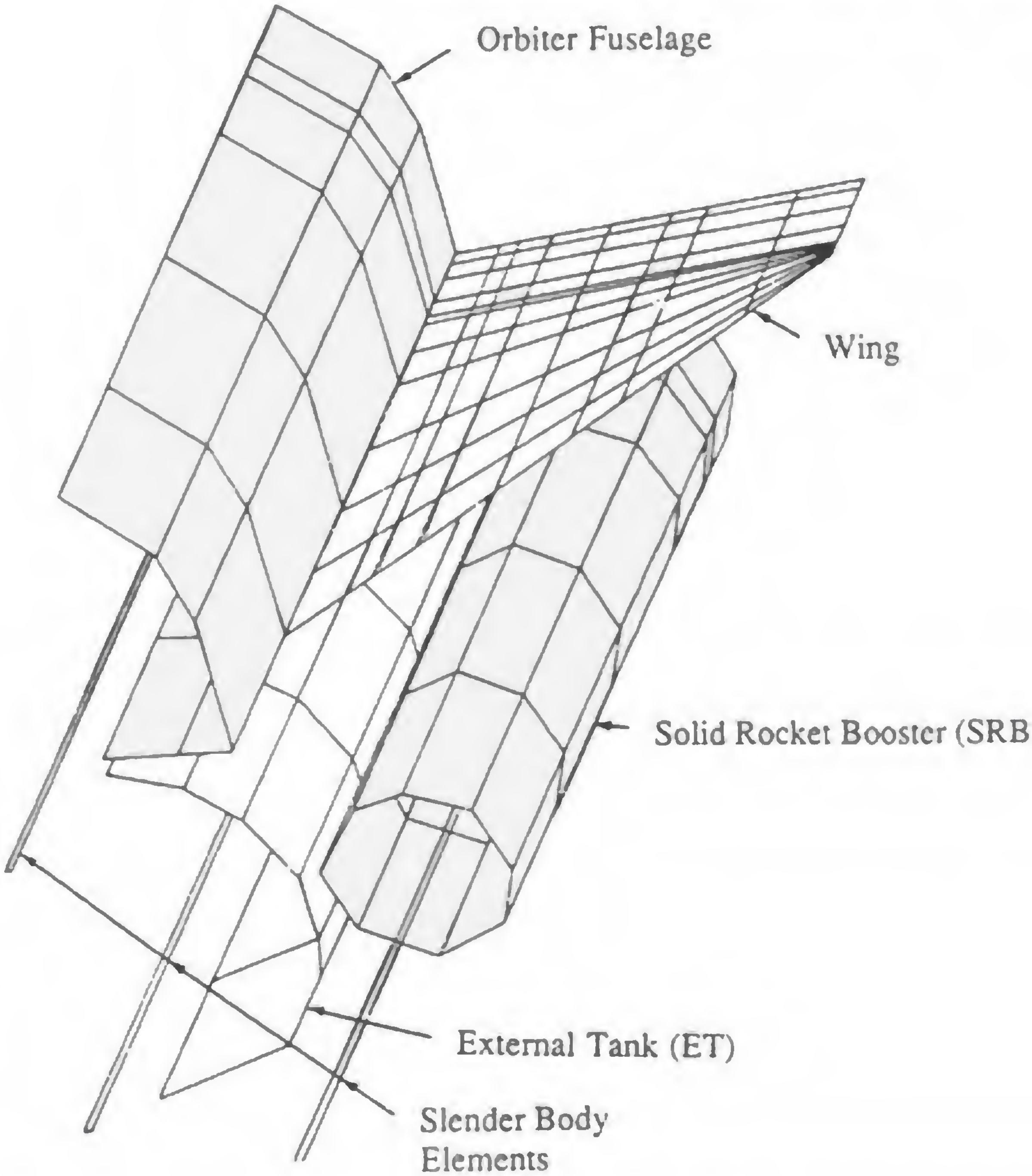



Figure 18. Aerodynamic Model of a Space Shuttle Using the Doublet-Lattice Method.

To obtain satisfactory pressure distributions, the lifting surface must be divided into strips of elements whose edges are parallel to the free stream. An example is shown in figure 19. Additionally, element edges should lie along surface edges, fold lines and control surface hinge lines. Three guidelines should be observed in subdivision:

- The leading and trailing edges of adjacent paris of elements should be aligned and located at a constant percent of the strip chord when possible.

- The dimensions of elements should be decreased in the directions and regions of large gradients in pressure and/or downwash, such as near hinge lines, leading edges and wing tips.
- The aspect ratio of each element should be unity or less. However, this is not always possible, especially in regions where a large gradient is expected. This is evident in the example shown in figure 14.

The optimum configuration is predicted by the need for keeping the number of elements to a minimum and at the same time generating generalized air force terms that are satisfactory for flutter analysis. It may become necessary to test a number of trial configurations and compare results before making a final analysis with a fixed element layout.

## 9.2 Modal Interpolation

For the above aerodynamic routine, the normal wash angle is required at specified aerodynamic grid points on the lifting surface:

$$w_{ij} = \frac{\bar{w}_{ij}}{U} = \alpha_{ij} + i\frac{k}{b_o}h_{ij} \qquad (14)$$

where $\alpha_{ij}$ and $h_{ij}$ are the streamwise slope and the displacement of point i in the $j^{th}$ vibration mode. since the modal data from the vibration analysis is specified on a dynamics grid which, in general does not coincide with the aerodynamics grid, an interpolation is required. The procedure used consists of representing the dynamics grid as a set of spanwise oriented lines connecting node points at which modal deflections are specified. These deflections are interpolated along the lines to each spanwise station at which aerodynamic grid points lie and then are interpolated chordwise along each such station to each aerodynamic grid point. This scheme is illustrated in Figure 19.

Using this set of lines and deflections, the interpolation proceeds according to the original scheme.

For a surface with a control surface or some other special region across the boundaries of which the modal deflections are not continuous, an option is available whereby the interpolation is performed over the main surface and over the control surfaces separately using separate sets of lines and node points. In this manner, modal discontinuities across the boundaries are preserved.

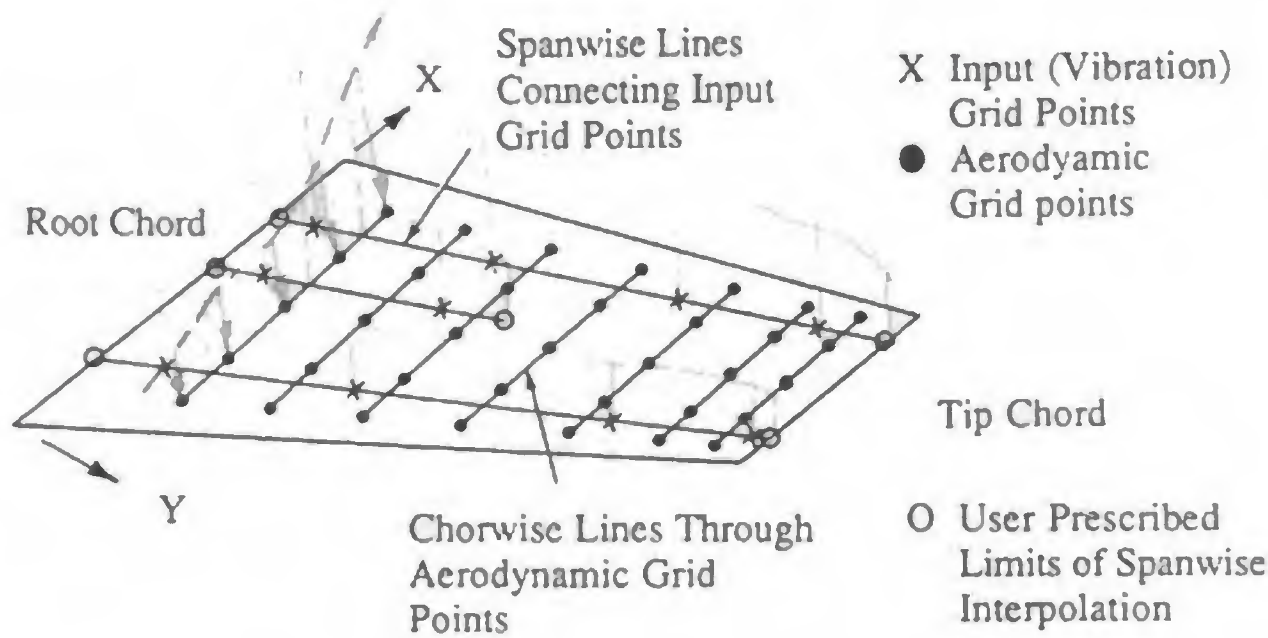



Figure 19. Modal Interpolation Scheme.

In a variation of this scheme, available as a program option, modal streamwise slopes, as well as deflections, are specified as input data along a single spanwise line. The program then creates a second line parallel to and at a specified streamwise distance from the specified line and transforms the modal slopes and deflections to a set of deflections along each line:

$$\begin{Bmatrix} h_1 \\ \hline h_2 \end{Bmatrix} = \begin{bmatrix} I : 0 \\ \hline I : \delta \end{bmatrix} \begin{Bmatrix} h_1 \\ \hline \alpha_1 \end{Bmatrix} \tag{15}$$

where,

$\{h_1\}$ and $\{\alpha_1\}$ are the deflections and streamwise slopes prescribed along the first line

$\{h_2\}$ are the deflections computed along the second line

$[I]$ is an identity matrix

$[\delta]$ is a diagonal matrix having the specified separation between the lines as each of its diagonal terms.

Using this set of lines and deflections, the interpolation proceeds according to the original scheme.

For a surface with a control surface or some other special region across the boundaries of which the modal deflections are not continuous, an option is available whereby the interpolation is performed over the main surface and over the control surfaces separately using separate set of lines and node points. In this manner modal discontinuities across the boundaries are preserved.

The calculations performed in the above schemes use the Lagrangian interpolation formula. Accordingly, a polynomial, g(x), is determined as an approximation to a function, f(x), the value of which is known at each of N+1 points, $\{x_0, x_1, \ldots, x_N\}$. This polynomial is computed by,

$$g(x) = \sum_{k=0}^{N} \frac{(x - x_0)\ldots(x - x_{k-1})\,(x - x_{k+1})\ldots(x - x_N)}{(x_k - x_0)\ldots(x_k - x_{k-1})\,(x_k - x_{k+1})\ldots(x_k - x_N)} f(x_k). \tag{16}$$

In the present program, the polynomial, g(x), is limited to degree N = 3 to minimize convolutions in the approximation. Consequently, in cases for which the modal deformations are specified at more than four points spanwise or chordwise, the user can further restrict the polynomial to a linear or parabolic function in regions where extrapolation is needed either spanwise or chordwise.

### 9.3 Generalized Aerodynamic Force Interpolation. Reference 3.

For a flutter analysis, generalized aerodynamic forces are required at several reduced velocities. To reduce the computation time needed to obtain these forces, interpolation is used to determine the generalized aerodynamic forces at all desired reduced velocities from forces directly computed at a small set of selected, references reduced velocities. A separate interpolation is performed on the real and on the imaginary part of each term of the generalized aerodynamic force matrix. As in the modal interpolation, Lagrangian interpolation is used and the approximating function is limited to a piecewise continuous second- or third-order polynomial.

In the present program, the user supplies a goodness-of-fit tolerance and six reference reduced velocities ordered by increasing value and distribute over the range required in the subsequent flutter analysis: $\{v_1, v_2, \ldots v_6\}$. Using generalized forces computed at three of these six, a generalized aerodynamic matrix at a fourth is determined by interpolation and compared with a matrix of generalized forces directly computed at that reduced velocity. The comparisons are performed for various arrays, C, formed from combinations of the terms of the generalized force matrix and take the form of the test:

$$\frac{\mathrm{abs}\left(\|C\|_{computed} - \|C\|_{interpolated}\right)}{\|C\|_{computed}} \leq \text{tolerance} \tag{17}$$

where. $\|C\|$.is the Euclidean norm - defined as the square root of the sum of the square of the terms of array, C. This test is made separately for C chosen as:

(1) The entire generalized force matrix, $[Q]$
(2) A vector of the main diagonal terms of $[Q]$ and
(3) Each successive two-by-two matrix formed about the main diagonal of $[Q]$.

If any of the above test fail, the computed forces at the fourth reduced velocity are added to the interpolation basis; and tests are made at a fifth reduced velocity. The procedure is continued for a sixth reduced velocity if necessary. After the goodness-of-fit tests have been made, the procedure for obtaining generalized forces at any selected reduced velocity is determined.

In choosing the reference reduced velocities, the following considerations should be observed:

(1) For the computed aerodynamic forces to most accurate, the lowest reduced velocity should be as high as possible consistent with the range of values needed for the flutter solutions.
(2) Generally, the first and sixth reference reduced velocities should span the expected range of v for which generalized aero forces are to be calculated. This precaution eliminates the need for extrapolation.
(3) The reference reduced velocities should emphasize regions where flutter is expected.

9.4 Vibration Analysis. Reference 3.

To perform a modal flutter analysis it is essential to have accurately determined mode shapes, natural frequencies, and generalized masses. This information may come from the finite element analysis or ground vibration tests and preferably both.

This subsection describes an analytical technique used by SAF for solving the eigenvalue problem,

$$\lambda[GM]\{X\} = [GK]\{X\} \tag{18}$$

where [GM] is the global mass matrix (including non-structural mass items), and [GK] is the global stiffness matrix. The solution to Equation (18) is given by ($\lambda_i$, $x_i$), i = 1,. . . .n, where $\lambda_i$ is the $i^{th}$ eigenvalue (natural frequency squared), and $x_i$ is the $i^{th}$ eigenvector (mode shape). The order of the system is n, and the lowest m (m < n) frequencies and corresponding mode shapes, and the generalized masses are desired. The generalized masses are defined as,

$$[GM_i] = \{x_i\}^T [GM]\{x_i\} \qquad i = 1, \ldots\ldots m < n \tag{19}$$

where T denotes matrix transposition.

SAF uses a simultaneous iteration algorithm called SIMIT to compute the modes and frequencies. This method was selected because of its excellent convergence characteristics and its relative simplicity. The procedure takes a minimum of storage and is easily adapted for the calculation of unrestrained nodes.

The key step in the SIMIT method is a matrix iteration of the form,

$$[A][U] = [V] \tag{20}$$

where [A] is a symmetric matrix, [U] is an n by m matrix of trial vectors, and [V] is an n by m matrix of iterated vectors. Note that all modes of interest are simultaneously iterated.

In practice it is necessary to use about twice as many trial vectors as the number of vectors desired. The iterated vectors are further processed as will be escribed below before they become the trial vectors in the next iteration. The SIMIT method differs from the usual matrix iteration method in that a sweeping matrix is not required. Also a dynamical matrix $[D] = [GK]^{-1} [GM]$ need not be formed.

As stated above, the matrix [A] must be symmetric. An important step in the SIMIT technique is to transform the eigenvalue problem in Equation (20) to the form

$$[A]\{q\}= \lambda \{Q\} \tag{21}$$

where [A] is symmetric, which has the eigenvalue eigenvector solutions ($\lambda_i$, $q_i$), i=1, . . .n. Note that this is the same equation as Eq (2).

This is accomplished by defining

$$[A] = [L]^{-1}[GM][L]^{-T} \tag{22}$$

$$\{q\} = [L]^{T}\{x\} \tag{23}$$

where,

$$[GK] = [L]\,[L]^{T} \tag{24}$$

and [L] is the lower triangular decomposition of the stiffness matrix. Note that [A] so defined is symmetric and that the eigenvalue problem described by Eq (21) and Eq (22) is equivalent to that of Eq (20).

In practice the [A] matrix is never explicitly formed. Instead, for a given set of trial vectors U, the product [A] [U] is formed in the following steps:

(1) Solve $[L]^{T}[Y] = [U]$ for [Y] by back substitution.
(2) Form [Z] = [GM] [Y] by premultiplication of [Y] by [GM].
(3) Solve [L] [V] = [Z] for [V] by forward substitution.

It is easily verified that the above steps which make use of properties of the [L] matrix are equivalent to forming the matrix product indicated in Equation (20). [L] is formed by Cholesky decomposition of the stiffness matrix which is stored in "skyline" i.e., variable bandwidth form.

The diagonal terms of $[U]^{T}[V]$ are used as the first estimates of the eigenvalues. The eigenvectors must be modified before the next step since they would otherwise all converge to the vector corresponding to the largest eigenvalue. An "interaction analysis" is performed on the vectors which decouples them. The decoupled vectors, [W], follow from the matrix multiplication,

$$[W] = [V]\,[T]^{*} \tag{25}$$

where $[T]^{*}$ has unit diagonal terms and skew-symmetric off-diagonal terms given by,

$$t_{ij}^{*} = -2b_{ij}/c_{ij}\,, \tag{26}$$

$b_{ij}$ are the elements of the "interaction matrix" as shown by Eq (27) and Eq (28).

$$[B] = [U]^T[V] \tag{27}$$

and, $$c_{ij} = b_{ii} - b_{jj} + S\sqrt{(b_{ii} - b_{jj})^2 + 16b_{ij}^2} \tag{28}$$

with S equal to the sign of $b_{ii}$ - $b_{jj}$. The [W] vectors are then made orthogonal by the usual Schmidt orthogonalization process. By efficient use of storage registers, it is not necessary to have separate storage for the [B] and [T]* matrices.

The orthogonalized vectors, [U], obtained from the [W] matrix are then stored in [U] and the iteration is continued. The process is completed by performing either a specified number of iterations or by achieving a given small error on the Euclidean norm of $[U] - [\bar{U}]$.

The final step of the procedure is to use back substitution to recover the mode shapes for the original vibration problem of Equation (18). This is accomplished by solving Equation (23) where the columns of the converged [U] are the $q_i$ vectors.

A number of investigators have used the SIMIT method on practical sized problems. Although the basis for the interaction analysis appears somewhat empirical, numerical results have been consistently good.

In the case of unrestrained aircraft modes, the [GK] matrix will not be positive definite and hence cannot be decomposed into the form $[L][L]^T$. It can be made positive definite by use of the eigenvalue shifting technique. This technique involves adding $\mu$ [GM] $x_i$ to both sides of Equation (18) where $\mu$ is a predetermined positive constant. Hence the "shifted problem" becomes

$$[GK][x] = \eta[GM][x] \tag{29}$$

where,

$$[GK] = [K] + \mu[GM] \tag{30}$$

$$\eta = 1 + \mu \tag{31}$$

For this explanation, $\mu$ is selected as

$$\mu = \min_j \, (GK_{jj}/GM_{jj}) \tag{32}$$

where $GK_{jj}$ and $GM_{jj}$ are the diagonal terms of the stiffness and mass matrices, respectively. Hence for unrestrained modal calculations, Equation (29) is solved by the SIMIT technique and the desired eigenvalues are then recovered by solving Equation (31) for $\lambda_i$. Zero frequency mode shapes are obtained in the solution of Equation (29).

# APPENDIX A

## *REFERENCES*


1. Advisory Circular No. 23.629-1A, "Means of Compliance with Section 23.629, Flutter, " Superintendent of Documents, U.S. Government Printing Office, Washington, DC 20402-9371.

2. Wilkinson, K., et al., "An Automated Procedure for Flutter and Strength Analysis and Optimization of Aerospace Vehicles," AFFDL-TR-75-137, Vols. I and II, December 1975.

3. Taylor, R. F., Miller, K. L., and Brockman, R. A., "A Procedure for Flutter Analysis of FASTOP-3 Compatible Mathematical Models," AFWAL-TR-81-3063, Vol. I and II, June 1981.

4. Taylor, R. S. " Improvements to the Fastex Flutter Analysis Computer Code UDRTR 8714," University of Dayton, Research Institute, Dayton, Ohio 45469

5. Brockman, R. A., "Modification of the Structural Optimization Code in FASTOP," UDR-TR-77-51, University of Dayton Research Institute, Dayton, Ohio, July 1987.

6. Brockman, R. A., "Modification of the Flutter Optimization Code in FASTOP," UDR-TR-77-60, University of Dayton Research Institute, Dayton, Ohio, October 1977.

7. Markowitz, J., and Isakson, G., "FASTOP-3: A Strength, Deflection, and Flutter Optimization Program for Metallic and Composite Structure," AFWAL-TR-78-50, Vols. I and II, May 1978.

8. Giesing, J. P., Kalman, T. P., Rodden, W. P., "Subsonic Unsteady Aerodynamics for General Configurations," AFFDL-TR-71-5, Part I, Vol. II, November 1971.

9. Scanlan and Rosenbaum, "Introduction to the Study of Aircraft Vibration and Flutter," McMillan Company, New York, 1951.

10. Smilg, B. and Wasserman, L., "Application of Three-Dimensional Flutter Theory to Aircraft Structures", Air Force Technical Report (A.F.T.R.) 4798, 1942.

11. Meirovitch, L., "Analytical Methods in Vibration," Macmillan Company, New York, 1967.

12. Albano, E., and Rodden, W. P., "A Double-Lattice Method for Calculating Lift Distributions on Oscillating Surfaces in Subsonic Flows," AIAA Journal, Vol. 7, No. 2, February 1969, pp. 279-285; errata, AIAA Journal, Vol. 7, No. 11, November 1969, p. 2192.

13. Charles C. Pate, Mahendra K. Punatar and Ray W. Winn, "Some Special Investigation Areas in Light Aircraft Flutter," Society of Automotive Engineers, Inc. Two Pennsylvania Plaza, New York, N.Y. 10001. 1972.

14. Robert Rosenbaum and A.A. Vollmecke, "Airframe and Equipment Engineering Report 45, Simplified Flutter Prevention Criteria for Personal Type Aircraft," Federal Aviation Administration.

15. G.S. Rasmussen, "Technical Memorandum No. 10, Critique of Airframe and Equipment Engineering Report No. 45," G.S. Rasmussen & Associates, 10760 Burbank, North Hollywood, CA.

16. Raymond L. Bisplinghoff, Holt Ashley, and Robert L. Halfman, "Aeroelasticity," Addison-Wesley Publishing Company, Menlo Park, CA. 1955.

17. Raymond L. Bisplinghoff and Holt Ashley, "Principles of Aeroelasticity." John Wiley and Sons, Inc., New York. 1962.

18. AGARD Conference Proceedings No. 278, "Low Cost Flutter Clearance," North Atlantic Treaty Organization, 7 rue Ancelle, 92200 Neuilly-sur-Seine, France. Paper presented at the 48th Meeting of the Structures and Materials Panel, Williamsburg, VA. 1979.

19. Goland, M.,"The Flutter of a Uniform Cantiflevered Wing," ASME, Journal of Applied Mechanics, Buffalo, N.Y. December 1945.

# APPENDIX B

## *NISA INPUT FILE FOR LANCAIR IV WING*

```
ANAL=EIGENVALUE
**EXEC=CHECK
FILE=SW
SAVE=26,27
EIGEN
EXTRACTION=SUBSPACE,ACCELERATED
MASS FORMULATION=CONSISTENT
AUTO=ON
*ECHO=OFF
*A1
LANCAIR IV WING SYMMETRIC MODES,
WITH FUEL, RUN 5 - 20 HZ AILERON
*C1
1,33,1
2,20,1
3,20,10
4,12,1
5,30,1
*D1
1,4
.02///
2,4
.25///
3,4
.03///
4,4
.04///
5,4
.40///
6,4
.18///
7,4
.125///
8,4
.10///
9,4
.06///
10,4
.03///
11,4
.05///
12,4
.04///
13,4
.02///
14,4
.024///
15,4
.14///
16,4
.14///
17,4
.2///
18,4
.12///
19,4
1.67///
20,4
.15///
21,4
.16///
22,4
.21///
23,4
.14///
24,4
.22///
25,4
.083///
26,4
.058///
27,4
.04///
28,4
0.48///
29,4
0.16///
30,4
0.095///
31,4
0.072///
32,4
0.32///
33,4
.01///
34,4
.24///
35,4
.05,2E-4,2E-4,4E-4
36,4
.09///
**FUEL, 40 GAL
37,6
.017//,0//
38,6
.011//,0//
```

```
39,6
.01//,0//
40,6
.0068//,0//
**1/2 FUS MASS, OP WT=2,800 LBS
41,6
2.6,2.6,2.6,81E3,81E3,0.75E3
**LINKS
42,4
12.6,3.14,3.14,6.28
**AILERON MASS BALANCE
43,4
.193,.02,.02,.04
*D2
1,4
0///
2,4
45///
*D3
3,0,1,2,1,2,1,2,1,3,1
3,0,3,2,3,2,1,2,1,3,1
3,0,4,2,1,2,1,2,1,3,1
3,0,2,19,2,2,1,2,2,3,2
3,0,3,2,1,2,1,2,1,3,1
3,0,1,2,1,2,1,2,2,3,2
3,0,1,7,33,2,1,2,1,3,1
3,0,3,7,33,2,1,2,1,3,1
*E1
**UPPER SKIN
$$,-3,32,21
1,44,43,11,12,$,1,1,0,21,1,1
$$,-3,32,21
64,172,171,139,140,$,1,1,0,21, 1,1
$$,-3,32,10
127,34,33,1,2,$,5,1,0,10,1,1
$$,-3,32,10
157,162,161,129,130,$,5,1,0,10,1,1
**LOWER SKIN
$$,-2,32,10
187,450,449,481,482,$,5,1,0,10,1,1
207,514,513,1,2,$,5,1,0,10,1,1
$$,-3,32,10
217,322,321,353,354,$,5,1,0,10,1,1
247,524,523,11,12,$,1,1,0,21,1,1
$$,-2,32,21
268,460,459,491,492,$,1,1,0,21,1,1
$$,-3,32,21
310,332,331,363,364,$,1,1,0,21,1,1
**UPPER CAP
373,130,129,97,98,$,4,2,28,2,1,1
375,132,131,99,100,$,4,2,34,3,1,1
378,135,134,102,103,$,4,2,6,2,1,1
380,137,136,104,105,$,4,2,29,2,1,1
382,139,138,106,107,$,4,2,16,3,1,1
385,142,141,109,110,$,4,2,7,2,1,1
387,144,143,111,112,$,4,2,31,5,1,1
392,149,148,116,117,$,4,2,4,11,1,1
**LOWER CAP
404,418,417,449,450,$,4,2,24,2,1,1
406,420,419,451,452,$,4,2,34,2,1,1
408,422,421,453,454,$,4,2,23,2,1,1
410,424,423,455,456,$,4,2,30,4,1,1
414,428,427,459,460,$,4,2,31,3,1,1
417,431,430,462,463,$,4,2,4,18,1,1
**INBD CAP
435,129,560,563,97,$,4,2,28
436,560,561,564,563,$,4,2,5
437,561,562,564,$,1,3,5
438,417,565,568,449,$,4,2,24
439,565,566,569,568,$,4,2,21
440,566,567,569,$,1,3,23
**SPAR SHEAR WEBS
441,450,449,97,98,$,2,2,17,6,1,1
447,456,455,103,104,$,2,2,21,3,1,1
450,459,458,106,107,$,2,2,25,22,1,1
472,449,568,563,97,$,2,2,32
473,568,569,564,563,$,2,2,32
474,569,567,562,564,$,2,2,32
475,418,417,129,130,$,2,2,17,7,1,1
506,417,565,560,129,$,2,2,32
507,565,566,561,560,$,2,2,32
508,566,567,562,561,$,2,2,32
**REAR SPAR
509,258,257,225,226,$,2,2,27,2,1,1
511,260,259,227,228,$,2,2,27,5,1,1
516,265,264,232,233,$,2,2,27,24,1,1
540,257,570,573,225,$,2,2,11
543,290,289,321,322,$,2,2,27,2,1,1
545,292,291,323,324,$,2,2,27,5,1,1
550,297,296,328,329,$,2,2,27,24,1,1
574,289,576,579,321,$,2,2,11
577,290,289,257,258,$,2,2,27,6,1,1
594,296,295,263,264,$,2,2,27,25,1,1
619,289,576,570,257,$,2,2,11
**RIBS
644,513,481,65,33,$,2,2,11
645,481,449,97,65,$,2,2,11
646,417,385,161,129,$,2,2,11
647,385,353,193,161,$,2,2,11
648,353,321,225,193,$,2,2,11
649,321,289,257,225,$,2,2,11
650,523,491,75,43,$,2,1,0
651,491,459,107,75,$,2,1,0
652,427,395,171,139,$,2,1,0
653,395,363,203,171,$,2,1,0
654,363,331,235,203,$,2,1,0
655,331,299,267,235,$,2,1,0
656,531,499,83,51,$,2,1,0
657,499,467,115,83,$,2,1,0
658,435,403,179,147,$,2,1,0
659,403,371,211,179,$,2,1,0
660,371,339,243,211,$,2,1,0
661,339,307,275,243,$,2,1,0
```

```
662,537,505,89,57,$,2,1,0
663,505,473,121,89,$,2,1,0
664,441,409,185,153,$,2,1,0
665,409,377,217,185,$,2,1,0
666,377,345,249,217,$,2,1,0
667,345,313,281,249,$,2,1,0
668,544,512,96,64,$,2,1,0
669,512,480,128,96,$,2,1,0
670,542,510,94,62,$,2,1,0
671,510,478,126,94,$,2,1,0
672,446,414,190,158,$,2,1,0
673,414,382,222,190,$,2,1,0
674,382,350,254,222,$,2,1,0
675,350,318,286,254,$,2,1,0
676,544,512,96,64,$,2,1,0
677,512,480,128,96,$,2,1,0
678,480,448,160,128,$,2,1,0
679,448,416,192,160,$,2,1,0
680,416,384,224,192,$,2,1,0
681,384,352,256,224,$,2,1,0
682,352,320,288,256,$,2,1,0
683,423,391,167,135,$,2,1,0
684,391,359,199,167,$,2,1,0
685,359,327,231,199,$,2,1,0
686,327,295,263,231,$,2,1,0
687,487,455,103,71,$,2,1,0
688,519,487,71,39,$,2,1,0
689,7,519,39,$,2,3,4
690,322,321,225,226,$,2,2,11,6,1,1
696,321,579,573,225,$,2,2,11
697,449,417,129,97,$,2,2,11
**FUEL WEB
698,354,353,193,194,$,6,1,0,31,1,1
**AILERON
729,589,588,580,581,$,7,1,0,3,1,1
732,589,588,584,585,$,7,1,0,3,1,1
735,585,584,580,581,$,5,1,0,3,1,1
738,585,584,592,593,$,8,1,0,2,1,1
827,581,580,592,593,$,8,1,0,2,1,1
829,592,593,$,5,4,43,2,1,1
741,584,588,580,$,2,3,7
742,592,584,580,$,2,3,7
743,585,589,581,$,2,3,7
744,593,585,581,$,2,3,7
745,586,590,582,$,2,3,7
746,594,586,582,$,2,3,7
747,587,591,583,$,2,3,7
748,595,587,583,$,2,3,7
**AILERON SUPPORTS
749,307,604,275,$,5,3,5
750,313,605,281,$,5,3,5
751,318,606,286,$,5,3,5
752,606,285,286,$,5,3,5
**FLAP
753,601,600,257,263,$,1,1,0
754,602,601,263,267,$,1,1,0
755,603,602,267,275,$,1,1,0
756,601,600,289,295,$,1,1,0
757,602,601,263,267,$,1,1,0
758,603,602,267,275,$,1,1,0
759,289,600,257,$,2,3,7
760,295,601,263,$,2,3,7
761,299,602,267,$,2,3,7
762,307,603,275,$,2,3,7
**CONTROL ROD
763,313,585,$,6,4,35
**FUEL
764,385,$,0,5,37,12,1,1
776,397,$,0,5,38,16,1,1
792,481,$,0,5,39,12,1,1
804,493,$,0,5,40,16,1,1
**FUS
820,607,$,0,5,41
821,607,563,$,5,4,42
822,607,562,$,5,4,42
823,607,568,$,5,4,42
824,607,567,$,5,4,42
825,607,570,$,5,4,42
826,607,576,$,5,4,42
*F1
1,0,$,155,3.3,0
32,0,1,$,0,1.45,3.6
33,0,$,155,4.4,1
64,0,1,$,0,1.9,4.1
65,0,$,155,5.65,4
96,0,1,$,0,2.6,6.3
97,0,$,155,6.6,8.3
128,0,1,$,0,3.05,8.80
129,0,$,155,7.3,13.2
160,0,1,$,0,3.4,13.8
161,0,$,155,7.5,18.2
192,0,1,$,0,3.35,16.6
193,0,$,155,7.4,23.2
224,0,1,$,0,3.2,19.3
225,0,$,155,6.4,31
256,0,1,$,0,2.85,22.7
257,0,$,155,6.1,33
288,0,1,$,0,2.6,24.6
289,0,$,155,1.4,33
320,0,1,$,0,0.85,24.6
321,0,$,155,1.1,31
352,0,1,$,0,0.65,22.7
353,0,$,155,0.35,23.2
384,0,1,$,0,0.35,19.3
385,0,$,155,0.2,18.2
416,0,1,$,0,0.18,16.6
417,0,$,155,0.4,13.2
448,0,1,$,0,0.1,13.8
449,0,$,155,0.8,8.3
```

```
480,0,1,$,0,0.3,8.8
481,0,$,155,1.45,4
512,0,1,$,0,0.55,6.3
513,0,$,155,2.4,1
544,0,1,$,0,1.1,4.1
560,0,$,162,7.3,13.2
562,0,1,$,198,7.3,13.2
563,0,$,162,6.6,8.3
564,0,$,180,7,10.75
565,0,$,162,0.4,13.2
567,0,1,$,198,0.4,13.2
568,0,$,162,0.8,8.3
569,0,$,180,0.6,10.8
570,0,$,163,6.1,33
573,0,$,163,6.4,31
576,0,$,163,1.4,33
579,0,$,163,1.1,31
**AILERON
580,0,$,65,3,33
581,0,$,35,2.6,30.5
582,0,$,10,2.2,28.5
583,0,$,0,2.1,27.5
584,0,$,65,1.2,33
585,0,$,35,1.2,30.5
586,0,$,10,1.2,28.5
587,0,$,0,1.2,27.5
588,0,$,65,2.3,38.5
589,0,$,35,2,36
590,0,$,10,1.6,33.5
591,0,$,0,1.4,32.2
592,0,$,65,1.2,31
593,0,$,35,1.2,29
594,0,$,10,1.2,27.5
595,0,$,0,1.2,26.2
**FLAP
600,0,$,155,3.4,47
601,0,$,124.5,3,44
602,0,$,105,2.8,42.2
603,0,$,65,2.3,38.5
604,0,$,64.9,3,33
605,0,$,34.9,2.6,30.5
606,0,$,9.9,2.2,28.5
**FUS
607,0,$,180,3.3,14
*H1
**GRAPHITE FABRIC
EX,1,0,4.2E6
EY,1,0,4.2E6
GXY,1,0,6E5
GYZ,1,0,10
GXZ,1,0,10
DENS,1,0,2.7E-4
**GLASS
EX,2,0,2.3E6
EY,2,0,2.3E6
GXY,2,0,1.25E5
GYZ,2,0,10
GXZ,2,0,10
DENS,2,0,3.6E-4
**CORE
EX,3,0,0
GXZ,3,0,2.2E3
GYZ,3,0,2.2E3
GXY,3,0,0
DENS,3,0,7.6E-6
**GRAPHITE TAPE
EX,4,0,9E6
EY,4,0,9E6
GXZ,4,0,10
GYZ,4,0,10
GXY,4,0,1.3E5
DENS,4,0,2.8E-4
**STEEL
EX,5,0,28E6
EY,5,0,28E6
GXZ,5,0,17E6
GXY,5,0,17E6
GYZ,5,0,17E6
DENS,5,0,7.3E-4
**CONTROL ROD
EX,6,0,68E3
EY,6,0,68E3
GXZ,6,0,34E3
GXY,6,0,34E3
GYZ,6,0,34E3
DENS,6,0,2.6E-4
*CPDISP
UXYZ,$,604,580
UXYZ,$,605,581
UXYZ,$,606,582
*EIGCNTL
13,0,25,,0,0,1.0E-3,-1
*SPDISP
607,UX,0
607,ROTY,0
607,ROTZ,0
*MODEOUT
3 $ 1
*EIGOUT
1,0,3,1,0,1
*ENDDATA
```

# APPENDIX C

## *AC NO. 23.629-1A, FLUTTER COMPLIANCE*

C.1. AIRPLANE CATEGORIES

1. GENERAL. Airplanes in the general category are those with a typical exterior configuration; i.e., high, mid, or low wing; single fin and single horizontal stabilizer aft-mounted on the fuselage; and tractor powerplant installations.

2. SPECIAL DESIGN. The special design category includes airplanes with certain design features that experience has shown warrant special consideration with regard to flutter. Flutter free operation for these special unconventional configurations may be shown by analyses which include an assessment of the effects of critical parameters. Flight flutter tests to supplement those analyses are recommended. Some of these special unconventional configurations are:

a. Any aircraft with a design dive speed of 260 knots (EAS) or more at altitudes below 14,000 feet and Mach 0.6 or more at altitudes at and above 14,000 feet.

b. Any aircraft approved for flight in icing conditions. (The effect of ice accretions on unprotected surfaces, including those which might occur during system malfunctions, should be considered).

c. Pusher powerplants.

d. Canard geometry.

e. T, V, X, H, or any other unusual tail configuration.

f. Any external pods or stores mounted to wing or other major aerodynamic surface.

g. Fuel tanks outboard of 50% semispan.

h. Tabs which to not meet the irreversibility criteria.

i. Spring tabs.

j. All-movable tails, i.e., stabilators.
k. Slender boom or twin-boom fuselages.

l. Multiple-articulated control surfaces.

m. Wing spoilers.

n. Hydraulic control systems with stability augmentation.

o. Full span flaps.

p. Leading edge devices (i.e., slots, etc.).

q. Geared tabs (servo or anti-servo, etc.).

3. SIMPLIFIED CRITERIA

a. Guidelines. Airframe and Equipment Engineering Report No. 45, Ref. 14, is intended to serve as a guide to the small airplane ($V_D$ less than 260 knot EAS at altitudes below 14,000 ft.) designed for the prevention of flutter, aileron reversal, and wing divergence. The material presented relies upon:

(1) A statistical study of the geometric, inertia, and elastic properties of those airplanes which had experienced flutter in flight, and the methods used to eliminate the flutter.

(2) Limited wind-tunnel tests conducted with semi-rigid models. These were solid models of high rigidity with motion controlled at the root by springs to simulate wing bending and torsion. Spring at the control surface were used to simulate rotation.

(3) Analytic studies based on the two-dimensional study of a representative section of an airfoil.

b. Wing and Aileron. Prevention of wing flutter is attempted through careful attention to three parameters; wing torsional flexibility, aileron balance, and aileron free play.

(1) The aileron balance criteria is obtained from the aileron product of inertia, , about the wing fundamental bending node line and the aileron hinge line; and the aileron mass moment of inertia, I, about its hinge line. A limit of the parameter, K/I, is set as a function of $V_D$.

(2) A wing torsional flexibility factor, F, is defined and a limit established as a function of $V_D$. In order to apply the criteria, one needs to know wing twist distribution per unit applied torque, wing planform, and limit dive speed.

(3) The total free play of each aileron with the other aileron clamped to the wing must not exceed the specified maximum.

c. Elevator and Rudder. Dynamic balance criteria for the elevator an rudder (similar to the K/I of the aileron) are defined and limits set as a function of limit dive speed. In order to utilize the criteria, the following information is required:

| | |
|---|---|
| (1) Geometry | - Horizontal tail semichord at the midspan<br>- Semispan of horizontal tail<br>- Distance from fuselage torsion axis to tip of fin<br>- Semichord of vertical tail measured at 70% span position |
| (2) Stiffness | - Fuselage vertical bending frequency<br>- Fuselage torsional frequency<br>- Fuselage lateral bending frequency |
| (3) Mass | - Elevator static balance about hinge line<br>- Elevator mass moment of inertia about hinge line<br>- Elevator: product of inertia referred to stabilizer centerline and elevator hinge line<br>- Rudder static balance about hinge line<br>- Product of inertia of rudder referred to fuselage torsion axis and rudder hinge line<br>- Rudder mass moment of inertia about hinge line |

d. Tabs. It is recommended that all reversible tabs be balanced about tab hinge line. The degree of static and dynamic balance should be determined by rational analyses. In practice, most tabs are irreversible, which means:

(1) For any position of the control surface and tab, no appreciable flection of the tab can be produced by means of a moment applied directly the tab when the control surface is held in a fixed position.

(2) The total free play at the tab trailing edge should be less than the following:

(i) If the tab span does not exceed 35 percent of the span of the supporting control surface, the total free play shall not exceed two percent of the distance from the tab hinge line to the trailing edge of the perpendicular to the tab hinge line.

(ii) If the tab span equals or exceeds 35 percent of the span of the supporting control surface, the total free play is not to exceed one percent of the distance from the tab hinge line to the trailing edge of the tab perpendicular to the tab hinge line.

(3) The tab natural frequency should be equal to or should exceed a calculated value and expressed as a function of tab and control surface geometry and airplane dive speed.

(4) Spring loaded tabs are free to rotate and thus are not reversible. Generally, these tabs will require dynamic as well as static lance. Extensive flutter analysis is always needed to define these requirements.

4. RATIONAL ANALYSIS.

a. Review of Past Analysis. Review of previous flutter analyses conducted upon similar aircraft can provide the engineer with useful information regarding trends, critical modes, etc. Although in general such a review is not used as a substantiation basis for a new aircraft, it can provide a useful tool in evaluating the effect of modifications to existing certified aircraft.

b. Two-Dimensional Analysis. The flutter characteristics of straight wings (or tails) of large aspect ratio can be predicted reasonably well by considering a "representative section" that has two or three degrees of freedom. Translation and pitch are always needed and, for control surfaces, the third freedom would be rotation about the hinge line.

c. Three-Dimensional Analysis. Current analysis is based upon consideration of total span, rather then "representative section" discussed in 4.b. above. The behavior is integrated over the whole structure being analyzed. Some idealization is always necessary; the most common being the division of the span into strips.

For FAR Part 23 airplanes, quite often the wing and empennage analyses are conducted separately; however, this is not always adequate for unconventional configurations. Both the symmetric and antisymmetric motions require investigation.

Calculated mass and stiffness distributions are generally used to calculate uncoupled modes and frequencies. These values are then used to conduct a coupled vibration analysis; the resulting coupled modes and frequencies are then usually compared with measured natural modes.

The calculated stiffness-related inputs are generally adjusted until good agreement is obtained with the test data. Once satisfactory agreement is achieved, the coupled vibration analysis is normally used for the flutter calculations.

It is suggested that one perform certain variations in the assumed input conditions to see which parameters are critical. Control surface balance conditions and system frequencies (especially tab frequencies) are often investigated parametrically. The effect of control system tension values at the low and high ends of the tolerance range should be assessed.

It may be advantageous to arbitrarily vary certain main surface frequencies (stiffness), especially torsional frequencies and engine mode frequencies, while leaving other frequencies constant.

Sometimes it is desirable to evaluate the effect of a slight shift in spanwise node location for a very massive item where the node is located very close to or within the item. (Test data may not be sufficiently accurate for this assessment.)

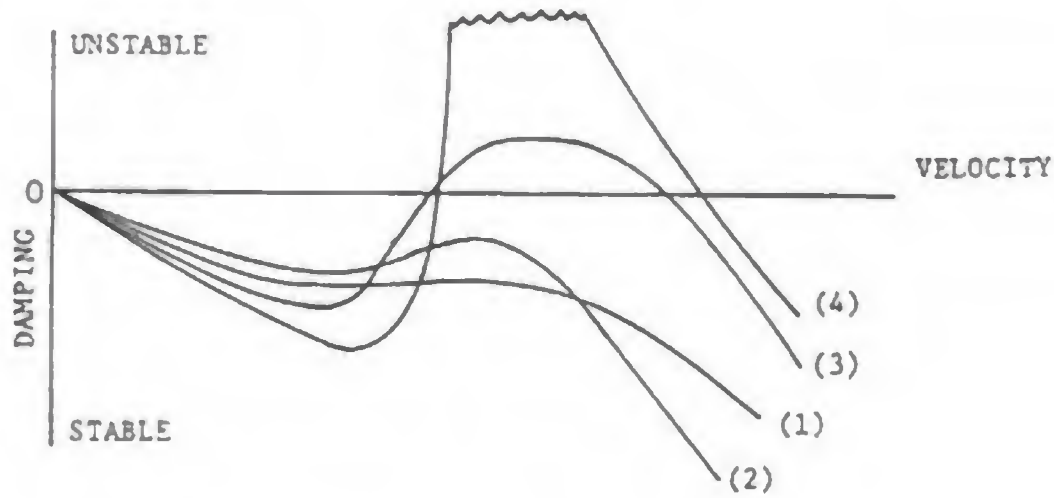
It is normal practice to run a density-altitude check to include near-sea-level, maximum and any other pertinent altitudes such as the knee of the airspeed-altitude envelope where the design dive speed becomes MACH-limited.

It is desirable to investigate combined wing-empennage modes for high performance ($V_D$ of 260 KEAS or above) airplanes, as well as for airplanes with unconventional configurations.

Flutter Analysis Evaluation: For a given set of input parameters, the resulting output generally consists of a number of theoretical damping values (g) with associated airspeeds and frequencies.

Various cross plots of these values among themselves and versus varied input parameters allow a study of trends. Common plots are: damping vs. equivalent airspeed (g-V plots), control surface balance vs. flutter speed, uncoupled frequency vs. flutter speed, altitude vs. flutter speed, etc. normally only the critical items will be extensively compared.

Of particular importance is an evaluation in the neighborhood of the crossing of a damping velocity (g-V) curve toward the unstable damping region through zero. The typical critical g-V curve will first become increasingly stable and with increasing speed will turn and rise toward or through g=O, then at some higher speed may again turn toward the stable region. Typical characteristics are discussed in the following examples:




Curves 1 and 2 show slight trends toward instability, but do not approach actual instability.

Curve 3 crosses the stability axis but, depending on the inherent structural damping, may or may not actually become unstable. Curve 4 is obviously unstable and probably violent, since its slope is steep as it passes through zero. In actual flight it may only be a mile an hour or so between completely stable and extremely unstable explosive flutter. Flight tests are not advisable when this type plot is observed inside or near the flight envelope.

Much can be learned from g-V curves. (Absolute values should be viewed with some reserve as there is no perfect one-to-one correspondence of the analytical parameters and flight parameters.) Where the critical curve crosses the axis (with respect to $V_D$ for the airplane) is important. Equally important is the rate of approach to instability (slope of curve).

The general practice is to use a damping value of g=0.03 at 1.2 $V_D$ as the flutter limit of the g-V plots. However, this value should be used with caution if the slope of the curve is large (damping decreases very rapidly with an increase in airspeed) between g = O and 0.03. In cases where the slope is steep, it is suggested that the g = O airspeed be at least 1.2 $V_D$.

If flight flutter testing is conducted to verify damping under the above circumstances, extreme caution should be exercised.

For damping curves such as (3), which peak out below 1.2 $V_D$, the predicted damping should be no more unstable than g = 0.02 unless justification is provided by other acceptable means.

5. ANALYSIS PLUS FLIGHT TEST
Although paragraph (c) of section 23.629 permits certification based upon flight test only, it is recommended that some analysis precede a flight flutter test. The results of any of the analysis procedures in paragraph 4 would be useful and could be used to provide guidance for formulating a flight flutter test plan. In all cases, as required by paragraph 23.629(a), the natural frequencies of main structural components should be determined by vibration tests or other approved methods prior to conducting any flight testing.

6. GROUND TESTING
Comparison of test data may be used in lieu of a totally new analysis in the case of dynamically similar aircraft. Comparison would usually be based upon geometry, mass and stiffness distributions, speed regime, and more importantly, upon a comparison of the measured couplet vibration modes.

a. Test data would normally include:

(1) Ground Vibration Testing
(2) Control Surfaces ant Tab Mass Property Determination
(3) Stiffness Tests
(4) Free Play Measurements of all Tabs
(5) Rotational Frequency for all Tabs
(6) Tab System Rotational Stiffness

b. Appendix 1 of AC23.629-1A presents some guidelines for recommended tests and procedures.

c. The degree of similarity between aircraft that is required for justification can vary greatly. Some of the factors which should be considered are the amount of safety margin available, flutter speed sensitivity to certain parameters, and the thoroughness of the original analysis.

7. WHIRL MODE. Beginning with Amendment 23-7, paragraph 23.629(e) required investigation of the whirl mode phenomena for multiengine turbopropeller planes only. The basis being these airplanes characteristically have mounted engines wherein the stability of a flexibly mounted engine/propeller on an elastic wing is of major concern. Amendment

23-31 of graph 23.629(e) now requires an investigation of the whirl mode phenomena for both single and multiengine turbopropeller airplanes. Though airframe influence may be negligible for fuselage mounted single engine tractor configurations, the potential for propeller whirl flutter still exists. For pusher configurations, empennage motion may be significantly affected by engine/propeller forces. Stability of either installation is dictated, in part, by engine mount stiffness, damping, mass properties, motion axes, propeller geometry and propeller advance ratio. Therefore, to assure freedom from whirl mode flutter, all turbopropeller installation investigations should include, in addition to the appropriate airframe degrees of freedom:

a. Whirl mode degree of freedom which takes into account the stability the plane of rotation of the propeller and significant elastic, inertial, aerodynamic forces.

b. Propeller, engine, engine mount, and airplane structure stiffness damping variations appropriate to the particular configuration; e.g., deterioration of engine isolators, large cantilevered engine installations.

Generalized mathematics are presented in appendix 2 of AC 23.629-1A.

C.3. MODIFICATIONS TO AIRCRAFT ALREADY CERTIFICATED

8. REEVALUATION. Considerable judgment is often required to determine the degree of reevaluation necessary. If the mass, mass distribution, or the stiffness distribution are affected sufficiently to result in possible significant changes in resonant frequencies of major modes, mode shapes, or mass coupling terms in the flutter equations, then some reevaluation, such pre-mod and post-mod GVT data comparison, or analysis may be required. Some examples of significant changes are:

a. Engine (Propeller). A change in mass or mass moment of inertia of powerplant or in its mounting system (bushings, etc.) or a c.g. shift should be investigated. On single-engine airplanes, such changes will most likely affect fuselage and empennage frequencies and mode shapes. For engines mounted on the wings, the entire airplane may be affected.

For changes in existing designs which entail significant increases in engine power and/or airplane speed, special assessments of the effect on primary and secondary control systems should be made. If tabs are exposed to the propeller slip stream, particularly on airplanes with a dive speed greater than 260 KEAS, it may be necessary to impose the fail-safe criterion.

b. Structural Cutouts. Severing or bridging across major structural members, such as fuselage bulkheads and ribs or stringers of aerodynamic surface, may produce discontinuities in stiffness parameters that significantly alter the vibratory response of the structure.

The significance of a change may be ascertained by its effect on the energy terms in the flutter modes being evaluated.

C.4. CONTROL SURFACES AND TABS

9. RESPONSE. The aerodynamic force on an airfoil is very sensitive to control surface displacement, which in turn is responsive to both control motions and aerodynamic forces from tab displacement. Control surface displacement may result from deflection of the control system, deflection of the control surface attachment, or structural deflection of the control surface itself under forces from control application, aerodynamic force due position or velocity of position change, and inertia force.

10. BALANCE. Control surfaces and tabs are balanced to prevent rotation bout their hinges resulting from inertial response to motion in any flutter ode. When the flutter mode consists of motion about some axis, perpendicular to the control surface hinge axis, a concentrated ballast is most efficiently used. Caution should be used to assure that its location is in a high response area of the vibratory mode, which is difficult when :he mode is complex. Caution should also be used to assure that its attachment is secure. Because the attachment is subjected to oscillatory Loads which cause fatigue failures and because a distributed ballast achieves balance against all flutter modes, it is conservative to distribute the ballast in accordance with the spanwise weight distribution of the surfaces. If less than static balance is provided, the effect of variations in the amount of balance should be evaluated. To guard against unintended balance changes in service, sealing and-proper drain holes should be provided to minimize the risk of water, ice, or dirt accumulation in a control surface or tab. Excessive accumulation of these substances could alter the static and/or dynamic balance of the control sufficiently to adversely affect flutter characteristics.

ll. VIBRATORY MODES. Control surface rotation about its hinge line is affected by various constraints. Control system stiffness and the rigidity of interconnection between control surfaces determine the primary rotational modes. Both symmetric and antisymmetric modes should be considered. Vibration mode changes resulting from the modifications to the control system such as the addition of a bob weight must be assessed for their effect on flutter. Secondary rotations may result from flexure of the attaching structure or bending of the control surface. This is a major consideration for long short-chord tabs and may affect their effective irreversible characteristics. When it is necessary to raise a tab frequency by redesign, consideration should be given to the contributions of: hinge bending perpendicular to the surface (especially near the horn-actuator station), horn length, axial stiffness of the push-pull rod or link, mounting flexibility and lateral stability at push-rod attachment of the tab actuating mechanism.

12. ANALYSES. In most cases involving control surfaces, the flutter speeds are largely governed by the mass balance vaLues and distributions. It is wise for the flutter analyst to cover a range of balance values and distributions to determine the most satisfactory ones. It is common t find that a change which improves one mode degrades another. When conducting a multi-degree-of-freedom analysis, it is advisable to investigate the effect of control system frequency from zero to about 1-1/2 times the system. Due to friction, etc., it may be difficult to excite and measure control system frequency accurately. The stiffness can measured at the surface with the control locked in the cockpit and, using inertia of the end items, the system frequency can be calculated.

Theoretical values of tab and primary control surface aerodynamic derivatives have, for some configurations, produced higher flutter speeds flutter model testing. Analytically derived tab and primary control aerodynamic coefficients based on strip theory have for some configurations produced higher flutter speeds than wind tunnel tests. Therefore, flutter speed sensitivity to variations in the theoretical coefficients should be evaluated in all control surface/tab investigations.

13. SAFE REQUIREMENTS. Amendment 23-23 of paragraph 23.629(f) requires flutter free operation after failure, malfunction, or disconnect of single tab element. This fail-safe requirement is extended to include a , malfunction, or disconnect of any element in the primary flight system or flutter damper on airplanes with a dive speed in excess of 260 KEAS below 14,000 feet, or MACH 0.6 above 14,000 feet.

Potential failures that require investigation include, but are not limited or primary control trim actuating system, primary control actuating (both of which includes bellcranks, pulleys, brackets, and their attachments), and control cables or push rods. Control surface hinges and tab hinges, their attachments, and local portions of structure need not be included as part of the control system in this investigation.

Possible means of compliance to actuating system failures (i.e., actuators, , rods) may be achieved by incorporating dual systems, mass balancing controls to counter the rotation of a zero stiffness free surface, or by incorporating a combination of the two. Proper mass balancing, particularly tabs, requires considerable care and knowledge of the flutter mechanism to assure adequacy of the design in suppressing flutter. Dual load path designs should include an assessment of residual strength with a single failure to assure that the remaining path will not fail before the single failure is detected during appropriate specified inspection intervals.

C.5. DIVERGENCE AND CONTROL REVERSAL

14. GENERAL. Steady state aeroelastic instabilities in an airfoil are avoided by providing adequate torsional rigidity. Methods to determine the adequacy of torsional rigidity are outlined in references 2 and 3 of appendix 4 in AC 23.629-1A.

15. AIRFOIL DIVERGENCE. Divergence occurs when the aerodynamic torque exceeds the torque resisting capability of the wing. Because the aerodynamic torque is a function of speed as well as deflection, whereas the resisting torque is a function of deflection only, there exists a limiting divergence speed. Divergence may occur with no warning.

16. CONTROL REVERSAL. Control reversal will often be preceded by pilot comments of "heavy" or "sluggish" ailerons. A limiting reversal speed is reached when the change in lift due to control surface rotation is nullified by the change in lift due to airfoil twist.

C. APPENDIX 1. GROUND TESTING

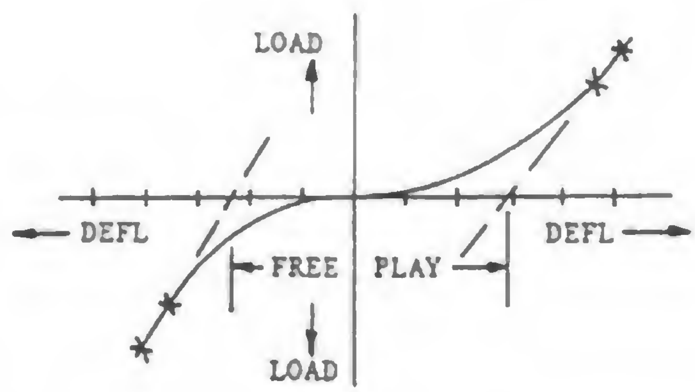
1. INTRODUCTION. The adequacy of the methods used to show compliance with section 23.629, as discussed in the main body of this document, is dependent upon the availability of reliable ground test data to verify the analytical data used and/or to serve as a basis for flutter substantiation per the simplified criteria of reference 14. This appendix, therefore, presents guidelines in conducting the more significant tests required to accomplish this objective. However, in keeping with the general purpose of this advisory circular, the information provided is not intended to be mandatory, nor is it to be considered an exhaustive treatment of the subject.

2. CONTROL SURFACE AND TAB MASS PROPERTIES. The experimental mass properties of control surfaces and tabs (weight, static moments, moments of inertia, and c.g.) are important ingredients in flutter substantiation. These properties form a basis for verification of the analytical data used in the rational analysis and provide the necessary parameters for use in the simplified criteria. Reference 1 presents a detailed procedure for the experimental determination of these properties.

3. TAB FREE PLAY. Free play tests provide the necessary data for determining the effectiveness of a tab in fulfilling the requirements for irreversibility as specified in the main body of this document. In addition to demonstrating the maximum free play available, these tests provide the stiffness of the actuating system for use in computing tab rotational frequency.

Free play and stiffness may best be measured by a simple static test wherein "upward" and "downward" (or "leftward" and "rightward") point forces are applied near the trailing edge of the tab at the spanwise attachment of the actuator (80 as not to twist the tab). The control surface should be blocked to its main surface. Rotational deflection readings are then taken near the tab trailing edge using an appropriate measuring device, such as a dial gauge. Several stepwise load and deflection readings should be taken using loads first applied in one direction, then in the opposite.

A plot of these load deflections typically appears as follows:




Free play is then defined by extending the best straight lines through zero. System stiffness may then be obtained from the slopes of the curves sway from the zero point.

4. INFLUENCE COEFFICIENT TESTS. Bending and/or torsion influence coefficient test results form the basis for the definition of component stiffness distributions. The extent of the tests depends on the intended use of the data. A full scale test program, wherein the coefficients of each spanwise mass strip are defined may be desired if experimental data is the primary source for defining component stiffness. In contrast, calculated influence coefficients, based on analytical bending (EI) and torsion (GJ) stiffness distributions, may be adjusted reliably with considerably less test data. A method is outlined below for determining influence coefficients for conventional structure, i.e., aspect ratio greater than four and unswept elastic axis.

The test article, wing, tailplane, or fin, is generally mounted at its root, without control surfaces, in a rigid test fixture for these tests. However, wing stiffness tests, particularly torsion as required for simplified criteria, may be successfully conducted with the wing mounted on the fuselage restrained in a cradle. This type of setup requires duplicate loading fixtures for right and left wing to balance the aircraft under load and thus minimize "jig rotation" effects.

The chordwise location of the elastic axis is determined by applying a torque load at selected stations and plotting the deflection vs. chord shear center or elastic axis at that station.

Torsional influence coefficients (radians twist about the elastic axis per unit torque load) are obtained by applying a pure torque load about the elastic axis at the tip and measuring the resulting spanwise twist. The twist per unit torque applied at intermediate inboard stations will be the same inboard of the load point. Thus, it is necessary to load only one additional inboard station, say 75% span, to check for data repeatability only. To insure that the load applied is a pure torque load, the deflections of the elastic axis should be monitored during the loading process. Zero deflections should result.

Bending influence coefficients (deflections per unit shear load) are obtained by applying shear load on the elastic axis at a selected station and measuring the resulting deflections at a sufficient number of spanwise locations to define the influence line for that load point. The procedure is repeated for each load station. To insure that the shear load is applied on the elastic axis, no appreciable chordwise variation in the measured deflections should be evident.

The experimental determination of fuselage stiffness properties can be accomplished essentially the same way as for the aerodynamic surfaces. In this test the fuselage is treated as two beams, forward and aft fuselage, each cantilevered from the wing-root attachment. It is extremely important that the fixture at this attachment be very rigid; and, any displacement of the test jig during loading must be monitored, regardless of how small, throughout the test for inclusion in the data analysis. Small displacement can be quite influential in a rather complex data reduction procedure, and if improperly done, can lead to erroneous and troublesome conclusions. On this basis it is often the practice to compute fuselage stiffness properties for the fuselage, then use ground vibration test results to tune calculate modes and, in turn, stiffness as required.

Thin-skinned structure may buckle at a very low load, reducing actual stiffness in flight considerably from that determined by the above procedure and the analyst is cautioned to investigate such conditions.

5. GROUND VIBRATION TESTS. Ground vibration testing has as its fundament objective the definition of vibration mode frequencies, mode shapes, and damping characteristics of an aircraft. These data then become the basis for the analytical development of a mathematical vibration model of the airplane or serve as a check on such a model once it is developed. The results ultimately become the basis for rational flutter analyses. If the simplified flutter prevention criteria of reference 14, discussed in the main body of this advisory circular, is used, then the results from these test are used directly to establish a predicted flutter speed of the airplane.

The degree of sophistication required to conduct a resonance test (techniques, recording equipment, suspension system, etc.,) depends upon the complexity of the structure being tested. Since it is impossible to cover all test situations that may arise, the discussions presented in this section are fundamental in nature, dealing specifically with sinusoidal methods of excitations. They are intended as guidelines for those persons concerned with general type aircraft, who have only the basic test facilities. Other procedures employing random or impulse excitations are being used more frequently. However, these methods are considered beyond the scope of this AC.

a. Test Article and Suspension System. The airplane should be supported in a level attitude such that the rigid body frequencies of the airplane on its support are less than one-half the frequencies of the lowest elastic wing or fuselage mode to be excited.

One of the following methods of support can generally be used:

(1) Support the airplane on its landing gear with the tires deflated sufficiently to achieve the above result. Fifty percent normal tire pressure usually achieves good results. It may be necessary to block the landing gear struts to eliminate damping in the oleos.

(2) Suspend the airplane on springs.

(3) Support the airplane on its landing gear resting on spring platforms.

(4) Support the airplane fuselage and wings on large air-filled flotation bags.

The airplane should be equipped with all items having appreciable mass such as engines and tip tanks. The weight and c.g. of the test article should be determined to enable proper correlation with the math model. Where fuel is located in the outboard 50% of the wing semispan, it may be desirable to test a full fuel condition in addition to the empty condition in order to provide additional data for math model correlation.

It is generally advantageous to block the control surfaces in their neutral position when obtaining airframe modes.

b. Equipment. Various types of shakers are available, i.e., inertia, elastic, airjet, electromagnetic, etc. Electromagnetic exciters are generally preferred and most commonly used. This type consists of a coil that is attached to the structure with a fixed drive rod, as opposed to a flexible shaft or spring for inertia or elastic type shakers. The coil is surrounded by a magnetic field and is set in motion by an alternating current. Electronic oscillators and amplifiers are used to control this type of system.

Vibration amplitude may be obtained by using either velocity pickups or accelerometers so long as transducer mass is insignificant. The output can be observed using a cathode ray oscilloscope and digital voltmeter. Phase relationship between two transducers can be noted with sufficient accuracy, and by exercising extra care, using an oscilloscope equipped with a grid screen.

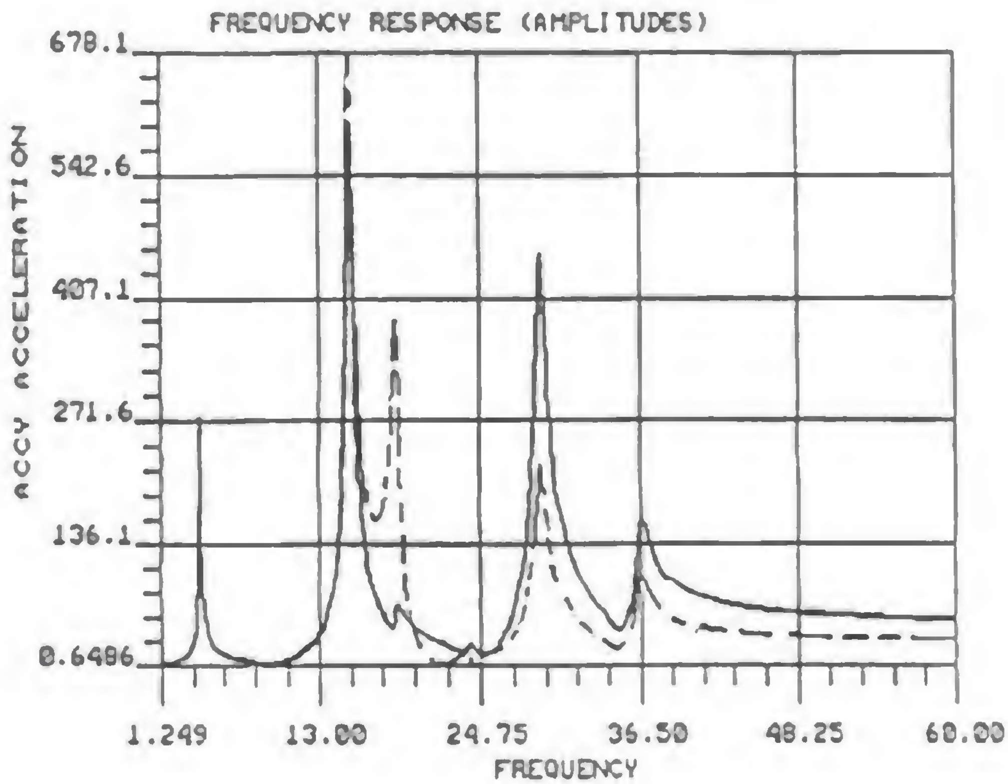
Data systems are available that provide the coincident, in-phase or real term, and the quadrature, the imaginary term, responses of the total response frequency (the product of the force and reference signal). Graphical representation of these terms is presented, providing a very accurate identification technique for resonant frequencies and phase relationships. Structural damping is also readily available from these data.

Whatever data system is used, uniformity is recommended. Piecemeal systems, using velocity pickups and accelerometers, or filters with different characteristics, etc., can give erroneous data and should not be used without careful regard to their calibrations and performance characteristics and limitations.

c. General Procedures for Airframe Modes. It is usually sufficient to apply a harmonic excitation force to the structure provided the force is not applied in the proximity of a node line. For this reason vibrators are usually attached at an extremity such as the nose and/or rear of the fuselage or near the tips of the wing or empennage surfaces where nodes are not likely to occur.

With the shakers and a reference pickup mounted at a selected location, frequency is varied upward through the range usually encountered in aircraft structures (2 to 100 Hz), With small increments of frequency, the response of the structure is recorded and the resulting plot of amplitude of response vs. forcing frequency is used to determine the resonant frequencies of the system.

A typical sweep is shown below:




Although duplication of peak responses will result, it is advantageous to obtain frequency response records with a reference pickup positioned on each of the main surfaces and fuselage at a specific shaker location. This will reduce the chances of overlooking modes.

There are several criteria for establishing that the excited response approximates a normal mode of vibration. The most commonly accepted approach requires that all of the criteria below be met:

(1) A relative maximum response per unit input exists.

(2) Accelerations at all points in the structure are either exactly in phase or 180° out of phase with each other. The accelerations measured at all points on the structure during resonance will be either in phase or out of phase with a reference location but will be at a + 90° phase angle with the force, for small values of damping.

(3) A decay record exhibits a single-frequency, non-beating, low-damped characteristic.

Having established the resonant frequencies, a survey of the aircraft is conducted with the shakers tuned to each frequency in-turn. A roving transducer is used to sense amplitude and phase angle relative to the reference pickup at each airplane location. An adequate number of points should be surveyed along the span and chord (typically on the spars) of each surface and along the fuselage to define the airplane modal displacements, and the associated node lines. To obtain proper phase relationship additional excitation may be necessary.

It may not be necessary to survey identical peak frequency responses although they occur at different locations. In all probability, the mode will be the same. This can be determined by checking only a few stations or simply by visually observing the motion of the aircraft.

Care should be exercised in defining component node lines for each mode. This is particularly important in evaluating the effectivity of balance weight locations.

d. Aircraft Structural Modes Usually Encountered. The modes excited during ground vibration depend on the type of configuration being tested. The vibration modes of an airplane that carries heavy mass on the wing, such as engines, tip tanks, etc., or has the stabilizer located high on the fin will be highly coupled and generally cannot be described except by diagrams that show the relative shape and phase of each part of the airplane. Airplanes that do not have these design characteristics usually have relatively uncoupled modes which can be described by naming the type of motion that is predominant. In general, the following predominant modes should be obtained insofar as is practicable.

(1) Wing Group Modes.

(i) For wings without engines, tip tanks, or heavy external or internal stores:

Wing vertical bending and wing torsion, fundamental and higher modes, symmetric and antisymmetric.

(ii) For wings carrying heavy masses outboard of the fuselage:

Wing bending coupled with wing torsion and flexible store (engines) modes, fundamental and higher modes, symmetric and antisymmetric.

(2) Fuselage - Empennage Group Modes.

(i) Fuselage Torsion (coupled with stabilizer antisymmetric bending).
(ii) Fuselage lateral bending and fin bending, fundamental and higher order consisting of two fundamental modes in which the fin tip and aft fuselage are in phase in one mode, and out of phase in the other.

(iii) Fin bending - symmetric and antisymmetric for multi-tail airplanes.

(iv) Fin torsion (generally highly coupled with stabilizer yawing if stabilizer is located at the outer span stations of the fin).

(v) Rudder bending and torsion.

(vi) Fuselage vertical bending and stabilizer bending, fundamental and higher order consisting of two fundamental modes in which the aft fuselage and stabilizer tips are in phase in one mode, and out of phase in the other.

(vii) Stabilizer torsion - symmetric and antisymmetric.

(viii) Stabilizer yawing for surface located at the outer span stations of the fin.

(ix) All movable horizontal tail - rotation coupled with bending, torsion.

(3) Engine or External Store Modes. For multiengine aircraft or aircraft carrying large pylon-mounted stores, the pitch, roll, yaw, and lateral and vertical translation modes should be defined. These modes should also be determined for all turbo propeller engine installations. It may be necessary to excite the engine fore and aft on the propeller blade to obtain the most critical pitch and yaw modes. If this method is used, consideration should be given to possible modal distortion due to propeller blade flexibility. Also, caution should be exercised and the engine manufacturer's instructions followed concerning possible damage to bearings when exciting the engine.

e. General Procedures for Control System Modes. The experimental determination of control surface and tab rotation modes about their hinge lines may be difficult due to, inherent friction within the system or the masking of these modes by structural interaction. On this basis, extra care is required for proper identification of the system's characteristics.

For conventional aileron or elevator systems, the rotation modes may be successfully measured by applying a single excitation force to either the righthand or lefthand surface. However, multiple shakers are preferred, particularly if the right and left surfaces are operated from separate control systems. Likely shaker positions are on the trailing edge at midspan or on the hom leading edge. Tab rotation may be determined from the control surface excitation but usually a direct excitation on the tab surface is required with the control surface (aileron, elevator, or rudder) blocked to its main surface.

A transducer paced on the control being excited is used to monitor the response and determine peak frequencies by the same technique described for airframe modes. To define the modes excited, it is generally necessary to follow any or all of the following procedures:

(1) Monitor the phase between the right and left surface, the control column, or the attaching structure.

(2) Conduct a detailed survey of the surface, spanwise and chordwise, to define any structural modes. If the surface has a very long span or wide chord, these modes, bending and torsion, are likely to be dominant.

(3) Visually monitor the surface under excitation.

(4) Simple rationalization to distinguish the excited modes from previously defined airframe modes.

In the performance of these tests, the shakers and/or transducers may contribute sufficient weight to the surface being tested to significantly affect the frequency of the surface. This is particularly true for tabs with very small mass and rotational inertias. Dunkerly's equation, presented in reference 8, provides an acceptable method for correcting the measured frequency to the true surface frequency.

A check on experimentally determined motes may be facilitated by calculating rotational frequencies from measured inertias and system stiffness properties obtained from static tests.

For extremely light weight structures, another method that may be used to eliminate the shaker influence is to use an air shaker or other device which does not directly attach to the control surface or tab.

f. Control Surface Rotation. Symmetric aileron rotation, the normal opposed operational mode, with control stick fixed or free, is defined as the peak frequency at which both ailerons are rotating in phase. Antisymmetric rotation, the normal operation mode, generally has zero or very low stiffness.

Rudder rotation in the normal operation mode with pedals free occurs when the rudder and pedals are out of phase.

Elevator and all movable tailplane rotation modes should be determined with the pilot's controls fixed and free. Elevator rotation with the stick fixed is defined as the peak frequency at which both elevators are in phase for symmetric rotation, and out of phase for antisymmetric rotation. For all moving tailplanes or elevators with stability augmentation systems (control column bob weights and down springs), normal opposed operation with stick free will occur when the control stick and elevator are responding out of phase.

The effect of variations in control cable tension should be investigated.

g. Tab Rotation. Rotational modes for irreversible trim and servo tabs are determined experimentally to supplement the calculated frequency obtained from measured stiffness in the free play tests. Tab rotation frequency will usually vary with angular deflection and is determined at maximum trailing edge up, neutral, and maximum trailing edge down positions to determine the range of tab frequencies. For geared tabs, the rotation frequency is usually determined with the control surface at maximum deflections and at neutral.

Large tabs, either wide chord or very long with a single actuator, often tend to be difficult to measure in a resonance test. Wide chord tabs often become significantly involved with "plate modes" of their carrying surfaces, while long narrow tabs may have their lowest frequency in a torsional mode rather than rotation. On this basis, it may be necessary to survey each response frequency rather extensively to properly define each mode.

Test requirements for spring tabs are dependent upon the tab control system design. In general, the following tests should be conducted to provide the required data for a mathematical representation of a spring tab system. (These tests are similar to those discussed in the previous paragraph for all moving tailplane systems.)

(1) For a preloading spring, tests should be performed for several amplitudes including complete removal of the preload, if practical.

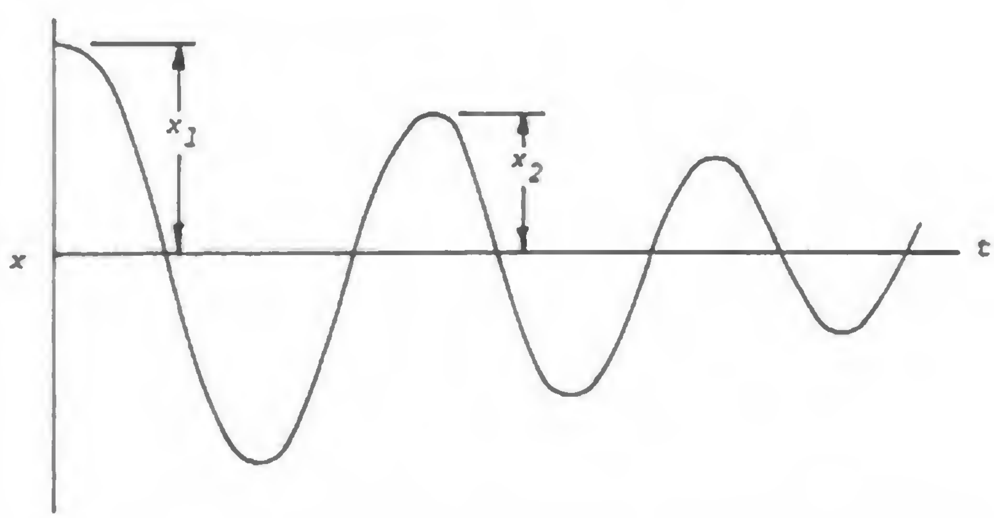
(2) Frequency of the control surface, with tab locked to surface and pilot's control column blocked, against the elastic restraint of the control system. A stick fixed mode.

(3) Frequency of the control column, with the control surface locked to its main surface, against the elastic restraint of the control system. A stick free mode.

(4) Frequency of the tab, with the control system cables disconnected and the control surface blocked to its supporting structure, against the elastic restraint of the springs in the tab system.

Spring loaded tabs are non-linear systems which are usually quite sensitive to small parameter changes making the design of these systems to preclude flutter most difficult. It is advisable to avoid their use unless extensive flutter analyses, including detail parameter evaluations, are conducted.

h. Structural Damping Measurements. Structural damping of each significant mode surveyed should be measured. The most commonly used procedure is based on the measurement of the rate of decay of oscillation. This is best expressed in terms of logarithmic decrement, the natural logarithm of the ratio of two successive amplitudes. Records of the response of a reference transduce-, while driving the structure at a specific frequency and obtained immediately before and after power to the shaker is cut off, provide the amplitude relationships required.




The log decrement, δ, is then equal to ln $(X_1/X_2)$; or as 0.693/n where n = no. of cycles to 1/2 amplitude; i.e.: $X_1/X_2 = 2$.

For small values of damping, the damping factor, ν or $C/C_0$, can be estimated as $\sigma/2\pi$ and the structural damping $g= \delta/\pi$.

i. Balance Weight Attachment. For control surfaces with balance weights mounted at one end of a cantilevered moment arm, the resonant frequency of the balance weight attachment arm should be at least 50% greater than the highest frequency of the fixed surface with which the control surface may couple. The control surface should be mounted in a jig and the vibrator attached to the balance weight. The input frequency is varied upward and the response of a reference transducer mounted on the balance weight is monitored to define the peak response.

All balance weight supporting structure should be deigned for a limit static load of 24g normal to a plane containing the hinge and the weight, and 12g within that plane parallel with the hinge. The balance weight load should be able to be carried by the control surface and by the fittings and their attachments on both sides of the hinge. Proof of these criteria can be accomplished by relatively simple static tests of the control surface mounted in a jig.

# APPENDIX D

## *STALLION WING ANALYSIS*

The "Stallion" is a four place, high performance, utility aircraft which is presently being built at High Tech Composites. Figure D-1 shows the overall aircraft configuration. The structural and flutter analysis method used in the design of the wing for this aircraft is summarized in the following pages. The wing is constructed of BASF X3288-50 3K-plain weave graphite prepreg fabric and 1/4 thick HRH 10 - 3/16- 3.0 Nomex honeycomb core.

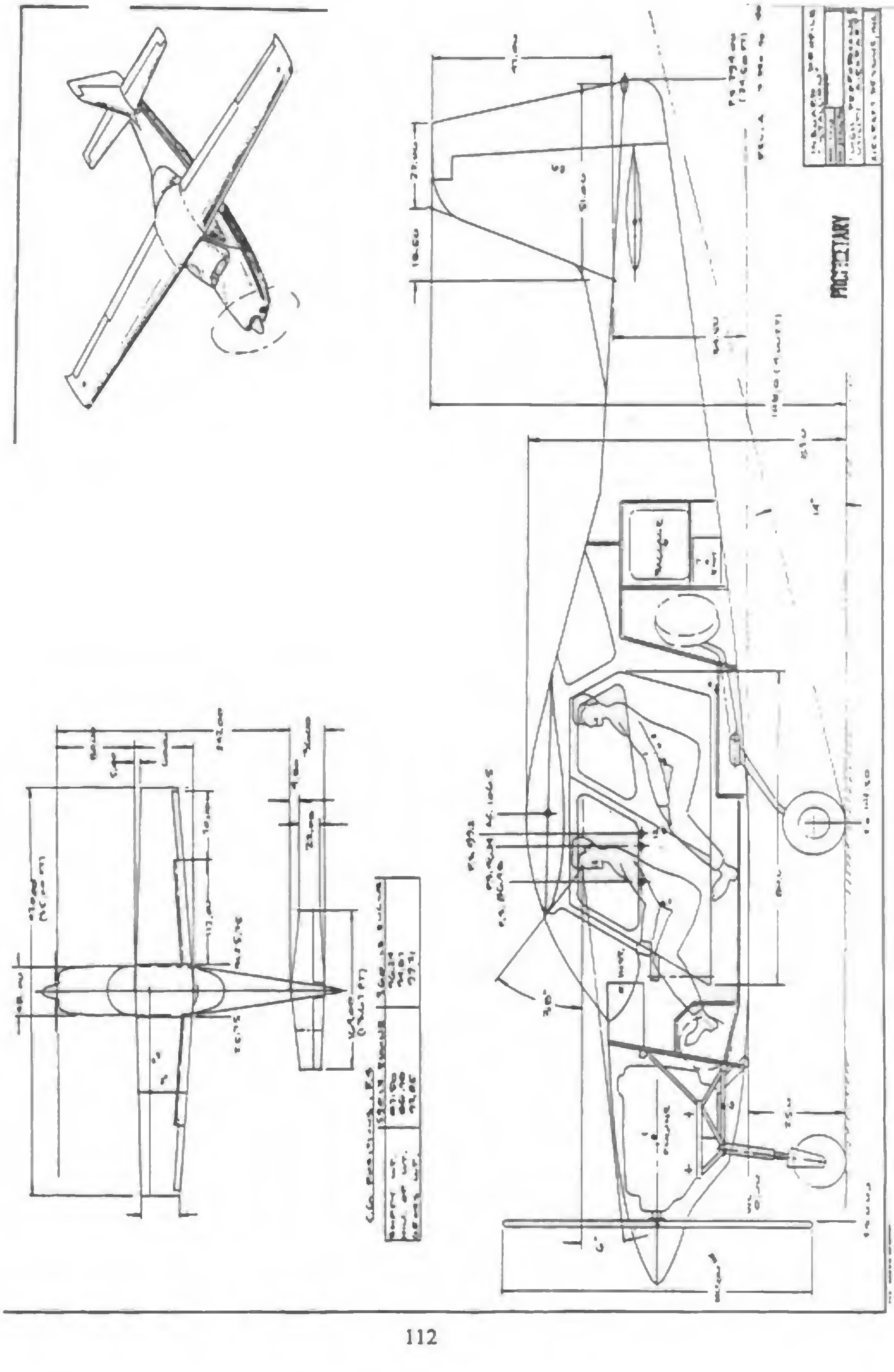
The wing is sized using the "Composite Wing Optimization" program as reported in "Modern Aircraft Design, Volume 1." This program allows quick sizing of wing spars and wing skins. It shows the planform of the wing as shown in figure D-2 and it calculates the airload distribution which is used in the fea. Using the structural sizes from "Composite Wing Optimization," a fem is set up using NISA386. Figure D-3 shows the feam and table D-1 shows the input file for the structural model. The results of this analysis shows if the correct core thickness and strength have been chosen. Wing deflection and skin stress contour plots are viewed to make certain that the structure is adequate to take the loads. Figure D-4 shows a deflected shape and figures D-5, D-6, D-7, and D-8 show typical stress contour plots.

A wing, upper panel is modeled using NISA386 and the air suction load and in-plane compression load is applied to this panel to check the buckling factor. Table D-2 shows the NISA input file for this panel. The results from the fea show that with a 4.4 g load on the wing a buckling load factor of 2.34 is realized. The margin of safety for panel buckling is,

$$M.S. = \frac{2.34}{1.5} - 1.0 = +0.56$$

Next, the NISA386 fem input file is changed to perform the modal (eigenvalue) analysis. Table D-3 shows this file and it is seen that very little modifications need be made to the static file shown in table D-1. The results from the eigenvalue run are shown in figure D-9. Note that the rigid body mode and other fea modes are calculated. Most of these modes are not included in the flutter analysis. The rigid body modes affect the value of the other frequencies. As such the rigid body modes and in-plane vibration modes which do not affect flutter are not included in the flutter analysis. The flutter modes are numbered sequently and seven modes are included for the flutter analysis. These mode numbers do not correspond to the eigenvalue mode numbers.

Finally, the flutter analysis is performed for the mode shapes shown in figure D-10. The input file for the flutter analysis is shown in table D-4 and the results are shown in figure D-11.


PROPRIETARY

A flutter speed of 520 mph is calculated as shown by the wing twisting mode crossing zero damping. Figure D-11 also shows that wing Mode 6 - twisting and Mode 1 - 1st wing bending couple at this speed. This speed is well above the 1.2 x $V_D$ of 360 mph. The aileron ballast weight was removed and the flutter speed was determined to by 160 mph. Obviously, we need to 100% balance the ailerons.

WING LOADS

| WS | Air Load, lb | Shear, lb | Moment, ft.lb | Torsion, ft-lb |
|---|---|---|---|---|
| 1 | 929 | 6160 | 42353 | 1655 |
| 2 | 887 | 5230 | 32386 | 1316 |
| 3 | 827 | 4343 | 24009 | 1006 |
| 4 | 757 | 3515 | 17133 | 735 |
| 5 | 679 | 2758 | 11643 | 506 |
| 6 | 598 | 2078 | 7411 | 321 |
| 7 | 514 | 1479 | 4298 | 177 |
| 8 | 425 | 965 | 2159 | 74 |
| 9 | 328 | 539 | 842 | 9 |
| 10 | 211 | 211 | 185 | -17 |
| 11 | 0 | 0 | 0 | 0 |

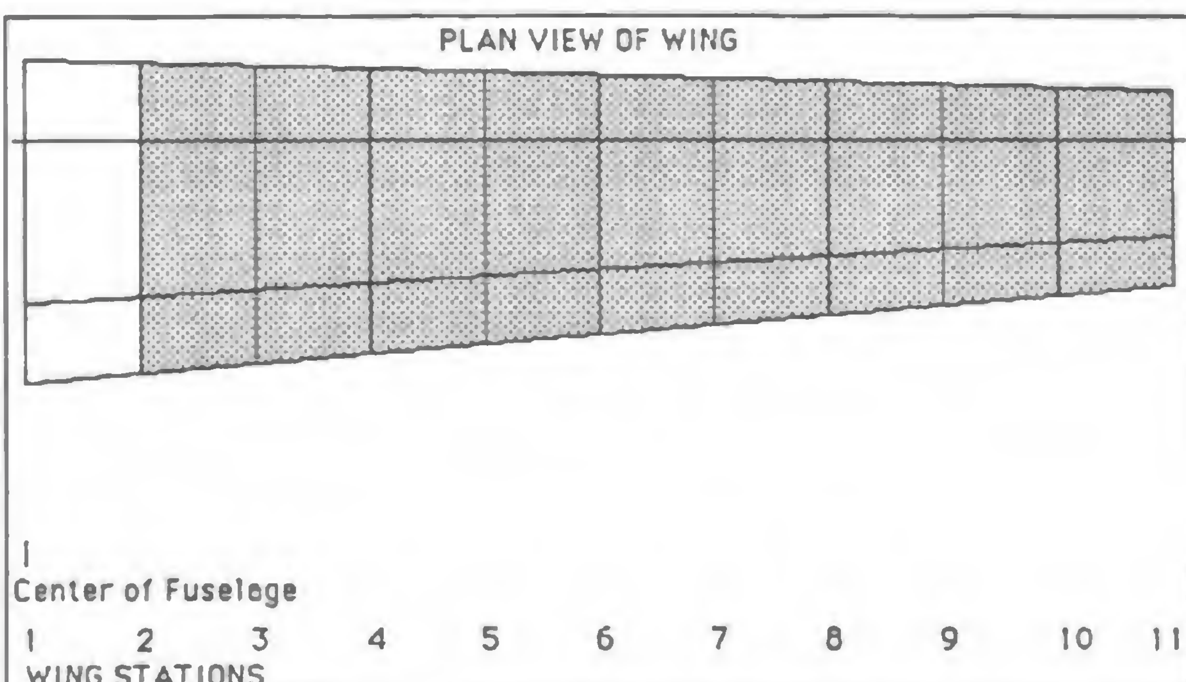
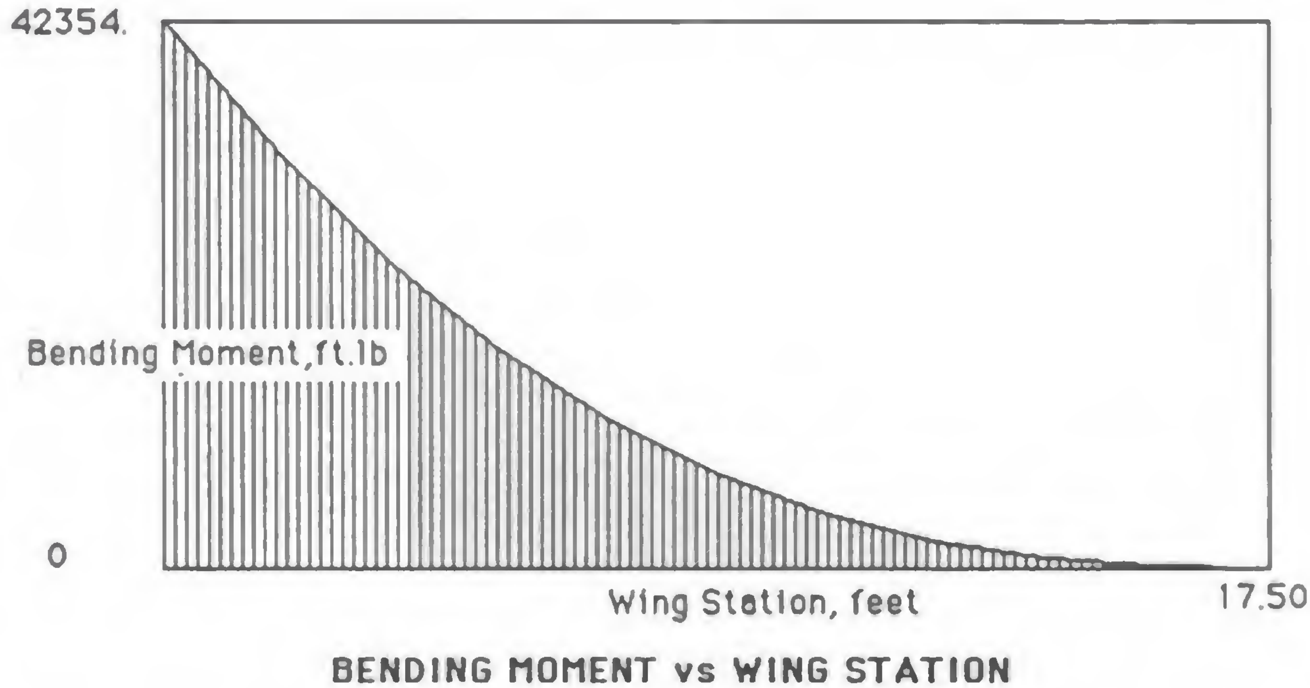



Figure D-2. "Composite Wing Optimization" Program Output Data.



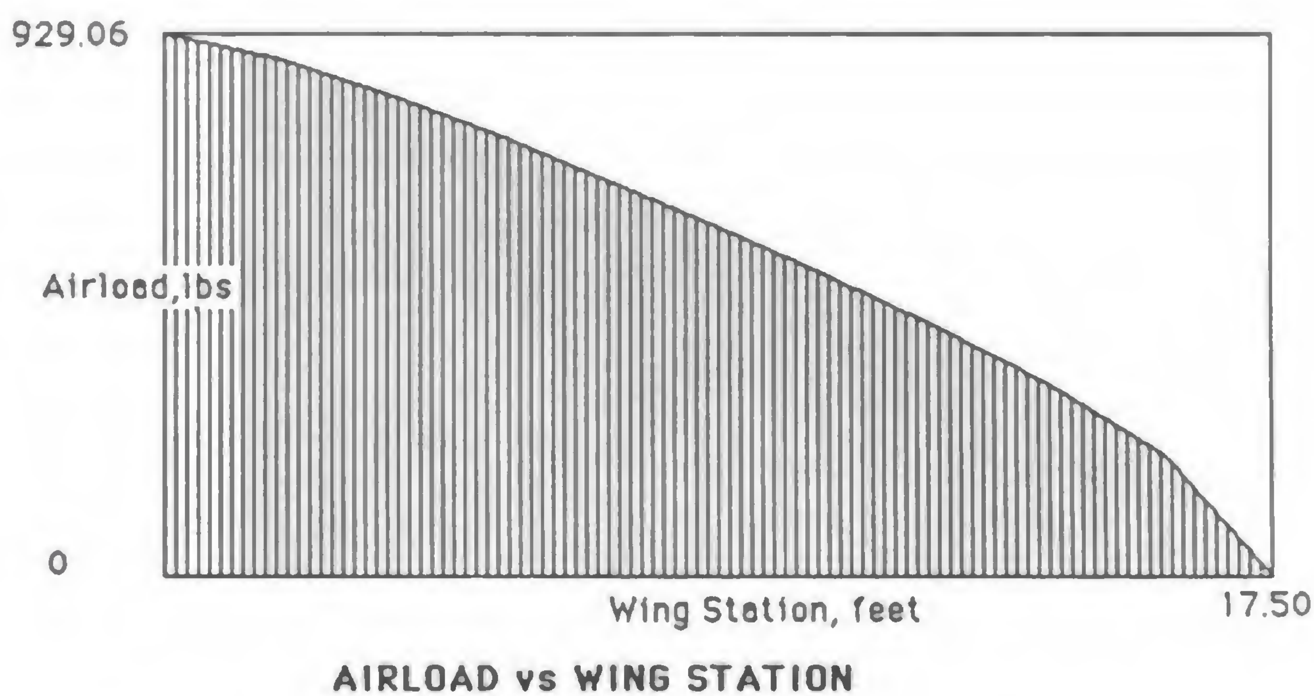




Figure D-2 Continued. "Composite Wing Optimization" Program Output Data.

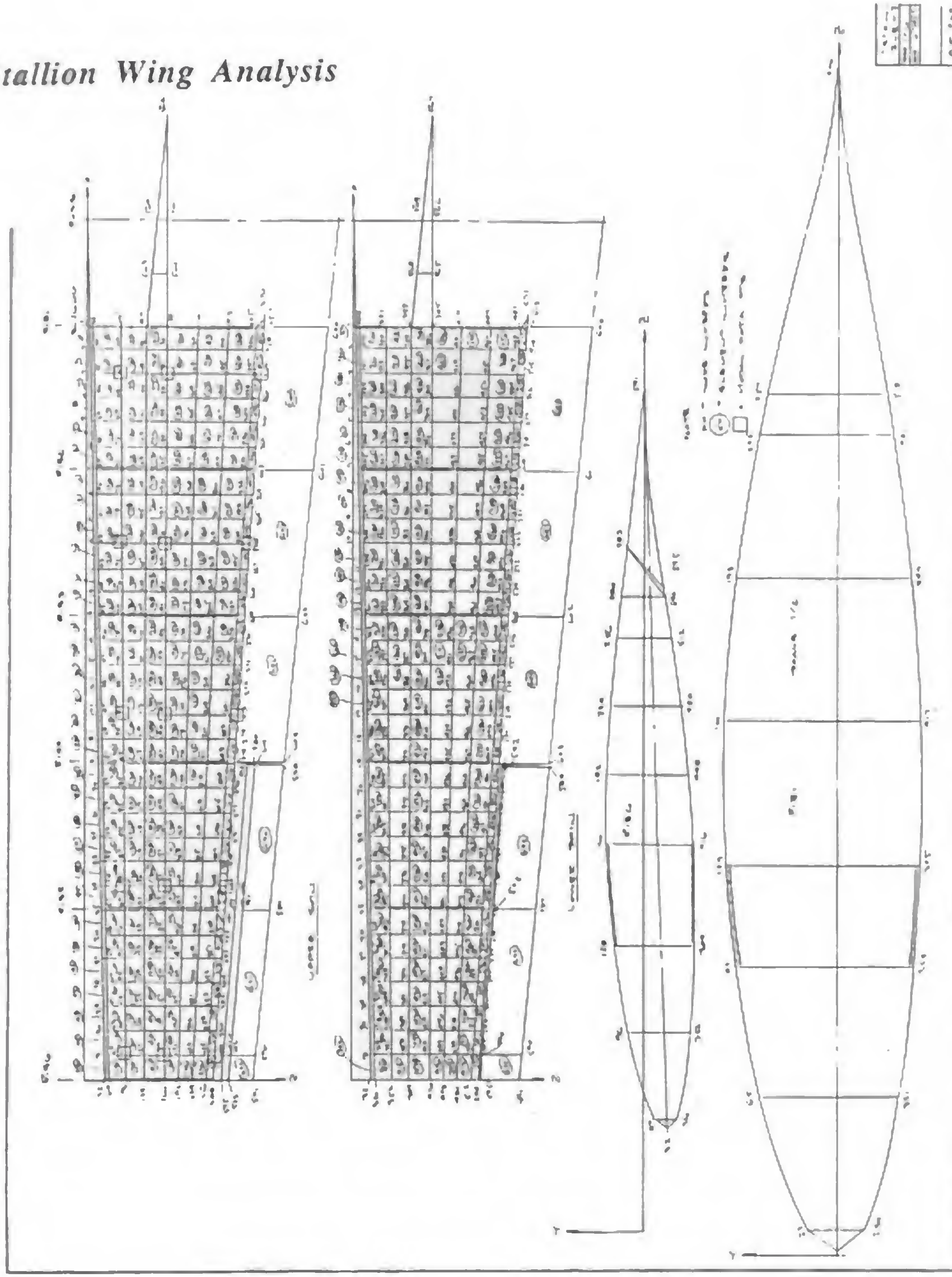


Figure D-3. Finite Element Model of Stallion Wing.

Table D-1. NISA386 Finite Element Model Static Input File for the Stallion Wing.

```
ANAL=STATIC
**EXEC=CHECK
FILE=W1
SAVE=26,27
AUTO=ON
*ECHO=OFF
*A1
STALLION WING
*C1
1,33,1
2,20,1
3,20,10
4,12,1
5,30,1
*D1
1,4
.02///
2,4
.25///
3,4
.03///
4,4
.01///
5,4
.30///
6,4
.18///
7,4
.14///
8,4
.12///
9,4
.07///
10,4
.05///
11,4
.02///
12,4
.02///
13,4
.25///
14,4
.16///
15,4
.05///
16,4
.02///
17,4
.02///
18,4
.02///
19,4
0.10///
20,4
.10///
21,4
.10///
22,4
.06///
23,4
.06///
24,4
.06///
25,4
.04///
26,4
.125///
**AILERON MASS BALANCE
27,4
0.135,0.02,0.02,0.04
**CONTROL ROD
28,4
0.05,2E-4,2E-4,4E-4
*D2
1,4
0///
2,4
45///
*D3
3,0,3,2,3,2,1,2,1,3,1
3,0,1,2,1,2,1,2,1,3,1
3,0,1,2,1,2,1,2,1,3,1
3,0,1,26,4,2,1,2,1,3,1
3,0,3,26,3,2,1,2,1,3,1
*E1
**UPPER SKIN
$$,-3,32,13
1,34,33,1,2,$,1,1,0,13,1,1
$$,-3,32,13
40,162,161,129,130,$,1,1,0,13,1,1
$$,-3,32,18
79,47,46,14,15,$,2,1,0,18,1,1
$$,-3,32,18
133,175,174,142,143,$,2,1,0,18,1,1
**LOWER SKIN
187,290,289,1,2,$,3,1,0,13,1,1
$$,-2,32,13
200,322,321,289,290,$,3,1,0,13,1,1
$$,-3,32,13
226,418,417,385,386,$,3,1,0,13,1,1
265,303,302,14,15,$,2,1,0,18,1,1
$$,-2,32,18
283,335,334,302,303,$,2,1,0,18,1,1
$$,-3,32,18
319,431,430,398,399,$,2,1,0,18,1,1
**UPPER CAP
373,130,129,97,98,$,4,2,5,2,1,1
375,132,131,99,100,$,4,2,6,2,1,1
377,134,133,101,102,$,4,2,7,2,1,1
379,136,135,103,104,$,4,2,8,3,1,1
382,139,138,106,107,$,4,2,9,3,1,1
385,142,141,109,110,$,4,2,10,6,1,1
391,148,147,115,116,$,4,2,11,6,1,1
397,154,153,121,122,$,4,2,12,7,1,1
```

```
**LOWER CAP
404,386,385,353,354,$,4,2,13,2,1,1
406,388,387,355,356,$,4,2,14,2,1,1
408,390,389,357,358,$,4,2,15,2,1,1
410,392,391,359,360,$,4,2,16,6,1,1
416,398,397,365,366,$,4,2,17,6,1,1
422,404,403,371,372,$,4,2,18,11,1,1
**INBD CAP
433,129,560,563,97,$,4,2,5
434,560,561,564,563,$,4,2,5
435,561,562,564,$,4,3,5
436,385,565,568,353,$,4,2,13
437,565,566,569,568,$,4,2,13
438,566,567,569,$,4,3,13
**SPAR SHEAR WEBS
439,353,568,563,97,$,2,2,19
440,568,569,564,563,$,2,2,19
441,569,567,562,564,$,2,2,19
442,385,565,560,129,$,2,2,19
443,565,566,561,560,$,2,2,19
444,566,567,562,561,$,2,2,19
445,354,353,97,98,$,2,2,19,3,1,1
448,357,356,100,101,$,2,2,20,4,1,1
452,361,360,104,105,$,2,2,21,6,1,1
458,367,366,110,111,$,2,2,22,18,1,1
476,386,385,129,130,$,2,2,19,3,1,1
479,389,388,132,133,$,2,2,20,6,1,1
485,395,394,138,139,$,2,2,21,6,1,1
**REAR SPAR
491,258,257,225,226,$,1,2,22,2,1,1
493,260,259,227,228,$,1,2,23,5,1,1
498,265,264,232,233,$,1,2,24,24,1,1
522,514,513,481,482,$,1,2,22,2,1,1
524,516,515,483,484,$,1,2,23,5,1,1
529,521,520,488,489,$,1,2,24,24,1,1
553,514,513,257,258,$,2,2,24,31,1,1
584,257,570,225,$,1,3,13
585,513,571,481,$,1,3,13
586,513,571,570,257,$,2,2,22
587,481,571,570,225,$,2,2,22
**RIBS
588,289,321,65,33,$,1,1,0,7,32,1
595,295,327,71,39,$,1,1,0,7,32,1
602,301,333,77,45,$,1,1,0,7,32,1
609,307,339,83,51,$,1,1,0,7,32,1
616,313,345,89,57,$,1,1,0,7,32,1
623,320,352,96,64,$,1,1,0,7,32,1
**AILERON
630,589,588,580,581,$,4,1,0,3,1,1
633,589,588,592,593,$,4,1,0,3,1,1
636,581,580,592,593,$,5,1,0,3,1,1
639,592,593,$,5,4,27,3,1,1
642,592,588,580,$,2,3,26
643,593,589,581,$,2,3,26
644,594,590,582,$,2,3,26
645,595,591,583,$,2,3,26
**AILERON SUPPORTS
646,531,604,275,$,5,3,5
647,537,605,281,$,5,3,5
648,543,606,287,$,5,3,5
649,606,286,287,$,5,3,5
**FLAP
650,601,600,257,263,$,2,1,0
651,602,601,263,269,$,2,1,0
652,603,602,269,275,$,2,1,0
653,601,600,513,519,$,2,1,0
654,602,601,519,525,$,2,1,0
655,603,602,525,531,$,2,1,0
656,513,600,257,$,2,3,26
657,519,601,263,$,2,3,26
658,525,602,269,$,2,3,26
659,531,603,275,$,2,3,26
**CONTROL ROD
660,249,593,$,6,4,28
*F1
1,0,$,184,0,0
32,0,1,$,0,-1.15,5
33,0,$,184,1.4,1.15
64,0,1,$,0,-.55,5.4
65,0,$,185,3.7,7.7
96,0,1,$,0,.75,9.7
97,0,$,184,4.9,14.1
128,0,1,$,0,1.4,14.0
129,0,$,184,5.3,19.1
160,0,1,$,0,1.7,19.0
161,0,$,184,5.5,26.15
192,0,1,$,0,1.75,22.35
193,0,$,184,4.9,33.1
224,0,1,$,0,1.6,25.65
225,0,$,184,3.8,40.1
256,0,1,$,0,1.25,29.0
257,0,$,184,3.4,42.1
288,0,1,$,0,1.1,31.0
289,0,$,184,-1.25,1.15
320,0,1,$,0,-1.6,5.4
321,0,$,184,-2.9,7.7
352,0,1,$,0,-2.3,9.7
353,0,$,184,-3.6,14.1
384,0,1,$,0,-2.4,14.0
385,0,$,184,-3.90,19.1
416,0,1,$,0,-2.4,19.0
417,0,$,184,-4.0,26.15
448,0,1,$,0,-2.1,22.35
449,0,$,184,-5,33.1
480,0,1,$,0,-1.75,25.65
481,0,$,184,-2.7,40.1
512,0,1,$,0,-1.30,29.0
513,0,$,184,-2.4,42.1
544,0,1,$,0,-1.05,31.0
560,0,$,197,5.3,19.1
561,0,$,210,5.3,19.1
562,0,$,236,5.3,19.11
563,0,$,197,5.3,15
564,0,$,210,5.3,16.5
565,0,$,197,-3.90,19.1
```

```
566,0,$,210,-3.9,19.1
567,0,$,236,-3.6,19.1
568,0,$,197,-3.6,15
569,0,$,210,-3.9,16.5
570,0,$,186,3.4,42.1
571,0,$,186,-2.4,42.1
**AILERON
580,0,$,76.5,1.5,41
581,0,$,41.55,1.15,38.5
582,0,$,5.935,0.8,35.6
583,0,$,0,0.8,35.0
588,0,$,76.5,0,48.2
589,0,$,41.55,0,45
590,0,$,5.935,0,41.5
591,0,$,0,0,41
592,0,$,76.5,-1.7,37.3
593,0,$,41.55,-1.35,34.8
594,0,$,5.935,-1,32.0
595,0,$,0,-1,31.5
**FLAP
600,0,$,184,0,58
601,0,$,148.39,0,55
602,0,$,112.8,0,51.8
603,0,$,77.16,0,48.5
604,0,$,76.52,1.5,41
605,0,$,41.57,1.15,38.5
606,0,$,5.955,0.8,35.6
*H1
**GRAPHITE FABRIC
EX,1,0,4.2E6
EY,1,0,4.2E6
GXY,1,0,6E5
GYZ,1,0,10
GXZ,1,0,10
DENS,1,0,2.7E-4
**GLASS
EX,2,0,2.3E6
EY,2,0,2.3E6
GXY,2,0,1.25E5
GYZ,2,0,10
GXZ,2,0,10
DENS,2,0,3.6E-4
**CORE
EX,3,0,0
GXZ,3,0,2.2E3
GYZ,3,0,2.2E3
GXY,3,0,0
DENS,3,0,7.6E-6
**GRAPHITE TAPE
EX,4,0,9E6
EY,4,0,9E6
GXZ,4,0,10
GYZ,4,0,10
GXY,4,0,1.3E5
DENS,4,0,2.8E-4
**STEEL
EX,5,0,28E6
EY,5,0,28E6
GXZ,5,0,17E6
GXY,5,0,17E6
GYZ,5,0,17E6
DENS,5,0,7.3E-4
**CONTROL ROD
EX,6,0,68E3
EY,6,0,68E3
GXZ,6,0,34E3
GXY,6,0,34E3
GYZ,6,0,34E3
DENS,6,0,2.6E-4
*CPDISP
UXYZ,$,604,580
UXYZ,$,605,581
UXYZ,$,606,582
*I3
0,1,3,0,0,2,1,0,0
*PRESS
1,39,1,1,0,0,1.42
79,132,1,1,0,0,1.15
40,66,1,1,0,0,1.0
133,186,1,1,0,0,0.74
373,385,1,1,0,0,1.42
386,403,1,1,0,0,0.74
491,521,1,1,0,0,1.0
*SPDISP
97,UXYZ,0
129,UXYZ,0
562,UXYZ,0
570,UXYZ,0
*BODYF
0,0,0,0,-1700,0
*PRINT
ELST,0
*ENDDATA
```

Table D-2. Buckling Analysis Input File.

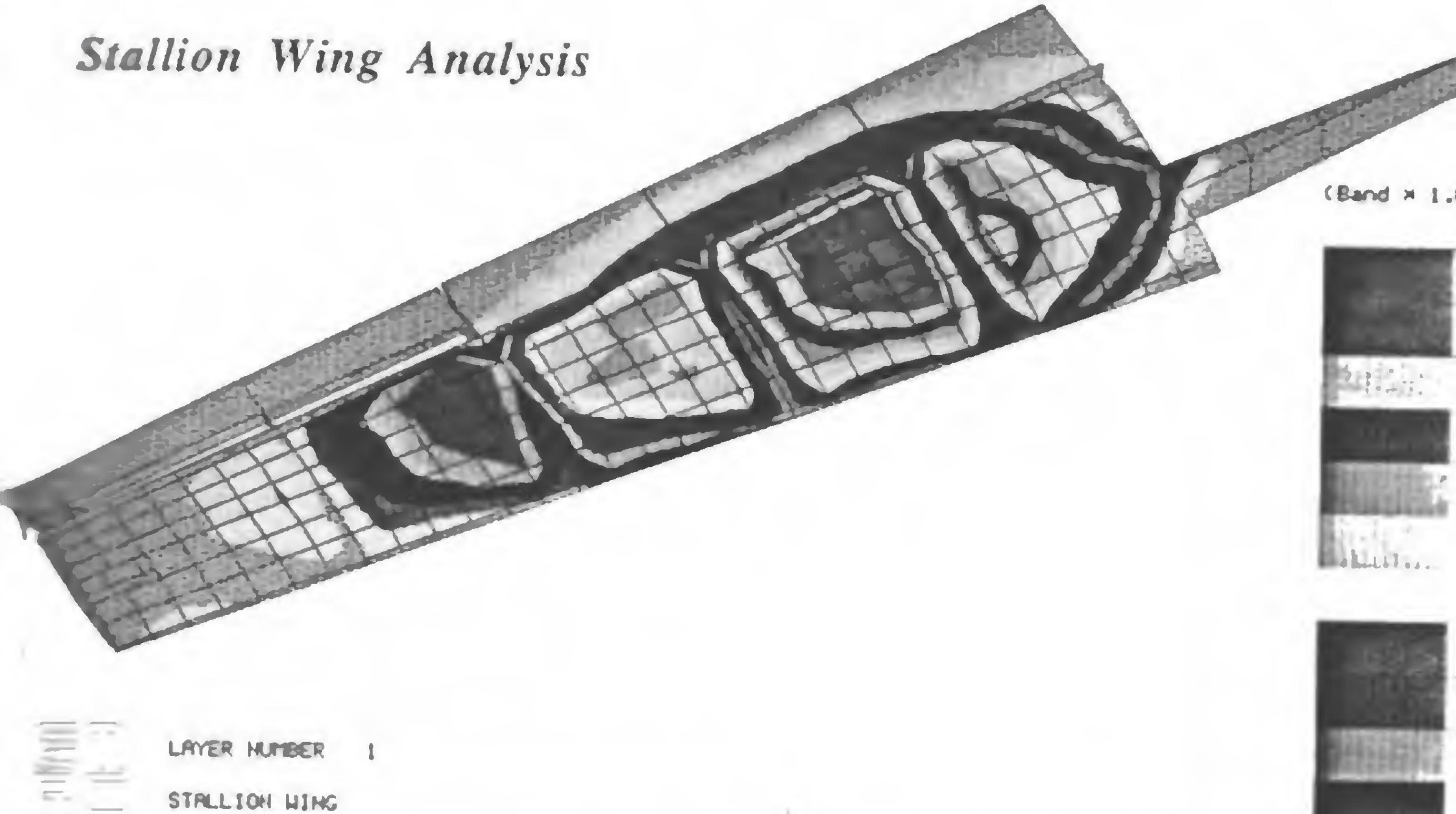
```
ANALYSIS=BUCKLING
AUTO CONSTRAINT=ON
RESEQUENCING OF ELEMENTS=ON
FILE=SB
SAVE=26,27
EIGEN=SUBS,CONV
*TITLE
STALLION WING, PANEL BUCKLING
*ELTYPE
1,33,1
*RCTABLE
1,4
.03///
2,4
.25///
3,4
.03///
*D2
1,4
0///
2,4
45///
*D3
3,0,1,2,1,2,1,2,1,2,1
*E1
$$,-9,11,10
1,22,21,10,11,$,1,1,0,10,-1,1
*F1
-11,1,11,10,0,0,3.8889
1,1,$,95,78.11,0
11,1,1,$,95,90,0
*H1
**GRAPHITE FABRIC
EX,1,0,4.2E6
EY,1,0,4.2E6
GXY,1,0,6E5
GYZ,1,0,80E3
GXZ,1,0,80E3
DENS,1,0,1.43E-4
**CORE
EX,2,0,0
GXZ,2,0,2.2E3
GYZ,2,0,2.2E3
GXY,2,0,0
DENS,2,0,6.0E-6
*LDCASE
0,1,3,0,0,1
*SPDISP
1,UXYZ,0,11,1
100,UY,0,110,1
12,UY,0,89,11
22,UY,0,99,11
104,UX,0
100,UX,0,110,1
12,UX,0,89,11
22,UX,0,99,11
*CFORCE
100,FZ,-473,110,1
*PRESS
**TOP SURFACE
1,90,1,1,0,0,-0.612
*EIGCNTL
3,0,0,0,0,0,1E-03
*ENDDATA
```

Table D-3. NISA386 Input File for Eigenvalue Run. Note Simularity with Static File.

```
ANAL=EIGENVALUE
**EXEC=CHECK
FILE=W2
SAVE=26,27
EIGEN EXTRACTION=SUBSPACE,ACCELERATED
MASS FORMULATION=CONSISTENT
AUTO=ON
*ECHO=OFF
*A1
STALLION WING, SYMMETRIC MODES, FUEL
*C1
1,33,1
2,20,1
3,20,10
4,12,1
5,30,1
*D1
1,4
.02///
2,4
.25///
3,4
.03///
4,4
.01///
5,4
.30///
6,4
.18///
7,4
.14///
8,4
.12///
9,4
.07///
10,4
.05///
11,4
.02///
12,4
.02///
13,4
.25///
14,4
.16///
15,4
.05///
16,4
.02///
17,4
.02///
18,4
.02///
19,4
0.10///
20,4
.10///
21,4
.10///
22,4
.06///
23,4
.06///
24,4
.06///
25,4
.04///
26,4
.125///
**AILERON MASS BALANCE
27,4
0.20,0.00317,0.00317,0.00633
**CONTROL ROD
28,4
0.05,2E-4,2E-4,4E-4
**FUEL
29,6
.0108//,0//
**1/2 FUS MASS, OP WT=3,200 LBS
30,6
2.9,2.9,2.9,87E3,87E3,0.80E3
**LINKS
31,4
24,6.3,6.3,12.6
*D2
1,4
0///
2,4
45///
*D3
3,0,3,2,3,2,1,2,1,3,1
3,0,1,2,1,2,1,2,1,3,1
3,0,1,2,1,2,1,2,1,3,1
3,0,1,26,4,2,1,2,1,3,1
3,0,3,26,3,2,1,2,1,3,1
*E1
**UPPER SKIN
$$,-3,32,13
1,34,33,1,2,$,1,1,0,13,1,1
$$,-3,32,13
40,162,161,129,130,$,1,1,0,13,1,1
$$,-3,32,18
79,47,46,14,15,$,2,1,0,18,1,1
$$,-3,32,18
133,175,174,142,143,$,2,1,0,18,1,1
**LOWER SKIN
187,290,289,1,2,$,3,1,0,13,1,1
$$,-2,32,13
200,322,321,289,290,$,3,1,0,13,1,1
$$,-3,32,13
226,418,417,385,386,$,3,1,0,13,1,1
265,303,302,14,15,$,2,1,0,18,1,1
$$,-2,32,18
283,335,334,302,303,$,2,1,0,18,1,1
$$,-3,32,18
319,431,430,398,399,$,2,1,0,18,1,1
**UPPER CAP
373,130,129,97,98,$,4,2,5,2,1,1
375,132,131,99,100,$,4,2,6,2,1,1
377,134,133,101,102,$,4,2,7,2,1,1
379,136,135,103,104,$,4,2,8,3,1,1
382,139,138,106,107,$,4,2,9,3,1,1
385,142,141,109,110,$,4,2,10,6,1,1
391,148,147,115,116,$,4,2,11,6,1,1
397,154,153,121,122,$,4,2,12,7,1,1
**LOWER CAP
```

```
404,386,385,353,354,$,4,2,13,2,1,1
406,388,387,355,356,$,4,2,14,2,1,1
408,390,389,357,358,$,4,2,15,2,1,1
410,392,391,359,360,$,4,2,16,6,1,1
416,398,397,365,366,$,4,2,17,6,1,1
422,404,403,371,372,$,4,2,18,11,1,1
**INBD CAP
433,129,560,563,97,$,4,2,5
434,560,561,564,563,$,4,2,5
435,561,562,564,$,4,3,5
436,385,565,568,353,$,4,2,13
437,565,566,569,568,$,4,2,13
438,566,567,569,$,4,3,13
**SPAR SHEAR WEBS
439,353,568,563,97,$,2,2,19
440,568,569,564,563,$,2,2,19
441,569,567,562,564,$,2,2,19
442,385,565,560,129,$,2,2,19
443,565,566,561,560,$,2,2,19
444,566,567,562,561,$,2,2,19
445,354,353,97,98,$,2,2,19,3,1,1
448,357,356,100,101,$,2,2,20,4,1,1
452,361,360,104,105,$,2,2,21,6,1,1
458,367,366,110,111,$,2,2,22,18,1,1
476,386,385,129,130,$,2,2,19,3,1,1
479,389,388,132,133,$,2,2,20,6,1,1
485,395,394,138,139,$,2,2,21,6,1,1
**REAR SPAR
491,258,257,225,226,$,1,2,22,2,1,1
493,260,259,227,228,$,1,2,23,5,1,1
498,265,264,232,233,$,1,2,24,24,1,1
522,514,513,481,482,$,1,2,22,2,1,1
524,516,515,483,484,$,1,2,23,5,1,1
529,521,520,488,489,$,1,2,24,24,1,1
553,514,513,257,258,$,2,2,24,31,1,1
584,257,570,225,$,1,3,13
585,513,571,481,$,1,3,13
586,513,571,570,257,$,2,2,22
587,481,571,570,225,$,2,2,22
**RIBS
588,289,321,65,33,$,1,1,0,7,32,1
595,295,327,71,39,$,1,1,0,7,32,1
602,301,333,77,45,$,1,1,0,7,32,1
609,307,339,83,51,$,1,1,0,7,32,1
616,313,345,89,57,$,1,1,0,7,32,1
623,320,352,96,64,$,1,1,0,7,32,1
**AILERON
630,589,588,580,581,$,4,1,0,3,1,1
633,589,588,592,593,$,4,1,0,3,1,1
636,581,580,592,593,$,5,1,0,3,1,1
639,592,593,$,6,4,27,3,1,1
642,592,588,580,$,2,3,26
643,593,589,581,$,2,3,26
644,594,590,582,$,2,3,26
645,595,591,583,$,2,3,26
**AILERON SUPPORTS
646,531,604,275,$,5,3,5
647,537,605,281,$,5,3,5
648,543,606,287,$,5,3,5
649,606,286,287,$,5,3,5
**FLAP
650,601,600,257,263,$,2,1,0
651,602,601,263,269,$,2,1,0
652,603,602,269,275,$,2,1,0
653,601,600,513,519,$,2,1,0
654,602,601,519,525,$,2,1,0
655,603,602,525,531,$,2,1,0
656,513,600,257,$,2,3,26
657,519,601,263,$,2,3,26
658,525,602,269,$,2,3,26
659,531,603,275,$,2,3,26
**CONTROL ROD
660,249,593,$,6,4,28
**FUEL
661,322,$,0,5,29,24,1,1
685,418,$,0,5,29,24,1,1
709,450,$,0,5,29,24,1,1
**FUS AND BEAMS
733,607,$,0,5,30
734,607,385,$,5,4,31
735,607,353,$,5,4,31
736,607,567,$,5,4,31
737,607,570,$,5,4,31
738,607,571,$,5,4,31
**DUMMY RIBS
739,292,324,68,36,$,7,2,3,7,32,1
746,298,330,74,42,$,7,2,3,7,32,1
753,304,336,80,48,$,7,2,3,7,32,1
760,310,342,86,54,$,7,2,3,7,32,1
767,316,348,92,60,$,7,2,3,7,32,1
*F1
1,0,$,184,0,0
32,0,1,$,0,-1.15,5
33,0,$,184,1.4,1.15
64,0,1,$,0,-.55,5.4
65,0,$,185,3.7,7.7
96,0,1,$,0,.75,9.7
97,0,$,184,4.9,14.1
128,0,1,$,0,1.4,14.0
129,0,$,184,5.3,19.1
160,0,1,$,0,1.7,19.0
161,0,$,184,5.5,26.15
192,0,1,$,0,1.75,22.35
193,0,$,184,4.9,33.1
224,0,1,$,0,1.6,25.65
225,0,$,184,3.8,40.1
256,0,1,$,0,1.25,29.0
257,0,$,184,3.4,42.1
288,0,1,$,0,1.1,31.0
289,0,$,184,-1.25,1.15
320,0,1,$,0,-1.6,5.4
321,0,$,184,-2.9,7.7
352,0,1,$,0,-2.3,9.7
353,0,$,184,-3.6,14.1
384,0,1,$,0,-2.4,14.0
385,0,$,184,-3.90,19.1
416,0,1,$,0,-2.4,19.0
417,0,$,184,-4.0,26.15
448,0,1,$,0,-2.1,22.35
449,0,$,184,-5,33.1
480,0,1,$,0,-1.75,25.65
481,0,$,184,-2.7,40.1
512,0,1,$,0,-1.30,29.0
513,0,$,184,-2.4,42.1
544,0,1,$,0,-1.05,31.0
560,0,$,197,5.3,19.1
```

```
561,0,$,210,5.3,19.1
562,0,$,236,5.3,19.11
563,0,$,197,5.3,15
564,0,$,210,5.3,16.5
565,0,$,197,-3.90,19.1
566,0,$,210,-3.9,19.1
567,0,$,236,-3.6,19.1
568,0,$,197,-3.6,15
569,0,$,210,-3.9,16.5
570,0,$,186,3.4,42.1
571,0,$,186,-2.4,42.1
**AILERON
580,0,$,76.5,1.5,41
581,0,$,41.55,1.15,38.5
582,0,$,5.935,0.8,35.6
583,0,$,0,0.8,35.0
588,0,$,76.5,0,48.2
589,0,$,41.55,0,45
590,0,$,5.935,0,41.5
591,0,$,0,0,41
592,0,$,76.5,-1.7,37.3
593,0,$,41.55,-1.35,34.8
594,0,$,5.935,-1,32.0
595,0,$,0,-1,31.5
**FLAP
600,0,$,184,0,58
601,0,$,148.39,0,55
602,0,$,112.8,0,51.8
603,0,$,77.16,0,48.5
604,0,$,76.52,1.5,41
605,0,$,41.57,1.15,38.5
606,0,$,5.955,0.8,35.6
**FUS
607,0,$,210,-24,16
*H1
**GRAPHITE FABRIC
EX,1,0,4.2E6
EY,1,0,4.2E6
GXY,1,0,6E5
GYZ,1,0,10
GXZ,1,0,10
DENS,1,0,2.7E-4
**GLASS
EX,2,0,2.3E6
EY,2,0,2.3E6
GXY,2,0,1.25E5
GYZ,2,0,10
GXZ,2,0,10
DENS,2,0,3.6E-4
**CORE
EX,3,0,0
GXZ,3,0,2.2E3
GYZ,3,0,2.2E3
GXY,3,0,0
DENS,3,0,7.6E-6
**GRAPHITE TAPE
EX,4,0,9E6
EY,4,0,9E6
GXZ,4,0,10
GYZ,4,0,10
GXY,4,0,1.3E5
DENS,4,0,2.8E-4
**STEEL
EX,5,0,28E6
EY,5,0,28E6
GXZ,5,0,17E6
GXY,5,0,17E6
GYZ,5,0,17E6
DENS,5,0,0
**CONTROL ROD
EX,6,0,20E3
EY,6,0,20E3
GXZ,6,0,10E3
GXY,6,0,10E3
GYZ,6,0,10E3
DENS,6,0,2.6E-4
**DUMMY RIBS
EX,7,0,1E6
EY,7,0,1E6
GXZ,7,0,.5E6
GXY,7,0,.5E6
GYZ,7,0,.5E6
DENS,7,0,0
*CPDISP
UXYZ,$,604,580
UXYZ,$,605,581
UXYZ,$,606,582
*EIGCNTL
13,0,25,,0,0,1E-3,-1
*SPDISP
607,UX,0
607,ROTZ,0
607,ROTY,0
*MODEOUT
3 $ 1
*EIGOUT
1,0,3,1,0,1
*ENDDATA
```



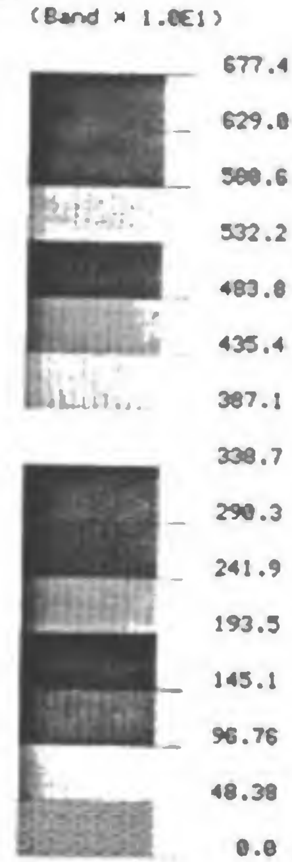
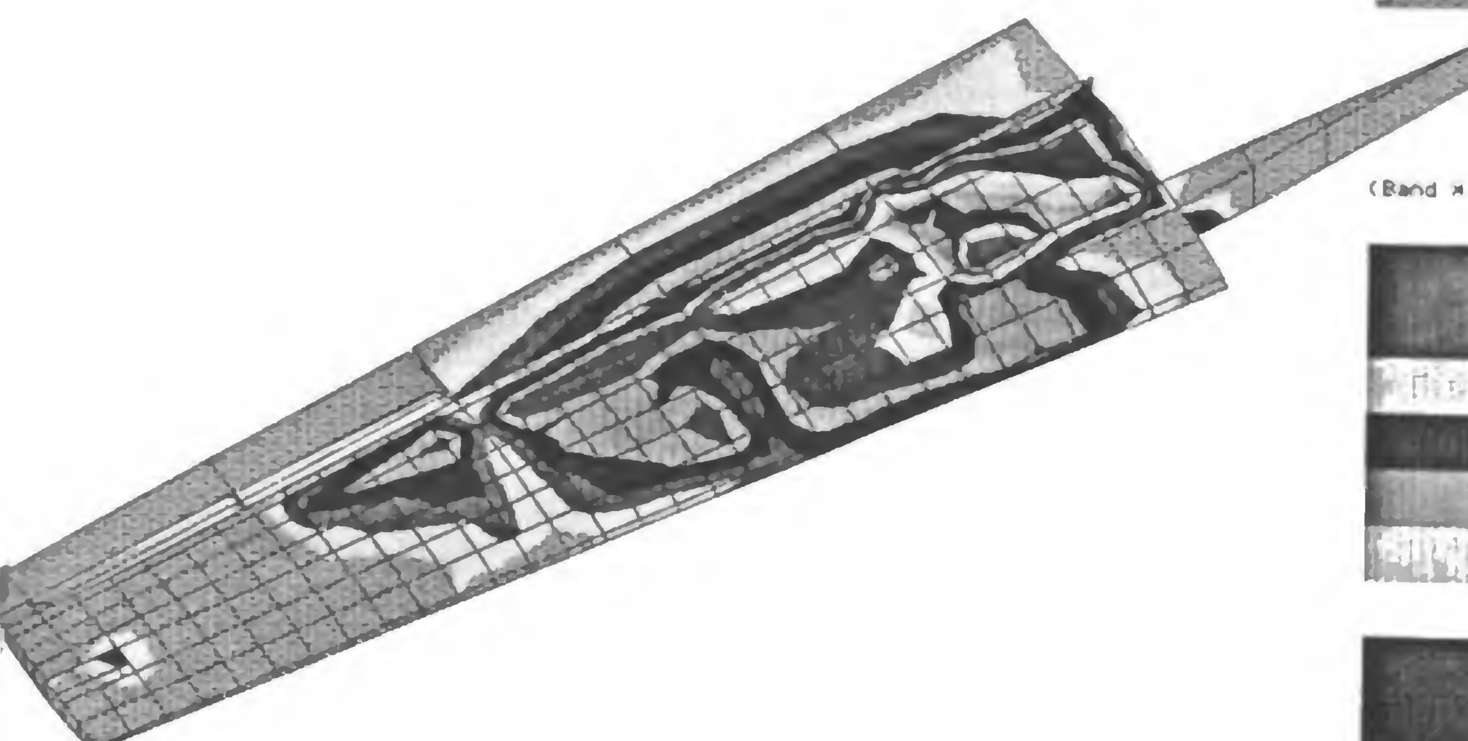




Figure D-4. Limit Load, Shear Stress Countours in the Upper, Top Ply, Wing Skin.



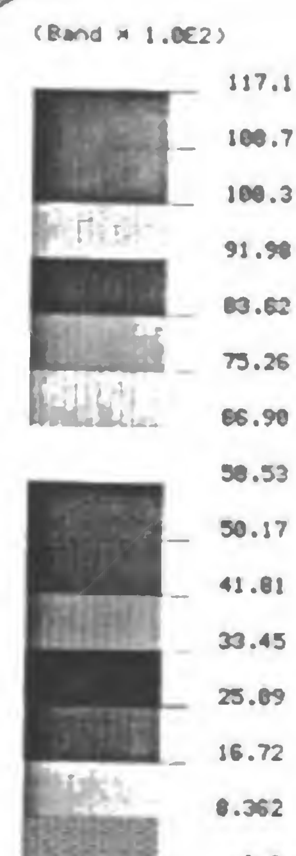



Figure D-5. Limit Load, Compression Stress Countours in the Upper, Top Ply, Wing Skin.

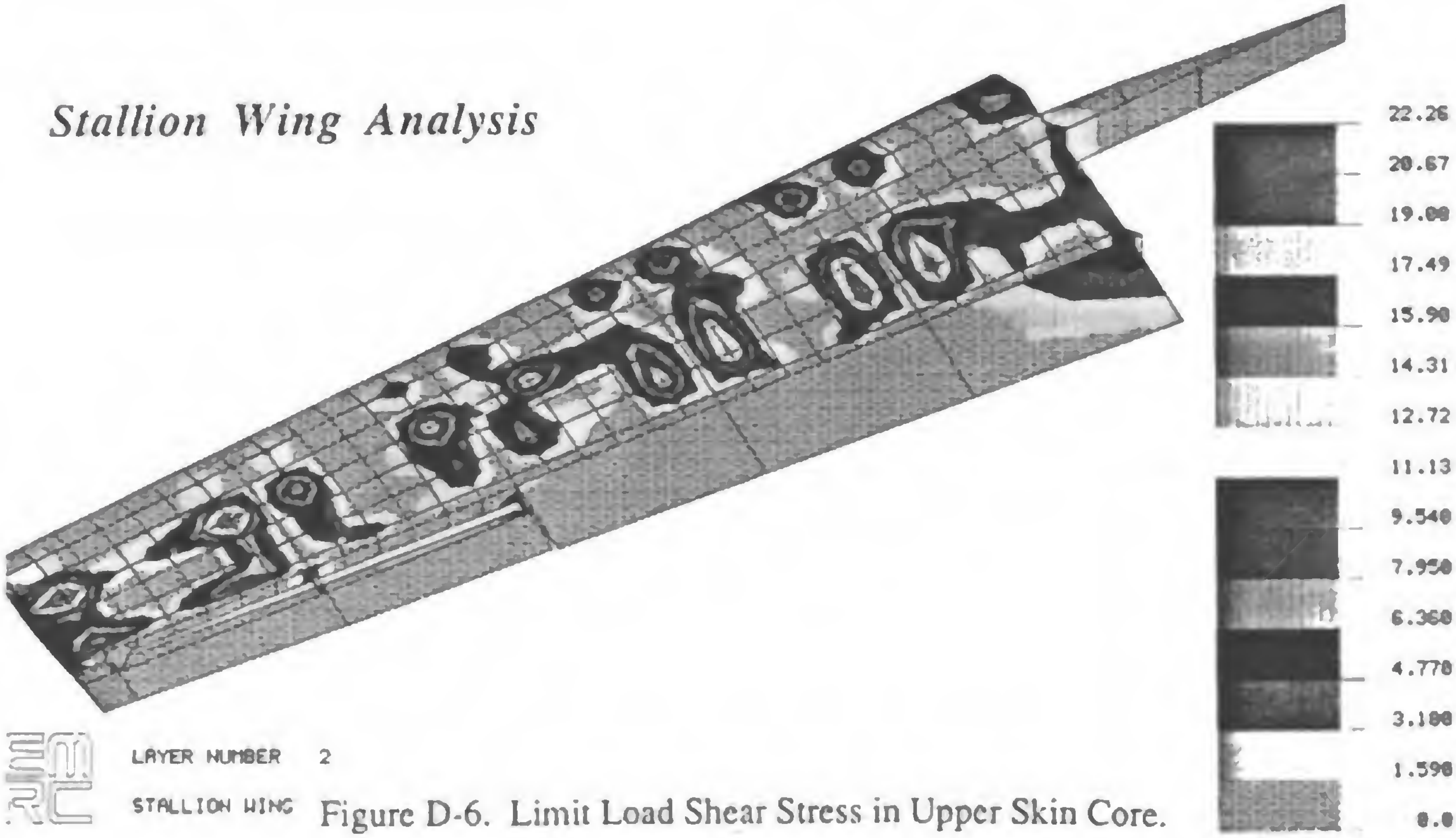



Figure D-6. Limit Load Shear Stress in Upper Skin Core.

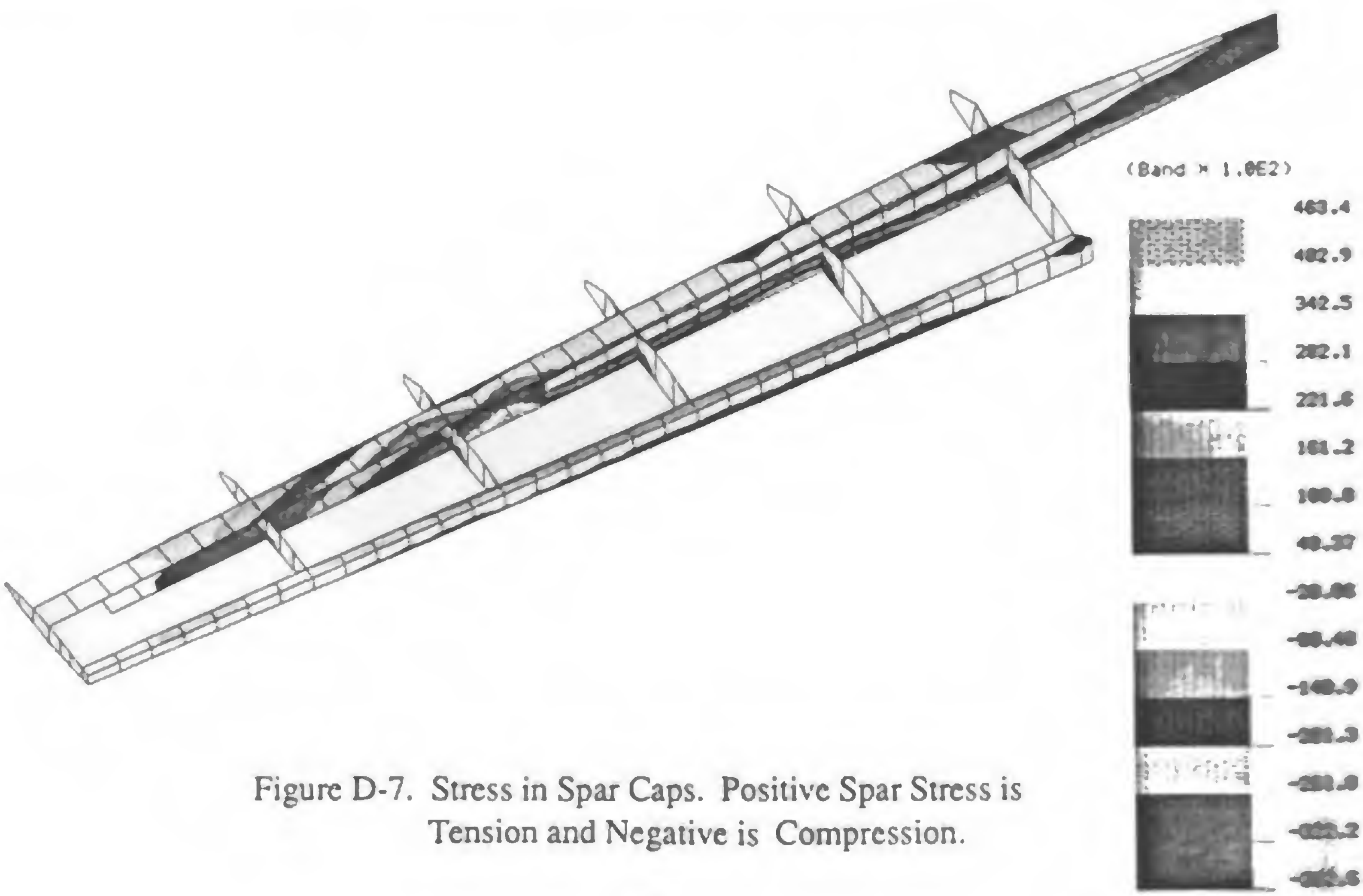
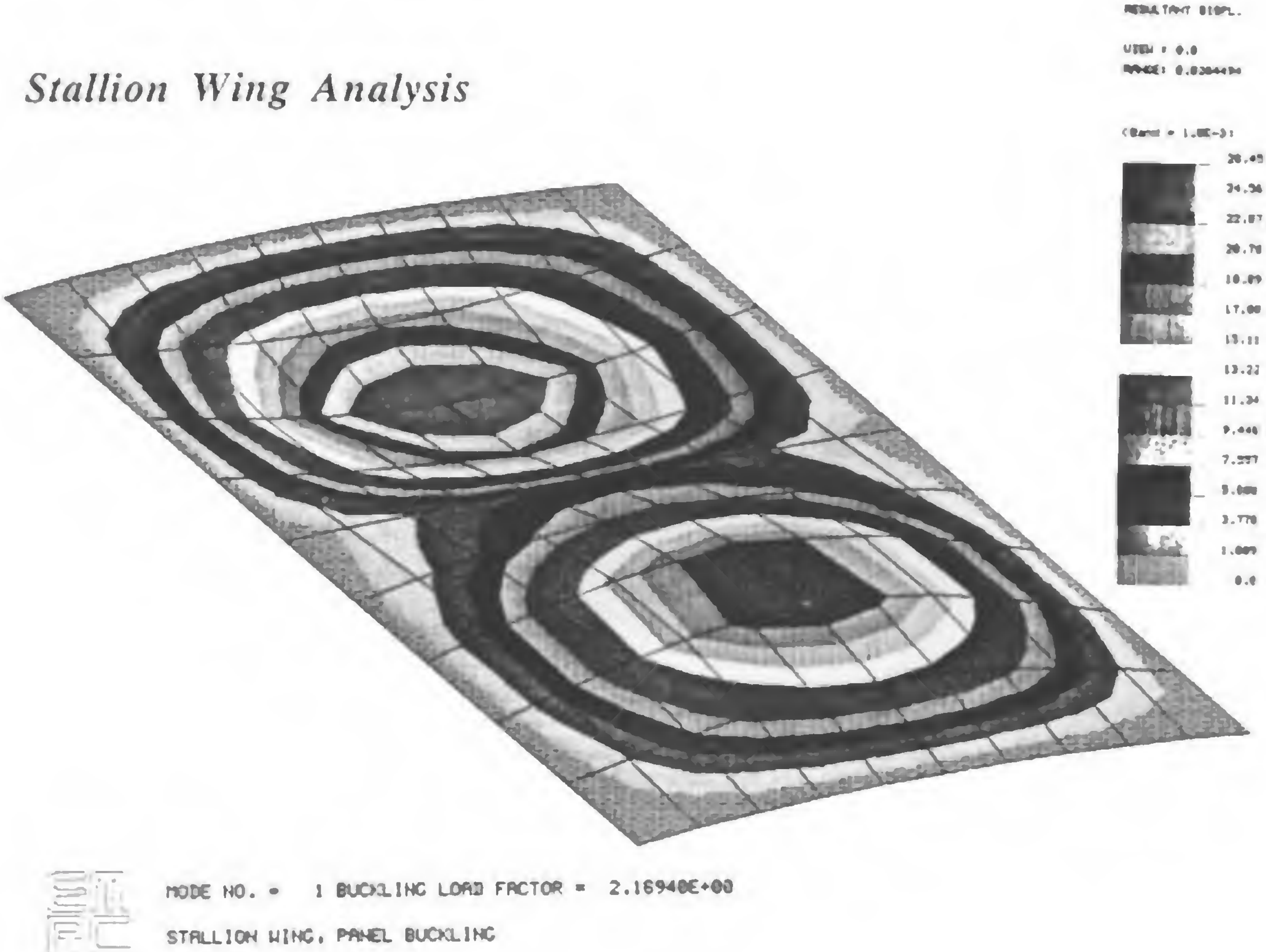



Figure D-7. Stress in Spar Caps. Positive Spar Stress is Tension and Negative is Compression.



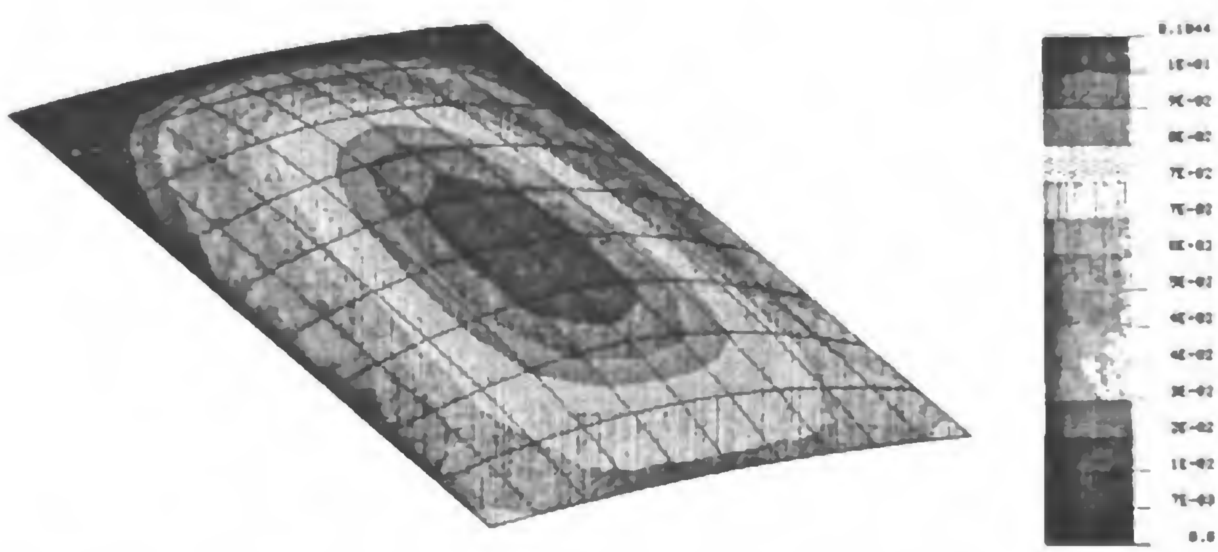
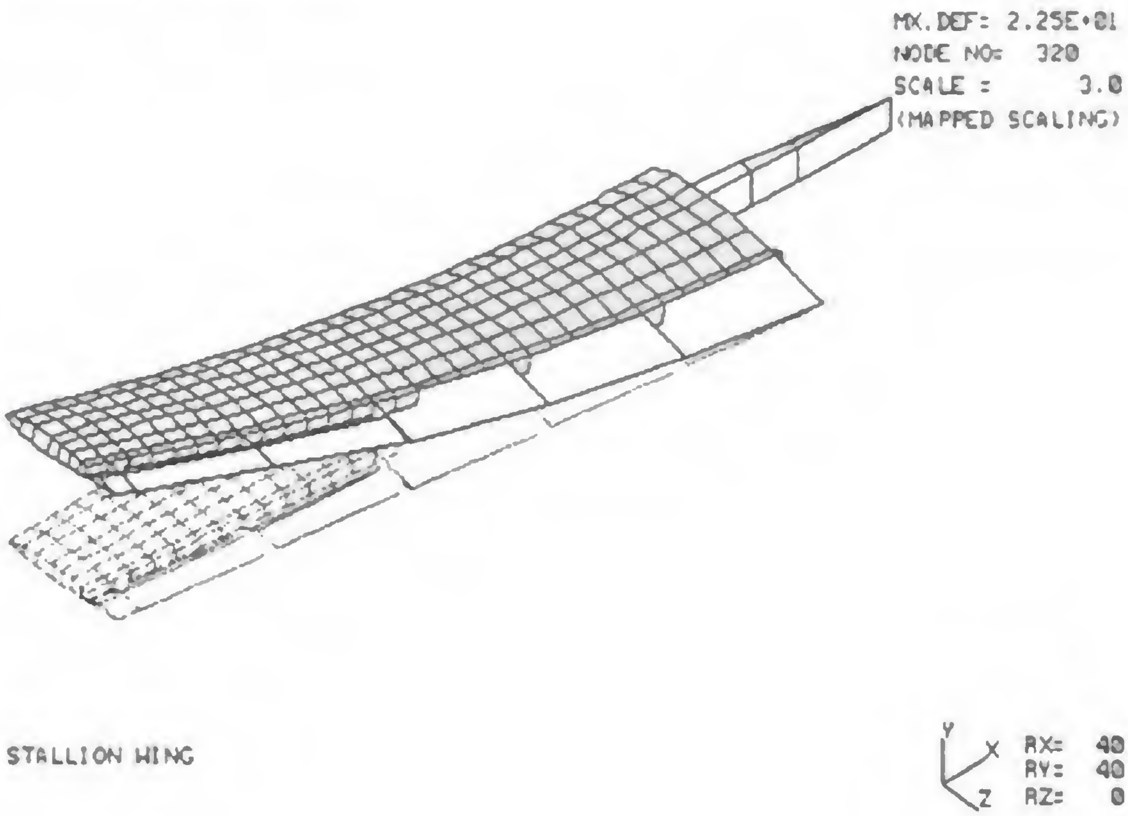
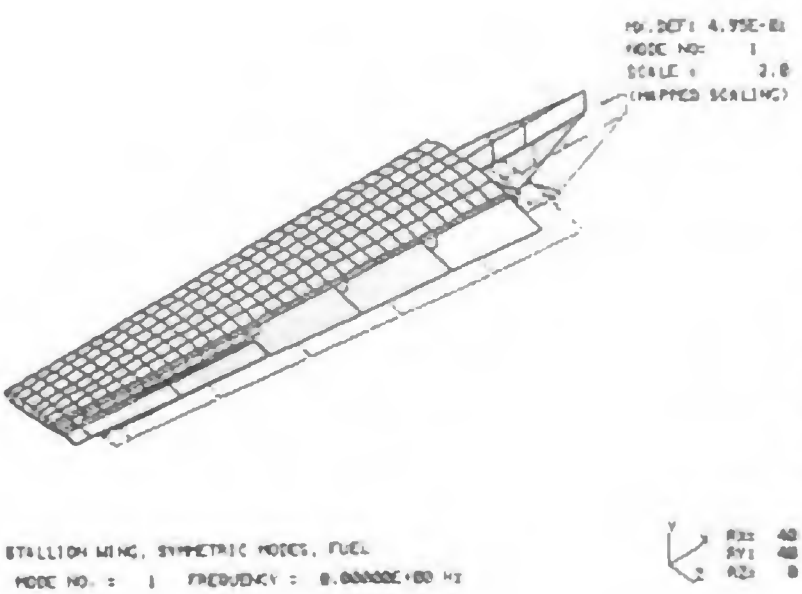
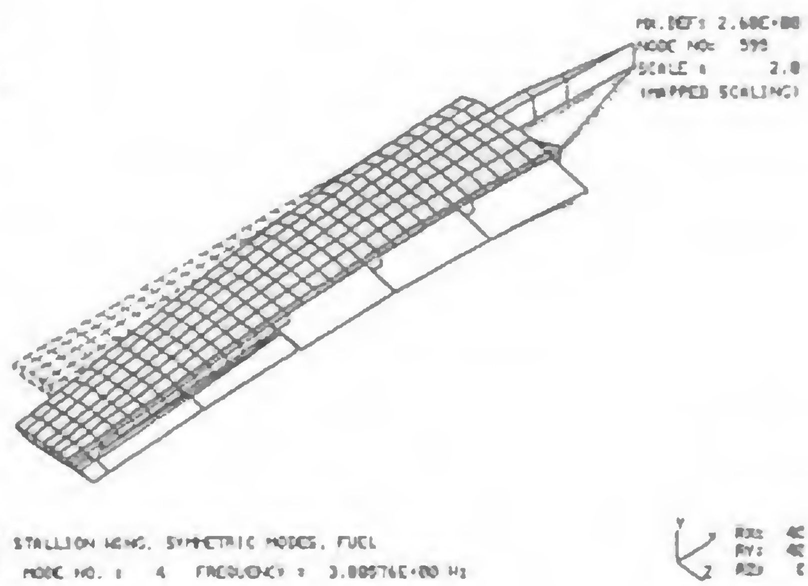



Figure D-8. Upper Picture shows Buckled Shape while the lower picture shows Static Panel Deflection.




Rigid Body Mode




Mode 1 - 1st wing bending

Figure D-10. Deflected Shape of Wing from Static Analysis and Mode Shapes from Eigenvalue Run.

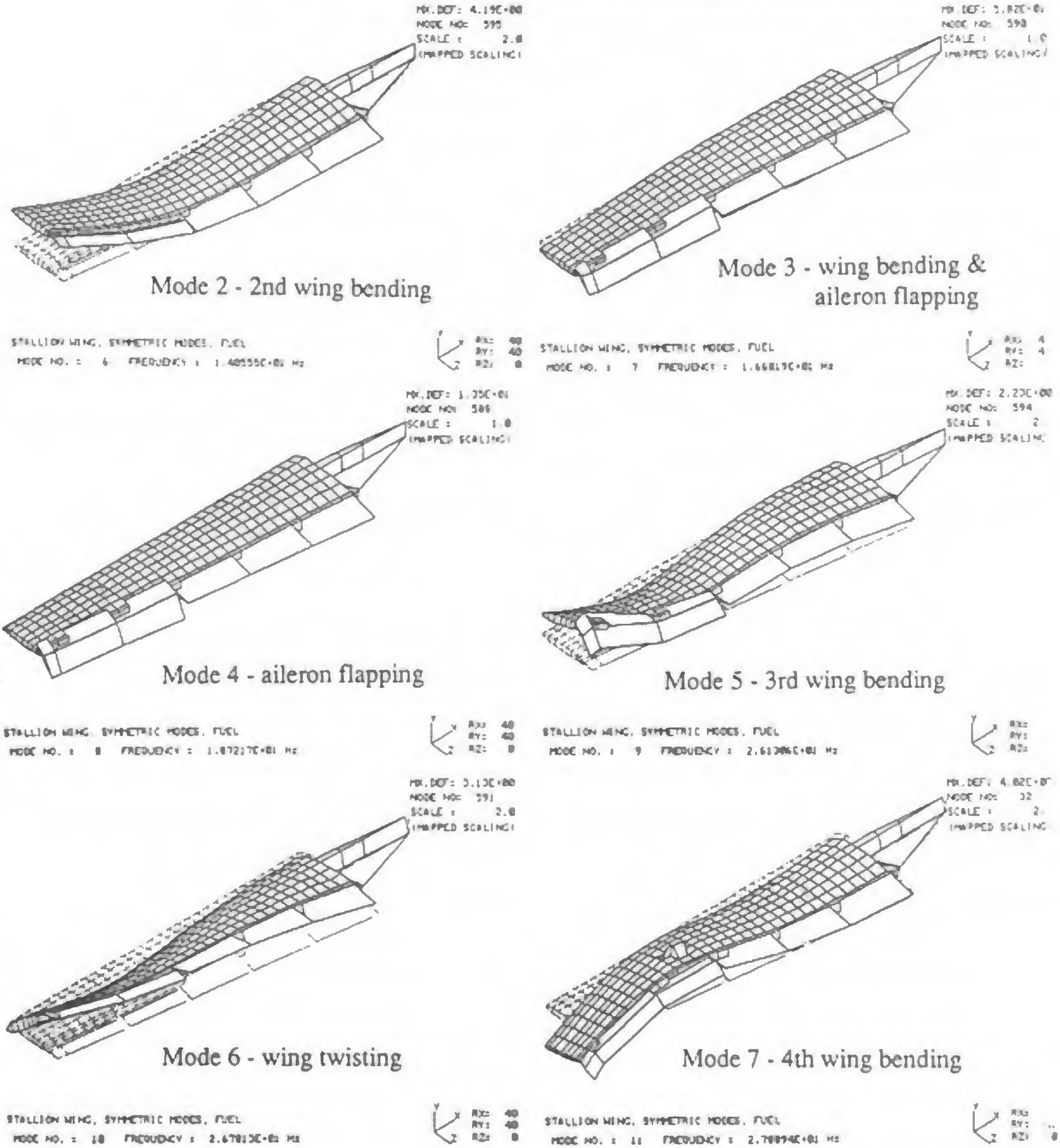


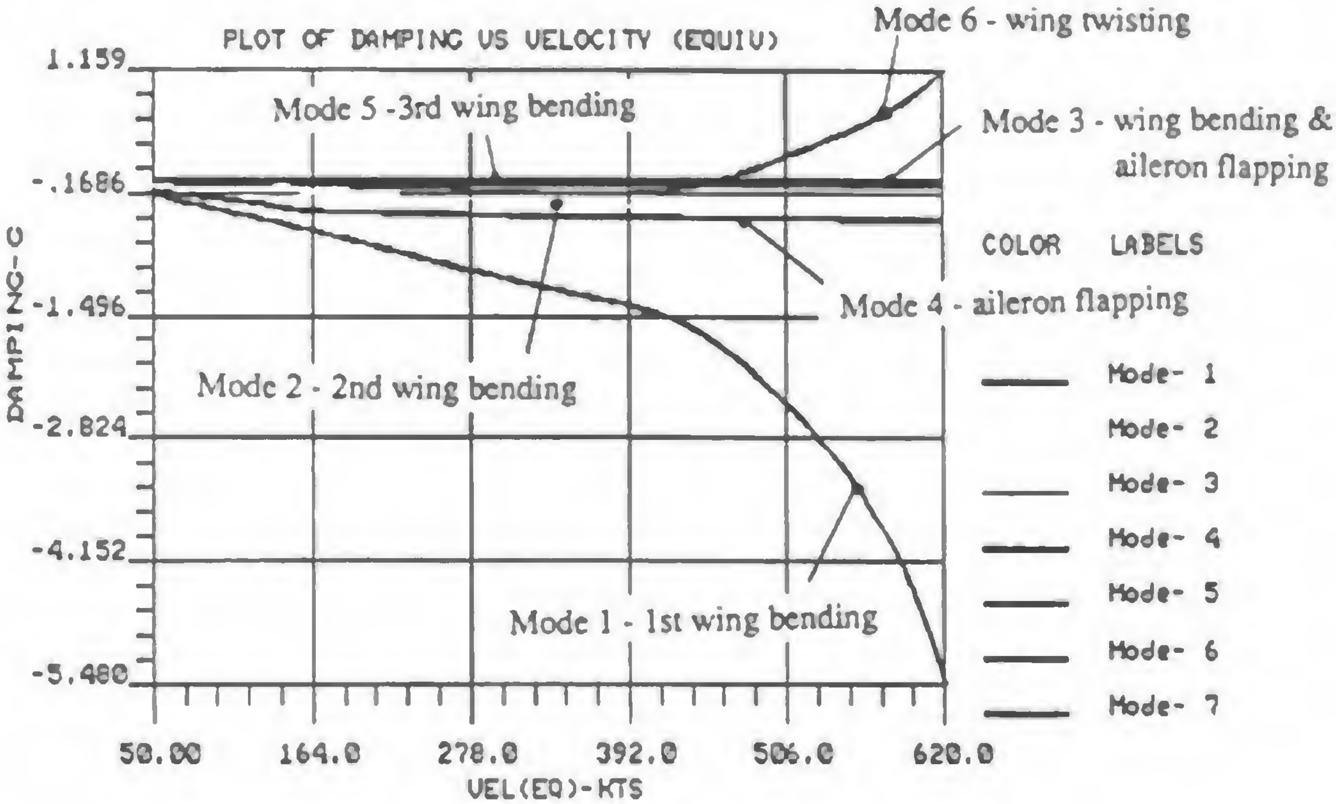

Figure D-10 Continued. Mode Shapes from Eigenvalue Run.

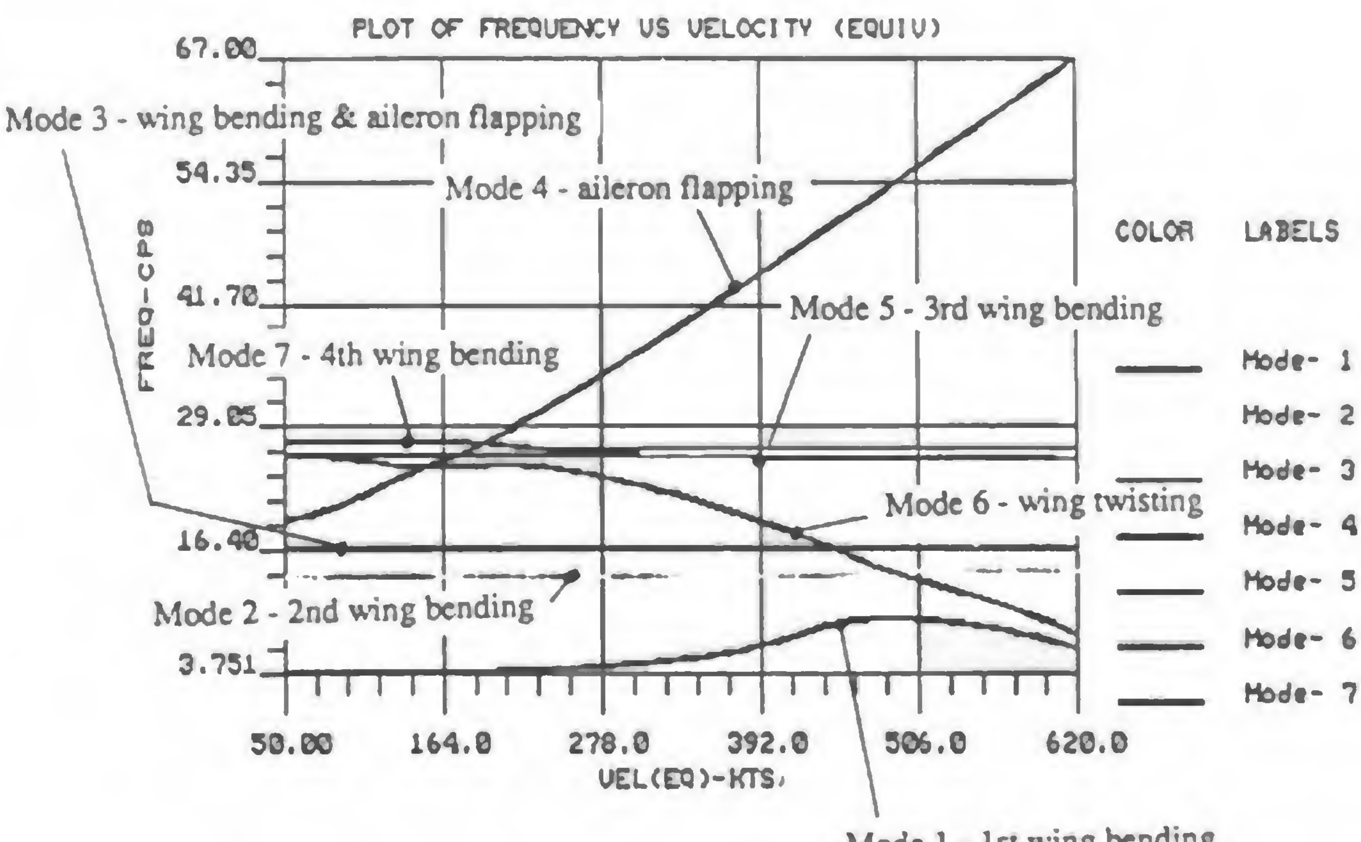
Table D-4. Flutter Analysis File for SAF.

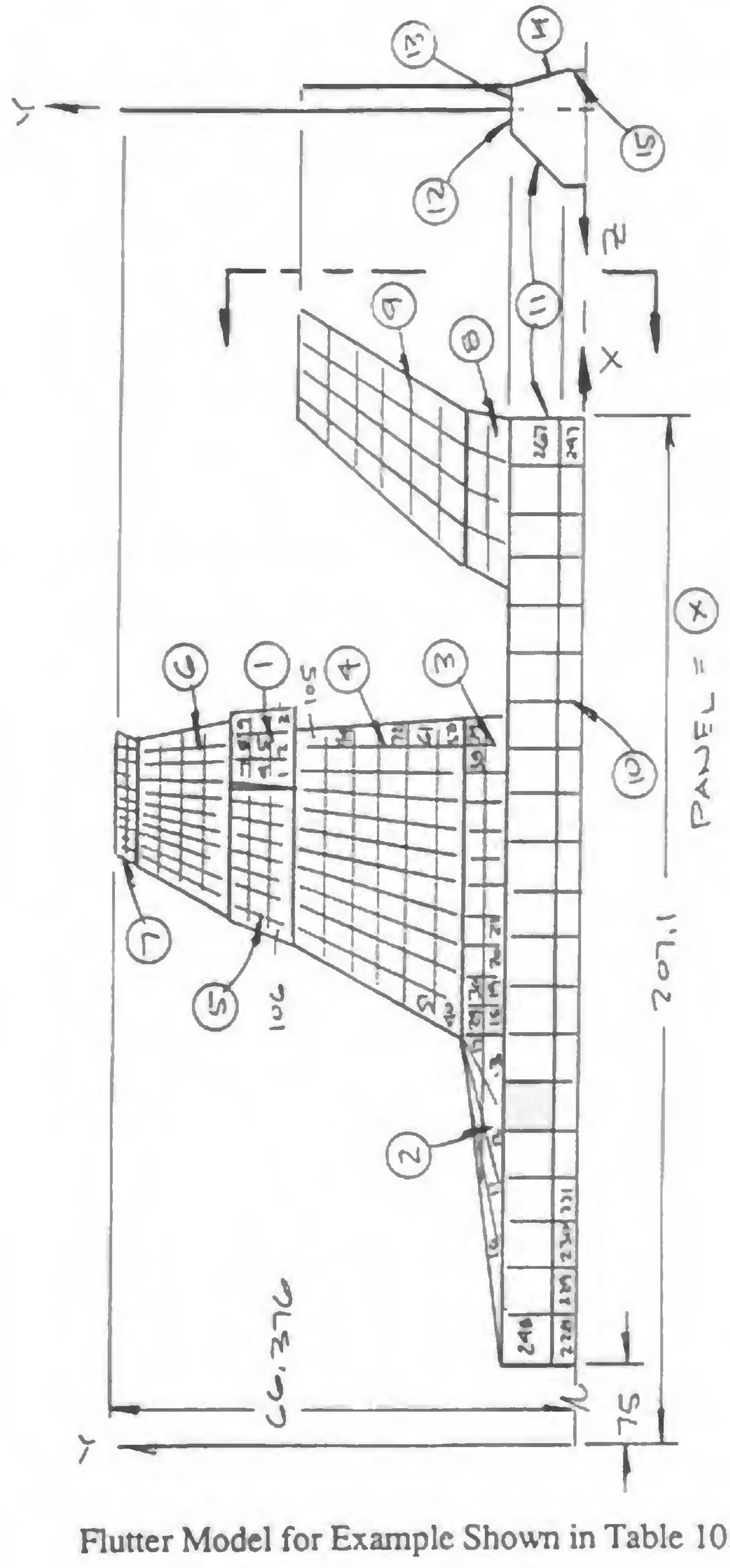
```
1
FLUTTER, STALLION WING
3200 LBS GROSS WITH 100 GALLONS FUEL
SYMMETRIC CASE
P-K METHOD
7 VIBRATION MODES
MACH=0.4
-1     7     1     6     1     0     0     0     0     0
1      0     1     0     0     1     0     0     1     0
1      0     0     1     0     0     0     0     0     0
0      0     0     0     0     0     0
21
0.1494    -0.1483   -0.73207  -1.5711   -2.5217   0.1438    -0.1604
-0.74506  -1.578    -2.527    0.150     -0.1569   -0.7403   -1.581
-2.528    -0.9655   -1.714    -2.530    -0.9597   -1.709    -2.524
-0.0846   -0.759    -0.975     0.528    3.4847    -0.0965   -0.7851
-0.9577   0.5395    3.5119    -0.1473   -0.8517   -1.059    0.4824
3.466     -0.8824   0.83677   3.475     -1.156    0.590     3.227
0.0128    0.0561    0.0579    -0.0401   -0.188    0.0105    0.0536
0.0521    -0.0462   -0.197    0.00925   0.0510    0.0477    -0.0539
-0.1986   0.02675   -0.0788   -0.1916   -0.4467   -0.496    -0.579
0.00956   0.0323    0.0525    0.0564    0.0487    0.00186   0.0205
0.0279    0.00445   0.00268   -0.00225  0.00046   -0.0122   -0.0733
-0.0647   -0.0399   -0.1534   -0.0865   -13.27    -11.867   -11.008
0.0733    0.2457    -0.2509   -0.488    0.8715    0.1352    0.3488
-0.0922   -0.2175   1.1834    0.2662    0.562     0.1657    0.0205
1.452     0.0529    0.2294    1.601     -0.4835   -0.4555   0.677
-0.4036   -0.5020   -1.1458   -1.446    -0.4603   0.01836   0.0768
-0.325    -0.415    0.647     0.6185    1.109     1.038     1.0789
1.966     1.4475    1.7027    2.461     1.511     1.8298    2.647
-0.524    -0.9161   -0.2038   -0.3005   -3.316    -0.3134   -0.6734
0.06898   -0.03828  -3.005    -0.1173   -0.3099   0.6315    0.7021
-2.408    1.1603    0.653     -2.251    0.6547    0.1107    -2.837
3
1     1     144.0     2     2     144.0     3     3     144.0
4     4     144.0     5     5     144.0     6     6     144.0
7     7     144.0
3.8057    14.055    16.601    18.72     26.13     26.781    27.889
24.00     0.4
```

```
20        50.0        30.0
0.02        0.50        1.0         5.0         10.0        15.0        50.0
0.03
0.5         -0.5         600.0      20.0
1.0
48.00       1.0
    1    1    0 1512     0     0     1
0.0         0.0         0.0         0.0
0.0         47.0        3.5         33.5        25.0        180.0
0.0         0.0           25 10     0.0
0.0         0.2         0.4         0.6         0.65        0.8
0.85        0.9         0.95        1.0
0.0         0.0417      0.0833      0.125       0.1667      0.208
0.25        0.2917      0.3333      0.375       0.4167      0.458
0.5         0.542       0.583       0.625       0.667       0.708
0.75        0.792       0.833       0.875       0.917       0.958
1.0
24    0    0    0    0    0
1,9,0,10,18,0,19,27,0,28,36,0,37,45,0,46,54,0
55,63,0,64,72,0,73,81,0,82,90,0,91,99,0,100,108,0
109,117,0,118,126,0,127,135,0,136,144,0,145,153,0,154,162,0
163,171,0,172,180,0,181,189,0,190,198,0,199,207,0,208,216,0
T  216     1
3     0     1     1
5  4.0        0.0        6.0        180.0
35.0        70.0        105.0       140.0       175.0
5 13.0        0.0        13.2        180.0
35.0        70.0        105.0       140.0       175.0
5 35.0        0.0        24.0        180.0
35.0        70.0        105.0       140.0       175.0
33.5        115.0       28.5        180.0
2     0     1     1
3 33.5       115.0       28.5        180.0
115.0       145.0       170.0
3 39.5       115.0       33.5        180.0
115.0       145.0       170.0
0
```

Figure D-11. Flutter Analysis Results showing Damping and Frequency vs Speed.






Flutter Model for Example Shown in Table 10.

# APPENDIX E

## *WHEELER EXPRESS TAIL FLUTTER*

The "Wheeler Express" is a four place, high performance kit aircraft for which over 300 kits have been sold. It is fabricated out of fiberglass fabric and impregnated with vinylester resin. The core for the sandwich structure of this clean composite aircraft is made out of 4.5 lbs/cu.ft. Clark foam. See figure E-1. This aircraft has had a series of accidents most of which can be blamed on pilot error. However there are two incidences that are of importance and that make flutter suspect. On the way to Oshkosh in 1990, after taking off from an airport at Casper, Wyoming, on the morning of July 25, the production prototype crashed killing the company test demonstration pilot, a salesman, and an engineer. NTSB said that there was no evidence of structural failure and the engine appeared to be operating normally until impact. The flap actuators had been extended which indicated that the pilot tried to slow the aircraft before impact. Todate no cause of failure has been found. Shortly thereafter, a Wheeler Express built by Mike Betts took off and encountered severe turbulence. The empennage began to vibrate and make a severe noise which scared both occupants. The aircraft was landed and an inspection of the tail showed severe delamination. The vibration occurred at a speed of 85 to 90 knots. Both of these aircraft had similar configurations and control surface weights. For example the rudder on the fatal aircraft had a weight of 19 lbs and that of Mike Betts 18 lbs.

One of the Wheeler Express builders, Bruce Decker, contacted Aircraft Designs, Inc. to perform a flutter analysis which is used as an example here. The first analysis was performed using data from Mike Betts. This data included control surface and ballast weights, control surface stiffnesses, and ply layup schedules.This analysis determined a critical flutter speed of 87.7 knots and 328 knots at sea level as shown in the damping and frequency curves of figure E-2. This analysis was performed without the knowledge of the flutter speed which Mike Betts had encountered.

The second analysis was performed after polling several builders and using the control surface weights of Ed Bernard's aircraft. Table E-1 shows the data that was obtained. Also, some additional plates were added into the finite element model of the vertical stabilizer to stiffen the horizontal tail to vertical tail attachment. This analysis showed that there if no flutter problem for these control surface weights. The same control stiffnesses and tail configuration of the first analysis were used.

The example shown on the next pages is for the new empennage that has no flutter. The analysis was performed for sea level up to 15,000 feet. Jim Warner and the new Wheeler Demo aircraft are presently flying and have demonstrated that indeed flutter is not a problem with the new control surface weights.

Figure E-1. The Wheeler Express is a sleek, all composite aircraft which is being built by a large number of homebuilders. The empennage flutter analysis is used as an example.

Table E-1. Wheeler Empennage Data

| Builder | Elevator Weight | Balance Weight | Total | Mom. Arm for Balance |
|---|---|---|---|---|
| Ed Bernard | 15.00 lbs | 13.875 lbs | 28.875 lbs | 3.70 inches |
| Wheeler Demo | - | - | 29.313 | - |
| Jim Warner | - | - | 32.00 | - |
| Mike Betts | 17.35 | 8.65* | 26.00 | 4.00 |

* Of this weight 4.2 lbs is attached to the elevator horn at the center of the elevator.

| Builder | Rudder Weight | Balance Weight | Total | Mom. Arm for Balance |
|---|---|---|---|---|
| Ed Bernard | 8.63 lbs | 5.00 lbs | 13.63 lbs | 4.8 inches |
| Wheeler Demo | - | - | - | 13.90 |
| Jim Warner | - | - | 15.00 | - |
| Mike Betts | 13.40 | 4.60 | 18.0 | 6.75 |

Control Surface Stiffness, Stick Fixed from Mike Betts

Load applied at trailing edge of elevator and deflection measured at t.e.

5.0 lbs = 3/8 inch     15.0 lbs = 3/4 inch

Load applied at trailing edge of rudder and deflection measured at t.e.

5.0 lbs = 3/4 inch     10.0 lbs = 1.00 inch

# Wheeler Express Tail Flutter

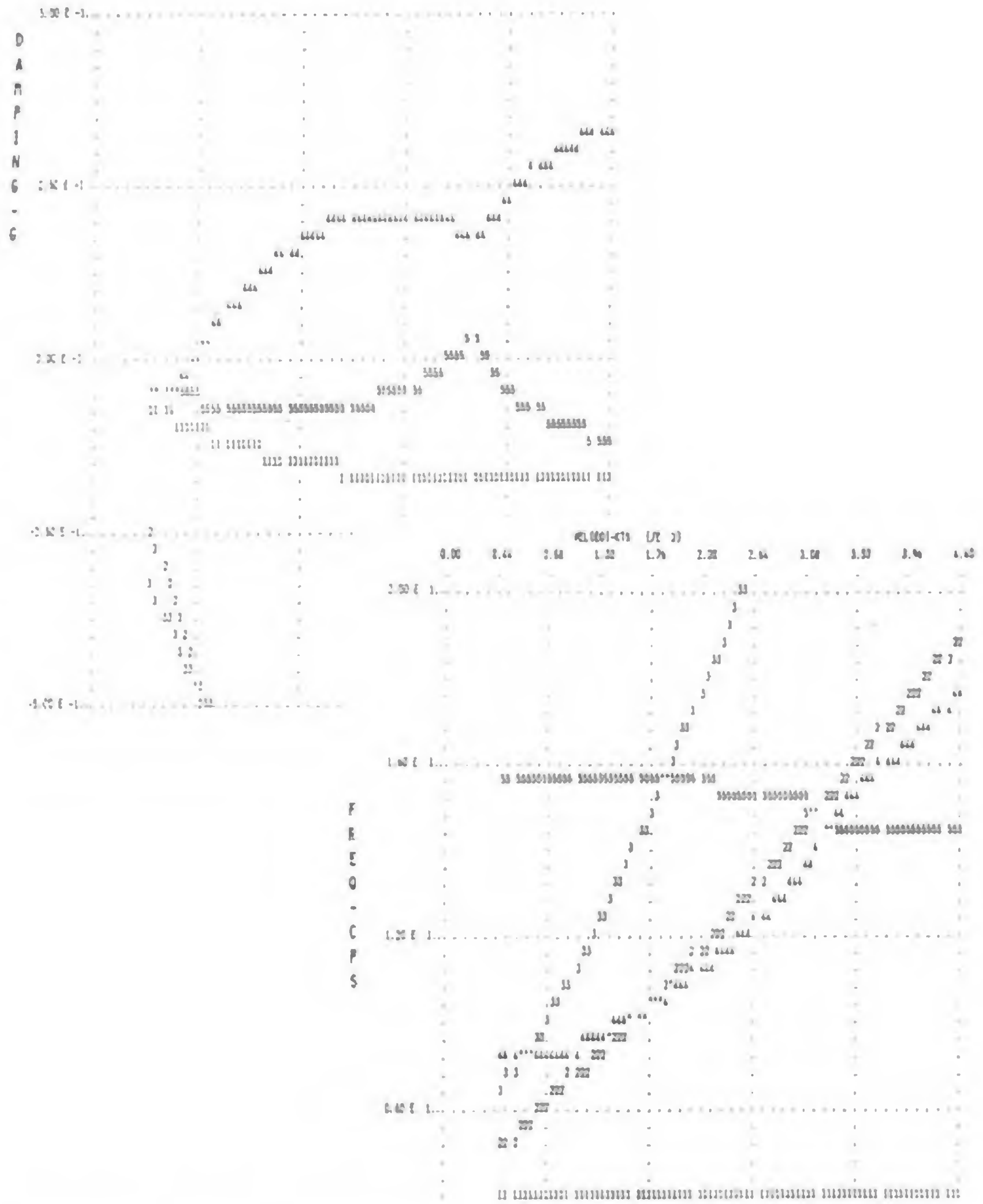


Figure E-2. Damping and Frequency Plots for Mike Betts Wheeler Express. Mode 4 crosses 0 Damping at 87.7 knots and Mode 5 crosses 0 Damping at 328 knots.

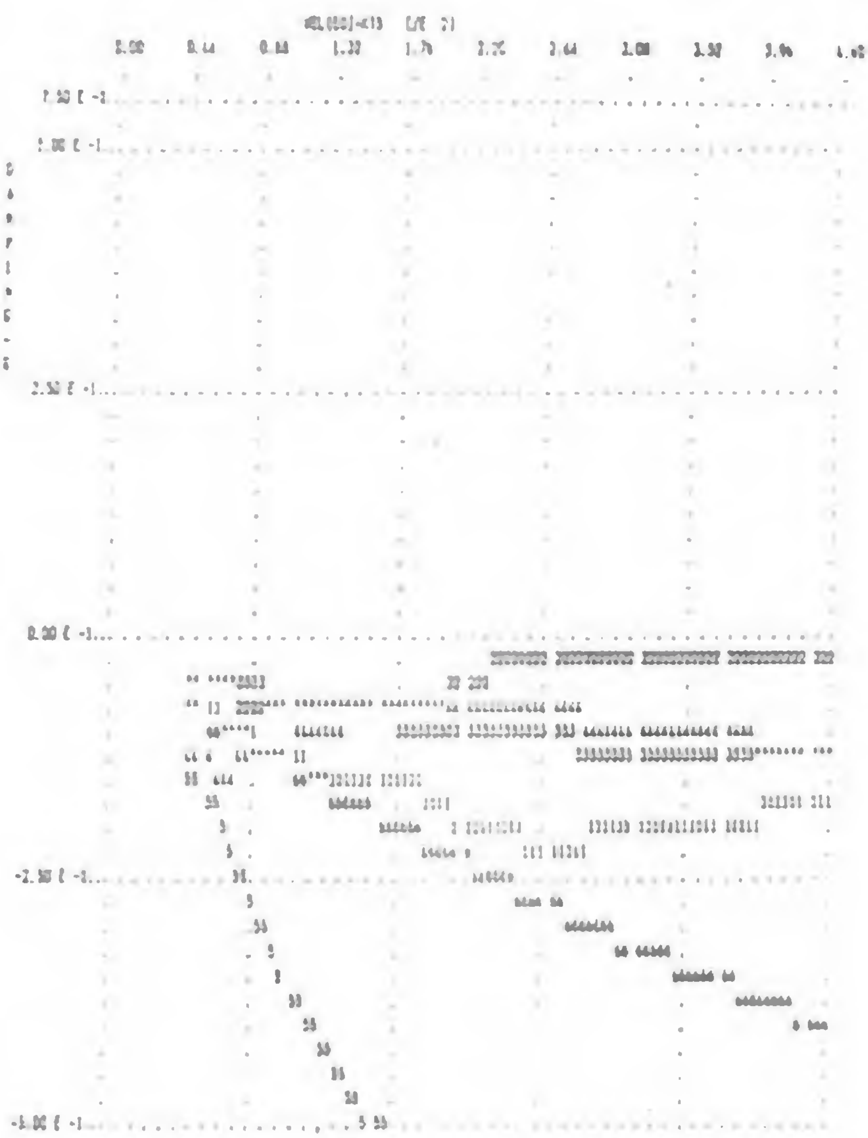
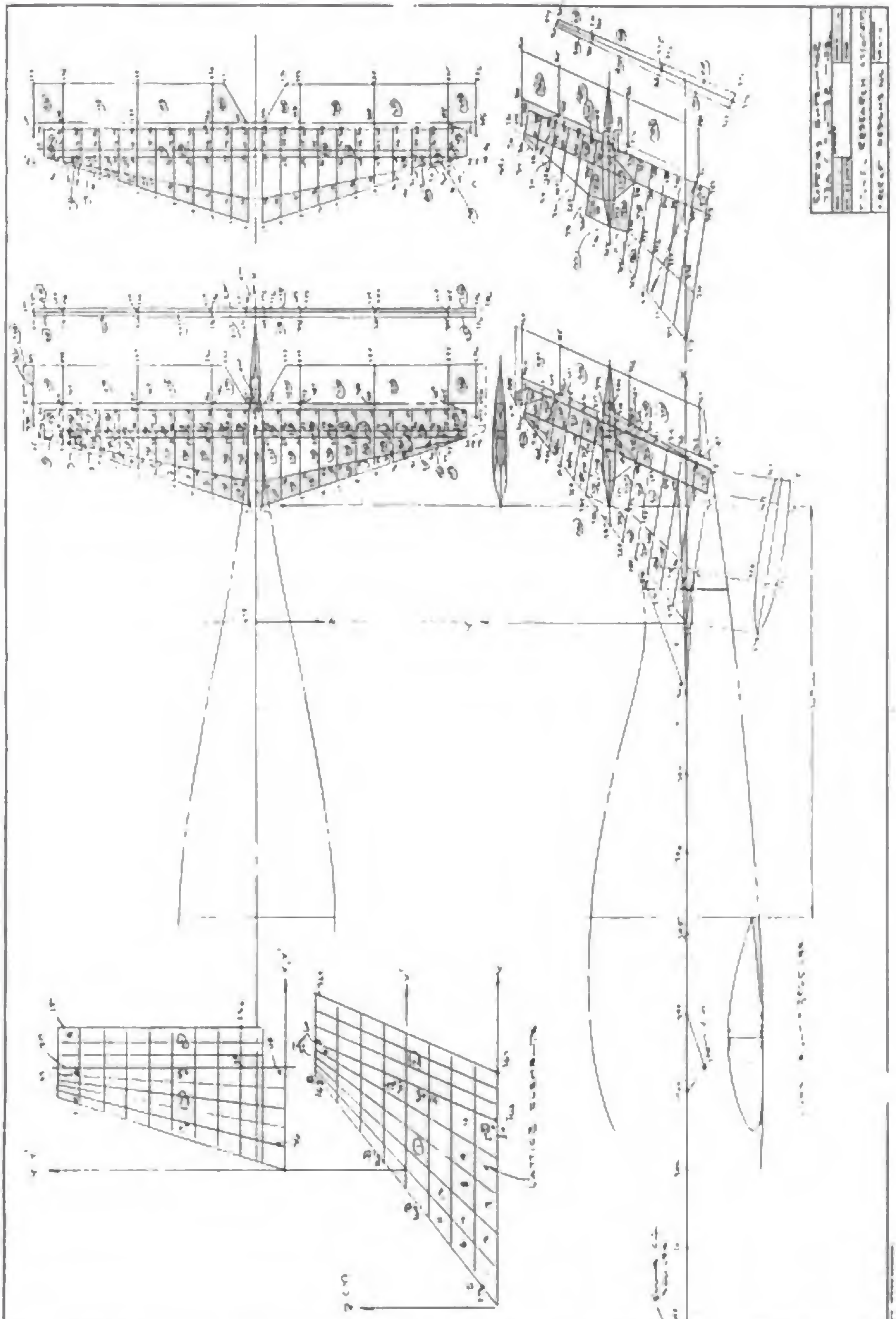


Figure E-3. Sea Level Damping Plot for the New Wheeler Express using Ed Bernard's aircraft data. Note that Mode 2 Approaches 0 Damping but does not cross it.

Figure E-4 on the next page shows the finite element model and flutter model of the Wheeler Express. In the fea the fuselage is modeled as a beam. Only one-half of the horizontal tail is modeled and the fuselage is not included.



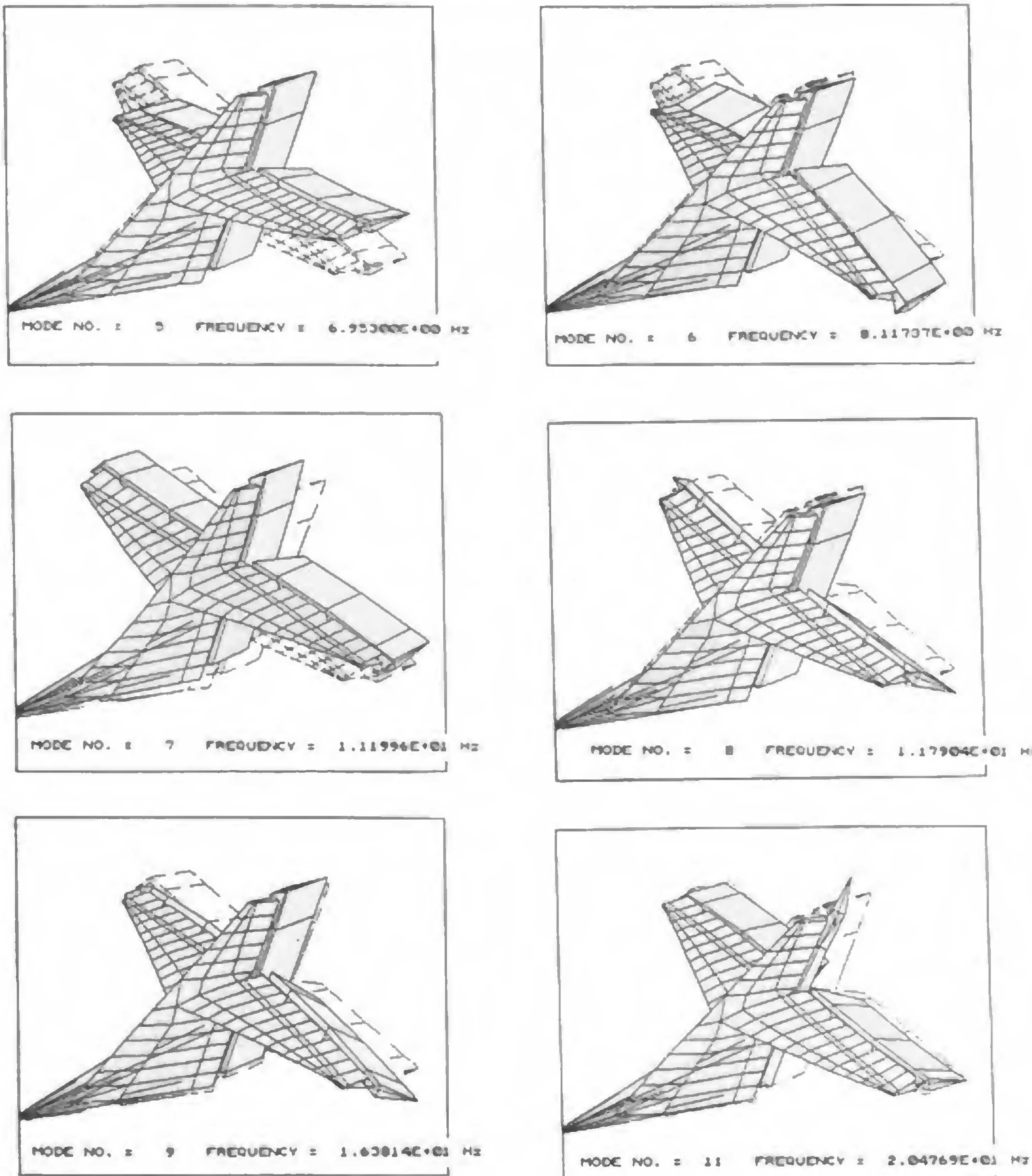



Figure E-5. Mode Shapes from the FEA Results. Rigid Body and In-Plane Bending Modes are not shown.

Table E-3. NISA FEA Input File.

```
ANAL=EIGENVALUE
**EXEC=CHECK
FILE=ET2
SAVE=26,27
EIGEN EXTRACTION=SUBSPACE,ACCELERATED
MASS FORMULATION=CONSISTENT
AUTO=ON
*ECHO=OFF
*A1
EXPRESS TAIL,DEMO
*C1
1,33,1
2,20,10
3,12,1
4,30,1
5,20,1
6,30,1
*D1
1,4
.020///
2,4
.25///
3,4
.032///
4,4
.032///
5,4
.20///
6,4
.06///
7,4
6.4,1700,800,2500
8,4
.293,.076,.076,.152
9,4
.1///
**ELEV CONTROL ROD
10,4
.00149,.001,.001,.002
**RUDDER CONTROL ROD
[illegible],4
[illegible]E-5,.0001,.0001,.0002
**[illegible]DER BALAST
[illegible],6
[illegible]294//
**ELEVATOR BALAST
[illegible],6
[illegible]779//
[illegible],6
[illegible]//
**[illegible]
[illegible],6
[illegible]9//
**[illegible]GS
[illegible],6
[illegible]//,35220,35220,1179
[illegible],4
[illegible]20,300,1020
**[illegible] WEB
[illegible],4
[illegible]5//
**[illegible] SKIN
[illegible],4
.01896///
**RUDDER WEB
20,4
.3687///
21,4
6,1200,600,1800
22,4
.02308///
*D2
1,4
0///
2,4
45///
*D3
3,0,1,2,1,2,1,2,1,2,1
3,0,3,2,3,2,1,2,1,2,1
3,0,4,2,4,2,1,2,1,2,1
3,0,9,2,9,2,1,2,1,2,1
3,0,19,2,19,2,1,2,1,2,1
3,0,22,2,22,2,1,2,1,2,1
*E1
**HORIZ. TAIL
$$,-2,24,21
1,3,27,26,2,$,2,1,0,21,1,1
43,26,50,49,$,1,2,4
44,26,49,1,$,1,2,4
45,2,26,1,$,1,2,4
46,122,2,1,$,1,2,4
47,145,122,1,$,1,2,4
48,146,122,145,$,1,2,4
49,72,71,47,$,1,2,4
50,24,72,47,$,1,2,4
51,24,47,23,$,1,2,4
52,24,23,143,$,1,2,4
53,168,24,143,$,1,2,4
54,168,143,167,$,1,2,4
55,74,98,97,73,$,2,1,0,23,1,1
78,3,123,122,2,$,2,1,0,21,1,1
99,123,147,146,122,$,2,1,0,21,1,1
120,170,194,193,169,$,2,1,0,23,1,1
**SPAR
143,50,74,73,49,$,3,5,6,7,1,1
150,57,81,80,56,$,3,5,5,9,1,1
159,66,90,89,65,$,3,5,6,7,1,1
166,146,170,169,145,$,3,5,6,7,1,1
173,153,177,176,152,$,3,5,5,9,1,1
182,162,186,185,161,$,3,5,6,7,1,1
189,51,147,146,50,$,2,1,0,21,1,1
**RIBS
$$,-3,4,4
210,2,122,26,$,1,2,4
211,122,146,50,26,$,1,1,0
212,146,170,74,50,$,1,1,0
213,170,194,98,74,$,1,1,0
$$,-3,4,4
222,15,135,39,$,1,2,4
223,135,159,63,39,$,1,1,0
224,159,183,87,63,$,1,1,0
225,183,207,111,87,$,1,1,0
**ELEVATOR
234,218,228,227,217,$,6,1,0,4,1,1
238,223,233,232,222,$,6,1,0,4,1,1
242,238,228,227,237,$,6,1,0,4,1,1
```

```
246,243,233,232,242,$,6,1,0,4,1,1
250,218,248,247,217,$,1,5,18,4,1,1
254,223,253,252,222,$,1,5,18,4,1,1
258,248,238,237,247,$,1,5,18,4,1,1
262,253,243,242,252,$,1,5,18,4,1,1
$$,-10,1,2
268,247,227,217,$,1,2,4
269,237,227,217,$,1,2,4
289,378,248,247,$,1,2,4
290,377,256,255,$,1,2,4
291,378,237,247,$,1,2,4
292,378,247,217,$,1,2,4
293,377,246,256,$,1,2,4
294,377,256,226,$,1,2,4
295,13,133,37,$,1,2,4
296,133,157,61,37,$,1,1,0
297,157,181,85,61,$,1,1,0
298,12,132,36,$,1,2,4
299,132,156,60,36,$,1,1,0
300,156,180,84,60,$,1,1,0
**ELEVATOR TUBE
301,221,400,$,6,3,8
302,400,222,$,6,3,8
303,400,379,$,6,3,8
304,400,380,$,6,3,8
**HINGE SUPPORTS
305,194,384,98,$,4,2,5
306,198,385,102,$,4,2,5
307,202,386,106,$,4,2,5
308,207,387,111,$,4,2,5
309,211,388,115,$,4,2,5
310,215,389,119,$,4,2,5
**VERTICAL STAB.
$$,-3,14,5
311,261,275,274,260,$,3,1,0,5,1,1
$$,-3,14,5
326,269,283,282,268,$,3,1,0,5,1,1
341,267,13,265,$,1,2,4
342,13,37,279,265,$,3,1,0
343,37,61,293,279,$,4,1,0
344,61,85,307,293,$,4,1,0
345,157,309,181,$,1,2,4
346,296,310,309,157,$,1,2,4
347,296,157,133,$,1,2,4
348,282,296,133,13,$,3,1,0
349,268,282,13,267,$,3,1,0
350,181,308,307,85,$,4,1,0
351,309,308,181,$,1,2,4
352,261,317,316,260,$,3,1,0,5,1,1
$$,-2,14,5
357,317,331,330,316,$,3,1,0,5,1,1
367,269,325,324,268,$,3,1,0,5,1,1
$$,-2,14,5
372,325,339,338,324,$,3,1,0,5,1,1
382,267,12,265,$,1,2,4
383,12,132,321,265,$,3,1,0
384,132,156,335,321,$,4,1,0
385,60,84,349,335,$,4,1,0
386,351,180,156,$,1,2,4
387,338,352,351,156,$,4,1,0
388,338,156,132,$,1,2,4
389,324,338,132,12,$,3,1,0
390,268,324,12,267,$,3,1,0
391,180,350,349,84,$,4,1,0
392,351,350,180,$,1,2,4
**RUDDER
393,361,365,364,360,$,5,1,0,3,1,1
396,369,365,364,368,$,5,1,0,3,1,1
399,361,374,373,360,$,1,5,20,3,1,1
402,374,369,368,373,$,1,5,20,3,1,1
$$,-4,1,2
405,360,364,373,$,1,2,4
406,373,364,368,$,1,2,4
503,372,360,373,$,1,2,4
504,372,373,368,$,1,2,4
505,374,373,372,$,1,2,4
**VERT. STAB. STRUCTURE
506,303,345,344,302,$,1,1,0,13,1,1
519,289,331,330,288,$,1,1,0,5,1,1
524,296,338,335,293,$,4,1,0
525,297,339,338,296,$,1,1,0,5,1,1
530,260,274,316,$,1,2,4
531,274,288,330,316,$,1,1,0
532,288,302,344,330,$,1,1,0
533,262,276,318,$,1,2,4
534,276,290,332,318,$,1,1,0
535,290,304,346,332,$,1,1,0
536,270,284,326,$,1,2,4
537,285,327,326,284,$,1,1,0,2,1,1
539,286,300,342,328,$,1,1,0,2,14,1
**RUDDER HING BRKTS
541,304,381,346,$,4,2,5
542,309,382,351,$,4,2,5
543,314,383,356,$,4,2,5
544,314,383,313,$,4,2,5
545,111,387,110,$,4,2,5
**FUSELAGE
551,390,391,$,5,3,7,6,1,1
557,396,397,$,5,3,21
558,397,398,$,5,3,17
559,398,273,$,5,3,17
560,273,285,$,5,3,17
561,398,270,$,7,3,8
562,398,286,$,7,3,8
563,398,328,$,7,3,8
564,393,399,$,7,3,8
565,399,394,$,7,3,8
**ELEVATOR CONTROL ROD
566,380,314,$,4,3,10
**RUDDER CONTROL ROD
567,363,314,$,4,3,11
**LUMP MASSES
571,372,$,,6,12
572,377,$,,6,13
573,378,$,,6,13
575,390,$,,6,15
576,399,$,,6,16
**HORIZ. TO VERT. STIFFENER
577,61,60,335,293,$,1,2,9
*F1
1,0,$,48.5,20,-55
12,0,1,$,30.5,20,-2
13,0,$,30.5,20,2
24,0,1,$,48.5,20,55
25,0,$,48.5,20.3,-55
36,0,1,$,37,21.5,-2
```

```
37,0,$,37,21.5,2
48,0,1,$,48.5,20.3,55
49,0,$,48.5,20.5,-55
60,0,1,$,48.5,21.8,-2
61,0,$,48.5,21.8,2
72,0,1,$,48.5,20.5,55
73,0,$,50.3,20.5,-55
84,0,1,$,50.3,21.8,-2
85,0,$,50.3,21.8,2
96,0,1,$,50.3,20.5,55
97,0,$,56,21.5,-55
108,0,1,$,56,21.5,-2
109,0,$,56,21.5,2
120,0,1,$,56,21.5,55
121,0,$,48.5,19.7,-55
132,0,1,$,37,18.5,-2
133,0,$,37,18.5,2
144,0,1,$,48.5,19.7,55
145,0,$,48.5,19.5,-55
156,0,1,$,48.5,18.5,-2
157,0,$,48.5,18.5,2
168,0,1,$,48.5,19.5,55
169,0,$,50.3,19.5,-55
180,0,1,$,50.3,18.5,-2
181,0,$,50.3,18.5,2
192,0,1,$,50.3,19.5,55
193,0,$,56,18.8,-55
204,0,1,$,56,18.5,-2
205,0,$,56,18.5,2
216,0,1,$,56,18.8,55
217,0,$,57.5,21.5,-57.5
218,0,$,57.5,21.5,-50.182
219,0,$,57.5,21.7,-30.91
220,0,$,57.5,21.8,-11.64
221,0,$,57.5,21.8,-2
222,0,$,57.5,21.8,2
223,0,$,57.5,21.8,11.64
224,0,$,57.5,21.7,30.91
225,0,$,57.5,21.2,50.182
226,0,$,57.5,21.2,57.5
227,0,$,69,20,-57.5
228,0,$,69,20,-50
229,0,$,69,20,-30.5
230,0,$,69,20,-11.5
231,0,$,69,20,-8
232,0,$,69,20,8
233,0,$,69,20,11.5
234,0,$,69,20,30.5
235,0,$,69,20,50
236,0,$,69,20,57.5
237,0,$,57.5,18.8,-57.5
238,0,$,57.5,18.8,-50.182
239,0,$,57.5,18.3,-30.91
240,0,$,57.5,18.2,-11.64
241,0,$,57.5,18.2,-2
242,0,$,57.5,18.2,2
243,0,$,57.5,18.2,11.64
244,0,$,57.5,18.3,30.91
245,0,$,57.5,18.8,50.182
246,0,$,57.5,18.1,57.5
247,0,$,57.5,20,-57.5
248,0,$,57.5,20,-50.182
249,0,$,57.5,20,-30.91
250,0,$,57.5,20,-11.64
251,0,$,57.5,20,-2
252,0,$,57.5,20,2
253,0,$,57.5,20,11.64
254,0,$,57.5,20,30.91
255,0,$,57.5,20,50.182
256,0,$,57.5,20,57.5
260,0,$,53.5,42.5,0
273,0,1,$,0,0,0
274,0,$,54.7,42.3,0.5
287,0,1,$,13,-2,2.5
288,0,$,55.4,42.25,0.6
301,0,1,$,34,-5.2,2.2
302,0,$,61.8,41.2,1
315,0,1,$,40.2,-6.5,2
316,0,$,54.7,42.3,-.5
329,0,1,$,13,-2,-2.5
330,0,$,55.4,42.25,-.6
343,0,1,$,34,-5.2,-2.2
344,0,$,61.8,41.2,-1
357,0,1,$,40.2,-6.5,-2
360,0,$,64.5,43,1
361,0,$,60.3,33,1.2
362,0,$,52.2,15,1.6
363,0,$,44,-3.6,2
364,0,$,76.5,43,0
365,0,$,72.3,33,0
366,0,$,64.2,15,0
367,0,$,56.0,-3.6,0
368,0,$,64.5,43,-1
369,0,$,60.3,33,-1.2
370,0,$,52.2,15,-1.6
371,0,$,44,-3.6,-2
372,0,$,59.7,43,0
373,0,$,64.5,43,0
374,0,$,60.3,33,0
375,0,$,52.2,15,0
376,0,$,44,-3.6,0
377,0,$,53.8,20,57.5
378,0,$,53.8,20,-57.5
379,0,$,51.75,20,0
380,0,$,55.5,20,0
381,0,$,60.3,32.98,0
382,0,$,52.2,14.98,0
383,0,$,44,-3.62,0
384,0,$,57.5,20,-50.16
385,0,$,57.5,20,-30.89
386,0,$,57.5,20,-11.61
387,0,$,57.5,20,11.61
388,0,$,57.5,20,30.89
389,0,$,57.5,20,50.16
**FUSELAGE
390,0,$,-185,0,0
398,0,1,$,-18.5,0,0
399,0,$,-117,-4.5,0
400,0,$,57.5,20,0
*H1
**B.D. FIBERGLASS
EX,1,0,2.3E6
EY,1,0,2.3E6
GXY,1,0,1.25E5
GYZ,1,0,10E3
GXZ,1,0,10E3
```

```
DENS,1,0,2.0E-4
**FOAM CORE
EX,2,0,0
GXY,2,0,2.0E3
GYZ,2,0,2.0E3
GXZ,2,0,0
DENS,2,0,7E-6
**U.D. FIBERGLASS
EX,3,0,4.5E6
EY,3,0,4.5E6
GXZ,3,0,1.25E5
GYZ,3,0,10E3
GXY,3,0,10E3
DENS,3,0,2.0E-4
**ALUMINUM
EX,4,0,10E6
EY,4,0,10E6
GXZ,4,0,6E6
GYZ,4,0,6E6
GXY,4,0,6E6
NUXY,4,0,0.3
DENS,4,0,2.6E-4
**FUSELAGE
EX,5,0,2.3E6
EY,5,0,2.3E6
NUXY,5,0,0.3
DENS,5,0,.00129
**STEEL
EX,6,0,29E6
EY,6,0,29E6
GXY,6,0,17.4E6
GXZ,6,0,17.4E6
GYZ,6,0,17.4E6
DENS,6,0,7.4E-4
**BEAMS
EX,7,0,10E10
EY,7,0,10E10
NUXY,7,0,.3
DENS,7,0,0
*CPDISP
UXYZ,$,248,384
UXYZ,$,249,385
UXYZ,$,250,386
UXYZ,$,253,387
UXYZ,$,254,388
UXYZ,$,389,255
UXYZ,$,374,381
UXYZ,$,375,382
UXYZ,$,376,383
*EIGCNTL
16,0,25,,0,0,1E-5,-1
*SPDISP
399,UX,0
399,UZ,0
*MODEOUT
3 $ 1
*EIGOUT
1,0,3,1,0,1
*ENDDATA
```

Table E-2. Modes and Frequencies used in the Flutter Analysis.

| Flutter Analysis Mode | FEA Mode | Frequency |
|---|---|---|
| 1 | 5 | 6.953 Hz |
| 2 | 6 | 8.117 |
| 3 | 7 | 11.20 |
| 4 | 8 | 11.79 |
| 5 | 9 | 16.38 |
| 6 | 11 | 20.477 |

Table E-4. Flutter Analysis File of Wheeler Express Empennage for SAF.

```
1
FLUTTER,DEMO EXPRESS
2900 LBS GROSS
5 PANELS, 2 CONTROL SURFACE
P-K METHOD
6 VIBRATION MODES
MACH=0.2
-1     6     4     6     6     0     0     0     0     0
1     0     1     0     0     1     0     0     1     0
1     0     0     1     0     0     0     0     0     0
0     0     0     1     0     0     0
22
0.164       0.418       0.1884      -0.149      0.2837      0.328       0.1865
-0.0966     0.3014      -0.2533     0.3847      -0.2903     -0.02668    -0.01848
-2.646      -0.03145    -1.1437     -3.0145     -0.4397     -3.4625     -0.1355
-3.1033
0.00361     0.00742     0.0014      -0.01662    0.00751     0.00238     -0.00303
-0.0174     0.00847     -0.01968    0.00558     -0.0241     -0.4217     -0.3784
-2.5346     -0.5613     -1.277      -3.167      -0.6148     -3.820      0.8908
-2.20
-0.4597     -0.395      -0.6954     -1.593      -0.9690     -1.2111     -1.379
-1.897      -1.0026     -2.1495     -1.1571     -2.3076     0.02665     0.0264
0.31093     0.0072      0.1219      0.4443      0.0524      0.5932      0.1909
0.7078
-0.01654    -0.02013    -0.0431     -0.07535    -0.02768    -0.03897    -0.0481
-0.0784     -0.0275     -0.09073    -0.01654    -0.0815     -0.9912     -0.9131
-0.3971     -1.239      -1.0839     -0.2054     -1.4249     0.1577      -7.7076
-6.453
0.0285      0.015       -0.02468    -0.08791    0.0684      0.0345      0.0091
-0.0744     0.0735      -0.09966    0.1738      0.00816     1.193       1.124
-1.270      1.3659      0.30177     -1.962      1.0609      -2.632      -6.6098
-10.754
-0.2835     -0.0694     0.3858      1.5825      -0.6890     -0.2617     0.06736
1.6833      -0.796      2.8197      -7.9015     -5.917      0.1365      0.1264
0.4256      0.1611      0.4958      -0.1829     0.3935      -0.8161     0.8066
-0.1528
2
1     1      144.0      2     2      144.0      3     3      144.0
4     4      144.0      5     5      144.0      6     6      144.0
6.953       8.1173      11.20       11.790      16.381      20.477
14.0        0.168
20        50.0        24.0
0.02        0.50        1.0         5.0         10.0        15.0        50.0
0.03
0.5         -0.5         440.0      20.0
1.0         0.915       0.836       0.762       0.693       0.6294
0     2
2     0.75
4     0.75
28.0        1.0
0     5     0  846     0     0     1
0.0         0.0         0.0         89.9
0.0         49.0        30.8        59.5        0.0         24.0
0.0         0.0            5    6   0.0
0.0         0.2         0.4         0.6         0.8         1.0
0.0         0.25        0.5         0.75        1.0
```

```
0.0       0.0       0.0       89.9
30.8      59.5      61.0      70.0      24.0      48.0
0.0       0.0         5   6   0.0
0.0       0.2       0.4       0.6       0.8       1.0
0.0       0.25      0.5       0.75      1.0
0.0       0.0       0.0       89.9
49.0      62.0      70.0      83.0      0.0       48.0
0.0       0.0         9   4   0.0
0.0       0.3333    0.6667    1.0
0.0       0.125     0.25      0.375     0.5       0.625
0.75      0.875     1.0
36.0      0.0       24.0      0.0
0.0       27.0      19.5      27.0      0.0       60.0
0.0       0.0        11   6   0.0
0.0       0.2       0.4       0.6       0.8       1.0
0.0       0.1       0.2       0.3       0.4       0.5
0.6       0.7       0.8       0.9       1.0
36.0      0.0       24.0      0.0
27.0      37.0      27.0      37.0      6.0       60.0
0.0       0.0        10   4   0.0
0.0       0.3333    0.6667    1.0
0.0       0.1111    0.2222    0.3333    0.4444    0.5556
0.6667    0.7778    0.8889    1.0
1    0    0    0    0    0
1 141  0
F   40    0
2    0    0    0
4  0.0     0.0       61.0      48.0
4.0       20.5      30.0      46.0
4  46.5    0.0       68.0      48.0
1.5       19.5      27.0      45.0
F   24    0
2    0    0    0
2  49.0    0.0       70.0      48.0
0.5       47.0
2  61.5    0.0       83.0      48.0
0.5       47.0
F   50    0
2    0    0    1
3  6.0     0.0       19.0      60.0
2.0       26.0      55.0
3  25.3    0.0       25.3      60.0
2.0       26.0      55.0
F   27    0
2    0    0    0
2  27.0    6.0       27.0      60.0
11.5      58.0
2  37.0    6.0       37.0      60.0
11.5      58.0
0
```

# APPENDIX F

## *CYGNET WING FLUTTER*

The CYGNET is a single seat sail plane designed to enter the World Class Glider Design Competition in Bielefeld, Germany in September, 1992. It is the only entry from the USA. See figure F-1 for a three view drawing.

This example shows the flutter analysis of the wing for the CYGNET using multiple panels. An aircraft weight of 601 lbs gross and a wing span of 44.00 feet and a weight of 240 lbs is used. Both symmetric and antisymmetric flutter modes were evaluated. However, only the symmetric modes are critical and, as such are shown here. The critical flutter speeds for the symmetric modes are determined and shown in table F-1.

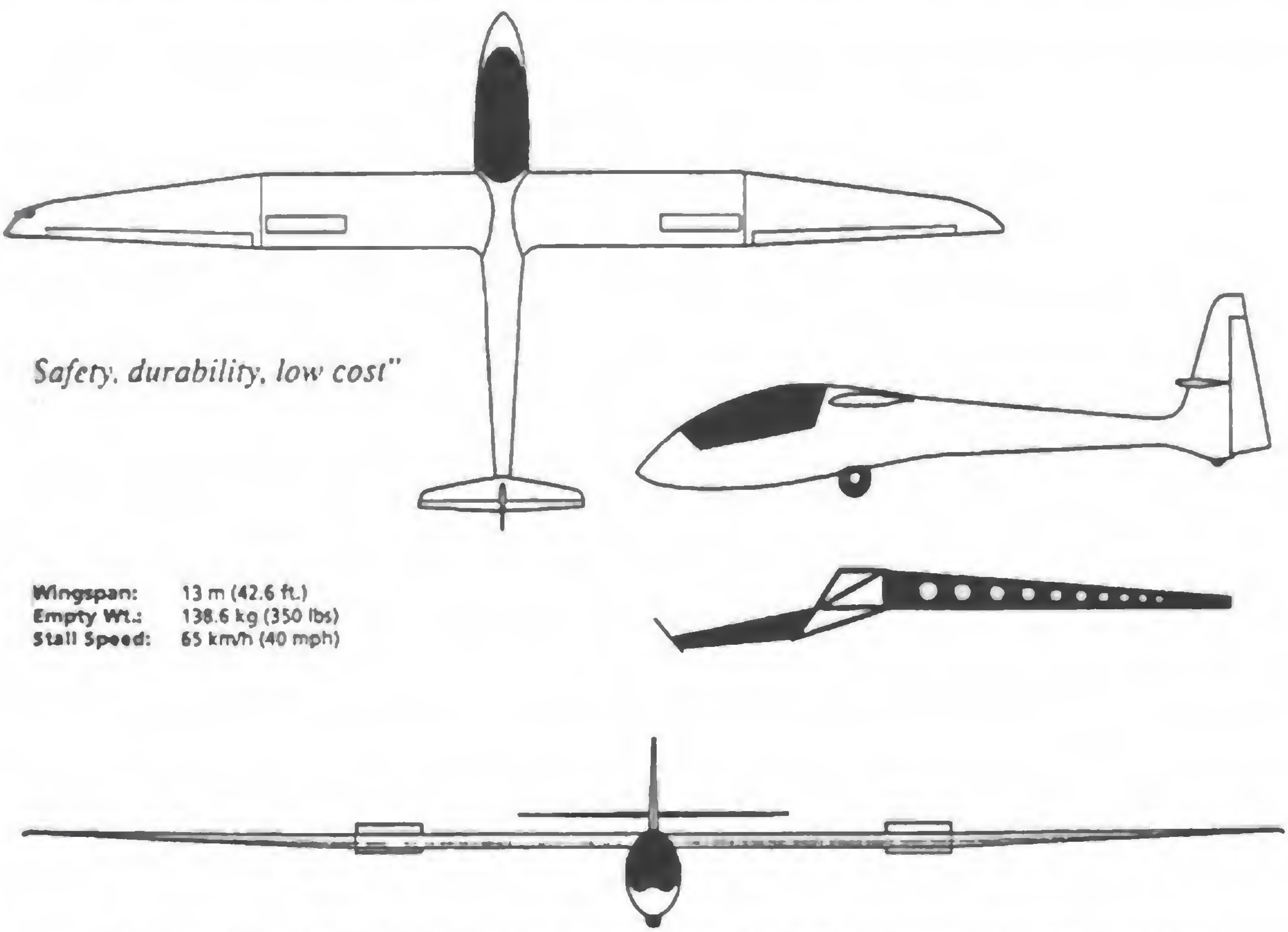



Figure F-1. The CYGNET single seat glider. First flight was in September 1992.

Table F-1. WING FLUTTER SPEEDS for the CYGNET

| Altitude | Sea Level | 10K feet | 20K feet | 30K feet |
|---|---|---|---|---|
| Eq. Airspeed, knots | 138.6 | 129.59 | 122.24 | 116.5 |
| True Airspeed, knots | 138.5 | 150.8 | 167.44 | 189.58 |

Half of the wing is modeled using NISA386 finite element analysis code. The weight of the wing for the fem was checked against the actual weight of the fabricated wing and a wing bending stiffness check was made on the actual wing by applying a 50 lb weight at the tip and comparing the results to the fem. The modulus of the aluminum for the wing was changed to so that the fem had the same static deflection. The fuselage was modeled as a lump mass which is attached to the wing fittings so that rigid body modes could be determined.

The mode shapes and natural frequencies from the fea were input into the SAF program and the damping and frequencies were determined as a function of velocity.

Mode shapes from the fea are shown on the attached sheets to show the actual deflected shapes.

The results from the SAF program are shown in the form of damping and frequency plots.

Table F-2. SYMMETRIC MODES

| Mode Number | FEA MODE | Frequency, Hz | Shape |
|---|---|---|---|
| 1 | 4 | 3.924 | 1st Wing Bending |
| 2 | 6 | 13.058 | 2nd Wing Bending |
| 3 | 7 | 23.42 | Aileron Bending + flapping |
| 4 | 8 | 24.81 | Aileron flapping |
| 5 | 9 | 29.05 | 3rd Wing Bending + aileron |
| 6 | 10 | 31.26 | Pitching + Aileron |

It is interesting to note that this wing was modeled as a single panel with a control surface and named the test wing as shown in the lattice model on a following page. The answer thus obtained is within 5% of the speed determined by the multipanel lattice model.

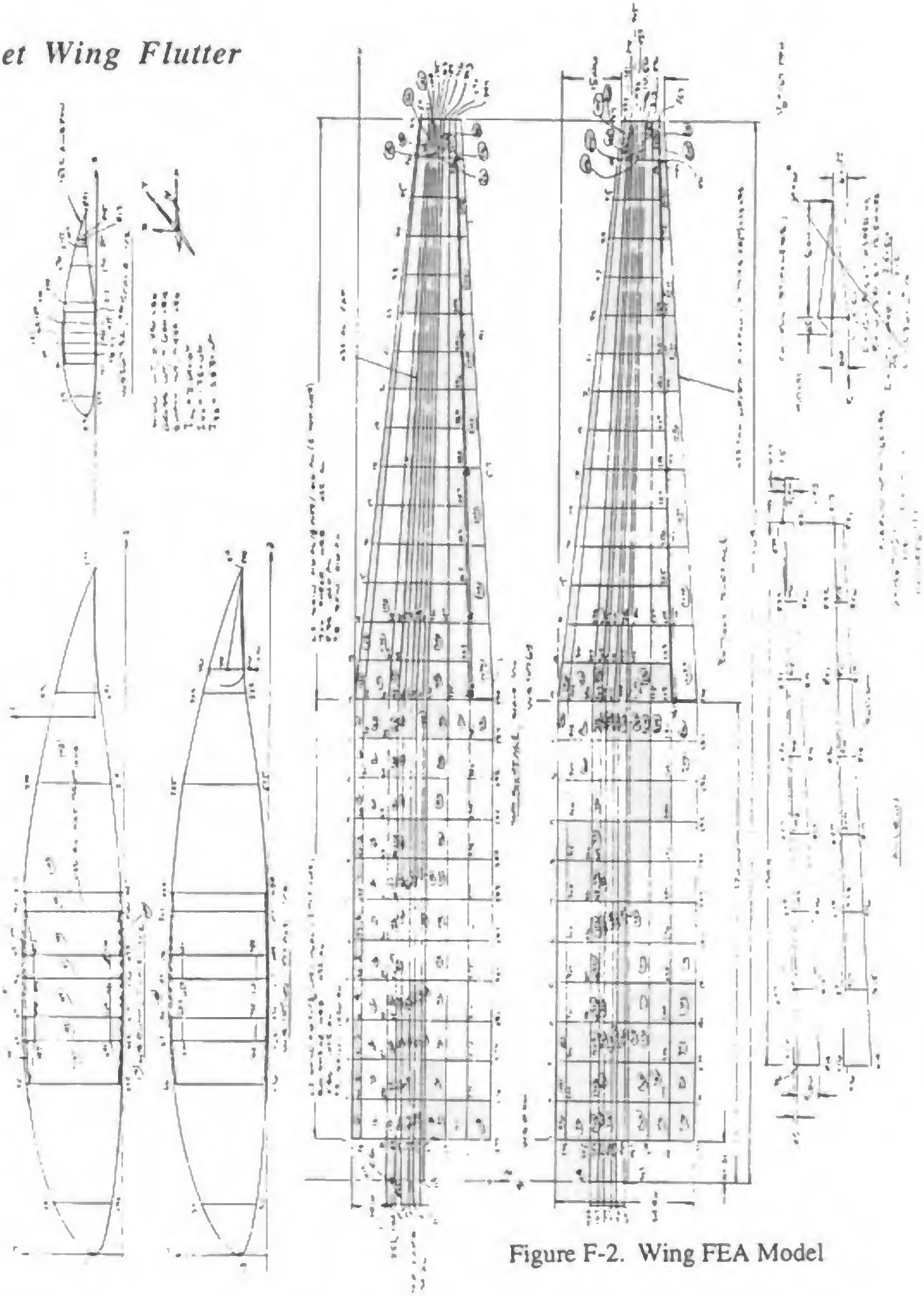


Figure F-2. Wing FEA Model

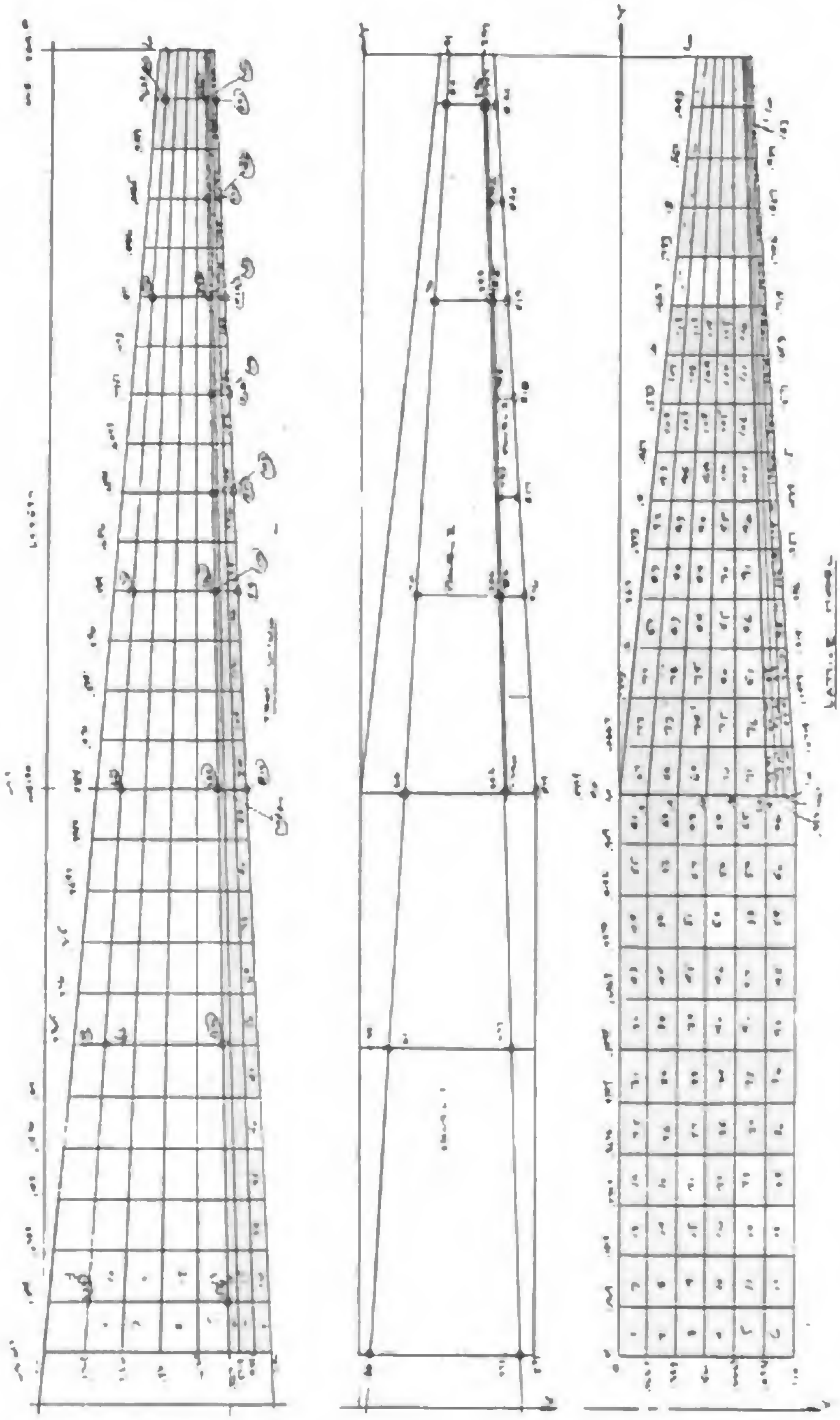


Figure F-3. Wing Lattice Model

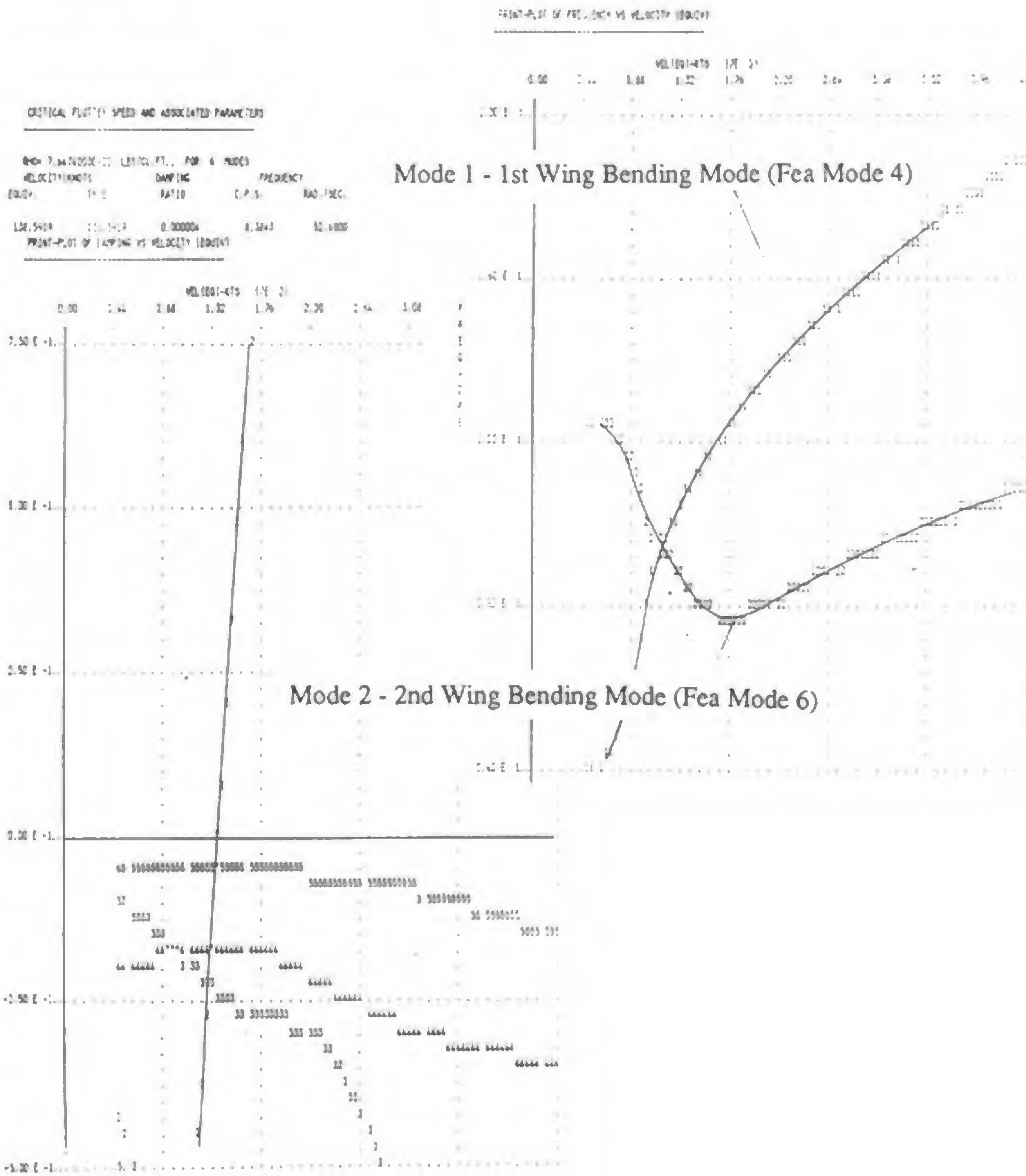

Mode 1 - 1st Wing Bending Mode (Fea Mode 4)
Mode 2 - 2nd Wing Bending Mode (Fea Mode 6)

Table F-3. NISA386 Finite Element Model of Wing

```
ANAL=EIGENVALUE
**EXEC=CHECK
FILE=C1
SAVE=26,27
EIGEN EXTRACTION=SUBSPACE,ACCELERATED
MASS FORMULATION=CONSISTENT
AUTO=ON
*ECHO=OFF
*A1
CYGNET WING,SYM MODES
*C1
1,20,1
2,33,1
3,20,10
4,30,1
5,12,1
*D1
1,4
.016///
2,4
.01///
3,4
.032///
4,4
.025///
5,4
.020///
6,4
.13///
7,4
.174///
8,4
.11///
9,4
.048///
10,4
.05///
**FUSELAGE MASS
11,6
.56,.56,.56,1.6E3,1.6E3,.5E3
**CONTROL ROD,1/2 DIA, 5.1 LONG
12,4
.20,.006,.006,.012
**BEAMS
13,4
.74,.32,.32,.64
**AILERON MASS
14,6
.0014597//,0//
15,6
0//,0//
16,4
.025///
*D2
1,4
0///
*D3
3,0,1,5,1,1,1,1,1,2,1
3,0,1,6,2,1,1,1,1,2,1
```

```
*E1
**L.E. (CENTER TOP)
$$,-2,27,11
1,28,29,2,1,$,1,2,0,11,1,1
**SPAR (CENTER TOP)
$$,-6,27,3
23,82,83,56,55,$,1,1,7,3,1,1
$$,-6,27,3
41,85,86,59,58,$,1,1,8,3,1,1
$$,-6,27,5
59,88,89,62,61,$,1,1,9,5,1,1
**T.E. (CENTER TOP)
$$,-2,27,11
89,244,245,218,217,$,1,1,5,11,1,1
111,577,578,272,271,$,1,1,5,11,1,1
**L.E. (TIP TOP)
$$,-2,27,15
122,39,40,13,12,$,2,2,0,15,1,1
**SPAR (TIP TOP)
$$,-6,27,15
152,93,94,67,66,$,1,1,3,15,1,1
**T.E. (TIP TOP)
$$,-2,27,15
242,255,256,229,228,$,1,1,1,15,1,1
**L.E. (CENTER BOTTOM)
280,298,299,2,1,$,1,2,0,11,1,1
291,325,326,299,298,$,1,2,0,11,1,1
**SPAR (CENTER BOTTOM)
$$,-6,27,3
302,352,353,326,325,$,1,1,7,3,1,1
$$,-6,27,3
320,355,356,329,328,$,1,1,8,3,1,1
$$,-6,27,5
338,358,359,332,331,$,1,1,9,5,1,1
**T.E. (CENTER BOTTOM)
$$,-2,27,11
368,514,515,488,487,$,1,1,5,11,1,1
390,577,578,542,541,$,1,1,5,11,1,1
**L.E. (TIP BOTTOM)
401,309,310,13,12,$,2,2,0,15,1,1
416,336,337,310,309,$,2,2,0,15,1,1
**SPAR (TIP BOTTOM)
$$,-6,27,15
431,363,364,337,336,$,1,1,3,15,1,1
**T.E. (TIP BOTTOM)
$$,-2,27,15
521,525,526,499,498,$,1,1,1,15,1,1
**BOX SHEAR WEB
560,325,326,56,55,$,1,1,3,11,1,1
571,336,337,67,66,$,1,1,4,15,1,1
586,460,461,191,190,$,1,1,3,11,1,1
597,471,472,202,201,$,1,1,4,15,1,1
**AFT WEB
612,541,542,272,271,$,1,1,4,26,1,1
**HAT SECTIONS
638,589,590,83,82,$,1,1,10,11,1,1
649,616,617,590,589,$,1,1,10,11,1,1
660,616,617,110,109,$,1,1,10,11,1,1
671,643,644,137,136,$,1,1,10,11,1,1
682,163,164,137,136,$,1,1,10,11,1,1
693,670,671,164,163,$,1,1,10,11,1,1
704,352,353,698,697,$,1,1,10,11,1,1
715,724,725,698,697,$,1,1,10,11,1,1
726,379,380,725,724,$,1,1,10,11,1,1
```

```
737,406,407,752,751,$,1,1,10,11,1,1
748,778,779,752,751,$,1,1,10,11,1,1
759,433,434,779,778,$,1,1,10,11,1,1
**RIBS
$$,-12,1,14
780,325,697,589,55,$,1,1,4
781,55,589,82,$,1,3,4
782,325,352,697,$,1,3,4
783,697,724,616,589,$,1,1,4
784,724,751,643,616,$,1,1,4
785,616,643,136,109,$,1,1,4
786,379,406,751,724,$,1,1,4
787,751,778,670,643,$,1,1,4
788,778,460,190,670,$,1,1,4
789,670,190,163,$,1,3,4
790,433,460,778,$,1,3,4
791,460,487,217,190,$,1,1,4
792,487,514,244,217,$,1,1,4
793,514,541,271,244,$,1,1,4
$$,-7,2,8
950,338,365,95,68,$,1,1,5,8,27,1
**TIP RIB
1010,324,351,81,54,$,1,1,5,9,27,1
**AILERON
1020,814,815,791,790,$,3,1,16,7,1,1
1027,814,815,807,806,$,3,1,16,7,1,1
1034,798,799,791,790,$,3,1,16,7,1,1
1041,806,807,799,798,$,3,1,16,7,1,1
$$,-8,1,2
1048,798,814,790,$,3,3,4
1049,806,814,798,$,3,3,4
**HINGE BRKTS
1064,552,822,282,$,1,3,6
1065,558,823,288,$,1,3,6
1066,562,824,292,$,1,3,6
**FUS MASS
1067,825,$,0,4,11
1068,825,55,$,4,5,13
1069,825,217,$,4,5,13
1070,825,325,$,4,5,13
1071,825,487,$,4,5,13
1072,825,82,$,4,5,13
1073,825,352,$,4,5,13
**CONTROL ROD
1080,806,525,$,5,5,12
**AILERON STUFF
1081,822,283,282,$,1,3,6
1082,798,799,827,826,$,3,1,8,7,1,1
1089,826,798,790,$,3,3,8,7,1,1
1096,826,$,0,4,14,8,1,1
1106,833,813,805,$,3,3,8
**HINGE BRKT
1108,566,835,296,$,1,3,6
*F1
1,0,$,10.3,1.4,0
11,0,1,$,110,1.4,0
12,0,$,120,1.4,0
27,0,1,$,264,.4,15
28,0,$,10.3,3.3,2.5
38,0,1,$,110,3.3,2.5
39,0,$,120,3.3,2.5
54,0,1,$,264,1.15,15.95
55,0,$,10.3,4.7,8.5
```

```
65,0,1,$,110,4.7,8.5
66,0,$,120,4.7,8.5
81,0,1,$,264,1.4,17.55
82,0,$,10.3,4.85,10.65
92,0,1,$,110,4.85,10.65
93,0,$,120,4.85,10.65
108,0,1,$,264,1.5,18.1
109,0,$,10.3,4.9,11.88
119,0,1,$,110,4.9,11.88
120,0,$,120,4.9,11.88
135,0,1,$,264,1.55,18.6
136,0,$,10.3,4.9,13.8
146,0,1,$,110,4.9,13.8
147,0,$,120,4.9,13.8
162,0,1,$,264,1.55,19.1
163,0,$,10.3,4.9,15
173,0,1,$,110,4.9,15
174,0,$,120,4.9,15
189,0,1,$,264,1.55,19.6
190,0,$,10.3,4.85,17.15
200,0,1,$,110,4.85,17.15
201,0,$,120,4.85,17.15
216,0,1,$,264,1.5,20.15
217,0,$,10.3,4.8,18.1
227,0,1,$,110,4.8,18.1
228,0,$,120,4.8,18.1
243,0,1,$,264,1.5,20.6
244,0,$,10.3,4.2,23.5
254,0,1,$,110,4.2,23.5
255,0,$,120,4.2,23.5
270,0,1,$,264,1.2,22.45
271,0,$,10.3,3.3,27.5
281,0,1,$,110,3.3,27.5
282,0,$,120,3.3,27.5
297,0,1,$,264,1,23
298,0,$,10.3,.5,2.5
308,0,1,$,110,.5,2.5
309,0,$,120,.5,2.5
324,0,1,$,264,.1,15.95
325,0,$,10.3,.1,8.5
335,0,1,$,110,.1,8.5
336,0,$,120,.1,8.5
351,0,1,$,264,0,17.55
352,0,$,10.3,0,10.65
362,0,1,$,110,0,10.65
363,0,$,120,0,10.65
378,0,1,$,264,0,18.1
379,0,$,10.3,0,11.88
389,0,1,$,110,0,11.88
390,0,$,120,0,11.88
405,0,1,$,264,0,18.6
406,0,$,10.3,0,13.8
416,0,1,$,110,0,13.8
417,0,$,120,0,13.8
432,0,1,$,264,0,19.1
433,0,$,10.3,0,15
443,0,1,$,110,0,15
444,0,$,120,0,15
459,0,1,$,264,0,19.6
460,0,$,10.3,.1,17.15
470,0,1,$,110,.1,17.15
471,0,$,120,.1,17.15
486,0,1,$,264,.05,20.15
487,0,$,10.3,.13,18.1
```

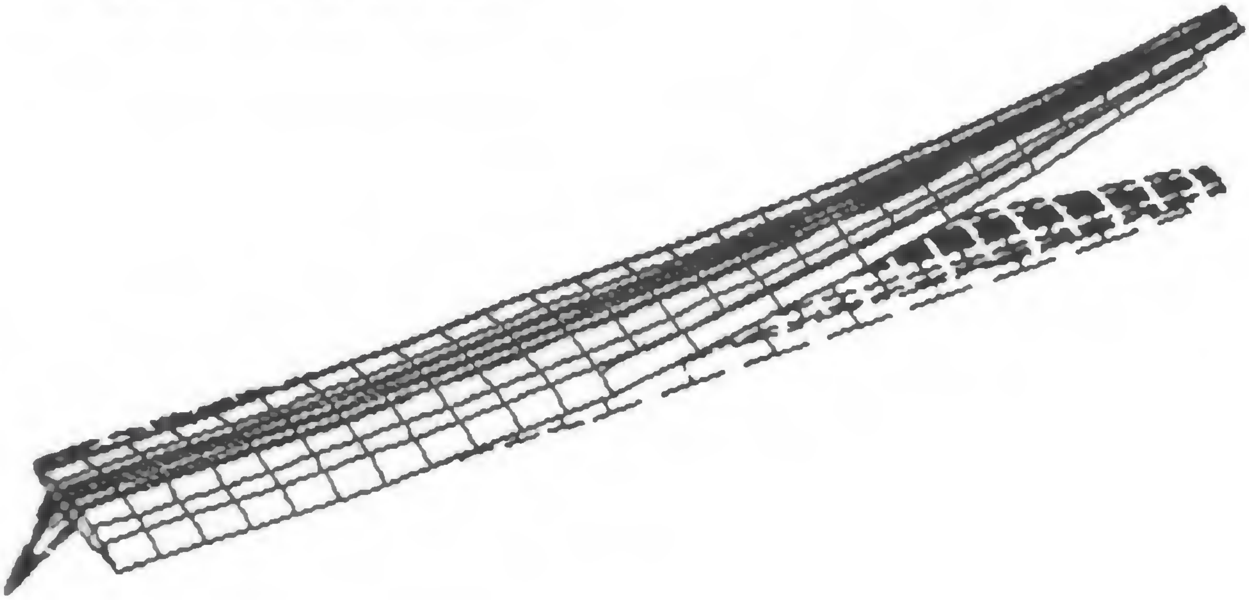
```
497,0,1,$,110,.13,18.1
498,0,$,120,.13,18.1
513,0,1,$,264,.1,20.6
514,0,$,10.3,.65,23.5
524,0,1,$,110,.65,23.5
525,0,$,120,.65,23.5
540,0,1,$,264,.3,22.45
541,0,$,10.3,1.25,27.5
551,0,1,$,110,1.25,27.5
552,0,$,120,1.25,27.5
567,0,1,$,264,.45,23
577,0,$,10.3,1.4,34.4
588,0,1,$,120,1.4,34.4
589,0,$,10.3,4.2,10.65
600,0,1,$,120,4.2,10.65
616,0,$,10.3,4.2,11.88
627,0,1,$,120,4.2,11.88
643,0,$,10.3,4.2,13.8
654,0,1,$,120,4.2,13.8
670,0,$,10.3,4.2,15
681,0,1,$,120,4.2,15
697,0,$,10.3,.6,10.65
708,0,1,$,120,.6,10.65
724,0,$,10.3,.6,11.88
735,0,1,$,120,.6,11.88
751,0,$,10.3,.6,13.8
762,0,1,$,120,.6,13.8
778,0,$,10.3,.6,15
789,0,1,$,120,.6,15
**AILERON
790,0,$,120.1,3,28.6
797,0,1,$,254.3,.9,24.1
798,0,$,120.1,2.1,28.6
805,0,1,$,254.3,.65,24.1
806,0,$,120.1,1.3,28.6
813,0,1,$,254.3,.46,24.1
814,0,$,120.1,1.4,34.4
821,0,1,$,254.3,.5,26.1
**HINGE
822,0,$,120.15,2.1,28.6
823,0,$,177.664,1.4786,26.671
824,0,$,216.007,1.0643,25.386
**FUS C.G.
825,0,$,0,-19,10
** AILERON BALANCE
826,0,$,120.05,2.1,27.75
833,0,1,$,254.3,.65,23.85
**HINGE
835,0,$,254.35,.65,24.1
*H1
**ALUMINUM
EX,1,0,10E6
EY,1,0,10E6
NUXY,1,0,.3
DENS,1,0,4.15E-4
**CORE
EX,2,0,0
GXZ,2,0,3E3
GYZ,2,0,3E3
GXY,2,0,3E3
DENS,2,0,6.7E-6
**GR/EP
EX,3,0,4.5E6
```

```
EY,3,0,4.5E6
NUXY,3,0,.3
DENS,3,0,1.3E-4
**CONTROL ROD
EX,5,0,5.36E4
EY,5,0,5.36E4
NUXY,5,0,.3
DENS,5,0,2.6E-4
**BEAM
EX,4,0,30E6
EY,4,0,30E6
NUXY,4,0,.3
DENS,4,0,0
*CPDISP
UXYZ,5,822,798
UXYZ,5,823,801
UXYZ,5,824,803
UXYZ,5,835,805
*EIGCNTL
15,0,35,,0,0,1E-3,-1
*SPDISP
825,UX,0
825,ROTZ,0
825,ROTY,0
55,UX,0,217,27
55,ROTZ,0,217,27
325,UX,0,487,27
325,ROTZ,0,487,27
*MODEOUT
3 5 1
*EIGOUT
0,0,0,0,0,0
*ENDDATA
```

## Table F-4. Flutter Analysis File for SAF

```
1
FLUTTER, CYGNET-WORLD CLASS
601 LBS OF WEIGHT
SYMMETRIC CASE
P-K METHOD
6 VIBRATION MODES
MACH=0.5
   -1    6    2    6    4    0    0    0    0    0
    1    0    1    0    0    1    0    0    1    0
    1    0    0    1    0    0    0    0    0    0
    0    0    0    0    0    0    0
26
-.456, .178, .989, -.4824, .1332, .95, .989
2.332, 3.497, 4.809, .95, 2.31, 3.487, 4.468
.9507, 1.86, 2.312, 2.84, 3.487, 4.209, 4.8
.9387, 1.85, 2.304, 2.83, 3.476, 4.197, 4.794
.283, -1.25, -2.286, .276, -1.184, -2.196, -2.286
-1.461, 1.264, 5.71, -2.196, -1.417, 1.341, 4.575
-2.2, -2.347, -1.415, -.397, 1.346, 4.171, 5.786
-2.524, -2.644, -1.704, -.654, 1.101, 3.924, 5.59
-.078, -.458, -.635, .121, .0403, .064, -.635
-.691, -.788, -.755, .064, -.031, -.251, -.352
.0928, 9.23, .0056, -7.02, -.215, 10.13, -.323
17.06, 24.17, 13.816, 5.232, 10.22, 18.81, 6.897
.026, -.0916, .0439, -.028, -.177, -.1259, .0439
.521, .739, .533, -.1259, .3172, .498, .397
-.133, -5.259, .308, 5.69, .4816, -9.113, .2917
26.65, 18.18, 21.66, 24.84, 17.39, 5.228, 11.37
```

```
-.111, -2.47, -1.063, .078, -1.446, -.48, -1.063
2.338, 1.77, -3.78, -.48, 2.77, 1.92, -1.987
-.455, 5.49, 2.81, 3.895, 1.935, -6.786, -3.761
-5.716, .95, -1.23, .108, -1.247, -9.22, -5.877
.428, .414, .672, -.546, -.813, -.827, .672
.825, .673, .0259, -.827, -.309, -.268, -.486
-.886, 1.37, -.366, -1.134, -.331, -.652, -.683
-1.767, .607, -1.038, -1.78, -.943, -1.135, -1.081
    2
    1    1     144.0    2    2     144.0    3    3     144.0
    4    4     144.0    5    5     144.0    6    6     144.0
3.924, 13.058, 23.42, 24.81, 29.05, 31.26
17.2, 0.5
   20      50.0      24.0
0.02      0.50      1.0       5.0       10.0      15.0      50.0
0.03
0.5       -0.5      440.0     20.0
**S.L., 10K, 20K, 30K feet alt.
1.0, .7386, .533, .3773
34.4, 1.0
1,3,0,1098,0,0,1
**PRIMARY SURFACE, PANEL 1
0.0, 0.0, 0.0, 0.0
0.0, 34.4, 0.0, 34.4, 10.3, 120.0
0.0, 0.0,12,7,0.0
0.0, 0.1667, 0.333, 0.5, 0.667, 0.833
1.0
0.0, 0.0909, .1818, .2727, .3636, .4545
.5454, .6364, .7273, .8182, .909, 1.0
**PRIMARY SURFACE, PANEL 2
0.0, 0.0, 0.0, 0.0
0.0, 28.66, 15.0, 24.1, 120.0, 264.0
0.0, 0.0, 16, 6, 0.0
0.0, 0.2, 0.4, 0.6, 0.8, 1.0
0.0, 0.0667, .1333, .20, .2667, .333
.4, .4667, .5333, .60, .6667, 0.7333
.80, .8667, .9333, 1.0
**CONTROL SURFACE, PANEL 3
0.0, 0.0, 0.0, 0.0
28.66, 34.4, 24.1, 26.1, 120., 254.4
0.0, 0.0, 15, 4, 0.0
0.0, .333, .6666, 1.0
0.0, .0714, .1429, .2143, .286, .3571
.429, .5, .571, .643, .714, .786
.857, .929, 1.0
1, 0, 0, 0, 0, 0
1, 183, 0
F, 141, 0
2, 0, 1, 1
6, 1.5, 0.0, 17., 264.
10.3, 70.5, 120., 158.5, 216., 254.5
6, 32., 0.0, 23.5, 264.
10.3, 70.5, 120., 158.5, 216., 254.5
F, 42, 0
2, 0, 1, 1
7, 28.66, 120., 24.1, 254.5
120., 158.5, 177.5, 196.5, 216., 235., 254.5
7, 34.4, 120., 26.1, 254.5
120., 158.5, 177.5, 196.5, 216., 235., 254.5
0
```

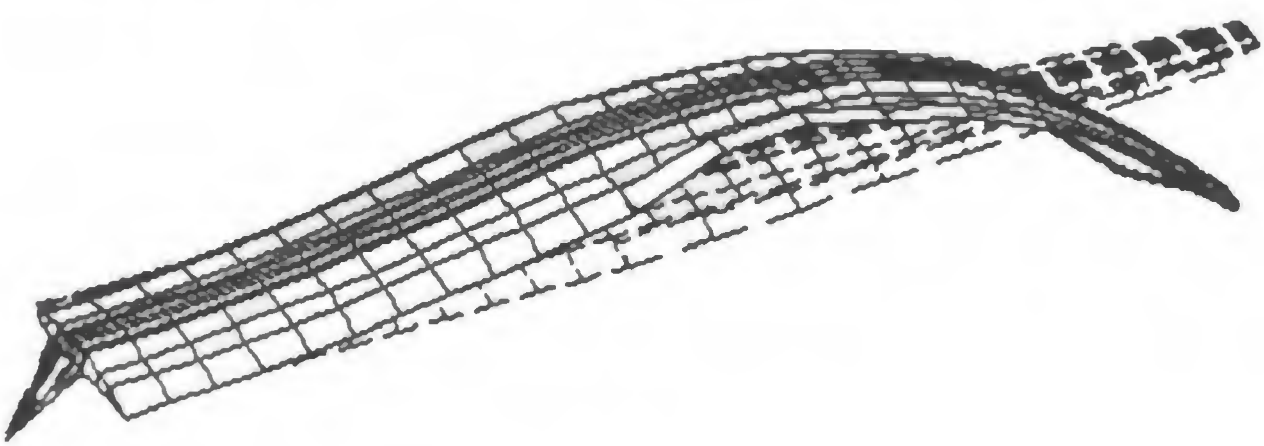


CYGNET WING,SYM MODES

MODE NO. = 4 FREQUENCY = 4.70359E+00 Hz

Y X Z

RX= 42
RY= 29
RZ= -6

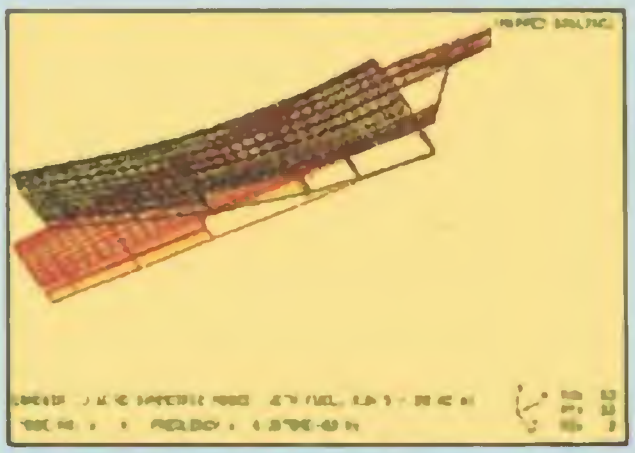


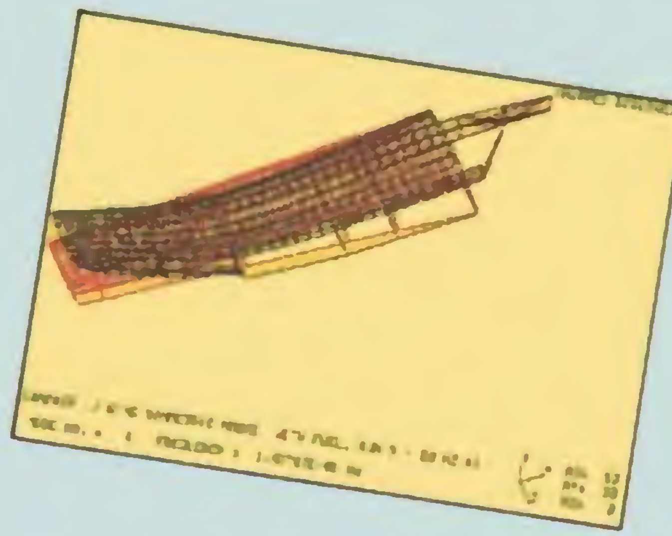
CYGNET WING,SYM MODES

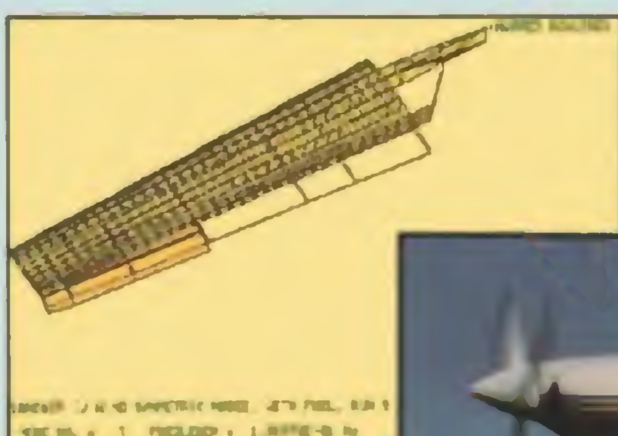
MODE NO. = 6 FREQUENCY = 1.55379E+01 Hz

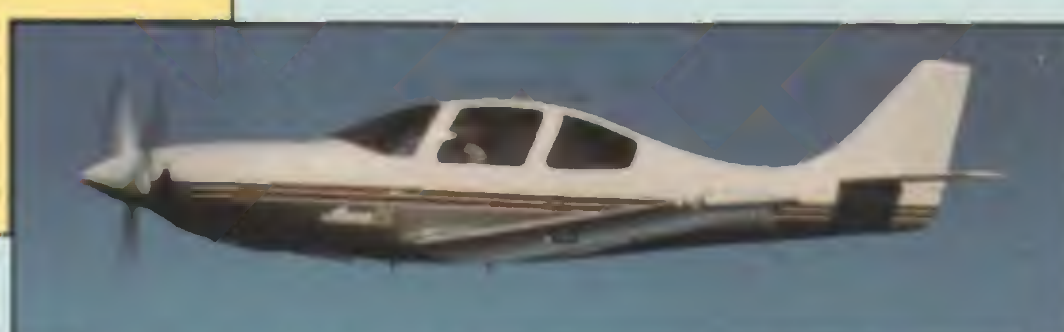
Y X Z

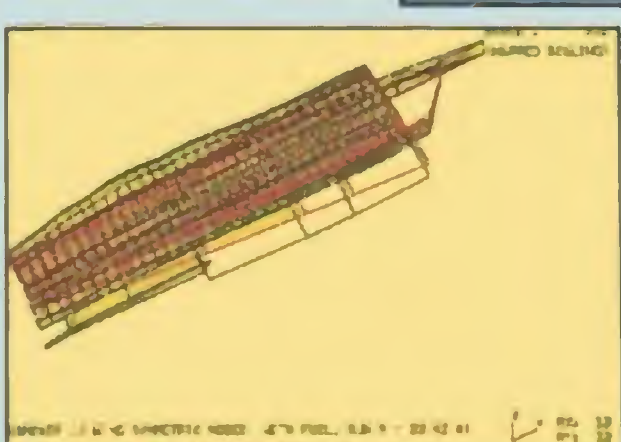
RX= 42
RY= 29
RZ= -6

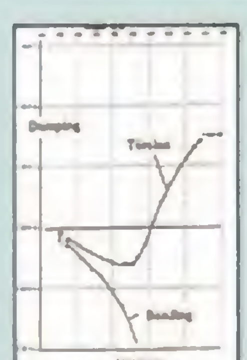


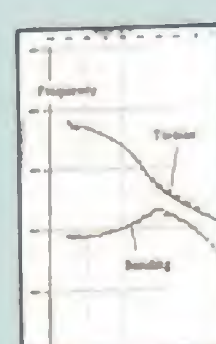









Damping
Torsion
Bending
Velocity


Frequency
Torsion
Bending